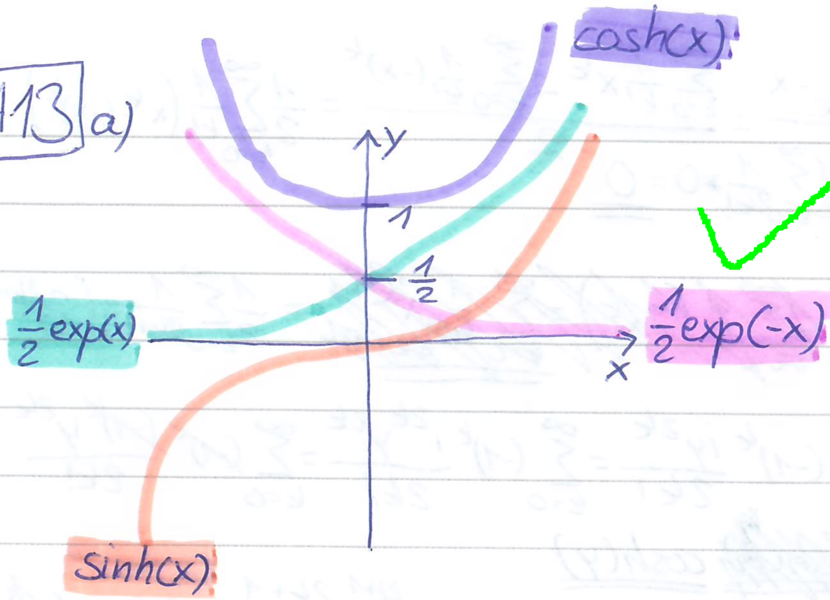
14/14*30 = 30

A13 a)

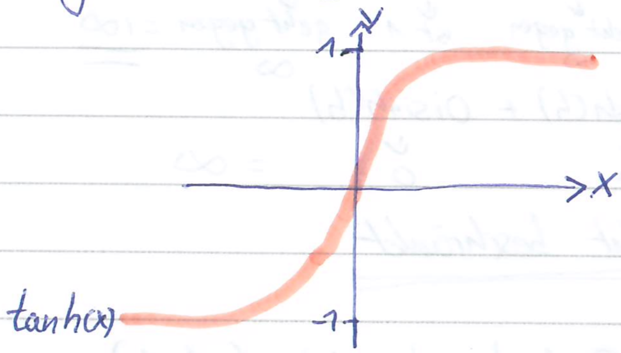


b) $\lim\limits_{x\to\infty} \tanh(x) = \lim\limits_{x\to\infty} \frac{\sinh(x)}{\cosh(x)} = \lim\limits_{x\to\infty} \frac{\frac{e^x - e^{-x}}{2}}{\frac{e^x+e^{-x}}{2}} = \lim\limits_{x\to\infty} \frac{e^x-e^{-x}}{2} \cdot \frac{2}{e^x+e^{-x}}$

$= \lim\limits_{x\to\infty} \frac{\cancel{2}(e^x-e^{-x})}{\cancel{2}(e^x+e^{-x})} = \lim\limits_{x\to\infty} \frac{\cancel{e^x}(1-e^{-2x})}{\cancel{e^x}(1+e^{-2x})} \longrightarrow \frac{1}{1} = \underline{\underline{1}}$

$\lim\limits_{x\to-\infty} \tanh(x) = \lim\limits_{x\to-\infty} \frac{e^x-e^{-x}}{e^x+e^{-x}} = \lim\limits_{x\to-\infty} \frac{\cancel{e^{-x}}(e^{2x}-1)}{\cancel{e^{-x}}(e^{2x}+1)} \longrightarrow \frac{-1}{1} = \underline{\underline{-1}}$

Funktionsgrenzwerte $= \{1, -1\}$



c) $\cosh^2(x) - \sinh^2(x) = \left(\frac{\exp(x)+\exp(-x)}{2}\right)^2 - \left(\frac{\exp(x)-\exp(-x)}{2}\right)^2$

$= \left(\frac{e^x+e^{-x}}{2}\right)^2 - \left(\frac{e^x-e^{-x}}{2}\right)^2 = \frac{(e^x+e^{-x})(e^x+e^{-x}) - (e^x-e^{-x})(e^x-e^{-x})}{4}$

