1 Introduction

Hall Effect Thrusters (HETs) are a form of electric propulsion that utilises both magnetic fields and electrostatic forces to accelerate a propellant and, in turn, produce thrust. For this reason, it can be seen as being part of both the electromagnetic and electrostatic families within electric propulsion. The propellant used in HETs is a neutral gas which is then ionised to become a plasma. Ions from the plasma are accelerated through a potential gradient to reach speeds of around 10-20 km/s[3].

Electrons are fed into the system using a hollow cathode which functions like an electron gun. These electrons are electrostatically attracted to the internal anode and so move towards the interior of the annular channel. Upon entering the channel, the electrons become trapped by a cross (ExB) field. There is a radial magnetic field generated as well as an axial electric field which causes a cross field acting azimuthally around the channel. The electrons are accelerated to high speeds by the Hall Effect, creating a Hall current. Collisions between the high energy electrons and the gas causes a plasma to be created. From this quasi-neutral plasma, ions are acted upon by the electric field and are accelerated out of the channel before being neutralised by the electrons coming out of the hollow cathode.

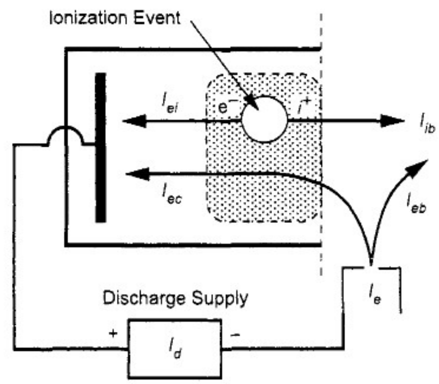

Figure 1: Diagram showing dynamics of currents during operation of Hall Effect Thruster.

With regards to designing a lower power HET with an input power of 100W, the geometries of the system will be scaled down from a larger system using methods outlined by Dannenmayer et al[2]. There may well be difficulties in manufacturing small components for this miniature system as they will have small scales and tolerances. Additionally, scaling down causes the power loss in the system to be dominated by interactions between the plasma and the channel wall because of the smaller volume that the plasma is confined to. However these losses are overcome by using data from high power HETs to scale down from.[6]

The miniature HET was experimentally tested using input powers between 30 W and 810 W with five different propellants. At 100 W, Krypton was fed into the system and thrust was measured to be 2 mN. The anode efficiency was found to be 5% at this power level[6]. A larger HET tested by Georgia Institute of Technology at 600V, using Krypton, at >2kW. Measured thrust was 70-90 mN and anode efficiency, 42-47%[9]. The discrepancy between the performance of this larger HET and a smaller 100W HET, is clear and significant. An efficiency of 5% is not feasible to be used in real-world applications as the costs would be too large and would likely outweigh the benefits, as well as other thrusters operate at this level more effectively, such as ion thrusters.

To prolong the life of a HET, magnetic shielding is used which effectively shields the channel walls from erosion from the ions. This shielding may reduce erosion or stop it altogether, but may cause a decrease in performance due to some of the potential plasma propellant being used in the shielding mechanism. This decrease in performance is small in larger systems but this effect will become more detrimental as the scale is reduced[1].

2 Theoretical Calculations

Cyclotron frequency is calculated using the following formula:

$$\omega_c = \frac{q\vec{B}}{m}$$

Where q is the charge of the particle (C), $\vec{B}$ is the magnetic field strength (T) and m is the mass of the particle (kg).

Using the image showing the measured magnetic field strength over a cross section of the channel, I determined the value of 200 G (0.02 T) would be suitable. This seems to be a good approximation for the overall magnetic strength at the end section of the channel where the cyclotron will be located. Furthermore, I have chosen Krypton for the subject of my calculations because this is the primary propellant that the thruster is designed to use. Using these additional values of fundamental charge (1.6×10^{-19} C), mass of Krypton (1.39×10^{-25} kg) and electron (9.11×10^{-31}kg); the cyclotron frequencies of Krypton ions and electrons are $\omega_{c,k} = \frac{1.6\times10^{-19}*0.02}{1.39\times10^{-25}} = 2.30 \times 10^4 s^{-1}$ and $\omega_{c,e} = \frac{1.6\times10^{-19}*0.02}{9.11\times10^{-31}} = 3.51 \times 10^9 s^{-1}$, respectively.

