



The Application of Reactor Kinetics to the Analysis of Experiments

A. F. Henry

To cite this article: A. F. Henry (1958) The Application of Reactor Kinetics to the Analysis of Experiments, Nuclear Science and Engineering, 3:1, 52-70, DOI: [10.13182/NSE58-1](https://doi.org/10.13182/NSE58-1)

To link to this article: <https://doi.org/10.13182/NSE58-1>



Published online: 12 May 2017.



Submit your article to this journal [↗](#)



Article views: 123



View related articles [↗](#)



Citing articles: 1 View citing articles [↗](#)

The Application of Reactor Kinetics to the Analysis of Experiments¹

A. F. HENRY

Bettis Atomic Power Division, Westinghouse Electric Corporation, Pittsburgh, Pennsylvania

Received April 10, 1957

The conventional kinetics equations for reactors in which fuel is stationary are derived without approximation from the time-dependent transport equation. The utility of the precise form which results is discussed, first generally, then with reference to the detailed analysis of two typical experiments. It is shown that the analysis required for a precise interpretation of kinetics experiments can generally be made using the same techniques employed in the computation of criticality for stationary cases.

INTRODUCTION

In recent years there has been an increasing tendency in reactor experimentation to employ techniques based on the kinetic behavior of critical assemblages. Interpretation of experiments has ranged from being quite direct—for example one measures a temperature coefficient of reactivity and compares with theory—to being highly inferential—a negative temperature coefficient is interpreted in terms of the extra fuel needed to operate a core in a hot condition. Under these circumstances precision with regard to the significance of the reactor kinetics equations becomes a matter of some importance. Accordingly it is the purpose of this paper to review the derivation and interpretation of the reactor kinetics equations and to point out by discussing several examples certain pitfalls which must be avoided if they are to be applied properly towards the development of accurate information. The word “review” is used advisedly. Much of what follows is well known to those who have worked in the field for some time. On the other hand, there are occasional striking instances of misuse of the kinetics equations. It is hoped that the following critical examination may help to avoid errors in interpretation which can otherwise make the best experimental data quite useless.

The body of the report is divided into two main sections. In the first, the conventional kinetics equations for reactors in which fuel is stationary are de-

¹ Work performed under the auspices of the U. S. Atomic Energy Commission.

rived without approximation from the time dependent transport equation. The derivation being exact, precise definitions of reactivity, prompt neutron lifetime, and effective delayed neutron fractions are thereby obtained. The utility of this method of treating time dependency—particularly for cases in which the time and space-energy part of the neutron flux are not separable—is discussed in the latter part of the section.

The second main section discusses application of the kinetics equations. Cases involving small reactivity changes present no formal problem. However, those in which the change is large require careful examination. To illustrate this fact, two specific experiments, the measurement of reactor shutdown by a rod drop technique and the determination of available excess reactivity from the change in reactivity with core height, are analyzed in detail.

THE KINETICS EQUATIONS

DERIVATION

For reactor systems in which the fuel is stationary, time behavior is described by the following equations

$$\begin{aligned}
 -\mathbf{\Omega} \cdot \nabla \mathfrak{F}(\mathbf{r}, \mathbf{\Omega}, u, t) - \Sigma(\mathbf{r}, u, t) \mathfrak{F}(\mathbf{r}, \mathbf{\Omega}, u, t) + \int_{\Omega'} \int_0^\infty d\Omega' du' \\
 \cdot \left[\Sigma_s(\mathbf{r}, u', u, \mathbf{\Omega}' \cdot \mathbf{\Omega}, t) + \frac{\nu}{\nu_0} \sum_j \{f^j(u) \nu^j(u') (1 - \beta^j)\} \Sigma_f^j(\mathbf{r}, u', t) \right] \mathfrak{F}(\mathbf{r}, \mathbf{\Omega}', u', t) \quad (1) \\
 + S(\mathbf{r}, \mathbf{\Omega}, u, t) + \sum_i \lambda_i C_i(\mathbf{r}, t) f_i(u) = \frac{\partial}{\partial t} \left[\frac{\mathfrak{F}(\mathbf{r}, \mathbf{\Omega}, u, t)}{v(u)} \right]
 \end{aligned}$$

$$\frac{\partial C_i(\mathbf{r}, t)}{\partial t} = \int_{\Omega'} \int_0^\infty d\Omega' du' \frac{\nu}{\nu_0} \sum_j \nu^j(u') \beta_i^j \Sigma_f^j(\mathbf{r}, u', t) \mathfrak{F}(\mathbf{r}, \mathbf{\Omega}', u', t) - \lambda_i C_i(\mathbf{r}, t). \quad (2)$$

Here $\mathfrak{F}(\mathbf{r}, \mathbf{\Omega}, u, t)$ is the directional flux density at point \mathbf{r} , and time t , for neutrons of lethargy u traveling in direction $\mathbf{\Omega}$.

$\Sigma(\mathbf{r}, u, t)$ is the total macroscopic interaction cross section.

$\Sigma_s(\mathbf{r}, u', u, \mathbf{\Omega}' \cdot \mathbf{\Omega}, t)$ is the total macroscopic cross section for scattering from lethargy u' and direction $\mathbf{\Omega}'$ to u and $\mathbf{\Omega}$.

$$\frac{\nu}{\nu_0} \sum_j [f^j(u) \nu^j(u') (1 - \beta^j)] \Sigma_f^j(\mathbf{r}, u', t)$$

is the fission source term for prompt neutrons, ν/ν_0 being an eigenvalue introduced for later mathematical convenience ($\nu/\nu_0 = 1$ for physically realizable systems). Superscript j distinguishes the various fissionable isotopes. Thus $\nu^j(u')$ is the total number of neutrons emitted from a fission of the j th element; these have a spectrum $f^j(u)$ such that $\int_u f^j(u) = 1$; $\Sigma_f^j(\mathbf{r}, u', t)$ is the macroscopic fission cross section.

$S(\mathbf{r}, \mathbf{\Omega}, u, t)$ is the external source density.

$C_i(\mathbf{r}, t)$ and λ_i represent the concentration and decay constant of the i th group of delayed neutron emitters.

$v(u)$ is the neutron speed corresponding to lethargy u .

$$\frac{\nu}{\nu_0} \sum_j \nu^j(u') \beta_i^j \Sigma_f^j(\mathbf{r}, u', t)$$

represents the source term for the i th group of delayed neutron emitters. Subscript i refers to the various groups of delayed neutron emitters, the fraction of neutrons emitted from the i th group due to a fission in element j being β_i^j ($\beta^j = \sum_i \beta_i^j$); $f_i(u)$ represents the spectrum (normalized to unity) of delayed neutrons.

