Problem Set 0 — Prior Knowledge Warm-Up

Goal. This notebook helps you self-check the prerequisite topics:

- Python / Jupyter / NumPy
- Linear Algebra
- Calculus
- Probability / Statistics
- Optimization (Convex function, Quadratic Program, KKT condition)

Each section has short tasks. Some cells include light tests. It's okay if you can't finish everything right away—use this to guide your review.

0) Jupyter notebook

Different size of title

Different size of title

Different size of title

Different size of title

$$x^2$$

$$\begin{aligned} & \min_x f(x) \\ s.t. \quad & g(x) = 0 \end{aligned}$$

```
# Data type
integar_a = 1
float_b = 1.5
string_c = "Hello"
print(integar_a, float_b, string_c)
```

```
1 1.5 Hello
```

```
# List
l = [integar_a, '123', [1, 2, 3, 4 ], 1 + 1j]
print(l)
print("The first element of list l is", l[0])
l.append(5)
print("After append, l is", l)
```

```
[1, '123', [1, 2, 3, 4], (1+1j)]
The first element of list l is 1
After append, l is [1, '123', [1, 2, 3, 4], (1+1j), 5]
```

```
# index
l = [0,1,2,3,4,5,6,7,8,9]
print(l[2:5]) # [2, 3, 4]
print(l[2:])  # [2, 3, 4, 5, 6, 7, 8, 9]
print(l[:5])  # [0, 1, 2, 3, 4]
print(l[-3:]) # [7, 8, 9]
print(l[:-3]) # [0, 1, 2, 3, 4, 5, 6]
print(l[::2])   # [0, 2, 4, 6, 8]
```

```
[2, 3, 4]
[2, 3, 4, 5, 6, 7, 8, 9]
[0, 1, 2, 3, 4]
```

```
[7, 8, 9]
[0, 1, 2, 3, 4, 5, 6]
[0, 2, 4, 6, 8]
```

1) Python / Jupyter / NumPy

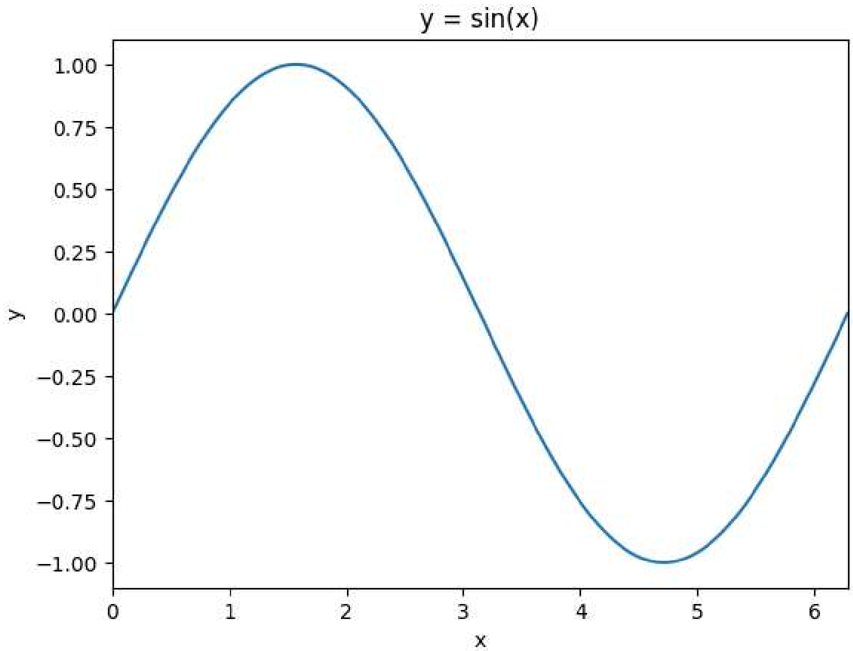
Warm-up tasks to get comfortable with arrays, broadcasting, vectorization, and plotting.

```
# --- Setup & helpers (run this first) ---
import math
import numpy as np
import matplotlib.pyplot as plt

np.set_printoptions(precision=4, suppress=True)
rng = np.random.default_rng(0)
```

```
#### CODING EXERCISES ####
# 1.a) Create a length-5 NumPy array `a` with values [0, 1, 2, 3, 4]
# TODO: set a = ...
a = np.array([0, 1, 2, 3, 4])

# 1.b) Create another array `b` of the same length where b[i] = a[i]^2 (vectorized).
# TODO: set b = ...
b = a ** 2

# 1.c) Compute the dot product of `a` and `b` and store in `ab_dot`.
# TODO: set ab_dot = ...
ab_dot = np.dot(a, b)

# 1.d) Use boolean indexing to select elements of `b` that are > 5. Store in `b_gt_5`.
# TODO: set b_gt_5 = ...
b_gt_5 = b[b > 5]

print(a, b, ab_dot, b_gt_5)
```

```
[0 1 2 3 4] [ 0  1  4  9 16] 100 [ 9 16]
```

```
#### CODING EXERCISES ####
# 1.e) Plotting (Matplotlib only — no seaborn).
# TODO: Plot y = sin(x) for x in [0, 2π]. Create a single figure.
# Hint: Use np.linspace, plt.figure, plt.plot, plt.title
x = np.linspace(0, 2*np.pi, 300)
y = np.sin(x)

plt.figure()
plt.plot(x,y)
plt.title('y = sin(x)')
plt.xlabel("x")
plt.ylabel("y")
plt.xlim(0,2*np.pi)
plt.show()
```



