Deckblatt für die Abgabe der Übungsaufgaben
IngMathC2

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Blatt-Nummer: 5

Übungsgruppen-Nr: 7

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A13, A14, ____, ____

10/14 *30 = 21

A13

$$\cosh(x) = \frac{e^x + e^{-x}}{2}, \quad \sinh(x) = \frac{e^x - e^{-x}}{2}, \quad \tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

a)

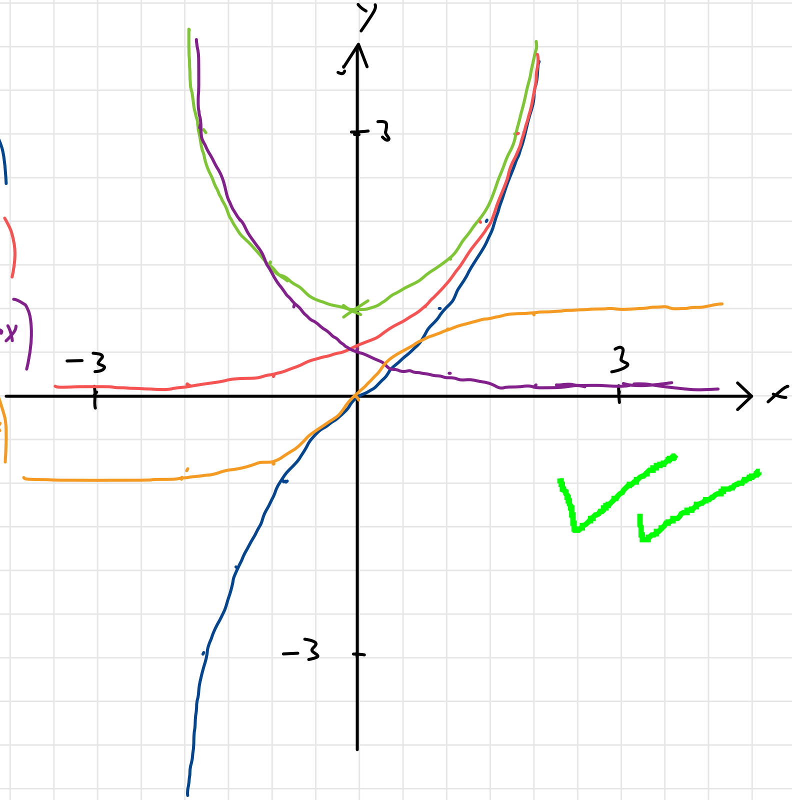
$f(x) = \cosh(x)$

$f(x) = \sinh(x)$

$f(x) = \frac{1}{2}\exp(x)$

$f(x) = \frac{1}{2}\exp(-x)$

$f(x) = \tanh(x)$



b) $\boxed{\lim_{x \to +\infty} \tanh(x)}$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{2(\exp(x) - \exp(-x))}{2(\exp(x) + \exp(-x))} = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)} =$$

$$\lim_{x \to +\infty} \tanh(x) = \lim_{x \to \infty} \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)} = \lim_{x \to \infty} \frac{\exp(x)\left[1 - \frac{\exp(-x)}{\exp(x)}\right]}{\exp(x)\left[1 + \frac{\exp(-x)}{\exp(x)}\right]} =$$

$$= \lim_{x \to \infty} \frac{1 - \frac{\exp(-x)}{\exp(x)}}{1 + \frac{\exp(-x)}{\exp(x)}} = \lim_{x \to \infty} \frac{1 - \frac{1}{\exp(x)^2}}{1 + \frac{1}{\exp(x)^2}} = \frac{1 - \frac{1}{"\infty"}}{1 + \frac{1}{"\infty"}} = 1$$

$$\lim_{x \to -\infty} \tanh(x) = \lim_{x \to -\infty} \frac{\exp(-x)\left[\frac{\exp(x)}{\exp(-x)} - 1\right]}{\exp(-x)\left[\frac{\exp(x)}{\exp(-x)} + 1\right]} = \lim_{x \to -\infty} \frac{\exp(x)^2 - 1}{\exp(x)^2 + 1} = \frac{0 - 1}{0 + 1} = -1$$

