Deckblatt für die Abgabe der Übungsaufgaben
IngMathC2

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Blatt-Nummer: 01

Übungsgruppen-Nr: 07

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A1, A2, A3, ______

23/24 * 33=31.5

(11)

a) $M = [\sqrt{3}, \sqrt{5})$ $\inf(M) = \sqrt{3}$, $\sup(M) = \sqrt{5}$,

$\min(M) = \sqrt{3}$, $\max(M)$ existiert nicht.

b) $M = \{\frac{1}{1+x} \mid \frac{1}{2} \le x \le 3\}$ $\inf(M) = \frac{1}{4}$, $\sup(M) = \frac{2}{3}$,

$\min(M) = \frac{1}{4}$, $\max(M) = \frac{2}{3}$.

c) $M = \{\frac{1}{1+n} \mid n \in \mathbb{N}\}$ $\inf(M) = 0$, $\sup(M) = \frac{1}{2}$,

$\min(M)$ existiert nicht, $\max(M) = \frac{1}{2}$.

d) $M = \{x \in \mathbb{R} \mid x^2 - 2x + 3 > 0\}$ $\inf(M) = -\infty$, $\sup(M) = +\infty$

$\min(M)$ und $\max(M)$ existieren nicht.

e) $M = \{\frac{r}{q} \mid r, q \in \mathbb{N}, r \le q\}$ $\inf(M) = 0$, $\sup(M) = 1$,

$\min(M)$ existiert nicht, $\max(M) = 1$,

f) $M = \{n - \frac{1}{3n} \mid n \in \mathbb{N}\}$ $\inf(M) = \frac{2}{3}$, $\sup(M) = +\infty$,

$\min(M) = \frac{2}{3}$, $\max(M)$ existiert nicht.

g) $M = \{n + \frac{1}{3m} \mid m, n \in \mathbb{N}\}$ $\inf(M) = \frac{2}{3}$ 1 $\sup(M) = +\infty$,

~~(abhängig von der Folge)~~ $\min(M) = \frac{2}{3}$, ~~$\sup(M)$~~

$\max(M)$ existiert nicht.

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(2) $2n \le m \le 3n$

(i) $\dfrac{3n+4m}{5n^2+10} \le \dfrac{3n+12n}{5n^2+10} = \dfrac{15n}{5n^2+10}$

(ii) $\dfrac{5n-m}{2n} \le \dfrac{5n-2n}{2n} = \dfrac{3}{2}$

(iii) $\dfrac{n}{n+m} \le \dfrac{n}{n+2n} = \dfrac{n}{3n} = \dfrac{1}{3}$

(iv) $\dfrac{n+m}{\frac{1}{2}-n} \le \dfrac{n+2n}{\frac{1}{2}-n} = \dfrac{3n}{\frac{1}{2}-n}$

$\leftarrow$ (da negativ)

(v) $\dfrac{5n-m+3\cdot 2^m}{3n^3-m+3} \le \dfrac{5n-2n+3\cdot 2^m}{3n^3-m+3} \le \dfrac{5n-2n+3\cdot 2^{3n}}{3n^3-m+3}$

$\le \dfrac{3\cdot(n+2^{3n})}{3n^3-3n+3} = \dfrac{n+2^{3n}}{n^3-n+1}$

$\leftarrow$ ((da $3n^3+3 \ge m$, samlet positiv) $\wedge$ $3n^3 \ge 3n$)

(vi) $m+n+\sin(m)-\sin(17m^2)+2^m+2^{-m} \le$

$3n+n+1-\sin(17m^2)+2^m+2^{-m} \le$

$4n+1-(-1)+2^m+2^{-m} \le$

$4n+2+2^{3n}+2^{-2n}$

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(13)

a) i) $\frac{a_{n+1}}{a_n} = \frac{\frac{2(n+1)}{n+1+3}}{\frac{2n}{n+3}} = \frac{2(n+1)\cdot(n+3)}{(n+4)\cdot 2n}$

$= \frac{n^2+3n+n+3}{n^2+4n} = \frac{(n^2+4n)+3}{(n^2+4n)} = 1+\frac{3}{n^2+4n} \geq 1$

$\Rightarrow$ monoton steigend

ii) $b_{n+1} - b_n = \frac{(n+1)}{4^{(n+1)}} - \frac{n}{4^n} = \frac{4^n(n+1) - 4^{(n+1)}(n)}{4^{(n+1)}4^n}$

