

Tidy Time Series & Forecasting in R

8. ARIMA models



Outline

- 1 ARIMA models
- 2 Lab Session 16
- 3 Seasonal ARIMA models
- 4 Lab Session 17
- 5 Forecast ensembles

Outline

- 1 ARIMA models
- 2 Lab Session 16
- 3 Seasonal ARIMA models
- 4 Lab Session 17
- 5 Forecast ensembles

ARIMA models

- AR:** autoregressive (lagged observations as inputs)
- I:** integrated (differencing to make series stationary)
- MA:** moving average (lagged errors as inputs)

ARIMA models

- AR:** autoregressive (lagged observations as inputs)
- I:** integrated (differencing to make series stationary)
- MA:** moving average (lagged errors as inputs)

An ARIMA model is rarely interpretable in terms of visible data structures like trend and seasonality. But it can capture a huge range of time series patterns.

Stationarity

Definition

If $\{y_t\}$ is a stationary time series, then for all s , the distribution of (y_t, \dots, y_{t+s}) does not depend on t .

Stationarity

Definition

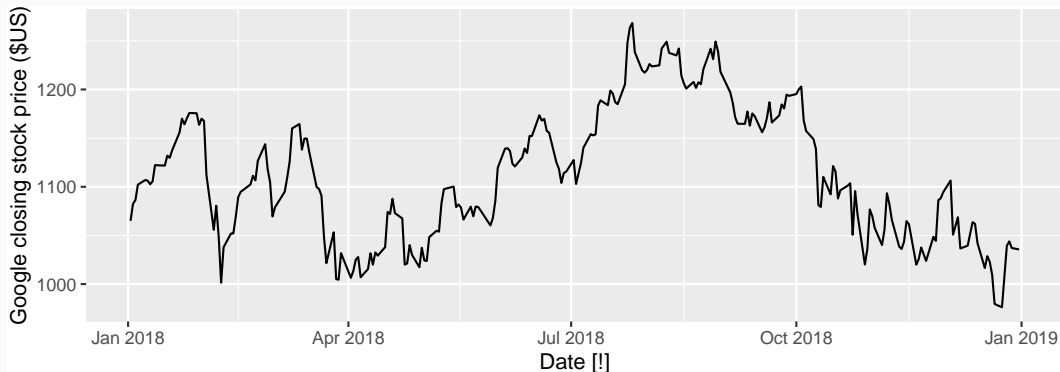
If $\{y_t\}$ is a stationary time series, then for all s , the distribution of (y_t, \dots, y_{t+s}) does not depend on t .

A **stationary series** is:

- roughly horizontal
- constant variance
- no patterns predictable in the long-term

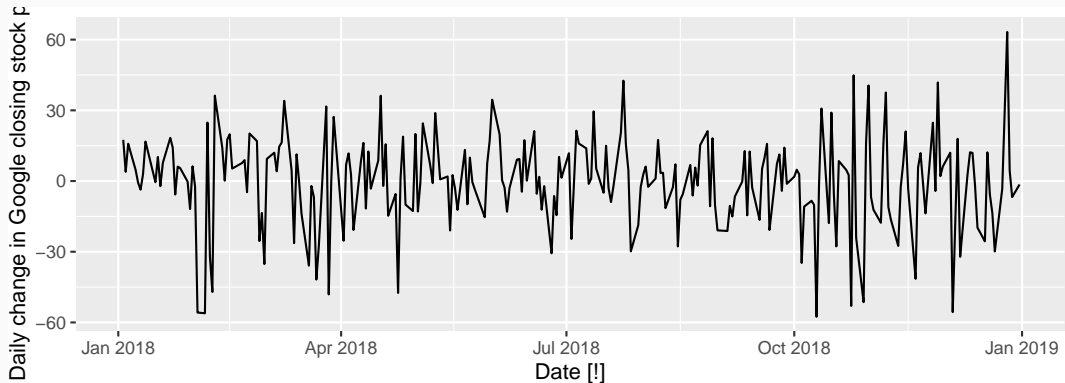
Stationary?

```
gafa_stock ▷  
  filter(Symbol = "GOOG", year(Date) = 2018) ▷  
  autoplot(Close) +  
  ylab("Google closing stock price ($US)")
```



Stationary?

```
gafa_stock >  
  filter(Symbol = "GOOG", year(Date) = 2018) >  
  autoplot(difference(Close)) +  
  ylab("Daily change in Google closing stock price")
```



Differencing

- Differencing helps to **stabilize the mean**.
- The differenced series is the *change* between each observation in the original series.
- Occasionally the differenced data will not appear stationary and it may be necessary to difference the data a second time.
- In practice, it is almost never necessary to go beyond second-order differences.

