Deckblatt für die Abgabe der Übungsaufgaben
IngMathC2

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Blatt-Nummer: 4

Übungsgruppen-Nr: 7

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A10, A11, A12, ____

6/10*30 = 18

$\boxed{A10}$ a) i) Cauchy-Produkt $\sum_{k=0}^{\infty} kq^k \sum_{k=0}^{\infty} q^k$, für $|q|<1$

$\Rightarrow$ sind safe absolut konvergent

$$\left(\sum_{k=0}^{\infty} kq^k\right)\left(\sum_{k=0}^{\infty} q^k\right) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} kq^k \cdot q^{n-k} =$$

$$= \sum_{n=0}^{\infty}\sum_{k=0}^{n} k\cdot q^n = \sum_{n=0}^{\infty}\left(q^n \sum_{k=0}^{n} k\right) \overset{\text{Gauss}}{=} \sum_{n=0}^{\infty} q^n \frac{n(n+1)}{2}$$

und jetzt? ... ?

ii) $\sum_{k=0}^{\infty} k^2 q^k = 2\left(\sum_{k=0}^{\infty}\left(\frac{1}{2}k^2q^k + \frac{1}{2}kq^k\right)\right) - \sum_{k=0}^{\infty} kq^k$

$\sum_{k=0}^{\infty} k^2 q^k$ für $|q|<1$, $\sum_{k=0}^{\infty} q^k = \frac{1}{1-q}$ (1)

$$\sum_{k=0}^{\infty} kq^k = \frac{q}{(1-q)^2} \quad (2)$$

$$\sum_{k=0}^{\infty}\left(\frac{k^2q^k}{2} + \frac{kq^k}{2}\right) = \sum_{k=0}^{\infty} kq^k \cdot \sum_{k=0}^{\infty} \quad (3)$$

$$\Rightarrow \sum_{k=0}^{\infty} k^2q^k = 2\left(\sum_{k=0}^{\infty}\left(\frac{1}{2}k^2q^k + \frac{1}{2}kq^k\right)\right) - \sum_{k=0}^{\infty} kq^k =$$

$$\overset{(3)}{=} 2\left(\sum_{k=0}^{\infty} kq^k \cdot \sum_{k=0}^{\infty} q^k\right) - \sum_{k=0}^{\infty} kq^k =$$

$$\overset{(1),(2)}{=} 2\cdot\left(\frac{q}{(1-q)^2}\cdot\frac{1}{1-q}\right) - \frac{q}{(1-q)^2} =$$

$$= 2\cdot\frac{q}{(1-q)^3} - \frac{q-q^2}{(1-q)^3} = \frac{q^2+q}{(1-q)^3}$$

b) $P_n = \sum_{k=0}^{n} \frac{1}{(k+1)(k+2)} = \sum_{k=0}^{n} \frac{A}{k+1} + \frac{B}{k+2} =$

NR:

$1 = A(k+2) + B(k+1)$

$1 = (A+B)k + 2A + B \Rightarrow A+B=0,\ 2A+B=1$

$\Rightarrow A=1 \quad B=-1$

$$= \sum_{k=0}^{n} \frac{1}{k+1} - \frac{1}{k+2} = \sum_{k=0}^{n}\frac{1}{k+1} - \sum_{k=0}^{n}\frac{1}{k+2}$$

Indextransformation

$$\sum_{k=-1}^{n-1}\frac{1}{k+2} - \sum_{k=0}^{n}\frac{1}{k+2} = \frac{1}{-1+2} - \frac{1}{n+2} = 1 - \frac{1}{n+2}$$

A11 a) i) $\sum_{k=0}^{\infty} \frac{5^k}{k} x^k$, $R = \frac{1}{\limsup_k \sqrt[k]{\left|\frac{5^k}{k}\right|}} = \frac{1}{\limsup_k \sqrt[k]{5^k k^{-1}}} =$

$= \frac{1}{\limsup_k 5^k \sqrt[k]{k^{-1}}} = \frac{1}{5}$

ii) $\sum_{k=0}^{\infty} (\sqrt{k+1} - \sqrt{k-\sqrt{k}})^{2k} \cdot x^k$

$\sqrt[k]{|a_k|} = \sqrt[k]{|(\sqrt{k+1} - \sqrt{k-\sqrt{k}})^{2k}|} = (\sqrt{k+1} - \sqrt{k-\sqrt{k}})^2 =$

