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Two-Region Kinetic Model for Reflected Reactors

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Abstract—Reflected reactors constitute one of the most important classes of nuclear reactors. Yet, experimental data clearly shows that the kinetic behavior of some types of reflected systems cannot be adequately characterized using the standard point kinetic model. In this work, a two-region, nodal kinetic model for reflected systems is developed that appears to explain the anomalous behavior of these systems in terms of integral parameters that are both measurable and calculable. The model is based on previous work by Avery and Cohn. We augment the Avery–Cohn model with the introduction of simple probability relationships essential to calculating the coupling parameters between the core and the reflector. Using these probability relationships, we then derive the definition of the effective multiplication factor k_{eff}, in terms of the multiplication factor of the core region, k_c, and the reflector return fraction, f. We also derive the reflected-core inhour equation and introduce expressions to calculate the neutron lifetimes characterizing the system's static and dynamic behavior. The model is then tested using Rossi-α data from the University of New Mexico's AGN-201 reactor. Copyright © 1996 Elsevier Science Ltd

I. INTRODUCTION

Reflected reactors constitute one of the most important classes of nuclear reactors. Yet, during the past 50 years, a plethora of experimental data involving reflected systems has been reported in the literature[1-32] that cannot be satisfactorily explained using the *standard* (i.e., one-region) point kinetic model.[a] In particular, many have observed that the zero-power transfer function of some types of reflected systems can depart significantly from the one-region, point kinetic model at high frequencies, or that the prompt decay α curves obtained from Rossi-α[33] and pulsed-neutron[34] experiments can exhibit multiple decay modes in the vicinity near delayed critical.[1-12] When analyzed using theories based on the standard point-kinetic model, these data have yielded system lifetimes that do not always agree well with the lifetimes predicted by numerical solutions of the multigroup, multidimensional diffusion or transport equations.[13-21] In several cases, when the longest-lived decay mode (i.e., the dominant root) was plotted as a function of reactivity, the α curve intercepted the reactivity axis at a reactivity significantly greater than 1$.[22-24] Brunson[24]

[a] The references in this work represent only a relatively small sample of the published literature dealing with reflected systems. Because of the vastness of this literature, it would be impossible for us to mention everyone. Therefore, we apologize in advance to those authors who have performed noteworthy work on reflected systems but who are not mentioned herein.

dubbed this seemingly inexplicable behavior as the *dollar discrepancy.* Furthermore, it has also been observed that the kinetic behavior of some reflected, fast-burst assemblies exhibit a very pronounced nonlinear relationship between reactivity and the initial inverse period for reactivity insertions greater than 1$.[25-32]

The multiple decay modes and the anomalous behavior of the transfer function at high frequencies were, of course, attributed to the presence of higher spatial harmonics in the system. The mismatches in neutron lifetime were often blamed on potential shortcomings in the cross-section sets, incorrect neutron spectra,[15-17] and/or the method used to calculate the neutron lifetime.[21] In most cases, however, no explanation for the observed discrepancies was even offered. With regard to the dollar discrepancy, it was generally concluded that die-away experiments in some reflected systems were noninterpretable based on a simple, one-region, lumped-parameter model, which obviously treats the neutronic characteristics of the core and reflector regions as equals. Clearly the neutronic characteristics of a reflected system can exhibit large differences between the core and the reflector regions. For example, in *fast-thermal breeder* reactors, the neutron lifetime in the fast core region can be several orders of magnitude less than the neutron lifetime in the thermal reflector region.[1] Hence, the time-scale for a fission chain to propagate can be significantly increased by neutrons that leak into the reflector region, and, after some relatively long delay, are scattered back into the core where they continue to propagate the same fission chain that spawned them. To adequately describe the time-dependent behavior of these complex systems, the various time constants that govern the propagation of the prompt-neutron fission chains must be accounted for in the kinetic model. In the past, this accounting was usually accomplished by assuming that the reflector-return neutrons behaved similarly to a delayed-neutron group but with a much shorter mean lifetime. When incorporated into the kinetic model, this assumption seemed to explain the non-linear relationship between reactivity and inverse period at reactivities near or above prompt critical.[2, 25-27, 29-30]

Numerous theories and models have been developed for reflected systems.[35-51] Most of these theories and models have been used to explain one or two of the aforementioned anomalies with varying degrees of success. Only one of the models, however, seems to explain all of the observed anomalies. We now recognize that the multiple decay modes, the observed differences between measured and calculated lifetimes, the dollar discrepancy, and the non-linear behavior between reactivity and inverse period can all be explained using a simple, two-region, *nodal* model based on the theory of coupled systems proposed by Avery and subsequently adapted to reflected systems by Cohn. In accordance to Avery's theory, Cohn separated the total neutron loss rate from the integral system into two loss rates—one from the core region and one from the reflector region. The two regions were then coupled together using coupling parameters that represent the probability that a neutron lost from one region will appear in the other region. Even though this sort of spatial representation is very crude, the Avery-Cohn two-region model is still very useful in that it readily explains all of the aforementioned anomalies. Surprisingly, however, their model has seldom been used.

We speculate that the lack of use of the Avery-Cohn model has occurred primarily because of the complexity involved in the calculation of the coupling parameters and neutron lifetimes. In accordance with Avery's formalism, the total fission neutron source in each region must be divided into a series of source components, S_{jk}, defined as the number of fission neutrons in region j which result from fissions caused by neutrons which originate in region k. To evaluate S_{jk}, it is first necessary to calculate the *partial* fluxes and adjoint fluxes in region j resulting from neutrons born in region k. It is not obvious how these partial fluxes are calculated. Furthermore, we note that the neutron lifetime for each region, as defined by Avery, appears to be a mathematical convenience rather than quantities that are physically interpretable and, more importantly, physically *measurable*.

In this work, we review the Avery-Cohn model, and we present a simplified methodology for determining the coupling parameters and the neutron lifetimes based on simple probability relationships that describe the aggregate migration of neutrons between the core and the reflector regions. Using these relationships, we define the effective multiplication factor of the integral system in terms of the multiplication factor of the core region and the reflector return fraction. We also derive a new version of the reflected-core inhour equation that differs in form from the inhour equations proposed by Avery and Cohn, and we discuss its solution. We then demonstrate the calculation of the coupling parameters and the neutron lifetimes for a simple reflected system and compare experimental Rossi-α data with the results obtained from a deterministic transport solution.

II. PROBABILITY RELATIONSHIPS

In a very general sense, there are only two ways[b] in which a neutron can be lost from a system—it can be absorbed,[c] or it can leak. However, in a reflected system, neutrons can be lost from the core and appear in the reflector, or lost from the reflector and reappear in the core. Therefore, with respect to the core region in the partially reflected system represented in Fig. 1, a neutron born (or reappearing) in the core can do one of three things—it can be absorbed in the core, which we represent by the probability f_{ca}; it can leak directly from the core to infinity (e.g., through the top and bottom axial surfaces of the system shown in Fig. 1), which we represent by the probability f_{ci}; or it can leak from the core into the reflector, which we represent by the probability f_{cr}.[d] By definition, the sum of these three probabilities must equal one. Accordingly,

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[b] We have ignored a third possibility of radioactive decay because the life expectancy of a neutron within a reactor system is extremely short relative to its 10.4 minute half-life.

[c] In this work, we have maintained the convention that an absorption event represents any non-scattering process. That is, $\sigma_a = \sigma_t - \sigma_s$ where σ_s includes both elastic and inelastic scattering processes.

[d] In our nomenclature, the first subscript refers to where the neutron originates, and the second subscript refers to where the neutron ends up or how the neutron is lost. For example, f_{cr} represents the fraction of neutrons in the core that leak into the reflector, f_{rc} represents the fraction of neutrons in the reflector that scatter back into the core, f_{ca} represents the fraction of neutrons in the core that are absorbed in the core, etc.

$$1 = f_{ca} + f_{ci} + f_{cr} \quad , \tag{1}$$

where it is understood that f_{ca}, f_{ci}, and f_{cr} represent average values for the core region.

If a neutron leaks from the core into the reflector, that neutron can also do one of three things—it can be absorbed in the reflector, which we represent by the probability f_{ra}; it can leak from the reflector to infinity, which we represent by the probability f_{ri}; or it can be scattered back into the core, which we represent by the probability f_{rc}. Again, the sum of these three probabilities must equal one. So,

$$1 = f_{ra} + f_{ri} + f_{rc} \quad , \tag{2}$$

where it is understood that f_{ra}, f_{ri}, and f_{rc} represent average values for the reflector region.

The quantities on the right-hand side of Eqs. (1) and (2) shall hereafter be referred to as the average *single-pass* probabilities because they represent the average probability of a particular outcome on *each* pass through a given region. We fully recognize that the *instantaneous* probability of a neutron being absorbed in the core, for example, during its i^{th} trip through the core region is a strong function of its position, energy, and the direction it is traveling when it appears in the core. For example, if a neutron is born at the edge of the core and is traveling in a direction towards the reflector region, the chances of it being absorbed in the core on its initial pass may be considerably smaller than the probability of it being absorbed in the core on its second pass since, in all likelihood, its energy will be appreciably lower due to multiple scatterings in the reflector, and it may well be traveling back towards the center of the core when it returns. Even though

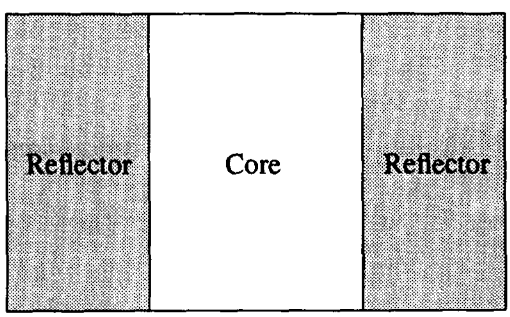

Fig. 1. Cylindrical, Radially-Reflected System.

arge differences in the instantaneous probabilities can occur among the various passes through a given region, when averaged over all space, energy, direction, and potential neutron histories, we postulate that there exists an integral quantity that characterizes the average probability that a neutron will be involved in a particular outcome per pass through a given region. These, of course, are the single-pass probabilities defined in Eqs. (1) and (2). However, it must be stressed that these single-pass probabilities do not represent the fraction of neutrons ultimately lost by absorption or leakage in a particular region. Because neutrons can be reflected in and out of the core and the reflector regions many times before being irrevocably lost, the fraction of neutrons ultimately absorbed in or leaked from a particular region becomes a function of the average single-pass probabilities and the average number of times a neutron passes through that region.

When viewed from the standpoint of the integral system, we can write a simple neutron balance equation that accounts for all the various outcomes a neutron can exhibit in a reflected system (see Fig. 2). Assuming that the neutron population has been normalized to one neutron lost per unit time, we can write

$$1 = P_{ca} + P_{ci} + P_{ra} + P_{ri} \quad , \tag{3}$$

where P_{ca} is the fraction of system neutrons absorbed in the core, P_{ci} is the fraction of system neutrons leaked from the core directly to infinity, P_{ra} is the fraction of system neutrons absorbed in the reflector, and P_{ri} is the fraction of system neutrons leaked from the reflector directly to infinity. (Note that the neutron flow rates at the core/reflector interface do not appear in this equation because the neutron balance is on the integral system as opposed to the core or the reflector region. At the end of this section, we present the neutron balance equations for these individual regions.)

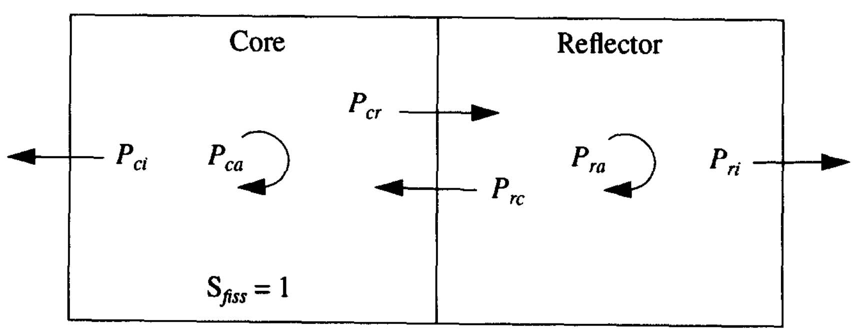

Fig. 2. Synoptic of Neutron Balance.

