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THE BRUCKLYN APARTMENTS

Mathe Übung

8/10*30=24

A12) a) i)

$\exp(ix)^3 = \exp(3ix) \Leftrightarrow i\sin(3x) + \cos(3x) = (\cos x + i\sin x)^3 \Leftrightarrow$

$\cos(3x) + i\sin(3x) = \cos^3 x + 3\cos^2 x\, i\sin x - 3\cos x \sin^2 x - i\sin^3 x \Leftrightarrow$

$\cos(3x) = \cos^3 x - 3\cos x \sin^2 x \wedge \sin(3x) = 3\cos^2 x \sin x - \sin^3 x$

$\cos(3x) = \cos^3 x - 3\cos x \sin^2 x \quad , \quad \sin(3x) = 3\cos^2 x \sin x - \sin^3 x$

ii) $\sin(3x) = \sin(x+2x) = \sin x \cos 2x + \cos x \sin 2x =$

$= \sin x(\cos(x+x)) + \cos x(\sin(x+x)) = \sin x(\cos^2 x - \sin^2 x) + \cos x(2\sin x \cos x) =$

$= -\sin^3 x + \sin x \cos^2 x + 2\sin x \cos^2 x = -\sin^3 x + 3\sin x \cos^2 x$

~~cos 3x~~ $\cos(3x) = \cos(x+2x) = \cos x \cos 2x - \sin x \sin 2x = \cos x \cos(x+x) - \sin x \sin(x+x) =$

$= \cos x(\cos^2 x - \sin^2 x) - \sin x(2\sin x \cos x) = \cos^3 x - \cos x \sin^2 x - 2\cos x \sin^2 x$

$= \cos^3 x - 3\cos x \sin^2 x$

b) i) $\sin(3\cdot\frac{\pi}{3}) = \sin\pi = 0 = -\sin^3\frac{\pi}{3} + 3\sin\frac{\pi}{3}\cos^2\frac{\pi}{3} = \sin\frac{\pi}{3}\left(-\sin^2\frac{\pi}{3} + 3\cos\frac{\pi}{3}\right) =$

$= \sin\frac{\pi}{3}\left(-\sin^2\frac{\pi}{3} + 3\left(1 - \sin^2\frac{\pi}{3}\right)\right) = \sin\frac{\pi}{3}\left(3 - 4\sin^2\frac{\pi}{3}\right)$

da $\sin\frac{\pi}{3} \neq 0$: $\quad 4\sin^2\frac{\pi}{3} = 3 \quad \sin\frac{\pi}{3} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\sin x(3\cos^2 x - \sin^2 x) = 0 \quad \rightarrow \frac{\sqrt{3}}{2}\left(3\cos^2\frac{\pi}{3} - \sin^2\frac{\pi}{3}\right) =$

$= \frac{\sqrt{3}}{2}\left(-1 + 4\cos^2\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} + \frac{\sqrt{3}\cdot 4}{2}\cos^2\frac{\pi}{3} = 0$

$\Leftrightarrow \frac{\sqrt{3}\cdot 4}{2}\cos^2\frac{\pi}{3} = \frac{\sqrt{3}}{2} \Leftrightarrow \sqrt{3}\cdot 2\cos^2\frac{\pi}{3} = \frac{\sqrt{3}}{2} \Leftrightarrow \cos^2\frac{\pi}{3} = \frac{1}{4} \Leftrightarrow \cos\frac{\pi}{3} = \frac{1}{2}$

ii) $\cos\frac{\pi}{3} = \frac{1}{2} = 1 - 2\sin^2\frac{\pi}{6} \Leftrightarrow \frac{1}{2} = 2\sin^2\frac{\pi}{6} \Leftrightarrow \sin\frac{\pi}{6} = \frac{1}{2}$

$\cos(2\cdot\frac{\pi}{6}) = \frac{1}{2} = \cos^2\frac{\pi}{6} - \sin^2\frac{\pi}{6} = \cos^2\frac{\pi}{6} - 1 + \cos^2\frac{\pi}{6} = 2\cos^2\frac{\pi}{6} - 1$

$\Leftrightarrow \frac{3}{4} = \cos^2\frac{\pi}{6} \Leftrightarrow \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

iii) $\cos 2\cdot\frac{\pi}{12} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} = 1 - 2\sin^2\frac{\pi}{12}$

$\Leftrightarrow 1 - \frac{\sqrt{3}}{2} = 2\sin^2\frac{\pi}{12} \Leftrightarrow \sqrt{\frac{1}{2} - \frac{\sqrt{3}}{4}} = \sin\frac{\pi}{12} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

$\frac{\sqrt{3}}{2} = \cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12} = 2\cos^2\frac{\pi}{12} - 1 \Leftrightarrow \cos^2\frac{\pi}{12} = \frac{\sqrt{3}}{2} + \frac{1}{2}$

$\Leftrightarrow \cos\frac{\pi}{12} = \sqrt{\frac{\sqrt{3}+2}{4}} = \frac{\sqrt{\sqrt{3}+2}}{2}$

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A11) a) i) $\sum_{k=0}^{\infty} \frac{5^k}{k} x^k$ $R = \frac{1}{\limsup_{k\to\infty} \sqrt[k]{|a_k|}} = \frac{1}{5}$ ✓

