Deckblatt für die Abgabe der Übungsaufgaben
IngMathC2

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Blatt-Nummer: 01

Übungsgruppen-Nr: 07

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A1, A2, A3, ______

23/24 *33=31.5

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A1)

	inf(M)	sup(M)	min(M)	max(M)
a)	$\sqrt{3}$	$\sqrt{5}$	$\sqrt{3}$	nicht existend
b)	0,25	$\frac{2}{3}$	0,25	$\frac{2}{3}$
c)	0	$\frac{1}{2}$	n. e.	$\frac{1}{2}$
d)	~~2~~ $-\infty$	$+\infty$	~~2~~	n. e.
e)	0	1	n. e.	1
f)	$\frac{2}{3}$	$+\infty$	$\frac{2}{3}$	n. e.
g)	1	$+\infty$	n. e.	n. e.

A2)

i) $$\frac{3n+4m}{5n^2+10} \leq \frac{3n+4\cdot 3n}{5n^2+10} = \frac{15n}{5n^2+10}$$

ii) $$\frac{5n-m}{2n} \leq \frac{5n-2n}{2n} = \frac{3n}{2n} = 1,5$$

iii) $$\frac{n}{n+m} \leq \frac{n}{n+2n} = \frac{1}{3}$$

iv) $$\frac{n+m}{\frac{1}{2}-m} \leq \frac{n+2n}{\frac{1}{2}-n} = \frac{3n}{\frac{1}{2}-n}$$

v) $$\frac{5n-m+3\cdot 2^m}{3n^3-m+3} \leq \frac{5n-2n+3\cdot 2^{3n}}{3n^3-3n+3} = \frac{3n+3\cdot 2^{3n}}{3n^3-3n+3}$$

vi) $$m+n+\sin(m)-\sin(17m^2)+2^m+2^{-m} \leq 3n+n+1-0+2^{3n}+2^{-2n}$$

$$= 4n+1+2^{3n}+2^{-2n}$$

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A3)

a) i) $a_{n+1} - a_n = \frac{2(n+1)}{n+4} - \frac{2n}{n+3} = \frac{(2n+2)\cdot(n+3) - 2n\cdot(n+4)}{(n+4)\cdot(n+3)} =$

$= \frac{2n^2+8n+6-2n^2-8n}{n^2+2n+12} = \frac{6}{n^2+7n+12} \geq 1$

$\Rightarrow$ monoton steigend

ii) $a_{n+1} - a_n = \frac{n+1}{4^{n+1}} - \frac{n}{4^n}$

$\frac{1}{x} = 0{,}x$

$\frac{1}{0{,}x} = x$

$\frac{1}{x} = \frac{13}{4}$

$1 \cdot \frac{4}{3} =$

$\frac{a_{n+1}}{a_n} = \frac{\frac{n+1}{4^{n+1}}}{\frac{n}{4^n}} = \frac{n+1}{4^{n+1}} \cdot \frac{4^n}{n} = \frac{n+1}{4} \cdot \frac{1}{n} = \frac{n+1}{4n} =$

$= \frac{n}{4n} + \frac{1}{4n} = \frac{1}{4} + \frac{1}{4n} \leq 1$

≤ 1, da $n \geq 1$

$\Rightarrow$ monoton fallend

b) i) $\frac{2\cdot 100}{100+3} \approx 1{,}94$ ii) $\frac{100}{4^{10}} = \frac{5}{524{,}288}$

Grenzwert: $a = 2$ $b = 0$

c) Sei $\varepsilon > 0$ beliebig vorgegeben:

i) $n \geq n_0 : |a_n - a| = \left|\frac{2n}{n+3} - 2\right| = \left|\frac{2n-(2n+6)}{n+3}\right| = \frac{6}{n+3} \leq \ldots = \varepsilon$

$\frac{6}{n+3} \leq \varepsilon \Leftrightarrow \frac{6}{\varepsilon} \leq n+3 \Leftrightarrow n \geq \frac{6}{\varepsilon} - 3$

$\Rightarrow$ setze $n_0 := \lceil \frac{6}{\varepsilon} \rceil$

$\frac{6}{n+3} \leq \frac{6}{n_0+3} = \frac{6}{\lceil \frac{6}{\varepsilon} \rceil + 3} \leq \frac{6}{\frac{6}{\varepsilon}} = \varepsilon \quad \square$

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ii) Sei $\varepsilon > 0$ beliebig vorgegeben.

$n \geq n_0 : |b_n - b| = \left|\frac{n}{\varphi^n} - 0\right| = \frac{n}{\varphi^n} \leq \ldots = \varepsilon$

$\frac{n}{\varphi^n} \leq \varepsilon \iff \frac{n}{\varepsilon} \leq \varphi^n$

$2^3 = 2 \cdot 2 \cdot 2$

$2^{2 \cdot 3} = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$= 2 \cdot 2 \cdot 2 = 2^3$

$\cancel{\frac{n_0}{\varphi^{n_0}} = \frac{n_0}{2^{2n_0}} \leq \frac{2^{n_0}}{2^{2n_0}} = \frac{1}{2^{n_0}} \leq \varepsilon \iff \frac{1}{\varepsilon} \leq 2^{n_0} \iff n_0 \geq \log_2\left(\frac{1}{\varepsilon}\right)}$

~~setze $n_0 :=$~~

$\cancel{\frac{n_0}{\varphi^{n_0}} \overset{!}{\leq} \varepsilon \iff \frac{n_0}{\varepsilon} \leq \varphi^{n_0}}$

$\frac{n_0}{\varphi^{n_0}} \leq \frac{2^{n_0}}{2^{2n_0}} = \frac{1}{2^{n_0}} \overset{!}{\leq} \varepsilon \iff \frac{1}{\varepsilon} \leq 2^{n_0} \iff n_0 \geq ld\left(\frac{1}{\varepsilon}\right)$

$\Rightarrow$ setze $n_0 := \left\lceil ld\left(\frac{1}{\varepsilon}\right)\right\rceil$

$\frac{n}{\varphi^n} \leq \frac{n_0}{\varphi^{n_0}} = \frac{\left\lceil ld\left(\frac{1}{\varepsilon}\right)\right\rceil}{\varphi^{\left\lceil ld\left(\frac{1}{\varepsilon}\right)\right\rceil}} \leq \frac{ld\left(\frac{1}{\varepsilon}\right)}{\varphi^{ld\left(\frac{1}{\varepsilon}\right)}} \leq \frac{2^{ld\left(\frac{1}{\varepsilon}\right)}}{2^{2 ld\left(\frac{1}{\varepsilon}\right)}} = \frac{1}{2^{ld\left(\frac{1}{\varepsilon}\right)}} =$

$= \frac{1}{2^{-ld(\varepsilon)}} = \frac{1}{\frac{1}{2^{ld(\varepsilon)}}} = 2^{ld(\varepsilon)} = \varepsilon \quad \square$