Problem_1

The following problem is from both the 2023 AMC 10A #1 and 2023 AMC 12A #1, so both problems redirect to this page.

Problem

Cities A and B are 45 miles apart. Alicia lives in A and Beth lives in B. Alicia bikes towards B at 18 miles per hour. Leaving at the same time, Beth bikes toward A at 12 miles per hour. How many miles from City A will they be when they meet?
$\textbf{(A) }20\qquad\textbf{(B) }24\qquad\textbf{(C) }25\qquad\textbf{(D) }26\qquad\textbf{(E) }27$

Solution 1

This is a $d=st$ problem, so let x be the time it takes to meet. We can write the following equation:

$$12x+18x=45$$

Solving gives us $x=1.5$. The $18x$ is Alicia so $18\times1.5=\boxed{\textbf{(E) 27}}$

~zhenghua

Solution 2

The relative speed of the two is $18+12=30$, so $\frac{3}{2}$ hours would be required to travel 45 miles. $d=st$, so $x=18\cdot\frac{3}{2}=\boxed{\textbf{(E) 27}}$

~walmartbrian

Solution 3

Since 18 mph is $\frac{3}{2}$ times 12 mph, Alicia will travel $\frac{3}{2}$ times as far as Beth. If x is the distance Beth travels,

$$\frac{3}{2}x+x=45$$

$$\frac{5}{2}x=45$$

$$x=18$$

Since this is the amount Beth traveled, the amount that Alicia traveled was

$$45-18=\boxed{\textbf{(E) 27}}$$

~daniel luo

Solution 4

Alice and Barbara close in on each other at 30mph. Since they are 45 miles apart, they will meet in t = d/s = 45miles / 30mph = 3/2 hours. We can either calculate the distance Alice travels at 18mph or the distance Barbara travels at 12mph; since we want the distance from Alice, we go with the former. Alice (and Barbara) will meet in 1 1/2 hours at 18mph x 3/2 hours = 27 miles from A.
$\boxed{\textbf{(E) 27}}$

~Dilip

Solution 5 (Under 20 seconds)

We know that Alice approaches Beth at 18 mph and Beth approaches Alice at 12 mph. If we consider that if Alice moves 18 miles at the same time Beth moves 12 miles –> 15 miles left. Alice then moves 9 more miles at the same time as Beth moves 6 more miles. Alice has moved 27 miles from point A at the same time that Beth has moved 18 miles from point B, meaning that Alice and Beth meet 27 miles from point A.

~MC_ADe ~SP

Solution 6 (simple linear equations)

We know that Beth starts 45 miles away from City A, let's create two equations: Alice-> $18t = d$ Beth-> $-12t + 45 = d$ [-12 is the slope; 45 is the y-intercept]

Solve the system:

$$18t = -12t + 4530t = 45t = 1.5$$

So, $18(1.5) = \boxed{\textbf{(E) } 27}$

~Education, the Study of Everything

~Math-X

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

Problem_2

The following problem is from both the 2023 AMC 10A #2 and 2023 AMC 12A #2, so both problems redirect to this page.

Problem

The weight of $\frac{1}{3}$ of a large pizza together with $3\frac{1}{2}$ cups of orange slices is the same as the weight of $\frac{3}{4}$ of a large pizza together with $\frac{1}{2}$ cup of orange slices. A cup of orange slices weighs $\frac{1}{4}$ of a pound. What is the weight, in pounds, of a large pizza?

$\textbf{(A) } 1\frac{4}{5} \qquad \textbf{(B) } 2 \qquad \textbf{(C) } 2\frac{2}{5} \qquad \textbf{(D) } 3 \qquad \textbf{(E) } 3\frac{3}{5}$

Solution 1 (Substitution)

Use a system of equations. Let x be the weight of a pizza and y be the weight of a cup of orange slices. We have

$$\frac{1}{3}x + \frac{7}{2}y = \frac{3}{4}x + \frac{1}{2}y.$$

Rearranging, we get

$$\begin{aligned} \frac{5}{12}x &= 3y, \\ x &= \frac{36}{5}y. \end{aligned}$$

Plugging in $\frac{1}{4}$ pounds for y by the given gives $\frac{9}{5} = \boxed{\textbf{(A) } 1\frac{4}{5}}$.

~ItsMeNoobieboy ~walmartbrian

Solution 2

Let: p be the weight of a pizza. o be the weight of a cup of orange.

From the problem, we know that $o = \frac{1}{4}$.

Write the equation below:

$$\frac{1}{3}p + \frac{7}{2} \cdot \frac{1}{4} = \frac{3}{4}p + \frac{1}{2} \cdot \frac{1}{4}$$

Solving for p: $\frac{5}{12}p = \frac{3}{4}$

$p = \frac{9}{5} = \boxed{\textbf{(A) } 1\frac{4}{5}}$.

~d_code

Solution 3

$\frac{P}{3}+\frac{7}{2}R=\frac{3}{4}P+\frac{R}{2}$ where P is the pizza weight and R is the weight of cup of oranges Since oranges weigh $\frac{1}{4}$ pound per cup, the oranges on the LHS weigh $\frac{7}{2}$ cups x $\frac{1}{4}$ pounds/cup = $\frac{7}{8}$ pound, and those on the RHS weigh $\frac{1}{2}$ cup x $\frac{1}{4}$ pounds/cup = $\frac{1}{8}$ pound.

So $\frac{P}{3}$ + $\frac{7}{8}$ pound = $\frac{3}{4}P$ + $\frac{1}{8}$ pound; $\frac{P}{3}$ + $\frac{3}{4}$ pound = $\frac{3}{4}P$.

Multiplying both sides by $\text{lcm}(3,4)=12$, we have $4P+9=9P$; $5P=9$; P = weight of a large pizza = $\frac{9}{5}$ pounds = $\boxed{\textbf{(A)} 1\frac{4}{5}}$ pounds.

~Dilip ~$\LaTeX$ by A_MatheMagician

~Education, the Study of Everything

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

See Also

Problem_3

The following problem is from both the 2023 AMC 10A #3 and 2023 AMC 12A #3, so both problems redirect to this page.

Problem

How many positive perfect squares less than 2023 are divisible by 5?

$\textbf{(A) } 8 \qquad \textbf{(B) } 9 \qquad \textbf{(C) } 10 \qquad \textbf{(D) } 11 \qquad \textbf{(E) } 12$

Solution 1

Note that $40^2=1600$ but $45^2=2025$ (which is over our limit of 2023). Therefore, the list is $5^2, 10^2, 15^2, 20^2, 25^2, 30^2, 35^2, 40^2$. There are 8 elements, so the answer is $\boxed{\textbf{(A) } 8}$.

~zhenghua ~walmartbrian (Minor edits for clarity by Technodoggo)

Solution 2 (slightly refined)

Since $\left\lfloor\sqrt{2023}\right\rfloor = 44$, there are $\left\lfloor\frac{44}{5}\right\rfloor = \boxed{\textbf{(A) } 8}$ perfect squares less than 2023.

~not_slay

Solution 3 (the best)

Since 5 is prime, each solution must be divisible by $5^2=25$. We take $\left\lfloor\frac{2023}{25}\right\rfloor = 80$ and see that there are $\boxed{\textbf{(A) } 8}$ positive perfect squares no greater than 80.

~sohan

Solution 4

We know the highest value would be at least 40 but less than 50 so we check 45, prime factorizing 45. We get $3^2 \cdot 5$. We square this and get $81 \cdot 25$. We know that $80 \cdot 25 = 2000$, then we add 25 and get 2025, which does not satisfy our requirement of having the square less than 2023. The largest multiple of 5 that satisfies this is 40 and the smallest multiple of 5 that works is 5 so all multiples of 5 from 5 to 40 satisfy the requirements. Now we divide each element of the set by 5 and get $1-8$ so there are $\boxed{\textbf{(A) } 8}$ solutions.

~kyogrexu (minor edits by vadava_lx)

Solution 5

If we want to have the square be divisible by 5 we must have it such that it is at least divisible by 25, since every prime in it's prime factorization must have an even power.

So, we must have $0 < 25x^2 < 2023$, and we see the range of x is $1 \leq x \leq 8$.

Therefore, there are $\boxed{\textbf{(A) } 8}$ solutions.

~ESAOPS

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

~Education, the Study of Everything

See Also

Problem_4

Problem

A quadrilateral has all integer sides lengths, a perimeter of 26, and one side of length 4. What is the greatest possible length of one side of this quadrilateral?

$\textbf{(A) }9\qquad\textbf{(B) }10\qquad\textbf{(C) }11\qquad\textbf{(D) }12\qquad\textbf{(E) }13$

Solution 1

Let's use the triangle inequality. We know that for a triangle, the sum of the 2 shorter sides must always be longer than the longest side. This is because if the longest side were to be as long as the sum of the other sides, or longer, we would only have a line.

Similarly, for a convex quadrilateral, the sum of the shortest 3 sides must always be longer than the longest side. Thus, the answer is $\dfrac{26}{2}-1=13-1=\boxed{\textbf{(D) }12}$

~zhenghua

Solution 2

Say the chosen side is a and the other sides are b, c, d.

By the Generalised Polygon Inequality, $a < b+c+d$. We also have $a+b+c+d=26 \Rightarrow b+c+d=26-a$.

Combining these two, we get $a<26-a \Rightarrow a<13$.

The smallest length that satisfies this is $a=\boxed{\textbf{(D) }12}$

~not_slay

Solution 3 (Fast)

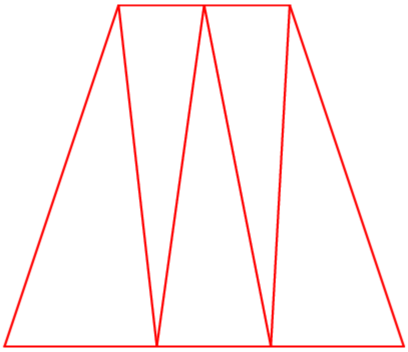
By Brahmagupta's Formula, the area of the quadrilateral is defined by $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where s is the semi-perimeter. If the perimeter of the quadrilateral is 26, then the semi-perimeter will be 13. The area of the quadrilateral must be positive so the difference between the semi-perimeter and a side length must be greater than 0 as otherwise, the area will be 0 or negative. Therefore, the longest a side can be in this quadrilateral is $\boxed{\textbf{(D) }12}$

~

(Also, why is the quadrilateral cyclic? Brahmagupta's Formula only applies to cyclic quadrilaterals.) ~ Technodoggo Quadrilateral is not necessarily cyclic - sepehr2010

Solution 4

This is an AMC 10 problem 4, so there is no need for any complex formulas. The largest singular side length from a quadrilateral comes from a trapezoid. So we can set the 2 sides of the trapezoid equal to 4. Next we can split the trapezoid into 5 triangles, where each base length of the triangle equals 4. So the top side equals 8, and the bottom side length equals $4+4+4=$ $\boxed{\textbf{(D) }12}$ ~ kabbybear



~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

See Also

Problem_5

The following problem is from both the 2023 AMC 10A #5 and 2023 AMC 12A #4, so both problems redirect to this page.

Problem

How many digits are in the base-ten representation of $8^5 \cdot 5^{10} \cdot 15^5$?

$\textbf{(A) } 14 \qquad \textbf{(B) } 15 \qquad \textbf{(C) } 16 \qquad \textbf{(D) } 17 \qquad \textbf{(E) } 18$

Solution 1

Prime factorizing this gives us $2^{15} \cdot 3^5 \cdot 5^{15} = 10^{15} \cdot 3^5 = 243 \cdot 10^{15}$.

10^{15} gives us 15 digits and 243 gives us 3 digits. $15 + 3 = \boxed{\textbf{(E) } 18}$

~zhenghua

Solution 2 (Bash)

Multiplying it out, we get that $8^5 \cdot 5^{10} \cdot 15^5 = 243000000000000000$. Counting, we have the answer is $\boxed{\textbf{(E) } 18}$
~andliu766

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

~Education, the Study of Everything

See Also

Problem_6

Problem

An integer is assigned to each vertex of a cube. The value of an edge is defined to be the sum of the values of the two vertices it touches, and the value of a face is defined to be the sum of the values of the four edges surrounding it. The value of the cube is defined as the sum of the values of its six faces. Suppose the sum of the integers assigned to the vertices is 21. What is the value of the cube? $\textbf{(A) }42 \qquad \textbf{(B) }63 \qquad \textbf{(C) }84 \qquad \textbf{(D) }126 \qquad \textbf{(E) }252$

Solution 1

Each of the vertices is counted 3 times because each vertex is shared by three different edges. Each of the edges is counted 2 times because each edge is shared by two different faces. Since the sum of the integers assigned to all vertices is 21, the final answer is $21 \times 3 \times 2 = \boxed{\textbf{(D) }126}$

~Mintylemon66

Solution 2

Note that each vertex is counted $2 \times 3 = 6$ times. Thus, the answer is $21 \times 6 = \boxed{\textbf{(D) }126}$.

~Mathkiddie

Solution 3

Just set one vertice equal to 21, it is trivial to see that there are 3 faces with value 42, and $42 \cdot 3 = \boxed{\textbf{(D) }126}$.

~SirAppel

Solution 4

Since there are 8 vertices in a cube, there are $\frac{21}{4}$ vertices for two edges. There are 4 edges per face, and 6 faces in a cube, so the value of the cube is $\frac{21}{4} \cdot 24 = \boxed{\textbf{(D) }126}$.

~DRBStudent ~Failure.net

(Minor formatting by Technodoggo)

Solution 5 (use an example)

Set each vertex to value 1, so the sum of the vertices is 8. We find that the value of the cube, if all vertices are 1, is 48. We conclude that the value of the cube is 6 times the value of the sum of the vertices. Therefore, we choose $21 \times 6 = \boxed{\textbf{(D) }126}$

~milquetoast

Solution 6

The wording of the problem implies that the answer should hold for any valid combination of integers. Thus, we choose the numbers $21, 0, 0, 0, 0, 0, 0, 0$, which are indeed 8 integers that add to 21. Doing this, we find three edges that have a value of 21, and from there, we get three faces with a value of 42 (while the other three faces have a value of 0). Adding the three faces together, we get $42 + 42 + 42 = \boxed{\textbf{(D) }126}$.

