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Studon: em89inym Blatt: 4

Übung: Gruppe 7 (Mi 12-14 Uhr)

Freigegebene Aufgaben: A12, A10, A11 7/10*30=21

A12 a) i)

$\exp(3ix) = \exp(ix)^3$

$\cos(3x) + i\sin(3x) = (\cos(x) + i\sin(x))^3 =$

$= (\cos(x)^2 + i\cos(x)\sin(x) - \sin(x)^2)(\cos(x) + i\sin(x)) =$

$= \cos(x)^3 + 3i\cos(x)^2\sin(x) - 3\sin(x)^2\cos(x) - i\sin(x)^3$

Re: $\cos(3x) = \cos(x)^3 - 3\sin(x^2)\cos(x) \quad / \cos(x)^2 = 1 - \sin^2 x$

$\cos(3x) = \cos(x)^3 - 3(1 - \cos(x)^2)\cos(x)$

$\cos(3x) = \cos(x)^3 - 3\cos(x) + 3\cos(x)^3$

$\underline{\cos(3x) = 4\cos(x)^3 - 3\cos(x)}$

Im: $\sin(3x) = 3\cos(x)^2\sin(x) - \sin(x)^3 \quad / \cos(x)^2 = 1 - \sin^2 x$

$\sin(3x) = 3(1 - \sin(x)^2)\sin(x) - \sin(x)^3$

$\sin(3x) = 3\sin(x) - 3\sin(x)^3 - \sin x^3$

$\underline{\sin(3x) = 3\sin(x) - 4\sin(x)^3}$

ii)

$\sin(3x) = \sin(2x + x) = \sin(2x)\cos(x) + \cos(2x)\sin(x) =$

$= \sin(x+x)\cos(x) + \cos(x+x)\sin(x) =$

$= [(\sin(x)\cos(x) + \cos(x)\sin(x))\cos(x)] + [(\cos(x)\cos(x) - \sin(x)\sin(x))\sin(x)]$

$= 2\sin(x)\cos(x)^2 + \cos(x)^2\sin(x) - \sin(x)^3$

$= 3\sin(x)\cos(x)^2 - \sin(x)^3 \quad / \cos(x)^2 = 1 - \sin^2 x$

$= 3\sin(x)(1 - \sin(x)^2) - \sin(x)^3 = \underline{3\sin(x) - 4\sin(x)^3}$

$\cos(3x) = \cos(2x)\cos(x) - \sin(2x)\sin(x) =$

$= [(\cos(x)^2 - \sin(x)^2)\cos(x)] - [(2\sin(x)\cos(x))\sin(x)] =$

$= \cos(x)^3 - \sin(x)^2\cos(x) - 2\sin(x)^2\cos(x) \quad / \cos(x)^2 = 1 - \sin^2 x$

$= \cos(x)^3 - 3(1 - \cos(x)^2)\cos(x) = \underline{4\cos(x)^3 - 3\cos(x)}$

b) i) $\sin(3x) = 3\sin(x) - 4\sin(x)^3 \quad x = \frac{\pi}{3}$

$\sin(3\frac{\pi}{3}) = 3\sin(\frac{\pi}{3}) - 4\sin(\frac{\pi}{3})^3$

$[\sin(\pi) = 0] = 3\sin(\frac{\pi}{3}) - 4\cdot\sin(\frac{\pi}{3})\cdot\sin(\frac{\pi}{3})\cdot\sin(\frac{\pi}{3}) \;/ \div \sin(\frac{\pi}{3})$

da $\sin(\frac{\pi}{3}) \neq 0$

$0 = 3 - 4\sin(\frac{\pi}{3})^2$

$4\sin(\frac{\pi}{3})^2 = 3 \Rightarrow \sin(\frac{\pi}{3})^2 = \frac{3}{4} \Rightarrow \sin(\frac{\pi}{3}) = \sqrt{\frac{3}{4}} = \underline{\underline{\frac{\sqrt{3}}{2}}}$

