Deckblatt für die Abgabe der Übungsaufgaben IngMathC2

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Blatt-Nummer: 8

Übungsgruppen-Nr: 7

Die folgenden Aufgaben gebe ich zur Korrektur frei:

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Aufgabenblatt 8:

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a) $\lim\limits_{x\to 0} \dfrac{\sin(x^2)}{x^3 + 8x^2}$, Typ $\frac{"0"}{"0"}$

$\overset{l'H}{=} \lim\limits_{x\to 0} \dfrac{2x \cdot \cos(x^2)}{3x^2 + 16x}$, Typ $\frac{"0"}{"0"}$

$\overset{l'H}{=} \lim\limits_{x\to 0} \dfrac{2\cdot \cos(x^2) + 2x(-\sin(x^2))\cdot 2x}{6x + 16} = \lim\limits_{x\to 0} \dfrac{2\cos(x^2) + 4x^2(-\sin(x^2))}{6x+16} =$

$= \dfrac{2\cdot 1 + 0}{0 + 16} = \dfrac{2}{16} = \underline{\dfrac{1}{8}}$ ✓✗

b) $\lim\limits_{x\to 0} \dfrac{\sin(x)}{x} = 1$

Substitution: $y = x^2$

$\lim\limits_{y\to 0} \dfrac{\sin(y)}{y^{3/2} + 8y} = \lim\limits_{y\to 0}\left(\left(\dfrac{1}{\sqrt{y}+8}\right)\left(\dfrac{\sin(y)}{y}\right)\right) = \dfrac{1}{8}\cdot \lim\limits_{y\to 0} \dfrac{\sin(y)}{y} = \dfrac{1}{8}\cdot 1$ ✓✗

$y\cdot\sqrt{y}$

also: $\lim\limits_{x\to 0} \dfrac{\sin(x^2)}{x^3 + 8x^2} = \lim\limits_{x\to 0}\left(\dfrac{1}{x+8}\cdot \dfrac{\sin(x^2)}{x^2}\right) = \lim\limits_{x\to 0}\dfrac{1}{x+8}\cdot \lim\limits_{x\to 0}\dfrac{\sin(x^2)}{x^2} = \dfrac{1}{8}\cdot 1 = \underline{\dfrac{1}{8}}$

c) $\lim\limits_{x\to 0} \sqrt{x}\, \ln x$, Typ $"0\cdot(-\infty)"$

$\overset{aut}{=} \lim\limits_{x\to 0} \dfrac{\sqrt{x}}{\frac{1}{\ln x}} \overset{l'H}{=} \lim\limits_{x\to 0} \dfrac{\frac{1}{2\sqrt{x}}}{\frac{1}{x}} = \lim\limits_{x\to 0} \dfrac{x}{2\sqrt{x}} \overset{l'H}{=} \dfrac{1}{\frac{1}{\sqrt{x}}} = \lim\limits_{x\to 0} \sqrt{x} = \underline{0}$ ✓✗

d) $\lim\limits_{x\to\infty} \sqrt{x}\cdot \ln x = \lim\limits_{x\to\infty}\sqrt{x}\cdot \lim\limits_{x\to\infty}\ln x = \infty\cdot\infty = \infty$ ✓

e) $\lim\limits_{x\to 0} \dfrac{\cos(6x)-1}{x^3 + 2x^2}$, Typ $\frac{"0"}{"0"}$

$\overset{l'H}{=} \lim\limits_{x\to 0} \dfrac{-\sin(6x)\cdot 6}{3x^2 + 4x} \overset{l'H}{=} \lim\limits_{x\to 0} \dfrac{(-6)\cos(6x)\cdot 6}{6x + 4} = \dfrac{\cos(0)\cdot(-36)}{6\cdot 0 + 4}$ ✓ $= -\dfrac{36}{4} = \underline{-9}$ ✓

f) $\lim\limits_{x\to 0} \dfrac{1-\cos(x)\cos(2x)}{e^{x^2}-1}$, Typ $\frac{"0"}{"0"}$

$\overset{l'H}{=} \lim\limits_{x\to 0} \dfrac{-\sin(x)\cdot(-\cos(2x)) - \cos(x)\cdot\sin(2x)\cdot 2}{e^{x^2}\cdot 2x} =$

$\overset{l'H}{=} \lim\limits_{x\to 0} \dfrac{\cos(x)\cdot(-\cos(2x)) + \sin(x)\sin(2x)\cdot 2 - (-\sin(x)\cdot\sin(2x)\cdot 2 + \cos(x)\cdot\cos(x)\cdot 4)}{e^{x^2}\cdot 2x + e^{x^2}\cdot 2}$

$= \dfrac{1\cdot(-1)2 + 0 - (0 + 4)}{1\cdot 0 + 1\cdot 2} = \dfrac{-2 - 4}{2} = -\dfrac{3}{1} = -3 \quad 2{,}5$ ✓

g) $\displaystyle\lim_{x\to\infty} \frac{\ln(1+\alpha x)}{\ln(\ln(e^{\beta x}+e^{-\beta x}))}$, $\alpha, \beta \in \mathbb{R}^+$, Typ "$\frac{\infty}{\infty}$"

$\displaystyle\overset{l'H}{=} \lim_{x\to\infty} \frac{(1+\alpha x)^{-1}\cdot \alpha}{(\ln(e^{\beta x}+e^{-\beta x}))^{-1}\cdot (e^{\beta x}+e^{-\beta x})^{-1}\cdot (e^{\beta x}\cdot\beta + e^{-\beta x}\cdot(-\beta))}$ = ✓

