Deckblatt für die Abgabe der Übungsaufgaben
IngMathC2

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Blatt-Nummer: 05

Übungsgruppen-Nr: 07

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A13, A14, ______, ______

13.5/14 * 30=28.5

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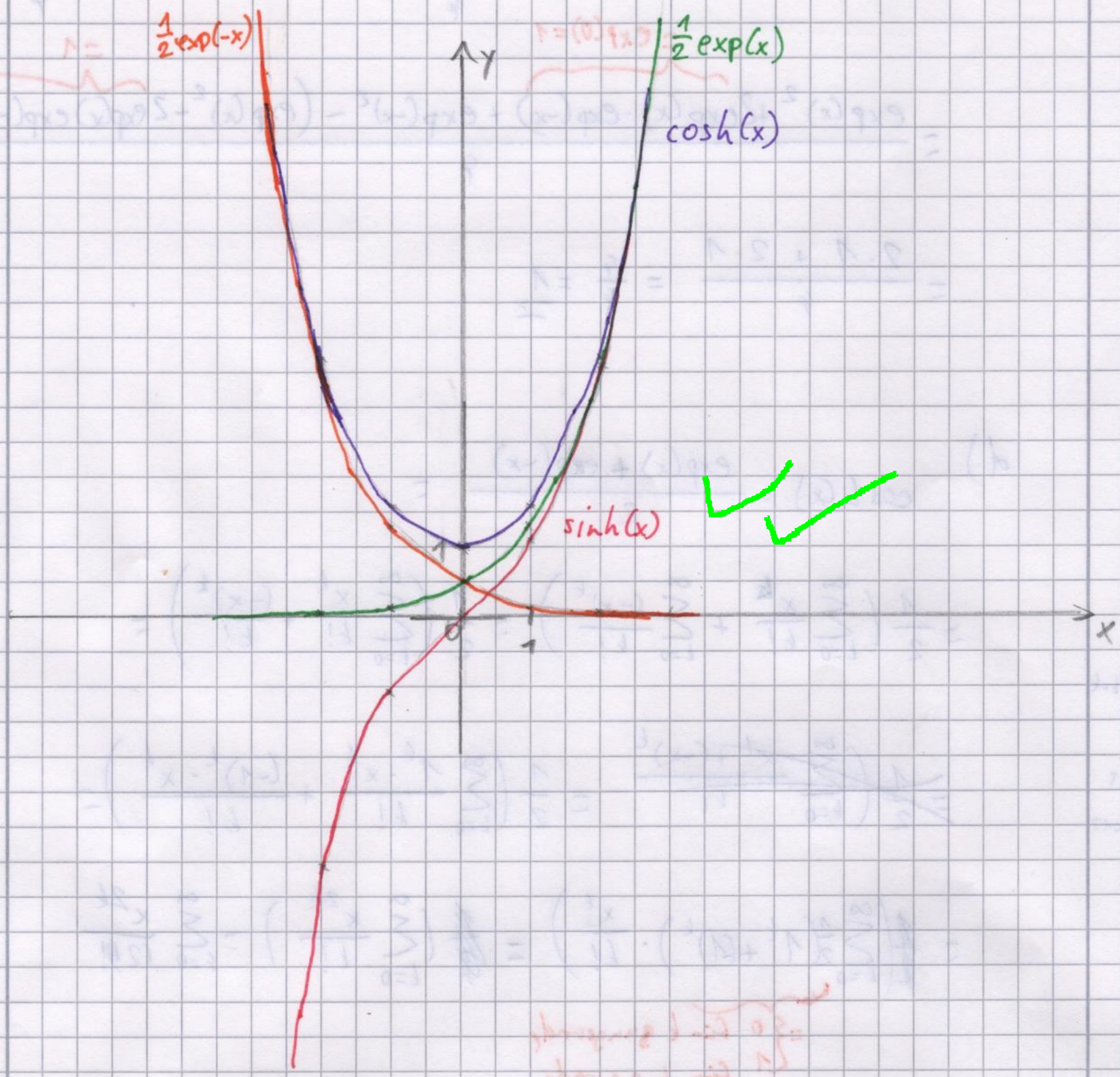
A13)

a)




b) ~~tan(h)~~ $\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{\frac{1}{2}(\exp(x) - \exp(-x))}{\frac{1}{2}(\exp(x) + \exp(-x))}$

$$\lim_{x \to +\infty} \tanh(x) = \lim_{x \to +\infty} \frac{\exp(x) \cdot \left(1 - \underbrace{\frac{\exp(-x)}{\exp(x)}}_{\to 0}\right)}{\exp(x) \cdot \left(1 + \underbrace{\frac{\exp(-x)}{\exp(x)}}_{\to 0}\right)} = \frac{1}{1} = \underline{\underline{1}}$$

$$\lim_{x \to -\infty} \tanh(x) = \lim_{x \to -\infty} \frac{\exp(-x) \cdot \left(\underbrace{\frac{\exp(x)}{\exp(-x)}}_{\to 0} - 1\right)}{\exp(-x) \cdot \left(\underbrace{\frac{\exp(x)}{\exp(-x)}}_{\to 0} + 1\right)} = \frac{-1}{1} = \underline{\underline{-1}}$$