2) Linear Algebra

Topics: norms, matrix multiplication, eigen/SVD.

Norms

Let V be a vector space (e.g. $\mathbb{R}^n$ or $\mathbb{C}^n$).
A function

$$\|\cdot\| : V \to [0, \infty)$$

is called a **norm** on V if for all $\mathbf{x}, \mathbf{y} \in V$ and scalars α, the following hold:

1. **Non-negativity and definiteness**

$$\|\mathbf{x}\| \geq 0, \quad \|\mathbf{x}\| = 0 \iff \mathbf{x} = \mathbf{0}$$

2. **Homogeneity**

$$\|\alpha \mathbf{x}\| = |\alpha| \, \|\mathbf{x}\|$$

3. **Triangle inequality**

$$\|\mathbf{x} + \mathbf{y}\| \leq \|\mathbf{x}\| + \|\mathbf{y}\|$$

A common class of norms are the ℓ_p norms, defined as

$$\|\mathbf{x}\|_p = \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p}, \quad (1 \leq p < \infty),$$

and

$$\|\mathbf{x}\|_\infty = \max_{1 \leq i \leq n} |x_i|.$$

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2.a) **TODO**: Give the formulation of l_2 norm of a vector $x = [x_1, x_2, x_3, x_4, x_5]$

$\|x\|_2 = \sqrt{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}$

2.b) Give the l_2 norm of a vector $y = [1, 5, 3, 1]$

$\|y\|_2 = \sqrt{1^2 + 5^2 + 3^2 + 1^2} = 6$

```
#### CODING EXERCISES ####
# 2.c) Norms in numpy  (Hint: np.linalg.norm)
x = np.array([1,2,3,4,5])
x_norm=np.linalg.norm(x)
print(x_norm)
```

```
7.416198487095663
```

Matrix Operations

A **matrix** is a rectangular array of numbers with m rows and n columns.
For example, $A \in \mathbb{R}^{m \times n}$ can be written as

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}.$$

- **Addition**: If $A, B \in \mathbb{R}^{m \times n}$, then
$$A + B = (a_{ij} + b_{ij})_{m \times n}.$$
- **Scalar multiplication**: For $\alpha \in \mathbb{R}$,
$$\alpha A = (\alpha \cdot a_{ij})_{m \times n}.$$
- **Matrix multiplication**: If $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{n \times p}$, then
$$C = AB \in \mathbb{R}^{m \times p}, \quad c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}.$$

```
#### CODING EXERCISES ####
# 2.d) Matrix multiplication in numpy
A = np.array([[1, 2],
              [3, 4]])

B = np.array([[5, 6],
              [7, 8]])

# TODO: C = AB
C = np.matmul(A,B)
print("AB =\n", C)
```

```
AB =
 [[19 22]
 [43 50]]
```

Eigenvalues and Eigenvectors

Let $A \in \mathbb{R}^{n \times n}$ be a square matrix.
A scalar $\lambda \in \mathbb{R}$ is called an **eigenvalue** of A if there exists a nonzero vector $\mathbf{v} \in \mathbb{R}^n$ such that
$$A\mathbf{v} = \lambda \mathbf{v}.$$
Here, $\mathbf{v}$ is called an **eigenvector** corresponding to λ.

