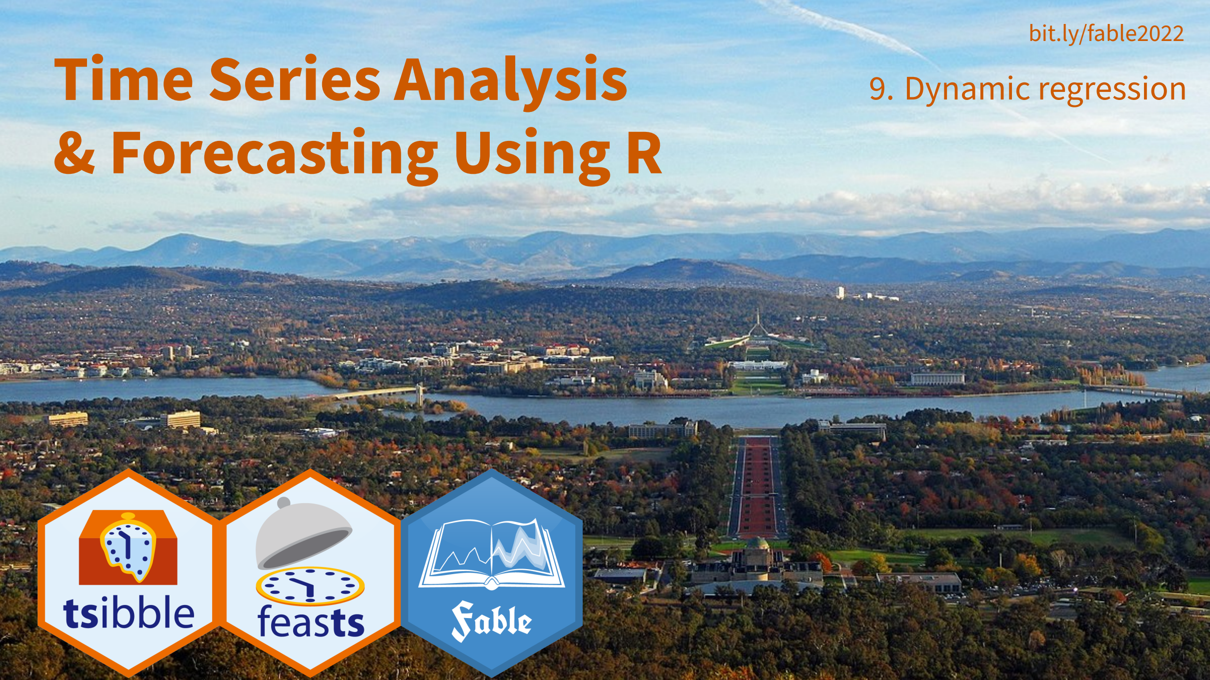
bit.ly/fable2022

Time Series Analysis & Forecasting Using R

9. Dynamic regression



tsibble feasts fable

Outline

Outline

Regression with ARIMA errors

Regression models

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \varepsilon_t,$$

- y_t modeled as function of k explanatory variables
- In regression, we assume that ε_t is white noise.

Regression with ARIMA errors

Regression models

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RegARIMA model

$$y_t = \beta_0 + \beta_1 x_{1,t} + \dots + \beta_k x_{k,t} + \eta_t,$$
$$\eta_t \sim \text{ARIMA}$$

- Residuals are from ARIMA model.
- Estimate model in one step using MLE
- Select model with lowest AICc value.

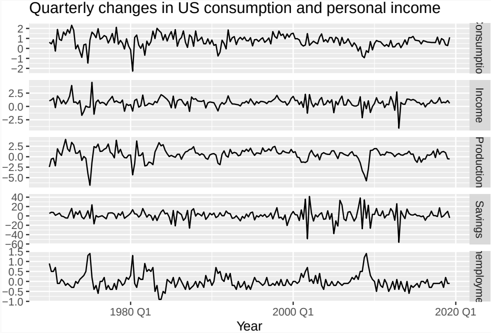
US personal consumption and income

```
us_change
```

```
## # A tsibble: 198 x 6 [1Q]
##    Quarter Consumption Income Production Savings Unemployment
##      <qtr>       <dbl>  <dbl>      <dbl>   <dbl>        <dbl>
##  1 1970 Q1       0.619  1.04      -2.45    5.30         0.9
##  2 1970 Q2       0.452  1.23      -0.551   7.79         0.5
##  3 1970 Q3       0.873  1.59      -0.359   7.40         0.5
##  4 1970 Q4      -0.272 -0.240     -2.19    1.17         0.700
##  5 1971 Q1       1.90   1.98       1.91    3.54        -0.100
##  6 1971 Q2       0.915  1.45       0.902   5.87        -0.100
##  7 1971 Q3       0.794  0.521      0.308  -0.406        0.100
##  8 1971 Q4       1.65   1.16       2.29   -1.49         0
##  9 1972 Q1       1.31   0.457      4.15   -4.29        -0.200
## 10 1972 Q2       1.89   1.03       1.89   -4.69        -0.100
## # ... with 188 more rows
```

