Deckblatt für die Abgabe der Übungsaufgaben
IngMathC2

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Blatt-Nummer: 2

Übungsgruppen-Nr: 7

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A4, A6, ____, ____

15/18 *18 = 15

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A4

$a_1 = 1 \qquad a_{n+1} = \frac{1}{2} a_n + \sqrt{a_n}$

a)

IA: $n = 1 \qquad a_1 = 1 \in (0,4)$ ✓

IV: $a_n \in (0,4)$ für ein $n \in \mathbb{N}$ ✓

(0,4) aber es steht nichts von a_n

IS: $n \to n+1$

$a_{n+1} = \frac{1}{2} a_n + \sqrt{a_n} \overset{IV}{<} \cancel{\frac{1}{2} \cdot 3 + \sqrt{3} = \frac{3\sqrt{3}}{2}}$ 4

$a_{n+1} = \frac{1}{2} a_n + \sqrt{a_n} > \frac{1}{2} \cdot \cancel{2} + \sqrt{2} = \sqrt{2}$ 0

$a_{n+1} \in \left(1, \frac{3\sqrt{3}}{2}\right) \ \forall n \in \mathbb{N}$

b)

$a_{n+1} \geq a_n$ ✓

$a_{n+1} - a_n = -\frac{1}{2} a_n + \sqrt{a_n}$ ✓

selber grund wie a)

$-\frac{1}{2} a_n + \sqrt{a_n} \overset{a)}{\geq} \cancel{-\frac{1}{2} \cdot 3 + \sqrt{3} \geq 0}$

$\Rightarrow a_{n+1} - a_n \geq 0 \qquad \square$

c)

Nach Satz der VL ist jede monoton wachsende, nach oben beschränkte Folge konvergent

$a_{n+1} = \frac{1}{2} a_n + \sqrt{a_n} \qquad | \lim_{n \to \infty}$

$a = \frac{1}{2} a + \sqrt{a} \qquad |^2$ ✓

~~$a^2 = \frac{1}{4} a^2 + a$~~

~~$\frac{5}{4} a^2 + a = 0$~~

~~$a\left(\frac{5}{4} a + 1\right) = 0$~~

~~$x_1 = 0$~~

~~$\frac{5}{4} a + 1 = 0$~~

~~$\frac{5}{4} a = -1 \quad | \cdot \frac{4}{5}$~~

~~$a = -\frac{4}{5}$~~

$\cancel{a^2 = \frac{1}{4} a^2 + a}$

a^2 = (1/2 a+sqrt(a))^2

$\frac{3}{4} a^2 - a = 0$

$a\left(\frac{3}{4} a - 1\right) = 0$

$a_1 = 0$

$\frac{3}{4} a - 1 = 0$ ✓

ff

$\frac{3}{4} a = 1$

$a = \frac{4}{3} \Rightarrow$ Da nur $\left\{\frac{4}{3}\right\} \in (0,4)$ folgt $a = \frac{4}{3}$

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A6

a) $$a_n = \frac{2n^3 - n}{n\cdot(3n^2+2)} = \frac{n^3\left(2-\frac{1}{n^2}\right)}{n^3\left(3+\frac{2}{n^2}\right)}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2-\underbrace{\frac{1}{n^2}}_{\to 0}}{3+\underbrace{\frac{2}{n^2}}_{\to 0}} = \frac{2}{3}$$

b) $$b_n = \left(\frac{5+2n}{1+n}\right)^3 = \left(\frac{n\left(\frac{5}{n}+2\right)}{n\left(\frac{1}{n}+1\right)}\right)^3$$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty}\left(\frac{\frac{5}{n}+2}{\frac{1}{n}+1}\right)^3 = \left(\lim_{n\to\infty}\left(\frac{\frac{5}{n}+2}{\frac{1}{n}+1}\right)\right)^3 = \left(\frac{2}{1}\right)^3 = 8$$

c) $$c_n = \sqrt{2n^2+n+1} - \sqrt{2n^2+9n}$$

$$= \frac{\left(\sqrt{2n^2+n+1}-\sqrt{2n^2+9n}\right)\left(\sqrt{2n^2+n+1}+\sqrt{2n^2+9}\right)}{\sqrt{2n^2+n+1}+\sqrt{2n^2+9n}}$$

