Pulses in Cables - PHY293 2 Weight Lab

Matthew Pihowich and Sencer Aksu

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1 Abstract

This experiment investigates the propagation characteristics of electrical pulses in transmission lines and coaxial cables. This study challenges the conventional assumption of instantaneous signal travel in DC and AC circuit theory, specifically over long distances and through non-ideal cables. This experiment approaches this challange by analyzing propagation through a simulation of a transmission line that is divided into units with known inductance and capacitance, and examining the behavior of pulses in coaxial cables of various lengths and with various load terminations. This study experimentally measured the speed of pulse propagation, characteristic impedance of a transmission line and coaxial cable, and the behavior of a signal in a coaxial cable with different load terminations and the attenuation factor of coaxial cables and compares these results with the theoretical values obtained by the definitions presented in the introduction. The experimental findings closely matched theoretical predictions, with the velocity of pulses in transmission lines and coaxial cables being within the expected range. The characteristic impedance for the transmission line slightly deviated from theoretical values, likely due to unaccounted resistance in the line. In coaxial cables, a characteristic impedance of 51±0.5 was found, and the attenuation factor was established at 0.0531±0.00280 dB/m. This experiment not only validates theoretical models for pulse propagation in non-ideal conditions but also provides practical insights crucial for long-distance communication and circuit design.

2 Introduction

DC and AC circuit theory typically assumes instantaneous travel of information through cables, however, long distances and non-ideal cables with internal capacitance and inductance causes that assumption to not hold true in real life. Therefore, it is important to understand how pulses in cables behave in non-ideal conditions work when trying to communicate signals over long distances. The purpose of this experiment is to investigate into those effects and verify the theory that will be introduced in this section.

2.1 Ideal Transmission Lines

Speed of which a signal travels: The theoretical value of the velocity of a travelling wave through a transmission line, $v_{tr,theor}$, can be determined using the equation below:

$$v_{tr,theor}^2 = \frac{1}{L_0 C_0} \tag{1}$$

L_0 is the inductance of the transmission line and C_0 is the capacitance of the transmission line.

The velocity of a pulse through a transmission line with known inductance and capacitance can be measured experimentally to verify this theory.

Characteristic Impedance: The theoretical value of the characteristic impedence for a given source and transmission line can be determined through the following equation:

$$Z_{0,theor} = \sqrt{\frac{L_0}{C_0}} \tag{2}$$

2.2 Coaxial Cables

The inductance per unit length of a coaxial cable can be written as:

$$L_0 = \frac{\mu}{2\pi} ln(\frac{R_2}{R_1}) \tag{3}$$

While the capacitance per unit length can be written as:

$$C_0 = \frac{2\pi\epsilon}{ln(\frac{R_2}{R_1})} \tag{4}$$

Where μ is the magnetic permeability of the wire, ϵ is the universal electrical permeability constant, and R_1 and R_2 are the radii of the inner and outer conductors.

When we take the products of L_0 and C_0 , radii, and all other constants conveniently cancel out, leaving us with the equation:

$$\frac{1}{L_0 C_0} = \frac{1}{\mu\epsilon} = v^2 \tag{5}$$

Where v is the speed at which a signal travels in a cable. Using this formula, we can determine the time it takes for a signal to travel through a cable, allowing us to account for the transmission line behavior.

2.3 Load Effect

In order to understand how a load with impedence Z_L attached at the end of the coaxial cable affects the reflected signal, we specify the components of variables that travel in the positive (+) or negative (-) direction. Because the characteristic impedance, Z_0 remains the same regardless of the direction:

$$\frac{V_+}{I_+} = Z_0 = \frac{V_-}{I_-} \tag{6}$$

Where V_- is the amplitude of the reflected signal and V_+ is the amplitude of the incident signal

At the end of the cable, where load Z_L is attached, the voltage and current combine so that

$$V = V_+ + V_- \tag{7}$$

$$I = I_+ + I_- \tag{8}$$

And thus the load impedence is:

$$Z_L = \frac{V}{I} \tag{9}$$

We can then define variables that would allow us to determine the characteristics of the cable:

- Reflection coefficient (r) is defined as:

  $$r = \frac{V_-}{V_+} \tag{10}$$

  and allows us to understand how much a signal is reflected.

  - its range is from -1 $\leq$ r $\leq$ 1, where a value of 0 means complete transmission and no reflection, and a value of -1 and 1 means total reflection with inverted or non-inverted phases respectively

- Transmission coefficient (t) is defined as:

  $$t = \frac{V}{V_+} \tag{11}$$

  and is a measure of how much of the signal is transmitted through the medium.

  - its range is from 0 $\leq$ t $\leq$ 1, where a value of 1 means all the signal was transmitted and there is no reflection and a value of 0 means all of the signal was reflected.

