CS 170 HW 12

Due on 2018-04-23, at 11:59 pm

1 (★) Study Group

List the names and SIDs of the members in your study group.

2 (—) Nostalgia

What's been your favorite homework problem this semester? Tell us the HW number and problem name/number, and briefly explain (a sentence or two) why you liked it.

3 (★★) Multiplicative Weights

(a) We are running Multiplicative Weights with $n = 2$ experts for $T = 10,000$ days, and we choose $\epsilon = 0.01$ Is this close to the optimum choice? (Hint: $\sqrt{\ln 2} \approx .8325$)

(b) In all of the first 140 days, Expert 1 has cost 0 and Expert 2 has cost 1. In the next day, with what probability will you play Expert 1? (Hint: You can assume that $0.99^{70} = \frac{1}{2}$)

4 (★★★★) Experts Alternatives

Recall the experts problem. Every day you must take the advice of one of n experts. At the end of each day t, if you take advice from expert i, the advice costs you some c_i^t in $[0, 1]$. You want to minimize regret R,

$$R = \frac{1}{T}(\sum_{t=1}^{T} c_{i(t)}^t - \min_{i \leq i \leq n} \sum_{t=1}^{T} c_i^t)$$

where $i(t)$ is the expert you choose on day t. Your strategy will be probabilities where p_i^t denotes the probability with which you choose expert i on day t. Assume an all powerful adversary can look at your strategy ahead of time and decide the costs associated with each expert on each day. Give the maximum possible expected regret that the adversary can guarantee in each of the following cases:

(a) Choose expert 1 at every step. That is, if $\forall t \; p_1^t = 1$, what is $\max_{C_i^t} R$? C_i^t is the set of costs for all experts and all days.

(b) Any deterministic strategy. Note that a "determinstic strategy" can be thought of as a probability distribution that satisfies the following: $\forall t \; \exists i \; p_i^t = 1$.

(c) Always choose an expert according to some fixed probability distribution at every time step. That is, if for some $p_1 ... p_n$, $\forall t, p_i^t = p_i$, what is $\max_{C_i^t}(\mathbb{E}[R])$?)

(d) In the last case, express your worst-case regret bound in terms of the $p_i's$, where $p_i \geq 0$ is the probability of choosing expert i and $\sum_i p_i = 1$. What is the best distribution? In other words, what is $\text{argmin}_{p_1 \dots p_n} \max_{C_i^t}(\mathbb{E}[R])$?)

This analysis should conclude that a good strategy for the problem must necessarily be randomized and adaptive.

5 (★★★★) [OPTIONAL] Knightmare

Give an algorithm to find the number of ways you can place knights on an N by M chessboard such that no two knights can attack each other (there can be any number of knights on the board, including zero knights). The runtime should be $O(2^{3M} \cdot N)$ (or symmetrically, switch the variables).

6 (★★★★) [OPTIONAL] Min Cost Flow

In the max flow problem, we just wanted to see how much flow we could send between a source and a sink. But in general, we would like to model the fact that shipping flow takes money. More precisely, we are given a directed graph G with source s, sink t, costs l_e, capacities c_e, and a flow value F. We want to find a nonnegative flow f with minimum cost, that is $\sum_e l_e f_e$, that respects the capacities and ships F units of flow from s to t.

(a) Show that the minimum cost flow problem can be solved in polynomial time.

(b) Show that the shortest path problem can be solved using the minimum cost flow problem

(c) Show that the maximum flow problem can be solved using the minimum cost flow problem.

7 (★★★★) [OPTIONAL] Graph Game

Given an undirected, unweighted graph, with each node having a certain value, consider the following game.

1. All nodes are initially *unmarked* and your score is 0.
2. First, you choose an unmarked node u. You look at the neighbors of u and add to your score the sum of the values of the *marked* neighbors v of u.
3. Then, mark u.
4. You repeat the last two steps for as many turns as you like (you *do not* have to mark all the nodes. Each node can be marked at most once).

For instance, suppose we had the graph $A - B - C$ with A, B, C having values $3, 2, 3$ respectively. Then, the optimal strategy is to mark A then C then B giving you a score of $0 + 0 + 6$. We can check that no other order will give us a better score.

a Suppose all the node values are nonnegative. Give an efficient greedy algorithm to determine the order to mark nodes to maximize your score. Justify your answer with an exchange argument.

b Now, node values can be negative. Show this problem is NP-hard by giving a reduction from INDEPENDENT SET