~MathHafiz

~Math-X

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

See Also

Problem_7

The following problem is from both the 2023 AMC 10A #7 and 2023 AMC 12A #5, so both problems redirect to this page.

Problem

Janet rolls a standard 6-sided die 4 times and keeps a running total of the numbers she rolls. What is the probability that at some point, her running total will equal 3?

$\textbf{(A) }\frac{2}{9}\qquad\textbf{(B) }\frac{49}{216}\qquad\textbf{(C) }\frac{25}{108}\qquad\textbf{(D) }\frac{17}{72}\qquad\textbf{(E) }\frac{13}{54}$

Solution 1 (Casework)

There are 3 cases where the running total will equal 3; one roll; two rolls; or three rolls:

Case 1: The chance of rolling a running total of 3 in exactly one roll is $\frac{1}{6}$.

Case 2: The chance of rolling a running total of 3 in exactly two rolls is $\frac{1}{6} \cdot \frac{1}{6} \cdot 2 = \frac{1}{18}$ since the dice rolls consist of a singular 2 and a singular 1 and vice versa.

Case 3: The chance of rolling a running total of 3 in exactly three rolls is $\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$ since the dice rolls would consist of 3 ones.

Using the rule of sum we have $\frac{1}{6} + \frac{1}{18} + \frac{1}{216} = \boxed{\textbf{(B) }\frac{49}{216}}$.

~walmartbrian ~andyluo ~DRBStudent ~MC_ADe

Solution 2 (Brute Force)

Because there is only a maximum of 3 rolls we must count (running total = 3 means there can't be a fourth roll counted), we can simply list out all of the probabilities.

If we roll a 1 on the first, the rolls that follow must be 2 or {1,1}, with the following results not mattering. This leaves a probability of $\frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{7}{216}$.

If we roll a 2 on the first, the roll that follows must be 1, resulting in a probability of $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

If we roll a 3 on the first, the following rolls do not matter, resulting in a probability of $\frac{1}{6}$. Any roll greater than three will result in a running total greater than 3 no matter what, so those cases can be ignored. Summing the answers, we have $\frac{7}{216} + \frac{1}{36} + \frac{1}{6} = \frac{7+6+36}{216} = \boxed{\textbf{(B) }\frac{49}{216}}$.

~Failure.net

Solution 3

Consider sequences of 4 integers with each integer between 1 and 6, the number of permutations of 6 numbers is $6^4 = 1296$.

The following 4 types of sequences that might generate a running total of the numbers to be equal to 3 (x, y, or z denotes any integer between 1 and 6).

Sequence #1, (1, 1, 1, x): there are 6 possible sequences.

Sequence #2, (1, 2, x, y): there are $6^2 = 36$ possible sequences.

Sequence #3, (2, 1, x, y): there are $6^2 = 36$ possible sequences.

Sequence #4, (3, x, y, z): there are $6^3 = 216$ possible sequences.

Out of 1296 possible sequences, there are a total of $6 + 36 + 36 + 216 = 294$ sequences that qualify. Hence, the probability is $294/1296 = \boxed{\textbf{(B) } \frac{49}{216}}$

~sqroot

~Math-X

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

~Education, the Study of Everything

See Also

Problem_8

Problem

Barb the baker has developed a new temperature scale for her bakery called the Breadus scale, which is a linear function of the Fahrenheit scale. Bread rises at 110 degrees Fahrenheit, which is 0 degrees on the Breadus scale. Bread is baked at 350 degrees Fahrenheit, which is 100 degrees on the Breadus scale. Bread is done when its internal temperature is 200 degrees Fahrenheit. What is this in degrees on the Breadus scale?

$\textbf{(A) }33\qquad\textbf{(B) }34.5\qquad\textbf{(C) }36\qquad\textbf{(D) }37.5\qquad\textbf{(E) }39$

Solution 1 (Substitution)

To solve this question, you can use $y=mx+b$ where the x is Fahrenheit and the y is Breadus. We have $(110,0)$ and $(350,100)$. We want to find the value of y in $(200,y)$ that falls on this line. The slope for these two points is $\frac{5}{12}$; $y=\frac{5}{12}x+b$. Solving for b using $(110,0)$, $\frac{550}{12}=-b$. We get $b=\frac{-275}{6}$. Plugging in $(200,y)$, $\frac{1000}{12}-\frac{550}{12}=y$. Simplifying, $\frac{450}{12}=\boxed{\textbf{(D) }37.5}$

~walmartbrian

Solution 2 (Faster)

Let $^\circ B$ denote degrees Breadus. We notice that $200^\circ F$ is $90^\circ F$ degrees to $0^\circ B$, and $150^\circ F$ to $100^\circ B$. This ratio is $90:150=3:5$; therefore, $200^\circ F$ will be $\frac{3}{3+5}=\frac{3}{8}$ of the way from 0 to 100, which is $\boxed{\textbf{(D) }37.5.}$

~Technodoggo

Solution 3 (Intuitive)

From 110 to 350 degrees Fahrenheit, the Breadus scale goes from 1 to 100. 110 to 350 degrees is a span of 240, and we can use this to determine how many Fahrenheit each Breadus unit is worth. 240 divided by 100 is 2.4, so each Breadus unit is 2.4 Fahrenheit, starting at 110 Fahrenheit. For example, 1 degree on the Breadus scale is $110+2.4$, or 112.4 Fahrenheit. Using this information, we can figure out how many Breadus degrees 200 Fahrenheit is. $200-110$ is 90, so we divide 90 by 2.4 to find the answer, which is $\boxed{\textbf{(D) }37.5}$

~MercilessAnimations

Solution 4

We note that the range of F temperatures that $0-100\text{ Br}^\circ$ represents is $350-110=240\text{ F}^\circ$. 200 F° is $(200-110)=90\text{ F}^\circ$ along the way to getting to 240 F°, the end of this range, or $90/240=9/24=3/8=0.375$ of the way. Therefore if we switch to the Br scale, we are 0.375 of the way to 100 from 0, or at $\boxed{\textbf{(D) }37.5}\text{ Br}^\circ$.

~Dilip -missmango ~Minor edits by FutureSphinx

Solution 5

We have the points $(0, 110)$ and $(100, 350)$. We want to find $(x, 200)$. The equation of the line is $y = \frac{12}{5}x + 110$. We use this to find $x = \frac{75}{2} = 37.5$, or $\boxed{D}$. ~MC413551

~Math-X

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

See Also

Problem_9

The following problem is from both the 2023 AMC 10A #9 and 2023 AMC 12A #7, so both problems redirect to this page.

Problem

A digital display shows the current date as an 8-digit integer consisting of a 4-digit year, followed by a 2-digit month, followed by a 2-digit date within the month. For example, Arbor Day this year is displayed as 20230428. For how many dates in 2023 will each digit appear an even number of times in the 8-digital display for that date?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Solution 1 (Casework)

Do careful casework by each month. In the month and the date, we need a 0, a 3, and two digits repeated (which has to be 1 and 2 after consideration). After the casework, we get $\boxed{\textbf{(E)}\ 9}$. For curious readers, the numbers (in chronological order) are: 20230113, 20230131, 20230223, 20230311, 20230322, 20231013, 20231031, 20231103, 20231130.

Solution 2

There is one 3, so we need one more (three more means that either the month or units digit of the day is 3). For the same reason, we need one more 0.

If 3 is the units digit of the month, then the 0 can be in either of the three remaining slots. For the first case (tens digit of the month), then the last two digits must match ($11, 22$). For the second (tens digit of the day), we must have the other two be 1, as a month can't start with 2 or 0. There are 3 successes this way.

If 3 is the tens digit of the day, then 0 can be either the tens digit of the month or the units digit of the day. For the first case, 1 must go in the other slots. For the second, the other two slots must be 1 as well. There are 2 successes here.

If 3 is the units digit of the day, then 0 could go in any of the 3 remaining slots again. If it's the tens digit of the day, then the other digits must be 1. If 0 is the units digit of the day, then the other two slots must both be 1. If 0 is the tens digit of the month, then the other two slots can be either both 1 or both 2. In total, there are 4 successes here.

Summing through all cases, there are $3+2+4=\boxed{\textbf{(E)}\ 9}$ dates.

-Benedict T (countmath1)

Solution 3

We start with $2023----$ we need an extra 0 and an extra 3. So we have at least one of those extras in the days, except we can have the month 03. We now have 6 possible months $01, 02, 03, 10, 11, 12$. For month 1 we have two cases, we now have to add in another 1, and the possible days are $13, 31$. For month 2 we need an extra 2 so we can have the day 23 note that we can't use 32 because it is to large. Now for month 3 we can have any number and multiply it by 11 so we have the solution $11, 22$. For October we need a 1 and a 3 so we have $13, 31$ as our choices. For November we have two choices which are $03, 30$.Now for December we have 0 options. Summing $2+1+2+2+2$ we get $\boxed{\textbf{(E)}\ 9}$ solutions.

~kyogrexu

~Math-X

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

See Also

Problem_10

The following problem is from both the 2023 AMC 10A #10 and 2023 AMC 12A #8, so both problems redirect to this page.

Problem

Maureen is keeping track of the mean of her quiz scores this semester. If Maureen scores an 11 on the next quiz, her mean will increase by 1. If she scores an 11 on each of the next three quizzes, her mean will increase by 2. What is the mean of her quiz scores currently? $\textbf{(A) }4\qquad\textbf{(B) }5\qquad\textbf{(C) }6\qquad\textbf{(D) }7\qquad\textbf{(E) }8$

Solution 1

Let a represent the amount of tests taken previously and x the mean of the scores taken previously.

We can write the following equations:

$$\frac{ax+11}{a+1}=x+1\qquad (1)$$

$$\frac{ax+33}{a+3}=x+2\qquad (2)$$

Multiplying (1) by $(a+1)$ and solving, we get:

$$ax+11=ax+a+x+1$$

$$11=a+x+1$$

$$a+x=10\qquad (3)$$

Multiplying (2) by $(a+3)$ and solving, we get:

$$ax+33=ax+2a+3x+6$$

$$33=2a+3x+6$$

$$2a+3x=27\qquad (4)$$

Solving the system of equations for (3) and (4), we find that $a=3$ and $x=\boxed{\textbf{(D) }7}$.

~walmartbrian ~Shontai ~andyluo ~megaboy6679

Solution 2 (Variation on Solution 1)

Suppose Maureen took n tests with an average of m.

If she takes another test, her new average is $\dfrac{(nm+11)}{(n+1)}=m+1$

Cross-multiplying: $nm+11=nm+n+m+1$, so $n+m=10$.

If she takes 3 more tests, her new average is $\dfrac{(nm+33)}{(n+3)}=m+2$

Cross-multiplying: $nm+33=nm+2n+3m+6$, so $2n+3m=27$.

But $2n+3m$ can also be written as $2(n+m)+m=20+m$. Therefore $m=27-20=\boxed{\textbf{(D) } 7}$

~Dilip ~megaboy6679 (latex)

Solution 3

Let s represent the sum of Maureen's test scores previously and t be the number of scores taken previously.

So, $\frac{s+11}{t+1}=\frac{s}{t}+1$ and $\frac{s+33}{t+3}=\frac{s}{t}+2$

We can use the first equation to write s in terms of t.

We then substitute this into the second equation: $\frac{-t^2+10t+33}{t+3}=\frac{-t^2+10}{t}+2$

From here, we solve for t, getting $t=3$.

We substitute this to get $s=21$.

Therefore, the solution to the problem is $\frac{21}{3}=\boxed{\textbf{(D) } 7}$

~milquetoast

Solution 4 (Trial and Error)

Let's consider all the answer choices. If the average is 8, then, we can assume that all her test choices were 8. We can see that she must have gotten 8 twice, in order for another score of 11 to bring her average up by one. However, adding three 11's will not bring her score up to 10. Continuing this process for the answer choices, we see that the answer is $\boxed{\textbf{(D) } 7}$ ~andliu766

~Math-X

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

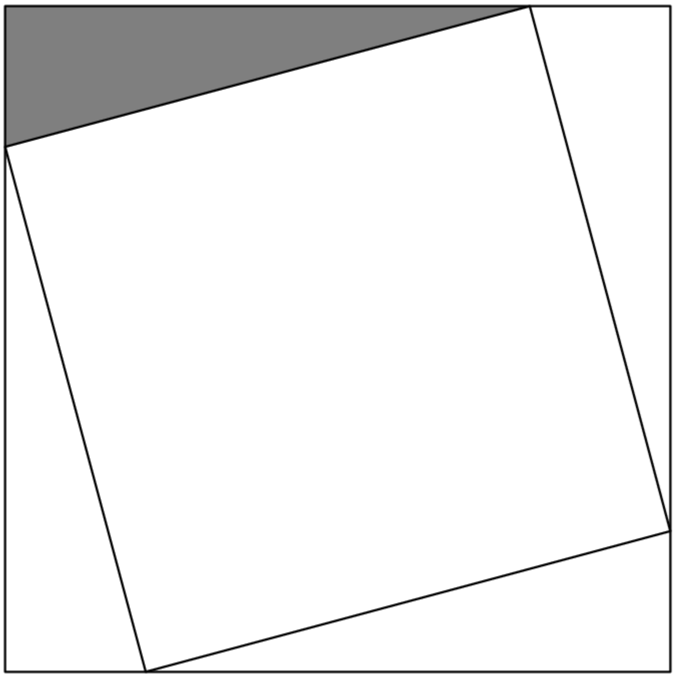
See Also

Problem_11

The following problem is from both the 2023 AMC 10A #11 and 2023 AMC 12A #9, so both problems redirect to this page.

Problem

A square of area 2 is inscribed in a square of area 3, creating four congruent triangles, as shown below. What is the ratio of the shorter leg to the longer leg in the shaded right triangle?