Skizze

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c)

$$\cosh^2 x - \sinh^2 x = \frac{(\exp(x) + \exp(-x))^2}{4} - \frac{(\exp(x) - \exp(-x))^2}{4} =$$

$$= \frac{\exp(x)^2 + 2\underbrace{\exp(x)\cdot\exp(-x)}_{=\exp(0)=1} + \exp(-x)^2 - \left(\exp(x)^2 - 2\underbrace{\exp(x)\exp(-x)}_{=1} + \exp(-x)^2\right)}{4} =$$

$$= \frac{2\cdot 1 + 2\cdot 1}{4} = \frac{4}{4} = \underline{\underline{1}}$$

d)

$$\cosh(x) = \frac{\exp(x) + \exp(-x)}{2} =$$

$$= \frac{1}{2}\left(\sum_{k=0}^{\infty} \frac{x^k}{k!} + \sum_{k=0}^{\infty} \frac{(-x)^k}{k!}\right) = \frac{1}{2}\left(\sum_{k=0}^{\infty} \frac{x^k}{k!} + \frac{(-x)^k}{k!}\right) =$$

$$\cancel{= \frac{1}{2}\left(\sum_{k=0}^{\infty} \frac{x^k + (-x)^k}{k!}\right)} = \frac{1}{2}\left(\sum_{k=0}^{\infty} \frac{1^k \cdot x^k}{k!} + \frac{(-1)^k \cdot x^k}{k!}\right) =$$

$$= \cancel{1}\left(\sum_{k=0}^{\infty} \underbrace{\frac{1}{2}(1^k + (-1)^k)}_{=\begin{cases} 0 & \text{für } k \text{ ungerade} \\ 1 & \text{für } k \text{ gerade} \end{cases}} \cdot \frac{x^k}{k!}\right) = \cancel{1}\left(\sum_{k=0}^{\infty} \frac{x^{2k}}{k!}\right) = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$$

$\Rightarrow$ nur gerade $k$ werden addiert (mit Auswirkung)

$$\sum_{k=0}^{n} k + \sum_{k=0}^{n} k = \sum_{k=0}^{n} k+k$$

$1+1+2+2+3+3$

$2+4+6=12$

$$\sinh(x) = \frac{1}{2}\left(\sum_{k=0}^{\infty} \frac{x^k}{k!} - \frac{(-x)^k}{k!}\right) = \cancel{1} \sum_{k=0}^{\infty} \underbrace{\frac{1}{2}(1^k - (-1)^k)}_{=\begin{cases} 0 & k \text{ gerade} \\ 1 & k \text{ ungerade} \end{cases}} \frac{x^k}{k!} =$$

$$= \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$$

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e)

$$\cancel{\cos(iy) = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{(iy)^{2k}}{(2k)!} = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{-1 \cdot y^{2k}}{(2k)!} =}$$

$$\cancel{= \sum_{k=0}^{\infty} (-1)^k \cdot (-1)^{2k+1} \cdot \frac{y^{2k}}{(2k)!} = \sum_{k=0}^{\infty} (-1)^{3k+1}}$$

$$\cos(iy) = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{iy^{2k}}{(2k)!} = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{(i^2)^k \cdot y^{2k}}{(2k)!} =$$

$$= \sum_{k=0}^{\infty} (-1 \cdot (-1))^k \cdot \frac{y^{2k}}{(2k)!} = \sum_{k=0}^{\infty} \frac{y^{2k}}{(2k)!} = \cosh(y)$$

$$\sin(iy) = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{iy^{2k+1}}{(2k+1)!} = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{i \cdot (i^2)^k \cdot y^{2k+1}}{(2k+1)!} =$$

$$= i \cdot \sum_{k=0}^{\infty} (-1 \cdot (-1))^k \cdot \frac{y^{2k+1}}{(2k+1)!} = i \sinh(y)$$

f) $\sin(x+iy) = \sin(x) \cdot \cos(iy) + \cos(x) \cdot \sin(iy) =$

$$= \sin(x) \cdot \cosh(y) + \cos(x) \cdot i \sinh(y)$$

g) $\sin(0+iy) = i \sinh(y)$

$\sinh(y) \xrightarrow{(y \to +\infty)} \infty$ (siehe Skizze aus a))

