ONE-POINT KINETICS EQUATIONS USING IMPORTANCE FUNCTION

AND GENERALIZED PERTURBATION THEORY

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Abstract

The purpose of the present work is to develop efficient solution methods to solve the time dependent multi-group diffusion equations for subcritical systems with external source by making use of the one-point kinetics equations. In the first part, applying the generalized perturbation theory to the one-point kinetics equations with delayed neutrons which are derived using an importance function to produce fission neutrons, a method to calculate the kinetics parameters, where the first order change of the flux due to the perturbation is taken into account, is shown.

In the second part, a method to derive one-point kinetics equations for the quasistatic method is given. In this method, it is shown that the use of $\omega-$eigenfunctions of the adjoint equation for the weight function can eliminate an error resulting from ignoring the term of first order change of the shape function to solve subcriticality problems, and it gives more accurate results than the use of conventional $k-$eigenfunctions of the critical adjoint equation.

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Introduction

The purpose of the present work is to develop efficient solution methods to solve the time dependent multi-group diffusion equations for subcritical system with external source by making use of the one-point kinetics equations.

In the first part, applying the generalized perturbation theory to the one-point kinetics equations with delayed neutrons which are derived using an importance function to produce fission neutrons, a method to calculate the kinetics parameters is shown, where the first order change of the flux due to the perturbation is taken into account.

Usually, the multiplication factor for a subcritical system which is expressed by inhomogeneous equations with an external source is calculated using the criticality equations of homogeneous equations without source term. Therefore, the system expressed by the homogeneous equation with a criticality factor k is a fictitious system which does not exist. As the system deviates significantly from the critical state, the difference of the flux distribution will be changed considerably, and then the discrepancy of the multiplication rate of neutrons from the real subcritical system will become larger.

It has been shown that the rigorous kinetics equations can be derived by using the importance function to produce fission neutrons [1,2]. An advantage of these kinetics equations is in that all kinetics parameters have clear physical meanings [3]. It has been shown that using these equations for a subcritical system at the steady state, a multiplication factor can be calculated which depends on the position of the external source [4].

In order to apply these kinetics equations to perturbed systems, it is necessary for the first order terms due to the perturbation to be taken into account. It has been shown that this can be done by using the generalized perturbation theory (GPT) for one- and two-point kinetics equations [5,6]. But, in the previous works, the delayed neutrons have been neglected. In the present work, the generalized perturbation theory is applied to the kinetics equations including delayed neutrons.

At the second part, a method to derive one-point kinetics equations to solve multi-group time dependent diffusion equations using the quasistatic method is given. There have been many studies on the quasistatic method[7-9], however, they are for systems which are close to criticality. Coppa et al.[10] and Ravetto et al.[11] applied the quasistatic method to subcritical problems, and they investigated the accuracy of one- and two-point kinetics equations for some weight functions, which shows that it is desirable to make further theoretical investigation to find out what kind of weight functions should be used to make one- and two-point kinetics equations more accurate.

In the second part, it is shown that the use of conventional $k-$eigenfunctions for an adjoint equation as the weight function to solve subcriticality problems with external source would give a first order error of neglecting a change of the shape function due to a perturbation, if the system is not close to the criticality. Then it is shown that the use of the $\omega-$eigenfunctions for an adjoint equation as the weight function can eliminate the term of first order change of the shape function due to perturbation, and the error of neglecting this term can be eliminated. It is numerically confirmed that the use of the $\omega-$eigenfunctions gives more accurate results than the use of current $k-$eigenfunctions to solve subcriticality problems as could be expected from the theory.

Cacuci stated as follows [12]: "It is shown that the exact ADS(Accelerator Driven Sub-critical System)-model is basically non-perturbative; consequently, the exact model cannot be obtained by using the traditional perturbation theory-based approaches as developed originally for critical reactors. In particular, the point-kinetics equations obtained in previous works by using traditional perturbation theory methods are shown to be inadequate for describing the space- and time-behavior of the neutron distribution in the target and the dynamic phenomena at the interface between the ADS target and the ADS sub-critical core regions".

The present method is another approach to subcritical problems using one-point kinetics equations instead of the traditional perturbation theory based method.

One-Point Kinetics Equations Using Importance Function

We assume that neutron flux change due to a perturbation is expressed by the following time dependent multi-group diffusion equations;

$$\frac{1}{v_g}\frac{\partial \phi_g(\boldsymbol{r},t)}{\partial t} = \left(-A' + \frac{1}{k}B^{p'}\right)\phi_g(\boldsymbol{r},t) + \sum_i \chi_{ig}^d \lambda_i C_i(\boldsymbol{r},t) + Q_g(\boldsymbol{r},t), \tag{1}$$

where $\phi_g(\boldsymbol{r},t)$, $Q_g(\boldsymbol{r},t)$, v_g, $C_i(\boldsymbol{r},t)$ and λ_i are the neutron flux, an external source, neutron velocity of $g-$th group, delayed neutron precursor density of i-th group and its decay constant, respectively, and prime denotes quantities for the perturbed system. The constant k is the criticality factor, when $Q_g(\boldsymbol{r},t) = 0$, and $k = 1$, when $Q_g(\boldsymbol{r},t) \neq 0$. The destruction and production operators A and B^p are defined by

$$A = -\nabla D_g \nabla + \Sigma_{rg} - \sum_{g' \neq g} \Sigma_{sg \leftarrow g'}, \quad B^p = \chi_g^p \sum_{g'} \nu_p \Sigma_{fg'} = \chi_g^p(1-\beta)\sum_{g'} \nu\Sigma_{fg'},$$

$$\beta = \sum_i \beta_i, \quad \beta_i = \frac{\nu_i}{\nu}, \quad \nu = \nu^p + \sum_i \nu_i, \tag{2}$$

where D_g, χ_g^p, Σ_{rg}, Σ_{fg}, $\Sigma_{sg \leftarrow g'}$, β, ν^p, ν_i and ν are the diffusion coefficient, normalized fission spectrum for prompt neutrons, removal cross section, fission cross section, scattering cross section from group g' to group g, the total fraction of delayed neutrons, the number of the prompt fission neutrons, number of delayed neutrons of i-th group and total number of fission neutrons per fission, respectively. Delayed neutron precursor density of i-th group is expressed by the equation

$$\frac{\partial C_i(\boldsymbol{r},t)}{\partial t} = -\lambda_i C_i(\boldsymbol{r},t) + \frac{1}{k}\beta_i F' \phi_g(\boldsymbol{r},t). \tag{3}$$