$$= \frac{2n^2+n+1-2n^2+9n}{\sqrt{2n^2+n+1}+\sqrt{2n^2+9n}} = \frac{-8n+1}{\sqrt{2n^2+n+1}+\sqrt{2n^2+9n}}$$

$$\lim_{n\to\infty} c_n = \lim_{n\to\infty} \frac{-8n+1}{n\cdot\sqrt{2+\frac{1}{n}+\frac{1}{n^2}}+n\sqrt{2+\frac{9}{n}}}$$

$$= \lim_{n\to\infty} \frac{-8+\frac{1}{n}}{\sqrt{2+\frac{1}{n}+\frac{1}{n^2}}+\sqrt{2+\frac{9}{n}}} = \frac{-8}{\sqrt{2}\cdot 2} = -2\sqrt{2}$$

d) $$d_n = n^3 - \sqrt{n^6+n^2+1}$$

$$= \frac{\left(n^3-\sqrt{n^6+n^2+1}\right)\left(n^3+\sqrt{n^6+n^2+1}\right)}{n^3+\sqrt{n^6+n^2+1}}$$

$$= \frac{n^6-(n^6+n^2+1)}{n^3+\sqrt{n^6+n^2+1}} = \frac{-n^2-1}{n^3+\sqrt{n^6+n^2+1}}$$

$$= -\frac{\frac{1}{n}+\frac{1}{n^3}}{1+\sqrt{1+\frac{1}{n^4}+\frac{1}{n^6}}}$$

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A6)

d)

$$\lim_{n\to\infty} d_n = \lim_{n\to\infty} -\frac{1}{n}\cdot\frac{1+\frac{1}{n^2}}{1+\sqrt{1+\frac{1}{n^4}+\frac{1}{n^6}}} = 0$$ ✓ ✓+

e)

$$e_n = \sqrt[4]{n^4+n^3} - \sqrt[4]{n^4-n^3}$$

$$= \frac{\sqrt{n^4+n^3}-\sqrt{n^4-n^3}}{\sqrt[4]{n^4+n^3}+\sqrt[4]{n^4-n^3}}$$

$$= \frac{(n^4+n^3)-(n^4-n^3)}{\left(\sqrt[4]{n^4+n^3}+\sqrt[4]{n^4-n^3}\right)\left(\sqrt{n^4+n^3}+\sqrt{n^4-n^3}\right)}$$

$$= \frac{2n^3}{n\cdot\left(\sqrt[4]{1+\frac{1}{n}}+\sqrt[4]{1-\frac{1}{n}}\right)\cdot\left(n^2\cdot\left(\sqrt{1+\frac{1}{n}}+\sqrt{1-\frac{1}{n}}\right)\right)}$$

$$= \frac{2}{\left(\sqrt[4]{1+\frac{1}{n}}+\sqrt[4]{1-\frac{1}{n}}\right)\cdot\left(\sqrt{1+\frac{1}{n}}+\sqrt{1-\frac{1}{n}}\right)}$$

$$\lim_{n\to\infty} e_n = \lim_{n\to\infty}\frac{2}{\left(\sqrt[4]{1+\frac{1}{n}}+\sqrt[4]{1-\frac{1}{n}}\right)\left(\sqrt{1+\frac{1}{n}}+\sqrt{1-\frac{1}{n}}\right)}$$

$$= \frac{2}{(1+1)(2)} = \frac{1}{2}$$ ✓ ✓+

f)

$$f_n = \frac{n^2}{n+2} - \frac{n^2}{n+1} = \frac{n^2(n+1)-n^2(n+2)}{(n+2)\cdot(n+1)}$$

$$= \frac{n^2\cdot(1+n-(2+n))}{n^2\cdot\left(1+\frac{2}{n}\right)\cdot\left(1+\frac{1}{n}\right)} = \frac{-1}{\left(1+\frac{2}{n}\right)\cdot\left(1+\frac{1}{n}\right)}$$

$$\lim_{n\to\infty} f_n = \lim_{n\to\infty}\frac{-1}{\underbrace{\left(1+\frac{2}{n}\right)}_{1}\cdot\underbrace{\left(1+\frac{1}{n}\right)}_{1}} = -1$$ ✓ ✓+