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Spatial neutronic coupling aspects in nuclear reactors

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ABSTRACT

In this paper, an effort is made to gain insights about neutronic coupling and decoupling phenomena of nuclear reactors and its consequences on their safety and stability. The neutronic coupling and decoupling aspects are investigated using eigenvalue separation (EVS) methodology. Higher harmonic eigenvalues are calculated by the method of mode subtraction. The eigenvalue separation for a typical 1000 MWe PWR is calculated and its relations with reactor core shape and size and consequent effects on spatial stability are investigated. It is demonstrated quantitatively that it is necessary to optimize height to diameter (H/D) ratio to suppress the susceptibility to multimode oscillations and to enable ease in designing spatial control algorithm. Consequences of extreme H/D ratio are also addressed. Optimum shape of the reactor core is investigated and the evaluation of upper limit of about 1.3 for H/D ratio has been carried out for large PWR cores. Safety implications of neutronic loose coupling on departure from nucleate boiling ratio (DNBR) are also addressed.

1. Introduction

Large sized nuclear reactors are preferred to achieve economy of scale in power production. However large sized reactors show spatial instability, i.e., these reactors show deviation in power distribution under certain transients. Xenon induced oscillation is a practical problem encountered in almost all large thermal reactors. A few early studies (Henry and German, 1956; Randall et al., 1958) attracted the researchers to investigate the basic cause of this spatial instability. Neutronic decoupling among various parts of the core was found to be the fundamental cause of the spatial instability of large power reactors (Wiberg, 1965). Commonly used methodology to understand the neutronic coupling phenomena in nuclear reactors is based on comparison of characteristic sizes (Stacy, 2007). The characteristic size of the reactor core is the core size expressed in units of neutron migration length (mean travel distance of neutrons between production and absorption) of the core. Beyond certain threshold value of characteristic size, the core tends to become neutronically decoupled. The characteristic size method gives a gross idea of spatial coupling of the core and does not reveal the details of decoupling and its significance in any transient. To understand the phenomenon in detail, more sophisticated techniques have come into vogue. One of such technique is eigenvalue separation technique. In this technique, higher harmonics of the neutron diffusion equation are evaluated and their relative influence in total neutron diffusion equation forms the basis of degree of neutronic coupling. This needs evaluation and analysis of higher flux harmonics. In this regards, Kaplan (1961) devised a few techniques to analyze space-time problems using modal expansion methods. Later natural mode approximation was used (Foulke and Gyftopoulos, 1967) to simplify a few selected space dependent reactor dynamics problems. A series of experiments were conducted (Rydin, 1971) to assess the noise response of loosely coupled cores. Higher flux mode effects were analyzed to check the accuracy of single flux mode stability criteria (Rydin, 1973). An accurate, higher order relationship between reactor power tilts and eigenvalue separation was developed and verified experimentally (Bechner and Rydin, 1975). This study connected the asymmetric reactivity perturbations to static axial power tilt and eigenvalue separation. Later, the same was experimentally verified for fast reactors (Brumbach et al., 1988; Sanda et al., 1993).

In the present paper the eigenvalue separation and its connection with flux tilt and reactivity perturbation has been presented using simplified analytical expressions. The eigenvalue separation is calculated as a function of core size and shape and an optimized cylindrical core shape has been evaluated to minimize neutronic decoupling. Effect of flux tilt arising from higher harmonics, on departure from nuclear boiling ratio (DNBR) has also been reported.

In Section 2, fundamental mathematics, describing EVS as a parameter to understand neutronic coupling is explained. In Section 3, the methodology of calculating EVS and higher harmonics is reported. Section 4 presents the results of shape optimization studies for PWR core. In Section 5, the effect of flux tilt arising due to disturbances in a large PWR core on the margin to

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critical heat flux is reported. Section 6 summarizes the conclusions.

