Deckblatt für die Abgabe der Übungsaufgaben
IngMathC2

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Blatt-Nummer: 4

Übungsgruppen-Nr: 7

Die folgenden Aufgaben gebe ich zur Korrektur frei:

10, 11, 12, (alle)

7/10*30 = 21

A10

a) i) $\sum_{k=0}^{\infty} kq^k$ $\quad |q|<1$

$\sum_{k=0}^{\infty} q^k$

$$\left(\sum_{k=0}^{\infty} kq^k\right)\left(\sum_{k=0}^{\infty} q^k\right) = \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} kq^k q^{n-k}\right)$$

Innere Summenformel

$$\left(\sum_{k=0}^{n} kq^k q^{n-k}\right) = \sum_{k=0}^{n} k\cdot q^n = q^n \cdot \sum_{k=0}^{n} k = q^n\left(\frac{n^2+n}{2}\right)$$

$$\Rightarrow \left(\sum_{k=0}^{\infty} kq^k\right)\cdot\left(\sum_{k=0}^{\infty} q^k\right) = \sum_{n=0}^{\infty}\left(q^n\left(\frac{n^2+n}{2}\right)\right) = \frac{1}{1-q}\sum_{n=0}^{\infty}\underbrace{\left(\frac{n^2+n}{2}\right)}_{=\sum_{k=0}^{n} k}$$

ii) $\sum_{k=0}^{\infty} q^k = \frac{1}{1-q}$, $\sum_{k=0}^{\infty} k q^k = \frac{1}{(1-q)^2} - \frac{1}{1-q} = \frac{q}{(1-q)^2}$

Annahme: $\left(\sum_{k=0}^{\infty} k q^k\right)\left(\sum_{k=0}^{\infty} q^k\right) = \sum_{k=0}^{\infty} k^2 q^k$

$$\Rightarrow \left(\sum_{k=0}^{\infty} k q^k\right)\left(\sum_{k=0}^{\infty} q^k\right) = \left(\frac{q}{(1-q)^2}\right)\left(\frac{1}{(1-q)}\right) =$$

$$= \frac{q}{(1-q)^3} = \sum_{k=0}^{\infty} k^2 q^k \qquad |q|<1$$

b) n-te Partialsumme: $p_n = \sum_{k=0}^{n} \frac{1}{(k+1)(k+2)} = \sum_{k=0}^{n} \left(\frac{1}{k+1} - \frac{1}{k+2}\right) =$

$$= \sum_{k=0}^{n} \frac{1}{k+1} - \sum_{k=0}^{n} \frac{1}{k+2} = \sum_{k=-1}^{n-1} \frac{1}{k+2} - \sum_{k=0}^{n} \frac{1}{k+2} =$$

$$= \sum_{k=-1}^{n-1} \frac{1}{k+2} - \sum_{k=0}^{n} \frac{1}{k+2} = \frac{1}{-1+2} - \frac{1}{n+2} = 1 - \frac{1}{n+2}$$

$$\xrightarrow{n \to \infty} 1 - 0 = \underline{1}$$

A11

a) (i) $\sum_{k=0}^{\infty} \frac{5^k}{k} x^k =: \sum_{k=0}^{\infty} a_k x^k$

$\limsup_k \sqrt[k]{\left|\frac{5^k}{k}\right|} = \lim_{k\to\infty} \frac{5}{\sqrt[k]{k}} = 5$ ✓

was ist R

(ii) $\sum_{k=0}^{\infty} \left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^{2k} x^k$

$\limsup_k \sqrt[k]{\left|\left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^{2k}\right|} =$

$= \limsup_k \left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^2 =$

$= \limsup_k \left(k+1 + k - \sqrt{k} - 2\sqrt{k+1}\sqrt{k-\sqrt{k}}\right) =$

$= \limsup_k \left(2k - \sqrt{k} + 1 - 2\sqrt{(k+1)(k-\sqrt{k})}\right) =$

$= \limsup_k \left(2k - \sqrt{k} + 1 - 2\sqrt{k^2 - k\sqrt{k} + k - \sqrt{k}}\right) =$ ✓

$= \limsup_k \left(2k - \sqrt{k} + 1 - 2k\sqrt{1 - \underbrace{\frac{\sqrt{k}}{k}}_{k\to\infty: 0} + \underbrace{\frac{1}{k}}_{\to 0} - \underbrace{\frac{\sqrt{k}}{k^2}}_{\to 0}}\right) =$

$= \limsup_k \left(2k - \sqrt{k} + 1 - 2k\right) = \limsup_k \left(-\sqrt{k} + 1\right) = 0$

