bit.ly/fable2022

Tidy Time Series & Forecasting in R

7. Exponential smoothing

tsibble feasts fable

Outline

Outline

Pharmaceutical Benefits Scheme

Pharmaceutical Benefits Scheme

The Pharmaceutical Benefits Scheme (PBS) is the Australian government drugs subsidy scheme.

- Many drugs bought from pharmacies are subsidised to allow more equitable access to modern drugs.
- The cost to government is determined by the number and types of drugs purchased. Currently nearly 1% of GDP.
- The total cost is budgeted based on forecasts of drug usage.

Pharmaceutical Benefits Scheme

ABC News Online
AUSTRALIAN BROADCASTING CORPORATION

NewsRadio
Streaming audio news
LISTEN: WMP | Real

Select a Topic
from the list below

Click "Refresh" or "Reload"
on your browser for the latest edition.

This Bulletin: **Wed, May 30 2001 6:22 PM AEST**

- Top Stories
- Just In
- World
- Asia-Pacific
- Business
- Sport
- Arts
- Sci Tech
- Indigenous
- Weather
- Rural
- Local News
- Broadband

Search

SPECIALS

- Federal Election

POLITICS

Opp demands drug price restriction after PBS budget blow-out

The Federal Opposition has called for tighter controls on drug prices after the Pharmaceutical Benefits Scheme (PBS) budget blew out by almost $800 million.

The money was spent on two new drugs including the controversial anti-smoking aid Zyban, which dropped in price from $220 to $22 after it was listed on the PBS.

the Public Record
For full election coverage

FEATURES

the Public Record
Federal Election 2001

For a fresh perspective on the federal election, reach into ABC Online's campaign weblog, The Poll Vault.

Audio News Online

Pharmaceutical Benefits Scheme

- In 2001: $4.5 billion budget, under-forecasted by $800 million.
- Thousands of products. Seasonal demand.
- Subject to covert marketing, volatile products, uncontrollable expenditure.
- Although monthly data available for 10 years, data are aggregated to annual values, and only the first three years are used in estimating the forecasts.
- All forecasts being done with the `FORECAST` function in MS-Excel!

Historical perspective

- Developed in the 1950s and 1960s as methods (algorithms) to produce point forecasts.
- Combine a “level”, “trend” (slope) and “seasonal” component to describe a time series.
- The rate of change of the components are controlled by “smoothing parameters”:
  α, β and γ respectively.
- Need to choose best values for the smoothing parameters (and initial states).
- Equivalent ETS state space models developed in the 1990s and 2000s.

A model for levels, trends, and seasonalities

We want a model that captures the level (ℓ_t), trend (b_t) and seasonality (s_t).

How do we combine these elements?

A model for levels, trends, and seasonalities

We want a model that captures the level (ℓ_t), trend (b_t) and seasonality (s_t).

How do we combine these elements?

Additively?

$y_t = \ell_{t-1} + b_{t-1} + s_{t-m} + \varepsilon_t$

A model for levels, trends, and seasonalities

We want a model that captures the level (ℓ_t), trend (b_t) and seasonality (s_t).

How do we combine these elements?

Additively?

$$y_t = \ell_{t-1} + b_{t-1} + s_{t-m} + \varepsilon_t$$

Multiplicatively?

$$y_t = \ell_{t-1} b_{t-1} s_{t-m} (1 + \varepsilon_t)$$

A model for levels, trends, and seasonalities

We want a model that captures the level (ℓ_t), trend (b_t) and seasonality (s_t).

How do we combine these elements?

Additively?

$$y_t = \ell_{t-1} + b_{t-1} + s_{t-m} + \varepsilon_t$$

Multiplicatively?

$$y_t = \ell_{t-1} b_{t-1} s_{t-m}(1 + \varepsilon_t)$$

Perhaps a mix of both?

$$y_t = (\ell_{t-1} + b_{t-1}) s_{t-m} + \varepsilon_t$$

A model for levels, trends, and seasonalities

We want a model that captures the level (ℓ_t), trend (b_t) and seasonality (s_t).

How do we combine these elements?

Additively?

$$y_t = \ell_{t-1} + b_{t-1} + s_{t-m} + \varepsilon_t$$

Multiplicatively?

$$y_t = \ell_{t-1} b_{t-1} s_{t-m} (1 + \varepsilon_t)$$

Perhaps a mix of both?

$$y_t = (\ell_{t-1} + b_{t-1}) s_{t-m} + \varepsilon_t$$

How do the level, trend and seasonal components evolve over time?

ETS models

General notation $\quad$ E T S : **E**xponen**T**ial **S**moothing

$\nearrow \quad \uparrow \quad \nwarrow$

Error **T**rend **S**eason

Error: Additive (`"A"`) or multiplicative (`"M"`)

ETS models

General notation E T S : **E**xponen**T**ial **S**moothing

Error **T**rend **S**eason

Error: Additive (`"A"`) or multiplicative (`"M"`)

Trend: None (`"N"`), additive (`"A"`), multiplicative (`"M"`), or damped (`"Ad"` or `"Md"`).

