

# CSI 2110 Tutorial (Section A)

Yiheng Zhao

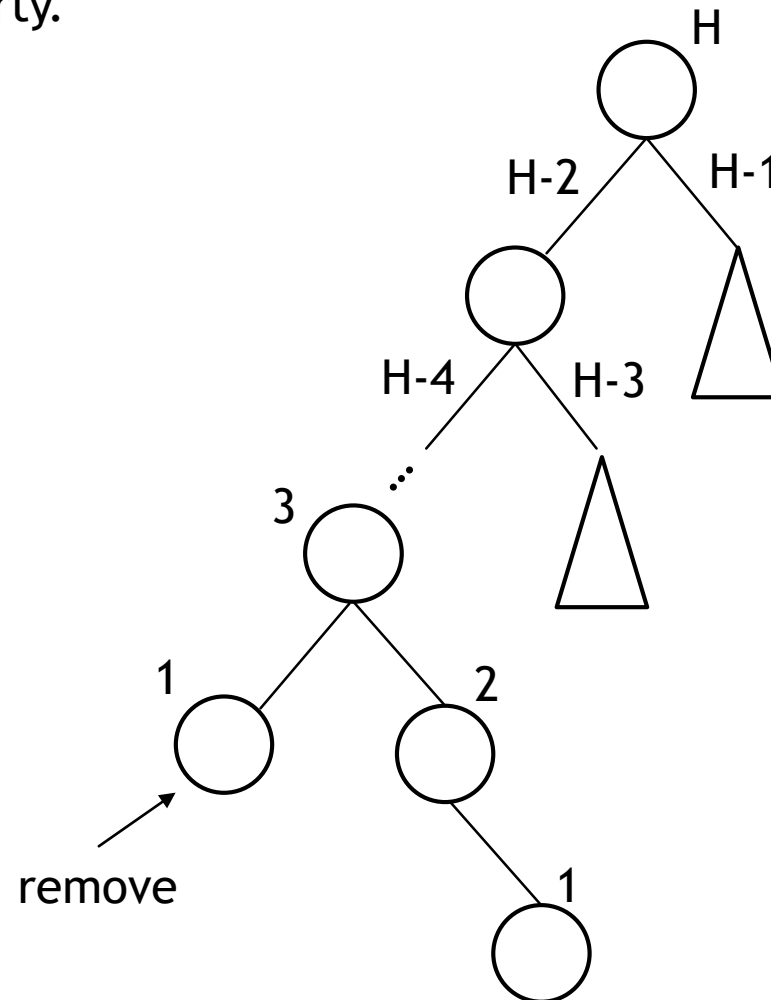
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Office Hour: Fri 13:00-14:00

Place: STE 5000G

## Previous Exercise

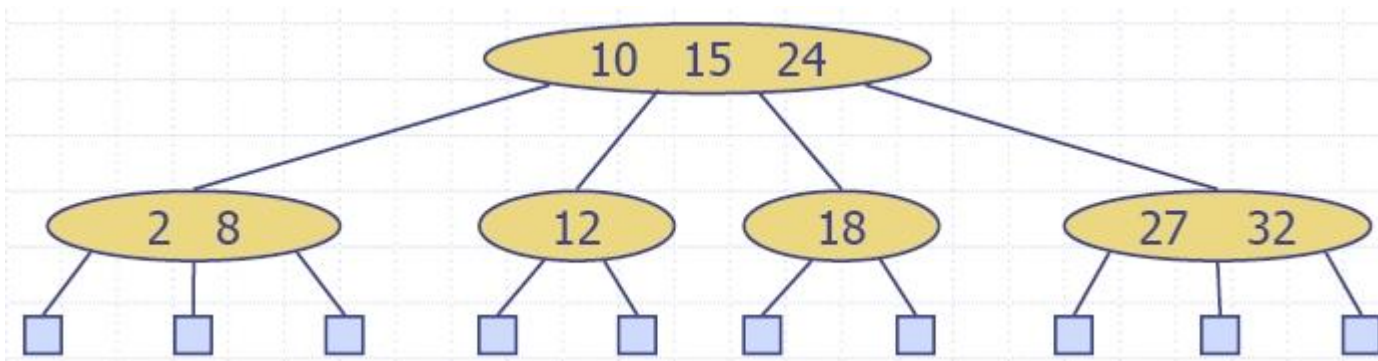
Draw a schematic of an AVL tree such that a single **remove** operation could require  $\Omega(\log n)$  trinode restructurings (or rotations) from a leaf to the root in order to restore the height-balance property.