$\sin(x)^2 = 1 - \cos(x)^2$

$\cos(x) = \sqrt{1 - \sin(x)^2} \Rightarrow \cos(\frac{\pi}{3}) = \sqrt{1 - \sin(\frac{\pi}{3})^2} = \sqrt{\frac{1}{4}} = \underline{\underline{\frac{1}{2}}}$

ii) $\cos(2x) = 1 - 2\sin^2 x$

$\cos(2\frac{\pi}{6}) = 1 - 2\sin(\frac{\pi}{6})^2 \Rightarrow \sin(\frac{\pi}{6}) = \sqrt{\frac{1}{2}(1 - \cos(\frac{\pi}{3}))} = \sqrt{\frac{1}{2}(1 - \frac{1}{2})} = \underline{\underline{\frac{1}{2}}}$

$\cos(\frac{\pi}{6}) = \sqrt{1 - \sin(\frac{\pi}{6})^2} = \sqrt{\frac{3}{4}} = \underline{\underline{\frac{\sqrt{3}}{2}}}$

iii) $\cos(2\frac{\pi}{12}) = 1 - 2\sin(\frac{\pi}{12})^2 \Rightarrow \sin(\frac{\pi}{12}) = \sqrt{\frac{1}{2}(1 - \cos(\frac{\pi}{6})} = \sqrt{\frac{2 - \sqrt{3}}{4}} =$

$= \frac{\sqrt{2 - \sqrt{3}}}{\sqrt{4}} = \underline{\underline{\frac{\sqrt{6} - \sqrt{2}}{4}}}$

$\cos(\frac{\pi}{12}) = \sqrt{1 - \sin(\frac{\pi}{12})^2} = \sqrt{\frac{2 + \sqrt{3}}{4}} = \underline{\underline{\frac{\sqrt{6} + \sqrt{2}}{4}}}$

A10 a) i) $\sum_{k=0}^{\infty} kq^k \cdot \sum_{k=0}^{\infty} q^k = \sum_{n=0}^{\infty}\sum_{k=0}^{n} kq^k \cdot q^{n-k} = \sum_{n=0}^{\infty}\sum_{k=0}^{n} k \cdot q^n =$

$= \sum_{n=0}^{\infty} q^n \cdot \sum_{k=0}^{n} k = \sum_{n=0}^{\infty} q^n \cdot (0 + \frac{n^2+n}{2}) =$

Gaußsche Summenformel: $\sum_{k=1}^{n} k = \frac{n^2+n}{2}$

$= \frac{1}{2}\sum_{n=0}^{\infty} q^n \cdot (n^2+n)$

ii) $\sum_{k=0}^{\infty} kq^k \cdot \sum_{k=0}^{\infty} q^k = \frac{1}{2}\sum_{n=0}^{\infty} q^n \cdot (n^2+n)$

$\frac{q}{(1-q)^2} \cdot \frac{1}{(1-q)} = \frac{1}{2}\sum_{n=0}^{\infty} n^2q^n + nq^n \quad \Big| - \sum_{n=0}^{\infty} nq^n$

$\frac{q}{(1-q)^3} - \frac{q}{(1-q)^2} = \frac{1}{2}\sum_{n=0}^{\infty} n^2q^n$

$$\sum_{n=0}^{\infty} n^2 q^n = 2\cdot\left(\frac{q}{(1-q)^3} - \frac{q(1-q)}{(1-q)^3}\right) = 2\cdot\frac{q-(q-q^2)}{(1-q)^3} = \frac{2q^2}{(1-q)^3}$$

b) $\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+2)}$

$$p_n = \sum_{k=0}^{n} \frac{1}{k+1} - \frac{1}{k+2} = \sum_{k=0}^{n} \frac{1}{k+1} - \sum_{k=0}^{n} \frac{1}{k+2}$$

$$k+1 = \hat{k}+2 \Rightarrow \hat{k} = k-1$$

$$\sum_{\hat{k}=-1}^{n-1} \frac{1}{\hat{k}+2} - \sum_{k=0}^{n} \frac{1}{k+2} = \frac{1}{1} - \frac{1}{n+2} \xrightarrow{n\to\infty} 1$$