We can now combine the definition of r and t in equation (10) and (11) with definitions for Z_L and Z_0 in equations (6) and (9) to get the following definitions:

$$r = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{12}$$

$$t = \frac{2Z_0}{Z_L + Z_0} \tag{13}$$

We can check two extreme and one critical value of Z_L to uncover the behavior for different terminations:

- Open circuit, where $Z_L = \infty$, would cause $r = 1, t = 0$, which means that all the signal would be reflected back from the boundary.

- Short circuit, where $Z_L = 0$, would cause $r = -1, t =$ undefined. This means that all of the signal will be transmitted with no reflections on the cable. The transmission coefficient is undefined because there is essentially no obstruction or change of medium in the cable, and therefore nothing to transmit into.

- The load is the same as characteristic impedance of the transmission line, where $Z_L = Z_0$. In this case, $r = 0$ and $t = 1$, meaning that none of the signal is reflected and all of it is transmitted perfectly to the next medium.

Replicating the third condition, or matching the characteristic impedance, is important for analog circuit design like audio equipment or connectors as it means that signal will not be reflected in the cable and will not cause any constructive or destructive interference that may cause information to be lost or disrupted.

In this experiment, we assume that only the termination load that we attach to the coaxial cables are significant, and therefore our final equations are:

$$r = \frac{R_L - Z_0}{R_L + Z_0} \tag{14}$$

$$t = \frac{2Z_0}{R_L + Z_0} \tag{15}$$

This means that in the experiment, we expect for the reflection amplitude to be 0 when the termination load we attach is equal to Z_0 of the coaxial cables.

The signal is always propagated with losses, with the most visible one being attenuation factor of a cable. It can be measured experimentally and is calculated using the following definition:

$$Attenuation(dB) = 10log\frac{OutputIntensity(W)}{InputIntensity(W)} = 10log\frac{V_{reflected}}{V_{initial}}^2 \tag{16}$$

For a given cable, attenuation value tells how much power a signal is going to lose as it travels from one end of the cable to the other end.

3 Materials and Methods

3.1 Materials:

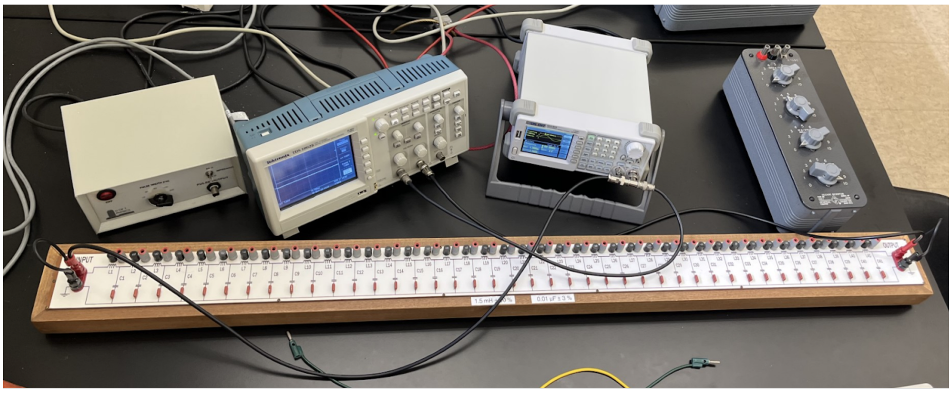


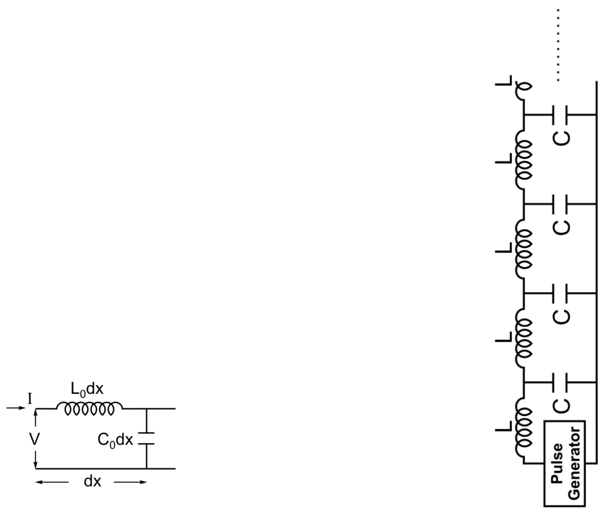
Figure 1: Image of all the equipment used for this experiment

The equipment used for the experiment are as follows:

- Pulse Generators:
  - SIGLENT SDG805: A lab bench style function generator used to study the transmission line
  - U of T Physics Pulse Generator: Custom pulse generator that can output pulses much shorter (50-200 ns) than SIGLENT pulse generator. Used to study coaxial cables
- Digital Oscilloscope:

– Used to visualize the signals in the coaxial cables and perform measurements

- Transmission Line:
  – A circuit that houses 41 units of basic transmission line with inductance of 1.5mH and capacitance of 0.01 F (Figure 1). With all 41 units are combined, it is used to mimic the behavior of a long transmission line (Figure 2)



(a) Transmission Line Unit (b) Transmission Line

3.2 Methods:

This experiment was split into two parts: one focusing on the travelling of waves through a representative transmission line, and one focusing on waves traveling through coaxial cables.

