Deckblatt für die Abgabe der Übungsaufgaben
IngMath C2

Name, Vorname: Dieringer, Nico

StudOn-Kennung: yb68ecaj

Blatt-Nummer: 4

Übungsgruppen-Nr.: 7

Die folgenden Aufgaben gebe ich zur Korrektur frei:

10, 11, 12

5.5/10*30 = 16.5

A10) a) i)

$$\sum_{k=0}^{\infty} k q^k \cdot \sum_{k=0}^{\infty} q^k = \sum_{n=0}^{\infty} \sum_{k=0}^{n} k q^k \cdot q^{n-k} = \sum_{n=0}^{\infty} \sum_{k=0}^{n} k \cdot q^n =$$

$$= \sum_{n=0}^{\infty} q^n \cdot \sum_{k=0}^{n} k = \sum_{n=0}^{\infty} q^n \cdot \left(0 + \frac{n^2+n}{2}\right) \quad \sum_{k=1}^{\infty} k$$

$$= \sum_{n=0}^{\infty} q^n \cdot \frac{n^2+n}{2}$$

ii) $$\frac{q}{(1-q)^2} \cdot \frac{1}{(1-q)} = \frac{1}{2} \sum_{n=0}^{\infty} q^n \cdot (n^2+n) = \frac{1}{2} \sum_{n=0}^{\infty} n^2 q^n + n q^n$$

$$\rightarrow \sum_{n=0}^{\infty} n^2 q^n = \underline{\underline{2\left(\left(\frac{q}{(1-q)^2} \cdot \frac{1}{(1-q)}\right) - \sum_{n=0}^{\infty} n q^n\right)}}$$

b) $$\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+2)} \qquad \frac{1}{(k+1)(k+2)} = \frac{A}{k+1} + \frac{B}{k+2} = \frac{A \cdot (k+2) + B \cdot (k+1)}{(k+1)(k+2)}$$

$$= \frac{k(A+B) + 2A + 1B}{(k+1)(k+2)} \rightarrow \begin{array}{l} A+B=0 \\ 2A+B=1 \end{array} \rightarrow \begin{array}{l} B=-A \\ 2A-A=1 \end{array} \rightarrow \begin{array}{l} B=-1 \\ A=1 \end{array}$$

$$\sum_{k=0}^{n} \frac{1}{(k+1)(k+2)} = \sum_{k=0}^{n} \left(\frac{1}{k+1}\right) - \sum_{k=0}^{n} \left(\frac{1}{k+2}\right) =$$

$$= \left(\sum_{k=0}^{n} \left(\frac{1}{k+1}\right)\right) - \left(\sum_{k=1}^{n+1} \left(\frac{1}{k+1}\right)\right) = \frac{1}{0+1} - \frac{1}{(n+1)+1} =$$

$$= \frac{1}{1} - \frac{1}{n} \quad \rightarrow \underline{\underline{\lim_{n\to\infty} = 1}}$$

1.1) a) (i) $\sum_{k=0}^{\infty} \frac{5^k}{k} x^k \rightarrow \sqrt[k]{\left|\frac{5^k}{k}\right|} = \frac{5}{\sqrt[k]{k}} = \frac{5}{1}$ ✓

$R = \frac{1}{5}$ ✓

ii) $\rightarrow \sqrt[k]{\left|\left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)\right|^{2k}} = \left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^2$

warum?

iii) $\sqrt[k]{|(k!+2)|} \xrightarrow{k\to\infty} \infty \Rightarrow R = „\frac{1}{\infty}“ \rightarrow 0$ ✓

iv) $\rightarrow \sum_{\tilde{k}=0}^{\infty} a_{\tilde{k}} x^{\tilde{k}} \rightarrow K = \frac{1}{4}\tilde{k}$

$\rightarrow a_{\tilde{k}} = \begin{cases} 0, & \tilde{k} \text{ nicht durch 4 teilbar} \\ \frac{2^{\frac{1}{4}\tilde{k}}}{\left(\frac{1}{4}\tilde{k}\right)^2}, & \tilde{k} \text{ – '' '' ''} \end{cases}$ ✓

$\rightarrow \sqrt[\tilde{k}]{\left|\frac{2^{\frac{1}{4}\tilde{k}}}{\left(\frac{1}{4}\tilde{k}\right)^2}\right|} = \frac{2^{\frac{1}{4}}}{\cancel{\sqrt[\tilde{k}]{\left(\frac{1}{4}\tilde{k}\right)^2}}} \xrightarrow{\tilde{k}\to\infty} 0$

geht gegen 1

$R = „\frac{1}{0}“ \rightarrow \infty$

b) $\rightarrow \sqrt[k]{|a_k|} = \sqrt[k]{\left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1\right)^k} = \sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1$

