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ARTICLE

Formulation for the Calculation of Reactivity Without Nuclear Power History

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This paper presents a new method for the solution of the inverse point kinetics equation. This method is based on the integration by parts of the integral of the inverse point kinetics equation, which results in a power series in terms of the nuclear power in time dependence. With the imposition of conditions to the nuclear power, the reactivity is represented as first and second derivatives of this nuclear power. This new calculation method for reactivity has very special characteristics, amongst which the possibility of using longer sampling period, and the possibility of restarting the calculation, after its interruption, allowing the calculation of reactivity in a non-continuous way. Beside that, the reactivity can be obtained independent of the nuclear power memory.

KEYWORDS: inverse point kinetics equation, reactivity, nuclear power history

I. Introduction

The time behavior of a nuclear reactor is a relevant issue in the operation of a nuclear power plant. Safety analysis also relies on the knowledge regarding reactivity coefficients and the characteristics of the dynamic responses that result from the variations of the operating conditions of such a power plant. During physics tests for the start-up of a nuclear reactor, a reactivity meter is needed, that is, a computational system to calculate reactivity, using the inverse method of point kinetics. In the existing literature,[1–4] the works carried out with this goal are based on the discretization of the term related to the integral of the equation of the inverse method known as nuclear power history. That is, the value for present reactivity depends of given a nuclear power history.

Most reactivity meters work with a sampling interval of 0.01 s. In this work an alternative form will be presented for the calculation of reactivity in a simpler and precise way that contains only first and second-derivatives of nuclear power, with the possibility of interruption and restart of the calculation of reactivity, making possible the implementation of a reactivity meter that can be used continuously or not at any time instance.

II. Inverse Kinetics

The expression for reactivity can be easily obtained from the point kinetics equations. It is a system with seven coupled ordinary differential and non-linear equations, which describe the evolution in time of the distribution of neutrons and of the concentrations of the delayed neutron precursors in the core of a nuclear reactor.

The formulation of point kinetics essentially admits that the shape of the neutron flux in space and energy remains unchanged through time. Therefore, it is only necessary to concentrate on the time dependence of the neutron population. It is known that it is frequently possible to separate the time behavior of the neutron flux in a nuclear reactor, in a transient phase, during which the neutron flux redistributes in the space and the energy, and in an asymptotic phase during which the neutron flux maintains its shape in the space and in the energy. In many applications of reactor kinetics, either only if it is interested in the asymptotic behavior of the neutron flux, or the flux redistribution in the space and in the energy can be negligible. For this reason, the system of point kinetics equation, which has a very simple structure, forms the base of the majority of the transients analysis carried out in operating nuclear reactors.

Through the use of the adiabatic approximation that supposes that the neutron flux can be factored in a time part and another spatial part, and assuming that the spatial dependence is the solution of the Helmholtz equation, one arrives at the classical form of point kinetics equations,

$$\frac{dP(t)}{dt} = \left[\frac{\rho(t) - \beta}{\Lambda}\right]P(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \tag{1}$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} P(t) - \lambda_i C_i(t), \quad i = 1, 2, \ldots, 6. \tag{2}$$

With the following initial conditions,

$$P(t = 0) = P_0 \tag{3}$$

$$C_i(t = 0) = \frac{\beta_i}{\Lambda \lambda_i} P_0 \tag{4}$$

where,

$P(t)$ = nuclear power

$C_i(t)$ = concentration of the i-th group of delayed neutrons precursors

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$\rho(t)$ = reactivity
Λ = prompt neutron generation time
β_i = effective fraction of the i-th group of delayed neutrons
β = total effective fraction of delayed neutrons ($\beta = \sum_i \beta_i$)
λ_i = decay constant of the i-th group of delayed neutrons precursors

In order to obtain the reactivity $\rho(t)$ in terms of nuclear power P(t), the equation (2) must be integrated until the time t, subject to the condition that the population of precursors was null in a time infinitely before to the time t, thus:

$$C_i(t) = \frac{\beta_i}{\Lambda}\int_{-\infty}^{t} e^{-\lambda_i(t-t')}P(t')dt' = \frac{\beta_i}{\Lambda}\int_{0}^{\infty} e^{-\lambda_i t'}P(t-t')dt' \quad (5)$$

Taking the first integral, supposing that $P(t<0) = \langle P_0\rangle$, that is, the reactor is critical, one has:

$$C_i(t) = \frac{\beta_i\langle P_o\rangle}{\lambda_i\Lambda}e^{-\lambda_i t} + \frac{\beta_i}{\Lambda}\int_0^t e^{-\lambda_i(t-t')}P(t')dt' \quad (6)$$

