Orbital Mechanics Assignment

Preliminary analysis of geostationary orbit transfer for UK launched satellites

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1. Definitions and Assumptions

1.1. GEO Target Orbit

Geostationary orbit is a protected region of space with the following characteristics:

- GEO Protected Zone
  - $r_{GEO} = 42164$ km $\pm$ 200 km
  - $i = 0\,°\pm 15\,°$

Geostationary satellites are most commonly designed with the purpose of observing a particular face of the Earth's surface at all times. This is achieved by orbiting Earth at an orbital radius of 41264 km. Additionally, any inclination will cause the satellite to sweep north and south along a particular longitudinal line; only useful for specialised spacecraft. So, because the most likely mission design for a GEO satellite will require orbiting at the "centre" of the GEO zone, this will be the specific target orbit for this analysis. Therefore, the final target orbit is defined as the "centre" of the GEO protected zone.

- GEO Target Orbit
  - $r_{GEO} = 42164$ km
  - $i = 0\,°$

1.2. LEO Parking Orbit

The minimum inclination for the parking orbit of any satellite launched from Sutherland Spaceport is 58.5107 °. Therefore, the most desirable parking orbit, to maximise efficiency for the transfer to GEO, will be at this inclination. Further LEO parking orbit details below:

- LEO Orbit
  - $r_{LEO} = 6871$ km
  - $i = 58.5107\,°$
  - $e = 0$

1.3. Further Assumptions

Assuming that the satellite is relatively small and is equipped with a chemical propulsion system; the additional satellite characteristics are as follows:

- Satellite Characteristics
  - $m_0 = 1700$ kg
  - $I_{sp} = 230$ s

Lastly, the burns used to change the spacecraft's velocity are modelled as impulsive.

2. Analytical Methodology

The following analysis of direct Hohmann transfers to GEO have been performed using the set mission parameters and standard formulae. Three transfers, varying by the inclination of the transfer orbit, are presented. Lastly, values used for the gravitational constant and the semi-major axis of the transfer orbit are listed below:.

- Constants
  - $\mu = 3.986 \times 10^5\ \text{km}^3/\text{s}^2$
  - $a = \frac{r_{LEO}+r_{GEO}}{2} = 24517.5$ km

2.1. Hohmann Transfer #1 : plane change at departure

2.1.1. Combined Plane Change and Departure Burn

$$v_0 = \sqrt{\frac{\mu}{r_{LEO}}} \tag{1}$$

$$v_1 = \sqrt{2\mu\left(\frac{1}{r_{LEO}} - \frac{1}{2a}\right)} \tag{2}$$

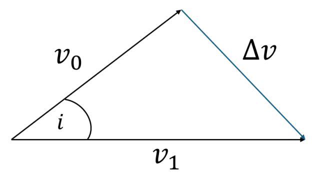

Figure 1. Illustration of spacecraft vectors to calculate ΔV burn with cosine rule.

$$\Delta V_1 = \sqrt{v_0^2 + v_1^2 - 2v_0v_1cos(i)} \tag{3}$$

2.1.2. Insertion Burn

$$\Delta V_2 = \sqrt{\frac{\mu}{r_{GEO}}}\left(1 - \sqrt{\frac{2r_{LEO}}{r_{LEO}+r_{GEO}}}\right) \tag{4}$$

2.2. Hohmann Transfer #2 : plane change at insertion

2.2.1. Departure Burn

$$\Delta V_1 = \sqrt{\frac{\mu}{r_{LEO}}}\left(\sqrt{\frac{2r_{GEO}}{r_{LEO}+r_{GEO}}} - 1\right) \tag{5}$$

2.2.2. Combined Plane Change and Insertion Burn

$$v_0 = \sqrt{2\mu\left(\frac{1}{r_{GEO}} - \frac{1}{2a}\right)} \tag{6}$$

$$v_1 = \sqrt{\frac{\mu}{r_{GEO}}} \tag{7}$$

$$\Delta V_2 = \sqrt{v_0^2 + v_1^2 - 2v_0v_1cos(i)} \tag{8}$$

2.3. Hohmann Transfer #3: Split Plane Change

This method found the optimal value for ΔV by varying the fraction of the plane change, α, performed with the departure burn.

