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One-group analytical solution to two-region reactor kinetic model

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ABSTRACT

The accurate modeling of nuclear reactor physics is an evolving topic of research that advances as improved methods and models are developed. Contemporary efforts to model nuclear reactors involve computer codes that utilize Monte Carlo and deterministic methods; however, analytical solutions may be just as valid depending on the application. In fact, a set of differential equations, known as the point reactor kinetics equations (PRKEs), can be analytically solved to predict reactor power based on kinetics parameters and applied to instances of rapid reactivity transients by assuming a one effective delayed neutron group point reactor kinetics solution, traditionally termed the prompt jump (PJ) approximation. Recently, analyses performed with a two-region model of the PRKEs investigated the influence of a reactor's reflector as compared to the one-region PRKEs. Reflectors provide improved neutron economy by decreasing neutron leakage, and are especially important in reactors with small core geometries, such as research reactors. However, the response to a rapid transient in the two-region PRKE model has yet to be investigated despite its potential application, since research reactors often perform rapid transients as part of normal operations and the inclusion of the reflector may significantly alter the modeled response during a rapid reactivity transient.

The derivation of a new, one-group solution to the two-region PRKEs is presented and compared with the traditional PJ approximation along with an analytic uncertainty analysis and a discussion on the strengths and weaknesses of each respective model.

1. Introduction

The accurate modeling of nuclear reactor physics is an evolving topic of research that advances as improved methods and models are developed to produce more accurate results with reduced uncertainty. Contemporary efforts to model nuclear reactors involve computer codes that utilize Monte Carlo and/or deterministic methods; however, analytical solutions may be just as valid depending on the application. In fact, a set of differential equations known as the point reactor kinetics equations (PRKEs) (Duderstadt and Hamilton, 1976) can be analytically solved to predict reactor power based on known kinetics parameters, such as the delayed neutron fraction and prompt neutron lifetime, and applied to instances of rapid reactivity transients by assuming a one effective delayed neutron group point reactor kinetics solution, traditionally termed the prompt jump (PJ) approximation. Typically, the PJ approximation models an assumed step change in reactivity, and does not include any temperature feedback effects, affecting the accuracy of the approximation at large, positive reactivity values.

Cohn (1962), Van Dam (1996) and Spriggs et al. (1997), have investigated the expansion of the PRKEs to a two-region model, which requires an additional term in the PRKEs to model the population of neutrons in the core and reflector separately. Reflectors provide improved neutron economy by decreasing neutron leakage, and are especially important in reactors with small reactor core geometries, such as research reactors. However, the response to a rapid transient in the two-region PRKE model has yet to be investigated despite its potential application, since research reactors often perform rapid transients as part of normal operations and the inclusion of the reflector may significantly alter the modeled response during a rapid reactivity transient. The inclusion of reflector properties causes an increase in the population of neutrons in the core region, and consequently reactor power, as compared to the traditional PRKEs.

The derivation of a new, one effective delayed neutron group solution to the two-region PRKEs is presented and compared with the traditional PJ approximation along with an analytic uncertainty analysis and a discussion on the strengths and weaknesses of each respective model.

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Nomenclature

Symbols

f	fraction of neutrons that enter reflector and return to core
f_{rc}	fraction of neutrons that move from reflector to core
f_{cr}	fraction of neutrons that move from core to reflector
l	prompt neutron lifetime
l_c	prompt neutron lifetime in core for two-region model
l_r	prompt neutron lifetime in reflector for two-region model
k	multiplication factor
k_c	core multiplication factor for two-region model
k_{eff}	total effective multiplication factor for two-region model
s	inverse reactor period
s_1	inverse reactor period (stable)
s_2	inverse reactor period (transient)
s_3	inverse reactor period (only for two-region model)
t	time
C	precursor group concentration for single delayed neutron group
C_i	precursor group concentration for each delayed neutron group
N_c	neutron population in core for two-region model
N_r	neutron population in reflector for two-region model
P	power
P_1	first coefficient term in simplified one-region PJ approximation
P_2	second coefficient term in simplified one-region PJ approximation
$\$$	dollar value of reactivity
β	delayed neutron fraction
β_i	delayed neutron fraction for each delayed neutron precursor group
λ	decay constant
λ_i	precursor group decay constant
ρ	reactivity
ρ_c	reactivity in core for two-region model
ρ_{eff}	reactivity in core and reflector for two-region model
σ_y	uncertainty in variable of interest, y
Λ	mean neutron generation time
Λ_c	mean neutron generation time in core for two-region model
Λ_r	mean neutron generation time in reflector for two-region model

