Deckblatt für die Abgabe der Übungsaufgaben IngMathC2

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Blatt- Nummer: 04

Übungsgruppe- Nr. 7

Die Folgenden Aufgaben gebe ich zur Korrektur frei: Alle

9/10*30=27

A 10) (i) $\left(\sum_{k=0}^{\infty} k \cdot q^k\right)\left(\sum_{k=0}^{\infty} q^k\right) = \sum_{n=0}^{\infty}\left(\sum_{k=0}^{\infty} k \cdot q^k \cdot q^{n-k}\right) = \sum_{n=0}^{\infty}\left(\sum_{k=0}^{\infty} k \cdot q^n\right) = \sum_{n=0}^{\infty} \underbrace{\frac{n(n+1)}{2}}_{\frac{1}{2}n^2+\frac{1}{2}n^2} \cdot q^n$

$$= \left(\frac{1}{(1-q)^2} - \frac{1}{1-q}\right) \cdot \frac{1}{1-q} = \frac{1}{(1-q)^3} - \frac{1}{(1-q)^2} \quad \Big| \cdot 2, \; - \sum_{n=0}^{\infty} n \cdot q^n$$

$$\Rightarrow \sum_{n=0}^{\infty} n^2 q^n = \frac{2}{(1-q)^3} - \frac{2}{(1-q)^2} - \left(\frac{1}{(1-q)^2} - \frac{1}{(1-q)}\right) = \frac{2}{(1-q)^3} - \frac{3}{(1-q)^2} + \frac{1}{1-q}$$

(ii) $\sum_{k=0}^{\infty} k^2 q^k = \frac{1}{(1-q)^3} - \frac{1}{(1-q)^2} - \frac{1}{(1-q)} = \frac{1}{(1-q)^3} - \frac{1-q}{(1-q)^3} - \frac{(1-q)^2}{(1-q)^3}$

$$= \frac{1-1+q-1+2q-q^2}{(1-q)^3} = \frac{-1+3q-q^2}{(1-q)^3}$$

b) $\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+2)} \stackrel{!}{=} \frac{A}{(k+1)} - \frac{B}{(k+2)} \stackrel{!}{=} A(k+2) - B(k+1)$

$0 = A - B$, $A = B$, $1 = 2A - B$, $1 = B$, $A = 1$

$$= \sum_{k=0}^{\infty}\left(\frac{1}{k+1} - \frac{1}{k+2}\right) = \sum_{k=0}^{\infty} \frac{1}{k+1} - \sum_{k=0}^{\infty} \frac{1}{k+2} = \left[\text{Indextransformation: } k+1 \stackrel{!}{=} \tilde{k}+2, \; \tilde{k} := k-1\right]$$

$$= \sum_{\tilde{k}=-1}^{\infty} \frac{1}{\tilde{k}+2} - \sum_{k=0}^{\infty} \frac{1}{k+2} = \frac{1}{-1+2} = \underline{\underline{1}}$$

A 11) (i) $\sum_{k=0}^{\infty} \frac{5^k}{k} \cdot x^k$, $\sqrt[k]{|a_k|} = \sqrt[k]{\left|\frac{5^k}{k}\right|} = \frac{5}{\sqrt[k]{k}} = \frac{5}{1} \Rightarrow R = \underline{\underline{\frac{1}{5}}}$

(ii) $\sum_{k=0}^{\infty} \left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^{2k} \cdot x^k = (k+1 - 2\sqrt{k+1} \cdot \sqrt{k-\sqrt{k}} + k - \sqrt{k}$ (x^k fällt weg, da unwichtig)

$= 2k+1-\sqrt{k} - 2\sqrt{k^2 - k\cdot\sqrt{k} + k - \sqrt{k}} = \frac{\left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^2 \cdot \left(\sqrt{k+1} + \sqrt{k-\sqrt{k}}\right)^2}{\left(\sqrt{k+1} + \sqrt{k-\sqrt{k}}\right)^2}$

