Deckblatt für die Abgabe der Übungsaufgaben
IngMathC(1+1)

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Blatt-Nummer: 04

Übungsgruppen-Nr: 07

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A10, A11, A12, ______

7/10 *30 = 21

A10

a) i) $\sum_{k=0}^{\infty} k q^k \cdot \sum_{k=0}^{\infty} q^k = \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} k q^k q^{n-k}\right)$

$= \sum_{n=0}^{\infty}\sum_{k=0}^{n} k q^n = \sum_{n=0}^{\infty} q^n \underbrace{\sum_{k=0}^{n} k}_{\text{nach dieser Gaußformel oder } \left(\frac{n}{2}\cdot n+1\right)} = \sum_{n=0}^{\infty} \frac{n}{2}(n+1) q^n$

$(0+10)+(9+1)+(8+2)\ldots+5$
Bei ungeraden läuft man ohne 0 an
z.B. $(1+11)+(2+10)\ldots$

ii) $\sum_{k=0}^{\infty} k^2 q^k = 2\sum_{k=0}^{\infty} \frac{k^2}{2} q^k = 2\left(\sum\left(\frac{k^2}{2}q^k + \frac{k}{2}q^k\right)\right) - \sum_{k=0}^{\infty} kq$

(nach i))

$= 2\left(\frac{1}{1-q}\cdot\frac{q}{(1-q)^2}\right) - \frac{(1-q)}{1-q} = \frac{2q}{(1-q)^3} - \frac{1}{1-q}$

$= \frac{2q}{(1-q)^3} - \frac{(1-q)^2}{(1-q)^3} = \frac{2q-(1-2q+q^2)}{(1-q)^3} = \frac{1+4q+q^2}{(1-q)^3}$

b) $\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+2)}$ (da die Reihe konvergent ist, darf sie umgeformt werden.)

$\Leftrightarrow \left(\frac{1}{1}\cdot\frac{1}{2}\right)+\left(\frac{1}{2}\cdot\frac{1}{3}\right)+\left(\frac{1}{3}\cdot\frac{1}{4}\right)+\left(\frac{1}{4}\cdot\frac{1}{5}\right)+\left(\frac{1}{5}\cdot\frac{1}{6}\right)+\left(\frac{1}{6}\cdot\frac{1}{7}\right)+$

$\Leftrightarrow \frac{1}{2}\left(\frac{1}{1}+\frac{1}{3}\right)+\frac{1}{4}\left(\frac{1}{3}+\frac{1}{5}\right)+\frac{1}{6}\left(\frac{1}{5}+\frac{1}{7}\right)+\ldots$

$\Leftrightarrow \sum_{k=0} \frac{1}{3}\left(\frac{1}{2}\right.$

(WH am Donnerstag)

Vasily [illegible]

b) $\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+2)} \Leftrightarrow \sum_{k=0}^{\infty} \left(\frac{1}{(k+1)} - \frac{1}{(k+2)} \right)$

$\Leftrightarrow \sum_{k=0}^{\infty} \frac{1}{(k+1)} - \sum_{k=0}^{\infty} \frac{1}{(k+2)}$ (der Bezug hier muss konvergent umstellen)

$\Leftrightarrow \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots$

$\Leftrightarrow \frac{1}{1} + \underbrace{\left(\frac{1}{2} - \frac{1}{2}\right)}_{0} + \underbrace{\left(\frac{1}{3} - \frac{1}{3}\right)}_{0} + \underbrace{\left(\frac{1}{4} - \frac{1}{4}\right)}_{0} + \ldots$

$\Leftrightarrow \underline{1}$, d.h. der Grenzwert ist $\underline{1}$

A11

a) i) $\sum_{k=0}^{\infty} \frac{5^k}{k} x^k$, $\sqrt[k]{\left|\frac{5^k}{k}\right|} = \frac{5}{\sqrt[k]{k}}$

$\lim_{k \to \infty} \frac{5}{\sqrt[k]{k}} = \frac{5}{1} = 5 \Rightarrow R_x = \underline{\frac{1}{5}}$

ii) $\sum_{k=0}^{\infty} \left(\sqrt{k+1} - \sqrt{k - \sqrt{k}}\right)^{2k} x^k$, $\sqrt[k]{\left|\left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^{2k}\right|}$

