Deckblatt für die Abgabe der Übungsaufgaben IngMathC2

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Blatt-Nummer: 04

Übungsgruppen-Nr: 07

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A10, A11, A12, ____

9/10*30 = 27

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A10)

a) 1) $\sum_{k=0}^{\infty} k q^k \cdot \sum_{k=0}^{\infty} k \cdot q^k =$

$$= \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} k \cdot q^k \cdot (n-k) \cdot q^{n-k}\right) = \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} k \cdot (n-k) \cdot q^n\right) =$$

$$= \sum_{n=0}^{\infty}\left(q^n \sum_{k=0}^{n} kn - k^2\right) = \sum_{n=0}^{\infty}\left(q^n \sum_{k=0}^{n} kn - \sum_{k=0}^{n} k^2\right)$$

$$\sum_{k=0}^{\infty} q^k \cdot \sum_{k=0}^{\infty} q^k = \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} q^k \cdot q^{n-k}\right) = \sum_{n=0}^{\infty} q^n = \frac{1}{1-q}$$

$$\sum_{n=0}^{\infty} \sum_{k}^{n} = \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} q^n\right) = \sum_{n=0}^{\infty} (n+1) \cdot q^n$$

A10)

b) $\sum_{k=0}^{\infty} \frac{1}{(k+1)\cdot(k+2)}$

$\boxed{\text{a) siehe letztes Blatt}}$

$$\frac{1}{(k+1)(k+2)} = \frac{A}{(k+1)} + \frac{B}{(k+2)}$$

$$1 = A(k+2) + B(k+1)$$

$$1 = Ak + 2A + Bk + B$$

$$1 = k(A+B) + 2A + B$$

$$\rightarrow \left.\begin{array}{l}(A+B) = 0 \\ 2A + B = 1\end{array}\right\} \Rightarrow \left.\begin{array}{l}B = -A \\ 2A - A = 1\end{array}\right\} = \begin{array}{l}B = -1 \\ A = 1\end{array}$$

$k+2 \overset{!}{=} \tilde{k}+1$

$\tilde{k} = k+1$

$$p_n = \sum_{k=0}^{n} \frac{1}{k+1} - \frac{1}{k+2} = \sum_{k=0}^{n} \frac{1}{k+1} - \sum_{k=0}^{n} \frac{1}{k+2} = \sum_{k=0}^{n} \frac{1}{k+1} - \sum_{\tilde{k}=1}^{n+1} \frac{1}{\tilde{k}+1} =$$

$$= \frac{1}{0+1} - \frac{1}{(n+1)+1} = 1 - \frac{1}{n+2} \xrightarrow{(n \to \infty)} \underline{\underline{1}}$$

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A11)

a) i) $\sum_{k=0}^{\infty} \frac{5^k}{k} x^k$

$$\sqrt[k]{|a_k|} = \sqrt[k]{\left|\frac{5^k}{k}\right|} = \frac{\sqrt[k]{5^k}}{\sqrt[k]{k}} = \frac{5}{\sqrt[k]{k}} \xrightarrow{(k\to\infty)} 5$$

$\Rightarrow$ Konvergenzradius $R = \frac{1}{5}$

$\boxed{\text{ii) siehe letztes Blatt}}$

~~ii)~~ $\sum_{k=0}^{\infty} \left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^{2k} x^k$

$$\sqrt[k]{|a_k|} = \sqrt[k]{\left|\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right|^{2k}} = \left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^2 =$$

$$= \left(\frac{k+1-(k-\sqrt{k})}{\sqrt{k+1} + \sqrt{k-\sqrt{k}}}\right)^2 = \frac{1+2\sqrt{k}+k}{k+1+2\sqrt{k-\sqrt{k}}+k-\sqrt{k}} =$$

$$= \frac{k\left(\frac{1}{k} + \frac{2\sqrt{k}}{k} + 1\right)}{2k+1-\sqrt{k}+2\sqrt{k\cdot\left(1-\frac{1}{\sqrt{k}}\right)}} \longrightarrow$$

$10 - \sqrt{10}$
$10\cdot\left(1-\frac{\sqrt{10}}{10}\right) =$
$= \frac{1}{\sqrt{10}}$

$(1+\sqrt{k})^2$
$1+2\sqrt{k}+k$

iii) $\sum_{k=0}^{\infty} (k!+2) x^k$

$$\sqrt[k]{|a_k|} = \sqrt[k]{|(k!+2)|} \geq \sqrt[k]{k!} \xrightarrow{(k\to\infty)} \infty$$

