$$\mathcal{H} = \lambda \langle \psi_i | \sigma_z | \psi_i \rangle - \sum_{1}^{3} J_j \sum_{1}^{N-1} \langle \psi_i | \sigma_j | \psi_i \rangle \langle \psi_{i+1} | \sigma_j | \psi_{i+1} \rangle$$

↑↑↑↑↑ ↗↗↗↗ ↑↑↑↑

$\psi_i \equiv \psi_{i+1}$ $\quad \langle \psi_i | \sigma_z | \psi_i \rangle = z$

$// \sigma_x // = x$

$\sigma_y = y$

$$\begin{cases} \mathcal{H} = \lambda z - J_x \cdot x^2 - J_y \cdot y^2 - J_z z^2 \\ x^2 + y^2 + z^2 = 1 \end{cases}$$

$$\frac{\partial \mathcal{H}}{\partial z} = \lambda - 2 J_z \cdot z \qquad \nabla \mathcal{H} = \begin{pmatrix} -2J_x x \\ -2J_y y \\ \lambda - 2J_z z \end{pmatrix}$$

$$\frac{\partial \mathcal{H}}{\partial x} = -2J_x x$$

$$\frac{\partial \mathcal{H}}{\partial y} = -2J_y y$$

$$\begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \overset{!}{=} 0$$

$$x \cdot \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{matrix} -y\beta - z\gamma \\ x\beta \\ x\gamma \end{matrix} = \beta \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} -z \\ 0 \\ x \end{pmatrix} \quad x\alpha + y\beta + z\gamma = 0$$

$$\alpha = -\frac{y\beta + z\gamma}{x}$$

$$v = \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix} \qquad u = \begin{pmatrix} -z \\ 0 \\ x \end{pmatrix} \qquad \nabla H: \begin{matrix} -2J_x x \\ -2J_y y \\ \lambda - 2J_z z \end{matrix}$$

$$+ \nabla H \cdot v = \begin{cases} 2J_x xy - 2J_y yx = 0 \\ 2J_x xz + \lambda x - 2J_z zx = 0 \\ x^2 + y^2 + z^2 = 1 \end{cases}$$

$x = 0$ or $y = 0$ $\qquad J_x > J_y \qquad y = 0$

$$\begin{cases} 2J_x xz + \lambda x - 2J_z zx = 0 \\ x^2 + z^2 = 1 \end{cases}$$

$$2Jx^2 + \lambda - 2J_z z^2 = 0 \rightarrow$$

$$2z(J_x - J_z) + \lambda = 0$$

$$z = \frac{\lambda^2}{4(J_z - J_x)^2}$$

$$x^2 = 1 - \frac{\lambda^2}{4(J_z - J_x)^2}$$

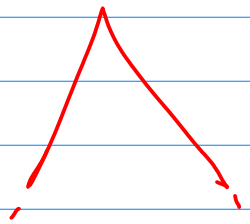
$$z = \frac{\lambda}{(J_z - J_x)2}$$

$$H = \lambda z - J_x x^2 - J_z z^2$$

$$= \frac{2\lambda^2 (J_z - J_x)}{4(J_z - J_x)^2} - J_x + \frac{J_x \lambda^2}{4(J_z - J_x)^2} - J_z \frac{\lambda^2}{4(J_z - J_x)^2}$$

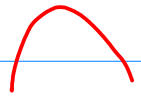
$$= \frac{2\lambda^2 J_z - 2J_x \lambda^2 + J_x \lambda^2 - J_z \lambda^2}{4(\Delta J)^2} - J_x$$

$$= \frac{\lambda^2 J_z - \lambda^2 J_x}{4(\Delta J)^2} - J_x$$



$$= \frac{\lambda^2}{4}\frac{1}{(J_z - J_x)} - J_x = \frac{\lambda^2}{4}\frac{1}{(J_z - J^*)} - J^*$$

$$\overset{!}{=} -\frac{\lambda^2}{4}\frac{1}{(J^* - J_z)} - J^*$$



$$H = \lambda \cdot 1 - J_z \cdot 1 \qquad z = \pm 1$$

$$\overset{!}{=} \lambda - J_z$$

$$H_{\pm 1} = \pm \lambda_z - J_z$$

$$\overset{!}{=} -|\lambda_z| - J_z$$