PAPER and PEN EXERCISES

2.e) **TODO**: Give an eigenvalue and corresponding eigenvector of matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}.$$

To find eigenvalues, set $det(A - \lambda I) = 0$:

$$(1 - \lambda)(1 - \lambda) - 4 = 0 \Leftrightarrow \lambda^2 - 2\lambda - 3 = 0$$

$\lambda = \{-1, 3\}$

To find eigenvectors, set $(A - \lambda I)v = 0$:

For $\lambda_1 = 3$, we get $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. For $\lambda_2 = -1$, we get $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

$v = (\lambda_1 = -1,\ v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}), \quad (\lambda_2 = 3,\ v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix})$

```
#### CODING EXERCISES ####
# 2.f) Eigen value in numpy
A = np.array([[1, 2],
              [2, 1]])

eigvals, eigvecs = np.linalg.eig(A)

print("Eigenvalues:", eigvals)
print("Eigenvectors:\n", eigvecs)
```

```
Eigenvalues: [ 3. -1.]
Eigenvectors:
 [[ 0.7071 -0.7071]
 [ 0.7071  0.7071]]
```

Positive Semidefinite (PSD) Matrices

Definition. A symmetric matrix $A \in \mathbb{R}^{n \times n}$ is **positive semidefinite (PSD)** if
$$\mathbf{x}^\top A\,\mathbf{x}\ \geq 0 \quad \text{for all } \mathbf{x} \in \mathbb{R}^n.$$
It is **positive definite (PD)** if the inequality is strict for all $\mathbf{x} \neq \mathbf{0}$.

Equivalent characterizations (PSD).

- All eigenvalues of A are nonnegative: $\lambda_i(A) \geq 0$.
- A is a Gram matrix: there exists $B \in \mathbb{R}^{m \times n}$ with $A = B^\top B$.
- All principal submatrices of a PSD matrix are PSD.

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2.g) **TODO**:

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix},$$

- (g.1) Compute eigenvalues of each matrix and decide PSD/PD/indefinite.

For matrix A: $\lambda_i(A) = \{1, 3\}$. Since $\lambda_i(A) > 0$, matrix A is PD.

For matrix B: $\lambda_i(B) = \{0, 1, 2\}$. Since $\lambda_i(B) \geq 0$, B is PSD.

- (g.2) For B, verify directly that $\mathbf{x}^\top B\,\mathbf{x} \geq 0$ for all $\mathbf{x}$.

Let $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, then

$\mathbf{x}^\top B\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 & 0 & 2x_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1^2 + 2x_3^2 \geq 0 \text{ for all } \mathbf{x} \in \mathbb{R}^3.$

3) Calculus

Topics: gradients, Hessians;

Gradient & Hessian

- **Gradient.** For a differentiable scalar field $f : \mathbb{R}^n \to \mathbb{R}$,

$$\nabla f(\mathbf{x}) = \begin{bmatrix} \frac{\partial f}{\partial x_1}(\mathbf{x}) \\ \vdots \\ \frac{\partial f}{\partial x_n}(\mathbf{x}) \end{bmatrix},$$

- **Hessian.** If second partials exist,

$$\nabla^2 f(\mathbf{x}) = \left[\frac{\partial^2 f}{\partial x_i\, \partial x_j}(\mathbf{x})\right]_{i,j=1}^{n}.$$

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3.a) **TODO**: For $f(\mathbf{x}) = \frac{1}{2}\,\mathbf{x}^\top A\,\mathbf{x} + \mathbf{b}^\top\mathbf{x} + c$ with symmetric (A),

$$\nabla f(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$$
$$\nabla^2 f(\mathbf{x}) = A$$

Convex Functions

Definition (convex). Let $f : \mathbb{R}^n \to (-\infty, +\infty]$ with a **convex domain** D

f is **convex** if for all $x, y \in D$ and $t \in [0,1]$,

$$f(tx + (1-t)y) \;\le\; t\,f(x) + (1-t)\,f(y).$$

It is **strictly convex** if the inequality is strict whenever $x \neq y$ and $t \in (0,1)$.

Second-order condition (C^2 on a convex domain). f is convex iff

$$\nabla^2 f(x) \;\succeq\; 0 \quad \text{for all } x \in D.$$

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3.b) **TODO**: Show the condition when $f(\mathbf{x}) = \frac{1}{2}\,\mathbf{x}^\top A\,\mathbf{x} + \mathbf{b}^\top\mathbf{x} + c$ is convex.

