CS 170 Dis 10

Released on 2017-04-04

1 Optimization versus Search

Recall the following definition of the Traveling Salesman Problem, which we will call TSP. We are given a complete graph G of whose edges are weighted and a budget b. We want to find a tour (i.e., path) which passes through all the nodes of G and has length $\leq b$, if such a tour exists.

The optimization version of this problem (which we call TSP-OPT) asks directly for the shortest tour.

(a) Show that if TSP can be solved in polynomial time, then so can TSP-OPT.

(b) Do the reverse of (a), namely, show that if TSP-OPT can be solved in polynomial time, then so can TSP.

2 A Faulty Reduction

In the Redrata path problem (AKA the Hamiltonian Path Problem), we are given a graph G and want to find if there is a path in G that uses each vertex exactly once.

What is wrong with the following argument?

We will show that Undirected Rudrata Path can be reduced to Longest Path in a DAG. Given a graph G, use DFS to find a traversal of G and assign directions to all the edges in G based on this traversal (i.e. edges will point in the same direction they were traversed and back edges will be omitted). This gives a DAG. If the longest path in this DAG has $|V| - 1$ edges then there is a Rudrata path in G since any simple path with $|V| - 1$ edges must visit every vertex.

3 Hitting Set

In the Hitting Set Problem, we are given a family of sets $\{S_1, S_2, \ldots, S_n\}$ and a budget b, and we wish to find a set H of size $\leq b$ which intersects every S_i, if such an H exists. In other words, we want $H \cap S_i \neq \emptyset$ for all i.
Show that the Hitting Set Problem is NP-complete.