

Time Series Analysis & Forecasting Using R

10. Forecast reconciliation



Outline

- 1 Hierarchical and grouped time series
- 2 Forecast reconciliation
- 3 Example: Australian tourism
- 4 Lab Session 20

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Australian Pharmaceutical Benefits Scheme



PBS sales

PBS

```
## # A tsibble: 67,596 x 9 [1M]
## # Key:      Concession, Type, ATC1, ATC2 [336]
##      Month Concession  Type  ATC1  ATC1_~1 ATC2  ATC2_~2 Scripts  Cost
##      <mt> <chr>      <chr> <chr> <chr>   <chr> <chr>   <dbl> <dbl>
##  1 1991 Jul Concession~ Co-p~ A      Alimen~ A01    STOMAT~ 18228 67877
##  2 1991 Aug Concession~ Co-p~ A      Alimen~ A01    STOMAT~ 15327 57011
##  3 1991 Sep Concession~ Co-p~ A      Alimen~ A01    STOMAT~ 14775 55020
##  4 1991 Oct Concession~ Co-p~ A      Alimen~ A01    STOMAT~ 15380 57222
##  5 1991 Nov Concession~ Co-p~ A      Alimen~ A01    STOMAT~ 14371 52120
##  6 1991 Dec Concession~ Co-p~ A      Alimen~ A01    STOMAT~ 15028 54299
##  7 1992 Jan Concession~ Co-p~ A      Alimen~ A01    STOMAT~ 11040 39753
##  8 1992 Feb Concession~ Co-p~ A      Alimen~ A01    STOMAT~ 15165 54405
##  9 1992 Mar Concession~ Co-p~ A      Alimen~ A01    STOMAT~ 16898 61108
## 10 1992 Apr Concession~ Co-p~ A      Alimen~ A01    STOMAT~ 18141 65356
## # ... with 67,586 more rows, and abbreviated variable names
## # 1: ATC1 desc, 2: ATC2 desc
```

ATC drug classification

- A Alimentary tract and metabolism
- B Blood and blood forming organs
- C Cardiovascular system
- D Dermatologicals
- G Genito-urinary system and sex hormones
- H Systemic hormonal preparations, excluding sex hormones and insulins
- J Anti-infectives for systemic use
- L Antineoplastic and immunomodulating agents
- M Musculo-skeletal system
- N Nervous system
- P Antiparasitic products, insecticides and repellents
- R Respiratory system
- S Sensory organs

ATC drug classification

ATC1: 14 classes

A

Alimentary tract and metabolism

ATC2: 84 classes

A10

Drugs used in diabetes

A10B

Blood glucose lowering drugs

A10BA

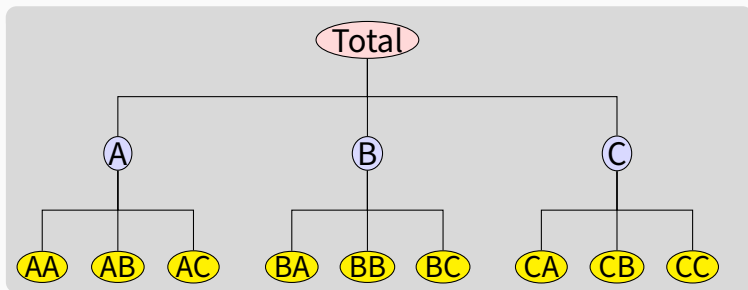
Biguanides

A10BA02

Metformin

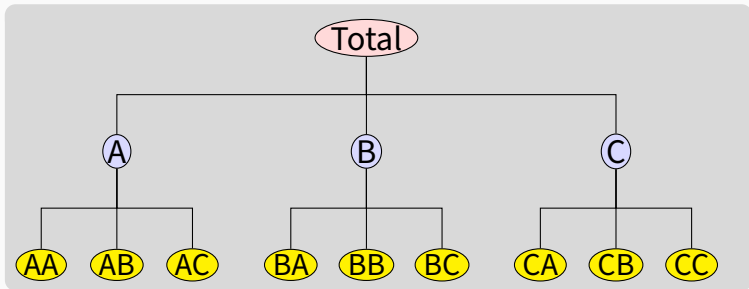
Hierarchical time series

A **hierarchical time series** is a collection of several time series that are linked together in a hierarchical structure.