$= \left(\sqrt{k+1} - \sqrt{k - \sqrt{k}}\right)^2$

$\lim_{k \to \infty} \left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^2 = \left(\lim_{k \to \infty} \left(\sqrt{k+1} - \sqrt{k - \sqrt{k}}\right)\right)^2$

$= \left(\lim_{k \to \infty} \frac{(k+1) - (k - \sqrt{k})}{\sqrt{k+1} + \sqrt{k - \sqrt{k}}}\right)^2 = \left(\lim_{k \to \infty} \frac{1 - \sqrt{k}}{\sqrt{k+1} + \sqrt{k - \sqrt{k}}}\right)^2$

$= \left(\lim_{k \to \infty} \frac{\sqrt{k}\left(\frac{1}{\sqrt{k}} - 1\right)}{\sqrt{k}\left(\sqrt{1 + \frac{1}{k}} + \sqrt{1 - \frac{1}{\sqrt{k}}}\right)}\right)^2 = \left(\frac{-1}{1+1}\right)^2 = \underline{\frac{1}{4}}$

$\Rightarrow R_x = \frac{1}{\frac{1}{4}} = \underline{4}$

iii) $\sum_{k=0}^{\infty}(k!+2)x^k$, $\sqrt[k]{|k!+2|} \geq \sqrt[k]{|k!|}$

$\lim_{k\to\infty} \sqrt[k]{k!} = \infty \Rightarrow R_x = 0$

iv) $\sum_{k=0}^{\infty} \frac{2^k}{k^2} x^{4k}$, Substitution $\tilde{k}_x = 4k$

$\sum_{\tilde{k}=0}^{\infty} \frac{2^{\frac{\tilde{k}}{4}}}{(\frac{\tilde{k}}{4})^2} x^{\tilde{k}}$, $\sqrt[\tilde{k}]{\frac{2^{\frac{\tilde{k}}{4}}}{(\frac{\tilde{k}}{4})^2}} = \frac{2^{\frac{1}{4}}}{\sqrt[\tilde{k}]{\frac{\tilde{k}^2}{16}}} = \frac{1}{2^{4}\sqrt[\tilde{k}-1]{\frac{\tilde{k}}{4}}}$

$\xrightarrow{\tilde{k}\to\infty} \frac{1}{2^{4}\cdot 1} = \frac{1}{2^4}$

$\Rightarrow R_x = \frac{1}{\frac{1}{2^4}} = 2^4 = 16$

Wenn du das machst, musst du eine Fallunter

b) $S(x) := \sum_{k=0}^{\infty}\left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1\right)^k \left(\frac{1}{x+3}\right)^k$

Substitution $y = \frac{1}{x+3}$

$\sqrt[k]{|(\ldots)^k|} = \sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1 \xrightarrow{k\to\infty} 1 + 0 + 1 = 2$

$R_y = \frac{1}{2}$, da $y = \frac{1}{x+3} \iff \frac{1}{2} > \left|\frac{1}{x+3}\right|$

$\Rightarrow (x > -1) \wedge (x < -5)$

Auf der Menge $(-\infty, -5) \cup (-1, +\infty)$ ist die Reihe konvergent.

das gilt nur im inneren, die Ränder gelten erstmal nicht.(Hier muss man sich die Ränder auch nich

b) (i) $\overbrace{\sin\left(3\frac{\pi}{3}\right)}^{0} = -\sin^3\left(\frac{\pi}{3}\right) + 3\sin\left(\frac{\pi}{3}\right)\cos^2\left(\frac{\pi}{3}\right)$

$= \sin\left(\frac{\pi}{3}\right)\left(-\sin^2\left(\frac{\pi}{3}\right) + 3\cos^2\left(\frac{\pi}{3}\right)\right) = \sin\left(\frac{\pi}{3}\right)\left(-\sin^2\left(\frac{\pi}{3}\right) + 3\left(1 - \sin^2\left(\frac{\pi}{3}\right)\right)\right)$

$= \underbrace{\sin\left(\frac{\pi}{3}\right)}\left(-4\sin^2\left(\frac{\pi}{3}\right) + 3\right)$

nicht 0 $\Rightarrow -4\sin^2\left(\frac{\pi}{3}\right) + 3 = 0 \Leftrightarrow \frac{3}{4} = \sin^2\left(\frac{\pi}{3}\right)$, da positiv