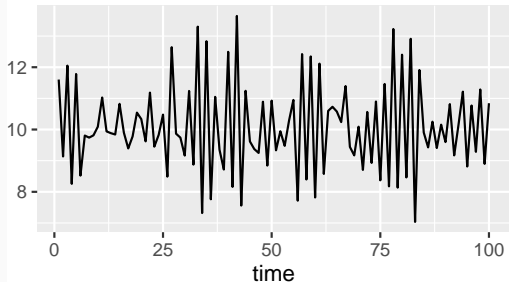
Autoregressive models

Autoregressive (AR) models:

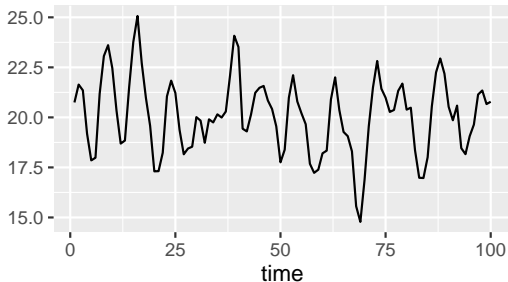
$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t,$$

where ε_t is white noise. A multiple regression with **lagged values** of y_t as predictors.

AR(1)



AR(2)



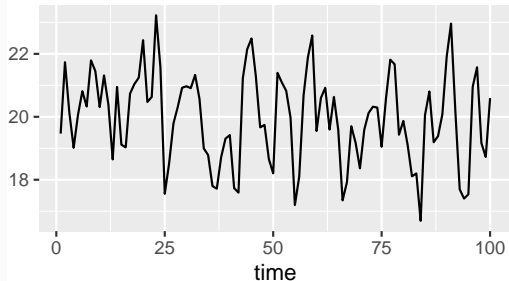
Moving Average (MA) models

Moving Average (MA) models:

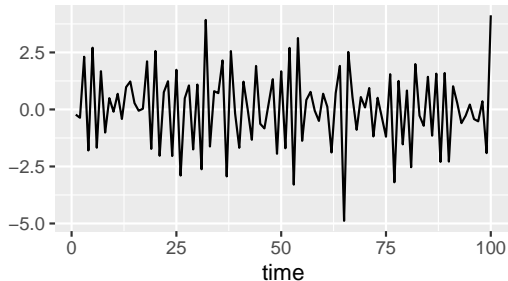
$$y_t = c + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \cdots + \theta_q\varepsilon_{t-q},$$

where ε_t is white noise. A multiple regression with **lagged errors** as predictors. *Don't confuse with moving average smoothing!*

MA(1)



MA(2)



ARIMA models

Autoregressive Moving Average models:

$$y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} \\ + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t.$$

ARIMA models

Autoregressive Moving Average models:

$$y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} \\ + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t.$$

- Predictors include both **lagged values of y_t and lagged errors.**

ARIMA models

Autoregressive Moving Average models:

$$y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} \\ + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t.$$

- Predictors include both **lagged values of y_t and lagged errors.**

Autoregressive Integrated Moving Average models

- Combine ARMA model with **differencing.**
- d -differenced series follows an ARMA model.
- Need to choose p, d, q and whether or not to include c .

ARIMA models

ARIMA(p, d, q) model

AR: p = order of the autoregressive part

I: d = degree of first differencing involved

MA: q = order of the moving average part.

- White noise model: ARIMA(0,0,0)
- Random walk: ARIMA(0,1,0) with no constant
- Random walk with drift: ARIMA(0,1,0) with const.
- AR(p): ARIMA($p,0,0$)
- MA(q): ARIMA(0,0, q)

Example: National populations

```
fit <- global_economy %>%  
  model(arima = ARIMA(Population))  
fit
```

```
## # A mable: 263 x 2  
## # Key:      Country [263]  
##   Country          arima  
##   <fct>           <model>  
## 1 Afghanistan    <ARIMA(4,2,1)>  
## 2 Albania         <ARIMA(0,2,2)>  
## 3 Algeria         <ARIMA(2,2,2)>  
## 4 American Samoa <ARIMA(2,2,2)>  
## 5 Andorra         <ARIMA(2,1,2) w/ drift>  
## 6 Angola          <ARIMA(4,2,1)>  
## 7 Antigua and Barbuda <ARIMA(2,1,2) w/ drift>  
## 8 Arab World     <ARIMA(0,2,1)>
```

