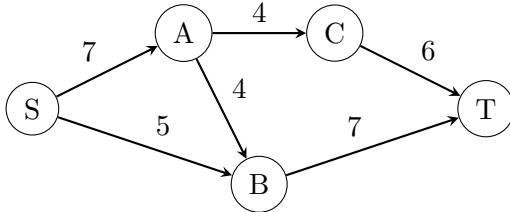


CS 170 DIS 08

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1 Residual in graphs

Consider the following graph with edge capacities as shown:



- Consider pushing 4 units of flow through $S \rightarrow A \rightarrow C \rightarrow T$. Draw the residual graph after this push.
- Compute a maximum flow of the above graph. Find a minimum cut. Draw the residual graph of the maximum flow.

2 A cohort of spies

A cohort of k spies resident in a certain country needs escape routes in case of emergency. They will be travelling using the railway system which we can think of as a directed graph $G = (V, E)$ with V being the cities. Each spy i has a starting point $s_i \in V$ and needs to reach the consulate of a friendly nation; these consulates are in a known set of cities $T \subseteq V$. In order to move undetected, the spies agree that at most c of them should ever pass through any one city. Our goal is to find a set of paths for each of the spies (or detect that the requirements cannot be met).

Hint: Model this problem as a flow network.

3 Repairing a Flow

In a particular network $G = (V, E)$ whose edges have integer capacities c_e , we have already found the maximum flow f from node s to node t . However, we now find out that one of the capacity values we used was wrong: for edge (u, v) we used c_{uv} whereas it should have been $c_{uv} - 1$. This is unfortunate because the flow f uses that particular edge at full capacity: $f_{uv} = c_{uv}$. We could redo the flow computation from scratch, but there's a faster way. Show how a new optimal flow can be computed in $O(|V| + |E|)$ time.