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Generalization of the analytical exponential model to solve the point kinetics equations of Be- and D_2O-moderated reactors

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ABSTRACT

A new mathematical formula of the period–reactivity relation for beryllium and heavy-water-moderated reactors has been presented. This formula is represented in a polynomial form with a degree $I+J+1$ for I-group of delayed neutrons and J-group of photoneutrons. A sample form for the coefficients of such a polynomial is presented which have a linear dependence on the step reactivity insertion. The analytical exponential model (AEM) is developed and generalized. The generalization of the analytical exponential model (GAEM) is analyzed and applied to solve point kinetics equations of the U^{235}-fuelled, Be- and D_2O-moderated reactors. The generalized method provides a fast and accurate computational technique for the point reactor kinetics equations of photoneutrons and delayed neutrons with step, ramp, sinusoidal and temperature feedback reactivity. Results of this method are presented for different types of reactivity and compared with other referenced methods.

Contents

1. Introduction

The dynamics of the fission chain-reacting system, in nuclear reactor, is determined primarily by the characteristics of the delayed emission of neutrons from the decay of fission products. Although there are a relatively large number of fission products which subsequently decay via neutron emission, the observed composite emission characteristics can be well represented by defining six effective groups of delayed neutron precursor fission products. Each group can be characterized by a decay constant, λ_i^{d}, and yield fraction, β_i^{d}. The total effective fraction of the total delayed fission neutrons is β^{d}.

In beryllium and heavy-water-moderated reactors, fission product characteristics must be extended to include the photoneutron precursor fission products which can be represented by nine effective groups of photoneutron precursor fission products. Each group can be characterized by a decay constant, λ_i^{p}, and yield fraction, β_i^{p}. The total effective fraction of the total photofission neutrons is β^{p}.

The photoneutrons are produced in a reactor in two ways. The first way is by gamma reactions (γ, n) which usually have high threshold energies (for Be 1.66 MeV, for D_2O 2.2 MeV). The sec-

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ond way is by photofission reactions (γ, f) taking place in heavy isotopes. The photoneutron periods, which are determined by β decay, are generally much longer than the delayed neutron periods; low-frequency kinetics behavior of a deuterium- or beryllium-moderated reactor can be quite sluggish compared to a system without photoneutrons.

Many authors have treated the point reactor kinetics equations of photoneutrons and delayed neutrons with different types of reactivity. Hainoun and Khamis (2000) presented an analytical solution for point reactor kinetics equations of average one-group of photoneutrons and delayed neutrons with step change of reactivity. Khamis (2002) treated this system with temperature feedback reactivity. Recently, Aboanber (2003b) presented a solution for point reactor kinetics equations of I-groups of delayed neutrons and J-groups of photoneutrons with constant reactivity using an analytical exponential model. The data of U^{235}-fuelled, beryllium- and heavy-water-moderated reactors were reported by many authors (Ash, 1979; Hainoun and Khamis, 2000; Khamis, 2002).

In this work, new sample mathematical formula of the period–reactivity equation, inhour equation, for beryllium and heavy-water-moderated reactors is presented, in Section 2.1, and its roots are calculated. The analytical exponential model (AEM) for solving the point reactor kinetics equations with constant reactivity (Aboanber, 2003a) which is representing neutronics with I-groups of delayed neutrons and J-groups of photoneutrons is introduced in Section 2.2. Generalization of the analytical exponential model is analyzed and applied for the point reactor kinetics equations of photoneutrons and delayed neutrons with step, ramp, sinusoidal and temperature feedback reactivity. The results of the generalized method are compared and discussed with different referenced methods in Section 3.

2. Analytical solutions

The core model consider space-average point kinetics representing neutronics with I-groups of delayed neutrons and J-groups of photoneutrons. The neutron kinetics equations are (Hetrick, 1971; Glasstone and Sesonske, 1981; Hainoun and Khamis, 2000; Stacey, 2001; Khamis, 2002; Jatuff et al., 2003)

$$\frac{dn(t)}{dt} = \left[\frac{\rho(t,n)-\beta}{\Lambda}\right] n(t) + \sum_{i=1}^{I} \lambda_i^{d} C_i^{d}(t) + \sum_{j=1}^{J} \lambda_j^{p} C_j^{p}(t) \tag{1}$$

