

Notes on NEWUOA

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Algorithm 0.1

^{?alg:newuoa)?} Input $\Delta_0 \in (0, +\infty)$, $m \in \{n+2, n+3, \dots, (n+1)(n+2)/2\}$, and $\mathcal{X}_0 \subset \mathbb{R}^n$ with $x_0 \in \mathcal{X}_0$ and $|\mathcal{X}_0| = m$. Set $Q_{-1} = 0$ and $k = 0$.

1. Pick $Q_k \in \{Q \in \mathcal{Q} : Q(x) = f(x) \text{ for all } x \in \mathcal{X}_k\}$.
2. Define $x_k = \operatorname{argmin}\{f(x) : x \in \mathcal{X}_k\}$, and calculate

$$x_k^+ \approx \operatorname{argmin}\{Q_k(x) : \|x - x_k\| \leq \Delta_k\}, \quad (0.1) \text{ ?eq: ?}$$

3. If $\|x_k^+ - x_k\| \geq \alpha \Delta_k$, then

$$x_k^- \approx \operatorname{argmin}\{\kappa(\mathcal{X}_k, x_k^+, x) : x \in \mathcal{X}_k\}, \quad (0.2) \text{ ?eq: ?}$$

$$\rho_k = [f(x_k) - f(x_k^+)]/[Q_k(x_k) - Q_k(x_k^+)]. \quad (0.3) \text{ \{?\}}$$

4. If $\|x_k^+ - x_k\| < \eta \Delta_k$ or $\rho_k \leq 0$ and $\kappa_k \geq \kappa_0$, then set $\mathcal{X}_{k+1} = \mathcal{X}_k \cup \{x_k^+\} \setminus \{x_k^-\}$. Otherwise, set $y_k^- = \operatorname{argmax}\{\|y - x_k\| : y \in \mathcal{X}_k\}$, $y_k^+ = \operatorname{argmin}\{\kappa(\mathcal{X}_k, y, y_k^-) : \|y - x_k\| \leq \Delta_k\}$, and $\mathcal{X}_{k+1} = \mathcal{X}_k \cup \{y_k^+\} \setminus \{y_k^-\}$. Increment k . Go to Step 1.
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How to terminate? Is $\|\nabla Q_k(x_k)\| \leq \eta \Delta_k$ attainable? What about $\|\nabla Q_k(x_k)\| \leq \epsilon$?
What about $\|x_k^+ - x_k\| \leq \eta \Delta_k$?

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