Deckblatt für die Abgabe der Übungsaufgaben IngMathC2

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Blatt-Nummer: 8

Übungsgruppen-Nr: 7

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A21, A22, ____, ____

19.5/23 * 23 = 19.5

Aufgabenblatt 8:

$\boxed{A21}$ a) $\lim\limits_{x\to 0} \frac{\sin(x^2)}{x^3+8x^2}$, Typ $\frac{"0"}{"0"}$

$\overset{l'H}{=} \lim\limits_{x\to 0} \frac{2x \cdot \cos(x^2)}{3x^2+16x}$, Typ $\frac{"0"}{"0"}$

$\overset{l'H}{=} \lim\limits_{x\to 0} \frac{2\cdot\cos(x^2) + 2x(-\sin(x^2))\cdot 2x}{6x+16} = \lim\limits_{x\to 0} \frac{2\cos(x^2)+4x^2(-\sin(x^2))}{6x+16} =$

$= \frac{2\cdot 1+0}{0+16} = \frac{2}{16} = \underline{\underline{\frac{1}{8}}}$ ✓

b) $\lim\limits_{x\to 0} \frac{\sin(x)}{x} = 1$

Substitution: $y = x^2$

$\lim\limits_{y\to 0} \frac{\sin(y)}{y\cdot\sqrt{y}+8y} = \lim\limits_{y\to 0}\left(\left(\frac{1}{\sqrt{y}+8}\right)\left(\frac{\sin(y)}{y}\right)\right) = \frac{1}{8}\cdot\lim\limits_{y\to 0}\frac{\sin(y)}{y} = \frac{1}{8}\cdot 1$ ✓

also: $\lim\limits_{x\to 0}\frac{\sin(x^2)}{x^3+8x^2} = \lim\limits_{x\to 0}\left(\frac{1}{x+8}\cdot\frac{\sin(x^2)}{x^2}\right) = \lim\limits_{x\to 0}\frac{1}{x+8}\cdot\lim\limits_{x\to 0}\frac{\sin(x^2)}{x^2} = \frac{1}{8}\cdot 1 = \underline{\underline{\frac{1}{8}}}$

c) $\lim\limits_{x\to 0}\sqrt{x}\ln x$, Typ „$0\cdot(-\infty)$"

$= \lim\limits_{x\to 0}\frac{\sqrt{x}}{\frac{1}{\ln x}} \overset{l'H}{=} \lim\limits_{x\to 0}\frac{\frac{1}{2\sqrt{x}}}{\frac{1}{x}} = \lim\limits_{x\to 0}\frac{x}{2\sqrt{x}} \overset{l'H}{=} \frac{1}{\frac{1}{\sqrt{x}}} = \lim\limits_{x\to 0}\sqrt{x} = \underline{\underline{0}}$ ✓

d) $\lim\limits_{x\to\infty}\sqrt{x}\cdot\ln x = \lim\limits_{x\to\infty}\sqrt{x}\cdot\lim\limits_{x\to\infty}\ln x = \infty\cdot\infty = \infty$ ✓

e) $\lim\limits_{x\to 0}\frac{\cos(6x)-1}{x^3+2x^2}$, Typ $\frac{„0"}{„0"}$

$\overset{l'H}{=} \lim\limits_{x\to 0}\frac{-\sin(6x)\cdot 6}{3x^2+4x} \overset{l'H}{=} \lim\limits_{x\to 0}\frac{(-6)\cos(6x)\cdot 6}{6x+4} = \frac{\cos(0)\cdot(-36)}{6\cdot 0+4}$ ✓ $= -\frac{36}{4} = \underline{\underline{-9}}$ ✓

f) $\lim\limits_{x\to 0}\frac{1-\cos(x)\cos(2x)}{e^{x^2}-1}$, Typ $\frac{„0"}{„0"}$

$\overset{l'H}{=} \lim\limits_{x\to 0}\frac{-\sin(x)\cdot(-\cos(2x)) - \cos(x)\cdot\sin(2x)\cdot 2}{e^{x^2}\cdot 2x} =$

$\overset{l'H}{=} \lim\limits_{x\to 0}\frac{-\cos(x)\cdot(-\cos(2x))2 + \sin(x)\sin(2x)\cdot 2 - (-\sin(x)\cdot\sin(2x)\cdot 2 + \cos(x)\cdot\cos(x)\cdot 4)}{e^{x^2}\cdot 2x + e^{x^2}\cdot 2}$

$= \frac{-1\cdot(-1)2 + 0 - (0+4)}{1\cdot 0 + 1\cdot 2} = \frac{-1-4}{2} = \cancel{-3}\ -2{,}5$ ff ✓

g) $\lim\limits_{x\to\infty} \dfrac{\ln(1+\alpha x)}{\ln(\ln(e^{\beta x}+e^{-\beta x}))}$, $\alpha, \beta \in \mathbb{R}^+$, Typ $\frac{"\infty"}{"\infty"}$

$$\overset{l'H}{=} \lim_{x\to\infty} \frac{(1+\alpha x)^{-1}\cdot \alpha}{(\ln(e^{\beta x}+e^{-\beta x}))^{-1}\cdot(e^{\beta x}+e^{-\beta x})^{-1}\cdot(e^{\beta x}\cdot\beta+e^{-\beta x}\cdot(-\beta))} =$$