Larmor radii for ions and electrons are calculated using the following formulae:

$$r_{L,i} = \frac{v_i}{\omega_c}$$

$$r_{L,e} = \frac{v_{th}}{\omega_c}$$

Where v_i is the velocity of the ion in the axial direction and v_{th} is the thermal velocity of the electron. These are both useful estimates of the velocity of each particle, perpendicular to the magnetic field.

$$v_i = \sqrt{\frac{2qV_b}{m}}$$

$$v_{th} = \sqrt{\frac{8kT_e}{\pi m}}$$

Where V_b is the beam voltage, which is the difference between the anode voltage and the sum of the cathode to ground voltage and the plasma potential. Using given estimations of plasma potential (10V) and cathode to ground voltage (30V), the beam voltage is found to be 560 Volts (V_b = 560V). T_e is the thermal velocity of (T_e = 40eV = 4.64×10^5K). From these formulae, I calculated the Larmor radii for Krypton ions and electrons to be $r_{L,i} = 1.56m$ and $r_{L,e} = 1.20mm$, with velocities of $v_i = 3.59 \times 10^4 ms^{-1}$ and $v_{th} = 4.23 \times 10^6 ms^{-1}$.

The Larmor radius of the ions is larger by a factor of 3 when compared to the electrons. This makes sense as their mass is much larger than the electrons and so they will feel the effects of the the Lorentz force less. Also using a propellant with a lower atomic mass will cause the Larmor radius to decrease with a rate of $\frac{1}{\sqrt{m}}$.

3 Experimental Calculations

First, I will outline the method used for performance parameter calculation based upon the experimental thrust measured using the thrust balance. Then, I will outline the first analytical method using a value of thrust calculated from operating parameters whilst accounting for ion characteristics and various losses due to plasma physics. Lastly, I will use a similar analytical formula to calculate the specific impulse and then derive thrust and efficiencies based upon this.

The following formula for thrust, $T(N)$, can be rearranged to find specific impulse, as shown below:

$$T = \dot{m} I_{sp} g_0$$

$$I_{sp} = \frac{T}{\dot{m} g_0}$$

Where $\dot{m}$ is mass flow rate (kg/s), I_{sp} is specific impulse (s) and g_0 is the gravitational constant ($9.81 m/s^2$).

Converting mass flow rate from standard cubic centimeter per minute (SCCM) to kg/s, for Krypton, uses the following procedure:

$$1sccm = 7.436 \times 10^{-4} * 83.798 \times 10^{-6}$$

$$55sccm = 3.427 \times 10^{-6} kg/s$$

Therefore, for an experimentally measured thrust of $72.182mN$ at 55sccm, specific impulse is found to be

$$I_{sp} = \frac{72.182 \times 10^{-3}}{3.427 \times 10^{-6} * 9.81} = 2147s$$

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Anode efficiency is calculated using the following formula:

$$\eta_{anode} = \frac{T^2}{2\dot{m}_{anode} P_{anode}}$$

Where $\dot{m}_{anode}$ is the mass flow rate at the anode and $P_{anode}(W)$ is the product of the voltage and current at the anode ($P_{anode} = V_{anode}I_{anode}$). Continuing with the same data point; $V_{anode} = 348V$, $I_{anode} = 7.29A$, $\dot{m}_{anode} = 50sccm = 3.116kg/s$ and $P_{anode} = 348 * 7.29 = 2533W$. Consequently,

$$\eta_{anode} = \frac{(72.182 \times 10^{-3})^2}{2 * 3.116 \times 10^{-6} * 2533} = 33\%$$

Total efficiency of the thruster is calculated using the following formula:

$$\eta_{Total} = \frac{T^2}{2\dot{m}(P_{anode} + P_{cathode} + P_{electromagnet})}$$

Where $P_{cathode}$ and $P_{electromagnet}$ are the power supplied to the cathode and electromagnets, respectively. In this instance, $P_{cathode} = 86.25W$ and $P_{electromagnet} = 48W$. As a result,

$$\eta_{Total} = \frac{(72.182 \times 10^{-3})^2}{2 * 3.427e - 6 * (2533 + 86.25 + 48)} = 28.5\%$$

Next, I will outline an analytical thrust calculation which will then be the basis for calculating performance parameters using the previous meethod shown above for the experimental value.