Hierarchical time series

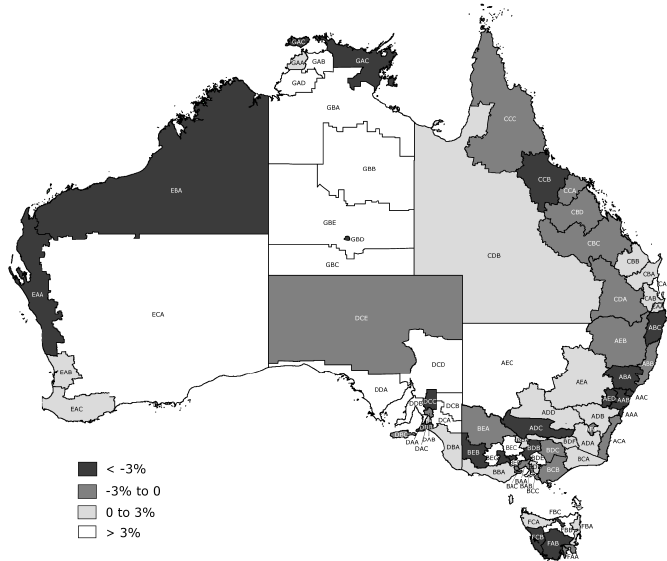
A **hierarchical time series** is a collection of several time series that are linked together in a hierarchical structure.



Examples

- PBS sales by ATC groups
- Tourism demand by states, zones, regions

Australian tourism



Australian tourism

tourism

A tsibble: 24,320 x 5 [1Q]

Key: Region, State, Purpose [304]

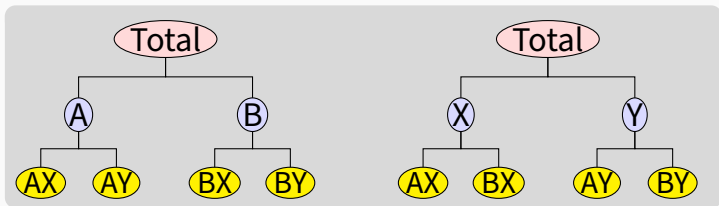
##	Quarter	Region	State	Purpose	Trips
##	<qtr>	<chr>	<chr>	<chr>	<dbl>
##	1	1998 Q1	Adelaide	South Australia Business	135.
##	2	1998 Q2	Adelaide	South Australia Business	110.
##	3	1998 Q3	Adelaide	South Australia Business	166.
##	4	1998 Q4	Adelaide	South Australia Business	127.
##	5	1999 Q1	Adelaide	South Australia Business	137.
##	6	1999 Q2	Adelaide	South Australia Business	200.
##	7	1999 Q3	Adelaide	South Australia Business	169.
##	8	1999 Q4	Adelaide	South Australia Business	134.
##	9	2000 Q1	Adelaide	South Australia Business	154.

Australian tourism

- Quarterly data on visitor night from 1998:Q1 – 2013:Q4
- From: *National Visitor Survey*, based on annual interviews of 120,000 Australians aged 15+, collected by Tourism Research Australia.
- Split by 7 states, 27 zones and 76 regions (a geographical hierarchy)
- Also split by purpose of travel
 - ▶ Holiday
 - ▶ Visiting friends and relatives (VFR)
 - ▶ Business
 - ▶ Other
- 304 bottom-level series

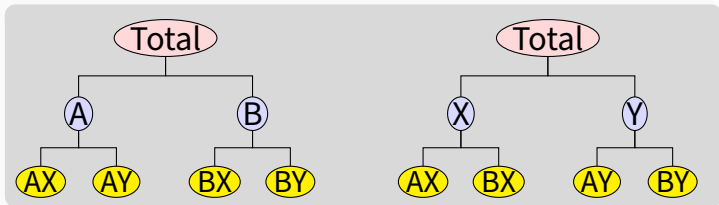
Grouped time series

A **grouped time series** is a collection of time series that can be grouped together in a number of non-hierarchical ways.



