Deckblatt für die Abgabe der Übungsaufgaben
IngMathC2

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Blatt-Nummer: 4

Übungsgruppen-Nr: 7

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A10, A11, A12, ______

6.5/10*30 = 19.5

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/10 a)

(i) $$\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} k\cdot q^{k}\cdot q^{n-k}\right)=\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} k\cdot q^{n}\right)=\left(\sum_{n=0}^{\infty} q^{n}\right)\cdot\left(\sum_{k=0}^{n} k\right)$$

(ii) -> Bekannt aus Vorlesung:

$$\lim_{k\to\infty}\sum_{k=0}^{\infty} q^{k}=\frac{1}{1-q} \qquad \lim_{k\to\infty}\sum_{k=0}^{\infty} k\cdot q^{k}=\frac{q}{(1-q)^{2}}$$

Cauchy Produkt:

$$\frac{1}{1-q}\cdot\frac{q}{(1-q)^{2}}=\frac{q}{(1-q)^{3}}$$

b) $$\sum_{k=0}^{\infty}\frac{1}{(k+1)(k+2)}=\sum_{k=1}^{\infty}\frac{1}{k}\cdot\sum_{k=2}^{\infty}\frac{1}{k}=$$

$$=\sum_{k=1}^{\infty}\left(\sum_{k=2}^{\infty}\frac{1}{k}\cdot\frac{1}{k}\right)$$

$\frac{1}{k}$ -> harmonische Reihe => divergent

/A11

(i) $\sum_{k=0}^{\infty}\frac{5^k}{k}x^k$

$$R=\frac{1}{\limsup_{k\to\infty}\sqrt[k]{\frac{5^k}{k}}}=\lim_{k\to\infty}\frac{1}{\frac{\sqrt[k]{5^k}}{\sqrt[k]{k}}}=\lim_{k\to\infty}\frac{1}{\frac{5}{\underbrace{\sqrt[k]{k}}_{\to 1}}}=\frac{1}{5}$$

(ii) $\sum_{k=0}^{\infty}(\sqrt{k+1}-\sqrt{k-\sqrt{k}})^{2k}x^k$

bei typ infty-inftykann man nicht auf 0 schließen

$$\lim_{k\to\infty}\sqrt{k}=+\infty \qquad \sqrt{k}<\sqrt{k+1}$$

$\Rightarrow 0$

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(iii) $\lim_{k\to\infty} \frac{1}{\sqrt[4]{k!}+2} = \frac{1}{\text{"}\infty\text{"}} = 0$

warum?

$\limsup_{k\to\infty} k! = +\infty$

$\limsup \sqrt[4]{k!} = +\infty$ $\quad R=0$

(iv) $\sum_{k=0}^{\infty} \frac{2^k}{k^2} x^{4k}$ $\quad y = x^4$

$\limsup_{k\to\infty} \sqrt[k]{\frac{2^k}{k^2}} = \lim_{k\to\infty} \frac{2}{\sqrt[k]{k^2}} = \frac{2}{\lim_{k\to\infty} \sqrt[k]{k^2}} = \frac{2}{1^2} = 2$

Potenzreihe konvergent für alle $y \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

Rücksubstituieren. $|y| < \frac{1}{2} \Rightarrow |x| = \sqrt[4]{y} < \frac{1}{\sqrt[4]{2}} = R_x$

b) $S(x) = \sum_{k=0}^{\infty} \left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1\right)^k \left(\frac{1}{x+3}\right)^k$ $\quad y = \frac{1}{x+3}$

$\limsup_{k} \sqrt[k]{\left|\left(\sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1\right)^k\right|} = \lim_{k\to\infty} \sqrt[k]{3k} + \frac{4}{\sqrt[k]{k!}} + 1 =$

$= \lim_{k\to\infty} \sqrt[k]{3} \cdot \sqrt[k]{k} + \lim_{k\to\infty} \frac{4}{\sqrt[k]{k!}} + 1 = 1 \cdot 1 + 0 + 1 = 2$ $\quad R_y = \frac{1}{2}$

Potenzreihe für alle $y \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ konvergent

$y = \frac{1}{x+3} \Rightarrow x \in (-\infty, -5) \cup (-1, +\infty)$

$\alpha = -5 \quad \beta = -1$

die Umformung hier genau machen:|y|<R_y ==> |1/(x+3)|<1/2 ==> |x+3|>2Das ist ja der eigentliche twist an der Aufgabe la 8 Lcyxy Almir Vučetović

1/2 a)