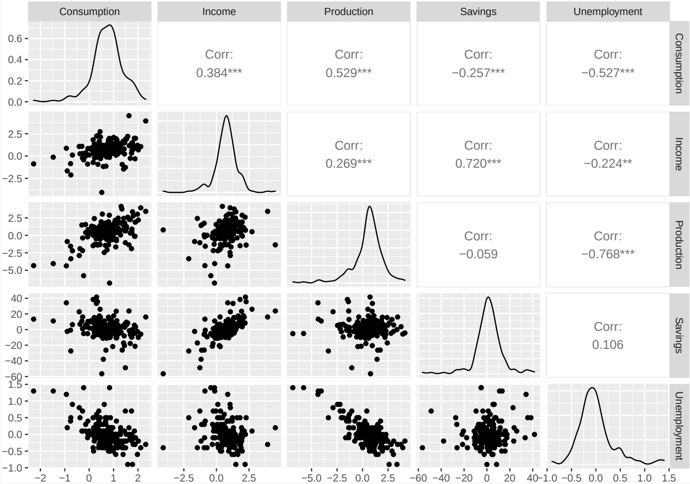
US personal consumption and income

```
us_change ▷
  pivot_longer(-Quarter, names_to = "variable", values_to = "value") ▷
  ggplot(aes(y = value, x = Quarter, group = variable)) +
  geom_line() + facet_grid(variable ~ ., scales = "free_y") +
  labs(x = "Year", y = "",
       title = "Quarterly changes in US consumption and personal income")
```



US personal consumption and income

```
us_change ▷ as_tibble() ▷ select(-Quarter) ▷ GGally::ggpairs()
```



US personal consumption and income

- No need for transformations or further differencing.
- Increase in income does not necessarily translate into instant increase in consumption (e.g., after the loss of a job, it may take a few months for expenses to be reduced to allow for the new circumstances). We will ignore this for now.

US personal consumption and income

```r
fit <- us_change |>
  model(regarima = ARIMA(Consumption ~ Income + Production + Savings + Unemployment))
report(fit)
```

```
## Series: Consumption
## Model: LM w/ ARIMA(0,1,2) errors
##
## Coefficients:
##           ma1     ma2  Income  Production  Savings  Unemployment
##       -1.0882  0.1118  0.7472      0.0370  -0.0531       -0.2096
## s.e.   0.0692  0.0676  0.0403      0.0229   0.0029        0.0986
##
## sigma^2 estimated as 0.09588:  log likelihood=-47.1
## AIC=108   AICc=109   BIC=131
```

US personal consumption and income

```r
fit <- us_change |>
  model(regarima = ARIMA(Consumption ~ Income + Production + Savings + Unemployment))
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```
## Series: Consumption
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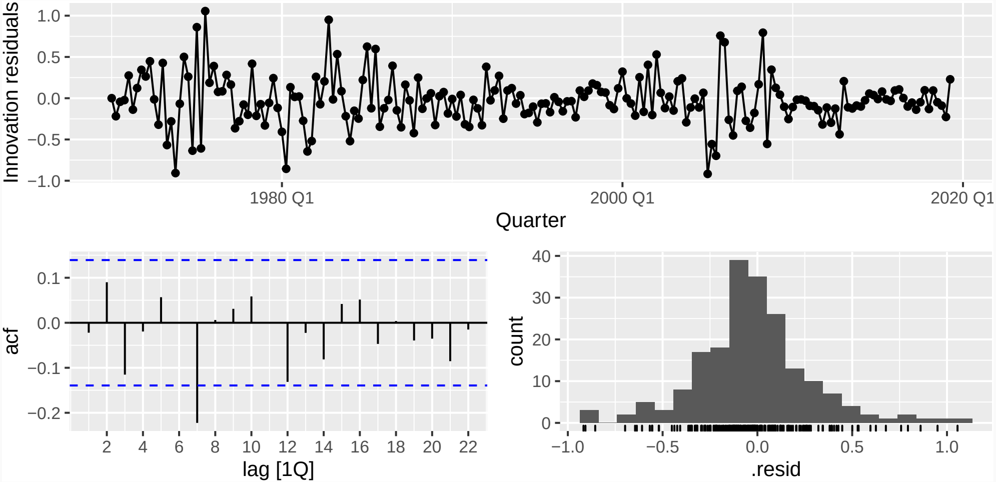
##
## sigma^2 estimated as 0.09588:  log likelihood=-47.1
## AIC=108   AICc=109   BIC=131
```

Write down the equations for the fitted model.

US personal consumption and income

```
gg_tsresiduals(fit)
```



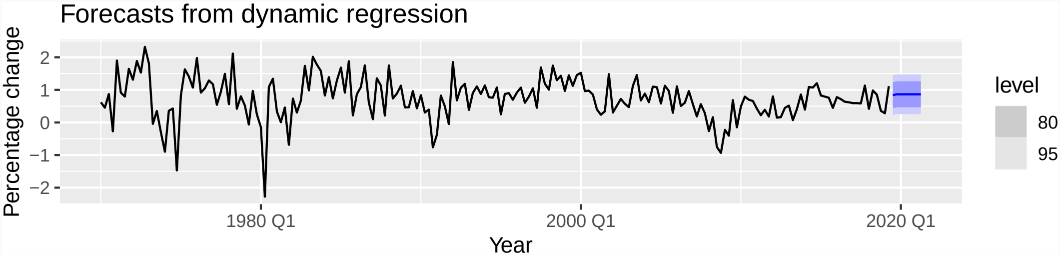
US personal consumption and income

```
augment(fit) |>
  features(.resid, ljung_box, dof = 6, lag = 12)
```

```
## # A tibble: 1 x 3
##   .model   lb_stat lb_pvalue
##   <chr>      <dbl>     <dbl>
## 1 regarima    20.0   0.00274
```

US personal consumption and income

```
us_change_future <- new_data(us_change, 8) ▷
  mutate(Income = tail(us_change$Income, 1),
         Production = tail(us_change$Production, 1),
         Savings = tail(us_change$Savings, 1),
         Unemployment = tail(us_change$Unemployment, 1))
forecast(fit, new_data = us_change_future) ▷
  autoplot(us_change) +
  labs(x = "Year", y = "Percentage change", title = "Forecasts from dynamic regression")
```



