Deckblatt für die Abgabe der Übungsaufgaben
IngMathC1

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Blatt-Nummer: 04

Übungsgruppen-Nr: 07

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A10, A11, A12,

5/10*30 = 15

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A10) a)

i)

$$\sum_{k=0}^{\infty} k\cdot q^k \cdot \sum_{k=0}^{\infty} q^k \quad \text{für } |q|<1$$

$$= \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} k\cdot q^n\cdot q^{n-k}\right) = \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} q^n\cdot k\right)$$

$$= \sum_{n=0}^{\infty}(n+1)\cdot q^n\cdot\sum_{k=0}^{n} k = \left(\frac{1}{(1-q)^2}-\frac{1}{1-q}\right)\cdot\frac{1}{1-q}$$

$$= \frac{1}{(1-q)^3}-\frac{1}{(1-q)^2} = \frac{q}{(1-q)^3}$$

ii)

$$\sum_{k=0}^{\infty} k\cdot q^k\cdot k = \frac{q}{(1-q)^3}\cdot\sum_{k=0}^{\infty} k = \frac{q}{(1-q)^3}\cdot\frac{1}{1-q} = \frac{q}{(1-q)^4}$$

b)

$$\sum_{k=0}^{\infty}\frac{1}{(k+1)(k+2)}$$

$$\frac{1}{(k+1)(k+2)} = \frac{A}{k+1}+\frac{B}{k+2} = \frac{A(k+2)+B(k+1)}{(k+2)(k+1)}$$

$$= \frac{Ak+2A+Bk+B}{(k+1)(k+2)} = \frac{k(A+B)+2A+B}{(k+1)(k+2)}$$

$$\Rightarrow \left\{\begin{matrix} A+B=0 \\ 2A+B=1 \end{matrix}\right\} \begin{matrix} B=-A \\ 2A-A=1 \end{matrix} \Rightarrow \begin{matrix} B=-1 \\ A=1 \end{matrix}$$

$$\sum_{k=0}^{n}\frac{1}{(k+1)(k+2)} = \sum_{k=0}^{n}\left(\frac{1}{k+1}+\frac{-1}{k+2}\right)$$

$$=\left(\sum_{k=0}^{n}\frac{1}{k+1}\right)-\left(\sum_{k=0}^{n}\frac{1}{k+2}\right)$$

$$=\sum_{k=0}^{n}\frac{1}{k+1}-\sum_{k=1}^{n+1}\frac{1}{k+1}$$

$$=\frac{1}{0+1}-\frac{1}{(n+1)+1}$$

$$=1-\frac{1}{n+2}\xrightarrow{n\to\infty}$$

$$\sum_{k=0}^{\infty}\frac{1}{(k+1)(k+2)} = \lim\sum_{k=0}^{n}\frac{1}{(k+1)(k+2)} = \lim_{n\to\infty} 1-\frac{1}{n+2} = 1$$

A11

a) i) $\sum_{k=0}^{\infty} \underbrace{\frac{5^k}{k}}_{a_k} x^k$

$$\Rightarrow \sqrt[k]{|a_k|} = \sqrt[k]{\left|\frac{5^k}{k}\right|} = \frac{\sqrt[k]{5^k}}{\underbrace{\sqrt[k]{k}}_{\to 1}} = \frac{5}{1} \xrightarrow{k\to\infty} 5 \Rightarrow R = \frac{1}{5}$$

ii) $\sum_{k=0}^{\infty} (\underbrace{\sqrt{k+1} - \sqrt{k-\sqrt{k}}}_{a_k})^{2k} x^k \Rightarrow \sqrt[k]{|a_k|} = \sqrt[k]{|(\sqrt{k+1} - \sqrt{k-\sqrt{k}})|^{2k}}$

3.Binom. Formel ist:(a-b)

$$= (\sqrt{k+1} - \sqrt{k-\sqrt{k}})^2$$

$$= \frac{((\sqrt{k+1} - \sqrt{k-\sqrt{k}})\cdot(\sqrt{k+1} + \sqrt{k-\sqrt{k}}))^2}{(\sqrt{k+1} + \sqrt{k-\sqrt{k}})^2} = \frac{(\overbrace{k+1-(k-\sqrt{k})}^{1+\sqrt{k}})^2}{(\sqrt{k+1} + \sqrt{k-\sqrt{k}})^2}$$