- $v_{0,t}$ = Transfer orbit perigee velocity
- $v_{2,t}$ = Transfer orbit apogee velocity

$$h_t = \sqrt{2\mu\frac{r_{LEO}r_{GEO}}{r_{LEO} + r_{GEO}}} \tag{9}$$

$$v_{0,t} = \frac{h_t}{r_{LEO}} \tag{10}$$

$$v_{2,t} = \frac{h_t}{r_{GEO}} \tag{11}$$

$$\Delta V_0 = \sqrt{v_{LEO}^2 + v_{0,t}^2 - 2v_{LEO}v_{0,t}cos(\delta_0)} \tag{12}$$

$$\Delta V_2 = \sqrt{v_{2,t}^2 + v_{GEO}^2 - 2v_{2,t}v_{GEO}cos(\delta_2)} \tag{13}$$

$$\delta_0 = \alpha i \tag{14}$$

$$\delta_2 = (1 - \alpha)i \tag{15}$$

These formulae were inputted into Matlab with a varying α value between 0 and 1 and plotted against ΔV, as shown in Fig 2. The minimum ΔV value for a split plane change corresponds to $\alpha = 0.052$, implying the plane change should be changed 5.2% at departure and the remaining 94.8% at insertion.

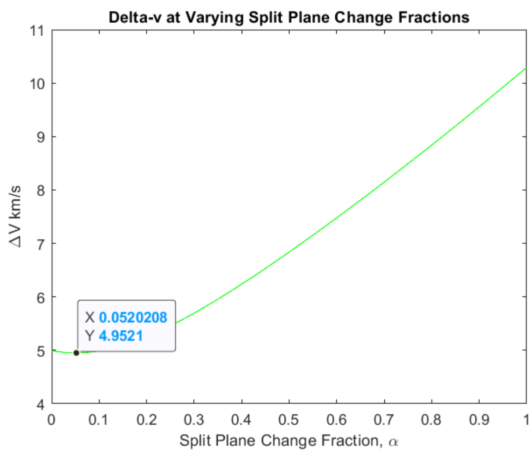

Figure 2. Plot showing split plane change fraction for GEO transfer with optimal value.

3. Analytical Results

3.1. Hohmann Transfer #1

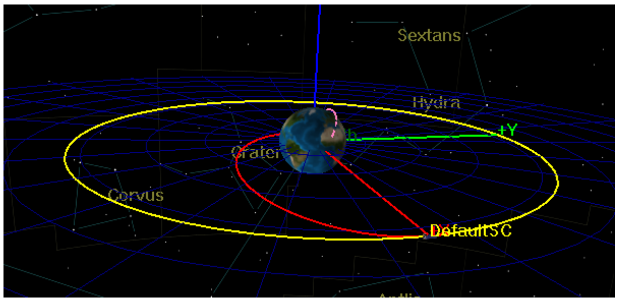

Figure 3. Graphical representation of Hohmann transfer with plane change combined with departure burn

- Cost and location of burns
  - $\Delta V_1 = 8.48878$ km/s @ r_{LEO}
  - $\Delta V_2 = 1.44698$ km/s @ r_{GEO}
  - ΔV_{total} **= 10.29586 km/s**

3.2. Hohmann Transfer #2

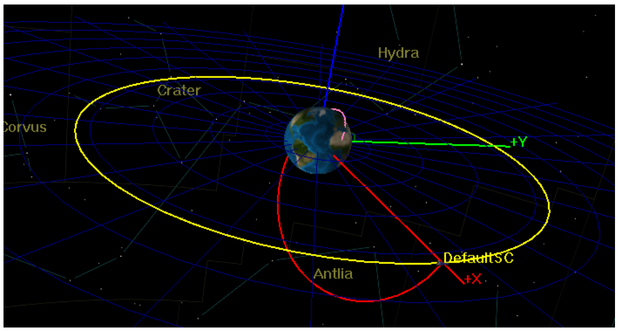

Figure 4. Graphical representation of Hohmann transfer with plane change combined with insertion burn.

