Deckblatt für die Abgabe der Übungsaufgaben
IngMathC2

Name, Vorname: Schleifer, Max

StudOn-Kennung: an66iboj

Blatt-Nummer: 7

Übungsgruppen-Nr: 7

Die folgenden Aufgaben gebe ich zur Korrektur frei:

18, 19, 20, (alle)

18.5/20*30 = 27.5

A18

a) $f(x) = x^2 + x + \sqrt{x} + 1 + \frac{1}{\sqrt{x}} + \frac{1}{x} + \frac{1}{x^2} \quad \Big|\; x \setminus \{0\}$

$f'(x) = 2x + 1 + \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x}^3} - \frac{1}{x^2} - \frac{2}{x^3}$ ✓

b) $f(x) = \left(x^2 + \sqrt{2x}\right)^4 \quad \Big|\; x > 0$

$f'(x) = 4\left(x^2 + \sqrt{2x}\right)^3 \cdot \left(2x + \frac{1}{\sqrt{2x}}\right)$ ✓

c) $f(x) = xe^{x^2} \ln(2+3x) \quad \Big|\; x > -\frac{2}{3}$

$f'(x) = e^{x^2} \ln(2+3x) + x \cdot \frac{d}{dx}\left[e^{x^2} \ln(2+3x)\right] =$

$= e^{x^2} \ln(2+3x) + x\left[2xe^{x^2} \ln(2+3x) + e^{x^2} \frac{1}{2+3x} \cdot 3\right] =$

$= e^{x^2}\left[\ln(2+3x) + 2x^2 \ln(2+3x) + \frac{3x}{2+3x}\right] =$

$= e^{x^2}\left[\ln(2+3x)(1+2x^2) + \frac{3x}{2+3x}\right]$ ✓

d) $f(x) = \arccos(\sqrt{x}) \quad \Big|\; 0 < x < 1$

$\arccos' x = \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow f'(x) = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{1-x}\sqrt{x}}$ ✓

e) $f(x)=\frac{\sin 2x}{\ln(x^2+1)} \quad \Big|\, x \neq 0$

$$f'(x)=\frac{\cos(2x)\cdot 2\cdot\ln(x^2+1)-\sin(2x)\frac{1}{x^2+1}\cdot 2x}{\ln^2(x^2+1)}=$$

$$=\frac{2\cos(2x)}{\ln(x^2+1)}-\frac{2x\sin(2x)}{(x^2+1)\ln^2(x^2+1)}$$

f) $f(x)=x^\alpha \quad \Big|\, x>0,\ \alpha\in\mathbb{R}\setminus\{0\}$

$$f'(x)=\alpha x^{\alpha-1}$$

warum? Ihr habt das nur für alpha als ganze zahl gezeigt

g) $f(x)=x^{-x^2}=e^{-x^2\ln x} \quad \Big|\, x>0$

$$f'(x)=e^{-x^2\ln x}\cdot\left(-2x\ln x-\frac{x^2}{x}\right)=x^{-x^2}\left(-2x\ln x-x\right)=$$

$$=-x^{1-x^2}(2\ln x+1)$$

h) $f(x)=\ln(x+\ln(2\ln x))$

$$f'(x)=\frac{1}{x+\ln(2\ln x)}\cdot\left[1+\frac{1}{2\ln x}\cdot\frac{2}{x}\right]=\frac{1}{x+\ln(2\ln x)}\cdot\left[1+\frac{\cancel{2}\,1}{\cancel{2}x\ln x}\right]$$

A13

a) Z.z.: $\frac{d}{dx}\cos x = -\sin x \qquad f(x) := \cos x$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h}$$

$$= \lim_{h\to 0}\frac{\cos x\cdot\cos h - \sin x\sin h - \cos x}{h} = \lim_{h\to 0}\frac{\cos x(\underbrace{\cos h - 1}_{\to 0}) - \sin x \sin h}{h} =$$

$$= \lim_{h\to 0}\frac{\cos x\cdot 0}{h} - \sin x\frac{\sin h}{h} = 0 - \sin x \underbrace{\frac{\sin h}{h}}_{\substack{\to 1 \\ \text{(Vorlesung)}}} = -\sin x$$

b) $\tan x = \frac{\sin x}{\cos x} =: f(x)$

$$f'(x) = \frac{\cos x\cos x + \sin x \sin x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$\sin' = \cos$