ETS models

General notation E T S : **E**xponen**T**ial **S**moothing

$\nearrow \quad \uparrow \quad \nwarrow$

Error **T**rend **S**eason

Error: Additive (`"A"`) or multiplicative (`"M"`)

Trend: None (`"N"`), additive (`"A"`), multiplicative (`"M"`), or damped (`"Ad"` or `"Md"`).

Seasonality: None (`"N"`), additive (`"A"`) or multiplicative (`"M"`)

ETS(A,N,N): SES with additive errors

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T$
Measurement equation	$y_t = \ell_{t-1} + \varepsilon_t$
State equation	$\ell_t = \ell_{t-1} + \alpha\varepsilon_t$

where $\varepsilon_t \sim \text{NID}(0, \sigma^2)$.

ETS(A,N,N): SES with additive errors

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T$
Measurement equation	$y_t = \ell_{t-1} + \varepsilon_t$
State equation	$\ell_t = \ell_{t-1} + \alpha\varepsilon_t$

where $\varepsilon_t \sim \text{NID}(0, \sigma^2)$.

- "innovations" or "single source of error" because equations have the same error process, ε_t.
- Measurement equation: relationship between observations and states.
- Transition/state equation(s): evolution of the state(s) through time.

ETS(M,N,N): SES with multiplicative errors

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T$
Measurement equation	$y_t = \ell_{t-1}(1+\varepsilon_t)$
State equation	$\ell_t = \ell_{t-1}(1+\alpha\varepsilon_t)$

where $\varepsilon_t \sim \text{NID}(0,\sigma^2)$.

ETS(M,N,N): SES with multiplicative errors

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T$
Measurement equation	$y_t = \ell_{t-1}(1 + \varepsilon_t)$
State equation	$\ell_t = \ell_{t-1}(1 + \alpha\varepsilon_t)$

where $\varepsilon_t \sim \text{NID}(0, \sigma^2)$.

- Models with additive and multiplicative errors with the same parameters generate the same point forecasts but different prediction intervals.

Outline

Holt's linear trend

Additive errors: ETS(A,A,N)

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T + hb_T$
Measurement equation	$y_t = \ell_{t-1} + b_{t-1} + \varepsilon_t$
State equations	$\ell_t = \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t$
	$b_t = b_{t-1} + \beta\varepsilon_t$

Holt's linear trend

Additive errors: ETS(A,A,N)

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T + hb_T$
Measurement equation	$y_t = \ell_{t-1} + b_{t-1} + \varepsilon_t$
State equations	$\ell_t = \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t$
	$b_t = b_{t-1} + \beta\varepsilon_t$

Multiplicative errors: ETS(M,A,N)

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T + hb_T$
Measurement equation	$y_t = (\ell_{t-1} + b_{t-1})(1 + \varepsilon_t)$
State equations	$\ell_t = (\ell_{t-1} + b_{t-1})(1 + \alpha\varepsilon_t)$
	$b_t = b_{t-1} + \beta\varepsilon_t$

Example: Australian population

```r
aus_economy <- global_economy %>%
  filter(Code == "AUS") %>%
  mutate(Pop = Population / 1e6)
fit <- aus_economy %>% model(AAN = ETS(Pop))
report(fit)
```

```
## Series: Pop
## Model: ETS(A,A,N)
##   Smoothing parameters:
##     alpha = 1
##     beta  = 0.327
##
##   Initial states:
##  l[0]  b[0]
##  10.1 0.222
##
##   sigma^2:  0.0041
##
##   AIC  AICc   BIC
```

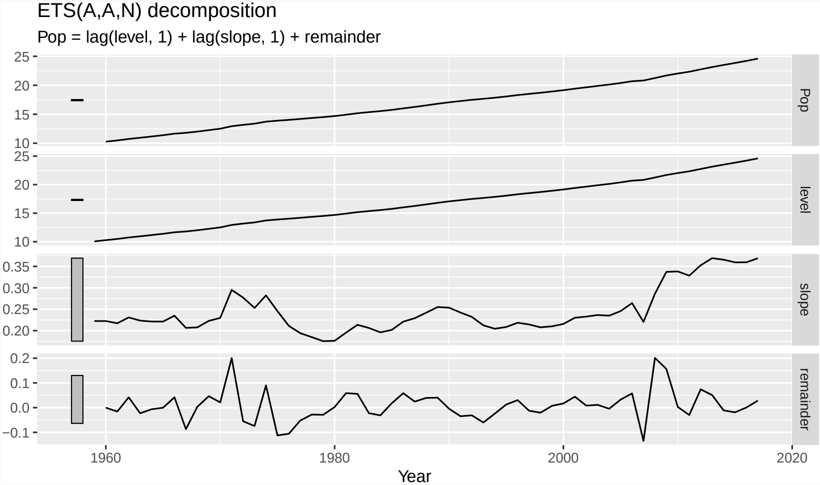
Example: Australian population

```
components(fit)
```

```
## # A dable: 59 x 7 [1Y]
## # Key:     Country, .model [1]
## # :        Pop = lag(level, 1) + lag(slope, 1) + remainder
##    Country   .model  Year   Pop level slope remainder
##    <fct>     <chr>  <dbl> <dbl> <dbl> <dbl>     <dbl>
##  1 Australia AAN     1959  NA    10.1 0.222 NA
##  2 Australia AAN     1960  10.3  10.3 0.222 -0.000145
##  3 Australia AAN     1961  10.5  10.5 0.217 -0.0159
##  4 Australia AAN     1962  10.7  10.7 0.231  0.0418
##  5 Australia AAN     1963  11.0  11.0 0.223 -0.0229
##  6 Australia AAN     1964  11.2  11.2 0.221 -0.00641
##  7 Australia AAN     1965  11.4  11.4 0.221 -0.000314
##  8 Australia AAN     1966  11.7  11.7 0.235  0.0418
##  9 Australia AAN     1967  11.8  11.8 0.206 -0.0869
```

