Deckblatt für die Abgabe der Übungsaufgaben
IngMathC2

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Blatt-Nummer: 04

Übungsgruppen-Nr: 07

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A10, A11, A12, ______

Du hattest mich jetzt angeschrieben, deswegen kein Punktabzug, aber denk dran immer nur eine Lösung abzugeben, s

A10

a) i) $a_k = k\cdot q^k$, $b_k = q^k$ für $|q|<1$

$$\left(\sum_{k=0}^{\infty} k\cdot q^k\right)\cdot\left(\sum_{k=0}^{\infty} q^k\right) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} \underbrace{k\cdot \cancel{q^k}}_{a_k}\cdot \underbrace{q^{n-k}}_{b_{n-k}} = \sum_{n=0}^{\infty}\sum_{k=0}^{n} k\cdot q^n = \sum_{n=0}^{\infty} \frac{n(n+1)}{2} q^n$$

ii) $$\left(\sum_{k=0}^{\infty} k\cdot q^k\right)\left(\sum_{k=0}^{\infty} q^k\right) = \sum_{n=0}^{\infty} \frac{1}{2}\cdot n(n+1)\, q^n = \frac{1}{2}\sum_{n=0}^{\infty} \underbrace{(n^2+n)\,q^n}_{n^2q^n+nq^n} = \frac{q}{(1-q)^2}\cdot\frac{1}{(1-q)} = \frac{q}{(1-q)^3} \quad \Big|\cdot 2$$

$$\sum_{n=0}^{\infty} n^2q^n + nq^n = \frac{2q}{(1-q)^3} \quad \Big| -\sum_{n=0}^{\infty} nq^n \Rightarrow \sum_{n=0}^{\infty} n^2 q^n = \frac{2q}{(1-q)^3} - \frac{q}{(1-q)^2}$$

b) $$P_n = \sum_{k=0}^{n} \frac{1}{(k+1)(k+2)} = \sum_{k=0}^{n}\left(\frac{1}{k+1} - \frac{1}{k+2}\right) = \sum_{k=0}^{n}\frac{1}{k+1} - \sum_{k=0}^{n}\frac{1}{k+2} \overset{\tilde{k}=k+1}{=} \sum_{k=0}^{n}\frac{1}{k+1} - \sum_{\tilde{k}=1}^{n+1}\frac{1}{\tilde{k}+1} = \frac{1}{0+1} - \boxed{\frac{1}{n+2}} \xrightarrow{n\to\infty} 1-0 = 1$$

($\frac{1}{n+2} \xrightarrow{n\to\infty} 0$)

A11

a) i) $$\sum_{k=0}^{\infty} \underbrace{\frac{5^k}{k}}_{a_k} x^k \longrightarrow \limsup_k \sqrt[k]{|a_k|} = \sqrt[k]{\left|\frac{5^k}{k}\right|} \overset{k\geq 0}{=} \frac{\sqrt[k]{5^k}}{\underbrace{\sqrt[k]{k}}_{\to 1}} \xrightarrow{k\to\infty} \frac{5}{1} = 5 \qquad R = \frac{1}{5}$$

ii) $$\sum_{k=0}^{\infty} \underbrace{\left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^{2k}}_{a_k} x^k \longrightarrow \limsup_k \sqrt[k]{\left(\sqrt{k+1}-\sqrt{k-\sqrt{k}}\right)^{2k}} = \limsup_k \left(\sqrt{k+1}-\sqrt{k-\sqrt{k}}\right)^2 = \left(\frac{\left(\sqrt{k+1}-\sqrt{k-\sqrt{k}}\right)\left(\sqrt{k+1}+\sqrt{k-\sqrt{k}}\right)}{\sqrt{k+1}+\sqrt{k-\sqrt{k}}}\right)^2$$

$$= \left(\frac{\cancel{k}+1-\cancel{k}+\sqrt{k}}{\sqrt{k+1}+\sqrt{k-\sqrt{k}}}\right)^2 = \left(\frac{\cancel{k}\left(\frac{1}{k}+\frac{1}{\sqrt{k}}\right)}{\cancel{k}\left(\sqrt{\frac{1}{k^4}+\frac{1}{k^2}}+\sqrt{\frac{1}{k}-\frac{1}{k^4}}\right)}\right)^2 = 0 \Longrightarrow R = \frac{1}{0} = \infty$$