2. One-region point reactor kinetics model

The one-region PRKEs are coupled differential equations (Duderstadt and Hamilton, 1976) of the form

$$\frac{dP}{dt} = \frac{\rho - \beta}{\Lambda} P(t) + \sum_{i=1}^{n} \lambda_i C_i(t), \tag{1}$$

and

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} P(t) - \lambda_i C_i(t), \tag{2}$$

where the reactivity (ρ) and reactor and isotopic information such as the delayed neutron fraction (β), mean neutron lifetime (Λ), and decay constant for each delayed neutron group (λ) are used to couple the time dependent impact of reactor power (P) and neutron precursor group concentration (C). The mean neutron generation time is dependent on the prompt neutron lifetime (l) and the reactivity inserted,

$$\Lambda = l(1 - \rho). \tag{3}$$

The derivation of the PJ approximation begins from the PRKEs in (1) and (2) with a single delayed neutron precursor group (λ). The PJ approximation assumes that at the time of reactivity insertion, the delayed precursor group concentration does not change, and the solutions for reactor power and precursor concentration are a linear combination of exponential terms of the form

$$\psi(t) = \psi_o e^{st} \tag{4}$$

where the initial state value of the parameter considered is ψ_o, and the solution time-constant is $(1/s)$. The quantity being characterized, ψ, may take on the value of P or C. The exponential solution forms may be applied to the one-group forms of (1) and (2) to obtain a second-order polynomial equation. From this, the solution for the inverse of the solution time-constant, or s-value, in terms of the kinetics parameters is found through the quadratic formula.

$$s_{1,2} = \frac{-(\lambda\Lambda + \beta - \rho) \pm \left((\lambda\Lambda + \beta - \rho)^2 - 4(\rho\lambda)\Lambda\right)^{\frac{1}{2}}}{2\Lambda}, \tag{5}$$

where the larger calculated value (s_1) is the inverse of the stable reactor period following reactivity insertion and the smaller value (s_2) is the inverse of the transient reactor period. Then, it is possible to obtain the remaining coefficients (P_1 and P_2, since there are two s-values) using the power and precursor equilibrium assumptions at $t = 0$ to provide additional relations from the one-group forms of (1) and (2), resulting in (6) and (7). Generally, the power is normalized such that the sum of P_1 and P_2 is equal to unity.

$$P_1 = \frac{s_2(s_1 + \lambda)}{\lambda(s_2 - s_1)} \tag{6}$$

$$P_2 = -\frac{s_1(s_2 + \lambda)}{\lambda(s_2 - s_1)}. \tag{7}$$

Combining the exponential solutions yields the "one-region PJ approximation"

$$P(t) = \frac{s_2(s_1 + \lambda)e^{s_1 t} - s_1(s_2 + \lambda)e^{s_2 t}}{\lambda(s_2 - s_1)}. \tag{8}$$

However, there is a more common form of the PJ approximation, with a slightly different treatment of the s-values. Returning to the quadratic formula that represents the solution for the s-values in (5), the $\lambda\Lambda$ term may often be ignored due to its relatively small magnitude compared to the other terms, and an explicit formula in terms of reactor kinetics parameters may be found for s_2. The resulting positive root, under this assumption, yields a trivial solution, so the value for s_1 is determined by returning to the original second-order polynomial used to define (5). Using the fact that $s_1{}^* s_2$ is equal to the constant term in the second-order polynomial $(-\lambda\rho/\Lambda)$, s_1 may be found.

$$s_1 = \frac{\lambda\rho}{\beta - \rho}. \tag{9}$$

$$s_2 = \frac{\rho - \beta}{\Lambda}. \tag{10}$$

Additionally, the coefficients P_1 and P_2 are found by using the simplified s-values from (9) and (10), reducing the expressions in (6) and (7) to the terms in (11) and (12).

$$P_1 = \frac{\beta}{\beta - \rho} \tag{11}$$

$$P_2 = \frac{-\rho}{\beta - \rho} \tag{12}$$

Therefore, the "simplified one-region PJ approximation" becomes

$$P(t) = \frac{\beta}{\beta - \rho} e^{\frac{\lambda\rho}{\beta - \rho}t} - \frac{\rho}{\beta - \rho} e^{\frac{\rho - \beta}{\Lambda}t}, \tag{13}$$

with the first term representing the power contribution of the stable reactor period while the second term represents an "approach" to the first term dependent on the reactor's transient period. This form of the PJ approximation is most common since the effect of individual kinetics parameters may be directly observed, though the only difference between the one-region and simplified one-region PJ approximations is the treatment of the *s*-values.