$R = 0$

iv) $\sum_{k=0}^{\infty} \frac{2^k}{k^2} x^{4k}$ $\quad y := x^4$

$\rightarrow \sum_{k=0}^{\infty} \frac{2^k}{k^2} y^k$

$$\sqrt[k]{|a_k|} = \sqrt[k]{\left|\frac{2^k}{k^2}\right|} = \frac{2}{\sqrt[k]{k^2}} \xrightarrow{(k\to\infty)} \frac{2}{1^2} = 2$$

$\Rightarrow R = \frac{1}{2}$

geg. Reihe: $R = \left(\sqrt[4]{\frac{1}{2}}\right) = \frac{1}{\sqrt[4]{2}}$

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b) $S(x) = \sum_{k=0}^{\infty} \Big(\underbrace{\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1}\Big)^k \left(\frac{1}{x+3}\right)^k \qquad y := \frac{1}{x+3}$

$\sqrt[k]{|a_k|} = \sqrt[k]{\quad} = \sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1 = \sqrt[k]{3} \cdot \sqrt[k]{k} + \frac{4}{\sqrt[k]{k!}} + 1$

$\xrightarrow{k \to \infty} 1 \cdot 1 + 0 + 1 = 2 \quad R_{subst} = \frac{1}{2}$

$\cancel{|y| = \left|\frac{1}{x+3}\right| \to |x| = \left|\frac{1}{y} - 3\right| = +1}$

|y| < R_y ==> |1/(x+3)|<R_y ==>....man

$\left(\frac{1}{x}\right) \Rightarrow$ Reihe konvergent für $\cancel{M = (-\infty, -4) \cup (-2, +\infty)}$

A12)

a) i) $\exp(3ix) = \exp(ix)^3$

$\Leftrightarrow \cos(3x) + i\sin(3x) = (\cos(x) + i\sin(x))^3$

$= ((\cos x)^2 + i\cos x \cdot \sin x + (\sin x)^2) \cdot (\cos x + i\sin x)$

$= (\cos x)^3 + i(\cos x)^2 \sin x \cdot 2 - \cos x (\sin x)^2 \cdot 2 - i(\sin x)^2$

$(\cos x + i\sin x)^3 = ((\cos x)^2 + 2 \cdot i\cos x \sin x - (\sin x)^2) \cdot (\cos x + i\sin x) =$

$= (\cos x)^3 + i(\cos x)^2 \sin x + 2 \cdot i(\cos x)^2 \sin x - 2\cos x (\sin x)^2$

$- \cos x (\sin x)^2 - i(\sin x)^3$

$\Rightarrow \cos(3x) = (\cos x)^3 - 3\cos x (\sin x)^2 \quad$ (Realteil = Realteil)

$\sin(3x) = 3(\cos x)^2 \sin x - (\sin x)^3 \quad$ (Imaginärteil = Imaginärteil

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ii) $\sin(3x) = \sin(x+2x) = \sin x \cos(2x) + \cos x \sin(2x) =$

$= \sin x \cdot ((\cos x)^2 - (\sin x)^2) + \cos x \cdot (\sin x \cos x + \cos x \sin x) =$

$= \sin x (\cos x)^2 - (\sin x)^3 + \sin x (\cos x)^2 + (\cos x)^2 \sin x =$

$= 3(\cos x)^2 \sin x - (\sin x)^3$

$\cos(3x) = \cos(x+2x) = \cos x \cdot \cos(2x) - \sin x \sin(2x) =$

$= \cos x \cdot ((\cos x)^2 - (\sin x)^2) - \sin x \cdot (\sin x \cos x + \cos x \sin x) =$

$= (\cos x)^3 - \cos x (\sin x)^2 - \cos x (\sin x)^2 - \cos x (\sin x)^2 =$

$= (\cos x)^3 - 3 \cos x (\sin x)^2$

b)

i) $\sin(3x) = 3 \cos x^2 \sin x - (\sin x)^3$

$x = \frac{\pi}{3}$: $\sin(\pi) = 3(\cos \frac{\pi}{3})^2 \sin \frac{\pi}{3} - (\sin \frac{\pi}{3})^3 =$

$= (\sin \frac{\pi}{3}) \cdot (3(\cos \frac{\pi}{3})^2 - (\sin \frac{\pi}{3})^2) \quad | : \sin \frac{\pi}{3} \neq 0$

$\frac{\sin \pi}{\sin \frac{\pi}{3}} = 3 \cdot (\cos \frac{\pi}{3})^2 - (\sin \frac{\pi}{3})^2$