$\rightarrow R = \infty$

Das problem hab ich in der Übung einmal angesprochen:du machs

(iii) $\sum_{k=0}^{\infty} (k!+2) x^k$

$\limsup_k \sqrt[k]{|k!+2|} = \infty \longrightarrow R = 0$

$\sqrt[k]{k!} < \sqrt[k]{k!+2}$

$\lim_{k\to\infty} \sqrt[k]{k!} = \infty \Rightarrow \sqrt[k]{k!+2} = \infty$

(iv) $\sum_{k=0}^{\infty} \frac{2^k}{k^2} x^{4k}$ mit $y := x^4$ folgt: $\sum_{k=0}^{\infty} \frac{2^k}{k^2} y^k$

$\limsup_k \sqrt[k]{\left|\frac{2^k}{k^2}\right|} = \limsup_k \frac{2}{\sqrt[k]{k^2}} = \limsup_k \frac{2}{(\sqrt[k]{k})^2} = \frac{2}{1^2} = 2$

$k\to\infty: 1$

$\Rightarrow R_y = \frac{1}{2}$

$\Rightarrow y = x^4 \rightarrow x = \sqrt[4]{y}$

$R_x = \sqrt[4]{R_y} = \sqrt[4]{\frac{1}{2}}$

b) $S(x) := \sum_{k=0}^{\infty} \left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1 \right)^k \left(\frac{1}{x+3} \right)^k$

Subst.: $y := \frac{1}{x+3}$

$$S(y) = \sum_{k=0}^{\infty} \left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1 \right)^k \left(\frac{1}{x+3} \right)^k$$

$$\limsup_k \sqrt[k]{\left| \sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1 \right|^k} = \limsup_k \left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1 \right) =$$

$$= \limsup_k \left(\sqrt[k]{3} \sqrt[k]{k} + \frac{4}{\sqrt[k]{k!}} + 1 \right) = \sqrt{3} \cdot 1 + \frac{4}{„\infty"} + 1 =$$

$$= \sqrt{3} + 0 + 1 = \sqrt{3} + 1$$ ✓ f f

c) $R_y = \frac{1}{\sqrt{3}+1} \cancel{=} \frac{1}{R_x+3} \Longleftrightarrow \frac{R_x+3}{\sqrt{3}+1} = 1 \Longleftrightarrow R_x = \sqrt{3} - 2 < 0$ ⚡

r. subst.

$M = \{ \}$

denn $|x| \overset{!}{<} \sqrt{3} - 2 < 0$

→ Widerspruch

Du hast gezeigt:Reihe konvergiert für:|y|<R_y ==> |1/(x+3)| <R_y =

A12

a) $x \in \mathbb{R}$, gesucht: Formeln für $\sin(3x)$ und $\cos(3x)$

Eulersche Formel: $e^{ix} = \cos x + i\sin x \quad \forall x \in \mathbb{R}$

(i) $\exp(3ix) \overset{\text{Euler}}{=} \cos(3x) + i\sin(3x)$

$\exp(ix)^3 \overset{\text{Euler}}{=} (\cos x + i\sin x)^3$

$(\cos x + i\sin x)^3 = \cos^3 x - 3\sin^2 x\cos x + i(3\cos^2 x\sin x - \sin^3 x)$

s. NR

$\Longrightarrow \cos(3x) + i\sin(3x) = \cos^3 x - 3\sin^2 x\cos x + i(3\cos^2 x\sin x - \sin^3 x)$

$\hookrightarrow \cos(3x) = \cos^3 x - 3\sin^2 x\cos x$

$\hookrightarrow \cancel{i}\sin(3x) = \cancel{i}(3\cos^2 x\sin x - \sin^3 x)$

NR

$(\cos x + i\sin x)(\cos x + i\sin x) = \cos^2 x + 2i\sin x\cos x - \sin^2(x)$

$(\cos^2 x + 2i\sin x\cos x - \sin^2(x))(\cos x + i\sin x) =$

$= \cancel{\cos^3 x} + i\sin x\cos^2 x + 2i\sin x\cos^2 x - 2\cancel{\sin^2 x}\cos x$

$- \cancel{\cos x\sin^2 x} - i\sin^3 x =$

$= \cos^3 x - 3\sin^2 x\cos x + i(3\cos^2 x\sin x - \sin^3 x)$

(ii) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \quad (*)$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \quad (*)$

ges.: Formeln f. $\sin(3x)$ und $\cos(3x)$
$\sin(2x+x)$ $\cos(2x+x)$

$$\sin(3x) = \sin(2x+x) \overset{*}{=} \sin(2x)\cos x + \sin x\cos(2x) =$$

$$\overset{*}{\underset{*}{=}} [\sin x\cos x + \sin x\cos x]\cos x + \sin x[\cos x\cos x - \sin x\sin x] =$$