c) $\boxed{\cosh^2(x) - \sinh^2(x)}$

$$\cosh^2(x) = \left(\frac{\exp(x) + \exp(-x)}{2}\right)^2 = \frac{\exp(x)^2 + \exp(-x)^2 + 2\exp(x)\exp(-x)}{4} =$$

$$= \frac{\exp(x)^2 + \exp(-x)^2 + 2}{4}$$

$$\sinh(x)^2 = \left(\frac{\exp(x) - \exp(-x)}{2}\right)^2 = \frac{\exp(x)^2 + \exp(-x)^2 - 2\exp(x)\exp(-x)}{4} =$$

$$= \frac{\exp(x)^2 + \exp(-x)^2 - 2}{4}$$

$$\cosh^2(x) - \sinh^2(x) = \frac{1}{4}\left[\cancel{\exp^2(x)} + \cancel{\exp^2(-x)} + 2 - \cancel{\exp^2(x)} - \cancel{\exp^2(-x)} + 2\right] =$$

$$= 1$$

d) $\exp(x) = \sum_{k=0}^{\infty} \frac{1}{k!} x^k$

$$\cosh(x) = \frac{\exp(x) + \exp(-x)}{2} = \frac{\sum_{k=0}^{\infty} \frac{1}{k!} x^k + \sum_{k=0}^{\infty} \frac{1}{k!} (-x)^k}{2} =$$

$$= \frac{\sum_{k=0}^{\infty} \frac{1}{k!} x^k + \sum_{\substack{k=0 \\ k \text{ gerade}}}^{\infty} \frac{1}{k!} x^k - \sum_{\substack{k=1 \\ k \text{ ungerade}}}^{\infty} \frac{1}{k!} x^k}{2} =$$

wie kommst du von oben nach unten.Warum ist di

$$= \sum_{\substack{k=0 \\ k \text{ gerade}}}^{\infty} \frac{1}{k!} x^k \qquad \left(\Leftrightarrow \sum_{k=0}^{\infty} \frac{1}{(2k)!} x^{2k} \right)$$ ✓

$$\sinh(x) = \frac{\exp(x) - \exp(-x)}{2} = \frac{\sum_{k=0}^{\infty} \frac{1}{k!} x^k + \sum_{\substack{k=1 \\ k \text{ ungerade}}}^{\infty} \frac{1}{k!} x^k - \sum_{\substack{k=0 \\ k \text{ gerade}}}^{\infty} \frac{1}{k!} x^k}{2} =$$

$$= \sum_{\substack{k=1 \\ k \text{ ungerade}}}^{\infty} \frac{1}{k!} x^k \qquad \left(\Leftrightarrow \sum_{k=0}^{\infty} \frac{1}{(2n+1)!} x^{2n+1} \right)$$ ✓

e) $\cos(iy)$ und $\sin(iy)$ als Potenzreihe

$$\cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k} \qquad \sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}$$

$$\Rightarrow \cos(iy) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} (iy)^{2k} = 1 - \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k)!} y^{2k} = 1 - \cosh(y)$$

$$\Rightarrow \sin(iy) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} (iy)^{2k+1} = -i \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} y^{2k+1} = -i \sinh(y)$$

selbst wenn diese umformung richtig wäre, sind die sun

f) $\sin(x+y) = \sin x \cos y + \cos x \sin y$

$$\Rightarrow \sin(x+iy) = \sin x \cos(iy) + \cos x \sin(iy) =$$

$$= \sin x (1 - \cosh y) + \cos x (-i \sinh y) =$$

ff

$$= \sin x (1 - \cosh y) - i \cos x \sinh y$$

g) $\sin(x+iy) = \sin x (1 - \cosh y) - i \cos x \sinh y$

mit $x = 0$ folgt: $\sin(iy) = -i \sinh y$

Da $\sinh(y)$ nicht beschränkt ist, ist $i \sinh y$ auch nicht beschränkt und damit auch $\sin(x+yi)$ nicht.