$\cancel{k}+1-\cancel{k}+\sqrt{k}$

$$= \frac{1+2\sqrt{k}+k}{k+1+2\sqrt{k+1}\cdot\sqrt{k-\sqrt{k}}+k-\sqrt{k}} = \frac{1+2\sqrt{k}+k}{2k+1-\sqrt{k}+2\sqrt{k+1}\cdot\sqrt{k-\sqrt{k}}}$$

ff

$$= \frac{\cancel{k}(\overbrace{\underbrace{\tfrac{1}{k}+2\sqrt{\tfrac{1}{k}}}_{\to 0}+1}^{\to 1})}{\cancel{k}(\underbrace{2+\underbrace{\tfrac{1}{k}-\sqrt{\tfrac{1}{k}}+2\sqrt{\tfrac{1}{k}+\tfrac{1}{k^2}}\cdot\sqrt{\tfrac{1}{k}-\sqrt{\tfrac{1}{k^3}}}}_{\to 0}}_{\to 2})} \xrightarrow{k\to\infty} \frac{1}{2} \qquad R = \frac{1}{\frac{1}{2}} = 2$$

iii) $\sum_{k=0}^{\infty} (k!+2)x^k$

Beweis

$$\sqrt[k]{|a_k|} = \sqrt[k]{|(k!+2)|} \xrightarrow{k\to\infty} \infty$$

$R =$ „$\frac{1}{\infty}$" $= 0$

iv) $\sum_{k=0}^{\infty} \frac{2^k}{k^2} x^{4k}$

$$y = x^4 \leadsto \sum_{k=1}^{\infty} \frac{2^k}{k^2} \cdot y^k$$

$$\sqrt[k]{|a_k|} = \sqrt[k]{\left|\frac{2^k}{k^2}\right|} = \frac{2}{\underbrace{\sqrt[k]{k^2}}_{\to 1}} = 2$$

„Re-substitution" $\Rightarrow R = \sqrt[4]{\frac{1}{2}}$

b)

$$s(x) := \sum_{k=0}^{\infty} \left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1 \right)^k \left(\frac{1}{x+3} \right)^k \Rightarrow \left(\frac{1}{x+3} \right)^k \Rightarrow (x^{-3})^k = x^{-3k}$$

????

$$y = x^{-3}$$

$$\sqrt[k]{|a_k|} = \left| \left(\underbrace{\sqrt[k]{3k}}_{\to 1} + \frac{4}{\underbrace{\underbrace{\sqrt[k]{k!}}_{\to\infty}}_{\to 0}} + \underbrace{1}_{\to 1} \right) \right| \xrightarrow{k\to\infty} 2 \qquad \text{Resubstitution: } R = \sqrt[\frac{1}{3}]{2} = 8$$

Σ ist konvergent für $-8 < a < 8$

Es war die Form:(-infty, a) U (b,inft

A12)

a) i) $\exp(3ix) = \exp(ix)^3$

$$\Leftrightarrow \cos(3x) + i\sin(3x) = \exp(ix) \cdot \exp(ix) \cdot \exp(ix) \qquad \exp(3x) =$$

$$\Leftrightarrow \quad '' \quad = \exp(ix + ix + ix)$$

$$\Leftrightarrow \quad '' \quad = \exp(3ix)$$

$$\Leftrightarrow \quad '' \quad = \cos(3x) + i\sin(3x)$$

ii) $\sin(\alpha+\beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

$$\sin(x+2x) = \sin(x)\cos(2x) + \cos(x)\sin(2x)$$

$$= \sin(x)\,(\cos^2(x) - \sin^2(x)) + \cos(x) \cdot 2\sin(x)\cos(x)$$

$$\cos(x+2x) = \cos(x)\cos(2x) - \sin(x)\sin(2x)$$

$$= -\sin(x)\,(\sin(x)\cos(x) + \cos(x)\sin(x)) + \cos\,(\cos^2(x) - \sin^2(x))$$

b) i) $\sin(\pi) = 0$

$$\sin\left(\frac{\pi}{3}\right)\left(3\cos^2\frac{\pi}{3} - \sin^2\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)\left(3 - 4\sin^2\frac{\pi}{3}\right)$$

$$\Rightarrow \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \Rightarrow \cos\frac{\pi}{3} = \frac{1}{2}$$

ii) $\frac{1}{2} = \cos\left(\frac{\pi}{3}\right) = 1 - 2\sin^2\left(\frac{\pi}{6}\right)$

$$\Rightarrow \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{6}\right) = \left(1 - \sin^2\left(\frac{\pi}{6}\right)\right)^{\frac{1}{2}} = \frac{\sqrt{3}}{2}$$

iii) $\frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right) = 1 - 2\sin^2\left(\frac{\pi}{12}\right)$

$$\Rightarrow \sin\left(\frac{\pi}{12}\right) = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{1}{2}\sqrt{2-\sqrt{3}}$$

$$\cos\left(\frac{\pi}{12}\right) = \left(1 - \sin^2\left(\frac{\pi}{12}\right)\right)^{\frac{1}{2}} = \left(\frac{3}{4}\right)^{\frac{1}{2}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$$