At steady state without perturbation, Eqs.(1) and (3) become

$$A\psi_g(\boldsymbol{r}) = \frac{1}{k}B\psi_g(\boldsymbol{r}) + Q_g^0(\boldsymbol{r}), \quad B = \chi_g F, \quad F = \sum_{g'} \nu\Sigma_{fg'}, \tag{4}$$

where $\psi_g(\boldsymbol{r})$ is the neutron flux and the average fission neutron spectrum χ_g for prompt and delayed neutrons is given by

$$\chi_g = (1-\beta)\chi_g^p + \sum_i \beta_i \chi_{ig}^d. \tag{5}$$

Eqs.(1) and (3) can be written in the form

$$\frac{\partial}{\partial t}\begin{pmatrix} n_g(\boldsymbol{r},t) \\ C_i(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} \left(-A' + \frac{1}{k}B^{p'}\right)v_g & \sum_i \chi_{ig}^d \lambda_i \\ \frac{1}{k}\beta_i F' v_g & -\lambda_i \end{pmatrix}\begin{pmatrix} n_g(\boldsymbol{r},t) \\ C_i(\boldsymbol{r},t) \end{pmatrix} + \begin{pmatrix} Q_g(\boldsymbol{r},t) \\ 0 \end{pmatrix}, \tag{6}$$

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where $n_g(\boldsymbol{r}, t)$ is the neutron density, namely

$$\phi_g(\boldsymbol{r}, t) = v_g n_g(\boldsymbol{r}, t). \tag{7}$$

Eq.(6) can be written in a form

$$\frac{\partial \boldsymbol{n}}{\partial t} = H' \boldsymbol{n} + \boldsymbol{Q}, \tag{8}$$

where

$$H' = \begin{pmatrix} \left(-A' + \frac{1}{k}B^{p'}\right)v_g & \sum_i \chi_{ig}^d \lambda_i \\ \frac{1}{k}\beta_i F' v_g & -\lambda_i \end{pmatrix}, \quad \boldsymbol{n} = \begin{pmatrix} n_g(\boldsymbol{r}, t) \\ C_i(\boldsymbol{r}, t) \end{pmatrix}, \quad \boldsymbol{Q} = \begin{pmatrix} Q_g(\boldsymbol{r}, t) \\ 0 \end{pmatrix}. \tag{9}$$

In order to derive one-point kinetics equations, we define importance functions $G_g(\boldsymbol{r})$ to produce fission neutrons by the equation,

$$A^\dagger G_g(\boldsymbol{r}) = \nu \Sigma_{fg} \delta(\boldsymbol{r}), \tag{10}$$

where $A^\dagger$ is an adjoint operator of operator A. Multiplying Eq.(1) by this importance function and integrating it over the whole space V, summing up over all energy groups and using notations from Eqs.(12) to (16), the following one-point kinetics equations can be derived[1,2];

$$l(t)\frac{ds(t)}{dt} = (\frac{1}{k}(1 - \beta(t))k^p(t) + \Delta k^F(t)\delta - \Delta k^A(t) - 1)s(t) + \sum_i k_i^d(t)\lambda_i C_i(t) + Q(t), \tag{11}$$

where $s(t)$ and $l(t)$ are the integrated fission source and mean generation time for the whole region V defined by

$$s(t) = \int_V s(\boldsymbol{r}, t)d\boldsymbol{r}, \quad s(\boldsymbol{r}, t) = F'\phi_g(\boldsymbol{r}, t) = (F + \delta F)\phi_g(\boldsymbol{r}, t), \tag{12}$$

and

$$l(t) = \frac{\int_V d\boldsymbol{r} \sum_g G_g(\boldsymbol{r})\dfrac{1}{v_g}\dfrac{\partial \phi_g(\boldsymbol{r}, t)}{\partial t}}{\int_V d\boldsymbol{r}\dfrac{\partial F'\phi_g(\boldsymbol{r}, t)}{\partial t}}, \tag{13}$$

respectively. $C_i(t)$ and $Q(t)$ are the total fission and delayed neutron precursor density over the whole region V defined by

$$C_i(t) = \int_V C_i(\boldsymbol{r}, t)d\boldsymbol{r}, \quad Q(t) = \int_V d\boldsymbol{r} \sum_g G_g(r)Q_g(\boldsymbol{r}, t). \tag{14}$$

The multiplication factor for prompt neutrons $k^p(t)$ and direct changes of multiplication factors $\Delta k^A(t)$ and $\Delta k^F(t)$ due to the perturbation are defined by

$$k^p(t) = \frac{\int_V d\boldsymbol{r} \sum_g G(\boldsymbol{r}, g)\chi_g^p F'\phi_g(\boldsymbol{r}, t)}{\int_V d\boldsymbol{r} F'\phi_g(\boldsymbol{r}, t)}, \quad k_i^d(t) = \frac{\int_V d\boldsymbol{r} \sum_g G(\boldsymbol{r}, g)\chi_{ig}^d C_i(\boldsymbol{r}, t)}{\int_V d\boldsymbol{r} C_i(\boldsymbol{r}, t)}, \tag{15}$$

$$\Delta k^A(t) = \frac{\int_V d\boldsymbol{r} \sum_g G_g(\boldsymbol{r})\delta A\phi_g(\boldsymbol{r}, t)}{\int_V d\boldsymbol{r} F'\phi_g(\boldsymbol{r}, t)}, \quad \text{and} \quad \Delta k^F(t) = \frac{\int_V d\boldsymbol{r} \delta F\phi_g(\boldsymbol{r}, t)}{\int_V d\boldsymbol{r} F'\phi_g(\boldsymbol{r}, t)}, \tag{16}$$

respectively.

Integrating Eq.(3) over V results in the following equations for delayed neutron precursors:

$$\frac{dC_i(t)}{dt} = \frac{1}{k}\beta_i s(t) - \lambda_i C_i(t). \tag{17}$$

Application of Generalized Perturbation Theory

Hereafter, we write exact functions and approximate ones in the form

$$\phi_g^e(\boldsymbol{r}, t) = \psi_g(\boldsymbol{r}, t) + \delta\phi_g(\boldsymbol{r}, t) = \psi_g(\boldsymbol{r})T(t) + \delta\phi_g(\boldsymbol{r}, t),$$
$$n_g^e(\boldsymbol{r}, t) = n_g(\boldsymbol{r}, t) + \delta n_g(\boldsymbol{r}, t) = n_g(\boldsymbol{r})T(t) + \delta n_g(\boldsymbol{r}, t),$$
$$C_i^e(\boldsymbol{r}, t) = C_i(\boldsymbol{r}, t) + \delta C_i(\boldsymbol{r}, t) = C_i(\boldsymbol{r})T_i(t) + \delta C_i(\boldsymbol{r}, t), \tag{18}$$

where $\phi_g^e(\boldsymbol{r}, t)$, for example, is the exact solution of Eq.(1), $\psi_g(\boldsymbol{r})$ is the unperturbed solution obtained from Eq.(4), and $\delta\phi_g(\boldsymbol{r}, t)$ is the difference of the approximate flux $\psi_g(\boldsymbol{r}, t)$ from the exact solution. The external source can be expressed by constant and time dependent terms of the form:

$$Q_g(\boldsymbol{r}, t) = Q_g^0(\boldsymbol{r}) + \delta Q_g(\boldsymbol{r}, t). \tag{19}$$