2. Eigenvalue separation (EVS) and neutronic coupling

Time dependence of neutron population in a nuclear reactor core can be studied by using neutron diffusion equation, which in operator form, can be written as:

$$\upsilon^{-1}\frac{\partial\phi}{\partial t} = (-A+B)\phi + \sum_s \chi_s^d \lambda_s C_s \tag{1}$$

where A is the destruction operator and B is the production operator. Other symbols in the equations have their usual meanings. Time dependent neutron flux, $\phi(r,t)$ can be expressed (Stacy, 2007) in terms of unperturbed neutron flux distribution, $\phi_0(r)$ and its eigenfunction, $\psi(r)$ as:

$$\phi(r,t) = \phi_0(r) + \Delta\phi(r,t) \tag{2}$$

where

$$\Delta\phi(r,t) = \sum_{n=1}^{\infty} a_n(t)\psi_n(r) \tag{3}$$

represents the deviation of neutron flux from steady state. a_n represents magnitude part of the eigenfunction and its value depends upon the magnitude of reactivity perturbations. The functions, $\psi_n(r)$ are based on higher harmonics of neutron diffusion equation. They could be any combination of trigonometric and Bessel's functions. For cylindrical shape, these harmonics could be axial or azimuthal harmonics or mixed (Fig. 1).

Neutron flux, at any instant is the sum of fundamental mode and higher harmonics with different weightage. Therefore, Eq. (2) can be written as:

$$\phi(r,t) = \text{steady state flux} + \text{contribution of higher harmonics}$$
$$(\text{axial} + \text{azimuthal}) \tag{4}$$

Ideally, contribution of these higher harmonics should be fixed under all operating conditions to maintain non-varying power distribution in course of time. This can be achieved in small cores to a great extent but it is not possible in large cores. This will be discussed in later section of this paper. Therefore in large core design, efforts are made to minimize the contribution of higher harmonics to gain maximum inherent stability. In an operating reactor, neutron flux shape could get disturbed because of several reasons like insertion/removal of reactivity devices, xenon effects and localized perturbations due to reactivity feedbacks etc. The effect of such perturbations on power transients is different for different size of reactors. This can be demonstrated with the help of few basic equations. For ease in understanding, simplest geometry of slab reactor is considered.

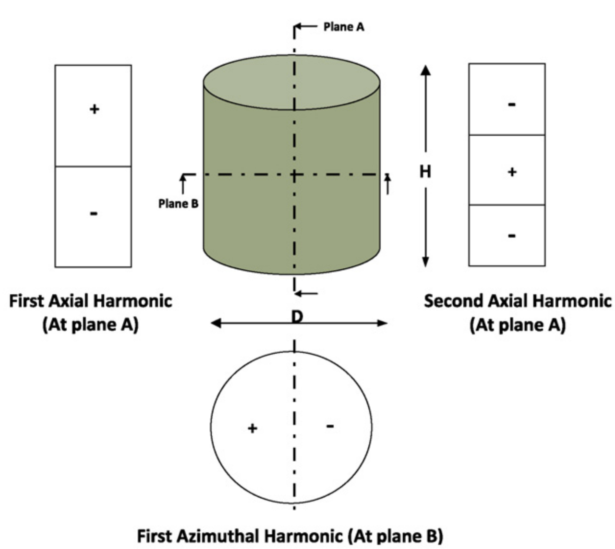


Fig. 1. Schematic of first and second harmonics.

2.1. Eigenvalue separation and core size

Higher harmonics eigenvalue (λ_n) of Eq. (1), can be written as:

$$\lambda_n = \frac{k_\infty}{1+M^2B_n^2} \tag{5}$$

where geometric buckling, $B_n = (n+1)\pi/a$ for a slab reactor of core thickness a. k_∞ is infinite multiplication factor, M^2 is neutron migration area, $M^2 = L^2 + \tau$, where L and τ are diffusion length and 'age to thermal neutron' respectively. Eigenvalue separation ($\in_1$), between the first mode and fundamental mode is expressed as:

$$\in_1 = \frac{1}{\lambda_1} - \frac{1}{\lambda_0} \tag{6}$$

Substituting the expressions of λ_1 and λ_0 from Eq. (5), one gets:

$$\in_1 = \frac{1+M^2B_1^2}{k\infty} - \frac{1+M^2B_0^2}{k_\infty}$$

or

$$\in_1 = \frac{M^2[(2(\pi/a))^2 - (\pi/a)^2]}{k\infty}$$

That is:

$$\in_1 \sim 3\left(\frac{M\pi}{a}^2\right) \tag{7}$$

Thus, eigenvalue separation is inversely proportional to the square of the size of the reactor core. This means, a large sized reactor core has small eigenvalue separation.