The one-region and simplified one-region PJ approximations are valid for all values of reactivity insertion; however, it is important to recognize that no terms are time or temperature dependent. More complex derivations are required to include these effects, and will not be investigated within this body of work.

3. Two-region point reactor kinetics model

The two-region kinetic model for reflected reactors, also known as the Avery-Cohn model (Cohn, 1962), is re-derived by Spriggs et al. (1997) using probabilistic relationships developed for neutrons moving between the core region and the reflector region. The result is a set of differential equations similar to the PRKEs, but with additional terms to include reflector properties. The source-free version of the two-region PRKEs is shown in (14), (15), and (16),

$$\frac{dN_c}{dt} = \frac{\rho - \beta - f(1-\beta)}{\Lambda_c(1-f)} N_c(t) + \frac{f_{rc}(1-\rho)}{\Lambda_r(1-f)} N_r(t) + \sum_{i=1}^{n} \lambda_i C_i(t) \tag{14}$$

$$\frac{dN_r}{dt} = \frac{f_{cr}(1-\rho)}{\Lambda_c(1-f)} N_c(t) - \frac{(1-\rho)}{\Lambda_r(1-f)} N_r(t) \tag{15}$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda_c} N_c(t) - \lambda_i C_i(t) \tag{16}$$

where N_c is the core neutron population (reactor power), N_r is the reflector neutron population, and C_i is the precursor concentration for each delayed neutron group. However, for the two-region model, additional variables must be defined. A separate mean neutron generation time is used for the core and reflector, Λ_c and Λ_r, respectively. As formulated by Spriggs et al. (1997), the form of the mean neutron generation time is shown in (17) and (18), where l_c and l_r represent the prompt neutron lifetime in the core and reflector, respectively,

$$\Lambda_c = \frac{l_c(1-\rho)}{1-f} \tag{17}$$

$$\Lambda_r = \frac{l_r(1-\rho)}{1-f}. \tag{18}$$

Finally, the fraction of neutrons that move from the core to the reflector is represented by f_{cr}, and f_{rc} is the fraction of neutrons that move from the reflector to the core, allowing the fraction of core neutrons that scatter in the reflector and return to the core to be defined by (19),

$$f = f_{cr} f_{rc}. \tag{19}$$

When including a reflector in the two-region model, the reactivity term must also be altered. The relationship provided by Spriggs et al. (1997) for the effective system multiplication factor k_{eff}, shown in (20), is a function of the core multiplication factor (k_c) and the reflector return fraction (f). Then, (20) may be reformulated in terms of reactivity (ρ), with (21), to form an expression for the effective reactivity by including the reflector contribution, ρ_{eff}, in (22) as a function of the core reactivity, ρ_c.

$$k_{eff} = \frac{k_c}{1-f} \tag{20}$$

$$k = \frac{1}{1-\rho} \tag{21}$$

$$\rho_{eff} = \rho_c + f(1-\rho_c) \tag{22}$$

It is important to recognize that f does not represent an albedo, a parameter traditionally used to describe a reactor's reflector. Instead, f represents the reflector's contribution to reactivity. From observation, a reactor with no reflector ($f = 0$) reduces to the multiplication factor and reactivity in the core. However, a reflector with maximum reactivity benefit as defined by Spriggs et al. (1997) ($f = 1$) causes a reactivity value of unity ($\rho = 1$) and a multiplication factor of *infinity* (not the infinite multiplication factor of the system). Reactivity in dollars ($) is defined as the ratio of inserted reactivity and delayed neutron fraction (ρ/β); therefore, the reflector's benefit in dollars is quantified by dividing (22) by the delayed neutron fraction.

For the application of a PJ approximation to the two-region point kinetics model, the relationship in (22) indicates that a reactivity benefit results from the inclusion of the reflector, and, as expected, the core neutron population in a reflected reactor will be larger following a rapid reactivity insertion than the same reactivity insertion in an unreflected reactor, due to the effective reactivity of the system (ρ_{eff}) caused by a fraction of the core leakage neutrons reflected back into the core. This phenomenon is investigated further in Section 4.