Equation (1) is the mathematical statement of the fact that, for an infinitesimal volume containing neutrons of lethargy u moving in direction $\mathbf{\Omega}$, the rate at which neutrons are removed from the volume (by leakage, scattering, or absorption) subtracted from the rate at which they are introduced (by scattering, fissioning, external source, or decay of delayed neutron emitters) equals the rate of increase of the number of this class of neutrons in the infinitesimal volume.

The time dependency allowed in the definition of the various cross sections is present primarily to account for changes in molecular motion (temperature) and moderator density during a transient. The cross sections as written are already averages over molecular motion and hence in the thermal region depend on material temperatures.

The eigenvalue ν/ν_0 provides a means of adjusting Eq. (1) so that time behavior predicted from it will not be merely the result of using incorrect cross sections or too gross an approximation. Thus if (1) with $\nu/\nu_0 = 1$ fails to predict a stationary state for a reactor known to be critical, ν/ν_0 can be adjusted until the solution is stationary; then the adjusted value of ν/ν_0 can be used in (1) to predict time behavior if the system is subsequently perturbed.

Equations in the time alone can be obtained from Eqs. (1) and (2) by the simple expedient of integrating over all space, direction, and lethargy; however, this procedure is impractical unless the time dependence of $\mathcal{F}(\mathbf{r}, \mathbf{\Omega}, u, t)$ is separable and even then yields results dependent on the difference of very large numbers. Thus, a certain amount of mathematical manipulation is required to convert Eqs. (1) and (2) to a more conventional and tractable form.

It is, first of all, of great practical help to introduce a source free adjoint equation (1-4) appropriate to some critical condition in the reactor (generally the initial or final state). This equation is

$$\begin{aligned} \mathbf{\Omega} \cdot \nabla \psi_0^*(\mathbf{r}, \mathbf{\Omega}, u) - \Sigma_0(\mathbf{r}, u) \psi_0^*(\mathbf{r}, \mathbf{\Omega}, u) + \int_{\mathbf{\Omega}'} \int_0^\infty d\mathbf{\Omega}' du' \\ \cdot \left[\Sigma_{s_0}(\mathbf{r}, u, u', \mathbf{\Omega}' \cdot \mathbf{\Omega}) + \frac{\nu}{\nu_0} \sum_j f_i^j(u') \nu^j(u) \Sigma_{f_0}^j(\mathbf{r}, u) \right] \psi_0^*(\mathbf{r}, \mathbf{\Omega}', u') = 0 \end{aligned} \quad (3)$$

where subscripts zero indicate the quantities correspond to a steady-state condition and

$$f_i^j(u') \equiv \sum_i [f^j(u')(1 - \beta^j) + f_i(u')\beta_i^j].$$

The adjoint flux $\psi_0^*(\mathbf{r}, \mathbf{\Omega}, u)$ has the physical significance (4) of being proportional to the over-all power level which will result asymptotically if neutrons of lethargy u having direction $\mathbf{\Omega}$ are introduced at point \mathbf{r} in a critical reactor initially at zero power.

Next, it is convenient to partition the function $\mathfrak{F}(\mathbf{r}, \mathbf{\Omega}, u, t)$ into a shape function $\psi(\mathbf{r}, \mathbf{\Omega}, u, t)$ and a time function $T(t)$, such that

$$\mathfrak{F}(\mathbf{r}, \mathbf{\Omega}, u, t) \equiv \psi(\mathbf{r}, \mathbf{\Omega}, u, t)T(t) \quad (4)$$

with the additional requirement that

$$\frac{\partial}{\partial t} \int_u \int_V \frac{\varphi_0^*(\mathbf{r}, u) \varphi(\mathbf{r}, u, t)}{v(u)} du dV = 0 \quad (5)$$

where

$$\varphi_0^*(\mathbf{r}, u) \equiv \int_{\mathbf{\Omega}} \psi_0^*(\mathbf{r}, \mathbf{\Omega}, u)$$

$$\varphi(\mathbf{r}, u, t) \equiv \int_{\mathbf{\Omega}} \psi(\mathbf{r}, \mathbf{\Omega}, u, t)$$

and the integration is over all space and lethargy.

Mathematically, these definitions are made to permit replacement of

$$\frac{\partial}{\partial t} \int_u \int_V \frac{\varphi_0^*(\mathbf{r}, u) \varphi(\mathbf{r}, u, t) T(t)}{v(u)} du dV$$

by

$$\frac{dT(t)}{dt} \left[\int_u \int_V \frac{\varphi_0^*(\mathbf{r}, u) \varphi(\mathbf{r}, u, t)}{v(u)} du dV \right].$$

Physically, since $\varphi(\mathbf{r}, u, t)/v(u)$ is just the number density of neutrons at point \mathbf{r} having lethargy u , the integral in (5) is proportional to the power level which will result asymptotically from the presence of these neutrons in the core for which

φ_0^* is the adjoint. It follows that for a small perturbation any asymptotic change in power level is expressed by variation of the time function $T(t)$.

It is important to note that Eq. (4) is purely a definition and embodies no physical assumption. Equation (5) is a restrictive condition placed on the time-dependent shape function ψ and again requires no physical assumption. If the time dependence of \mathfrak{F} is separable (for example, if the flux is rising on an asymptotic period), ψ will be independent of time and Eq. (5) will be obeyed automatically.

With the symbols $\langle 1 \rangle$, $\langle 2 \rangle$, and $\langle 3 \rangle$ representing the corresponding equations, the operations

$$\int_V \int_u \left[\varphi_0^*(\mathbf{r}, u) \int_{\Omega} \langle 1 \rangle d\Omega - T(t) \varphi(\mathbf{r}, u, t) \int_{\Omega} \langle 3 \rangle d\Omega \right] dV du$$

and

$$\int_V \int_u \varphi_0^*(\mathbf{r}, u) f_i(u) \langle 2 \rangle dV du$$

(integration being over all volume and lethargy), yield

$$\left. \begin{aligned} \frac{dT}{dt} &= \frac{\rho - \bar{\beta}}{\Lambda} T + \sum_i \lambda_i c_i + Q \\ \frac{dc_i}{dt} &= \frac{\bar{\beta}_i}{\Lambda} T - \lambda_i c_i \end{aligned} \right\} \quad (6)$$

where

$$\begin{aligned} \rho = \rho(t) &\equiv \frac{1}{F} \int_V \int_u \left[-\varphi_0^*(\mathbf{r}, u) \int_{\Omega} \mathbf{\Omega} \cdot \nabla \psi(\mathbf{r}, \mathbf{\Omega}, u, t) d\Omega \right. \\ &\quad - \varphi(\mathbf{r}, u, t) \int_{\Omega} \mathbf{\Omega} \cdot \nabla \psi_0^*(\mathbf{r}, \mathbf{\Omega}, u) d\Omega - \delta \Sigma(\mathbf{r}, u, t) \varphi_0^*(\mathbf{r}, u) \varphi(\mathbf{r}, u, t) \\ &\quad + \int_{u'} \delta \left\{ \Sigma_s^0(\mathbf{r}, u', u, t) + \frac{\nu}{\nu_0} \sum_j f_i^j(u) \nu^j(u') \Sigma_f^j(\mathbf{r}, u', t) \right\} \varphi_0^*(\mathbf{r}, u) \\ &\quad \left. \times \varphi(\mathbf{r}, u', t) du' \right] dV du \end{aligned} \quad (7)$$