$$\frac{dC_i^{d}(t)}{dt} = \frac{\gamma^{d}\beta_i^{d}}{\Lambda} n(t) - \lambda_i^{d} C_i^{d}(t), \quad i = 1, 2, \ldots, I \tag{2}$$

$$\frac{dC_j^{p}(t)}{dt} = \frac{\gamma^{p}\beta_j^{p}}{\Lambda} n(t) - \lambda_j^{p} C_j^{p}(t), \quad j = 1, 2, \ldots, J \tag{3}$$

where $n(t)$ is the neutron density, β is total effective fraction of delayed neutrons and photoneutrons, Λ is prompt neutron generation time, $\rho(t, n)$ is the reactivity which, in general, is a function of time and neutron density n. $C_i^{d}(t)$ and $C_j^{p}(t)$ are the concentration of precursor delayed neutrons and photoneutrons respectively. γ^{d} and γ^{p} are the effective coefficients of delayed neutrons and photoneutrons with estimated theoretical values for Miniature Neutron Source Reactor (MNSR) of 1.23 and 0.246 respectively (MNSR, 1992; Hainoun and Khamis, 2000).

Let us rewrite the Eqs. (1)–(3) as following

$$\frac{dn(t)}{dt} = \left[\frac{\rho(t,n)-\beta}{\Lambda}\right] n(t) + \sum_{k=1}^{K} \lambda_k C_k(t) \tag{4}$$

$$\frac{dC_k(t)}{dt} = \mu_k n(t) - \lambda_k C_k(t), \quad k = 1, 2, \ldots, K \tag{5}$$

where

$$C_k(t) = \begin{cases} C_k^{d}(t), & k = 1, 2, \ldots, I \\ C_{k-I}^{p}(t), & k = I+1, I+2, \ldots, K \end{cases},$$

$$\lambda_k = \begin{cases} \lambda_k^{d}, & k = 1, 2, \ldots, I \\ \lambda_{k-I}^{p}, & k = I+1, I+2, \ldots, K \end{cases},$$

$$\beta_k = \begin{cases} \gamma^{d}\beta_k^{d}, & k = 1, 2, \ldots, I \\ \gamma^{p}\beta_{k-1}^{p}, & k = I+1, I+2, \ldots, K \end{cases},$$

$$\beta = \sum_{k=1}^{K} \beta_k = \sum_{i=1}^{I} \gamma^{d}\beta_i^{d} + \sum_{j=1}^{J} \gamma^{p}\beta_j^{p}, \quad \mu_k = \frac{\beta_k}{\Lambda}, \quad \text{and} \quad K = I + J.$$

The Eqs. (4) and (5) are, in general, a system of stiff coupled nonlinear ordinary differential equations for the reactivity as a function of time and neutron density.

2.1. Period reactivity relations

Let us assume that the solution of the Eqs. (4) and (5) with step reactivity takes the form

$$n(t) = A\exp(\omega t) \tag{6}$$

and

$$C_k(t) = B_k \exp(\omega t) \tag{7}$$

where A and B_k are constants.

Substituting the Eqs. (6) and (7) into Eqs. (4) and (5) to determine the value of ω, it found that the value of ω

$$\rho = \Lambda\omega + \omega \sum_{k=1}^{K} \frac{\beta_k}{\omega + \lambda_k} \tag{8}$$

This equation is called inhour equation. For a fixed value of reactivity ρ this equation is a $(k+1)^{\text{th}}$ degree algebraic equation with $(K+1)$ roots.

To determine the roots of this equation, let us simplified this equation as:

$$\Lambda\omega \prod_{k=1}^{K}(\omega+\lambda_k) - \rho \prod_{k=1}^{K}(\omega+\lambda_k) + \omega \sum_{k=1}^{K} \beta_k \prod_{\substack{l=1 \\ l \neq k}}^{K} (\omega+\lambda_l) = 0 \tag{9}$$