- Cost and location of burns
  - $\Delta V_1 = 2.37174$ km/s @ r_{LEO}
  - $\Delta V_2 = 2.62197$ km/s @ r_{GEO}
  - ΔV_{total} **= 4.99371 km/s**

3.3. Hohmann Transfer #3

- Cost and location of burns
  - $\Delta V_0 = 2.41657$ km/s @ r_{LEO}
  - $\Delta V_2 = 2.53553$ km/s @ r_{GEO}
  - ΔV_{total} **= 4.95210 km/s**
- Split change fraction = 5.20%/94.80%

4. Computational Methodology

To set up the correct initial conditions for the LEO parking orbit, the following values were used, shown in the figure below. These were used for all computational analyses. The eccentricity of the parking orbit can not equal exactly zero in the software and so a value approaching zero is used instead. The value used here was approximately 10^{-11}. Also, it should be noted that the coordinate system used for the impulsive burns is a local VNB (Velocity, Normal, Binormal) reference axes.

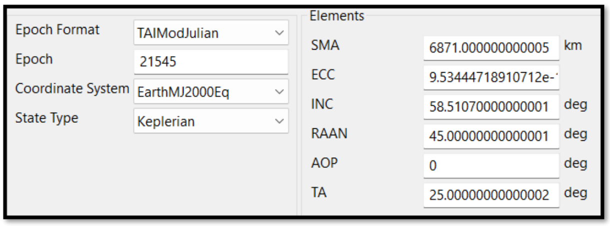

Figure 5. Initial conditions used for GMAT.

4.1. Hohmann Transfer #3

To reproduce and confirm the split plane change maneuver computationally, a script was written in GMAT to optimise the transfer, utilising the Yukon solver. A variable, named iTrans, was created to represent the inclination of the transfer orbit and varied with an initial value of 58.5107°with a perturbation of 0.2. This method starts at 0%, adding approximatley 0.3% each iteration until an optimal value is found. For each iteration of this value, target functions would find the ΔV values for each burn element. There are two burns; insertion and departure, with three orthogonal elements, corresponding to the VNB axes. However, for this transfer trajectory, it was sufficient to only vary element 1 and 2, of the velocity and normal axes. The departure burn was given two targets to achieve: RMAG = 42164 (tol = 0.1) and INC = iTrans (tol = 0.001), propagating to apoapsis. Once propagated to apoapsis, the following targets were found by varying the insertion burn elements: ECC = 0.0 (tol = 0.0001) and INC = (tol = 0.001). Once all burn elements are found, they are added vectorially:

$$\Delta V_{total} = \sqrt{\Delta V_{dep,1}^2 + \Delta V_{dep,2}^2} + \sqrt{\Delta V_{ins,1}^2 + \Delta V_{ins,2}^2} \tag{16}$$

The optimiser is programmed to return the value for this ΔV variable when it is minimised as a result of finding the values for each burn element that satisfies the targets for each new inclination of the transfer orbit.

4.2. Bi-Elliptic Transfer

Firstly, a third burn is added to this transfer, to be performed at the apoapsis of the first transfer orbit, with the goal of performing the required plane change at this intermediate radius. The inclination change was designated to be performed fully within the second burn in the bi-elliptic transfer maneuver because this is the furthest point from the primary and so, velocity will be at a minimum, with regards to the whole mission. This leads to the reasonable designation of changing inclination of the second transfer orbit to zero at $r = r_B$.

A feasible bi-elliptic transfer orbit was obtained through iterating over β values from $\frac{r_B}{r_A}$ to 15, at moderate intervals of 1, and then from 8 to 8.5, at intervals of 0.05. The optimal β value was found to be 8.30, corresponding to an intermediate radius of $r_B = 57029\,km$.