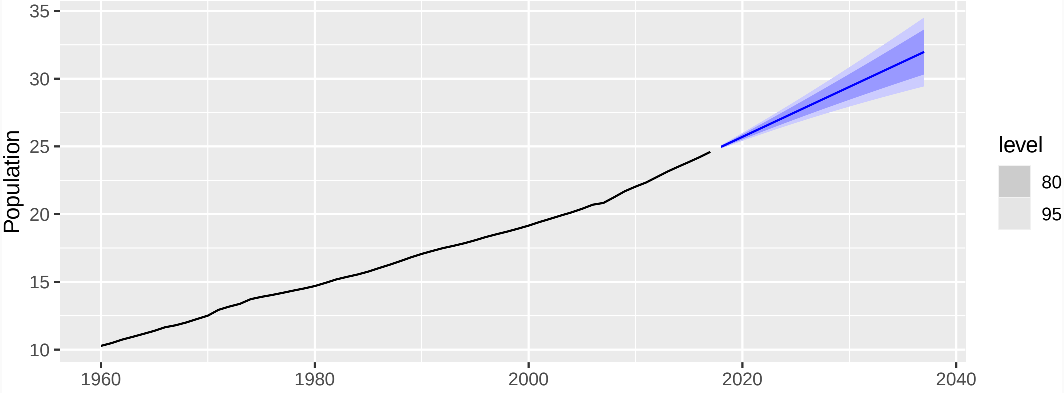
Example: Australian population

```
components(fit) %>% autoplot()
```



Example: Australian population

```
fit %>%
  forecast(h = 20) %>%
  autoplot(aus_economy) +
  ylab("Population") + xlab("Year")
```



ETS(A,Ad,N): Damped trend method

Additive errors

$$\begin{aligned}
\text{Forecast equation} \quad & \hat{y}_{T+h|T} = \ell_T + (\phi + \cdots + \phi^{h-1})b_T \\
\text{Measurement equation} \quad & y_t = (\ell_{t-1} + \phi b_{t-1}) + \varepsilon_t \\
\text{State equations} \quad & \ell_t = (\ell_{t-1} + \phi b_{t-1}) + \alpha\varepsilon_t \\
& b_t = \phi b_{t-1} + \beta\varepsilon_t
\end{aligned}$$

ETS(A,Ad,N): Damped trend method

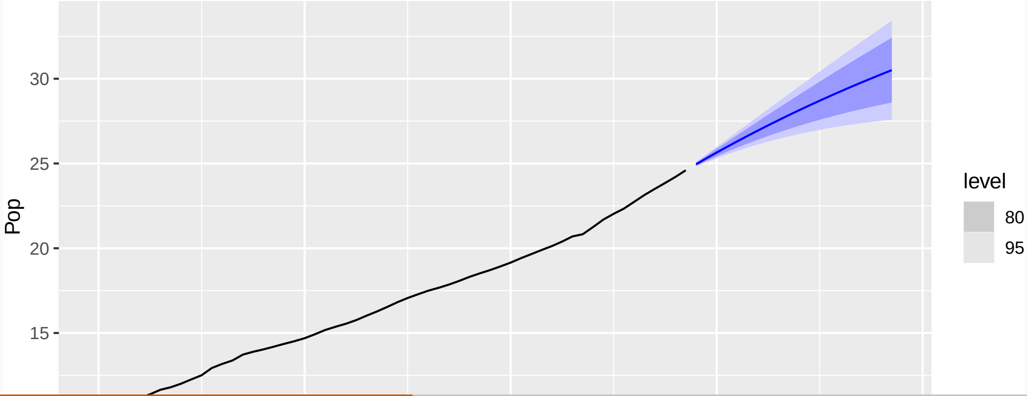
Additive errors

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T + (\phi + \cdots + \phi^{h-1})b_T$
Measurement equation	$y_t = (\ell_{t-1} + \phi b_{t-1}) + \varepsilon_t$
State equations	$\ell_t = (\ell_{t-1} + \phi b_{t-1}) + \alpha\varepsilon_t$
	$b_t = \phi b_{t-1} + \beta\varepsilon_t$

- Damping parameter $0 < \phi < 1$.
- If $\phi = 1$, identical to Holt's linear trend.
- As $h \to \infty$, $\hat{y}_{T+h|T} \to \ell_T + \phi b_T/(1-\phi)$.
- Short-run forecasts trended, long-run forecasts constant.