The comparison of neutron populations in the core and reflector has been previously investigated by Spriggs et al. (1997). In the interest of a PJ approximation for the two-region model, it is only necessary to recognize the increased core neutron population (reactor power) resulting from the contribution of the reflector neutrons. To solve for the PJ approximation of the two-region model, it is assumed that at the time of reactivity insertion, the delayed neutron precursor group concentration and neutron population in the reflector are not changing. At $t = 0$, reactor power is normalized and the initial reflector neutron population and initial precursor concentration are given by equilibrium values. It is assumed that the solutions for core neutron population (reactor power), reflector neutron population, and precursor concentration are the linear combination of exponential terms, described previously in (4).

By solving for equilibrium reflector neutrons and precursor concentration populations and substituting each into the one delayed neutron group versions of (14), (15), and (16) using the equations for the core and reflector mean neutron generation times in (17) and (18), it is possible to rearrange the expression for the inverse time constants into a third-order polynomial,

$$\begin{aligned} 0 = {} & s^3 l_c l_r (1-\rho) + s^2 (l_c(1-\rho)(1+\lambda l_r) + l_r(\beta - \rho + f(1-\beta))) \\ & + s(l_c\lambda(1-\rho) + (\rho-\beta)(f-1) + \lambda l_r(f-\rho)) + \rho\lambda(f-1). \end{aligned} \tag{23}$$

From the form of (23), the *s*-values are computed by solving the cubic equation and three real roots are obtained; however, finding an analytical solution for the *s*-values is cumbersome, as will be

detailed in Section 6. Similar to the one region solution, the s-values represent contributions to reactor period from different sources, where $s_1 > s_2 > s_3$. In the two-region model, the additional root not present in the one-region model (s_3) represents the transient reactor period associated with the additional reflector neutrons and is relatively large in magnitude compared to the other roots.

Similar to the one-region model, the two-region PJ approximation coefficients are determined in terms of the s-values, the reflector neutron lifetime, and the one-group precursor group decay constant, resulting in the fully developed expression for core neutron population (reactor power) in the form of (24), where the s-values are the roots found from (23).

$$N_c(t) = \frac{s_2 s_3 (s_1+\lambda)(s_1 l_r + 1) e^{s_1 t}}{\lambda (s_1 - s_2)(s_1 - s_3)} + \frac{s_1 s_3 (s_2+\lambda)(s_2 l_r + 1) e^{s_2 t}}{\lambda (s_2 - s_1)(s_2 - s_3)} + \frac{s_1 s_2 (s_3+\lambda)(s_3 l_r + 1) e^{s_3 t}}{\lambda (s_3 - s_1)(s_3 - s_2)} \quad (24)$$

This is a similar form of the one-region PJ approximation, derived in (8). In fact, by setting l_r to zero, and allowing s_3 to become large compared to s_2 and s_1 (representing a null contribution from the reflector to reactor period) in (24), the expression reduces to the relationship to the one-region PJ approximation (8). The two-region model is very sensitive to the reflected neutron fraction, but relatively insensitive to changes in the reflector neutron lifetime, which minimally affects the transient reactor period following the reactivity insertion.

4. Reactivity in one- and two-region models

As discussed briefly in Section 3, the reactivity term in the two-region model is altered from the one-region and simplified one-region models by including the effect of increased reactivity due to the reflector neutron fraction. Additionally, reactivity insertions are differentiated when they are greater or less than the delayed neutron fraction ($\rho = \beta$). Reactivity insertions less than this threshold are deemed "square waves," while insertions greater than the threshold are "pulses." The separate terms indicate a drastic difference in the rate of increase in reactor power, where pulses are much greater than square waves. If the value of f is increased beyond the threshold by setting ρ_{eff} equal to β in (22), then an intended square wave modeled with the one-region PJ approximation will result in an unintended pulse modeled with the two-region PJ approximation. Fig. 1 depicts the relationship shown in (22), and it can be observed that an intended pulse ($ = 1) does not require any reflected neutrons to create a pulse. However, if a reflector with a value of f equivalent to the delayed neutron fraction is instantly introduced to a critical system, an unintended pulse occurs. For the purposes of this paper, a $0.50 square wave will be evaluated, and a value of f equal to 4e−3 will be used to model the unintended pulse scenario, since this is slightly beyond the threshold for a $0.50 reactivity insertion.

