CS 170 HW 8

Due on 2018-03-18, at 11:59 pm

1 (★) Study Group

List the names and SIDs of the members in your study group.

2 (★★) Linear Programming Warm-ups

1. Find necessary and sufficient conditions on the reals a and b under which the linear program

$$\begin{aligned} \max \ & x + y \\ & ax + by \leq 1 \\ & x, y \geq 0 \end{aligned}$$

   i Is infeasible.

   ii Is unbounded.

   iii Has a unique optimal solution.

2. Write the dual to following linear program.

$$\begin{aligned} \max \, & x + y \\ & 2x + y \leq 3 \\ & x + 3y \leq 5 \\ & x, y \geq 0 \end{aligned}$$

   Find optimal solutions to both primal and dual LPs.

3 (★★★) Provably Optimal

For the linear program

$$\begin{aligned} \max \ & x_1 - 2x_3 \\ & x_1 - x_2 \leq 1 \\ & 2x_2 - x_3 \leq 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

show that the solution $(x_1, x_2, x_3) = (3/2, 1/2, 0)$ is optimal **using its dual**. You should not have to directly solve for the optimum of the dual.

(*Hint:* Recall that any feasible solution of the dual is an upper bound on any feasible solution of the primal)

4 (★★★) Major Key

You are a locksmith tasked with producing keys $k_1, ..., k_n$ that sell for $p_1, ..., p_n$ respectively. Each key k_i takes g_i grams of gold and s_i grams of silver. You have a total of G gold and S silver to work with, and can produce as many keys of any type as you want within the time and material constraints.

1. Unfortunately, integer linear programming is an NP-complete problem. Fortunately, you have found someone to instead buy the alloys at an equivalent price! Instead of selling keys, you have decided to focus on melting the prerequisite metals together, and selling the mixture. Formulate the linear program to maximize the profit of the locksmith, and explain your decision variables, objective function, and constraints.

2. Formulate the dual of the linear program from part (a), and explain your decision variables, objective function, and constraints. The explanations provide economic intuition behind the dual. We will only be grading the dual formulation.
   Hint: Formulate the dual first, then think about it from the perspective of the locksmith when negotiating prices for buying G gold and S silver if they had already signed a contract for the prices for the output alloys p_i. Think about the breakeven point, from which the locksmith's operations begin to become profitable for at least one alloy.

5 (★★) Zero-Sum Battle

Two Pokemon trainers are about to engage in battle! Each trainer has 3 Pokemon, each of a single, unique type. They each must choose which Pokemon to send out first. Of course each trainer's advantage in battle depends not only on their own Pokemon, but on which Pokemon their opponent sends out.

The table below indicates the competitive advantage (payoff) Trainer A would gain (and Trainer B would lose). For example, if Trainer B chooses the fire Pokemon and Trainer A chooses the rock Pokemon, Trainer A would have payoff 2.

		Trainer B: ice	grass	fire
Trainer A:	dragon	-10	3	3
	steel	4	-1	-3
	rock	6	-9	2

Feel free to use an online LP solver to solve your LPs in this problem.
Here is an example of a Python LP Solver and its Tutorial.

1. Write an LP to find the optimal strategy for Trainer A. What is the optimal strategy and expected payoff?

2. Now do the same for Trainer B. What is the optimal strategy and expected payoff?