$\delta \Sigma(\mathbf{r}, u, t)$, etc., representing the difference between perturbed quantities [as in (1)] and unperturbed [as in (3)],

$$F = F(t) \equiv \int_V \int_u \int_{u'} \sum_j f_i^j(u) \nu^j(u') \Sigma_f^j(\mathbf{r}, u', t) \varphi_0^*(\mathbf{r}, u) \varphi(\mathbf{r}, u', t) dV du du'. \quad (8)$$

$$\Lambda = \Lambda(t) \equiv \frac{1}{F} \int_V \int_u \frac{\varphi_0^*(\mathbf{r}, u) \varphi(\mathbf{r}, u, t)}{v(u)} dV du. \quad (9)$$

$$\bar{\beta}_i = \bar{\beta}_i(t) \equiv \frac{1}{F} \int_V \int_V \int_u \int_{u'} \frac{\nu}{\nu_0} \sum_j \nu^j(u') f_i(u) \beta_i^j \Sigma_f^j(\mathbf{r}, u', t) \varphi_0^*(\mathbf{r}, u) \varphi(\mathbf{r}, u', t) dV du du'. \quad (10)$$

$$c_i = c_i(t) \equiv \frac{1}{\Lambda F} \int_V \int_u C_i(\mathbf{r}, t) f_i(u) \varphi_0^*(\mathbf{r}, u) dV du. \quad (11)$$

$$Q = Q(t) \equiv \frac{1}{\Lambda F} \int_V \int_u \int_{\Omega} \varphi_0^*(\mathbf{r}, u) s(\mathbf{r}, \Omega, u, t) dV du d\Omega. \quad (12)$$

The definition (8) of F is arbitrary. That is, although the magnitudes of ρ , $\bar{\beta}$, and Λ are altered by changes in F , the kinetics equations (6) are entirely independent of it. The particular definition used here is chosen so that the "perturbation" associated with fictitious alteration of the eigenvalue ν is $\delta\nu/\nu_0$ (see Eq. 17). Had there been a desire to make this change equal $\delta\nu/\nu$, F would have been defined with ν_0 replaced by ν . In elementary terms, this question reduces to the perennial one of whether reactivity is more appropriately defined as $(k_{\text{eff}} - 1)/k_{\text{eff}}$ or as $(k_{\text{eff}} - 1)$. Evidently it makes no difference, provided $\bar{\beta}$ and Λ are defined consistently with ρ .

More fundamentally, the fact that the present formalism allows an arbitrary choice of F implies that attempts to define ρ , Λ , or $\bar{\beta}$ as physical quantities have been abandoned. They have an approximate significance, but it is difficult to make this precise except when the reactor system is rising or falling on an asymptotic period.

Elementary derivations of the kinetics equations generally fail to distinguish between $\bar{\beta}_i$, the effective delayed neutron fraction, and β_i^j the actual fraction from that isotope (j) in which most of the fissions occur. Since the energy spectrum (and hence, leakage probability) of delayed neutrons is much lower than that of prompt neutrons, and since the $\nu^j \beta_i^j$ differ considerably for different isotopes, the ratios $\bar{\beta}_i/\beta_i^j$ approach unity only in large cores containing a single isotope. Even for thermal reactors $\bar{\beta}_i/\beta_i^j$ may differ from unity by twenty or thirty per cent.

Reduction of the expressions (6) through (11) to corresponding diffusion theory, group theory, and bare core approximations is discussed in WAPD-142 (5). Methods of computing Λ and $\bar{\beta}_i$ based on the fact that these quantities are equivalent respectively to the reactivities associated with introducing an absorber $\delta\Sigma(\mathbf{r}, u, t) = -1/v(u)$ and a source change

$$\delta \left[\frac{\nu}{\nu_0} \sum_j f_i^j(u) \nu^j(u') \Sigma_f^j(\mathbf{r}, u', t) \right] = \frac{\nu}{\nu_0} \sum_j \nu^j(u') f_i(u) \beta_i^j \Sigma_f^j(\mathbf{r}, u', t)$$

are also outlined. It is rarely necessary to consider the slight variation in $\bar{\beta}$ and Λ arising from the time behavior of $\psi(\mathbf{r}, \mathbf{\Omega}, u, t)$ or the fission cross sections.

THE UTILITY OF THE KINETICS EQUATIONS

Having been obtained without approximation from transport theory, Eqs. (6) are quite accurate for reactors in which fuel is stationary. Their utility depends on the possibility of computing ρ , Λ , $\bar{\beta}_i$, and Q . Fortunately, this problem, while very difficult in general, can for a great many practical cases be treated to a high order of accuracy by relatively simple approximations.

The principal difficulty in the way of general treatment is that of computing the time-dependent shape function $\psi(\mathbf{r}, \mathbf{\Omega}, u, t)$. In fact, unless ψ or some accurate approximation to it can be found readily, the recasting of (1) and (2) into (6) becomes a mere formality.

1. The Time Separable Case

The most fruitful approach to the investigation of ψ seems to be first to examine cases where its determination involves no formal difficulty. This is the situation whenever ψ is independent of time, that is, whenever the time behavior of the directional flux, $\mathfrak{F}(\mathbf{r}, \mathbf{\Omega}, u, t)$, is a separable function.

