

A2.1 a) $\lim_{x \rightarrow 0} \frac{\sin(x^2)}{x^3 + 8x^2} \xrightarrow{\text{Hos}} \lim_{x \rightarrow 0} \frac{\cos(x^2) \cdot 2x}{3x^2 + 16x} \xrightarrow{\text{Hos}} \lim_{x \rightarrow 0} \frac{-\sin(x^2) \cdot 2x \cdot 2x + \cos(x^2) \cdot 2}{6x + 16}$

$= \frac{2}{16} = \frac{1}{8}$ ✓ ✓

13.5/22 * 30 = 18

b) $\lim_{x \rightarrow 0} \frac{\sin(x^2)}{x^3 + 8x^2} = \lim_{x \rightarrow 0} \left(\frac{\sin(x^2)}{x^2} \right) \cdot \lim_{x \rightarrow 0} \frac{1}{x+8} = 1 \cdot \frac{1}{8} = \frac{1}{8}$ ✓ ✓

c) $\lim_{x \rightarrow 0} \sqrt{x} \cdot \ln(x) = \lim_{x \rightarrow 0} \frac{\sqrt{x}}{\frac{1}{\ln(x)}} \xrightarrow{\text{Hos}} \lim_{x \rightarrow 0} \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{x}} = \lim_{x \rightarrow 0} \frac{1}{2\sqrt{x}} = +\infty$ ✓

d) $\lim_{x \rightarrow \infty} \sqrt{x} \cdot \ln(x) = +\infty$ ✓ ✓

e) $\lim_{x \rightarrow 0} \frac{\cos(6x) - 1}{x^3 + 2x^2} \xrightarrow{\text{Hos}} \lim_{x \rightarrow 0} \frac{-\sin(6x) \cdot 6}{3x^2 + 4x} \xrightarrow{\text{Hos}} \lim_{x \rightarrow 0} \frac{-\cos(6x) \cdot 6 \cdot 6 + \sin(6x) \cdot 0}{6x + 4} = \frac{-36}{4} = -9$ ✓

f) $\lim_{x \rightarrow 0} \frac{1 - \cos(x) \cos(2x)}{e^{x^2} - 1} \xrightarrow{\text{Hos}} \lim_{x \rightarrow 0} \frac{\sin(x) \cdot \cos(2x) + \cos(x) \sin(2x) \cdot 2}{e^{2x} \cdot 2x}$

$= \lim_{x \rightarrow 0} \frac{\sin(x)}{x} \cdot \lim_{x \rightarrow 0} \frac{\cos(2x)}{2e^{2x}} + \lim_{x \rightarrow 0} \frac{\sin(2x)}{2x} \cdot \lim_{x \rightarrow 0} \frac{2\cos(x)}{e^{x^2} - 1}$

$= 1 \cdot \frac{1}{2} + 1 \cdot 2 = 2.5$ ✓

g) $\lim_{x \rightarrow \infty} \frac{\ln(1+\alpha x)}{\ln(\ln(e^{\beta x} + e^{-\beta x}))} \xrightarrow{\text{Hos}} \lim_{x \rightarrow \infty} \frac{\frac{1}{1+\alpha x} \cdot \alpha}{\frac{1}{\ln(e^{\beta x} + e^{-\beta x})} \cdot \beta \left(\frac{e^{\beta x} - e^{-\beta x}}{e^{\beta x} + e^{-\beta x}} \right)}$ ✓

$= \frac{\alpha}{\beta} \lim_{x \rightarrow \infty} (1+\alpha x)^{-1} \cdot (\ln(e^{\beta x} + e^{-\beta x}))^{-1} \cdot \left(\frac{e^{\beta x} - e^{-\beta x}}{e^{\beta x} + e^{-\beta x}} \right)^{-1}$ ✓

