COMP3702 Tutorial 5

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Logic

- Logic is a formal language used to represent a set of states
- A convenient
- A convenient abstraction for dealing with many states
- Regardless of whether there's a natural notion of "near" or not (i.e. not a metric space), we can use logic to group different states together
- For example:
  - I have a laptop ⇒ Includes any brand and model
  - There is a laptop on the table ⇒ Can be at any position on the table.

Logic Definitions

- Interpretation Assignment of values to all variables
- Model An interpretation (assignment of values) that satisfies the constraints
  - Often we don't want to just find a model, but we want to know what is true in all models
- Proposition A statement that is true or false in each interpretation

- The formal language representation and reasoning system is made up of:
  - Syntax Describes what sentences are legal / what sentences are illegal
  - Semantics Specifies the meaning of symbols
  - Atom A symbol, starting with a lowercase letter
  - Definite Clause An atom or rule of the form $\text{atom} \Leftarrow \textit{sentence}$
  - Knowledge Base A set of definite clauses A knowledge base is true if and only if every definite clause within it is TRUE in every model (interpretation; assignment).

Syntax of Propositional Logic

Complex propositions (sentences) can be built from simpler propositions using logical connectives, such as:

- Brackets ()
- Negation ¬
- And; Conjunction ∧
- Or; Disjunction ∧
- Implication ⇒
- Biconditional / Equivalence ⇔

Model Example

For each I_n, describe whether it is a model of the knowledge base.

$$KB = \begin{cases} p \Leftarrow q \\ q \\ r \Leftarrow s \end{cases}$$

	p	q	r	s
I_1	TRUE	TRUE	TRUE	TRUE
I_2	FALSE	FALSE	FALSE	FALSE
I_3	TRUE	TRUE	FALSE	FALSE
I_4	TRUE	TRUE	TRUE	FALSE
I_5	TRUE	TRUE	FALSE	TRUE

Model Example

$$KB = \begin{cases} p \Leftarrow q \\ q \\ r \Leftarrow s \end{cases}$$

For I_1, we have p=TRUE, q=TRUE, r=TRUE and s=TRUE

	p	q	r	s
I_1	TRUE	TRUE	TRUE	TRUE

Model Example

$$KB = \begin{cases} p \Leftarrow q \\ q \\ r \Leftarrow s \end{cases}$$

For I_1, we have p=TRUE, q=TRUE, r=TRUE and s=TRUE

$$\begin{array}{ccc} p \Leftarrow q & TRUE \Leftarrow TRUE & TRUE \\ q & TRUE & TRUE \\ r \Leftarrow s & TRUE \Leftarrow TRUE & \text{TRUE} \end{array}$$

Since each clause is true, I_1 is a model of the knowledge base KB

	p	q	r	s
I_1	TRUE	TRUE	TRUE	TRUE

Model Example

$$KB = \begin{cases} p \Leftarrow q \\ q \\ r \Leftarrow s \end{cases}$$

For I_2, we have p=FALSE, q=FALSE, r=FALSE, s=FALSE

	p	q	r	s
I_2	FALSE	FALSE	FALSE	FALSE

Model Example

$$KB = \begin{cases} p \Leftarrow q \\ q \\ r \Leftarrow s \end{cases}$$

For I_2, we have p=FALSE, q=FALSE, r=FALSE, s=FALSE

$$\begin{array}{ccc} p \Leftarrow q & FALSE \Leftarrow FALSE & TRUE \\ q & FALSE & FALSE \\ r \Leftarrow s & FALSE \Leftarrow FALSE & TRUE \end{array}$$

