

# Tidy Time Series & Forecasting in R

## 10. Forecast reconciliation



# Outline

- 1 Hierarchical and grouped time series
- 2 Forecast reconciliation
- 3 Example: Australian tourism
- 4 Lab Session 20

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# Australian Pharmaceutical Benefits Scheme



# PBS sales

PBS

```
## # A tsibble: 67,596 x 9 [1M]
## # Key:      Concession, Type, ATC1, ATC2 [336]
##      Month Concession  Type  ATC1  ATC1_desc ATC2  ATC2_desc Scripts
##      <mt> <chr>        <chr> <chr> <chr>      <chr> <chr>      <dbl>
##  1 1991 Jul Concessional Co-pa~ A      Alimenta~ A01    STOMATOL~ 18228
##  2 1991 Aug Concessional Co-pa~ A      Alimenta~ A01    STOMATOL~ 15327
##  3 1991 Sep Concessional Co-pa~ A      Alimenta~ A01    STOMATOL~ 14775
##  4 1991 Oct Concessional Co-pa~ A      Alimenta~ A01    STOMATOL~ 15380
##  5 1991 Nov Concessional Co-pa~ A      Alimenta~ A01    STOMATOL~ 14371
##  6 1991 Dec Concessional Co-pa~ A      Alimenta~ A01    STOMATOL~ 15028
##  7 1992 Jan Concessional Co-pa~ A      Alimenta~ A01    STOMATOL~ 11040
##  8 1992 Feb Concessional Co-pa~ A      Alimenta~ A01    STOMATOL~ 15165
##  9 1992 Mar Concessional Co-pa~ A      Alimenta~ A01    STOMATOL~ 16898
## 10 1992 Apr Concessional Co-pa~ A      Alimenta~ A01    STOMATOL~ 18141
## # ... with 67,586 more rows, and 1 more variable: Cost <dbl>
```

# ATC drug classification

- A Alimentary tract and metabolism
- B Blood and blood forming organs
- C Cardiovascular system
- D Dermatologicals
- G Genito-urinary system and sex hormones
- H Systemic hormonal preparations, excluding sex hormones and insulins
- J Anti-infectives for systemic use
- L Antineoplastic and immunomodulating agents
- M Musculo-skeletal system
- N Nervous system
- P Antiparasitic products, insecticides and repellents
- R Respiratory system
- S Sensory organs

# ATC drug classification

**ATC1: 14 classes**

A

Alimentary tract and metabolism

**ATC2: 84 classes**

A10

Drugs used in diabetes

A10B

Blood glucose lowering drugs

A10BA

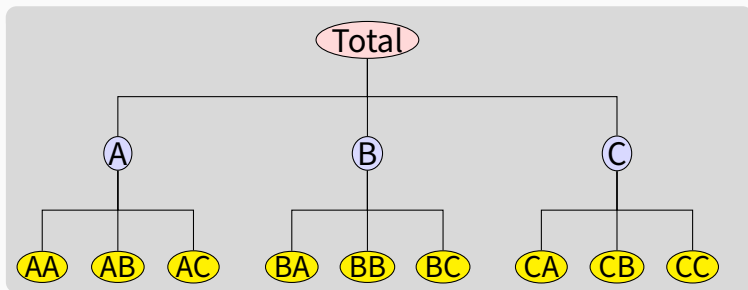
Biguanides

A10BA02

Metformin

# Hierarchical time series

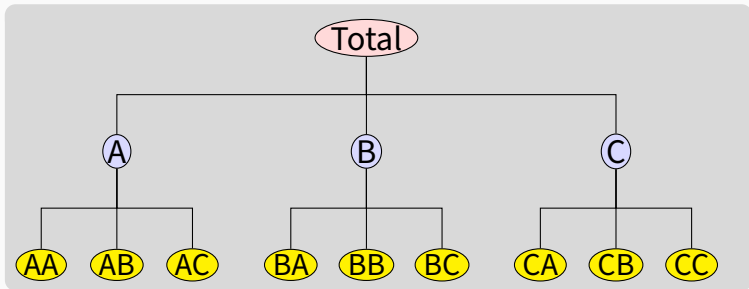
A **hierarchical time series** is a collection of several time series that are linked together in a hierarchical structure.





