$$T(n) = 2T(n/2) + \frac{n}{\lg n}$$

Guess: $O(n \cdot \lg(\lg n))$

Assume: $T(k) \leq C \cdot n \cdot (\lg(\lg n))$

for $k < n$, $C > 0$

$$T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\lg n}$$

$$\leq 2\left(\frac{n}{2} \cdot \lg\left(\lg\left(\frac{n}{2}\right)\right)\right)$$

$$= \cancel{2}\frac{n}{\cancel{2}} \cdot 2\lg\left(\lg\left(\frac{n}{2}\right)\right)$$

$$= n \cdot 2\lg(\lg n - \lg 2)$$

$$= n \cdot 2\lg(\lg n - 1)$$

$$\leq n \cdot \lg(\lg n)$$

Thus, $T(n) \leftarrow O(n \cdot \lg(\lg n))$

$T(n) = 7T(n/2) + n^2$

$a = 7 \quad b = 2$

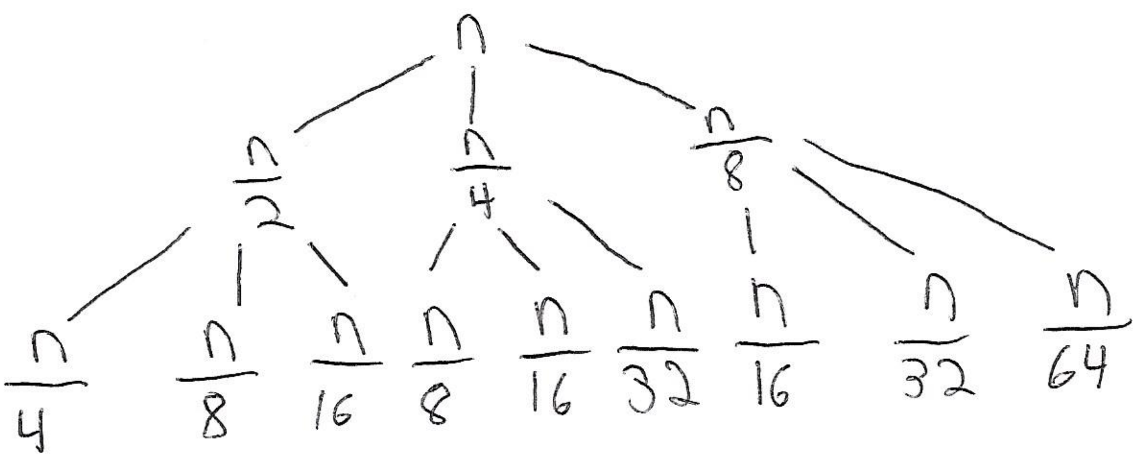
If $f(n) = \Theta(n^{\log_b a - \varepsilon})$, for some $\varepsilon > 0$

Yes! $\varepsilon = 3$

$T(n) = \cancel{\Theta}(n^2)$

Thus, $T(n) \leftarrow \Theta(n^2)$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + T\left(\frac{n}{8}\right) + n$$



#nodes	cost
$3^0 = 1$	n
$3^1 = 3$	$\frac{7}{8}n$
$3^2 = 9$	$\frac{49}{64}n = \left(\frac{7}{8}\right)^2 n$
$\vdots$	$\vdots$

For each row i: $\left(\frac{7}{8}\right)^i n$

Infinite Geometric series

$$\sum_{i=0}^{\infty} a r^i n = \frac{a_1}{1-r} n = \ldots$$

$$\sum_{i=0}^{\log_3 n} 1\left(\frac{7}{8}\right)^i n = \frac{1}{1-\frac{7}{8}} \cdot n = 8n$$

Thus, $T(n) \longleftarrow \Theta(n)$

$T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$

$a = 2 \qquad b = 4$

If $f(n) = O\left(n^{\log_b a - \varepsilon}\right)$, for some $\varepsilon > 0$

No!

If $f(n) = \Theta\left(n^{\log_b a}\right)$

Yes!

$T(n) = \Theta\left(n^{\log_b a} \log n\right) = \Theta(\sqrt{n} \log n)$

Thus, $T(n) \leftarrow \Theta(\sqrt{n} \log n)$

Assignment 1

Question 2:

n!	n!
2^n	N^71+5^n +17n
	4^n
	2^n
	n71+5^n +17n
n^2	¾n^4
	N^3
	√n^3
	N^3-log n
	N^2
n	18n
log(n)	log10 n
	3 $\log_2$ n
C^log(n)	2^$\log_2$ n

Question 3:

Upper bound is O(nlogn)