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Studon: em89inym Blatt: 1

Übung: Gruppe 7 (Mi 12–14 Uhr)

23.5/24 *33=32

Freigegebene Aufgaben: A1, A2, A3

A1	Inf	Sup	Min	Max
a)	$\sqrt{3}$	$\sqrt{5}$	$\sqrt{3}$	/
b)	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{2}{3}$
c)	0	$\frac{1}{2}$	/	$\frac{1}{2}$
d)	$\cancel{2}$	$+\infty$	$\cancel{2}$	/
e)	0	1	/	1
f)	$\frac{2}{3}$	$+\infty$	$\frac{2}{3}$	/
g)	1	$+\infty$	/	/

-infty / existiert nicht

A2

i) $\frac{3n+4m}{5n^2+10} \leq \frac{3n+12n}{5n^2+10} = \frac{3n}{n^2+2}$

ii) $\frac{5n-m}{2n} \leq \frac{5n-2n}{2n} = \frac{3n}{2n} = \frac{3}{2}$

iii) $\frac{n}{n+m} \leq \frac{n}{n+2n} = \frac{n}{3n} = \frac{1}{3}$

iv) $\frac{n+m}{\frac{1}{2}-n} \leq \frac{n+3n}{\frac{1}{2}-n} = \frac{8n}{1-2n}$

v) $\frac{5n-m+3\cdot 2^m}{3n^3-m+3} \leq \frac{5n-2n+3\cdot 2^{3n}}{3(n^3-n+1)} = \frac{n+2^{3n}}{n^3-n+1}$

vi) $m+n+\sin(m)-\sin(17m^2)+2^m+2^{-m} \leq 3n+n+1-(-1)+2^{3n}+\frac{1}{2^{2n}} =$

$= 4n+2+2^{3n}+\frac{1}{4^n}$

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A3 i) $a_n = \frac{2n}{n+3}$ ii) $b_n = \frac{n}{4^n} = \frac{n}{2^{2n}}$

a) i)
$$a_{n+1} - a_n = \frac{2(n+1)}{(n+1)+3} - \frac{2n}{n+3} = \frac{2n+2}{n+4} - \frac{2n}{n+3} =$$
$$= \frac{(2n+2)(n+3) - 2n\cdot(n+4)}{(n+4)(n+3)} = \frac{2n^2+6n+2n+6-2n^2-8n}{(n+4)(n+3)}$$
$$= \frac{6}{(n+4)(n+3)} \geq 0 \Rightarrow \text{monoton wachsend}$$

ii)
$$b_{n+1} \div b_n = \frac{\frac{n+1}{4^{n+1}}}{\frac{n}{4^n}} = \frac{(n+1)\cdot 4^n}{n\cdot 4^{n+1}} = \frac{(n+1)\cdot \cancel{4^n}}{n\cdot \cancel{4^n}\cdot 4} =$$

besser als bruch aufschreiben...

$$= \frac{n+1}{4n} = \frac{n}{4n} + \frac{1}{4n} \overset{n\geq 1}{\leq} \frac{1}{4} + \frac{1}{4\cdot 1} \leq 1 \Rightarrow \text{monoton fallend}$$

b) i) $\lim\limits_{n\to\infty} a_n = 2$

ii) $\lim\limits_{n\to\infty} b_n = 0$

c) i) $\forall \varepsilon > 0\ \exists n_0 \in \mathbb{N}\ \forall n \geq n_0 : |a_n - a| \leq \varepsilon$

Problem: für e=4 ist das negativ

Sei $\varepsilon > 0$ belibig vorgegeben. Setze $n_0 \overset{NR1}{:=} \lceil \frac{6}{\varepsilon} - 3 \rceil$.

Dann gilt für alle $n \geq n_0$:
$$|a_n - a| = \left|\frac{2n}{n+3} - 2\right| = \left|\frac{2n-2n-6}{n+3}\right| = \frac{6}{n+3} \overset{n\geq n_0}{\leq} \frac{6}{n_0+3}$$
NR1
$$= \frac{6}{\lceil \frac{6}{\varepsilon} - 3 \rceil + 3} \leq \frac{6}{\frac{6}{\varepsilon} - 3 + 3} = \frac{6}{\frac{6}{\varepsilon}} = \varepsilon$$

NR1: $\frac{6}{n+3} \leq \varepsilon \Leftrightarrow \frac{6}{\varepsilon} \leq n+3 \Leftrightarrow n \geq \frac{6}{\varepsilon} - 3$

ii) Sei $\varepsilon > 0$ belibig vorgegeben. Setze $n_0 \overset{NR2}{:=} \lceil \log_2(\frac{1}{\varepsilon}) \rceil$

Dann gilt für alle $n \geq n_0$:
$$|b_n - b| = \left|\frac{n}{2^{2n}} - 0\right| \overset{n\leq 2^n}{\leq} \frac{2^n}{2^{2n}} = 2^{n-2n} = \frac{1}{2^n} \overset{n\geq n_0}{\leq} \frac{1}{2^{n_0}}$$
NR2
$$= \frac{1}{2^{\log_2(\frac{1}{\varepsilon})}} = \frac{1}{\frac{1}{\varepsilon}} = \varepsilon$$

NR2: $\frac{1}{2^n} \leq \varepsilon \Leftrightarrow \frac{1}{\varepsilon} \leq 2^n \Leftrightarrow \log_2(\frac{1}{\varepsilon}) \leq n$

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