A11 a)

i) $\sqrt[k]{|a_k|} = \sqrt[k]{\left|\frac{5^k}{k}\right|} = \frac{\sqrt[k]{5^k}}{\sqrt[k]{k}} = \frac{5}{\sqrt[k]{k}} \xrightarrow{k\to\infty} \frac{5}{1} = 5 \quad R = \frac{1}{5}$

ii) $\sqrt[k]{|a_k|} = \sqrt[k]{\left|\left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^{2k}\right|} = \left|\left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^2\right|$

$\xrightarrow{k\to\infty} \left(\frac{1}{2}\right)^2 = \frac{1}{4} \quad R = \frac{1}{\frac{1}{4}} = 4$

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iii) $\sqrt[k]{|k!+2|} \geq \sqrt[k]{k!} \xrightarrow{k\to\infty} \infty \quad R = "\frac{1}{\infty}" = 0$

iv) $\sum_{k=0}^{\infty} \frac{2^k}{k^2} x^{4k}$, $\sum_{\tilde{k}=0}^{\infty} a_{\tilde{k}} \lambda^{\tilde{k}}$ mit $k = \frac{1}{4}\tilde{k}$

und $a_{\tilde{k}} = \begin{cases} 0 & \tilde{k} \text{ nicht durch 4 teilbar} \\ \frac{2^{\frac{1}{4}\tilde{k}}}{(\frac{1}{4}\tilde{k})^2} & \tilde{k} \text{ durch 4 teilbar} \end{cases}$

$$\sqrt[\tilde{k}]{|a_{\tilde{k}}|} = \begin{cases} 0 & , \tilde{k} \text{ nicht durch 4 teibar} \\ \frac{\sqrt[\tilde{k}]{2^{\frac{1}{4}\tilde{k}}}}{\sqrt[\tilde{k}]{(\frac{1}{4}\tilde{k})^2}} & , \tilde{k} \text{ durch 4 teibar} \end{cases} = \begin{cases} 0 \\ \frac{2^{\frac{1}{4}} \rightarrow \text{const}}{\underbrace{\sqrt[\tilde{k}]{\frac{1}{16}}}_{\Downarrow\, 1} \underbrace{\sqrt[\tilde{k}]{k^2}}_{\Downarrow\, \cancel{\infty}\ 1}} \end{cases} = \begin{cases} 0 \\ 0 \end{cases}$$

$$\Rightarrow R = "\frac{1}{0}" \Rightarrow \infty$$

(k^2)^1/k = k^1/k * k^1/k = 1*1 = 1

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b) $S_{(x)} := \sum_{k=0}^{\infty} \left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1\right)^k \left(\frac{1}{x+3}\right)^k \quad y = \frac{1}{x+3} \quad (*)$

$$\sqrt[k]{\left|\left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1\right)^k\right|} = \sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1 =$$

$$\underbrace{\sqrt[k]{3}}_{\downarrow\, 1} \cdot \underbrace{\sqrt[k]{k}}_{\downarrow\, 1} + \frac{4}{\underbrace{\sqrt[k]{k!}}_{\downarrow\, \infty}} + 1 \xrightarrow{k \to \infty} 1 \cdot 1 + \text{"}\frac{4}{\infty}\text{"} + 1 = 2$$

Die Reihe mit (*) hat $R = \frac{1}{2}$ d.h. Konvergent für $|y| < \frac{1}{2}$

Für $\boxed{|x| > -1 \text{ ist } |y| < \frac{1}{|x|+3} < \frac{1}{4}}$ d.h. die geg. Reihe ist konvergent

$\Rightarrow$ Die Reihe ist konvergent in den Intervallen $(-\infty, 0) \cup (0, +\infty)$

|y|< 1/2 ==> |1/(x+3)| < 1/2 ==> |x+3|> 2==>x<-5 oder x>-1