$= \frac{\cancel{e^{x^2}} + 2e^xe^{-x} + \cancel{e^{-x^2}} - \cancel{e^{x^2}} + 2e^xe^{-x} - \cancel{e^{-x^2}}}{4} = \frac{2(1)+2(1)}{4} = \frac{4}{4} = \underline{\underline{1}}$

d) $\cosh(x) = \frac{e^x+e^{-x}}{2} = \frac{\sum\limits_{k=0}^{\infty} \frac{1}{k!}x^k + \sum\limits_{k=0}^{\infty} \frac{1}{k!}(-x)^k}{2} = \frac{1}{2}\sum\limits_{k=0}^{\infty} \frac{1}{k!}\left(x^k + (-x)^k\right)$

Fall k = gerade $\frac{1}{2}\sum\limits_{k=0}^{\infty} \frac{1}{2k!}(2x^{2k}) = \underline{\underline{\sum\limits_{k=0}^{\infty} \frac{1}{2k!}x^{2k}}}$

Fall k = ungerade $\frac{1}{2}\sum\limits_{k=0}^{\infty} \frac{1}{(2k-1)!}(0) = \underline{\underline{0}}$

A13 d) weiter

$$\sinh(x) = \frac{e^x - e^{-x}}{2} = \frac{\sum_{k=0}^{\infty} \frac{1}{k!} x^k - \sum_{k=0}^{\infty} \frac{1}{k!}(-x)^k}{2} = \frac{1}{2}\sum_{k=0}^{\infty} \frac{1}{k!}\left(x^k - (-x)^k\right)$$

x^{2k+1}

Fall k = gerade: $\frac{1}{2}\sum_{k=0}^{\infty} \frac{1}{2k!} * 0 = \underline{0}$

Fall k = ungerade: $\cancel{\frac{1}{2}\sum_{k=0}^{\infty} \frac{1}{k!}(2x^k) = \sum_{k=0}^{\infty} \frac{1}{(2k+1)!} x^{2k+1}} = \frac{1}{2}\sum_{k=0}^{\infty} \frac{1}{(2k+1)!}\left(2x^{2k+1}\right) = \underline{\sum_{k=0}^{\infty} \frac{1}{(2k+1)!} x^{2k+1}}$

e) $\cos(iy) = \sum_{k=0}^{\infty} (-1)^k \frac{iy^{2k}}{2k!} = \sum_{k=0}^{\infty} (-1)^k \frac{i^{2k} y^{2k}}{2k!} = \sum_{k=0}^{\infty} \cancel{(-1)^k} \frac{\cancel{(-1)^k} y^{2k}}{2k!}$

$= \sum_{k=0}^{\infty} \frac{y^{2k}}{2k!} = \cancel{\cosh(y)}\ \underline{\cosh(y)}$

$\sin(iy) = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{iy^{2k+1}}{(2k+1)!} = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{i^{2k+1} y^{2k+1}}{(2k+1)!} = \sum_{k=0}^{\infty} \cancel{(-1)^k} \frac{\cancel{(-1)^k} \cdot i \cdot y^{2k+1}}{(2k+1)!}$

$\cancel{i\sinh(y)} = \underline{i\sinh(y)}$

f) $\sin(x+iy) = \sin(x)\cos(iy) + \cos(x)\sin(iy) = \sin(x)\cosh(y) + \cos(x)i\sinh(y)$

g) $\sin(z) = \sin(a+bi) = \sin(a)\cosh(b) + \cos(a)i\sinh(b)$

Fall $a=0$ $\lim_{b\to\infty} \underbrace{\sin(0)}_{\text{ist } 0}\underbrace{\cosh(b)}_{\text{geht gegen } \infty} + \underbrace{\cos(0)}_{\text{ist } 1}\underbrace{i\sinh(b)}_{\text{geht gegen } \infty} = \underline{i\infty}$

Fall $a=\frac{\pi}{2}$ $\lim_{b\to\infty} 1\underbrace{\cosh(b)}_{\infty} + \underbrace{0 i\sinh(b)}_{0} = \infty$

Eins reicht schon

$\sin\ \mathbb{C} \to \mathbb{C}$ ist $\underline{\text{nicht beschränkt}}$

A14 a) $f(x)\frac{1-x}{\sqrt{1-x^2}}$ Definitionsbereich $= (-1,1)$

Randpunkte $= \{-1, 1\}$

$$\lim_{x\to -1} \frac{1-x}{\sqrt{1-x^2}} = \frac{\lim_{x\to -1} 1-x}{\sqrt{1-x}\sqrt{1+x}} = \frac{\lim_{x\to -1}\sqrt{1-x}^{\cancel{2}}}{\cancel{\sqrt{1-x}}\sqrt{1+x}} = \lim_{x\to -1} \frac{\sqrt{1-x}^{\to\sqrt{2}}}{\sqrt{1+x}_{\to 0}} \longrightarrow \underline{\infty}$$

$$\lim_{x\to 1} \frac{1-x}{\sqrt{1-x^2}} = \lim_{x\to 1} \frac{\sqrt{1-x}^{\to 0}}{\underbrace{\sqrt{1+x}}_{\sqrt{2}}} \longrightarrow \underline{0}$$