Grouped time series

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Examples

- Tourism by state and purpose of travel
- Retail sales by product groups/sub groups, and by countries/regions

Creating aggregates

PBS ▷

```
aggregate_key(ATC1 / ATC2, Scripts = sum(Scripts)) ▷  
filter(Month = yearmonth("1991 Jul")) ▷  
print(n = 18)
```

```
## # A tsibble: 98 x 4 [1M]  
## # Key:      ATC1, ATC2 [98]  
##      Month ATC1      ATC2      Scripts  
##      <mt> <chr*>    <chr*>    <dbl>  
## 1 1991 Jul <aggregated> <aggregated> 8090395  
## 2 1991 Jul A      <aggregated> 799025  
## 3 1991 Jul B      <aggregated> 109227  
## 4 1991 Jul C      <aggregated> 1794995  
## 5 1991 Jul D      <aggregated> 299779  
## 6 1991 Jul G      <aggregated> 300931  
## 7 1991 Jul H      <aggregated> 112114  
## 8 1991 Jul J      <aggregated> 1151681  
## 9 1991 Jul L      <aggregated> 24580  
## 10 1991 Jul M     <aggregated> 562956  
## 11 1991 Jul N     <aggregated> 1546023  
## 12 1991 Jul P     <aggregated> 47661  
## 13 1991 Jul R     <aggregated> 859273  
## 14 1991 Jul S     <aggregated> 391639
```

Creating aggregates

```
tourism ▷  
  aggregate_key(Purpose * (State / Region), Trips = sum(Trips)) ▷  
  filter(Quarter = yearquarter("1998 Q1")) ▷  
  print(n = 15)
```

```
## # A tsibble: 425 x 5 [1Q]  
## # Key:      Purpose, State, Region [425]  
##   Quarter Purpose      State      Region      Trips  
##   <qtr> <chr*>      <chr*>      <chr*>      <dbl>  
## 1 1998 Q1 <aggregated> <aggregated> <aggregated> 23182.  
## 2 1998 Q1 Business <aggregated> <aggregated> 3599.  
## 3 1998 Q1 Holiday <aggregated> <aggregated> 11806.  
## 4 1998 Q1 Other <aggregated> <aggregated> 680.  
## 5 1998 Q1 Visiting <aggregated> <aggregated> 7098.  
## 6 1998 Q1 <aggregated> ACT <aggregated> 551.  
## 7 1998 Q1 <aggregated> New South Wales <aggregated> 8040.  
## 8 1998 Q1 <aggregated> Northern Territory <aggregated> 181.  
## 9 1998 Q1 <aggregated> Queensland <aggregated> 4041.  
## 10 1998 Q1 <aggregated> South Australia <aggregated> 1735.  
## 11 1998 Q1 <aggregated> Tasmania <aggregated> 982.  
## 12 1998 Q1 <aggregated> Victoria <aggregated> 6010.
```


Creating aggregates

- Similar to `summarise()` but using the key structure
- A grouped structure is specified using `grp1 * grp2`
- A nested structure is specified via `parent / child`.
- Groups and nesting can be mixed:

```
(country/region/city) * (brand/product)
```
- All possible aggregates are produced.
- These are useful when forecasting at different levels of aggregation.

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The problem

- 1 How to forecast time series at all nodes such that the forecasts add up in the same way as the original data?
- 2 Can we exploit relationships between the series to improve the forecasts?

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- 2 Can we exploit relationships between the series to improve the forecasts?