The *neutron utilization fractions*, P_{ca}, P_{ct}, P_{ra}, and P_{rt}, can be related to the average single-pass probabilities by summing the fraction of neutrons involved in a particular outcome over every potential path leading to that outcome. In the case of absorption in the core, the first path corresponds to those neutrons born and immediately absorbed in the core (see Fig. 3). This quantity corresponds to the probability that a neutron is born in the core times the single-pass probability f_{ca}. Because the reflector is assumed to be source-free, then the probability that a neutron is born in the core is identically equal to 1.0. The second path corresponds to those neutrons that are born in the core, leak into the reflector, scatter back into the core, and are then absorbed. This fraction corresponds to $f_{cr}f_{rc}f_{ca}$. The third path corresponds to those neutrons that are born in the core, leak into the reflector, scatter back into the core, leak back into the reflector, scatter back into the core, and are then absorbed. This fraction corresponds to $(f_{cr}f_{rc})^2 f_{ca}$. By continuing this process, we obtain the following infinite series.

$$P_{ca} = f_{ca}\,[1 + f_{cr}\,f_{rc} + (f_{cr}\,f_{rc})^2 + (f_{cr}\,f_{rc})^3 + \ldots] \quad . \tag{4}$$

The product $f_{cr}f_{rc}$ appears quite often in the two-region kinetic model. From a physical standpoint, it represents the fraction of core neutrons returned to the core after having leaked into the reflector. For convenience, we define this special product as the *reflector return fraction*, f.

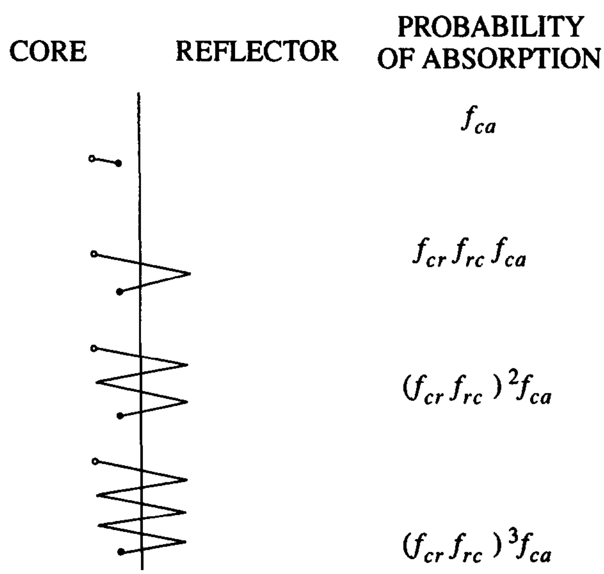

Fig. 3. First four terms of the potential neutron histories resulting in an absorption in the core.

$$f = f_{cr} f_{rc} \quad . \tag{5}$$

Because f is always less than 1.0, the infinite series defined by Eq. (4) converges to

$$P_{ca} = f_{ca}(1 + f + f^2 + f^3 + \ldots) = \frac{f_{ca}}{1 - f} \quad . \tag{6}$$

Following analogous procedures for all other potential outcomes, we obtain

$$P_{ci} = \frac{f_{ci}}{1 - f} \quad , \tag{7}$$

$$P_{ra} = \frac{f_{cr} f_{ra}}{1 - f} \quad , \tag{8}$$

and

$$P_{ri} = \frac{f_{cr} f_{ri}}{1 - f} \quad . \tag{9}$$

Equation (3) can now be rewritten in terms of the single-pass probabilities and the reflector return fraction as

$$1 = \frac{f_{ca}}{1 - f} + \frac{f_{ci}}{1 - f} + \frac{f_{cr} f_{ra}}{1 - f} + \frac{f_{cr} f_{ri}}{1 - f} \quad . \tag{10}$$

We can also obtain similar expressions for the net neutron flow at the interface between the reflector and the core by performing a neutron balance on the core region or on the reflector region.[52] For the core region, the net flow rate is equal to

$$(P_{cr} - P_{rc}) = 1 - (P_{ca} + P_{ci}) \quad , \tag{11}$$

where P_{cr} is the flow out of the core region and P_{rc} is the flow into the core region from the reflector. For the reflector region, the net flow is equal to

$$(P_{cr} - P_{rc}) = P_{ra} + P_{ri} \quad . \tag{12}$$

We can express either of these two equations in terms of the single-pass probability f_{cr} and the reflector return fraction as

$$(P_{cr} - P_{rc}) = \frac{f_{cr}}{1-f} - \frac{f}{1-f} \quad , \tag{13}$$

from which we infer that the neutron flow leaving the core region and entering the reflector is

$$P_{cr} = \frac{f_{cr}}{1-f} \quad , \tag{14}$$

and the flow returning to the core from the reflector is

$$P_{rc} = \frac{f}{1-f} \quad . \tag{15}$$

As an aside, the ratio of P_{rc} to P_{cr} is, by definition, the albedo of the reflector and is equal to the single-pass probability f_{rc}.

III. EFFECTIVE MULTIPLICATION FACTOR

By definition, the effective multiplication factor of the integral system is the number of neutrons produced per neutron lost. If the number of neutrons in the integral system is N_s and the average lifetime of those neutrons is τ_s, then we can write

$$k_{eff} = \frac{\int\limits_{sys}\int \bar{\nu}_t(\mathbf{r}, v)\,\Sigma_f(\mathbf{r}, v)\,\phi(\mathbf{r}, v)\,d\mathbf{r}dv}{\left(\dfrac{N_s}{\tau_s}\right)} \quad , \tag{16}$$

where ϕ is the spatial- and velocity-dependent neutron flux, Σ_f is the spatial- and velocity-dependent macroscopic fission cross section, and $\bar{\nu}_t$ is the spatial- and velocity-dependent average total number of neutrons (prompt plus delayed) released per fission. (In this particular definition of k_{eff}, we have ignored the neutron production sources arising from other types of reactions, such as (n,2n), (n,3n), (γ,n), etc., since the major neutron production source in most reactor systems is fission. Nevertheless, we could easily include these other minor neutron production source terms into the Avery-Cohn model by simply replacing the fission source in all equations where it appears with

$$\int_{sys}\int \bar{\nu}_t(\mathbf{r}, v)\,\Sigma_f(\mathbf{r}, v)\,\phi(\mathbf{r}, v)\,d\mathbf{r}dv + \int_{sys}\int \nu_x \Sigma_x(\mathbf{r}, v)\,\phi(\mathbf{r}, v)\,d\mathbf{r}dv + \ldots \quad ,$$

where Σ_x is the spatial- and velocity-dependent macroscopic cross section for a (n,xn) reaction, and ν_x is equal to x. If, however, any of these neutron production sources should appear in the reflector region, such as a (γ,n) source that arises in beryllium-reflected systems, then the probability relationships derived in the previous section must be modified to account for the neutrons born in the reflector. Furthermore, we have assumed that the angular dependence of the flux and the macroscopic cross sections, etc. is negligible. If angular dependence is to be considered, we simply replace dv with $d\mathbf{v}$ where $\mathbf{v} = v \cdot \Omega$.)

We can also define the multiplication of the core region, k_c, as the number of neutrons produced in the core per neutron lost *from the core*. If the number of neutrons in the core region is N_c and the average lifetime of those neutrons is τ_c, then we can write

$$k_c = \frac{\int_{core}\int \bar{\nu}_t(\mathbf{r}, v)\,\Sigma_f(\mathbf{r}, v)\,\phi(\mathbf{r}, v)\,d\mathbf{r}dv}{\left(\frac{N_c}{\tau_c}\right)} \quad . \tag{17}$$

In this particular two-region model it is assumed that there is no fissioning occurring in the reflector; so all neutrons born in the system are born in the core region. Thus, the numerators in Eqs. (16) and (17) are identical. Consequently,

$$k_{eff}\left(\frac{N_s}{\tau_s}\right) = k_c\left(\frac{N_c}{\tau_c}\right) \quad . \tag{18}$$

The loss rate from the core region can also be expressed in terms of the neutron utilization fractions defined in Section II as,

$$\frac{N_c}{\tau_c} = (P_{ca} + P_{ci} + P_{cr})\frac{N_s}{\tau_s} = \left(\frac{1}{1-f}\right)\frac{N_s}{\tau_s} \quad . \tag{19}$$

Hence, Equation. (18) now becomes

$$k_{eff} = \frac{k_c}{1-f} \quad . \tag{20}$$

(Note that this expression differs from the definition of k_{eff} used in our earlier attempts to formulate a two-region model for reflected systems.[e])

IV. POINT KINETIC EQUATIONS FOR A REFLECTED SYSTEM

In 1958, Avery presented a general point kinetic model to describe the time-dependent behavior of multiplying systems, composed of an arbitrary number of regions, each characterized by a multiplication factor k_i and a neutron lifetime τ_i. For a two-region system consisting of a core surrounded by a non-multiplying, source-free reflector, Cohn reduced Avery's model to the following set of coupled differential equations.

$$\frac{dN_c}{dt} = [k_c(1-\beta)-1]\left(\frac{N_c}{\tau_c}\right)+f_{rc}\left(\frac{N_r}{\tau_r}\right)+\sum\lambda_i C_i+S_c \quad , \tag{21}$$

$$\frac{dN_r}{dt} = f_{cr}\left(\frac{N_c}{\tau_c}\right)-\left(\frac{N_r}{\tau_r}\right) \quad , \tag{22}$$

and

$$\frac{dC_i}{dt} = k_c\beta_i\left(\frac{N_c}{\tau_c}\right)-\lambda_i C_i \quad \text{for } i=1 \text{ to } m, \tag{23}$$

where

N_c = adjoint-weighted total number of neutrons in the core region,
N_r = adjoint-weighted total number of neutrons in the reflector region,
k_c = multiplication factor of the core region,
β = effective delayed neutron fraction,

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[e] In Refs. 53, 54, and 55, we mistakenly deduced that k_{eff} was equal to k_c+f. As a consequence, we defined several other system parameters incorrectly and made several erroneous conclusions concerning differences between reflected systems and equivalent bare systems. In particular, the system lifetimes were defined incorrectly, and we erroneously concluded that prompt critical in reflector systems occurs at a lower value of k_{eff} than in equivalent bare systems. In spite of the wrong definition of k_{eff}, the reflected-core inhour equation in those references is still correct, and the definition of reactivity is unaltered as are the definitions of the mean static and dynamic prompt-neutron generation times. Furthermore, the probability relationships used to determine the coupling parameters are still correct.

τ_c = adjoint-weighted neutron lifetime in the core region,
τ_r = adjoint-weighted neutron lifetime in the reflector region,
f_{cr} = fraction of neutrons that leak from the core into the reflector,
f_{rc} = fraction of neutrons that leak from the reflector back into the core,
C_i = adjoint-weighted concentration of the i^{th} precursor group,
β_i = delayed neutron fraction of the i^{th} precursor group,
λ_i = decay constant of the i^{th} precursor group, and
S_c = adjoint-weighted intrinsic/external neutron source rate in the core.

The interpretation of the individual terms in the above set of differential equations is as follows:

- N_c/τ_c represents the instantaneous rate at which neutrons are lost from the core region,
- N_r/τ_r represents the instantaneous rate at which neutrons are lost from the reflector region,
- $k_c(1-\beta)N_c/\tau_c$ represents the instantaneous rate at which prompt neutrons are produced in the core region,
- $k_c\beta_i N_c/\tau_c$ represents the instantaneous rate at which delayed neutron precursors in the ith group are produced in the core region,
- $f_{cr}N_c/\tau_c$ represents the instantaneous rate at which neutrons from the core region leak into the reflector region (i.e., the partial current from the core to the reflector),
- $f_{rc}N_r/\tau_r$ represents the instantaneous rate at which neutrons from the reflector re-enter the core region (i.e., the partial current from the reflector to the core), and
- $\sum \lambda_i C_i$ represents the rate at which delayed neutrons appear in the core region.

