Deckblatt für die Abgabe der Übungsaufgaben
IngMathC2

Name, Vorname: Pleasance Benno

StudOn-Kennung: gi86jyhy

Blatt-Nummer: 1

Übungsgruppen-Nr: 7

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A10, A11, A12, ______

5/10*30 = 15

A10)

a) (i) $\left(\sum_{k=0}^{\infty} a_k\right)\left(\sum_{k=0}^{\infty} b_k\right) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} a_k b_{n-k}$ $a_k = k\cdot q^k$ $b_k = q^k$ für $|q|<1$

$\left(\sum_{k=0}^{\infty} k\cdot q^k\right)\left(\sum_{k=0}^{\infty} q^k\right) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} k\cdot q^k\cdot q^{n-k} = \sum_{n=0}^{\infty}\sum_{k=0}^{n} k\cdot q^n = \sum_{n=0}^{\infty} q^n\cdot\sum_{k=0}^{n} k$

$= \sum_{n=0}^{\infty} q^n\cdot\frac{n^2+n}{2}$ für $|q|<1$

ii) $\sum_{k=0}^{\infty} k^2\cdot q^k$ für $|q|<1$ $\sum_{k=0}^{n} q^n\cdot\frac{n^2+n}{2} = \sum_{k=0}^{\infty}\frac{1}{2}n^2q^n + \frac{1}{2}nq^n$

$\sum_{n=0}^{\infty} n^2q^n = \left(\sum_{n=0}^{\infty} q^n\cdot\frac{n^2+n}{2} - \sum_{n=0}^{\infty}\frac{1}{2}nq^n\right)\cdot 2 = \left(\frac{q}{(1-q)^3} - \frac{1}{2}\cdot\frac{1}{1-q}\right)\cdot 2 = \frac{2q}{(1-q)^3} - \frac{1}{1-q}$

$= \frac{2q-(1-q)^2}{(1-q)^3}$

b) $\sum_{k=0}^{\infty}\frac{1}{(k+1)(k+2)} = \sum_{k=0}^{n}\frac{1}{k+1} - \frac{1}{k+2} = \sum_{k=0}^{n}\frac{1}{k+1} - \sum_{k=0}^{n}\frac{1}{k+2} = \sum_{k=0}^{n-1}\frac{1}{k+1} - \sum_{\tilde{k}=1}^{n}\frac{1}{\tilde{k}+1}$

$= 1 - \frac{1}{n+1} = 1$
∞

A11) a) i) $\sum_{k=0}^{\infty}\frac{5^k}{k}x^k$ $\lim_k\sup\sqrt[k]{\left|\frac{5^k}{k}\right|} = \lim_{k\to\infty}\frac{5}{\sqrt[k]{k}} = 5$ $R = \frac{1}{5}$

ii) $\sum_{k=0}^{\infty}\left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^{2k}x^k$ $\lim_k\sup\sqrt[k]{\left|\left(\sqrt{k+1}-\sqrt{k-\sqrt{k}}\right)^{2k}\right|}$

$= \lim_{k\to\infty}\left(\frac{\left(\sqrt{k+1}-\sqrt{k-\sqrt{k}}\right)\cdot\left(\sqrt{k+1}+\sqrt{k-\sqrt{k}}\right)}{\sqrt{k+1}+\sqrt{k-\sqrt{k}}}\right)^2 = \lim_{k\to\infty}\left(\frac{k+1-k+\sqrt{k}}{\sqrt{k+1}+\sqrt{k-\sqrt{k}}}\right)^2$

$= \lim_{k\to\infty}\left(\frac{1+\sqrt{k}}{\sqrt{k+1}+\sqrt{k-\sqrt{k}}}\right)^2 = \lim_{k\to\infty}\left(\frac{\sqrt{k}\left(\frac{1}{\sqrt{k}}+1\right)}{\sqrt{k}\left(\sqrt{1+\frac{1}{k}}+\sqrt{1-\frac{1}{\sqrt{k}}}\right)}\right)^2 = \lim_{k\to\infty}\left(\frac{\sqrt{k}}{2\sqrt{k}}\right)^2 = \left(\frac{1}{2}\right)^2$ $R = 4$

iii) + iv) ? Ich raffs nicht

das gleiche wie du bei der b) machst: y =x^4 substituieren.+ Ge

b) $S(x) = \sum_{k=0}^{\infty}\left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1\right)^k\left(\frac{1}{x+3}\right)^k$ $y = \frac{1}{x+3}$ $\lim_k\sup\sqrt[k]{\left|\left(\sqrt[k]{3k}+\frac{4}{\sqrt[k]{k!}}+1\right)^k\right|}$

$= \lim_{k\to\infty}\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1 = \lim_{k\to\infty}\sqrt[k]{3}\cdot\sqrt[k]{k} + \lim_{k\to\infty}\frac{4}{\sqrt[k]{k!}} + 1$

