The American Mathematical Monthly

ISSN: 0002-9890 (Print) 1930-0972 (Online) Journal homepage: https://maa.tandfonline.com/loi/uamm20

The May Meeting of the Indiana Section

P. M. Pepper (Secretary)

To cite this article: P. M. Pepper (Secretary) (1949) The May Meeting of the Indiana Section, The American Mathematical Monthly, 56:3, 217-220, DOI: 10.1080/00029890.1949.11999364

To link to this article: https://doi.org/10.1080/00029890.1949.11999364

Published online: 18 Apr 2018.

A method of proving that the Riemann integral of a continuous function on a closed interval exists without introducing the notion of uniform continuity was presented. The upper and lower integrals are shown to be additive. Then the difference of the upper and lower integrals, considered as a function of one end-point, is shown to have a derivative identically zero. It then follows that the upper and lower integrals are equal.

11. *Trends in the teaching of algebra*, by Professor K. H. Bracewell, Hamline University.

One hundred thirty-three replies received from a selected list of leading universities and colleges to whom a questionnaire was sent indicate a downward trend in pre-college preparation in algebra. Considerable variation exists among all higher institutions in both the amount of credit and content of courses in college algebra. Very few cover the chapters on theory of investment and infinite series. Permutations and combinations, probability and mathematical induction are in somewhat greater, but still limited, use. Sixty per cent of all schools reporting divide their algebra sections into elementary and advanced classes. No marked difference in practice was distinguished between universities and colleges.

12. *Concrete expression of mathematical ideas: an exhibit*, by Mr. E. J. Berger, Monroe High School, introduced by the Secretary.

The exhibit included about forty articles made by students at Monroe High School. Some of the articles illustrated properties of various algebraic curves, such as the focal properties of the ellipse and parabola. Others were measuring devices such as Jacob's staff, transit, clinometer, and sextant.

L. E. BUSH, *Secretary*

THE MAY MEETING OF THE INDIANA SECTION

The spring meeting of the Indiana Section of the Mathematical Association of America was held at Purdue University, Lafayette, Indiana, on May 8, 1948.

Eighty-two persons attended the meeting including the following forty-seven members of the Association: J. L. Beal, Stanley Bolks, C. F. Brumfiel, Lee Byrne, G. E. Carscallen, K. W. Crain, H. E. Crull, Rev. H. F. De Baggis, M. W. De Jonge, R. H. Downing, Sister M. Virgilia Dragowski, F.O.S.F., P. D. Edwards, Ky Fan, E. L. Godfrey, Noel Gottesman, S. H. Gould, G. H. Graves, W. S. Gustin, Smith Higgins, Jr., Carl Holtom, H. K. Hughes, H. F. S. Jonah, P. S. Jones, M. W. Keller, E. L. Klinger, Florence Long, Sister Mary Ferrer McFarland, R.S.M., Karl Menger, G. T. Miller, P. M. Nastucoff, Paul Overman, Philip Peak, J. C. Polley, P. M. Pepper, C. K. Robbins, Arthur Rosenthal, A. E. Ross, G. X. Saltarelli, L. S. Shively, R. B. Stone, Raimond Struble, Anna K. Suter, G. L. Walker, M. S. Webster, A. M. Welchons, K. P. Williams and M. A. Zorn.

At the business meeting it was decided that the fall meetings which for the past several years have been held jointly with the Mathematics Section of the Indiana Academy of Science will be discontinued. The spring meeting of 1949 will be held at the University of Notre Dame.

P. D. Edwards, Ball State Teachers College, was elected Section Governor. Other officers elected at the meeting are: Chairman, H. E. Wolfe, Indiana University; Vice-chairman, A. E. Ross, University of Notre Dame. P. M. Pepper, University of Notre Dame, continues as Secretary. P. D. Edwards was appointed chairman of a committee of three, the other members to be chosen by him, to investigate the possibility of the Indiana Section compiling a report on the curricula of the colleges of Indiana similar to the report of a committee of the Michigan Section.

Professor Karl Menger of the Illinois Institute of Technology gave an interesting hour lecture entitled *Are Variables Necessary in Calculus?* Professor Menger's paper is to be published in the MONTHLY.

Professor P. S. Jones, University of Michigan, on invitation of the Section, gave a report entitled *Report of a Study of the High School Mathematics Prerequisite to Various College Curricula in Michigan College.* He described the activities and findings of the Committee on High School Mathematics of the Michigan Section of the Mathematical Association of America composed of Professors H. W. Alexander, C. C. Richtmeyer, and the speaker. Further information on this subject is presented in the February, 1949, issue of the MONTHLY in the paper by C. C. Richtmeyer.

The following papers were presented:

1. *On the use of a single axis, and of the unit circle in the teaching of trigonometry*, by Professor J. C. Polley, Wabash College.