The solution

- 1 Forecast all series at all levels of aggregation using an automatic forecasting algorithm.
(e.g., ETS, ARIMA, ...)
- 2 Reconcile the resulting forecasts so they add up correctly using least squares optimization (i.e., find closest reconciled forecasts to the original forecasts).
- 3 This is available using `reconcile()`.

Forecast reconciliation

```
tourism ▷  
  aggregate_key(Purpose * (State / Region), Trips = sum(Trips)) ▷  
  model(ets = ETS(Trips)) ▷  
  reconcile(ets_adjusted = min_trace(ets)) ▷  
  forecast(h = 2)
```

```
## # A tibble: 1,700 x 7 [1Q]  
## # Key:   Purpose, State, Region, .model [850]  
##   Purpose State      Region      .model Quarter      Trips .mean  
##   <chr*>  <chr*>      <chr*>      <chr>    <qtr>      <dist> <dbl>  
## 1 Business ACT      Canberra ~ ets     2018 Q1 N(144, 1119) 144.  
## 2 Business ACT      Canberra ~ ets     2018 Q2 N(203, 2260) 203.  
## 3 Business ACT      Canberra ~ ets_a~ 2018 Q1 N(157, 539) 157.  
## 4 Business ACT      Canberra ~ ets_a~ 2018 Q2 N(214, 951) 214.  
## 5 Business ACT      <aggregated> ets     2018 Q1 N(144, 1119) 144.  
## 6 Business ACT      <aggregated> ets     2018 Q2 N(203, 2260) 203.  
## 7 Business ACT      <aggregated> ets_a~ 2018 Q1 N(157, 539) 157.  
## 8 Business ACT      <aggregated> ets_a~ 2018 Q2 N(214, 951) 214.
```

Hierarchical and grouped time series

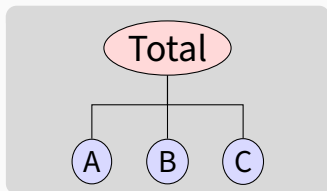
Every collection of time series with aggregation constraints can be written as

$$\mathbf{y}_t = \mathbf{S}\mathbf{b}_t$$

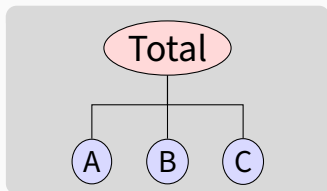
where

- \mathbf{y}_t is a vector of all series at time t
- \mathbf{b}_t is a vector of the most disaggregated series at time t
- \mathbf{S} is a “summing matrix” containing the aggregation constraints.

Hierarchical time series



Hierarchical time series

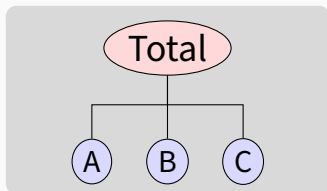


y_t : observed aggregate of all series at time t .

$y_{X,t}$: observation on series X at time t .

\mathbf{b}_t : vector of all series at bottom level in time t .

Hierarchical time series



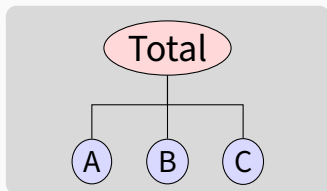
y_t : observed aggregate of all series at time t .

$y_{X,t}$: observation on series X at time t .

\mathbf{b}_t : vector of all series at bottom level in time t .

$$\mathbf{y}_t = \begin{pmatrix} y_t \\ y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix}$$

Hierarchical time series



y_t : observed aggregate of all series at time t .

$y_{X,t}$: observation on series X at time t .

\mathbf{b}_t : vector of all series at bottom level in time t .

$$\mathbf{y}_t = \begin{pmatrix} y_t \\ y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{S}} \underbrace{\begin{pmatrix} y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix}}_{\mathbf{b}_t}$$

$$\mathbf{y}_t = \mathbf{S}\mathbf{b}_t$$

Forecasting notation

Let $\hat{\mathbf{y}}_n(h)$ be vector of initial h -step forecasts, made at time n , stacked in same order as \mathbf{y}_t .

