Deckblatt für die Abgabe der Übungsaufgaben
IngMathC2

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Blatt-Nummer: 07

Übungsgruppen-Nr: 07

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A18, A19, A20, ______

20/20*30

A18

a) $f'(x) = 2x + 1 + \frac{1}{2\sqrt{x}} + 0 - \frac{1}{2x^{3/2}} - \frac{1}{x^2} - \frac{2}{x^3}$

b) $f'(x) = 4\left(x^2 + \sqrt{2x}\right)^3 \cdot \left(2x + \frac{\sqrt{2}}{2\sqrt{x}}\right)$

c) $f'(x) = 1 \cdot e^{x^2} \ln(2+3x) + 2xe^{x^2} \cdot x \cdot \ln(2+3x) + x \cdot e^{x^2} \cdot \frac{3}{2+3x} = e^{x^2} \ln(2+3x) + 2x^2 e^{x^2} \ln(2+3x) + \frac{3xe^{x^2}}{2+3x}$

d) $f'(x) = \frac{1}{2\sqrt{x}} \cdot \frac{-1}{\sqrt{1-x}} = -\frac{1}{2\sqrt{x}\sqrt{1-x}}$

$y = e^x \quad y = \ln(x)$

$\ln e^x = x$

e) $f'(x) = \frac{2 \cdot \cos(2x) \cdot \ln(x^2+1) - 2x \cdot \frac{1}{x^2+1} \cdot \sin 2x}{[\ln(x^2+1)]^2} = \frac{2\cos(2x)}{\ln(x^2+1)} - \frac{2x\sin(2x)}{(x^2+1)\ln^2(x^2+1)}$

f) $f(x) = x^\alpha = e^{\alpha \ln x} \Rightarrow f'(x) = e^{\alpha \ln(x)} \cdot \left(\alpha \cdot \frac{1}{x}\right) = x^\alpha \cdot \alpha \cdot \frac{1}{x} = \alpha \cdot x^{\alpha-1}$

g) $f'(x) = e^{-x^2 \ln(x)} \cdot \left[-2x \cdot \ln(x) + \overbrace{\frac{1}{x} \cdot -x^2}^{-x}\right] = \underbrace{e^{-x^2 \ln(x)}}_{= x^{-x^2}} \cdot -x(2\ln(x)+1) = x^{-x^2} \cdot (-x) \cdot (2\ln(x)+1) = -x^{1-x^2} \cdot (2\ln(x)+1)$

h) $f'(x) = \frac{1 + \frac{1}{x\ln(x)}}{x + \ln(2\ln x)}$

A19

a) $f'(x) = \lim_{h\to 0} \frac{\cos(x+h) - \cos(x)}{h} = \lim_{h\to 0} \frac{(\cos(x).\cos(h) - \sin(x)\sin(h)) - \cos(x)}{h} = \lim_{h\to 0} \frac{\cos(x)(\cos(h)-1) - \sin(x)\sin(h)}{h} =$

$\lim_{h\to 0}\left(\frac{\cos(x).(\cos(h)-1)}{h}\right) - \lim_{h\to 0}\left(\frac{\sin(x).\sin(h)}{h}\right) = \cos(x).\lim_{h\to 0}\left(\frac{\cos(h)-1}{h}\right) - \sin(x).\lim_{h\to 0}\left(\frac{\sin(h)}{h}\right) = \cos(x).0 - \sin(x).1 = -\sin x$

b) $\tan'(x) = \frac{\sin'(x).\cos(x) - \sin(x).\cos'(x)}{\cos^2(x)} = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$

$\tan'(x) = \frac{\sin'(x)\cos(x) - \sin x \cos'(x)}{\cos^2(x)} = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = 1 + \frac{\sin^2(x)}{\cos^2(x)} = 1 + \tan^2(x)$

c) $\arctan'(\tan(x)) = \frac{1}{\tan'(x)} \Rightarrow \arctan(y) = \frac{1}{1+y^2} \quad \left[\text{Da } (f^{-1})'(y) = \frac{1}{f'(x)} = \frac{1}{f'(f^{-1}(y))}\right]$