Or

$$\sum_{m=0}^{K+1} \left(\Lambda D_{m,0} - \rho D_{m-1,0} + \sum_{l=1}^{K} \beta_l D_{m-1,l} \right) \omega^{K+1-m} = 0 \tag{10}$$

where

$$D_{m,l} = \begin{cases} 0, & \text{when } m < 0 \text{ or } m > K \\ 1, & \text{when } m = 0 \\ \displaystyle\sum_{\substack{l_1 = l \\ l_1 \neq 1}}^{K} \sum_{\substack{l_2 = l_1+1 \\ l_2 \neq l}}^{K} \cdots \sum_{\substack{l_m = l_{m-1}+1 \\ l_m \neq l}}^{K} \lambda_{l_1}\lambda_{l_2}\cdots\lambda_{l_m}, & \text{when } 0 < m \leq K \end{cases} \tag{11}$$

and value of $\displaystyle\sum_{\substack{l_m = l_{m-1}+1 \\ l_m \neq l}}^{K} \lambda_{l_1}\lambda_{l_2}\cdots\lambda_{l_m} = 0$ when $l_m > K$ or $l_m = l = K$.

This is a new mathematical form of inhour equation which is sample.

2.2. Analytical exponential model

After solved the inhour equation, there are $K+1$ solutions of the point kinetics equations. Then the general solution is the linear combination of all solutions.

$$n(t) = n(0)\sum_{m=1}^{K+1} A_m \exp(\omega_m t) \tag{12}$$

and

$$C_k(t) = C_k(0)\sum_{m=1}^{K+1} B_{m,k} \exp(\omega_m t) \tag{13}$$

where A_m and $B_{m,k}$ are constants that can be determined from initial conditions.

The initial conditions are $n(0)=n_0$, $C_k(0)=C_{k_0}$, and $(\mathrm{d}C_k/\mathrm{d}t)_{t=0}=0$.

Using these conditions yields

$$\sum_{m=1}^{K+1} A_m = 1, \sum_{m=1}^{K+1} B_{m,k} = 1 \quad \text{and} \quad C_{k_0} = \frac{\mu_k}{\lambda_k} n_0 \quad \text{where } k = 1, 2, \ldots, K \tag{14}$$

Substituting the Eqs. (12) and (13) into equation (5) at $t=0$ and using the conditions (14) yields

$$B_{m,k} = \frac{\lambda_k}{\omega_m + \lambda_k} A_m, \; m = 1, 2, \ldots, K+1, \quad \text{and} \quad k = 1, 2, \ldots, K \tag{15}$$

$$\sum_{m=1}^{K+1} A_m = 1 \quad \text{and} \quad \sum_{m=1}^{K+1} \frac{A_m}{\omega_m + \lambda_k} = \frac{1}{\lambda_k}, \quad k = 1, 2, \ldots, K \tag{16}$$

The solution of these algebraic equations gives the value of constants. Then, we have a general solution of point kinetics equations which called an analytical exponential model. This solution gives a good results for a constant reactivity only (Aboanber, 2003a).

2.3. Generalization of the analytical exponential model

Let us rewrite the point kinetics Eqs. (4) and (5), in matrix form as

$$\frac{\mathrm{d}\Psi(t)}{\mathrm{d}t} = F(t,n)\Psi(t) \tag{17}$$

where

$$\Psi(t) = \begin{pmatrix} n(t) \\ C_1(t) \\ C_2(t) \\ \vdots \\ C_K(t) \end{pmatrix}, \quad F(t,n) = \begin{pmatrix} \dfrac{\rho(t,n)-\beta}{\Lambda} & \lambda_1 & \lambda_2 & \cdots & \lambda_K \\ \mu_1 & -\lambda_1 & 0 & \cdots & 0 \\ \mu_2 & 0 & -\lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mu_K & 0 & 0 & \cdots & -\lambda_K \end{pmatrix} \tag{18}$$

The solution of the Eq. (17) take the form

$$\Psi(t) = \sum_{k=1}^{K+1} A_k \exp(\omega_k t) U_k \tag{19}$$

where A_k are constant matrices, ω_k are the eigenvalues of matrix $F(t,n)$ and therefore the roots of inhour Eq. (8), and U_k are the eigenvectors of matrix F which correspond to eigenvalues ω_k (Quarteroni et al., 2000).

Using the initial condition $\Psi(0)=\Psi_0$ to determine constant matrix $A_k = V_k^{\mathrm{T}}\Psi_0$, the Eq. (19) become

$$\Psi(t) = \sum_{k=1}^{K+1} \exp(\omega_k t) U_k V_k^{\mathrm{T}} \Psi_0 \tag{20}$$

where V_k are the eigenvectors of matrix F^{T} which correspond to eigenvalues ω_k and satisfies the normalization condition $U_k^{\mathrm{T}} V_k = 1$.

