Annals of Nuclear Energy 170 (2022) 108979

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

Runge–Kutta type-2 method for solving reactor point kinetics equations and its validation by analysing thermal and fast reactor benchmarks

P. Ravindra Babu [a,*], Usha Pal [b], R.N. Sen [a], R. Karthikeyan [b]

[a] ALWR, BARC(V), Visakhapatnam, Andhra Pradesh, India
[b] RPDD, BARC (M), Mumbai, Maharashtra, India

ARTICLE INFO

Article history:
Received 22 June 2021
Received in revised form 3 January 2022
Accepted 14 January 2022
Available online 1 February 2022

Keywords:
RK2
Modified ETD
Predictor
Corrector
Cohen's

ABSTRACT

Runge – Kutta type-2 method is used for solving point kinetic equations. Time step size chosen ranging from 10^{-3}-10^{-4} sec. RK-2 method is applied to estimate power transients of thermal reactor and fast breeder reactors with step, ramp and sinusoidal neutron reactivity perturbations. Results are compared with the standard problem results at different time steps with different numerical methods ie. Modified ETD and Cohen's methods. This comparison of transient power values shows that RK-2 method is predicting accurately for fast and thermal reactors for longer duration of time.

1. Introduction

The power transient taking place in nuclear reactor depends on amount of the neutron reactivity inserted in the core, type of reactor core, and power level at which the reactor is operating. It is important to estimate power transients due to reactivity perturbations, during normal power operations, and accidental conditions. Though space time kinetics gives time dependent behaviour of reactor during power transient conditions more accurately, estimating the neutron flux at each time interval is a difficult task compared to estimating reactor power. For neutronically tightly coupled reactors, point kinetics equations gives accurate results in the estimation of transient power levels.

Point kinetic equations (PKEs) with multi - group delayed neutrons are stiff in nature. Because of stiffness term, smaller time step has to be chosen for good accuracy of estimated power values. There are different numerical methods available in literature to study reactor power transients. Hennart and Barrios (1976) uses Paade and Chebyshev type of approximations to solve PKEs. Hermite polynomial method can also be used for solving the equations. Aboanber and Nahla (2002a, 2002b) used analytical inversion method for estimating power transients by solving point kinetic equations with multi-group delayed neutrons. Recently Nahla (2011) developed Taylor's series expansion method for solving PKEs.

In this present paper, Runge kutta Type-2 method is used for solving PKEs with multi-group delayed neutrons. This method is used for solving first order coupled differential equations. It uses two different steps to solve the equation. In this method precursor concentrations and neutronic power estimated initially at n^{th} and $n + 1^{th}$ fuel burn up time step are considered as predictor step. Same way in corrector step, power and precursor concentrations estimated at n^{th} and $(n + 1)^{th}$ predictor step are used to estimate the values at $(n + 1)^{th}$ step. This method is applied to evaluate power transient values of different thermal reactors and results are compared with other methods, which are referred by Nahla (2011) and Li et al. (2009). This method is also used to estimate power transients of Indian Prototype Fast Breeder Reactor (PFBR) at IGCAR Kalpakkam. Estimated results are compared with Cohen's Method (1958). From the comparison of results, it is seen that RK-2 method estimates power transients accurately for thermal and fast reactors with multi-group of delayed neutrons.

2. Runge – Kutta Order-2 method and point Kinetic equations

The PKEs are two coupled equations which has to be solved simultaneously. Runge – Kutta type 2 methods are similar to Euler's method. But this method will have two different solution steps.

* Corresponding author.
E-mail address: ravindrab@barc.gov.in (P. Ravindra Babu).

https://doi.org/10.1016/j.anucene.2022.108979
0306-4549/

Nomenclature

P(t)	neutronic power at time t (watts)	$\rho(t)$	Reactivity of the core at time 't' (k)
Λ	Mean neutron generation time (sec)	t	Time in sec
β_i	i^{th} group delayed neutron fraction	$\lambda_{max}(t)$	largest eigenvalue of matrix A(t) (sec^{-1})
β	total effective fraction of delayed neutrons	h	time step size (sec)
λ_i	decay constant (sec^{-1})	h_{max}	maximum value of time step size
C_i	precursor concentration ($atoms/cm^3$)	π	Radians to degrees conversion factor

Predictor step
Corrector step

In the Predictor, step Euler's method is used to estimate the neutron density and precursor concentrations at $(n + 1)^{th}$ step. In the Corrector step, the estimated neutron density and concentrations of precursor at n^{th} step and $(n + 1)^{th}$ predictor step are used to estimate final concentrations and power values at $(n + 1)^{th}$ step.

2.1. Procedure used in the Runge – Kutta Type-2 method with general kind of first order coupled equations

Consider the following first order differential equations,

$\frac{dy_1(t)}{dt}$ = f1(t,y1(t),$y_{2,i}$(t)) where (i = 1,6) and

$\frac{dy_{2i}(t)}{dt}$ = f2(t,y1,$y_{2,i}$) where (i = 1,6) are two coupled first order differential equations

Withinitial boundary conditions $y_1(0) = y_10$, $y_{2,i}(0) = y_{2,i}0$ (i = 1,6)

Consider the Point kinetics coupled equations given by

$$\frac{dp(t)}{dt} = \frac{(\rho(t)-\beta)}{\Lambda}\mathrm{p(t)} + \sum_{i=1}^{6}\lambda_i C_i(t)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda}p(t) - \lambda_i C_i(t) (\mathrm{i} = 1, 2\cdots 6)$$

If we write the above equations in matrix form

$$\begin{pmatrix} dp(t)/dt \\ dC1(t)/dt \\ dC2(t)/dt \\ dC3(t)/dt \\ dC4(t)/dt \\ dC5(t)/dt \\ dC6(t)/dt \end{pmatrix} = \begin{pmatrix} \frac{(\rho(t)-\beta)}{\Lambda} & \lambda1 & \lambda2 & \lambda3 & \lambda4 & \lambda5 & \lambda6 \\ \beta_1/\lambda1 & -\lambda1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ \beta_2/\lambda2 & 0.0 & -\lambda2 & 0.0 & 0.0 & 0.0 & 0.0 \\ \beta_3/\lambda3 & 0.0 & 0.0 & -\lambda3 & 0.0 & 0.0 & 0.0 \\ \beta_4/\lambda4 & 0.0 & 0.0 & 0.0 & -\lambda4 & 0.0 & 0.0 \\ \beta_5/\lambda5 & 0.0 & 0.0 & 0.0 & 0.0 & -\lambda5 & 0.0 \\ \beta_6/\lambda6 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -\lambda6 \end{pmatrix} \begin{pmatrix} p(t) \\ C1(t) \\ C2(t) \\ C3(t) \\ C4(t) \\ C5(t) \\ C6(t) \end{pmatrix}$$