5. Model comparison and discussion

The difference in the three PJ models (one-region, simplified one-region, and two-region), shown in their final form in (8), (13), and (24), respectively, is most effectively shown graphically in Fig. 2. Reference values were used for the Oregon State TRIGA Reactor (OSTR) from Marcum et al. (2012), with a nominal reflector neutron lifetime provided by Spriggs et al. (1997). The uncertainty in the reflector neutron lifetime is maintained at 10% for all uncertainty calculations in the two-region model, since the model is insensitive to this parameter and the value of uncertainty is consistent with known uncertainties in the core neutron lifetime. The parameter values, shown in Table 1, are representative and provide the reader with the ability to observe trends within the plots. The PJ approximations are plotted up to fifty milliseconds, which provide the reader an opportunity to observe the "approach" to a new power level that occurs during a rapid reactivity insertion. The reactivity insertion used for this analysis was $0.50, chosen to cause a doubling of reactor power in a short time following the reactivity insertion. From discussions with reactor operators at OSTR, it was determined a conservative value for the uncertainty of reactivity insertion was approximately $0.02.

The one-region, simplified one-region, and two-region PJ approximations are plotted in Fig. 2. All plots are the result of a

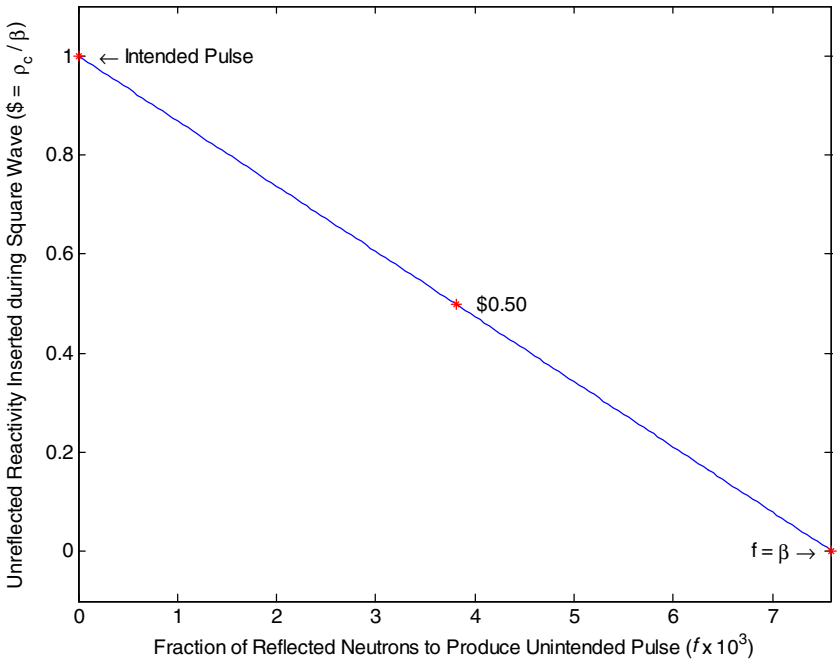

Fig. 1. Intended reactivity insertion as a result of increasing reflected neutron fraction.

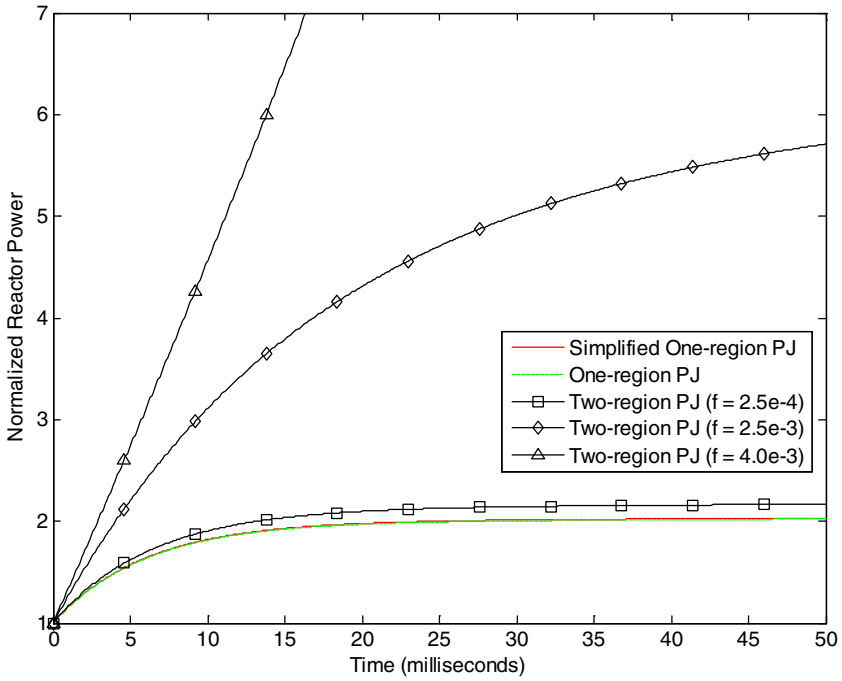

Fig. 2. One-region, simplified one-region, and two-region PJ approximations with varying f for \$0.50 reactivity insertion.