Example: National populations

```
fit ▷  
  filter(Country = "Australia") ▷  
  report()
```

```
## Series: Population
```

```
## Model: ARIMA(0,2,1)
```

```
##
```

```
## Coefficients:
```

```
##          ma1
```

```
##        -0.661
```

```
## s.e.    0.107
```

```
##
```

```
## sigma^2 estimated as 4.063e+09:  log likelihood=-699
```

```
## AIC=1401   AICc=1402   BIC=1405
```

Example: National populations

```
fit >  
  filter(Country = "Australia") >  
  report()
```

```
## Series: Population
```

```
## Model: ARIMA(0,2,1)
```

```
##
```

```
## Coefficients:
```

```
##          ma1
```

```
##        -0.661
```

```
## s.e.    0.107
```

```
##
```

```
## sigma^2 estimated as 4.063e+09:  log likelihood=-699
```

```
## AIC=1401   AICc=1402   BIC=1405
```

$$y_t = 2y_{t-1} - y_{t-2} - 0.7\varepsilon_{t-1} + \varepsilon_t$$
$$\varepsilon_t \sim \text{NID}(0, 4 \times 10^9)$$

Understanding ARIMA models

- If $c = 0$ and $d = 0$, the long-term forecasts will go to zero.
- If $c = 0$ and $d = 1$, the long-term forecasts will go to a non-zero constant.
- If $c = 0$ and $d = 2$, the long-term forecasts will follow a straight line.
- If $c \neq 0$ and $d = 0$, the long-term forecasts will go to the mean of the data.
- If $c \neq 0$ and $d = 1$, the long-term forecasts will follow a straight line.
- If $c \neq 0$ and $d = 2$, the long-term forecasts will follow a quadratic trend.

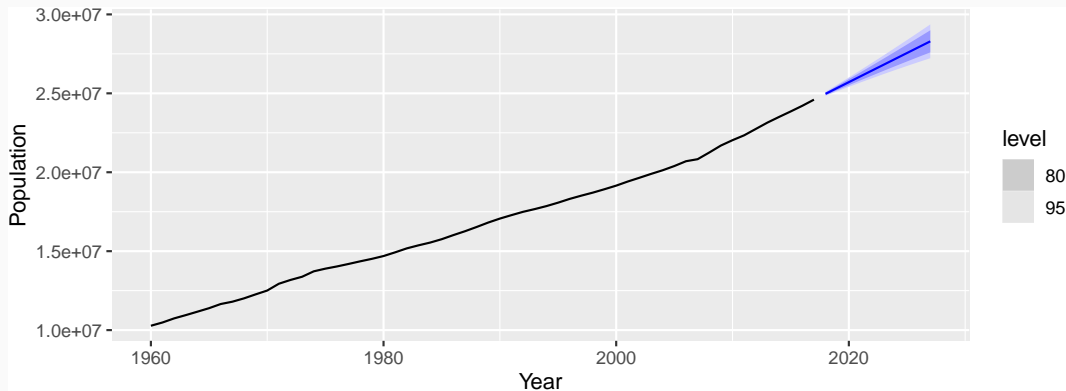
Understanding ARIMA models

Forecast variance and d

- The higher the value of d , the more rapidly the prediction intervals increase in size.
- For $d = 0$, the long-term forecast standard deviation will go to the standard deviation of the historical data.

Example: National populations

```
fit ▷  
  forecast(h = 10) ▷  
  filter(Country = "Australia") ▷  
  autoplot(global_economy)
```



How does ARIMA() work?

Hyndman and Khandakar (JSS, 2008) algorithm:

- Select no. differences d via KPSS test.
- Select p, q and inclusion of c by minimising AICc.
- Use stepwise search to traverse model space.

How does ARIMA() work?

Hyndman and Khandakar (JSS, 2008) algorithm:

- Select no. differences d via KPSS test.
- Select p, q and inclusion of c by minimising AICc.
- Use stepwise search to traverse model space.

$$\text{AICc} = -2 \log(L) + 2(p + q + k + 1) \left[1 + \frac{(p + q + k + 2)}{T - p - q - k - 2} \right].$$

where L is the maximised likelihood fitted to the *differenced* data,
 $k = 1$ if $c \neq 0$ and $k = 0$ otherwise.