# Hierarchical time series

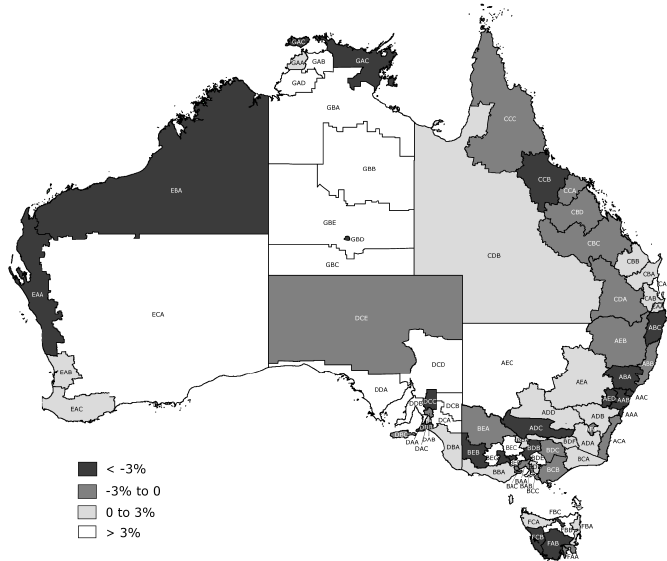
A **hierarchical time series** is a collection of several time series that are linked together in a hierarchical structure.



## Examples

- PBS sales by ATC groups
- Tourism demand by states, zones, regions

# Australian tourism



# Australian tourism

tourism

```
## # A tsibble: 24,320 x 5 [1Q]
```

```
## # Key:           Region, State, Purpose [304]
```

```
##   Quarter Region   State           Purpose   Trips
```

```
##   <qtr> <chr>      <chr>           <chr>    <dbl>
```

```
## 1 1998 Q1 Adelaide South Australia Business 135.
```

```
## 2 1998 Q2 Adelaide South Australia Business 110.
```

```
## 3 1998 Q3 Adelaide South Australia Business 166.
```

```
## 4 1998 Q4 Adelaide South Australia Business 127.
```

```
## 5 1999 Q1 Adelaide South Australia Business 137.
```

```
## 6 1999 Q2 Adelaide South Australia Business 200.
```

```
## 7 1999 Q3 Adelaide South Australia Business 169.
```

```
## 8 1999 Q4 Adelaide South Australia Business 134.
```

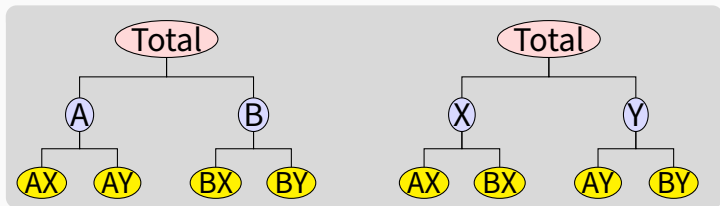
```
## 9 2000 Q1 Adelaide South Australia Business 154.
```

# Australian tourism

- Quarterly data on visitor night from 1998:Q1 – 2013:Q4
- From: *National Visitor Survey*, based on annual interviews of 120,000 Australians aged 15+, collected by Tourism Research Australia.
- Split by 7 states, 27 zones and 76 regions (a geographical hierarchy)
- Also split by purpose of travel
  - ▶ Holiday
  - ▶ Visiting friends and relatives (VFR)
  - ▶ Business
  - ▶ Other
- 304 bottom-level series

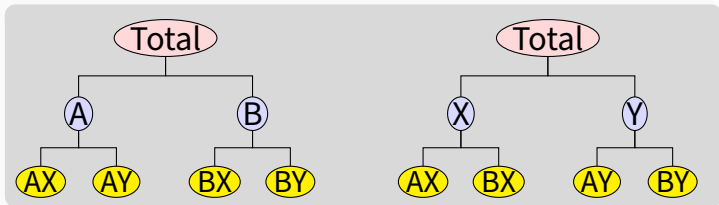
# Grouped time series

A **grouped time series** is a collection of time series that can be grouped together in a number of non-hierarchical ways.