$\textbf{(A) }\frac{1}{5}\qquad\textbf{(B) }\frac{1}{4}\qquad\textbf{(C) }2-\sqrt3\qquad\textbf{(D) }\sqrt3-\sqrt2\qquad\textbf{(E) }\sqrt2-1$

Solution

Note that each side length is $\sqrt{2}$ and $\sqrt{3}.$ Let the shorter side of our triangle be x, thus the longer leg is $\sqrt{3}-x$. Hence, by the Pythagorean Theorem, we have

$$(\sqrt3-x)^2+x^2=2$$

$$2x^2-2x\sqrt3+1=0$$

.

By the quadratic formula, we find $x=\frac{\sqrt3\pm1}{2}$. Hence, our answer is $\frac{\sqrt3-1}{\sqrt3+1}=\boxed{\textbf{(C) }2-\sqrt3}$.

~SirAppel ~ItsMeNoobieboy

Solution 2 (Area Variation of Solution 1)

Looking at the diagram, we know that the square inscribed in the square with area 3 has area 2. Thus, the area outside of the small square is $3-2=1.$ This area is composed of 4 congruent triangles, so we know that each triangle has an area of $\frac{1}{4}$.

From solution 1, the base has length x and the height $\sqrt{3}-x$, which means that $\frac{x\left(\sqrt{3}-x\right)}{2}=\frac{1}{4}$.

We can turn this into a quadratic equation: $x^2 - x\sqrt{3} + \frac{1}{2} = 0$.

By using the quadratic formula, we get $x = \frac{\sqrt{3}\pm 1}{2}$.Therefore, the answer is $\frac{\sqrt{3}-1}{\sqrt{3}+1} = \boxed{\textbf{(C) } 2-\sqrt{3}}$.

~ghfhgvghj10 (If I made any mistakes, feel free to make minor edits)

(Clarity & formatting edits by Technodoggo)

Solution 3

Let x be the ratio of the shorter leg to the longer leg, and y be the length of longer leg. The length of the shorter leg will be xy.

Because the sum of two legs is the side length of the outside square, we have $xy + y = \sqrt{3}$, which means $(xy)^2 + y^2 + 2xy^2 = 3$. Using the Pythagorean Theorem for the shaded right triangle, we also have $(xy)^2 + y^2 = 2$. Solving both equations, we get $2xy^2 = 1$. Using $y^2 = \frac{1}{2x}$ to substitute y in the second equation, we get $x^2 \cdot \frac{1}{2x} + \frac{1}{2x} = 2$. Hence, $x^2 - 4x + 1 = 0$. By using the quadratic formula, we get $x = 2 \pm \sqrt{3}$. Because x be the ratio of the shorter leg to the longer leg, it is always less than 1. Therefore, the answer is $\boxed{\textbf{(C) } 2-\sqrt{3}}$.

~sqroot

Solution 4

The side length of the bigger square is equal to $\sqrt{3}$, while the side length of the smaller square is $\sqrt{2}$. Let x be the shorter leg and y be the longer one. Clearly, $x + y = \sqrt{3}$, and $xy = \frac{1}{2}$. Using Vieta's to build a quadratic, we get

$$x^2 - \sqrt{3}x + \frac{1}{2} = 0.$$

Solving, we get $x = \frac{\sqrt{3}-1}{2}$ and $y = \frac{\sqrt{3}+1}{2}$. Thus, we find $\frac{x}{y} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)\cdot(\sqrt{3}-1)} = \frac{4-2\sqrt{3}}{2} = \boxed{\textbf{(C) } 2-\sqrt{3}}$.

~vadava_lx

Solution 5

Let θ be the angle opposite the smaller leg. We want to find $\tan\theta$.

The area of the triangle is $\frac{1}{2}\left(\sqrt{2}\sin\theta\right)\left(\sqrt{2}\cos\theta\right) = \frac{1}{2}\sin 2\theta = \frac{1}{4}$, which implies $\sin 2\theta = \frac{1}{2}$, or $\theta = 15^\circ$. Therefore $\tan\theta = \boxed{\textbf{(C) } 2-\sqrt{3}}$

Solution 6

Allow a, b to be the sides of a triangle. WLOG, suppose $a > b$We want to find $\frac{b}{a}$. Notice that the area of a triangle is $\frac{3-2}{4}$, which results in $\frac{1}{4}$. Thus, $ab = \frac{1}{2}$. However, the square of the hypotenuse of this triangle is $a^2 + b^2$, but also 2. We can write b as $\frac{1}{2a}$, and then plug it in. We get $a^2 + \frac{1}{4a^2} = 2$, so $4a^4 - 8a^2 + 1 = 0$. Applying the quadratic formula,

$a^2 = \dfrac{8 \pm 4\sqrt{3}}{2}$, or $4 \pm 2\sqrt{3}$. However, since a and b must both be solutions of the quadratic, since both equations were cyclic. Since $a > b$, then $a^2 = 4 + 2\sqrt{3}$, and $b^2 = 4 - 2\sqrt{3}$. To find $\dfrac{b}{a}$, we can simply find the square root of $\dfrac{b^2}{a^2}$. This is $\sqrt{\dfrac{4-2\sqrt{3}}{4+2\sqrt{3}}} = \sqrt{\dfrac{2-\sqrt{3}}{2+\sqrt{3}}} = \sqrt{(2-\sqrt{3})(2-\sqrt{3})} = \boxed{2-\sqrt{3}}$, so the answer is $\boxed{C}$. - Sepehr2010

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

~Math-X

See Also

Problem_12

Problem

How many three-digit positive integers N satisfy the following properties?

- The number N is divisible by 7.
- The number formed by reversing the digits of N is divisible by 5.

$\textbf{(A) } 13 \qquad \textbf{(B) } 14 \qquad \textbf{(C) } 15 \qquad \textbf{(D) } 16 \qquad \textbf{(E) } 17$

Note

One thing to note is the number 560. When it is flipped, the result is 065, which is a number but has a leading zero. Since the problem doesn't say anything about 560, it is assumed to be a valid N. HamstPan38825 provides a good explanation on why this problem is wrong, "Define f to be the digit-reversal function in question, and suppose for the sake of contradiction that $f(560)$ is a strictly defined number, hence $f(560) = 065 \equiv 65$, as was assumed when 560 was included in the count. Thus 65 and 065 are equivalent under input to f too, so

$$56 = f(65) = f(065) = 560$$

which is a contradiction as f is a function. Hence $f(560) = 065$ is not a strictly-defined number, and it cannot be divisible by 5."

~A_MatheMagician ~ESAOPS

Solution 1

Multiples of 5 will always end in 0 or 5, and since the numbers have to be a three-digit numbers (otherwise it would be a two-digit number), it cannot start with 0, narrowing our choices to 3-digit numbers starting with 5. Since the numbers must be divisible by 7, all possibilities have to be in the range from $7 \cdot 72$ to $7 \cdot 85$ inclusive.

$85 - 72 + 1 = 14. \boxed{\textbf{(B) } 14}.$

~walmartbrian ~Shontai ~andliu766 ~andyluo ~ESAOPS

Solution 2 (solution 1 but more thorough)

Let $N = \overline{cab} = 100c + 10a + b.$ We know that $\overline{bac}$ is divisible by 5, so c is either 0 or 5. However, since c is the first digit of the three-digit number N, it can not be 0, so therefore, $c = 5$. Thus, $N = \overline{5ab} = 500 + 10a + b.$ There are no further restrictions on digits a and b aside from N being divisible by 7.

The smallest possible N is 504. The next smallest N is 511, then 518, and so on, all the way up to 595. Thus, our set of possible N is $\{504, 511, 518, \dots, 595\}$. Dividing by 7 for each of the terms will not affect the cardinality of this set, so we do so and get $\{72, 73, 74, \dots, 85\}$. We subtract 71 from each of the terms, again leaving the cardinality unchanged. We end up with $\{1, 2, 3, \cdots, 14\}$, which has a cardinality of 14. Therefore, our answer is $\boxed{\textbf{(B) } 14.}$

~ Technodoggo

Solution 3 (modular arithmetic)

We first proceed as in the above solution, up to $N = 500 + 10a + b$. We then use modular arithmetic:

$$\begin{aligned} 0 &\equiv N \pmod 7 \\ &\equiv 500 + 10a + b \pmod 7 \\ &\equiv 3 + 3a + b \pmod 7 \\ 3a + b &\equiv -3 \pmod 7 \\ &\equiv 4 \pmod 7 \end{aligned}$$

We know that $0 \leq a, b < 10$. We then look at each possible value of a:

If $a = 0$, then b must be 4.

If $a = 1$, then b must be 1 or 8.

If $a = 2$, then b must be 5.

If $a = 3$, then b must be 2 or 9.

If $a = 4$, then b must be 6.

If $a = 5$, then b must be 3.

If $a = 6$, then b must be 0 or 7.

If $a = 7$, then b must be 4.

If $a = 8$, then b must be 1 or 8.

If $a = 9$, then b must be 5.

Each of these cases are unique, so there are a total of $1 + 2 + 1 + 2 + 1 + 1 + 2 + 1 + 2 + 1 = \boxed{\textbf{(B) } 14.}$

~ Technodoggo

Solution 4

The key point is that when reversed, the number must start with a 0 or a 5 based on the second restriction. But numbers can't start with a 0.

So the problem is simply counting the number of multiples of 7 in the 500s.

$7 \times 72 = 504$, so the first multiple is 7×72.

$7 \times 85 = 595$, so the last multiple is 7×85.

Now, we just have to count $7 \times 72, 7 \times 73, 7 \times 74, \cdots, 7 \times 85$.

We have a set that numbers $85 - 71 = \boxed{\textbf{(B) 14}}$

~Dilip ~boppitybop ~ESAOPS (LaTeX)

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

-paixiao

~Math-X

See Also

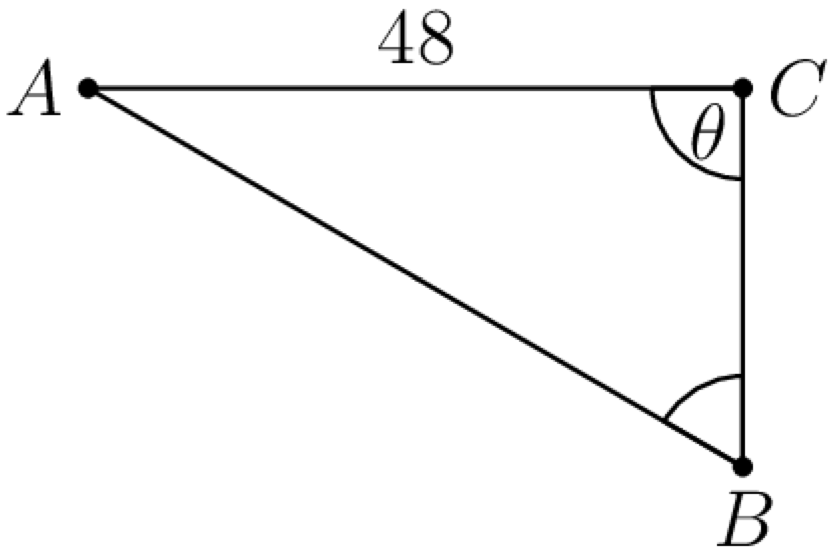
Problem_13

Problem

Abdul and Chiang are standing 48 feet apart in a field. Bharat is standing in the same field as far from Abdul as possible so that the angle formed by his lines of sight to Abdul and Chiang measures 60°. What is the square of the distance (in feet) between Abdul and Bharat?

$\textbf{(A) }1728\qquad\textbf{(B) }2601\qquad\textbf{(C) }3072\qquad\textbf{(D) }4608\qquad\textbf{(E) }6912$

Solution 1



Let $\theta = \angle ACB$ and $x = \overline{AB}$.

By the Law of Sines, we know that $\frac{\sin\theta}{x} = \frac{\sin 60^\circ}{48} = \frac{\sqrt{3}}{96}$. Rearranging, we get that $x = \frac{\sin\theta}{\frac{\sqrt{3}}{96}} = 32\sqrt{3}\sin\theta$ where x is a function of θ. We want to maximize x.

We know that the maximum value of $\sin\theta = 1$, so this yields $x = 32\sqrt{3} \implies x^2 = \boxed{\textbf{(C) }3072.}$

A quick check verifies that $\theta = 90^\circ$ indeed works.

~Technodoggo ~(minor grammar edits by vadava_lx)

Solution 2 (no law of sines)

Let us begin by circumscribing the two points A and C so that the arc it determines has measure 120. Then the point B will lie on the circle, which we can quickly find the radius of by using the 30-60-90 triangle formed by the radius and the midpoint of segment $\overline{AC}$. We will find that $r = 16 \times \sqrt{3}$. Due to the triangle inequality, $\overline{AB}$ is maximized when B is on the diameter passing through A, giving a length of $32 \times \sqrt{3}$ and when squared gives $\boxed{\textbf{(C) }3072}$.

Solution 3

It is quite clear that this is just a 30-60-90 triangle as an equilateral triangle gives an answer of $48^2 = 2304$, which is not on the answer choices. Its ratio is $\frac{48}{\sqrt{3}}$, so $\overline{AB} = \frac{96}{\sqrt{3}}$.

Its square is then $\frac{96^2}{3} = \boxed{\textbf{(C) }3072}$

~not_slay

~wangzrpi

Solution 4

We use A, B, C to refer to Abdul, Bharat and Chiang, respectively. We draw a circle that passes through A and C and has the central angle $\angle AOC = 60^\circ \cdot 2$. Thus, B is on this circle. Thus, the longest distance between A and B is the diameter of this circle. Following from the law of sines, the square of this diameter is

$$\left(\frac{48}{\sin 60^\circ} \right)^2 = \boxed{\textbf{(C) } 3072}.$$

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

Solution 5 (Straightforward)

We can represent Abdul, Bharat and Chiang as A, B, and C, respectively. Since we have $\angle ABC = 60^\circ$ and $\angle BCA = 90^\circ$, this is obviously a $30-60-90$ triangle, and it would not matter where B is. By the side ratios of a $30-60-90$ triangle, we can infer that $AB = \dfrac{48 \times 2}{\sqrt{3}}$. Squaring AB we get $\boxed{\textbf{(C) } 3072}$.