V. THE REFLECTED-CORE INHOUR EQUATION

The Laplace transform of $N_c(t)$ is given by

$$\Gamma_c(s) = \frac{\dfrac{\tau_c}{k_c}\left[N_{co} + \dfrac{f_{rc}N_{ro}}{(\tau_r s + 1)} + \sum \dfrac{\lambda_i C_{io}}{s+\lambda_i} + \Gamma_s\right]}{s\left[\dfrac{\tau_c}{k_c} + \dfrac{f\tau_r}{k_c(\tau_r s + 1)}\right] + \sum \dfrac{\beta_i s}{s+\lambda_i} - \left(\dfrac{k_c + f - 1}{k_c}\right)} , \tag{24}$$

where Γ_s is the Laplace transform of the source term, and C_{io} is the equilibrium precursor density for the i^{th} delayed neutron group.

The denominator of Eq. (24) is the inhour equation and has $m+2$ distinct real roots where m is the number of delayed neutron groups. So, $\Gamma_c(s)$ has $m+2$ simple poles on the real s-axis. At each root $s=\omega_j$, the denominator of Eq. (24) is zero, which means ω_j satisfies the equation

$$\frac{k_c+f-1}{k_c} = \omega\frac{\tau_c}{k_c} + \omega\frac{f\tau_r}{k_c(\tau_r\omega+1)} + \sum\frac{\beta_i\omega}{\omega+\lambda_i} \ . \tag{25}$$

From Eq. (20), we note that the left-hand side of Eq. (25) can be written as,

$$\frac{k_c+f-1}{k_c} = \frac{(\frac{k_c}{1-f})-1}{(\frac{k_c}{1-f})} = \frac{k_{eff}-1}{k_{eff}} = \rho \ . \tag{26}$$

If we substitute k_c on the right-hand side of Eq. (25) with $k_{eff}(1-f)$, then we can write the reflected-core inhour equation as

$$\rho = \omega\frac{\tau_c}{k_{eff}(1-f)} + \omega\frac{f\tau_r}{k_{eff}(1-f)\,(\tau_r\omega+1)} + \sum\frac{\beta_i\omega}{\omega+\lambda_i} \ . \tag{27}$$

Furthermore, if we define the core's prompt-neutron *generation* time as

$$\Lambda_c = \frac{\tau_c}{k_{eff}(1-f)} \ , \tag{28}$$

and the reflector's prompt-neutron *generation* time as

$$\Lambda_r = \frac{\tau_r}{k_{eff}(1-f)} \ , \tag{29}$$

then Equation (27) can be written more compactly as

$$\rho = \omega\Lambda_c + \frac{\omega f\Lambda_r}{\tau_r\omega+1} + \sum\frac{\beta_i\omega}{\omega+\lambda_i} \ . \tag{30}$$

Similar to the single-pass probabilities, τ_c and τ_r represent the average life expectancy of a neutron in the core region and the reflector region, respectively, per pass through that region. On an average, each neutron in the system will appear in those two regions $1/(1-f)$ times. Hence, the quantities $\tau_c/(1-f)$ and $\tau_r/(1-f)$ in Eqs. (28) and (29) represent the *total* length of time that a neutron will spend in the core region and the reflector region, respectively, before being lost.

Also note that when f approaches zero Eq. (27) collapses to the inhour equation for a bare reactor. However, for f greater than zero, an extra term associated with the reflector appears in the inhour equation. When grouped according to time scale, this extra reflector term is more likely to be similar to the term containing the core's prompt-neutron generation time than to any of the delayed neutron terms. Consequently, treating reflector return neutrons as an extra delayed neutron group may not be advisable and, in most situations, would complicate the interpretation of results obtained from pulsed-neutron and Rossi-α experiments.

VI. EQUIVALENT ONE-REGION MODEL

In most reflected systems the reflector lifetime will be sufficiently small such that $\tau_r\omega_j \ll 1$ for the first m roots of the reflected-core inhour equation. When this occurs, the core and reflector generation times can be readily combined to yield an equivalent one-region model given by

$$\rho = \omega\Lambda_m + \sum \frac{\beta_i\omega}{\omega+\lambda_i} , \tag{31}$$

where the system's mean *dynamic* prompt-neutron generation time, Λ_m, is defined as

$$\Lambda_m = \Lambda_c + f\Lambda_r = \frac{\tau_c + f\tau_r}{k_{eff}(1-f)} . \tag{32}$$

Equation (31) is identical in form to the standard, one-region inhour equation. For reactivities below prompt critical, Eq. (31) and the standard inhour equation yield nearly identical roots for roots 1 through m. This result shows that the kinetic behavior of a reflected system will be essentially identical to the kinetic behavior of an equivalent[f] bare system providing the reflector lifetime is sufficiently small. However, two important changes in the model have occurred which should not be forgotten. First, the core's prompt-neutron generation time that appears in the standard inhour equation has been replaced by the mean dynamic prompt-neutron generation time defined by Eq. (32). And second, by making the assumption $\tau_r\omega_j \ll 1$, the order of the reflected-

[f] Equivalent, as used in this context, means that the core's prompt-neutron generation time is equal to the mean generation time of the reflected system and that the effective delayed neutron fraction for both systems is identical.

core inhour equation is reduced by one. This assumption, in effect, eliminates the short-lived prompt decay root associated with the core region. Notwithstanding these two changes, Eq. (31) is still useful in that it can be used to predict the value of the first m roots of the reflected-core inhour equation for all reactivities << 1$ for reasonably small values of the reflector lifetime.

VII. SOLUTION OF THE REFLECTED-CORE INHOUR EQUATION

In deriving the reflected-core inhour equation, we did not place any restrictions on how k_{eff} is controlled. Nevertheless, we know that k_{eff} cannot be altered without changing k_c and/or f. These changes are usually attained by inserting or removing control rods in the core region or by changing the reflector configuration. Regardless of how k_{eff} is changed, we desire an inhour equation in which the reactivity is a function of only one variable—the inverse period, ω. In the standard inhour equation, this one-variable dependence is achieved by assuming that the system lifetime, τ_s, varies in direct proportion to k_{eff}. If that assumption is valid, then the neutron generation time, defined by τ_s/k_{eff}, will be nearly constant in the vicinity of delayed critical and will be numerically equal to the value of τ_s at delayed critical. In the reflected-core inhour equation, we make a similar assumption. We force Eq. (27) to be only a function of ω by assuming that the core and reflector lifetimes vary in direct proportion to k_{eff} such that the core and reflector neutron generation times are nearly constant in the vicinity of delayed critical.

If six groups of delayed neutrons are assumed, the reflected-core inhour equation will have eight roots. A qualitative plot of these roots is shown in Fig. 4. Similar to the standard inhour equation, the reflected-core inhour equation has six asymptotes corresponding to the negative values of the decay constants of each of the delayed neutron groups (i.e., $-\lambda_i$ for i=1 through 6), and an additional asymptote at $-1/\tau_r$.

As with the standard inhour equation, Equation (30) also has an asymptote that is a linear function of reactivity with a slope inversely proportional to the core's prompt-neutron generation time. This function is obtained from Eq. (30) by assuming that $|\omega| >> \lambda_i$ and $|\tau_r\,\omega| >> 1$, which leads to

$$\omega = \frac{\rho - \beta - \dfrac{f}{k_c}}{\Lambda_c} \quad . \tag{33}$$

For reactivities in the vicinity of delayed critical, the m+1 root of the reflected-core inhour equation follows a linear function of reactivity with a slope inversely proportional to the system's mean dynamic prompt-neutron generation time.

$$\omega = \frac{\rho - \beta}{\Lambda_m} \quad . \tag{34}$$

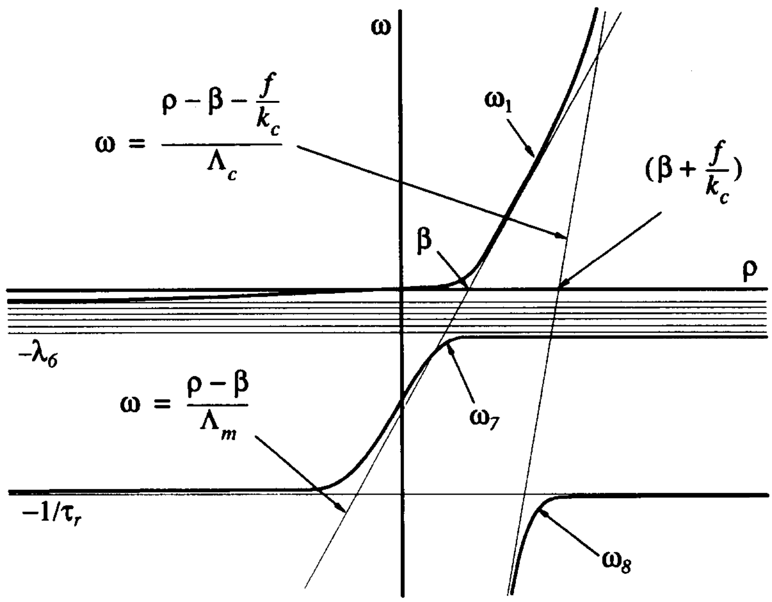

Fig. 4. Qualitative plot of the roots of the reflected-core inhour equation. (Not drawn to scale.)

This function is obtained by assuming that the magnitude of ω is some intermediate value which is much larger than the largest decay constant, yet much smaller than $1/\tau_r$. That is, we assume $|\omega| >> \lambda_i$ and $|\tau_r \omega| << 1$. This function, however, is not, in a strict sense, an asymptote since none of the m+2 roots converge to this function as ρ approaches 1.0 (i.e., the maximum positive reactivity) or minus infinity.[56]

Roots 2 through 6 of the reflected-core inhour equation are essentially identical to the roots of the standard inhour equation. However, as can be observed from Fig. 4, the explicit treatment of the reflector as a unique region has added one more α-eigenvalue to the fundamental mode k-eigenvalue. This additional α-eigenvalue is bounded by the asymptote at $-1/\tau_r$ and $-\lambda_m$. For reactivities in the vicinity of delayed critical, the additional α-eigenvalue closely follows the linear function defined by Eq. (34).

The first root of the reflected-core inhour equation is bounded by the asymptote defined by Eq. (33) and $-\lambda_1$, but for reactivities greater than 1\$, ω_1 closely follows the linear function defined by Eq. (34). Hence, ω_1 is inversely proportional to the mean prompt-neutron generation time for reactivities just above prompt critical and is inversely proportional to the core's prompt-neutron generation time for $\rho >> \beta + f/k_c$.

Because ω_7 and ω_8 are bounded by the asymptote at $-1/\tau_r$, eventually both ω_7 and ω_8 must curve away from the linear functions defined by Eqs. (33) and (34). The reactivities at which ω_1 and ω_7 begin to depart from the linear function defined by Eq. (34) provide a qualitative measure of the reactivity range where the equivalent one-region inhour equation can be used in lieu of the two-region model. In reflected systems where the reflector lifetime is small enough such that $\tau_r \omega_j << 1$, then the reactivity range can be quite large. An example of this situation is shown in Fig. 5a. In reflected systems with small core lifetimes and relatively larger reflector lifetimes, ω_7 can depart from Eq. (34) at reactivities very close to delayed critical. An example of this situation is shown in Fig. 5b.