When Eq. (6) is replaced in Eq. (1), one gets that:

$$\rho(t) = \beta + \frac{\Lambda}{P(t)}\frac{dP(t)}{dt} - \frac{1}{P(t)}\sum_{i=1}^{6}\lambda_i\beta_i\left[\frac{\langle P_0\rangle}{\lambda_i}e^{-\lambda_i t} + \int_0^t e^{-\lambda_i(t-t')}P(t')dt'\right] \quad (7)$$

The expression above represents the reactivity through the inverse method and it is frequently used to program the control rod motion to attain the desired nuclear power variation and during the physical startup tests of a nuclear power plant. It is also the basic equation for the development of a digital reactivity meter.

III. New Formulation for Reactivity

Integrating by parts, the integral of Eq. (7) can be written in terms of the n-order derivatives of nuclear power $[P^{(n)}(t)]$, with $P^{(0)}(t) = P(t)$,

$$\int_0^t e^{-\lambda_i(t-t')}P(t')dt' = -\sum_{n=0}^{k}(-1)^{n+1}\frac{1}{\lambda_i^{n+1}}P^{(n)}(t) + \sum_{n=0}^{k}(-1)^{n+1}\frac{1}{\lambda_i^{n+1}}P^{(n)}(0)e^{-\lambda_i t} + (-1)^{k+1}\frac{1}{\lambda_i^{k+1}}\int_0^t e^{-\lambda_i(t-t')}P^{(k+1)}(t')dt' \quad (8)$$

with the following approximations assumed in order to rewrite the nuclear power history in a more convenient form,

$$P^{(2n-1)}(t) = P^{(1)}(t)\left[\frac{P^{(2)}(t)}{P(t)}\right]^{n-1}, \quad n\epsilon N \quad (9)$$

$$P^{(2n)}(t) = P(t)\left[\frac{P^{(2)}(t)}{P(t)}\right]^{n}, \quad n\epsilon N \quad (10)$$

where,

$$\frac{P^{(2)}(t)}{P(t)} = a_o \quad (11)$$

where a_o is a constant value for any time value.

If k is odd, then $k+1$ is even, that is, $k+1 = 2m$ or,

$$k = 2m-1, \quad (12)$$

with m belong to the set of positive integers (Z^+).

Thus the integral of the Eq. (8) can be written as:

$$\int_0^t e^{-\lambda_i(t-t')}P(t')dt' = S_1(t) + S_2(t) \quad (13)$$

where,

$$S_1(t) \equiv \frac{1}{1-\left[\frac{P^{(2)}(t)}{P(t)\lambda_i^2}\right]^m}\sum_{n=0}^{2m-1}(-1)^{n+1}\frac{1}{\lambda_i^{n+1}}e^{-\lambda_i t}P^{(n)}(0) \quad (14)$$

$$S_2(t) \equiv -\frac{1}{1-\left[\frac{P^{(2)}(t)}{P(t)\lambda_i^2}\right]^m}\sum_{n=0}^{2m-1}(-1)^{n+1}\frac{1}{\lambda_i^{n+1}}P^{(n)}(t) \quad (15)$$

Detailing at first the calculation of $S_2(t)$, represented by Eq. (15), the summation of Eq. (15) can be written as follows:

$$\sum_{n=0}^{2m-1}(-1)^{n+1}\frac{1}{\lambda_i^{n+1}}P^{(n)}(t) = \sum_{n=0}^{m-1}\frac{1}{\lambda_i^{2n+2}}P^{(2n+1)}(t) - \sum_{n=0}^{m-1}\frac{1}{\lambda_i^{2n+1}}P^{(2n)}(t) \quad (16)$$

Using Eqs. (9) and (10) in Eq. (16) one finds that,

$$\sum_{n=0}^{2m-1}(-1)^{n+1}\frac{1}{\lambda_i^{n+1}}P^{(n)}(t) = \left[\frac{P^{(1)}(t)}{\lambda_i^2} - \frac{P(t)}{\lambda_i}\right]\sum_{n=0}^{m-1}\left[\frac{P^{(2)}(t)}{P(t)\lambda_i^2}\right]^n \quad (17)$$

