Goals

We have a dataset contain axon diameters of neurons from the optic nerve of control and mutant zebrafish. We'd like to know if the mean axon diameter, or the distribution of axon diameters, differs between groups. We want to implement tests that take account of within and between animal variance.

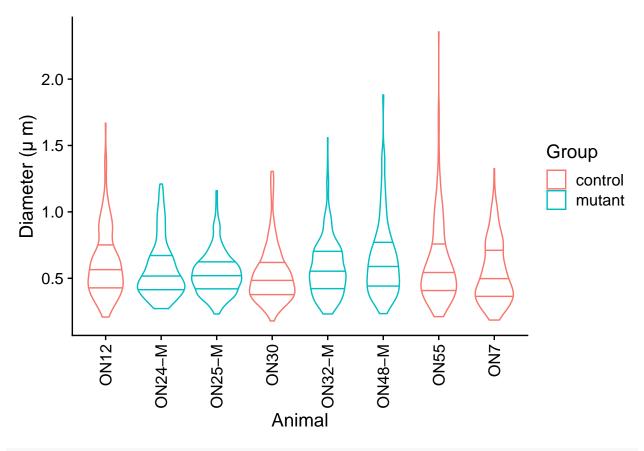
Here, we focus on the myRF data set.

Load and format data

Plot the data

Focus here on plot of individual mice, colour coded by group.

```
(plot_by_id <- ggplot(data = df, aes(name, value)) +
    geom_violin(aes(colour = group), draw_quantiles = c(0.25, 0.5, 0.75)) +
    theme_cowplot(font_size = 14) +
    theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust=1)) +
    labs(x = 'Animal', y = 'Diameter (\u00085 m)', colour = "Group"))</pre>
```



```
ggsave('Plots/violins.jpeg', plot_by_id)
```

Saving 6.5×4.5 in image

Tests for differences in means

```
mm <- lmer(value ~ group + (1 | name), data = df)
mm_null <- lmer(value ~ (1 | name), data = df)</pre>
summary(mm)
## Linear mixed model fit by REML ['lmerMod']
## Formula: value ~ group + (1 | name)
##
      Data: df
##
## REML criterion at convergence: -75.2
##
## Scaled residuals:
##
       Min
                1Q Median
                                 3Q
                                        Max
##
   -1.6912 -0.7023 -0.1939 0.4556 7.3979
##
## Random effects:
##
    Groups
                         Variance Std.Dev.
             Name
##
    name
             (Intercept) 0.001786 0.04226
                         0.055828 0.23628
    Residual
## Number of obs: 2160, groups: name, 8
##
```

```
## Fixed effects:
##
              Estimate Std. Error t value
## (Intercept) 0.571517 0.022377 25.540
## groupmutant 0.007131
                          0.031583
                                   0.226
## Correlation of Fixed Effects:
               (Intr)
## groupmutant -0.709
anova(mm, mm_null)
## refitting model(s) with ML (instead of REML)
## Data: df
## Models:
## mm_null: value ~ (1 | name)
## mm: value ~ group + (1 | name)
                           BIC logLik deviance Chisq Df Pr(>Chisq)
         npar
                 AIC
## mm null
             3 -80.980 -63.946 43.490 -86.980
              4 -79.046 -56.335 43.523 -87.046 0.0664 1
                                                              0.7966
mm_t <- lmer(log(value) ~ group + (1 | name), data = df)</pre>
mm_t_null <- lmer(log(value) ~ (1 | name), data = df)</pre>
summary(mm_t)
## Linear mixed model fit by REML ['lmerMod']
## Formula: log(value) ~ group + (1 | name)
     Data: df
##
## REML criterion at convergence: 1997.2
##
## Scaled residuals:
##
      Min
            1Q Median
                                3Q
                                       Max
## -2.6621 -0.7026 -0.0217 0.6423 3.8142
##
## Random effects:
## Groups Name
                        Variance Std.Dev.
## name
           (Intercept) 0.003713 0.06094
## Residual
                         0.145934 0.38201
## Number of obs: 2160, groups: name, 8
##
## Fixed effects:
              Estimate Std. Error t value
                          0.03271 -19.777
## (Intercept) -0.64693
## groupmutant 0.03399
                           0.04615
##
## Correlation of Fixed Effects:
##
               (Intr)
## groupmutant -0.709
anova(mm_t, mm_t_null)
## refitting model(s) with ML (instead of REML)
## Data: df
## Models:
## mm_t_null: log(value) ~ (1 | name)
```