$$= \lim_{x\to\infty} \frac{\alpha\,(\ln(e^{\beta x}+e^{-\beta x}))\cdot(e^{\beta x}+e^{-\beta x})}{(1+\alpha x)\,(\beta e^{\beta x}-\beta e^{-\beta x})} = \frac{\alpha}{\beta}\lim_{x\to\infty} \frac{\ln(e^{\beta x}+e^{-\beta x})\,(e^{\beta x}+e^{-\beta x})}{(1+\alpha x)\,(e^{\beta x}-e^{-\beta x})}$$

h) $\lim\limits_{x\to\infty} \dfrac{6\sqrt{x}+5+\frac{4}{\sqrt{x}}}{3\sqrt{x}+e^{-2x}+\frac{1}{\sqrt{x}}}$, Typ $\frac{"\infty"}{"\infty"}$

$$\overset{l'H}{=} \lim_{x\to\infty} \frac{3x^{-\frac{1}{2}}-2x^{-\frac{3}{2}}}{\frac{3}{2}x^{-\frac{1}{2}}-2e^{-2x}-\frac{1}{2}x^{-\frac{3}{2}}} \overset{l'H}{=} \lim_{x\to\infty} \frac{-\frac{3}{2}x^{-\frac{3}{2}}+3x^{-\frac{5}{2}}}{-\frac{3}{4}x^{-\frac{3}{2}}+4e^{-2x}+\frac{3}{4}x^{-\frac{5}{2}}} = \ldots\ldots$$

Einschachteln, ab $x > 1$ gilt

$$\frac{6\sqrt[3]{x}+5+\frac{4}{\sqrt[3]{x}}}{3\sqrt[3]{x}+e^{-2x}+\frac{1}{\sqrt[3]{x}}} < \frac{6\sqrt{x}+5+\frac{4}{\sqrt{x}}}{3\sqrt{x}+e^{-2x}+\frac{1}{\sqrt{x}}} < \frac{6x+5+\frac{4}{x}}{3x+e^{-2x}+\frac{1}{x}}$$

das geht auch einfacher: sqrt(x) kürz

$$\lim_{x\to\infty} \frac{6\sqrt[3]{x}+5+\frac{4}{\sqrt[3]{x}}}{3\sqrt[3]{x}+e^{-2x}+\frac{1}{\sqrt[3]{x}}} =$$

$$= \lim_{x\to\infty} \frac{6+\frac{5}{\sqrt[3]{x}}+\frac{4}{\sqrt[3]{x^2}}}{3+\frac{e^{-2x}}{\sqrt[3]{x}}+\frac{1}{\sqrt[3]{x^2}}} =$$

$$= \frac{6+0+0}{3+0+0} = \frac{6}{3} = 2$$

$$\lim_{x\to\infty} \frac{6x+5+\frac{4}{x}}{3x+e^{-2x}+\frac{4}{x}} =$$

$$= \lim_{x\to\infty} \frac{6+\frac{5}{x}+\frac{4}{x^2}}{3+\frac{e^{-2x}}{x}+\frac{1}{x^2}} =$$

$$= \frac{6+0+0}{3+0+0} = 2$$

$$\rightarrow \lim_{x\to\infty} \frac{6\sqrt[3]{x}+5+\frac{4}{\sqrt[3]{x}}}{3\sqrt[3]{x}+e^{-2x}+\frac{1}{\sqrt[3]{x}}} = \lim_{x\to\infty} \frac{6x+5+\frac{4}{x}}{3x+e^{-2x}+\frac{1}{x}} = \lim_{x\to\infty} \frac{6\sqrt{x}+5+\frac{4}{\sqrt{x}}}{3\sqrt{x}+e^{-2x}+\frac{1}{\sqrt{x}}} = 2$$

A22

a) $f: [-\frac{\pi}{2}, \pi] \to \mathbb{R}$ $f(x) = \cos x - \cos^2 x$

$f'(x) = -\sin x - 2\cos x \cdot (-\sin x)$

$-\sin x - 2\cos x \cdot (-\sin x) = 0$

$1 = 2 \cos x$

$x_1 = \frac{\pi}{3}$ in $f: [\frac{-\pi}{2}, \pi]$

$x_2 = \frac{-\pi}{3}$ in $f: [\frac{-\pi}{2}, \pi]$

$\to f(\frac{-\pi}{2}) = 0 - 0^2 = 0$

$f(\frac{-\pi}{3}) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$f(\frac{\pi}{3}) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

Maxima in $\{(\frac{-\pi}{3}, \frac{1}{4}), (\frac{\pi}{3}, \frac{1}{4})\}$

$f(\pi) = -1 - (-1)^2 = -2$ $\to$ Minimum in $(\pi, -2)$

b) $f: (0, \infty)$, $f(x) = \frac{\ln x}{x}$

$f'(x) = \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$

$\frac{1 - \ln x}{x^2} = 0$

$1 = \ln x$

$x = e$

$\to \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\ln x}{x} \overset{l'H}{=} \lim_{x \to 0} \frac{1}{x} = +\infty$

$f(e) = \frac{1}{e} = e^{-1}$

$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{\ln x}{x} \overset{l'H}{=} \lim_{x \to \infty} \frac{1}{x} = 0$

~~$\to$ Maximum am linken Rand von $f(x)$~~

~~Minimum am rechten Rand von $f(x)$~~

maximum bei 1/eDer Rand ist ausgeschlossen, deshalb gibt es nur ein