## Review (2,4) Tree

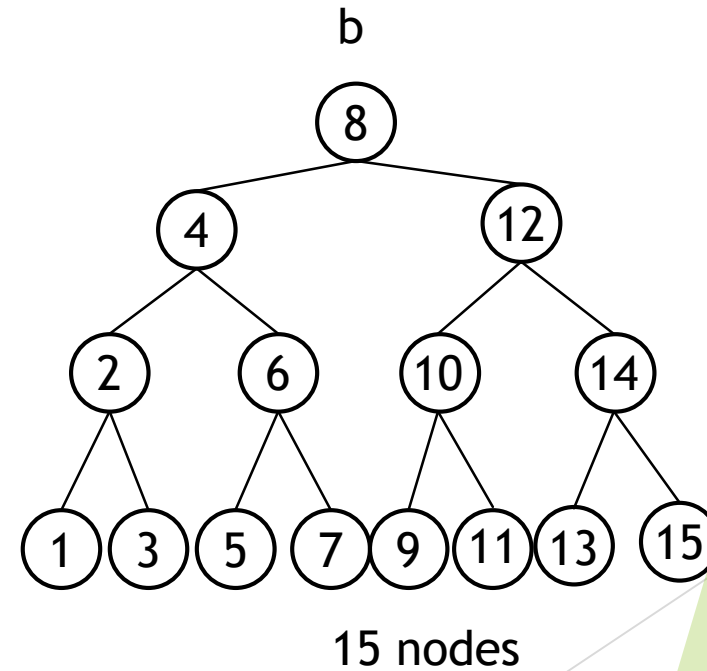
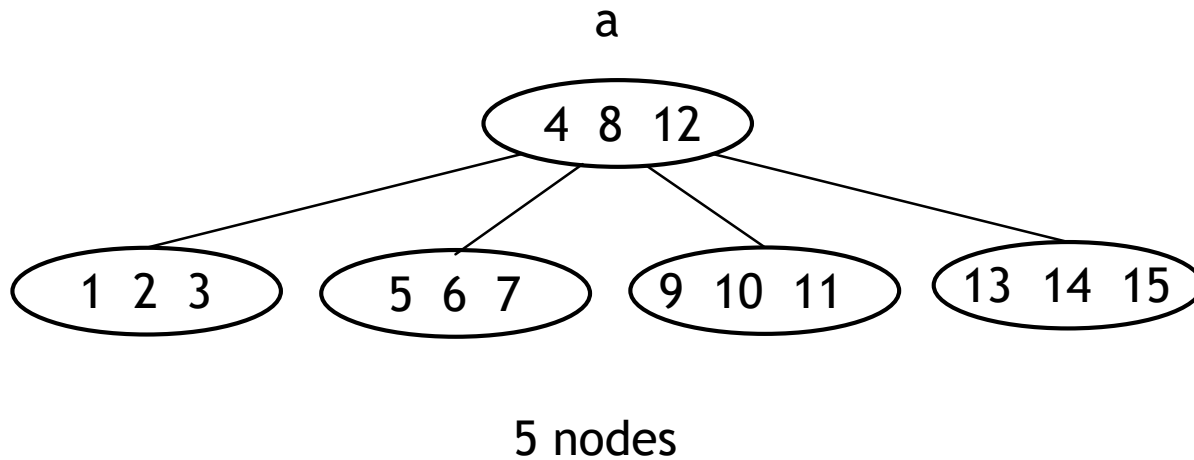
A **multi-way search tree** such that:

- the number of children must be 2, 3, or 4.
- the number of stored elements must be  $d-1$  ( $d$  is the number of children)
- for a node with children  $v_1, v_2, \dots, v_d$ , storing keys  $k_1, k_2, \dots, k(d-1)$ 
  - keys in the subtree of  $v_1$  are **less** than  $k_1$
  - keys in the subtree of  $v_i$  are **between**  $k(i-1)$  and  $k_i$  ( $i=2, \dots, d-1$ )
  - keys in the subtree of  $v_d$  are **greater** than  $k(d-1)$
- the leaves store no items and serve as placeholders
- all the external nodes have the same depth