The eigenvectors U_k and V_k of matrix F and F^{T}, respectively, take the form Aboanber and Nahla, 2002

$$U_k = \begin{pmatrix} 1 \\ \dfrac{\mu_1}{\omega_k+\lambda_1} \\ \dfrac{\mu_2}{\omega_k+\lambda_2} \\ \vdots \\ \dfrac{\mu_K}{\omega_k+\lambda_K} \end{pmatrix}, \quad V_k = \frac{1}{1+\sum_{l=1}^{K}(\mu_l\lambda_l/(\omega_k+\lambda_l))} \begin{pmatrix} 1 \\ \dfrac{\lambda_1}{\omega_k+\lambda_1} \\ \dfrac{\lambda_2}{\omega_k+\lambda_2} \\ \vdots \\ \dfrac{\lambda_K}{\omega_k+\lambda_K} \end{pmatrix} \tag{21}$$

where $\left(1+\sum_{l=1}^{K}(\mu_l\lambda_l/(\omega_k+\lambda_l))\right)^{-1}$ is the normalization factor.

The Eq. (20) gives a good results for a constant reactivity only, but for ramp, sinusoidal and temperature feedback reactivity the Eq. (20) can take the form

$$\Psi(t_{m+1}) = \sum_{k=1}^{K+1} \exp(\omega_k \Delta t_m) U_k V_k^{\mathrm{T}} \Psi(t_m) \tag{22}$$

where $\Delta t_m = t_{m+1} - t_m$ is the time step interval.

The Eqs. (20) and (22) are the solution of point kinetics equations. This solution is considered a generalization of the analytical exponential model for ramp, sinusoidal and feedback reactivities.

3. Results and discussions

The generalization of the analytical exponential model (GAEM) is applied to solve the point kinetics equations with six-groups delayed neutrons. This model is extended to solve the point kinetics equations with six-groups delayed neutrons and nine-groups photoneutrons. The four types of problems step, ramp, sinusoidal, and feedback reactivities are presented. The values for β_i and λ_i of U^{235}-fuelled, beryllium- and heavy-water-moderated reactors, in Table 1, are taken from the references Lewins (1978), Hainoun and Khamis (2000) and Keepin (1962). All results started from the equilibrium conditions with $n(0)=1.0$ and the effective coefficients of photoneutrons and delayed neutrons taken values $\gamma^{\mathrm{p}}=1.0$ and $\gamma^{\mathrm{d}}=1.0$. In the following each example will be discussed separately.

3.1. Step reactivity

To check the accuracy of the GAEM, it is applied to the U^{235}-fuelled, beryllium- and heavy-water-moderated reactors, in Table 1, with step reactivity. Four different generation time (10^{-3}, 10^{-4}, 10^{-5}, 10^{-6} s) are presented in two cases of the step reactivity +0.5\$ and −0.5\$ in Tables 2 and 3, respectively. In most cases, the results show a large correction obtained at large transients. The GAEM is a fast and accurate computational technique for the point reactor kinetics equations. There are differences between neutron density with and without photoneutrons which confirm the importance of photoneutrons and its effects on the neutron density.

Table 1
The data of U^{235}-fuelled, Be- and D_2O-moderated reactors

Delayed neutrons [a]		Photoneutrons [b] of Be		Photoneutrons [c] of D_2O	
$\beta_i^d \times 10^{-3}$	λ_i^d	$\beta_i^p \times 10^{-6}$	λ_i^p	$\beta_i^p \times 10^{-5}$	λ_i^p
0.246	0.0127	20.7	2.265×10^{-2}	65.1	0.27726
1.363	0.0317	36.6	8.886×10^{-3}	20.4	1.691×10^{-2}
1.203	0.115	18.5	3.610×10^{-3}	7.00	4.813×10^{-3}
2.605	0.311	36.8	7.453×10^{-4}	3.36	1.500×10^{-3}
0.819	1.40	3.60	2.674×10^{-4}	2.07	4.279×10^{-4}
0.167	3.87	32.0	6.191×10^{-5}	2.34	1.167×10^{-4}
		2.60	1.591×10^{-5}	0.323	4.376×10^{-5}
		0.38	2.478×10^{-6}	0.103	3.633×10^{-6}
		0.57	6.098×10^{-7}	0.05	6.267×10^{-7}
$\beta^d = 6.4 \times 10^{-3}$		$\beta^p = 1.5175 \times 10^{-4}$		$\beta^p = 1.0075 \times 10^{-3}$	

[a] From Lewins (1978).
[b] From Hainoun and Khamis (2000).
[c] From Keepin (1962).