($\frac{1}{k}\to 0$, $\frac{1}{\sqrt{k}}\to 0$, $\frac{1}{k^4}\to 0$, $\frac{1}{k^2}\to 0$, $\frac{1}{k}\to 0$, $\frac{1}{k^4}\to 0$)

warum teilst du hier durch k^4 ?Wurzelumfor

iii) $$\sum_{k=0}^{\infty} \overbrace{(k!+2)}^{a_k} x^k \longrightarrow \limsup_k \sqrt[k]{|(k!+2)|} = \infty \Rightarrow R = \frac{1}{\infty} = 0$$

$\sqrt[k]{k!} \xrightarrow{k\to\infty} \infty$

$\sqrt[k]{k!+2} \geq \sqrt[k]{k!} \longrightarrow \infty$

iv) $$\sum_{k=0}^{\infty} \frac{2^k}{k^2} x^{4k} \longrightarrow y := x^4 \ (\geq 0) \rightarrow \sum_{k=0}^{\infty} \frac{2^k}{k^2} y^k \longrightarrow R_y = \frac{1}{\limsup_k \sqrt[k]{\frac{2^k}{k^2}}} = \frac{1}{\limsup_k \frac{2}{\sqrt[k]{k^2}}} = \frac{1}{\frac{2}{1}} = \frac{1}{2}$$

$\sqrt[k]{k^2} \xrightarrow{k\to\infty} 1$

Damit ist die Reihe für alle $|y| < \frac{1}{2}$ konvergiert,

somit für alle x mit $|x| = \sqrt[4]{y} < \sqrt[4]{\frac{1}{2}} \Rightarrow R_x = \sqrt[4]{\frac{1}{2}}$

b) $y := \frac{1}{x+3}$ $(x \neq -3)$

$$\rightarrow \sum_{k=0}^{\infty}\left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1\right)^k y^k \rightarrow \limsup_k \sqrt[k]{\left|\left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1\right)^k\right|} = \limsup_k \overbrace{\sqrt[k]{3k}}^{\to 1} + \overbrace{\frac{4}{\sqrt[k]{k!}}}^{\to 0} + 1 = 2 \Rightarrow R_y = \frac{1}{2}$$

$\sqrt[k]{3k} \xrightarrow{k\to\infty} 1$

$\sqrt[k]{k!} \xrightarrow{k\to\infty} \infty$

Damit ist die Reihe für alle $|y| < \frac{1}{2}$ konvergiert, $\left|\frac{1}{x+3}\right| < \frac{1}{2}$

$$\Leftrightarrow |x+3| > 2 \Leftrightarrow \begin{cases} ① \ x+3 > 2 \rightarrow x > -1 \\ ② \ x+3 < -2 \rightarrow x < -5 \end{cases} \Rightarrow (-\infty, -5) \cup (-1, +\infty) \qquad \begin{matrix}\alpha = -5 \\ \beta = -1\end{matrix}$$

A12

a) i) $$\exp(3ix) = \exp(ix+ix+ix) = \exp(ix)\cdot\exp(ix)\cdot\exp(ix) = (\cos(x)+i\sin(x))(\cos(x)+i\sin(x))(\cos(x)+i\sin(x))$$

$$= \left(\cos^2(x) + 2i\sin(x)\cos(x) - \sin^2(x)\right)\left(\cos(x)+i\sin(x)\right) = \cos^3(x) + i\sin(x)\cos^2(x) + 2i\sin(x)\cos^2(x) -$$

$$2\sin^2(x)\cos(x) - \sin^2(x)\cos(x) - i\sin^3(x) \quad \left[\left(\sin^2(x) = 1-\cos^2(x)\right) \text{ und } \left(\cos^2(x) = 1-\sin^2(x)\right)\right]$$

Realteil auflösen: $\cos^3(x) - 3\cos(x) + 3\cos^3(x) = 4\cos^3(x) - 3\cos(x) = \cos(3x)$

Im. auflösen: $3i\sin(x)\cos^2(x) - i\sin^3(x) = 3i\sin(x)(1-\sin^2(x)) - i\sin^3(x) = 3i\sin(x) - 4i\sin^3(x)$

