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Blatt: 07
Gruppe: 7
Aufgaben: alle

19.5/20 * 30 = 29

A18)

a.) $f'(x) = 2x + 1 + \frac{1}{2\sqrt{x}} - \frac{1}{2x^{\frac{3}{2}}} - \frac{1}{x^2} - \frac{2}{x^3}$

b.) $f'(x) = 4(x^2 + \sqrt{2x})^3 \cdot (2x + \frac{\sqrt{2}}{2\sqrt{x}})$

c.) $f'(x) = 1 \cdot e^{x^2} \cdot \ln(2+3x) + x \cdot e^{x^2} \cdot 2x \ln(2+3x) + x\, e^{x^2} \cdot \frac{1}{2+3x} \cdot 3$

$= e^{x^2} \cdot \ln(2+3x) + 2x^2 \cdot e^{x^2} \ln(2+3x) + x \cdot e^{x^2} \frac{3}{2+3x}$

d.) $f'(x) = \frac{-1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$

e.) $f'(x) = \frac{2 \cdot \cos 2x \cdot \ln(x^2+1) - \frac{2x}{x^2+1} \cdot \sin 2x}{\ln^2(x^2+1)}$

f.) $f'(x) = (e^{\ln x \cdot \alpha})' = e^{\ln x \cdot \alpha} \cdot \frac{\alpha}{x} = \alpha \cdot x^{\alpha - 1}$

g.) $f'(x) = (e^{\ln x \cdot (-x^2)})' = e^{\ln x \cdot (-x^2)} \cdot (\frac{1}{x}(-x^2) - 2x \cdot \ln x) =$

$= x^{-x^2} \cdot (-x - 2x \cdot \ln x) = -x^{-x^2+1}(1 + 2\ln x)$

h.) $f'(x) = (\ln(x + \ln(2\ln x)))' = \frac{1}{x + \ln(2\ln x)} \cdot (x + \ln(2\ln x))' =$

$= \frac{1}{x+\ln(2\ln x)} \cdot (1 + \frac{1}{2\ln x} \cdot (2\ln x)') = \frac{1}{x+\ln(2\ln x)} \cdot (1 + \frac{1}{x\ln x})$

A19)

a.) $\frac{d}{dx}\cos x = \lim_{h\to 0} \frac{\cos(x+h) - \cos(x)}{h} = \lim_{h\to 0} \frac{\cos x \cdot \cos h - \sin x \cdot \sinh - \cos x}{h} =$

$= \lim_{h\to 0} (\frac{\cos x \cdot \cos h - \cos x}{h} - \sin x \cdot \frac{\sin h}{h}) = \lim_{h\to 0} (\frac{\cos x \cdot (\cos h - 1)}{h} - \sin x \cdot \frac{\sin h}{h})$

$= \cos x \cdot 0 - \sin x \cdot 1 = -\sin x$

b.) $\tan'(x) = (\frac{\sin x}{\cos x})' = \frac{\cos x \cdot \cos x + \sin x \cdot \sin x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$ (i) $=$

$= 1 + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x$ (ii)

c.) (i) $\arctan' x = \frac{1}{1 + \tan^2(\arctan x)} = \frac{1}{1+x^2}$

(ii) $\tan'' x = (\tan' x)' = (1 + \tan^2 x)' = 2\tan x \cdot (1 + \tan^2 x) =$

$= 2\tan x + 2\tan^3 x$

$\tan''' x = (\tan'' x)' = (1 + 2\tan x + 2\tan^3 x)' = 2(1 + \tan^2 x) + 2 \cdot 3 \cdot \tan^2 x \cdot (1 + \tan^2 x)$

$= 2 + 8\tan^2 x + 6\tan^4 x$

A10)

a.) $f'(x) = \left(x^{\alpha} \cdot \sin\frac{1}{x^2}\right)' = \alpha\, x^{\alpha-1} \cdot \sin\frac{1}{x^2} + \cos\frac{1}{x^2} \cdot \frac{-2}{x^3} \cdot x^{\alpha} =$

$= \alpha \cdot x^{\alpha-1} \sin\frac{1}{x^2} - 2 \cdot x^{\alpha-3} \cdot \cos\frac{1}{x^2}$

b.) $f'(0) = \lim_{h \to 0} \frac{(0+h)^{\alpha} \cdot \sin\frac{1}{(0+h)^{\alpha}} - 0}{h} = \lim_{h \to 0} h^{\alpha-1} \cdot \sin\frac{1}{h^{\alpha}} =$

$= \lim_{h \to 0} \left(\frac{\frac{1}{h} \cdot \sin\frac{1}{h^{\alpha}}}{\frac{1}{h^{\alpha}}} \right) = \lim_{h \to 0} h^{\alpha-1} \cdot \sin\frac{1}{h^{\alpha}}$

$\lim_{h \to 0} h^{\alpha-1} = \begin{cases} 0 & \text{für } \alpha > 1 \\ 1 & \text{für } \alpha = 1 \\ \text{existiert nicht} & \text{für } \alpha < 1 \end{cases}$

$\sin\frac{1}{h^{\alpha}}$ begrenzt $[-1, 1]$

$\Rightarrow f'(0)$ existiert für $\alpha \geq 1$

c.) f' an der stelle $x = 0$ stetig

$\Leftrightarrow$ ~~$\Leftrightarrow f'(0) = 0$~~ * $\Leftrightarrow \alpha > 1$

$\alpha = 1$ f' an der stelle $x = 0$ nicht stetig

$\alpha > 1$ f' an der stelle $x = 0$ stetig

d.) $f''(x) = (f'(x))' = \left(\alpha\, x^{\alpha-1} \cdot \sin\frac{1}{x^2}\right)' - \left(2 \cdot x^{\alpha-3} \cdot \cos\frac{1}{x^2}\right)' =$

$= \alpha \cdot \left((\alpha-1) \cdot x^{\alpha-2} \cdot \sin\frac{1}{x^2} + \cos\frac{1}{x^2} \cdot \frac{-2}{x^3} \cdot x^{\alpha-1} \right) - 2 \cdot$

$\left((\alpha-3)\, x^{\alpha-4} \right) \cos\frac{1}{x^2} + \sin\frac{1}{x^2} \cdot \frac{2}{x^3} \cdot x^{\alpha-3} \big) = \alpha \cdot (\alpha-1)\, x^{\alpha-2} \sin\frac{1}{x^2}$

$-2\alpha \cdot x^{\alpha-4} \cos\frac{1}{x^2} - 2(\alpha-3) \cdot x^{\alpha-4} \cdot \cos\frac{1}{x^2} - 4 x^{\alpha-6} \cdot \sin\frac{1}{x^2}$

* $\lim_{x \to 0} f'(x) = f'\left(\lim_{x \to 0} x\right)$

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