$= \frac{(k+1-k+\sqrt{k})^2}{\left(\sqrt{k+1} + \sqrt{k-\sqrt{k}}\right)^2} = \frac{k\left(1 + \frac{2}{\sqrt{k}} + \frac{1}{k}\right)}{k\left(2 + 2\sqrt{1 - \frac{1}{\sqrt{k}} + \frac{1}{k} - \frac{1}{k\sqrt{k}}} - \frac{1}{\sqrt{k}}\right)}$, $\lim_{k\to\infty} \ldots = \frac{1+0+0}{2+2\cdot\sqrt{1-0+0-0}-0} = \frac{1}{4} \Rightarrow R = \underline{\underline{4}}$

(iii) $\sum_{k=0}^{\infty} (k!+2)\, x^k$, $\sqrt[k]{k!+2} \geq \sqrt[k]{k!}$ ($\hat{=}$ laut Vorlesung ∞) $\Rightarrow \lim_{k\to\infty} \sqrt[k]{k!} = \infty \Rightarrow R = \frac{1}{\infty} = \underline{\underline{0+0}}$

(iv) $\sum_{k=0}^{\infty} \underbrace{\frac{2^k}{k^2}}_{\hat{=} a_{\tilde{k}}} \cdot x^{4k} = \sum_{\tilde{k}=0}^{\infty} a_{\tilde{k}} \cdot x^{4\tilde{k}}$, wobei $a_{\tilde{k}} \begin{cases} 0, & a_{\tilde{k}}\ \tilde{k} \bmod 4 \neq 0 \\ \frac{2^{\frac{1}{4}\tilde{k}}}{\left(\frac{1}{4}\tilde{k}\right)^2}, & \tilde{k} \bmod = 0 \end{cases}$

$= \sqrt[k]{\frac{2^{\frac{1}{4}k}}{\left(\frac{1}{4}k\right)^2}} = \frac{\sqrt[4]{2}}{\sqrt[k]{\left(\frac{1}{4}k\right)^2}} = \sqrt[4]{2} \Rightarrow R = \frac{1}{\sqrt[4]{2}}$

$\sqrt[k]{\left(\frac{1}{4}k\right)^2} \to 1$

b) $\sum_{k=0}^{\infty} \left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1\right)^k \cdot \left(\frac{1}{x+3}\right)^k$, Subst.: $y = \frac{1}{x+3}$, $\sum_{k=0}^{\infty} \underbrace{\left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1\right)^k}_{} \cdot y^k$

der Konvergenzradius heißt, dass für alle|y|<R_yDie

$\underbrace{\sqrt[k]{3k}}_{=1} + \underbrace{\frac{4}{\sqrt[k]{k!}}}_{=0} + 1 \Rightarrow 1+0+1 = 2 \Rightarrow R = \frac{1}{2}$

Die Reihe (x) hat $R \frac{1}{2}$, d.h.: Für ~~$|x| > 1$~~ ist $|y| = \frac{1}{x+3} < \frac{1}{2} \Rightarrow$ Die Reihe ist konvergent

Für ~~$|x| < 1$~~ ist $|y| = \frac{1}{x+3} > \frac{1}{2} \Rightarrow$ Die Reihe ist divergent

$\Rightarrow$ Die gegebene Reihe hat Konvergenzradius $R = -1$

~~$(-\infty, -1) \cup (1, +\infty)$~~

A 12 a) (i) $e^{3ix} = \cos(3x) + i\sin(3x)$

$$\sin(3x) = \frac{e^{3ix} - \cos(3x)}{i}$$

$$\cos(3x)\ e^{3ix} - i\sin(3x)$$

b) $\sin(3x) = \sin(x)(3\cos(x)^2 - \sin(x)^2)$

$$= \sin(x)(3(1-\sin(x)^2 - \sin(x)^2)$$

$$= \sin(x)(3 - 4\sin(x)^2)$$

$$\sin\left(3 \cdot \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)\left(3 - 4\sin\left(\frac{\pi}{3}\right)^2\right) \quad \Big| : \sin\left(\frac{\pi}{3}\right)$$