3.2.1 Transmission Line:

The setup process for this part

- **Electrical Connections:**
  – Connect the first channel of the oscilloscope to the output of the SIGLENT function generator and the input of the transmission line using the TEE connector.
  – Connect the output of the Transmission Line to the second channel of the oscilloscope
  – Connect the variable resistor load across the transmission line
- **Function Generator Settings:** Set the function generator to PULSE mode and adjust the frequency of the pulse to 300Hz and the width of the pulse to 75 μm

If the setup is correct, the reflection of the pulse should be visible in the second channel of the oscilloscope.

Determining the impedence match: Vary the resistor across the transmission line so that the reflection disappears. The final resistance is the impedance match between the source and the load.

Determining the speed of pulse propagation: In order to determine how the speed of the pulse propagates, through the transmission line, measure the delay of the reflection per number of transmission line units. The slope of the linear fit for the Delay vs. number of Transmission Line Units would give how the change of speed of the pulse per transmission line unit. (With a unit of number of LC units / time)

3.2.2 Coaxial Cable:

The goal of this section of the experiment is to determine how the speed of the pulse compares to that of speed of light, as well as calculate the attenuation factor of the cables and investigate into the behavior of different terminations.

The setup process for this part

- **Electrical Connections:**
  - Connect the first channel of the oscilloscope to the output of the U of T Physics pulse generator and a terminal of a coaxial cable using the TEE connector.
- **Pulse Generator Settings:** Set the impedance of the pulse generator to 50 Ω and the pulse width to 100ns

If the setup is correct, both the initial pulse and the reflection of it should be visible in the first channel of the oscilloscope.

Determining how the speed of the pulse compares to the speed of light:

4 Data and Analysis

4.1 Transmission Line

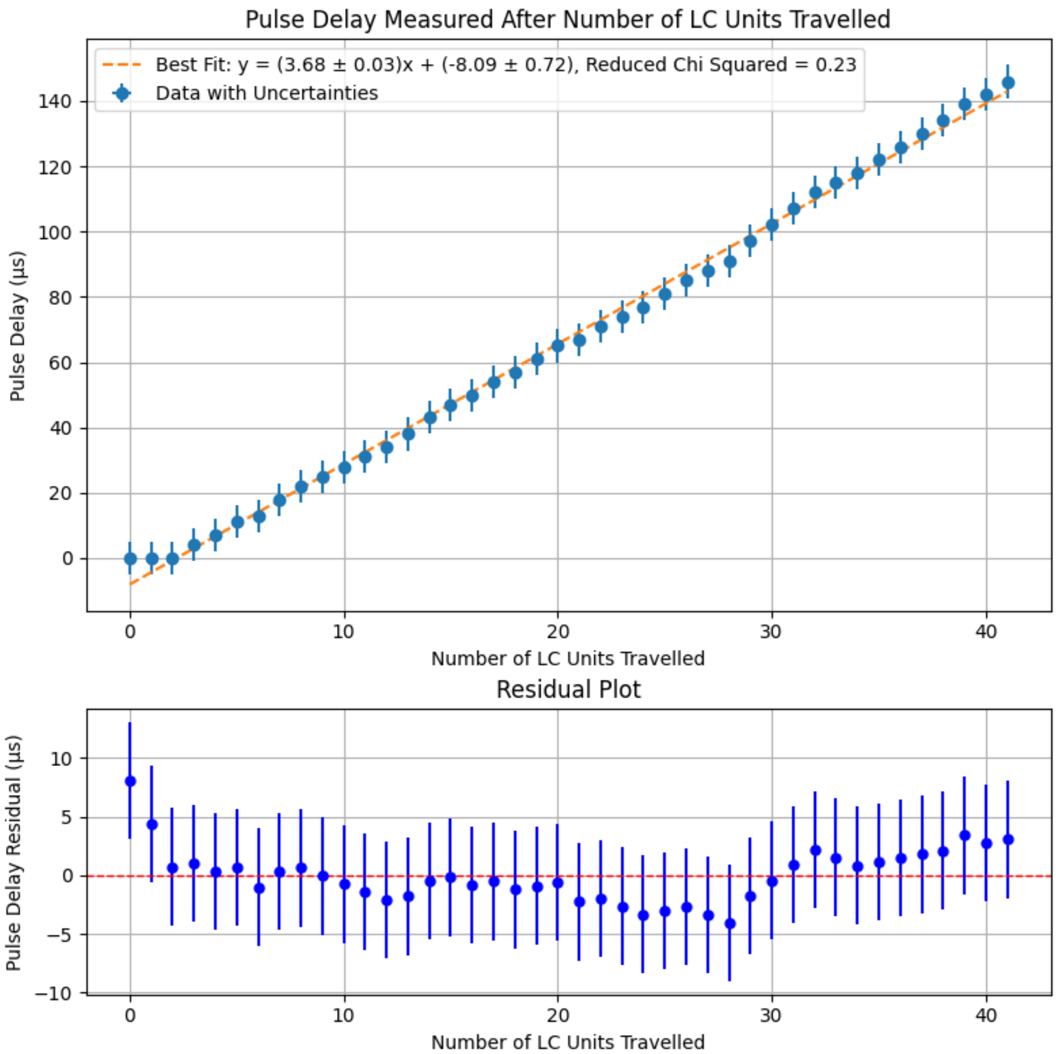


Figure 3: The linear fit for Pulse Delay after Number of LC Units Travelled by the pulse is given by the parameters $m = 3.68 \pm 0.03 \mu s$/LC Unit and $b = -8.09 \pm 0.72 \mu s$ with a reduced chi-squared value of 0.23 for the fit. The lower plot shows the residuals for this plot and uncertainties are shown on both graphs. No uncertainty was reported for the number of LC Units because they are quantized values. Python code for the fit can be seen in Appendix A.

The linear fit described in Figure 3 yielded a reduced chi-squared value of 0.23, which is near but lower than the ideal value of 1. This means that, while the actual linear fit between the two variables is reasonable, the reported uncertainty in the measurement for the dependent variable was likely too

Resistance Load of No Reflection	$353.5 \pm 0.05\Omega$

Table 1: Resistance Load Applied to the Transmission Line Observed to Cause No Reflection.

high given the resulting data. In addition, the residuals plot is decently ideal, with residuals being within uncertainty values to 0 and generally well distributed. It is possible to see that the data was generally above or generally low the lines at some points, which is most likely a result of the limited display size and resolution of the measurement for the oscilloscope. Hence, it overestimated certain values for several measurements before correcting itself lower. However, for the purposes of this lab, this fit is valid, useful, and can be compared to the theoretical equations.

The slope m of this linear fit corresponds to the inverse of the velocity as described as seen below, and as such it is possible to find the experimental velocity of the traveling wave. The derivation and calculation for the uncertainty in this value and all others going forward can be seen in Appendix B.

$$v_{tr,exp} = \frac{1}{m} = 0.2717 \pm 0.0022 \text{ Units}/\mu s$$

4.2 Coaxial Cables

The linear fit described in Figure 4 yielded a reduced chi-squared value of 0.10, which is near but lower than the ideal value of 1. Similarly to the previous experiment, this means that the uncertainty reported in measurement was likely too high, but that the actual fit is very good. The residuals plot is ideal, with residuals being within uncertainty values to 0 and generally well distributed, although more data points would strengthen the dataset.

As seen in Figure 4 and similar to the analysis on the transmission line above, the slope $m = 4.984 \pm 0.017$ ns/m corresponds to the inverse of the velocity as given below. So $v_{co,exp}$ for the electricity in the coaxial cables can be calculated.

$$v_{co,exp} = \frac{1}{m} = 0.200642 \pm 0.00068 \text{ m}/ns$$

Furthermore, by using recorded initial and reflected voltages, it is possible to calculate the attenuation for each cable using the relation from Equation 16 above. Using this relationship, the attenuation factor over the distance the pulse travels through the cable can be plotted, as seen in Figure 5.

The relevant linear fit seen in Figure 5 yielded a reduced chi-squared of 14.72, which is higher than ideal but still relatively close to one. The residuals' uncertainties also do not quite overlap with the zero axis but are well-distributed and decently small. This means that the fit may not be fully representative of the data, but it is fair for the purposes of this lab and the fit will be used moving forward for further analysis. This gives the value of $0.0531 \pm 0.0028 dB/m$ for the Attenuation per meter of coaxial cable.

Finally, Z_0 can be determined using the data as seen in Table 2. Z_0, as seen in Equation 14 is load impedance that returns no reflection through the cable. Through experiment, this value has been determined to be $51 \pm 0.5\Omega$ for each cable. in addition, it can be noted that Constructive Interference was only noted for a Cable Length of 15.09 m.