We write the multiplication factor $k^p(t)$ of Eq.(15) as:

$$k^p(t) = \frac{\int_V d\boldsymbol{r} \sum_{g'} G_{g'}(\boldsymbol{r})\chi_{g'}^p F'\phi_g(\boldsymbol{r}, t)}{\int_V d\boldsymbol{r} F'\phi_g(\boldsymbol{r}, t)} = \frac{\int_V d\boldsymbol{r} \sum_g \Sigma'_{\alpha g}(\boldsymbol{r})\phi_g(\boldsymbol{r}, t)}{\int_V d\boldsymbol{r} \sum_g \Sigma'_{\beta g}\phi_g(\boldsymbol{r}, t)}, \tag{20}$$

where

$$\Sigma'_{\alpha g}(\boldsymbol{r}) = \sum_{g'} G_{g'}(\boldsymbol{r})\chi_{g'}^p \nu\Sigma'_{fg}(\boldsymbol{r}), \quad \text{and} \quad \Sigma'_{\beta g}(\boldsymbol{r}) = \nu\Sigma'_{fg}(\boldsymbol{r}). \tag{21}$$

Using these notations and retaining first order terms of the perturbation, we write Eq.(20) as

$$k^p(t) = \frac{\langle(\Sigma_\alpha + \delta\Sigma_\alpha)(\psi + \delta\phi)\rangle}{\langle(\Sigma_\beta + \delta\Sigma_\beta)(\psi + \delta\phi)\rangle} = \frac{\langle\Sigma_\alpha\psi + \Sigma_\alpha\delta\phi + \delta\Sigma_\alpha\psi + \delta\Sigma_\alpha\delta\phi\rangle}{\langle\Sigma_\beta\psi + \Sigma_\beta\delta\phi + \delta\Sigma_\beta\psi + \delta\Sigma_\beta\delta\phi)\rangle}$$
$$\doteq \frac{\langle\Sigma_\alpha\psi\rangle}{\langle\Sigma_\beta\psi\rangle}\left(1 + \frac{\langle\Sigma_\alpha\delta\phi\rangle}{\langle\Sigma_\alpha\psi\rangle} + \frac{\langle\delta\Sigma_\alpha\psi\rangle}{\langle\Sigma_\alpha\psi\rangle} - \frac{\langle\Sigma_\beta\delta\phi\rangle}{\langle\Sigma_\beta\psi\rangle} - \frac{\langle\delta\Sigma_\beta\psi\rangle}{\langle\Sigma_\beta\psi\rangle}\right), \tag{22}$$

where $\langle\Sigma_\alpha\psi\rangle = \int_V d\boldsymbol{r} \sum_g \Sigma_{\alpha g}(\boldsymbol{r})\psi_g(\boldsymbol{r})$. In order to calculate the first order terms of the change of the flux due to the perturbation, $\langle\Sigma_\alpha\delta\phi\rangle$ and $\langle\Sigma_\beta\delta\phi\rangle$, we use the GPT.

We define the following quantity:

$$q_g(\boldsymbol{r}) = \frac{\Sigma_{\alpha g}(\boldsymbol{r})v_g}{\langle\Sigma_{\alpha g}\psi_g\rangle} - \frac{\Sigma_{\beta g}(\boldsymbol{r})v_g}{\langle\Sigma_{\beta g}\psi_g\rangle}. \tag{23}$$

From this definition, we can know that the following equation holds:

$$\int_V d\boldsymbol{r} \sum_g n_g(\boldsymbol{r})q_g(\boldsymbol{r}) = \langle n_g(\boldsymbol{r})q_g(\boldsymbol{r})\rangle = 0. \tag{24}$$

Using Eq.(23), Eq.(22) can be written in a form,

$$k^p(t) \doteq \frac{\langle\Sigma_\alpha\psi\rangle}{\langle\Sigma_\beta\psi\rangle}\left(1 + \frac{\langle\delta\Sigma_\alpha\psi\rangle}{\langle\Sigma_\alpha\psi\rangle} - \frac{\langle\delta\Sigma_\beta\psi\rangle}{\langle\Sigma_\beta\psi\rangle} + \langle q\delta n\rangle\right). \tag{25}$$

Using an adjoint operator $H^\dagger$ of operator H of Eq.(9) and $q_g(\boldsymbol{r})$ of Eq.(23), we consider the following equation:

$$H^\dagger\boldsymbol{n}^\dagger = \boldsymbol{q}, \quad \boldsymbol{n}^\dagger = \begin{pmatrix} n_g^\dagger(\boldsymbol{r}) \\ C_i^\dagger(\boldsymbol{r}) \end{pmatrix}, \quad \boldsymbol{q} = \begin{pmatrix} q_g(\boldsymbol{r}) \\ 0 \end{pmatrix}. \tag{26}$$

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Using Eq.(18) in Eq.(8), we obtain

$$\frac{\partial(\boldsymbol{n}+\delta\boldsymbol{n})}{\partial t} = (H + \delta H)(\boldsymbol{n} + \delta\boldsymbol{n}) + \boldsymbol{Q} \doteq H\boldsymbol{n} + \delta H\boldsymbol{n} + H\delta\boldsymbol{n} + \boldsymbol{Q}^0 + \delta\boldsymbol{Q}, \tag{27}$$

where

$$\boldsymbol{n} = \begin{pmatrix} n_g(\boldsymbol{r})T(t) \\ C_i(\boldsymbol{r})T_i(t) \end{pmatrix}, \quad \delta\boldsymbol{n} = \begin{pmatrix} \delta n_g(\boldsymbol{r},t) \\ \delta C_i(\boldsymbol{r},t) \end{pmatrix}. \tag{28}$$

Multiplying Eq.(27) by $\boldsymbol{n}^\dagger$ and integrating it over V, we obtain

$$\frac{\partial\left\langle \boldsymbol{n}^\dagger(\boldsymbol{n}+\delta\boldsymbol{n})\right\rangle}{\partial t} = \left\langle \boldsymbol{n}^\dagger H\boldsymbol{n}\right\rangle + \left\langle \boldsymbol{n}^\dagger\delta H\boldsymbol{n}\right\rangle + \left\langle \boldsymbol{n}^\dagger H\delta\boldsymbol{n}\right\rangle + \left\langle \boldsymbol{n}^\dagger\boldsymbol{Q}^0\right\rangle + \left\langle \boldsymbol{n}^\dagger\delta\boldsymbol{Q}\right\rangle. \tag{29}$$