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## Examples

- Tourism by state and purpose of travel
- Retail sales by product groups/sub groups, and by countries/regions

# Creating aggregates

PBS ▷

```
aggregate_key(ATC1 / ATC2, Scripts = sum(Scripts)) ▷  
filter(Month = yearmonth("1991 Jul")) ▷  
print(n = 18)
```

```
## # A tsibble: 98 x 4 [1M]  
## # Key:      ATC1, ATC2 [98]  
##   Month ATC1      ATC2      Scripts  
##   <mt> <chr*>    <chr*>    <dbl>  
## 1 1991 Jul <aggregated> <aggregated> 8090395  
## 2 1991 Jul A      <aggregated> 799025  
## 3 1991 Jul B      <aggregated> 109227  
## 4 1991 Jul C      <aggregated> 1794995  
## 5 1991 Jul D      <aggregated> 299779  
## 6 1991 Jul G      <aggregated> 300931  
## 7 1991 Jul H      <aggregated> 112114  
## 8 1991 Jul J      <aggregated> 1151681  
## 9 1991 Jul L      <aggregated> 24580  
## 10 1991 Jul M     <aggregated> 562956  
## 11 1991 Jul N     <aggregated> 1546023  
## 12 1991 Jul P     <aggregated> 47661  
## 13 1991 Jul R     <aggregated> 859273  
## 14 1991 Jul S     <aggregated> 391639
```

# Creating aggregates

```
tourism ▷  
  aggregate_key(Purpose * (State / Region), Trips = sum(Trips)) ▷  
  filter(Quarter = yearquarter("1998 Q1")) ▷  
  print(n = 15)
```

```
## # A tsibble: 425 x 5 [1Q]  
## # Key:      Purpose, State, Region [425]  
##   Quarter Purpose      State      Region      Trips  
##   <qtr> <chr*>      <chr*>      <chr*>      <dbl>  
## 1 1998 Q1 <aggregated> <aggregated> <aggregated> 23182.  
## 2 1998 Q1 Business <aggregated> <aggregated> 3599.  
## 3 1998 Q1 Holiday <aggregated> <aggregated> 11806.  
## 4 1998 Q1 Other <aggregated> <aggregated> 680.  
## 5 1998 Q1 Visiting <aggregated> <aggregated> 7098.  
## 6 1998 Q1 <aggregated> ACT <aggregated> 551.  
## 7 1998 Q1 <aggregated> New South Wales <aggregated> 8040.  
## 8 1998 Q1 <aggregated> Northern Territory <aggregated> 181.  
## 9 1998 Q1 <aggregated> Queensland <aggregated> 4041.  
## 10 1998 Q1 <aggregated> South Australia <aggregated> 1735.  
## 11 1998 Q1 <aggregated> Tasmania <aggregated> 982.  
## 12 1998 Q1 <aggregated> Victoria <aggregated> 6010.
```



# Creating aggregates

- Similar to `summarise( )` but using the key structure
- A grouped structure is specified using `grp1 * grp2`
- A nested structure is specified via `parent / child`.
- Groups and nesting can be mixed:  

```
(country/region/city) * (brand/product)
```
- All possible aggregates are produced.
- These are useful when forecasting at different levels of aggregation.

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# The problem

- 1 How to forecast time series at all nodes such that the forecasts add up in the same way as the original data?
- 2 Can we exploit relationships between the series to improve the forecasts?

