Deckblatt für die Abgabe der Übungsaufgaben
IngMathC2

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Blatt-Nummer: 3

Übungsgruppen-Nr: 7

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A7, A8, ____, ____

9.5/10*20=19

A7)

a) i) $a_n \frac{5+(-1)^n + \frac{1}{n}\sin n}{n^2}$ $\frac{5-1-\frac{1}{n}}{n^2} \leq a_n \leq \frac{5+1+\frac{1}{n}}{n^2}$

$\lim_{n\to\infty} \frac{4-\frac{1}{n}}{n^2} = 0$ $\lim_{n\to\infty} \frac{6+\frac{1}{n}}{n^2} = 0$

$\lim_{n\to\infty} a_n = 0$

ii) $b_n = \frac{n}{n^2+1} \cdot \frac{5\sin(2n) - 2\sin(3n)}{6+\cos(4n)-\cos(5n)}$

$\frac{-5-2}{6-1-1} \cdot \frac{n}{n^2+1} \leq b_n \leq \frac{5+2}{6-1-1} \cdot \frac{n}{n^2+1}$

$-\frac{7}{4} \cdot \frac{n}{n^2+1} \leq b_n \leq \frac{7}{4} \cdot \frac{n}{n^2+1}$

$\lim_{n\to\infty} -\frac{7}{4} \frac{n}{n^2+1} = -\frac{7}{4} \cdot 0 = 0$ $\lim_{n\to\infty} \frac{7}{4} \cdot \frac{n}{n^2+1} = 0$ $\lim_{n\to\infty} b_n = 0$

b) i) $a_n = ((-1)^n + 1)n$ $M = \{0, +\infty\}$ $\lim_n \sup a_n = +\infty$

ii) $a_n = \sin\left(\frac{\pi n}{2}\right) + \cos\left(\frac{\pi n}{2}\right)$ $M = \{-1, 1\}$ $\lim_n \inf a_n = -1$ $\lim_n \sup a_n = 1$

iii) $a_n = \{-n \text{ wenn } n \leq 17, n \text{ wenn } n > 17\}$ $M = \{+\infty\} = \lim_n \inf = \lim_n \sup$

iv) $a_n = q^n$ $q < -1$ $M = \{-\infty, +\infty\}$ $\lim_n \inf = -\infty$ $\lim_n \sup +\infty$

$-1 < q < 0$ $M = \{0\}$ $\lim_n \inf = \lim_n \sup = 0$

$q = -1$ $M = \{-1, 1\}$ $\lim_n \inf = -1$ $\lim_n \sup = 1$

$q = 0$ $\lim_n \inf = \lim_n \sup = 0$ $M = \{0\}$

$0 < q < 1$ $M = \{0\}$ $\lim_n \inf = 0 = \lim_n \sup$

$q = 1$ $M = \{1\}$ $\lim_n \inf = \lim_n \sup = 1$

$q \geq 1$ $M = \{+\infty\}$ $\lim_n \inf = \lim_n \sup = +\infty$

A8) a) $\sum_{k=0}^{\infty} \frac{k}{2+k}$ $\quad \lim_{k\to\infty} \frac{k}{2+k} = \lim_{k\to\infty} \frac{1}{1+\underbrace{\frac{2}{k}}_{0}} = 1$

a_k Keine Nullfolge $\Rightarrow$ divergent

b) $\sum_{k=2}^{\infty} \left(\frac{k-1}{3k^2+2k}\right)^{\frac{k}{2}}$ $\quad$ Wurzelkriterium $\left(\sqrt[k]{\left|\frac{k-1}{3k^2+2k}\right|}\right)^{\frac{k}{2}}$

$\Rightarrow$ Reihe konvergent $\quad = \sqrt{\frac{k-1}{3k^2+2k}} \le \frac{1}{4} \quad (k=2) \le 1$

c) $\sum_{k=0}^{\infty} \frac{\sin k}{k^k}$ $\quad$ Wurzelkrit. $\sqrt[k]{\left|\frac{\sin k}{k^k}\right|} = \frac{\sqrt[k]{|\sin k|}}{k} \le \frac{1}{k} \le 1$

$\Rightarrow$ Reihe konvergent

d) $\sum_{k=1}^{\infty} \frac{\sqrt{k+2} - \sqrt{k-1}}{2^k} = \sum_{k=1}^{\infty} \frac{3}{2^k\left(\sqrt{k+2}+\sqrt{k-1}\right)}$ $\quad$ Quotientkriterium

$$\left|\frac{3}{2^{k+1}\left(\sqrt{k+3}+\sqrt{k}\right)} \cdot \frac{2^k\left(\sqrt{k+2}+\sqrt{k-1}\right)}{3}\right| = \frac{1}{2} \cdot \frac{\sqrt{k+2}+\sqrt{k-1}}{\sqrt{k+3}+\sqrt{k}}$$

$$= \frac{1}{2} \cdot \frac{\sqrt{k}\left(\sqrt{1+\frac{2}{k}}+\sqrt{1-\frac{1}{k}}\right)}{\sqrt{k}\left(1+\sqrt{1+\frac{3}{k}}\right)}$$

$k\to\infty \quad \frac{1}{2}\cdot 1 = \frac{1}{2} < 1 \Rightarrow$ R konvergent