Mathe Übung   zur Korrektur: nur A19 und A18   0174 umad

20/20*30

A18

a) $f'(x) = 2x + 1 + \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-1{,}5} - \frac{1}{x^2} - 2x^{-3} =$

$= 2x + 1 + \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x}^3} - \frac{1}{x^2} - \frac{2}{x^3}$

b) $f'(x) = 4(x^2 + \sqrt{2x})^3 \cdot (2x + \frac{1}{2}\frac{1}{\sqrt{2x}} \cdot 2) = 4(x^2 + \sqrt{2x})^3 \cdot (2x + \frac{1}{\sqrt{2x}})$

c) $f'(x) = e^{x^2}\ln(2+3x) + x e^{x^2} \cdot 2x \ln(2+3x) + x e^{x^2} \cdot \frac{3 \cdot 1}{2+3x} =$

$= e^{x^2}\left(\ln(2+3x) + 2x^2 \ln(2+3x) + \frac{3x}{2+3x}\right)$

d) $f(x) = \arccos(\sqrt{x})$ $f'(x) = -\frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} = -\frac{1}{\sqrt{1-x}\,2\sqrt{x}}$

e) $f(x) = \frac{\sin(2x)}{\ln(x^2+1)}$ $f'(x) = \frac{\ln(x^2+1) \cdot \cos 2x \cdot 2 - (\sin 2x \cdot \frac{1}{x^2+1} \cdot 2x)}{(\ln(x^2+1))^2} =$

$= \frac{\frac{2\cos 2x}{\ln(x^2+1)} - \frac{2x \sin 2x}{x^2+1}}{(\ln(x^2+1))^2} = \frac{2\cos 2x}{\ln(x^2+1)} - \frac{2x \sin 2x}{(x^2+1)(\ln(x^2+1))}$

f) $f(x) = x^\alpha$ $f'(x) = e^{\alpha \ln x} \cdot \frac{\alpha}{x} = x^\alpha \cdot \frac{\alpha}{x} = x^{\alpha - 1} \cdot \alpha$

g) $f(x) = x^{-x^2}$ $f'(x) = e^{\ln x(-x^2)} \cdot (\frac{1}{x}(-x^2) + \ln x(-2x)) = x^{-x^2} \cdot (-x - 2x\ln x)$

h) $f(x) = \ln(x + \ln(2\ln x))$ $f'(x) = \frac{1}{x + \ln(2\ln x)} \cdot (1 + \frac{1}{2\ln x} \cdot \frac{2}{x}) =$

$= \frac{1}{x + \ln(2\ln x)} + \frac{2}{(x + \ln 2\ln x)(2\ln x \cdot x)}$

A19)

a) $\frac{d}{dx}\cos x = \lim_{h \to 0} \frac{\cos(x_0 + h) - \cos(x_0)}{h} = \lim_{h \to 0} \frac{\cos x_0 \cos h - \sin x_0 \sin h - \cos x_0}{h} =$

$= \lim_{h \to 0} \frac{\cos(x_0)(\cosh - 1) - \sin x_0 \cdot \sinh}{h} = \lim_{h \to 0} \cos(x_0)\underbrace{\frac{\cosh - 1}{h}}_{\text{„gegen 0"}} - \sin x_0 \underbrace{\frac{\sinh}{h}}_{\text{„gegen 1"}}$

$= -\sin x_0$

b) $\tan x = \frac{\sin x}{\cos x}$

i) $(\tan x)' = \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

ii) $(\tan x)' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = 1 + \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\cos x} = 1 + (\tan x)^2$

c) i) $\arctan(y)' = \frac{1}{1 + \tan^2 x} = \frac{1}{1 + y^2}$ da $\tan x = y \Rightarrow \tan^2 x = y^2$

ii) $\tan''(x) = (1 + \tan^2 x)' = 2\tan x \cdot (1 + \tan^2 x) = 2\tan x + 2\tan^3 x$

$\tan'''(x) = (2\tan x + 2\tan^3 x)' = 2 + 2\tan^2 x + 6\tan^2 x \cdot (1 + \tan^2 x) =$

$= 2 + 8\tan^2 x + 6\tan^4 x$