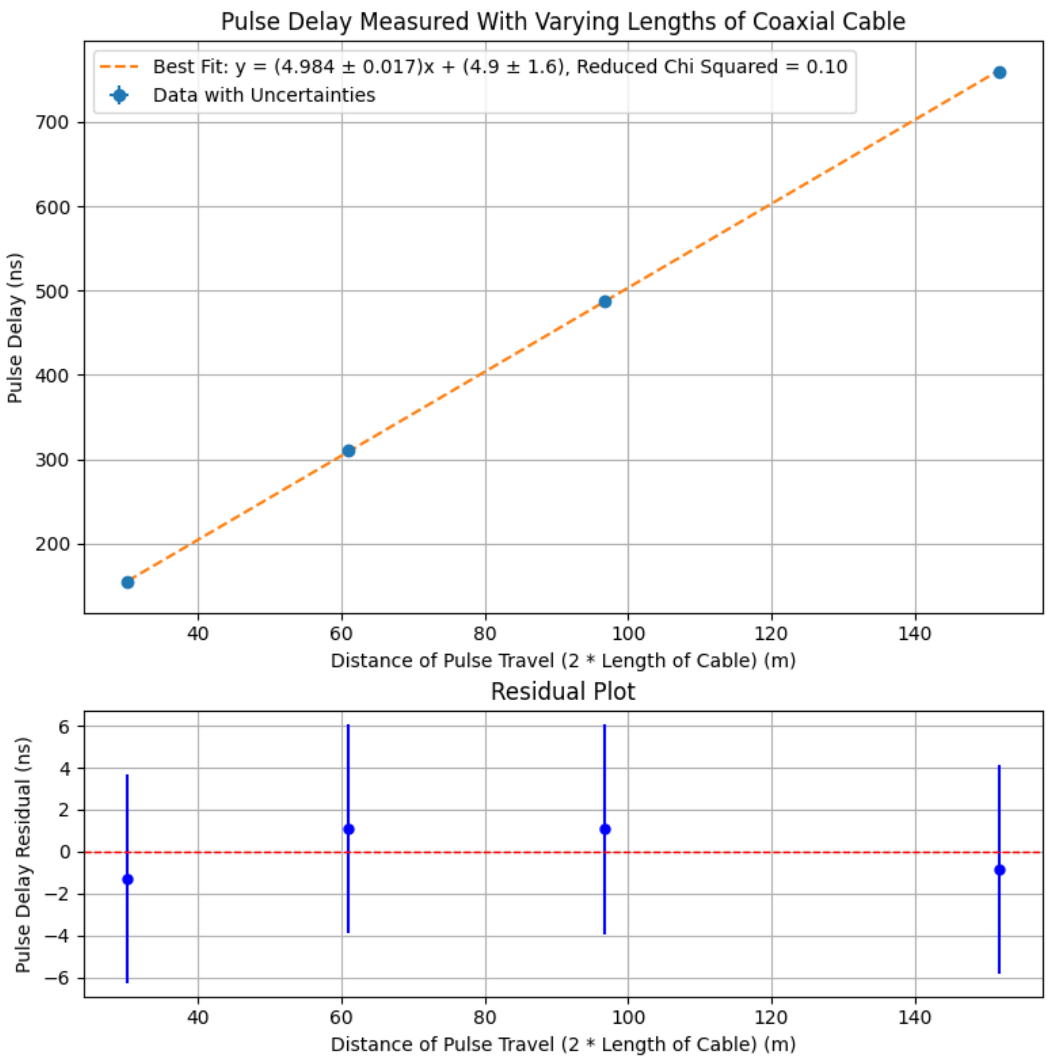


Figure 4: The linear fit for Pulse Delay with Varying Lengths of Cables graphed over the distance of the pulse travel is given by the parameters $m = 4.984 \pm 0.017 ns/m$ and $b = 4.9 \pm 1.6 ns$ with a reduced chi-squared value of 0.10 for the fit. The lower plot shows the residuals for this plot and uncertainties are shown on both graphs. Python code for the fit can be seen in Appendix A.

Coaxial Cable Length (m)	$51 \pm 0.5\Omega$ Termination Behavior	200ns Pulse Width Behaviour
15.09 ± 0.02	No Reflection	Constructive Interference
30.50 ± 0.02	No Reflection	None Reported
48.36 ± 0.02	No Reflection	None Reported
75.84 ± 0.02	No Reflection	None Reported

Table 2: Further Data on $51 \pm 0.5\Omega$ Termination Behavior and 200ns Pulse Width Behavior for Varying Lengths of Coaxial Cables.