$$\mathrm{A(t)} = \begin{pmatrix} \frac{(\rho(t)-\beta)}{\Lambda} & \lambda1 & \lambda2 & \lambda3 & \lambda4 & \lambda5 & \lambda6 \\ \beta_1/\lambda1 & -\lambda1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ \beta_2/\lambda2 & 0.0 & -\lambda2 & 0.0 & 0.0 & 0.0 & 0.0 \\ \beta_3/\lambda3 & 0.0 & 0.0 & -\lambda3 & 0.0 & 0.0 & 0.0 \\ \beta_4/\lambda4 & 0.0 & 0.0 & 0.0 & -\lambda4 & 0.0 & 0.0 \\ \beta_5/\lambda5 & 0.0 & 0.0 & 0.0 & 0.0 & -\lambda5 & 0.0 \\ \beta_6/\lambda6 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -\lambda6 \end{pmatrix}$$

In the above point kinetics equations p is power, Λ is prompt neutron generation time, β_i is i^{th} group delayed neutron fraction, β is total effective fraction of delayed neutrons ($\beta = \sum_{i=1}^{6}\beta_i$), λ_i and C_i are decay constant and precursor concentrations of i^{th} group of delayed neutrons. Initial conditions of point kinetics equations are chosen as p(t = 0), $C_i(t = 0) = \frac{\beta_i}{\Lambda\lambda i}$

If we write PKEs in general form of first order differential equations then

$y_1(t) = p(t)$, $y_{2i}(t) = C_i(t)$,$y_1(0) = p(0)$, $y_{2,i}(0) = C_i(0)$ (i = 1,6)

Functions f1(t,y1(t),y2i(t)) and f2(t,y1(t),y2i(t)) (Venkateshan and Swaminathan, 2014) are defined as shown below

f1(t,y1(t),$y2_i$(t))=$\frac{(\rho(t)-\beta)}{\Lambda}$p(t) + $\sum_{i=1}^{6}\lambda_i C_i(t)$

f2i(t,y1(t),$y2_i$(t))=$\frac{\beta_i}{\Lambda}p(t) - \lambda_i C_i(t)$ (i = 1,2…6)

y1(t),y2i(t) values at predictor and corrector step are solved as shown below

1. Predictor step:

In predictor step y1(t),y2i(t) estimated at $(n + 1)^{th}$ step are as shown below

$t_{n+1} = t_n + h$
$y^p_{1,n+1} = y_{1,n} + h^*f1(t_n, y_{1,n},y_{2i,n})$
$y^p_{2i,n+1} = y_{2,n} + h^*f2(t_n, y_{1,n},y_{2i,n})$

2. Corrector step:

$t_{n+1} = t_n + h$
$y_{1,n+1} = y_{1,n}+(h/2.0)^*(f1(t_n, y_{1,n},y_{2i,n}) + f1(t_{n+1}, y^p_{1,n+1},y^p_{2i,n+1}))$
$y_{2i,n+1} = y_{2,n}+(h/2.0)^*(f2(t_n, y_{1,n},y_{2,n}) + f2(t_{n+1}, y^p_{1,n+1},y^p_{2i,n+1}))$

For smaller step size of time (of the order of 10^{-2} sec) RK type 2 method used to give good accuracy, in the case of non - stiff equations. In the case of stiff equations like PKEs, product of largest Eigen value of A(t) (λ_{max}) and 'dt' (λ_{max}*dt) should be <2.00 for getting better accuracy of the results. So a time step of the order of 10^{-3}-10^{-4}sec is required for better accuracy of results.

The main advancement in this work is that, instead of directly implementing RK2 method, Largest Eigen value of the coefficient matrix 'A' is estimated at each time step. We can choose the time step such that 0.0<|Z_{max}|<2.0 is satisfied. Here, Z_{max} = h* λ_{max}.|Z_{max}| is chosen such that, it is slightly<2.0 (∼1.75), so that

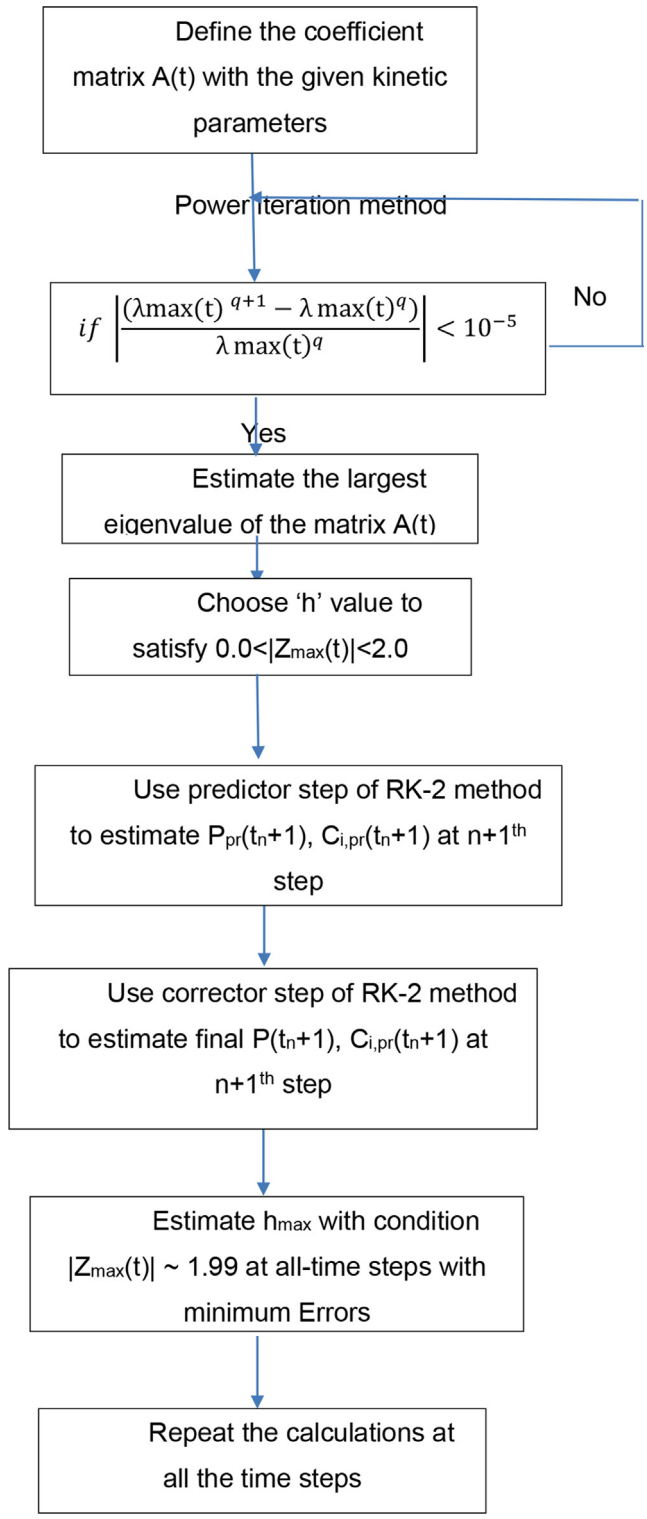

Fig. 1. Flow chart of Solution of PKE with Rk-2 Method.

the maximum time step size (h) can be achieved without losing the accuracy.

