Notes on NEWUOA

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Algorithm 0.1

(alg:newuoa)? $\frac{1}{\text{Input }\Delta_0 \in (0,+\infty), m \in \{n+2,n+3,\ldots,(n+1)(n+2)/2\}, \text{ and } \mathcal{X}_0 \subset \mathbb{R}^n \text{ with } x_0 \in \mathcal{X}_0}}{\text{and } |\mathcal{X}_0| = m. \text{ Set } Q_{-1} = 0 \text{ and } k = 0.}$

- 1. $Q_k = \operatorname{argmin}\{\|\nabla^2 Q \nabla^2 Q_{k-1}\|_F : Q \in \mathcal{Q} \text{ and } Q(x) = f(x) \text{ for } x \in \mathcal{X}_k\}.$
- 2. $x_k = \operatorname{argmin}\{f(x) : x \in \mathcal{X}_k\}$
- 3. $x_k^+ = \operatorname{argmin}\{Q_k(x) : ||x x_k|| \le \Delta_k\}, x_k^- = \operatorname{argmin}\{\kappa(\mathcal{X}_k, x_k^+, x) : x \in \mathcal{X}_k\}$
- 4. $\rho_k = [f(x_k) f(x_k^+)]/[Q_k(x_k) Q_k(x_k^+)];$ update Δ_k according to ρ_k .
- 5. If $\kappa(\mathcal{X}_k, x_k^+, x_k^-) \leq \kappa_0$, then set $\mathcal{X}_{k+1} = \mathcal{X}_k \cup \{x_k^+\} \setminus \{x_k^-\}$. Increment k. Go to Step 1.
- 6. $y_k^- = \operatorname{argmax}\{\|y x_k\| : y \in \mathcal{X}_k\}, \ y_k^+ = \operatorname{argmin}\{\kappa(\mathcal{X}_k, y, y_k^-) : \|y x_k\| \leq \bar{\Delta}_k\}.$ Set $\mathcal{X}_{k+1} = \mathcal{X}_k \cup \{y_k^+\} \setminus \{y_k^-\}.$ Increment k. Go to Step 1.

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