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Reconciled forecasts must be of the form:

$$\tilde{\mathbf{y}}_n(h) = \mathbf{S}\mathbf{G}\hat{\mathbf{y}}_n(h)$$

for some matrix \mathbf{G} .

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Reconciled forecasts must be of the form:

$$\tilde{\mathbf{y}}_n(h) = \mathbf{S}\mathbf{G}\hat{\mathbf{y}}_n(h)$$

for some matrix \mathbf{G} .

- \mathbf{G} extracts and combines base forecasts $\hat{\mathbf{y}}_n(h)$ to get bottom-level forecasts.
- \mathbf{S} adds them up

Optimal combination forecasts

Main result

The best (minimum sum of variances) unbiased forecasts are obtained when $\mathbf{G} = (\mathbf{S}'\Sigma_h^{-1}\mathbf{S})^{-1}\mathbf{S}'\Sigma_h^{-1}$, where Σ_h is the h -step base forecast error covariance matrix.

Optimal combination forecasts

Main result

The best (minimum sum of variances) unbiased forecasts are obtained when $\mathbf{G} = (\mathbf{S}'\Sigma_h^{-1}\mathbf{S})^{-1}\mathbf{S}'\Sigma_h^{-1}$, where Σ_h is the h -step base forecast error covariance matrix.

$$\tilde{\mathbf{y}}_n(h) = \mathbf{s}(\mathbf{S}'\Sigma_h^{-1}\mathbf{S})^{-1}\mathbf{S}'\Sigma_h^{-1}\hat{\mathbf{y}}_n(h)$$

Problem: Σ_h hard to estimate, especially for $h > 1$.

Solutions:

- Ignore Σ_h (OLS) [`min_trace(method='ols')`]
- Assume $\Sigma_h = k_h \Sigma_1$ is diagonal (WLS)
[`min_trace(method='wls')`]
- Assume $\Sigma_h = k_h \Sigma_1$ and estimate it (GLS)

Features

- Covariates can be included in initial forecasts.
- Adjustments can be made to initial forecasts at any level.
- Very simple and flexible method. Can work with *any* hierarchical or grouped time series.
- Conceptually easy to implement: regression of base forecasts on structure matrix.

Outline

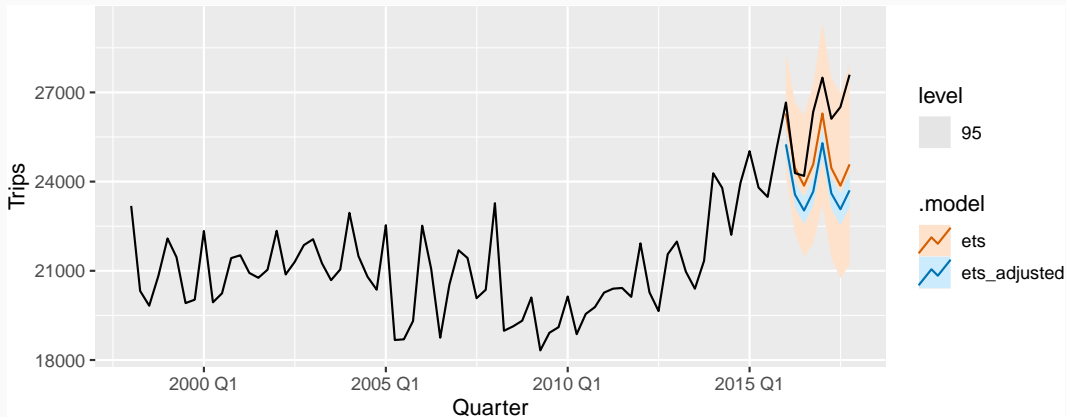
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Example: Australian tourism

```
tourism_agg <- tourism ▷  
  aggregate_key(Purpose * (State / Region),  
    Trips = sum(Trips)  
  )  
fc <- tourism_agg ▷  
  filter_index(. ~ "2015 Q4") ▷  
  model(ets = ETS(Trips)) ▷  
  reconcile(ets_adjusted = min_trace(ets)) ▷  
  forecast(h = "2 years")
```