VIII. STATIC NEUTRON GENERATION TIME

Unlike the standard point kinetic equations, another prompt-neutron generation time appears in the solution of the Avery-Cohn equations. This generation time is associated with the equilibrium condition of the integral system and is derived by solving for the equilibrium conditions of Eqs. (21), (22), and (23).

$$N_{co} = \frac{\tau_c S_c}{1 - f - k_c} , \tag{35}$$

and

$$N_{ro} = \frac{f_{cr} \tau_r}{\tau_c} N_{co} , \tag{36}$$

where N_{co} is the number of neutrons in the core region at equilibrium, and N_{ro} is the number of neutrons in the reflector region at equilibrium. The total neutron population of the integral system at equilibrium, N_s, is given by

$$N_s = N_{co} + N_{ro} = \frac{(\tau_c + f_{cr} \tau_r) S_c}{1 - f - k_c} . \tag{37}$$

When written in terms of k_{eff} as defined by Eq. (20), Eq. (37) becomes

$$N_s = \frac{\left[\frac{\tau_c + f_{cr} \tau_r}{1 - f} \right] S_c}{1 - k_{eff}} , \tag{38}$$

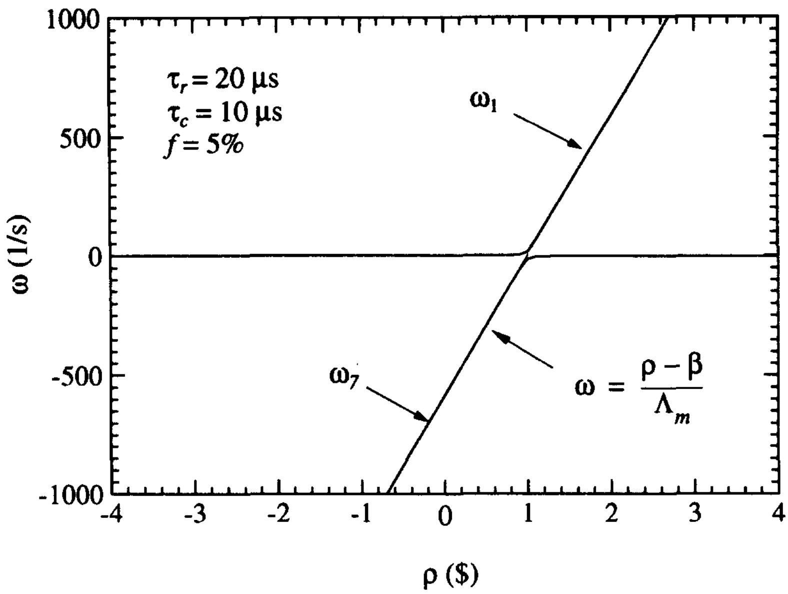

Fig. 5a. Plot of the first and seventh root of the reflected-core inhour equation for a short reflector lifetime.

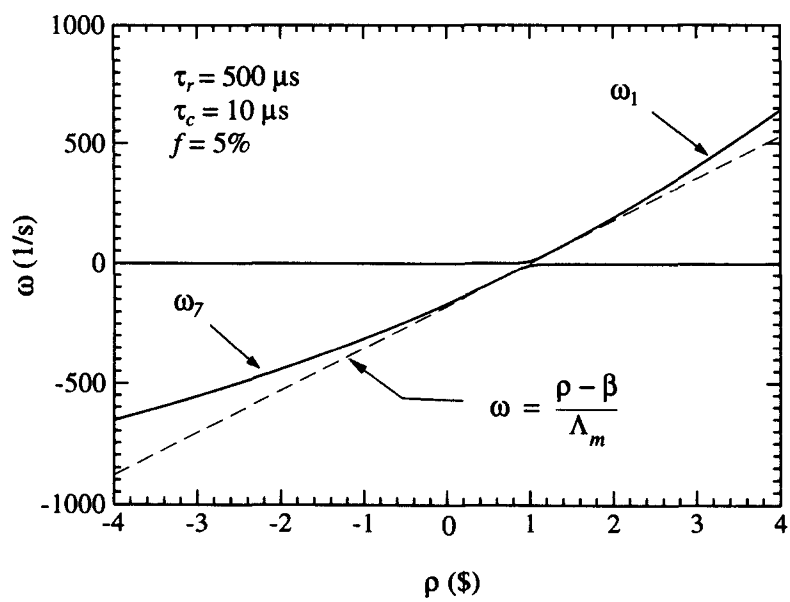

Fig. 5b. Plot of the first and seventh root of the reflected-core inhour equation for a long reflector lifetime.

from which we infer that the system's *static* lifetime is

$$\tau_s = \frac{\tau_c + f_{cr}\tau_r}{1-f} \quad . \tag{39}$$

By dividing through by k_{eff}, we obtain the system's static generation time,

$$\Lambda_s = \frac{\tau_c + f_{cr}\tau_r}{k_{eff}(1-f)} \quad , \tag{40}$$

which can also be written as

$$\Lambda_s = \Lambda_c + f_{cr}\Lambda_r \quad . \tag{41}$$

This same expression can also be obtained from the numerator of the steady-state portion of the combined Laplace transform of the core and reflector neutron population, Γ_c and Γ_r.

$$\Gamma_c(\omega) + \Gamma_r(\omega) = \frac{\left[\frac{\tau_c}{k_c} + \frac{f_{cr}\tau_r}{k_c(\tau_r\omega+1)}\right]\Gamma_s}{\omega\left[\frac{\tau_c}{k_c} + \frac{f\tau_r}{k_c(\tau_r\omega+1)}\right] + \sum\frac{\beta_i\omega}{\omega+\lambda_i} - \left(\frac{k_c+f-1}{k_c}\right)} \quad . \tag{42}$$

Because ω_1 is zero at steady state, the term in the brackets in the numerator reduces to Eq. (40).

When compared to Eq. (32), we see that Λ_s includes all neutrons that leak into the reflector (i.e., τ_r is weighted by f_{cr}), whereas, Λ_m includes only those neutrons that are returned to the core after leaking into the reflector (i.e., τ_r is weighted by f). The dynamic generation time, Λ_m, is a measure of the average time between the appearance of fission-source neutrons based on the fraction of the neutron population that can potentially propagate the fission chains. In contrast, the static generation time, Λ_s, is a measure of the average time between the appearance of fission-source neutrons based on the total neutron population.

IX. REACTIVITY FORM OF THE AVERY-COHN MODEL

The Avery-Cohn differential equations, as written in the form of Eqs (21), (22), and (23), are not very convenient from an operations standpoint since they are written in terms of the core's ıultiplication factor, k_c, which cannot be measured directly in an operating system. It is much ıore convenient to rewrite the differential equations in terms of ρ so that the system reactivity cale can be expressed in terms of *dollars*. With the use of Eqs. (20), (28), and (29), the Avery-ohn differential equations can be rewritten as

$$\frac{dN_c}{dt} = \frac{\rho - \beta - f(1-\beta)}{(1-f)}\left(\frac{N_c}{\Lambda_c}\right) + f_{rc}\frac{(1-\rho)}{(1-f)}\left(\frac{N_r}{\Lambda_r}\right) + \sum \lambda_i C_i + S \ , \tag{43}$$

$$\frac{dN_r}{dt} = f_{cr}\frac{(1-\rho)}{(1-f)}\left(\frac{N_c}{\Lambda_c}\right) - \frac{(1-\rho)}{(1-f)}\left(\frac{N_r}{\Lambda_r}\right) \ , \tag{44}$$

nd

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda_c}N_c - \lambda_i C_i \quad \text{for } i\text{=1 to } m, \tag{45}$$

vhere we have maintained the exact relationship between τ_c and Λ_c, and τ_r and Λ_r by noting that $_{eff}$ is

$$k_{eff} = \frac{1}{1-\rho} \ . \tag{46}$$

)bviously, in the vicinity of delayed critical, k_{eff} will be close to 1.0; thus, ρ will be very small. Ience, the term (1 − ρ) in Eqs. (43) and (44) can be set equal to 1.0 without introducing any sigıificant error in the solution of that system of equations.

As with the standard point kinetic equations, reactivity feedback effects can be incorpoated into the Avery-Cohn model by coupling Eqs. (43), (44), and (45) with the thermodynamic nd heat-transfer equations that describe the flow and deposition of energy in the core and reflecor regions. However, it must be remembered that N_c, N_r, and C_i in Eqs. (43), (44), and (45) correspond to neutron populations (or i^{th}–group precursor populations) in the core and reflector egions. A given neutron population in the reflector, however, does not produce the same *power* as t would in the core because there are no fissions occurring in the reflector. Consequently, there is ıo conversion factor between neutron population and power that can be simultaneously applied to oth the core and the reflector which allow us to interpret N_c and N_r in Eqs. (43) and (44) as the ower being produced in those regions. Nevertheless, conversion of the neutron populations in hose regions to absolute power is necessary if reactivity feedback effects are to be incorporated. o accomplish this conversion, we retain the interpretation that N_c, N_r, and C_i correspond to neuon populations, and we convert individually N_c and N_r to actual power levels as they appear in ıe thermodynamic and heat-transfer equations using the appropriate conversion factors.

X. PROMPT CRITICALITY IN REFLECTED SYSTEMS

There are two prompt criticals in a reflected reactor. The first prompt critical occurs when the reactivity of the system is high enough that a chain reaction can be sustained without delayed neutrons. As in an unreflected system, this occurs at $\rho=\beta$. The second prompt critical occurs when the reactivity of the system is high enough that a chain reaction can be sustained without delayed neutrons and reflector return neutrons. This occurs at

$$\rho = \beta + \frac{f}{k_c} \quad . \tag{47}$$

In highly reflected systems, the second prompt critical is somewhat academic because it occurs at such a high reactivity; therefore, it may not be physically obtainable and/or very prudent from a safety standpoint. Nevertheless, it is conceivable that the second prompt critical could be obtained in reflected systems in which the reflector is worth only a few dollars or less. As an example, the Godiva fast-burst assembly at the Los Alamos National Laboratory behaves like a reflected system because of long-lived, *room-return* neutrons. The reactivity worth of the room-return neutrons has been previously measured to be approximately 0.05$. Hence, Godiva reaches the second prompt critical at a reactivity of approximately 1.05$.

At both prompt criticals, the characteristic time constant of the system decreases. Before reaching the first prompt critical, the characteristic time constant is governed by the mean decay time of the delayed neutron precursors. After passing through the first prompt critical, the characteristic time constant is greatly reduced to the mean prompt-neutron generation time (assuming $\tau_r\omega_j<<1$). As shown in Fig. 4, another decrease in the characteristic time constant will occur when the system reactivity exceeds the second prompt critical. The characteristic time constant associated with this prompt critical is the core's prompt-neutron generation time, which can be significantly smaller than the mean prompt-neutron generation time in fast cores surrounded by thermal reflectors.

Obviously, the change in characteristic time constant is much more dramatic when reactivity exceeds the first prompt critical than when it exceeds the second prompt critical. Therefore, the first prompt critical is still viewed as the most important reactivity. In the case of Godiva, the characteristic time constant changes from about 0.45 seconds to 6.6 ns at the first prompt critical, but only changes from 6.6 ns to 6.3 ns at the second prompt critical.

XI. CALCULATION OF THE COUPLING PARAMETERS

The coupling parameters for the Avery-Cohn model can be calculated using the probability relationships presented in Section II and a Monte Carlo or deterministic transport analysis of the system. Calculation of these parameters requires that a transport solution be obtained for two cases: (1) a bare-core solution and (2) an integral-system solution.

The reflector return fraction is determined as follows. From the bare-core solution, we obtain the value of k_c from a k-eigenvalue calculation. From the integral-system solution, we obtain k_{eff} from another k-eigenvalue calculation. Using Eq. (20), the reflector return fraction corresponds to:

$$f = 1 - \frac{k_c}{k_{eff}} \quad . \tag{48}$$

The coupling parameter f_{cr} is obtained from the fraction of neutrons associated with the total absorption rate in the core region, P_{ca}, and the fraction of neutrons that leak directly from the core to infinity, P_{ci}. (Note that in a fully reflected system, the leakage rate from the core to infinity is zero because it is assumed that all neutrons must leak into the reflector before leaking from the system.) From the first two terms on the right-hand side of Eq. (10), we note that

$$f_{ca} = (1-f)\,P_{ca} \quad , \tag{49}$$

and

$$f_{ci} = (1-f)\,P_{ci} \quad . \tag{50}$$

P_{ca} can usually be obtained directly from one of the transport solution summary tables listing the integral amount of absorptions (both fission and capture) in the core, and P_{ci} can be obtained by integrating the leakage rates over the unreflected area of the core.