Potenzreihe für $y\in\left(-\frac{1}{2},\frac{1}{2}\right)$ konvergent

hier brauchst du wesentlich mehr beg

$y = \frac{1}{x+3} \to x\in(-\infty,-5)\cup(-1,+\infty)$ $\alpha = -5$ $\beta = -1$

|1/(x+3)| <R_y ==> |1/(x+3)| <1/2 ==>|x+3|> 2und dann kann mar

A12) a) i)

$\exp(3ix) = \exp(ix)^3 \qquad \cos(3x) + i\sin(3x) = (\cos(x) + i\sin(x))^3$

$e^{3ix} = e^{ix^3}$

$\cos(3x) + i\sin(3x) = \cos^3(x) + 3\cos^2(x)\, i\sin(x) + 3\cos(x)\,(i\sin(x))^2 + (i\sin(x))^3$

$= \cos^3(x) + i\cdot 3\cos^2(x)\sin(x) - 3\cos(x)\sin^2(x) - i\sin^3(x)$

$\cos(3x) = \cos^3(x) - 3\cos(x)\sin^2(x) = \cos^3(x) - 3\cos(x)(1-\cos^2(x)) = 4\cos^3(x) - 3\cos(x)$

$\sin(3x) = 3\cos^2(x)\sin(x) - \sin^3(x) = 3\cdot(1-\sin^2(x))\sin(x) - \sin^3(x) = -4\sin^3(x) + 3\sin(x)$

ii) $\sin(x+2x) = \sin(x)\cos(2x) + \cos(x)\cdot\sin(2x) = \sin(x)\cdot(\cos^2(x) - \sin^2(x)) + \cos(x)\cdot 2\sin(x)\cos(x)$

$= -\sin^3(x) + \sin(x)\cos^2(x) + 2\sin\cos^2(x) = -\sin^3(x) + 3\sin(x)\cdot(1-\sin^2 x) = -4\sin^3(x) + 3\sin(x)$

$\cos(x+2x) = \cos(x)\cdot\cos(2x) - \sin(x)\cdot\sin(2x) = \cos(x)\cdot(\cos^2(x) - \sin^2(x)) - \sin(x)\cdot 2\sin(x)\cdot\cos(x) =$

$\cos^3(x) - \sin^2(x)\cos(x) - 2\sin^2(x)\cos(x) = \cos^3(x) - (1-\cos^2(x))\cos(x) - 2(1-\cos^2(x))\cos(x)$

$= \cos^3(x) + \cos^3(x) - \cos(x) - 2\cos(x) + 2\cos^3(x) = 4\cos^3(x) - 3\cos(x)$

$\sin(2x) = \sin(x+x) = \sin(x)\cos(x) + \cos(x) + \cos(x)\sin(x) = 2\sin(x)\cos(x)$

$\cos(2x) = \cos(x+x) = \cos^2(x) - \sin^2(x)$

b) $\sin(3x) = -4\sin^3(x) + 3\sin(x) \qquad \cos(3x) = 4\cos^3(x) - 3\cos(x)$

$\sin\left(3\frac{\pi}{3}\right) = -4\sin^3(x) + 3\sin(x) \qquad \cos(3x) = 4\cos^3(x) - 3\cos(x)$

$\sin\left(3\frac{\pi}{3}\right) = -4\sin^3\left(\frac{\pi}{3}\right) + 3\sin\left(\frac{\pi}{3}\right) \qquad 0 = -4\sin^3\left(\frac{\pi}{3}\right) + 3\sin\left(\frac{\pi}{3}\right)$

$0 = -4\sin^2\left(\frac{\pi}{3}\right) + 3$

$\frac{3}{4} = \sin^2\left(\frac{\pi}{3}\right) \qquad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \qquad \cos^2\left(\frac{\pi}{3}\right) = 1 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4}$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

Sorry

ii) $\cos\left(2\cdot\frac{\pi}{6}\right) = 1 - 2\sin^2\left(\frac{\pi}{6}\right) \qquad \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\frac{1}{2} = 1 - 2\sin^2\left(\frac{\pi}{6}\right) \qquad \frac{1}{2} = 2\sin^2\left(\frac{\pi}{6}\right) \qquad \frac{1}{4} = \sin^2\left(\frac{\pi}{6}\right) \qquad \cos\left(\frac{\pi}{6}\right) = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$

iii) $\cos\left(2\cdot\frac{\pi}{12}\right) = 1 - 2\sin^2\left(\frac{\pi}{12}\right) \qquad \sin^2\left(\frac{\pi}{12}\right) = \frac{2-\sqrt{3}}{4}$

$\frac{\sqrt{3}}{2} = 1 - 2\sin^2\frac{\pi}{12} \qquad \sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{2-\sqrt{3}}}{2}$

$\frac{\sqrt{3}-2}{2} = 2\sin^2\frac{\pi}{12}$

$\cos^2\frac{\pi}{12} = 1 - \left(\frac{\sqrt{2-\sqrt{3}}}{2}\right)^2$

$= 1 - \frac{2-\sqrt{3}}{4} = \frac{2+\sqrt{3}}{4}$

$\cos\frac{\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2}$

nochmal sorry