How does ARIMA() work?

Hyndman and Khandakar (JSS, 2008) algorithm:

- Select no. differences d via KPSS test.
- Select p, q and inclusion of c by minimising AICc.
- Use stepwise search to traverse model space.

$$\text{AICc} = -2 \log(L) + 2(p + q + k + 1) \left[1 + \frac{(p + q + k + 2)}{T - p - q - k - 2} \right].$$

where L is the maximised likelihood fitted to the *differenced* data,
 $k = 1$ if $c \neq 0$ and $k = 0$ otherwise.

Note: Can't compare AICc for different values of d .

How does ARIMA() work?

Step1: Select current model (with smallest AICc) from:

ARIMA(2, d , 2)

ARIMA(0, d , 0)

ARIMA(1, d , 0)

ARIMA(0, d , 1)

How does ARIMA() work?

Step1: Select current model (with smallest AICc) from:

ARIMA(2, d , 2)

ARIMA(0, d , 0)

ARIMA(1, d , 0)

ARIMA(0, d , 1)

Step 2: Consider variations of current model:

- vary one of p, q , from current model by ± 1 ;
- p, q both vary from current model by ± 1 ;
- Include/exclude c from current model.

Model with lowest AICc becomes current model.

How does ARIMA() work?

Step1: Select current model (with smallest AICc) from:

ARIMA(2, d , 2)

ARIMA(0, d , 0)

ARIMA(1, d , 0)

ARIMA(0, d , 1)

Step 2: Consider variations of current model:

- vary one of p , q , from current model by ± 1 ;
- p , q both vary from current model by ± 1 ;
- Include/exclude c from current model.

Model with lowest AICc becomes current model.

Repeat Step 2 until no lower AICc can be found.

Outline

- 1 ARIMA models
- 2 Lab Session 16
- 3 Seasonal ARIMA models
- 4 Lab Session 17
- 5 Forecast ensembles

Lab Session 16

For the United States GDP data (from `global_economy`):

- Fit a suitable ARIMA model for the logged data.
- Produce forecasts of your fitted model. Do the forecasts look reasonable?

Outline

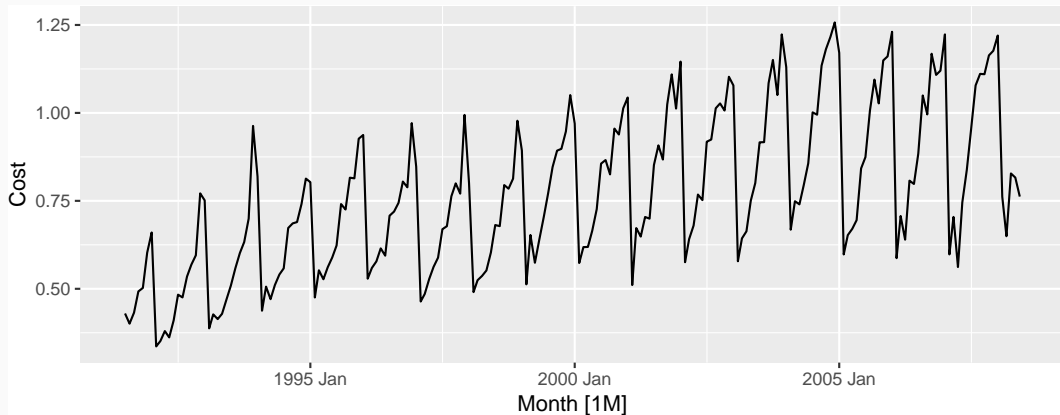
- 1 ARIMA models
- 2 Lab Session 16
- 3 Seasonal ARIMA models
- 4 Lab Session 17
- 5 Forecast ensembles

Corticosteroid drug sales

```
h02 <- PBS %>%  
  filter(ATC2 = "H02") %>%  
  summarise(Cost = sum(Cost)/1e6)
```