2.2. Computation methodology

Computation is performed using Compaq visual Fortran in windows 10.0 operating system. The computational details are presented in the form of flow chart as shown in Fig. 1.

3. Results and discussion

Runge – Kutta Type 2 method is applied for solving the PKEs with six group delayed neutrons for both thermal and fast reactors. The main advantage of the RK2 method are they are easy to implement, very stable and self-starting. Current work is lesser in complexity, it takes less computational time for estimating transient power values, when compared to HPM, modified ETD and modified TSM methods, which involves estimation of expansion coefficients of higher derivatives of Y(t) at each time step.

Power transients for few different types of reactivity perturbations i.e., step, ramp and oscillatory reactivity are computed and results are discussed. In Benchmark problems 1 and 2, exact values are estimated using analytical exponential method, which is very effective technique for solving stiff differential equations.

3.1. Power transients of thermal and fast reactors caused by step reactivity

The RK-2 method is applied to estimate the power transients of thermal reactor described by Nahla (2011) (bench mark problem-1). step reactivity of $r_0 = -1.0$ \$ is applied and power transients are analysed. The decay constants and kinetic parameters are as shown in Table 1.

Table 2 shows the power transients obtained from this method using a time step of h = 0.0002 sec along with exact values referred by Nahla (2011). Relative errors (Pcal – Pexact)/Pexact, are shown in of Table 2. The comparison is made upto 10 sec, because exact values are available up to 10.0 sec. Comparison of power transients initiated due to insertion of −1.0 \$ step reactivity using RK-2 and exact method is as shown in Fig. 2. It can be seen from Fig. 2 that the power value predicted by RK2 method agrees very closely with reported values. All these calculations are performed with Intel ®, Core ™ i9-10900 CPU @ 2.81 GHZ, RAM of 32.0 GB with 64- bit.

Thermal reactor described by Li et al. (2009) (bench mark problem-2) is considered as another example. Kinetic parameters of this reactor are as shown in the Table 3. A step reactivity r0 = 0.003 k is inserted and power transients are computed using RK-2 method (h = 0.0002 sec). Results are compared with exact values and TPM values referred by Li et al. (2009) in Table 4. It is observed that power transient values estimated by this method is more accurate than TPM values, up to the time of 1.00 sec and in good agreement with exact values. Comparison of results is made up to 1.0 sec because exact values are available up to 1.0 sec only. The power transients obtained with 1.0 \$ step reactivity using RK-2 and exact method is as shown in Fig. 3.

As an example of fast reactor, Indian prototype fast breeder reactor (PFBR) at IGCAR, kalpakkam is considered. PFBR is a 500 MWe reactor which contains181 fuel Assemblies (Riyas and Mohanakrishnanan (2008)) step reactivity of 50 pcm is inserted in PFBR. Kinetic parameters are as shown in Table 5. In this case time step chosen as h = 0.0002 sec, the power transient is computed using RK-2 method and results are compared with Cohen's

Table 1
Kinetic Parameters of Thermal Reactor Benchmark Problem1 (Nahla (2011)).

Group no.	$\lambda_{i\ (sec-1)}$ (decay constant of i th group)	β_i (delayed neutron fraction of i^{th} group)	Mean neutron generation time Λ(sec)
1	0.0127	0.0002850	5.000E-04
2	0.0317	0.0015975	
3	0.1150	0.0014100	
4	0.3110	0.0030525	
5	1.4000	0.0009600	
6	3.8700	0.0001950	

Table 2
Comparison of Power transients initiated due to Step Reactivity Insertion of −1.0$ (Thermal Reactor Benchmark Problem1 (NAHLA (2011).

Time (sec)	Power (Ref.) (watts)	Power (watts) RK-2 method (Time step = 0.0002 sec)	Relative error
0.0	1.0000000	1.0000000	0.0000000
0.1	0.5205643	0.5205646	5.76298E-07
0.2	0.4880113	0.4880114	2.04913E-07
0.4	0.4702262	0.4702265	6.37991E-07
0.6	0.4562794	0.4562792	−4.38328E-07
0.8	0.4441506	0.4441518	2.70179E-06
1.0	0.4333335	0.4333339	9.23077E-07
10.0	0.2361207	0.2361207	0.00000000

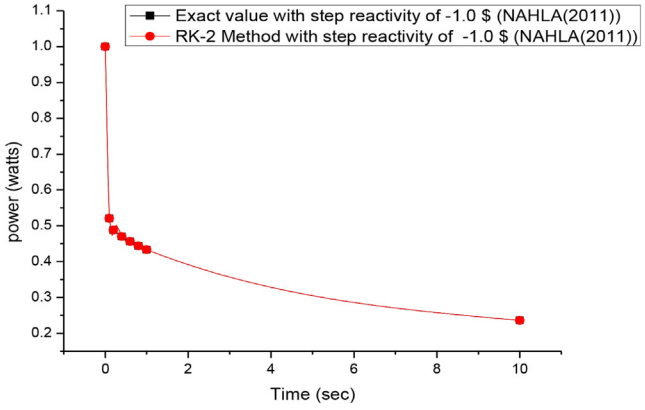

Fig. 2. Comparison of Power transients initiated due to Step Reactivity Insertion of −1.0$ (Thermal Reactor Benchmark Problem1 (NAHLA (2011)).

Table 3
Kinetic Parameters of Thermal Reactor Benchmark Problem-2 (Li et al. (2009)).