Example: Australian population

```
aus_economy %>%
  model(holt = ETS(Pop ~ trend("Ad"))) %>%
  forecast(h = 20) %>%
  autoplot(aus_economy)
```



Example: National populations

```
fit <- global_economy %>%
  mutate(Pop = Population / 1e6) %>%
  model(ets = ETS(Pop))
fit
```

```
## # A mable: 263 x 2
## # Key:     Country [263]
##    Country                       ets
##    <fct>                     <model>
##  1 Afghanistan          <ETS(A,A,N)>
##  2 Albania              <ETS(M,A,N)>
##  3 Algeria              <ETS(M,A,N)>
##  4 American Samoa       <ETS(M,A,N)>
##  5 Andorra              <ETS(M,A,N)>
##  6 Angola               <ETS(M,A,N)>
##  7 Antigua and Barbuda  <ETS(M,A,N)>
##  8 Arab World           <ETS(M,A,N)>
##  9 Argentina            <ETS(A,A,N)>
## 10 Armenia              <ETS(M,A,N)>
```

Example: National populations

```r
fit %>%
  forecast(h = 5)
```

```
## # A fable: 1,315 x 5 [1Y]
## # Key:     Country, .model [263]
##    Country     .model  Year              Pop .mean
##    <fct>       <chr>  <dbl>           <dist> <dbl>
##  1 Afghanistan ets     2018    N(36, 0.012) 36.4
##  2 Afghanistan ets     2019    N(37, 0.059) 37.3
##  3 Afghanistan ets     2020     N(38, 0.16) 38.2
##  4 Afghanistan ets     2021     N(39, 0.35) 39.0
##  5 Afghanistan ets     2022     N(40, 0.64) 39.9
##  6 Albania     ets     2018 N(2.9, 0.00012)  2.87
##  7 Albania     ets     2019   N(2.9, 6e-04)  2.87
##  8 Albania     ets     2020  N(2.9, 0.0017)  2.87
```

Outline

Lab Session 14

Try forecasting the Chinese GDP from the `global_economy` data set using an ETS model.

Experiment with the various options in the `ETS()` function to see how much the forecasts change with damped trend, or with a Box-Cox transformation. Try to develop an intuition of what each is doing to the forecasts.

[Hint: use `h=20` when forecasting, so you can clearly see the differences between the various options when plotting the forecasts.]

Outline

ETS(A,A,A): Holt-Winters additive method

$$
\begin{aligned}
\text{Forecast equation} \quad \hat{y}_{t+h|t} &= \ell_t + hb_t + s_{t+h-m(k+1)} \\
\text{Observation equation} \quad y_t &= \ell_{t-1} + b_{t-1} + s_{t-m} + \varepsilon_t \\
\text{State equations} \quad \ell_t &= \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t \\
b_t &= b_{t-1} + \beta\varepsilon_t \\
s_t &= s_{t-m} + \gamma\varepsilon_t
\end{aligned}
$$

- k = integer part of $(h-1)/m$.
- $\sum_i s_i \approx 0$.
- Parameters: $0 \le \alpha \le 1$, $0 \le \beta^* \le 1$, $0 \le \gamma \le 1-\alpha$ and m = period of seasonality (e.g. m = 4 for quarterly data).

ETS(M,A,M): Holt-Winters multiplicative method

$$\begin{aligned}
\text{Forecast equation} \quad \hat{y}_{t+h|t} &= (\ell_t + hb_t)s_{t+h-m(k+1)} \\
\text{Observation equation} \quad y_t &= (\ell_{t-1} + b_{t-1})s_{t-m}(1 + \varepsilon_t) \\
\text{State equations} \quad \ell_t &= (\ell_{t-1} + b_{t-1})(1 + \alpha\varepsilon_t) \\
b_t &= b_{t-1}(1 + \beta\varepsilon_t) \\
s_t &= s_{t-m}(1 + \gamma\varepsilon_t)
\end{aligned}$$

- k is integer part of $(h-1)/m$.
- $\sum_i s_i \approx m$.
- Parameters: $0 \le \alpha \le 1$, $0 \le \beta^* \le 1$, $0 \le \gamma \le 1-\alpha$ and $m =$ period of seasonality (e.g. $m = 4$ for quarterly data).