From the previous exercise, $\nabla^2 f(\mathbf{x}) = A$. Hence, $f(\mathbf{x})$ will be convex if $A \succeq\; 0 \text{ for all } x \in D$, which as previously studied, requires $\lambda_i(A) \geq 0$.

4) Statistic

Topics: Normal & Uniform distribution, Expectation, Variance

Basic definition

- **Expectation (mean).**
  Discrete:

$$\mathbb{E}[X] = \sum_x x\,\mathbb{P}(X = x)$$

  Continuous:

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x\,f_X(x)\,dx$$

- **Variance.**

$$\mathrm{Var}(X) = \mathbb{E}\big[(X - \mathbb{E}[X])^2\big] = \mathbb{E}[X^2] - \big(\mathbb{E}[X]\big)^2.$$

- **Normal (Gaussian) distribution.** Define $X \sim \mathcal{N}(\mu, \sigma^2)$:
$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \qquad \mathbb{E}[X] = \mu, \quad \mathrm{Var}(X) = \sigma^2.$$
- **Uniform distribution on ([a,b]).** Define $X \sim \mathcal{U}(a, b)$,
$$f(x) = \frac{1}{b-a} \quad (a \le x \le b), \qquad \mathbb{E}[X] = \frac{a+b}{2}, \quad \mathrm{Var}(X) = \frac{(b-a)^2}{12}$$
.

```
#### CODING EXERCISES ####
# 4.a) TODO: Normal distribution

# Normal N(mu, sigma^2) (Hint: np.random.normal)
mu, sigma = 2.0, 3.0
z = np.random.normal(mu, sigma,100000)
print("Normal sample mean/var ~", z.mean(), z.var())

# 4.b) TODO: Uniform distribution

# Uniform U(a, b) (Hint: np.random.uniform)
a, b = -1.0, 5.0
u = np.random.uniform(a,b,100000)
print("Uniform sample mean/var ~", u.mean(), u.var())

# Theoretical checks
print("Normal theory mean/var =", mu, sigma**2)
print("Uniform theory mean/var =", (a+b)/2, (b-a)**2/12)
```

```
Normal sample mean/var ~ 1.9880137612942752 9.030038956568484
Uniform sample mean/var ~ 1.9981920165483056 2.9999870837933327
Normal theory mean/var = 2.0 9.0
Uniform theory mean/var = 2.0 3.0
```

Bayes' Theorem

Statement For events A and B with $\mathbb{P}(B) > 0$,
$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(B \mid A)\,\mathbb{P}(A)}{\mathbb{P}(B)}.$$
Here, $\mathbb{P}(A)$ is the **prior**, $\mathbb{P}(B \mid A)$ the **likelihood**, $\mathbb{P}(A \mid B)$ the **posterior**, and $\mathbb{P}(B)$ the **evidence**.

Total probability If $\{H_i\}$ is a partition of the sample space,
$$\mathbb{P}(B) = \sum_i \mathbb{P}(B \mid H_i)\,\mathbb{P}(H_i).$$

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4.c) **TODO**:

You pick one of two boxes at random (each with probability $1/2$):

- Box A: 3 red ball, 1 blue ball.
- Box B: 1 red ball, 3 blue ball.

You then draw one ball and observe it is red. What is $\mathbb{P}(A \mid \mathrm{red})$?

$\mathbb{P}(A \mid \mathrm{red}) = \frac{\mathbb{P}(\mathrm{red}|A)\mathbb{P}(A)}{\mathbb{P}(\mathrm{red})} = \frac{0.75 \times 0.5}{0.5} = 0.75$

5) Optimization

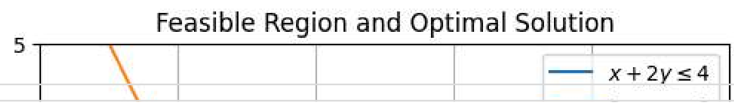
Topics: cvxpy, KKT condition.