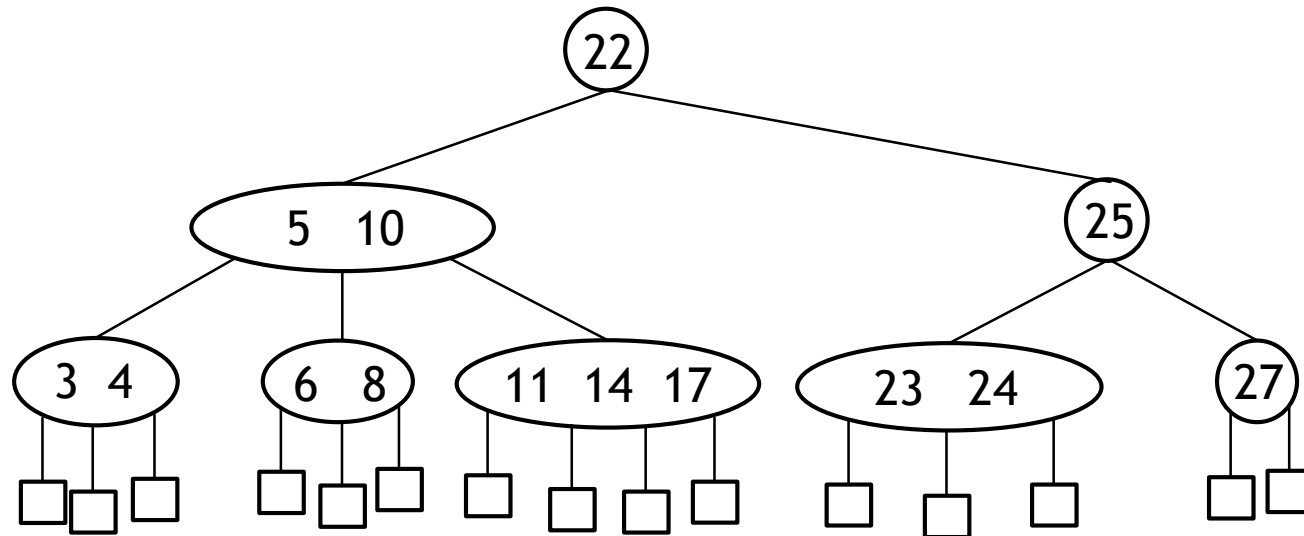


# Exercise

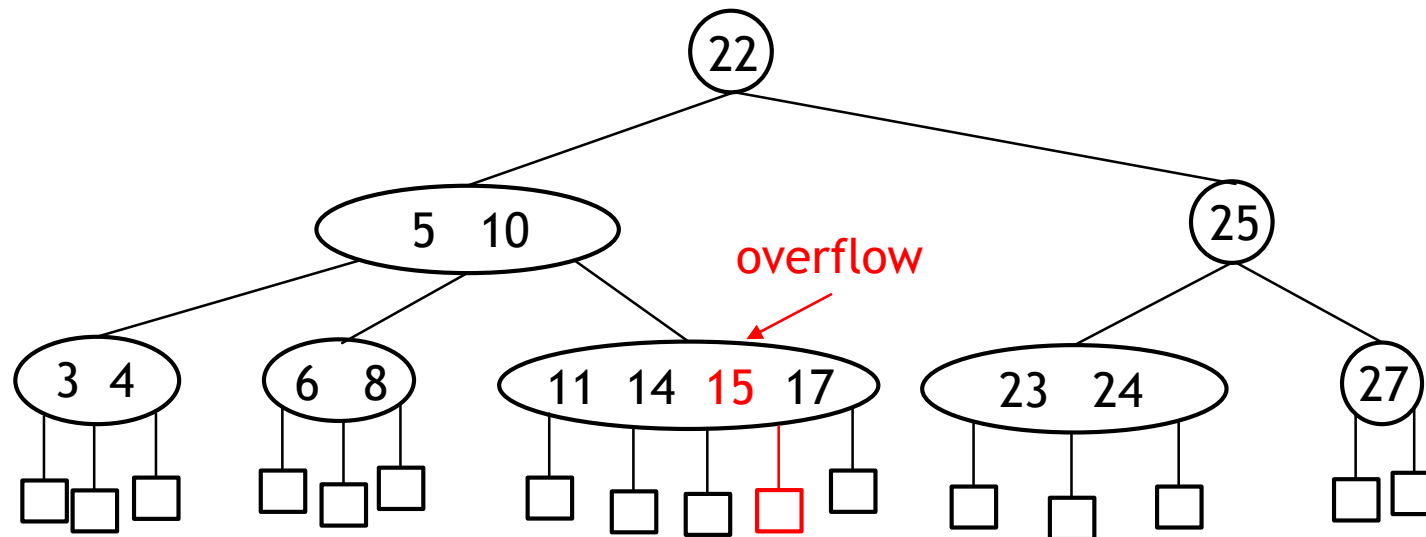
1. Consider the set of keys  $k=\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$ .
- a. Draw a (2,4) tree storing K as its keys using the **fewest** number of nodes
  - b. Draw a (2,4) tree storing K as its keys using the **greatest** number of nodes.



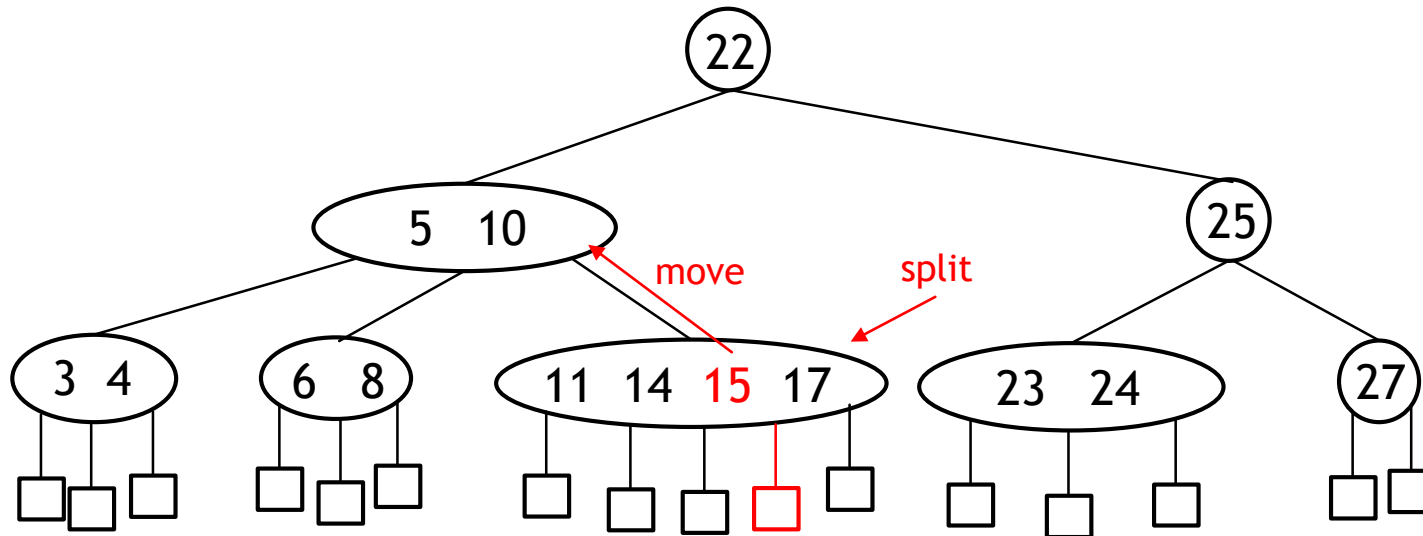
2. Insert 15 in the tree of the picture and show the resulting tree.