(i)

$$\exp(3ix) = \exp(ix)^3$$

$$e^{3ix} = e^{ix^3}$$

$$\cos(3x) + i\sin(3x) = (\cos(x) + i\sin(x))^3$$

$$\cos(3x) + i\sin(3x) = \cos^3(x) + 3\cos^2(x)\, i\sin(x) + 3\cos(x)(i\sin(x))^2 + (i\sin(x))^3 =$$

$$= \cos^2(x) + i \cdot 3\cos^2(x)\sin(x) - 3\cos(x)\sin^2(x) - i\sin^3(x)$$

$$\cos(3x) = \cos^3(x) - 3\cos(x)\sin^2(x) = \cos^2(x) - 3\cos(x)(1-\cos^2(x)) =$$

$$= 4\sin^2(x) + 3\sin(x)$$

$$\sin(3x) = 3\cos^2(x)\sin(x) - \sin^3(x) = 3(1-\sin^2(x))\sin(x) - \sin^3(x) =$$

$$= -4\sin^2 + 3\sin(x)$$

(ii)

$$\sin(x+2x) = \sin(x)\cos(2x) + \cos(x)\cdot\sin(2x) =$$

$$= \sin(x)\cdot((\cos^2(x) - \sin^2(x)) + \cos(x)\cdot 2\sin(x)\cos(x) =$$

$$= -\sin^3(x) + \sin(x)\cos^2(x) + 2\sin\cos^2(x) =$$

$$= -\sin^3(x) + 3\sin(x)(1-\sin^2(x)) =$$

$$= -4\sin^3(x) + 3\sin(x)$$

$$\cos(x+2x) = \cos(x)\cdot\cos(2x) - \sin(x)\cdot\sin(2x) =$$

$$= \cos(x)\cdot(\cos^2(x) - \sin^2(x)) - \sin(x)\cdot 2\sin(x)\cdot\cos(x) =$$

$$= \cos^3(x) - \sin^2(x)\cos(x) - 2\sin^2(x)\cos(x) =$$

$$= \cos^3(x) - (1-\cos^2(x))\cos(x) - 2(1-\cos^2(x))\cos(x) =$$

$$= \cos^3(x) + \cos^3(x) - \cos(x) - 2\cos(x) + 2\cos^3 =$$

$$= 4\cos^3(x) - 3\cos(x)$$

$$\sin(2x) = \sin(x+x) = \sin(x)\cos(x) + \cos(x)\sin(x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos(x+x) = \cos^2(x) - \sin^2(x)$$

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b) $\sin(3x) = -4\sin^3(x) + 3\sin(x)$

$\cos(3x) = 4\cos^3(x) - 3\cos(x)$

$\sin\left(3\frac{\pi}{3}\right) = -4\sin^3\left(\frac{\pi}{3}\right) + 3\sin\left(\frac{\pi}{3}\right)$

$0 = -4\sin^3\left(\frac{\pi}{3}\right) + 3\sin\left(\frac{\pi}{3}\right)$

$0 = -4\sin^2\left(\frac{\pi}{3}\right) + 3$

$\frac{3}{4} = \sin^2\left(\frac{\pi}{3}\right)$

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\cos^2\left(\frac{\pi}{3}\right) = 1 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4}$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

(ii)

$\cos\left(2\cdot\frac{\pi}{6}\right) = 1 - \sin^2\left(\frac{\pi}{6}\right)$ $\qquad \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\frac{1}{2} = 1 - \sin^2\left(\frac{\pi}{6}\right)$

$\frac{1}{2} = 2\sin^2\left(\frac{\pi}{6}\right)$

$\frac{1}{4} = \sin^2\left(\frac{\pi}{6}\right)$

$\cos\left(\frac{\pi}{6}\right) = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$

(iii)

$\cos\left(2\cdot\frac{\pi}{12}\right) = 1 - \sin^2\left(\frac{\pi}{12}\right)$ $\qquad \sin^2\left(\frac{\pi}{12}\right) = \frac{2-\sqrt{3}}{4}$

$\frac{\sqrt{3}}{2} = 1 - \sin^2\left(\frac{\pi}{12}\right)$

$\frac{\sqrt{3}-2}{2} = -2\sin^2\left(\frac{\pi}{12}\right)$ $\qquad \sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{2-\sqrt{3}}}{2}$

$\cos^2\left(\frac{\pi}{12}\right) = 1 - \left(\frac{\sqrt{2-\sqrt{3}}}{2}\right)^2 = 1 - \frac{2-\sqrt{3}}{4} = \frac{2+\sqrt{3}}{4}$

$\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{2+\sqrt{3}}}{2}$