Deckblatt f. d. Abgabe d. Übungsaufgaben
Ing. Mathe C2

Name, Vorname: Gauert, Vasily

Studen. Kennung: og65akyh

Blatt-Nummer: 07

Übungsgruppe-Nr.: 07

Die folgenden Aufgaben gebe ich zur Korrektur frei:
A18, A19, A20

16.5/20*30=24.5

(A 11)

$$\left(f = x^{-\frac{1}{2}} \quad f' = -\frac{1}{2} x^{-1\frac{1}{2}} \qquad -\frac{1}{x^2} \qquad -\frac{2x}{x^4} = -\frac{2}{x^3} \right)$$

a) $f'(x) = 2x + 1 + \frac{1}{2x^{\frac{3}{2}}} + \left(-\frac{1}{2\sqrt{x^3}}\right) + \left(-\frac{1}{x^2}\right) + \left(-\frac{2}{x^3}\right)$ ✓

$$\left(\cancel{x^{-\frac{1}{2}}} \ \frac{1}{\sqrt{x}} = \frac{-\sqrt{x}'}{x} = -\frac{1}{2\sqrt{x} \cdot x} = \frac{1}{2\sqrt{x^3}} \right)$$

b) $f'(x) = \Big((\underbrace{x^2 + \sqrt{2x}}_{g(x)})^4 \Big)' = 4(x^2 + \sqrt{2x})^3 \cdot (2x + \sqrt{2} \cdot \sqrt{x}')$

$= 4(x^2 + \sqrt{2x})^3 \cdot \left(2x + \frac{\sqrt{2}}{2\sqrt{x}}\right)$

$= 4(x^2 + \sqrt{2x})^3 \cdot \left(2x + \frac{1}{\sqrt{2x}}\right)$ ✓

c) $f'(x) = e^{x^2} \ln(2+3x) + x e^{x^2} \cdot 2x \cdot \ln(2+3x) + x e^{x^2} \cdot \frac{1}{2+3x} \cdot 3$

$= e^{x^2} \ln(2+3x) + 2x^2 e^{x^2} \cdot \ln(2+3x) + \frac{3x}{2+3x} \cdot e^{x^2}$

$= e^{x^2} \left(\ln(2+3x) + 2x^2 \cdot \ln(2+3x) + \frac{3x}{2+3x} \right)$ ✓

d) $f(x) = \arccos(\sqrt{x}) \quad (0 < x < 1)$

$$\left((f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))} \right)$$

$\Rightarrow f'(x) = \frac{1}{-\sin(\arccos(x))} \cdot \ldots$ P24-a verwenden

$= \frac{1}{-\sqrt{1 - \cancel{x^2}(x^2+1)^2}} \cdot \cancel{2x} \ 2x$

$= \frac{2x}{-\sqrt{x^4 + 2x^2}} = \frac{2x}{\sqrt{x^2(x^2+2)}} = \frac{2}{\sqrt{x^2+2}}$

e) $f(x) = \frac{\sin 2x}{\ln(x^2+1)}, \quad x \neq 0 \Rightarrow f' = \frac{f'(x) g(x) - f(x) g'(x)}{g(x)^2}$

$= \frac{\cancel{\sin} \cos 2x \cdot 2 \cdot \ln(x^2+1) - \sin 2x \frac{1}{(x^2+1)} \cdot 2x}{\ln(x^2+1)^2} = \cancel{2(2(\cos(2x) \cdot \ln(x}$

$= 2\left(\frac{\cos(2x)}{\ln(x^2+1)} - \frac{\sin(2x)\,x}{(x^2+1)(\ln(x^2+1))^2} \right)$ ✓ ✗

f) $f(x) = x^{\alpha}$

$= e^{\alpha \ln x}$ $\Rightarrow f'(x) = e^{\alpha \ln x} \cdot \alpha \cdot \frac{1}{x}$

$= x^{\alpha} \cdot \alpha \cdot \frac{1}{x} = \alpha \cdot \frac{x^{\alpha}}{x} = \underline{\underline{\alpha \cdot x^{\alpha - 1}}}$

g) $f(x) = x^{-x^2} = e^{-x^2 \ln x}$

$f'(x) = e^{-x^2 \ln x} \cdot \left(-2x \ln x + \cancel{\ln} - \frac{x^2}{x}\right)$