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## The solution

- 1 Forecast all series at all levels of aggregation using an automatic forecasting algorithm.  
(e.g., ETS, ARIMA, ...)
- 2 Reconcile the resulting forecasts so they add up correctly using least squares optimization (i.e., find closest reconciled forecasts to the original forecasts).
- 3 This is available using `reconcile()`.

# Forecast reconciliation

```
tourism ▷  
  aggregate_key(Purpose * (State / Region), Trips = sum(Trips)) ▷  
  model(ets = ETS(Trips)) ▷  
  reconcile(ets_adjusted = min_trace(ets)) ▷  
  forecast(h = 2)
```

```
## # A fable: 1,700 x 7 [1Q]
```

```
## # Key:      Purpose, State, Region, .model [850]
```

##	Purpose	State	Region	.model	Quarter	Trips	.mean
##	<chr*>	<chr*>	<chr*>	<chr>	<qtr>	<dist>	<dbl>
##	1 Business	ACT	Canberra	~ ets	2018 Q1	N(144, 1119)	144.
##	2 Business	ACT	Canberra	~ ets	2018 Q2	N(203, 2260)	203.
##	3 Business	ACT	Canberra	~ ets_a~	2018 Q1	N(157, 539)	157.
##	4 Business	ACT	Canberra	~ ets_a~	2018 Q2	N(214, 951)	214.
##	5 Business	ACT	<aggregated>	ets	2018 Q1	N(144, 1119)	144.
##	6 Business	ACT	<aggregated>	ets	2018 Q2	N(203, 2260)	203.
##	7 Business	ACT	<aggregated>	ets_a~	2018 Q1	N(157, 539)	157.
##	8 Business	ACT	<aggregated>	ets_a~	2018 Q2	N(214, 951)	214.

# Hierarchical and grouped time series

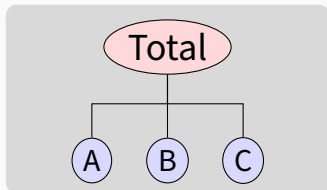
Every collection of time series with aggregation constraints can be written as

$$\mathbf{y}_t = \mathbf{S}\mathbf{b}_t$$

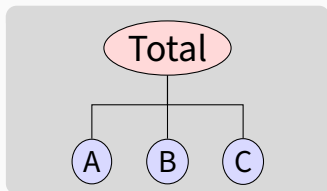
where

- $\mathbf{y}_t$  is a vector of all series at time  $t$
- $\mathbf{b}_t$  is a vector of the most disaggregated series at time  $t$
- $\mathbf{S}$  is a “summing matrix” containing the aggregation constraints.

# Hierarchical time series



# Hierarchical time series



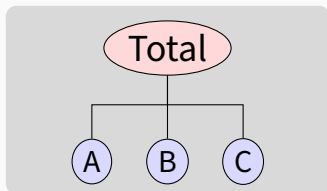
$y_t$  : observed aggregate of all series at time  $t$ .

$y_{X,t}$  : observation on series  $X$  at time  $t$ .

$\mathbf{b}_t$  : vector of all series at bottom level in time  $t$ .



# Hierarchical time series



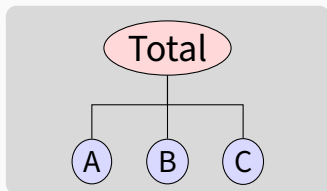
$y_t$  : observed aggregate of all series at time  $t$ .

$y_{X,t}$  : observation on series  $X$  at time  $t$ .

$\mathbf{b}_t$  : vector of all series at bottom level in time  $t$ .

$$\mathbf{y}_t = \begin{pmatrix} y_t \\ y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix}$$

# Hierarchical time series



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$$\mathbf{y}_t = \begin{pmatrix} y_t \\ y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{S}} \underbrace{\begin{pmatrix} y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix}}_{\mathbf{b}_t}$$

$$\mathbf{y}_t = \mathbf{S}\mathbf{b}_t$$

## Forecasting notation

Let  $\hat{\mathbf{y}}_n(h)$  be vector of initial  $h$ -step forecasts, made at time  $n$ , stacked in same order as  $\mathbf{y}_t$ .