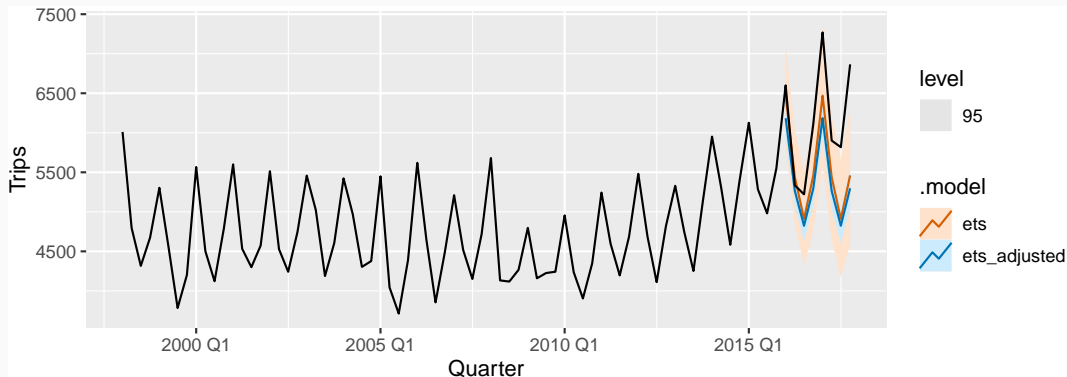
Example: Australian tourism

```
fc ▷  
  filter(is_aggregated(Purpose) & is_aggregated(State)) ▷  
  autoplot(tourism_agg, level = 95)
```



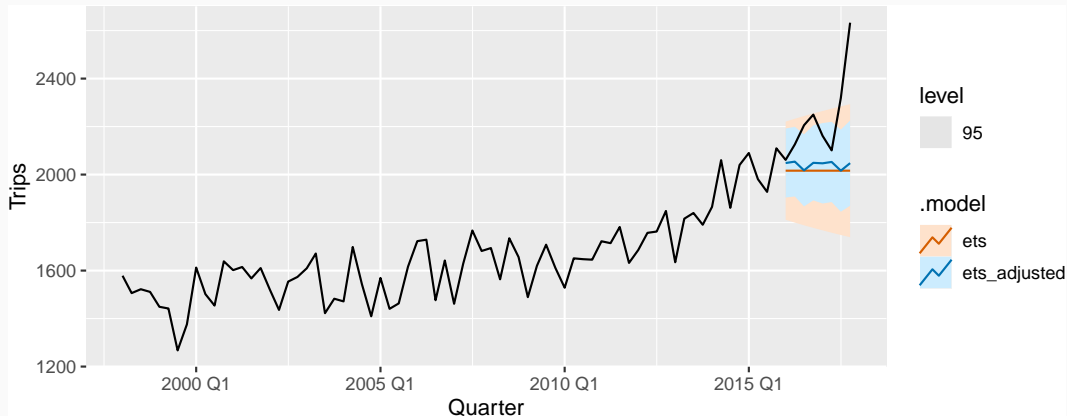
Example: Australian tourism

```
fc ▷  
  filter(is_aggregated(Purpose) & State = "Victoria" &  
    is_aggregated(Region)) ▷  
  autoplot(tourism_agg, level = 95)
```