The condition of time separability exists for constant values of reactivity when the source term is negligible in comparison with the fission rate. Strictly speaking, this is the only time when it exists. If these conditions are fulfilled, the neutron and delayed emitter populations will eventually behave as $e^{\omega t}$ where ω is given by the largest root of the inhour equation, [obtained from (6) by taking $C_i \sim T \sim e^{\omega t}$].

$$\rho = \omega\Lambda + \sum_i \frac{\bar{\beta}_i \omega}{\omega + \lambda_i}. \quad (13)$$

Under such circumstances the stationary shape function, $\psi(\mathbf{r}, \mathbf{\Omega}, u)$ is [from Eqs. (1) and (2)] one of the solutions of

$$\begin{aligned} -\mathbf{\Omega} \cdot \nabla \psi_\omega(\mathbf{r}, \mathbf{\Omega}, u) - \left[\Sigma(\mathbf{r}, u) + \frac{\omega}{v} \right] \psi_\omega(\mathbf{r}, \mathbf{\Omega}, u) \\ + \int_{\Omega'} \int_0^\infty d\Omega' du' \Sigma_s(\mathbf{r}, u', u, \mathbf{\Omega}' \cdot \mathbf{\Omega}) \psi_\omega(\mathbf{r}, \mathbf{\Omega}', u') \\ + \sum_{j,i} \int_{\Omega'} \int_0^\infty d\Omega' du' \left[\nu^j(u') (1 - \beta^j) f^j(u) \right. \\ \left. + \frac{\lambda_i}{\omega + \lambda_i} \nu^j(u') \beta_i^j f_i^j(u) \right] \Sigma_f^j(\mathbf{r}, u') \psi_\omega(\mathbf{r}, \mathbf{\Omega}', u') = 0. \end{aligned} \quad (14)$$

The values of ω which yield stationary solutions of Eq. (14) being the same as the roots of Eq. (13), and the particular solution desired being that corre-

sponding to the largest ω . Except for a prompt critical situation, the largest value of ω is such that $|\omega/v| \ll \Sigma$. Thus, except during a runaway, the stationary solution is one corresponding to a delayed neutron yield differing from the equilibrium value. The term $\lambda_i/(\omega + \lambda_i)$ shows the yield at any instant to be decreased for a supercritical situation ($\omega > 0$) and increased (generally substantially) for a subcritical situation. For very large ω (prompt critical cases), the stationary solution is one corresponding to an increase in the absorption cross section by an amount ω/v , and the complete absence of any delayed neutrons ($\lambda_i/(\omega + \lambda_i) \rightarrow 0$).

If Eq. (14) is actually solved,² there is no need to determine ρ since ω will already have been found. However, because of the inconvenience of having ω as an eigenvalue, two approximate methods of computing ρ (and thence ω) are commonly employed. Careful examination of these will suggest means of obtaining the shape function for nonseparable situations.

The first method is to use in the expression (7) for ρ the flux, $\psi_0(\mathbf{r}, \boldsymbol{\Omega}, u)$, which is the stationary solution before the perturbation. This procedure requires that the integrals in (7) be evaluated and becomes of questionable accuracy if—as can occasionally happen—a small change in reactivity leads to a major change in flux shape.

The second method consists of solving the stationary equation

$$\begin{aligned}
 -\boldsymbol{\Omega} \cdot \nabla \psi_\nu(\mathbf{r}, \boldsymbol{\Omega}, u) - \Sigma(\mathbf{r}, u) \psi_\nu(\mathbf{r}, \boldsymbol{\Omega}, u) + \int_{\boldsymbol{\Omega}'} \int_0^\infty d\boldsymbol{\Omega}' du' \left[\Sigma_s(\mathbf{r}, u', u, \boldsymbol{\Omega}' \cdot \boldsymbol{\Omega}) \right. \\
 \left. + \frac{\nu}{\nu_0} \sum_j f_i^j(u) \nu^j(u) \Sigma_f(\mathbf{r}, u') \right] \psi_\nu(\mathbf{r}, \boldsymbol{\Omega}', u') = 0
 \end{aligned}
 \tag{15}$$

where the constants in the equation are the perturbed ones and the eigenvalue ν is adjusted to obtain a stationary solution. For excursions in reactivity smaller in magnitude than prompt critical ψ_ω [from Eq. (14)] and ψ_ν [from Eq. (15)] will be virtually identical since the equations which lead to them then differ only in that a small fraction of the source (roughly equal to the magnitude of the reactivity change being computed) consists in one case of prompt neutrons and in the other case of delayed neutrons. Thus, use of ψ_ν in Eq. (7) for the reactivity should yield almost the same result as the use of ψ_ω .

Once this fact has been established, there is no need actually to compute ψ_ω and perform all numerical integrations in (7), since the desired reactivity change can be found indirectly from the eigenvalue ν required to make Eq. (15) stationary. The argument which justifies this procedure is based on the fact that (15) describes a physical system differing from the one under consideration only

² Statements about solving the transport equation should be interpreted as meaning solving some tractable but unspecified approximation to it. We talk in terms of transport theory for the sake of generality.

in that it is maintained critical by a fictitious value of ν . Since this system is stationary, the reactivity change between it and the original, critical, unperturbed system is zero. But this zero change is made up of the reactivity change ρ which is sought and that change (designated by ρ_ν) due to the fictitious alteration of ν . Thus $\rho = -\rho_\nu$. For a steady-state value of ν/ν_0 of unity we find

$$\begin{aligned} \delta \left[\frac{\nu}{\nu_0} \sum f_i^j(u) \nu^j(u') \Sigma_f^j(\mathbf{r}, u') \right] &\equiv \frac{\nu}{\nu_0} \left[\sum_j f_i^j(u) \nu^j(u') \Sigma_f^j(\mathbf{r}, u') \right] \\ &- \sum_j f_i^j(u) \nu^j(u') \Sigma_{f_0}^j(\mathbf{r}, u') = \frac{\nu - \nu_0}{\nu_0} \left[\sum_j f_i^j(u) f^j(u') \Sigma_f^j(\mathbf{r}, u') \right] \\ &+ \delta \left[\sum_j f_i^j(u) \nu^j(u') \Sigma_f^j(\mathbf{r}, u') \right]. \end{aligned}$$

Hence

$$\rho_\nu = \frac{\nu - \nu_0}{\nu_0} \frac{\int_V \int_u \int_{u'} \sum_j f_i^j(u) \nu^j(u') \Sigma_f^j(\mathbf{r}, u') \varphi_0^*(\mathbf{r}, u) \varphi_\nu(\mathbf{r}, u') dV du du'}{\int_V \int_u \int_{u'} \sum_j f_i^j(u) \nu^j(u') \Sigma_f^j(\mathbf{r}, u') \varphi_0^*(\mathbf{r}, u) \varphi_\nu(\mathbf{r}, u') dV du du'}. \quad (16)$$

Thus for small changes in reactivity, Eq. (7) may be replaced by

$$\rho = -\rho_\nu = \frac{\nu_0 - \nu}{\nu_0} \quad (17)$$

and the desired reactivity change may be computed directly from the eigenvalues ν associated with the perturbed and unperturbed states.

2. Core Properties Changing Continually

There appears, then, to be some utility to casting the time-dependent transport equation into the form (6) even when $\mathfrak{F}(\mathbf{r}, \mathbf{\Omega}, u, t)$ may be found directly by solution of (14). Moreover, the procedure which gives rise to this utility (i.e., replacing ψ_ω by ψ_ν) can now be applied as an approximation to the nonseparable case. Thus, for the source free situation in which the core properties are changing continuously, the time behavior of the flux is given approximately by $\psi_\nu(\mathbf{r}, \mathbf{\Omega}, u, t)T(t)$, where $T(t)$ is obtained from solution of (6) and $\psi_\nu(\mathbf{r}, \mathbf{\Omega}, u, t)$ from (15), the time dependence of ψ_ν being due to variation in core properties as reflected through changes in the cross sections in (15).