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(In general, they will not “add up”.)

Reconciled forecasts must be of the form:

$$\tilde{\mathbf{y}}_n(h) = \mathbf{S}\mathbf{G}\hat{\mathbf{y}}_n(h)$$

for some matrix  $\mathbf{G}$ .

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Reconciled forecasts must be of the form:

$$\tilde{\mathbf{y}}_n(h) = \mathbf{S}\mathbf{G}\hat{\mathbf{y}}_n(h)$$

for some matrix  $\mathbf{G}$ .

- $\mathbf{G}$  extracts and combines base forecasts  $\hat{\mathbf{y}}_n(h)$  to get bottom-level forecasts.
- $\mathbf{S}$  adds them up

# Optimal combination forecasts

## Main result

The best (minimum sum of variances) unbiased forecasts are obtained when  $\mathbf{G} = (\mathbf{S}'\Sigma_h^{-1}\mathbf{S})^{-1}\mathbf{S}'\Sigma_h^{-1}$ , where  $\Sigma_h$  is the  $h$ -step base forecast error covariance matrix.

# Optimal combination forecasts

## Main result

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$$\tilde{\mathbf{y}}_n(h) = \mathbf{s}(\mathbf{S}'\Sigma_h^{-1}\mathbf{S})^{-1}\mathbf{S}'\Sigma_h^{-1}\hat{\mathbf{y}}_n(h)$$

**Problem:**  $\Sigma_h$  hard to estimate, especially for  $h > 1$ .

## Solutions:

- Ignore  $\Sigma_h$  (OLS) [`min_trace(method='ols')`]
- Assume  $\Sigma_h = k_h \Sigma_1$  is diagonal (WLS)  
[`min_trace(method='wls')`]
- Assume  $\Sigma_h = k_h \Sigma_1$  and estimate it (GLS)



# Features

- Covariates can be included in initial forecasts.
- Adjustments can be made to initial forecasts at any level.
- Very simple and flexible method. Can work with *any* hierarchical or grouped time series.
- Conceptually easy to implement: regression of base forecasts on structure matrix.

# Outline

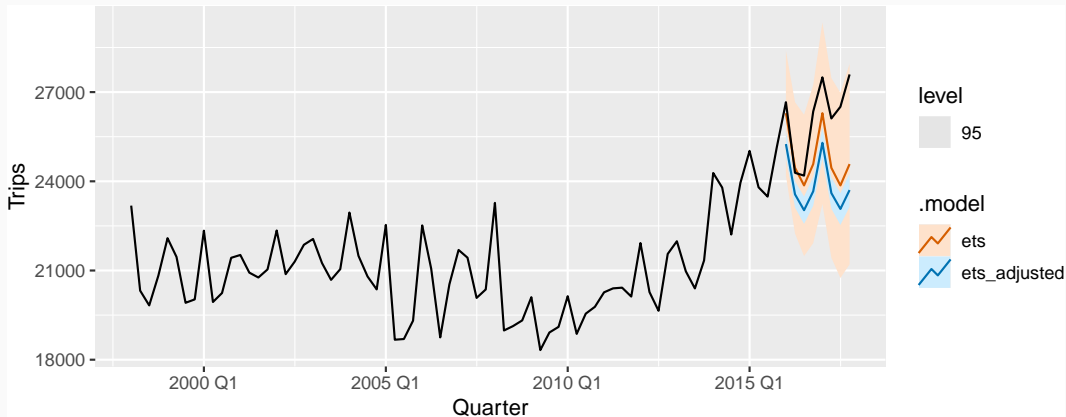
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# Example: Australian tourism

```
tourism_agg <- tourism ▷  
  aggregate_key(Purpose * (State / Region),  
    Trips = sum(Trips)  
  )  
fc <- tourism_agg ▷  
  filter_index(. ~ "2015 Q4") ▷  
  model(ets = ETS(Trips)) ▷  
  reconcile(ets_adjusted = min_trace(ets)) ▷  
  forecast(h = "2 years")
```