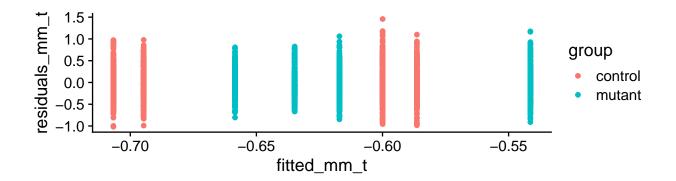
Test again with more complicated random effects stuctures

```
vmm_t_2 <- lmer(log(value) ~ group + (1 | litter/name), data = df)</pre>
mm_t_null_2 <- lmer(log(value) ~ (1 | litter/name), data = df)</pre>
summary(mm_t_2)
## Linear mixed model fit by REML ['lmerMod']
## Formula: log(value) ~ group + (1 | litter/name)
      Data: df
##
## REML criterion at convergence: 1996.8
##
## Scaled residuals:
       Min
               1Q Median
                                3Q
                                        Max
## -2.6577 -0.7005 -0.0188 0.6393 3.8127
##
## Random effects:
## Groups
                            Variance Std.Dev.
                Name
## name:litter (Intercept) 0.002358 0.04856
             (Intercept) 0.001445 0.03802
## litter
## Residual
                            0.145938 0.38202
## Number of obs: 2160, groups: name:litter, 8; litter, 5
## Fixed effects:
               Estimate Std. Error t value
## (Intercept) -0.64303
                         0.03472 -18.522
## groupmutant 0.04355
                           0.04461
##
## Correlation of Fixed Effects:
               (Intr)
## groupmutant -0.654
anova(mm_t_2, mm_t_null_2)
## refitting model(s) with ML (instead of REML)
## Data: df
## Models:
## mm_t_null_2: log(value) ~ (1 | litter/name)
## mm_t_2: log(value) ~ group + (1 | litter/name)
                       AIC BIC logLik deviance
                                                     Chisq Df Pr(>Chisq)
               npar
## mm_t_null_2
                  4 1995.5 2018.2 -993.77
                                            1987.5
                  5 1996.6 2025.0 -993.31
                                             1986.6 0.9227 1
## mm_t_2
                                                                   0.3368
mm_t_3 \leftarrow lmer(log(value) \sim group + (1 \mid processed/name), data = df)
mm_t_null_3 <- lmer(log(value) ~ (1 | processed/name), data = df)</pre>
summary(mm_t_3)
## Linear mixed model fit by REML ['lmerMod']
## Formula: log(value) ~ group + (1 | processed/name)
```

```
Data: df
##
##
## REML criterion at convergence: 1996.5
##
## Scaled residuals:
##
      Min
           1Q Median
                            3Q
                                    Max
## -2.6643 -0.7015 -0.0129 0.6368 3.8046
##
## Random effects:
## Groups
                             Variance Std.Dev.
                 Name
## name:processed (Intercept) 0.002588 0.05087
                  (Intercept) 0.001885 0.04342
## processed
                            0.145936 0.38202
## Residual
## Number of obs: 2160, groups: name:processed, 8; processed, 2
##
## Fixed effects:
##
             Estimate Std. Error t value
## groupmutant 0.02162
                         0.04047 0.534
##
## Correlation of Fixed Effects:
##
             (Intr)
## groupmutant -0.500
anova(mm_t_2, mm_t_null_3)
## refitting model(s) with ML (instead of REML)
## Data: df
## Models:
## mm_t_null_3: log(value) ~ (1 | processed/name)
## mm_t_2: log(value) ~ group + (1 | litter/name)
                     AIC
                          BIC logLik deviance Chisq Df Pr(>Chisq)
             npar
                4 1995.1 2017.8 -993.54
                                        1987.1
## mm t null 3
## mm t 2
                5 1996.6 2025.0 -993.31
                                       1986.6 0.474 1
```

Evaluate residuals

```
df$residuals_mm_t <- resid(mm_t)
df$fitted_mm_t <- fitted(mm_t)
(res_t_plot <- ggplot(data = df, aes(x = fitted_mm_t, y = residuals_mm_t)) +
    geom_point(aes(colour = group)) +
    theme_cowplot())</pre>
```