Example: Australian holiday tourism

```r
holidays <- tourism %>%
  filter(Purpose == "Holiday")
fit <- holidays %>% model(ets = ETS(Trips))
fit
```

```
## # A mable: 76 x 4
## # Key:     Region, State, Purpose [76]
##    Region                   State              Purpose          ets
##    <chr>                    <chr>              <chr>        <model>
##  1 Adelaide                 South Australia Holiday <ETS(A,N,A)>
##  2 Adelaide Hills           South Australia Holiday <ETS(A,A,N)>
##  3 Alice Springs            Northern Terri~ Holiday <ETS(M,N,A)>
##  4 Australia's Coral Coast  Western Austra~ Holiday <ETS(M,N,A)>
##  5 Australia's Golden Outback Western Austra~ Holiday <ETS(M,N,M)>
##  6 Australia's North West   Western Austra~ Holiday <ETS(A,N,A)>
##  7 Australia's South West   Western Austra~ Holiday <ETS(M,N,M)>
##  8 Ballarat                 Victoria        Holiday <ETS(M,N,A)>
##  9 Barkly                   Northern Terri~ Holiday <ETS(A,N,A)>
```

Example: Australian holiday tourism

```r
fit %>%
  filter(Region == "Snowy Mountains") %>%
  report()
```

```
## Series: Trips
## Model: ETS(M,N,A)
##   Smoothing parameters:
##     alpha = 0.157
##     gamma = 1e-04
##
##   Initial states:
##  l[0] s[0] s[-1] s[-2] s[-3]
##   142  -61   131 -42.2 -27.7
##
##   sigma^2:  0.0388
##
##  AIC AICc  BIC
##  852  854  869
```

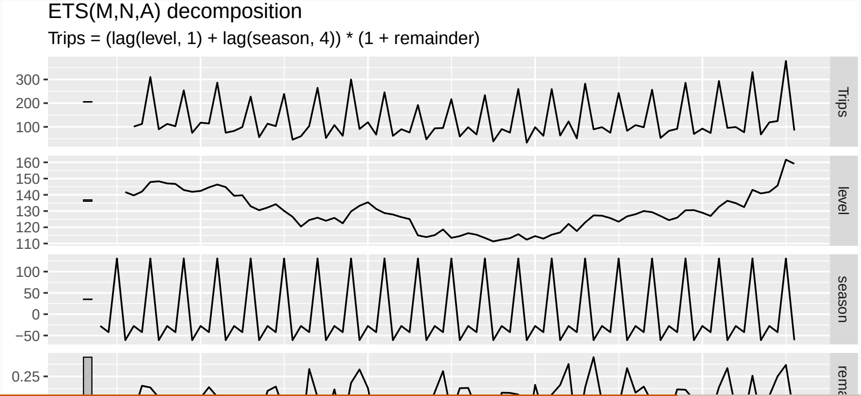
Example: Australian holiday tourism

```
fit %>%
  filter(Region == "Snowy Mountains") %>%
  components(fit)
```

```
## # A dable: 84 x 9 [1Q]
## # Key:     Region, State, Purpose, .model [1]
## # :        Trips = (lag(level, 1) + lag(season, 4)) * (1 +
## #   remainder)
##    Region  State Purpose .model Quarter Trips level season remai~1
##    <chr>   <chr> <chr>   <chr>    <qtr> <dbl> <dbl>  <dbl>   <dbl>
##  1 Snowy ~ New ~ Holiday ets    1997 Q1  NA      NA   -27.7 NA
##  2 Snowy ~ New ~ Holiday ets    1997 Q2  NA      NA   -42.2 NA
##  3 Snowy ~ New ~ Holiday ets    1997 Q3  NA      NA   131.  NA
##  4 Snowy ~ New ~ Holiday ets    1997 Q4  NA     142.  -61.0 NA
##  5 Snowy ~ New ~ Holiday ets    1998 Q1 101.    140.  -27.7 -0.113
##  6 Snowy ~ New ~ Holiday ets    1998 Q2 112.    142.  -42.2  0.154
##  7 Snowy ~ New ~ Holiday ets    1998 Q3 310.    148.  131.   0.137
##  8 Snowy ~ New ~ Holiday ets    1998 Q4  89.8   148.  -61.0  0.0335
```

Example: Australian holiday tourism

```
fit %>%
  filter(Region == "Snowy Mountains") %>%
  components(fit) %>%
  autoplot()
```