2.2. Eigenvalue separation and flux tilt

Consider a case, in which core gets subjected to asymmetric reactivity perturbation. These perturbations will disturb the neutron flux distribution. A classical way to represent such spatial perturbations is to represent them by higher mode reactivity. First mode reactivity (ρ_1) can be written as (Sanda et al., 1993):

$$\rho_1 = \frac{\langle\psi_1^*, ((\delta B/\lambda_0) - \delta A)\psi_0\rangle}{\langle\psi_1^*, B\psi_1\rangle} \tag{8}$$

where δA and δB represent changes in destruction and production operator respectively due to the reactivity perturbation, $\psi_1\psi_1^*$ are first mode eigenfunction and adjoint eigenfunction respectively and λ_0 is fundamental mode eigenvalue. The induced flux tilt (τ) in the core due to this perturbation (ρ_1) can be written as (Sanda et al., 1993):

$$\tau = \frac{\langle\phi'(r,E)\rangle_T - \langle\phi'(r,E)\rangle_B}{\langle\phi'(r,E)\rangle} \tag{9}$$

where $\langle\,\rangle_T$ and $\langle\,\rangle_B$ and $\langle\,\rangle$ are integrals respectively over the top half, the bottom half and the total core volume. Under asymmetric perturbation, the tilt equation (Eq. (9)) can be approximated in terms of reactivity perturbation and EVS as (Sanda et al., 1993):

$$\tau = \frac{\rho_1\langle|\psi_1(r,E)|\rangle}{\in_1\langle\psi_0(r,E)\rangle} \tag{10}$$

where, ψ_0 is fundamental mode eigenfunction and ψ_1 is first harmonic eigenfunction. As the eigenfunctions are normalized as $\langle \psi_1^*, B\psi_1 \rangle = \lambda_1$ and $\langle |\psi_1(r,E)| \rangle = 1$, the Eq (10) can be simplified as (Sanda et al., 1993):

$$\tau \cong \frac{\rho_1}{\in_1} \tag{11}$$

From Eqs. (7) and (11), it can be said that a large sized reactor core will exhibit more flux tilt for the same magnitude of asymmetric reactivity perturbation.

2.3. Eigenvalue separation and higher harmonics

Time dependent part of Eq. (2) can be shown to be proportional to eigenvalue separation (Lamarsh, 1966) as:

$$\Delta\phi(r,t) \propto \frac{\lambda_n}{\lambda_n - 1} \exp\left[\frac{\lambda_n - 1}{\lambda_n}\frac{t}{l_0}\right] - \frac{\lambda_n}{\lambda_n - 1} \tag{12}$$

where l_0 is the prompt neutron lifetime. It can be seen that larger is the eigenvalue separation, more rapidly the higher harmonics contribution to the flux will decrease and more quickly the fundamental mode flux get established. In other words, the local disturbance at one place will be realized more quickly in the entire core. Thus a large EVS can be considered to represent the degree of neutronic coupling of the core. This feature articulates the reason of higher susceptibility of large sized cores against spatial instability.

3. Methodology of EVS evaluation

Evaluation of EVS for real core geometries is relatively a difficult job. In the present analysis, 'Method of Mode Subtraction' (described below), has been used. Solution of higher order eigenvalue equation of neutron diffusion equation is required for EVS evaluation. The n^{th} order of a lambda mode eigenvalue equation, in operator form can be written as follows:

$$A\phi_n = \frac{1}{\lambda_n} B\phi_n \quad (n = 0, 1, 2, 3 \ldots) \tag{13}$$

ϕ_n is n^{th} order eigen vector of higher flux mode, and λ_n is the corresponding eigenvalue. One of the widely known method to evaluate higher flux modes is the method of mode subtraction which is also known as deflation method. In this method, higher mode flux is obtained through source iterations by removing already calculated lower order mode components from an initially assumed neutron flux distribution using simple power iteration method, That is:

$$A\phi_n^{(m)} = \frac{1}{\lambda_n^{(m-1)}} B\phi_n^{(m-1)} - \sum_{n=0}^{n-1} \frac{1}{\lambda_n^{(m-1)}} B\phi_n^{(m-1)} \times \frac{\langle\langle \phi_n^+ B\phi_n^{(m-1)} \rangle\rangle}{\langle\langle \phi_n^+ B\phi_n \rangle\rangle} \quad (n \geq 1) \tag{14}$$

where$\phi_n^{(m)}$: n^{th} order of higher mode flux in m^{th} outer iteration, $\lambda_n^{(m-1)}$: n^{th} order of eigenvalue in m^{th} outer iteration, ϕ_n^+: n^{th} order adjoint function.