The validity of this approximation depends entirely on the similarity of the actual shape function $\psi(\mathbf{r}, \mathbf{\Omega}, u, t)$ at any instant to the shape $\psi_\nu(\mathbf{r}, \mathbf{\Omega}, u)$. Differences in the two shapes will arise primarily from behavior of the prompt neutrons, since these account for over 99 % of the source term in a critical core. Accordingly, only prompt neutrons need be considered and, this being the case, some notion

of the time required for the flux shape to readjust after an arbitrary change in core properties can be acquired from experience with machine solution of reactor equations.

Most machine methods operate by guessing at an initial source distribution computing the resultant fission rate distribution and using this as the next source guess. Thus, successive iterations correspond roughly to successive generations of neutrons, all of which may be considered prompt. Accordingly, the time to reach an asymptotic shape can be obtained from the number of iterations required for convergence of the machine problem. Unfortunately, no categorical statements can be made concerning this number. Perturbation of the critical core will give rise to transients which die away as the flux shape approaches ψ_* . Thus the convergence rate will depend on the difference between ψ_* and the original flux shape and on the leakage characteristics of the core, and certain configurations may yield flux shapes which change significantly but relatively slowly after a small perturbation. In any event, the number of iterations required by the machine, multiplied by the prompt neutron lifetime Λ , is a good estimate of time between the perturbation and the time at which ψ_* is a good representation of the shape function.

An accurate description of time behavior by the product $\psi_* T$ then seems possible, provided the change in core properties which gives rise to this behavior is small in times smaller than $N\Lambda$, where N is the number of iterations required for the perturbed flux to approach ψ_* . For example, with $N = 10$ and one-tenth prompt critical considered a small change in reactivity, the above method should be accurate for reactivity insertion rates up to 100%/sec in a core where $\Lambda = 7.5 \times 10^{-5}$ sec. Control rod oscillations under shock can lead to such rates, but occurrence under normal conditions is unusual.

3. Large Reactivity Changes

For reactivity surges of magnitude in excess of prompt critical, ψ_* may differ somewhat from ψ . However, precision is not particularly significant in this range, since so many uncalculable higher order effects enter the picture. Thus, power behavior during a runaway is adequately described by Eq. (6).

Problems in analyzing cases involving large negative reactivity changes are discussed as a special example in a later section.

4. Source Term Significant

It remains to consider the utility of the kinetics equations when the source term is significant. This situation is encountered during startup or when pulsed source techniques are used. The shape function $\psi(\mathbf{r}, \Omega, u, t)$ changes continually with time, and application of the kinetics equations is useful only if some simple means of computing ψ is available.

As a first approximation it is common to ignore the presence of the source and to use ψ_s from Eq. (15). For startup problems where the greatest need for validity is in the region close to criticality, this procedure is usually satisfactory, since ψ_s and the true shape function are similar near criticality. If greater accuracy is desired, a second approximation is to ignore the terms

$$\sum_i \lambda_i C_i f_i \quad \text{and} \quad \partial(\bar{\tau}/v)/\partial t$$

in (1) and get the resulting shape function at any instant from the inhomogeneous equation which results. This approximation amounts to neglecting $\partial(\bar{\tau}/v)/\partial t$ in comparison with $\Sigma \bar{f}$ and to assuming that *shape* (as opposed to criticality) is independent of the delayed neutron emitter distribution. Both conditions appear to be valid for startup conditions.

Computation of the time-dependent shape function appropriate to pulsed source experiments is generally difficult. Frequently, as in the work of Von Dardel (6) or Poole (7), there is a fortunate lack of interest in the details, of the reactor kinetics; all that need be established is that there exists a single decaying flux mode. However, in work such as that reported by Sjöstrand (8) the purpose of the experiment is actually to measure $\rho/\bar{\beta}$. Comparison with theory here requires that good approximations to the shape function be obtained; the sizable effect of the delayed neutrons when the pulse is turned off makes this process difficult and may require use of an approximation the inaccuracy of which is difficult to assess. Such experiments must be carefully designed if any really exact comparison with theory is to be made.

APPLICATION TO EXPERIMENTS

SMALL REACTIVITY CHANGES

It is convenient to classify experiments into two categories dependent on whether the reactivity change being determined is small or large.

The measurement of small reactivity changes provides extremely valuable information with regard to the validity of particular mathematical models and allows direct measurement of certain reactor or nuclear constants.

Particular mathematical models describe reactor behavior in terms of specific calculable or measurable parameters. If values of these constants are thought to be correct, experimental determination of the reactivity change associated with varying one or more of them serves to test the validity of the model. Conversely, if the model is believed correct, such experiments serve to determine one or more of the parameters.

For example, a critical equation for a large bare thermal reactor is

$$\rho = 0 = 1 - \frac{1 + M^2 B^2}{k} \quad (18)$$

where M^2 is the migration area, B^2 the geometrical buckling, and k the infinite medium multiplication constant. The buckling, being purely a geometrical quantity, is known and can be varied by changing core dimensions. Thus measurements of dimensions for the critical core and of the asymptotic period associated with a slight increase in these dimensions gives B^2 and $\partial\rho/\partial B^2$; these quantities together with (18) yield values of M^2 and k . It is important to note even in this very simple example that what is measured is the period ω ; the associated reactivity must be obtained from the inhour equation (13), and this requires that correct values of $\bar{\beta}_i$ be used (for small ω , the term $\Lambda\omega$ in (13) is negligible). The assumption that $\bar{\beta}_i = \beta_i^{25}$ will rarely be valid, either because of small core size or because of the presence of delayed neutrons from U^{238} . Failure to account for this fact will lead to incorrect values of k and M^2 or, if these are already known, an unjustified conclusion with regard to the validity of the model. Fortunately, any mathematical formulation which can be used to determine criticality can also be applied to the accurate computation of $\bar{\beta}_i$ and Λ (5). Thus there is in principle no difficulty in the interpretation of experiments involving small changes in reactivity.

LARGE REACTIVITY CHANGES

Experiments designed to measure large reactivity changes fall into two categories, depending on whether the large change is introduced as a step or whether it is inferred by adding together a series of small changes. Both categories require extreme care in interpretation. The difficulties and methods by which they may be overcome can best be illustrated by considering in more detail two specific examples.

1. *The Rod Drop Experiment*

The problem of measuring reactor shutdown by adjusting control rods so that the reactor is in an equilibrium critical condition and then suddenly fully inserting all rods illustrates the first kind of measurement of a large reactivity change. It is instructive to examine what is actually measured in this experiment, what interpretation in terms of reactivity can be made from the measurements, and what use can be made of the inferred reactivity values.