Group no.	$\lambda_{i\ (sec-1)}$ (decay constant of i th group)	β_i (delayed neutron fraction of i^{th} group)	Mean neutron generation time Λ(sec)
1	0.0127	0.000266	2.00E-05
2	0.0317	0.001491	
3	0.1150	0.001316	
4	0.3110	0.002849	
5	1.4000	0.000896	
6	3.8700	0.000182	

method (1958) for an initial period of 100 s. Results are shown in Table 6. The power transients estimated with this method is in good agreement with Cohen's method (1958). This confirms that RK-2 method is also capable of evaluating the power transients accurately even for longer duration of time for a typical fast breeder reactor. The power transients estimated with 50 pcm step reactivity using RK-2 and cohen's method is as shown in Fig. 4.

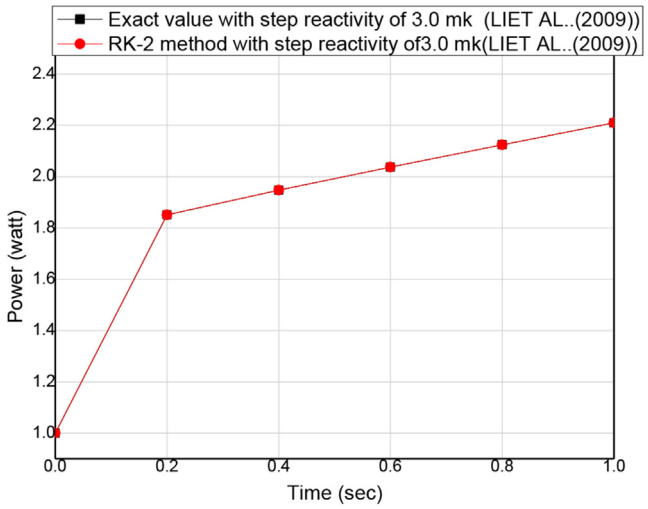

Fig. 3. Comparison of Power transients initiated due to Step Reactivity Insertion of + 3.0 mk (Thermal Reactor Benchmark Problem-2 (Li et al. (2009))).

Table 5
Kinetic Parameters of Indian Prototype Fast Breeder Reactor.

Group no.	$\lambda_{i\ (sec-1)}$ (decay constant of i th group)	β_i (delayed neutron fraction of i^{th} group)	Mean neutron generation time Λ(sec)
1	0.0127	0.00008246	4.100E-07
2	3.1200	0.00076817	
3	0.1344	0.00066296	
4	0.3448	0.00128490	
5	1.3922	0.00057615	
6	3.7491	0.00017213	

3.2. Power transients initiated by ramp reactivity addition in thermal and fast reactors

In this section we consider the power transients, caused by positive and negative ramp reactivity, of thermal reactor described by Li et al. (2009) (BM prob-2). Here we first consider the positive ramp reactivity r(t) = 0.1 t $/sec. Power transients, following the ramp reactivity, are estimated using RK-2 method using time step of h = 0.005 sec. The results are compared with exact values and TSM values, which are referred by Li et al. (2009). Kinetic parameters are shown in Table 4. Results and relative errors are mentioned in Table 7. It is observed that RK-2 results are in good agreement with exact values and TSM estimated values. Even though TSM values are very accurate upto 8.0 sec, at t = 10.0 sec RK2 method estimated values are very close to exact value with larger step of (h = 0.005 sec). RK-2 method takes 0.12 sec of CPU time for estimation of power transients, which is lesser than the CPU time taken by SCS method (0.16sec, (Hamada, 2018)). Power transients obtained with RK-2 and exact values are as shown in Fig. 5.

The power transient of thermal reactor (Nahla, 2011) (BM prob-1) caused by the negative ramp reactivity r(t) = −0.1 t $/sec is also

Table 4
Comparison of Power transients initiated due to Step Reactivity Insertion of +3.0 mk (Thermal Reactor Benchmark Problem-2 (LI et al. (2009)).

Time (sec)	Power (watt) Exact value	Power (watt) Runge - Kutta −2 method (h = 0.008 sec)	Power (watt) TPM (Li et al., 2009)	Relative Error
0.0	1.000000	1.000000	1.000000	0.0000E + 00
0.2	1.851268	1.851261	1.851246	−3.7812E-06
0.4	1.947593	1.947591	1.947581	−1.0269E-06
0.6	2.037922	2.037921	2.037915	−4.9070E-07
0.8	2.124832	2.124830	2.124827	−9.4125E-07
1.0	2.209841	2.209840	2.209838	−4.5252E-07

Table 6
Comparison of Power transients initiated due to Step Reactivity Insertion of + 50 pcm Indian Prototype Fast Breeder.

Time (sec)	Power (watt) Cohen's Method (h = 0.0001 sec)	Power (watt) Runge – Kutta –2 method (h = 0.0002 sec)	Relative Error
0.0	1.000000	1.000000	0.0000E + 00
10.0	2.451120	2.452462	5.4750E-04
20.0	4.523096	4.525801	5.9804E-04
30.0	8.237786	8.242696	5.9603E-04
40.0	14.927707	14.936330	5.7765E-04
50.0	26.988310	27.003202	5.5172E-04
60.0	48.739982	48.765472	5.2294E-04
70.0	87.977112	88.020031	4.8783E-04
80.0	158.761943	158.833326	4.4948E-04
90.0	286.465074	286.583245	4.1237E-04
100.0	516.859342	517.053526	3.7565E-04

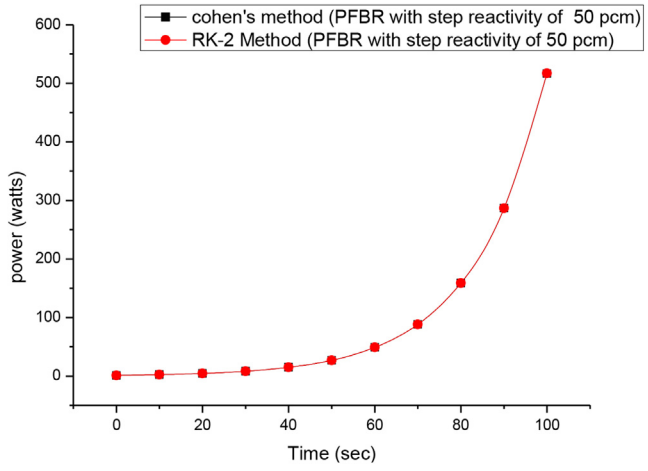

Fig. 4. Comparison of Power transients initiated due to Step Reactivity Insertion of + 50 pcm Indian Prototype Fast Breeder.

computed using RK-2 method with time step of h = 0.005 sec. Kinetic parameters of benchmark problem1 are as shown in Table 1 since the exact values of the power transients are not presented in this case, results are compared with modified ETD method values (Mohideen Abdul Razak et al., 2015). It is observed from Table 8 that power transients calculated by this method are in good agreement with modified ETD method. Power transients obtained with RK-2 and modified ETD methods are as shown in Fig. 6.