$\displaystyle = \lim_{x\to\infty} \frac{\alpha\,(\ln(e^{\beta x}+e^{-\beta x}))\cdot(e^{\beta x}+e^{-\beta x})}{(1+\alpha x)(\beta e^{\beta x}-\beta e^{-\beta x})} = \frac{\alpha}{\beta}\lim_{x\to\infty}\frac{\ln(e^{\beta x}+e^{-\beta x})(e^{\beta x}+e^{-\beta x})}{(1+\alpha x)(e^{\beta x}-e^{-\beta x})}$ ✗

h) $\displaystyle\lim_{x\to\infty}\frac{6\sqrt[3]{x}+5+\frac{4}{\sqrt[3]{x}}}{3\sqrt[3]{x}+e^{-2x}+\frac{1}{\sqrt[3]{x}}}$, Typ "$\frac{\infty}{\infty}$"

$\displaystyle\overset{l'H}{=}\lim_{x\to\infty}\frac{3x^{-\frac{1}{2}}-2x^{-\frac{3}{2}}}{\frac{3}{2}x^{-\frac{1}{2}}-2e^{-2x}-\frac{1}{2}x^{-\frac{3}{2}}}\overset{l'H}{=}\lim_{x\to\infty}\frac{-\frac{3}{2}x^{-\frac{3}{2}}+3x^{-\frac{5}{2}}}{-\frac{3}{4}x^{-\frac{3}{2}}+4e^{-2x}+\frac{3}{4}x^{-\frac{5}{2}}}= \ldots$

Einschachteln, ab $x>1$ gilt

$$\frac{6\sqrt[3]{x}+5+\frac{4}{\sqrt[3]{x}}}{3\sqrt[3]{x}+e^{-2x}+\frac{1}{\sqrt[3]{x}}} < \frac{6\sqrt[3]{x}+5+\frac{4}{\sqrt[3]{x}}}{3\sqrt[3]{x}+e^{-2x}+\frac{1}{\sqrt[3]{x}}} < \frac{6x+5+\frac{4}{x}}{3x+e^{-2x}+\frac{1}{x}}$$

$\displaystyle\lim_{x\to\infty}\frac{6\sqrt[3]{x}+5+\frac{4}{\sqrt[3]{x}}}{3\sqrt[3]{x}+e^{-2x}+\frac{1}{\sqrt[3]{x}}}=$

$\displaystyle=\lim_{x\to\infty}\frac{6+\frac{5}{\sqrt[3]{x}}+\frac{4}{\sqrt[3]{x^2}}}{3+\frac{e^{-2x}}{\sqrt[3]{x}}+\frac{1}{\sqrt[3]{x^2}}}=$

$\displaystyle=\frac{6+0+0}{3+0+0}=\frac{6}{3}=2$

$\displaystyle\lim_{x\to\infty}\frac{6x+5+\frac{4}{x}}{3x+e^{-2x}+\frac{1}{x}}=$

$\displaystyle=\lim_{x\to\infty}\frac{6+\frac{5}{x}+\frac{4}{x^2}}{3+\frac{e^{-2x}}{x}+\frac{1}{x^2}}=$

$\displaystyle=\frac{6+0+0}{3+0+0}=2$ ✓

$\displaystyle\to\lim_{x\to\infty}\frac{6\sqrt[3]{x}+5+\frac{4}{\sqrt[3]{x}}}{3\sqrt[3]{x}+e^{-2x}+\frac{1}{\sqrt[3]{x}}}=\lim_{x\to\infty}\frac{6x+5+\frac{4}{x}}{3x+e^{-2x}+\frac{1}{x}}=\lim_{x\to\infty}\frac{6\sqrt[3]{x}+5+\frac{4}{\sqrt[3]{x}}}{3\sqrt[3]{x}+e^{-2x}+\frac{1}{\sqrt[3]{x}}}=2$ ✓

A22

a) $f : [-\frac{\pi}{2}, \pi] \rightarrow \mathbb{R}$ $f(x) = \cos x - \cos^2 x$

$f'(x) = -\sin x - 2\cos x \cdot (-\sin x)$

$-\sin x - 2\cos x \cdot (-\sin x) = 0$

$1 = 2\cos x$

$x_1 = \frac{\pi}{3}$ in $f : [-\frac{\pi}{2}, \pi]$

$x_2 = \frac{-\pi}{3}$ in $f : [-\frac{\pi}{2}, \pi]$

$\rightarrow f\left(\frac{-\pi}{2}\right) = 0 \cdot 0^2 = 0$

$f\left(\frac{-\pi}{3}\right) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$f\left(\frac{\pi}{3}\right) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ } Maxima in $\{ (\frac{-\pi}{3}, \frac{1}{4}), (\frac{\pi}{3}, \frac{1}{4}) \}$

$f(\pi) = -1 - (-1)^2 = -2$ $\rightarrow$ Minimum in $(\pi, -2)$

b) $f : (0, \infty)$, $f(x) = \frac{\ln x}{x}$

$f'(x) = \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$

$\frac{1 - \ln x}{x^2} = 0$

$1 = \ln x$

$x = e$

$\rightarrow \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\ln x}{x} \overset{l'H}{=} \lim_{x \to 0} \frac{1}{x} = +\infty$

$f(e) = \frac{1}{e} = e^{-1}$

$\lim_{x_2 \to \infty} f(x) = \lim_{x_2 \to \infty} \frac{\ln x}{x} \overset{l'H}{=} \lim_{x_2 \to \infty} \frac{1}{x} = 0$

$\rightarrow$ Maximum am linken Rand von $f(x)$

Minimum am rechten Rand von $f(x)$