~ESAOPS

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

```
~Math-X
```

See Also

Problem_14

A number is chosen at random from among the first 100 positive integers, and a positive integer divisor of that number is then chosen at random. What is the probability that the chosen divisor is divisible by 11?

$\textbf{(A) } \frac{4}{100} \qquad \textbf{(B) } \frac{9}{200} \qquad \textbf{(C) } \frac{1}{20} \qquad \textbf{(D) } \frac{11}{200} \qquad \textbf{(E) } \frac{3}{50}$

Solution 1

In order for the divisor chosen to be a multiple of 11, the original number chosen must also be a multiple of 11. Among the first 100 positive integers, there are 9 multiples of 11; 11, 22, 33, 44, 55, 66, 77, 88, 99. We can now perform a little casework on the probability of choosing a divisor which is a multiple of 11 for each of these 9, and see that the probability is 1/2 for each. The probability of choosing these 9 multiples in the first place is $\frac{9}{100}$, so the final probability is $\frac{9}{100} \cdot \frac{1}{2} = \frac{9}{200}$, so the answer is $\boxed{\textbf{(B) } \frac{9}{200}}$.

$$11 = 11 - 1/2$$
$$22 = 2 * 11 : 11, 22 - 1/2$$
$$33 = 3 * 11 : 11, 33 - 1/2$$
$$44 = 2^2 * 11 : 11, 22, 44 - 1/2$$
$$55 = 5 * 11 : 11, 55 - 1/2$$
$$66 = 2 * 3 * 11 : 11, 22, 33, 66 - 1/2$$
$$77 = 7 * 11 : 11, 77 - 1/2$$
$$88 = 2^3 * 11 : 11, 22, 44, 88 - 1/2$$
$$99 = 3^2 * 11 : 11, 33, 99 - 1/2$$

~vaisri ~walmartbrian ~Shontai

Solution 2

As stated in Solution 1, the 9 multiples of 11 under 100 are $11, 22, 33, 44, 55, 66, 77, 88, 99$. Because all of these numbers are multiples of 11 to the first power and first power only, their factors can either have 11 as a factor (11^1) or not have 11 as a factor (11^0), resulting in a $\frac{1}{2}$ chance of a factor chosen being divisible by 11. The chance of choosing any factor of 11 under 100 is $\frac{9}{100}$, so the final answer is $\frac{9}{100} \cdot \frac{1}{2} = \boxed{\textbf{(B) } \frac{9}{200}}$.

~Failure.net

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

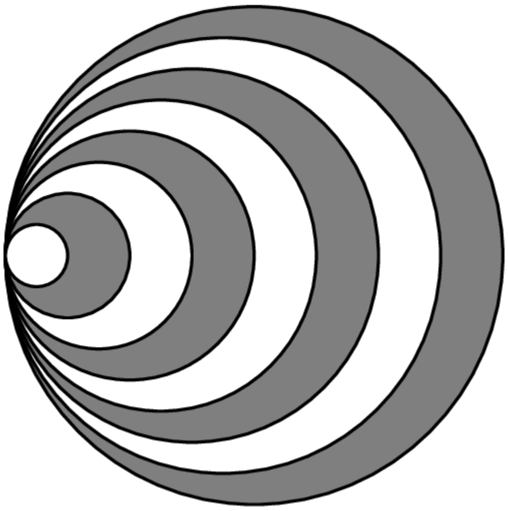
```
~Math-X
```

See Also

Problem_15

Problem

An even number of circles are nested, starting with a radius of 1 and increasing by 1 each time, all sharing a common point. The region between every other circle is shaded, starting with the region inside the circle of radius 2 but outside the circle of radius $1.$ An example showing 8 circles is displayed below. What is the least number of circles needed to make the total shaded area at least 2023π?



$\textbf{(A) } 46 \qquad \textbf{(B) } 48 \qquad \textbf{(C) } 56 \qquad \textbf{(D) } 60 \qquad \textbf{(E) } 64$

Solution 1

Notice that the area of the shaded region is $(2^2\pi-1^2\pi)+(4^2\pi-3^2\pi)+(6^2\pi-5^2\pi)+ \cdots + (n^2\pi-(n-1)^2\pi)$ for any even number n.

Using the difference of squares, this simplifies to $(1+2+3+4+\cdots+n)\pi$. So, we are basically finding the smallest n such that $\frac{n(n+1)}{2}>2023 \Rightarrow n(n+1)>4046$. Since $60(61)>60^2=3600$, the only option higher than 60 is $\boxed{\textbf{(E) } 64}$.

~MrThinker

Solution 2

After first observing the problem, we can work out a few of the areas.

1st area = $4\pi - \pi = 3\pi$

2nd area = $16\pi - 9\pi = 7\pi$

3rd area = $36\pi - 25\pi = 11\pi$

4th area = $64\pi - 49\pi = 15\pi$

We can see that the pattern is an arithmetic sequence with first term 3 and common difference 4. From here, we can start from the bottom of the answer choices and work our way up. As the question asks for the least number of circles needed total, we have to divide the number of total circles by 2.

We can find the sum of the first 32 terms of the arithmetic sequence by using the formula.

The last term is: $3 + 4 \cdot (32 - 1) = 127$.

Then, we can find the sum: $(3 + 127)/2 \cdot 32 = 65 \cdot 32 = 2080$. It is clear that 64 works.

The next answer choice is 60, which we have to divide by 2 to get 30.

The last term is: $3 + 4 \cdot (30 - 1) = 119$.

The sum is: $(3 + 119)/2 \cdot 30 = 61 \cdot 30 = 1830$. This does not work.

As answer choice D does not work and E does, we can conclude that the answer is $\boxed{\textbf{(E) } 64}$.

~zgahzlkw

Solution 3 (Similar to Solution 2)

We can easily see that all of the answer choices are even, which helps us solve this problem a little.

Lets just not consider the π, since it is not that important, so let's just cancel that out.

When we plug in 64, we get $64^2 - 63^2 + 62^2 - 61^2 + \cdots + 4^2 - 3^2 + 2^2 - 1^2$. By difference of squares, we get $1 + 2 + 3 + \cdots + 62 + 63 + 64$, which by the sum of an arithmetic sequence, is $\frac{64(64+1)}{2}$, which is 2080.

Similarly, we can use this for answer choice D, and we have $\frac{60(60+1)}{2}$ which is 1830.

So, we see that answer choice D is too small to satisfy the requirements, so we conclude the answer is $\boxed{\textbf{(E) } 64}$.

~ESAOPS

Solution 4

We can consider making a table.

If there is 1 circle, the area of the shaded region is 0π. (We can write this as 0π.)

If there are 2 circles, the area of the shaded region is 3π. (We can write this as (1+2)π).

If there are 3 circles, the area of the shaded region is 3π. (We can write this as (1+2)π).

If there are 4 circles, the area of the shaded region is 10π. (We can write this as (1+2+3+4)π).

If there are 5 circles, the area of the shaded region is 10π. (We can write this as (1+2+3+4)π).

If there are 6 circles, the area of the shaded region is 21π. (We can write this as (1+2+3+4+5+6)π).

Now the pattern emerges. When n is even, the area of the shaded region is $(1 + 2 + 3 + \cdots + n)\pi$, or $\left(\frac{n(n+1)}{2}\right)\pi$. But remember that the problem stated that there are an even number of circles. So now we are solving the equation

$$\left(\frac{n(n+1)}{2}\right)\pi \geq 2023\pi.$$

Dividing by π and multiplying by 2 on both sides, we get $n(n+1) \geq 4046$. Now we can plug in the answer choices, and we start off with 60 because it is the only answer choice that is a multiple of 10. Plugging in we get $60 \cdot 61 = 3660$, and this is not quite yet more than 4046. But only option (E) is bigger, so we know that the solution can only be $\boxed{\textbf{(E) } 64}$.

~

(P.S. I did this in the test and I solved it in 2 minutes)

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

~Math-X

See Also

Problem_16

The following problem is from both the 2023 AMC 10A #16 and 2023 AMC 12A #13, so both problems redirect to this page.

Problem

In a table tennis tournament every participant played every other participant exactly once. Although there were twice as many right-handed players as left-handed players, the number of games won by left-handed players was 40% more than the number of games won by right-handed players. (There were no ties and no ambidextrous players.) What is the total number of games played?

$\textbf{(A) }15\qquad\textbf{(B) }36\qquad\textbf{(C) }45\qquad\textbf{(D) }48\qquad\textbf{(E) }66$

Solution 1 (3 min solve)

We know that the total amount of games must be the sum of games won by left and right handed players. Then, we can write $g = l + r$, and since $l = 1.4r$, $g = 2.4r$. Given that r and g are both integers, $g/2.4$ also must be an integer. From here we can see that g must be divisible by 12, leaving only answers B and D. Now we know the formula for how many games are played in this tournament is $n(n-1)/2$, the sum of the first $n-1$ triangular numbers. Now, setting 36 and 48 equal to the equation will show that two consecutive numbers must have a product of 72 or 96. Clearly $72 = 8*9$, so the answer is $\boxed{\textbf{(B) }36}$.

~~ Antifreeze5420

Solution 2

First, we know that every player played every other player, so there's a total of $\binom{n}{2}$ games since each pair of players forms a bijection to a game. Therefore, that rules out D. Also, if we assume the right-handed players won a total of x games, the left-handed players must have won a total of $\frac{7}{5}x$ games, meaning that the total number of games played was $\frac{12}{5}x$. Thus, the total number of games must be divisible by 12. Therefore leaving only answer choices B and D. Since answer choice D doesn't satisfy the first condition, the only answer that satisfies both conditions is $\boxed{\textbf{(B) }36}$

Solution 3

Let r be the amount of games the right-handed won. Since the left-handed won $1.4r$ games, the total number of games played can be expressed as $(2.4)r$, or $12/5r$, meaning that the answer is divisible by 12. This brings us down to two answer choices, B and D. We note that the answer is some number x choose 2. This means the answer is in the form $x(x-1)/2$. Since answer choice D gives $48 = x(x-1)/2$, and $96 = x(x-1)$ has no integer solutions, we know that $\boxed{\textbf{(B) }36}$ is the only possible choice.

Solution 4

Here is the rigid way to prove that 36 is the only result. Let the number of left-handed players be n, so the right-handed player is 2n. The number of games won by the left-handed players comes in two ways: (1) the games played by two left-left pairs, which is $\frac{n(n-1)}{2}$, and (2) the games played by left-right pairs, which is x. And $x \le 2n^2$. so

$$\frac{\frac{n(n-1)}{2}+x}{\frac{2n(2n-1)}{2}+2n^2-x} = 1.4$$

which gives

$$x = \frac{17n^2}{8} - \frac{3n}{8}$$

By setting up the inequality $x \le 2n^2$, it comes $n \le 3$. So the total number of players can only be 3, 6, and 9. Then we can rule out all other possible values for the total number of games they played.

~ ~ ggao5uiuc ~ ~ yingkai_0_ (minor edits)

Solution 5 (Cheese)

If there are n players, the total number of games played must be $\binom{n}{2}$, so it has to be a triangular number. The ratio of games won by left-handed to right-handed players is $7 : 5$, so the number of games played must also be divisible by 12. Finally, we notice that only $\boxed{\textbf{(B) }36}$ satisfies both of these conditions.

~MathFun1000

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

```
~Math-X
```

See Also

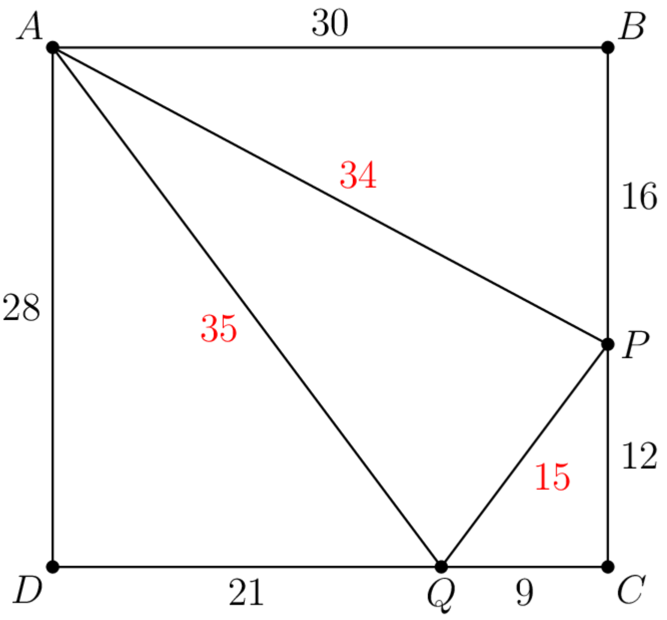
Problem_17

Problem

Let $ABCD$ be a rectangle with $AB = 30$ and $BC = 28$. Point P and Q lie on $\overline{BC}$ and $\overline{CD}$ respectively so that all sides of $\triangle ABP, \triangle PCQ,$ and $\triangle QDA$ have integer lengths. What is the perimeter of $\triangle APQ$?

$\textbf{(A)}\ 84 \qquad \textbf{(B)}\ 86 \qquad \textbf{(C)}\ 88 \qquad \textbf{(D)}\ 90 \qquad \textbf{(E)}\ 92$

Solution 1



We know that all side lengths are integers, so we can test Pythagorean triples for all triangles.

First, we focus on $\triangle ABP$. The length of AB is 30, and the possible Pythagorean triples $\triangle ABP$ can be are $(3, 4, 5), (5, 12, 13), (8, 15, 17),$ where the value of one leg is a factor of 30. Testing these cases, we get that only $(8, 15, 17)$ is a valid solution because the other triangles result in another leg that is greater than 28, the length of $\overline{BC}$. Thus, we know that $BP = 16$ and $AP = 34$.

Next, we move on to $\triangle QDA$. The length of AD is 28, and the possible triples are $(3, 4, 5)$ and $(7, 24, 25)$. Testing cases again, we get that $(3, 4, 5)$ is our triple. We get the value of $DQ = 21$, and $AQ = 35$.

