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# The Sixth Annual Meeting of The Indiana Section

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deformation are equal. Since the quantity on the right hand side should be a minimum, the problem is to find the equation of the elastic curve which will make it so. An approximate solution may be found by setting up a series of the form

$$y = a_1 f(x_1) + a_2 f(x_2) + a_3 f(x_3) + \cdots,$$

in which each term satisfies the boundary condition of the problem and where $a_1$, $a_2$, $a_3$, etc. are parameters to be determined so that the curve produces a minimum value for $P$. This value of $P$ is a critical value and is on the boundary between stable and unstable equilibrium.

6. In this paper Miss Carlson pointed out some of the differences in methods used in teaching classes of about one hundred students as compared with methods used in teaching small classes. Also, she gave figures showing that the grade of work done by the students in the large classes compared favorably with the grade of work done by the students in the small classes.

7. Mr. Jackson spoke briefly about the organization and purposes of a committee on college entrance requirements in geometry, with regard to which a more detailed announcement appeared in the August-September number of this Monthly.

A. L. UNDERHILL, *Secretary*

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## THE SIXTH ANNUAL MEETING OF THE INDIANA SECTION

The sixth annual meeting of the Indiana Section of the Mathematical Association of America was held May 3–4, 1929 at Culver Military Academy, Culver, Indiana.

There were sixty present at the meeting including the following twenty-nine members of the Association: W. C. Arnold, Gladys Banes, Stanley Bolks, H. T. Davis, S. C. Davisson, C. S. Doan, J. E. Dotterer, P. D. Edwards, E. D. Grant, H. E. H. Greenleaf, G. E. Happell, C. T. Hazard, F. H. Hodge, H. K. Hughes, Juna M. Lutz, William Marshall, T. E. Mason, H. R. Mathias, G. T. Miller, J. A. Reising, C. K. Robbins, L. S. Shively, J. R. K. Stauffer, R. B. Stone, R. O. Virts, C. J. Waits, K. P. Williams, W. A. Zehring, H. A. Zinszer.

On Friday afternoon at 5:30 a reception was given to the visiting members and their guests. At 6:30 a complimentary banquet which was held in the mess hall was attended by approximately ninety guests of the academy. General L. R. Gignilliat, superintendent of the academy, presided at the banquet and made a brief address of welcome. Entertainment was furnished by Major Norman Imrie, head of the public speaking department of the academy, who regaled the guests with stories appropriate to the occasion. Music was furnished during the banquet by the cadet band.

At eight o'clock a public lecture under the auspices of the academy was given in the gymnasium by Professor Warren Weaver of the University of Wisconsin on the subject, "Science and Imagination." Professor Weaver presented the new view of mathematical and physical science which is emerging from modern speculations. Ancient mathematics, according to the speaker, made use of defined elements and self-evident axioms. Modern mathematics makes use of undefined elements and assumed postulates. The theorems of modern mathematics are thus creations, not discoveries. Since mathematics is now a product of the creative imagination, it deserves consideration as an art. A somewhat similar change has come about in the logical structure of physical theories. The older model theories of physics explaining by analogy are comparable to the older mathematics; while the modern more abstract physical theories are more closely related with modern postulational mathematics. In the development of physical theories, therefore, the imagination now plays a more significant rôle than formerly and the theories have become more artistic in structure.

At 8:30 Saturday morning, a military review was held in honor of the visitors and this was followed by a tour of the academy buildings.

At the session at 10:00 a.m. in the Memorial Building of the Academy presided over by Professor H. E. H. Greenleaf, De Pauw University, chairman, the following officers were elected: Professor H. A. Zinszer, Hanover College, Chairman; Professor E. D. Grant, Earlham College, Vice-chairman; Professor H. T. Davis, Indiana University, Secretary-treasurer.

A chairman's address was made by Professor Greenleaf on the subject, "Mathematics in the Fundamentals of Music." Professor Greenleaf, considering the musical scale and the principal intervals of music as fundamental, discussed the changes in the scale from earliest times to the present. The speaker pointed out the mathematical basis of the Pythagorean, the diatonic, the mean-tone temperament, and the equal temperament scales and intervals and made a comparison of the four in regard to tonality.

The remainder of the program consisted of the following papers:

1. "The sectioning of freshman engineering students in mathematics," by Professor William Marshall, Purdue University.