3.2. Ramp reactivity

In this case, the GAEM is applied to solve the point reactor kinetics equations with a positive change in reactivity as linear function of time. The neutron density of U^{235}-fuelled, Be- and D_2O-moderated reactors with reactivity ramp of 0.25$/s and 0.5$/s is introduced in Table 4 with a generation time of 10^{-5} s. The GAEM is a fast and accurate computational technique for the point reactor kinetics equations compared to Padé approximation with treatment of inhour equation roots. There are differences between neutron density with and without photoneutrons which confirm the importance of photoneutrons and its effects on the neutron density.

3.3. Sinusoidal reactivity

In this case, the point kinetics equations of the U^{235}-fuelled, beryllium-moderated reactors with delayed neutrons and photoneutrons are solved with an oscillatory perturbation in reactivity which is performed according to the form

$$\rho(t) = a \sin\left(\frac{\pi t}{\tau}\right)$$

where a is constant.

Table 2
The neutron density of U^{235}-fuelled, Be- and D_2O-moderated reactors with +0.5$ reactivity

Λ (s)	Time (s)	Delayed neutrons only		Delayed neutrons and Be-photoneutrons		Delayed neutrons and D_2O-photoneutrons	
		AEM	GAEM	AEM	GAEM	AEM	GAEM
10^{-3}	1.0	2.255399	2.255399	2.257078	2.257078	2.288550	2.288550
	5.0	5.026116	5.026116	4.905631	4.905630	4.941124	4.941124
	10.0	11.70712	11.70712	11.02691	11.02691	11.17554	11.17554
	20.0	59.02105	59.02105	51.36781	51.36781	52.69074	52.69074
10^{-4}	1.0	2.682980	2.682980	2.663643	2.663643	2.639445	2.639445
	5.0	6.287728	6.287727	6.039466	6.039465	5.935175	5.935175
	10.0	16.13041	16.13041	14.78766	14.78766	14.47796	14.47796
	20.0	99.52817	99.52817	82.45804	82.45803	79.99799	79.99799
10^{-5}	1.0	2.736489	2.736489	2.713641	2.713641	2.681562	2.681562
	5.0	6.462109	6.462109	6.192960	6.192961	6.065003	6.065003
	10.0	16.77494	16.77494	15.32133	15.32133	14.92747	14.92747
	20.0	106.0960	106.0960	87.31406	87.31407	84.04431	84.04431
10^{-6}	1.0	2.742005	2.742005	2.718783	2.718783	2.685878	2.685878
	5.0	6.480224	6.480224	6.208859	6.208860	6.078388	6.078389
	10.0	16.84233	16.84233	15.37693	15.37693	14.97405	14.97405
	20.0	106.7921	106.7921	87.82606	87.82607	84.46789	84.46790

Table 3
The neutron density of U^{235}-fuelled, Be- and D_2O-moderated reactors with −0.5$ reactivity

Λ (s)	Time (s)	Delayed neutrons only		Delayed neutrons and Be-photoneutrons		Delayed neutrons and D_2O-photoneutrons	
		AEM	GAEM	AEM	GAEM	AEM	GAEM
10^{-3}	1.0	0.613980	0.613980	0.614989	0.614989	0.615600	0.615600
	5.0	0.486539	0.486539	0.491809	0.491809	0.491740	0.491740
	10.0	0.399048	0.399048	0.408076	0.408076	0.406635	0.406635
	20.0	0.300732	0.300732	0.314792	0.314792	0.313986	0.313986
10^{-4}	1.0	0.603473	0.603473	0.605039	0.605040	0.607056	0.607056
	5.0	0.480605	0.480605	0.486276	0.486276	0.486763	0.486763
	10.0	0.394793	0.394793	0.404160	0.404160	0.403128	0.403128
	20.0	0.298001	0.298001	0.312311	0.312311	0.311822	0.311822
10^{-5}	1.0	0.602472	0.602472	0.604091	0.604091	0.606232	0.606232
	5.0	0.480016	0.480016	0.485728	0.485728	0.486269	0.486269
	10.0	0.394371	0.394371	0.403772	0.403772	0.402780	0.402780
	20.0	0.297730	0.297730	0.312064	0.312064	0.311607	0.311607
10^{-6}	1.0	0.602372	0.602372	0.603996	0.603996	0.606150	0.606150
	5.0	0.479957	0.479957	0.485673	0.485673	0.486220	0.486220
	10.0	0.394329	0.394329	0.403733	0.403733	0.402745	0.402745
	20.0	0.297703	0.297703	0.312039	0.312039	0.311586	0.311586

