14.5/22 *30 = 19.5

A21 a) $\lim_{x\to0}\frac{\sin(x^2)}{x^3+8x^2} \overset{Hos}{\Rightarrow} \lim_{x\to0}\frac{\cos(x^2)\cdot2x}{3x^2+16x} \overset{Hos}{\Rightarrow} \lim_{x\to0}\frac{-\sin(x^2)\cdot2x\cdot2x+\cos(x^2)\cdot2}{6x+16}$

$=\frac{2}{16}=\frac{1}{8}$

b) $\lim_{x\to0}\frac{\sin(x^2)}{x^3+8x^2} = \lim_{x\to0}\left(\frac{\sin(x^2)}{x^2}\right)\cdot\lim_{x\to0}\frac{1}{x+8} = 1\cdot\frac{1}{8}=\frac{1}{8}$

c) $\lim_{x\to0}\sqrt{x}\cdot\ln(x) = \lim_{x\to0}\frac{\sqrt{x}}{\frac{1}{\ln(x)}} \overset{Hos}{\Rightarrow} \lim_{x\to0}\frac{\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{\frac{1}{x}}} = \lim_{x\to0}\frac{1}{2x\sqrt{x}} = +\infty$

d) $\lim_{x\to\infty}\sqrt{x}\cdot\ln(x) = +\infty$

e) $\lim_{x\to0}\frac{\cos(6x)-1}{x^3+2x^2} \overset{hos}{\Rightarrow} \lim_{x\to0}\frac{-\sin(6x)6}{3x^2+4x} \overset{Hos}{\Rightarrow} \lim_{x\to0}\frac{-\cos(6x)\cdot6\cdot6+\sin(6x)\cdot0}{6x+4} = \frac{36}{4}$

$=-9$

f) $\lim_{x\to0}\frac{1-\cos(x)\cos(2x)}{e^{x^2}-1} \overset{Hos}{\Rightarrow} \lim_{x\to0}\frac{\sin(x)\cdot\cos(2x)+\cos(x)\sin(2x)\cdot2}{e^{x^2}\cdot2x}$

$=\lim_{x\to0}\frac{\sin(x)}{x}\cdot\lim_{x\to0}\frac{\cos(2x)}{2e^{x^2}}+\lim_{x\to0}\frac{\sin(2x)}{2x}\cdot\lim_{x\to0}\frac{2\cos(x)}{e^{x^2}}$

$=1\cdot\frac{1}{2}+1\cdot2=2{,}5$

g) $\lim_{x\to\infty}\frac{\ln(1+\alpha x)}{\ln(\ln(e^{\beta x}+e^{-\beta x}))} \overset{Hos}{\Rightarrow} \lim_{x\to\infty}\frac{\frac{1}{1+\alpha x}\cdot\alpha}{\frac{1}{\ln(e^{\beta x}+e^{-\beta x})}\cdot\beta\left(\frac{e^{\beta x}-e^{-\beta x}}{e^{\beta x}+e^{-\beta x}}\right)}$

$=\frac{\alpha}{\beta}\lim_{x\to\infty}(1+\alpha x)^{-1}\cdot(\ln(e^{\beta x}+e^{-\beta x}))^{\cancel{-1}}\cdot\left(\frac{e^{\beta x}-e^{-\beta x}}{e^{\beta x}+e^{-\beta x}}\right)^{-1}$

$=\frac{\alpha}{\beta}\lim_{x\to\infty}\frac{\ln(e^{\beta x}+e^{-\beta x})}{1+\alpha x} \overset{Hos}{\Rightarrow} \frac{\alpha}{\beta}\frac{\frac{1}{(e^{\beta x}+e^{-\beta x})}(e^{\beta x}\cdot\beta-e^{-\beta x}\cdot\beta)}{\alpha}=\frac{\alpha\cdot\beta}{\beta}\cdot\frac{1}{\alpha}=1$