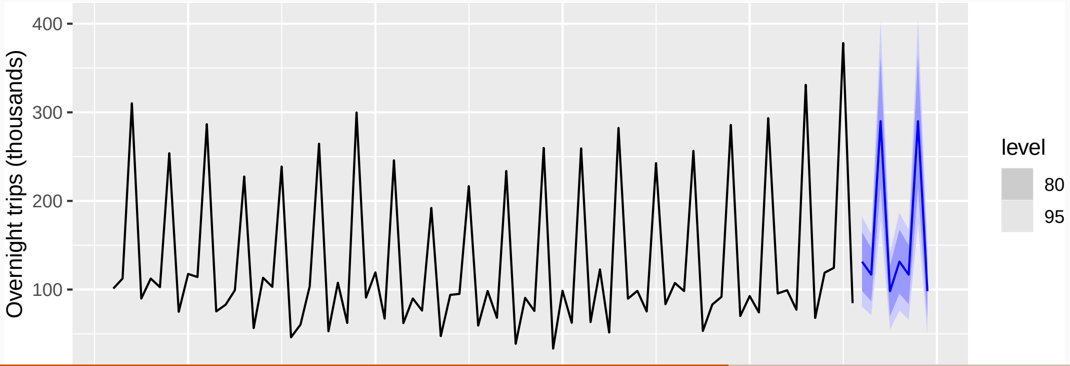
Example: Australian holiday tourism

```
fit %>% forecast()
```

```
## # A fable: 608 x 7 [1Q]
## # Key:     Region, State, Purpose, .model [76]
##    Region         State   Purpose .model Quarter        Trips .mean
##    <chr>          <chr>   <chr>   <chr>    <qtr>       <dist> <dbl>
##  1 Adelaide       South ~ Holiday ets    2018 Q1 N(210, 457) 210.
##  2 Adelaide       South ~ Holiday ets    2018 Q2 N(173, 473) 173.
##  3 Adelaide       South ~ Holiday ets    2018 Q3 N(169, 489) 169.
##  4 Adelaide       South ~ Holiday ets    2018 Q4 N(186, 505) 186.
##  5 Adelaide       South ~ Holiday ets    2019 Q1 N(210, 521) 210.
##  6 Adelaide       South ~ Holiday ets    2019 Q2 N(173, 537) 173.
##  7 Adelaide       South ~ Holiday ets    2019 Q3 N(169, 553) 169.
##  8 Adelaide       South ~ Holiday ets    2019 Q4 N(186, 569) 186.
##  9 Adelaide Hills South ~ Holiday ets    2018 Q1   N(19, 36)  19.4
## 10 Adelaide Hills South ~ Holiday ets    2018 Q2   N(20, 36)  19.6
## # ... with 598 more rows
```

Example: Australian holiday tourism

```
fit %>%
  forecast() %>%
  filter(Region == "Snowy Mountains") %>%
  autoplot(holidays) +
  xlab("Year") + ylab("Overnight trips (thousands)")
```



Outline

Exponential smoothing models

Additive Error		**Seasonal Component**		
Trend Component		N (None)	A (Additive)	M (Multiplicative)
N	(None)	A,N,N	A,N,A	~~A,N,M~~
A	(Additive)	A,A,N	A,A,A	~~A,A,M~~
A_d	(Additive damped)	A,A_d,N	A,A_d,A	~~A,A_d,M~~

Multiplicative Error		**Seasonal Component**		
Trend Component		N (None)	A (Additive)	M (Multiplicative)
N	(None)	M,N,N	M,N,A	M,N,M
A	(Additive)	M,A,N	M,A,A	M,A,M
A_d	(Additive damped)	M,A_d,N	M,A_d,A	M,A_d,M

Estimating ETS models

- Smoothing parameters α, β, γ and ϕ, and the initial states ℓ_0, b_0, $s_0, s_{-1}, \dots, s_{-m+1}$ are estimated by maximising the "likelihood" = the probability of the data arising from the specified model.
- For models with additive errors equivalent to minimising SSE.
- For models with multiplicative errors, **not** equivalent to minimising SSE.

Model selection

Akaike's Information Criterion

$$\mathrm{AIC} = -2\log(\mathrm{L}) + 2k$$

where L is the likelihood and k is the number of parameters initial states estimated in the model.

Model selection

Akaike's Information Criterion

$$\text{AIC} = -2\log(\text{L}) + 2k$$

where L is the likelihood and k is the number of parameters initial states estimated in the model.

Corrected AIC

$$\text{AIC}_\text{c} = \text{AIC} + \frac{2(k+1)(k+2)}{T-k}$$

which is the AIC corrected (for small sample bias).

Model selection

Akaike's Information Criterion

$$\text{AIC} = -2\log(\text{L}) + 2k$$

where L is the likelihood and k is the number of parameters initial states estimated in the model.

Corrected AIC

$$\text{AIC}_\text{c} = \text{AIC} + \frac{2(k+1)(k+2)}{T-k}$$

which is the AIC corrected (for small sample bias).

Bayesian Information Criterion

$$\text{BIC} = \text{AIC} + k(\log(T) - 2).$$

AIC and cross-validation

Minimizing the AIC assuming Gaussian residuals is asymptotically equivalent to minimizing one-step time series cross validation MSE.

Automatic forecasting

From Hyndman et al. (IJF, 2002):

1. Apply each model that is appropriate to the data. Optimize parameters and initial values using MLE.
2. Select best method using AICc.
3. Produce forecasts using best method.
4. Obtain forecast intervals using underlying state space model.

- Method performed very well in M3 competition.
- Used as a benchmark in the M4 competition.