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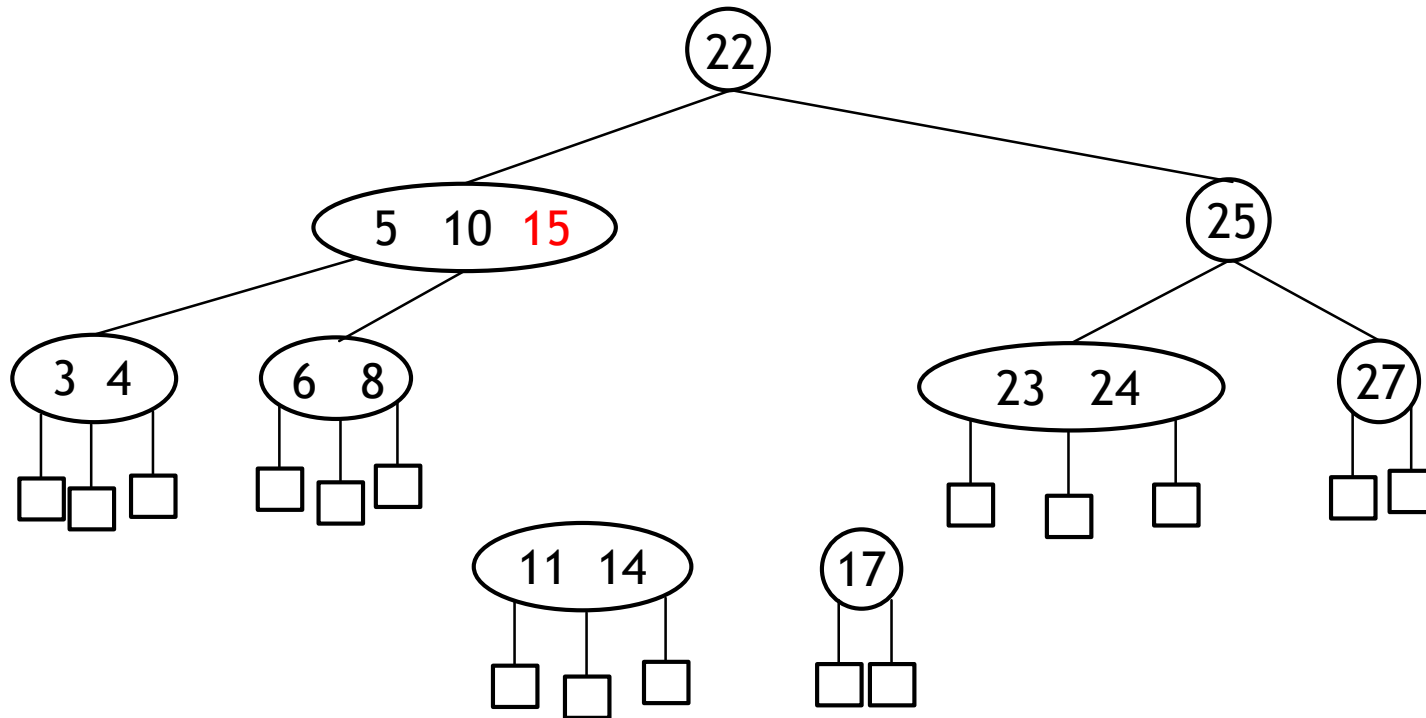


2. Insert 15 in the tree of the picture and show the resulting tree.



Split the node to 3-node and 2-node  
Move the third element to parent

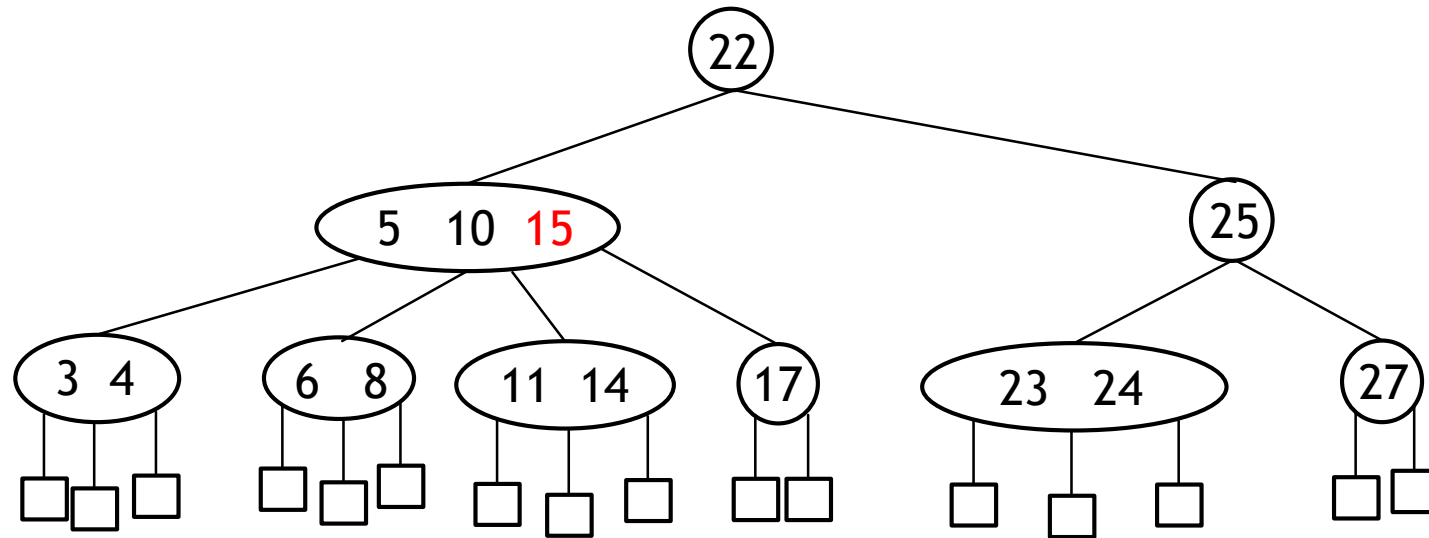
2. Insert 15 in the tree of the picture and show the resulting tree.