2. Extracts from a discourse of J. F. Hennert (Utrecht, 1765): "On the necessity of including the study of mathematics in a good education," by Professor T. E. Mason, Purdue University.

3. "Invariance under the symmetric group of order three of a functional equation due to Abel," by Professor P. D. Edwards, Ball State Teacher's College.

4. "A solution of the biquadratic equation," by Professor E. D. Grant, Earlham College.

5. "A certain general type of contact transformation in three dimensions," by Professor C. K. Robbins, Purdue University.

6. "Three methods for finding the shortest distance between two skew lines" by Margaret L. Darragh, Hanover College (Introduced by Professor Zinszer).

7. "Transformations by reciprocal rays," by Mr. James Avas Cooley, Indiana University (Introduced by Professor H. E. Wolfe).

8. "Some properties of the circles that can be connected with the complete quadrilateral," by Mr. Maurice M. Lemme, De Pauw University (Introduced by Professor Greenleaf).

9. "Present status of the theory of the Volterra integral equation of the second kind," by Professor H. T. Davis, Indiana University.

10. "Notes on quantum mechanics," by Professor H. A. Zinszer, Hanover College.

Abstracts of the papers follow:

1. In the fall of 1928 the mathematics department of Purdue University divided the incoming freshmen in engineering mathematics into three groups: a sub-collegiate group, a normal group, and an advanced or honor group. Professor Marshall set forth in some detail the reasons for this sectioning, how it was done, how the various groups were handled, and the results of the experiment in so far as they are apparent at the present time.

2. Professor Mason's paper consisted of a translation of the inaugural address of J. F. Hennert at Utrecht on a subject of perennial interest to mathematicians. The striking feature of this discourse lies in the fact that the criticisms of students made by this professor in 1765 sound very modern. Apparently students have not changed much in the last century and a half. There are some reasons advanced to the people of the commercial town of Utrecht for the study of mathematics and physics which we should put today under the heading of reasons for the study of engineering.

3. In order that $F(x, y)F(y, z)F(z, x)$ be identical with $F(y, x)F(z, y)F(x, z)$ it is obviously sufficient that $F(x, y)$ be composed of factors which are (1) functions of $x$ only, or (2) functions of $y$ only, or (3) symmetric functions of $x$ and $y$. That the condition is also necessary is not evident. Proof is given that if $F$ is an algebraic function which is rational, or if irrational, one that belongs to a realm in which the unique factorization law holds, then the conditions named are necessary. Extension is made to the invariance of the function $F(x_1, x_2)$ $F(x_2, x_3) \cdots F(x_n, x_1)$ under the symmetric group of order $n$.

4. The biquadratic equation is first reduced to the form lacking the term in $x^3$. The roots are assumed to be $a \pm \sqrt{b}$, $-a \pm \sqrt{c}$. If we express the relation between the roots and the coefficients, there are three equations to solve for $a$, $b$, and $c$. The elimination of $b$ and $c$ leads to an equation of the sixth degree in $a$, containing the terms $a^2$, $a^4$, and $a^6$. This equation may be solved by Cardan's method in any numerical case; $b$ and $c$ may then be obtained, and the four roots written out.

5. If the transformation $x' = f_1(x, y, z, p, q)$, $y' = f_2(x, y, z, p, q)$, $z' = f_3(x, y, z, p, q)$, $p' = f_4(x, y, z, p, q)$, $q' = f_5(x, y, z, p, q)$ transforms a union of plane elements into a union of plane elements, it is a contact transformation. The analytical condition is that the vanishing of $p'dx' + p'dy' - dz'$ is a consequence of the vanishing of $pdx + qdy - dz$. This leads to a set of four partial differential equations, the integration of which can be determined (theoretically) according to the general theory of such equations. The actual application of this condition

seems to lead to insurmountable difficulties, but certain special cases are of interest. In particular if $f_4$ and $f_5$ are functions of $p$ and $q$ only, that is if the orientation of the plane in the transformed element depends only on the orientation of the plane in the original element, the system of partial differential equations can be completely solved. If $f_4=p$ and $f_5=q$, and the arbitrary function introduced by integration assumes a certain value, the transformation becomes the well known dilation.

6. The following methods were discussed: I. Through each of the lines any plane is passed perpendicularly. At some position of these planes their line of intersection will intersect each of the skew lines. The distance between these points is the shortest distance between the two lines. II. A plane is constructed perpendicular to one of the lines and through each line a plane is passed perpendicular to it. By expressing the equations for these two planes in their normal form and adding the right members the distance is obtained. III. The last method finds the distance directly by the minimizing process.