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$= \frac{4^n(n+1) - 4^n(4n)}{4^{(n+1)}4^n} = \frac{4^n((n+1)-4n)}{4^{(n+1)}4^n}$

$= \frac{n+1-4n}{4^{(n+1)}} = \frac{1-3n}{4^{(n+1)}} \leq 1$ da $n \geq 1$

$\Rightarrow 1-3n =$ negativ

$\Rightarrow$ monoton fallend

b) i) $\left[\lim_{n\to\infty} a_n = 2\right]$, ii) $\left[\lim_{n\to\infty} b_n = 0\right]$

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(A3)

c) $\forall \epsilon > 0 \; \exists n_0 \in \mathbb{N} \; \forall n \geq n_0 : |a_n - a| \leq \epsilon$

i) $|a_n - a| = \left|\frac{2n}{n+3} - 2\right| = \left|\frac{2n - 2(n+3)}{n+3}\right| = \left|\frac{-6}{n+3}\right|$

$= \frac{6}{n+3} \leq \epsilon \quad \Rightarrow \frac{6}{\epsilon} \leq n+3 \Leftrightarrow \frac{6}{\epsilon} - 3 \leq n$

$\Rightarrow n_0 = \frac{6}{\epsilon}$ ✓

In diesem Fall besser aufschreiben, obauf o

Sei $\epsilon > 0$ beliebig vorgegeben. Setze $n_0 = \frac{6}{\epsilon}$

$\Rightarrow n \geq n_0 : |a_n - a| = \left|\frac{2n}{n+3} - 2\right| = \left|\frac{2n - 2(n+3)}{n+3}\right|$

$= \left|\frac{-6}{n+3}\right| = \frac{6}{n+3} \overset{(1)}{\leq} \frac{6}{n_0+3} = \frac{6}{\frac{6}{\epsilon}+3} \leq \frac{6}{\frac{6}{\epsilon}} = \epsilon \cdot \frac{6}{6} = \epsilon$ ✓

ii) $|b_n - b| = \left|\frac{n}{4^n} - 0\right| = \frac{n}{4^n} \leq \epsilon$

, da $n \leq 2^n \Rightarrow \frac{2^n}{4^n} \leq \epsilon$

$\Leftrightarrow \left(\frac{1}{2}\right)^n \leq \epsilon \Leftrightarrow \frac{1}{2^n} \leq \epsilon \Leftrightarrow \frac{1}{\epsilon} \leq 2^n$

$\Leftrightarrow \operatorname{ld} \frac{1}{\epsilon} \leq n$

$\Rightarrow n_0 = \lceil \operatorname{ld} \frac{1}{\epsilon} \rceil$ ✓

Sei $\epsilon > 0$ beliebig vorgegeben. Setze $n_0 = \lceil \operatorname{ld} \frac{1}{\epsilon} \rceil$

$\Rightarrow n \geq n_0 : |b_n - b| = \frac{n}{4^n} = \frac{n_0}{4^{n_0}} = \frac{n_0}{2^{2n_0}}$

$\cancel{= \frac{\lceil \operatorname{ld} \frac{1}{\epsilon} \rceil}{2^{2 \cdot \lceil \operatorname{ld} \frac{1}{\epsilon} \rceil}} \leq \frac{\lceil \operatorname{ld} \frac{1}{\epsilon} \rceil}{2^{2 \cdot \operatorname{ld} \frac{1}{\epsilon}}} = \frac{\lceil \operatorname{ld} \frac{1}{\epsilon} \rceil}{2^{\operatorname{ld}(\frac{1}{\epsilon})^2}} = \frac{\lceil \operatorname{ld} \frac{1}{\epsilon} \rceil}{(\frac{1}{\epsilon})^2}}$

$\leq \frac{2^{n_0}}{2^{2n_0}} = \frac{2^{\lceil \operatorname{ld} \frac{1}{\epsilon} \rceil}}{2^{2\lceil \operatorname{ld} \frac{1}{\epsilon} \rceil}} \leq \frac{2^{\operatorname{ld} \frac{1}{\epsilon}}}{4^{\operatorname{ld} \frac{1}{\epsilon}}} = \frac{\frac{1}{\epsilon}}{2^{\operatorname{ld}(\frac{1}{\epsilon})^2}} = \frac{\frac{1}{\epsilon}}{\frac{1}{\epsilon^2}} = \epsilon$ ✓

das ist viel komplizierte als es sein muss: 2^n/2^(2n) = 1/2^n <= 1/2