By substituting (17) in Eq. (15), one obtains:

$$S_2(t) = -\frac{\frac{P^{(1)}(t)}{\lambda_i^2} - \frac{P(t)}{\lambda_i}}{1-\left[\frac{P^{(2)}(t)}{P(t)\lambda_i^2}\right]^m}\sum_{n=0}^{m-1}\left[\frac{P^{(2)}(t)}{P(t)\lambda_i^2}\right]^n \quad (18)$$

As,

$$\sum_{n=0}^{k} ar^n = \frac{a(1-r^{k+1})}{1-r}, \quad (19)$$

where r is known as being the geometric summation common ratio,

$$|r| = \frac{P^{(2)}(t)}{P(t)\lambda_i^2} \quad (20)$$

By using Eq. (19), Eq. (18) can be written as

$$S_2(t) = -\frac{P(t)P^{(1)}(t) - \lambda_i(P(t))^2}{P(t)\lambda_i^2 - P^{(2)}(t)} = \frac{\lambda_i P(t) - P^{(1)}(t)}{P(t)\lambda_i^2 - P^{(2)}(t)}P(t) \quad (21)$$

By proceeding similarly to what was done for term $S_2(t)$, for Eq. (14), one obtains that:

$$S_1(t) = -\frac{\lambda_i P(0) - P^{(1)}(0)}{P(0)\lambda_i^2 - P^{(2)}(0)}e^{-\lambda_i t}P(0), \quad (22)$$

using the fact that the constant of Eq. (11) is admitted equal to $\frac{P^{(2)}(0)}{P(0)}$. By replacing Eqs. (21) and (22) in Eq. (13), one obtains that:

$$\int_0^t e^{-\lambda_i(t-t')}P(t')dt' = -\frac{\lambda_i P(0) - P^{(1)}(0)}{P(0)\lambda_i^2 - P^{(2)}(0)}e^{-\lambda_i t}P(0) + \frac{\lambda_i P(t) - P^{(1)}(t)}{P(t)\lambda_i^2 - P^{(2)}(t)}P(t) \quad (23)$$

The same result is obtained for the case in which k is even. Thus, replacing Eq. (23) in Eq. (7), produces:

$$\rho(t) = \beta + \Lambda\frac{P^{(1)}(t)}{P(t)} - \frac{1}{P(t)}\sum_{i=1}^{6}\lambda_i\beta_i\left[\frac{\langle P_0\rangle}{\lambda_i}e^{-\lambda_i t} + \frac{\lambda_i P(t) - P^{(1)}(t)}{P(t)\lambda_i^2 - P^{(2)}(t)}P(t) - \frac{\lambda_i P(0) - P^{(1)}(0)}{\lambda_i^2 P(0) - P^{(2)}(0)}P(0)e^{-\lambda_i t}\right]. \quad (24)$$

Regrouping Eq. (24), allows the obtaining of the final expression of the reactivity associated to a variation of nuclear power P(t),

$$\rho(t) = \frac{P^{(1)}(t)}{P(t)}\left(\Lambda + \sum_{i=1}^{6}\beta_i\left(\frac{\lambda_i - \frac{P^{(2)}(t)}{P^{(1)}(t)}}{\lambda_i^2 - \frac{P^{(2)}(t)}{P(t)}}\right)\right) - \frac{P^{(1)}(0)}{P(t)}\left(\sum_{i=1}^{6}\beta_i e^{-\lambda_i t}\left(\frac{\lambda_i - \frac{P^{(2)}(0)}{P^{(1)}(0)}}{\lambda_i^2 - \frac{P^{(2)}(0)}{P(0)}}\right)\right) - \frac{\langle P_0\rangle}{P(t)}\sum_{i=1}^{6}\beta_i e^{-\lambda_i t} + \frac{P(0)}{P(t)}\sum_{i=1}^{6}\beta_i e^{-\lambda_i t} \quad (25)$$

Equation (25) shows that the reactivity has the form $\frac{P^{(1)}(t)}{P(t)}$, that is, the form known as inverse reactor period.[5)]