$$= 2\cos^2 x\sin x + \cos^2 x\sin x - \sin^3 x = 3\cos^2 x\sin x - \sin^3 x$$

$$\cos(3x) = \cos(2x+x) \overset{*}{=} \cos(2x)\cos x - \sin(2x)\sin x =$$

$$\overset{*}{\underset{*}{=}} [\cos x\cos x - \sin x\sin x]\cos x - [\sin x\cos x + \sin x\cos x]\sin x =$$

$$= \cos^3 x - \cos x\sin^2 x - \cos x\sin^2 x - \cos x\sin^2 x =$$

$$= \cos^3 x - 3\cos x\sin^2 x$$

b) (i) $\sin\frac{\pi}{3}, \cos\frac{\pi}{3} \quad \left(x = \frac{\pi}{3}\right)$

mit $\cos^2 x = 1 - \sin^2 x$

$$\sin(3x) = \sin(x)\left(3\cos^2 x - \sin^2 x\right) = \sin(x)\left[3\left(1-\sin^2 x\right) - \sin^2 x\right] =$$

$$= \sin(x)\left(3 - 4\sin^2 x\right) = 3\sin x - 4\sin^3 x$$

mit $\sin(\pi) = 3\sin\frac{\pi}{3} - 4\sin^3\frac{\pi}{3}$

$\sin\frac{\pi}{3} \neq 0$ $\sin\pi = 0$

$$\sin(3x) = 3\sin x - 4\sin^3 x$$

mit $x = \frac{\pi}{3}$ und $\sin\pi = 0$ folgt:

$$0 = 3\sin\frac{\pi}{3} - 4\sin^3\frac{\pi}{3}$$

$$\Leftrightarrow 4\sin^3\frac{\pi}{3} = 3\sin\frac{\pi}{3}$$

$$\Leftrightarrow 4\sin^2\frac{\pi}{3} = 3$$

$$\Leftrightarrow \sin^2\frac{\pi}{3} = \frac{3}{4}$$

$$\Leftrightarrow \sin\frac{\pi}{3} = \sqrt[2]{\frac{3}{4}} = \frac{\sqrt[2]{3}}{2}$$

$$\cos^2(x) + \sin^2(x) = 1$$

mit $x = \frac{\pi}{3}$ und $\sin\frac{\pi}{3} = \frac{\sqrt[2]{3}}{2}$ folgt:

$$\cos^2\frac{\pi}{3} + \frac{3}{4} = 1$$

$$\Leftrightarrow \cos^2\frac{\pi}{3} = \frac{1}{4}$$

$$\Leftrightarrow \cos\frac{\pi}{3} = \frac{1}{2}$$

(ii) $\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

mit $x = \frac{\pi}{6}$ folgt:

$\cos\left(\frac{\pi}{3}\right) = 1 - 2\sin^2 \frac{\pi}{6}$

$\Leftrightarrow \frac{1}{2} = 1 - 2\sin^2 \frac{\pi}{6}$

$\Leftrightarrow \sin^2 \frac{\pi}{6} = \frac{1}{4}$

$\Leftrightarrow \sin \frac{\pi}{6} = \frac{1}{2}$

$\cos^2 x + \sin^2 x = 1$

mit $x = \frac{\pi}{6}$ und $\sin^2 x = \frac{1}{4}$ folgt:

$\cos^2 \frac{\pi}{6} + \frac{1}{4} = 1$

$\Leftrightarrow \cos^2 \frac{\pi}{6} = \frac{3}{4}$

$\Leftrightarrow \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

(iii)

$\cos(2x) = 1 - 2\sin^2 x$

mit $x = \frac{\pi}{12}$ und $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$ folgt

$\frac{\sqrt{3}}{2} = 1 - 2\sin^2 \frac{\pi}{12}$

$\Leftrightarrow -\frac{\sqrt{3}}{2} + 1 = 2\sin^2 \frac{\pi}{12}$

$$\Longleftrightarrow -\frac{\sqrt{3}}{4} + \frac{1}{2} = \sin^2\frac{\pi}{12}$$

$$\Longleftrightarrow \frac{2-\sqrt{3}}{4} = \sin^2\frac{\pi}{12}$$

$$\Longleftrightarrow \sin\frac{\pi}{12} = \sqrt{\frac{2-\sqrt{3}}{4}} = \sqrt{\frac{1}{4}\left(2-\sqrt{3}\right)} =$$

$$= \frac{1}{2}\sqrt{2-\sqrt{3}}$$

$$\cos^2 x + \sin^2 x = 1$$

mit $x = \frac{\pi}{12}$ und $\sin^2 x = \frac{2-\sqrt{3}}{4}$ folgt:

$$\cos^2\frac{\pi}{12} = 1 - \frac{2-\sqrt{3}}{4}$$

$$\Longleftrightarrow \cos\frac{\pi}{12} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{1}{2}\sqrt{2+\sqrt{3}}$$