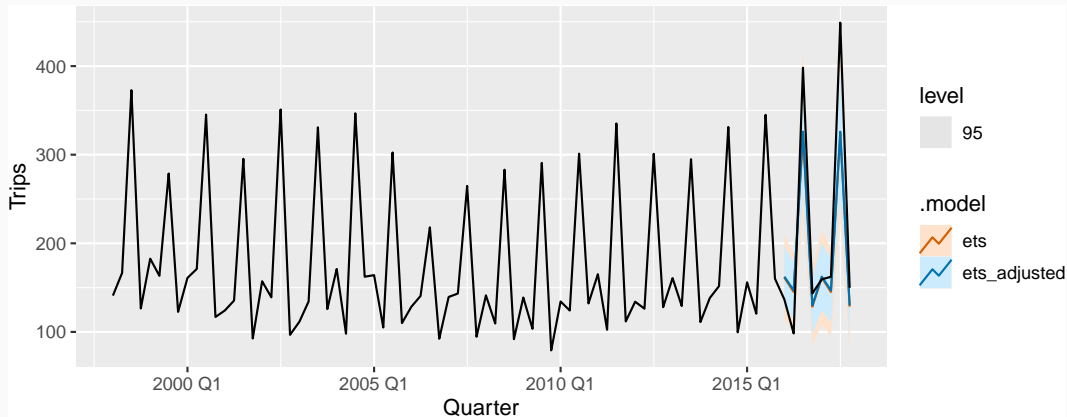
Example: Australian tourism

```
fc ▷  
  filter(is_aggregated(Purpose) & Region = "Melbourne") ▷  
  autoplot(tourism_agg, level = 95)
```



Example: Australian tourism

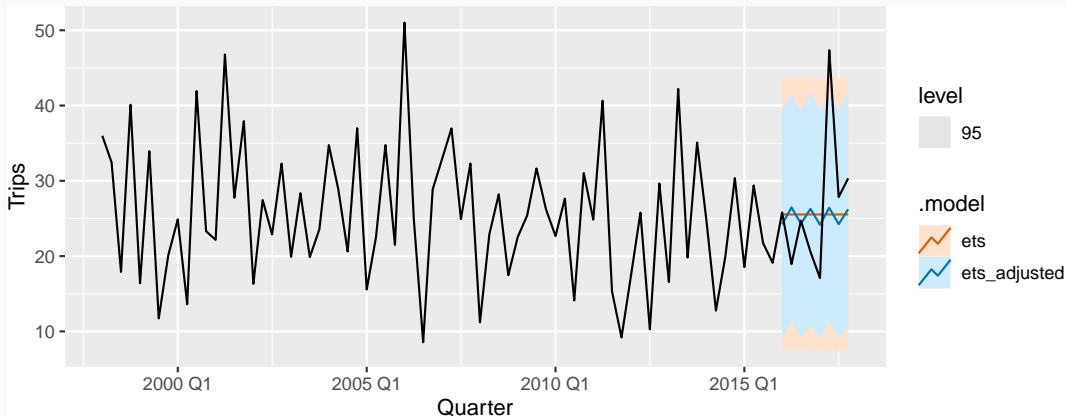
```
fc ▷  
  filter(is_aggregated(Purpose) & Region = "Snowy Mountains") ▷  
  autoplot(tourism_agg, level = 95)
```