It must be pointed that, if there is continuity in time $t = 0$, the last two terms of Eq. (25) annul each other instantly, that is, if $\langle P_0\rangle = P(0)$. If there is a discontinuity, they do not annul each other, but with the increase of time they disappear due to the attenuation factor $e^{-\lambda_i t}$. If it is admitted that $\langle P_0\rangle = P(0)$, Eq. (25) can be simplified even further,

$$\rho(t) = \frac{P^{(1)}(t)}{P(t)}\left(\Lambda + \sum_{i=1}^{6}\beta_i\left(\frac{\lambda_i - \frac{P^{(2)}(t)}{P^{(1)}(t)}}{\lambda_i^2 - \frac{P^{(2)}(t)}{P(t)}}\right)\right) - \frac{P^{(1)}(0)}{P(t)}\left(\sum_{i=1}^{6}\beta_i e^{-\lambda_i t}\left(\frac{\lambda_i - \frac{P^{(2)}(0)}{P^{(1)}(0)}}{\lambda_i^2 - \frac{P^{(2)}(0)}{P(0)}}\right)\right) \equiv \rho_1(t) + \rho_2(t) \quad (26)$$

The terms of Eq. (26) are mathematically similar, but with different physical meanings. The second term ($\rho_2(t)$) is the memory due to the initial condition (critical reactor), but disappears in time (transient), and the first term ($\rho_1(t)$) is out of this memory, being the value for reactivity that refers to the present power, which remains in time (stationary). This can be named asymptotic solution, this solution is determinate for the roots of the "inhour" equation. Concluding, the general solution for reactivity can be written by the summation of two solutions, the stationary solution and the transient solution.

It can easily be seen that $\rho_1(t)$ given by Eq. (26) can be written as,

$$\rho_1(t) = \beta + \Lambda\frac{P^{(1)}(t)}{P(t)} - \frac{1}{P(t)}\sum_{i=1}^{6}\lambda_i\beta_i\left[\frac{P(t)P^{(1)}(t) - \lambda_i[P(t)]^2}{P^{(2)}(t) - P(t)\lambda_i^2}\right] \quad (27)$$

It can also be seen that at the limit where t tends to zero, the expression within brackets in the Eq. (27) represents the Laplace Transform of P(-t), but with the replacing of the "s" space of Laplace by the space λ_i. This can also be seen in the second integral of Eq. (5) taking $t = 0$.

It can be easily demonstrated that Eq. (26) or Eq. (24) are reduced to:

$$\rho(t) = \beta + \Lambda\frac{P^{(1)}(t)}{P(t)} + \sum_{i=1}^{6}\lambda_i\beta_i\left[\frac{P^{(1)}(t) - P(t)\lambda_i}{P(t)\lambda_i^2 - P^{(2)}(t)}\right] - \frac{P(0)}{P(t)}\sum_{i=1}^{6}\lambda_i\beta_i e^{-\lambda_i t}\left[\frac{P^{(1)}(0) - P(0)\lambda_i}{P(0)\lambda_i^2 - P^{(2)}(0)}\right] - \frac{P(0)}{P(t)}\sum_{i=1}^{6}\beta_i e^{-\lambda_i t} \quad (28)$$

As the third term on the right side of Eq. (28) can present discontinuities, for the case of a nuclear power that is not well behaved, that is, that does not obey the conditions represented by Eqs. (9) and (10), to prevent these discontinuities, the condition given by Eq. (11) is used, in the particular case where $t = 0$, to write Eq. (28) as follows:

$$\rho(t) = \Lambda\frac{P^{(1)}(t)}{P(t)} + \frac{P(0)}{P(t)}\sum_{i=1}^{6}\beta_i\left[\frac{\lambda_i P^{(1)}(t) - P^{(2)}(t)}{P(0)\lambda_i^2 - P^{(2)}(0)}\right] - \frac{P(0)}{P(t)}\sum_{i=1}^{6}\lambda_i\beta_i e^{-\lambda_i t}\left[\frac{P^{(1)}(0) - P(0)\lambda_i}{P(0)\lambda_i^2 - P^{(2)}(0)}\right] - \frac{P(0)}{P(t)}\sum_{i=1}^{6}\beta_i e^{-\lambda_i t} \quad (29)$$

Equation (29) apparently is a complex expression, but it is not so as it only needs the first and second derivatives, which

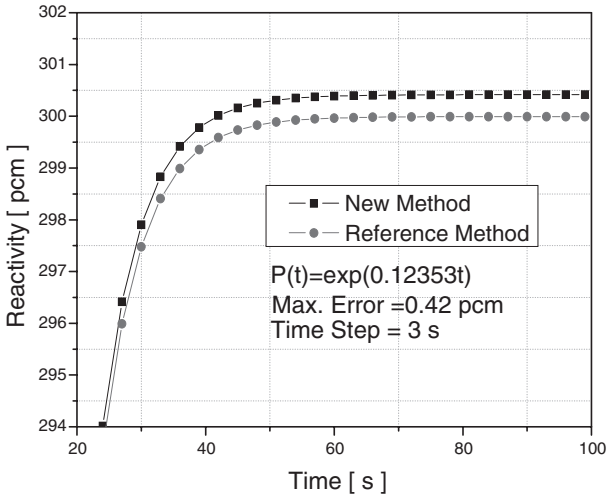

Fig. 1 Variation of reactivity needed to reproduce the variation of nuclear power, in the form of $P(t) = \exp(\omega t)$, with ω equal to 0.12353, with sampling step equal to 3 s.