Link 3-node and 2-node to parent

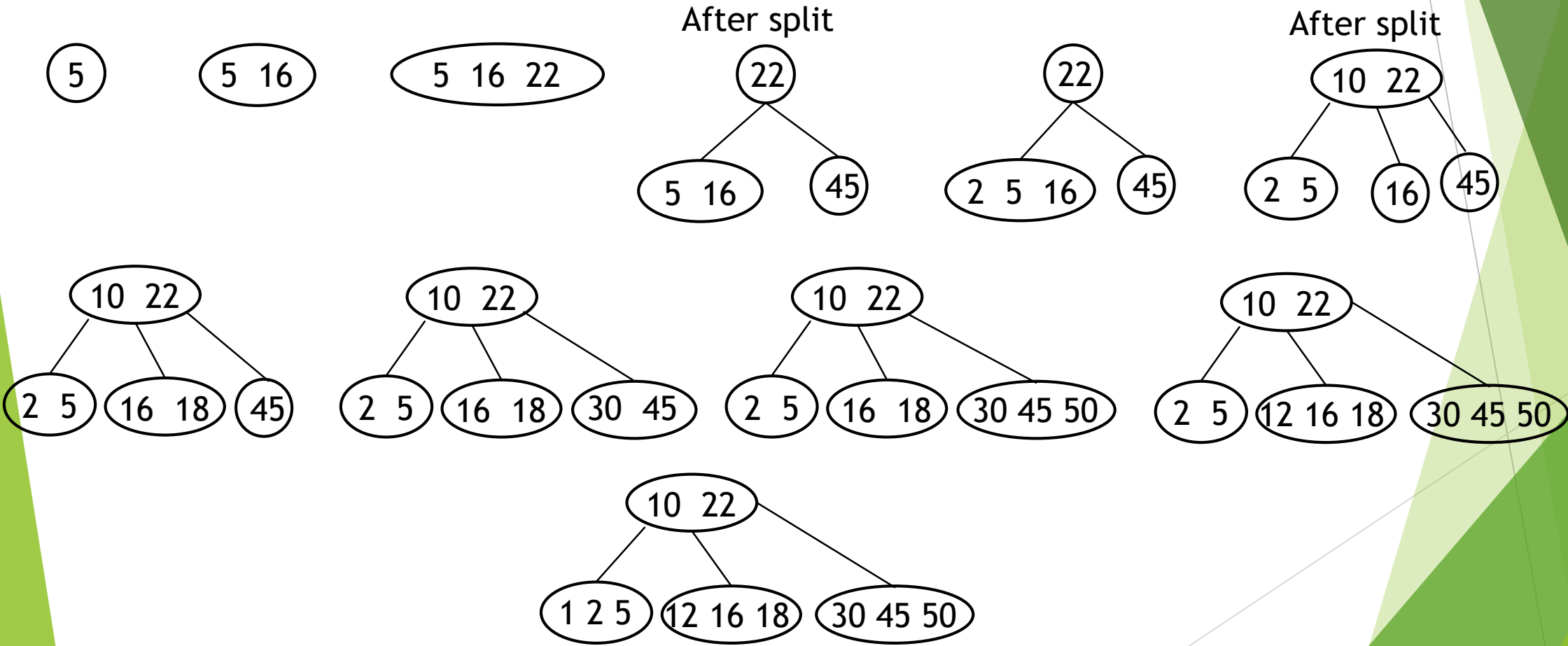


2. Insert 15 in the tree of the picture and show the resulting tree.

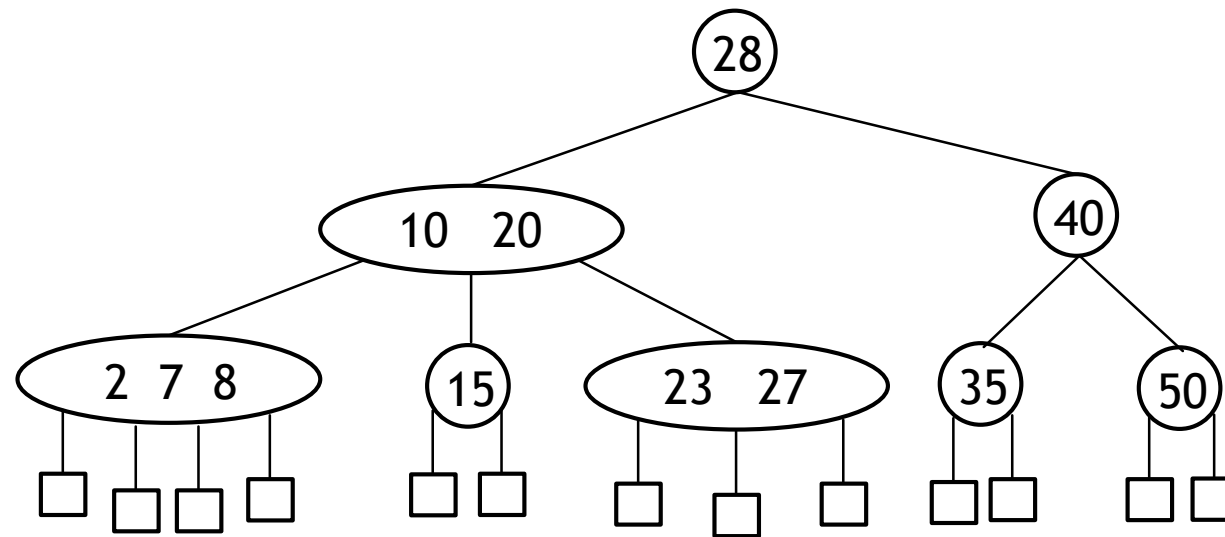


Exercise

3. Consider the sequence of keys (5,16,22,45,2,10,18,30,50,12,1). Draw the Result of inserting entries with