Analytical thrust is found using the following formaula:

$$T = \alpha\eta_{cos}\sqrt{\frac{2M_{ion}}{q}} I_b\sqrt{V_b}$$

Where α represents the losses caused by double ionisation, η_{cos} is the loss due to beam divergence, M_{ion} is the ion mass, I_b is the beam current and V_b is the beam voltage. These factors are constant and can be found using the following formulae:

$$\alpha = \frac{1 + \frac{1}{\sqrt{2}}\frac{I^{++}}{I^{+}}}{1 + \frac{I^{++}}{I^{+}}}$$

$$\eta_{cos} = \frac{1}{2}(1 + cos^2\theta)$$

As the proportion of doubly charged ions $\frac{I^{++}}{I^{+}} = 0.2$, α is found to be 0.951, and $\eta_{cos} = \frac{7}{8}$ for a beam divergence half-angle of 30 degrees. Furthermore, using the beam current fraction given (0.77) and ignoring effects of backstreaming electrons on the beam current, I can estimate the beam current to be $I_b = 0.77I_d$. Assuming $I_{anode} = I_d$ and using the same data point from the previous method: $I_b = 0.77 * 7.29 = 5.61A$. Similarly to find beam voltage, I assume $V_{anode} = V_d$ and then taken $40V$ away from it to account for the cathode to ground voltage and plasma potential to find $V_b = 348 - 40 = 308V$.

The resulting analytical thrust is

$$T = 0.951 * \frac{7}{8} * \sqrt{\frac{2 * 1.39 \times 10^{-25}}{1.6 \times 10^{-19}}} * 5.61 * \sqrt{308} = 108mN$$

The exact same method used for the experimental value is then used to find the specific impulse and efficiencies. These are found to be: $I_{sp} = 3212s$, $\eta_{anode} = 73.8\%$ and $\eta_{total} = 63.8\%$.

The second analytical method starts with analytically calculating a specific impulse value and then finding the thrust using $T = \dot{m}I_{sp}g_0$.

$$I_{sp} = \frac{\alpha\eta_{cos}\eta_m}{g_0}\sqrt{\frac{2qV_b}{M_{ion}}}$$

η_m is the mass utilisation efficiency; given as 0.88. All the other values used are unchanged from the previous analytical method.

$$I_{sp} = \frac{0.951 * \frac{7}{8} * 0.88}{9.81} * \sqrt{\frac{2 * 1.6 \times 10^{-19} * 308}{1.39 \times 10^{-25}}} = 1987s$$

Using this specific impulse, thrust is to be $T = 3.427 \times 10^{-6} * 9.81 * 1987 = 66.79mN$. Using the same method for finding efficiencies yields $\eta_{anode} = 28.6\%$ and $\eta_{total} = 24.4\%$.

4 Data Analysis

In this section I will be presenting various graphs showing relationships between different operating and performance parameters, and discussing any implications arising from these observations. As there are nine potential datasets to plot together, I have omitted the dataset pertaining to the 50 sccm flow rate (except for Figure 2) as this had a limited number of data points. The aim of doing this was to make the relationship between the experimental and analytical methods clear.

