

Notes on NEWUOA

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Algorithm 0.1

^{alg:newuoa}? Input $\Delta_0 \in (0, +\infty)$, $m \in \{n+2, n+3, \dots, (n+1)(n+2)/2\}$, and $\mathcal{X}_0 \subset \mathbb{R}^n$ with $x_0 \in \mathcal{X}_0$ and $|\mathcal{X}_0| = m$. Set $Q_{-1} = 0$ and $k = 0$.

1. Pick $Q_k \in \{Q \in \mathcal{Q} : Q(x) = f(x) \text{ for all } x \in \mathcal{X}_k\}$.
2. Define $x_k = \operatorname{argmin}\{f(x) : x \in \mathcal{X}_k\}$. Calculate

$$x_k^+ \approx \operatorname{argmin}\{Q_k(x) : \|x - x_k\| \leq \Delta_k\}. \quad (0.1) \text{ ?eq: ?}$$

3. If $\|x_k^+ - x_k\| \leq \alpha\Delta_k$, then set $\Delta_{k+1} = \theta\Delta_k$. Otherwise, calculate

$$x_k^- \approx \operatorname{argmin}\{\kappa(\mathcal{X}_k, x_k^+, x) : x \in \mathcal{X}_k\}, \quad (0.2) \text{ ?eq: ?}$$

$$\rho_k = [f(x_k) - f(x_k^+)]/[Q_k(x_k) - Q_k(x_k^+)], \quad (0.3) \text{ ?eq: ?}$$

update Δ_k to Δ_{k+1} according to ρ_k , and set $\mathcal{X}_{k+1} = \mathcal{X}_k \cup \{x_k^+\} \setminus \{x_k^-\}$ if $\rho_k > 0$ or $\kappa(\mathcal{X}_k, x_k^+, x_k^-) < \kappa_0$.

4. If $\|x_k^+ - x_k\| \leq \alpha\Delta_k$, or $\|x_k^+ - x_k\| > \alpha\Delta_k$ but $\rho_k \leq 0$ and $\kappa(\mathcal{X}_k, x_k^+, x_k^-) \geq \kappa_0$, then calculate

$$y_k^- = \operatorname{argmax}\{\|y - x_k\| : y \in \mathcal{X}_k\}, \quad (0.4) \text{ ?eq: ?}$$

$$y_k^+ \approx \operatorname{argmin}\{\kappa(\mathcal{X}_k, y, y_k^-) : \|y - x_k\| \leq \Delta_k\}, \quad (0.5) \text{ ?eq: ?}$$

and set $\mathcal{X}_{k+1} = \mathcal{X}_k \cup \{y_k^+\} \setminus \{y_k^-\}$.

How to terminate? Is $\|\nabla Q_k(x_k)\| \leq \eta\Delta_k$ attainable? What about $\|\nabla Q_k(x_k)\| \leq \epsilon$?
What about $\|x_k^+ - x_k\| \leq \eta\Delta_k$?

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