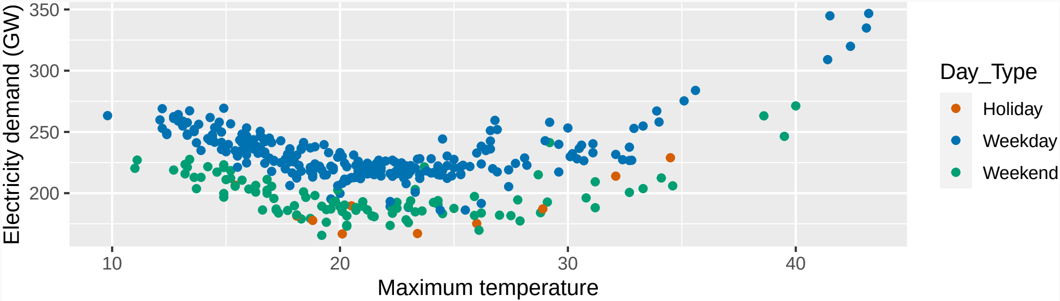
Forecasting

- To forecast a regression model with ARIMA errors, we need to forecast the regression part of the model and the ARIMA part of the model and combine the results.
- Some predictors are known into the future (e.g., time, dummies).
- Separate forecasting models may be needed for other predictors.
- Forecast intervals ignore the uncertainty in forecasting the predictors.

Daily electricity demand

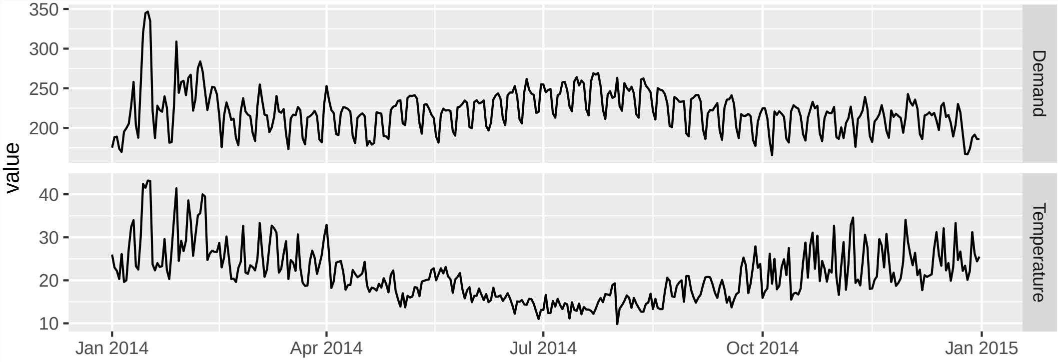
Model daily electricity demand as a function of temperature using quadratic regression with ARMA errors.

```
vic_elec_daily |>
  ggplot(aes(x = Temperature, y = Demand, colour = Day_Type)) +
  geom_point() +
  labs(x = "Maximum temperature", y = "Electricity demand (GW)")
```



Daily electricity demand

```
vic_elec_daily ▷
  pivot_longer(c(Demand, Temperature)) ▷
  ggplot(aes(x = Date, y = value)) +
  geom_line() +
  facet_grid(vars(name), scales = "free_y")
```



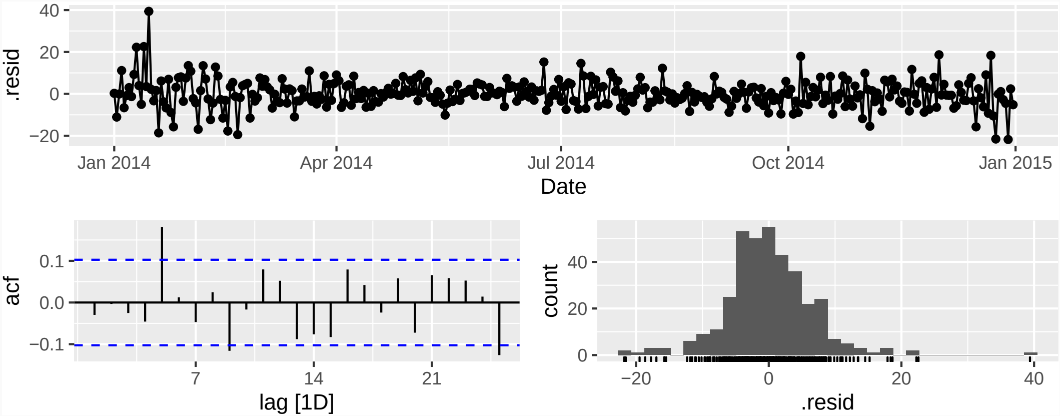
Daily electricity demand

```
fit <- vic_elec_daily |>
  model(fit = ARIMA(Demand ~ Temperature + I(Temperature^2) +
    (Day_Type == "Weekday")))
report(fit)
```

```
## Series: Demand
## Model: LM w/ ARIMA(2,1,2)(2,0,0)[7] errors
##
## Coefficients:
##           ar1     ar2      ma1      ma2    sar1   sar2  Temperature
##       -0.1093  0.7226  -0.0182  -0.9381  0.1958  0.417       -7.614
## s.e.   0.0779  0.0739   0.0494   0.0493  0.0525  0.057        0.448
##       I(Temperature^2)  Day_Type == "Weekday"TRUE
##                 0.1810                      30.40
## s.e.            0.0085                       1.33
##
## sigma^2 estimated as 44.91:  log likelihood=-1206
## AIC=2432   AICc=2433   BIC=2471
```