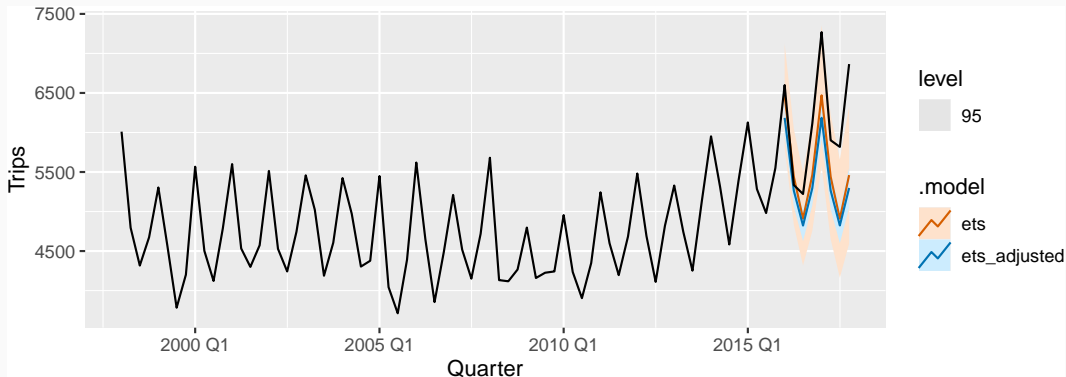
# Example: Australian tourism

```
fc ▷  
  filter(is_aggregated(Purpose) & is_aggregated(State)) ▷  
  autoplot(tourism_agg, level = 95)
```



# Example: Australian tourism

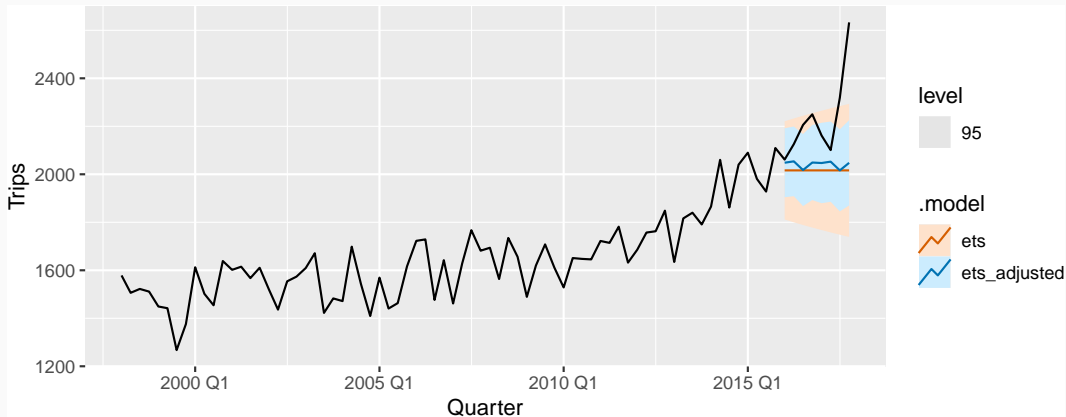
```
fc ▷  
  filter(is_aggregated(Purpose) & State = "Victoria" &  
    is_aggregated(Region)) ▷  
  autoplot(tourism_agg, level = 95)
```



