$\boxed{A18}$

17.5/20*30 = 26

a) $f(x) = x^2 + x + \sqrt{x} + 1 + \frac{1}{\sqrt{x}} + \frac{1}{x} + \frac{1}{x^2}$

$f'(x) = 2x + 1 + \frac{1}{2\sqrt{x}} + 0 - \frac{1}{2}x^{-\frac{3}{2}} - \frac{1}{x^2} - 2\frac{1}{x^3}$ ✓

b) $f(x) = (x^2 + \sqrt{2x})^4$

$f'(x) = 4(x^2 + \sqrt{2x})^3 \left(2x + \frac{1}{\sqrt{2x}}\right)$ ✓✗

c) $f(x) = xe^{x^2}\ln(2+3x)$

$f'(x) = 1 \cdot e^{x^2} \cdot \ln(2+3x) + x \cdot e^{x^2} \cdot 2x \cdot \ln(2+3x) + x \cdot e^{x^2} \cdot \frac{1}{2+3x} \cdot 3$ ✓

d) $\arccos(\sqrt{x}) = f(x)$

$f'(x) = \frac{1}{\sqrt{1-\sqrt{x}^2}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$ ✓

e) $f(x) = \frac{\sin(2x)}{\ln(x^2+1)}$

$f'(x) = \frac{2\cos(2x) \cdot \ln(x^2+1) - \sin 2x \cdot \frac{2x}{x^2+1}}{\ln(x^2+1)^2}$ ✓✗

f) $f(x) = x^{\alpha}$

$\cancel{f'(x) = \alpha x^{\alpha - 1}}$

das ist bekannt für N, aber nicht für R

g) $f(x) = x^{-x^2} = e^{-x^2 \cdot \ln(x)}$

$f'(x) = x^{-x^2} \cdot \left(-2x \cdot \ln(x) - x^2 \cdot \frac{1}{x}\right)$ ✓

zwischenschritte

h) $f(x) = \ln(x + \ln(2\ln(x)))$

$f'(x) = \frac{1}{x + \ln(2\ln(x))} \cdot \left(1 + \frac{1}{2\ln x} \cdot \frac{2}{x}\right)$ ✓✗

$\boxed{A19}$

a) $f(x) = \cos(x)$

$f'(x) = \lim\limits_{h \to 0} \frac{\cos(x+h) - \cos(x)}{h}$ ✓ $= \lim\limits_{h \to 0} \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h}$

$= \lim\limits_{h \to 0} \cos(x) \cdot \underbrace{\frac{\cos(h) - 1}{h}}_{0} - \sin(x) \cdot \underbrace{\frac{\sin(h)}{h}}_{1} = \cos(x) \cdot 0 - \sin(x) \cdot 1 = -\sin(x)$ ✓

b) $f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}$

$f'(x) = \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos(x^2)}$ ✓ $= \frac{\cos^2(x) + \sin^2(x)}{\cos(x^2)}$ i) $= \frac{1}{\cos(x^2)}$ ✓

ii) $1 + \tan^2(x)$ ✓

$\boxed{A19}$ c) $\boxed{(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}}$

i) $\arctan'(y) = \frac{1}{1+\tan^2(\arctan(y))} = \frac{1}{1+y^2}$ ✓ ✓

ii) ~~tan~~ $f'(x) = 1 + \tan^2(x)$

$f''(x) = 0 + 2\tan(x)\cdot(1+\tan^2(x)) = 2\tan(x) + 2\tan^3(x)$ ✓ ✓

$f'''(x) = 2\cdot(1+\tan^2(x)) + 2(3\tan^2(x)\cdot(1+\tan^2(x))$

$= 2 + 2\tan^2(x) + 2\cancel{\tan^4(x)}(3\tan^2(x) + 3\tan^4(x))$

$= \underline{2 + 8\tan^2(x) + 6\tan^4(x)}$ ✓ ✓

$\boxed{A20}$ a) $f'(x) = \underline{\alpha x^{\alpha-1}\sin(x^{-2}) + x^{\alpha}\cos(x^{-2})\cdot(-2x^{-3})}$

b) $x=0$ $f'(0) = \lim\limits_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim\limits_{h\to 0} \frac{h^{\alpha}\sin(\frac{1}{h^2}) - 0}{h}$

$= \lim\limits_{h\to 0} h^{\alpha-1}\cdot \underbrace{\sin(\tfrac{1}{h^2})}_{\text{beschränkt zwischen } 1\ \&\ -1}$

1. Fall $\alpha < 1$ $\to \lim\limits_{h\to 0} h^{\alpha-1}$ existiert nicht $\Rightarrow f'(0)$ existiert nicht

2. Fall $\alpha = 1$ $\to \lim\limits_{h\to 0} h^{1-1} = \lim\limits_{h\to 0} 1 = 1$ existiert nicht $\Rightarrow f'(0)$ existiert nicht

3. Fall $\alpha > 1$ $\to \lim\limits_{h\to 0} h^{\alpha-1} = 0$ $f'(0)$ existreier und ist 0

d) $f''(x) = \alpha x^{\alpha-1}\cos(x^{-2})(-2x^{-3}) + (\alpha-1)(\alpha)x^{\alpha-2}\sin(x^{-2})$

$+ \alpha x^{\alpha-1}\cos(x^{-2})\cdot(-2x^{-3}) + x^{\alpha}(-\sin(x^{-2}))(-2x^{-3})(-2x^{-3})$

$+ x^{\alpha}\cos(x^{-2}) + 6x^{-4}$

$= \underline{[(\alpha x^{\alpha-1}\cos(x^{-2})(-2x^{-3})) + (x^{\alpha})(-\sin(x^{-2}))(-2x^{-3})(-2x^{-3})}$

$\underline{+ (x^{\alpha}\cos x^{-2} + 6x^{-4})]}$