$\sinh(y) \xrightarrow{y \to -\infty} -\infty$

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A14) $f(x) = \frac{1-x}{\sqrt{1-x^2}}$

a) ~~$D_f = \mathbb{R} \setminus \{-1, +1\}$~~

$D_f = (-1; 1)$

$$\lim_{x \to -1} \frac{1-x}{\sqrt{1-x^2}} = \lim_{x \to -1} \frac{1-x}{\sqrt{(1-x)(1+x)}} = \lim_{x \to -1} \frac{\overbrace{\sqrt{1-x}}^{\to \sqrt{2}}}{\underbrace{\sqrt{1+x}}_{\to \sqrt{0}}} = \underline{\underline{+\infty}}$$

$$\lim_{x \to +1} \frac{\sqrt{1-x}}{\sqrt{1+x}} = \underline{\underline{0}}$$

b) i) f für $x \neq 0$ stetig (Verkettung stetiger Funktionen)

$x = 0$:

$$\left.\begin{aligned} \lim_{x \nearrow 0} f(x) &= \lim_{x \nearrow 0} 0 = 0 \\ \lim_{x \searrow 0} f(x) &= \lim_{x \searrow 0} e^{1+x-\frac{1}{x}} = \text{"}e^{-\infty}\text{"} = 0 \end{aligned}\right\} \text{beide gleich} \to f \text{ stetig}$$

ii) g für $x \neq 0$ stetig (Verkettung stetiger Funktionen)

$x = 0$

$$\left.\begin{aligned} \lim_{x \nearrow 0} g(x) &= \lim_{x \nearrow 0} e^{1+x-\overbrace{\frac{1}{x}}^{\text{"}-0\text{"}}} = +\infty \\ \lim_{x \searrow 0} g(x) &= \lim_{x \searrow 0} e^{1+x-\frac{1}{x}} = 0 \end{aligned}\right\} \neq \to g \text{ unstetig an Stelle } x = 0$$

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c)

i) $\lim_{x\to 0} \sqrt{x^2+x+1} - x = „\sqrt{1} - 0" = \underline{\underline{1}}$

ii) $\lim_{x\to +\infty} \sqrt{x^2+x+1} - x = \lim_{x\to+\infty} \frac{x^2+x+1-x^2}{\sqrt{x^2+x+1}+x} =$

$= \lim_{x\to+\infty} \frac{x(1+\frac{1}{x})}{x\cdot\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1\right)} = \frac{1}{1+1} = \underline{\underline{\frac{1}{2}}}$

iii) $\lim_{x\to-\infty} \sqrt{x^2+x+1} - x = \cancel{\lim_{x\to-\infty} x\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-1\right)} =$

$= \lim_{x\to-\infty} \sqrt{x(x+\frac{1}{x}+1)} - x = „\sqrt{-\infty\cdot(-\infty)} + \infty" = \underline{\underline{+\infty}}$

iv) $\lim_{x\to+\infty} x\overbrace{|\sin \pi x|}^{\text{divergent}} =$

~~Sei $x_n := n\frac{\pi}{2} \xrightarrow{n\to+\infty} +\infty \Rightarrow f(x) = n\frac{\pi}{2}\cdot(\sin \pi\cdot\frac{\pi}{2}\cdot n)$~~

Sei $x_n := \frac{1}{2}(2n+1) = n+\frac{1}{2} \xrightarrow{n\to+\infty} +\infty \Rightarrow f(x_n) = (n+\frac{1}{2})\cdot\overbrace{|\sin(\pi\cdot(n+\frac{1}{2}))|}^{=+1}$

$\xrightarrow{n\to+\infty} +\infty$

Sei $\tilde{x}_n := n \xrightarrow{n\to+\infty} +\infty \Rightarrow f(\tilde{x}_n) = n\cdot\underbrace{|\sin(\pi\cdot n)|}_{=0 \text{ (genau 0)}} = 0 \xrightarrow{n\to+\infty} 0$

$\hookrightarrow 0 \neq +\infty$

~~$\Rightarrow$ konvergiert nicht~~

$\Rightarrow$ kein Grenzwert

v) $\lim_{x\to 0} x\underbrace{|\sin \pi x|}_{\to 0\cdot 0} = 0$

vi) $\lim_{x\to 0} \cos x \cdot \cos^2 \frac{2}{x}$

Sei $x_n := \frac{2}{2\pi n} \xrightarrow{n\to+\infty} 0 \Rightarrow f(x_n) = \cos\frac{2}{2\pi n}\cdot\cos^2 2\pi n \xrightarrow{n\to+\infty}$ existiert nicht $\Rightarrow$ Funktion hat keinen Grenzwert