h) $\lim_{x\to\infty} \frac{6\sqrt{x}+5+\frac{4}{\sqrt{x}}}{3\sqrt{x}+e^{-2x}+\frac{1}{\sqrt{x}}} = \lim_{x\to\infty} \frac{\cancel{\sqrt{x}}\left(6+\frac{5}{\sqrt{x}}+\frac{4}{\sqrt{x}^2}\right)}{\cancel{\sqrt{x}}\left(3+\frac{e^{-2x}}{\sqrt{x}}+\frac{1}{\sqrt{x}^2}\right)} = \lim_{x\to\infty} \frac{\overbrace{6+\frac{5}{\sqrt{x}}+\frac{4}{\sqrt{x}}}^{\to 6}}{3+\frac{e^{2x}}{\sqrt{x}}+\frac{1}{x}} \to 3$

$\frac{6}{3} = \underline{\underline{2}}$

$\boxed{A22}$ a) $f: [-\frac{\pi}{2}, \pi] \Rightarrow \mathbb{R}$ $f(x) = \cos x - \cos^2 x$

$f'(x) = -\sin x + 2\cos(x)\cdot\sin(x)$

$0 = (\sin x)(-1+2\cos(x))$

$0 = \sin x$

$x = 0$

$1 = 2\cos(x)$

$\frac{1}{2} = \cos(x)$

$x = \pm\frac{\pi}{3}$

$f(-\frac{\pi}{2}) = \cos(-\frac{\pi}{2}) - \cos^2(-\frac{\pi}{2})$
$= 0$

$f(\pi) = \cos(\pi) - \cos^2(\pi)$
$= -2$

$f(\frac{\pi}{3}) = \cos(\frac{\pi}{3}) - \cos^2(\frac{\pi}{3})$
$= \frac{1}{4}$

$f(-\frac{\pi}{3}) = \cos(-\frac{\pi}{3}) - \cos^2(-\frac{\pi}{3})$
$= \frac{1}{4}$

$f(0) = \cos(0) - \cos^2(0)$
$= 0$

glob. min $= -2$ $f(\pi)$

glob. max $= \frac{1}{4}$ $f(\frac{\pi}{3})$ $f(-\frac{\pi}{3})$

b) $f(0,\infty) \Rightarrow \mathbb{R}$ $f(x) = \frac{\ln x}{x}$

$f'(x)$ $\frac{\frac{1}{x}\cdot x - \ln x \cdot 1}{x^2} = 0$

$= \cancel{1}(1-\ln x)x^{-2} = 0$ $0 = 1-\ln x$ $x^{-2} = 0$ ↯

$\ln x = 1$

$x = e$

$f(e) = \frac{1}{e}$

$\lim_{x\searrow 0} \frac{\ln(x)}{x} = -\infty$

$\lim_{x\to\infty} \frac{\overbrace{\ln(x)}^{\to\infty}}{\underbrace{x}_{\to\infty}} \overset{\text{Hos}}{\Rightarrow} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$

glob max $= \frac{1}{e}$ $f(e)$

glob min ist nicht im werte bereich

$\boxed{A23}$ a)

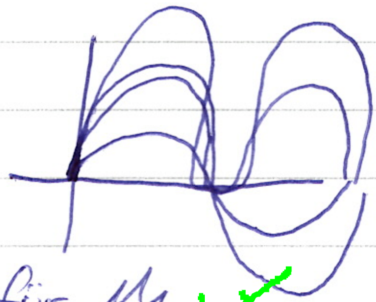
$$\lim_{n \to \infty} (2 \sin x)^n$$

$$f = \begin{cases} 0 & \text{wenn } [0, \frac{\pi}{6}), [\frac{5\pi}{6}, \frac{7\pi}{6}] \\ \cancel{\infty} & \cancel{\text{wenn } (\frac{\pi}{6}, \frac{5\pi}{6})} \\ 1 & \text{wenn } \frac{\pi}{6} \end{cases}$$

(11pi/6,2pi]

f(x) hat nie den Wert infty

x=5pi/6



eine skizze soll schon sowas wie einheiten habe

b) nein für M_1

~~ja für M_2~~

~~ja für M_3~~

ja für M_4