For some transport codes, the summary tables may combine the absorption rates for the core and the reflector into one quantity. When this occurs, P_{ca} can be determined by numerical integration of the following integral:

$$P_{ca} = \frac{\int_{core}\int \Sigma_a(\mathbf{r}, \mathrm{v})\,\phi(\mathbf{r}, \mathrm{v})\,d\mathbf{r}d\mathrm{v}}{\left(\frac{N_s}{\tau_s}\right)} \quad , \tag{51}$$

where Σ_a is the spatial- and velocity-dependent macroscopic absorption cross section, and N_s / τ_s is total neutron loss rate, which is equal to 1.0 if the neutron population has been normalized to a total loss rate of one neutron per unit time.

Once f_{ca} and f_{ci} have been determined, f_{cr} is obtained directly from Eq. (1).

$$f_{cr} = 1 - (f_{ca} + f_{ci}) \quad . \tag{52}$$

In general, the value of f_{cr} calculated from Eq. (52) for a fully reflected system will not agree exactly with the value of the total leakage rate, f_{ci}, ascertained from the bare-core solution. This difference is a function of two opposing effects that occur when a reflector is added to the system. The first effect is associated with a *softening* of the energy spectrum of the reflector return neutrons which usually leads to an increase in the absorption probability in the core region. The second effect is associated with an increase in the average leakage probability from the core because of a change in the fission source distribution; that is, more neutrons are born near the outer edge of the core as a result of an increase in neutron flux in the vicinity of the core/reflector interface. Because of these two effects, it is desirable that f_{ca}, f_{ci}, and f_{cr} be obtained using the above procedure rather than estimating these parameters from a bare-core calculation with the optimistic assumption that these parameters remain constant when a reflector is added.

Following the calculation of f and f_{cr}, the single-pass probability f_{rc} is then readily obtained from the definition of f:

$$f_{rc} = \frac{f}{f_{cr}} \quad . \tag{53}$$

Although the single-pass probabilities f_{ra} and f_{ri} are not explicitly used in the Avery-Cohn model, we nevertheless include the calculation of these parameters for completeness. From Eq. (10), we note that

$$f_{ra} = \frac{(1-f)}{f_{cr}} P_{ra} \quad , \tag{54}$$

in which P_{ra} can be obtained from an output summary table or calculated by

$$P_{ra} = \frac{\int\limits_{ref}\int \Sigma_a(\mathbf{r}, v)\,\phi(\mathbf{r}, v)\, d\mathbf{r}dv}{\left(\dfrac{N_s}{\tau_s}\right)} \quad . \tag{55}$$

Once f_{ra} is known, f_{ri} is determined from Eq. (2) as:

$$f_{ri} = 1 - (f_{ra} + f_{rc}) \quad . \tag{56}$$

Using the procedure outlined in this section, we easily obtain the coupling parameters f_{cr} and f_{rc}. All that remains to complete the model is to calculate the neutron lifetimes τ_c and τ_r. It is very important to remember that N_c, N_r, τ_c, and τ_r that appear in the Avery-Cohn model are adjoint-weighted quantities. As such, an adjoint solution must be obtained prior to evaluating these terms. However, not all transport codes are capable of yielding an adjoint solution. Most notably, Monte Carlo codes normally predict *unweighted* neutron lifetimes (i.e., a straight, unweighted average value that represents the actual life expectancy of all neutrons within the boundaries of a pre-defined system) rather than an *effective* lifetime (i.e., an average value in which the lifetime of each neutron is weighted by its probability of propagating a fission chain). Therefore, before presenting the algorithms used to calculate the neutron lifetimes, we first want to discuss the meaning of the unweighted neutron lifetimes and show how these quantities can be calculated using either Monte Carlo or deterministic transport codes.

XII. UNWEIGHTED NEUTRON LIFETIMES

As previously mentioned, in any system, there are only two ways in which a neutron can be lost—absorption and leakage. If we were able to perform an experiment where we could measure the length of time required for an individual neutron to be absorbed after being born at time $t=0$ and we repeated this experiment for a large number of neutrons, we could calculate the average life span, t_a, of those neutrons destined to be absorbed. By performing a similar experiment, we could also calculate the average life span, t_l, of those neutrons destined to leak from the system. Based on these two quantities, we could then define a unweighted *system* neutron lifetime by

$$\tau_s^* = (P_{ca}+P_{ra})\,t_a + (P_{ci}+P_{ri})\,t_i \quad , \tag{57}$$

which is often referred to as the removal lifetime since it represents the mean time between the removal of a neutron from that system via absorption or leakage. From a physical standpoint, this quantity corresponds to the *actual* neutron lifetime of the integral system since the life span of the individual neutrons are not weighted by their importance in accordance to the position where they were born and their initial energy; τ_s^* is merely a straight average of life spans of all the neutrons in the system.

When using Monte Carlo codes, the hypothetical experiment described above is performed in a numerical fashion using random numbers to determine the eventual fate of each neutron born in the system. However, some care must be taken when interpreting the results of such calculations. Because of the manner in which t_l is determined, the system lifetime becomes a strong function of how the system is defined. For example, calculations performed on a bare core in which the system is defined by the outer surface of the core will produce the same k_{eff} as a calculation performed on a similar core surrounded by several meters of a very low density gas;

however, the system lifetime would be substantially larger for the second case because of the additional time required for leakage neutrons to traverse the gas region and cross the outer system boundary.

The unweighted neutron lifetime of a system can also be calculated from a transport solution of the neutron flux distribution. By noting that the ratio N_s^*/τ_s^* represents the rate at which neutrons are lost from the integral system as the result of both absorption and leakage, and that k_{eff} is the number of neutrons produced per neutron lost, we can write the following equality for the neutron production rate.

$$k_{eff}\left(\frac{N_s^*}{\tau_s^*}\right) = \int_{sys}\int \bar{\nu}_t(\mathbf{r}, \mathrm{v})\,\Sigma_f(\mathbf{r}, \mathrm{v})\,\phi(\mathbf{r}, \mathrm{v})\,d\mathbf{r}d\mathrm{v} \quad , \tag{58}$$

where, as before, we have ignored potential angular dependence of the flux. By further noting that the total neutron population in the integral system is related to the neutron flux by

$$N_s^* = \int_{sys}\int \frac{\phi(\mathbf{r}, \mathrm{v})}{\mathrm{v}}\,d\mathbf{r}d\mathrm{v} \quad , \tag{59}$$

we can rewrite Eq. (58) as

$$\tau_s^* = \frac{k_{eff}\displaystyle\int_{sys}\int \frac{\phi(\mathbf{r}, \mathrm{v})}{\mathrm{v}}\,d\mathbf{r}d\mathrm{v}}{\displaystyle\int_{sys}\int \bar{\nu}_t(\mathbf{r}, \mathrm{v})\,\Sigma_f(\mathbf{r}, \mathrm{v})\,\phi(\mathbf{r}, \mathrm{v})\,d\mathbf{r}d\mathrm{v}} \quad . \tag{60}$$

Using an expression analogous to Eq. (39), the unweighted system lifetime can be separated into its two constituents, τ_c^* and τ_r^* .

$$\tau_s^* = \frac{\tau_c^*}{1-f} + \frac{f_{cr}\tau_r^*}{1-f} \quad . \tag{61}$$

If the numerator of Eq. (60) is broken into two integrals—one over the core region and the other over the reflector region—and we equate the two integrals to the two terms in Eq. (61), then we obtain

$$\tau_c^* = \frac{k_c \int\limits_{core}\int \frac{\phi(\mathbf{r}, v)}{v} d\mathbf{r} dv}{\int\limits_{sys}\int \bar{v}_t(\mathbf{r}, v)\,\Sigma_f(\mathbf{r}, v)\,\phi(\mathbf{r}, v)\, d\mathbf{r} dv} , \tag{62}$$

ınd

$$\tau_r^* = \frac{k_c \int\limits_{ref}\int \frac{\phi(\mathbf{r}, v)}{v} d\mathbf{r} dv}{f_{cr} \int\limits_{sys}\int \bar{v}_t(\mathbf{r}, v)\,\Sigma_f(\mathbf{r}, v)\,\phi(\mathbf{r}, v)\, d\mathbf{r} dv} . \tag{63}$$

Hence, by defining the core and reflector lifetimes in this manner, we simultaneously naintain the equality of Eq. (39) and Eq. (60). Also note that Eq. (62) is in total agreement with Eq. (17).

XIII. EFFECTIVE NEUTRON LIFETIMES

As derived from the transport equation,[57] the effective neutron lifetime of any system, vhether it be reflected or unreflected, involves the adjoint weighting of the spatial- and velocity-lependent neutron population and neutron production rate. Accordingly, the *effective* system life-ime is defined as

$$\tau_s = \frac{k_{eff} \int\limits_{sys}\int \frac{\phi^+(\mathbf{r}, v)\,\phi(\mathbf{r}, v)}{v} d\mathbf{r} dv}{\int\limits_{sys}\iint \phi^+(\mathbf{r}, v)\,\chi_t(v)\,\bar{v}_t(\mathbf{r}, v')\,\Sigma_f(\mathbf{r}, v')\,\phi(\mathbf{r}, v')\, d\mathbf{r} dv' dv} , \tag{64}$$

vhere ϕ^+ is the spatial- and velocity-dependent adjoint flux, and the total fission spectrum, χ_t, is given by

$$\chi_t(v) = (1-\beta_o)\,\chi(v) + \sum \beta_{oi}\chi_i(v) , \tag{65}$$

vhere

β_o = unweighted delayed neutron fraction,

χ = energy spectrum of prompt neutrons,

β_{oi} = unweighted delayed neutron fraction of i^{th} group, and
χ_i = energy spectrum of i^{th} delayed neutron group.

By analogy with Eqs. (62) and (63), the effective system lifetime can also be separated into an *effective* core lifetime and an *effective* reflector lifetime.

$$\tau_c = \frac{k_c \int\limits_{core}\int \frac{\phi^+(\mathbf{r},v)\,\phi(\mathbf{r},v)}{v}\,d\mathbf{r}\,dv}{\int\limits_{sys}\iint \phi^+(\mathbf{r},v)\,\chi_t(v)\,\bar{v}_t(\mathbf{r},v')\,\Sigma_f(\mathbf{r},v')\,\phi(\mathbf{r},v')\,d\mathbf{r}\,dv'\,dv}\,, \tag{66}$$

and

$$\tau_r = \frac{k_c \int\limits_{ref}\int \frac{\phi^+(\mathbf{r},v)\,\phi(\mathbf{r},v)}{v}\,d\mathbf{r}\,dv}{f_{cr}\int\limits_{sys}\iint \phi^+(\mathbf{r},v)\,\chi_t(v)\,\bar{v}_t(\mathbf{r},v')\,\Sigma_f(\mathbf{r},v')\,\phi(\mathbf{r},v')\,d\mathbf{r}\,dv'\,dv}\,. \tag{67}$$

We again stress that the adjoint-weighted lifetimes calculated using Eqs. (66) and (67) are the correct parameters to be used in the Avery-Cohn model and that N_c and N_r are adjoint-weighted neutron populations. And, as before, we have again chosen to ignore the angular dependence of the neutron flux and adjoint flux.