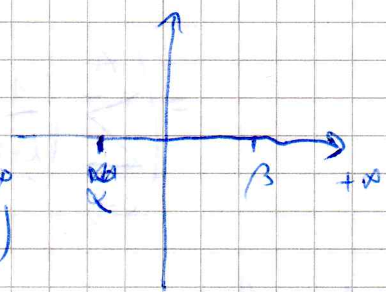
$= \sqrt[k]{3} \cdot \sqrt[k]{k} + \frac{4}{\sqrt[k]{k!}} + 1$

$\quad\ \ 1 \quad + \quad 1 \quad + \quad 0 \quad = \underline{\underline{2}}$ ✓

R_y != y

$R_y = \frac{1}{2} \qquad y = \frac{1}{x+3} \rightarrow \cancel{x = \frac{1}{y-3}}$

$R_x = \frac{1}{\frac{1}{2}-3} = -0{,}4$




$M = (-\infty; -0{,}4) \cup \left(\frac{1}{2}; \infty\right)$

A12)

a) (i) $\exp(3ix) = \exp(ix)^3$

$\cos(3x) + i\sin(3x) = (\cos(x) + i\sin(x))^3$

$= (\cos(x)^2 + 2i\cos(x)\sin(x) - \sin(x)^2)(\cos(x) + i\sin(x))$

$= \cos(x)^3 + i\cos(x)^2\sin(x) + 2i\cos(x^2)\sin x + 2i\sin(x)^2\cos(x)^2 -$

$- i\sin(x)^3 - \sin(x)^2\cos(x) = \cos(x)^2 + 3i\cos(x)^2\sin(x) - 3\sin(x)^2\cos(x)$

$- i\sin(x)^3$

Re: $\cos(3x) = \cos(x)^3 - 3\sin(x)^2\cos(x) = \dots = 4\cos(x)^3 - 3\cos(x)$

Im: $\sin(3x) = 3\cos(x)^2\sin(x) - \sin(x)^3 = \dots = 3\sin(x) - 4(\sin x)^3$

ii) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

~~cos~~ $\sin(3x) = \sin(2x)\cos(x) + \cos(2x)\sin(x) = (\sin(x)\cos(x) +$

$\cos(x)\sin(x))\cos(x)\sin(x) + \dots$

$\cos(3x) = \cos(2x)\cos(x) - \sin(2x)\sin(x) = (\cos(x)^2 - \sin(x)^2)\cdot$

$\cos(x) - (2\sin(x)\cos(x))\sin(x) = \cos(x)^3 - \sin(x)^2\cos(x) -$

$- 2\sin(x)^2\cos(x) = \cos(x)^3 - 3\sin(x)^2\cos(x) =$

$= \cos(x)^3 - 3(1-\cos(x)^2)\cos(x) = 4\cos(x)^3 - 3\cos(x)$

b) (i) $\sin(3\frac{\pi}{3}) = 3\sin(\frac{\pi}{3}) - 4\sin(\frac{\pi}{3})^3$

$\sin(\pi) = 0 = 3\sin(\frac{\pi}{3}) - 4\sin(\frac{\pi}{3})^3$

$0 = 3 - 4\sin(\frac{\pi}{3})^2 \rightarrow 4\sin(\frac{\pi}{3})^2 = 3 \quad \sin(\frac{\pi}{3})^2 = \frac{3}{4}$

$\sin(\frac{\pi}{3}) = \sqrt{\frac{3}{4}}$

$\cos(\frac{\pi}{3}) \quad \sin(x)^2 = 1 - \cos(x)^2$

$\cos(\frac{\pi}{3}) = \sqrt{1 - \sin(\frac{\pi}{3})^2}$

ii) $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

$\cos(\frac{\pi}{3}) = 1 - 2\sin\frac{\pi}{6}^2$

$\sin\frac{\pi}{6} = \sqrt{\frac{1}{2}(1 - \cos\frac{\pi}{3})}$

$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

$\cos(\frac{\pi}{3}) = 1 - 2\sin\frac{\pi}{6}^2 \qquad \sin\frac{\pi}{6} = \sqrt{\frac{1}{2}(1 - \cos\frac{\pi}{3})} = \frac{1}{2}$

iii) $\cos\left(\frac{\pi}{12}\right) = \sqrt{1-\sin\left(\frac{\pi}{12}\right)^2} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\cos\left(2\frac{\pi}{12}\right) = 1-2\sin^2\left(\frac{\pi}{12}\right) \Rightarrow \sin\left(\frac{\pi}{12}\right) = \sqrt{\frac{1}{2}\left(1-\cos\left(\frac{\pi}{6}\right)\right)} =$

$\sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{6}-\sqrt{2}}{4}$