Since not every clause of I_2 is true, this is not a model of the knowledge base KB

	p	q	r	s
I_2	FALSE	FALSE	FALSE	FALSE

Model Example

$$KB = \begin{cases} p \Leftarrow q \\ q \\ r \Leftarrow s \end{cases}$$

For I_3, we have p=TRUE, q= TRUE, r=FALSE, s=FALSE

	p	q	r	s
I_3	TRUE	TRUE	FALSE	FALSE

Model Example

$$KB = \begin{cases} p \Leftarrow q \\ q \\ r \Leftarrow s \end{cases}$$

For I_3, we have p=TRUE, q= TRUE, r=FALSE, s=FALSE

$$\begin{array}{ccc} p \Leftarrow q & TRUE \Leftarrow TRUE & TRUE \\ q & TRUE & TRUE \\ r \Leftarrow s & FALSE \Leftarrow FALSE & TRUE \end{array}$$

Since every clause of I_3 is true, this is a model of the knowledge base KB

	p	q	r	s
I_3	TRUE	TRUE	FALSE	FALSE

Conjugative Normal Form

Conjunctions of Disjunctions

Statements of the form $(\neg A \lor B) \land (C \lor D) \land (E \lor F)$.

In CNF:

- <u>Clause</u> A disjunction of literals $(\neg A \lor B)$
- <u>Literals</u> Variables, or the negation of variables, e.g. $A, \neg A, B$

Conjugative Normal Form

Conjunctions of Disjunctions

Statements of the form $(\neg A \vee B) \wedge (C \vee D) \wedge (E \vee F)$.

Suppose we have the statement $(A \vee B) \Rightarrow (C \Rightarrow D)$ that we want to convert to CNF – to do this, we need to follow three key steps

$$(A \vee B) \Rightarrow (C \Rightarrow D)$$

1. Eliminate arrows, using the rule $A \Rightarrow B \equiv \neg A \vee B$

   $\neg(A \vee B) \vee (\neg C \vee D)$

2. Drive in negations

   $(\neg A \wedge \neg B) \vee (\neg C \vee D)$

3. Distribute OR over AND

   $(\neg A \vee \neg C \vee D) \wedge (\neg B \vee \neg C \vee D)$

Exercise 5.1

Exercise 6.2. Mr Jones finds three trunks A, B, and C in a cave. Based on studying the history of where these trunks came about, he knows that one trunk contains gold, while two are empty. On the wall of the cave, he found three clues: "A is empty", "B is empty", and "gold is in B". From studying the historical social norm of the villages around the cave, Mr Jones knows that only one of the clues is true, while the other two are false". Which trunk has the gold? (source: `http://disi.unitn.it/~ldkr/ml2014/ExercisesBooklet.pdf`)

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Hint: Break down the problem into the following steps:

1. Convert to atoms (symbols, that we want to evaluate the truth of)
2. Build logical statements
3. Convert to Conjugative Normal Form
4. Evaluate the CNF sentences

Exercise 5.1 - Solution

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We want to encode the clauses (atoms / rules in the problem) that are true in the intended interpretation by *axiomatizing the domain*.

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We want to encode the clauses (atoms / rules in the problem) that are true in the intended interpretation by *axiomatizing the domain*. We want to:

1. Associate an atom with the situation we wish to validate (i.e., gold in trunk)
2. Encode Mr Jones' knowledge of the situation into a set of logical sentences
3. Use these sentences to determine if there is a solution, where one of the chests is guaranteed to contain gold.

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We begin by associating a variable for each of the trunks containing gold:

A: Trunk A contains gold B: Trunk B contains gold C: Trunk C contains gold

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3. Use these sentences to determine if there is a solution, where one of the chests is guaranteed to contain gold.

We begin by associating a variable for each of the trunks containing gold:

A: Trunk A contains gold B: Trunk B contains gold C: Trunk C contains gold

$\neg A$ – *Trunk A does not contain gold*

Exercise 5.1 - Solution

A: Trunk A contains gold   B: Trunk B contains gold   C: Trunk C contains gold

We then want to encode Mr Jones' knowledge

One trunk contains gold, while the other two are empty

Exercise 5.1 - Solution

A: Trunk A contains gold   B: Trunk B contains gold   C: Trunk C contains gold

We then want to encode Mr Jones' knowledge

One trunk contains gold, while the other two are empty

$$S_1 = (A \wedge \neg B \wedge \neg C) \vee (\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge C)$$