Using the aforementioned definitions of the core and reflector neutron generation times, Eqs. (28) and (29), in conjunction with Eqs. (66) and (67), we can define Λ_c and Λ_r as

$$\Lambda_c = \frac{\int\limits_{core}\int \frac{\phi^+(\mathbf{r},v)\,\phi(\mathbf{r},v)}{v}\,d\mathbf{r}\,dv}{\int\limits_{sys}\iint \phi^+(\mathbf{r},v)\,\chi_t(v)\,\bar{v}_t(\mathbf{r},v')\,\Sigma_f(\mathbf{r},v')\,\phi(\mathbf{r},v')\,d\mathbf{r}\,dv'\,dv}\,, \tag{68}$$

and

$$\Lambda_r = \frac{\int\limits_{ref}\int \frac{\phi^+(\mathbf{r},v)\,\phi(\mathbf{r},v)}{v}\,d\mathbf{r}\,dv}{f_{cr}\int\limits_{sys}\iint \phi^+(\mathbf{r},v)\,\chi_t(v)\,\bar{v}_t(\mathbf{r},v')\,\Sigma_f(\mathbf{r},v')\,\phi(\mathbf{r},v')\,d\mathbf{r}\,dv'\,dv}\,. \tag{69}$$

From Eqs. (68) and (69), we can formulate Λ_s and Λ_m.

$$\Lambda_s = \Lambda_c + f_{cr}\Lambda_r \ ,$$

and

$$\Lambda_m = \Lambda_c + f\Lambda_r \ .$$

As an aside, the effective system lifetime, τ_s, can also be calculated directly from the time-dependent transport equation using the α-eigenvalue technique. In this technique, it is assumed that the system is on an asymptotic period, α, thereby, allowing the time derivative term in the transport equation to be replaced by its asymptotic value, $\alpha\phi/v$, and the value iterated to solution. In some codes, the k-eigenvalue algorithm is used and α is adjusted until k is 1.0. It can be shown that the system lifetime is related to α by

$$\alpha = \frac{k-1}{\tau_s} \ , \tag{70}$$

where k is either the effective multiplication factor, k_{eff}, or the prompt multiplication factor, K, depending on whether the cross section set utilizes the total number of neutrons produced per fission, ν_t, or the number of prompt neutrons born per fission, ν_p. In all cases, the α-eigenvalue solution should yield a more accurate value for the instantaneous *system* neutron lifetime since the flux shape corresponding to the α-eigenvalue solution is a much better representation of the asymptotic flux shape that will occur when $k_{eff} \neq 1.0$. And furthermore, the α-eigenvalue technique is a lot simpler since it does not involve performing numerical integrals. Unfortunately, α-eigenvalue solutions are difficult to obtain in subcritical systems, particularly fast systems. Notwithstanding this inherent difficulty, the α-eigenvalue technique can occasionally be used successfully in some subcritical thermal systems.

When comparing the lifetimes obtained from the α-eigenvalue solutions (for both supercritical and subcritical configurations) to the lifetimes obtained from Eq. (64), we note the following. In bare systems, the system lifetime predicted by the α-eigenvalue technique agrees relatively well with the effective system lifetime calculated using Eq. (64) over a surprisingly large range of k_{eff}. The difference between the two lifetimes was less than a few percent for multiplication factors as low as $k_{eff} = 0.75$. However, in reflected systems operating in the vicinity of delayed critical, we found that these two methods did not yield the same result when the fluxes in Eq. (64) were obtained by a k-eigenvalue solution. To match the α-eigenvalue lifetime using Eq. (64), we found it necessary to use the adjoint fluxes from the k-eigenvalue solution (i.e., the unperturbed configuration at k_{eff}) and the neutron fluxes from the α-eigenvalue solution (i.e, the perturbed configuration). That is,

$$\tau_s = \frac{k_{eff} \int\limits_{sys}\int \frac{\phi_k^+(\mathbf{r}, v)\,\phi_\alpha(\mathbf{r}, v)}{v} d\mathbf{r} dv}{\int\limits_{sys}\int\int \phi_k^+(\mathbf{r}, v)\,\chi_t(v)\,\bar{\nu}_t(\mathbf{r}, v')\,\Sigma_f(\mathbf{r}, v')\,\phi_\alpha(\mathbf{r}, v')\, d\mathbf{r} dv' dv} \quad . \tag{71}$$

Interestingly enough, Eq. (71) is consistent with what is known as *exact* perturbation theory.[58] In exact perturbation theory, the perturbed fluxes resulting from some small perturbation in the system are weighted by the unperturbed adjoint fluxes.

XIV. VALIDATION OF MODEL

To test the Avery-Cohn two-region model—as interpreted in this work—a Rossi-α experiment was performed on the University of New Mexico's AGN-201 reflected system. The core of the AGN-201 system is a right circular cylinder and is a homogeneous mixture of polyethylene and UO_2 enriched to 20% ^{235}U. It is fully reflected by a three-zone reflector. The first zone is a 20-cm-thick graphite reflector; the second zone is a 10-cm-thick lead shield; and the third zone is a 55-cm-thick water shield. A summary of the atom densities and dimensions is given in Table I.

We analyzed four cases for this system: (1) the bare core, (2) the core plus the graphite reflector, (3) the core plus the graphite reflector and lead shield, and (4) the core plus the graphite reflector, and the lead and water shields. In each case, the core was considered to be the first region in the Avery-Cohn model and everything outside of the core was taken to be the reflector region. Each case was analyzed using the Monte Carlo code MCNP[59] and the two-dimensional deterministic transport code TWODANT.[60]

Coupling Parameters

Values of the coupling parameters for each case are summarized in Table II. These results were obtained from TWODANT using 16-group, Hansen & Roach cross sections. As an independent check, we also calculated the coupling parameters from the MCNP solutions using the continuous energy cross section data. It was found that the MCNP results invariably differed somewhat from the TWODANT results, but, for all cases analyzed, were still within a few percent.

From Table II, we note that the fraction of neutrons that leak from the bare core is 48.57%. When the graphite reflector is added, the single-pass probability, f_{cr}, increases slightly to 48.72%, and increases slightly more as the lead and water shields are added. As previously discussed, this change in the leakage probability from the core region is the net result of two opposing effects—a redistribution of the fission source neutrons in the core and a change in spectrum of the reflector

Table I: AGN-201 Reactor Description

Region/Dimension	Material	Atom Density (x 10^{-24}) cm^{-3}
Core (12.8 cm radius x 24 cm height)	^{235}U	1.379 x 10^{-4}
	^{238}U	5.552 x 10^{-4}
	Carbon	3.701 x 10^{-2}
	^{16}O	1.386 x 10^{-3}
	Hydrogen	7.402 x 10^{-2}
Graphite Reflector (20 cm thick)	Carbon	8.420 x 10^{-2}
	Cadmium (impurity)	4.400 x 10^{-8}
Lead Shield (10 cm thick)	Lead	3.300 x 10^{-2}
Water Shield (55 cm thick)	^{16}O	3.346 x 10^{-2}
	Hydrogen	6.691 x 10^{-2}

eturn neutrons. Apparently the redistribution effect in this particular case is slightly stronger than he increased absorption caused by spectral softening.

Also observe from Table II that k_{eff} increases with the addition of each new zone of reflecor. These increases are the direct result of more neutrons being scattered back into the core as ndicated by an increase in the reflector return fraction, f. Because f_{cr} changed only slightly, this ncrease in f was due almost entirely to an increase in the albedo from the reflector, f_{rc}, which ncreased from 44.29% for the graphite reflector to 48.77% for the full, three-zone reflector.

Although these increases in f_{rc} are somewhat expected, it was unexpected that the lead hield would have more of an effect on the reflector return fraction than the addition of the water hield. Lead is normally considered to be a relatively poor reflecting material, whereas water is onsidered to be a relatively good reflecting material. We attribute the disproportionately small ncrease in f produced by the water shield to the fact that the lead shield is closer to the core than he water shield and, therefore, is better able to scatter more neutrons back into the core based olely on geometric factors. Furthermore, we note that the vast majority of the absorptions in the eflector region (i.e., P_{ra}) occur in the water and that the majority of the remaining absorptions ccur in the lead. We surmise that the lead is allowing fast neutrons to escape, while simultaeously acting as an *absorption shield* for those thermal neutrons trying to return from the water. Vhen combined with the geometric factors, the reflective property of the water appears to be less

Table II: Coupling Parameters

Coupling Parameters	Core	Core + Graphite	Core + Graphite + Lead	Core + Graphite + Lead + Water
k_c	0.7846	0.7846	0.7846	0.7846
k_{eff}	0.7846	1.0005	1.0203	1.0297
$f = 1 - k_c/k_{eff}$	*NA*	0.2158	0.2310	0.2380
P_{ca}	0.5143[a]	0.6539	0.6662	0.6719
P_{ci}	0.4857[b]	*NA*[c]	*NA*	*NA*
P_{ra}	*NA*	0.0062	0.0459	0.3281
$P_{ri} = 1 - P_{ca} - P_{ra} - P_{ci}$	*NA*	0.3399	0.2878	≈0.0
$f_{ca} = (1 - f)P_{ca}$	0.5143	0.5128	0.5123	0.5120
$f_{cr} = 1 - f_{ca}$	*NA*	0.4872	0.4877	0.4880
$f_{ci} = (1 - f)P_{ci}$	0.4857	*NA*	*NA*	*NA*
$f_{rc} = f / f_{cr}$	*NA*	0.4429	0.4737	0.4877
$f_{ra} = (1 - f)P_{ra}/f_{cr}$	*NA*	0.0100	0.0724	0.5123
$f_{ri} = 1 - f_{rc} - f_{ra}$	*NA*	0.5471	0.4539	≈0.0

a. $P_{ca} = f_{ca}$ for a bare core.
b. $P_{ci} = f_{ci}$ for a bare core.
c. System is fully reflected.

effective than that of the lead in this particular instance.

The coupling parameters f_{cr} and f_{rc} in Table II correctly estimate the *net* neutron flows at the core/reflector interface, as predicted by Eq. (13). However, when they are substituted into Eqs. (14) and (15), these coupling parameters yield partial currents at that interface that are approximately 20% lower than the partial currents predicted by the two transport codes. Our explanation for this discrepancy is based on a purely heuristic argument.

As previously stated, the coupling parameters used in the Avery-Cohn model are average values that, for all intents and purposes, are based solely on three integral quantities: k_c, k_{eff}, and P_{ca}. From these three quantities, the average values of the coupling parameters f, f_{cr}, and f_{rc} are readily determined as described in Section XI. However, it is known that these single-pass probabilities are spatially dependent. For example, in systems where the dimensions of the core are much larger than the mean free path of a typical core neutron, neutrons born near the edge of the core are expected to have a much higher probability of leaking from the core than those neutrons born in the center. Hence, it is expected that in the immediate vicinity of the core/reflector interface the local single-pass probability for leakage and the local reflector return probability will be significantly greater than the average values that characterize the integral core and reflector. As a consequence, the local reflector return fraction is expected to be much higher, which, in turn, results in higher partial currents. In contrast, when the core is small and compact, the probability of a neutron leaking from the center of the core and the probability of a neutron leaking from the outer regions of the core are approximately the same. For this situation, the local reflector return fraction is expected to be approximately the same as the system-average reflector return fraction. Therefore, the partial currents, as predicted by Eqs. (14) and (15), are expected to be in good agreement with those ascertained from the transport codes.

Using TWODANT, various reflected systems of different core dimensions were analyzed. When the dimensions of the core were large, large discrepancies between the model and the transport solution for the partial currents were observed. (Despite these large discrepancies in the partial currents, however, the net flow was always correct.) As the dimensions of the core were decreased, these discrepancies also decreased and matched almost perfectly for small metal systems. The observed discrepancies in the partial currents in large systems could possibly explain the failure of the Avery-Cohn model to adequately describe the experimental data reported by Bergstrom et al.[18] In their effort to calculate the coupling parameters, they relied heavily on the partial currents at the core/reflector interface as predicted by their computer code. Based on our studies, we suspect that the partial currents they calculated could have been wrong by as much as a factor of two.

