Deckblatt für die Abgabe der Übungsaufgaben
IngMathC2

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Blatt-Nummer: 4

Übungsgruppen-Nr: 7

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A10, A11, A12, _____

2.5/10*30 = 7.5

A10)

a) i)

$$\sum_{k=0}^{\infty} kq^k \cdot \sum_{k=0}^{\infty} q^k = \sum_{n=0}^{\infty} \sum_{k=0} kq^k q^{n-k} = \sum_{n=0}^{\infty} q^n \sum_{k=0}^{n} k$$

$$= \sum_{n=0}^{\infty} \frac{1}{2} n(n+1) q^n = \left(\frac{1}{(1-q)^2} - \frac{1}{1-q}\right)\left(\frac{1}{1-q}\right)$$

$$= \frac{1}{(1-q)^3} - \frac{1}{(1-q)^2}$$

ii)

$$\sum_{k=0}^{\infty} k^2 \cdot q^k = 2\left(\sum_{k=0}^{\infty} \frac{1}{2} k(k+1) q^k - \frac{1}{2} \sum_{n=0}^{\infty} kq^k\right)$$

$$= 2\left(\frac{1}{(1-q)^3} - \frac{1}{(1-q)^2} - \frac{1}{2} \frac{q}{(1-q)^2}\right) = \frac{2}{(1-q)^3} - \frac{2+q}{(1-q)^2}$$

b)
$$\sum_{k=0}^{\infty} \frac{1}{(n+1)(n+2)} = \sum_{k=1}^{\infty} \frac{1}{k(k+1)} = \sum_{k=1} \left(\frac{1}{k} - \frac{1}{k+1}\right)$$

$$\Rightarrow \lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{1}{k} - \sum_{k=2}^{n+1} \frac{1}{k}\right)$$

$$\Rightarrow 1 - \lim_{n\to\infty} \frac{1}{n+1} = 1$$

A12)

a) i)

$$\cos 3x + i \sin = e^{i3x} = (e^{ix})^3 = (\cos x + i \sin x)^3$$

$$(\cos x)^3 \cdot 3 \sin^2 x \cos x \qquad + 3 i \cdot \cos^2 x \sin x - i \sin^3 x$$

ii)

$$\sin 3x = \sin(x + 2x) = \sin x \cos 2x + \cos x \sin 2x$$

$$\sin x (\cos^2 x - \sin^2 x) + \cos x \, 2 \sin x \cos x$$

$$\cos 3x = \cos(x + 2x) = \cos x \cos 2x - \sin x \sin 2x$$

$$= \cos x (\cos^2 x - \sin^2 x) - \sin x (\sin x \cos x + \cos x \sin x)$$

b) i)

$$\sin \pi = 0 = \sin \frac{\pi}{3} \left(3 \cos^2 \frac{\pi}{3} - \sin^2 \frac{\pi}{3}\right)$$

$$= \sin \frac{\pi}{3} \left(3 - 4 \sin^2 \frac{\pi}{3}\right)$$

$$\sin \frac{\pi}{3} > 0 \Rightarrow \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow \cos \frac{\pi}{3} = \frac{1}{2}$$

ii)

$$\frac{1}{2} = \cos\frac{\pi}{3} = 1 - 2\sin^2\frac{\pi}{6} \Rightarrow \sin\frac{\pi}{6} = \frac{1}{2}$$

$$\cos\frac{\pi}{6} = \left(1 - \sin^2\frac{\pi}{6}\right)^{\frac{1}{2}} = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$$

iii)

$$\frac{\sqrt{3}}{2} = \cos\frac{\pi}{6} = 1 - 2\sin^2\frac{\pi}{12} \Rightarrow \sin\frac{\pi}{12} = \sqrt{\frac{2-\sqrt{3}}{4}}$$

$$= \frac{1}{2}\sqrt{2-\sqrt{3}}$$

$$\cos\frac{\pi}{12} = \left(1 - \sin^2\frac{\pi}{12}\right)^{\frac{1}{2}} = \left(\frac{1}{2} + \frac{\sqrt{3}}{4}\right)^{\frac{1}{2}} = \frac{1}{2}\sqrt{2+\frac{\sqrt{3}}{1}}$$

AM)

a) i)

Notation, beides in den nenner

$$\sum_{k=0}^{\infty} \frac{5^{-k}}{k} x^k \Rightarrow \frac{1}{\limsup_k \sqrt[k]{a_k}} = R$$

$$= \frac{1}{5 \limsup_k \sqrt[k]{\frac{1}{k}}} = \frac{1}{5}$$

ii)

iii) $\sum_{k=0}^{\infty} (k!+2)x^k \quad R = (\limsup_{\cancel{n}\, k} \sqrt[\cancel{n}\, k]{k!+2})^{-1} = 0$

iv) $\sum_{n=0}^{\infty} \frac{2^n}{\cancel{k}^2} x^{4n} \to \sum_{n=0}^{\infty} \frac{2^n}{n^2} 4^n$

du musst dich entscheiden, n oder k

$$R_4 = (\limsup_{\cancel{n}\, k} \sqrt[k]{\frac{1}{\cancel{n}^2}} \cdot 2) = \frac{1}{2}$$

$$R_x = \sqrt[4]{\frac{1}{2}}$$

Schau dir das mit der substitution nochmal an. Du kannst nicht einfach mit einer k

b)