$$0 = 3 - 4\sin\left(\frac{\pi}{3}\right)^2$$

$$\sin\left(\frac{\pi}{3}\right)^2 = \frac{3}{4}$$

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

(ii) $\cos\left(2^x \cdot \frac{\pi}{6}\right) = \cos(x)^2 - \sin(x)^2 = 1 - 2\sin(x)^2$

$$\cos\left(2 \cdot \frac{\pi}{6}\right) = 1 - 2\sin\left(\frac{\pi}{6}\right)^2$$

$$\frac{1}{2} + 2\sin\left(\frac{\pi}{6}\right)^2 = 1$$

$$\sin\left(\frac{\pi}{6}\right)^2 = \frac{1}{4}$$

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\sin(2x) = 2\sin(x) \cdot \cos(x)$$

$$\cos(x) = \frac{\sin(2x)}{2\sin(x)}$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sin\left(2 \cdot \frac{\pi}{6}\right)}{2\sin\left(\frac{\pi}{6}\right)}$$

$$= \frac{\frac{\sqrt{3}}{2}}{2 \cdot \frac{1}{2}} = \frac{\sqrt{3}}{2}$$

(ii) $\sin(2x) = \sin(x+x) = \sin(x) \cdot \cos(x) + \cos(x) \cdot \sin(x)$

$$\sin(3x) = \sin(2x+x) = \sin(x) \cdot \cos(x)^2 + \cos(x)^2 \cdot \sin(x) + \cos(x)^2 \cdot \sin(x) - \sin(x)^2 - \sin(x)$$

$$= 3\sin(x) \cdot \cos(x)^2 - \sin(x)^3$$

$$\cos(2x) = \cos(x+x) = \cos(x)^2 - \sin(x)^2$$

$$\cos(3x) = \cos(2x+x) = (\cos(x)^2 - \sin(x)^2) \cdot \cos(x) - (\sin(x) \cdot \cos(x) + \cos(x) \cdot \sin(x)) \cdot \sin(x)$$

$$= \cos(x)^3 - 3\sin(x)^2 \cdot \cos(x)$$

b) $\cos\left(3 \cdot \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right)^3 - 3\sin\left(\frac{\pi}{3}\right)^2 \cdot \cos\left(\frac{\pi}{3}\right)$

$$-1 = \cos\left(\frac{\pi}{3}\right)\left(\cos\left(\frac{\pi}{3}\right)^2 - 3\sin\left(\frac{\pi}{3}\right)^2\right)$$

$$0 = \cos\left(\frac{\pi}{3}\right)\left(1 - \sin\left(\frac{\pi}{3}\right)^2 - 3\sin\left(\frac{\pi}{3}\right)\right) + 1$$

$$0 = \cos\left(\frac{\pi}{3}\right)\left(1 - 4\sin\left(\frac{\pi}{3}\right)^2\right) + 1$$

$$0 = \cos\left(\frac{\pi}{3}\right)\left(1 - 4 \cdot \frac{3}{4}\right) + 1$$

$$0 = \cos\left(\frac{\pi}{3}\right) \cdot (-2) + 1$$

$$-1 = -2\cos\left(\frac{\pi}{3}\right)$$

$$\frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$

(iii) $\cos\left(2\ \frac{\pi}{12}\right) = 1 - 2\sin\left(\frac{\pi}{12}\right)^2$

$$\frac{\sqrt{3}}{2} + 2\sin\left(\frac{\pi}{12}\right)^2 = 1$$

$$\sin\left(\frac{\pi}{12}\right)^2 = \frac{2-\sqrt{3}}{4}$$

$$\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{2-\sqrt{3}}}{2}$$

$$\cos(x) = \frac{\sin(2x)}{2\sin(x)}$$

$$= \frac{\frac{1}{2}}{2 \cdot \frac{1}{2} \cdot \sqrt{2-\sqrt{3}}}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{2-\sqrt{3}}}$$

$$= \frac{1}{2\sqrt{2-\sqrt{3}}}$$