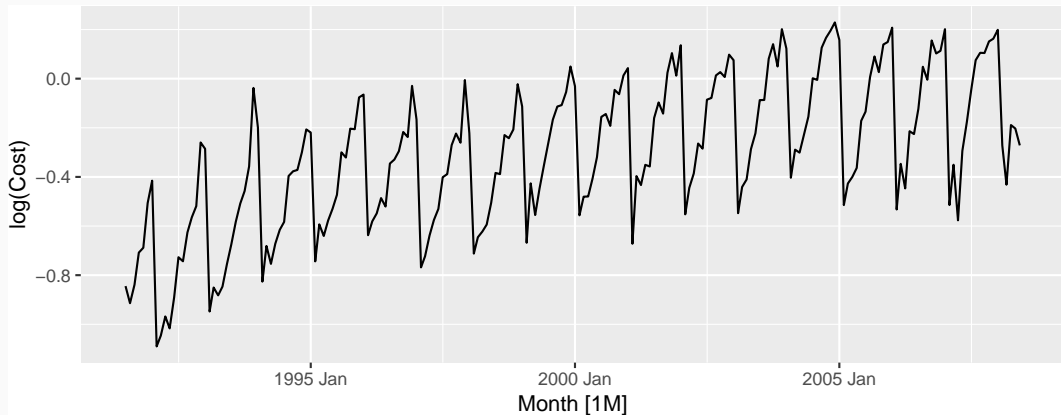

Corticosteroid drug sales

```
h02 %>% autoplot(  
  Cost  
)
```



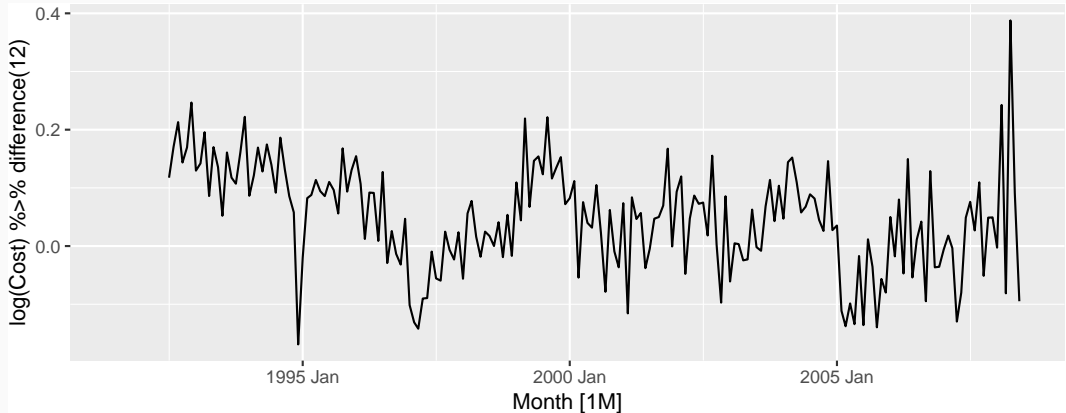
Corticosteroid drug sales

```
h02 %>% autoplot(  
  log(Cost)  
)
```



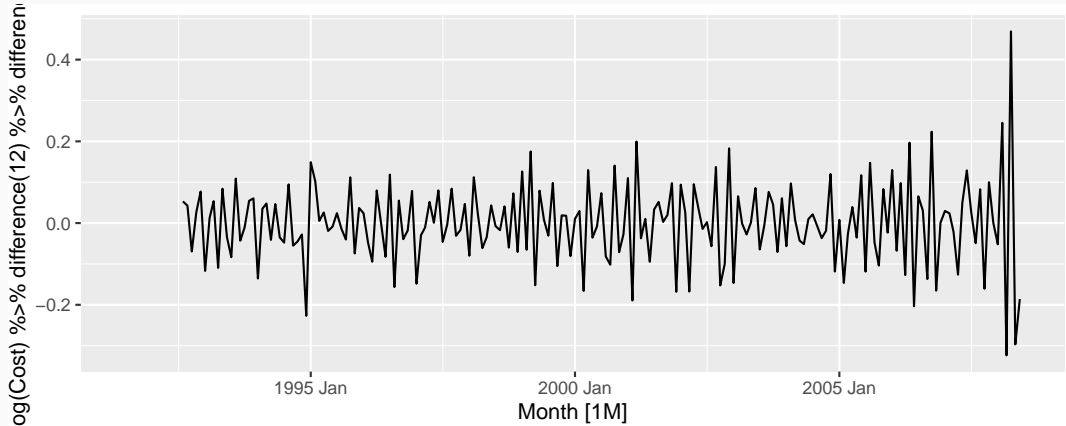
Corticosteroid drug sales

```
h02 %>% autoplot(  
  log(Cost) %>% difference(12)  
)
```