$(A \wedge \neg B \wedge \neg C)$ Trunk A contains gold, while trunk B and trunk C are empty

$(\neg A \wedge B \wedge \neg C)$ Trunk B contains gold, while trunk A and trunk C are empty

$(\neg A \wedge \neg B \wedge C)$ Trunk C contains gold, while trunk A and trunk B are empty

Exercise 5.1 - Solution

A: Trunk A contains gold B: Trunk B contains gold C: Trunk C contains gold

Additionally, Mr Jones knows of three clues, but additionally knows that only one of the clues is true, with the other two being false. We also want to encode this knowledge using logic.

Exercise 5.1 - Solution

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Additionally, *Mr Jones knows of three clues, but additionally knows that only one of the clues is true, with the other two being false. We also want to encode this knowledge using logic.*

We begin by representing the three clues:

$$\neg A$$

$$\neg B$$

$$B$$

Exercise 5.1 - Solution

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$$\neg A$$

$$\neg B$$

$$B$$

Since Mr Jones knows that only one of these are true, and the other two are false, we now need to consider each possibility to form our second logical sentence.

Exercise 5.1 - Solution

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We begin by representing the three clues:

$$\neg A$$

$$\neg B$$

$$B$$

Since Mr Jones knows that only one of these are true, and the other two are false, we now need to consider each possibility to form our second logical sentence.

$$\boldsymbol{S_2} = (\neg \boldsymbol{A} \land \neg(\neg \boldsymbol{B}) \land \neg \boldsymbol{B}) \lor (\neg(\neg \boldsymbol{A}) \land \neg \boldsymbol{B} \land \neg \boldsymbol{B}) \lor (\neg(\neg \boldsymbol{A}) \land \neg(\neg \boldsymbol{B}) \land \boldsymbol{B})$$

Exercise 5.1 - Solution

A: Trunk A contains gold B: Trunk B contains gold C: Trunk C contains gold

We now use both sentences to determine if there is a situation where one of the chests is guaranteed to contain gold.

$$S_1 = (A \land \neg B \land \neg C) \lor (\neg A \land B \land \neg C) \lor (\neg A \land \neg B \land C)$$

$$\boldsymbol{S_2} = (\neg \boldsymbol{A} \land \neg(\neg \boldsymbol{B}) \land \neg \boldsymbol{B}) \lor (\neg(\neg \boldsymbol{A}) \land \neg \boldsymbol{B} \land \neg \boldsymbol{B}) \lor (\neg(\neg \boldsymbol{A}) \land \neg(\neg \boldsymbol{B}) \land \boldsymbol{B})$$

Exercise 5.1 - Solution

A: Trunk A contains gold B: Trunk B contains gold C: Trunk C contains gold

We now use both sentences to determine if there is a situation where one of the chests is guaranteed to contain gold.

$$S_1 = (A \land \neg B \land \neg C) \lor (\neg A \land B \land \neg C) \lor (\neg A \land \neg B \land C)$$

$$S_2 = (\neg A \land \neg(\neg B) \land \neg B) \lor (\neg(\neg A) \land \neg B \land \neg B) \lor (\neg(\neg A) \land \neg(\neg B) \land B)$$

We begin by simplifying the second sentence

1. Through the elimination of double-negatives, we get:
   $$S_2 = (\neg A \land B \land \neg B) \lor (A \land \neg B \land \neg B) \lor (A \land B \land B)$$
2. We know that $X \land \neg X = FALSE$ (as we can't have the same variable / atom be true and false at the same time)
   $$S_2 = (\neg A \land FALSE) \lor (A \land \neg B \land \neg B) \lor (A \land B \land B)$$
3. We additionally know that $A \land A = A$ and can use this fact to simplify our expression:
   $$S_2 = FALSE \lor (A \land \neg B) \lor (A \land B)$$
4. Additionally, we know that $FALSE \lor A = A$
   $$S_2 = (A \land \neg B) \lor (A \land B)$$

