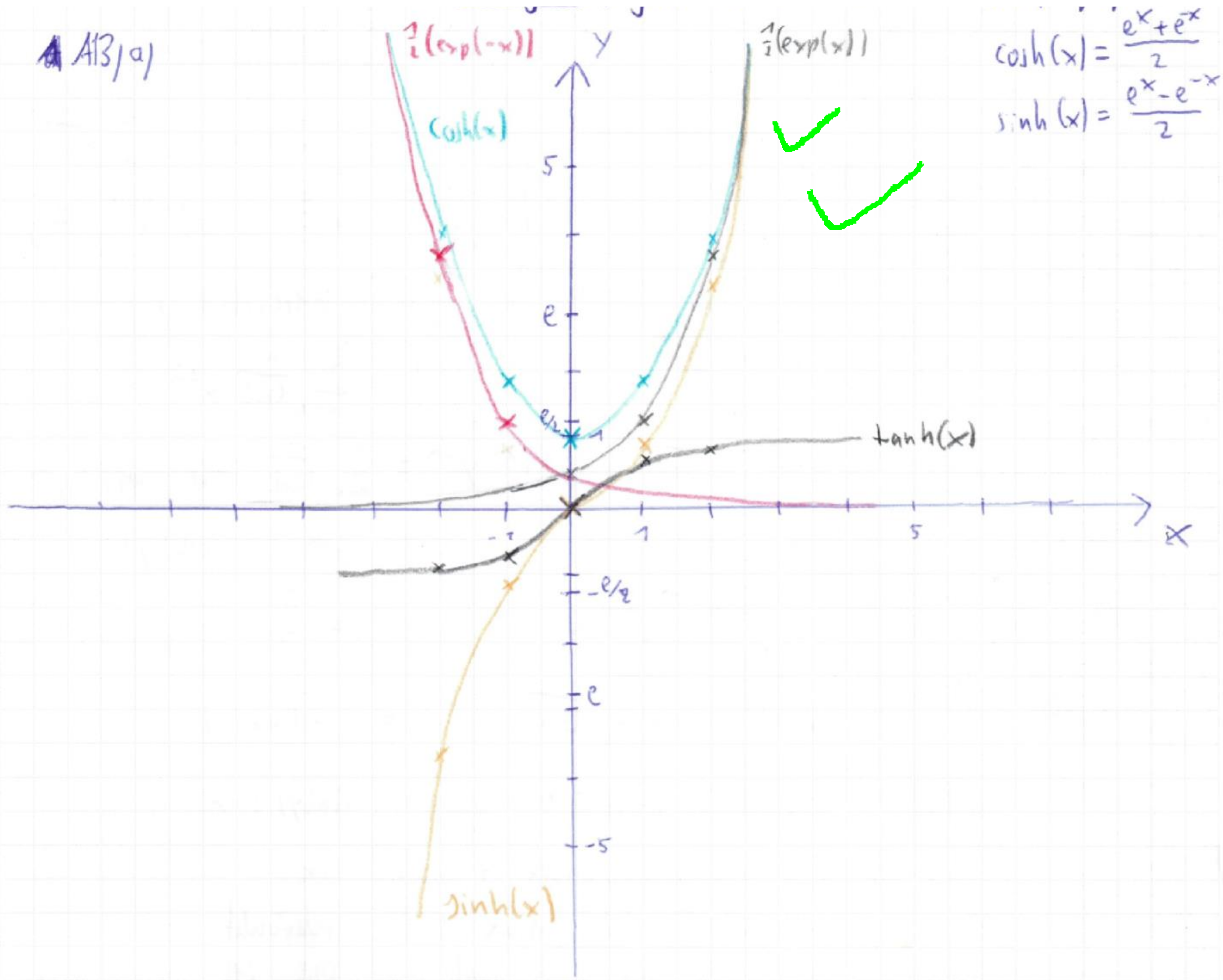
14/14*30 =30


A13) a)
$\frac{1}{2}(\exp(-x))$
$\frac{1}{2}(\exp(x))$
$\cosh(x) = \frac{e^x + e^{-x}}{2}$
$\sinh(x) = \frac{e^x - e^{-x}}{2}$
cosh(x)
tanh(x)
sinh(x)
y
x
5
e
e/2
1
-1
5
-e/2
-e
-5

b) $\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{\frac{e^x - e^{-x}}{2}}{\frac{e^x + e^{-x}}{2}} = \frac{2e^x - e^{-x}}{2e^x + 2e^{-x}} = \frac{2e^x(1-e^{-2x})}{2e^x(1+e^{-2x})} =$