We know that $CQ = CD - DQ$ which is 9, and $CP = BC - BP$ which is 12. $\triangle CPQ$ is therefore a right triangle with side length ratios $3, 4, 5$, and the hypotenuse is equal to 15. $\triangle APQ$ has side lengths $34, 35,$ and $15,$ so the perimeter is equal to $34 + 35 + 15 = \boxed{\textbf{(A)}\ 84}$.

~Gabe Horn ~ItsMeNoobieboy

Solution 2

Let $BP = y$ and $AP = z$. We get $30^2 + y^2 = z^2$. Subtracting y^2 on both sides, we get $30^2 = z^2 - y^2$. Factoring, we get $30^2 = (z-y)(z+y)$. Since y and z are integers, both $z - y$ and $z + y$ have to be even or both have to be odd. We also have $y < 31$. We can pretty easily see now that $z - y = 18$ and $z + y = 50$. Thus, $y = 16$ and $z = 34$. We now get $CP = 12$. We do the same trick again. Let $DQ = a$ and $AQ = b$. Thus, $28^2 = (b+a)(b-a)$. We can get $b + a = 56$ and $b - a = 14$. Thus, $b = 35$ and $a = 21$. We get $CQ = 9$ and by the Pythagorean Theorem, we have $PQ = 15$. We get $AP + PQ + AQ = 34 + 15 + 35 = 84$. Our answer is A.

If you want to see a video solution on this solution, look at Video Solution 1.

-paixiao

-paixiao

VIdeo Solution 2

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

See Also

Problem_18

Problem

A rhombic dodecahedron is a solid with 12 congruent rhombus faces. At every vertex, 3 or 4 edges meet, depending on the vertex. How many vertices have exactly 3 edges meet?

$\textbf{(A) }5\qquad\textbf{(B) }6\qquad\textbf{(C) }7\qquad\textbf{(D) }8\qquad\textbf{(E) }9$

Solution 1

Note Euler's formula where $V+F-E=2$. There are 12 faces and the number of edges is 24 because there are 12 faces each with four edges and each edge is shared by two faces. Now we know that there are 14 vertices on the figure. Now note that the sum of the degrees of all the points is twice the number of edges. Now we know $\frac{3x+4(14-x)}{2}=24$. Solving this system of equations gives $x=8$ so the answer is $\boxed{\textbf{(D) }8}$. ~aiden22gao ~zgahzlkw (LaTeX) ~ESAOPS (Symplified)

Solution 2 (Quick solution)

Let x be the number of vertices with three edges, and y be the number of vertices with four edges. Since there are $\frac{4*12}{2}=24$ edges on the polyhedron, we can see that $\frac{3x+4y}{2}=24$. Then, $3x+4y=48$. Notice that by testing the answer choices, (D) is the only one that yields an integer solution for y. Thus, the answer is $\boxed{\textbf{(D) }8}$.

~Mathkiddie

Solution 3

With 12 rhombi, there are $4\cdot 12=48$ total boundaries. Each edge is used as a boundary twice, once for each face on either side. Thus we have $\frac{48}{2}=24$ total edges.

Let A be the number of vertices with 3 edges (this is what the problem asks for) and B be the number of vertices with 4 edges. We have $3A+4B=48$.

Euler's formula states that, for all convex polyhedra, $V-E+F=2$. In our case, $V-24+12=2 \implies V=14.$ We know that $A+B$ is the total number of vertices as we are given that all vertices are connected to either 3 or 4 edges. Therefore, $A+B=14.$

We now have a system of two equations. There are many ways to solve for A; choosing one yields $A=\boxed{\textbf{(D) }8}$.

Even without Euler's formula, we can do a bit of answer guessing. From $3A+4B=48$, we take mod 4 on both sides.

$$3A+4B\equiv 48 \pmod{4}$$

$$3A\equiv 0 \pmod{4}$$

We know that $3A$ must be divisible by 4. We know that the factor of 3 will not affect the divisibility by 4 of $3A$, so we remove the 3. We know that A is divisible by 4. Checking answer choices, the only one divisible by 4 is indeed $A=\boxed{\textbf{(D) }8}$.

~Technodoggo ~zgahzlkw (small edits) ~ESAOPS (LaTeX)

Solution 4

Note that Euler's formula is $V+F-E=2$. We know $F=12$ from the question. We also know $E=\frac{12\cdot 4}{2}=24$ because every face has 4 edges and every edge is shared by 2 faces. We can solve for the vertices based on this information.

Using the formula we can find:

$$V + 12 - 24 = 2$$

$$V = 14$$

Let t be the number of vertices with 3 edges and f be the number of vertices with 4 edges. We know $t + f = 14$ from the question and $3t + 4f = 48$. The second equation is because the total number of points is 48 because there are 12 rhombuses of 4 vertices. Now, we just have to solve a system of equations.

$$3t + 4f = 48$$

$$3t + 3f = 42$$

$$f = 6$$

$$t = 8$$

Our answer is simply just t, which is $\boxed{\textbf{(D) } 8}$ ~musicalpenguin

Solution 5

Each of the twelve rhombi has two pairs of angles across from each other that must be congruent. If both pairs of angles occur at 4-point intersections, we have a grid of squares. If both occur at 3-point intersections, we would have a cube with six square faces. Therefore, two of the points must occur at a 3-point intersection and two at a 4-point intersection.

Since each 3-point intersection has 3 adjacent rhombuses, we know the number of 3-point intersections must equal the number of 3-point intersections per rhombus times the number of rhombuses over 3. Since there are 12 rhombuses and two 3-point intersections per rhombus, this works out to be:

$$\frac{2 \cdot 12}{3}$$

Hence: $\boxed{\textbf{(D) } 8}$ ~hollph27 ~Minor edits by FutureSphinx

Solution 6 (Based on previous knowledge)

Note that a rhombic dodecahedron is formed when a cube is turned inside out (as seen), thus there are 6 4-vertices (corresponding to each face of the cube) and 8 3-vertices (corresponding to each corner of the cube). Thus the answer is $\boxed{\textbf{(D) } 8}$

Solution 7 (Using Answer Choices)

Let m be the number of 4-edge vertices, and n be the number of 3-edge vertices. The total number of vertices is $m + n$. Now, we know that there are $4 \cdot 12 = 48$ vertices, but we have overcounted. We have overcounted m vertices 3 times and overcounted n vertices 2 times. Therefore, we subtract $3m$ and $2n$ from 48 and set it equal to our original number of vertices.

$$48 - 3m - 2n = m + n$$

$$4m + 3n = 48$$

From here, we reduce both sides modulo 4. The $4m$ disappears, and the left hand side becomes $3n$. The right hand side is 0, meaning that $3n$ must be divisible by 4. Looking at the answer choices, this is only possible for $n = \boxed{8}$.

-DEVSAXENA

(Isn't this the same as the last half of Solution 2?)

Solution 8 (Dual)

Note that a rhombic dodecahedron is the dual of a cuboctahedron. A cuboctahedron has 8 triangular faces, which correspond to $\boxed{\textbf{(D) } 8}$ vertices on a rhombic dodecahedron that have 3 edges.

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

See Also

Problem_19

Problem

The line segment formed by $A(1,2)$ and $B(3,3)$ is rotated to the line segment formed by $A'(3,1)$ and $B'(4,3)$ about the point $P(r,s)$. What is $|r-s|$?

$\textbf{(A)}\ \frac{1}{4} \qquad \textbf{(B)}\ \frac{1}{2} \qquad \textbf{(C)}\ \frac{3}{4} \qquad \textbf{(D)}\ \frac{2}{3} \qquad \textbf{(E)}\ 1$

Solution 1

Due to rotations preserving an equal distance, we can bash the answer with the distance formula. $D(A,P)=D(A',P)$, and $D(B,P)=D(B',P)$. Thus we will square our equations to yield: $(1-r)^2+(2-s)^2=(3-r)^2+(1-s)^2$, and $(3-r)^2+(3-s)^2=(4-r)^2+(3-s)^2$. Cancelling $(3-s)^2$ from the second equation makes it clear that r equals 3.5. Now substituting will yield $(2.5)^2+(2-s)^2=(-0.5)^2+(1-s)^2$. $6.25+4-4s+s^2=0.25+1-2s+s^2$ $2s=9$, $s=4.5$. Now $|r-s|=|3.5-4.5|=\boxed{\textbf{(E)}\ 1}$.

-Antifreeze5420

Solution 2

Due to rotations preserving distance, we have that $BP=B'P$, as well as $AP=A'P$. From here, we can see that P must be on the perpendicular bisector of $\overline{BB'}$ due to the property of perpendicular bisectors keeping the distance to two points constant.

From here, we proceed to find the perpendicular bisector of $\overline{BB'}$. We can see that this is just a horizontal line segment with midpoint at $(3.5,3)$. This means that the equation of the perpendicular bisector is $x=3.5$.

Similarly, we find the perpendicular bisector of $\overline{AA'}$. We find the slope to be $\frac{1-2}{3-1}=-\frac{1}{2}$, so our new slope will be 2. The midpoint of A and A' is $(2,\frac{3}{2})$, which we can use with our slope to get another equation of $y=2x-\frac{5}{2}$.

Now, point P has to lie on both of these perpendicular bisectors, meaning that it has to satisfy both equations. Plugging in the value of x we found earlier, we find that $y=4.5$. This means that $|r-s|=|3.5-4.5|=\boxed{\textbf{(E)}\ 1}$.

-DEVSAXENA

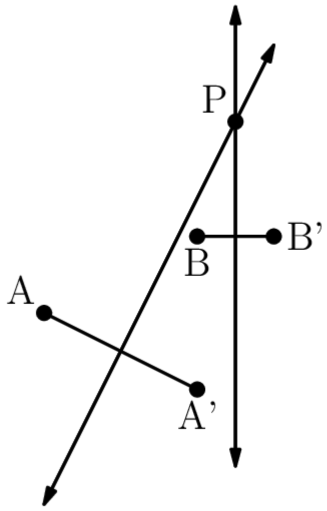
Solution 3 (Coordinate Geometry)

To find the center of rotation, we find the intersection point of the perpendicular bisectors of $\overline{AA'}$ and $\overline{BB'}$.

We can find that the equation of the line $\overline{AA'}$ is $y=-\frac{1}{2}x+\frac{5}{2}$, and that the equation of the line $\overline{BB'}$ is $y=3$.

When we solve for the perpendicular bisector of $y=-\frac{1}{2}x+\frac{5}{2}$, we determine that it has a slope of 2, and it runs through $(2,1.5)$. Plugging in $1.5=2(2)-n$, we get than $n=\frac{5}{2}$. Therefore our perpendicular bisector is $y=2x-\frac{5}{2}$. Next, we solve for the perpendicular of $y=3$. We know that it has an undefined slope, and it runs through $(3.5,3)$. We can determine that our second perpendicular bisector is $x=3.5$.

Setting the equations equal to each other, we get $2x-\frac{5}{2}=3.5$. Solving for x, we get that $x=\frac{9}{2}$. Therefore, $|r-s|=|3.5-4.5|=\boxed{\textbf{(E)}\ 1}$.



~aydenlee & wuwang2002

Solution

We use the complex numbers approach to solve this problem. Denote by θ the angle that AB rotates about P in the counterclockwise direction.

Thus, $A' - P = e^{i\theta} \left(A - P \right)$ and $B' - P = e^{i\theta} \left(B - P \right)$.

Taking ratio of these two equations, we get

$$\frac{A' - P}{A - P} = \frac{B' - P}{B - P}.$$

By solving this equation, we get $P = \frac{7}{2} + i \frac{9}{2}$. Therefore, $|s - t| = \left| \frac{7}{2} - \frac{9}{2} \right| = \boxed{\textbf{(E) 1}}$.

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

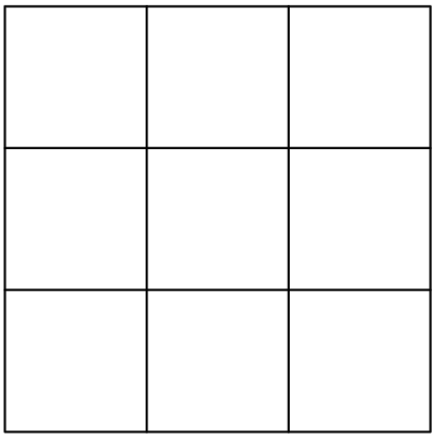
~IceMatrix

See Also

Problem_20

Problem

Each square in a 3×3 grid of squares is colored red, white, blue, or green so that every 2×2 square contains one square of each color. One such coloring is shown on the right below. How many different colorings are possible?



B	R	B
G	W	G
R	B	R

$\textbf{(A) } 24 \qquad \textbf{(B) } 48 \qquad \textbf{(C) } 60 \qquad \textbf{(D) } 72 \qquad \textbf{(E) } 96$

Solution 1

Let a "tile" denote a 1×1 square and "square" refer to 2×2.

We first have $4! = 24$ possible ways to fill out the top left square. We then fill out the bottom right tile. In the bottom right square, we already have one corner filled out (from our initial coloring), and we now have 3 options left to pick from.

We then look at the right middle tile. It is part of two squares: the top right and top left. Among these squares, 3 colors have already been used, so we only have one more option for it. Similarly, every other square only has one more option, so we have a total of $3 \cdot 4! = \boxed{\textbf{(D) } 72}$ ways.

~Technodoggo

Solution 2

B	R	B
G	W	G
R	B	R

B	R	B
G	W	G
R	B	R

We can split this problem into 2 cases as shown above. We can swap a set of equal colors for another set of equal colors to create a new square.

Square 1: The first square can be rotated to create another square so we have to multiply the number of arrangements by 2. We have $4! = 24$ arrangements without rotating and $24 \cdot 2 = 48$ arrangements in total for the first square.

Square 2: There are $4! = 24$ ways to arrange the colors.

In total, we have $48 + 24 = \boxed{\textbf{(D) } 72}$ arrangements.

~ (LaTeX and Solution)

Solution 3

Let’s call the top-right corner color A, the top-middle color B, the top-right color C, and so on, with color D being the middle row, and right corner square, and color G being the bottom-left square’s color. WLOG A=Red, B=White, D=Blue, and E=Green. We will now consider squares C and F’s colors. Case 1 : C=Red and F=Blue In this case, we get that G and H have to be Red and White in some order, and the same for H and I. We can color this in 2 ways. Case 2 : C=Blue and F=Red In this case, one of G and H needs to be White and Red, and H and I needs to be White and Blue. There is 1 way to color this. In total, we get 24*(2+1)=72 ways to color the grid. $\boxed{\textbf{(D) } 72}$.