System Lifetimes

Using MCNP, the neutron life spans for the four cases previously mentioned were calculated for the AGN-201. These results are summarized in Table III. As can be noted, there are three different life spans that are tracked by MCNP—a capture life span, a fission life span, and a leakage life span. The capture life span, $t_{capture}$, is actually a misnomer since it includes both nonfission absorption [i.e., (n,γ),(n,α), and (n,p)] and fission absorption. Although the fission life span, $t_{fission}$, is not used in the calculation of the system lifetime—which is referred to as the removal lifetime in MCNP—it is included in the code output for the sake of additional information about the fission process. Using t_{escape} and $t_{capture}$, and an estimate obtained from one of the summary tables of the respective probabilities of occurrence, the unweighted system lifetime per Eq. (57) corresponds to

$$\tau_s = (0.5143)\,(33.6) + (0.4857)\,(2.9) = 18.8\ \mu s \quad .$$

Table III: Neutron Life Spans from MCNP

	Core	Core + Graphite	Core + Graphite + Lead	Core + Graphite + Lead + Water
	Lifetime(μs)			
$t_{capture}$	33.6	57.1	115.8	297.6
$t_{fission}$	35.8	57.2	74.4	91.8
t_{escape}	2.9	160.5	317.0	1.7[a]
$\tau_{removal}$ $(=\tau_s)$	18.8	91.4	170.9	297.6

a. See text for explanation.

The unweighted system lifetimes for the three reflected cases are also determined in a similar fashion. These values are shown in Table III and are repeated in Table IV along with the unweighted system lifetimes determined using TWODANT. Note that the escape life spans predicted by MCNP appear to have a noticeable inconsistency for the fourth case. When the water shield is added, t_{escape} decreases sharply from 317.0 to 1.7 μs. This decrease is explained as follows.

The addition of the graphite reflector and lead shield reduced the overall system leakage fraction from 49% for the bare core case to 29% for case III. This reduction in leakage was the result of more neutrons being scattered back into the core and more neutrons being absorbed in the reflector region. Concurrently, the addition of more reflector increased the number of scattering interactions each neutron had before leaving the region which, in turn, greatly increased the average life span of those neutrons that eventually leaked; hence, the average escape life span steadily increased to 317.0 μs as more reflector was added. When the water reflector was added, however, the leakage fraction dropped to approximately 10^{-6} indicating that once the neutrons thermalized, it became nearly impossible for them to traverse the relatively thick water-shield zone without being absorbed. We speculate that the few neutrons that were able to escape must have corresponded to relatively high-energy neutrons that had just a few scattering interactions before reaching the outer surface of the system. An average fission neutron born in energy group 3 would traverse the 91 cm thick system in 62 ns, whereas a neutron born in energy group 6 would require 0.34 μs. These escape times are on the correct order of magnitude to explain the rapid decrease in the escape lifetime calculated by MCNP.

When we compare the unweighted system lifetimes from MCNP with the unweighted system lifetimes calculated using fluxes from TWODANT (see Table IV), we find that the TWODANT values are consistently 5 to 10% higher. The source of this discrepancy has not yet been identified.

The effective neutron lifetimes and generation times calculated from the TWODANT fluxes are also presented in Table IV. For comparison, we have also included the effective system lifetime as determined from an α-eigenvalue solution.

As can be noted, the effective lifetimes for the reflected cases are significantly less than the unweighted lifetimes. Apparently, the effective neutron lifetimes are dominated by those high-importance, short-lived neutrons born near the core center and are relatively insensitive to those low-importance, long-lived neutrons that leak into the reflector.

Although the effective core generation time, Λ_c, decreases 24% with the addition of all reflectors, the static and dynamic prompt-neutron generation times, Λ_s and Λ_m, are seen to increase. This occurs primarily because of significant increases in the reflector generation time. For case II, Λ_r is calculated to be 37.5 μs, and for case IV, Λ_r is calculated to be 91.2 μs. This increase, of course, is due to the additional time required for the neutrons to traverse the extra reflector zones.

From Table IV, we also note that Λ_m increases more slowly with additional reflector than does Λ_s. This result is also expected since the addition of more reflector increases the number of neutrons in the integral system but only slightly increases the number of neutrons returning to the core. (Note that there is an inflection point in the values of Λ_m. The bare core Λ_m is higher than case II because of the low value of k_{eff} corresponding to the bare core configuration.)

And finally, we note that the effective system lifetime determined from the α-eigenvalue solution agrees relatively well with the adjoint-weighted system lifetime obtained from Eq. (64) for the bare-core case, but is lower for the three reflected cases. As previously mentioned, the neutron and adjoint fluxes from an α-eigenvalue solution differ from a k-eigenvalue solution. In our effort to understand the consequences of this difference, we used all possible combinations of the k- and α-eigenvalue fluxes in Eq. (64) to predict a neutron lifetime for case III. The results of this exercise are shown in Table V. As can be noted, when the adjoint fluxes corresponded to the k-eigenvalue solution and the neutron fluxes corresponded to the α-eigenvalue solution, the two lifetimes were found to be in fair agreement.

XV. COMPARISON WITH EXPERIMENTAL RESULTS

A Rossi-α experiment was performed on the AGN-201 reactor at the University of New Mexico to measure β/Λ_m at delayed critical.[61] As predicted by the model presented in this work, when the system was close to delayed critical, two noticeable prompt-decay modes were measured (see Fig. 6). (Note, when the initial portion of the Rossi-α curve was more closely examined using a smaller channel width on the correlator, it was discovered that at least two more

Table IV: Neutron Lifetimes per TWODANT Solution

		Core	Core + Graphite	Core + Graphite + Lead	Core + Graphite + Lead + Water
		Lifetime (μs)[a]			
τ_s	M	18.8	91.4	170.9	297.6
	U	20.4	102.3	177.9	329.3
	W	40.0	58.0	70.1	85.7
	α[b]	39.6	56.9	65.2	74.8
τ_c	U	20.4	20.4	20.4	20.4
	W	40.0	31.2	30.5	30.3
τ_r	*U*	*NA*	122.7	238.6	472.3
	W	*NA*	29.3	48.0	71.8
$\Lambda_c = \frac{\tau_c}{k_{eff}(1-f)}$	U	26.0	26.0	26.0	26.0
	W	51.0	39.8	38.9	38.6
$\Lambda_r = \frac{\tau_r}{k_{eff}(1-f)}$	*U*	*NA*	156.2	304.3	602.1
	W	*NA*	37.5	61.5	91.2
$\Lambda_s = \Lambda_c + f_{cr}\Lambda_r$	U	26.0	102.2	174.3	319.8
	W	51.0	58.0	68.8	83.2
$\Lambda_m = \Lambda_c + f\Lambda_r$	U	26.0	59.7	96.3	169.3
	W	51.0	47.9	53.1	60.3

a. M = MCNP *removal* lifetime, U = unweighted (i.e., actual) neutron lifetime or generation time, W = adjoint-weighted neutron lifetime or generation time.
b. Calculated from an α-eigenvalue solution.

Table V: System Lifetime for Case III

Combination	Lifetime (μs) per	
	Eq. (64)	α-Eigenvalue
ϕ_k^+, ϕ_k	70.1	–
ϕ_α^+, ϕ_k	66.2	–
ϕ_k^+, ϕ_α	64.9	65.2
$\phi_\alpha^+, \phi_\alpha$	61.1	–

decay constants could be resolved. However, the value of these two additional decay modes were judged to be unreliable for this particular measurement because of detector deadtime effects. Therefore, the values of the extra decay constants have not been included in this work. A new experiment is currently being designed in which the deadtime will be greatly reduced, thereby, allowing us to resolve as many as four decay constants for this system.)

As discussed in Ref. 62, the dominant prompt-decay mode is expected to be fairly linear with $1/C$, where C is the count rate of the detector used to obtain the Rossi-α measurement. This assumes that τ_r is on the order of 500 μs or less and that the mean prompt-neutron generation time remains relatively constant as a function of subcritical reactivity. Furthermore, it is also expected that the product of ω_7 and its corresponding amplitude, A_7, will be nearly constant over a small range of reactivity in the vicinity of delayed critical. However, in this experiment, it was observed that ω_7 did not follow a linear relationship with $1/C$ and that the magnitude of the product $\omega_7 A_7$ decreased significantly with subcriticality. These variations indicated to us that the mean prompt-neutron generation time was changing as a function of subcritical reactivity.

We postulated that some of the observed nonlinearities in our data could be negated by normalizing the measured ω_7 data to a constant mean prompt-neutron generation time using the inherent property that the product $\omega_7 A_7$ is inversely proportional to the square of the mean prompt-neutron generation time. Hence, we normalized each measured ω_7 by the factor

$$\omega_7^* = \omega_7 \sqrt{\frac{\omega_{7o} A_{7o}}{\omega_7 A_7}} \,, \tag{72}$$

where $\omega_{7o} A_{7o}$ is the measured product at delayed critical.

A plot of both the ω_7 and ω_7* is shown in Fig. 7. From this plot, we note that the adjusted

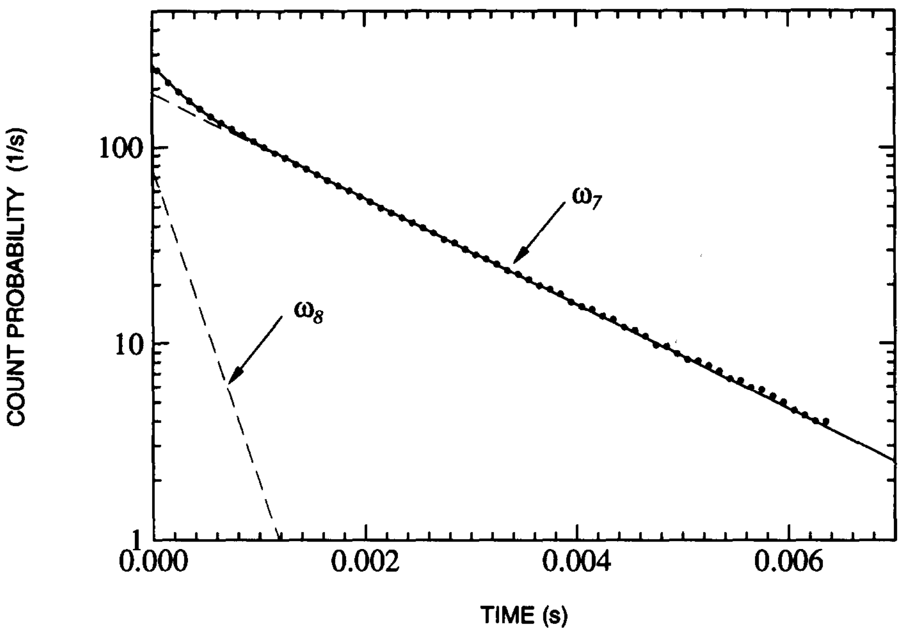

Fig. 6. Typical Rossi-α curve for an AGN-201 reflected system.

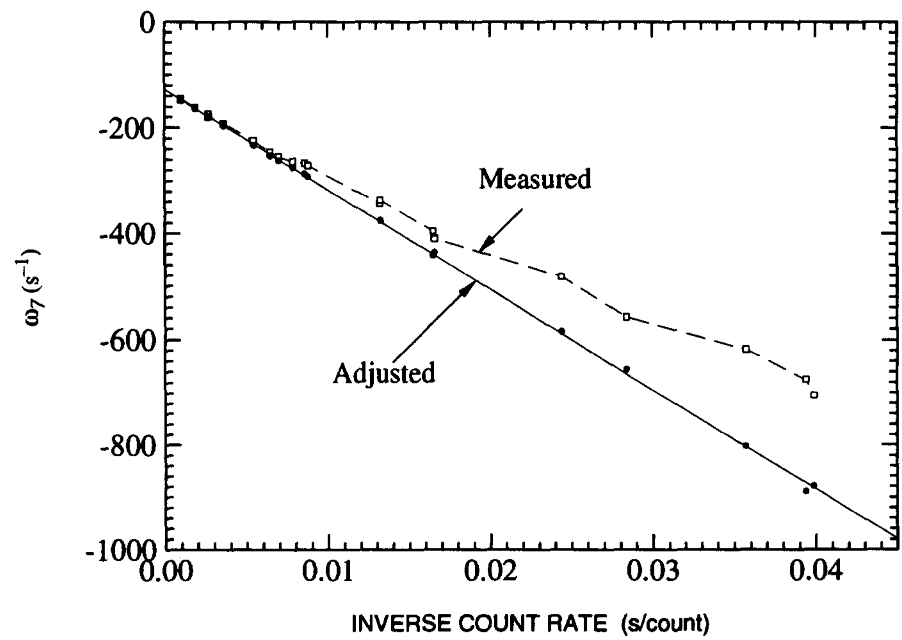

Fig. 7. Measured and adjusted dominant prompt-decay root.

decay constants are markedly more linear with $1/C$ than the measured decay constants. And, furthermore, when used to estimate the reactivity of the system, the adjusted decay constants agreed to within a few percent of the reactivity determined by a control rod calibration. Even though the adjusted decay constants satisfactorily corrected for changes in the neutron generation time, the change in ω_{7o} was still negligible. When either the measured or adjusted data were extrapolated to delayed critical, ω_{7o} corresponded to -125.9 ($\pm 1\%$) s^{-1}.