$= \frac{1-\frac{1}{e^{2x}}}{1+\frac{1}{e^{2x}}} = \frac{\frac{e^{2x}-1}{e^{2x}}}{\frac{e^{2x}+1}{e^{2x}}} = \frac{e^{2x}(e^{2x}-1)}{e^{2x}(e^{2x}+1)} = \frac{e^{2x}-1}{e^{2x}+1}$

$\lim_{x\to\infty} \tanh = \lim \frac{1-\frac{1}{e^{2x}}}{1+\frac{1}{e^{2x}}} = \frac{1-0}{1+0} = 1$ ✓

$\lim_{x\to-\infty} \tanh = \lim_{x\to-\infty} \frac{e^{2x}-1}{e^{2x}+1} = \frac{0-1}{0+1} = -1$ ✓

Skizze: siehe vorherige Zeichnung

c) $\cosh^2 x - \sinh^2 x = \frac{(\exp(x)+\exp(-x))^2}{2^2} - \frac{(\exp(x)-\exp(-x))^2}{2^2} =$

$= \frac{\cancel{e^{2x}} + 2e^x e^{-x} + \cancel{e^{-2x}} - \cancel{e^{2x}} + 2 e^x e^{-x} - \cancel{e^{-2x}}}{4}$ ✓ $= \frac{2\cdot 1 + 2\cdot 1}{4} = 1$ ✓

d) $\sinh(x) = \frac{e^x - e^{-x}}{2} = \frac{1}{2}\left(\sum_{n=0}^{\infty} \frac{1}{n!}x^n - \sum_{n=0}^{\infty}\frac{1}{n!}(-x)^n\right) =$

$= \frac{1}{2}\left(\sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n} + \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n+1} - \sum_{n=0}^{\infty}\frac{1}{(2n)!}(-x)^{2n} - \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}(-x)^{2n+1}\right) =$

$= \frac{1}{2}\left(\sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n} + \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n+1} - \sum_{n=0}^{\infty}\frac{1}{(2n)!}(x^2)^n - \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}(-1)(x)\cdot(x^2)^n\right) =$

$= \frac{1}{2}\left(\sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n+1} - (-1)\cdot\sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n+1}\right) = \frac{1}{2}\cdot 2 \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n+1} = \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n+1}$ ✓

$\cosh(x) = \frac{e^x + e^{-x}}{2} = \frac{1}{2}\left(\sum_{n=0}^{\infty}\frac{1}{n!}x^n + \sum_{n=0}^{\infty}\frac{1}{n!}(-x)^n\right) = \frac{1}{2}\left(\sum_{n=0}^{\infty}\frac{1}{n!}x^n + \sum_{n=0}^{\infty}\frac{1}{(2n)!}(-x)^{2n} + \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}(-x)^{2n+1}\right) =$

$= \frac{1}{2}\left(\sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n} + \sum_{n=0}^{\infty}\frac{1}{(2n)!}(x^2)^n + \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n+1} + \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}(-1)(x)(x)^{2n}\right) =$

$= \frac{1}{2}\left(2\sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n} + \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n+1} - \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n+1}\right) = \sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n}$ ✓

e) $\cos(iy) = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(iy)^{2n} = \sum_{n=0}^{\infty}\frac{1}{(2n)!}(-1)^n i^{2n} y^{2n} = \sum_{n=0}^{\infty}\frac{1}{(2n)!}(-1)^n (-1)^n y^{2n} =$

$= \sum_{n=0}^{\infty}\frac{1}{(2n)!}(-1\cdot(-1))^n y^{2n} = \sum_{n=0}^{\infty}\frac{1}{(2n)!}y^{2n}\cdot 1^n = \sum_{n=0}^{\infty}\frac{1}{(2n)!}y^{2n} = \cosh(y)$ ✓

$\sin(iy) = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}(yi)^{2n+1} = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!} i^{2n+1} y^{2n+1} = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!} i\cdot i^{2n} y^{2n+1} =$

$= \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}y^{2n+1}\cdot\overbrace{(-1)^n\cdot(-1)^n}^{(-1(-1))^n = 1^n = 1}\cdot i = i\sum_{n=0}^{\infty}\frac{1}{(2n+1)!}y^{2n+1} = i\cdot\sinh(y)$ ✓

f) $\sin(iy+x) = \sin(iy)\cos(x) + \cos(iy)\sin(x) = i\sinh(y)\cos x + \cosh(y)\sin x$ ✓

g) $\sin: \mathbb{C}\to\mathbb{C}$ $\quad \sin(x+iy) = i\sinh(y)\cos x + \cosh(y)\sin x$

sinh, cosh sind nicht beschränkt, cos x, sin x sind beschränkt.