$= x^{-x^2} \cdot \left(-2x \ln x + \cancel{\ln} - x\right)$

$= x^{-x^2} \cdot (-x)(2\ln x + 1) = \underline{\underline{-x^{1-x^2} \cdot (2\ln x + 1)}}$

h) $f(x) = \ln(x + \ln(2\ln x))$ $\quad x \geq \sqrt{e}$

$f'(x) = \frac{1}{x + \ln(2\ln x)} \cdot \left(1 + \frac{1}{2\ln x} \cdot \frac{2}{x}\right) = \frac{1}{x + \ln(2\ln x)} \cdot \left(1 + \frac{1}{x\ln x}\right)$

$= \frac{1}{x + \ln(2\ln x)} + \frac{1}{x^2 \ln(x) + x\ln(x)\ln(2\ln x)} =$

$= \frac{1}{x + \ln(2\ln x)} + \frac{1}{x(\ln x)(x + \ln(2\ln x))}$

$= \frac{x(\ln(x)) + 1}{x(\ln(x))(x + \ln(2\ln(x)))}$

Gank

A19

$$\frac{d}{dx}\cos(x) = \lim_{h\to 0}\left(\frac{\cos(x+h)-\cos(x)}{h}\right) = \lim$$

$$= \lim_{h\to 0}\left(\frac{\cos(x)\cos(h)-\cos(x)-\sin(x)\sin(h)}{h}\right)$$

$$= \lim_{h\to 0}\left(\cos(x)\cdot\left(\frac{\cos h - 1}{h}\right) - \sin(x)\cdot\left(\frac{\sin(h)}{h}\right)\right)$$

$\stackrel{!}{=}$ [~~beschränkt~~ beschränkt] $\to 0$ $\qquad \to 1$ (nach Vorlesung)

$\stackrel{!}{=}$ ~~cos(x)~~

$$= -\sin(x)\cdot 1 \,(+\cos(x)\cdot 0) = \underline{\underline{-\sin(x)}}$$

b) (I) $\tan'(x) = \left(\frac{\sin(x)}{\cos(x)}\right)' = \frac{(\cos(x))^2 + (\sin(x))^2}{(\cos(x))^2} = \underline{\underline{\frac{1}{(\cos(x))^2}}}$

(II) $\Leftrightarrow \frac{(\cos(x))^2}{(\cos(x))^2} + \left(\frac{\sin(x)}{\cos(x)}\right)^2 = \underline{\underline{1+(\tan(x))^2}}$

c) $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$

(I) $\Rightarrow \arctan'(x) = \frac{1}{\tan'(\arctan(x))} = \frac{1}{1+(\tan(\arctan(x)))^2}$

$$= \underline{\underline{\frac{1}{1+x^2}}}$$

d) $\tan'' = \left(\frac{1}{\cos^2(x)}\right)' = \frac{-2}{(\cos(x))^3}\cdot(-\sin(x)) = \underline{\underline{\frac{2\sin(x)}{(\cos(x))^3}}}$ $= \cancel{\frac{2\tan(x)}{\cos(x)^2}}$

$$\tan''' = \left(\frac{2\sin(x)}{(\cos(x))^3}\right)' = 2\cdot\left(\frac{\sin(x)}{(\cos(x))^3}\right)'$$

$$= 2\cdot\left(\frac{\cos(x)(\cos(x))^3 + \sin(x)\,3(\cos(x))^2\overbrace{\sin(x)}^{((-)\cdot(-))=(+)}}{(\cos(x))^6}\right)$$

$$= \frac{2}{\cos(x)^2} + \frac{6(\sin(x))^2}{\cos(x)^4} \Leftrightarrow \underline{\underline{\frac{2\cos(x)+6(\sin(x))^2}{4\cos(x)^4}}}$$

Gerd

A20

a) $f'(x) = \alpha x^{\alpha-1} \cdot \sin\left(\frac{1}{x^2}\right) + x^{\alpha} \cos\left(\frac{1}{x^2}\right) \cdot \frac{(-2)}{x^3}$