Example: Australian tourism

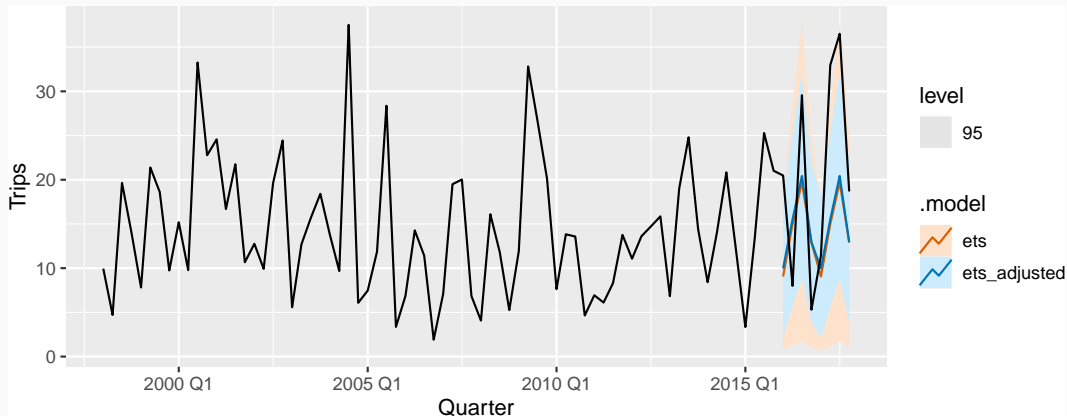
fc ▷

```
filter(Purpose = "Holiday" & Region = "Barossa") ▷  
autoplot(tourism_agg, level = 95)
```



Example: Australian tourism

```
fc ▷  
  filter(is_aggregated(Purpose) & Region = "MacDonnell") ▷  
  autoplot(tourism_agg, level = 95)
```



Example: Australian tourism

```
fc <- tourism_agg ▷  
  filter_index(. ~ "2015 Q4") ▷  
  model(  
    ets = ETS(Trips),  
    arima = ARIMA(Trips)  
  ) ▷  
  mutate(  
    comb = (ets + arima) / 2  
  ) ▷  
  reconcile(  
    ets_adj = min_trace(ets),  
    arima_adj = min_trace(arima),  
    comb_adj = min_trace(comb)  
  ) ▷  
  forecast(h = "2 years")
```

Forecast evaluation

```
fc ► accuracy(tourism_agg)
```

```
## # A tibble: 2,550 x 13
```

```
##   .model Purpose State      Region      .type    ME  RMSE  MAE  MPE
##   <chr>  <chr*>  <chr*>    <chr*>    <chr> <dbl> <dbl> <dbl> <dbl>
## 1 arima Business ACT          Canberra ~ Test  35.9  45.7  35.9  16.9
## 2 arima Business ACT          <aggregat~ Test  35.9  45.7  35.9  16.9
## 3 arima Business New South Wales Blue Moun~ Test   1.93  10.6   8.52 -18.0
## 4 arima Business New South Wales Capital C~ Test   8.08  15.6  10.4  11.8
## 5 arima Business New South Wales Central C~ Test  10.0  14.5  10.8  26.9
## 6 arima Business New South Wales Central N~ Test  17.7  31.9  28.2  12.0
## 7 arima Business New South Wales Hunter    ~ Test  35.3  43.9  35.3  24.2
## 8 arima Business New South Wales New Engla~ Test  23.1  31.8  26.8  19.5
## 9 arima Business New South Wales North Coa~ Test  24.8  40.1  36.8  11.5
## 10 arima Business New South Wales Outback N~ Test   6.87  11.0   7.76  13.7
## # ... with 2,540 more rows, and 4 more variables: MAPE <dbl>, MASE <dbl>,
```

Forecast evaluation

```
fc ▷  
  accuracy(tourism_agg) ▷  
  group_by(.model) ▷  
  summarise(MASE = mean(MASE)) ▷  
  arrange(MASE)
```

```
## # A tibble: 6 x 2  
##   .model      MASE  
##   <chr>      <dbl>  
## 1 ets_adj    1.02  
## 2 comb_adj   1.02  
## 3 ets        1.04  
## 4 comb       1.04  
## 5 arima_adj  1.07  
## 6 arima      1.09
```

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Lab Session 20

- Prepare aggregations of the PBS data by Concession, Type, and ATC1.
- Use forecast reconciliation with the PBS data, using ETS, ARIMA and SNAIVE models, applied to all but the last 3 years of data.
- Which type of model works best?
- Does the reconciliation improve the forecast accuracy?
- Why doesn't the reconciliation make any difference to the SNAIVE forecasts?

Feedback form

bit.ly/fable2022feedback