Power transients of Indian PFBR (Riyas and Mohanakrishnanan, 2008), described in section 3.1, caused by positive ramp reactivity of 1.0 pcm/s. Table 9 shows the power transients caused by RK-2 method and the cohen' s method (1958) for an initial period of 100 s. time step chosen for this step is h = 0.0002 sec. The relative errors are also shown in Table 9. Even though step size is chosen h = 0.0002 sec in present method, which is less than step size h = 0.001 sec chosen in the cohen's method, The power transients estimated in this method are in good agreement with cohen's method (1958). A maximum relative error of −2.7994E-03 is observed during transient time. From results, it is observed that this method is capable of estimating power transients for longer duration of time. Fig. 7 shows power transients computed using RK-2 method and Cohen's method. Even Though, time step chosen for RK-2 method is (h = 0.0002 sec) is greater than Cohen's method (h = 0.0001 sec), Results are in good agreement with Cohen's values.

3.3. Power transients of initiated by oscillatory form of reactivity

A sinusoidal formof reactivity addition is considered in this case:

$$\rho(t) = \rho_0^* \, \mathrm{Sin}(\pi \, t/T)$$

Where T is half period and $\rho_0 = 8.0\beta/(8 + \lambda T)$. This is fast reactor-1 problem which is taken from reference (Hamada, 2018). Kinetic parameters corresponding to this problem are mentioned in Table 10. time step chosen for this problem is h = 0.0001 sec. Fig. 8 shows power transients computed using RK-2 method and CATS method. Transient power levels estimated using RK-2 method is in good agreement with CATS method. Calculations are performed up to 50.0 sec, Results are shown in Table 11. This shows that RK-2 method can estimate power transients due to sinusoidal reactivity insertion for longer duration of time. Maximum relative error observed during all the time steps is 3.9671E-02. The total computational effort was 0.5 M evaluations at 1.2E-06 sec per evaluation, which is lesser than the effort made in CATS method (2.9 M evaluations at 1.1E-05 sec per evaluation).

3.4. Innovation in this work

The main innovation in this work is that estimation of Eigen values of coefficient matrix A(t) at different time steps and choose the time step (h), such that $\lambda_{max}(t)^*h$ will be close 1.75, which leads minimum errors in the computation of transient power values. By computing the largest Eigen value with the above condition, h_{max} (sec) can be achieved with minimum errors and minimum computational time. So, this method is different from the normal RK-2 method.

4. Error analysis of RK-2 method with different 'h' values for slowly varying reactivity insertions

A second order RK2 method requires that it to be equivalent to Taylor expansion that it includes terms proportional to h^2. Hence we have

$$Y_{n+1} = Y_n + Yn^{(1)}\, h + \frac{Yn^{(2)}h^2}{2.0} + O(h^3) \text{---} (7)$$

Here the subscript 'n' indicates calculations made at time t_n. Local Truncation Error (LTE) is proportional to h^3. However, GTE

Table 7
Comparison of Power transients initiated due to Ramp Reactivity Insertion of + 0.1 t$/sec (Thermal Reactor Benchmark Problem-2 (LI et al. (2009)).

Time (sec)	Power (watt) Exact value	Power (watt) Runge - kutta method-2(h = 0.005 sec)	Power (watt) TSM (h = 0.0001 sec)	Relative Error
0.0	1.00000E + 00	1.00000E + 00	1.00000E + 00	0.0000E + 00
2.0	1.33820E + 00	1.33760E + 00	1.33820E + 00	−4.4836E-04
4.0	2.22844E + 00	2.22932E + 00	2.22844E + 00	3.9490E-04
6.0	5.58205E + 00	5.58945E + 00	5.58205E + 00	1.3257E-03
8.0	4.27863 E + 01	4.29642E + 01	4.27890E + 01	4.1579E-03
10.0	4.51163E + 05	4.51085E + 05	4.51430E + 05	−7.9350E-05

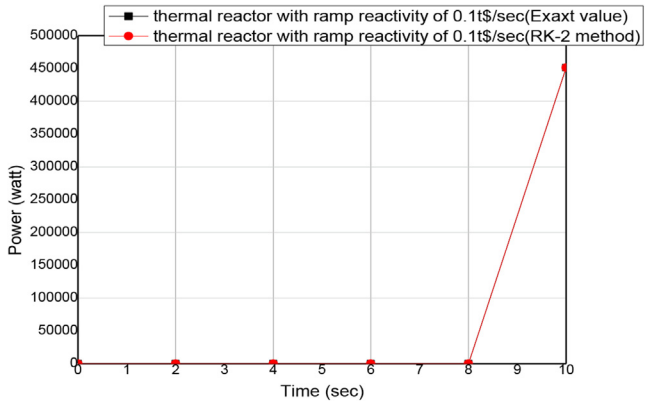

Fig. 5. Comparison of Power transients initiated due to Ramp Reactivity Insertion of + 0.1 t$/sec (Thermal Reactor Benchmark Problem-2 (LI et al. (2009))).

Table 8
Comparison of Power transients initiated due to Ramp Reactivity Insertion of (-0.1 t$/sec) (Thermal Reactor Benchmark Problem1 (Nahla (2011)).

Time (sec)	Power (Ref.) (watts) (Modified ETD Method)	Power (watts) RK-2 method (Time step = 0.0001 sec)	Relative error
0.0	1.000000	1.000000	0.0000E + 00
2.0	0.792005	0.799683	9.6944E-03
4.0	0.613016	0.619356	1.0342E-02
6.0	0.474057	0.478875	1.0163E-02
8.0	0.369168	0.372720	9.6216E-03
10.0	0.290654	0.293230	8.8628E-03

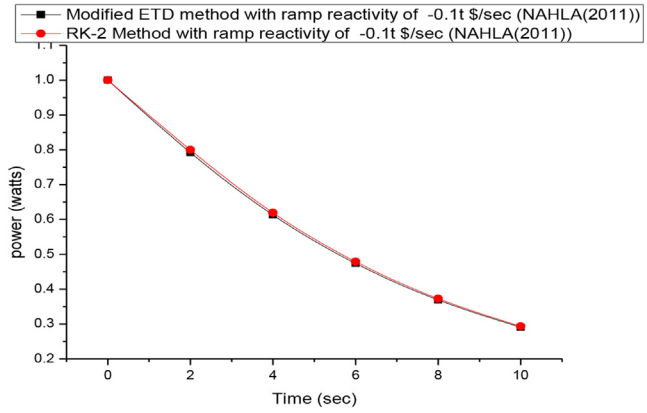

Fig. 6. Comparison of Power transients initiated due to ramp reactivity of -0.0.1 t$/sec (Thermal Reactor Benchmark Problem1 (NAHLA (2011).

