CS 170 Dis 5

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1 Minimum Spanning Trees

For each of the following statements, either prove or supply a counterexample. Always assume $G = (V, E)$ is undirected and connected. Do not assume the edge weights are distinct unless specifically stated.

1. Let e be any edge of minimum weight in G. Then e must be part of some MST.

2. If e is part of some MST of G, then it must be a lightest edge across some cut of G.

3. If G has a cycle with a unique lightest edge e, then e must be part of every MST.

4. For any $r > 0$, define an r-path to be a path whose edges all have weight less than r. If G contains an r-path from s to t, then every MST of G must also contain an r-path from s to t.

2 Divide and Conquer for MST?

Is the following algorithm correct? If so, prove it. Otherwise, give a counterexample and explain why it doesn't work.

procedure FindMST(G: graph on n vertices)
 If $n = 1$ return the empty set
 $T_1 \leftarrow$ FindMST(G_1: subgraph of G induced on vertices $\{1, \ldots, n/2\}$)
 $T_2 \leftarrow$ FindMST(G_2: subgraph of G induced on vertices $\{n/2 + 1, \ldots, n\}$)
 $e \leftarrow$ cheapest edge across the cut $\{1, \ldots, \frac{n}{2}\}$ and $\{\frac{n}{2} + 1, \ldots, n\}$.
 return $T_1 \cup T_2 \cup \{e\}$.

3 Huffman Proofs

1. Prove that in the Huffman coding scheme, if some character occurs with frequency more than $\frac{2}{5}$, then there is guaranteed to be a codeword of length 1. Also prove that if all characters occur with frequency less than $\frac{1}{3}$, then there is guaranteed to be no codeword of length 1.

2. Under a Huffman encoding of n symbols with frequencies $f_1, f_2, \ldots, f_n$, what is the longest a codeword could possibly be? Give an example set of frequencies that would produce this case, and argue that it is the longest possible.

4 Horn Formula Practice

Find the variable assignment that solves the following horn formulas:

1. $(w \wedge y \wedge z) \Rightarrow x, (x \wedge z) \Rightarrow w, x \Rightarrow y, \Rightarrow x, (x \wedge y) \Rightarrow w, (\bar{w} \vee \bar{x}, \vee \bar{y}), (\bar{z})$

2. $(x \wedge z) \Rightarrow y, z \Rightarrow w, (y \wedge z) \Rightarrow x, \Rightarrow z, (\bar{z} \vee \bar{x}), (\bar{w} \vee \bar{y} \vee \bar{z})$