$$\psi = \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \cos\frac{\phi}{2} \\ \sin\frac{\phi}{2} e^{i\vartheta} \end{pmatrix} = \cos\frac{\phi}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sin\frac{\phi}{2} e^{i\vartheta}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\stackrel{!}{=} \cos\frac{\phi}{2}|1\rangle + \sin\frac{\phi}{2} e^{i\vartheta}|0\rangle$$

$$\|\psi\| = 1$$

$$\bar{\psi}\psi = \cos^2\frac{\phi}{2} + \sin^2\frac{\phi}{2}$$

$$\langle\psi|\sigma_x|\psi\rangle = \begin{pmatrix} \cos\frac{\phi}{2} \\ \sin\frac{\phi}{2} e^{-i\vartheta} \end{pmatrix}^T \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \begin{pmatrix} \cos\frac{\phi}{2} \\ \sin\frac{\phi}{2} e^{i\vartheta} \end{pmatrix}$$

$$= \cos\frac{\phi}{2}\sin\frac{\phi}{2} e^{i\vartheta} + \cos\frac{\phi}{2}\sin\frac{\phi}{2} e^{-i\vartheta}$$

$$= 2\cos\frac{\phi}{2}\sin\frac{\phi}{2}\left(\frac{e^{i\vartheta} + e^{-i\vartheta}}{2}\right)$$

$$= \sin\phi\cos\vartheta$$

$$\langle\psi|\sigma_y|\psi\rangle = \sin\phi\sin\vartheta$$

$$\langle\psi|\sigma_z|\psi\rangle = \cos^2\frac{\phi}{2} - \sin^2\frac{\phi}{2} = \cos\phi$$