$f(x) = 1 + \tan^2 x \Rightarrow f'(x) = 2\tan(x) \cdot \tan'(x) = 2\frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos^2(x)} = \frac{2\sin(x)}{\cos^3(x)} \to \tan''(x)$

$\frac{d}{dx}\left(\frac{2\sin(x)}{\cos^3(x)}\right) = 2 \cdot \frac{\cancel{\cos(x)}\cos^{\cancel{3}\,2}(x) + 3\cos^{\cancel{2}}(x)\sin^2(x)}{\cos^{\cancel{6}\,4}(x)} = \frac{4\sin^2(x) + 2}{\cos^4(x)} \to \tan'''(x)$

A20

a) $f'(x) = \alpha x^{\alpha-1} \cdot \sin\frac{1}{x^2} + x^{\alpha} \cdot \frac{-2}{x^3} \cdot \cos\frac{1}{x^2} = x^{\alpha-3}\left(\alpha x^2 \sin\frac{1}{x^2} - 2\cos\frac{1}{x^2}\right)$

b) $f'(0) = \lim\limits_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim\limits_{h\to 0} \frac{h^{\alpha}\sin\frac{1}{h^2} - 0}{h} = \lim\limits_{h\to 0} \overbrace{h^{\alpha-1}}^{\to 0} \overbrace{\sin\frac{1}{h^2}}^{\text{beschränkt}} = 0$ existiert für $\alpha > 1$

Fall $\alpha = 1$: $\lim\limits_{h\to 0} \overbrace{h^{\alpha-1}}^{\to 1} \sin\frac{1}{h^2}$ existiert nicht

Fall $\alpha < 1$: $\lim\limits_{h\to 0} h^{\alpha-1} \sin\frac{1}{h^2}$ existiert nicht.

c) (Fall $\alpha>1$): $\lim\limits_{x\to 0} f'(x) = \lim\limits_{x\to 0} x^{\alpha-3}\left(\alpha x^2 \sin\frac{1}{x^2} - 2\cos\frac{1}{x^2}\right)$

$$\lim_{x\to 0} \overbrace{x^{\alpha-3}}^{\to 0}\Big(\underbrace{\overbrace{\alpha x^2}^{\to 0} \overbrace{\sin\frac{1}{x^2}}^{\text{beschränkt}}}_{\to 0} - 2\overbrace{\cos\frac{1}{x^2}}^{\text{divergent}}\Big) \longrightarrow \text{existiert nicht}$$

f' ist an der Stelle $x=0$ unstetig.

d) $f''(x) = (\alpha-3)x^{\alpha-4} \cdot \left(\alpha x^2 \sin\frac{1}{x^2} - 2\cos\frac{1}{x^2}\right) + x^{\alpha-3}\left[\left(2\alpha x \sin\frac{1}{x^2} + \alpha x^2 \cos\left(\frac{1}{x^2}\right)\cdot\left(\frac{-2}{x^3}\right)\right) - \left(-\sin\left(\frac{1}{x^2}\right)\right)\cdot\left(\frac{-2}{x^3}\right)\right]$

$= (\alpha-3)x^{\alpha-4}\left(\alpha x^2 \sin\frac{1}{x^2} - 2\cos\frac{1}{x^2}\right) + x^{\alpha-3}\left[\frac{2}{x^3}\left(\alpha x^2\left(x^2 \sin\left(\frac{1}{x^2}\right) - \cos\left(\frac{1}{x^2}\right)\right) - 2\sin\left(\frac{1}{x^2}\right)\right)\right]$

$= x^{\alpha-6}\left(\alpha x^4(\alpha-1)\sin\frac{1}{x^2} - 4\alpha x^2 \cos\frac{1}{x^2} + 6x^2\cos\left(\frac{1}{x^2}\right) - 4\sin\left(\frac{1}{x^2}\right)\right)$