The departure burn varies element 1 and 2 with the target of RMAG = $r_B = \beta r_{LEO}$, propagating to apoapsis. Next, the second burn (Ichange) varies all three elements of the burn to achieve INC = 0 (tol = 0.001) and RMAG = 42164 (tol = 0.1), propagating to periapsis. Thirdly, the insertion burn circularises by achieving targets of ECC = 0 (tol = 0.0001) and INC = 0 (tol = 0.001), propagating for one orbital period for visual confirmation. All three elements were subject to variation for insertion. All burn elements are added vectorially, as above, and total cost is returned via a write function.

The data gathered from computational analysis was plotted in the figures below.

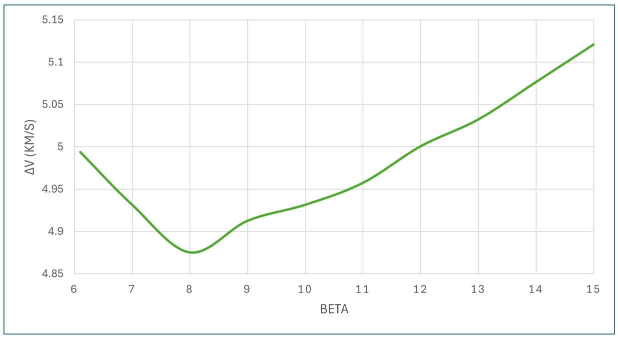

Figure 6. ΔV cost for bi-elliptic transfer, varying r_B moderately.

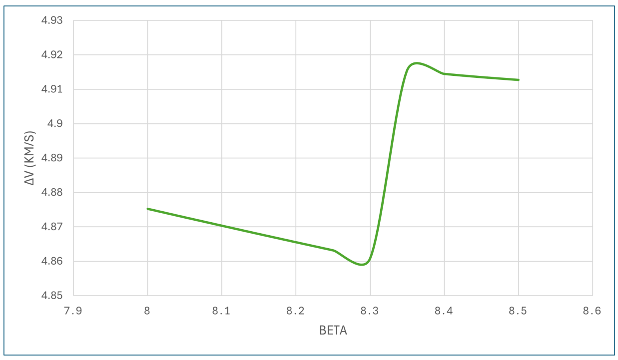

Figure 7. ΔV cost for bi-elliptic transfer, zoomed in at minimum, showing optimal value ($\beta = 8.3$).

5. Computational Results

5.1. Hohmann Transfer #3

As shown by Fig 8, the computational analysis yielded an optimal transfer orbit inclination of 55.4817°. This is 94.82% of the initial inclination of 58.5107°, meaning that 5.18% of the plane change has been performed at departure. This is extremely close to agreeing with the analytical analysis of 5.2%. The computational analysis is more sophisticated and offers results with a higher level of accuracy. The ΔV cost is indistinguishable from the analytical result up to an accuracy of 0.01 m/s.

```
*** Optimization Completed in 6 iterations and 7 function evaluations
*** The Optimizer Converged!
Final Variable values:
   iTrans = 55.4817305893                              DVtot =
Yukon1 converged to within target accuracy.            4.952104563790691
```

Figure 8. Results from optimisation of split plane change via GMAT.

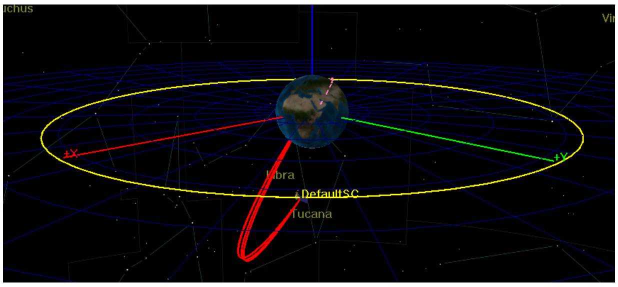

Figure 9. Graphical representation of Hohmann transfer with various split plane change fractions simulated to find the optimal value.