As in deriving conventional one-point kinetics equations, we use the following approximation

$$\frac{\partial\left\langle \boldsymbol{n}^\dagger\delta\boldsymbol{n}\right\rangle}{\partial t} \doteq 0. \tag{30}$$

When $\boldsymbol{n}^\dagger$ is obtained from Eq.(26), we use the following condition

$$\left\langle \boldsymbol{n}^\dagger\boldsymbol{n}\right\rangle = \left\langle n_g^\dagger(\boldsymbol{r})n_g(\boldsymbol{r})\right\rangle + \left\langle C_i^\dagger(\boldsymbol{r})C_i(\boldsymbol{r})\right\rangle = 0. \tag{31}$$

Then, from Eq.(29), we obtain

$$\left\langle \boldsymbol{n}^\dagger H\boldsymbol{n}\right\rangle + \left\langle \boldsymbol{n}^\dagger\delta H\boldsymbol{n}\right\rangle + \left\langle \boldsymbol{n}^\dagger H\delta\boldsymbol{n}\right\rangle + \left\langle \boldsymbol{n}^\dagger\boldsymbol{Q}^0\right\rangle + \left\langle \boldsymbol{n}^\dagger\delta\boldsymbol{Q}\right\rangle = 0. \tag{32}$$

Using the following equation

$$\left\langle \boldsymbol{n}^\dagger H\boldsymbol{n}\right\rangle + \left\langle \boldsymbol{n}^\dagger\boldsymbol{Q}^0\right\rangle = 0, \tag{33}$$

which is derived from Eq.(4), we obtain

$$\left\langle \boldsymbol{n}^\dagger H\delta\boldsymbol{n}\right\rangle = \left\langle \delta\boldsymbol{n}H^\dagger\boldsymbol{n}^\dagger\right\rangle = \left\langle \delta\boldsymbol{n}\boldsymbol{q}\right\rangle = -\left\langle \boldsymbol{n}^\dagger\delta H\boldsymbol{n}\right\rangle - \left\langle \boldsymbol{n}^\dagger\delta\boldsymbol{Q}\right\rangle. \tag{34}$$

Using Eq.(9), (26) and (28), the explicit forms of each term of Eq.(34) are

$$\left\langle \delta\boldsymbol{n}\boldsymbol{q}\right\rangle = \left\langle (\delta n_g(\boldsymbol{r},t)\ \delta C_i(\boldsymbol{r},t))\begin{pmatrix} q_g(\boldsymbol{r}) \\ 0 \end{pmatrix}\right\rangle = \left\langle \delta n_g(\boldsymbol{r},t)q_g(\boldsymbol{r})\right\rangle$$

$$\left\langle \boldsymbol{n}^\dagger\delta H\boldsymbol{n}\right\rangle = \left\langle (n_g^\dagger(\boldsymbol{r})\ C_i^\dagger(\boldsymbol{r}))\begin{pmatrix} \delta H^p & 0 \\ \frac{1}{k}\beta_i\delta Fv_g & 0 \end{pmatrix}\begin{pmatrix} n_g(\boldsymbol{r},t) \\ C_i(\boldsymbol{r},t) \end{pmatrix}\right\rangle \tag{35}$$

$$= \left\langle (n_g^\dagger(\boldsymbol{r})\ C_i^\dagger(\boldsymbol{r}))\begin{pmatrix} \delta H^p n_g(\boldsymbol{r},t) \\ \frac{1}{k}\beta_i\delta Fv_g n_g(\boldsymbol{r},t) \end{pmatrix}\right\rangle$$

$$= \left\langle (n_g^\dagger(\boldsymbol{r})\delta H^p n_g(\boldsymbol{r},t)\right\rangle + \left\langle C_i^\dagger(\boldsymbol{r})\frac{1}{k}\beta_i\delta Fv_g n_g(\boldsymbol{r},t)\right\rangle \tag{36}$$

$$\left\langle \boldsymbol{n}^\dagger\delta\boldsymbol{Q}\right\rangle = \left\langle (n_g^\dagger(\boldsymbol{r})\ C_i^\dagger(\boldsymbol{r}))\begin{pmatrix} \delta Q_g(\boldsymbol{r},t) \\ 0 \end{pmatrix}\right\rangle = \left\langle n^\dagger(\boldsymbol{r})\delta Q_g(\boldsymbol{r},t)\right\rangle, \tag{37}$$

from which we obtain

$$\langle \delta n_g(\boldsymbol{r},t)q_g(\boldsymbol{r})\rangle = -\left\langle (n_g^\dagger(\boldsymbol{r})\delta H^p n_g(\boldsymbol{r},t)\right\rangle - \left\langle C_i^\dagger(\boldsymbol{r})\frac{1}{k}\beta_i\delta Fv_g n_g(\boldsymbol{r},t)\right\rangle - \left\langle n^\dagger(\boldsymbol{r})\delta Q_g(\boldsymbol{r},t)\right\rangle, \tag{38}$$

where

$$\delta H^p = -(\delta A + \frac{1}{k}\delta B^p)v_g. \tag{39}$$

Using Eq.(38), we can obtain kinetics parameter of Eq.(25) including first order term of the perturbation.

One-Point Kinetics Equations for the Quasistatic Method

In the quasi-static method, the neutron flux is expressed as the product of the amplitude function $T(t)$ which changes quickly with time, and the shape function $\psi_g(\boldsymbol{r}, t)$ which changes slowly as:

$$\phi_g(\boldsymbol{r}, t) = T(t)\psi_g(\boldsymbol{r}, t). \tag{40}$$

Substitution of Eq.(40) into Eq.(1) gives

$$\frac{1}{v_g}\frac{dT(t)}{dt}\psi_g(\boldsymbol{r}, t) + \frac{1}{v_g}T(t)\frac{\partial\psi_g(\boldsymbol{r}, t)}{\partial t} = (-A' + B^{p'})T(t)\psi_g(\boldsymbol{r}, t) + \sum_i \chi_{ig}^d \lambda_i C_i(\boldsymbol{r}, t) + Q_g(\boldsymbol{r}, t). \tag{41}$$

Multiplying Eq.(41) by an appropriate weight function $w_g(\boldsymbol{r})$, we obtain

$$\left\langle w_g(\boldsymbol{r})\frac{1}{v_g}\psi_g(\boldsymbol{r}, t)\right\rangle \frac{dT(t)}{dt} + \left\langle w_g(\boldsymbol{r})\frac{1}{v_g}\frac{\partial\psi_g(\boldsymbol{r}, t)}{\partial t}\right\rangle T(t)$$

$$= \left\langle w_g(\boldsymbol{r})(-A' + B^{p'})\psi_g(\boldsymbol{r}, t)\right\rangle T(t) + \sum_i \lambda_i \left\langle w_g(\boldsymbol{r})\chi_{ig}^d C_i(\boldsymbol{r}, t)\right\rangle + \left\langle w_g(\boldsymbol{r})Q_g(\boldsymbol{r}, t)\right\rangle. \tag{42}$$

