Deckblatt für die Abgabe der Übungsaufgaben
IngMathC2

Name, Vorname: Vucetovic, Almir

StudOn-Kennung: la82cyxy

Blatt-Nummer: 1

Übungsgruppen-Nr: 7

Die folgenden Aufgaben gebe ich zur Korrektur frei:

A1, A2, A3, ______

23.5/24=32

Almir Vucetović 1a82cyxy

/A1

	sup	inf	max	min
a)	$\sqrt{3}$	$\sqrt{3}$	n.e.	$\sqrt{3}$
b)	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{1}{4}$
c)	$\frac{1}{2}$	0	$\frac{1}{2}$	n.e.
d)	$+\infty$	$-\infty$	n.e.	n.e.
e)	1	0	1	n.e.
f)	$+\infty$	$\frac{2}{3}$	n.e.	$\frac{2}{3}$
g)	$+\infty$	1	n.e.	n.e.

/A2 $\quad 2n \leq m \leq 3n$

i) $\frac{3n+4m}{5n^2+10} \leq \frac{3n+12n}{5n^2+10} = \frac{15n}{5n^2+10}$

ii) $\frac{5n-m}{2n} \leq \frac{5n-2n}{2n} = \frac{3}{2}$

iii) $\frac{n}{n+m} \leq \frac{n}{3n} = \frac{1}{3}$

iv) $\frac{n+m}{\frac{1}{2}-n} \leq \frac{4n}{\frac{1}{2}-n}$

v) $\frac{5n-m+3\cdot 2^m}{3n^3-m+3} \leq \frac{3n^2+3\cdot 2^{3n}}{3n^3-3n+3} = \frac{n+8^n}{n^3-n-1}$

vi) $m+n+\sin(m)-\sin(12m^2)+2^m+2^{-m} \leq 4n+2^{3n}+\frac{1}{2^{2n}}+\sin(m)-\sin(12m^2)$

$\leq 4n+2^{3n}+\frac{1}{2^{2n}}+2$

Almir Vucetovic, la82cyxy

(A3 i) $a_n = \frac{2n}{n+3}$ ii) $b_n = \frac{n}{4^n} = \frac{n}{2^{2n}}$

a) i) $a_{n+1} - a_n = \frac{2n+2}{n+4} - \frac{2n}{n+3} = \frac{2n^2+6n+2n+6-2n^2-8n}{(n+4)(n+3)}$

$= \frac{6}{(n+4)(n+3)} \geq 0$ => monoton wachsend

ii) $b_{n+1} - b_n = \frac{n+1}{4^{n+1}} - \frac{n}{4^n} = \frac{-3n+1}{4^{n+1}} \leq 0$ $(1 \leq n)$ => monoton fallend

b) Vermutungen:

$\lim\limits_{n\to\infty} (a_n) = 2$ $\lim\limits_{n\to\infty} (b_n) = 0$

c) i) z.z. $\forall \epsilon > 0 \; E_{n_0} \in \mathbb{N} \; \forall n \geq n_0 : |a_n - a| \leq \epsilon$

$\left|\frac{2n}{n+3} - 2\right| = \left|\frac{2n-2n-6}{n+3}\right| = \left|\frac{6}{n+3}\right| = \frac{6}{n+3} \leq \epsilon$

$n \geq \frac{6}{\epsilon} - 3 \rightarrow n_0 = \lceil \frac{6}{\epsilon} - 3 \rceil$

Problem: für e=4 wird dein n_0 negativ

Sei $\epsilon > 0$ beliebig und $n_0 \lceil \frac{6}{\epsilon} - 3 \rceil$, dann gilt

$\forall n \geq n_0 : |a_n - a| = \left|\frac{2n}{n+3} - 2\right| = \frac{6}{n+3} \leq \frac{6}{\frac{6}{\epsilon}} = \epsilon$ $\square$

ii) z.z. $\forall \epsilon > 0 \; \exists n_0 \in \mathbb{N} \; n \geq n_0 : |b_n - 0| \leq \epsilon$

$\left|\frac{n}{4^n}\right| = \frac{n}{4^n} \leq \frac{2^n}{4^n} = \frac{2^n}{4^n} = \frac{1}{2^n} \leq \epsilon$

$\frac{1}{\epsilon} \leq 2^n \Rightarrow n_0 = \lceil \log_2(\frac{1}{\epsilon}) \rceil$

Sei $\epsilon > 0$ beliebig und $n_0 = \lceil \log_2(\frac{1}{\epsilon}) \rceil$, dann gilt $\forall n \geq n_0 : |b_n - 0| \leq$

$\frac{1}{2^n} \leq \frac{1}{2^{\log_2(\frac{1}{\epsilon})}} = \epsilon$ $\square$