Daily electricity demand

```
augment(fit) ▷
  gg_tsdisplay(.resid, plot_type = "histogram")
```



Daily electricity demand

```
augment(fit) |>
  features(.resid, ljung_box, dof = 9, lag = 14)
```

```
## # A tibble: 1 x 3
##   .model lb_stat lb_pvalue
##   <chr>    <dbl>     <dbl>
## 1 fit       28.4 0.0000304
```

Daily electricity demand

```r
# Forecast one day ahead
vic_next_day <- new_data(vic_elec_daily, 1) |>
  mutate(Temperature = 26, Day_Type = "Holiday")
forecast(fit, vic_next_day)
```

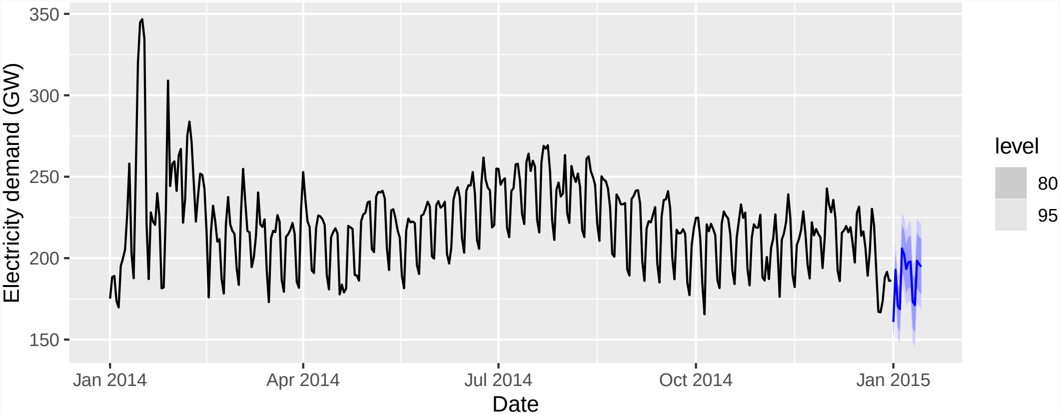
```
## # A fable: 1 x 6 [1D]
## # Key:     .model [1]
##   .model Date           Demand .mean Temperature Day_Type
##   <chr>  <date>         <dist> <dbl>       <dbl> <chr>
## 1 fit    2015-01-01 N(161, 45)  161.          26 Holiday
```

Daily electricity demand

```
vic_elec_future <- new_data(vic_elec_daily, 14) |>
  mutate(
    Temperature = 26,
    Holiday = c(TRUE, rep(FALSE, 13)),
    Day_Type = case_when(
      Holiday ~ "Holiday",
      wday(Date) %in% 2:6 ~ "Weekday",
      TRUE ~ "Weekend"
    )
  )
```

Daily electricity demand

```
forecast(fit, vic_elec_future) |>
  autoplot(vic_elec_daily) + labs(y = "Electricity demand (GW)")
```



Outline

Lab Session 18

Repeat the daily electricity example, but instead of using a quadratic function of temperature, use a piecewise linear function with the "knot" around 20 degrees Celsius (use predictors `Temperature` & `Temp2`). How can you optimize the choice of knot?

```
vic_elec_daily <- vic_elec |>
  filter(year(Time) == 2014) |>
  index_by(Date = date(Time)) |>
  summarise(Demand = sum(Demand) / 1e3,
            Temperature = max(Temperature),
            Holiday = any(Holiday)
  ) |>
  mutate(Temp2 = I(pmax(Temperature - 20, 0)),
         Day_Type = case_when(
           Holiday ~ "Holiday",
           wday(Date) %in% 2:6 ~ "Weekday",
           TRUE ~ "Weekend")
  )
```

Outline

1. Regression with ARIMA errors
2. Lab Session 18
3. Dynamic harmonic regression
4. Lab Session 19
5. Lagged predictors

Dynamic harmonic regression

Combine Fourier terms with ARIMA errors

Advantages

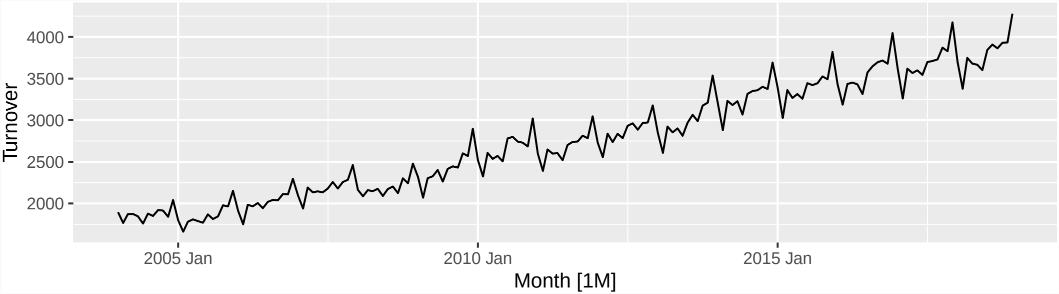
- it allows any length seasonality;
- for data with more than one seasonal period, you can include Fourier terms of different frequencies;
- the seasonal pattern is smooth for small values of K (but more wiggly seasonality can be handled by increasing K);
- the short-term dynamics are easily handled with a simple ARMA error.

