Math 3P40 final project

Deadline: April 20, 2017 (at 1pm in the assignment box)

You must select a topic for the final project from the list at the end. There is also a possibility of choosing a topic which is not on that list, but related to the course material, but in that case you must obtain my approval first.

The complete project should include two parts:

- written report
- supporting software (printout of the code if it is short - up to 3 or 4 pages, or, if it is something longer, email me and attach sources).

Written report should be well-organized, and it should normally include the following elements (not necessarily in that order):

- Introduction and discussion of the underlying mathematics
- Description of the method you used to verify your claims
- Description of the supporting software (may be brief)
- Analysis of the evidence generated by your program: curve fitting, graphs, calculations, etc.
- Conclusion: is the evidence you presented convincing? What could be done to improve your results? What did you learn in this project?
- Bibliography: if you used books, articles, web sites, or other sources, this should be placed here.

Important notes:

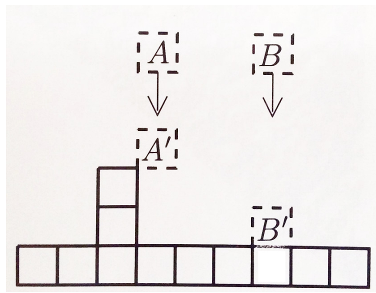
1) Your written report **should not exceed 10 pages**, including all figures. The length of the report does not determine your mark – the quality does.
2) Your software program will be graded based on its correctness and readability (inlcude comments in all non-obvious places, given descriptive names to variables)
3) All figures/plots/graphs must have numbered captions. In the written report, refer to them as Fig. 1, Fig. 2, etc.
4) All graphs must have clearly labeled axes.
5) Your report should be neat and well organized. Pay attention to correct spelling and grammar.

IMPORTANT: **I will not be helping you to debug your programs for the final project. You will have to debug your programs on your own.**

Possible topics for the final project

Ballistic deposition with "corner" sticking

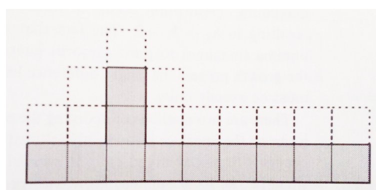
Simulate the model of ballistic deposition where particles can stick to a diagonal neighbour as well, as shown in the picture below.



Find the exponents α and β using the same method as in assignment 3, and use scaling relationship to find exponent z.

Single step deposition model

Simulate the growth of a surface in which every time step a single particle is added to a site randomly selected among all sites (dotted sites) adjacent to the existing surface (shaded sites).



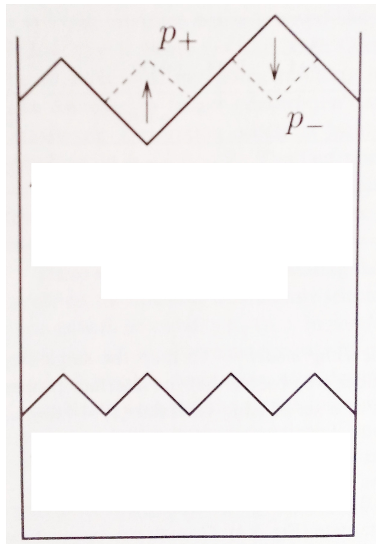
This is a version of a classic growth model introduced in 1961 by M. Eden. Estimate the exponents α and β using the same method as in assignment 3, and use scaling relationship to find exponent z.

Restricted SOS model

Simulate the surface growth in the restricted SOS model, introduced by J. M. Kim and J. M. Kosterlitz in 1989. In this model, a randomly selected site i is selected, and the surface height is increased by one, $h_i \to h_i + 1$, provided that after the deposition the difference of height between column i and each of the neighbouring columns satisfies $|\Delta h| \in \{0, 1\}$. Estimate the exponents α and β using the same method as in assignment 3, and use scaling relationship to find z.

Single step model, also called kinetic Ising model

Simulate the surface growth in a model similar to rule 184, where all surface minima are filled with deposited particles with probability p_+, and all maxima are erased with probability p_-, as shown below.



The initial surface is as shown at the bottom of the above picture. Assume $p_+ = p_- = 1/2$. Find the relevant exponents α, β, z. Two possible update schemes are possible here: either all minima and maxima are simultaneously updated (parallel update) or one site is randomly selected and update per time step (sequential update). You man want to study both update schemes and check if critical exponents depend on the update scheme.

Cluster distribution in critical percolation

Study percolation model of your choice (site or bond, square, triangular or hexagonal lattice) and verify that at the percolation threshold, the number of clusters of size s, to be denoted as n_s, behaves as a power law, $n_s \sim s^{-\tau}$. Estimate the Fisher exponent τ. Note: this is a bit more ambitious project, as you need to devise a procedure for counting clusters of a given size.

Glauber dynamics

Implement Glauber dynamics for Ising model and produce a plot of magnetization as a function of β. Verify that the phase transition happens at the point predicted by Onsager (formula given in class).

Sand pile model

Implement the sand pile model of Bak et al. and verify that $N(s)$, the number of avalanches of size s, behaves as $s^{-\tau}$, where τ is close to 1. Again, this topic will require quite a bit of programming, because you need to devise a method for counting avalanches.

Forest fire model

Implement the Drossel-Schwab forest fire model and verify some of the power laws believed to be present in the model. Programming component will be significant.