Das Produkt einer unbeschränkten mit einer beschränkten Fkt. ist unbeschränkt. ✓

Die Addition zweier unb. Fkt. ist ebenfalls unb.. $\Rightarrow \sin:\mathbb{C}\to\mathbb{C}$ ist unbeschränkt. ✓

Mathe Übung   d74umay

A 14, a)

$f(x) = \frac{1-x}{\sqrt{1-x^2}}$ $1-x^2 > 0$ $x^2 < 1$ $x \gtrless \pm 1$

$D_f = -1 < x < 1$

$\lim\limits_{x\to -1} f(x) = \lim\limits_{x\to -1} \frac{\overbrace{1-x}^{\to 2}}{\underbrace{\sqrt{1-x^2}}_{\to 0}} = \infty$

$\frac{1-x}{\sqrt{1-x^2}} = \frac{1-x}{(1-x^2)^{1/2}} = \frac{1-x}{((1+x)(1-x))^{1/2}} = \frac{1-x}{\sqrt{1+x}\sqrt{1-x}} = \frac{\sqrt{1-x}\sqrt{1-x}}{\sqrt{1+x}\sqrt{1-x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}}$

$\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} \frac{\sqrt{1-x}}{\sqrt{1+x}} = \frac{\sqrt{0}}{\sqrt{2}} = 0$

b) i) „Teilstücke" stetig (Konstante und e-Fkt.) $\to x' = 0$

$f(0) = 0$ $\lim\limits_{x\nearrow 0} f(x) = 0$ $\lim\limits_{x\searrow 0} f(x) = \lim\limits_{x\searrow 0} e^{\underbrace{1+x-\frac{1}{x}}_{-\infty}} = 0$

$\to$ f stetig, weil $f(x) \longrightarrow f(x')$ für $x \longrightarrow x'$

ii) „Teilstücke" stetig (s.o.) $\to x' = 0$

$f(0) = 0$ $\lim\limits_{x\searrow 0} f(x) = 0$ (s.o.) $\lim\limits_{x\nearrow 0} f(x) = \lim\limits_{x\nearrow 0} e^{\underbrace{1+x-\frac{1}{x}}_{+\infty}} = \infty$ „$\neq 0$"

$\to$ f nicht stetig, weil $f(x) \not\to f(x')$ für $x \to x'$

c) i) $\lim\limits_{x\to 0} \sqrt{x^2+x+1} - x = \sqrt{0+0+1} - 0 = \sqrt{1} = 1$

ii) $\lim\limits_{x\to\infty} \sqrt{x^2+x+1} - x = \lim\limits_{x\to\infty} \frac{x^2+x+1-x^2}{\sqrt{x^2+x+1}+x} = \lim\limits_{x\to\infty} \frac{x(1+\frac{1}{x})}{x(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1)} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$

iii) $\lim\limits_{x\to-\infty} \sqrt{x^2+x+1} - x = \lim\limits_{x\to-\infty} \frac{1+\frac{1}{x}}{\sqrt{1+\frac{1}{x^2}+\frac{1}{x}}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$

iv) $\lim\limits_{x\to\infty} \overset{\infty}{x}\,|\sin \pi x| = 0$ sin von Vielfachen von π immer 0

v) $\lim\limits_{x\to 0} \overset{\to 0}{x}\,|\sin \overset{\to 0}{\pi x}| = 0$

vi) $\lim\limits_{x\to 0} \underbrace{\cos x}_{\text{konv. gg. } 1} \underbrace{\cos^2 \frac{2}{x}}_{\text{unbestimmt divergent}}$ Limes existiert nicht, da Teilfolgen mit versch. Grenzwerten existieren

$x_n = \frac{1}{2\pi n} \xrightarrow{n\to\infty} 0 \Rightarrow f(x_n) = \left(1+\frac{1}{2\pi n}\right)\left(\cos\left(2\pi n\right)\right)^2 \xrightarrow{n\to\infty} 1 \cdot 1^2 = 1$

$x_n' = \frac{1}{\pi n+\frac{\pi}{4}} \xrightarrow{n\to\infty} 0 \Rightarrow f(x_n') = \left(1+\frac{1}{2\pi n+\frac{\pi}{4}}\right)\left(\cos\left(2\pi n+\frac{\pi}{2}\right)\right)^2 \xrightarrow{n\to\infty} 1\cdot 0^2 = 0$