Compare residual distributions between groups, for the untransformed data first and then for the log-transformed data.

```
### We could use a KS test:
ks.test(residuals_mm_t ~ group, data = df)

## Warning in ks.test.default(x = DATA[[1L]], y = DATA[[2L]], ...): p-value will be
## approximate in the presence of ties

##
## Asymptotic two-sample Kolmogorov-Smirnov test
##
## data: residuals_mm_t by group
## D = 0.071709, p-value = 0.007785
## alternative hypothesis: two-sided
```

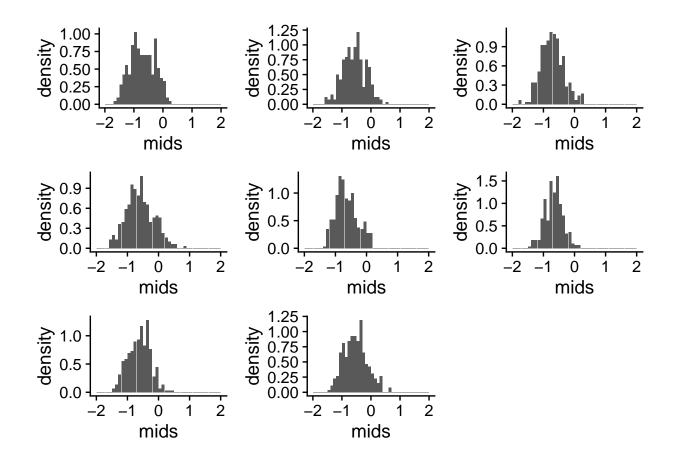
This suggests differences between groups in the variance, but doesn't account for within vs between subject effects in the residuals. But worry here is that we don't treat animals as indepwndent. Instead calculate SD for each animal and compare groups:

```
gr_t <- df %>% group_by(group, name) %>% summarize(avg_r = sd(residuals_mm_t))
## `summarise()` has grouped output by 'group'. You can override using the `.groups`
## argument.
t.test(avg_r ~ group, data = gr_t)
##
##
   Welch Two Sample t-test
##
## data: avg_r by group
## t = 1.9909, df = 5.4795, p-value = 0.09813
## alternative hypothesis: true difference in means between group control and group mutant is not equal
## 95 percent confidence interval:
  -0.01468272 0.12853729
## sample estimates:
## mean in group control mean in group mutant
##
               0.4089329
                                     0.3520056
```

Hard to interpret because group sizes are small. No firm evidence for difference in distributions.

Generate histograms for all animals

```
### Make histograms for each animal
### Use log transformed data
names = unique(df$name)
### If submean = 1 then will substract means before making histograms
histfun <- function(name, df, submean = 0) {
    sub <- ifelse(submean == 1, mean(df$value[df$name==name]), 0)</pre>
    hist(log(df$value[df$name==name]-sub), seq(-2,2,0.1), plot = FALSE)
hists <- sapply(names, histfun, df, submean=0)
### Convert results to tibble for use with tidyverse functions
rns <- rownames(hists)</pre>
hists_tib <- as_tibble(hists) %>%
    rownames_to_column(var = "rowname") %>%
    pivot_longer(-rowname, names_to = "column", values_to = "value") %>%
    pivot_wider(names_from = rowname, values_from = value)
## Warning: The `x` argument of `as_tibble.matrix()` must have unique column names if `.name_repair`
## is omitted as of tibble 2.0.0.
## i Using compatibility `.name_repair`.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was generated.
colnames(hists_tib) <- c("animal", rns)</pre>
ggconvfun <- function(mids, density) {</pre>
    gghist <- cbind(mids, density)</pre>
    ggplot(gghist, aes(mids, density)) +
        geom_col() +
        theme_cowplot()
}
gghists <- map2(hists_tib$mids, hists_tib$density, ggconvfun)</pre>
plot_grid(plotlist = gghists)
```