A14

$1-x^2 \overset{!}{\geq} 0$

a) $f(x) = \frac{1-x}{2\sqrt{1-x^2}}$

$1 \geq x^2 \rightarrow x \leq 1$

$x^2 \overset{!}{<} 1$

$D = (-1, 1)$

ges: $\lim_{x \to -1} f(x)$, $\lim_{x \to 1} f(x)$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{1-x}{2\sqrt{1-x^2}} = \lim_{x \to 1^-} \frac{1-x}{2\sqrt{(1+x)(1-x)}} =$$

$$= \lim_{x \to 1^-} \frac{\cancel{(1-x)}\, 2\sqrt{(1+x)(1-x)}}{(1+x)\cancel{(1-x)}} \underset{\lim_{x\to 1^-}}{=} \frac{2\sqrt{(1+x)(1-x)}}{1+x} = \frac{0}{2} = 0$$

für $x \to 1^+$ ist $f(x)$ nicht definiert

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \frac{1-x}{2\sqrt{1-x^2}} = \frac{2}{0} = \infty$$

b) (i) $f(x) = \begin{cases} e^{1+x-\frac{1}{x}} & , x > 0 \\ 0 & , x \leq 0 \end{cases}$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} e^{1+x-\frac{1}{x}} = e^{\text{"}-\infty\text{"}} = 0 = f(0)$$

$$\lim_{x\to 0^-} f(x) = 0 = f(0)$$

$$\hookrightarrow \lim_{x\to 0} f(x) = 0 = f(0) \rightarrow f(x) \text{ ist stetig}$$

(da auch $f(x) = e^x$,

$f(x) = -\frac{1}{x}$,

und $f(x) = x$ stetig sind)

(ii) $g(x) = \begin{cases} e^{1+x-\frac{1}{x}} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

$$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} e^{1+x-\frac{1}{x}} = e^{"-\infty"} = 0$$

$$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} e^{1+x-\frac{1}{x}} = e^{"\infty"} = \infty$$

$g(x)$ ist also nicht stetig, da

$$\infty = \lim_{x\to 0^-} g(x) \neq \lim_{x\to 0^+} g(x) = g(0) = 0$$

c)

(i) $\lim_{x\to 0} \sqrt{x^2+x+1} - x = \sqrt{0+0+1} - 0 = 1$

(ii) $\lim_{x\to\infty} \sqrt{x^2+x+1} - x = \lim_{x\to\infty} = \frac{x^2+x+1-x^2}{\sqrt{x^2+x+1}+x} =$

$= \lim_{x\to\infty} \frac{\cancel{x}\left(1+\frac{1}{x}\right)}{\cancel{x}\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1\right)} = \frac{1+0}{\sqrt{1+0+0}+1} = \frac{1}{2}$

(iii) $\lim_{x\to -\infty} \sqrt{x^2+x+1} - x = \lim_{x\to-\infty} \sqrt{\underbrace{x\underbrace{(x+1)}_{\to -\infty}}_{\to +\infty}+1} \underbrace{-x}_{\to +\infty} =$

$= \infty$

(iv) $\lim_{x\to+\infty} \overbrace{x|\sin \pi x|}^{f(x)}$ hat keinen Grenzwert

$x_n = \frac{1}{2}n \xrightarrow{n\to\infty} \infty \qquad f(x_n) = \frac{1}{2}n\left|\sin\left(\pi\frac{1}{2}n\right)\right| \xrightarrow{n\to\infty} 1$

$\tilde{x}_n = n \xrightarrow{n\to\infty} \infty \qquad f(\tilde{x}_n) = n|\sin(\pi n)| \xrightarrow{n\to\infty} 0$ $\quad ↯$

(v) $\lim_{x\to 0} x|\sin \pi x| = 0\cdot 0 = 0$

(vi) $\lim\limits_{x \to 0} \overbrace{\cos x \cos^2\left(\frac{2}{x}\right)}^{f(x)}$

$$x_n := \frac{1}{2\pi n} \xrightarrow{n \to \infty} 0 \qquad f(x_n) = \cos\frac{1}{2\pi n} \cos^2\left(4\pi n\right) \xrightarrow{n \to \infty} 1$$

$$\tilde{x}_n := \frac{1}{2\pi n + \frac{\pi}{2}} \xrightarrow{n \to \infty} 0 \qquad f(\tilde{x}_n) = \cos\frac{1}{2\pi n + \frac{\pi}{2}} \cos^2\left(2\pi n + \frac{\pi}{2}\right) \xrightarrow{n \to \infty} 0 \quad \lightning$$

$\Rightarrow$ Grenzwert existiert nicht