Swinburne University of Technology

Faculty of Science, Engineering and Technology

LABORATORY COVER SHEET

Subject Code: COS30008
Subject Title: Data Structures and Patterns
Lab number and title: 3, Solution Design in C++
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However difficult life may seem, there is always something you can do and succeed at.

Steven Hawking

Solution Design in C++

The goal of this laboratory session is to build a C++ console application, called `Polynomials`, that allows users to specify the degree and coefficients of simple polynomials, multiply two polynomials, and output a human-readable representation.

A polynomial with a single variable x can be written in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x^1 + a_0 x^0$$

where $0, \ldots, n$ are integers, $a_0, \ldots, a_n$ are real numbers, and x is the variable of the polynomial. Programmatically in C++, we can represent polynomial as an array of double values with length n+1. The degree of a polynomial is the largest exponent of any one term with a non-zero coefficient. For the purpose of this tutorial, we limit the maximum degree of user-specified polynomials to 10.

A polynomial can be expressed more concisely by using summation notation, which allows for a straightforward mapping to a standard for-loop in C++:

$$f(x) = \sum_{i=0}^{n} a_i x^i = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x^1 + a_0 x^0$$

In C++, we map this summation to a for-loop:

```
for ( size_t i = 0; i <= n; i++ ) { … }
```

In addition to representing polynomials, we also wish to support polynomial multiplication. Given two polynomials

$$f(x) = \sum_{i=0}^{n} a_i x^i \quad \text{and} \quad g(x) = \sum_{j=0}^{m} b_j x^j$$

the product is defined as

$$f(x)g(x) = \sum_{i=0}^{n} a_i x^i \sum_{j=0}^{m} b_j x^j = \sum_{i=0}^{n} \sum_{j=0}^{m} a_i b_j x^{i+j}$$

In order words, the product of two polynomials can be realized as a nested for-loop that aggregates the respective ith and jth polynomial terms. The maximum degree of the resulting polynomial is i+j. Since we allow 10 as the maximum user-specified degree for polynomials, our implementation must support polynomials up to degree 20 = 10 + 10.

To facilitate the implementation, we shall use fixed-size arrays of double values to represent polynomials. All elements in the array have to be initialized to 0.0. For all non-zero coefficients a_i the array contains at index i the value a_i. As a result, the array arranges a given polynomial from right to left, that is, in increasing degree order.

The application should consist of two parts: a class `Polynomial` that implements the desired functionality plus the equivalence operator `==` and a `main` function that declares, reads, multiplies polynomials, and outputs the results to the Console. Please note that two polynomials are structural equivalent, if and only if they have the same degree and are pairwise equivalent for each component:

$$\sum_{i=0}^{n} a_i x^i == \sum_{j=0}^{m} b_j x^j \iff n == m \ \land \ \forall i, 0 \le i \le n : a_i == b_i$$

The specification of class `Polynomial` is shown below:

```cpp
#pragma once

#include <iostream>

#define MAX_POLYNOMIAL 10                  // max degree for input
#define MAX_DEGREE MAX_POLYNOMIAL*2        // max degree = 10 + 10 = 20

class Polynomial
{
private:
  size_t fDegree;                      // the maximum degree of the polynomial
  double fCoeffs[MAX_DEGREE+1];        // the coefficients (0..10, 0..20)

public:

  Polynomial();

  // binary operator* to multiple to polynomials
  // arguments are read-only, signified by const
  // the operator* returns a fresh polynomial with degree i+j
  Polynomial operator*( const Polynomial& aRHS ) const;

  // binary operator== to compare two polynomials
  // arguments are read-only, signified by const
  // the operator== returns true if this polynomial is
  // structurally equivalent to the aRHS polynomial
  bool operator==( const Polynomial& aRHS ) const;

  // input operator for polynomials (highest to lowest)
  friend std::istream& operator>>( std::istream& aIStream,
                                   Polynomial& aObject );

  // output operator for polynomials (highest to lowest)
  friend std::ostream& operator<<( std::ostream& aOStream,
                                   const Polynomial& aObject );
};
```

To implement the class `Polynomial` follow the process outlined in the lecture notes. First implement the constructor. Then implement `operator>>` and `operator<<`. The input operator requires two types of information: the degree (an integer value) and the corresponding number (degree+1) of coefficients (floating-point values) in decreasing order of degree.

The output operator should only print the polynomial terms with non-zero coefficients with decreasing order of degree.

Finally, define the equivalence and multiplication operators.