-paixiao

Solution 4

Let us start with choosing the colors of the top middle square and the center square. There is 4 ways to choose the color of the top middle square, and 3 ways to choose the color of the center square, since these two squares must have two different colors. Then, from here, we have two remaining colors for us to put in the top left and middle left, and the same two colors to put the top right and middle right squares. Now, we have two cases:

Case 1: The middle left and middle right squares are the same color. There are 2 ways to choose the color, then there are also 2 ways to choose the color of the bottom middle square, because it has two things it cannot be.

Case 2: is that the middle left and middle right squares are different colors. There are $2!$ ways to order the two different colors, and then there is 1 way to choose the two different colors.

Thus, once we choose the middle squares (in 12 possible ways), we have 6 ways to color the squares. Thus, we have $6 \cdot 12 = 72$, and so the answer is $48 + 24 = \boxed{\textbf{(D) } 72}$.

Solution 5

Note that there can be no overlap between colors in each square. Then, we can choose 1 color to be in the center. $\binom{4}{1}$= 4

Now, we have some casework: Case 1: 1 color is placed in 4 corners and then others are placed on opposite edges. 232 414 232 There's $3! = 6$ ways to do this.

Case 2: 2 colors are placed with 2 in adjacent corners and 1 edge opposite them. The final color is placed in the remaining 2 edges. 232 414 323 The orientation of the 2 colors has 2 possibilities, and there are $3!$ color permutations. $2 * 3! = 12$

There can't be any more ways to do this, as we have combined all cases such that each color is used once and only once per $2 * 2$ square. We multiply the start with the sum of the 2 cases: $4(6 + 12) = \boxed{\textbf{(D) } 72}$.

Solution 6

3	1	2
2	3	1
2	1	1

Note that we could fill the 3 by 3 square with numbers of choices, rather than letters or color names. The top-left corner receives a 3 because there are 3 total choices to choose from: R, G and B. The squares bordering them has values of 2 and 1, regardless of order. 2 indicates that the small square can have any color excluding the one existing color, 1 indicates the remaining color of the 2 by 2 square. Finally, the middle square receives 3, since the first 2 by 2 square has RGB already, and it does not matter what color it has. Moving onto the next 2 by 2 square, we see that there are already 2 decided colors, so the other squares must be 2 and 1, again, regardless of the order. The same applys to the third 2 by 2 square, and finally the last square has the value of one, as it is the only square left. Multiplying the numbers, $2 \times 2 \times 2 \times 3 \times 3$ = $\boxed{\textbf{(D) } 72}$

-MEZE_RUN

-paixiao

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

~IceMatrix Solution 5.

```
Let's name the cells A,B,C,D,E,F,G,H,I from the top left to the bottom right.
```

Case 1. Cell B and cell H have the same color. The middle one cell E has 4 choices, cell B has 3 choices, then cell E has 2 choices and cell F has 2 choices, this gives $4 \cdot 3 \cdot 2 \cdot 2 = 48$ ways.

Case 2. Cell B and cell H have different colors. The middle one cell E has 4 choices, cell B has 3 choices, cell H has 2 choices, then cell D and F each can only have one choice(different from B,E,H). This gives $4 \cdot 3 \cdot 2 = 24$ ways.

The answer= 48+24=72. (D)

See Also

Problem_21

Problem

Let $P(x)$ be the unique polynomial of minimal degree with the following properties:

- $P(x)$ has a leading coefficient 1,
- 1 is a root of $P(x) - 1$,
- 2 is a root of $P(x-2)$,
- 3 is a root of $P(3x)$, and
- 4 is a root of $4P(x)$.

The roots of $P(x)$ are integers, with one exception. The root that is not an integer can be written as $\frac{m}{n}$, where m and n are relatively prime integers. What is $m+n$?

$\textbf{(A) } 41 \qquad \textbf{(B) } 43 \qquad \textbf{(C) } 45 \qquad \textbf{(D) } 47 \qquad \textbf{(E) } 49$

Solution 1

From the problem statement, we know $P(2-2)=0$, $P(9)=0$ and $4P(4)=0$. Therefore, we know that 0, 9, and 4 are roots. So, we can factor $P(x)$ as $x(x-9)(x-4)(x-a)$, where a is the unknown root. Since $P(x)-1=0$, we plug in $x=1$ which gives $1(-8)(-3)(1-a)=1$, therefore $24(1-a)=1 \implies 1-a=1/24 \implies a=23/24$. Therefore, our answer is $23+24=\boxed{\textbf{(D) } 47}$

~aiden22gao

~cosinesine

~walmartbrian

~sravya_m18

~ESAOPS

Solution 2

We proceed similarly to solution one. We get that $x(x-9)(x-4)(x-a)=1$. Expanding, we get that $x(x-9)(x-4)(x-a)=x^4-(a+13)x^3+(13a+36)x^2-36ax$. We know that $P(1)=1$, so the sum of the coefficients of the cubic expression is equal to one. Thus $1+(a+13)+(13a+36)-36a=1$. Solving for a, we get that a=23/24. Therefore, our answer is $23+24=\boxed{\textbf{(D) } 47}$

~Aopsthedude

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

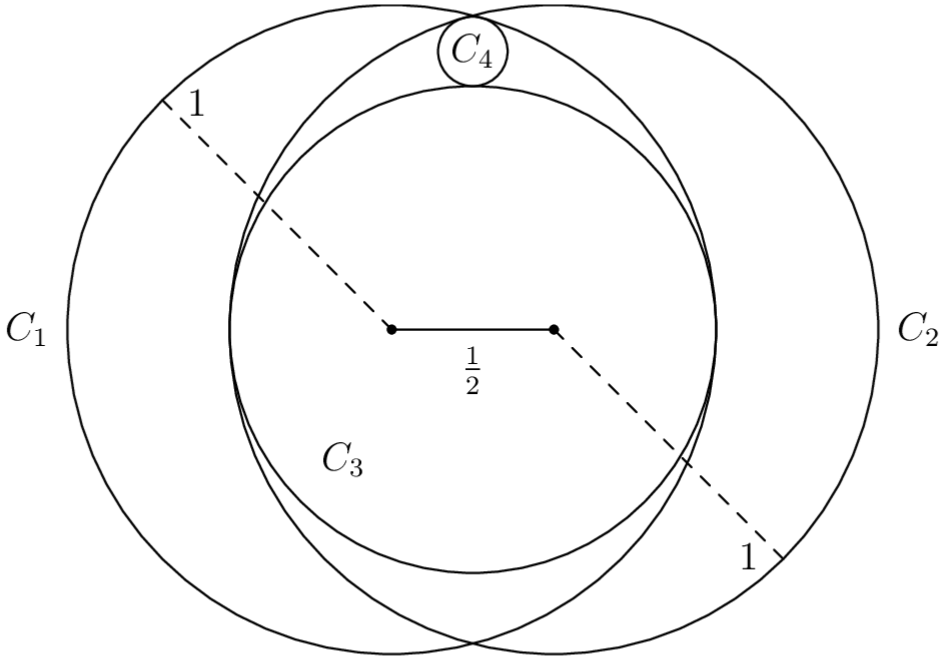
See Also

Problem_22

The following problem is from both the 2023 AMC 10A #22 and 2023 AMC 12A #18, so both problems redirect to this page.

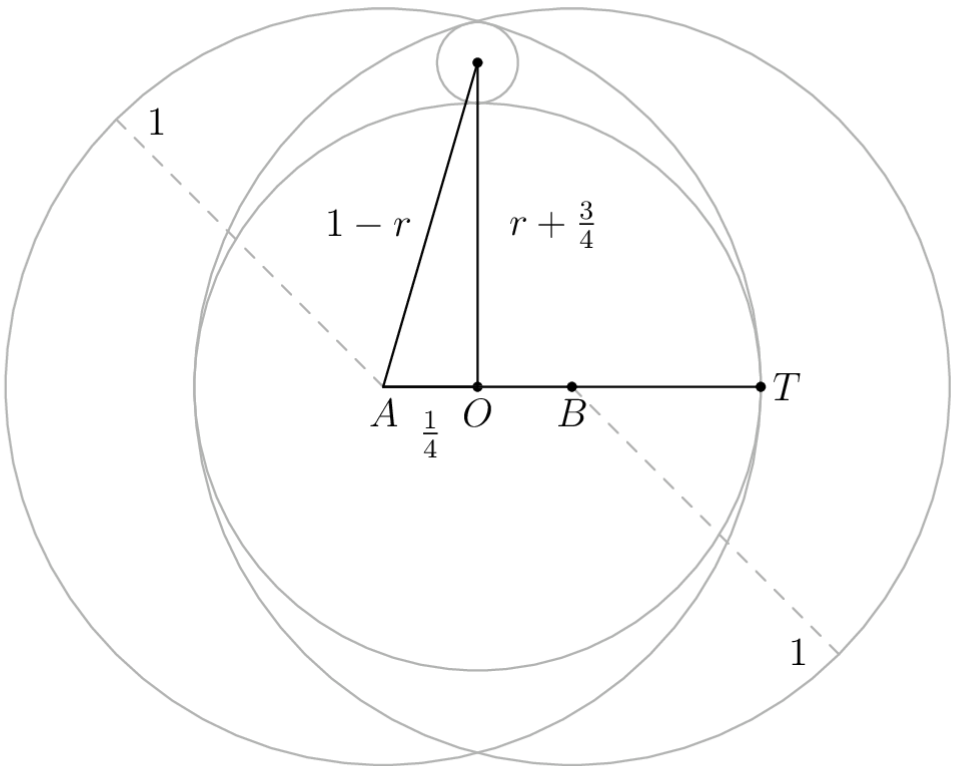
Problem

Circle C_1 and C_2 each have radius 1, and the distance between their centers is $\frac{1}{2}$. Circle C_3 is the largest circle internally tangent to both C_1 and C_2. Circle C_4 is internally tangent to both C_1 and C_2 and externally tangent to C_3. What is the radius of C_4?



$\textbf{(A) } \frac{1}{14} \qquad \textbf{(B) } \frac{1}{12} \qquad \textbf{(C) } \frac{1}{10} \qquad \textbf{(D) } \frac{3}{28} \qquad \textbf{(E) } \frac{1}{9}$

Solution



Let O be the center of the midpoint of the line segment connecting both the centers, say A and B.

Let the point of tangency with the inscribed circle and the right larger circles be T.

Then $OT = BO + BT = BO + AT - \frac{1}{2} = \frac{1}{4} + 1 - \frac{1}{2} = \frac{3}{4}$.

Since C_4 is internally tangent to C_1, center of C_4, C_1 and their tangent point must be on the same line.

Now, if we connect centers of C_4, C_3 and C_1/C_2, we get a right angled triangle.

Let the radius of C_4 equal r. With the pythagorean theorem on our triangle, we have

$$\left(r + \frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = (1-r)^2$$

Solving this equation gives us

$$r = \boxed{\textbf{(D) } \frac{3}{28}}$$

~lptoggled

~ShawnX (Diagram)

~ap246 (Minor Changes)

~EpicBird08

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

~IceMatrix

See Also

Problem_23

Problem

If the positive integer n has positive integer divisors a and b with $n = ab$, then a and b are said to be $complementary$ divisors of n. Suppose that N is a positive integer that has one complementary pair of divisors that differ by 20 and another pair of complementary divisors that differ by 23. What is the sum of the digits of N?

$\textbf{(A) } 9 \qquad \textbf{(B) } 13 \qquad \textbf{(C) } 15 \qquad \textbf{(D) } 17 \qquad \textbf{(E) } 19$

Solution 1

Consider positive integers a, b with a difference of 20. Suppose $b = a - 20$. Then, we have that $(a)(a-20) = n$. If there is another pair of two integers that multiply to 30 but have a difference of 23, one integer must be greater than a, and one must be smaller than $a - 20$. We can create two cases and set both equal. We have $(a)(a-20) = (a+1)(a-22)$, and $(a)(a-20) = (a+2)(a-21)$. Starting with the first case, we have $a^2 - 20a = a^2 - 21a - 22$,or $0 = -a - 22$, which gives $a = -22$, which is not possible. The other case is $a^2 - 20a = a^2 - 19a - 42$, so $a = 42$. Thus, our product is $(42)(22) = (44)(21)$, so $c = 924$. Adding the digits, we have $9 + 2 + 4 = \boxed{\textbf{(C) } 15}$. wait but how do we know the two positive integers multiply to thirty? -Sepehr2010

Solution 2

We have 4 integers in our problem. Let's call the smallest of them a. $a(a+23) =$ either $(a+1)(a+21)$ or $(a+2)(a+22)$. So, we have the following:

$a^2 + 23a = a^2 + 22a + 21$ or

$a^2 + 23a = a^2 + 24a + 44.$

The second equation has negative solutions, so we discard it. The first equation has $a = 21$, and so $a + 23 = 44$. If we check $(a+1)(a+21)$ we get $22 \cdot 42 = 21 \cdot 44$. 44 is 2 times 22, and 42 is 2 times 21, so our solution checks out. Multiplying 21 by 44, we get 924 => $9 + 2 + 4 = \boxed{\textbf{(C) 15}}$.

~Arctictурn ~Mathkiddie

Solution 3

From the problems, it follows that

$$\begin{aligned}
x(x+20) &= y(y+23) = N \\
x^2 + 20x &= y^2 + 23y \\
4x^2 + 4 \cdot 20x &= 4y^2 + 4 \cdot 23y \\
4x^2 + 4 \cdot 20x + 20^2 - 20^2 &= 4y^2 + 4 \cdot 23y + 23^3 - 23^2 \\
(2x+20)^2 - 20^2 &= (2y+23)^2 - 23^2 \\
23^2 - 20^2 &= (2y+23)^2 - (2x+20)^2 \\
(23+20)(23-20) &= (2y+23+2x+20)(2y+23-2x-20) \\
43 \cdot 3 &= (2y+2x+43)(2y-2x+3) \\
129 \cdot 1 &= (2y+2x+43)(2y-2x+3)
\end{aligned}$$

Since both $(2y+2x+43)$ and $(2y-2x+3)$ must be integer, we get two equations.

$$
\begin{align}
129 or 43 &= (2y + 2x + 43) \tag{1}\\
1 or 3 &= 2y - 2x + 3 \tag{2}\\
& \tag{3}
\end{align}
$$

43 & 1 yields (0,0) which is not what we want. 129 & 1 yields (22,21) which is more interesting.