Differential terms in the eigenvalue Eq. (14) can be treated by either simple finite difference method (FDM) or by more refined nodal method. In the present work, FDM has been used. The higher mode flux is calculated with a finer mesh width than the fundamental mode, as distribution of higher mode flux spatially oscillates more widely between positive and negative values than that of a fundamental mode. Though, this method is relatively tedious in terms of computational effort, but with present day computers, it is possible to evaluate the first few harmonics in few minutes.

4. EVS calculations and results

A typical cylindrical PWR core of 1000 MWe, as shown in Fig. 1 is considered for calculations and analysis. The reactor core is represented by homogenized one neutron energy group cross-section. A 3D diffusion theory code, AARTEEZ (Jain et al., 1990) which solves Eq. (14) in r-theta-z geometry, has been used to evaluate higher harmonics. AARTEEZ is based on FDM and assumes neutron flux to be uniform in every finite mesh. The calculations are performed in double precision to avoid truncation errors while subtracting previous modes. A reasonably fine grid of 50, 24 and 30 meshes in r, theta and z direction respectively has been used and maintained in the entire analysis. Acceleration for convergence in iterations is achieved by the successive over relaxation method. Schematic of fundamental mode flux shape and first harmonic shapes in azimuthal and axial modes are shown in Fig. 2 for the understanding purpose but the actual results are presented in Table 1 and will be discussed latter.

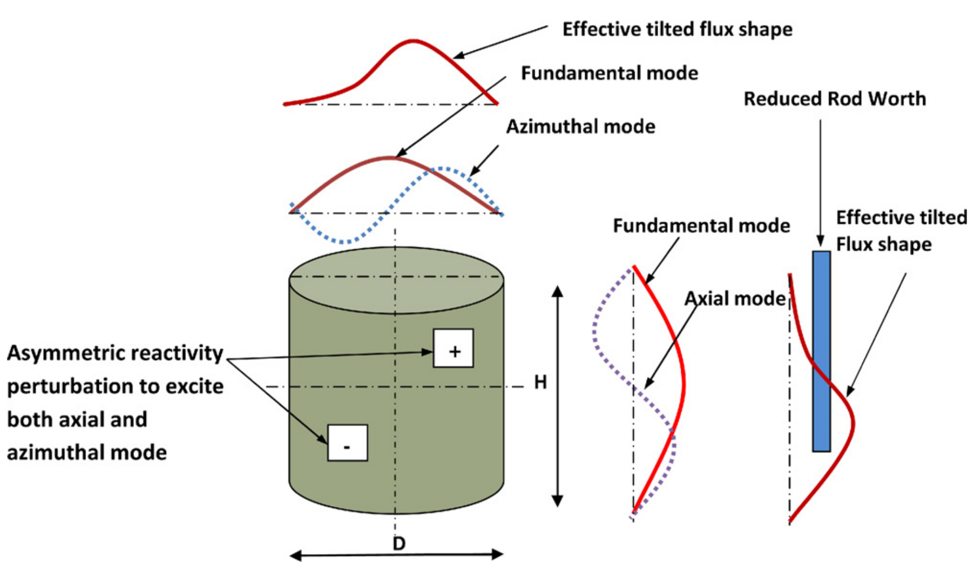
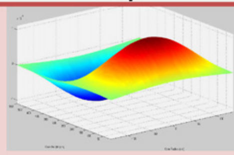
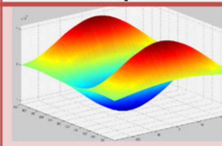
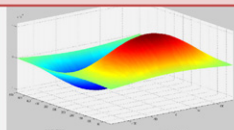
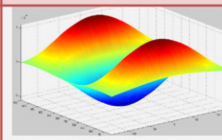
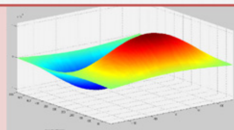
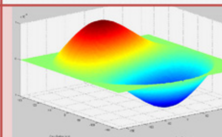
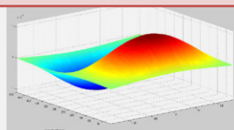
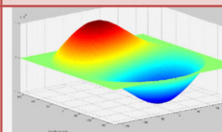


Fig. 2. Typical core shape.