In the quasi static method, the following is assumed for shape function:

$$\left\langle w_g(\boldsymbol{r})\frac{1}{v_g}\psi_g(\boldsymbol{r}, t)\right\rangle = \text{const.} \tag{43}$$

The shape function can be written as the sum of the initial function $\psi_g(\boldsymbol{r}, t_0)$ and its difference from the initial function $\delta\psi_g(\boldsymbol{r}, t)$ as

$$\psi_g(\boldsymbol{r}, t) = \psi_g(\boldsymbol{r}, t_0) + \delta\psi_g(\boldsymbol{r}, t). \tag{44}$$

Using Eq.(44) in Eq.(43), we obtain

$$\left\langle w_g(\boldsymbol{r})\frac{1}{v_g}\psi_g(\boldsymbol{r}, t)\right\rangle = \left\langle w_g(\boldsymbol{r})\frac{1}{v_g}\psi_g(\boldsymbol{r}, t_0)\right\rangle + \left\langle w_g(\boldsymbol{r})\frac{1}{v_g}\delta\psi_g(\boldsymbol{r}, t)\right\rangle = \text{const.} \tag{45}$$

Therefore the difference of the shape function from the initial shape satisfies the following equation:

$$\left\langle w_g(\boldsymbol{r})\frac{1}{v_g}\delta\psi_g(\boldsymbol{r}, t)\right\rangle = 0. \tag{46}$$

We define the following quantities:

$$l(t) = \left\langle w_g(\boldsymbol{r})\frac{1}{v_g}\psi_g(\boldsymbol{r}, t)\right\rangle, \quad \rho(t) = \left\langle w_g(\boldsymbol{r})(-A' + B')\psi_g(\boldsymbol{r}, t)\right\rangle,$$

$$\hat{C}_i(t) = \left\langle w_g(\boldsymbol{r})\chi_{ig}^d C_i(\boldsymbol{r}, t)\right\rangle, \quad \beta(t) = \left\langle w_g(\boldsymbol{r})\beta B\psi_g(\boldsymbol{r}, t)\right\rangle, \quad \hat{Q}(t) = \langle w_g(\boldsymbol{r})Q_g(\boldsymbol{r}, t)\rangle. \tag{47}$$

Using these quantities and Eq.(43), Eq.(42) can be written in the form,

$$l(t)\frac{dT(t)}{dt} = (\rho(t) - \beta(t))T(t) + \sum_i \lambda_i \hat{C}_i(t) + \hat{Q}(t), \tag{48}$$

When use is made of the initial condition that the initial flux at $t = t_0$ is given by the steady state flux of Eq.(4), namely

$$\phi_g(\boldsymbol{r}, t_0) = \psi_g(\boldsymbol{r}, t_0) = \psi_g(\boldsymbol{r}). \tag{49}$$

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the initial condition for $T(t)$ becomes

$$T(t_0) = 1. \tag{50}$$

Multiplying Eq.(3) by $w_g(\boldsymbol{r})\chi_{ig}^d$ and using quantities of Eq.(47), we obtain

$$\frac{d\hat{C}_i(t)}{dt} = -\lambda_i \hat{C}_i(t) + \beta_i \hat{F}' T(t), \tag{51}$$

where

$$\hat{F}' = \left\langle w_g(\boldsymbol{r})\chi_{ig}^d F' \psi_g(\boldsymbol{r}, t) \right\rangle. \tag{52}$$

Eq.(41) can be rewritten as

$$\frac{1}{v_g}\frac{\partial \psi_g(\boldsymbol{r}, t)}{\partial t} = (-A' + B^{p'} - \frac{1}{v_g T(t)}\frac{dT(t)}{dt})\psi_g(\boldsymbol{r}, t) + \frac{1}{T(t)}(\sum_i \lambda_i \chi_{ig}^d C_i(\boldsymbol{r}, t) + Q_g(\boldsymbol{r}, t)). \tag{53}$$

Using Eq.(40), Eq.(3) becomes

$$\frac{\partial C_i(\boldsymbol{r}, t)}{\partial t} = -\lambda_i C_i(\boldsymbol{r}, t) + \beta_i F' \psi_g(\boldsymbol{r}, t)T(t). \tag{54}$$

Solving the one-point kinetics equations, Eq.(48) and Eq.(51), the amplitude function $T(t)$ can be obtained. Using the amplitude function $T(t)$ thus obtained in Eqs.(53) and (54), new shape function $\psi_g(\boldsymbol{r}, t)$ can be obtained.

Weight Function

Let us consider how the weight function of Eq.(42) should be chosen to make numerical errors small. For any weight functions, the following equation holds;

$$\left\langle w_g(\boldsymbol{r})A'\psi_g(\boldsymbol{r}, t) \right\rangle = \left\langle \psi_g(\boldsymbol{r}, t)A'^\dagger w_g(\boldsymbol{r}) \right\rangle + \text{Boundary Term}, \tag{55}$$

where

$$\text{Boundary Term} = \sum_g \int_S (\psi_g(\boldsymbol{r}, t)D_g'\boldsymbol{n}\nabla w_g(\boldsymbol{r}) - w_g(\boldsymbol{r})D_g'\boldsymbol{n}\nabla\psi_g(\boldsymbol{r}, t))dS, \tag{56}$$

from which the boundary condition for $w_g(\boldsymbol{r})$ at outer boundary S of the system is obtained. If we use the boundary condition

$$\psi_g(\boldsymbol{r}, t)\Big|_{\boldsymbol{r}\in S} = 0. \tag{57}$$

for the shape function, we should use the boundary condition for the weight function

$$w_g(\boldsymbol{r})|_{\boldsymbol{r}\in S} = 0, \tag{58}$$

in order for the boundary term of Eq.(56) to vanish. Using Eq.(44), the first term of the right hand side of Eq.(8) becomes

$$\left\langle w_g(\boldsymbol{r})(-A' + B^{p'})\psi_g(\boldsymbol{r}, t) \right\rangle = \left\langle w_g(\boldsymbol{r})(-A' + B^{p'})\psi_g(\boldsymbol{r}, t_0) \right\rangle + \left\langle w_g(\boldsymbol{r})(-A' + B^{p'})\delta\psi_g(\boldsymbol{r}, t) \right\rangle$$
$$= \left\langle w_g(\boldsymbol{r})(-A' + B^{p'})\psi_g(\boldsymbol{r}, t_0) \right\rangle + \left\langle \delta\psi_g(\boldsymbol{r}, t)(-A'^\dagger + B^{p'\dagger})w_g(\boldsymbol{r}) \right\rangle. \tag{59}$$