Based on a TWODANT model for this system, we calculate an effective delayed neutron fraction of $\beta = 0.00754$ assuming the delayed neutron spectra of Brady.[63] The uncertainty of β, however, is on the order of $\pm 10\%$ due to the uncertainty of Brady's[63] semi-empirical delayed neutron spectra, which differs somewhat from previously measured delayed neutron spectra. Notwithstanding this uncertainty, when coupled with the measurement of ω_{7o} (i.e., $-\beta/\Lambda_m$), the mean value of the mean prompt-neutron generation time, Λ_m, was determined to be 59.9 ($\pm 10\%$) μs. This value agrees well with the calculated value of 60.3 μs (see Table IV).

XVI. DISCUSSION

The Avery-Cohn model represents an exact neutron balance on the two neutronically-distinct regions of a reflected system–the core and the reflector. These two regions are coupled to each other by partial neutron currents that are based on the average probability of a neutron flowing from the core region to the reflector region or vice versa. We have made no attempts to derive this model starting from the transport equation and, as such, are uncertain at this time as to how the coupling parameters in the Avery-Cohn model are related to fundamental quantities such as the spatial- and velocity-dependent angular fluxes. Nevertheless, using the approach outlined in Section XI, we can easily surmise these average probabilities from a static transport solution, and we can relate the remaining macroscopic parameters appearing in the model to quantities that can be derived from the transport equation.

There is one idiosyncrasy associated with this "force-fit" approach that should be kept in mind when modeling a reflected system. When the spatial variation of the single-pass leakage probability across the core region and/or reflector region is large, the local values of the partial currents at the core/reflector interface do not represent the average leakage rate from either region. This disparity is most unfortunate because it precludes calculating k_c and f directly from an integral system k-eigenvalue solution using

$$k_c = \frac{\int\limits_{core} \int \bar{\nu}_t(\mathbf{r}, v)\, \Sigma_f(\mathbf{r}, v)\, \phi(\mathbf{r}, v)\, d\mathbf{r} dv}{1 + P_{rc}} , \tag{73}$$

and

$$f = \frac{P_{rc}}{1 + P_{rc}} \quad , \tag{74}$$

in which the partial current, P_{rc}, is ascertained directly from the transport solution. As a result of this idiosyncrasy, we are forced to calculate f from a k-eigenvalue solution of an integral-system model where k_c is independently determined from a k-eigenvalue solution of a bare-core model. From a purely intuitive standpoint, we suspect that the values of k_c and f are probably slightly in error when determined in this fashion. We think that the value of k_c increases somewhat when the reflector is added because of changes in the flux shape and neutron spectrum. Although the extent of this increase is unknown, for the present we presume it to be small enough to ignore.

In Section I we mentioned four anomalies that have been observed in some types of reflected systems: (1) multiple decay modes in both the time and frequency domain, (2) the dollar discrepancy, (3) a nonlinear relationship between reactivity and the initial inverse period at reactivities greater than 1\$, and (4) an inordinately large disparity between measured and calculated neutron lifetimes. All four of these anomalies can now be explained, at least qualitatively, using the Avery-Cohn model.

Multiple Decay Modes

As shown in the plot of the reflected–core inhour equation (see Fig. 4), there are two prompt decay modes in a reflected reactor—ω_7 and ω_8. For these two decay modes to appear during a die-away experiment, the mean prompt-neutron generation time must be sufficiently larger than the core's neutron generation time. From the definition of the mean prompt-neutron generation time, $\Lambda_m = \Lambda_c + f\Lambda_r$, we note that for $\Lambda_m >> \Lambda_c$, the reflector return fraction must be large and/or the reflector lifetime must be much, much larger than the core lifetime. The first condition usually occurs when a reflector is relatively thick, or has a large albedo; the second condition usually occurs in reflected systems comprised of a fast core surrounded by a weakly-absorbing, moderating reflector (i.e., a thermal reflector).

An experiment performed by Coats[29] on the SPR-II fast-burst assembly at Sandia clearly illustrates the effect of increasing the reflector return fraction. In his experiment, Coats[29] changed the reflector return fraction using a 1-inch-thick, polyethylene reflector. As the polyethylene reflector was placed closer to the assembly, the observed mean prompt-neutron generation time was seen to significantly increase relative to the bare system.

Long[28] performed a similar experiment on SPR-II that demonstrated the effect of the reflector lifetime on the mean prompt-neutron generation time. In his experiment, Long[28] measured the dominant root of the Rossi-α corresponding to various reflector configurations in which moderating and nonmoderating reflectors were placed around the assembly. When surrounded by a moderating material, the mean prompt-neutron generation time increased by more than a factor of two relative to the bare system. When surrounded by a nonmoderating reflector of approximately equal reactivity worth, the mean prompt-neutron generation time still increased, but to a much lesser extent.

Both sets of experimental results are consistent with the definition of Λ_m.

Dollar Discrepancy

In fast, moderator-reflected systems employing thick, weakly-absorbing reflectors (such as graphite), the lifetime of the reflector can be relatively large. If $\tau_r\omega_7$ is not $<< 1.0$, then ω_7 will depart from the linear function defined by Eq. (34)—even in the vicinity of delayed critical (see Fig. 5b). Consequently, if a series of subcritical die-away measurements are performed in a reflected reactor of this type and the dominant root is plotted as a function of reactivity, the curve will extrapolate to a reactivity that exceeds prompt critical by an amount that cannot be accounted for by statistical uncertainties in the measurements.

Brunson[24] reviewed the data from 11 different reflected systems in which this discrepancy had been observed. Using the two-region, Rossi-α model proposed by Kistner[64] (which is based on a modified version of the Avery-Cohn model in which only prompt neutrons are considered), Brunson correctly concluded that the dollar discrepancy was somehow related to the nonlinear relationship of one of the roots with reactivity. This is consistent with the behavior of the mean prompt-neutron generation root of the reflected-inhour equation.

We submit that the dollar discrepancy is not really a discrepancy; it is merely the consequence of trying to fit a straight line to a nonlinear function, and then extrapolating beyond the region of the fit.

Nonlinear Behavior Above Prompt Critical

As shown from the Avery-Cohn model, the nonlinear relationship between the initial inverse period and reactivity near or above prompt critical arises primarily because of a relatively small reflector return fraction in conjunction with large differences between the mean prompt-neutron generation time and the core's neutron generation time. When the reflector return fraction in a fast, moderator-reflected system is small, the system will pass through a second prompt critical shortly beyond the first prompt critical. As reactivity is further increased, the characteristic neutron generation time of the system will rapidly decrease from the mean prompt-neutron generation time to that corresponding to the core's neutron generation time. This effect is shown in Fig. 4.

Coats[29] observed this nonlinearity during the aforementioned experiment in which he changed the reflector return fraction in the SPR-II burst reactor using a 1-inch-thick, polyethylene reflector. As the reflector was moved closer to the assembly, the nonlinear relationship between the initial inverse period and reactivity became more pronounced.

As expected, the nonlinear relationship between reactivity and initial inverse period has not been observed in systems that have a high reflector return fraction, or in thermal, nonmoderator-reflected systems. In systems that have high reflector return fractions, it is nearly impossible (and not very prudent from a safety standpoint) to insert enough reactivity to exceed the second prompt critical; and, in thermal, nonmoderator-reflected systems, the difference between the mean prompt-neutron generation time and the core's neutron generation time is expected to be small. Hence, even if the second prompt critical were exceeded, there would be very little change in the slope of the initial inverse period vs. reactivity.

Discrepancies in the Neutron Lifetimes

The Avery-Cohn model clearly demonstrates that certain types of reflected systems cannot be characterized by a single neutron generation time. For example, at subcritical, source equilibrium, the total neutron population of a fast, moderator-reflected system is characterized by the static neutron generation time, $\Lambda_s = \Lambda_c + f_{cr}\Lambda_r$. That is,

$$N_t = -\frac{\Lambda_s S}{\rho} \quad . \tag{75}$$

The dynamic response of that same reflected system, however, will be characterized by time constants involving other combinations of Λ_c, Λ_r, and f, depending on the operating regime the system. For positive reactivities in the vicinity of the first prompt critical and below, the dynamic response of the system is characterized by the mean prompt-neutron generation time, $\Lambda_m = \Lambda_c + f\Lambda_r$, providing $\tau_r\omega$<<1.0; whereas, for reactivities in the vicinity of the second prompt critical and above, the dynamic response of the system becomes a sole function of the core's neutron generation time, which can be significantly smaller than Λ_m. Furthermore, for negative reactivities, the prompt decay modes of the system can exhibit considerable nonlinearities as a function of reactivity if $\tau_r\omega$ is not <<1.0. Consequently, when performing static and dynamic measurements in a reflected system of this type, it is essential to understand what is being measured and how these measured quantities are related by the individual neutronic properties of the core and reflector regions.

This lack of understanding has been a key factor in the observed discrepancies between measured and calculated neutron lifetimes. In accordance with the Avery-Cohn model, we now understand that the dominant decay mode that is measured during a die-away experiment in a fast, moderator-reflected system corresponds to approximately $-\beta/\Lambda_m$ at delayed critical. In principle, Λ_m can be determined from this measurement if β is known. The value of Λ_m determined from such a measurement, however, does not correspond to the effective system lifetime that is inferred from an α-eigenvalue solution of a transport code. As demonstrated in this work, the neutron lifetime obtained from an α-eigenvalue solution for the AGN-201 reflected system was 24% larger than the mean prompt-neutron generation time formulated from the results of Eqs. (66) and (67).

Because the lifetime inferred from an α-eigenvalue solution does not correspond to the mean prompt-neutron generation time measured during a die-away experiment explains, in part, the inordinately large discrepancies reported by Davey.[17] When calculating the system lifetimes for 23 fast, moderator-reflected systems tested in ZPR-III, Davey used an α-eigenvalue solution. When compared to the lifetimes obtained from a series of die-away experiments performed in these same systems, he noted an average discrepancy of 34%. It is our contention that some of this discrepancy is simply a misinterpretation of the quantities that were measured during the die-away experiments and how these measured quantities are properly calculated from a transport solution.

XVII. CONCLUSIONS

The Avery-Cohn differential equations, coupled with the probability relationships and the definitions of the neutron lifetime/generation times introduced in this work, represent a more sophisticated, lumped-parameter, kinetic model for reflected systems. Although the model does not explicitly treat spatial- and velocity-dependent variations in the neutron population, the macroscopic parameters used in the model are appropriately weighted over space and velocity such that the average behavior of a core neutron and a reflector neutron match a static, deterministic transport solution. Despite its simplicity, we believe that the Avery-Cohn model is particularly useful in that it (1) provides a basic understanding of the neutron economy and time-dependent behavior of neutron populations in reflected systems, (2) provides a theoretical basis for formulating and calculating integral quantities in reflected systems, and (3) provides a simple model that can be used to interpret integral results obtained from pulsed-neutron and Rossi-α experiments.

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