You can use as main program the following code:

```cpp
#include <iostream>

#include "Polynomial.h"

using namespace std;

int main()
{
  Polynomial A;
  cout << "Specify first polynomial (degree followed by coefficients):" << endl;
  cin >> A;
  cout << "A = " << A << endl;

  Polynomial AA = A;

  if ( AA == A )
  {
    cout << "Comparison successful." << endl;
  }
  else
  {
    cout << "Comparison failed." << endl;
  }

  Polynomial B;
  cout << "Specify second polynomial (degree followed by coefficients):" << endl;
  cin >> B;
  cout << "B = " << B << endl;

  Polynomial C = A * B;
  cout << "C = A * B = " << C << endl;

  return 0;
}
```
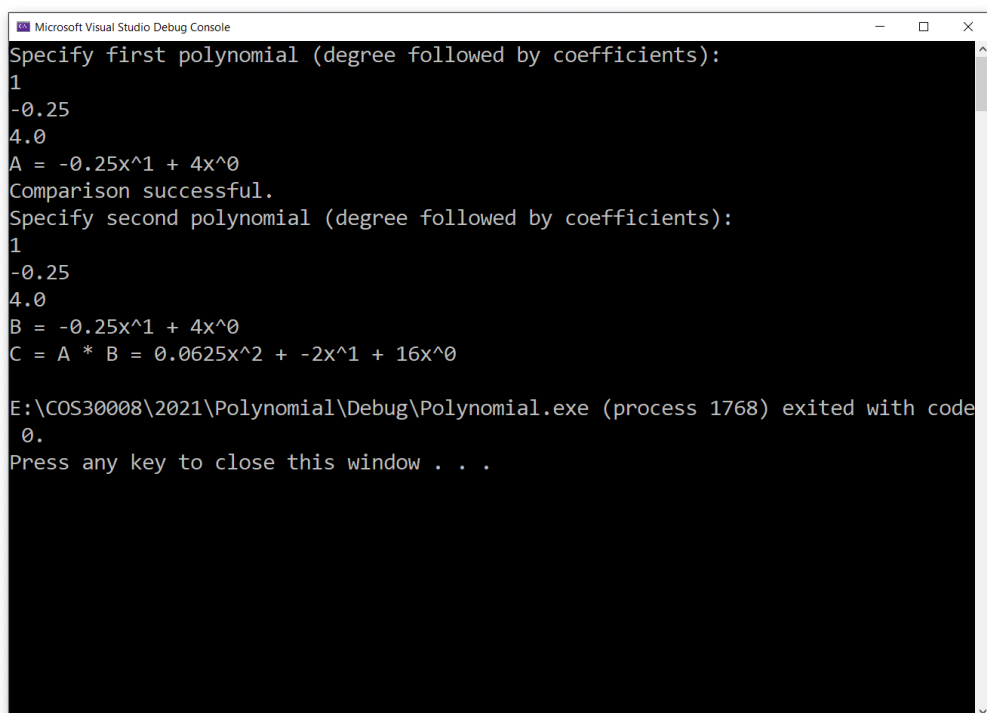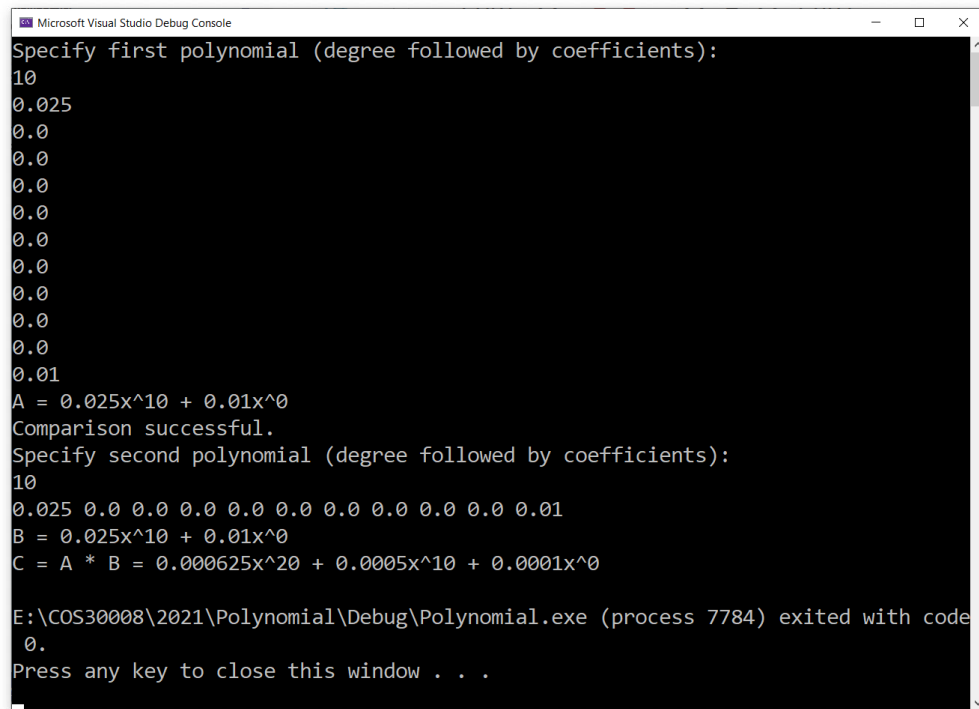
Naturally, you can comment-out parts that you have not yet implemented. Once the implementation is complete, test your code as shown below (e.g., $-0.25x + 4.0$):

Your solution must support polynomials up to the 10th degree. For example, the polynomial $0.025x^{10} + 0.01$ must produce a result as show below:

```
Microsoft Visual Studio Debug Console                                    —   □   ×
Specify first polynomial (degree followed by coefficients):
10
0.025
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.01
A = 0.025x^10 + 0.01x^0
Comparison successful.
Specify second polynomial (degree followed by coefficients):
10
0.025 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.01
B = 0.025x^10 + 0.01x^0
C = A * B = 0.000625x^20 + 0.0005x^10 + 0.0001x^0

E:\COS30008\2021\Polynomial\Debug\Polynomial.exe (process 7784) exited with code
 0.
Press any key to close this window . . .
```

You need to input:

```
10
```

```
0.01  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.025
```

The result of the multiplication is a polynomial of the 20th degree:

$$0.000625x^{20} + 0.0005x^{10} + 0.0001$$

The solution requires 100-150 lines of low density C++ code.