Exercise 5.1 - Solution

A: Trunk A contains gold B: Trunk B contains gold C: Trunk C contains gold

1. Through the elimination of double-negatives, we get:
   $S_2 = (\neg A \wedge B \wedge \neg B) \vee (A \wedge \neg B \wedge \neg B) \vee (A \wedge B \wedge B)$
2. We know that $X \wedge \neg X = FALSE$ (as we can't have the same variable / atom be true and false at the same time)
   $S_2 = (\neg A \wedge FALSE) \vee (A \wedge \neg B \wedge \neg B) \vee (A \wedge B \wedge B)$
3. We additionally know that $A \wedge A = A$ and can use this fact to simplify our expression:
   $S_2 = FALSE \vee (A \wedge \neg B) \vee (A \wedge B)$
4. Additionally, we know that $FALSE \vee A = A$.
   $S_2 = (A \wedge \neg B) \vee (A \wedge B)$
5. By the distributivity law, we know that, the above statement is equivalent to:
   $S_2 = A \wedge (\neg B \wedge B)$
6. We know that $(\neg X \wedge X) = T$

   $S_2 = A$ Therefore, to satisfy S_2, A must be true (i.e. the gold is in trunk A)

Exercise 5.1 - Solution

A: Trunk A contains gold B: Trunk B contains gold C: Trunk C contains gold

We can perform validation of rule S_1 through the construction of a truth table:

A	B	C	$S_1 = (A \land \neg B \land \neg C) \lor (\neg A \land B \land \neg C) \lor (\neg A \land \neg B \land C)$	$\boldsymbol{S_2 = A}$
F	F	F	F	F
F	F	T	T	F
F	T	F	T	F
F	T	T	F	F
T	F	F	T	T
T	F	T	F	T
T	T	F	F	T
T	T	T	F	T

Exercise 5.1 - Solution

A: Trunk A contains gold   B: Trunk B contains gold   C: Trunk C contains gold

We can perform validation of rule S_1 through the construction of a truth table:

A	B	C	$S_1 = (A \land \neg B \land \neg C) \lor (\neg A \land B \land \neg C) \lor (\neg A \land \neg B \land C)$	$\boldsymbol{S_2 = A}$
F	F	F	F	F
F	F	T	T	F
F	T	F	T	F
F	T	T	F	F
T	F	F	T	T
T	F	T	F	T
T	T	F	F	T
T	T	T	F	T

Exercise 5.2

Exercise 5.2. Are the following entailments correct? Please provide the proof.

a) $(A \wedge B) \models (A \Leftrightarrow B)$

b) $(A \Leftrightarrow B) \models (A \wedge B)$

Hint: When we are asked to determine if an entailment is correct (or holds, or is true) we can convert the entailment into an implication, and check if the implication is valid. Checking whether or not the implication is valid means solving a validity problem. Remember that a sentence is valid when every combination of variable assignments in the sentence causes it to be true.

Exercise 5.2a - Solution

$(A \land B) \vDash (A \Leftrightarrow B)$

We can convert the entailment $(A \land B) \vDash (A \Leftrightarrow B)$ to an implication of the form $(A \land B) \rightarrow (A \Leftrightarrow B)$

By constructing a truth table, we can determine whether the implication is valid.

Exercise 5.2a - Solution

$(A \wedge B) \vDash (A \Leftrightarrow B)$

We can convert the entailment $(A \wedge B) \vDash (A \Leftrightarrow B)$ to an implication of the form $(A \wedge B) \rightarrow (A \Leftrightarrow B)$

By constructing a truth table, we can determine whether the implication is valid.

A	B	$A \wedge B$	$\boldsymbol{A \Leftrightarrow B}$	$(\boldsymbol{A} \wedge \boldsymbol{B}) \rightarrow (\boldsymbol{A} \Leftrightarrow \boldsymbol{B})$
F	F	F	T	T
F	T	F	F	T
T	F	F	F	T
T	T	T	T	T

Since all of the rows of the implication column of the truth table is true, the implication is valid, and thus the original entailment is correct.