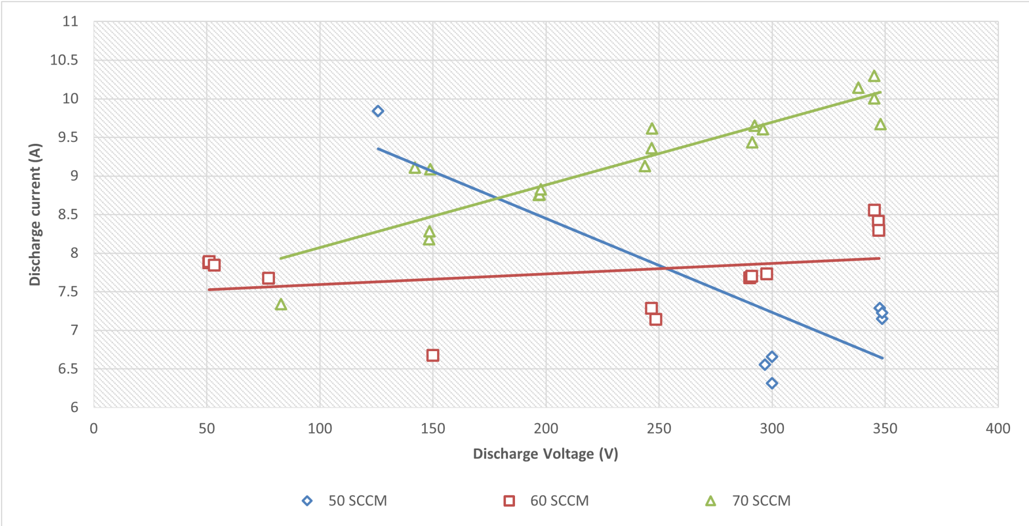
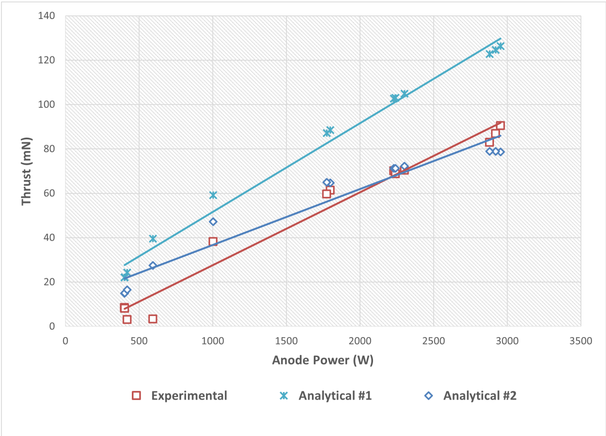


Figure 2: A graph showing the relationship between discharge current and discharge voltage at different flow rates.



(a) Thrust vs Power at 60 sccm

(b) Thrust vs Power at 70 sccm

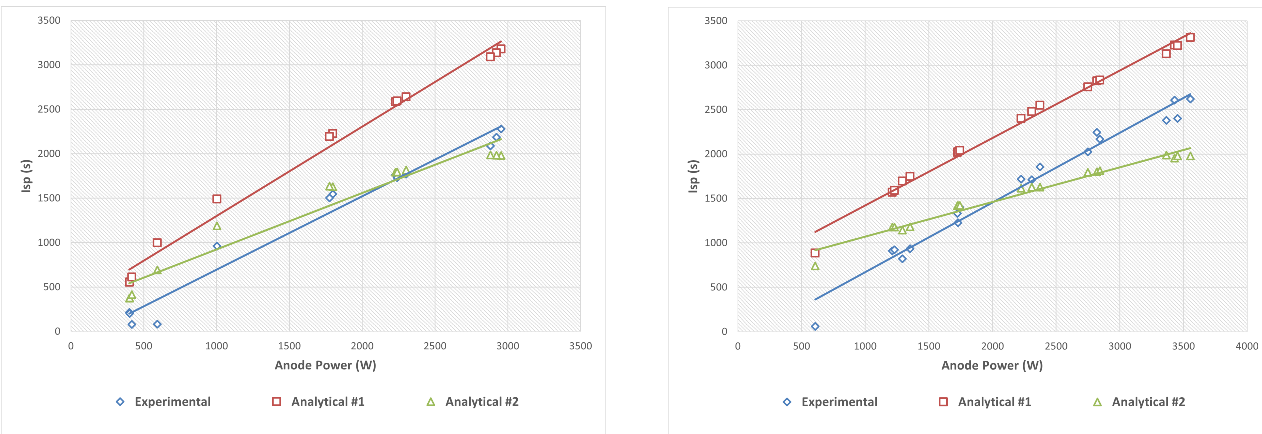
Figure 3: Plots showing experimental and analytical thrust trends at varying mass flow rates.

The discharge current is a measure of the current running through the discharge supply at any one time. There are two sources to this electron flow: 1) electrons from ionisation events and 2) upstream electrons coming from the cathode. The anode and cathode are connected via this supply and electrons traveling upstream through the channel to meet the anode complete the circuit. On the other hand, the voltage measured at the anode is varied manually and will affect the strength of the electric field that accelerates the ions causing thrust.