Evaluate distributions

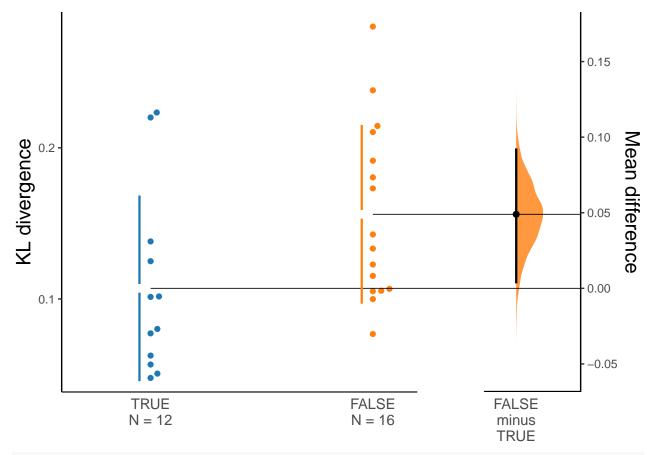
```
fit.lqmm <- lqmm(fixed = value ~ group, random = ~1, group = name, tau = c(0.25,0.5, 0.75), nK = 7, typ
summary(fit.lqmm, R = 500, seed = 2)
## Call: lqmm(fixed = value ~ group, random = ~1, group = name, tau = c(0.25,
      0.5, 0.75), nK = 7, type = "normal", data = df, control = c(method = "df",
##
      LP_{max_iter} = 5000)
##
## tau = 0.25
##
## Fixed effects:
                  Value Std. Error lower bound upper bound Pr(>|t|)
  (Intercept) 0.372273
                          0.025789
                                      0.321605
                                                    0.4229 < 2.2e-16 ***
  groupmutant 0.105470
                          0.037054
                                      0.032669
                                                    0.1783 0.004604 **
##
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## tau = 0.5
##
## Fixed effects:
##
                    Value Std. Error lower bound upper bound Pr(>|t|)
## (Intercept) 0.5722731 0.0355788
                                       0.5023704
                                                      0.6422
                                                               <2e-16 ***
## groupmutant 0.0054705 0.0361782 -0.0656098
                                                               0.8799
                                                      0.0766
## ---
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## tau = 0.75
##
## Fixed effects:
##
                  Value Std. Error lower bound upper bound Pr(>|t|)
## (Intercept) 0.692258
                          0.031817
                                      0.629747
                                                    0.7548
                                                             <2e-16 ***
## groupmutant -0.010545
                          0.041330
                                     -0.091747
                                                    0.0707
                                                             0.7987
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## AIC:
## [1] -443.5 (df = 4) -63.3 (df = 4) 768.8 (df = 4)
fit.log.lqmm \leftarrow lqmm(fixed = log(value) \sim group, random = \sim1, group = name, tau = c(0.25,0.5, 0.75), nK
summary(fit.log.lqmm, R = 500, seed = 2)
## Call: lqmm(fixed = log(value) ~ group, random = ~1, group = name, tau = c(0.25,
      0.5, 0.75), nK = 7, type = "normal", data = df, control = c(method = "df", f)
      LP_{max_iter} = 5000)
##
##
## tau = 0.25
##
## Fixed effects:
                   Value Std. Error lower bound upper bound Pr(>|t|)
## (Intercept) -0.9393497 0.0344783 -1.0070903
                                                 -0.8716 < 2e-16 ***
## groupmutant 0.0676887 0.0393206 -0.0095657
                                                     0.1449 0.08579 .
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## tau = 0.5
##
## Fixed effects:
                   Value Std. Error lower bound upper bound Pr(>|t|)
## (Intercept) -0.666737
                          0.037753
                                    -0.740912
                                                   -0.5926
                                                             <2e-16 ***
                          0.049686
                                     -0.055927
                                                    0.1393
                                                             0.4018
## groupmutant 0.041693
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## tau = 0.75
##
## Fixed effects:
                  Value Std. Error lower bound upper bound Pr(>|t|)
## (Intercept) -0.366817
                          0.046582
                                     -0.458338
                                                   -0.2753 2.148e-14 ***
## groupmutant -0.016329
                          0.060507
                                     -0.135210
                                                    0.1026
                                                              0.7874
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## AIC:
## [1] 2372 (df = 4) 2264 (df = 4) 2633 (df = 4)
```