Corticosteroid drug sales

```
h02 %>% autoplot(  
  log(Cost) %>% difference(12) %>% difference(1)  
)
```



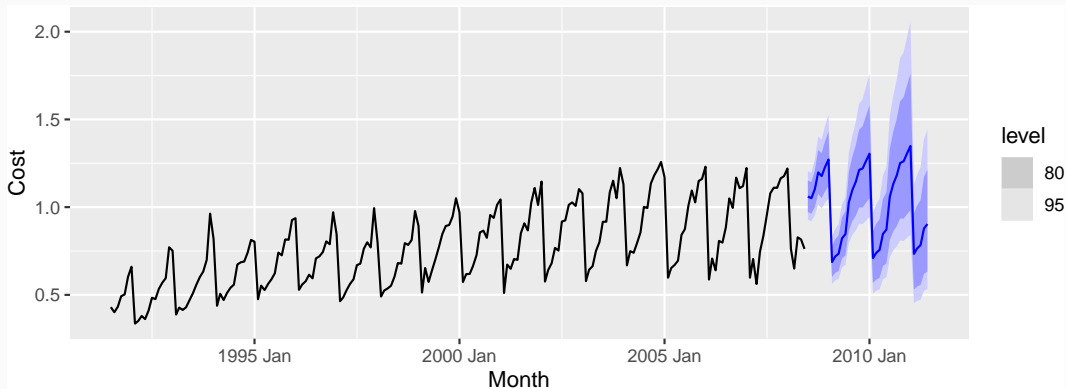
Example: US electricity production

```
h02 ▷  
  model(arima = ARIMA(log(Cost))) ▷  
  report()
```

```
## Series: Cost  
## Model: ARIMA(2,1,0)(0,1,1)[12]  
## Transformation: log(Cost)  
##  
## Coefficients:  
##           ar1      ar2      sma1  
##      -0.8491  -0.4207  -0.6401  
## s.e.   0.0712   0.0714   0.0694  
##  
## sigma^2 estimated as 0.004387:  log likelihood=245  
## AIC=-483   AICc=-483   BIC=-470
```

Example: US electricity production

```
h02 ▷  
  model(arima = ARIMA(log(Cost))) ▷  
  forecast(h = "3 years") ▷  
  autoplot(h02)
```



Seasonal ARIMA models

ARIMA	$\underbrace{(p, d, q)}$	$\underbrace{(P, D, Q)_m}$
	↑	↑
	Non-seasonal part of the model	Seasonal part of of the model

- m = number of observations per year.
- d first differences, D seasonal differences
- p AR lags, q MA lags
- P seasonal AR lags, Q seasonal MA lags

Seasonal and non-seasonal terms combine multiplicatively

Common ARIMA models

The US Census Bureau uses the following models most often:

ARIMA(0,1,1)(0,1,1)_m with log transformation

ARIMA(0,1,2)(0,1,1)_m with log transformation

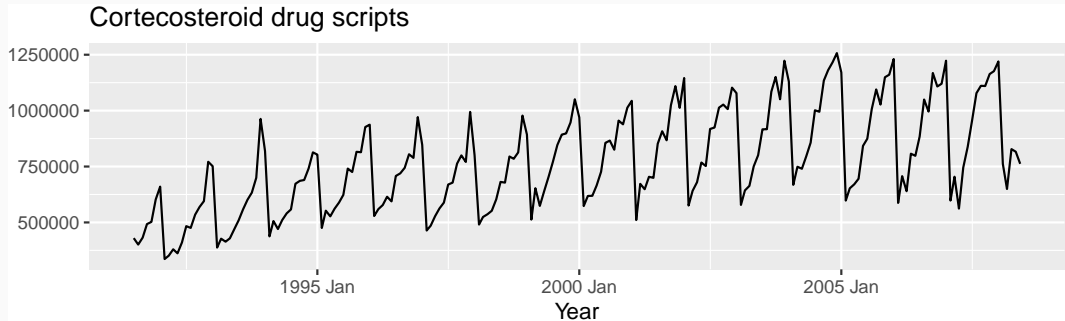
ARIMA(2,1,0)(0,1,1)_m with log transformation