The first part of the paper was a discussion emphasizing the lack of both utility and theoretical importance in the so-called vertical axis in the development of trigonometric theory, concluding with the opinion that it might better be abandoned in favor of a system in which the coordinates of points are defined relative to a single axis and a point thereon. The rest of the paper was devoted to a discussion on the more extensive use of the unit circle in trigonometry. In illustration the forms for the sine and cosine of the sum and difference of two angles, and those for the sum and difference of the sines and of the cosines of two angles were derived.

2. *On the teaching of determinants*, by Professor A. E. Ross, University of Notre Dame.

It was the purpose of the speaker to derive the usual properties of determinants from a set of assumptions connected as directly as possible with the solution of systems of linear equations. He considered an $n\times n$ matrix $A=(a_{ij})$ and the related systems of equations (I) $a_jx_j=\beta$. He showed that if a function $V(A)=V(\alpha_1\cdots\alpha_n)$ has the properties

$$V(\alpha_1\cdots c\cdot\alpha_k\cdots\alpha_n)=cV(\alpha_1\cdots\alpha_n) \tag{1}$$

$$V\left(\alpha_1,\cdots,\alpha_k+\sum_{j\neq k}c_j\alpha_j,\cdots,\alpha_n\right)=V(\alpha_1\cdots\alpha_n) \tag{2}$$

$$V(I)=V(e_1\cdots e_n)=1 \tag{3}$$

then

$$\begin{aligned}V(\alpha_1\cdots\alpha_n)x_k&=V(\alpha_1\cdots a_kx_k\cdots\alpha_n)=V(\alpha_1,\cdots,\textstyle\sum a_jx_j\cdots\alpha_n)\\&=V(\alpha_1\cdots\beta\cdots\alpha_n)=V_k.\end{aligned} \tag{II}$$

Thus if $V(A)\neq 0$, then system (I) has solutions x_j and $x_j=V_k/V$ (Cramer's rule). Following Artin, he proved the "product" formula, and, specializing one of the factors, showed that $V(\alpha_1\cdots\alpha_n)$ is the desired multilinear form with the correct rule of signs for the individual terms. The existence of V with the properties (1), (2), and (3) is proved by induction. In teaching, one may employ areas of parallelograms and volumes of parallelopipeds (which do have properties (1), (2), (3)), together with (II) to derive an equivalent of Cramer's rule without the use of determinants in the usual elementary sense, and thus pave the way for the general geometrical theory.

3. *Determination of the area of a triangle from its sides*, by Professor E. L. Godfrey, Defiance College, Defiance, Ohio.

The presentation of the theory of simultaneous equations and determinants may well include a method of determining the area of a triangle from the equations of its sides, as well as that commonly given using its vertices.

4. *Exponent laws for integral powers*, by Professor M. A. Zorn, Indiana University.

The exponent laws for integral powers of the same base are derived by means of a modified induction principle.

5. *An introduction to a new theory of elementary complex geometry*, by Mr. E. L. Klinger, Purdue University.

To each point in the kth, three-dimensional complex space are assigned coordinates of the form $(x+ik, u+iv)$, or briefly (z, w), where x, u and v are real variables and k is any real constant. After a discussion of distances, it was shown that the equation of any line, except one lying in planes that are perpendicular to the z-axis, had the form $Ax+Bw+C=0$, where A, B and C are real or complex constants, except when $B=0$. In this case any line through (Z, W) may be represented by the system $z=x+ik$, $w=L(w)+ia$, if L and a are real constants and L an arbitrary one. Certain derived formulas involving the angle γ of intersection of two lines were discussed.

6. *Simplicially interlocking spheres*, by Professor William Gustin, Indiana University.

In an n-dimensional euclidean space let there be given $n+1$ closed spheres S_k such that the simplex T spanning the $n+1$ centers of these spheres is non-degenerate, and such that the simplex spanning any subset of the centers is covered by the spheres with those centers. According to a known theorem, due jointly to Knaster, Kuratowski, and Masurkiewicz, there exists a point common to all the spheres S_k and the simplex T. In this note such a point is found by elementary means.

7. *Short formulations of Boolean algebra, using ring operations*, by Dr. Lee Byrne, Purdue University.

Much interest has attached to recent formulations of Boolean algebras intended to emphasize their character as rings, and thus featuring especially ring operations. Most of these are relatively long, and Dr. Byrne's note was concerned with the question whether a simple formulation of this type might show appreciably more brevity. Leaving closure (and non-emptiness) assumptions tacit, he presented four "transformation" postulates, followed by ten theorems, which suffice to show the system to be a ring, a Boolean ring (i.e. one in which every element is idempotent), and a Boolean algebra (i.e., a Boolean ring with unit). The number of transformation axioms appears to be about two less than in previous versions with a similar approach.