Table 1
Parameter values used for one- and two-region PJ approximations.

Parameter	Value	Uncertainty Value	References
Delayed Neutron Fraction (β_{eff})	0.0076	0.0001	Marcum et al. (2012)
Core Neutron Lifetime (l_c)	22.6 μs	2.9 μs	Marcum et al. (2012)
Precursor Group Decay Constants	Six-group U-235 (λ_4) for one-group	Six-group U-235 (λ_4) for one-group	Ott and Neuhold (1985)
Reflector Neutron Lifetime (l_r)	71.8 μs	10%	Spriggs et al. (1997)
Reflected Neutron Fraction (f)	Varies	1%, 10%	
Reactivity Insertion (\$)	\$0.50	\$0.02	

\$0.50 transient, with the two-region models possessing different reflected neutron fractions as specified in the plot legend. As expected, the one-region and simplified one-region models produce very similar results, with a maximum difference of less than 0.5% during the duration of the rapid reactivity insertion, demonstrating the validity of using the simplified one-region model in predicting reactor power.

Fig. 2 also shows the two-region prompt jump approximation with varying f. As observed in Fig. 2, increasing the value of f causes a dramatic increase in the reactor power as compared to the one-region PJ approximation. Recall Fig. 1 and the value of f required to produce an unintended pulse in a \$0.50 reactivity insertion. Below this threshold, a value of f equal to 2.5e−4 results in a benefit of approximately \$0.03 and the rate of increase in reactor power is similar to the one-region model. However, as f is increased to 2.5e−3, the reactivity benefit is approximately \$0.30 and the resulting change in reactor power and rate of increase in reactor power are much larger than the one-region model. Finally, if f is increased beyond the threshold shown in Fig. 1 for a \$0.50 reactivity insertion to a value of 4e−3, corresponding to a reactivity benefit of \$0.53 and total reactivity insertion in excess of \$1.00, the two-region model predicts an unintended pulse and a drastic change in the rate of increase in reactor power. Additionally, at reactivity insertion values that cause a reactor pulse, temperature feedback mechanisms may drastically alter the shape of the solution and invalidating the use of the derived analytical solution to model the system.

In reactor operations, it is difficult to assess the value of f in the reactor system and would require the reactor to operate prior to the installation of the reflector for calculation of its fraction of reflected neutrons. Therefore, though Fig. 2 with f equal to 4e−3 presents an unintended pulse, it is not likely that this can occur in an operating research reactor. In reality, reactor operators intending a \$0.50 square wave will remove control rods a distance that corresponds to a reactivity value that already includes the reflector's contribution.

The two-region model provides an adequate number of parameters to characterize the contribution from the reflector during a rapid reactivity insertion despite the difficulty to obtain accurate values for the added parameters. Though the two-region PJ approximation is novel, the simplified models may be of better utility to describe reactor behavior in a simple, preliminary manner in circumstances of rapid reactivity insertion.

6. Model uncertainty analysis

A method of calculating uncertainty through error propagation known as the Taylor series expansion method, described in Figliola et al. (2011), combines the uncertainty of each independent variable with a root mean square approach to determine the uncertainty in the calculated variable by evaluating the partial derivative with respective to each independent variable, where σ_y is the individual uncertainty of each variable, y.

