3/10*30=9

A10)

i) $\sum_{k=0}^{\infty} k \cdot q^k \qquad \sum_{k=0}^{\infty} q^k$

$$\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} k \cdot \cancel{q^k} \cdot q^{n-\cancel{k}}\right) = \sum_{n=0}^{\infty} \cdot \left(q^n \sum_{k=0}^{n} k\right)$$

ii) -> Bekannt aus Vorlesun:

$$\lim_{k\to\infty} \sum_{k=0}^{\infty} q^k = \frac{1}{1-q} \qquad \lim_{k\to\infty} \sum_{k=0}^{\infty} k \cdot q^k = \frac{q}{(1-q)^2}$$

(Geometrische Reihe)

Cauchy-Produkt: $\frac{1}{1-q} \cdot \frac{q}{(1-q)^2} = \frac{q}{(1-q)^3}$

b) $\sum_{k=0}^{\infty} \frac{1}{(k+1)\cdot(k+2)} = \sum_{k=0}^{\infty}\left(\frac{1}{k+1} \cdot \frac{1}{k+2}\right)$

$$= \sum_{\substack{k=0 \\ \tilde{k}=1}}^{\infty} \frac{1}{k+1} \cdot \sum_{\substack{k=0 \\ \tilde{k}=2}}^{\infty} \frac{1}{k+2} = \sum_{\tilde{k}=1}^{\infty} \frac{1}{k} \cdot \sum_{\tilde{k}=2}^{\infty} \frac{1}{k}$$

$$= \sum_{n=1}^{\infty} \cdot \left(\sum_{k=2}^{\infty} \frac{1}{k} \cdot \frac{1}{k}\right)$$

$\frac{1}{k}$ -> harmonische Reihe = divergent gegen ∞

A11)

a) i) $\sum_{k=0}^{\infty} \frac{5^k}{k} \cdot x^k \qquad \lim_{k\to\infty} \sqrt[k]{\frac{5^k}{k}} = \frac{\sqrt[k]{5^k}}{\sqrt[k]{k}} = \frac{5}{\sqrt[k]{k}} = 5$

$\to 1$

$R = \frac{1}{5}$

ii) $\sum_{k=0}^{\infty} \left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^{2k} \cdot x^k \qquad \lim_{k\to\infty} \sqrt[k]{\left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^{2k}}$

$$\lim_{k\to\infty} \left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^{k}$$

$$\lim_{k\to\infty} \underbrace{\sqrt{k}}_{\infty} \leq \sqrt{k+1}$$?

iii) $\sum_{k=0}^{\infty} (k!+2)\cdot x^{k}$

$$\lim_{k\to\infty} \sqrt[k]{k!+2} \cancel{=} \lim_{k\to\infty} \underbrace{\sqrt[k]{k!}}_{\infty} + \underbrace{\sqrt[k]{2}}_{1} = \infty$$

$R = 0$

A12)

a) I) $\exp(3ix) = \exp(ix)^3$

$$e^{3ix} = e^{ix^{3}}$$

$$\cos(3x) + i \cdot \sin(3x) = (\cos(x) + i \cdot \sin(x))^3$$

$$\cos(3x) + i \cdot \sin(3x) =$$

$$\cos^3(x) + 3 \cdot \cos^2(x) \cdot i \sin(x) + 3 \cdot \cos(x) \cdot (i \cdot \sin(x))^2 + (i \cdot \sin(x))^3 =$$

$$\cos^3(x) + i \cdot 3 \cdot \cos^2(x) \cdot \sin(x) - 3 \cdot \cos(x) \cdot (1 - \cos^2(x) - i \cdot \sin^3(x)$$

$$\cos(3x) = \cos^3(x) - 3 \cdot \cos(x) \cdot \sin^2(x)$$

$$= 4 \cdot \cos^3(x) - 3\cos(x)$$

$$\sin(3x) = 3 \cdot \cos^2(x) \cdot \sin(x) - \sin^3(x) = -4 \cdot \sin^3(x) + 3 \cdot \sin(x)$$

b) I) $\sin(3x) = -4 \cdot \sin^3(x) + 3 \cdot \sin(x)$

$$\cos(3x) = 4 \cdot \cos^3(x) - 3 \cdot \cos(x)$$

$$\sin\left(3 \frac{\pi}{3}\right) = -4 \cdot \sin^3\left(\frac{\pi}{3}\right) + 3 \cdot \sin\left(\frac{\pi}{3}\right)$$

$$0 = -4 \cdot \sin^3\left(\frac{\pi}{3}\right) + 3 \cdot \sin\left(\frac{\pi}{3}\right)$$

$$0 = -4 \cdot \sin^2\left(\frac{\pi}{3}\right) + 3$$

$$\frac{3}{4} = \sin^2\left(\frac{\pi}{3}\right)$$

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\cos^2\left(\frac{\pi}{3}\right) = 1 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4}$$