Table 1
Nature, shape and EVS of first and second harmonic.

H/D Ratio	First Harmonic Shape	Plane[a] of View	EVS (ε_1) (pcm)	Remark	Second Harmonic Shape	Plane[a] of View	EVS (ε_2) (pcm)	Remark
2.0		A	422	First Axial		A	1122	Second Axial
1.4		A	680	First Axial		A	1600	Second Axial
1.3		A	750	First Axial		B	1526	First Azimuthal
1.2		A	834	First Axial		B	1450	First Azimuthal

[a]Refer to Fig. 1 for planes A and B.

4.1. Size effect

To understand the effect of core size, a case has been studied by keeping the shape of the reactor core constant, i.e., by keeping height to diameter ratio constant. Core power density of the reactor is also maintained constant (100 kW/lit) and core size is expressed in reactor power (MWe). Higher harmonics are calculated for each size. The results, in terms of eigenvalue separation as a function of core size are presented in Fig. 3. It can be seen that for reactor size increasing from 200 MWe to 1000 MWe, the EVS decreases from 2600 pcm to 200 pcm. This is because for very large core of about 1000 MWe, the fundamental mode and first harmonic are very close to each other and therefore contribution of first harmonic to the total neutron flux will be significant in such cores and such cores will be more susceptible to axial mode of oscillations. Such systems will need special control system to maintain the desired power distribution. For small cores, axial mode EVS is large enough and therefore excited axial mode will die out quickly. Such systems will not require any spatial control system to maintain the desired power distribution.

4.2. Shape effect

To understand the effect of shape, a study is carried out by conserving the volume of core (1000 MWe PWR core) and changing H/D ratios. Axial and azimuthal eigenvalue separation have been calculated. The results, in terms of EVS as a function of H/D ratio are plotted in Fig. 4. It can be seen that axial harmonic EVS decreases and azimuthal EVS increases with increase in H/D ratio. Fig. 4 also shows that at some value of H/D ratio ($H/D \approx 0.92$), EVS of axial and azimuthal modes is same. In this situation, if reactor core is subjected to an asymmetric perturbation of nature as shown in Fig. 2, any one or both of these modes will grow and it will be very difficult to set a simple spatial control procedure for operator to correct power distribution to desired level. Therefore a large H/D

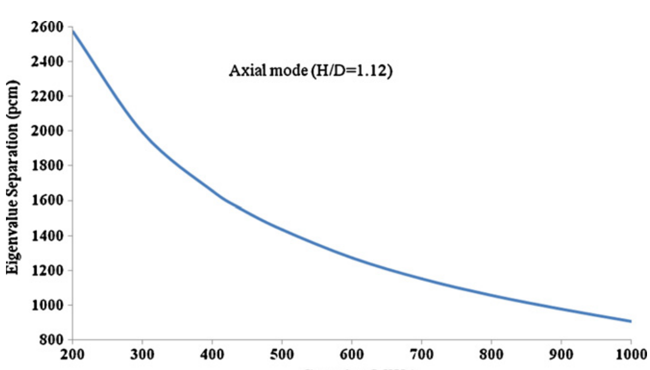


Fig. 3. Core size effect.

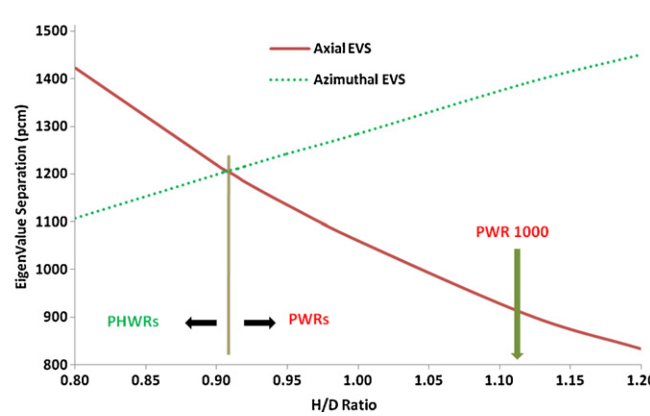


Fig. 4. Core shape effect.

ratio ($H/D > 0.92$) would be preferred if it is easy to correct axial modes of oscillations. In case of large PWRs, group of control rods or partial rods can be efficiently used to suppress axially excited power density oscillations (Christie and Poncelet, 1973). Thus a H/D ratio slightly larger than 1.0 is a common practice in large PWR core design.