Table 4
The neutron density of U^{235}-fuelled, Be- and D_2O-moderated reactors with ramp reactivity and the generation time 10^{-5}

a (\$/s)	Time (s)	Delayed neutron only		Delayed neutrons and Be-photoneutrons		Delayed neutron and D_2O-photoneutrons	
		Padé	GAEM	Padé	GAEM	Padé	GAEM
0.25	0.25	1.069840	1.069541	1.069772	1.069474	1.069618	1.069363
	0.50	1.157065	1.156694	1.156727	1.156356	1.156063	1.155741
	0.75	1.265795	1.265331	1.264926	1.264476	1.263362	1.262974
	1.0	1.402562	1.401981	1.400780	1.400217	1.397818	1.397332
0.5	0.25	1.149544	1.149200	1.149394	1.149052	1.149109	1.148814
	0.50	1.369438	1.368927	1.368570	1.368076	1.366943	1.366515
	0.75	1.708411	1.707600	1.705677	1.704905	1.700876	1.700207
	1.0	2.276692	2.275271	2.269353	2.267997	2.257237	2.256093

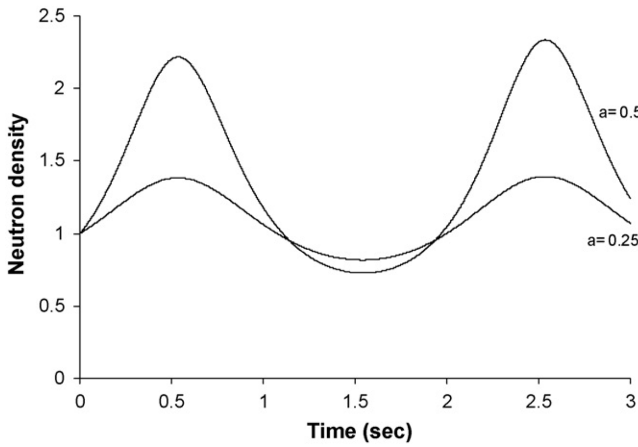

Fig. 1. The neutron density of delayed neutrons and Be-photoneutrons for sinusoidal reactivity.

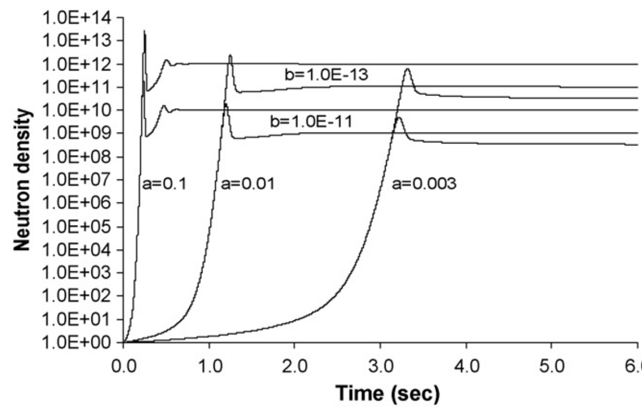

Fig. 2. The neutron density of delayed neutrons and D_2O-photoneutrons for feedback reactivity.

The generation time and the half-period time are taken the values 10^{-5} and 1.0 s, respectively, in this reactor. The time history of the neutron density over 3.0 s is illustrated, in Fig. 1, for two different cases $a = 0.25\$$ and 0.5\$.