Linear Programming Example

We want to solve the following linear program:

$$\begin{aligned} \max_{x,y} \quad & x + y \\ \text{s.t.} \quad & x + 2y \leq 4, \\ & 2x + y \leq 6, \\ & x \geq 0,\ y \geq 0. \end{aligned}$$

```
# visualization
x = np.linspace(0, 5, 200)

y1 = (4 - x) / 2
y2 = 6 - 2*x

# Feasible region: intersection
plt.figure(figsize=(6,6))

# Plot constraint lines
plt.plot(x, y1, label=r"$x+2y \leq 4$")
plt.plot(x, y2, label=r"$2x+y \geq 6$")

# Fill feasible region
y_upper = np.minimum(y1, y2)
y_upper = np.maximum(y_upper, 0)  # ensure non-negative
plt.fill_between(x, 0, y_upper, where=(y_upper>=0), color="lightblue", alpha=0.5)

# Mark optimal point (from CVXPY solution)
opt_x, opt_y = 8/3, 2/3
plt.plot(opt_x, opt_y, "ro", label="Optimal")

# Labels and formatting
plt.xlim(0, 5)
plt.ylim(0, 5)
plt.xlabel("$x$")
plt.ylabel("$y$")
plt.title("Feasible Region and Optimal Solution")
plt.legend()
plt.grid(True)
plt.show()
```




```
import cvxpy as cp

#### CODING EXERCISES ####

# 4.a) TODO: Define variables
# Variables
x = cp.Variable()
y = cp.Variable()

# 4.b) TODO: Define objectives
# Objective: maximize x + y
# (Hint: cp.Maximize)
objective = cp.Maximize(x+y)

# 4.c) TODO: Define constraints
# Constraints:
# x + 2y <= 4
# 4x + 2y <= 12
# x >= 0
# y >= 0
constraints = [
    x + 2*y <= 4,
    4*x + 2*y <= 12,
    x >= 0,
    y >= 0
]

# Problem definition
# (Hint: cp.Problem)
problem = cp.Problem(objective, constraints)

# Solve
problem.solve()

print("status:", problem.status)
print("optimal value:", problem.value)
print("optimal (x,y):", x.value, y.value)
```

```
status: optimal
optimal value: 3.333333332201236
optimal (x,y): 2.6666666676628243 0.6666666645384114
```

KKT condition

You don't need to fully understand but learn to use it as a tool.

The **Karush–Kuhn–Tucker (KKT) conditions** are necessary conditions for a solution x^* to be optimal in a constrained optimization problem of the form:

$$\begin{aligned} \min_x \quad & f(x) \\ \text{s.t.} \quad & g_i(x) \leq 0, \quad i = 1, \dots, m, \\ & h_j(x) = 0, \quad j = 1, \dots, p. \end{aligned}$$

KKT conditions (assuming regularity conditions hold):

1. **Primal feasibility**

$$g_i(x^*) \leq 0, \quad h_j(x^*) = 0$$

2. **Dual feasibility**

$$\lambda_i \geq 0, \quad i = 1, \dots, m$$

3. **Stationarity**

$$\nabla f(x^*) + \sum_{i=1}^{m} \lambda_i \nabla g_i(x^*) + \sum_{j=1}^{p} \mu_j \nabla h_j(x^*) = 0$$

4. **Complementary slackness**

$$\lambda_i g_i(x^*) = 0, \quad i = 1, \ldots, m$$

Here:

- λ_i is defined as **Lagrange multipliers** for inequality constraints.
- μ_j is defined as **Lagrange multipliers** for equality constraints.

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5. a) **TODO**: Solve KKT by hand

We solve

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & f(x) = x^\top x + c^\top x, \quad c = \begin{bmatrix} -2 \\ -6 \end{bmatrix} \\ \text{s.t.} \quad & a^\top x = 3, \quad a = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\ & x \geq 0 \end{aligned}$$

KKT (by hand) :

$f(x) = x_1^2 + x_2^2 - 2x_1 - 6x_2$ subject to:

1. $a^\top x = 3 \Rightarrow x_1 + x_2 = 3$
2. $x \geq 0 \Rightarrow x_1 \geq 0, x_2 \geq 0$

We introduce the lagrange multipliers λ_1, λ_2 for inequality constraints and μ for equality constraints.