is proportional to h^2 (According to the procedure used in the Euler Scheme)

$O(h^3) = Y_{exact} - Y_{RK2}$

Equation (7) can be rewritten in eigenvalue form as shown below:

$$Y_{n+1} = Y_n\left(1.0+\lambda(t)h + \frac{(\lambda(t)h)^2}{2.0}\right)$$

Here $\lambda(t)$ = Eigen value of Matrix A(t)

In general, stability of the Rk2 method is governed by magnification factor $\left|\frac{Yn+1}{Yn}\right| < 1$ or $\left|1.0 + z(t) + z(t)^2\right| < 1$ where z(t) stands for $\lambda(t) * h$. Stability condition for 'Z' is as shown below:

$$0.0<|Z(t)|<2.0 \text{ ------ } (8)$$

Table 9
Comparison of Power transients initiated due to Ramp Reactivity Insertion of + 1.0 pcm /sec Indian Prototype Fast Breeder.

Time (sec)	Power (watt) Cohen's method Time step = 0.0001 sec	Power (watt) Runge – kutta type-2 method (h = 0.0002 sec)	Relative Error
0.0	1.000000	1.000000	0.0000E + 00
10.0	1.102051	1.102665	5.5714E-04
20.0	1.353832	1.355709	1.3864E-03
30.0	1.860560	1.866395	3.1362E-03
40.0	2.895566	2.909805	4.9175E-03
50.0	5.190173	5.221714	6.0771E-03
60.0	10.939741	11.011923	6.5979E-03
70.0	27.771452	27.945814	6.2780-E-03
80.0	87.208655	87.631585	4.8496E-03
90.0	348.972575	349.647821	1.9349E-03
100.0	1839.939692	1834.789725	−2.7994E-03

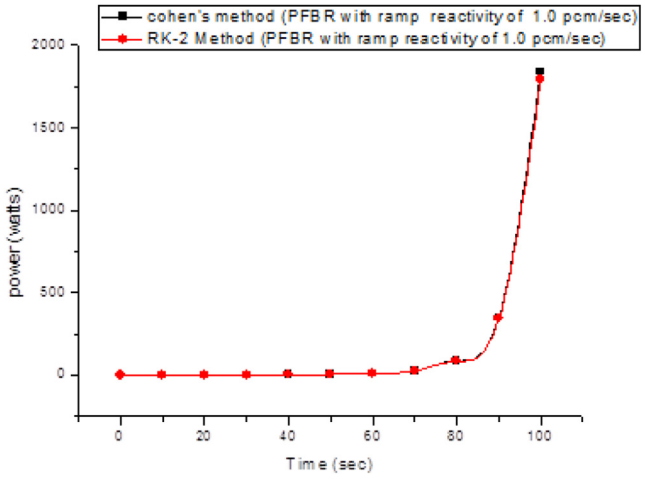

Fig. 7. Comparison of Power transients initiated due to positive ramp reactivity of 1.0 pcm/sec in Indian Prototype Fast Breeder.

Table10
Kinetic Parameters of Benchmark Problem with Sinusoidal Reactivity Insertion.

Reactivity ρ (t)	Time period of sinusoidal reactivity(sec)	β - value	Mean generation time (sec)
ρ_0* Sin(π t/T) ρ_0 = 5.333 mk	50	0.0079	10^{-6}

When we insert step reactivity in a reactor, the largest Eigen value of matrix 'A" will be constant at any time, correspondingly, Z_{max} (t) = constant. Whereas, in ramp reactivity insertion cases, Z_{max} (t) will be varying with respect to time, correspondingly errors also will changes with time. To study the errors as a function of time, two cases are studied.

Case 1. Error Estimation in the Thermal Reactor with ramp reactivity insertion of 0.1 t $/sec (Thermal Reactor Benchmark Problem-1 (Nahla (2011)):

Errors are estimated corresponding to different step sizes i.e. h = 0.002, 0.005, 0.0002 and 0.00002 sec.it is observed that good accuracy of power transient values is achieved with 0.005, 0.0002 sec step size values. For both step sizes, condition (8) is satisfied. If we choose Z(t) just below 2.00 at time t = 0.0, then as a function of time most of the cases, a(t) will decrease with increase in time and consequently condition of (8) will be satisfied at all the time steps. Finally, time step h = 0.005 sec was estimating transient

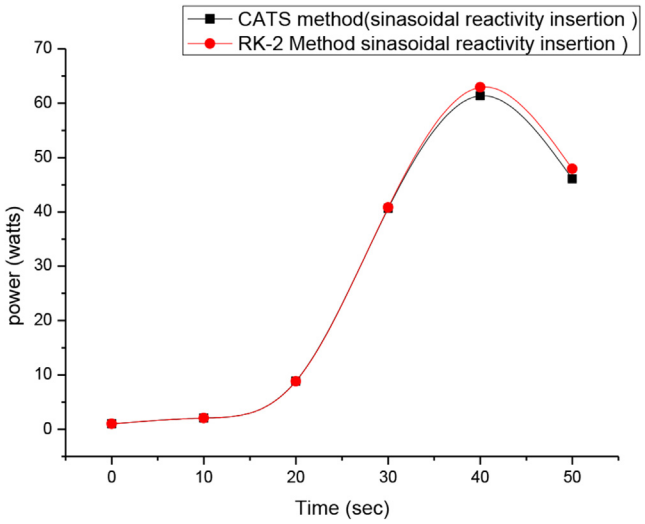

Fig. 8. Comparison of Power transients initiated due to Sinusoidal Reactivity Insertion with Time Period of 50 sec.

Table 11
Reactor Operation with Sinusoidal Reactivity.

