Project One Template

MAT350: Applied Linear Algebra

Matthew Muller

Date: 11/11/21

Problem 1

Develop a system of linear equations for the network by writing an equation for each router (A, B, C, D, and E). Make sure to write your final answer as A**x**=**b** where A is the 5x5 coefficient matrix, **x** is the 5x1 vector of unknowns, and **b** is a 5x1 vector of constants.

Solution:

Router A : 2x1 + x2 = 100 (Router A recieves 100 and outputs x1, x1, and x2)

Router B : x1 + x2 - x3 - x5 = 0 (Router B recieves x1 and x2 and outputs x3 and x5)

Router C : -x1 + x3 + x5 = 50 (Router C recieves 50 and x1 and outputs x3 and x5)

Router D : -x2 + x4 + x5 = 120 (Router D recieves x4 and x5 and outputs x2 and 120)

Router E : x2 + x3 - x4 + x5 = 0 (Router E recieves x2, x3, and x5 and outputs x4)

I solved for all of these by setting up equations with inputs on the left and outputs on the right and rearranging to the correct format. (I could not figure out how to output the matrices in an Ax = b format)

```
%code
A = [2 1 0 0 0; 1 1 -1 0 -1; -1 0 1 0 1; 0 -1 0 1 1; 0 1 1 -1 1]
```

```
A = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
    -1     0     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
syms x1 x2 x3 x4 x5
x = [x1; x2; x3; x4; x5]
```

x =

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$$

```
b = [100; 0; 50; 120; 0]
```

```
b = 5×1
```

```
   100
     0
    50
   120
     0
```

Problem 2

Use MATLAB to construct the augmented matrix [A **b**] and then perform row reduction using the rref() function. Write out your **reduced matrix and identify the free and basic variables of the system**.

Solution:

x1 and x2 are basic variables since they are pivot columns. Since the remaining columns are not pivot columns, x3, x4, and x5 are free variables.

```
%code
%Create augmented matrix
Ab = [A b]
```

```
Ab = 5×6
     2     1     0     0     0   100
     1     1    -1     0    -1     0
    -1     0     1     0     1    50
     0    -1     0     1     1   120
     0     1     1    -1     1     0
```

```
%Reduce augmented matrix
[rowreducedAb, pivotvarsAb] = rref(Ab)
```

```
rowreducedAb = 5×6
     1     0     0     0     0    25
     0     1     0     0     0    50
     0     0     1     0     0    30
     0     0     0     1     0   125
     0     0     0     0     1    45
pivotvarsAb = 1×5
     1     2     3     4     5
```

Problem 3

Use MATLAB to **compute the LU decomposition of A**, i.e., find A = LU. For this decomposition, find the transformed set of equations L**y** = **b**, where **y** = U**x**. Solve the system of equations L**y** = **b** for the unknown vector **y**.

Solution:

```
%code
%Compute the LU decomposition of A. Store the lower and upper matrices in L and U
```

```
[L, U] = lu(A)
```

```
L = 5×5
    1.0000         0         0         0         0
    0.5000   -0.5000   -1.0000    1.0000         0
   -0.5000   -0.5000    1.0000         0         0
         0    1.0000         0         0         0
         0   -1.0000    1.0000   -0.5000    1.0000
U = 5×5
    2.0000    1.0000         0         0         0
         0   -1.0000         0    1.0000    1.0000
         0         0    1.0000    0.5000    1.5000
         0         0         0    1.0000    1.0000
         0         0         0         0    1.0000
```

```
%Solve the system using LU decomposition. The intermediary solution is stored in y.
y = L\b
```

```
y = 5×1
   100
   120
   160
   170
    45
```

Problem 4

Use MATLAB to **compute the inverse** of U using the inv() function.

Solution:

```
%code
%Compute the inverse of U
invU = inv(U)
```

```
invU = 5×5
    0.5000    0.5000         0   -0.5000         0
         0   -1.0000         0    1.0000         0
         0         0    1.0000   -0.5000   -1.0000
         0         0         0    1.0000   -1.0000
         0         0         0         0    1.0000
```

Problem 5

Compute the solution to the original system of equations by transforming **y** into **x**, i.e., compute **x** = inv(U)**y**.

Solution:

```
%code
%Use the inverse of U and the intermediary solution y to solve for x
x = invU * y
```

```
x = 5×1
    25
    50
```

```
   30
  125
   45
```

Problem 6

Check your answer for x_1 using Cramer's Rule. Use MATLAB to compute the required determinants using the det() function.

Solution:

```
%code
%First, initialize matrix A1 equal to matrix A
A1 = A
```

```
A1 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
    -1     0     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
%Replace column 1 with the column matrix of constants b
A1(:, 1) = b
```

```
A1 = 5×5
   100     1     0     0     0
     0     1    -1     0    -1
    50     0     1     0     1
   120    -1     0     1     1
     0     1     1    -1     1
```

```
%Find the solution to x1 with the ratio of determinants
x1 = det(A1)/det(A)
```

```
x1 = 25.0000
```

Problem 7

The Project One Table Template, provided in the Project One Supporting Materials section in Brightspace, shows the recommended throughput capacity of each link in the network. Put your solution for the system of equations in the third column so it can be easily compared to the maximum capacity in the second column. In the fourth column of the table, provide recommendations for how the network should be modified based on your network throughput analysis findings. The modification options can be No Change, Remove Link, or Upgrade Link. In the final column, explain how you arrived at your recommendation.

Solution:

*Fill out the table in the original project document and export your table as an image. Then, use the **Insert** tab in the MATLAB editor to insert your table as an image.*

Network Link	Recommended Capacity (Mbps)	Solution	Recommendation	Explanation
x_1	60	25	Remove Link	The solution is much less than the recommended capacity, so it would be beneficial to remove the link and replace it with one that won't be wasted on a small required capacity.
x_2	50	50	No Change	The solution is exactly the capacity of the link so there is no need to upgrade it or remove it.
x_3	100	30	Remove Link	The solution is much less than the recommended capacity, so it would be beneficial to remove the link and replace it with one that won't be wasted on a small required capacity.
x_4	100	125	Upgrade Link	The solution exceeds the capacity so it is necessary for the company to upgrade this link.
x_5	50	45	No Change	The solution is very close to the recommended capacity so there is no need to upgrade or remove the link.