Exercise 5.2b - Solution

$(A \Leftrightarrow B) \vDash (A \wedge B)$

We can convert the entailment$(A \Leftrightarrow B) \vDash (A \wedge B)$to an implication of the form $(A \Leftrightarrow B) \rightarrow (A \vee B)$

By constructing a truth table, we can determine whether the implication is valid.

Exercise 5.2b - Solution

$(A \Leftrightarrow B) \vDash (A \land B)$

We can convert the entailment $(A \Leftrightarrow B) \vDash (A \land B)$ to an implication of the form $(A \Leftrightarrow B) \to (A \land B)$

By constructing a truth table, we can determine whether the implication is valid.

A	B	$\boldsymbol{A \Leftrightarrow B}$	$A \land B$	$(A \Leftrightarrow B) \to (A \lor B)$
F	F	T	F	F
F	T	F	F	T
T	F	F	F	T
T	T	T	T	T

Since not every row of the implication column is true, the implication is not valid, and thus the original entailment is not correct.

Resolution Refutation

Resolution Refutation is a technique for simplifying logical propositions.

There are three steps to perform resolution refutation

1. Convert all sentences to Conjugative Normal Form (CNF)
2. Negate the desired conclusion
3. Apply the resolution rule, until we either:
    1. Derive false (a contradiction)
    2. Can’t apply the rule anymore.

The resolution refutation rule is sound and complete (for propositional logic):

- If we derive a contradiction, the conclusion follows from the axioms (proof by contradiction)
- If we can’t apply any more, the conclusion cannot be proven from the axioms (no entailment; invalid statement)

Resolution Refutation:

$$\begin{array}{r} A \lor B \\ \neg B \lor C \\ \hline A \lor C \end{array}$$

Exercise 5.3

Exercise 5.3. Please use resolution refutation to show

$$(P \land \neg P) \vDash R$$

(Note: negation can be represented by "¬" or "∼" or "!" or just plain "not")

Use the resolution refutation rule to show that $(P \land \neg P) \vDash R$

Refutation is like a proof by contradiction – assume that the statement that we want to prove is false, and then derive a contradiction (show that the knowledge base cannot be true, for the negated statement we want to prove)

Exercise 5.3 - Solution

Exercise 5.3. Please use resolution refutation to show

$$(P \land \neg P) \vDash R$$

(Note: negation can be represented by "$\neg$" or "$\sim$" or "!" or just plain "not")

The first step of resolution refutation is to convert all sentences in our axioms to Conjugative Normal Form. In this case, we only have a single sentence, which is straightforward to convert.

$$(P \land \neg P) = (F \lor P) \land (\neg P \lor F)$$

Exercise 5.3 - Solution

Exercise 5.3. Please use resolution refutation to show

$$(P \wedge \neg P) \vDash R$$

(Note: negation can be represented by "¬" or "∼" or "!" or just plain "not")

The first step of resolution refutation is to convert all sentences in our axioms to Conjugative Normal Form. In this case, we only have a single sentence, which is straightforward to convert.

$$(P \wedge \neg P) = (F \vee P) \wedge (\neg P \vee F)$$

Secondly, we negate the conclusion, to get

$$\neg R$$

We then apply the resolution refutation rule:

Our first and second clauses are $F \vee P$ and $\neg P \vee F$ with conclusion $\neg R$

Applying the resolution refutation rule (right) yields $F \vee F = F$. We have derived False, which is a contradiction (which interestingly doesn't depend on P, or our conclusion).

Since False entail anything, including R in this case, so it is valid.

