Useful Linear Algebra Tricks for Statistics

Matthew Cocci

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1 Definitions

Suppose we have two matrices, A which is $m \times n$ and B which is $p \times q$. Then the *Kronecker Product* of A and B is

$$A \otimes B = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{pmatrix}$$

which implies that the new matrix is $(mp) \times (nq)$.

Next, the *vec operator* takes any matrix A that is $m \times n$ and stacks to columns on top of each other (left to right) to form a column vector of length mn. Supposing that a_i are column vectors to simplify notation:

$$\text{if } A = \begin{pmatrix} a_1 & \cdots & a_n \end{pmatrix} \qquad a_i \in \mathbb{R}^{n \times 1}$$
$$\text{then } \mathbf{vec}A = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}$$

2 Properties with Proofs, Kronecker Product

Property 1 Let A be $m \times n$, B be $p \times q$, C be $n \times r$, and D be $q \times s$. Then

$$(A \otimes B)(C \otimes D) = AC \otimes BD \tag{1}$$

Proof. We start by writing:

$$(A \otimes B)(C \otimes D) = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{pmatrix} \begin{pmatrix} c_{11}D & \cdots & c_{1r}D \\ \vdots & \ddots & \vdots \\ c_{n1}D & \cdots & c_{nr}D \end{pmatrix}$$

Since the matrix D has the same number of rows as B has columns, we can carry out the multiplication to get

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