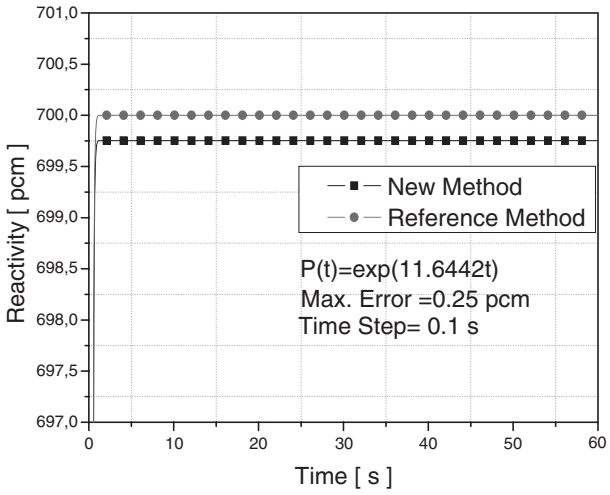

Fig. 2 Variation of reactivity needed to reproduce the variation of nuclear power, in the form of $P(t) = \exp(\omega t)$, with ω equal to 11.6442, with sampling step equal to 0.1 s.

can be numerically implemented, for example, using five point progressive or regressive derivatives or with centered derivatives. This equation shows that in the case of a possible equipment malfunction, the calculation of the reactivity it can be restarted, due to the fact that all the terms of the right side are known. To restart the calculation for reactivity one only needs to consider how long it was kept interrupted, storing the initial value of the power and using the present value of the nuclear power. On the other hand, should it be the case that one does not happen to interrupt the calculation of reactivity, one simply uses the present value of the nuclear power. In both cases the physical condition, imposed by the supposition that the reactor was critical is maintained and therefore does not affect the value for reactivity. But there is another option in which one does not need to consider the condition of criticality. This condition occurs when a sufficiently long time ensues for the transient solution of Eq. (28) to disappear and then it will be possible to restart the calculation for reactivity and to obtain its present value for any time, but with the possibility of having discontinuities for forms of nuclear power that do not satisfy the conditions represented by Eqs. (9–11), may be named without memory system. Equation (29) represents a system that depends on the memory, which are the terms of the previous values of nuclear power. In this case corresponding only in time zero, can be named of one-memory length system.

IV. Results

Figures 1, **2** and **3** show the variations of reactivity corresponding to the nuclear power in the form of $P(t) = \exp(\omega t)$, with the values for ω equal to 0.12353, 11.64420 and 52.80352. With ω being the value of the positive root of the "inhour" equation considering the decay-constants λ_i: (0.0127, 0.0317, 0.115, 0.311, 1.4 and 3.87 s^{-1}) and delayed neutron fractions β_i: (0.000266, 0.001491, 0.001316, 0.002849, 0.000896 and 0.000182). These variations for nuclear power imply reactivity (ρ) values of 300 pcm, 700 pcm and 800 pcm, being the time step of 3 s, 0.1 s and 0.01 s, respectively. Reference values were obtained by analytically calculating the expression of Eq. (7).

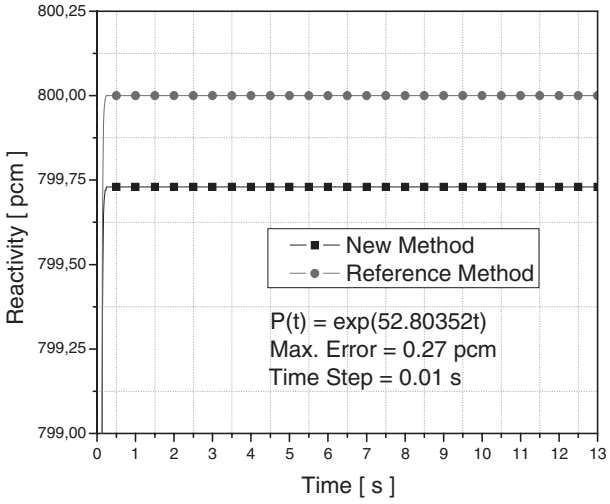

Fig. 3 Variation of reactivity needed to reproduce the variation of the nuclear power, in the form of $P(t) = \exp(\omega t)$, with ω equal to 52.80352, with sampling step equal to 0.01 s.