8. *On the eigenvalues of symmetric kernels*, by Professor Ky Fan and Mr. Norman Haaser, University of Notre Dame.

Let the kernel $K(s, t)$ be real symmetric in $a\leqq s$, $t\leqq b$, and such that the classical Hilbert-Schmidt's theory is applicable. (I) Let ξ be a real number and let the eigenvalues λ_i of K be so arranged that $|\lambda_i-\xi|\leqq|\lambda_{i+1}-\xi|$, $(i=1, 2, \cdots)$. Then for any fixed integer $m>0$, $|\lambda_j-\xi|$ is the greatest value which can be taken by the *G. L. B.* of the expression $(\|(\xi K-I)^m f\|/\|K^m f\|^{1/m}$ when f is orthogonal to $j-1$ arbitrarily fixed functions. (II) From the case $m=1, j=1$ of (I) one obtains directly the inclusion theorem of D. H. Weinstein (*Proc. Nat. Acad. Sci.*, vol. 20, 1934, pp. 529–532. (III) If the eigenvalues of K are bounded from below and so arranged that $\lambda_1 \leqq \lambda_2 \leqq \cdots \leqq \lambda_i \leqq \cdots$, then as limiting case $\xi = -\infty$ of (I), for any fixed even integer m, λ_j is the greatest value which can be taken by the *G. L. B.* of $(K^{m-1}f, f)/(K^m f, f)$ when f is orthogonal to $j-1$ arbitrarily fixed functions. If, in addition, K is positive definite, then it can be shown, as was proved by L. Collatz (*Math. Zeitschr.*, vol. 46, 1940, pp. 692–708) and R. Iglisch (*Math. Ann.*, vol. 118, 1942, pp. 263–275), that the above characterization of λ_j holds for any integer m, even or odd, and $(K^{m-1}f, f)/(K^m f, f)$ is non-increasing with respect to m. (IV) If K, K', K'' are three real symmetric kernels such that

$$K(s, t) = \int_a^b K'(s, r)K''(r, t)dr,$$

and if their respective eigenvalues λ_i, λ_i', λ_i'' are so arranged that

$$|\lambda_i| \leqq |\lambda_{i+1}|, \qquad |\lambda_i'| \leqq |\lambda_{i+1}'|, \qquad |\lambda_i''| \leqq |\lambda_{i+1}''|,$$

then $|\lambda_{i+j+1}| \geqq |\lambda_{i+1}'| \cdot |\lambda_{j+1}''|$ holds for all $i, j \geqq 0$. This inequality implies that, for the composite kernel K, the series $\sum |\lambda_n|^{-1}$ converges. This is a particular case of a thorem due to Lalesco-Gheorghini (cf. Hille-Tamarkin, *Acta Math.*, vol. 57, 1931, p. 31).

P. M. PEPPER, *Secretary*

CALENDAR OF FUTURE MEETINGS

Joint Meeting with American Society for Engineering Education, Troy, New York, June 20–21, 1949.

Thirty-first Summer Meeting, Boulder, Colorado, August 29–30, 1949.

Thirty-third Annual Meeting, New York City, December 30, 1949.

ALLEGHENY MOUNTAIN, West Virginia University, Morgantown, May 7, 1949.

ILLINOIS, Bradley University, Peoria, May 13–14, 1949

INDIANA, University of Notre Dame, May 7, 1949

IOWA, Drake University, Des Moines, April 15–16, 1949

KANSAS, Kansas State College, Manhattan, April 2, 1949

KENTUCKY, Centre College, Danville, May 14, 1949

LOUISIANA-MISSISSIPPI, University of Mississippi, Oxford, April 8–9, 1949

MARYLAND-DISTRICT OF COLUMBIA-VIRGINIA, University of Virginia, Charlottesville, Spring, 1949

METROPOLITAN NEW YORK, Brooklyn College, April 9, 1949

MICHIGAN, Wayne University, Detroit, April 2, 1949

MINNESOTA, Gustavus Adolphus College, St. Peter, May 7, 1949

MISSOURI, University of Missouri, Columbia, April 9, 1949

NEBRASKA, Lincoln, May, 1949

NORTHERN CALIFORNIA

OHIO, Ohio State University, Columbus, April 2, 1949

OKLAHOMA

PACIFIC NORTHWEST, Oregon State College, Corvallis, March 25–26, 1949

PHILADELPHIA, Haverford College, November 26, 1949

ROCKY MOUNTAIN, Colorado School of Mines, Golden, April 22–23, 1949

SOUTHEASTERN, University of Alabama, University, March 18–19, 1949

SOUTHERN CALIFORNIA

SOUTHWESTERN

TEXAS, Denton, April 8–9, 1949

UPPER NEW YORK STATE, University of Buffalo, April 30, 1949

WISCONSIN, Lawrence College, Appleton, May 14, 1949