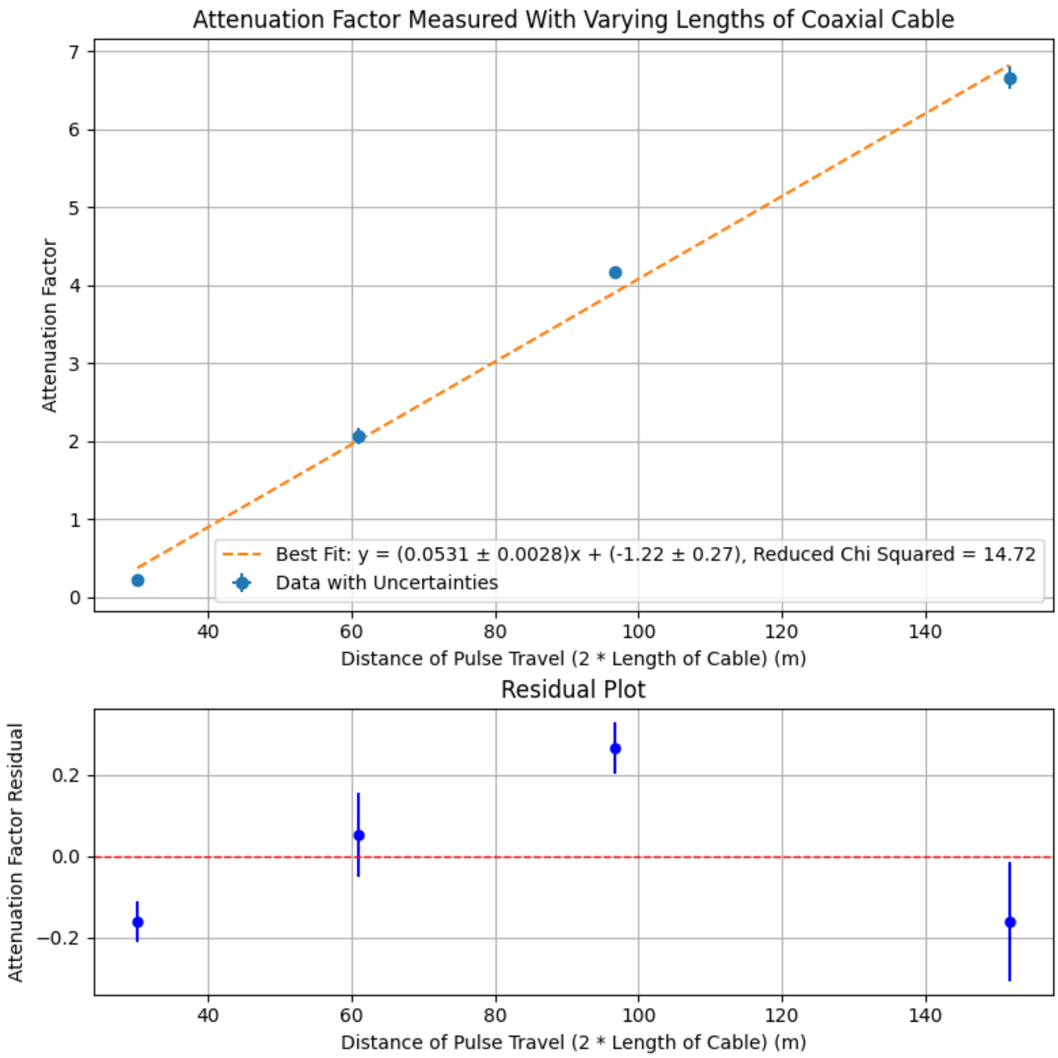


Figure 5: The linear fit for Attenuation Factor with Varying Pulse Travel Distance is given by the parameters $m = 0.0531 \pm 0.0028 dB/m$ and $b = -1.22 \pm 0.27 dB$ with a reduced chi-squared value of 14.72 for the fit. The lower plot shows the residuals for this plot and uncertainties are shown on both graphs.

5 Discussion and Conclusion

5.1 Transmission Line

The theoretical value of the velocity of the traveling wave can be determined from Equation 1 seen previously. Using Equation 1 below for the velocity of the wave with the values of $1.5 \pm 0.15mH$ and $0.01 \pm 0.0003\mu F$ for measured values of the inductance L_0 and capacitance C_0 of the circuit elements:

$$v_{tr,theor}^2 = \frac{1}{L_0 C_0}$$
$$v_{tr,theor} = \sqrt{\frac{1}{L_0 C_0}} = 0.258 \pm 0.013m/\mu s$$

This value matches up very well with the experimentally determined value of 0.2717 ± 0.0022 Units/μs, and the theoretical and experimental resultant values are within uncertainties of each other. Therefore, this theoretical model has been successfully verified by this experiment.

Further, it is possible to find the theoretical value of the characteristic impedance using the relation found in Equation 2 and the values for C_0 and L_0 previously mentioned.

$$Z_{0,theor} = \sqrt{\frac{L_0}{C_0}} = 387 \pm 20\Omega$$

This can then be compared to the experimentally determined value of this impedance which was $353.5 \pm 0.05\Omega$. These values are incredibly close, but unfortunately are not within uncertainties of each other. This means that either the theory is incorrect, there was some flaw in the equipment, or other external factors that led to this value being lower than expected. This relation assumes a perfect transmission line, so one possible explanation could be that the pulse is experiencing resistance within the transmission line before it ever reaches the load. This could have enough effect to slightly deviate the characteristic impedance from the theoretical value. Less resistive circuits could be used in the future to verify this reasoning and obtain a better understanding of this relationship.