The trend lines, from Figure 2, show that for a mass flow rate of 50 sccm, the discharge current decreases by $\sim$3A as the voltage is increased by 300V. For 60 sccm, there a very slight increase in the current of $\sim$0.5A over the ΔV of 300V, but this increases to $\sim$2A for 70 sccm. Overall, it can be seen that as the flow rate is increased, from 50 to 60 to 70 sccm, the relationship between discharge current and voltage changes from inversely correlated to directly correlated. However the range of data for voltage at 50 sccm is limited and the data point at 125V is the sole reason for this inverse correlation. So this trend has low reliability but nonetheless does not change the overall observation of the trend lines becoming steeper as flow rate increases.

Since the dominant contributor to the discharge current are the electrons coming from ionisation events, this relationship is suggesting that as the flow rate is increased, this flow of these ionisation electrons increases. Theoretically, this is logical as at a higher mass flow rate, there is a higher density of neutrals flowing into the channel, meaning a higher chance of collisions and ionisations. This reasoning is supported by the relationship between rate of ionisation and neutral number density: $v_i = n_n \langle \sigma_i v_e \rangle$.

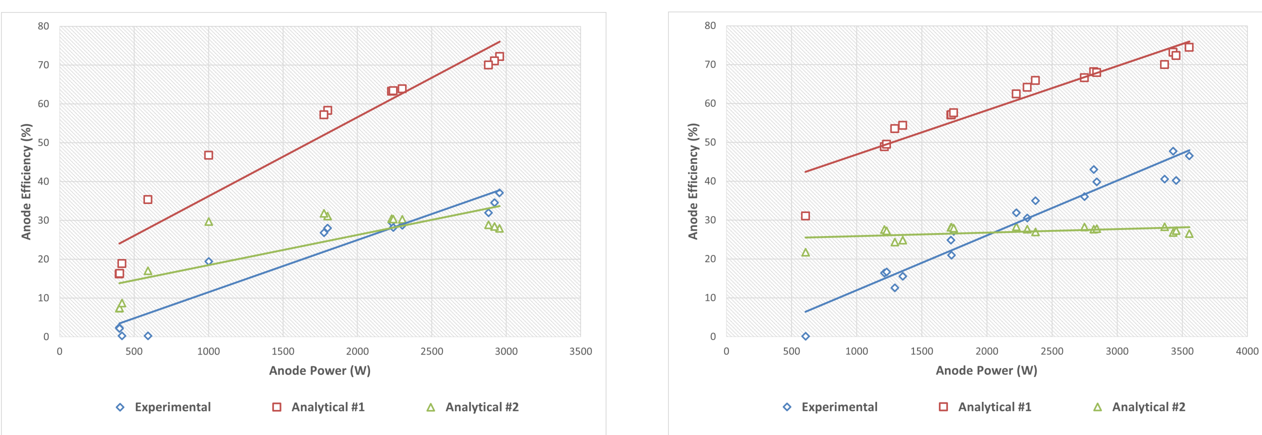
Figure 3 shows the fairly intuitive relationship between the power supplied to the thruster and how much thrust it produces. As more power is supplied to the anode, the electric field accelerating the ions will become stronger, due to the higher potential at the anode, causing the exhaust velocity of the thruster to increase. This is what is expected according to fundamental propulsion concepts - greater propellant velocity equals greater thrust. Furthermore the analytical calculations yield a higher thrust than the experimental values at the same power level.



(a) Specific Impulse vs Power at 60 sccm

(b) Specific Impulse vs Power at 70 sccm

Figure 4: Plots showing experimental and analytical specific impulse trends at varying mass flow rates.



