Mathe Übung ol74umay

11.5/22*30 =15.5

Blatt 8

und (5pi/6,7pi/6) und (11pi/6,2p

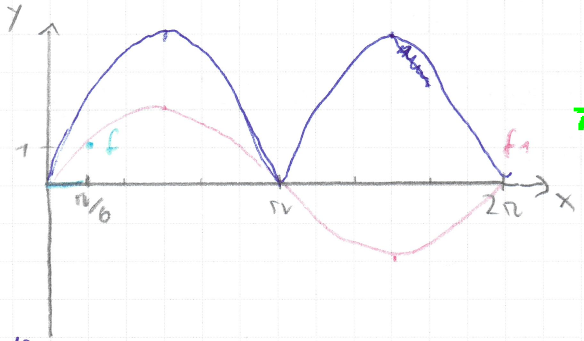
A23) a)

$D_f \subseteq [0, \frac{\pi}{6}]$ da $\lim_{n\to\infty}(2\sin x)^n = \begin{cases} 0 & \forall x < \frac{\pi}{6} \\ 1 & x = \frac{\pi}{6} \\ \cancel{\infty} & \cancel{\text{sonst}} \end{cases}$ im Intervall: $x \in [0, 2\pi]$

$f(x) \begin{cases} 0 & \forall x < \frac{\pi}{6} \\ 1 & x = \frac{\pi}{6} \\ \infty & \text{sonst} \end{cases}$ s.o.

b) gleichmäßig konvergent auf M_3, M_4

eine funktion kann nie une



ff

6

A21)

a) $\lim_{x\to0}\frac{\sin x^2}{x^3+8x^2} = \lim_{x\to0}\frac{\cos x^2\cdot 2x}{3x^2+16x} = 2\lim_{x\to0}\frac{\cos x^2}{3x+16} = 2\cdot 0 = 0$

2/16

b) $\lim_{x\to0}\frac{\sin x^2}{x^3+8x^2} = \lim_{x\to0}\frac{\sin x^2}{x^2}\cdot\lim_{x\to0}\frac{1}{x+8} = \infty$

1/8

=16

c) $\lim_{x\to0}\sqrt{x}\,\ln x = \frac{\ln x}{\frac{1}{\sqrt{x}}} = \lim_{x\to0}\frac{\frac{1}{x}}{-\frac{1}{2}x^{-1,5}} = \lim_{x\to0}\frac{\frac{1}{x}}{-\frac{1}{2\sqrt{x}^3}} = \lim_{x\to0}\frac{\frac{1}{x}}{-\frac{1}{2}\frac{1}{\sqrt{x}}\cdot x} =$

$= \lim_{x\to0}\frac{1}{-\frac{1}{2\sqrt{x}}} = 0$

typ infty*infty, man DARF kein hospital anwe

d) $\lim_{x\to\infty}\sqrt{x}\,\ln x = \lim_{x\to\infty}\frac{1}{\frac{1}{2\sqrt{x}}} = +\infty$

e) $\lim_{x\to0}\frac{\cos(6x)-1}{x^3+2x^2} = \lim_{x\to0}\frac{-\sin(6x)\cdot 6}{3x^2+4x} = \lim_{x\to0}\frac{-\cos(6x)\cdot 6^2}{6x+4} = -\frac{6^2}{4} = 9$

f) $\lim_{x\to0}\frac{1-\cos(x)\cos(2x)}{e^{x^2}-1} = \lim_{x\to0}\frac{\sin x\cos 2x + \cos x\sin 2x\cdot 2}{e^{x^2}\cdot 2x}$ Add.-theorem $\frac{\sin(3x)}{e^{x^2}\cdot 2x} = \lim_{x\to0}$

$= \lim_{x\to0}\frac{\cancel{\cos(3x)\cdot 3}}{e^{x^2}\cdot(2x)^2} = \cancel{\infty}$

g) $\lim_{x\to\infty}\frac{\ln(1+\alpha x)}{\ln(\ln(e^{\beta x}+e^{-\beta x}))} = \lim_{x\to\infty}\frac{\frac{\alpha}{1+\alpha x}}{\frac{1}{\ln(e^{\beta x}+e^{-\beta x})}\cdot\frac{1}{e^{\beta x}+e^{-\beta x}}(\beta e^{\beta x}-\beta e^{-\beta x})} =$