Table 1 shows the maximum reactivity differences in pcm, obtained between the reference value and the proposed method, for different time steps. It is possible to notice that when the reactivity is too high ($\rho = 800$ pcm) it is difficult to sample with time steps larger than 0.01 s. For smaller reactivity ($\rho = 300$ pcm) it is possible to increase the time step of the sampling. It is important to highlight that the reactor period, provided by expression $\frac{P^{(1)}(t)}{P(t)}$, is a constant.

Figure 4 shows the variation of reactivity for the variation of nuclear power of form $P(t) = \mathrm{Cosh}((\pi/180)t)$, with sampling period of 5 s.

There are forms of nuclear power that apparently do not satisfy the conditions represented by Eqs. (9–11). For some forms of nuclear power the reactivity may diverge, but the

Table 1 Differences in reactivity, in pcm, for different time steps

Step (s)	Power (MW)		
	P(0) exp(0.12353t)	P(0) exp(11.6442t)	P(0) exp(52.80352t)
0.01	2.0954e-6	1.4294e-4	2.7238e-1
0.1	5.3247e-7	2.4620e-1	—
3	4.2428e-1	—	—

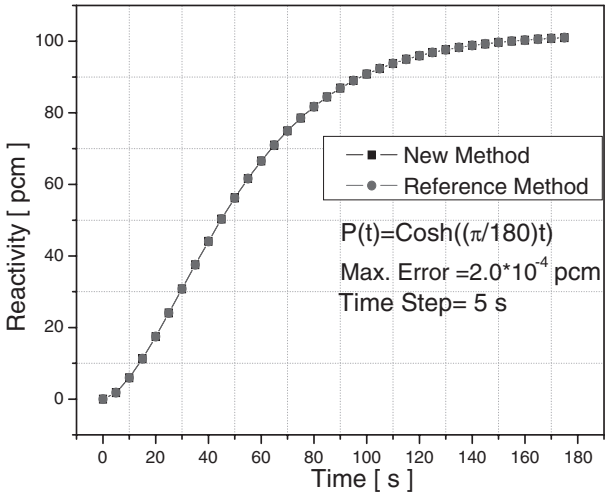

Fig. 4 Variation of reactivity needed to reproduce the variation of nuclear power, of form $P(t) = \text{Cosh}((\pi/180)t)$, with sampling step equal to 5 s.

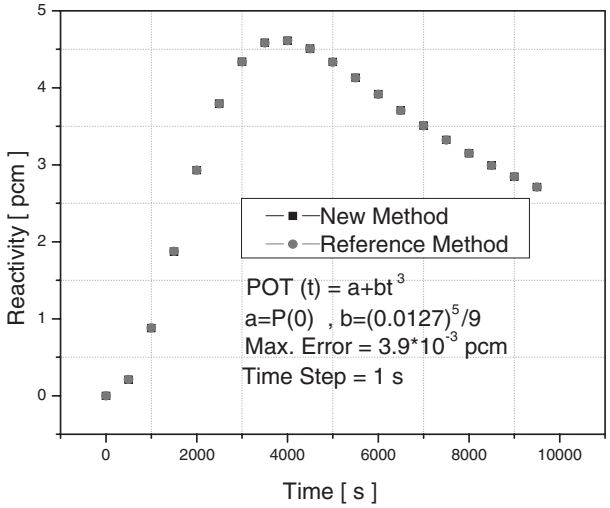

Fig. 5 Variation of reactivity needed to reproduce the variation of nuclear power, of form $P(t) = P(0) + bt^3$, with sampling step equal to the 1 s.

value for reactivity can be found if Eq. (20) is restricted to the cases where $\frac{P^{(2)}(t)}{P(t)\lambda_i^2} < 1$. In these cases it is possible to annul the contribution given by the right side integral of Eq. (8) and the summations of the same equation are converted in series with common ratio $|r| = \frac{P^{(2)}(t)}{P(t)\lambda_i^2} < 1$. With this it is possible to find the value of reactivity from a form of nuclear power that does not fulfill the conditions represented by Eqs. (9–11), but satisfies the criterion of convergence of the geometric series.