Simplifying the equations, we get:

$$
\begin{aligned}
x + y &= 43\\
x - y &= 1\\
x &= 22, y = 21\\
N &= (22)(22 + 20) = 924.
\end{aligned}
$$

So, the answer is $\boxed{\textbf{(C) } 15}$.

~Technodoggo

Solution 4

Say one factorization is $n(n+23)$. The two cases for the other factorization are $(n+1)(n+21)$ and $(n+2)(n+22)$. We know it must be the first because of AM-GM intuition: lesser factors are closer together. Thus, $n(n+23) = (n+1)(n+21)$ and we find that $n = 21, N = 924$ meaning the answer is $\boxed{\textbf{(C) } 15}$.

~DouDragon

Solution 5

Since we are given that some pairs of divisors differ by 20 and 23 and we can let the pair be $(x-10)$ and $(x+10)$ as well as $(y-\frac{23}{2})$ and $(y+\frac{23}{2})$. We also know the product of both the complementary divisors give the same number so $(x-10)(x+10) = (y-\frac{23}{2})(y+\frac{23}{2})$. Now we let $y = \frac{a}{2}$. Then we substitute and get $x^2 - 100 = \frac{(a^2-529)}{4}$. Finally we multiply by 4 and get $4x^2 - a^2 = -129, a^2 - 4x^2 = 129$. Then we use differences of squares and get a+$2x$=129, a-$2x$=1. We finish by getting a $=$65 and $x = 32$. So $(42)(22) = 924$ Adding the digits, we have $9+2+4 = \boxed{\textbf{(C) } 15}$.

~averageguy

Solution 6

N can be written $N = (a-10)\,(a+10)$ with a positive integer $a > 10$ and $N = \left(\frac{2b+1}{2} - \frac{23}{2} \right) \left(\frac{2b+1}{2} + \frac{23}{2} \right)$ with a positive integer $b > 11$.

The above equations can be reorganized as

$$
(2b+1+2a)\,(2b+1-2a) = 43 \cdot 3.
$$

The only solution is $2b+1+2a = 129$ and $2b+1-2a = 1$. Thus, $a = b = 32$. Therefore, $N = 924$. So the sum of the digits of N is $9+2+4 = \boxed{\textbf{(C) } 15}$.

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

Solution 7

We can write N as $a(a+20)$ or $b(b+23)$ where a and b are divisors of N. Since $a(a+20)=b(b+23)$, we know that $a^2+20a-b^2-23b=0$, and we can view this as a quadratic in a.

Since the solution for a must be an integer, the discriminant for this quadratic must be a perfect square and therefore $20^2-4(-b^2-23b)=(2c)^2=400+4b^2+92b$ so $b^2+23b-c^2+100=0.$

Since the discriminant of this quadratic in b must also be a perfect square we know that $23^2-4(-c^2+100)=d^2$ which we can simplify as $d^2-4c^2=(d-2c)(d+2c)=129.$ Since they are both positive integers $d-2c$ and $d+2c$ are factors of $129=3\cdot 43$ so $d-2c=1$ and $d+2c=129$ or $d-2c=3$ and $d-2c=43.$

These systems of equations give us $(c,d)=(32,65)$ and $(c,d)=(10,23)$ respectively, if we plug our values for c into the equation for b we get $b^2+23b-924=0$ and $b^2+23b=0$ respectively. The first equation gives us $b=21$ or $b=-44$ and the second gives us $b=0$ or $b=-23$, since b is positive we know that $b=21$ and $N=(21)(21+23)=924$, therefore the sum of the digits of N is $9+2+4=\boxed{\textbf{(C) }15}$.

~SailS

Solution 8 (Trial and Error)

Consider the numbers that are the product of two numbers that differ by twenty. From the condition of the problem, this number must be even. Thus, starting from 2, we consider all even numbers and multiply them by the number that is greater than them by twenty. Then, we check if it can also be represented as a product of numbers that differ by 23. Checking, we see that $22\cdot 42=21\cdot 44=924$ works. Thus, the answer is $9+2+4=\boxed{\textbf{(C) }15}$

~andliu766

~EpicBird08

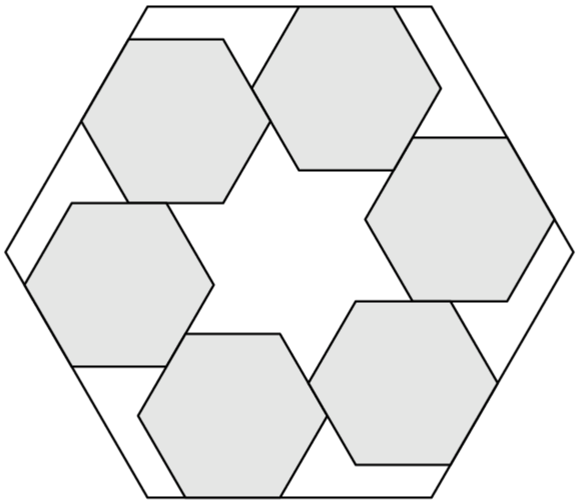
~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

See Also

Problem_24

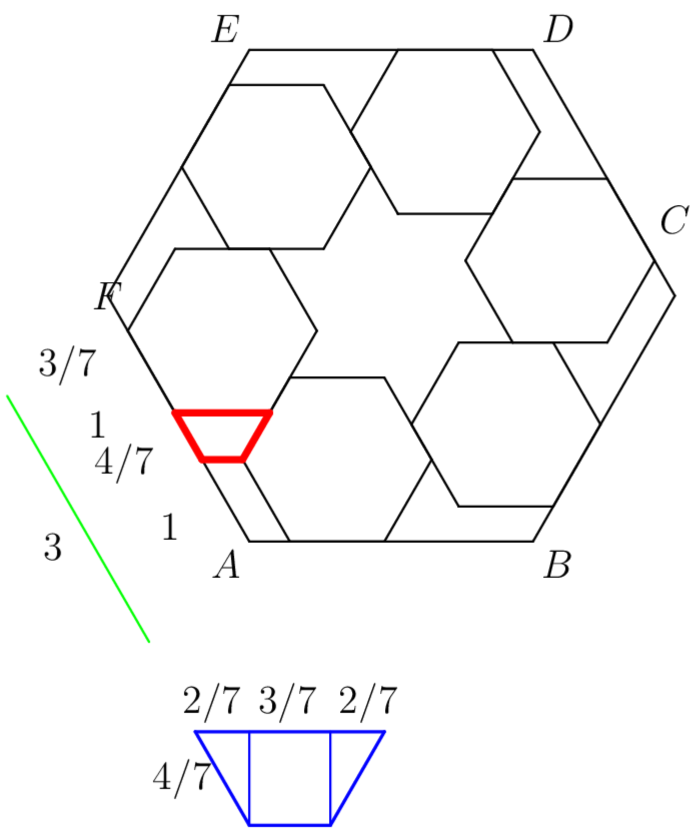
Problem

Six regular hexagonal blocks of side length 1 unit are arranged inside a regular hexagonal frame. Each block lies along an inside edge of the frame and is aligned with two other blocks, as shown in the figure below. The distance from any corner of the frame to the nearest vertex of a block is $\frac{3}{7}$ unit. What is the area of the region inside the frame not occupied by the blocks?



$\textbf{(A)}\ \frac{13\sqrt{3}}{3} \qquad \textbf{(B)}\ \frac{216\sqrt{3}}{49} \qquad \textbf{(C)}\ \frac{9\sqrt{3}}{2} \qquad \textbf{(D)}\ \frac{14\sqrt{3}}{3} \qquad \textbf{(E)}\ \frac{243\sqrt{3}}{49}$

Solution 1



Examining the red isosceles trapezoid with 1 and $\frac{3}{7}$ as two bases, we know that the side lengths are $\frac{4}{7}$ from $30-60-90$ triangle.

We can conclude that the big hexagon has side length 3.

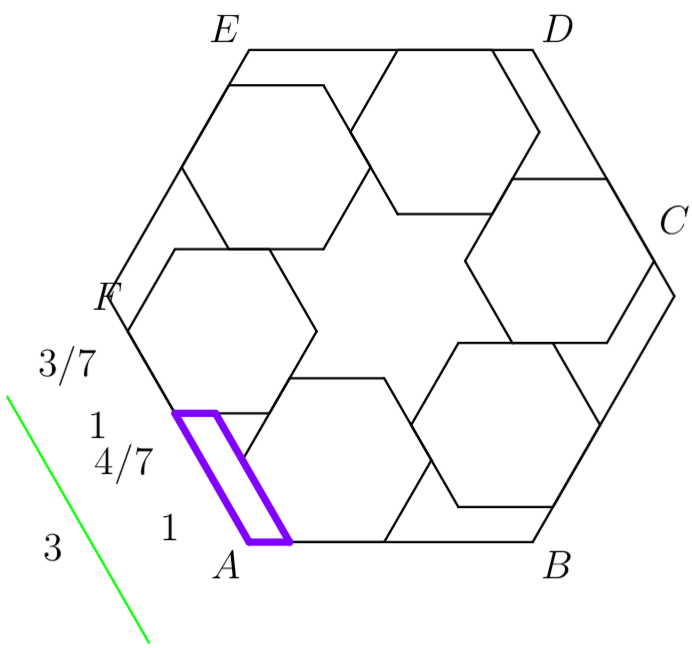
Thus the target area is: area of the big hexagon - 6 * area of the small hexagon.

$$\frac{3\sqrt{3}}{2}(3^2 - 6 \cdot 1^2) = \frac{3\sqrt{3}}{2}(3) = \boxed{\textbf{(C)}\ \frac{9\sqrt{3}}{2}}$$

~Technodoggo

Solution 1.1

We can extend the line of the parallelogram to the end until it touches the next hexagon and it will make a small equilateral triangle and a longer parallelogram. We can prove that one side of the tiny equilateral triangle is 4/7 by playing around with angles and the parallelogram because it is parallel, we can then use the whole side of the hexagon which is one and subtract 3/7 which is one side of the equilateral triangle which is 4/7. That means the whole side of the big hexagon length is 3 and we can continue with solution 1.



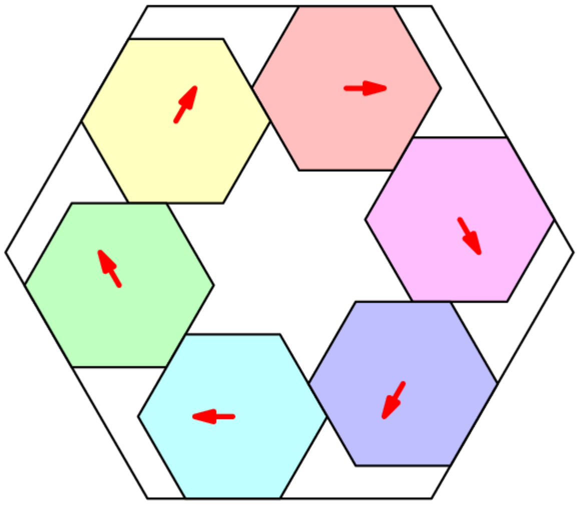
-bcMath-192343

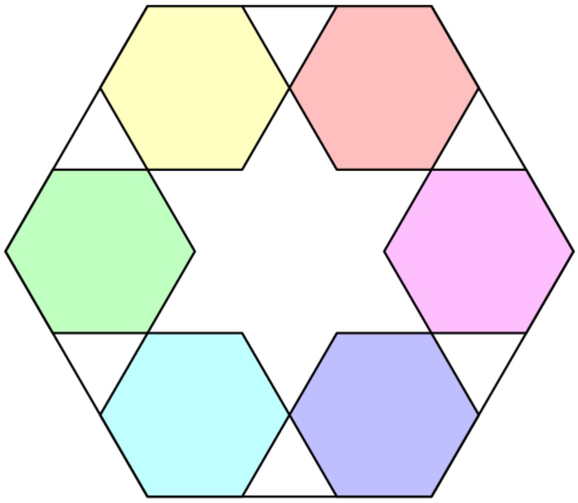
Solution 2 (Not rigorous)

Note that one can "slide' the small hexagons along their respective edges, and either by sliding them to the center or to the corners, and thus getting that the side length of the larger hexagon is 3. The rest proceeds the same as solution 1.

Solution 2.1 (Clarification)

Notice that when sliding the smaller hexagon along the edge, we see that the contact edge withe the smaller hexagon "in front" of it is 60°, thus meaning the hexagon "in front" is pushed at a speed $\sin 60^\circ$ times the actual speed of the hexagon. We can preform a similar analysis on the hexagon that is being pushed and get that the speed at which that hexagon is moving is $\frac{1}{\sin 60^\circ}$ times the speed it pushed by. As we can see, the two factors cancel out and by the same argument, every small hexagon can move at the same speed while mantaining an edge of contact with the two adjacent hexagons.





Note

The number $\frac{3}{7}$ is irrelevant to solve the problem. In fact, if the smaller hexagons have side length x, the side length of the large hexagon will always be $3x$.

Solution 2

We put the diagram to a complex plane, with the center of the outside hexagon at the origin. We denote by s the length of each side of the outside hexagon.