# Example: Australian tourism

fc ▷

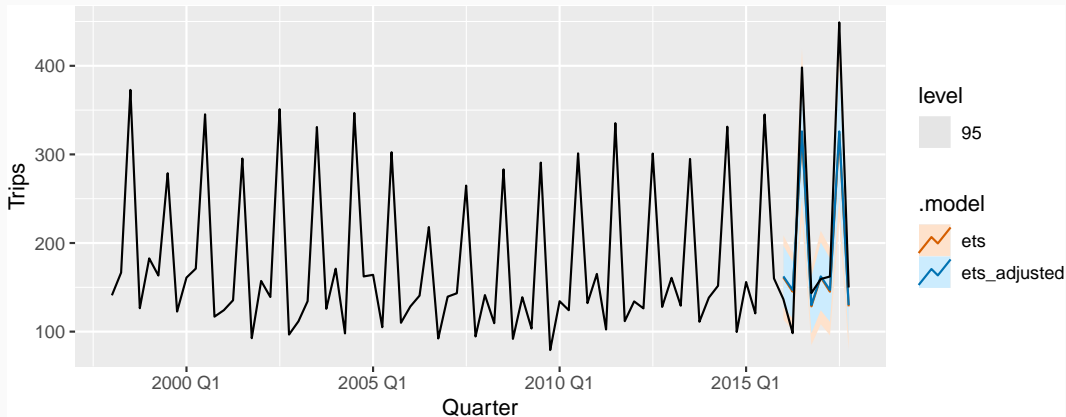
```
filter(is_aggregated(Purpose) & Region == "Melbourne") ▷  
autoplot(tourism_agg, level = 95)
```



# Example: Australian tourism

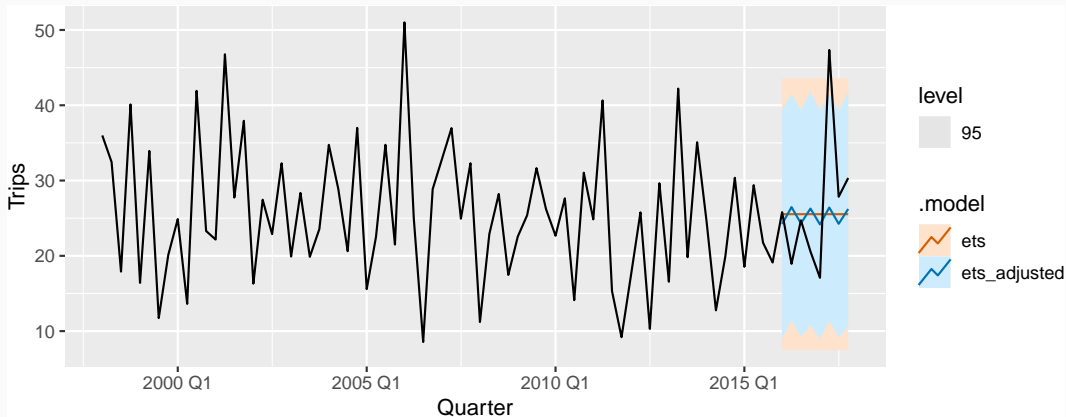
fc ▷

```
filter(is_aggregated(Purpose) & Region == "Snowy Mountains") ▷  
autoplot(tourism_agg, level = 95)
```



# Example: Australian tourism

```
fc ▷  
  filter(Purpose = "Holiday" & Region = "Barossa") ▷  
  autoplot(tourism_agg, level = 95)
```