What is measured is the counting rate in one or a number of counters, first under the initial equilibrium critical conditions and then just after the rods have been driven into the core, the particular time of the second measurement being such that the prompt neutrons have come into equilibrium with the new core conditions and the delayed neutron emitters have not yet decayed significantly from their initial values. Certain experimental difficulties involving the speed of rod insertion may be encountered in performing the experiment. We shall assume these to have been overcome, so that the results correspond to an in-

stantaneous step insertion of the rods. If subscripts 0 and 1 are used to label quantities associated with the two measurements, the rates measured are in the ratio

$$\frac{T_0 \int_{\Omega} \int_u \int_V \psi_0(\mathbf{r}, \mathbf{\Omega}, u) G(\mathbf{r}, \mathbf{\Omega}, u) d\Omega du dV}{T_1 \int_{\Omega} \int_u \int_V \psi_1(\mathbf{r}, \mathbf{\Omega}, u, t) G(\mathbf{r}, \mathbf{\Omega}, u) d\Omega du dV}$$

where $G(\mathbf{r}, \mathbf{\Omega}, u)$ represents the cross section in the counter for the reaction which causes counts.

Equations (6) can now be used to interpret this piece of data. Since equilibrium exists at t_0 ,

$$\frac{\beta_{i_0} T_0}{\Lambda_0} = c_{i_0} \lambda_i.$$

At t_1 , the $c_i \lambda_i$ to a good approximation still have their initial values; thus,

$$\frac{dT_1}{dt} = \frac{\rho_1 - \bar{\beta}_1}{\Lambda_1} T_1 + \frac{\beta_0 T_0}{\Lambda_0}$$

so that

$$\frac{\rho_1}{\bar{\beta}_1} = 1 - \left(\frac{\bar{\beta}_0/\Lambda_0}{\bar{\beta}_1/\Lambda_1} \right) \frac{T_0}{T_1} + \frac{\Lambda_1}{\bar{\beta}_1} \frac{1}{T_1} \frac{dT_1}{dt}$$

the derivative, $(1/T_1)(dT_1/dt)$ is approximately $-0.5 \rho_1/(\rho_1 - \bar{\beta}_1)$. Thus it can be neglected if $|0.5\Lambda_1/(\rho_1 - \bar{\beta}_1)| \ll 1$. Otherwise, this term can be used as a correction factor.

Equations (5), (8), and (9) show $\Lambda_0 F_0 = \Lambda_1 F_1$; thus, the ratio $(\bar{\beta}_0/\Lambda_0)/(\bar{\beta}_1/\Lambda_1)$ differs from unity because of differences in the shape functions ψ_0 and ψ_1 which enter into computation of the numerators of $\bar{\beta}_0$ and $\bar{\beta}_1$. Equation (14) shows that these shape functions differ in two respects: first, the source term for $\psi_0(\omega = 0)$ includes prompt neutrons and delayed neutrons in a ratio roughly $1:\beta_0$, whereas that for ψ_1 corresponds approximately to a prompt to delayed ratio, $1:(\bar{\beta}_1 - \rho_1)$; secondly, ψ_0 corresponds to control rods in a critical configuration, whereas ψ_1 is for rods fully inserted. The second effect will generally cause the greater difference between ψ_0 and ψ_1 . Since this difference must also be known at the counter positions in order to infer T_0/T_1 from the measurement, ψ_0 and ψ_1 should be computed if an accurate value of $\rho_1/\bar{\beta}_1$ is to be inferred.

The interpretation of T_0/T_1 in terms of a reactivity is thus involved. Fortunately, most reactor codes can be altered to allow introduction of an arbitrary source term and interruption after each iteration. Under these circumstances ψ_1 (and hence $\bar{\beta}_1$) can be computed and the correction factors which allow $\rho_1/\bar{\beta}_1$ to be found from the counting ratios can be calculated.

The significance of the quantity inferred from the measurement must also be determined with care. There is considerable utility in knowing a value of $\rho_1/\bar{\beta}_1$. However, it is important to distinguish this quantity, which for the particular experiment being discussed here is the reactivity (in dollars) just after all rods have been inserted, from the analogous quantity (call it $\rho_1/\bar{\beta}_1|_{\infty}$) which might be measured sometime later, after the concentration of delayed neutron emitters have attained an asymptotic shape. If comparison is made with theory, it is important to compute $\rho_1/\bar{\beta}_1$ not $\rho_1/\bar{\beta}_1|_{\infty}$. The difference in the two arises because of differences in the shape function, which in the first case is due to a source of delayed emitters distributed according to the preshutdown flux and in the second to a distribution appropriate to the core with all rods inserted. This situation is an illustration of the fact that the "shutdown" of a core (generally defined as the negative of the reactivity with all rods inserted) varies after the rods are inserted. In fact there are, in principle, an infinite number of shutdown values ($\rho_1/\bar{\beta}_1$) corresponds to an infinite number of rod configurations with which the reactor can be critical. These all approach $\rho_1/\bar{\beta}_1|_{\infty}$ at long times after rod insertion. Lack of agreement in the measurement of shutdown of the same core by several methods may thus be due to the fact that the shutdowns being measured are in actuality different.

The shutdown in the particular experiment under discussion is the number which determines the immediate decrease in power following a scram. The measurement thus allows a direct determination of that quantity. It also provides a test of the mathematical model being used to represent the core, since any particular model which yields the fluxes and adjoints needed to analyze the data can also be used to compute $\rho_1/\bar{\beta}_1$ itself. A very rough approximation is adequate for computation of the correction factors which give $\rho_1/\bar{\beta}_1$ from the data. However, comparison between calculated and measured values of this quantity constitutes a stringent test of theory, particularly of the methods being used to represent control rods.

The particular shutdown value under consideration cannot properly be used to answer such questions as how much extra fuel the fully inserted rods will hold down. The quantity of concern in this case is $\rho_1/\bar{\beta}_1|_{\infty}$; it may be significantly smaller than the shutdown measured, particularly if $\rho_1/\bar{\beta}_1$ is large ($\rho_1/\bar{\beta}_1$ and $\rho_1/\bar{\beta}_1|_{\infty}$ approach zero together so that no paradoxes can arise).

The rod drop experiment appears then to be meaningful and susceptible to precise interpretation. The procedure of neglecting experimental correction factors and of assuming that all measurements of shutdown will lead to the same value should be scrutinized carefully however.

2. Integrated Water Height Worth Experiments

It is frequently important in reactor design to know the reactivity which would be introduced were all control rods fully withdrawn from the core. If

this number is known, for example, for a cold core, it can be combined with calculated figures for the reactivity associated with temperature change and fuel depletion to give some notion of whether the total fuel loading is sufficient to meet design requirements. One of the experiments designed to measure this total excess reactivity provides an example of the determination of a large reactivity change by the summation of a series of small changes.

