Notes on NEWUOA

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Algorithm 0.1

?(alg:newuoa)? $\frac{\operatorname{Ingorithm} \operatorname{GI}}{\operatorname{Input} \Delta_0 \in (0, +\infty), \tau > 0, m \in \{n+2, n+3, \dots, (n+1)(n+2)/2\}, \text{ and } \mathcal{X}_0 \subset \mathbb{R}^n \text{ with } |\mathcal{X}_0| = m.}$ Set $Q_{-1} = 0$ and k = 0.

- 1. Model construction. Pick $Q_k \in \{Q \in \mathcal{Q} : Q(x) = f(x) \text{ for all } x \in \mathcal{X}_k\}$.
- 2. Trust-region step. Define $x_k = \operatorname{argmin}\{f(x) : x \in \mathcal{X}_k\}$. Calculate

$$x_k^+ \approx \operatorname{argmin}\{Q_k(x) : \|x - x_k\| \le \Delta_k\}. \tag{0.1) ?eq:?}$$

If $||x_k^+ - x_k|| \le \alpha \Delta_k$, then set $\Delta_{k+1} = \theta \Delta_k$. Otherwise, update Δ_k to Δ_{k+1} according to $\rho_k = [f(x_k) - f(x_k^+)]/[Q_k(x_k) - Q_k(x_k^+)].$

3. Interpolation set update. If $||x_k^+ - x_k|| \ge \alpha \Delta_k$, then calculate

$$x_k^- \approx \operatorname{argmin}\{\kappa(\mathcal{X}_k, x_k^+, x) : x \in \mathcal{X}_k\},$$
 (0.2) ?eq:?

and set $\mathcal{X}_{k+1} = \mathcal{X}_k \cup \{x_k^+\} \setminus \{x_k^-\}$ if $\rho_k > 0$ or $\kappa(\mathcal{X}_k, x_k^+, x_k^-) < \kappa_0$.

4. Geometry improvement. If $||x_k^+ - x_k|| \le \alpha \Delta_k$, or $||x_k^+ - x_k|| > \alpha \Delta_k$ but $\rho_k \le 0$ and $\kappa(\mathcal{X}_k, x_k^+, x_k^-) \geq \kappa_0$, then calculate

$$y_k^- = \operatorname{argmax}\{\|y - x_k\| : y \in \mathcal{X}_k\},$$
 (0.3) ?eq:?

$$y_k^+ \approx \operatorname{argmin}\{\kappa(\mathcal{X}_k, y, y_k^-) : ||y - x_k|| \le \Delta_k\},\tag{0.4} \ \text{?eq:?}$$

and set $\mathcal{X}_{k+1} = \mathcal{X}_k \cup \{y_k^+\} \setminus \{y_k^-\}.$

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