5.2 Coaxial Cables

The theoretical value of the wave travel velocity through a Coaxial cable can be determined using a similar relation from Equation 5, where ε is the permittivity and μ is the magnetic permeability of the dielectric material contained within the Coaxial Cable. In the cables used in the experiment, the relative magnetic permeability and relative permittivity are given by $\mu_{rel} = 1 \pm 0.5$ and $\varepsilon_{rel} = 2.25 \pm 0.005$. These correspond to magnetic permeability and permittivity values of $\mu = (2\pi \pm \pi) * 10^{-7}$ H/m and $\varepsilon = (1.9922 \pm 0.0044) * 10^{-11}$ F/m [1][2].

$$v_{co,theor} = \sqrt{\frac{1}{\mu\varepsilon}} = 0.28 \pm 0.16m/ns$$

The uncertainty in this value is quite high due to uncertainties in the provided constants, however, the experimental value of wave velocity $0.200642 \pm 0.00068m/ns$ does fall within uncertainties to this value. So it is possible to say that the experimental data reasonably supports the model, albeit with a large uncertainty. In addition, both the theoretical and experimental values are below the value for the speed of light, which is also consistent with the theory.

In addition, it is possible to characterize the attenuation per meter and the characteristic impedance of these Coaxial cables. The experimental value of this, as determined earlier in the

Data and Analysis section, is 0.0531 ± 0.0028 dB/m of Coaxial Cable. Then, found in Table 2, it can be seen that a resistive load of $51 \pm 0.5\Omega$ led to no reflection in the cable. Thus, the characteristic impedance of the coaxial cables is $51 \pm 0.5\Omega$. With the measured value of $50 \pm 0.5\Omega$ for the impedance of the coaxial cables, this means the relationship as seen in Equation 14 has been verified where the value of the reflection coefficient is zero. In addition, constructive interference was absent for all cables except for the 15.09 m cable.

5.3 Conclusion

This paper has successfully verified the pulse delay present in transmission lines with an experimental value of 0.2717 ± 0.0022 Units/μs. This value lies within uncertainties for the theoretical value given by $0.258 \pm 0.013 m/\mu s$ and only deviates by around 5 percent. The characteristic impedance of the line has been found to slightly vary with a value of $353.5 \pm 0.05\Omega$, compared to the theoretical value of $387 \pm 20\Omega$. This is most likely due to the resistance not considered in the theoretical model. In addition, this paper has provided some evidence that the given model for wave propagation in coaxial cables is valid with an experimental value of 0.200642 ± 0.00068 m/ns and a theoretical value of 0.28 ± 0.16 m/ns. However, large uncertainties were present. Finally, this paper was able to successfully characterize several behaviors of Coaxial Cables including the Attenuation per Meter and the Characteristic Impedance with values of 0.0531 ± 0.0028 dB/m and $51 \pm 0.5\Omega$ respectively.

6 References

[1] $https://www.engineeringtoolbox.com/relative-permittivity-d_1660.html$
[2] $https://www.engineeringtoolbox.com/permeability-d_1923.html$
[3] $https://q.utoronto.ca/courses/324674/files/27571499?module_item_id=5066359$

7 Appendices

7.1 Appendix A: Python Code for Linear Fit

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
from scipy.stats import pearsonr
from matplotlib import gridspec

# Sample data with uncertainties (x, y, x_err, y_err)
x_data = np.array([30.18,61,96.72,151.68])
y_data = np.array([154,310,488,760])
x_err = np.array([.04,.04,.04,.04])
y_err = np.array([5,5,5,5])

# Define the linear function for the line of best fit
def linear_fit(x, m, b):
    return m * x + b

# Perform the curve fit to find the best-fit parameters (m and b)
params, covariance = curve_fit(linear_fit, x_data, y_data)

# Extract the best-fit parameters and their uncertainties
m, b = params
m_err, b_err = np.sqrt(np.diag(covariance))
correlation, pvalue = pearsonr(x_data, y_data)

# Reduced Chi Squared
residuals = y_data - linear_fit(x_data, m, b)
chi_squared = np.sum((residuals / y_err) ** 2)
degrees_of_freedom = len(x_data) - len(params)
reduced_chi_squared = chi_squared / degrees_of_freedom

# Create a figure with two subplots
fig = plt.figure(figsize=(8, 8))
gs = gridspec.GridSpec(2, 1, height_ratios=[2, 1], width_ratios=[1])