$\limsup_{k\to\infty} \sqrt[k]{|a_k|} = \limsup_{k\to\infty} \frac{5}{\sqrt[k]{k}} = 5$ ✓

ii) $\sum_{k=0}^{\infty} (\sqrt{k+1} - \sqrt{k-\sqrt{k}})^{2k} x^k$ $R = 4$

$\limsup_k \sqrt[k]{|a_k|} = \limsup_k (\sqrt{k+1} - \sqrt{k-\sqrt{k}})^{2k \cdot \frac{1}{k}} = \limsup_k (\sqrt{k+1} - \sqrt{k-\sqrt{k}})^2$ ✓

$= \limsup_k \left(\frac{k+1-k-\sqrt{k}}{\sqrt{k+1}+\sqrt{k-\sqrt{k}}}\right)^2 = \limsup_k \left(\frac{1-\sqrt{k}}{\sqrt{k+1}+\sqrt{k-\sqrt{k}}}\right)^2 = \limsup_k \left(\frac{\sqrt{k}(1/\sqrt{k}-1)}{\sqrt{k}(\sqrt{1+\frac{1}{k}}+\sqrt{1-\frac{1}{\sqrt{k}}})}\right)^2 = \frac{1}{4}$ ✓

iii) $\sum_{k=0}^{\infty} (k!+2) x^k$ $R = 0$

$\limsup_{k\to\infty} \sqrt[k]{|a_k|} = \limsup_k \sqrt[k]{k!+2} = \infty$ ✓ da $k!+2 \geq k! \; \forall k$ und $\lim_{k\to\infty} \sqrt[k]{k!} = \infty$ ✓

iv) $\sum_{k=0}^{\infty} \frac{2^k}{k^2} x^{4k}$ $z = x^4$ $R_z = \frac{1}{2}$ $R_x = \sqrt[4]{\frac{1}{2}}$ ✓

$\limsup_k \sqrt[k]{|a_k|} = \limsup_k \sqrt[k]{\frac{2^k}{k^2}} = \limsup_k \frac{2}{\sqrt[k]{k^2}} = \limsup_k \frac{2}{\sqrt[k]{k}\sqrt[k]{k}} = 2$ ✓

b) $\sum_{k=0}^{\infty} \left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}}\right)^k \left(\frac{1}{x+3}\right)^k$ +1 $z = \frac{1}{x+3}$ $R_z = 1$

$\limsup_k \sqrt[k]{|a_k|} = \limsup_k \sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} \limsup_k \sqrt[k]{3} \cdot \sqrt[k]{k} + \frac{4}{\sqrt[k]{k!}} = 1 \cdot 1 + 0 = ~~1~~ 2$

$\sum_{k=0}^{\infty} \left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}}\right)^k z^k$ konvergiert für $z < 1$ und divergiert für $z > 1$

$|z| = \left|\frac{1}{x+3}\right|$ $\longrightarrow$ Konvergenz für $\left|\frac{1}{x+3}\right| < 1$ $\Rightarrow$ $x > -2 \quad \forall x > -3$

$x < -2 \quad \forall x < -3$

$\Longrightarrow$ (-infty,-4) U (-2,infty)

Das Intervall ist somit $(-\infty, -3) \cup (-2, \infty)$.

A10) a) i)

$\left(\sum_{k=0}^{\infty} k q^k\right)\left(\sum_{k=0}^{\infty} q^k\right) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} k \cdot q^k \cdot q^{n-k} = \sum_{n=0}^{\infty}\sum_{k=0}^{n} k \cdot q^n = \sum_{n=0}^{\infty} q^n \sum_{k=0}^{n} k = \sum_{n=0}^{\infty} q^n \cdot \frac{n^2+n}{2}$

ii) $\sum_{k=0}^{\infty} k^2 q^k = \sum_{k=0}^{\infty} q^k \left(\frac{k^2+k}{2} \cdot 2 - k\right) = \sum_{k=0}^{\infty} 2q^k \frac{k^2+k}{2} - k \cdot q^k = \sum_{k=0}^{\infty} 2 \cdot \left(\sum_{n=0}^{\infty} q^n\right)\left(\sum_{n=0}^{\infty} n \cdot q^n\right) - k \cdot q^k$

$= \sum_{k=0}^{\infty} 2 \cdot \frac{1}{1-q} \cdot \frac{q}{(1-q)^2} - k \cdot q^k = \frac{2}{1-q} \cdot \frac{q}{(1-q)^2} \cdot (-1) \sum_{k=0}^{\infty} k \cdot q^k = \frac{2q}{(1-q)^2} - \frac{q}{(1-q)^2} = \frac{2q - q + q^2}{(1-q)^3} = \frac{q+q^2}{(1-q)^3}$

b)

$p_n = \sum_{k=0}^{n} \frac{1}{(k+1)(k+2)} = \sum_{k=0}^{n} \frac{A}{k+1} + \frac{B}{k+2} = \sum_{k=0}^{n} \frac{1}{k+1} - \sum_{k=0}^{n} \frac{1}{k+2} = \sum_{k=1}^{n} \frac{1}{k+2} - \sum_{k=0}^{n} \frac{1}{k+2}$

$< 1 - \frac{1}{n+2} \xrightarrow{n\to\infty} 1$ Grenzwert der Reihe: 1

$\frac{1}{(k+1)(k+2)} \stackrel{!}{=} \frac{A}{k+1} + \frac{B}{k+2} \Leftrightarrow 1 = A(k+2) + B(k+1) \Leftrightarrow 1 = k(A+B) + 2A + B$

$\Leftrightarrow A + B = 0 \wedge 1 = 2A + B \Leftrightarrow A = 1 \wedge B = -1$