Deckblatt für die Abgabe der Übungsaufgaben IngMathC2

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Blatt-Nummer: 4

Übungsgruppen-Nr: 7

Die folgenden Aufgaben gebe ich zur Korrektur frei:

10, 11, 12,

10/10*30

Mathe Übung 4

A10

I) $\left(\sum_{k=0}^{\infty} a_k\right)\left(\sum_{k=0}^{\infty} b_k\right) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} a_k b_{n-k}$

$a_k = k\cdot q^k \quad b_k = q^k \quad$ für $|q|<1$

$$\left(\sum_{k=0}^{\infty} k\cdot q^k\right)\left(\sum_{k=0}^{\infty} q^k\right) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} k\cdot q^k \cdot q^{n-k} = \sum_{n=0}^{\infty}\sum_{k=0}^{n} k\cdot q^n = \sum_{n=0}^{\infty} q^n \cdot \sum_{k=0}^{n} k = \sum_{n=0}^{\infty} q^n \cdot \frac{n^2+n}{2} \quad \text{für } |q|<1$$

II) $\sum_{k=0}^{\infty} k^2 \cdot q^k$ für $|q|<1$

$$\sum_{k=0}^{n} q^n \cdot \frac{n^2+n}{2} = \sum_{k=0}^{\infty} \frac{1}{2} n^2 q^n + \frac{1}{2} n q^n$$

$$\Rightarrow \sum_{n=0}^{\infty} n^2 q^n = \left(\sum_{n=0}^{\infty} q^n \cdot \frac{n^2+n}{2} - \sum_{n=0}^{\infty} \frac{1}{2} n q^n\right)\cdot 2 = \left(\frac{q}{(1-q)^3} - \frac{1}{2}\cdot\frac{1}{1-q}\right)\cdot 2 = \frac{2q}{(1-q)^3} - \frac{1}{1-q} = \frac{2q-(1-q)^2}{(1-q)^3}$$

b) $$\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+2)} = \sum_{k=0}^{n} \frac{1}{k+1} - \frac{1}{k+2} = \sum_{k=0}^{n} \frac{1}{k+1} - \sum_{k=0}^{n} \frac{1}{k+2} = \sum_{k=0}^{n-1} \frac{1}{k+1} - \sum_{\tilde{k}=1}^{n} \frac{1}{\tilde{k}+1} = \frac{1}{1} - \frac{1}{n+1} \xrightarrow{n\to\infty} 1$$

A11

a) I) $\sum_{k=0}^{\infty} \frac{5^k}{k} x^k \qquad \limsup_k \sqrt[k]{\left|\frac{5^k}{k}\right|} \overset{\lim_{k\to\infty}}{=} \frac{5}{\sqrt[k]{k}} = 5 \Rightarrow R = \frac{1}{5}$

II) $\sum_{k=0}^{\infty} \left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^{2k} x^k$

$$\limsup_k \sqrt[k]{\left|\left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^{2k}\right|} = \lim_{k\to\infty}\left(\frac{\left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)\cdot\left(\sqrt{k+1} + \sqrt{k-\sqrt{k}}\right)}{\sqrt{k+1} + \sqrt{k-\sqrt{k}}}\right)^2 \overset{\lim_{k\to\infty}}{=} \left(\frac{k+1-k+\sqrt{k}}{\sqrt{k+1} + \sqrt{k-\sqrt{k}}}\right)^2 \overset{\lim_{k\to\infty}}{=} \left(\frac{1+\sqrt{k}}{\sqrt{k+1} + \sqrt{k-\sqrt{k}}}\right)^2 =$$

$$\lim_{k\to\infty}\left(\frac{\sqrt{k}\left(\overbrace{\frac{1}{\sqrt{k}}}^{0}+1\right)}{\sqrt{k}\left(\sqrt{1+\underbrace{\tfrac{1}{k}}_{0}} + \sqrt{1-\underbrace{\tfrac{1}{\sqrt{k}}}_{0}}\right)}\right)^2 = \lim_{k\to\infty}\left(\frac{\sqrt{k}}{2\sqrt{k}}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \qquad \Rightarrow R = 4$$

III) $\sum_{k=0}^{\infty} (k!+2)x^k$

$\limsup_k \sqrt[k]{|k!+2|}$ Minorante: $\sqrt[k]{k!}$ divergiert gegen ∞, also $\limsup_k \sqrt[k]{k!+2} = +\infty \Rightarrow R = 0$

IV) $\sum_{k=0}^{\infty} \frac{2^k}{k^2} x^{4k} \qquad y = x^4$

$$\limsup_k \sqrt[k]{\left|\frac{2^k}{k^2}\right|} = \lim_{k\to\infty} \frac{2}{\sqrt[k]{k^2}} = \frac{2}{\left(\lim_{k\to\infty} \sqrt[k]{k}\right)^2} = \frac{2}{1^2} = 2 \quad \Rightarrow \quad R_y = \frac{1}{2}$$

Potenzreihe konvergent für alle $y \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