Disadvantages

- seasonality is assumed to be fixed

Eating-out expenditure

```
aus_cafe <- aus_retail ▷
  filter(
    Industry == "Cafes, restaurants and takeaway food services",
    year(Month) %in% 2004:2018
  ) ▷
  summarise(Turnover = sum(Turnover))
aus_cafe ▷ autoplot(Turnover)
```

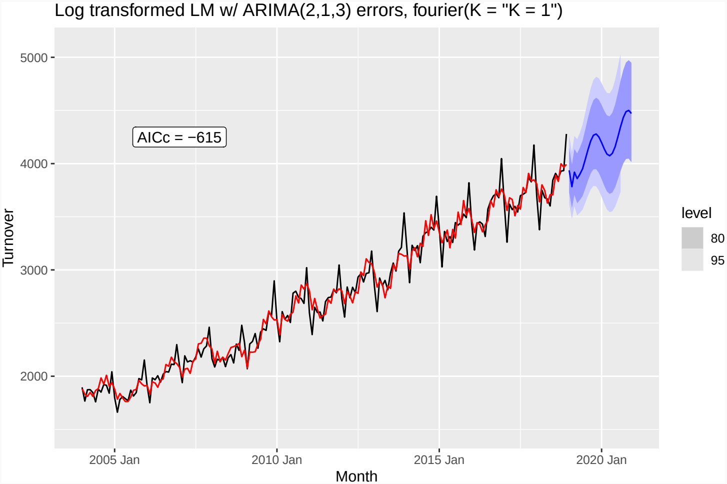


Eating-out expenditure

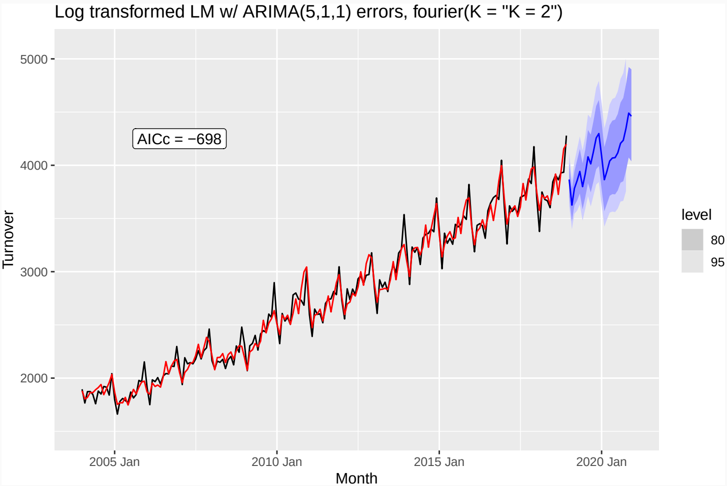
```r
fit <- aus_cafe |> model(
  `K = 1` = ARIMA(log(Turnover) ~ fourier(K = 1) + PDQ(0, 0, 0)),
  `K = 2` = ARIMA(log(Turnover) ~ fourier(K = 2) + PDQ(0, 0, 0)),
  `K = 3` = ARIMA(log(Turnover) ~ fourier(K = 3) + PDQ(0, 0, 0)),
  `K = 4` = ARIMA(log(Turnover) ~ fourier(K = 4) + PDQ(0, 0, 0)),
  `K = 5` = ARIMA(log(Turnover) ~ fourier(K = 5) + PDQ(0, 0, 0)),
  `K = 6` = ARIMA(log(Turnover) ~ fourier(K = 6) + PDQ(0, 0, 0))
)
glance(fit)
```

.model	sigma2	log_lik	AIC	AICc	BIC
K = 1	0.002	317	-616	-615	-588
K = 2	0.001	362	-700	-698	-661
K = 3	0.001	394	-763	-761	-725
K = 4	0.001	427	-822	-818	-771
K = 5	0.000	474	-919	-917	-875
K = 6	0.000	474	-920	-918	-875