$= \lim_{x\to\infty}\frac{\frac{\alpha}{1+\alpha x}}{\frac{1}{\ln(e^{\beta x}+e^{-\beta x})}\cdot\frac{\beta e^{\beta x}(1-e^{-2\beta x})}{e^{\beta x}(1+e^{-2\beta x})}} = \lim_{x\to\infty}\frac{\alpha(\ln(e^{\beta x}+e^{-\beta x}))\cdot(1+e^{-2\beta x})}{\beta(1-e^{-2\beta x})(1+\alpha x)}$

$$= \lim_{x\to\infty} \frac{\frac{\alpha(\beta e^{\beta x} - \beta e^{-\beta x})(1+e^{-2\beta x})}{e^{\beta x}+e^{-\beta x}} + \alpha \ln(e^{\beta x}+e^{-\beta x})(-2\beta e^{-2\beta x})}{\alpha\beta(1-e^{-2\beta x}) + (\alpha x+1)\beta(2e^{-2\beta x})} =$$

$$\lim_{x\to\infty} \frac{\frac{\alpha\beta \cancel{e^{\beta x}}(1-e^{-2\beta x})\cancel{(1+e^{-2\beta x})}}{\cancel{e^{\beta x}}\cancel{(1+e^{-2\beta x})}} + \alpha\ln(e^{\beta x}+e^{-\beta x})(-2\beta e^{-2\beta x})}{\alpha\beta(1-e^{-2\beta x}) + (\alpha x+1)\beta(2e^{-2\beta x})}$$

h) $$\lim_{x\to\infty} \frac{6\sqrt{x}+5+\frac{4}{\sqrt{x}}}{3\sqrt{x}+e^{-2x}+\frac{1}{\sqrt{x}}} = \lim_{x\to\infty} \frac{6\cdot\frac{1}{2}x^{-\frac{1}{2}} + 4\cdot(-\frac{1}{2}x^{-\frac{3}{2}})}{\frac{3}{2\sqrt{x}} + e^{-2x}(-2) - \frac{1}{2}x^{-3/2}}$$

→ Führt nicht zum Ziel, da e- und Wurzelterme unendlich oft abgeleitet werden könnten ohne „einfacher" zu werden.

Statt dessen:

$$\lim_{x\to\infty} \frac{\cancel{\sqrt{x}}\left(6+\underbrace{\frac{5}{\sqrt{x}}}_{\to 0}+\underbrace{\frac{4}{\sqrt{x}^2}}_{\to 0}\right)}{\cancel{\sqrt{x}}\left(3+\underbrace{\frac{e^{-2x}}{\sqrt{x}}}_{\to 0}+\underbrace{\frac{1}{\sqrt{x}}}_{\to 0}\right)} = \frac{6}{3} = \frac{1}{2}$$

A22) a)

$f(x) = \cos x - \cos^2 x \qquad f'(x) = -\sin x + 2\cos x \sin x$

$f'(x) = 0 \Leftrightarrow \sin x = 2\cos x \sin x \Leftrightarrow \cos x = \frac{1}{2} \Leftrightarrow x = \pm\frac{\pi}{3} \in D_f$

Untersuche Nullstellen der Ableitung und Randpunkte:

$f(-\frac{\pi}{2}) = 0 \quad f(-\frac{\pi}{3}) = \frac{1}{4} \quad f(\frac{\pi}{3}) = \frac{1}{4} \quad f(\pi) = -2$

globales Minimum: $(\pi \mid -2)$; Maxima $f(\pm\frac{\pi}{3}) = \frac{1}{4}$

b) $f(x) = \frac{\ln x}{x} \qquad f'(x) = \frac{x\cdot\frac{1}{x} - \ln x}{x^2} = \frac{1-\ln x}{x^2}$

$f'(x) = 0 \Leftrightarrow \frac{1-\ln x}{x^2} = 0 \Leftrightarrow 1 = \ln x \Leftrightarrow x = 0 \notin D_f$

Im geg. Intervall gibt es weder Nullstellen der Ableitung noch Randpunkte.

$\left(\lim_{x\searrow 0} \frac{\ln x}{x} = -\infty \qquad \lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0 \right)$

Es gibt keine Maxima oder Minima in D_f.