$= k+1 - 2\sqrt{(k+1)(k-\sqrt{k})} + k - \sqrt{k} =$

$= 2k+1 - 2\sqrt{k^2 - \sqrt{k^3} + k - \sqrt{k}} - \sqrt{k} =$

$= \cancel{2k+1 - 2k\sqrt{1 - \frac{\sqrt{k}}{k} + \frac{1}{k} - \sqrt{\frac{1}{k^3}}} - \sqrt{k} \xrightarrow{k\to\infty} \limsup_k 1 - \sqrt{k} = -\infty}$

$\Rightarrow R = \text{"}\frac{1}{-\infty}\text{"} = 0$

iii) $\sum_{k=0}^{\infty} (k!+2)x^k$, $\sqrt[k]{|a_k|} = \sqrt[k]{|k!+2|}$

$\limsup_k \sqrt[k]{|k!+2|} = \limsup_k \sqrt[k]{k!+2} > \limsup_k \sqrt[k]{k!} = +\infty$

$\Rightarrow R = \text{"}\frac{1}{\infty}\text{"} = 0$

iv) $\sum_{k=0}^{\infty} \frac{2^k}{k^2} x^{4k} = \sum_{\tilde{k}=0}^{\infty} a_{\tilde{k}} x^{\tilde{k}}$, wobei $a_k = \begin{cases} 0 & \text{für } \tilde{k} \text{ modulo } 4 \neq 0 \\ \frac{2^{\tilde{k}/4}}{(\frac{\tilde{k}}{4})^2} & \text{für } \tilde{k} \text{ modulo } 4 = 0 \end{cases}$

$\Rightarrow \sqrt[k]{\left|\frac{2^{\frac{k}{4}}}{(\frac{k}{4})^2}\right|} = \left|\frac{2^{\frac{1}{4}}}{\frac{\sqrt[k]{k^2}}{\sqrt[k]{16}}}\right| = \frac{\sqrt[4]{2} \cdot \sqrt[k]{16}}{\sqrt[k]{k^2}} \xrightarrow{k\to\infty} \cancel{0}\ \sqrt[4]{2}$

$\Rightarrow R = \text{"}\frac{1}{0}\text{"} = +\infty$

b) $\sum_{k=0}^{\infty} \left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1\right)^k \left(\frac{1}{x+3}\right)^k$, $y^k := \left(\frac{1}{x+3}\right)^k$

$\Rightarrow \sqrt[k]{|a_k|} = \sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1 \xrightarrow{k\to\infty} 1 + 0 + 1 = 2$

$\Rightarrow |y| = \left|\frac{1}{x+3}\right| < \cancel{2}\ \frac{1}{2}$ <=> Potenzreihe konvergiert

$\Rightarrow |x+3| > \frac{1}{2} \Rightarrow x_1 < -2{,}5 < x_2$

Die größtmögliche Menge auf der die Potenzreihe konvergent ist ist $(-\infty; -2{,}5) \cup (-2{,}5; +\infty)$

man konvergiert innerhalb des Pote

wenn das Richtig wäre, dann würde immernoch gelten:|x+3| > 1

$\boxed{A12}$ a) $\sin(3x)$, $\cos(3x)$

i) $\exp(3ix) = \exp(ix)^3$ | Euler

$\cos(3x) + i\sin(3x) = (\cos(x) + i\sin(x))^3 = (\cos x)^3 - 3\cos x(\sin x)^2 + i(-(\sin x)^3 + 3(\cos x)^2 \cdot \sin x)$

$\cancel{\cos^3 x^3 - 3\sin x \cdot (\cos x)^2 + i(-(\sin^3 x)^3 + 3(\sin x)^2(\cos x))}$

Real:

$\cos(3x) = \cos^3 x - 3\sin^2(x)\cdot\cos(x)$

$\sin(3x) = -\sin^3(x) + 3\sin(x)\cos^2(x)$

$\Rightarrow \cos(3x) = (\cos x)^3 - 3(\sin(x))^2 \cdot \cos x$

$\sin(3x) = -(\sin x)^3 + 3\sin x(\cos x)^2$

ii) $\sin 3x = \sin(2x + x) = \sin(2x)\cdot\cos x + \cos(2x)\sin x$

$\sin 2x = \sin(x+x) = \sin x\cos x + \cos x\sin x = 2\sin x\cos x$

$\cos 2x = \ldots = \cos^2 x - \sin^2 x$

$\sin 3x = 2\sin x\cos x\cdot\cos x + (\cos^2 x - \sin^2 x)\sin x =$

$= 2\sin x(\cos x)^2 + \sin x(\cos x)^2 - (\sin x)^3 =$

$= 3\sin x(\cos x)^2 - (\sin x)^3$

$\cos 3x = \cos(2x+x) = \cos 2x\cos x - \sin 2x\sin x$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 3x = (\cos^2 x - \sin^2 x)\cos x - 2\sin x\cos x\sin x =$