$(\sin \pi = 0:)\ 0 = 3 \cdot (1 - (\sin \frac{\pi}{3})^2) - (\sin \frac{\pi}{3})^2$

$0 = 3 - 4(\sin \frac{\pi}{3})^2$

$\sqrt{\frac{3}{4}} = \sin \frac{\pi}{3}$

$\frac{\sqrt{3}}{2} = \sin \frac{\pi}{3}$

$(\cos \frac{\pi}{3})^2 = 1 - (\sin \frac{\pi}{3})^2$

$(\cos \frac{\pi}{3})^2 = 1 - \frac{3}{4}$

$\cos \frac{\pi}{3} = \frac{1}{2}$

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ii) $\cancel{\cos\frac{\pi}{6} = \cos}$

$$\cos\left(2\cdot\frac{\pi}{6}\right) = \left(\cos\frac{\pi}{6}\right)^2 - \left(\sin\frac{\pi}{6}\right)^2 = 1 - 2\left(\sin\frac{\pi}{6}\right)^2$$

$$\left(\cos\frac{\pi}{3}\right) = 1 - 2\left(\sin\frac{\pi}{6}\right)^2$$

$$\frac{1}{2}\cdot\frac{1}{2} = \left(\sin\frac{\pi}{6}\right)^2$$

$$\frac{1}{4} = \sin\frac{\pi}{6}$$

$$\left(\cos\frac{\pi}{6}\right)^2 = 1 - \left(\sin\frac{\pi}{6}\right)^2$$

$$\cos\frac{\pi}{6} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

iii) $\cos\left(2\cdot\frac{\pi}{12}\right) = 1 - 2\left(\sin\frac{\pi}{12}\right)^2$

$$\cos\frac{\pi}{6} = 1 - 2\left(\sin\frac{\pi}{12}\right)^2$$

$$-\left(\frac{\sqrt{3}}{2} - 1\right) = 2\cdot\left(\sin\frac{\pi}{12}\right)^2$$

$$\frac{2-\sqrt{3}}{4} = \left(\sin\frac{\pi}{12}\right)^2$$

$$\sin\frac{\pi}{12} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

$$\left(\cos\frac{\pi}{12}\right)^2 = 1 - \left(\sin\frac{\pi}{12}\right)^2$$

$$\cos\frac{\pi}{12} = \sqrt{1 - \frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

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A10)

a) i) $\sum_{k=0}^{\infty} k \cdot q^k \cdot \sum_{k=0}^{\infty} q^k = \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} k \cdot q^k \cdot q^{n-k} \right) =$

$= \sum_{n=0}^{\infty} \left(q^n \left(\sum_{k=0}^{n} k \right) \right) = \sum_{n=0}^{\infty} q^n \cdot \frac{n(n+1)}{2}$

ii) ?

A11) a) ii)

$\sqrt[k]{|a_k|} = \sqrt[k]{\sqrt{k+1} - \sqrt{k-\sqrt{k}}}^{\,2k} = \left(\sqrt{k+1} - \sqrt{k-\sqrt{k}} \right)^2 =$

$= \left(\frac{k+1-(k-\sqrt{k})}{\sqrt{k+1} + \sqrt{k-\sqrt{k}}} \right)^2 = \frac{(1+\sqrt{k})^2}{k+1+2\sqrt{k+1}\sqrt{k-\sqrt{k}} + k - \sqrt{k}}$ ✓

$= \frac{1+2\sqrt{k}+k}{\cancel{k}+\cancel{1}1+2\sqrt{k}\cdot\sqrt{1+\frac{1}{k}}\cdot\sqrt{k}\cdot\sqrt{1-\frac{1}{\sqrt{k}}}+k-\sqrt{k}} =$

$= \frac{1+2\sqrt{k}+k}{k+1+2k\cdot\sqrt{1+\frac{1}{k}}\cdot\sqrt{1-\frac{1}{\sqrt{k}}}+k-\sqrt{k}} = \frac{k\cdot\left(\frac{1}{k}+\frac{2}{\sqrt{k}}+1\right)}{k\cdot\left(1+\frac{1}{k}+2\cdot\sqrt{1+\frac{1}{k}}\cdot\sqrt{1-\frac{1}{\sqrt{k}}}+1-\frac{1}{\sqrt{k}}\right)} =$

$\xrightarrow{k\to\infty} \frac{1}{1+0+2\cdot 1\cdot 1+1-0} = \frac{1}{4} \quad \Rightarrow \underline{\underline{R = 4}}$ ✓

Soll das so umständlich sein? :-)

Nein:Du kannst das Quadrat einfach außen stehen lassen:[(