Outline

Lab Session 15

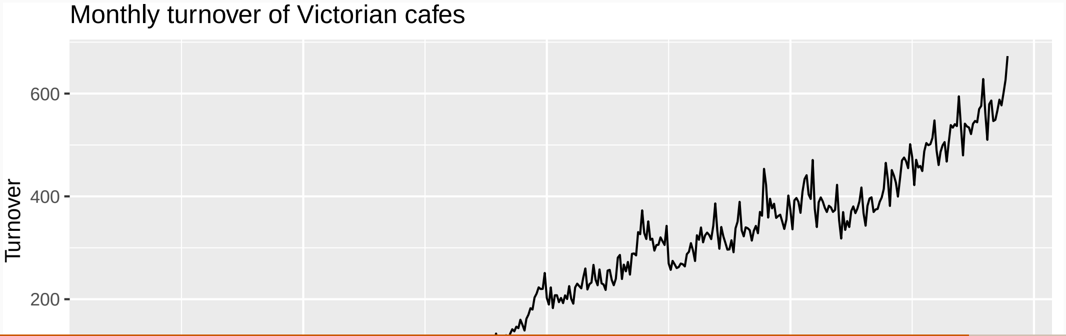
Find an ETS model for the Gas data from `aus_production`.

- Why is multiplicative seasonality necessary here?
- Experiment with making the trend damped. Does it improve the forecasts?

Outline

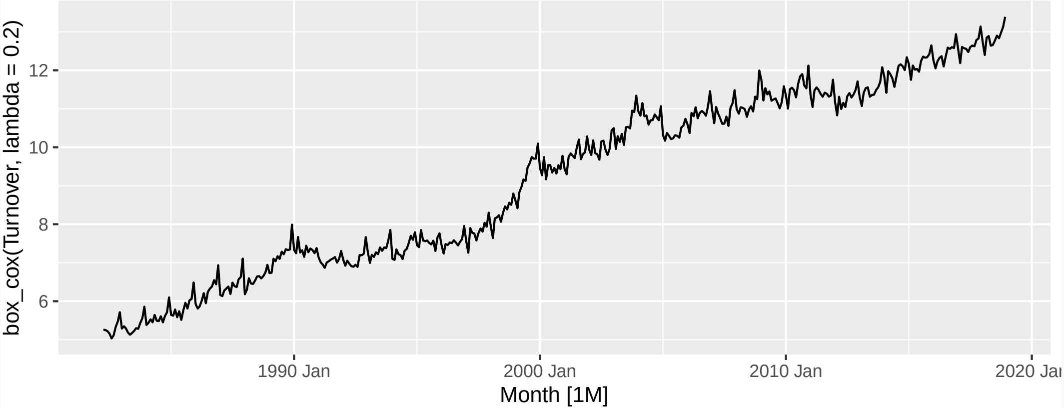
Non-Gaussian forecast distributions

```
vic_cafe <- tsibbledata::aus_retail %>%
  filter(State == "Victoria",
         Industry == "Cafes, restaurants and catering services") %>%
  select(Month, Turnover)
vic_cafe %>%
  autoplot(Turnover) + ggtitle("Monthly turnover of Victorian cafes")
```



Forecasting with transformations

```
vic_cafe %>% autoplot(box_cox(Turnover, lambda = 0.2))
```



Forecasting with transformations

```r
fit <- vic_cafe %>%
  model(ets = ETS(box_cox(Turnover, 0.2)))
fit
```

```
## # A mable: 1 x 1
##              ets
##          <model>
## 1 <ETS(A,A,A)>
```

```r
(fc <- fit %>% forecast(h = "3 years"))
```

```
## # A fable: 36 x 4 [1M]
## # Key:     .model [1]
##    .model    Month        Turnover .mean
##    <chr>     <mth>          <dist> <dbl>
##  1 ets    2019 Jan  t(N(13, 0.02))  608.
##  2 ets    2019 Feb t(N(13, 0.028))  563.
##  3 ets    2019 Mar t(N(13, 0.036))  629.
##  4 ets    2019 Apr t(N(13, 0.044))  615.
```

Forecasting with transformations

```
fit <- vic_cafe %>%
  model(ets = ETS(box_cox(Turnover, 0.2)))
fit
```

```
## # A mable: 1 x 1
##             ets
##         <model>
## 1 <ETS(A,A,A)>
```