$= (\cos x)^3 - (\sin x)^2\cos x - 2(\sin x)^2\cos x =$

$= (\cos x)^3 - 3(\sin x)^2\cos x$

b) (i) $\sin\frac{\pi}{3} = \cancel{\sin 3\frac{\pi}{9}}$

$\sin 3x = \cancel{\sin x\, 2\cos^2 x - \sin^2 x = \sin x(2}$

$3\sin x\cos^2 x - \sin^3 x = \sin x(3\cos^2 x - \sin^2 x) =$

$= \sin x(3 - 3\sin^2 x - \sin^2 x) = \sin x(3 - 4\sin^2 x) =$

$= 3\sin x - 4\sin^3 x$

$\frac{\pi}{3}$ einsetzen, i guess: $3\sin\frac{\pi}{3} - 4\sin^3\frac{\pi}{3} = \sin 3\frac{\pi}{3} = \sin\pi = 0$

$\sin\frac{\pi}{3} = y \quad\Rightarrow\quad 3\sin\frac{\pi}{3} - 4\sin^3\frac{\pi}{3} = 0$

$3y - 4y^3 = 0$

$3y = 4y^3 \quad |\ \text{da } y \neq 0,\ :y$

$3 = 4y^2 \quad |:4$

$\frac{3}{4} = y^2 \quad | \sqrt{\ldots}$

, da wir wissen (dürfen) dass $y > 0$ $\quad y = \sin\frac{\pi}{3} = \sqrt{3/4} = \frac{1}{2}\sqrt{3}$

$\cos\frac{\pi}{3} > 0$

$(\cos\frac{\pi}{3})^2 = 1 - (\sin\frac{\pi}{3})^2 = 1 - \frac{3}{4} = \frac{1}{4}$

$\Leftrightarrow \quad \cos\frac{\pi}{3} = \frac{1}{2}$

ii) $\sin\frac{\pi}{3} = \sin 2\frac{\pi}{6} = \frac{1}{2}\sqrt{3} \qquad \cos\frac{\pi}{3} = \cos 2\frac{\pi}{6} = \frac{1}{2}$

$\cos 2x = (\cos x)^2 - (\sin x)^2 = 1 - 2(\sin x)^2 = (\cos x)^2 - (1-(\cos x)^2) =$
$= 2(\cos x)^2 - 1$

$y = \cos\frac{\pi}{6}$

$\frac{1}{2} = 2(\cos\frac{\pi}{6})^2 - 1 = 2y^2 - 1$

$\Leftrightarrow \quad 2y^2 = \frac{3}{2} \quad | :2$

$y^2 = \frac{3}{4} \quad | \sqrt{}$

$y = \frac{1}{2}\sqrt{3}$

$\Rightarrow \quad \cos\frac{\pi}{6} = \frac{1}{2}\sqrt{3}$

$\sin\frac{\pi}{6} > 0$:

~~$(\cos x)^2 = 1 - (\sin x)^2$~~ $\quad (\sin\frac{\pi}{6})^2 = 1 - (\cos\frac{\pi}{6})^2 = 1 - \frac{3}{4} = \frac{1}{4}$

$\Leftrightarrow \quad \sin\frac{\pi}{6} = \frac{1}{2}$

iii) $\sin\frac{\pi}{6} = \sin 2\frac{\pi}{12} = \frac{1}{2} \qquad \cos\frac{\pi}{6} = \cos 2\frac{\pi}{12} = \frac{1}{2}\sqrt{3}$

$\cos 2x = 2(\cos x)^2 - 1 \qquad y = \cos\frac{\pi}{12}$

$\frac{\sqrt{3}}{2} = 2y^2 - 1 \quad | +1$

$\frac{\sqrt{3}+2}{2} = 2y^2 \quad | :2$

$y^2 = \frac{\sqrt{3}+2}{4} \quad | \sqrt{}$

$y = +\sqrt{\frac{\sqrt{3}+2}{4}} = \frac{\sqrt{\sqrt{3}+2}}{2} = \cos\frac{\pi}{12} = \frac{1}{2}\sqrt{\sqrt{3}+2}$

$\sin\frac{\pi}{12}$: $\quad (\sin\frac{\pi}{12})^2 = 1 - (\cos\frac{\pi}{12})^2 = 1 - \frac{\sqrt{3}+2}{4} = \frac{4-\sqrt{3}-2}{4} =$
$= \frac{2-\sqrt{3}}{4}$

$\Leftrightarrow \quad \sin\frac{\pi}{12} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2} = \frac{1}{2}\sqrt{2-\sqrt{3}}$