The complex number of the upper left vertex of the upper right small hexagon is

$$se^{i30^\circ} + \frac{3}{7}e^{i(90^\circ+60^\circ)} - \sqrt{3}.$$

The complex number of the upper right vertex of the top small hexagon is

$$si + \frac{3}{7}e^{i\left(\pi+\frac{\pi}{6}\right)} + e^{-i\frac{\pi}{6}}.$$

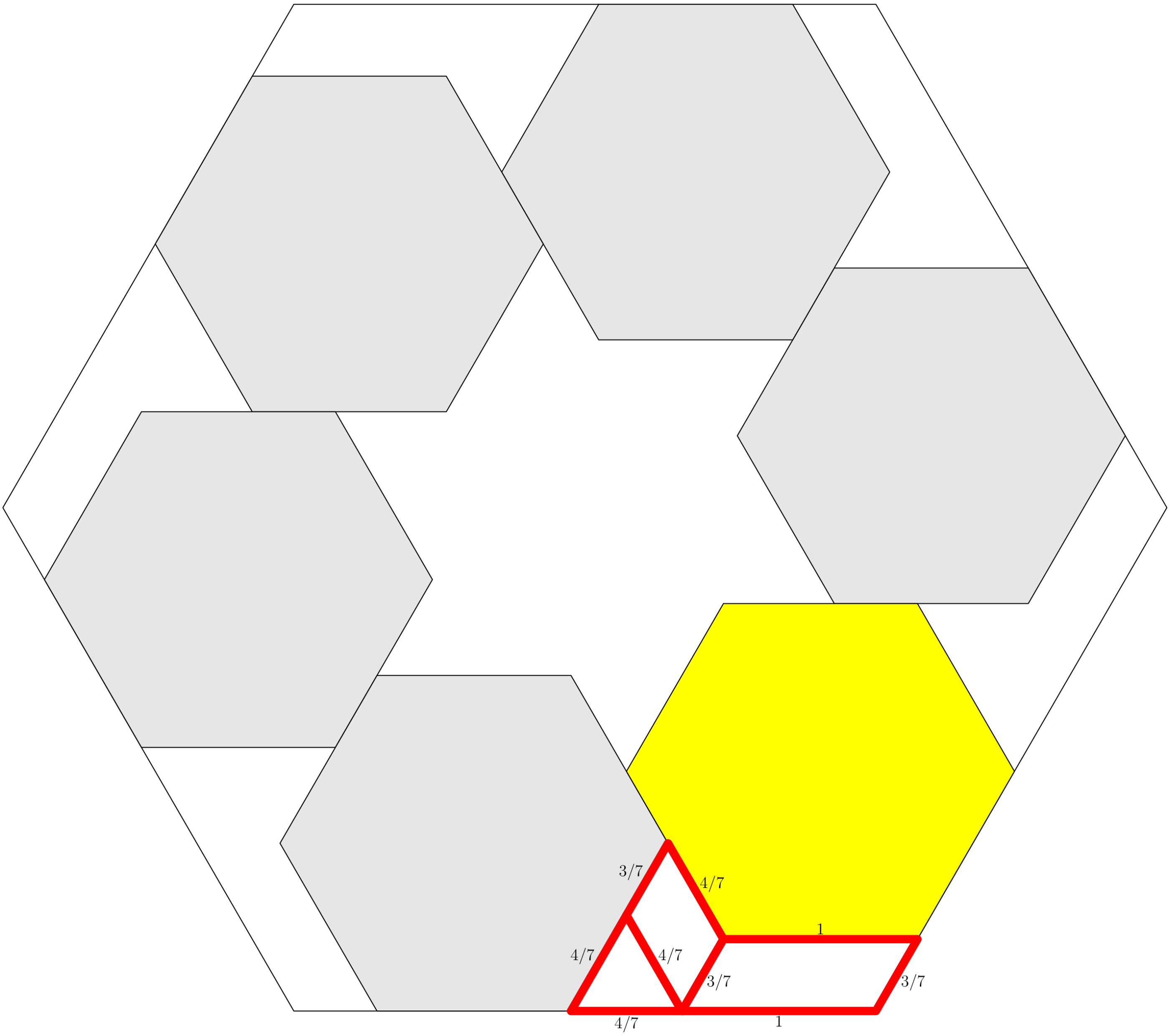
The above two vertices are on the same vertical line. So their real part values are the same. By solving this equation, we get $s = 3$.

Therefore, the area of the region not occupied by the blocks is

$$6 \cdot \frac{\sqrt{3}}{4}s^2 - 6 \cdot 6 \cdot \frac{\sqrt{3}}{4}1^2 = \boxed{\textbf{(C)}\ \frac{9\sqrt{3}}{2}}.$$

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

Solution 3 (almost no words)



$$(3^2 - 6 \times 1^2) \times \frac{3\sqrt{3}}{2} = (9 - 6 \times 1) \times \frac{3\sqrt{3}}{2} = (9 - 6) \times \frac{3\sqrt{3}}{2} = 3 \times \frac{3\sqrt{3}}{2} = \boxed{\frac{9\sqrt{3}}{2}(\text{A})}$$

~~By

~EpicBird08

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

~IceMatrix

See Also

Problem_25

The following problem is from both the 2023 AMC 10A #25 and 2023 AMC 12A #21, so both problems redirect to this page.

Problem

If A and B are vertices of a polyhedron, define the distance $d(A, B)$ to be the minimum number of edges of the polyhedron one must traverse in order to connect A and B. For example, if $\overline{AB}$ is an edge of the polyhedron, then $d(A, B) = 1$, but if $\overline{AC}$ and $\overline{CB}$ are edges and $\overline{AB}$ is not an edge, then $d(A, B) = 2$. Let Q, R, and S be randomly chosen distinct vertices of a regular icosahedron (regular polyhedron made up of 20 equilateral triangles). What is the probability that $d(Q, R) > d(R, S)$?

$\textbf{(A)}\ \frac{7}{22} \qquad \textbf{(B)}\ \frac{1}{3} \qquad \textbf{(C)}\ \frac{3}{8} \qquad \textbf{(D)}\ \frac{5}{12} \qquad \textbf{(E)}\ \frac{1}{2}$

Solution 1

To find the total amount of vertices we first find the amount of edges, and that is $\frac{20 \times 3}{2}$. Next, to find the amount of vertices we can use Euler's characteristic, $V - E + F = 2$, and therefore the amount of vertices is 12

So there are $P(12, 3) = 1320$ ways to choose 3 distinct points.

Now, the furthest distance we can get from one point to another point in a icosahedron is 3. Which gives us a range of $1 \leq d(Q, R), d(R, S) \leq 3$

With some case work, we get two cases:

Case 1: $d(Q, R) = 3; d(R, S) = 1, 2$

Since we have only one way to choose Q, that is, the opposite point from R, we have one option for Q and any of the other points could work for S.

Then, we get $12 \times 1 \times 10 = 120$ (ways to choose R × ways to choose Q × ways to choose S)

Case 2: $d(Q, R) = 2; d(R, S) = 1$

We can visualize the icosahedron as 4 rows, first row with 1 vertex, second row with 5 vertices, third row with 5 vertices and fourth row with 1 vertex. We set R as the one vertex on the first row, and we have 12 options for R. Then, Q can be any of the 5 points on the third row and finally S can be one of the 5 points on the second row.

Therefore, we have $12 \times 5 \times 5 = 300$ (ways to choose R × ways to choose Q × ways to choose S)

Hence, $P(d(Q, R) > d(R, S)) = \frac{120 + 300}{1320} = \boxed{\textbf{(A)}\ \frac{7}{22}}$

~lptoggled, edited by ESAOPS

Solution 2 (Cheese + Actual way)

In total, there are $\binom{12}{3} = 220$ ways to select the points. However, if we look at the denominators of B, C, D, they are $3, 8, 12$ which are not divisors of 220. Also $\frac{1}{2}$ is impossible as cases like $d(Q, R) = d(R, S)$ exist. The only answer choice left is $\boxed{\textbf{(A)}\ \frac{7}{22}}$

Note: this cheese is actually wrong because the total number of ways to select the points is actually $12 \times 11 \times 10 = 1320$ as order matters, so all denominators are possible. Rather, you can arrive at the same conclusion by fixing R WLOG, leading to $11 \times 10 = 110$ ways in total, which works for the original cheese. ~awesomeguy856

(Actual way)

Fix an arbitrary point, to select the rest 2 points, there are $\binom{11}{2}=55$ ways. To make $d(Q,R)=d(R,S), d=1/2$.

Which means there are in total $2\cdot\binom{5}{2}=20$ ways to make the distance the same. $\frac{1}{2}(1-\frac{20}{55})=\boxed{\textbf{(A) }\frac{7}{22}}$

~bluesoul

Solution 3

We can imagine the icosahedron as having 4 layers. 1 vertex at the top, 5 vertices below connected to the top vertex, 5 vertices below that which are 2 edges away from the top vertex, and one vertex at the bottom that is 3 edges away. WLOG because the icosahedron is symmetric around all vertices, we can say that R is the vertex at the top. So now, we just need to find the probability that S is on a layer closer to the top than Q. We can do casework on the layer S is on to get

$$\frac{5}{11}\cdot\frac{6}{10}+\frac{5}{11}\cdot\frac{1}{10}=\frac{35}{110}=\frac{7}{22}$$

So the answer is $\boxed{\textbf{(A) }\frac{7}{22}}$. -awesomeparrot

Solution 4

We can actually see that the probability that $d(Q,R)>d(R,S)$ is the exact same as $d(Q,R)<d(R,S)$ because $d(Q,R)$ and $d(R,S)$ have no difference. (In other words, we can just swap Q and S, meaning that can be called the same probability-wise.) Therefore, we want to find the probability that $d(Q,R)=d(R,S)$.

WLOG, we can rotate the icosahedron so that R is the top of the icosahedron. Then we can divide this into 2 cases:

1. They are on the second layer

There are 5 ways to put one point, and 4 ways to put the other point such that $d(Q,R)=d(R,S)=1$. So, there are $5\cdot 4=20$ ways to put them on the second layer.

2. They are on the third layer

There are 5 ways to put one point, and 4 ways to put the other point such that $d(Q,R)=d(R,S)=2$. So, there are $5\cdot 4=20$ ways to put them on the third layer.

The total number of ways to choose P and S are $11\cdot 10=110$ (because there are 12 vertices), so the probability that $d(Q,R)=d(R,S)$ is $\frac{20+20}{110}=\frac{4}{11}$.

Therefore, the probability that $d(Q,R)>d(R,S)$ is $\frac{1-\frac{4}{11}}{2}=\boxed{\textbf{(A) }\frac{7}{22}}$

~Ethanzhang1001

Solution 5

We know that there are 20 faces. Each of those faces has 3 borders (since each is a triangle), and each edge is used as a border twice (for each face on either side). Thus, there are $\frac{20\cdot 3}{2}=30$ edges.

By Euler's formula, which states that $v-e+f=2$ for all convex polyhedra, we know that there are $2-f+e=12$ vertices.

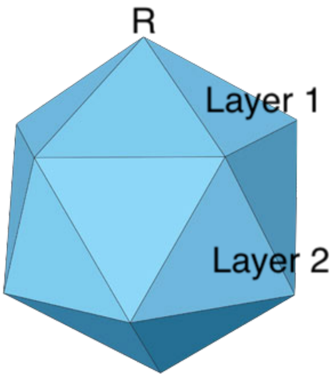
The answer can be counted by first counting the number of possible paths that will yield $d(Q,R) > d(R,S)$ and dividing it by $12 \cdot 11 \cdot 10$ (or $\binom{12}{3}$, depending on the approach). In either case, one will end up dividing by 11 somewhere in the denominator. We can then hope that there will be no factor of 11 in the numerator (which would cancel the 11 in the denominator out), and answer the only option that has an 11 in the denominator: $\boxed{\textbf{(A) } \frac{7}{22}}$.

~Technodoggo

Additional note by "Fruitz": Note that one can eliminate $1/2$ by symmetry if you swap the ineq sign.

Another note by "andliu766": A shorter way to find the number of vertices and edges is to use the fact that the MAA logo is an icosahedron. :)

Solution 6 (Case Work)



WLOG, let R be at the top-most vertex of the icosahedron. There are 2 cases where $d(Q,R) > d(R,S)$.

Case 1: Q is at the bottom-most vertex

If Q is at the bottom-most vertex, no matter where S is, $d(Q,R) > d(R,S)$. The probability that Q is at the bottom-most vertex is $\frac{1}{11}$

Case 2: Q is at the second layer

If Q is at the second layer, S must be at the first layer, for $d(Q,R) > d(R,S)$ to be true. The probability that Q is at the second layer, and S is at the first layer is $\frac{5}{11} \cdot \frac{5}{10} = \frac{5}{22}$

$$\frac{1}{11} + \frac{5}{22} = \boxed{\textbf{(A) } \frac{7}{22}}$$

~

Solution 7 (efficient)

Since the icosahedron is symmetric polyhedron, we can rotate it so that R is on the topmost vertex. Since Q and S basically the same, we can first count the probability that $d(Q,R) = d(R,S)$.

$\mathfrak{Case\ 1}: d(Q,R) = d(R,S) = 1$

There are 5 points P such that $d(Q,P) = 1$. There is $5 \times 4 = \boxed{20}$ ways to choose Q and S in this case.

$\mathfrak{Case\ 2}: d(Q,R) = d(R,S) = 2$

There are 5 points P such that $d(Q,P) = 2$. There is $5 \times 4 = \boxed{20}$ ways to choose Q and S in this case.

$\mathfrak{Case\ 3}: d(Q,R) = d(R,S) = 3$

There is 1 point P such that $d(Q,P) = 3$. There is $1 \times 0 = \boxed{0}$ ways to choose Q and S in this case.

$\mathfrak{Final\ solution}$

There are 11 points P that are distinct from R. There is $11 \times 10 = \boxed{110}$ ways to choose Q and S. There is $20 + 20 + 0 = \boxed{40}$ ways to choose Q and S such that $d(Q,R) = d(R,S)$. There is $\frac{110-40}{2} = 35$ ways to choose Q and S such that $d(Q,R) > d(R,S)$. The probability that $d(Q,R) > d(R,S)$ is therefore $\frac{35}{110} = \frac{7}{22}$ which corresponds to answer choice $\boxed{A}$

~~

~EpicBird08

Vide Solution by SpreadTheMathLove(Casework and Complementary)

~Steven Chen (Professor Chen Education Palace, www.professorchenedu.com)

~IceMatrix