The results of calculations for different H/D ratio, in terms of EVS and shapes of different harmonics are given in Table 1. It can be noticed that for H/D ratio of up to 1.3, the resultant perturbed neutron flux takes the form:

$$\phi'(r,t) = \text{steady state flux} + \text{first axial harmonics}\,(\varepsilon_1) + \text{first azimuthal harmonics}\,(\varepsilon_2)$$

where axial harmonic, being the first close (in terms of EVS) harmonic makes axial mode susceptible to oscillate, keeping azimuthal mode self correcting due large EVS ($\varepsilon_2 \gg \varepsilon_1$). For large H/D ratio, say 1.5 or more, perturbed neutron flux takes the form:

$$\phi'(r,t) = \text{steady state flux} + \text{first axial harmonics}\,(\varepsilon_1) + \text{second axial harmonics}\,(\varepsilon_2)$$

Thus, in such situations, first two (first and second) consecutive higher harmonics become the axial harmonic with very small EVS. Under such situation, more than one closer axial harmonic will be available and this situation makes axial mode highly susceptible to even minor perturbations. This feature articulates that beyond some value of H/D ratio, core will have two close axial harmonics, making axial mode highly unstable.

The parametric study (Table 1) carried out for larger values of H/D ratios (keeping core volume and power to be constant) indicates that an upper limit of 1.3 for H/D ratio can be devised for large PWRs to optimize core shape, size against neutronic decoupling. Present 900–1000 MWe large PWRs, are typically designed to have H/D ratio of 1.1. One of the main objectives of new generation reactor designs is to scale up power output from 1000 MWe to 1500 MWe. From the present analysis, it can be recommended that new large sized reactors can be designed with a larger height (maintaining $1.1 < H/D < 1.3$ limit). Such a shape will not pose any problem in azimuthal mode stability of core and axial mode, if excited, can be corrected with available existing spatial control measures. This H/D limit has been determined specific to PWRs (or VVERs). For other systems (CANDUs/PHWRs, FBRs, AGRs) this value will be different due to differences in the core physics design.

5. Neutronic decoupling and reactor safety

The degree of neutronic decoupling has significant impact on reactor safety. To demonstrate this, a case is considered where neutronic loose coupling deepens flux tilt under certain transients. PWR thermal design limit (Minimum Departure from Nucleate Boiling Ratio) is compared for controlled and uncontrolled situations against spatial instability which essentiality arise due to neutronic decoupling. DNBR is the ratio of critical heat flux (CHF) to the operating heat flux. In the present analysis, CHF is determined from W3-Tong's correlation, which is most widely used for PWRs (Todreas and Kazimi, 1990). For a pressurized heated channel, critical heat flux, q''_{cr} is given by:

$$\begin{aligned} q''_{cr} = {} & [(2.022 - 0.06238p) + (0.1722 - 0.001427p) \\ & \times \exp(18.177 - 0.5987p)x_e][(0.1484 - 1.596x_e \\ & + 0.1729x_e|x_e|)2.326G + 3271][1.157 - 0.869x_e] \\ & \times [0.2664 + 0.837\exp(-124.1D_h)] \\ & \times [0.8258 + 0.0003413(h_f - h_{in})] \end{aligned} \tag{15}$$

where q''_{cr} is in kW/m^2, p is coolant pressure in MPa, h is coolant enthalpy in kJ/kg, D_h is hydraulic diameter in meters, x_e is coolant quality.