We define the lagrangian: $\mathcal{L}(x_1, x_2, \lambda_1, \lambda_2, \mu) = x_1^2 + x_2^2 - 2x_1 - 6x_2 + \mu(x_1 + x_2 - 3) - \lambda_1 x_1 - \lambda_2 x_2$

KKT Conditions:

1. Stationarity (gradients = 0): $\frac{\partial \mathcal{L}}{\partial x_1} = 2x_1 - 2 + \mu - \lambda_1 = 0$, $\frac{\partial \mathcal{L}}{\partial x_2} = 2x_2 - 6 + \mu - \lambda_2 = 0$
2. Primal Feasibility: $x_1 + x_2 = 3$, $x_1 \geq 0$, $x_2 \geq 0$
3. Dual Feasibility: $\lambda_1 \geq 0$, $\lambda_2 \geq 0$
4. Complementary Slackness: $\lambda_1 x_1 = 0$, $\lambda_2 x_2 = 0$

Try case by case (ie. [$x_1 > 0$, $x_2 > 0$], [$x_1 > 0$, $x_2 = 0$], [$x_1 = 0$, $x_2 > 0$], [$x_1 = 0$, $x_2 = 0$]):

Case 1: $x_1 > 0$, $x_2 > 0$,

then, by complementary slackness $\lambda_1 = \lambda_2 = 0$, resulting in $x_2 - x_1 = 2$. Using $x_1 + x_2 = 3$, we get $x_1 = 0.5$ and $x_2 = 2.5$.

Solving for λ, we get $\lambda = 1$.

Hence, $f(x^*) = 0.5^2 + 2.5^2 - 2(0.5) - 6(2.5) = -9.5$ is the optimal solution.

```
#### CODING EXERCISES ####
# 5. b) Verify using cvxpy
c = np.array([-2.0, -6.0])
a = np.array([1.0, 1.0])


x = cp.Variable(2)
con_eq = a @ x == 3         # equality: x₁ + x₂ = 3
con_nn = x >= 0
objective = cp.Minimize(cp.sum_squares(x) + c @ x)
prob = cp.Problem(objective, [con_eq, con_nn])
prob.solve()

x_cvx = x.value
```

```
mu = con_eq.dual_value     # equality multiplier
nu = con_nn.dual_value     # inequality multipliers

print("status:", prob.status)
print("x* (cvxpy):", x_cvx)
```

```
status: optimal
x* (cvxpy): [0.5 2.5]
```

6) Gym environment

Just have fun interacting with gym!

```
import gymnasium as gym

# Build deterministic FrozenLake env
env = gym.make("FrozenLake-v1", map_name="4x4", is_slippery=False)
obs, info = env.reset(seed=42)

episode_return = 0.0
done = False

# --- Helpers ---
def _colors(desc):
    code_map = {b'H': 0, b'F': 1, b'S': 2, b'G': 3}
    return np.vectorize(lambda b: code_map[b])(desc)

def draw_board(desc, s):
    nrow, ncol = desc.shape
    grid = _colors(desc)
    from matplotlib.colors import ListedColormap
    cmap = ListedColormap([
        (0,0,0),        # hole
        (0.7,0.85,1.0), # ice
        (0.2,0.7,0.2),  # start
        (1.0,0.84,0.0)  # goal
    ])
    fig, ax = plt.subplots(figsize=(4,4))
    ax.imshow(grid, cmap=cmap, vmin=0, vmax=3)
    ax.set_xticks(np.arange(-0.5, ncol, 1), minor=True)
    ax.set_yticks(np.arange(-0.5, nrow, 1), minor=True)
    ax.grid(which="minor", color="white", linewidth=2)
    ax.set_xticks([])
    ax.set_yticks([])
    r, c = divmod(int(s), ncol)
    ax.plot(c, r, marker="o", markersize=18,
            markeredgecolor="white", markerfacecolor="red")
    plt.title("FrozenLake-v1 (4x4)")
    plt.show()

def step_once(action):
    global obs, done, episode_return
    if done:
        print("Episode finished. Please reset the env.")
        return
    step = env.step(action)
    if len(step) == 5:
        obs, reward, terminated, truncated, info = step
        done = terminated or truncated
    else:
        obs, reward, done, info = step
    episode_return += reward
    print(f"Action {action}, Reward {reward}, Return {episode_return}, Done {done}")
    draw_board(env.unwrapped.desc, env.unwrapped.s)

def reset_env(seed=None):
    global obs, done, episode_return
```

```
    obs, info = env.reset(seed=seed)
    done = False
    episode_return = 0.0
    print("Environment reset.")
```

```
# Run this when you start a new game
reset_env(seed=42)
```

```
Environment reset.
```

```
step_once(0)   # do action=2 (right)
# change the step yourself and see what happens
# 0: left, 1: down, 2: right, 3: up
```

```
Episode finished. Please reset the env.
```