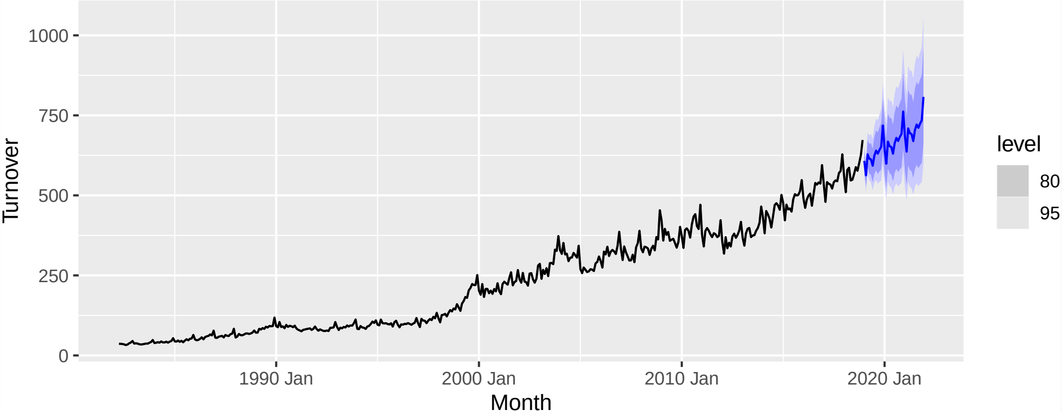
```
(fc <- fit %>% forecast(h = "3 y
```

```
## # A fable: 36 x 4 [1M]
## # Key:     .model [1]
##    .model    Month        Turnover .mean
##    <chr>     <mth>          <dist> <dbl>
##  1 ets    2019 Jan  t(N(13, 0.02))  608.
##  2 ets    2019 Feb t(N(13, 0.028))  563.
##  3 ets    2019 Mar t(N(13, 0.036))  629.
##  4 ets    2019 Apr t(N(13, 0.044))  615.
```

- `t(N)` denotes a transformed normal distribution.
- back-transformation and bias adjustment is done automatically.

Forecasting with transformations

```
fc %>% autoplot(vic_cafe)
```



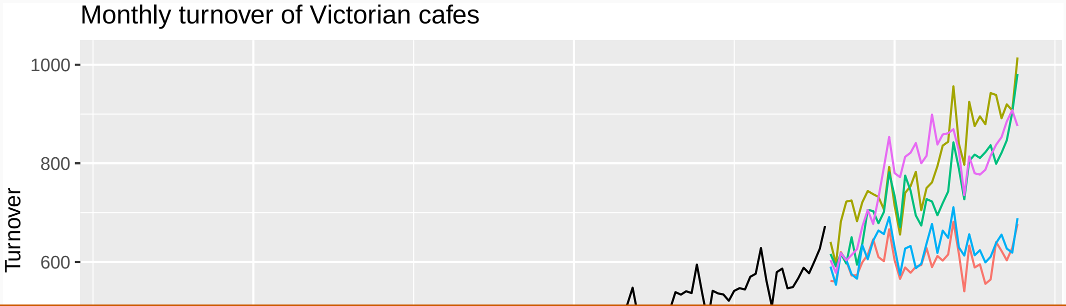
Bootstrapped forecast distributions

```r
sim <- fit %>% generate(h = "3 years", times = 5, bootstrap = TRUE)
sim
```

```
## # A tsibble: 180 x 4 [1M]
## # Key:       .model, .rep [5]
##    .model    Month .rep   .sim
##    <chr>     <mth> <chr> <dbl>
##  1 ets    2019 Jan 1      562.
##  2 ets    2019 Feb 1      560.
##  3 ets    2019 Mar 1      620.
##  4 ets    2019 Apr 1      602.
##  5 ets    2019 May 1      572.
##  6 ets    2019 Jun 1      573.
##  7 ets    2019 Jul 1      599.
##  8 ets    2019 Aug 1      615.
##  9 ets    2019 Sep 1      646.
## 10 ets    2019 Oct 1      610.
## # ... with 170 more rows
```

Bootstrapped forecast distributions

```
vic_cafe %>%
  filter(year(Month) >= 2008) %>%
  ggplot(aes(x = Month)) +
  geom_line(aes(y = Turnover)) +
  geom_line(aes(y = .sim, colour = as.factor(.rep)), data = sim) +
  ggtitle("Monthly turnover of Victorian cafes") +
  guides(col = FALSE)
```



Bootstrapped forecast distributions

```
fc <- fit %>% forecast(h = "3 years", bootstrap = TRUE)
fc
```

```
## # A fable: 36 x 4 [1M]
## # Key:     .model [1]
##    .model    Month          Turnover .mean
##    <chr>     <mth>            <dist> <dbl>
##  1 ets    2019 Jan t(sample[5000])  608.
##  2 ets    2019 Feb t(sample[5000])  563.
##  3 ets    2019 Mar t(sample[5000])  628.
##  4 ets    2019 Apr t(sample[5000])  614.
##  5 ets    2019 May t(sample[5000])  612.
##  6 ets    2019 Jun t(sample[5000])  592.
##  7 ets    2019 Jul t(sample[5000])  623.
##  8 ets    2019 Aug t(sample[5000])  640.
##  9 ets    2019 Sep t(sample[5000])  630.
## 10 ets    2019 Oct t(sample[5000])  642.
## # ... with 26 more rows
```

Bootstrapped forecast distributions

```
fc %>% autoplot(vic_cafe) +
  ggtitle("Monthly turnover of Victorian cafes")
```