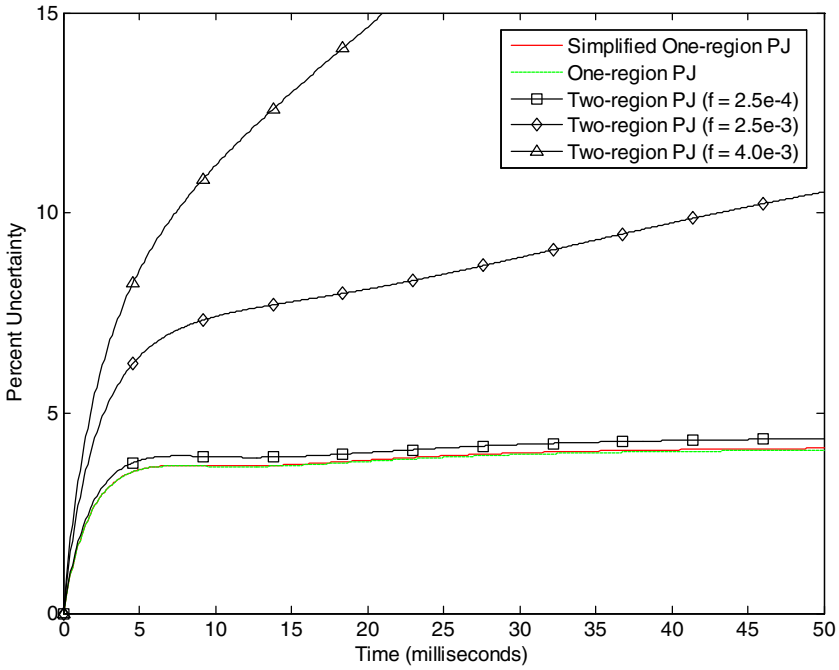

Fig. 3. Percent uncertainty for PJ approximations with varying f, $\sigma_f = 1\%$.

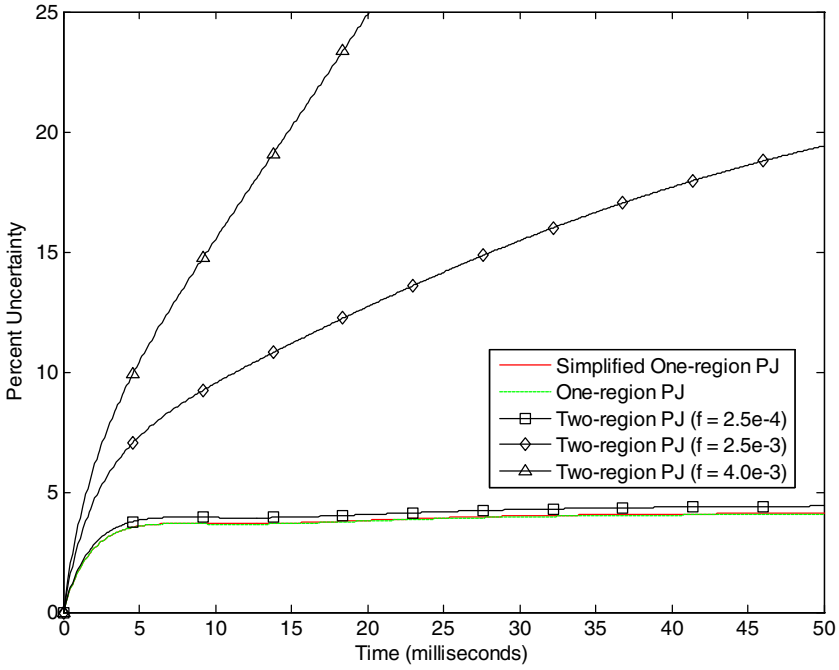

Fig. 4. Percent uncertainty for PJ approximations with varying f, $\sigma_f = 10\%$.

Recall the one-region and simplified one-region PJ approximations derived in (8) and (13), respectively. A root mean square method is applied to each model is shown in (25), which utilizes the parameter uncertainties provided in Table 1.

$$\sigma_P = \left(\left(\frac{\partial P}{\partial \beta} \sigma_\beta \right)^2 + \left(\frac{\partial P}{\partial \rho} \sigma_\rho \right)^2 + \left(\frac{\partial P}{\partial l} \sigma_l \right)^2 + \left(\frac{\partial P}{\partial \lambda} \sigma_\lambda \right)^2 \right)^{\frac{1}{2}} \tag{25}$$

In the simplified one-region model, the partial derivatives may be calculated directly, but the one-region model requires the partial derivatives to be calculated through a chain of partial derivatives incorporating the roots of the quadratic, s_1, s_2, whose solution is shown in (5), since an expression for the s-values explicitly in terms of kinetics parameters is not attainable. The total uncertainty for the simplified one-region and one-region models is displayed in Figs. 3 and 4 in percent uncertainty, corresponding to a single standard deviation in calculated reactor power.