Additional hacky attempts to evaluate distributions

```
### Make histograms compatible with comparison tools
hists_as_matrix <- function(counts, mids) {matrix(c(counts/sum(counts), mids), ncol=2)}
hists tib <- hists tib %>%
   mutate(mat_2 = map2(density, mids, hists_as_matrix),
           mat = map2(counts, mids, hists as matrix))
### List all combinations of histograms
combinations <- combn(1:length(names), 2, simplify = FALSE)</pre>
### Calculate KL divergence for all combinations
kl_on_comb <- function(comb, mat) {</pre>
   P <- mat[[comb[[1]]]][,1] / sum(mat[[comb[[1]]]][,1])
    Q <- mat[[comb[[2]]]][,1] / sum(mat[[comb[[2]]]][,1])
### check order so that control is left in x
### if(df\$group[[comb[[1]]]]=="mutant") \{x \leftarrow rbind(Q,P)\}\ else \{x \leftarrow rbind(P,Q)\}\
x <- rbind(P,Q)
### KL(x)
    distance(x, "taneja") #pearson(asymmetric), divergence (symmetric), clark(symmetric), taneja (symme
}
kls <- lapply(combinations, kl_on_comb, hists_tib$mat)</pre>
## Metric: 'taneja' using unit: 'log'; comparing: 2 vectors.
## Metric: 'taneja' using unit: 'log'; comparing: 2 vectors.
## Metric: 'taneja' using unit: 'log'; comparing: 2 vectors.
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## Metric: 'taneja' using unit: 'log'; comparing: 2 vectors.
## Metric: 'taneja' using unit: 'log'; comparing: 2 vectors.
```

```
### Put everything together for analysis
collected <- tibble(pairs = combinations, kl = unlist(kls))</pre>
### Add columns to identify animals
a1 func <- function(pair, names, pos) {
    names[pair[[pos]][1]]
    }
collected$pair1 <- sapply(collected$pairs, a1_func, names, 1)</pre>
collected$pair2 <- sapply(collected$pairs, a1_func, names, 2)</pre>
### Reorganise
get_group <- function(name, df) {as.character(df$group[[grep(name, df$name)[[1]]]])}</pre>
grouplist <- lapply(names, get_group, df)</pre>
group_lookup <- function(id, names, grouplist) {grouplist[[grep(id, names)]]}</pre>
compare_groups <- function(g1, g2) {g1 == g2}</pre>
collected <- collected %>%
    mutate(group1 = map_chr(pair1, group_lookup, names, grouplist),
           group2 = map_chr(pair2, group_lookup, names, grouplist),
           samegroup = map2_lgl(group1, group2, compare_groups))
### Compare KL divergece for pairs that are from the same group with pairs from different groups
wilcox.test(kl ~ samegroup, data = collected)
##
## Wilcoxon rank sum exact test
##
## data: kl by samegroup
## W = 143, p-value = 0.02918
\#\# alternative hypothesis: true location shift is not equal to 0
Make dabest plots for the above compariosns. Use dabest plot.
dabest_obj_kl.mean_diff <- load(</pre>
 data = collected,
 x = samegroup,
 y = kl,
 idx = c("TRUE", "FALSE")
) %>%
 mean_diff()
(de_kl <- dabest_plot(dabest_obj_kl.mean_diff, TRUE,</pre>
                       swarm_label = 'KL divergence'))
```

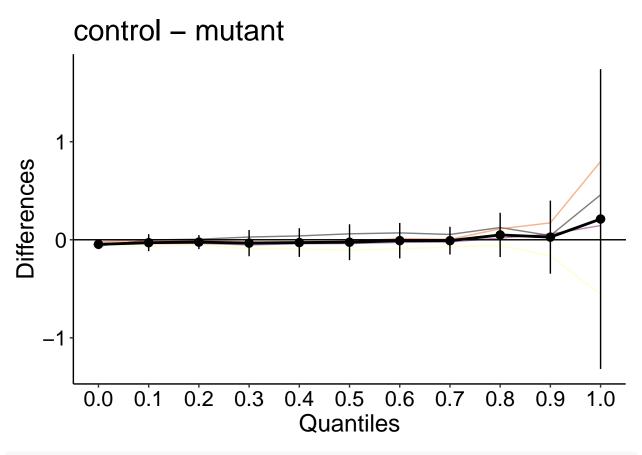


ggsave('Plots/KL_dabest.jpeg', de_kl)

Saving 6.5×4.5 in image

Another approach is to compare the distributions using a hierarchical shift function. See: $\frac{https:}{github.}$ $\frac{https:}{github.}$

```
out <- hsf(df, log(value) ~ group + name, qseq = c(seq(0, 1, 0.1)))
plot_hsf(out)</pre>
```



out\$adjusted_pvalues

[1] 0.6377587 0.8834981 0.8834981 0.8834981 0.8834981 0.8834981 0.8834981 0.8834981 ## [9] 0.8834981 0.8834981

Again, data size too small to be conclusive.