ARIMA(0,2,2)(0,1,1)_m with log transformation

ARIMA(2,1,2)(0,1,1)_m with no transformation

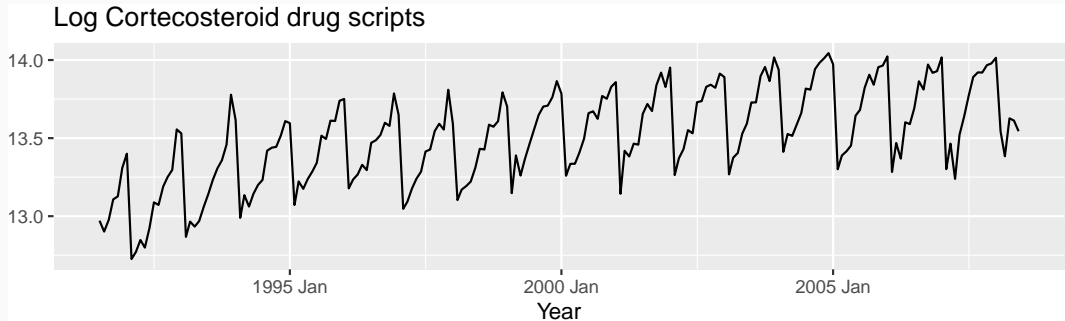
Corticosteroid drug sales

```
h02 <- PBS ▷  
  filter(ATC2 = "H02") ▷  
  summarise(Cost = sum(Cost))  
h02 ▷ autoplot(Cost) +  
  xlab("Year") + ylab("") +  
  ggtitle("Corticosteroid drug scripts")
```



Corticosteroid drug sales

```
h02 <- PBS ▷  
  filter(ATC2 = "H02") ▷  
  summarise(Cost = sum(Cost))  
h02 ▷ autoplot(log(Cost)) +  
  xlab("Year") + ylab("") +  
  ggtitle("Log Corticosteroid drug scripts")
```



Corticosteroid drug sales

```
fit <- h02 ▷  
  model(auto = ARIMA(log(Cost)))  
report(fit)
```

```
## Series: Cost  
## Model: ARIMA(2,1,0)(0,1,1)[12]  
## Transformation: log(Cost)  
##  
## Coefficients:  
##          ar1      ar2      sma1  
##      -0.8491  -0.4207  -0.6401  
## s.e.   0.0712   0.0714   0.0694  
##  
## sigma^2 estimated as 0.004399:  log likelihood=245  
## AIC=-483   AICc=-483   BIC=-470
```

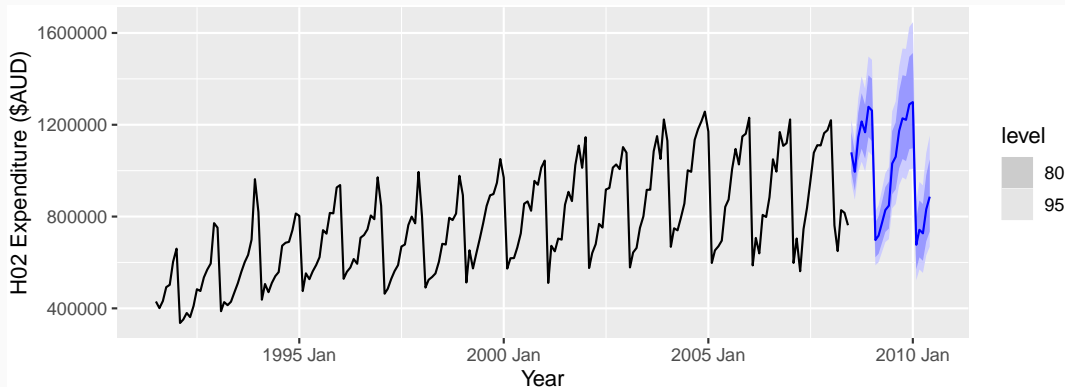
Corticosteroid drug sales

```
fit <- h02 ▷  
  model(best = ARIMA(log(Cost),  
    stepwise = FALSE,  
    approximation = FALSE,  
    order_constraint = p + q + P + Q ≤ 9  
  ))  
report(fit)
```

```
## Series: Cost  
## Model: ARIMA(4,1,1)(2,1,2)[12]  
## Transformation: log(Cost)  
##  
## Coefficients:  
##          ar1      ar2      ar3      ar4      ma1      sar1      sar2      sma1      sma2  
##      -0.0426  0.210  0.202  -0.227  -0.742  0.621  -0.383  -1.202  0.496  
## s.e.   0.2167  0.181  0.114   0.081   0.207  0.242   0.118   0.249  0.214  
##  
## sigma^2 estimated as 0.004061:  log likelihood=254  
## AIC=-489   AICc=-487   BIC=-456
```