$\cos' = -\sin$

(i) $f'(x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \cos^{-2} x$

(ii) $f'(x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = 1 + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x$

c) (i) ges: $f'(y) = \arctan' x$

$$(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}$$

$\hookrightarrow f(y) := \tan y,\ f^{-1}(y) = \arctan y$

$$\Rightarrow (f^{-1})'(y) = \frac{1}{1+\tan^2(\arctan y)} = \frac{1}{1+y^2} \Rightarrow \arctan'(x) = \frac{1}{1+x^2}$$

(iii)

$\tan'' x =: f''(x) = (1+\tan^2 x)' = 2\tan x \cdot [1+\tan^2 x] =$

$= 2\tan x + 2\tan^3 x$

$\tan''' x =: f'''(x) = (2\tan x + 2\tan^3 x)' =$

$= 2 + 2\tan^2 x + 6\tan^2 x \cdot (1+\tan^2 x) =$

$= 2 + 2\tan^2 x + 6\tan^2 x + 6\tan^4 x =$

$= 2 + 2\tan^2 x (4 + 3\tan^2 x)$

A20 $f: [0,\infty) \to \mathbb{R}$ $\quad f(x) = \begin{cases} x^\alpha \sin \frac{1}{x^2}, & x > 0 \\ 0, & x = 0 \end{cases}$

mit $\alpha \in (0, \infty)$

a) $f'(x) = \alpha x^{\alpha-1} \sin \frac{1}{x^2} + x^\alpha \cdot \cos \frac{1}{x^2} \cdot \left(\frac{-2}{x^3}\right) =$

$= \alpha x^{\alpha-1} \sin \frac{1}{x^2} - 2x^{\alpha-3} \cos \frac{1}{x^2}$

b) $f'(0) = \lim\limits_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim\limits_{h\to 0} \frac{h^\alpha \cdot \sin \frac{1}{h^2} - 0}{h} =$

$= \lim\limits_{h\to 0} h^{\alpha-1} \sin \frac{1}{h^2}$

Fall 1: $\alpha = 1$: $f'(0) = \lim\limits_{h \to 0} \sin\frac{1}{h^2}$

$\to$ existiert nicht

Fall 2: $\alpha < 1$: $f'(0) = \lim\limits_{h \to 0^+} \underbrace{h^{\alpha-1}}_{\to\infty} \underbrace{\sin\frac{1}{h^2}}_{0 \leq \sin\frac{1}{h^2} \leq 1} = \infty$ $\qquad \alpha \in (0, \infty)$

Fall 3: $\alpha > 1$: $f'(0) = \lim\limits_{h \to 0} \underbrace{h^{\alpha-1}}_{\to 0} \sin\frac{1}{h^2} = 0$

c) Fall 2: $\alpha < 1 \to f'(0) = \infty$

Nicht stetig, da $\lim\limits_{x \to 0} f(x) = \infty \neq f(0) = 0$

Fall 3: $\alpha > 1 \to f'(0) = 0$

Stetig, da $\lim\limits_{x \to 0} f(x) = 0 = f(0)$

d) $f'(x) = \alpha x^{\alpha-1} \sin\frac{1}{x^2} - 2x^{\alpha-3} \cos\frac{1}{x^2}$

$$f''(x) = \alpha(\alpha-1)x^{\alpha-2} \sin\frac{1}{x^2} + \alpha x^{\alpha-1} \cdot \cos\frac{1}{x^2} \cdot \left(-\frac{2}{x^3}\right)$$

$$- 2(\alpha-3)x^{\alpha-4} \cos\frac{1}{x^2} + 2x^{\alpha-3} \sin\frac{1}{x^2} \left(\frac{2}{x^3}\right) =$$

$$= (\alpha^2 - \alpha)x^{\alpha-2} \cdot \sin\frac{1}{x^2} - 2\alpha x^{\alpha-4} \cos\frac{1}{x^2}$$

$$- 2(\alpha-3)x^{\alpha-4} \cos\frac{1}{x^2} + 4x^{\alpha-6} \sin\frac{1}{x^2} =$$

$$= -\cos\frac{1}{x^2}\left(+2\alpha x^{\alpha-4} + 2(\alpha-3)x^{\alpha-4}\right)$$

$$+\sin\frac{1}{x^2}\left((\alpha-1)x^{\alpha-1}+4x^{\alpha-6}\right)=$$

$$=\sin\frac{1}{x^2}\left[(\alpha-1)x^{\alpha-1}+4x^{\alpha-6}\right]-2\cos\frac{1}{x^2}\left[2\alpha-3\right]x^{\alpha-4}$$