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In the conventional quasistatic method, the adjoint function for the following criticality equation of the adjoint equation to Eq.(4) is used for the weight function[10];

$$A^{\dagger}\psi_{kg}^{\dagger}(\boldsymbol{r}) = \frac{1}{k}B^{\dagger}\psi_{kg}^{\dagger}(\boldsymbol{r}). \tag{60}$$

Adjoint function $\psi_{kg}^{\dagger}(\boldsymbol{r})$ will be called as $k-$eigenfunction for simplicity. For this $k-$eigenfunction, we obtain the relation,

$$(-A^{\dagger} + B^{\dagger})\psi_{kg}^{\dagger}(\boldsymbol{r}) = \left(-A^{\dagger} + \frac{1}{k}B^{\dagger} - \frac{1}{k}B^{\dagger} + B^{\dagger}\right)\psi_{kg}^{\dagger}(\boldsymbol{r}) = \left(1 - \frac{1}{k}\right)B^{\dagger}\psi_{kg}^{\dagger}(\boldsymbol{r}). \tag{61}$$

Using this $k-$eigenfunctions $\psi_{kg}^{\dagger}(\boldsymbol{r})$ for the weight function, the last term of Eq.(59) becomes

$$\left\langle \delta\psi_g(\boldsymbol{r},t)(-A'^{\dagger} + B^{p'\dagger})\psi_{kg}^{\dagger}(\boldsymbol{r})\right\rangle = \left\langle \delta\psi_g(\boldsymbol{r},t)(-A'^{\dagger} + (1-\beta)B'^{\dagger})\psi_{kg}^{\dagger}(\boldsymbol{r})\right\rangle$$

$$\doteq \left\langle \delta\psi_g(\boldsymbol{r},t)(-A^{\dagger} + B^{\dagger})\psi_{kg}^{\dagger}(\boldsymbol{r})\right\rangle = \left(1 - \frac{1}{k}\right)\left\langle \delta\psi_g(\boldsymbol{r},t)B^{\dagger}\psi_{kg}^{\dagger}(\boldsymbol{r})\right\rangle. \tag{62}$$

If the criticality factor k is close to 1, the above term is small, and it can be neglected regarding it as the second order term. However, if the system is subcritical, and the criticality factor k is not close to 1, neglecting this term gives an error of ignoring the first order change of the shape function. This first order term of Eq.(62) for the subcritical system can be eliminated by using the following adjoint equation;

$$(-A^{\dagger} + B^{\dagger})\psi_{\omega g}^{\dagger}(\boldsymbol{r}) = \frac{\omega}{v_g}\psi_{\omega g}^{\dagger}(\boldsymbol{r}), \tag{63}$$

which is obtained by putting $\psi_{\omega g}^{\dagger}(\boldsymbol{r},t) = \psi_{\omega g}^{\dagger}(\boldsymbol{r})e^{\omega t}$ in time dependent adjoint equations. The eigenfunction $\psi_{\omega g}^{\dagger}(\boldsymbol{r})$ of Eq.(63) will be called as $\omega-$eigenfunction. Neglecting 2nd order terms due to the perturbation and using Eq.(63) for the weight function in Eq.(59) and the condition of Eq.(46), we obtain

$$\left\langle \delta\psi_g(\boldsymbol{r},t)(-A'^{\dagger} + B^{p'\dagger})\psi_{\omega g}^{\dagger}(\boldsymbol{r})\right\rangle \doteq \left\langle \delta\psi_g(\boldsymbol{r},t)(-A^{\dagger} + B^{\dagger})\psi_{\omega g}^{\dagger}(\boldsymbol{r})\right\rangle = \left\langle \delta\psi_g(\boldsymbol{r},t)\frac{\omega}{v_g}\psi_{\omega g}^{\dagger}(\boldsymbol{r})\right\rangle = 0. \tag{64}$$

Therefore, using the $\omega-$eigenfunctions for the weight function, the first order change of the shape function due to the perturbation has vanished, and it does not need be neglected. Then the accuracy of Eq.(48) with the use of $\omega-$eigenfunctions should be improved compared to using of $k-$eigenfunctions.

Slab Geometry with Two Groups without Delayed Neutrons

In order to investigate the accuracy of the equations derived in the preceding chapters, sample calculations were made for a simple geometry of a homogeneous slab with two groups. For simplicity, delayed neutrons are neglected. Perturbation of changing removal cross section was put at a local position ξ in a form

$$\delta A = \Delta\Sigma_{rg}\delta(x - \xi), \tag{65}$$

where $\delta(x)$ is the Dirac's delta function. Neglecting delayed neutrons, Eq.(1) for this slab geometry becomes

$$\frac{1}{v_g}\frac{\partial\phi_g(x,t)}{\partial t} = \left(D_g\frac{d^2}{dx^2} - \Sigma_{rg} + \Delta\Sigma_g\delta(x - \xi)\right)\phi_g(x,t) + \sum_{g'}\Sigma_{sg\leftarrow g'}\phi_{g'}(x,t)$$

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$$+\chi_g \sum_{g'} \nu \Sigma_{fg'} \phi_{g'}(x,t) + Q_g(x,t). \tag{66}$$

External source and fission neutrons is assumed to be given only at the first group, namely

$$Q_g(x) = Q_1^0 \delta_{g1} \cos B_1 x, \qquad \chi_g = \delta_{g1}, \tag{67}$$

where $\delta_{gg'}$ is the Kronecker's delta and $B_1 = \pi/a$, a being a thickness of the slab.

The initial flux at $t = t_0$ is given by the steady state equation of Eq.(4),

$$\left(-D_1 \frac{d^2}{dx^2} + \Sigma_{r1} \right) \psi_1(x) = \nu \Sigma_{f1} \psi_1(x) + \nu \Sigma_{f2} \psi_2(x) + Q_1^0 \cos B_1 x,$$

$$\left(-D_2 \frac{d^2}{dx^2} + \Sigma_{r2} \right) \psi_2(x) = \Sigma_{s2 \leftarrow 1} \psi_1(x). \tag{68}$$

The solution of Eq.(68) is given by

$$\psi_g(x) = C_{kg} \cos B_1 x, \qquad C_{k1} = \frac{Q_1^0}{\Sigma_{r1}(1 + L_1^2 B_1^2)(1 - k)},$$

$$C_{k2} = \frac{\Sigma_{s2 \leftarrow 1} Q_1^0}{\Sigma_{r1} \Sigma_{r2}(1 + L_1^2 B_1^2)(1 + L_2^2 B_1^2)(1 - k)}, \qquad L_g^2 = \frac{D_g}{\Sigma_{rg}}, \tag{69}$$

where the criticality factor is given by

$$k = \frac{\nu \Sigma_1}{\Sigma_{r1}(1 + L_1^2 B_1^2)} + \frac{\nu \Sigma_2 \Sigma_{s2 \leftarrow 1}}{\Sigma_{r1} \Sigma_{r2}(1 + L_1^2 B_1^2)(1 + L_2^2 B_1^2)}. \tag{70}$$