3.4. Feedback reactivity

In this case, the GAEM is applied to solve the point reactor kinetics equations of photoneutrons and delayed neutrons with feedback reactivity, which is dependent on time and neutron density, as

$$\rho = at - b\int_0^t n(t')\,dt'$$

where the first term represent the impressed reactivity variation and b represent the shutdown coefficient of the reactor system.

The neutron density of delayed neutrons, delayed neutrons with Be- and D_2O-photoneutrons at first peak and its time are introduced in Table 5. The value of generation time is 5×10^{-5}, a takes the values 0.1, 0.01, 0.001, and b takes the values 10^{-11}, 10^{-13}. The GAEM is a fast and accurate computational technique for the point reactor kinetics equations compared to Padé approximation with treatment of inhour equation roots. Finally, the neutron density of delayed neutrons and D_2O-photoneutron with the generation time 5×10^{-5}, $a = 0.1$, 0.01, 0.003, $b = 10^{-11}$, 10^{-13} is drawn in Fig. 2.

Table 5
The neutron density at first peak of U^{235}-fuelled, Be- and D_2O-moderated reactors with feedback reactivity $\rho = at - b\int_0^t n(t')\,dt'$ and the generation time 5×10^{-5}

a	b	Method	Delayed neutrons only		Delayed neutrons and Be-photoneutrons		Delayed neutrons and D_2O-photoneutrons	
			First peak	Time	First peak	Time	First peak	Time
0.1	10^{-11}	Padé	2.4197×10^{11}	0.224	2.4224×10^{11}	0.225	2.4063×10^{11}	0.233
		GAEM	2.4202×10^{11}	0.224	2.4222×10^{11}	0.225	2.4059×10^{11}	0.233
	10^{-13}	Padé	2.9055×10^{13}	0.238	2.8928×10^{13}	0.239	2.8727×10^{13}	0.248
		GAEM	2.9057×10^{13}	0.238	2.8919×10^{13}	0.239	2.8737×10^{13}	0.248
0.01	10^{-11}	Padé	2.0107×10^{10}	1.100	2.0079×10^{10}	1.115	1.9883×10^{10}	1.198
		GAEM	2.0103×10^{10}	1.100	2.0077×10^{10}	1.115	1.9879×10^{10}	1.198
	10^{-13}	Padé	2.4890×10^{12}	1.149	2.4864×10^{12}	1.164	2.4668×10^{12}	1.248
		GAEM	2.4882×10^{12}	1.149	2.4859×10^{12}	1.164	2.4672×10^{12}	1.248
0.001	10^{-11}	Padé	1.2645×10^{9}	7.425	1.2601×10^{9}	7.574	1.2026×10^{9}	8.389
		GAEM	1.2645×10^{9}	7.426	1.2601×10^{9}	7.576	1.2026×10^{9}	8.391
	10^{-13}	Padé	1.7088×10^{11}	7.621	1.7043×10^{11}	7.771	1.6419×10^{11}	8.590
		GAEM	1.7088×10^{11}	7.622	1.7043×10^{11}	7.772	1.6419×10^{11}	8.591

4. Conclusions

The prediction of the dynamic behavior for U^{235}-fuelled, Be- and D_2O-moderated reactors has required an appropriate mathematical model. So, a new mathematical formula of the period–reactivity inhour equation is presented. This formula is represented in a polynomial form with a degree $I+J+1$ for I-group of delayed neutrons and J-group of photoneutrons. The new formula of the period–reactivity equation has a linear dependence on the step reactivity insertion. The analytical exponential model is developed using this new formula of the period–reactivity equation. This method gives a good results for the point reactor kinetics equations with a constant reactivity only. So, the analytical exponential model is generalized using matrix formula of point reactor kinetics equations and the new formula of the period–reactivity equation.

Generalization of the analytical exponential model is analyzed and applied to solve the point reactor kinetics equations with six group of delayed neutrons and nine group of Be- and D_2O-photoneutrons. Results of this method are compared with AEM and Padé approximation with treatment of inhour equation roots. The relative between the CPU time of GAEM and Padé approximation methods is 0.5 for delayed neutron only and 0.8 for delayed neutrons and photoneutrons. So, the generalized method provides a fast and accurate computational technique for the point reactor kinetics equations of delayed neutrons and photoneutrons with step, ramp, sinusoidal, and feedback reactivities.

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