Notes on NEWUOA

Zaikun Zhang *

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Algorithm 0.1

?⟨alg:newuoa⟩? Input $\Delta_0 \in (0, +\infty)$, $m \in \{n+2, n+3, \dots, (n+1)(n+2)/2\}$, and $\mathcal{X}_0 \subset \mathbb{R}^n$ with $x_0 \in \mathcal{X}_0$ and $|\mathcal{X}_0| = m$. Set $Q_{-1} = 0$ and $k = 0$.

Pick $Q_k \in \{Q \in \mathcal{Q} : Q(x) = f(x) \text{ for all } x \in \mathcal{X}_k\}$. Define $x_k = \text{argmin}\{f(x) : x \in \mathcal{X}_k\}$. Calculate

$$x_k^+ \approx \text{argmin}\{Q_k(x) : \|x - x_k\| \le \Delta_k\}, \tag{0.1}$$?eq:?

If $\|x_k^+ - x_k\| \le \alpha\Delta_k$, then set $\Delta_{k+1} = \theta\Delta_k$. Otherwise, calculate

$$x_k^- \approx \text{argmin}\{\kappa(\mathcal{X}_k, x_k^+, x) : x \in \mathcal{X}_k\}, \tag{0.2}$$?eq:?

$$\rho_k = [f(x_k) - f(x_k^+)]/[Q_k(x_k) - Q_k(x_k^+)], \tag{0.3}$$?eq:?

update Δ_k to Δ_{k+1} according to ρ_k, and set $\mathcal{X}_{k+1} = \mathcal{X}_k \cup \{x_k^+\} \setminus \{x_k^-\}$ if $\rho_k > 0$ or $\kappa(\mathcal{X}_k, x_k^+, x_k^-) < \kappa_0$.

if $\|x_k^+ - x_k\| \le \theta\Delta_k$ or **then**

$$y_k^- = \text{argmax}\{\|y - x_k\| : y \in \mathcal{X}_k\}, \tag{0.4}$$?eq:?

$$y_k^+ \approx \text{argmin}\{\kappa(\mathcal{X}_k, y, y_k^-) : \|y - x_k\| \le \Delta_k\} \tag{0.5}$$?eq:?

$$\mathcal{X}_{k+1} = \mathcal{X}_k \cup \{y_k^+\} \setminus \{y_k^-\}. \tag{0.6}$$?eq:?

end if

How to terminate? Is $\|\nabla Q_k(x_k)\| \le \eta\Delta_k$ attainable? What about $\|\nabla Q_k(x_k)\| \le \epsilon$? What about $\|x_k^+ - x_k\| \le \eta\Delta_k$?

*Hong Kong Polytechnic University, zaikun.zhang@polyu.edu.hk