Figures 5 and **6** show the reactivity for forms of nuclear power $P(t) = P(0) + bt^3$ and $P(t) = 100 + \text{Sinh}(bt)$ which do not fulfill the conditions provided by Eqs. (9–11), but those satisfy the condition of the common ratio of the geometric series provided by Eq. (20), where constant b must be elected very small so that it fulfills this restriction. The sampling step was undertaken with t = 1 s and t = 20 s, respectively.

Table 2 shows the results of the calculations for reactivity regarding variations of nuclear power, represented by function $P(t) = P(0) + bt^3$. With P(0) being an arbitrary initial value of the nuclear power and b the parameter that was varied in order to validate the method proposed in comparison to the results of the reference method given by Eq. (7). The selection of the values for b shown in Table 2 had, as criterion, the expression provided by Eq. (20). What is inferred from these tests is that, depending on the value of b, an imprecision can be obtained in pcm units that vary directly with the maximum value for reactivity. That is, the largest

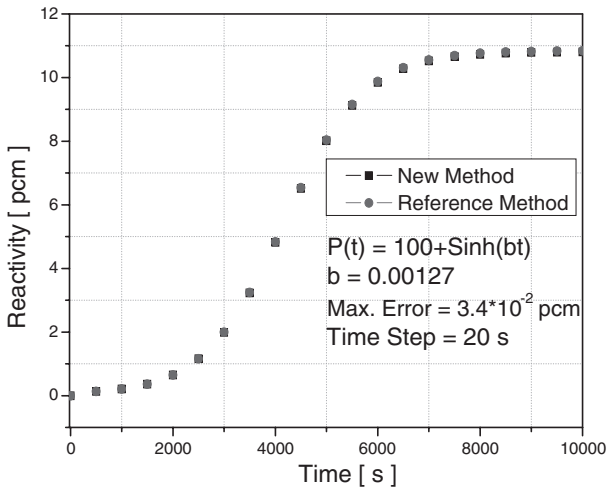

Fig. 6 Variation of reactivity needed to reproduce the variation of nuclear power, in the form $P(t) = 100 + \text{Sinh}(bt)$, with sampling step equal to 20 s.

Table 2 Factor b, maximum difference and reactivity

$P(t) = P(0) + bt^3$ (MW)	$b = \frac{\lambda_i^5}{9}P(0)$	$b = \frac{\lambda_i^4}{40}P(0)$	$b = \frac{\lambda_i^4}{4}P(0)$
Max. error (pcm)	3.8901e-3	6.2e-2	6.2059e-1
Max. reactivity (pcm)	4.6	12	24

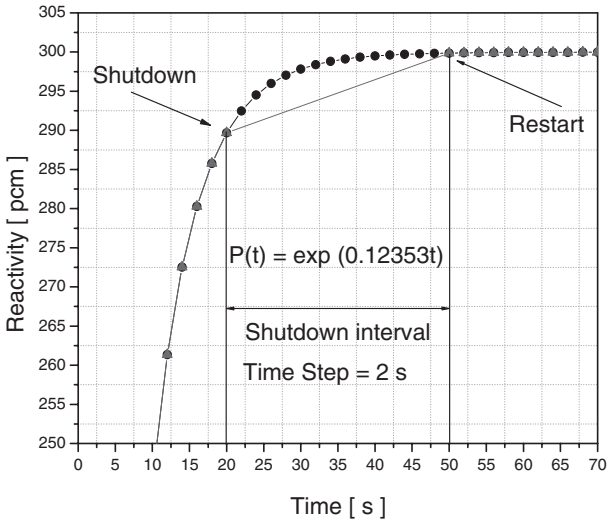

Fig. 7 Discontinuity of the reactivity calculation, for the variation of nuclear power of the form P(t) = exp(0.12353t).

the value for b is, the biggest will the maximum value for reactivity and consequently larger will be the difference between the method proposed and that of reference. The sampling step was carried out with $\Delta t = 1$ s for the cases in Table 2. The sampling step can be of up to $\Delta t = 100$ s for these cases of low reactivity values.

Figure 7 shows the alternative for discontinuing the calculation of reactivity, for the variation of nuclear power of form P(t) = exp(0.12353t). The interruption of the calculation was simulated, at the time t = 20 s and the restart of the calculation at the time t = 50 s. The sampling step was undertaken with t = 2 s. It was verified that the results match, as if no interruption on the calculation of reactivity had taken place.