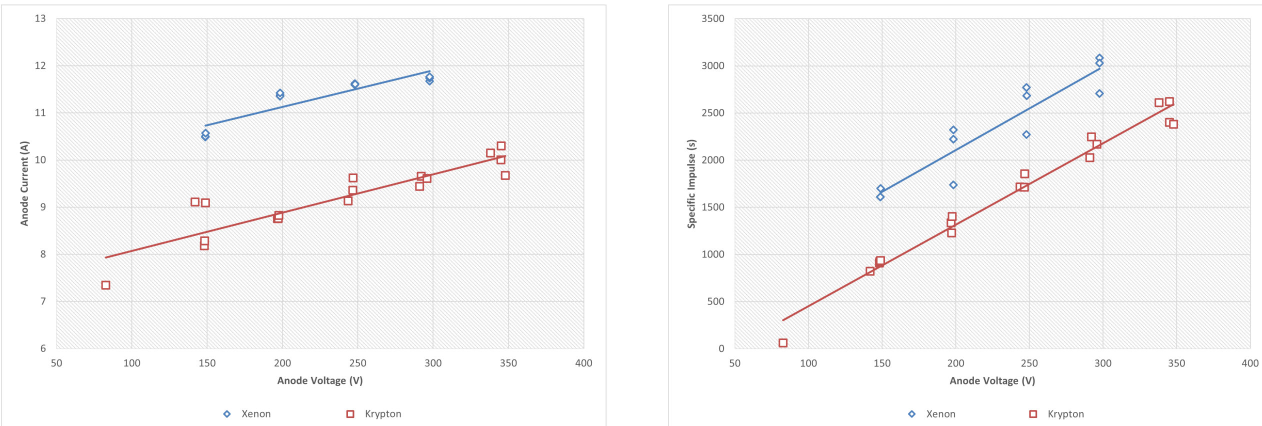
(a) Anode Efficiency vs Power at 60 sccm

(b) Anode Efficiency vs Power at 70 sccm

Figure 5: Plots showing experimental and analytical anode efficiencies trends at varying mass flow rates.

Following on from the previous plots, Figure 4 shows an almost identical trend for each method. Specific impulse is fundamentally the total impulse per unit weight of propellant. So it makes sense that as anode power is increased, thrust increases, along with specific impulse. When increasing the mass flow rate, the specific impulse appears to decrease slightly. This suggests that the extra thrust produced from a higher density of propellant may not be increasing specific impulse at a steady rate. In fact, if mass flow rate were to increase above 70 sccm, I would expect the gains in specific impulse to become less and less.

The first analytical method predicts anode efficiency to be much greater than the results gathered from experimental data. Figure 5 echoes the same implications from previous plots: the first analytical method significantly overestimates the performance of the thruster, whilst the second analytical method does not scale at the same rate with power to match the experimental data. Once again, the efficiency does seem to slightly decrease when increasing from 60 to 70 sccm. The linear trendline can make this difficult to discern. At 1000W and 60 sccm, an anode efficiency of 20% was obtained, whereas this was only attained at 70 sccm, past 1700W. At higher powers this discrepancy does seem to disappear but it supports the fact that the thruster was designed to run optimally at 1.35kW.



(a) Anode Voltage vs Anode Current

(b) Specific Impulse vs Anode Voltage

Figure 6: Performance comparison of Xenon & Krypton

Xenon is a larger atom than Krypton with approximately double the mass. With regards to neutral gas dynamics; its neutral velocity has decreased by a factor of $\frac{1}{\sqrt{2}}$. However, at an electron temperature of 40eV, it has a greater cross sectional area of ionisation of $\sim$7 Å compared to $\sim$4 Å for Krypton, leading to the reaction rate coefficient increasing from 1.1×10^{-13} to 1.7×10^{-13}. As a result, the ionisation rate of Xenon is higher than Krypton, with a quarter of the mean free path of ionisation. As ionisation events cause ions to be accelerated downstream and electrons to be sent to the anode[3], this explains the consistently higher anode current seen when using Xenon propellant compared to Krypton. The trendlines seem to run virtually parallel, with a separation of $\sim$2A.

Theoretically, a reduction in specific impulse when switching to Xenon would be a reasonable expectation as it has twice the mass of Krypton. The analytical formula states a relationship of $I_{sp} \propto \frac{1}{\sqrt{M_{ion}}}$. However, the mass utilisation efficiency of Xenon will be significantly greater because of its greater ionisation rate and lower ionisation cost; also causing a further reduction in losses due to double ionisation.

5 Discussion

The first analytical method seems to overestimate the performance of the thruster as can be seen in Fig. 3, 4 and 5. Although, the trendline does run virtually parallel to the experimental plot which indicates it is able to predict how the performance will scale with input power. On the other hand, the second analytical method gives a more realistic prediction of performance but does not scale accurately with power. At 70 sccm and below 2000W of anode power, it provides an overestimation of performance and an underestimation over 2000W.