Time (sec)	Neutron density (neutrons/ cm3) CATS method	Neutron density (neutrons/ cm3) R.K-2 method (h = 0.0001 sec)	Relative Error
0.0	1.000000000E + 00	1.000000E + 00	0.0000E + 00
10.0	2.065311114E + 00	2.065184E + 00	−6.1547E-05
20.0	8.852831171E + 00	8.839174E + 00	−1.5427E-03
30.0	4.063328342E + 01	4.082185E + 01	4.6407E-03
40.0	6.134134599E + 01	6.294450E + 01	2.6135E-02
50.0	4.609559630E + 01	4.792427E + 01	3.9671E-02

power values accurately with sufficiently larger time step with lesser amount of computational time. Table 12 Shows Abs. Errors in power values, corresponding to different time steps at various time intervals. Table 13 shows the variation of 'Z 'value at different time intervals. Fig. 9 shows variation of Abs.error in the power w.r.t. to time corresponding to different time step values. From the figure Error in power values corresponding to h = 0.005, 0.0002 sec are

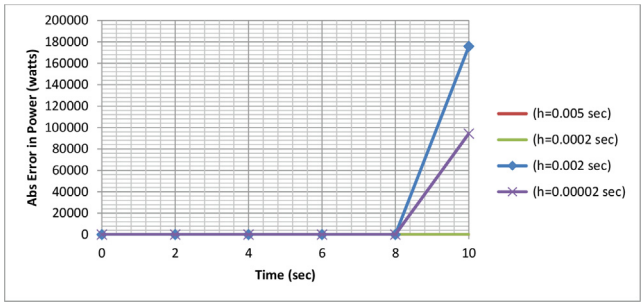

Fig. 9. Abs.Error (t) vs time plot corresponding to different 'h' values Of thermal reactor benchmark-1 with ramp reactivity Insertion.

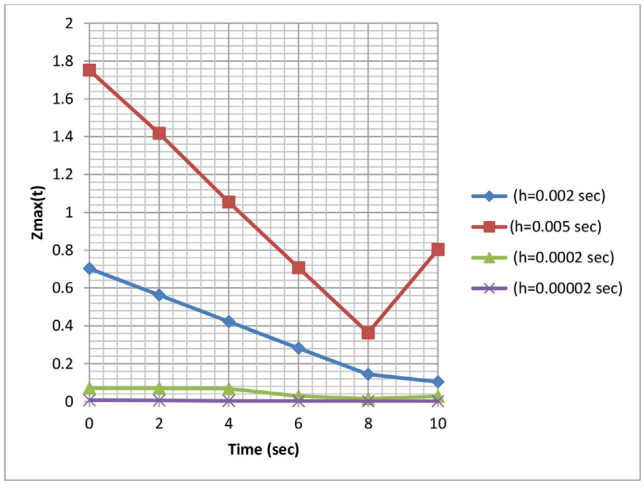

Fig. 10. $Z_{max}(t)$ vs time plot corresponding to different 'h' values Of thermal reactor benchmark-1 with ramp reactivity Insertion.

lesser than errors corresponding h = 0.002, 0.00002 sec values. Fig. 10 shows variation of 'Z' values from 1.75 to 0.80 in the time interval t = 0 to t = 10.0 sec with the time step size of (h = 0.005 sec).

Case 2. Error Estimation in the Indian Prototype Fast Breeder with 1.0 pcm/sec positive

Table 12
Abs.Error (t) values corresponding different 'h' values.

Time (sec)	Ab.Error (t) (h = 0.002 sec)	Ab.Error (t) (h = 0.005 sec)	Ab.Error (t) (h = 0.0002 sec)	Ab.Error (t) (h = 0.00002 sec)	Ab.Error (t) (h = 0.000002 sec)
0	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0015	0.0006	0.0004	0.0071	0.0018
4	0.0082	0.0008	0.0033	0.0048	0.0494
6	0.0578	0.0074	0.0251	0.0349	0.4387
8	1.5477	0.1779	0.6995	0.9208	11.7682
10	175695.3465	37.7163	77.8684	94283.6291	430850.0152

Table 13
$Z_{max}(t)$ values corresponding different 'h' values.

Time (sec)	$Z_{max}(t)$ (h = 0.002 sec)	$Z_{max}(t)$ (h = 0.005 sec)	$Z_{max}(t)$ (h = 0.0002 sec)	$Z_{max}(t)$ (h = 0.00002 sec)
0	0.7026	1.7517	0.0701	0.0068
2	0.5625	1.4173	0.0689	0.0056
4	0.4222	1.0551	0.0687	0.0042
6	0.2823	0.7072	0.0282	0.0028
8	0.1441	0.3634	0.0145	0.0014
10	0.1045	0.8033	0.0272	0.0013

Table 14
Abs.Error (t) values corresponding to different 'h' values.

Time (sec)	Abs.Error (t) h = 0.0001 sec	Abs.Error (t) h = 0.0002 sec	Abs.Error (t) h = 0.00023 sec	Abs.Error (t) h = 0.000232 sec	Abs.Error (t) h = 0.0000125 sec	Abs.Error (t) h = 0.00005 sec
0	0	0	0	0	0	0
10	0.0006	0.0006	0.0002	3.9E + 10	0.0008	0.0010
20	0.0019	0.0018	0.0003	4.4E + 10	0.0063	0.0015
30	0.0031	0.0058	0.0026	5.8E + 10	0.0351	0.0012
40	0.0039	0.0142	0.0118	8.9E + 10	0.0799	0.0012
50	0.0013	0.0315	0.0390	1.6E + 10	0.1167	0.0134
60	0.0193	0.0721	0.1308	3.5E + 11	0.0354	0.0633
70	0.1379	0.1743	0.4898	9.0E + 11	0.6619	0.1554
80	0.8415	0.4229	2.1623	2.9E + 11	0.3676	1.0056
90	5.6481	0.6852	11.7657	1.2E + 13	27.6214	17.5374
100	46.0497	5.1506	81.488	6.7E + 13	486.1153	219.2323

Table 15
$Z_{max}(t)$ values corresponding to different 'h' values.

Time (sec)	$Z_{max}(t)$ h = 0.0001 sec	$Z_{max}(t)$ h = 0.0002 sec	$Z_{max}(t)$ h = 0.00023 sec	$Z_{max}(t)$ h = 0.000232 sec	$Z_{max}(t)$ h = 0.0000125 sec	$Z_{max}(t)$ h = 0.00005 sec
0	0.8651	1.7324	1.9734	2.0115	0.1081	0.4325
10	0.8407	1.6863	1.9338	1.9505	0.1051	0.4193
20	0.8164	1.6327	1.8773	1.9024	0.1020	0.4088
30	0.7922	1.5839	1.8214	1.8436	0.0987	0.3901
40	0.7680	1.5354	1.7655	1.7814	0.0959	0.3841
50	0.7439	1.4871	1.7097	1.7218	0.0931	0.3720
60	0.7197	1.4387	1.6539	1.6756	0.0903	0.3599
70	0.6900	1.3903	1.5981	1.6162	0.0871	0.3473
80	0.6713	1.3419	1.5423	1.5527	0.0834	0.3343
90	0.6471	1.2936	1.4864	1.5062	0.0797	0.3212
100	0.6229	1.2452	1.4306	1.4432	0.0760	0.3080