7. Laguerre in Nouvelles Annales, 3rd series, volume 1, defined the transformation by reciprocal rays and gave some of its properties. Mr. Cooley introduced a new constant for the modulus of the transformation, defining it as a cross ratio. In terms of the constant, he developed relations between the angle which reciprocal rays make with each other and the axis of transformation. He also gave additional properties and applications of the transformation especially with regard to circles and their tangents.

8. This paper proved by methods of Euclidean geometry alone the following theorem given by Jakob Steiner in the Annales de Gergonne, vol. 18, p. 16: In each of the four triangles formed by the sides of a complete quadrilateral there is one circle inscribed and three circles escribed, making in all sixteen circles. The centers of these sixteen circles arrange themselves in groups such that each of the four circles of one group cuts orthogonally all the circles of the other group. The lines of centers of the two groups of circles are perpendicular to each other. The lines of centers of the two groups of circles intersect at the point of intersection of the circles circumscribed to the four triangles forming the quadrilateral. Various consequences of the theorem were also exhibited by the speaker.

9. The methods used to solve the Volterra integral equation, not only for the case of continuous kernels, but for various types of discontinuities, were discussed. Numerous properties of the equation of the closed cycle, namely the case of the kernel of the form $K(x-t)$, were exhibited.

10. Assuming a closed system consisting of a nucleus and an electron the former being a point charge located at the origin of coordinates, the principle of the conservation of energy expressed in Newtonian notation was imposed. Applying the principle of Maupertuis (least action) and assuming the resulting equation to describe a family of wave-fronts travelling with a speed $E/\sqrt{[2m(E-V)]}$, where $E$ is the total energy and $V$ the potential energy function, a particular form of de Broglie's wave equation finally resulted.

At the afternoon session a resolution was adopted by the members of the section expressing their appreciation of the welcome that had been given them by the academy and of the efforts of General Gignilliat and of Major G. H. Crandall, Captain L. R. Kellam and other members of the department of mathematics who had contributed to the success of the meeting.

H. T. DAVIS, *Secretary*

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# A MODIFICATION OF A PROOF BY STEINER

By OTTO DUNKEL, Washington University

INTRODUCTION. An elegant and elementary proof was given by Steiner of the theorem that the equilateral triangle has the greatest area of all triangles having the same perimeter.[1] This proof is interesting in that no use is made of either parallels or metrical expressions for the area; it applies therefore whether the sum of the angles of a triangle is supposed to be less than, equal to, or more than 180°, and Steiner showed that his proof applied to spherical triangles without essential change. His proof consists of two parts of which the first part is essentially the proof under Theorem I below, while the second part has been altered to the form of proof under Theorem II. This modified form of proof is applicable to other similar geometrical theorems, and two such theorems are proved in this way without the use of parallels or metrical expressions for the area. The following proofs are worded for spherical triangles since in a few places restrictions are required peculiar to this form of geometry. For the cases where the sum of the angles of the triangle is less than or equal to 180° the proofs are essentially the same but simpler. In conclusion two theorems are given which result from the consideration of a metrical expression for the area. In the discussions below when one side of a triangle is designated as a base the term side will be considered to apply only to the two remaining sides.

THEOREM I. *Two triangles which have equal perimeters and bases of equal lengths have unequal areas if they are neither congruent nor symmetric. The triangle having the smaller area has the smallest base angle, the greatest base angle, the shortest side and the longest side.*

PROOF: Let $ABC$ and $A'B'C'$ be two triangles which are neither congruent nor symmetric, but are such that $AB=A'B'$, $AC+BC=A'C'+B'C'$, $A\leqq B$, $A'\leqq B'$, where $A$ denotes the angle $BAC$ etc. The equality signs in the last relation are assumed to hold for only one triangle, for otherwise the two triangles would be congruent. Let the bases be made to coincide so that $A'$ falls at $A$ and $B'$ at $B$. If then $C$ and $C'$ fall on opposite sides of the common base, we shall replace one triangle by its symmetric triangle and we shall suppose that the lettering of the vertices of the new triangle is the same as that for the old.

[1] Steiner, *Sur le maximum et le minimum des figures dans le plan, sur la sphère et dans l'espace en général*, Crelle's Journal, vol. 24 (1842), pp. 96–99.