7/8*20 = 17.5

Abgabe Aufgaben A10 & A12

A10 a) i) $\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} k\cdot q^k\cdot q^{n-k}\right)=\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} k\cdot q^{k+n-k}\right)$

$$\sum_{n=0}^{\infty} q^n \sum_{k=0}^{n} k = \sum_{n=0}^{\infty} q^n\cdot\left(0+\frac{n^2+n}{2}\right)=\sum_{n=0}^{\infty} q^n\cdot\left(\frac{n^2+n}{2}\right)=\frac{1}{2}\sum_{n=0}^{\infty} q^n(n^2+n)$$

ii) $\frac{1}{2}\left[\sum_{n=0}^{\infty} n^2q^n+q^n n\right)\overset{GW}{=}\left(\frac{1}{1-q}\right)\left(\frac{q}{(1-q)^2}\right) \quad \Big| -\frac{1}{2}\sum_{n=0}^{\infty} nq^n$

der vorfaktor bezieht sich auf beid

$n=k$ $\quad \frac{1}{2}\sum_{k=0}^{\infty} k^2q^k=\frac{q}{(1-q)^3}-\frac{q}{2(1-q)^2} \quad \Big|\cdot 2$ ff

ff

$$\sum_{k=0}^{\infty} k^2q^k=2\left[\frac{q}{(1-q)^3}-\frac{q}{(1-q)^2}\right]=2\left[\frac{q}{(1-q)^3}-\frac{q(1-q)^1}{(1-q)^3}\right]=2\left(\frac{q-q+q^2}{(1-q)^3}\right)=\frac{2q^2}{(1-q)^3}$$

b) $\sum_{k=0}^{\infty}\frac{1}{(k+1)(k+2)}=\sum_{k=0}^{\infty}\frac{A}{(k+1)}+\frac{B}{(k+2)}$

$1=A(k+2)+B(k+1)$

$k=0$	$k=1$
$1=A(0+2)+B(0+1)$	$1=A(1+2)+B(1+1)$
$1=2A+B$	$1=3A+2B$
$1-2A=B$	$1=3A+2(1-2A)$
	$1=3A+2-4A)$
$B=1-2(1)$	$1=-A+2$
$B=-1$	$A=1$

$\sum_{k=0}^{\infty}\frac{1}{k+1}+\frac{-1}{k+2}$ $\qquad k+2=\tilde{k}+1$, $\quad k=\tilde{k}-1$

$\sum_{k=0}^{\infty}\frac{1}{k+1}-\sum_{k=0}^{\infty}\frac{1}{k+2}$

$$\sum_{k=0}^{\infty}\frac{1}{k+1}-\sum_{\tilde{k}=1}^{\infty}\frac{1}{\tilde{k}-1+2}=\sum_{k=0}^{\infty}\frac{1}{k+1}-\sum_{k=1}^{\infty}\frac{1}{k+1}=\frac{1}{0+1}=1$$

$\boxed{A12}$ a) i) $\exp(3ix) = \exp(ix)^3$

$$\cos(3x) + i\sin(3x) = [\cos(x) + i\sin(x)]^3$$

$$\cos(3x) + i\sin(3x) = (\cos(x) + i\sin(x))(\cos^2(x) + 2i\sin(x)\cos(x) - \sin^2(x)) = \cos^3(x) + 2i\sin(x)\cos^2(x) - \sin^2(x)\cos(x) + i\sin(x)\cos^2(x) - 2\sin^2(x)\cos(x) - i\sin^3(x)$$

RE $\cos(3x) = \cos^3(x) - \sin^2(x)\cos(x) - 2\sin^2(x)\cos(x)$

$\cancel{= \cos(x)(\cos^2(x) - \sin^2(x) - 2\sin^2(x))}$

$\cancel{= \cos(x)(\cos^2(x) + \cos^2(x) - 1 - 2(1 - \cos^2(x)))}$

$\cancel{= \cos(x)(2\cos^2(x) - 3 - 2\cos^2(x))}$

$= \cos^3(x) - 3\sin^2(x)\cos(x)$

$= \cos(x)(\cos^2(x) - 3(\cancel{\cos}\,1 - \cos^2(x)))$

$= \cos(x)(\cos^2(x) - 3 + 3\cos^2(x))$

$\underline{\underline{= 4\cos^3(x) - 3\cos(x)}}$

IM $\sin(3x) = 2\sin(x)\cos^2(x) + \sin(x)\cos^2(x) - \sin^3(x)$

$= 3\sin(x)\cos^2(x) - \sin^3(x)$

$= 3\sin(x)(1 - \sin^2(x)) - \sin^3(x)$

$\underline{\underline{= 3\sin(x) - 4\sin^3(x)}}$

ii) $\sin(2x) = \sin(x+x) = \sin(x)\cos(x) + \cos(x)\sin(x) = 2\sin(x)\cos(x)$

$\sin(3x) = \sin(x+2x) = \sin(x)\cos(2x) + \cos(x)\sin(2x)$

$= \sin(x)[\cos^2(x) - \sin^2(x)] + \cos(x)[2\sin(x)\cos(x)]$

$= \cancel{[1-\sin^2(x)][\sin^3(x)] + [2\sin(x)][1-\sin^2(x)]}\ \sin(x)[1-\sin^2(x)] - \sin^3(x) + [2\sin(x)][1-\sin^2(x)] = \sin(x) - \sin^3(x) - \sin^3(x) + 2\sin(x) - 2\sin^3(x) = \underline{3\sin(x) - 4\sin^3(x)}$

