Deckblatt für die Abgabe der Übungsaufgaben IngMathC2

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Blatt-Nummer: 4

Übungsgruppen-Nr: 7

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A10, A11, A12,

7/10 *30 = 21

A 10

a) i) $\sum_{k=0}^{\infty} k q^k \quad \sum_{k=0}^{\infty} q^k \quad , \quad |q| < 1$

$$\rightarrow \sum_{n=0}^{\infty} \sum_{k=0}^{n} k q^k \cdot q^{n-k} = \sum_{n=0}^{\infty} \sum_{k=0}^{n} k \cdot q^n = \sum_{n=0}^{\infty} q^n \sum_{k=0}^{n} k = \sum_{n=0}^{\infty} q^n \cdot \frac{n^2+n}{2}$$

ii) $\sum_{k=0}^{\infty} k^2 q^k$ für $|q| < 1$, $\sum_{k=0}^{\infty} q^k = \frac{1}{1-q}$ (1)

$$\sum_{k=0}^{\infty} k \cdot q^k = \frac{q}{(1-q)^2} \quad (2)$$

aus i): $\sum_{k=0}^{\infty} \left(\frac{k^2 q^k}{2} + \frac{k q^k}{2} \right) = \sum_{k=0}^{\infty} k q^k \cdot \sum_{k=0}^{\infty} q^k$ (3)

$$\rightarrow \sum_{k=0}^{\infty} k^2 q^k = 2 \cdot \left(\sum_{k=0}^{\infty} \left(\frac{1}{2} k^2 q^k + \frac{1}{2} k q^k \right) \right) - \sum_{k=0}^{\infty} k q^k$$

$$\overset{(3)}{=} 2 \cdot \left(\sum_{k=0}^{\infty} k q^k \cdot \sum_{k=0}^{\infty} q^k \right) - \sum_{k=0}^{\infty} k q^k \overset{(1),(2)}{=} 2 \cdot \left(\frac{q}{(1-q)^2} \cdot \frac{1}{1-q} \right) - \frac{q}{(1-q)^2}$$

$$= 2 \cdot \frac{q}{(1-q)^3} - \frac{q-q^2}{(1-q)^3} = \frac{q^2+q}{(1-q)^3}$$

b) $P_n = \sum_{k=0}^{n} \frac{1}{(k+1)(k+2)} = \sum_{k=0}^{n} \frac{A}{k+1} + \frac{B}{k+2}$

$\rightarrow 1 = A \cdot (k+2) + B \cdot (k+1)$

$1 = (A+B)k + 2A + B \quad \rightarrow A + B = 0 , \quad 2A + B = 1$

$\rightarrow A = 1 , \quad B = -1$

$$\rightarrow \sum_{k=0}^{n} \frac{1}{k+1} - \frac{1}{k+2} = \sum_{k=0}^{n} \frac{1}{k+1} - \sum_{k=0}^{n} \frac{1}{k+2}$$

nach Indextransformation:

$$\sum_{-1}^{n-1} \frac{1}{k+2} - \sum_{k=0}^{n} \frac{1}{k+2} = \frac{1}{-1+2} - \frac{1}{n+2} = 1 - \frac{1}{n+2}$$

A 11

a) i) $\sum_{k=0}^{\infty} \frac{5^k}{k} x^k$, $R = \frac{1}{\limsup_k \sqrt[k]{\left|\frac{5^k}{k}\right|}} = \frac{1}{\limsup_k \sqrt[k]{5^k \cdot k^{-1}}} = \frac{1}{\frac{5}{\sqrt[k]{k}}}$ ($\limsup_k$)

$= \frac{1}{5}$ ✓

Aufpassen, das gilt nur, weil der limes sup hier gleich ist, i.A.