Eqations (28) and (29) were applied to different representative forms of the nuclear power, without considering the presence of the noise. To make possible the use of this method for reactivity calculation when the nuclear power contains random noise, we may use the following procedure. We introduced a noise around the averaged power

$$\overline{P_i} = \frac{1}{N}\sum_{j=1}^{N} P_j \tag{30}$$

where P_j denotes the measured power signal and N the number of samples in a time step T. Assuming that the random noise has a Gaussian distribution with a relative standard deviation σ around the average value of the nuclear power,[6] we calculated the reactivity for different values of noise degree, for different time steps. To overcome the fact that the derivatives present problems for different forms of the nuclear power in the presence of noises, we used the standard least-squares method[7] to adjust the values of the nuclear power for an exponential form $P(t) = Ce^{At}$, being C and A constants adjusted by the least-squares method. The resulting function satisfies the conditions represented by the Eqs. (9–11).

As explained above, we assumed that the random noise has a Gaussian distribution with a relative standard deviation $\sigma_{\text{Reference method}}$ and $\sigma_{\text{New method}}$ around the average value of the nuclear power. Eqs. (28) and (29) using differential terms which do that the resistance to the noise is very weak, by one same degree of the fluctuation is needed that $\sigma_{\text{Reference method}}/\sigma_{\text{New method}} \approx 10^7$. The larger influence of the noise means that a noise reduction becomes more important in this new method in the application to actual reactors, because the differential terms involved.

For show the applicability of the new method to signal involving a random noise, we used the standard least-squares method.[7] In the **Table 3** shows the results of the maximum difference of the reactivity for the nuclear power P(t) = 100 + sinh(bt), with b = 0.00127, with a degree of the noise $\sigma = 0.001$ with samples numbers of 2 and 100, times steps of 20 s and 0.1 s. It is also presented with a degree of the noise $\sigma = 0.01$ with samples numbers of 10 and 200, times steps of 20 s and 0.1 s.

V. Conclusions

An alternative method was presented for the calculation of reactivity with special characteristics, such as, for example, the possibility of using different sampling frequencies, which is mentioned as one of the technical issues in recent works on the monitoring of the reactivity, in sub-critical systems, through a digital reactivity meter, during critical approximation.[8] Another feature is the possibility of interrupting the calculation of reactivity associated with a possible equipment malfunction and later restarting at any time moment, with the maintenance of the same precision that would have been obtained should the calculation not be interrupted.

Table 3 Maximum difference in reactivity for the nuclear power 100 + sinh(bt), degree of random noise (standard deviation) and time step

Nuclear Power P(t) (MW)	Standard Deviation σ	Samples Numbers N	Time Step T	Max. Difference (pcm) and time
100 + sinh(bt) b = 0.00127	0.001	2	20 s	1.67 in t = 9960 s
		100	0.1 s	0.87 in t = 4860 s
	0.01	10	20 s	1.11 in t = 7200 s
		200	0.1 s	2.85 in t = 5700 s

This characteristic will allow the measurement of reactivity in a non-continuous form, different from that which occurs in the existing reactivity meters. The reactivity can be measured at any moment after the reactor has been started up with any reactivity value.

The alternative method proposed for the calculation of reactivity is rather simple, relying only on first-derivatives and second-derivatives of the nuclear power. For its calculation simplicity, the method is appropriate for the implementation is real-time systems for measuring reactivity associated to the variation of the nuclear power.

Through the mathematical formulation presented in this work it is possible to establish a relationship between "inhour" equations and the inverse kinetics method, this relationship being determined by the stationary solution for reactivity given by Eqs. (28–29).

The possible limitation of Eq. (29) refers to the fact that it is exact only for functions, according to the conditions represented by Eqs. (9) and (10). At least seven characteristic functions of nuclear power attend to this condition. They are: a_0, $a_0 + b_0 t$, $a_0 \cos(b_0 t)$, $a_0 \sin(b_0 t)$, $a_0 e^{b_0 t}$, $a_0 \sinh(b_0 t)$ and $a_0 \cosh(b_0 t)$, where a_0 and b_0 are constants. It is possible to calculate the reactivity, for other functions, with reasonable accuracy, but the restriction of the ratio for the geometrical series, given by Eq. (20), that is, $|r| = \frac{P^{(2)}(t)}{P(t)\lambda_i^2} < 1$, must be satisfied.

The result of the tests carried out showed that the method proposed is sufficiently precise, allowing its use to monitor on-line the reactivity of the core of the reactor.

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