Resolution Refutation:

$$\begin{array}{c} A \vee B \\ \neg B \vee C \\ \hline A \vee C \end{array}$$

$$\begin{array}{c} F \vee P \\ \neg P \vee F \\ \hline \neg R \end{array}$$

Exercise 5.4

Exercise 5.4. UQPark is a theme park with 5 rides: Bumper cars, carousel, haunted class, roller coaster, and ferris wheel, where each ride can be turned on and off independently of the other rides. After performing a cost-benefit analysis, UQPark Management decided that only 3 rides should be open at any given day, and the set of rides that are open/closed must satisfy the following constraints:

1. Either bumper cars or carousel must be open.
2. If bumper cars is closed, then roller coaster must open.
3. If carousel is open, then either bumper cars or haunted class must be open too.
4. If haunted class is open, then ferris wheel must be open too.
5. Bumper cars and ferris wheel cannot both be open at the same day.
6. If roller coaster is open, then ferris wheel must be open too.
7. If roller coaster is closed, then either haunted class or ferris wheel must be open.

UQPark Facilities thinks there is no combination of the rides that can satisfy all of Management's constraints.

(a) Please frame this problem as a satisfiability problem with propositional logic representation.

(b) Please solve the problem in (a) using DPLL. If both Management and Facilities are correct or both are incorrect, please also explain why using DPLL.

TIP: Assign W = Ferris Wheel

Exercise 5.4a

We begin by assigning some atoms:

B Bumper Cars open

C Carousel open

H Haunted class open

R Roller Coaster

W Ferris Wheel Open

1. Either bumper cars or carousel must be open.
2. If bumper cars is closed, then roller coaster must open.
3. If carousel is open, then either bumper cars or haunted class must be open too.
4. If haunted class is open, then ferris wheel must be open too.
5. Bumper cars and ferris wheel cannot both be open at the same day.
6. If roller coaster is open, then ferris wheel must be open too.
7. If roller coaster is closed, then either haunted class or ferris wheel must be open.

Exercise 5.4a

We then express the constraints in the question as clauses of variables.

1. Either bumper cars or carousel must be open
   $B \lor C$
2. If bumper cars is closed, then roller coaster must be open
   $\neg B \rightarrow R \Rightarrow \neg(\neg B) \lor R \Rightarrow B \lor R$
3. If carousel is open, then either bumper cars or haunted class must be open
   $C \rightarrow (B \lor H) \Rightarrow \neg C \lor B \lor H$
4. If haunted class is open, then Ferris wheel must be open too
   $H \rightarrow W \Rightarrow \neg H \lor W$
5. Bumper cars and Ferris wheel cannot both be open at the same day.
   $\neg(B \land W) \Rightarrow \neg B \lor \neg W$
6. If roller coaster is open, then Ferris wheel must be open too.
   $R \rightarrow W \Rightarrow \neg R \lor W$
7. If roller coaster is closed, then either haunted class or Ferris wheel must be open.
   $\neg R \Rightarrow (H \lor W) \Rightarrow \neg(\neg R) \lor (H \lor W) \Rightarrow R \lor H \lor W$

1. Either bumper cars or carousel must be open.
2. If bumper cars is closed, then roller coaster must open.
3. If carousel is open, then either bumper cars or haunted class must be open too.
4. If haunted class is open, then ferris wheel must be open too.
5. Bumper cars and ferris wheel cannot both be open at the same day.
6. If roller coaster is open, then ferris wheel must be open too.
7. If roller coaster is closed, then either haunted class or ferris wheel must be open.

B Bumper Cars open

C Carousel open

H Haunted class open

R Roller Coaster

W Ferris Wheel Open

Exercise 5.4a

We then combine each of the clauses using the CNF syntax

$$(B \vee C) \wedge (B \vee R) \wedge (\neg C \vee B \vee H) \wedge (\neg H \vee W) \wedge (\neg B \vee \neg W) \wedge (\neg R \vee W) \wedge (R \vee H \vee W)$$

B Bumper Cars open

C Carousel open

H Haunted class open

R Roller Coaster

W Ferris Wheel Open

Exercise 5.4b

We count the occurrences of each variable in the CNF statement from part A, of this question:

$$(B \lor C) \land (B \lor R) \land (\neg C \lor B \lor H) \land (\neg H \lor W) \land (\neg B \lor \neg W) \land (\neg R \lor W) \land (R \lor H \lor W)$$