A 11 a)

ii) $\sum_{k=0}^{\infty} \left(\sqrt{k+1} - \sqrt{k - \sqrt{k}}\right)^{2k} x^k$

$\sqrt[k]{|a_k|} = \sqrt[k]{\left|\left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^{2k}\right|} = \left(\sqrt{k+1} - \sqrt{k-\sqrt{k}}\right)^2$

$= k+1 - 2\sqrt{(k+1)(k-\sqrt{k})} + k - \sqrt{k} = 2k + 1 - 2\sqrt{k^2 - \sqrt{k^3} + k - \sqrt{k}} - \sqrt{k}$

$= 2k + 1 - 2k\sqrt{1 - \frac{\sqrt{k}}{k} + \frac{1}{k} - \sqrt{\frac{1}{k^3}}} - \sqrt{k} \xrightarrow{k\to\infty} \limsup_k 1 - \sqrt{k}$

$= -\infty$

aus infty-infty kann man nichts folgern, weiter umform

$\rightarrow R = "\frac{1}{-\infty}" = \underline{\underline{0}}$

iii) $\sum_{k=0}^{\infty} (k!+2)\, x^k$, $\sqrt[k]{|a_k|} = \sqrt[k]{|k!+2|}$

$\limsup_k \sqrt[k]{|k!+2|} = \limsup_k \sqrt[k]{k!+2} > \limsup_k \sqrt[k]{k!} = +\infty$

$\rightarrow R = "\frac{1}{+\infty}" = \underline{\underline{0}}$

iv) $\sum_{k=0}^{\infty} \frac{2^k}{k^2} x^{4k} = \sum_{\bar{k}=0}^{\infty} a_{\bar{k}} x^{\bar{k}}$ wobei $a_{\bar{k}} = \begin{cases} 0 & \text{für } \bar{k} \text{ modulo } 4 \neq 0 \\ \frac{2^{\frac{\bar{k}}{4}}}{\left(\frac{\bar{k}}{4}\right)^2} & \text{für } \bar{k} \text{ modulo } 4 = 0 \end{cases}$

$\rightarrow \sqrt[\bar{k}]{\left|\frac{2^{\frac{\bar{k}}{4}}}{\left(\frac{\bar{k}}{4}\right)^2}\right|} = \left|\frac{2^{\frac{1}{4}}}{\frac{\bar{k}}{\sqrt[\bar{k}]{16}}}\right| = \frac{\sqrt[4]{2} \cdot \sqrt[\bar{k}]{16}}{\bar{k}} \xrightarrow{\bar{k}\to\infty} 0$

k^(2/k)

$\rightarrow R = "\frac{1}{0}" = \underline{\underline{+\infty}}$

b) $\sum_{k=0}^{\infty} \left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1\right)^k \left(\frac{1}{x+3}\right)^k$, $\left(\frac{1}{x+3}\right)^k = y^k$

$\rightarrow \sqrt[k]{|a_k|} = \sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1 \xrightarrow{k\to\infty} 1 + 0 + 1 = 2$

$\rightarrow |y| = \left|\frac{1}{x+3}\right| < \cancel{2}\ \frac{1}{2} \Leftrightarrow$ Potenzreihe konvergiert

konvergiert im konverger

$\rightarrow |x+3| > \frac{1}{2} \quad \rightarrow x_1 < -2{,}5 < x_2$

$\rightarrow$ Die größtmögliche Menge, auf der die Potenzreihe konvergent ist, ist $(-\infty, -2{,}5) \cup (-2{,}5, +\infty)$

A12 a) $\sin(3x)$, $\cos(3x)$

i) $\exp(3ix) = \exp(ix)^3$

-> Euler: $\cos(3x) + i\sin(3x) = (\cos(x) + i\sin(x))^3$

$= (\cos x)^3 - 3\cos x(\sin x)^2 + i(-(\sin x)^3 + 3(\cos x)^2 \sin x)$

Realteil:

$\cos(3x) = \cos^3 x - 3\sin^2 x \cos x$

$\sin(3x) = -\sin^3 x + 3\sin x \cos^2 x$

-> $\cos(3x) = \cos^3 x - 3\sin^2 x \cos x$

$\sin(3x) = -(\sin^3 x) + 3\sin x \cos^2 x$

ii) $\sin 3x = \sin(2x+x) = \sin(2x)\cdot\cos x + \cos(2x)\cdot\sin x$

$\sin 2x = \sin(x+x) = \sin x\cos x + \cos x \sin x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\sin 3x = 2\sin x\cos x\cdot\cos x + (\cos^2 x - \sin^2 x)\sin x$

$= 2\sin x\cos^2 x + \sin x\cos^2 x - \sin^3 x$

$= 3\sin x\cos^2 x - \sin^3 x$

$\cos 3x = \cos(2x+x) = \cos 2x\cos x - \sin 2x\sin x$

$= (\cos^2 x - \sin^2 x)\cos x - 2\sin^2 x\cos x$

$= \cos^3 x - \sin^2 x\cos x - 2\sin^2 x\cos x$

$= \cos^3 x - 3\sin^2 x\cos x$

b) i) $\sin\frac{\pi}{3}$, $\sin 3x = 3\sin x - 4\sin^3 x$

-> einsetzen: $3\sin\frac{\pi}{3} - 4\sin^3\frac{\pi}{3} = \sin 3\frac{\pi}{3} = 0$

$\sin\frac{\pi}{3} = y$, $3\sin\frac{\pi}{3} - 4\sin^3\frac{\pi}{3} = 0$

$3\cancel{\sin}\, y - 4\cancel{\sin}\, y^3 = 0$

$3y = 4y^3$

$3 = 4y^2$

$\frac{3}{4} = y^2$

-> da $y > 0$ laut Angabe, ist $\sin\frac{\pi}{3}$ also $\sqrt{y^2} = \frac{1}{2}\sqrt{3}$

$$\cos^2\frac{\pi}{3} = 1 - \sin^2\frac{\pi}{3} = 1 - \frac{3}{4} = \frac{1}{4} \iff \cos\frac{\pi}{3} = \frac{1}{2}$$

ii) $\sin\frac{\pi}{3} = \sin 2\frac{\pi}{6} = \frac{1}{2}\sqrt{3}$, $\cos\frac{\pi}{3} = \cos 2\frac{\pi}{6} = \frac{1}{2}$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

$\rightarrow \frac{1}{2} = 2\left(\cos\frac{\pi}{6}\right)^2 - 1 = 2y^2 - 1 \qquad \left(y = \cos\frac{\pi}{6}\right)$

$$2y^2 = \frac{3}{2}$$

$$y^2 = \frac{3}{4}$$

$$y = \frac{1}{2}\sqrt{3}$$

$\rightarrow \cos\frac{\pi}{6} = \frac{1}{2}\sqrt{3}$, $\sin\frac{\pi}{6} = \frac{1}{2}$

iii) $\sin\frac{\pi}{6} = \sin 2\frac{\pi}{12} = \frac{1}{2}$, $\cos\frac{\pi}{6} = \cos 2\frac{\pi}{12} = \frac{1}{2}\sqrt{3}$

$\cos 2x = 2\cos^2 x - 1$, $y := \cos\frac{\pi}{12}$

$$\frac{\sqrt{3}}{2} = 2y^2 - 1$$

$$y^2 = \frac{\sqrt{3}+2}{4}$$

$$y = \frac{\sqrt{\sqrt{3}+2}}{2} = \cos\frac{\pi}{12} = \frac{1}{2}\sqrt{\sqrt{3}+2}$$

$\sin\frac{\pi}{12}$: $\quad \sin^2\frac{\pi}{12} = 1 - \cos^2\frac{\pi}{12} = 1 - \frac{\sqrt{3}+2}{4} = \frac{2-\sqrt{3}}{4}$

$\rightarrow \sin\frac{\pi}{12} = \sqrt{\frac{2-\sqrt{3}}{4}} = \sqrt{\frac{2-\sqrt{3}}{\cancel{4}}}\,\frac{1}{2}$