Felix Damm 31,5 Mathe - Übung

0174umaq Gruppe 7

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A1, a) $M=[\sqrt{3},\sqrt{5})$ $\inf(M)=\min(M)=\sqrt{3}$ $\sup(M)=\sqrt{5}$ $\max(M)=$ n.e.

b) $M=\{\frac{1}{1+x} \mid \frac{1}{2}\le x\le 3\}$ $\inf(M)=\min(M)=\frac{1}{4}$ $\sup(M)=\max(M)=\frac{2}{3}$

c) $M=\{\frac{1}{1+n} \mid n\in\mathbb{N}\}$ $\max(M)=\sup(M)=\frac{1}{2}$ $\inf(M)=0$ $\min(M)=$ n.e.

d) $M=\{x\in\mathbb{R} \mid x^2-2x+3\}$ $\max(M)=$ n.e. $\sup(M)=+\infty$ $\min(M)=\inf(M)=2$ -\infty

e) $M=\{\frac{p}{q} \mid q,p\in\mathbb{N},\ p\le q\}$ $\max(M)=\sup(M)=1$ $\min(M)=$ n.e. $\inf(M)=0$

f) $M=\{n-\frac{1}{3^n} \mid n\in\mathbb{N}\}$ $\min(M)=\inf(M)=\frac{2}{3}$ $\max(M)=$ n.e. $\sup(M)=+\infty$

g) $M=\{n+\frac{1}{3^m} \mid m,n\in\mathbb{N}\}$ $\min(M)=$ n.e. $\inf(M)=1$ $\max(M)=$ n.e. $\sup(M)=+\infty$

A2) $2n\le m\le 3n$

i) $\frac{3n+4m}{5n^2+10}\le\frac{3n+12n}{5n^2+10}=\frac{3n}{n^2+2}$ ii) $\frac{5n-m}{2n}\le\frac{5n-2n}{2n}=\frac{3}{2}$

iii) $\frac{n}{n+m}\le\frac{n}{n+2n}=\frac{1}{3}$ iv) $\frac{n+m}{\frac{1}{2}-n}\le\frac{3n}{\frac{1}{2}-n}$

v) $\frac{5n-m+3\cdot2^m}{3n^3-m+3}\le\frac{5n-2n+3\cdot2^{3n}}{3n^3-3n+3}=\frac{2n+3\cdot2^{3n}}{3n^3-3n+3}$

vi) $m+n+\sin(m)-\sin(17m^2)+2^m+2^{-m}\le m+n+1-(-1)+2^m+2^{-m}\le$
$3n+n+2+2^{3n}+\frac{1}{2^{2n}}=4n+2+2^{3n}+\frac{1}{2^{2n}}$

A3) 10/10

i) $a_n=\frac{2n}{n+3}$ ii) $b_n=\frac{n}{4^n}$

a) $a_{n+1}-a_n=\frac{2n+2}{n+4}-\frac{2n}{n+3}=\frac{(2n+2)(n+3)-2n(n+4)}{(n+4)(n+3)}=\frac{2n^2+6n+2n+6-2n^2-8n}{(n+4)(n+3)}=$
$=\frac{6}{(n+3)(n+4)}\ge 0\Rightarrow$ monoton wachsend

$b_{n+1}-b_n=\frac{n+1}{4^{n+1}}-\frac{n}{4^n}=\frac{n+1-4n}{4^{n+1}}=\frac{-3n+1}{4^{n+1}}\overset{n\ge1}{\le}\frac{-3+1}{4^{n+1}}<0\Rightarrow$ monoton fallend

b) (a_n) konvergiert gegen 2, (b_n) konvergiert gegen 0

c) Sei $\varepsilon>0$ beliebig, $n_0=\lceil\frac{6}{\varepsilon}\rceil$ (aus NR.)
$|a_n-a|=\left|\frac{2n}{n+3}-2\right|=\left|\frac{2n-2n+6}{n+3}\right|=\frac{6}{n+3}\le\frac{6}{n_0+3}=\frac{6}{\frac{6}{\varepsilon}+3}\le\frac{6}{\frac{6}{\varepsilon}}=\varepsilon\quad\forall n\ge n_0$

Sei $\varepsilon>0$ beliebig, $n_0=\lceil\log_2(\frac{1}{\varepsilon})\rceil$ (aus NR.)
$|b_n-b|=\left|\frac{n}{4^n}-0\right|=\frac{n}{4^n}\le\frac{n_0}{4^{n_0}}\le\frac{2^{n_0}}{2^{2n_0}}=\frac{1}{2^{n_0}}\le\frac{1}{2^{\mathrm{ld}(\frac{1}{\varepsilon})}}=\frac{1}{\frac{1}{\varepsilon}}=\varepsilon\quad\forall n\ge n_0$