Rücksubstitution: $|y| < \frac{1}{2} \Leftrightarrow |x| = \sqrt[4]{|y|} < \frac{1}{\sqrt[4]{2}} = R_x$

b) $S(x) = \sum_{k=0}^{\infty} \left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1\right)^k \left(\frac{1}{x+3}\right)^k \qquad y = \frac{1}{x+3}$

$$\limsup_k \sqrt[k]{\left|\left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1\right)^k\right|} \overset{\lim_{k\to\infty}}{=} \sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1 = \lim_{k\to\infty} \sqrt[k]{3}\cdot\sqrt[k]{k} + \lim_{k\to\infty} \frac{4}{\sqrt[k]{k!}} + 1 =$$

$$1\cdot 1 + 0 + 1 = 2 \qquad R_y = \frac{1}{2}$$

Potenzreihe für alle $y \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ konvergent

$y = \frac{1}{x+3} \Rightarrow x \in (-\infty, -5) \cup (-1, +\infty)$

$\alpha = -5 \quad \beta = -1$

hier gerne nochmal explizit 1/|x+

A12

a) I) $\exp(3ix) = \exp(ix)^3$

$e^{3ix} = e^{ix^3}$

$\cos(3x) + i\sin(3x) = (\cos(x) + i\sin(x))^3$

$\cos(3x) + i\sin(3x) = \cos^3(x) + 3\cos^2(x)\, i\sin(x) + 3\cos(x)(i\sin(x))^2 + (i\sin(x))^3 =$

$= \cos^3(x) + i \cdot 3\cos^2(x)\sin(x) - 3\cos(x)\sin^2(x) - i\sin^3(x)$

$\cos(3x) = \cos^3(x) - 3\cos(x)\sin^2(x) = \cos^3(x) - 3\cos(x)(1-\cos^2(x)) = 4\cos^3(x) - 3\cos(x)$

$\sin(3x) = 3\cos^2(x)\sin(x) - \sin^3(x) = 3(1-\sin^2(x))\sin(x) - \sin^3(x) = -4\sin^3(x) + 3\sin(x)$

II) $\sin(x+2x) = \sin(x)\cos(2x) + \cos(x)\cdot\sin(2x) = \sin(x)\cdot(\cos^2(x) - \sin^2(x)) + \cos(x)\cdot 2\sin(x)\cos(x) =$

$= -\sin^3(x) + \sin(x)\cos^2(x) + 2\sin\cos^2(x) = -\sin^3(x) + 3\sin(x)(1-\sin^2 x)$

$= -4\sin^3(x) + 3\sin(x)$

$\cos(x+2x) = \cos(x)\cdot\cos(2x) - \sin(x)\cdot\sin(2x) = \cos(x)\cdot(\cos^2(x) - \sin^2(x)) - \sin(x)\cdot 2\sin(x)\cos(x) =$

$\cos^3(x) - \sin^2(x)\cos(x) - 2\sin^2(x)\cos(x) =$

$\cos^3(x) - (1-\cos^2(x))\cos(x) - 2(1-\cos^2(x))\cos(x) =$

$\cos^3(x) + \cos^3(x) - \cos(x) - 2\cos(x) + 2\cos^3(x) = 4\cos^3(x) - 3\cos(x)$

$\sin(2x) = \sin(x+x) = \sin(x)\cos(x) + \cos(x)\sin(x) = 2\sin(x)\cos(x)$

$\cos(2x) = \cos(x+x) = \cos^2(x) - \sin^2(x)$

b) $\sin(3x) = -4\sin^3(x) + 3\sin(x)$

$\cos(3x) = 4\cos^3(x) - 3\cos(x)$

$\sin\left(3\frac{\pi}{3}\right) = -4\sin^3\left(\frac{\pi}{3}\right) + 3\sin\left(\frac{\pi}{3}\right)$

$0 = -4\sin^3\left(\frac{\pi}{3}\right) + 3\sin\left(\frac{\pi}{3}\right)$ $\qquad \sin\frac{\pi}{3} \neq 0$

$0 = -4\sin^2\left(\frac{\pi}{3}\right) + 3$

$\frac{3}{4} = \sin^2\left(\frac{\pi}{3}\right)$

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\cos^2\left(\frac{\pi}{3}\right) = 1 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4}$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

II) $\cos\left(2\cdot\frac{\pi}{6}\right) = 1 - 2\sin^2\left(\frac{\pi}{6}\right)$ $\qquad \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\frac{1}{2} = 1 - 2\sin^2\left(\frac{\pi}{6}\right)$

$\frac{1}{2} = 2\sin^2\left(\frac{\pi}{6}\right)$

$\frac{1}{4} = \sin^2\left(\frac{\pi}{6}\right)$

$\cos\left(\frac{\pi}{6}\right) = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$

III) $\cos\left(2\cdot\frac{\pi}{12}\right) = 1 - 2\sin^2\left(\frac{\pi}{12}\right)$ $\qquad \sin^2\left(\frac{\pi}{12}\right) = \frac{2-\sqrt{3}}{4}$

$\frac{\sqrt{3}}{2} = 1 - 2\sin^2\frac{\pi}{12}$ $\qquad \sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{2-\sqrt{3}}}{2}$

$\frac{\sqrt{3}-2}{2} = -2\sin^2\frac{\pi}{12}$

$$\cos^2 \frac{\pi}{12} = 1 - \left(\frac{\sqrt{2-\sqrt{3}}}{2}\right)^2 = 1 - \frac{2-\sqrt{3}}{4} = \frac{2+\sqrt{3}}{4}$$

$$\cos \frac{\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2}$$