$\cos(2x) = \cos(x+x) = \cos(x)\cos(x) - \sin(x)\sin(x) = \cos^2(x) - \sin^2(x)$

$\cos(3x) = \cos(2x+x) = [\cos^2(x) - \sin^2(x)][\cos(x)] - [2\sin(x)\cos(x)][\sin(x)]$

$= \cos^3(x) - [1 - \cos^2(x)][\cos(x)] - 2\cos(x)[1 - \cos^2(x)]$

$= 2\cos^3(x) - \overset{\cos(x)}{\cancel{1}} - 2\cos(x) + 2\cos^3(x)$

$\underline{\underline{= 4\cos^3(x) - 3\cos(x)}}$

$\boxed{A12}$ b) i) $\cos(3\frac{\pi}{3}) = 4\cos^3(\frac{\pi}{3}) - 3\cos(\frac{\pi}{3})$

$$\sin(3\frac{\pi}{3}) = 3\sin(\frac{\pi}{3}) - 4\sin^3(\frac{\pi}{3})$$
$$0 = 3\sin(\frac{\pi}{3}) - 4\sin^3(\frac{\pi}{3})$$
$$0 = \sin(\frac{\pi}{3})[3 - 4\sin^2(\frac{\pi}{3})]$$
$$0 = 3 - 4\sin^2(\frac{\pi}{3})$$
$$4\sin^2(\frac{\pi}{3}) = 3$$
$$\sin(\frac{\pi}{3}) = \sqrt{\frac{3}{4}} = \underline{\underline{\frac{\sqrt{3}}{2}}}$$

$$\cos(3\frac{\pi}{3}) = 4\cos^3(\frac{\pi}{3}) - 3\cos(\frac{\pi}{3})$$
$$\frac{-1}{\cos(\frac{\pi}{3})} = 4\cos^2(\frac{\pi}{3}) - 3$$
$$\frac{-1}{\cos(\frac{\pi}{3})} = 4(1 - \frac{\sqrt{3}^2}{2^2}) - 3$$
$$= 4 - \frac{12}{4} - 3$$
$$= -2$$
$$\frac{-1}{-2} = \cos(\frac{\pi}{3}) = \underline{\underline{\frac{1}{2}}}$$

ii) $\cancel{\sin(3\frac{\pi}{6}) = 3\sin(\frac{\pi}{6}) - 4\sin^3(\frac{\pi}{6})}$

$$\cancel{\frac{\sqrt{3}}{2} = \sin(\frac{\pi}{6})(3 - 4(\sin^3(\frac{\pi}{6}))}$$

$$\cos(2\frac{\pi}{6}) = \cos^2(\frac{\pi}{6}) - \sin^2(\frac{\pi}{6}) = 1 - 2\sin^2(\frac{\pi}{6})$$
$$\cos(\frac{\pi}{3}) = 1 - 2\sin^2(\frac{\pi}{6})$$
$$\frac{1}{2} - 1 = -2\sin^2(\frac{\pi}{6})$$
$$-\frac{1}{4} = -\sin^2(\frac{\pi}{6})$$
$$\underline{\underline{\frac{1}{2}}} = \sin(\frac{\pi}{6})$$

$$\cos(2\frac{\pi}{6}) = \cos^2(\frac{\pi}{6}) - \sin^2(\frac{\pi}{6})$$
$$\frac{1}{2} = \cos^2(\frac{\pi}{6}) - \frac{1}{4}$$
$$\frac{3}{4} = \cos^2(\frac{\pi}{6})$$
$$\cos(\frac{\pi}{6}) = \underline{\underline{\frac{\sqrt{3}}{2}}}$$

b(iii) $\cos\left(2\frac{\pi}{12}\right) = \cos^2\left(\frac{\pi}{12}\right) - \sin^2\left(\frac{\pi}{12}\right)$

$$\cos\left(\frac{\pi}{6}\right) = 1 - \sin^2\left(\frac{\pi}{12}\right) - \sin^2\left(\frac{\pi}{12}\right)$$

$$-\frac{\sqrt{3}}{2} + 1 = 2\sin^2\left(\frac{\pi}{12}\right)$$

$$\frac{2-\sqrt{3}}{2} = 2\sin^2\left(\frac{\pi}{12}\right)$$

$$\frac{2-\sqrt{3}}{4} = \sin^2\left(\frac{\pi}{12}\right)$$

$$\frac{\sqrt{2-\sqrt{3}}}{2} = \sin\left(\frac{\pi}{12}\right)$$

$$\cos\left(\frac{\pi}{6}\right) = \cos^2\left(\frac{\pi}{12}\right) - \left(\frac{\sqrt{2-\sqrt{3}}}{2}\right)^2$$

$$\frac{\sqrt{3}}{2} = \cos^2\left(\frac{\pi}{12}\right) - \frac{2-\sqrt{3}}{4}$$

$$\frac{2\sqrt{3}}{4} + \frac{2-\sqrt{3}}{4} = \cos^2\left(\frac{\pi}{12}\right)$$

$$\frac{2+\sqrt{3}}{4} = \cos^2\left(\frac{\pi}{12}\right)$$

$$\frac{\sqrt{2+\sqrt{3}}}{2} = \cos\left(\frac{\pi}{12}\right)$$