these keys (in the given order) into an initially Empty (2, 4) tree

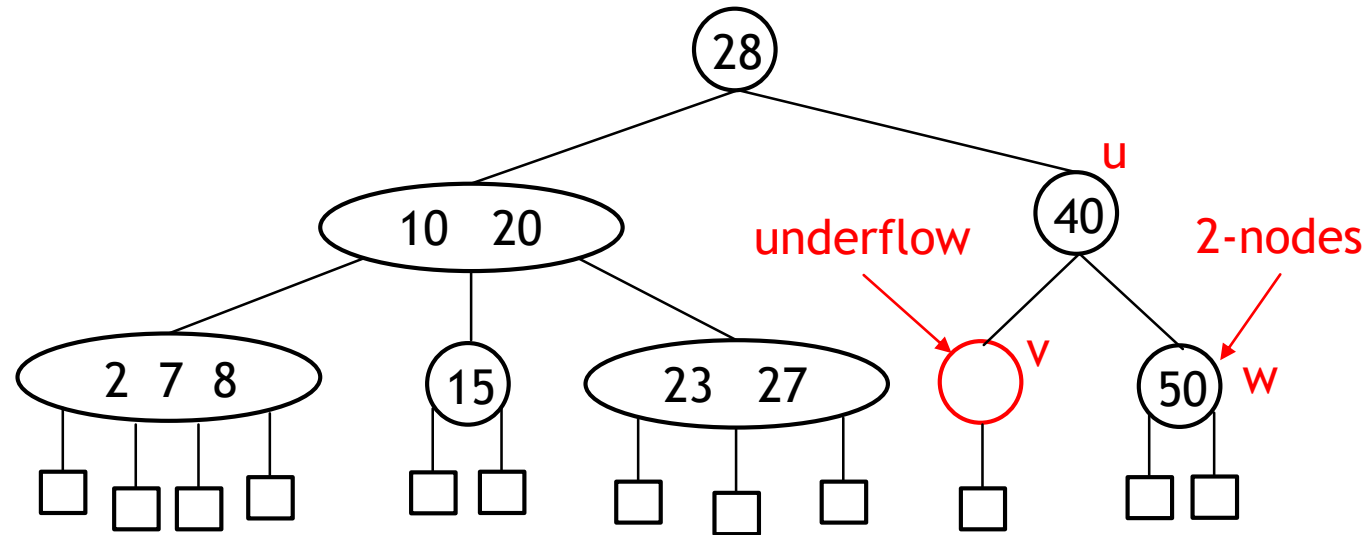


4. Delete 35 from the following (2,4) tree. Draw the resulting tree after rebalancing (if necessary)



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Case 1: adjacent siblings are 2-nodes