$\Leftrightarrow \sin\left(\frac{\pi}{3}\right) = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\sin^2\left(\frac{\pi}{3}\right) + \cos^2\left(\frac{\pi}{3}\right) = 1$

$\Leftrightarrow \cos^2\left(\frac{\pi}{3}\right) = 1 - \sin^2\left(\frac{\pi}{3}\right) = 1 - \frac{3}{4} = \frac{1}{4}$

da positiv

$\Leftrightarrow \cos\left(\frac{\pi}{3}\right) = \sqrt{\frac{1}{4}} = \frac{1}{2}$

ii) $\frac{\sqrt{3}}{2} \neq \cos \quad \frac{1}{2} = \cos 2\frac{\pi}{6} = 1 - 2\sin^2\frac{\pi}{6}$

$\Leftrightarrow \frac{1}{2} = 1 - 2\sin^2\frac{\pi}{6}$

$\Leftrightarrow 2\sin^2\frac{\pi}{6} = 1 - \frac{1}{2} = \frac{1}{2}$, da positiv

$\Leftrightarrow \sin\frac{\pi}{6} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\Rightarrow \sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{6} = 1 \Leftrightarrow \cos^2\frac{\pi}{6} = 1 - \frac{1}{4} = \frac{3}{4}$

(da pos.) $\Leftrightarrow \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

iii) $\frac{\sqrt{3}}{2} = \cos\frac{2\pi}{12} = 1 - 2\sin^2\frac{\pi}{12} \Leftrightarrow \sin^2\frac{\pi}{12} = \frac{1 - \frac{\sqrt{3}}{2}}{2} = \frac{2-\sqrt{3}}{4}$

$\Rightarrow \sin\frac{\pi}{12} = \frac{\sqrt{2-\sqrt{3}}}{2}$

$\Rightarrow \cos^2\frac{\pi}{12} = 1 - \frac{2-\sqrt{3}}{4} = \frac{4-2-\sqrt{3}}{4} = \frac{2+\sqrt{3}}{4}$

$\Rightarrow \cos\frac{\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2}$

(112) $e^{ix} = \cos(x) + i\sin(x)$ (54)

i) $\exp(3ix) = \exp(ix)^3$

$\Leftrightarrow e^{i3x} = (e^{ix})^3$

$\Leftrightarrow \cos(3x) + i\sin(3x) = \left(\cos(x) + i\sin(x)\right)^3$

$= \left(\cos^2(x) + 2i\cos(x)\sin(x) - \sin^2(x)\right)\cdot\left(\cos(x) + i\sin(x)\right)$

$= \cos^3(x) + 2i\cos^2(x)\sin(x) - \sin^2(x)\cos(x)$

$+ i\cos^2(x)\sin(x) - 2\cos(x)\sin^2(x) - i\sin^3(x)$

$= \cos^3(x) + 3i\cos^2(x)\sin(x) - 3\cos(x)\sin^2(x) - i\sin^3(x)$

$\Rightarrow \underline{\cos(3x) = \cos^3(x) - 3\cos(x)\sin^2(x)}$

$\wedge \underline{\sin(3x) = -\sin^3(x) + 3\cos^2(x)\sin(x)}$

ii) $3x = x + 2x$

$\sin(3x) = \sin(x+2x) = \sin(x)\cos(\underbrace{2x}_{x+x}) + \cos(x)\sin(\underbrace{2x}_{x+x})$

$= \sin(x)\left(\cos(x)\cos(x) - \sin(x)\sin(x)\right) + \cos(x)\left(\sin(x)\cos(x) + \cos($

$= \sin(x)\cos^2(x) - \sin^3(x) + \cos^2(x)\sin(x) + \cos^2(x)\sin(x)$

$= \underline{-\sin^3(x) + 3\sin(x)\cos^2(x)}$

$\cos(3x) = \cos(x+2x) = \cos(x)\cos(\underbrace{2x}_{x+x}) - \sin(x)\sin(\underbrace{2x}_{x+x})$

$= \cos(x)\left(\cos^2(x) - \sin^2(x)\right) - \sin(x)\left(\cos(x)\sin(x) + \cos(x)\sin(x)\right)$

$= \cos^3(x) - \cos(x)\sin^2(x) - \cos(x)\sin^2(x) - \cos(x)\sin^2(x)$

$= \underline{\cos^3(x) - 3\cos(x)\sin^2(x)}$