The number of occurrences of each variable are:

B=4 C=2 H=3 R=3 W=4

Since B is the variable with the most entries, we choose to assign it first (as it will lead to the most simple answer for this step)

We first choose to set B=True

$$\cancel{(T \lor C)} \land \cancel{(T \lor R)} \land \cancel{(\neg C \lor T \lor H)} \land (\neg H \lor W) \land (\neg T \lor \neg W) \land (\neg R \lor W) \land (R \lor H \lor W)$$

$$(\neg H \lor W) \land (F \lor \neg W) \land (\neg R \lor W) \land (R \lor H \lor W)$$

$$(\neg H \lor W) \land (\neg W) \land (\neg R \lor W) \land (R \lor H \lor W)$$

This doesn't lead to a case where the statement is false, so we proceed with the assignment. We once again count the occurrences of each variable in our statement

W=4 H=2 R=2

Since W is the variable with the next-most entries we choose to assign it next

Exercise 5.4b

Since W is the variable with the next-most entries we choose to assign it next. We first choose to assign W=True

$$(\neg H \vee W) \wedge (\neg W) \wedge (\neg R \vee W) \wedge (R \vee H \vee W)$$
$$\cancel{(\neg W \vee T)} \wedge (\neg T) \wedge \cancel{(\neg R \vee T)} \wedge \cancel{(R \vee H \vee T)}$$
$$F \quad (as\ \neg T = F)$$

Since this assignment doesn't hold (i.e. the statement isn't true for the assignments), we try to assign W=False.

$$(\neg H) \wedge (\neg R) \wedge (R \vee H)$$

Counting the variable occurrences, we get H=2 and R=2. We choose to assign H=True.

$$(\neg T) \wedge (\neg R) \wedge (R \vee T)$$
$$(F) \wedge (\neg R)$$

Since this variable assignment doesn't hold, we try to assign H=False as above.

$$(\neg F) \wedge (\neg R) \wedge (R \vee F)$$
$$(\neg R) \wedge (R) = F$$

Since this variable assignment doesn't hold, and we have tried setting it to both true and false, we backtrack to the assignment of W. However, we have already tried all possibilities of assigning to W so we backtrack to the assignment of B.

Exercise 5.4b

However, we have already tried all possibilities of assigning to W so we backtrack to the assignment of B, and try to assign it the value B=False

$$(B \lor C) \land (B \lor R) \land (\neg C \lor B \lor H) \land (\neg H \lor W) \land (\neg B \lor \neg W) \land (\neg R \lor W) \land (R \lor H \lor W)$$

$$(F \lor C) \land (F \lor R) \land (\neg C \lor F \lor H) \land (\neg H \lor W) \land \cancel{(\neg F \lor \neg W)} \land (\neg R \lor W) \land (R \lor H \lor W)$$

as $(\neg F \lor \neg W) = (T \lor W) = T$ and $T \land X = X$

$$(C) \land (R) \land (\neg C \lor H) \land (\neg H \lor W) \land (\neg R \lor W) \land (R \lor H \lor W)$$

Counting of variable occurrences gives us:

C=2 H=3 R=3 W=3

Since H is the variable with the next-most occurrences we choose to assign it. Assign H=True

$$(C) \land (R) \land \cancel{(\neg C \lor T)} \land (\cancel{\neg T} \lor W) \land (\neg R \lor W) \land \cancel{(R \lor T \lor W)}$$

$$(C) \land (R) \land (W) \land (\neg R \lor W)$$

Counting of variable occurrences gives us C=1, W=2, R=2. We choose to set W=True

$$(C) \land (R) \land (T) \land (\neg R \lor T)$$

$$(C) \land (R)$$

We set C=True

$$R$$

Exercise 5.4b

We set C=True

$$R$$

We set R=True

$$True$$

Exercise 5.4b

To determine whether the problem is satisfiable with only 3 variables being true, we traverse the DPLL Search Tree.