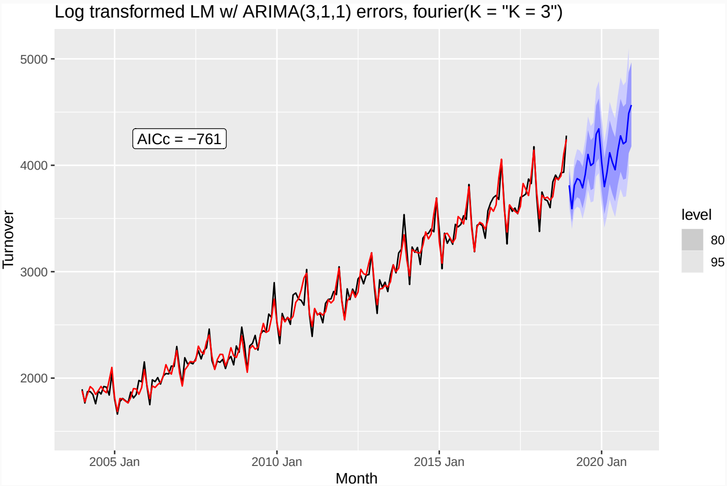
Eating-out expenditure



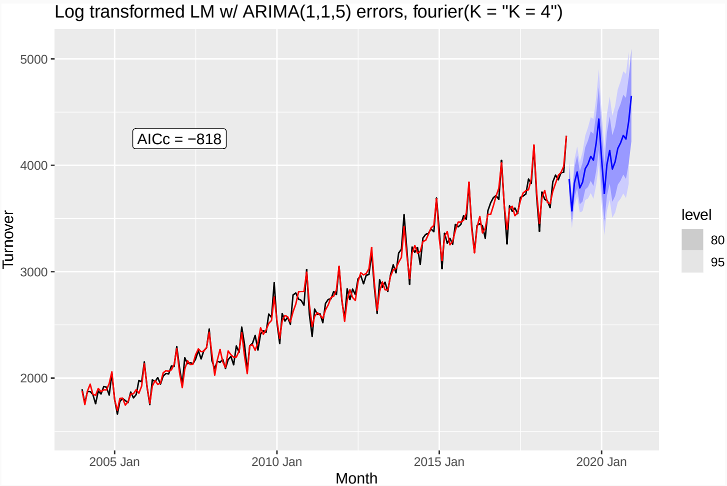
Eating-out expenditure



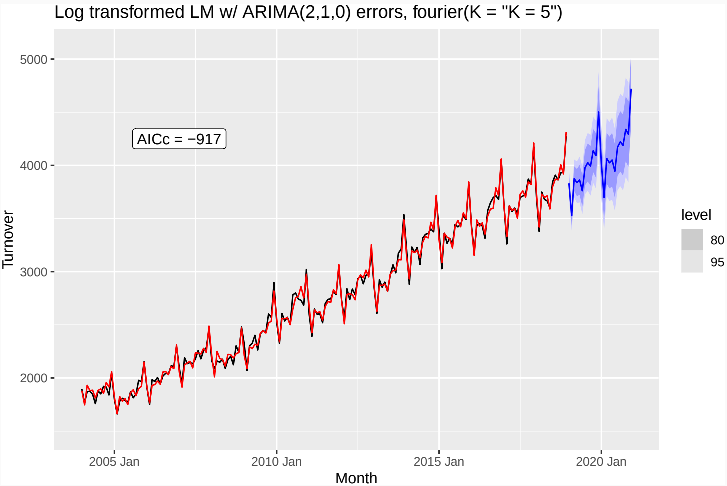
Eating-out expenditure



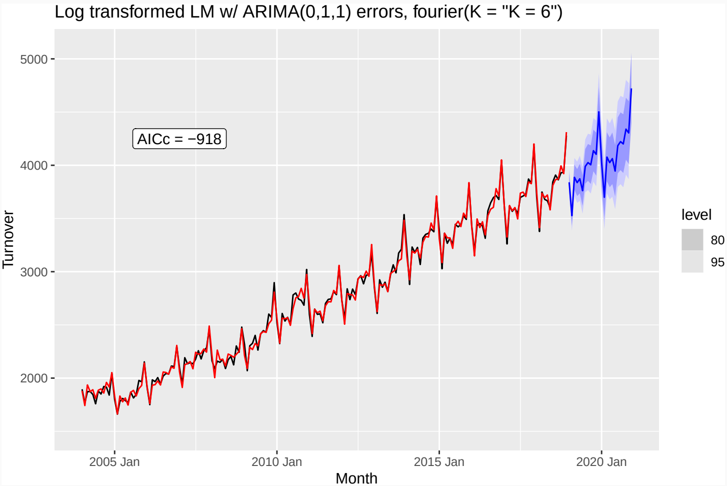
Eating-out expenditure



Eating-out expenditure



Eating-out expenditure



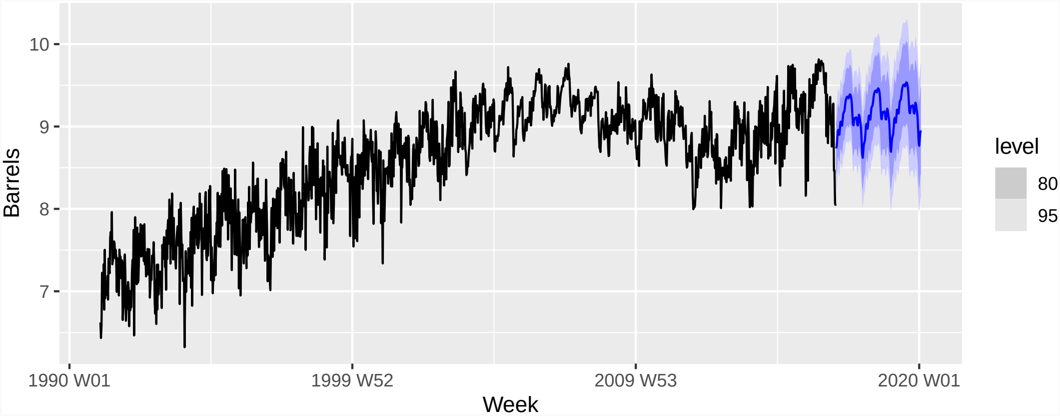
Example: weekly gasoline products

```
fit <- us_gasoline ▷ model(ARIMA(Barrels ~ fourier(K = 13) + PDQ(0, 0, 0)))
report(fit)
```

```
## Series: Barrels
## Model: LM w/ ARIMA(0,1,1) errors
##
## Coefficients:
##           ma1  fourier(K = 13)C1_52  fourier(K = 13)S1_52
##       -0.8934               -0.1121               -0.2300
## s.e.   0.0132                0.0123                0.0122
##       fourier(K = 13)C2_52  fourier(K = 13)S2_52  fourier(K = 13)C3_52
##                     0.0420                0.0317                0.0832
## s.e.                0.0099                0.0099                0.0094
##       fourier(K = 13)S3_52  fourier(K = 13)C4_52  fourier(K = 13)S4_52
##                     0.0346                0.0185                0.0398
## s.e.                0.0094                0.0092                0.0092
##       fourier(K = 13)C5_52  fourier(K = 13)S5_52  fourier(K = 13)C6_52
##                    -0.0315                0.0009               -0.0522
## s.e.                0.0091                0.0091                0.0090
##       fourier(K = 13)S6_52  fourier(K = 13)C7_52  fourier(K = 13)S7_52
##                      0.000               -0.0173                0.0053
## s.e.                 0.009                0.0090                0.0090
```

Example: weekly gasoline products

```
forecast(fit, h = "3 years") |>
  autoplot(us_gasoline)
```



Outline

Lab Session 19

Repeat Lab Session 18 but using all available data, and handling the annual seasonality using Fourier terms.