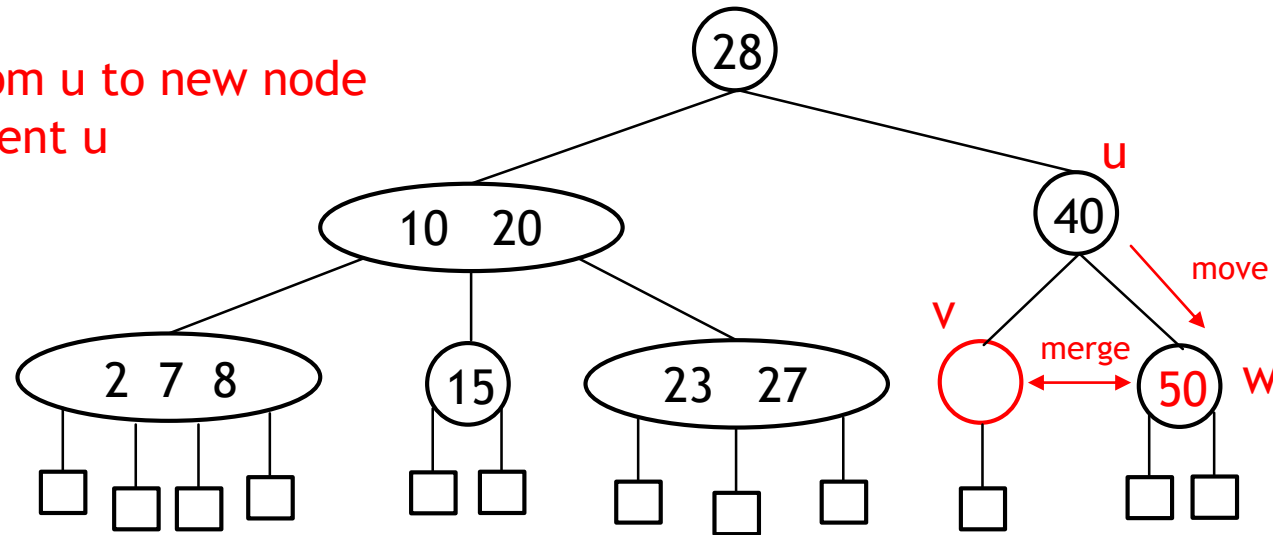
Underflow: node becomes a 1-node with one child and no keys

## Exercise

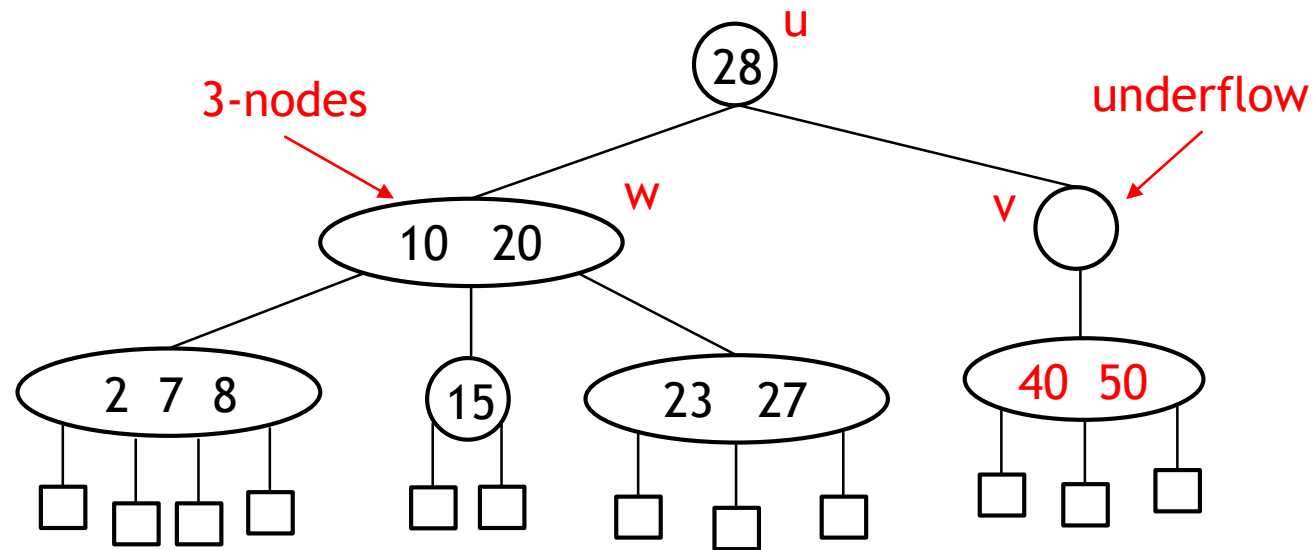
4. Delete 35 from the following (2,4) tree. Draw the resulting tree after rebalancing (if necessary)