$\xrightarrow{\text{Hos}} \frac{\alpha}{\beta} \lim_{x \rightarrow \infty} \frac{\ln(e^{\beta x} + e^{-\beta x})}{1+\alpha x} \xrightarrow{\text{Hos}} \frac{\frac{1}{e^{\beta x} + e^{-\beta x}} \cdot (\beta \cdot e^{\beta x} - e^{-\beta x})}{\alpha} = \frac{\alpha \cdot \beta}{\beta \cdot \alpha} = 1$ ✓

$$h) \lim_{x \rightarrow \infty} \frac{6\sqrt{x} + 5 + \frac{4}{\sqrt{x}}}{3\sqrt{x} + e^{-2x} + \frac{1}{\sqrt{x}}} = \lim_{x \rightarrow \infty} \frac{\sqrt{x} \left(6 + \frac{5}{\sqrt{x}} + \frac{4}{\sqrt{x^2}} \right)}{\sqrt{x} \left(3 + \frac{e^{-2x}}{\sqrt{x}} + \frac{1}{\sqrt{x^2}} \right)} = \lim_{x \rightarrow \infty} \frac{6 + \frac{5}{\sqrt{x}} + \frac{4}{\sqrt{x^2}}}{3 + \frac{e^{-2x}}{\sqrt{x}} + \frac{1}{\sqrt{x^2}}} \checkmark$$

$$\frac{6}{3} = \underline{\underline{2}} \checkmark$$

A22 a) $f: [-\frac{\pi}{2}, \pi] \rightarrow \mathbb{R} \quad f(x) = \cos x - \cos^2 x$

$$f'(x) = -\sin x + 2\cos(x) \cdot \sin(x)$$

$$0 = (\sin x)(-1 + 2\cos(x))$$

$$0 = \sin x$$

$$f(-\frac{\pi}{2}) = \cos(-\frac{\pi}{2}) - \cos^2(-\frac{\pi}{2}) \quad x = 0$$

$$= 0$$

$$1 = 2\cos(x)$$

$$f(\pi) = \cos(\pi) - \cos^2(\pi)$$

$$= -2$$

$$\frac{1}{2} = \cos(x)$$

$$x = -\frac{\pi}{3}$$

$$f(\frac{\pi}{3}) = \cos(\frac{\pi}{3}) - \cos^2(\frac{\pi}{3})$$

$$= \frac{1}{4}$$

$$f(-\frac{\pi}{3}) = \cos(-\frac{\pi}{3}) - \cos^2(-\frac{\pi}{3})$$

$$= \frac{1}{4}$$

$$\text{glob. min} = -2 \quad f(\pi)$$

$$\text{glob. max} = \frac{1}{4} \quad f(\frac{\pi}{3}) \quad f(-\frac{\pi}{3})$$

$$f(0) = \cos(0) - \cos^2(0)$$

$$= 0$$

b) $f: (0, \infty) \rightarrow \mathbb{R} \quad f(x) = \frac{\ln x}{x}$

$$f'(x) = \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2} = 0$$

$$= \frac{1 - \ln x}{x^2} = 0 \quad 0 = 1 - \ln x \quad x^{-2} = 0 \quad \checkmark$$

$$\ln x = 1$$

$$x = e$$

$$f(e) = \frac{1}{e}$$

$$\lim_{x \rightarrow \infty} \frac{\ln(x)}{x} = -\infty$$

$$\lim_{x \rightarrow \infty} \frac{\ln(x)}{x} \stackrel{\text{Hos}}{\sim} \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = \lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

glob max = $\frac{1}{e} \quad f(e)$
 glob min ist nicht im
 Werte bereich

A23 a)

$$\lim_{n \rightarrow \infty} (2 \sin x)^n$$

$$f = \begin{cases} 0 & \text{wenn } [0, \frac{\pi}{6}) \cup [\frac{5\pi}{6}, \frac{7\pi}{6}] \\ \infty & \text{wenn } (\frac{\pi}{6}, \frac{5\pi}{6}) \\ 1 & \text{wenn } \frac{\pi}{6} \end{cases}$$

$(11\pi/6, 2\pi]$

$f(x)$ hat nie den Wert ∞

$x = 5\pi/6$



eine skizze soll schon sowas wie einheiten habe

b) nein für M_1 ✓

~~Ja für M_2~~

~~Ja für M_3~~

Ja für M_4 ✓