

CS 170 Dis 1

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1 (★★) Recurrence Relations

- (a) $T(n) = 4T(n/2) + 42n$
- (b) $T(n) = 4T(n/3) + n^2$
- (c) $T(n) = 2T(2n/3) + T(n/3) + n^2$
- (d) $T(n) = 3T(n/4) + n \log n$

2 (★★) Sorted Array

Given a sorted array A of n (possibly negative) distinct integers, you want to find out whether there is an index i for which $A[i] = i$. Give a divide-and-conquer algorithm that runs in time $O(\log n)$. Provide only the main idea and the runtime analysis.

3 (★★) Counting inversions

This problem arises in the analysis of *rankings*. Consider comparing two rankings. One way is to label the elements (books, movies, etc.) from 1 to n according to one of the rankings, then order these labels according to the other ranking, and see how many pairs are “out of order”.

We are given a sequence of n distinct numbers a_1, \dots, a_n . We say that two indices $i < j$ form an inversion if $a_i > a_j$ that is if the two elements a_i and a_j are “out of order”. Provide a divide and conquer algorithm to determine the number of inversions in the sequence a_1, \dots, a_n in time $O(n \log n)$ (*Hint*: Modify merge sort to count during merging)

4 (★) Complex numbers review

Briefly justify your answers to parts (b) and (c).

- (a) Write each of the following numbers in the form $\rho(\cos \theta + i \sin \theta)$ (for real ρ and θ):
 - (i) $-\sqrt{3} + i$
 - (ii) The three 3-rd roots of unity
 - (iii) The sum of your answers to the previous item
- (b) Let $\text{sqrt}(x)$ represent one of the complex square roots of x , so that $\text{sqrt}(x) = \pm\sqrt{x}$. What are the possible values of $\text{sqrt}(\text{sqrt}(-1))$?

You can use any notation for complex numbers, e.g., rectangular, polar, or complex exponential notation.