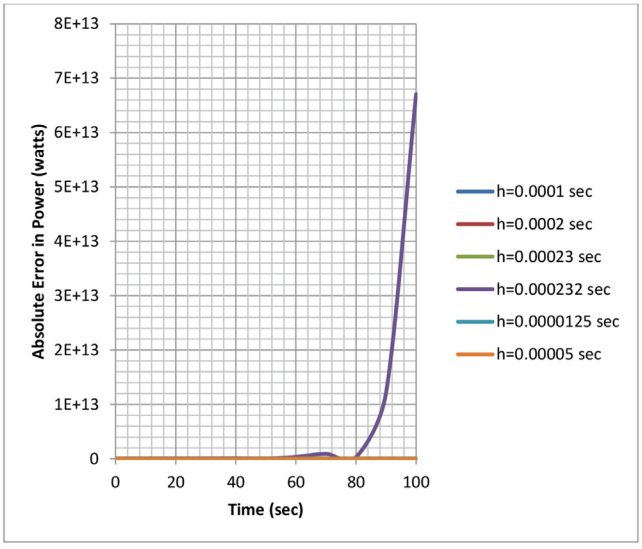

Fig. 11. Abs.Error (t) vs time plot corresponding to different 'h' values Of Fast reactor with ramp reactivity Insertion.

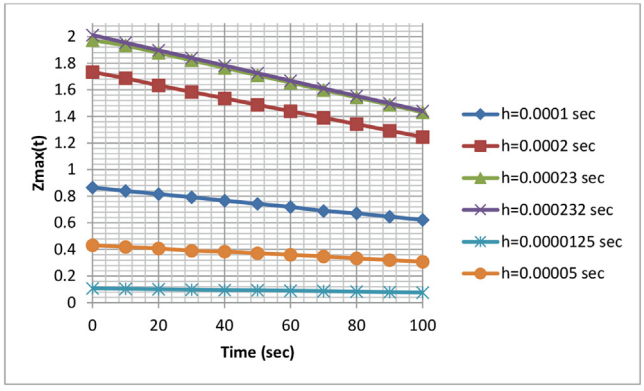

Fig. 12. $Z_{max}(t)$ vs time plot corresponding to different 'h' values of fastreactor with ramp reactivity Insertion.

4.1. Ramp reactivity insertion

Errors are estimated corresponding to six different time steps ranging from h = 0.000232 to 0.00005 sec.it is observed that good accuracy of power transient power values is achieved with 0.0001, 0.0002 sec step size values. For both step sizes, condition (8) is satisfied. If we choose Z just below 2.00 at time t = 0.0, then as a function of time most of the cases, Z(t) will decrease with increase in time and consequently condition of (8) will be satisfied at all the time steps. Finally, time step h = 0.0001, 0.0002 sec will estimates transient power values accurately. Table 14 Shows Abs. Errors in power values, corresponding to different time step sizes at various time intervals. Table 15 shows the variation of 'Z' value at different time intervals. Fig. 11 shows variation of Abs. Error in the power w. r.t. to time corresponding to different time step values. From the figure, Error in power values corresponding to h = 0.0001, 0.0002 sec are lesser than errors corresponding h = 0.00023, 0.000232, 0.0000125 and 0.00005 sec values. Fig. 12 shows variation of 'Z' values corresponding to different time step values at different time interval. Here we can see that at h = 0.0002 sec '$Z_{max}(t)$'value varies from 1.7324 to1.2452 sec which is well in the limit of condition (8), for this time step value with ramp reactivity insertion in fast reactor transient power values are in good agreement with exact values. In the case of h = 0.000232 sec Error is increasing drastically increased from t > 0 sec, this clearly shows that solution RK2 method is unstable, even if the 'Z' value is slightly above or equivalent to 2.0.

5. Conclusions

RK-2 method is used for solving PKEs with multi-group of delayed neutrons. The method is applied for analysing power transients of both thermal and fast reactors. The advantage of using this method is that from the largest Eigen value, optimum value of time step can be estimated. Most of the cases, time step chosen here is larger than TSM method. It is seen that there is good amount of accuracy in prediction of the power transients using time step of the order of 10^{-3}-10^{-4} sec. Three different types of reactivity perturbations i.e, step, ramp and oscillatory were studied and the results are compared with standard methods. It is seen that RK-2 method estimates the power transients accurately for both thermal and fast reactors. For most of the cases analysed, the power transient values at different steps are almost exactly matching with the power values computed using other method like analytical, Cohen's method and CATS method. Relative errors computed in all the cases are below the order of $10^{-2.}$

6. Funding Agency

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

CRediT authorship contribution statement

P. Ravindra Babu: Methodology, Software. **Usha Pal:** Investigation, Writing – original draft. **R.N. Sen:** Validation, Software. **R. Karthikeyan:** Visualization, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

Aboander, A.E., Nahla, A.A., 2002a. Generalization of the analytical inverse method for the solution of the point kinetics equations. J. Phys A: Math. Gen. 35, 3245–3263.

Aboander, A.E., Nahla, A.A., 2002b. Solution of point kinetics equation in presence of Newtonian temperature feedback by pade approximations via the analytical inversion method. J. Phys, A: Math. Gen. 35, 9609–9627.

Cohen, R., 1958. Proceedings of the second United Nations International conference on the Peaceful uses of atomic energy, held in Geneva, vol.11. pp. 302–309.

Hamada, Y.M., 2018. A new accurate numerical method based on shifted chebyshev series for nuclear reactor dynamical systems. Sci. Technol. Nucl. Install. 2018, 1–15.

Hennart, J.P., Barrios, B.T., 1976. Generalized Pade and Chebyshev approximations for point kinetics. Nucl. Sci. Eng. 59, 170–198.

Li, H., Chen, W., Luo, L., Zhu, Q., 2009. A new integral method for solving reactor neutron kinetics equations. Ann. Nucl Energy 36, 427–432.

Mohideen Abdul Razak, M., Devan, K., Shathiyasheela, T.. 2015. The modified exponential time differencing (ETD) method for solving the reactor point kinetics equations, Annals of nuclear energy, vol. 76, pages 193-199.

Nahla, A.A., 2011. Taylor series method for solving point kinetics equations. Nucl. Eng. Des. 241, 1592–1595.

Riyas, A., Mohanakrishnanan, P., 2008. Studies on physics parameters of metal (U-Pu-Zr) fuelled PBR cores. Ann. Nucl. Energy 35, 87–92.

Venkateshan, S.P., Swaminathan, P., 2014. Computational Methods in Engineering. Academic Press.