In order to simplify the discussion, we shall examine the case of a heterogeneous water-moderated reactor consisting of a uniform array of long fuel rods arranged in a form approximating a right circular cylinder of height H and radius R . It is desired to know the excess reactivity (ρ_{ex}) should the fuel rod array be fully immersed in water.

The procedure used is to lower the height of the water in which the fuel rods are immersed until the reactor is just critical with all control rods withdrawn. The water height is then raised slightly, and the resulting positive period is measured. Next some fuel rods are removed from the periphery of the assembly, such that a core of radius slightly smaller than R remains. The water height in this core is also adjusted so that it is just critical with all control rods withdrawn and again the period associated with increasing the water height is measured. This process is repeated until the water at criticality reaches a height H ; some magnitude which exceeds the fuel rod height by such an amount that increasing it further does not affect criticality. By inserting into the inhour formula the measured periods and computed values of the β_i , the reactivities $\rho(\Delta h_i)$ associated with the various water height increases (Δh_i) can be found. If the reactivity changes are small, these values of $\rho(\Delta h_i)/\Delta h_i$ can be interpreted as $\partial\rho/\partial h|_i$. Thus the experiment determines $\partial\rho/\partial h$ as a function of height for a reactor that is maintained critical by varying its radius. The question which must be examined is whether the integral

$$\int_{H_1}^H \frac{\partial\rho}{\partial h} dh$$

between the initial height H_1 , corresponding to radius R , and the final water height equals ρ_{ex} , the desired excess reactivity.

The answer is that it never does exactly—in fact it would not, even if the core were at all stages of the experiment bare and homogeneous, both axially and radially, with fluxes at all lethargies vanishing at the same extrapolated end point. Fortunately, under certain conditions, the difference between ρ_{ex} and $\int_{H_1}^H (\partial\rho/\partial h) dh$ is of order ρ_{ex}^2 . Such accuracy is generally quite sufficient for design purposes. The analysis of this and similar experiments must then be aimed

at determining whether conditions prevail under which the equality of ρ_{ex} and the integral is a satisfactory approximation.

The problem can be reduced to examining whether the reactivity change, ρ_{13} , between R_1H_1 and R_1H_3 (the core being critical at water height H_1 and radius R_1 and supercritical at some greater water height H_3) equals the sum of ρ_{12} , the change between R_1H_1 and R_1H_2 (H_2 being some intermediate water height) and ρ_{23} , the change between R_2H_2 and R_2H_3 (R_2 being the radius which gives criticality at H_2).

Equations (7) and (8) are the theoretical expressions for the reactivities under examination, but they are of very little practical help, since the most important terms are those involving the gradients of ψ and ψ_0^* , and determination of these requires solution of the transport equation.

Two approximations will serve to cast (7) into a more helpful form. The first of these is to employ a diffusion theory approximation to the transport equation; the second is to assume an equivalent bare core in the vertical (Z) direction. For the case under consideration both approximations are somewhat suspect because of the nest of water-free fuel rods and control rods above the active (water-immersed) part of the core. Actually, with computing codes which are available today, the experiment can be analyzed without making an assumption as severe as the second. It is made here because experience with the particular experiment under consideration indicates it is valid and because it allows the expressions for the various reactivities to be written so that the addition corrections required to establish the relationship of ρ_{13} to $(\rho_{12} + \rho_{23})$ can readily be seen. Because of the second approximation, only the first two terms of (7) survive. The reason is that, since the perturbation consists in extending the extrapolated boundary from $z = H_1$ to $z = H_2$,

$$\delta\Sigma(\mathbf{r}, u) \text{ and } \delta\left\{\Sigma_s^0(\mathbf{r}, u', u) + \frac{\nu}{\nu_0} \sum_j f_t^j(u) \nu^j(u') \Sigma_f^j(\mathbf{r}, u', t)\right\}$$

are finite only between H_1 and H_2 ; but φ_0^* being the preperturbation adjoint is zero in this region. The approximations then reduce (7) for the case of ρ_{12} to

$$\rho_{12} = \frac{1}{F} \int_0^\infty \int_0^{2\pi} \int_0^{H_2} \int_u [\varphi_0^* \nabla \cdot D \nabla \varphi - \varphi \nabla \cdot D \nabla \varphi_0^*] r \, dr \, d\theta \, dz \quad (19)$$

where $D[= D(u, r, \theta)]$ is the diffusion constant (5).

Because of the equivalent bare core approximation in the Z direction, $\nabla_z D \nabla_z \varphi = -DB_{z_2}^2 \varphi$ and $\nabla_z D \nabla_z \varphi_0^* = -DB_{z_1}^2 \varphi_0^*$, where $B_{z_2} = \pi/H_2$ and $B_{z_1} = \pi/H_1$. Moreover,

$$\begin{aligned}
& \int_0^\infty \int_0^{2\pi} \int_0^{H_2} [\varphi_0^* \nabla_{r,\theta} \cdot D \nabla_{r,\theta} \varphi - \varphi \nabla_{r,\theta} \cdot D \nabla_{r,\theta} \varphi_0^*] r \, dr \, d\theta \, dz \\
&= \int_0^\infty \int_0^{2\pi} \int_0^{H_2} [\nabla_{r,\theta} \cdot (\varphi_0^* D \nabla_{r,\theta} \varphi) - \nabla_{r,\theta} \cdot (\varphi D \nabla_{r,\theta} \varphi_0^*)] r \, dr \, d\theta \, dz \quad (20) \\
&= \int_0^{H_2} \int_{S_{r,\theta}} (\varphi_0^* D \nabla_{r,\theta} \varphi - \varphi D \nabla_{r,\theta} \varphi_0^*) \cdot d\mathbf{S}_{r,\theta} = 0
\end{aligned}$$

by Gauss's theorem, and since φ_0^* and φ vanish on the surface $S_{r,\theta}$ which bounds the reactor in the radial direction. Thus (19) becomes

$$\rho_{12} = \frac{(B_{z_1}^2 - B_{z_2}^2) \int_V \int_u D \varphi_0^* \varphi \, dV \, du}{\int_V \int_u \int_{u'} \nu f(u) \Sigma_f(u') \varphi_0^*(u) \varphi(u') \, dV \, du \, du'} \quad (21)$$

Analogous expressions can be written for ρ_{13} and ρ_{23} , φ_0^* in each case being the adjoint function for the unperturbed state, and φ the shape function [Eq. (14)] which rises in time after the water height is increased. If the coefficients of the buckling differences $(\rho_{ij})/(B_{z_i}^2 - B_{z_j}^2)$ were the same for all three reactivity changes, the relation $\rho_{13} = \rho_{12} + \rho_{23}$ would be exact within the limit of the two approximations made thus far. Actually, the coefficients differ slightly because of the different radii of the fuel regions and because of the different periods for which the ϕ 's are to be computed.