Corticosteroid drug sales

```
fit ▷  
  forecast() ▷  
  autoplot(h02) +  
  ylab("H02 Expenditure ($AUD)") + xlab("Year")
```



Outline

- 1 ARIMA models
- 2 Lab Session 16
- 3 Seasonal ARIMA models
- 4 Lab Session 17
- 5 Forecast ensembles

Lab Session 17

For the Australian tourism data (from `tourism`):

- Fit a suitable ARIMA model for all data.
- Produce forecasts of your fitted models.
- Check the forecasts for the “Snowy Mountains” and “Melbourne” regions. Do they look reasonable?

Outline

- 1 ARIMA models
- 2 Lab Session 16
- 3 Seasonal ARIMA models
- 4 Lab Session 17
- 5 Forecast ensembles

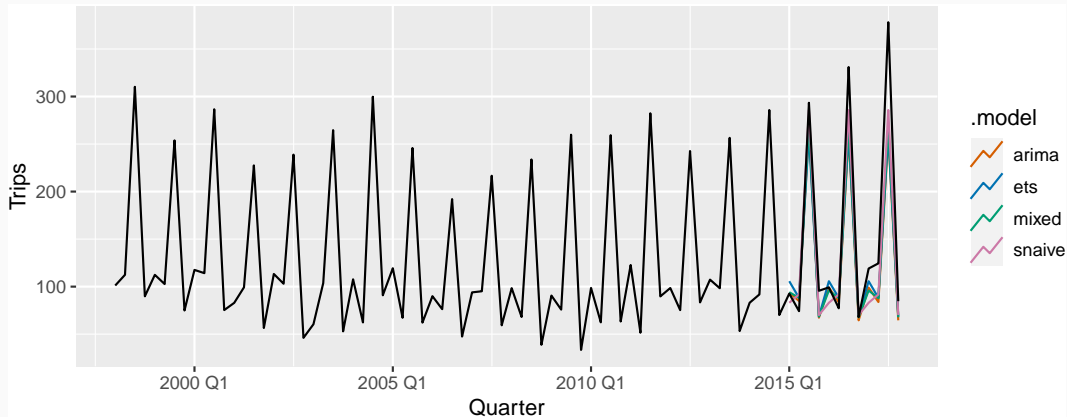
Forecast ensembles

```
train <- tourism ▷  
  filter(year(Quarter) ≤ 2014)  
fit <- train ▷  
  model(  
    ets = ETS(Trips),  
    arima = ARIMA(Trips),  
    snaive = SNAIVE(Trips)  
  ) ▷  
  mutate(mixed = (ets + arima + snaive) / 3)
```

- Ensemble forecast `mixed` is a simple average of the three fitted models.
- `forecast()` will produce distributional forecasts taking into account the correlations between the forecast errors of the component models.

Forecast ensembles

```
fc <- fit > forecast(h = "3 years")  
fc > filter(Region = "Snowy Mountains", Purpose = "Holiday") >  
  autoplot(tourism, level = NULL)
```



Forecast ensembles

```
accuracy(fc, tourism) ▷  
  group_by(.model) ▷  
  summarise(  
    RMSE = mean(RMSE),  
    MAE = mean(MAE),  
    MASE = mean(MASE)  
  ) ▷  
  arrange(RMSE)
```

```
## # A tibble: 4 x 4  
##   .model  RMSE    MAE  MASE  
##   <chr>  <dbl> <dbl> <dbl>  
## 1 mixed   19.8  16.0  0.997  
## 2 ets     20.2  16.4  1.00  
## 3 snaive  21.5  17.3  1.17  
## 4 arima   21.9  17.8  1.06
```

Forecast ensembles

Can we do better than equal weights?

Forecast ensembles

Can we do better than equal weights?

- Hard to find weights that improve forecast accuracy.
- Known as the “forecast combination puzzle”.
- Solution: FFORMA

Forecast ensembles

Can we do better than equal weights?

- Hard to find weights that improve forecast accuracy.
- Known as the “forecast combination puzzle”.
- Solution: FFORMA

FFORMA (Feature-based FOforecast Model Averaging)

- Vector of time series features used to predict best weights.
- A modification of xgboost is used.
- Method came 2nd in the 2018 M4 international forecasting competition.
- Main author: Pablo Montero-Manso (Monash U)
- Not (yet) available for fable.