Putting $B_1 = 0$ in the above equation, the infinite multiplication factor is obtained as

$$k_\infty = \frac{\nu \Sigma_1}{\Sigma_{r1}} + \frac{\nu \Sigma_2 \Sigma_{s2 \leftarrow 1}}{\Sigma_{r1} \Sigma_{r2}}. \tag{71}$$

k−Eigenfunction for the adjoint equation

The adjoint equation of homogeneous equations of Eq.(68), namely Eq.(60) becomes

$$\left(-D_1 \frac{d^2}{dx^2} + \Sigma_{r1} \right) \psi_{k1}^\dagger(x) = \frac{1}{k} \nu \Sigma_{f1} \psi_{k1}(x) + \Sigma_{s2 \leftarrow 1} \psi_{k2}^\dagger(x),$$

$$\left(-D_2 \frac{d^2}{dx^2} + \Sigma_{r2} \right) \psi_{k2}^\dagger(x) = \frac{1}{k} \nu \Sigma_{f2} \psi_{k1}^\dagger(x). \tag{72}$$

Solution of the above equation and kinetics parameters of Eqs.(47) are given by

$$\psi_{kg}^\dagger(x) = C_{kg}^\dagger \cos B_1 x, \qquad C_{k1}^\dagger = 1, \qquad C_{k2}^\dagger = \frac{\nu \Sigma_{f2}}{k \Sigma_{r2}(1 + L_2^2 B_1^2)} \tag{73}$$

$$l_k = \left\langle \psi_{kg}^\dagger(x) \frac{1}{v_g} \psi_g(x) \right\rangle = \frac{a}{2} \sum_g \frac{1}{v_g} C_{kg}^\dagger C_{kg}, \quad \rho_k = -\sum_g (\Delta \Sigma_{rg} C_{kg}^\dagger C_{kg} \cos 2B_1 \xi + Q_g^0 \delta_{g0}). \tag{74}$$

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ω−Eigenfunction for the adjoint equation

Explicit form of Eq.(63) is

$$\left(-D_1\frac{d^2}{dx^2} + \Sigma_{r1}\right)\psi_{\omega 1}^\dagger(x) = \nu\Sigma_{f1}\psi_{\omega 1}^\dagger(x) + \Sigma_{s2\leftarrow 1}\psi_{\omega 2}^\dagger(x) - \frac{\omega}{v_1}\psi_{\omega 1}^\dagger(x),$$

$$\left(-D_2\frac{d^2}{dx^2} + \Sigma_{r2}\right)\psi_{\omega 2}^\dagger(x) = \nu\Sigma_{f2}\psi_{\omega 1}^\dagger(x) - \frac{\omega}{v_2}\psi_{\omega 2}^\dagger(x). \tag{75}$$

Solution of Eq.(75) and kinetics parameters of Eq.(47) are given by

$$\psi_{\omega g}^\dagger(x) = C_{\omega g}^\dagger\cos B_1 x, \qquad C_{\omega 1}^\dagger = 1, \qquad C_{\omega 2}^\dagger = \frac{\nu\Sigma_{f2}}{\Sigma_{r2}(1 + L_2^2 B_1^2 + \omega^+ l_2)}, \tag{76}$$

$$l_\omega = \left\langle \psi_{\omega g}^\dagger(x)\frac{1}{v_g}\psi_g(x)\right\rangle = \frac{a}{2}\sum_g\frac{1}{v_g}C_{\omega g}^\dagger C_{kg}, \qquad \rho_\omega = -\sum_g(\Delta\Sigma_{rg}C_{\omega g}^\dagger C_{kg}\cos\, 2B_1\xi + Q_g^0\delta_{g0}),$$

$$\omega^\pm = \frac{-b\pm\sqrt{b^2 - 4l_{1ef}l_{2ef}(1-k)}}{2l_{1ef}l_{2ef}}, \qquad l_{gef} = \frac{l_g}{1 + L_g^2 B_1^2},$$

$$b = l_{1ef} + l_{2ef}\left(1 - \frac{\nu\Sigma_{f1}}{\Sigma_{r1}(1 + L_1^2 B_1^2)}\right), \qquad l_g = \frac{1}{v_g\Sigma_{rg}}. \tag{77}$$

Since ω^+ corresponds to the fundamental mode of the time change of the flux, this is used in Eq.(76).

If we neglect the delayed neutrons, the solution of Eq.(48) becomes

$$T(t) = T(0)e^{\frac{\rho}{l}t} - \frac{1}{\rho}\left(1 - e^{\frac{\rho}{l}t}\right)\hat{Q}, \tag{78}$$

using the kinetics parameters of Eqs.(74) or (77).

Exact Solution Depending on Space and Time

Exact solution of Eq.(66) can be obtained by expanding the flux into Fourier series as

$$\phi_g(x,t) = \sum_{m'}\phi_{gm'}(t)\sin B_{m'}x, \qquad B_m = \frac{m\pi}{a}. \tag{79}$$

Substituting Eqs.(79) into Eq.(66), we obtain equations for the expansion coefficients as

$$l_g\frac{d\phi_{gm}(t)}{dt} = -(L_g^2 B_m^2 + 1)\phi_{gm}(t) - \sum_{m'}\frac{2\Delta\Sigma_g}{a\Sigma_{rg}}\phi_{gm'}(t)\sin B_m(\xi + \frac{a}{2})\sin B_{m'}(\xi + \frac{a}{2})$$

$$+ \frac{1}{\Sigma_{rg}}\sum_{g'}\Sigma_{sg\leftarrow g'}\phi_{g'm}(t) + \frac{1}{\Sigma_{rg}}\chi_g\sum_{g'}\nu\Sigma_{fg'}\phi_{g'm}(t) + \frac{1}{\Sigma_{rg}}Q^0\delta_{g1}. \tag{80}$$

Integrating Eq.(80) with respect to time t numerically, expansion coefficients $\phi_{gm}(t)$ were obtained for $g = 1$ and 2.

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Numerical Example

Two group constants used are shown in Table 1, which were reduced into two groups from three group constants for fast lead-cooled system given by Coppa et al.[10] by simply averaging first and second group constants into one group. Velocities given in the Table correspond to the energy 1MeV and 100keV, which are chosen somewhat arbitrarily.