# Plot the data points with error bars and the line of best fit with uncertainty on the first subplot
ax1 = plt.subplot(gs[0])
ax1.errorbar(x_data, y_data, xerr=x_err, yerr=y_err, fmt='o', label='Data with Uncertainties')
x_fit = np.linspace(min(x_data), max(x_data), 100)
line_of_best_fit = lambda x: m * x + b
ax1.plot(x_fit, line_of_best_fit(x_fit), '--', label=f'Best Fit: y = ({m:.3f} ± {m_err:.3f})x +
({b:.1f} ± {b_err:.1f}), Reduced Chi Squared = {reduced_chi_squared:.2f}')
ax1.set_xlabel('Distance of Pulse Travel (2 * Length of Cable) (m)')
ax1.set_ylabel('Pulse Delay (ns)')
ax1.legend()
ax1.set_title('Pulse Delay Measured With Varying Lengths of Coaxial Cable')
```

Figure 6: Python Linear Fit Code: Part 1 of 2

```
ax1.grid(True)

# Create the residual plot on the second subplot
ax2 = plt.subplot(gs[1])
ax2.errorbar(x_data, residuals, yerr=y_err, xerr=x_err, linestyle='', marker='o', markersize=5,
color='b')
ax2.axhline(0, color='r', linestyle='--', linewidth=1)
ax2.set_xlabel('Distance of Pulse Travel (2 * Length of Cable) (m)')
ax2.set_ylabel('Pulse Delay Residual (ns)')
ax2.set_title('Residual Plot')
ax2.grid(True)

# Adjust the space between subplots
plt.tight_layout(h_pad=0.5)

# Show the combined figure
plt.show()

# Print the best-fit parameters and their uncertainties
print(f'Best-Fit Parameters:')
print(f'Slope (m) = {m:.2f} ± {m_err:.2f}')
print(f'Intercept (b) = {b:.2f} ± {b_err:.2f}')
print(f'Chi Squared = {reduced_chi_squared:.2f}')
```

Figure 7: Python Linear Fit Code: Part 2 of 2

7.2 Appendix B: Uncertainty Propagation Derivations

7.2.1 $\Delta v_{tr,exp}$

$$\frac{\Delta v_{tr,exp}}{v_{tr,exp}} = \sqrt{(\frac{\Delta m}{m})^2}$$
$$\Delta v_{tr,exp} = \frac{v_{tr,exp}\Delta m}{m}$$

7.2.2 $\Delta v_{co,exp}$

Similarly as $\Delta v_{tr,exp}$:
$$\Delta v_{co,exp} = \frac{v_{co,exp}\Delta m}{m}$$

7.2.3 Attenuation (dB) and ΔAttenuation

$$\frac{\partial A}{\partial V_r} = \frac{20}{V_r ln10}$$
$$\frac{\partial A}{\partial V_i} = -\frac{20}{V_i ln10}$$
$$\Delta A = \sqrt{(\frac{\partial A}{\partial V_r}\Delta V_r)^2 + (\frac{\partial A}{\partial V_i}\Delta V_i)^2}$$
$$\Delta A = \sqrt{(\frac{20}{V_r ln10}\Delta V_r)^2 + (-\frac{20}{V_i ln10}\Delta V_i)^2}$$

7.3 $\Delta v_{tr,theor}$

$$\frac{\partial v_{tr,theor}}{\partial L_0} = \frac{-1}{2L_0^{3/2}C_0^{3/2}}$$
$$\frac{\partial v_{tr,theor}}{\partial C_0} = \frac{-1}{2L_0^{3/2}C_0^{3/2}}$$
$$\Delta v_{tr,theor} = \sqrt{(\frac{-1}{2L_0^{3/2}C_0^{1/2}}\Delta L_0)^2 + (\frac{-1}{2L_0^{1/2}C_0^{3/2}}\Delta C_0)^2}$$

7.3.1 $\Delta Z_{0,theor}$

$$\frac{\partial \Delta Z_{0,theor}}{\partial L_0} = \frac{1}{2\sqrt{L_0 C_0}}$$
$$\frac{\partial \Delta Z_{0,theor}}{\partial C_0} = \frac{-\sqrt{L_0}}{2C_0^{3/2}}$$
$$\Delta Z_{0,theor} = \sqrt{(\frac{1}{2\sqrt{L_0 C_0}}\Delta L_0)^2 + (\frac{-\sqrt{L_0}}{2C_0^{3/2}}\Delta C_0)^2}$$

7.4 $\Delta v_{co,theor}$

Similarly as $\Delta v_{tr,theor}$:
$$\Delta v_{co,theor} = \sqrt{(\frac{-1}{2\mu^{3/2}\varepsilon^{1/2}}\Delta \mu)^2 + (\frac{-1}{2\mu^{1/2}\varepsilon^{3/2}}\Delta \varepsilon)^2}$$