Finally, recall the two-region PJ model derived in (24). Since three s-values may be found as roots in (23), a cubic solver is required. However, to perform an uncertainty analysis of the roots, the s-values must be computed analytically, and a cubic equation with three real roots is generally solved using trigonometric

functions. In the chosen method, the coefficient of the third order term to be unity, shown in (26).

$$s^3 + As^2 + Bs + C = 0 \tag{26}$$

The solutions for the s-values are found by the expression in (27), where n = 1,2,3 for the three desired values,

$$s_n = 2\left(\frac{A^2 - 3B}{9}\right)^{1/2} \cos\left(\frac{\theta + 2\pi(n-1)}{3}\right) - \frac{A}{3}, \tag{27}$$

where θ is defined as

$$\cos(\theta) = \frac{9AB - 27C - 2A^3}{54}\left(\frac{A^2 - 3B}{9}\right)^{-3/2}. \tag{28}$$

The uncertainty in the two-region model is found in a similar manner to the one-region model, by combining partial derivatives of the analytical solution and parameter uncertainties from Table 1 with the root mean square approach, shown in (29). Similar to the one-region model, the computation of the partial derivatives require the repeated application of the chain rule to acquire the desired partial derivatives.

$$\sigma_{N_c} = \left(\left(\frac{\partial N_c}{\partial \beta}\sigma_\beta\right)^2 + \left(\frac{\partial N_c}{\partial \rho}\sigma_\rho\right)^2 + \left(\frac{\partial N_c}{\partial l_c}\sigma_{l_c}\right)^2 + \left(\frac{\partial N_c}{\partial \lambda}\sigma_\lambda\right)^2 + \left(\frac{\partial N_c}{\partial l_r}\sigma_{l_r}\right)^2 + \left(\frac{\partial N_c}{\partial f}\sigma_f\right)^2\right)^{\frac{1}{2}} \tag{29}$$

Compared to the one-region and simplified one-region PJ approximations' uncertainty analysis, the number of terms in the two-region model increases substantially, and the calculated reactor power is very sensitive to the value of and uncertainty in the reflected neutron fraction. Rather than choosing an arbitrary value of f and associated uncertainty, Figs. 3 and 4 provide percent uncertainty in the reactor power using the two-region model and apply the values of reflected neutron fraction utilized previously for Fig. 2 with associated uncertainties of 1% and 10%, respectively.

In Figs. 3 and 4, the uncertainty of the one-region and simplified one-region PJ approximations is shown to be less than 5% for the duration of the \$0.50 reactivity insertion and nearly indistinguishable between the two models, with the one-region model slightly less. For the two-region model, the uncertainty is highly dependent on the value of the reflected neutron fraction. In the case of a small reflected neutron fraction (2.5e−4), the uncertainty is nearly the same as the one-region models, which may allow the application of the two-region model to adequately analyze a reactor system with a small reflector contribution following a reactivity insertion. However, drastic increase in the uncertainty with larger reflector values is caused by the exponential form of the PJ approximations. Uncertainty in the reflected neutron fraction causes uncertainty in the rapid reactivity insertion, and, ultimately, reactor power, which is exaggerated when the reactivity insertions are large, such as an unintended pulse scenario (f = 4e−3). This effect is exaggerated even more when the uncertainty in the reflected neutron fraction is increased in Fig. 4.

7. Summary

The derivation of a new, two-region PJ approximation is presented along with the one-region approximation to illustrate the effect of reflected neutrons on a rapid reactivity insertion as part of normal operations in a research reactor. The analytical solutions are valid in the absence of temperature and reactivity feedback; incorporating these feedback mechanisms would require additional analysis. Beyond a certain limit of the fraction of reflected neutrons, square waves, or reactivity insertions less than the delayed neutron fraction, can be found to present unintended pulses in the reactor. Accurate modeling of a research reactor is essential, and is a main area of investigation with the improvement in computational modeling capabilities. However, analytical solutions, under certain circumstances, may be just as valid when the correct model is used.

The one-region PJ approximation expresses reactor power in terms of stable and transient reactor periods, which does not allow for a quick identification of reactor power as compared to the simplified one-region PJ approximation. However, an uncertainty analysis of the one-region PJ approximation produces a slightly smaller value of uncertainty in reactor power. The simplified one-region PJ approximation is traditionally used during rapid reactivity insertion events to predict reactor behavior, and its simplicity allows the approximation to be expressed in terms of the kinetics parameters, permitting a quick identification of the relationship between reactor power and the kinetics parameters.

The two-region PJ approximation is previously unpublished and provides the ability to model reactor power following a rapid reactivity insertion in a reactor that possesses a reflector. However, it is difficult to obtain values of reflector properties with small uncertainty, which may cause relatively large uncertainty in reactor power following a rapid reactivity insertion, though for reactors that only receive a small reactivity benefit from the reflector, the two-region PJ approximation may provide a better estimate of reactor power with approximately the same uncertainty as the one-region or simplified one-region models.

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