$\boxed{A14}$ b) i) prüfe Stetigkeit an der stelle $x_* = 0$ da alle anderen Stellen "offensichtlich" stetig sind.

$f(0) = \underline{\underline{0}}$

$\lim_{x \nearrow 0} f(x) = \lim_{x \nearrow 0} 0 = \underline{\underline{0}}$

$\lim_{x \searrow 0} f(x) = \lim_{x \searrow 0} e^{1+x-\frac{1}{x}} \longrightarrow \underline{\underline{0}}$ alle gleich deswegen ist f(x) stetig

ii) prüfe Stetigkeit an der Stelle $x_* = 0$ da alle anderen Stellen "offensichtlich" stetig sind.

$g(0) = \underline{\underline{0}}$

$\lim_{x \searrow 0} g(x) = \lim_{x \searrow 0} e^{1+x-\frac{1}{x}} \longrightarrow \underline{\underline{0}}$

$\lim_{x \nearrow 0} g(x) = \lim_{x \nearrow 0} e^{1+x-\frac{1}{x}} \longrightarrow \underline{\underline{\infty}}$ alle Ergebnisse nicht gleich deswegen ist g(x) nicht stetig

c) i) $\lim_{x \to 0} \sqrt{x^2+x+1} - x = \sqrt{1} = \underline{\underline{1}}$

ii) $\lim_{x \to \infty} \sqrt{x^2+x+1} - x = \frac{(\sqrt{x^2+x+1} - x)(\sqrt{x^2+x+1} + x)}{\sqrt{x^2+x+1} + x} = \frac{\cancel{x^2}+x+1-\cancel{x^2}}{\sqrt{x^2+x+1} + x}$

$= \frac{x(1+\frac{1}{x})}{x(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}} + 1)} = \frac{1}{\sqrt{1}+1} = \underline{\underline{\frac{1}{2}}}$

iii) $\lim_{x \to -\infty} \sqrt{x^2+x+1} - x = \underline{\underline{\infty}}$

positiver Wert, $+\infty$

iv) $\lim_{x \to \infty} x|\sin(\pi x)|$

geht gegen ∞ ist 0

$x_n := 2n \xrightarrow{\infty} \infty$ $2n|\sin(\pi 2n)| \xrightarrow{\infty} \underline{0}$

$x_n := 2n+\frac{1}{2} \xrightarrow{\infty} \infty$ $2n+\frac{1}{2}|\sin((2n+\frac{1}{2})\pi)| \longrightarrow \underline{\underline{\infty}}$

∞ positiver wert

der $\lim_{x \to \infty}$ exsistiert nicht

v) $\lim\limits_{x \to 0} \underset{\downarrow\atop 0}{x} \, |\underbrace{\sin(\pi x)}_{\text{positive Zahl zwischen 1 und 0}}| \longrightarrow \underline{\underline{0}}$

vi) $\lim\limits_{x \to 0} \cos(x) \cos^2\left(\frac{2}{x}\right)$

$x_n = \frac{1}{\pi n} \xrightarrow{\infty} 0 \qquad \lim\limits_{x \to 0} \underset{\downarrow\atop 1}{\cos\left(\frac{1}{\pi n}\right)} \underset{\downarrow\atop 1}{\cos^2\left(\frac{2}{\frac{1}{\pi n}}\right)} = \underline{\underline{1}}$

$x_n = \frac{4}{\pi n} \xrightarrow{\infty} 0 \qquad \lim\limits_{x \to 0} \cancel{\cos\left(\frac{4}{\pi n}\right) \cos^2\left(\frac{2}{\frac{4}{\pi n}}\right)} \; \underbrace{\cos\left(\frac{4}{\pi n}\right)}_{\downarrow\atop 1} \underbrace{\cos^2\left(\frac{2}{\frac{4}{\pi n}}\right)}_{\downarrow\atop 0} = \underline{\underline{0}}$

der $\lim\limits_{x \to 0}$ exsistiert nicht