Case 1: adjacent siblings are 2-nodes

- merge v and w
- move an item from u to new node
- propagate to parent u

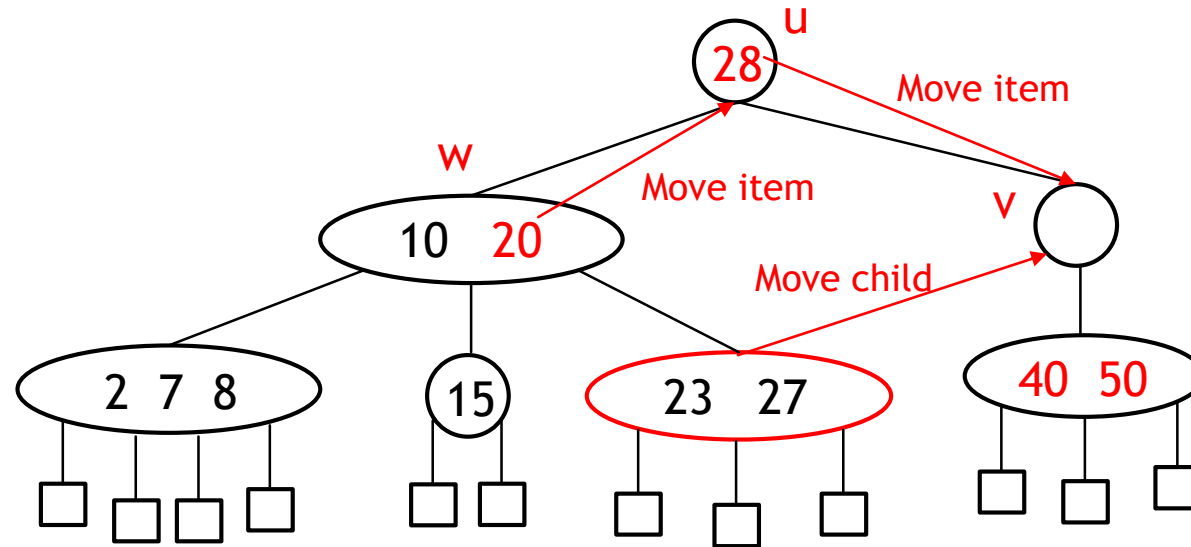


4. Delete 35 from the following (2,4) tree. Draw the resulting tree after rebalancing (if necessary)



Case 2: adjacent siblings are 3-nodes or 4-nodes

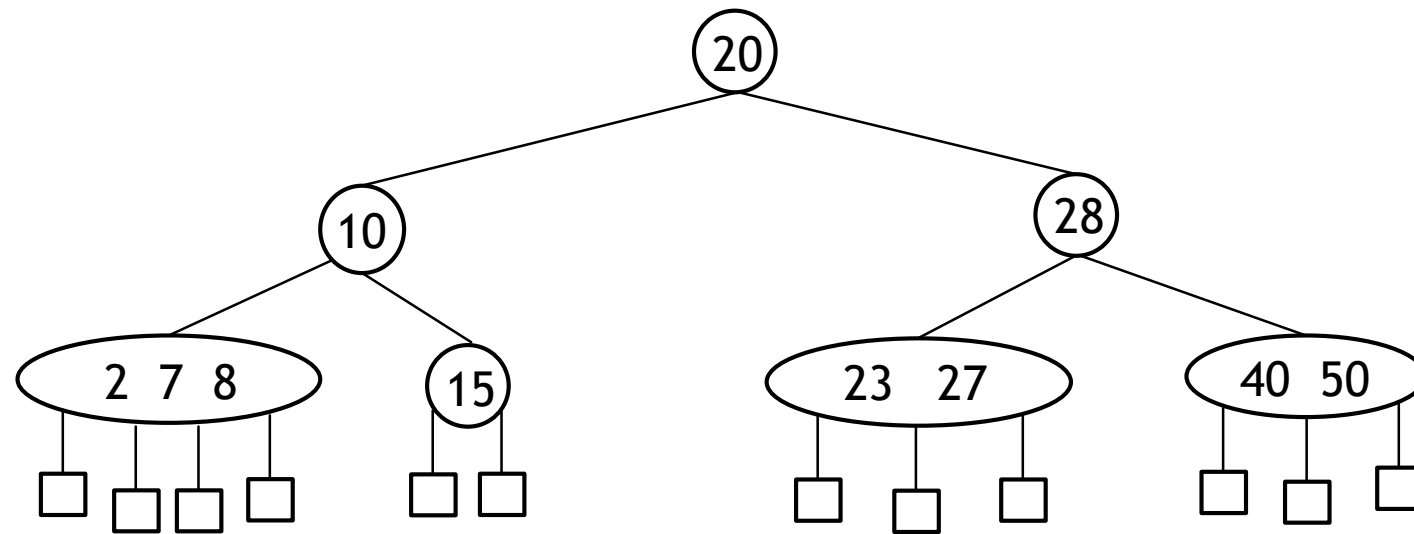
4. Delete 35 from the following (2,4) tree. Draw the resulting tree after rebalancing (if necessary)



Case 2: adjacent siblings are 3-nodes or 4-nodes

- move a child of w to v
- move an item from u to v
- move an item from w to u

4. Delete 35 from the following (2,4) tree. Draw the resulting tree after rebalancing (if necessary)

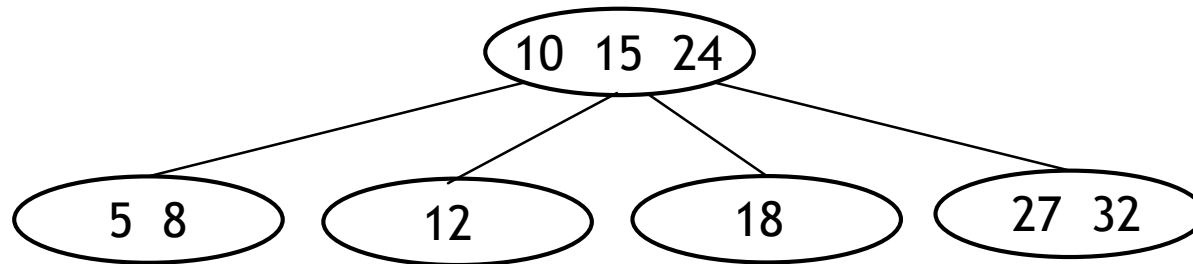




## Exercise

Consider the following 2-4 tree:

1. Insert 2 into the following 2-4 tree and show the resulting tree beside it.



2. Delete 12 from the following 2-4 tree and show the resulting tree beside it.