$= \alpha x^{\alpha-1} \cdot \sin\left(\frac{1}{x^2}\right) - x^{\alpha-1} \cos\left(\frac{1}{x^2}\right) \cdot \frac{(-2)}{x^2}$

$= x^{\alpha-1}\left(\alpha \sin\left(\frac{1}{x^2}\right) - 2\cos\left(\frac{1}{x^2}\right)\right)$

b) $f'(x)$ nach Def. bei $x = 0$, da [illegible] ist:

$\to 0$ $\to \alpha$ (beschränkt)

$\lim\limits_{h \to 0}\left(\frac{(0+h)^{\alpha} \sin\left(\frac{1}{(0+h)^2}\right) - 0 \sin\left(\frac{1}{0}\right)}{h}\right)$

$= \lim\limits_{h \to 0} \frac{h^{\alpha} \sin\left(\frac{1}{h^2}\right)}{h} = \lim\limits_{h \to 0} h^{\alpha-1} \cdot \sin \frac{1}{h^2}$

Sei $\alpha = 1$ $\Rightarrow$ $\left[h^0 \to 1 \wedge \lim\limits_{h \to 0} 1 \cdot \sin \frac{1}{h^2}\right.$ ~~ist~~ existiert nicht. $\left.\right]$

Sei $\alpha \in (0, 1)$ $\Rightarrow$ $\left[h^{\alpha-1}\right.$ hat nach Hinweis keinen Limes, existiert nicht. $\left.\right]$

Sei $\alpha \in (1, \infty)$ $\Rightarrow$ $\left[h^{\alpha-1} \xrightarrow{h \to 0} 0 \right.$

$\Rightarrow \lim\limits_{h \to 0} f(0) = 0 \cdot$ (etwas Beschränktes)

$\left. = 0 \right]$

c) ~~f'(0)~~ Existiert für $\alpha \in (1, \infty)$.

$\underline{f'(0) = 0.}$ Nun Betrachtung der Limes $x \to 0$

$$\lim_{x \to 0} f'(x) = \lim_{x \to 0} \underbrace{x^{\alpha-1}}_{\to 0 \text{ da } \alpha \in (1,\infty)} \underbrace{\Big(\underbrace{\alpha \sin\left(\tfrac{1}{x^2}\right)}_{\text{beschränkt}} - \underbrace{2\cos\left(\tfrac{1}{x^2}\right)}_{\text{beschränkt}} \cdot \tfrac{1}{x^2}\Big)}_{\to 0} \to 0$$

$\Rightarrow f'(0)$ ist stetig.

(*) Nach Umstellung $f'(0)$ stetig nur für $\alpha \in (3, \infty)$.

$\alpha \in (1, 3] \Rightarrow$ unstetig.

d) $f''(x) = (f')'(x) = \left(x^{\alpha-1}\left(\alpha \sin\left(\frac{1}{x^2}\right) - 2\cos\left(\frac{1}{x^2}\right)\cdot \frac{1}{x^2}\right)\right)'$

$= \left(x^{\alpha-1} \cdot \alpha \cdot \sin\left(\frac{1}{x^2}\right) - 2\cos\left(\frac{1}{x^2}\right) \cdot x^{\alpha-3}\right)'$

$= \alpha \cdot \left((\alpha-1)\, x^{\alpha-2} \cdot \sin\left(\frac{1}{x^2}\right) + x^{\alpha-1} \cdot \cos\left(\frac{1}{x^2}\right) \cdot \frac{(-2)}{x^3}\right)$

$- 2\left(\sin\left(\frac{1}{x^2}\right) \cdot \frac{(+2)}{x^3} \cdot x^{\alpha-3} + 2\cos\left(\frac{1}{x^2}\right) \cdot (\alpha-3)\, x^{\alpha-4}\right)$

$= \alpha x^{\alpha-2}\left((\alpha-1)\sin\left(\frac{1}{x^2}\right) - \frac{2\cos\left(\frac{1}{x^2}\right)}{x^2}\right)$

$- 2x^{\alpha-4}\left(\sin\left(\frac{1}{x^2}\right) \cdot \frac{2}{x^2} + 2\cos\left(\frac{1}{x^2}\right) \cdot (\alpha-3)\right)$

Gauert