Outline

Lagged predictors

Sometimes a change in x_t does not affect y_t instantaneously

- $y_t =$ sales, $x_t =$ advertising.
- $y_t =$ stream flow, $x_t =$ rainfall.
- $y_t =$ size of herd, $x_t =$ breeding stock.

Lagged predictors

Sometimes a change in x_t does not affect y_t instantaneously

- $y_t =$ sales, $x_t =$ advertising.
- $y_t =$ stream flow, $x_t =$ rainfall.
- $y_t =$ size of herd, $x_t =$ breeding stock.

- These are dynamic systems with input (x_t) and output (y_t).
- x_t is often a leading indicator.
- There can be multiple predictors.

Lagged predictors

The model include present and past values of predictor:

$x_t, x_{t-1}, x_{t-2}, \ldots.$

$$y_t = a + \nu_0 x_t + \nu_1 x_{t-1} + \cdots + \nu_k x_{t-k} + \eta_t$$

where η_t is an ARIMA process.

Lagged predictors

The model include present and past values of predictor: $x_t, x_{t-1}, x_{t-2}, \ldots.$

$$y_t = a + \nu_0 x_t + \nu_1 x_{t-1} + \cdots + \nu_k x_{t-k} + \eta_t$$

where η_t is an ARIMA process.

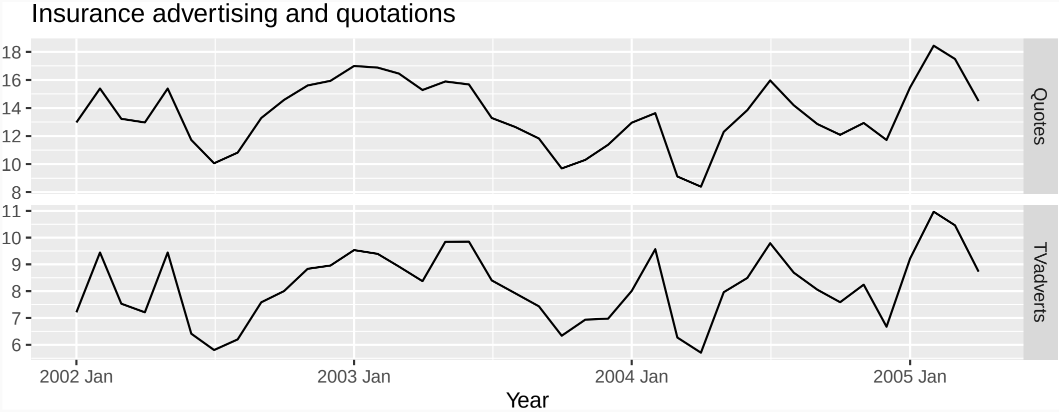
- x can influence y, but y is not allowed to influence x.

Example: Insurance quotes and TV adverts

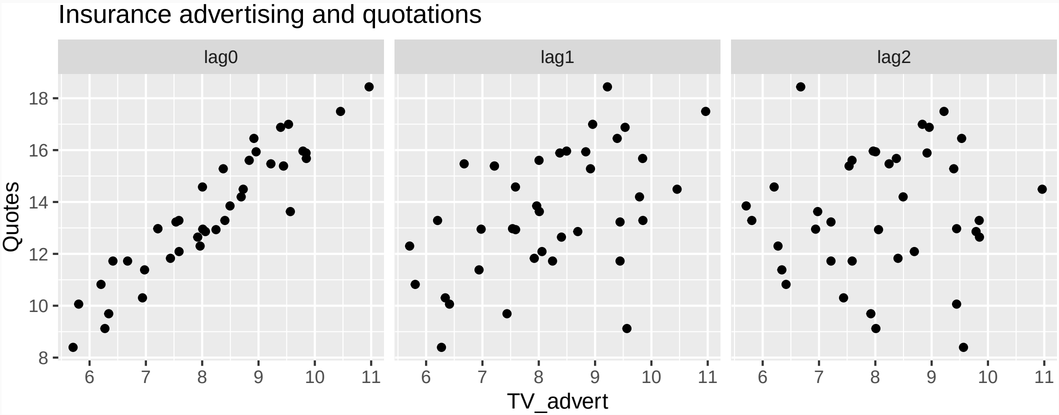
```
insurance
```

```
## # A tsibble: 40 x 3 [1M]
##       Month Quotes TV.advert
##       <mth>  <dbl>     <dbl>
##  1 2002 Jan   13.0      7.21
##  2 2002 Feb   15.4      9.44
##  3 2002 Mar   13.2      7.53
##  4 2002 Apr   13.0      7.21
##  5 2002 May   15.4      9.44
##  6 2002 Jun   11.7      6.42
##  7 2002 Jul   10.1      5.81
##  8 2002 Aug   10.8      6.20
##  9 2002 Sep   13.3      7.59
## 10 2002 Oct   14.6      8.00
```

Example: Insurance quotes and TV adverts



Example: Insurance quotes and TV adverts



Example: Insurance quotes and TV adverts

```r
fit <- insurance |>
  # Restrict data so models use same fitting period
  mutate(Quotes = c(NA, NA, NA, Quotes[4:40])) |>
  model(
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts),
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts +
      lag(TVadverts)),
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts +
      lag(TVadverts) +
      lag(TVadverts, 2)),
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts +
      lag(TVadverts) +
      lag(TVadverts, 2) +
      lag(TVadverts, 3))
  )
```