The reason for the differing accuracy of these analytical models is most likely due to factors which are considered in their respective formulae. The first method completely neglects the mass utilisation efficiency. By doing this, it essentially assumes a value of 100% for this parameter which may be why it has such a significant overestimation of thrust/performance. Whilst the second method does account for mass utilisation efficiency, it does lack to consider any effect that the temperature of the neutral gas may have on ionisation. As power at the anode is increased, temperature of the anode increases, causing a higher temperature of the neutrals being introduced to the channel. It would be reasonable to suggest that ionisation rates would increase if the neutral temperature increases. Furthermore, modeling an accurate variation of mass utilisation efficiency, as current varies, and factoring that into the formulae may provide stronger agreement for the second analytical method.

These analytical methods are a more accurate prediction of performance when applied to gridded ion thrusters because of the beam voltage and current being easier to derive. They can be directly measured. However in HETs, the plasma potential and cathode potential below ground are difficult to measure and so have to be estimated to be able to obtain a value for the beam voltage. This will introduce inherent errors as the estimate will not be reflecting the realistic plasma physics dynamics present in the thruster.

Even though the thruster is designed to perform at 1.35kW, anode efficiency gains did not drop after this point in the graphs. Instead it continued to climb, at a steady rate, so that the best efficiencies were at the highest power levels. The best experimental performance was seen at powers of ∼3500W. However, at 60 sccm, the efficiencies did noticeably improve, compared to 70 sccm. As the applications of these thrusters is on spacecraft, high power supplies do pose a problem because of weight and concerns due to large currents. Power plant mass on a spacecraft using electric propulsion does need to be kept within a limit as there is a point of diminishing returns for needing a more powerful power supply. More data collected around 1.35kW would be helpful in gaining insight into the level of optimisation at this power level.

6 Conclusion

Theoretical analysis is not accurate enough for predicting the performance of HETs based solely upon operating parameters and geometries. This is due to difficulties in estimating various parameters such as plasma potential which introduce errors when calculating key values; namely beam voltage and current. Furthermore, there are unknown areas of plasma physics dynamics, such as anomalous transport, that I have not explored in this report and which make the prediction of HET performance inaccurate[5]. This is an active area of research and adds to the intrigue of Hall Effect Thrusters.

I have evaluated the thrusters performance at 70 sccm as this seems to be the best mass flow rate for overall performance. Even though 70 sccm gives the thruster the best chance of achieving desired performance ratings, it still slightly underperforms. The thruster reaches a total efficiency of ∼45% at 3500W. Maximum thrust was measured to be 120mN at this power with a specific impulse of 2600s. At lower power levels I was able to compare it to other HETs that have been analysed for performace using Krypton. At an input power of 2000W, Szabo et al. were able to exceed a specific impulse of 2000s with an overall efficiency of 50%[9]. Furthermore, Hargus et al. were able to obtain a specific impulse of 1568s at 1356W[7], whereas the SHEKT-600V was unable to exceed 1000s at its designed power supply of 1350W. From the data presented, I would conclude that the thruster does not perform well enough at its designed power level of 1.35kW as there are other HETs that can far outperform it. Although it may be important to note that these thrusters I have compared it to were developed within more advanced research programmes attached to the United States Air Force.

At 1.35kW, thrust was measured to be 43mN, leading to a thrust to power ratio of $\frac{43}{1.35} = 31.9mN/kW$. Even at the highest operating power of 3.5kW, producing 120mN of thrust, the thrust to power ratio slightly increased to $\frac{120}{3.5} = 34.3mN/kW$. A reasonable, acceptable range of thrust to power ratios that have to be obtained to be considered for an achievable LEO orbit raise of 500km to 1100km is 40-60 mN/kW[8]. The SHEKT-600V falls short of this minimum of 40 mN/kW by approximatley 20%. Therefore, it would be not be considered for this mission. Also it would ideally need a specific impulse greater than 1000s at this power level.

Alternative fuels such as Xenon do perform better than Krypton as shown in figure 5. Using Xenon as a propellant for HETs is established and leads to good performance. However other fuels are being explored because of the scarcity of Xenon on Earth. Total global production is only about 53,000 kg per year which leads to high prices and low supply when there is a demand for it to be used as fuel in high quantities[4].

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