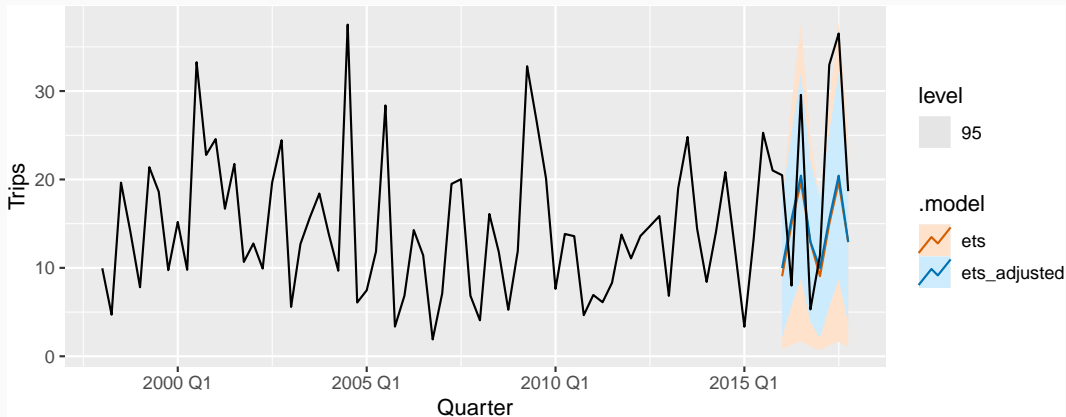




# Example: Australian tourism

fc ▷

```
filter(is_aggregated(Purpose) & Region = "MacDonnell") ▷  
autoplot(tourism_agg, level = 95)
```



# Example: Australian tourism

```
fc <- tourism_agg ▷  
  filter_index(. ~ "2015 Q4") ▷  
  model(  
    ets = ETS(Trips),  
    arima = ARIMA(Trips)  
  ) ▷  
  mutate(  
    comb = (ets + arima) / 2  
  ) ▷  
  reconcile(  
    ets_adj = min_trace(ets),  
    arima_adj = min_trace(arima),  
    comb_adj = min_trace(comb)  
  ) ▷  
  forecast(h = "2 years")
```

# Forecast evaluation

```
fc > accuracy(tourism_agg)
```

```
## # A tibble: 2,550 x 13
```

##	.model	Purpose	State	Region	.type	ME	RMSE	MAE	MPE	
##	<chr>	<chr*>	<chr*>	<chr*>	<chr>	<dbl>	<dbl>	<dbl>	<dbl>	
##	1	arma	Business	ACT	Canberra ~	Test	35.9	45.7	35.9	16.9
##	2	arma	Business	ACT	<aggregat~	Test	35.9	45.7	35.9	16.9
##	3	arma	Business	New South Wales	Blue Moun~	Test	1.93	10.6	8.52	-18.0
##	4	arma	Business	New South Wales	Capital C~	Test	8.08	15.6	10.4	11.8
##	5	arma	Business	New South Wales	Central C~	Test	10.0	14.5	10.8	26.9
##	6	arma	Business	New South Wales	Central N~	Test	17.7	31.9	28.2	12.0
##	7	arma	Business	New South Wales	Hunter ~	Test	35.3	43.9	35.3	24.2
##	8	arma	Business	New South Wales	New Engla~	Test	23.1	31.8	26.8	19.5
##	9	arma	Business	New South Wales	North Coa~	Test	24.8	40.1	36.8	11.5
##	10	arma	Business	New South Wales	Outback N~	Test	6.87	11.0	7.76	13.7
##	#	...	with 2,540 more rows, and 4 more variables: MAPE <dbl>, MASE <dbl>,							

# Forecast evaluation

```
fc ▷  
  accuracy(tourism_agg) ▷  
  group_by(.model) ▷  
  summarise(MASE = mean(MASE)) ▷  
  arrange(MASE)
```

```
## # A tibble: 6 x 2  
##   .model      MASE  
##   <chr>      <dbl>  
## 1 ets_adj    1.02  
## 2 comb_adj   1.02  
## 3 ets        1.04  
## 4 comb       1.04  
## 5 arima_adj  1.07  
## 6 arima      1.09
```

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## Lab Session 20

- Prepare aggregations of the PBS data by Concession, Type, and ATC1.
- Use forecast reconciliation with the PBS data, using ETS, ARIMA and SNAIVE models, applied to all but the last 3 years of data.
- Which type of model works best?
- Does the reconciliation improve the forecast accuracy?
- Why doesn't the reconciliation make any difference to the SNAIVE forecasts?

## Feedback form

**[bit.ly/fable2022feedback](https://bit.ly/fable2022feedback)**