The effect of different values of ω can be estimated by examining the case for a core which is bare radially (as well as axially) and uniform in the θ direction. Under these circumstances coefficients appropriate to particular reactivity changes differ only because of the dependence of φ on ω . They differ by a factor which is never greater than

$$\left(1 - \sum_i \frac{\omega \bar{\beta}_i}{\omega + \lambda_i}\right).$$

Thus, for this case the total reactivity change between H_1 and H will differ from the sum of smaller changes measured at different radii by a factor of at most $(1 - \bar{\beta})$. Since, in the particular experiment being examined, the equivalent bare core model is a good one, the error due to use of φ 's corresponding to incorrect periods is expected to be of the same order. This fact having been determined, it is then legitimate to investigate the variation in the coefficients by computing them using the critical eigenvalue method discussed earlier. For example, in the case of ρ_{12} the coefficient of $(B_{z_1}^2 - B_{z_2}^2)$ may be found by computing the eigenvalues ν_1 and ν_2 leading to criticality at (R_1, H_1) and (R_1, H_2) , and then finding the eigenvalue ν_2' corresponding to the core at (R_1, H_2) with a

negative poison $-\delta\Sigma$ everywhere proportional to the diffusion constant D . Comparison of (7) and (21) then shows the desired coefficient $[-\delta\Sigma(u, \mathbf{r}) = \alpha D(u, \mathbf{r})]$, where α is an arbitrary constant taken as small as the accuracy of the result will allow] to be $(\nu_2' - \nu_2)/(\nu_1\alpha)$. The coefficient in (21) appropriate to the change $(R_1, H_1)(R_1, H)$ can be found by treating ω in Eq. (14) as an eigenvalue and varying it until the solution for (R_1, H) is stationary with $\nu = \nu_1$. Keeping this value of ω and adding a negative poison $\alpha D(u, \mathbf{r})$, the ν value which again makes (14) stationary is found. The fractional difference between this ν value and ν_1 divided by α is then the desired coefficient. Comparison of the last coefficient and that appropriate to ρ_{12} , ρ_{23} , etc., allow correction of the data, $\partial\rho/\partial h$, so that the integral of this quantity between H_1 and H will give ρ_{ex} . To the extent that the equivalent bare core model is justified, these elaborate correction to the data are expected to be very small.

Water height worth experiments then appear to yield values of the available excess reactivity which are sufficiently accurate to serve as reliable design aids. Unfortunately, what has been found is the value of reactivity which, if inserted with appropriate $\bar{\beta}_i$ and Λ values into the inhour equation, would give the asymptotic period if all control rods were instantaneously withdrawn from the water-filled core of radius R_1 . What is desired from the design viewpoint is the magnitude of the fictitious eigenvalue ν , which would just give criticality for this core. The excess reactivity in terms of this eigenvalue is $(\nu_1 - \nu)/(\nu_1)$, and it is not strictly correct to equate this to the corrected integral of $\partial\rho/\partial h$. What must be done is to compute ρ_{ex} from the inhour equation and the value of ω which makes (14) stationary for the rod free core, to compute the value of ν which makes (15) stationary for the same core, and to use the theoretical ratio $(\nu_1 - \nu)/(\nu_1\rho_{\text{ex}})$ as a correction factor to the data. Again, insofar as the bare core approximation is valid, the correction factor will be close to unity.

Once the experiment is analyzed in terms of the ν value, application to design calculations requires merely that a generalization be made of the definition of reactivity so that we may speak consistently of the reactivity change between two states, neither of which is critical. The simplest definition for the change between any two states i and j seems to be $\rho_{ij} = (\nu_i - \nu_j)/(\nu_0)$, where ν_i and ν_j are fictitious values of ν which would make the states critical and ν_0 is the true physical value. In what may be more familiar terms, this definition corresponds to $\rho_{ij} = (k_{\text{eff}j} - k_{\text{eff}i})/(k_{\text{eff}i}k_{\text{eff}j})$, where $k_{\text{eff}}(\equiv \nu_0/\nu)$ is the usual effective multiplication constant of the system. If state (i) is critical, the result reduces to the usual $\rho_{ij} = (k_{\text{eff}j} - 1)/(k_{\text{eff}j})$. With this definition we can compute consistent values of the reactivity change due to temperature variation and fuel burnout and can thus acquire some notion of whether the core for which the excess reactivity has been measured will meet design requirements.

CONCLUSION

The great advantage of converting the time-dependent transport Eq. (1) into the form of the conventional kinetics equation (6) is that most space-times reactor problems are thereby reduced to a series of static spatial problems and a set of purely time-dependent equations. Cast in this form, the problems become tractable by current analog and digital machine methods. It has been the purpose of this paper to indicate, first generally, then by detailed analysis of specific experiments, just what conditions must be fulfilled for this mode of solution to be legitimate. In all cases the crucial quantity was found to be the time-dependent shape function, $\psi(\mathbf{r}, \mathbf{\Omega}, u, t)$, during or immediately after a perturbation. The fact that to a high order of accuracy, ψ readjusts to changing reactor conditions in several prompt neutron lifetimes, allows it to be calculated as a stationary quantity. Thus computation of the time dependency of the quantities, ρ , β , and Λ , appearing in the kinetics equations reduces to solving a series of static problems.

In the interests of generality the mathematical treatment has been couched in terms of transport theory. Also, in order to illustrate what degree of precision can be attained, a sizable amount of detail has been included in the discussion of experiments. In practice, gross approximations to transport theory are usually quite satisfactory. Moreover, refinements in interpretation as detailed as those discussed are frequently unnecessary. It is, nevertheless, important to recognize that great precision is attainable, generally by mathematical procedures already available. It is also reassuring to be fully aware of what constitutes complete rigor.

REFERENCES

1. A. WEINBERG, *Am. J. Phys.* **20**, 401 (1952).
2. R. EHRLICH AND H. HURWITZ, *Nucleonics* **12**, No. 2, 23 (1954).
3. E. D. PENDLEBURY, *Proc. Phys. Soc. (London)* **A68**, 474 (1955).
4. L. N. USSACHOFF, Geneva Conference Paper No. 656 (1955).
5. A. F. HENRY, WAPD-142-Navy (1955).
6. G. VON DARDEL AND N. G. SJÖSTRAND, *Phys. Rev.* **96**, 1245 (1954).
7. M. J. POOLE, A.E.R.E./N.R.D.C. 95 (1956).
8. N. G. SJÖSTRAND, Geneva Conference Paper No. 789 (1955).