$\Rightarrow \sin(3x) = 3\sin(x) - 4\sin^3(x)$

ii) aus der Vorlesung: 1. $\sin(2x) = 2\sin(x)\cos(x)$ 2. $\cos(2x) = \cos^2 x - \sin^2(x) = 1 - 2\sin^2(x)$ 3. $\cos^2(x) = 1 - \sin^2(x)$

$\sin(3x) = \sin(x+2x) = \sin(x)\cos(2x) + \cos(x)\sin(2x) \overset{1.}{\underset{2.}{=}} \sin(x)(\cos^2(x) - \sin^2(x)) + \cos(x)(2\sin(x)\cos(x))$

$= \sin(x)(1 - 2\sin^2(x)) + 2\sin(x)(1-\sin^2(x)) = \sin(x)(1 - 2\sin^2(x) + 2 - 2\sin^2(x)) =$

$= 3\sin(x) - 4\sin^3(x)$

$\cos(3x) = \cos(x+2x) = \cos x \cos 2x - \sin x \sin 2x = \cos(x)(2\cos^2 x - 1) - 2\sin^2(x)\cdot\cos(x)$

$= 2\cos^3(x) - \cos x - 2\cos(x)(1-\cos^2 x) = 2\cos^3(x) - \cos(x) - 2\cos(x) + 2\cos^3(x) = 4\cos^3(x) - 3\cos(x)$

b) i) $\sin(3x) = 3\sin(x) - 4\sin^3(x) \Rightarrow \sin(\pi) = 3\sin\left(\frac{\pi}{3}\right) - 4\sin^3\left(\frac{\pi}{3}\right) = 0 \Leftrightarrow \sin\left(\frac{\pi}{3}\right)\left(3 - 4\sin^2\left(\frac{\pi}{3}\right)\right) = 0$

$\xrightarrow{\sin\frac{\pi}{3} \neq 0} 3 - 4\sin^2\left(\frac{\pi}{3}\right) = 0 \Leftrightarrow \sin^2\left(\frac{\pi}{3}\right) = \frac{3}{4} \Leftrightarrow \sin\frac{\pi}{3} = \pm\sqrt{\frac{3}{4}} \overset{\sin\frac{\pi}{3} > 0}{\Leftrightarrow} \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos^2\left(\frac{\pi}{3}\right) = 1 - \sin^2\left(\frac{\pi}{3}\right) = 1 - \frac{3}{4} = \frac{1}{4} \Leftrightarrow \cos\frac{\pi}{3} = \pm\sqrt{\frac{1}{4}} \overset{\cos\frac{\pi}{3} > 0}{\Longrightarrow} \cos\frac{\pi}{3} = \frac{1}{2}$

ii) $\cos\left(\frac{\pi}{3}\right) = 1 - 2\sin^2\left(\frac{\pi}{6}\right) \Leftrightarrow \sin^2\frac{\pi}{6} = \frac{1-\frac{1}{2}}{2} = \frac{1}{4} \Rightarrow \sin\left(\frac{\pi}{6}\right) = \pm\sqrt{\frac{1}{4}} \overset{\sin\left(\frac{\pi}{6}\right) > 0}{=} \frac{1}{2}$

$\cos^2\left(\frac{\pi}{6}\right) = 1 - \sin^2\left(\frac{\pi}{6}\right) = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow \cos\frac{\pi}{6} = \pm\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$ $\left(\cos\frac{\pi}{6} > 0\right)$

iii) $\cos\left(\frac{\pi}{6}\right) = 1 - 2\sin^2\left(\frac{\pi}{12}\right) \Leftrightarrow \sin^2\frac{\pi}{12} = \frac{1-\frac{\sqrt{3}}{2}}{2} = \frac{2-\sqrt{3}}{4} \Rightarrow \sin\frac{\pi}{12} = \pm\sqrt{\frac{2-\sqrt{3}}{4}} \overset{>0}{=} \frac{\sqrt{2-\sqrt{3}}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\cos^2\frac{\pi}{12} = 1 - \sin^2\frac{\pi}{12} = 1 - \frac{2-\sqrt{3}}{4} = \frac{4-2+\sqrt{3}}{4} = \frac{2+\sqrt{3}}{4} \Leftrightarrow \cos\frac{\pi}{12} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$