Example: Insurance quotes and TV adverts

```
glance(fit)
```

Lag order	sigma2	log_lik	AIC	AICc	BIC
0	0.265	-28.3	66.6	68.3	75.0
1	0.209	-24.0	58.1	59.9	66.5
2	0.215	-24.0	60.0	62.6	70.2
3	0.206	-22.2	60.3	65.0	73.8

Example: Insurance quotes and TV adverts

```r
# Re-fit to all data
fit <- insurance |>
  model(ARIMA(Quotes ~ TVadverts + lag(TVadverts) + pdq(d = 0)))
report(fit)
```

```
## Series: Quotes
## Model: LM w/ ARIMA(1,0,2) errors
##
## Coefficients:
##         ar1    ma1    ma2  TVadverts  lag(TVadverts)  intercept
##       0.512  0.917  0.459     1.2527          0.1464       2.16
## s.e.  0.185  0.205  0.190     0.0588          0.0531       0.86
##
## sigma^2 estimated as 0.2166:  log likelihood=-23.9
## AIC=61.9   AICc=65.4   BIC=73.7
```

Example: Insurance quotes and TV adverts

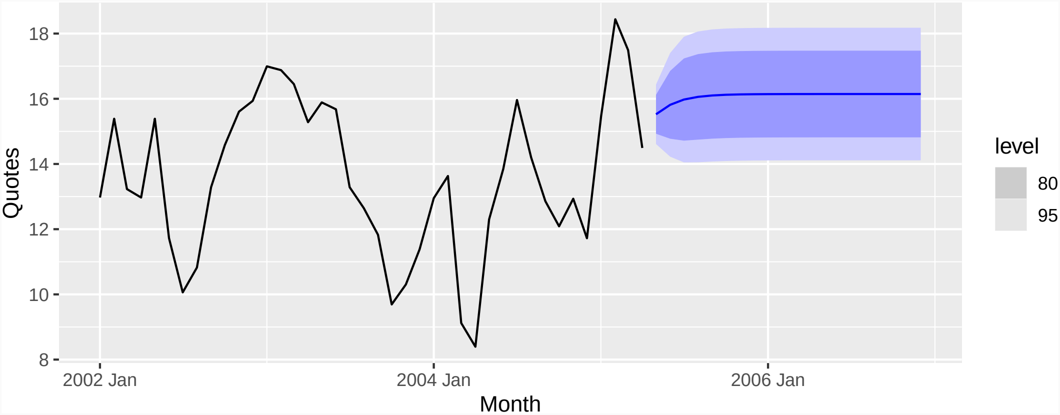
```r
# Re-fit to all data
fit <- insurance |>
  model(ARIMA(Quotes ~ TVadverts + lag(TVadverts) + pdq(d = 0)))
report(fit)
```

```
## Series: Quotes
## Model: LM w/ ARIMA(1,0,2) errors
##
## Coefficients:
##          ar1    ma1    ma2  TVadverts  lag(TVadverts)  intercept
##        0.512  0.917  0.459     1.2527          0.1464       2.16
## s.e.   0.185  0.205  0.190     0.0588          0.0531       0.86
##
## sigma^2 estimated as 0.2166:  log likelihood=-23.9
## AIC=61.9   AICc=65.4   BIC=73.7
```

$$y_t = 2.16 + 1.25x_t + 0.15x_{t-1} + \eta_t,$$
$$\eta_t = 0.512\eta_{t-1} + \varepsilon_t + 0.92\varepsilon_{t-1} + 0.46\varepsilon_{t-2}.$$

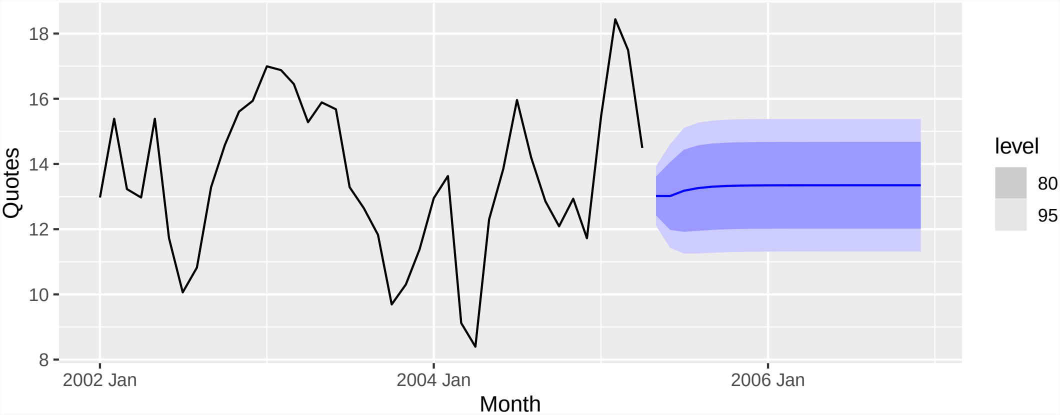
Example: Insurance quotes and TV adverts

```
advert_a <- new_data(insurance, 20) ▷
  mutate(TVadverts = 10)
forecast(fit, advert_a) ▷ autoplot(insurance)
```



Example: Insurance quotes and TV adverts

```
advert_b <- new_data(insurance, 20) |>
  mutate(TVadverts = 8)
forecast(fit, advert_b) |> autoplot(insurance)
```



Example: Insurance quotes and TV adverts

```
advert_c <- new_data(insurance, 20) |>
  mutate(TVadverts = 6)
forecast(fit, advert_c) |> autoplot(insurance)
```