Table I Two group constants

g	D_g(cm)	Σ_{rg}(cm^{-1})	$\Sigma_{s2\leftarrow1}$(cm^{-1})	$\nu\Sigma_{fg}$(cm^{-1})	v_g(cm/s)
1	1.710	7.27×10^{-3}	4.45×10^{-3}	3.68×10^{-3}	1.39×10^9
2	0.849	9.91×10^{-3}	0	9.32×10^{-3}	4.40×10^8

The thickness of the slab was assumed to be $a = 50$cm. The position of the perturbation $\xi = a/20$ cm, its magnitudes $\Delta\Sigma_{r1} = -0.5\Sigma_{r1}$cm^{-1}, and $\Delta\Sigma_{r2} = -0.5\Sigma_{r2}$cm^{-1}. The infinite multiplication factor of Eq.(71) $k_\infty = 1.08185$, and the criticality factor of Eq.(70) $k = 0.485517$. In Figs.1 and 2 are shown $k-$eigenfunctions and $\omega-$eigenfunctions of Eq.(73) and (76), respectively. Since both coefficients of eigenfunctions for the first group were normalized to unity, we can see In Figs.1 and 2 that the magnitude of the adjoint flux of the second group differs appreciably, which results the difference in reactivity ρ of Eqs.(74) and (77), when the perturbation is added.
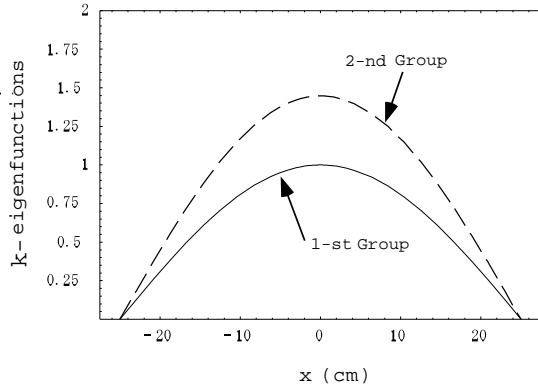

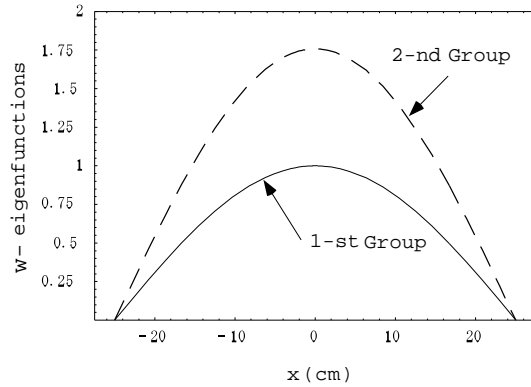Figure 1. $k-$eigenfunctions of Eq.(74) for the adjoint equation

Figure 2. $\omega-$eigenfunctons of Eq.(77) for the adjoint equation

The amplitude functions $T(t)$ calculated from Eq.(78) are shown in Figs.3 and 4. In order to know the accuracy of these amplitude functions, the weighted exact fluxes calculated by the following Eqs.(81) and (82),

$$\left\langle \psi^\dagger_{\omega g}(x)\frac{1}{v_g}\psi_g(x,t) \right\rangle = \left\langle \psi^\dagger_{\omega g}(x)\frac{1}{v_g}\sum_m \psi_{gm}(t)\sin B_m x \right\rangle, \tag{81}$$

and

$$\left\langle \psi^\dagger_{kg}(x)\frac{1}{v_g}\psi_g(x,t) \right\rangle = \left\langle \psi^\dagger_{kg}(x)\frac{1}{v_g}\sum_m \psi_{gm}(t)\sin B_m x \right\rangle. \tag{82}$$

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respectively, are also shown in Figs.3 and 4, where the exact time dependent flux of Eq.(80) was used for $\psi_g(x,t)$ in these equations. Therefore these quantities are designated as exact value in Figs.3 and 4, where those values are normalized to unity at $t = 0$.

As seen in Figs.3 and 4, the difference of the weighted exact flux between $\omega-$eigenfunctions and $k-$eigenfunctions of Eq.(81) and Eq.(82) is small. From these figures, we can see that the use of $\omega-$eigenfunctions gives more accurate results than the use of current $k-$eigenfunctions as could be expected from the preceding formulation of the one-point kinetics equations.
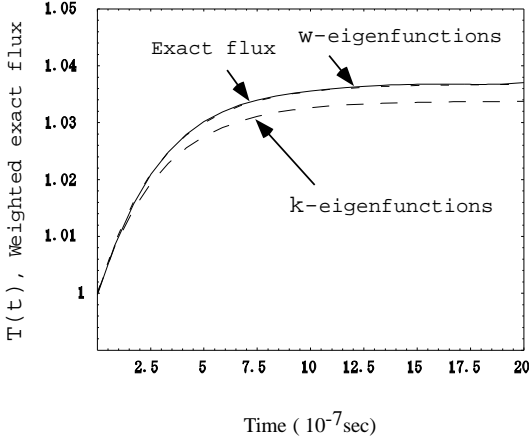

Figure 3. $T(t)$ and weighted exact flux with $\omega-$eigenfunctions of Eq.(81)
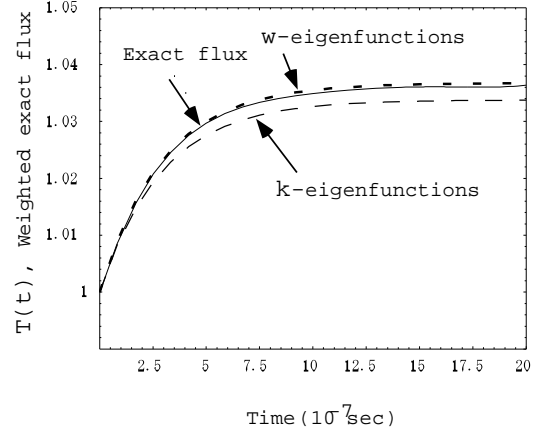
Figure 4. $T(t)$ and weighted exact flux with $k-$eigenfunctions of Eq.(82)

Conclusion

In the first part, one-point kinetics equations which take into account the first order change of the flux due to the perturbation are derived using the generalized perturbation theory. An advantage of these kinetics equations which are derived using an importance function to produce fission neutrons is in that all kinetics parameters and dependent functions have clear physical meanings, and they may be convenient to analyze experiments.

In the second part, it is shown that the use of $\omega-$eigenfunction as a weight function in the quasistatic method can eliminate the term of the first order change of the shape function, and the resulting one-point kinetics equations do not have an error of neglecting the change of the shape function. Simple numerical calculations show that the use of the $\omega-$eigenfunctions gives more accurate results than the use of current $k-$eigenfunctions as expected from the theory.

If a perturbation causes the increase of the flux in some energy groups, but at the same time the decrease in other energy groups, it may be better to assume the group dependent amplitude function in the form;

$$\phi_g(\boldsymbol{r},t) = T_g(t)\psi_g(\boldsymbol{r},t), \tag{83}$$

instead of Eq.(40), although the resulting group-wise multi-point kinetics equations become a little complicated.

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Acknowledgement

The author wishes to express his sincere thanks to Dr. E. Kiefhaber for his kind helps to improve the manuscript.

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