For non-uniform axial heating of the channel (considering axial profile) a correction, F is applied.

$$\text{CHF} = \frac{q''_{cr}}{F}$$

where F at any channel location, l is given:

$$F = \frac{C\int_0^l q''(z)\exp[-C(l-z')]dz'}{q''(l)[1-\exp(-Cl)]}$$

and

$$C = \frac{4.23 \times 10^6[1 - x_e(l)]^{7.9}}{G^{1.72}}$$

DNBR, at any point along the channel can be evaluated by:

$$\text{DNBR}(z) = \frac{\text{CHF}(z)}{q''(z)}$$

where $q''(z)$ is the operating heat flux at point z along the channel, which changes in course of time due to xenon induced spatial instability in the core. A case of axially unstable large PWR, operating at full power is studied in terms of changes in flux tilt. On calculation

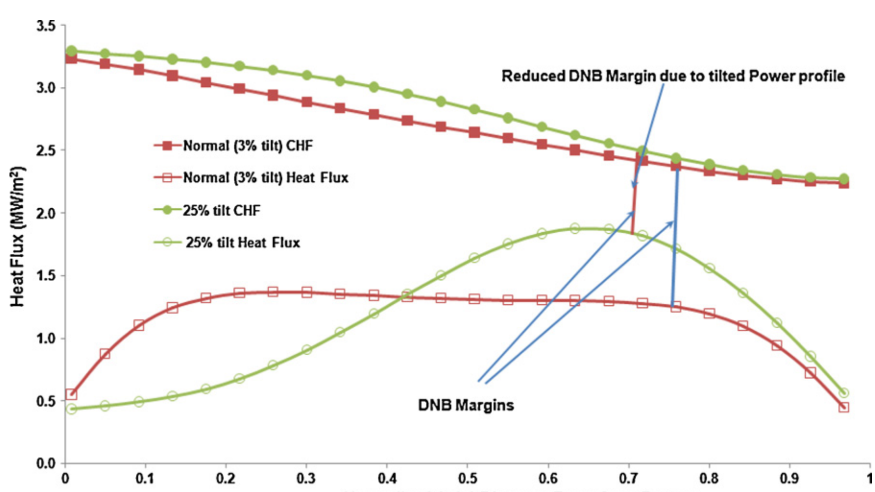


Fig. 5. Loss of DNB margin due to tilted power profile in large core.

of flux tilt it is observed that under normal operation, power distribution is flattened with an initial steady state axial tilt of 3%. A control rod induced perturbation may grow the axial tilt to 20–25% because of uncontrolled xenon oscillation transients (Onega and Kisner, 1977; Christie and Poncelet, 1973) in the reactor without effecting total reactor power. These two axial heat flux profiles (3% tilt and 25% tilt) are shown in Fig. 5. Corresponding critical heat fluxes are also shown. Fig. 5 clearly indicates that 25% tilted heat flux profile shortens the safety margin to critical heat flux (DNBR) from 1.94 to 1.35, though total reactor power remains unchanged.

It can also be observed that due to contribution of higher harmonics, there could be unwanted peaks or kinks in the total flux. If such kinks are at the control/absorber rod location, the worth of these rods would be affected (see Fig. 2). This is also a serious safety concern of loosely coupled cores.

6. Conclusions

- The EVS is a useful index to judge the degree of neutronic coupling of a reactor core.
- The eigenvalue separation is inversely proportional to the size of the reactor. The flux tilt due to asymmetric reactivity perturbation is inversely proportional to the eigenvalue separation. Therefore, large sized core will exhibit more tilt for the same magnitude of perturbations.
- Large reactor cores have small eigenvalue separation and are more susceptible to spatial instability due to dominance of higher harmonics. It is impossible to design a large core totally tightly coupled in all possible modes but design optimization could be used to minimize the susceptibility of core in more than one mode of oscillations.
- A critical height to diameter (H/D) ratio exists, at which both azimuthal and axial modes coexist with the same EVS. Such a situation should be avoided by design measures and core should be designed with a H/D that necessitates control of only one type of mode (either axial or azimuthal) to enable ease in designing spatial control algorithms.
- Very large H/D ratio reduces the probability of contribution of azimuthal modes but could lead to situations that make the core very susceptible to the axial mode. This is not desirable from the stability point of view. Therefore, while designing large cores, H/D ratio must be optimized to gain maximum core neutronic coupling and ease in design of spatial control algorithm. An upper limit of about 1.3 for H/D ratio is recommended for PWR core based on the analysis conducted.
- A spatially uncontrolled situation in loosely coupled reactor system could lead to safety consequences like reduction in DNB safety margin due to increase in flux tilt, reduced rod worth, etc. The calculations indicate that for flux tilt increase from 3% to 25%, the DNBR decreases from 1.94 to 1.35. Therefore neutronic coupling of core should be thoroughly understood and unstable modes of oscillations should be corrected by sufficient design and control measures.

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