- Cost and location of burns
  - $\Delta V_0 = 2.41657$ km/s @ r_{LEO}
  - $\Delta V_2 = 2.53553$ km/s @ r_{GEO}
  - ΔV_{total} = **4.95210 km/s**
- Split change fraction = 5.17%/94.83%

5.2. Bi-Elliptic Transfer

Through iteration of β within GMAT, the optimal value was found to be 8.3 ($r_B = 57029$ km), as shown Fig 7.

Control Variable	Current Value	Last Value	Difference
Departure.Element1	2.559304766710072	0.5	2.059304766710072
Constraints	**Desired**	**Achieved**	**Difference**
(==) DefaultSC.Earth.RMAG	57029	57029.00166286564	0.001662865644902922

CONVERGED

Control Variable	Current Value	Last Value	Difference
ichange.Element1	0.04725331710879799	0.04725331710879799	0
ichange.Element2	-2.078654414360817	-2.078654414360817	4.440892098500626e-16
ichange.Element3	0.0003077036366182201	0.0003077036366182201	0
Constraints	**Desired**	**Achieved**	**Difference**
(==) DefaultSC.EarthMJ2000Eq.INC	0	9.507220963046152e-06	9.507220963046152e-06
(==) DefaultSC.Earth.RMAG	42164	42164.15780993503	0.1578099350299453

CONVERGED

Control Variable	Current Value	Last Value	Difference
Insertion.Element1	-0.2223397832130963	-0.2223397832130963	0
Insertion.Element2	1.948322329285756e-06	1.948322329285756e-06	-4.235164736271502e-22
Insertion.Element3	-3.075056144709911e-05	-3.075056144709911e-05	-6.776263578034403e-21
Constraints	**Desired**	**Achieved**	**Difference**
(==) DefaultSC.Earth.ECC	0	1.005455654986522e-05	1.005455654986522e-05
(==) DefaultSC.EarthMJ2000Eq.INC	0	2.690403190900053e-05	2.690403190900053e-05

CONVERGED

Figure 10. GMAT convergence results for optimal bi-elliptic transfer.

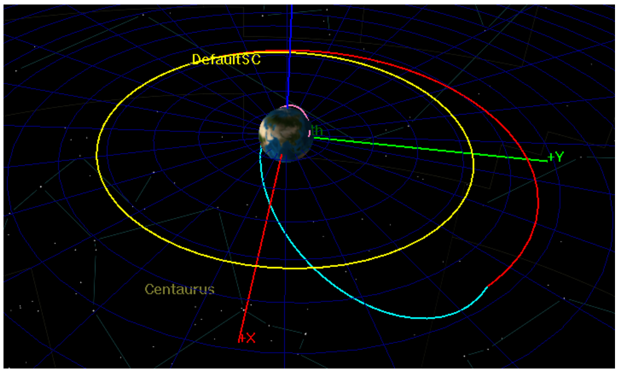

Figure 11. Graphical representation of optimal bi-elliptic transfer.

- Cost and location of burns
  - $\Delta V_0 = 2.55930$ km/s @ r_{LEO}
  - $\Delta V_1 = 2.07919$ km/s @ $r_B = 8.3 r_{LEO}$
  - $\Delta V_2 = 0.22234$ km/s @ r_{GEO}
  - ΔV_{total} = **4.86083 km/s**

	Total ΔV km/s	Required Fuel (kg)	Payload Mass (kg)	Duration
Hohmann #1	10.29586	1682.3	17.7	5hr18m
Hohmann #2	4.99371	1514.1	185.9	5hr18m
Split Plane Change (5.2/94.8)	4.95210	1510.7	189.3	5hr18m
Bi-elliptic @ β = 8.3	4.86084	1502.8	197.2	23hr10m
Hohmann from equatorial launch	3.81550	1386.6	313.4	5hr18m

Table 1. Results showing performance index of transfers.

6. Discussion

Hohmann Transfer #1 is not feasible due to the enormous cost of performing the inclination change at the beginning of the transfer - close to the Earth. It only allows for 17.7 kg of payload mass on a 1700 kg satellite - demonstrating that the costs of this transfer vastly outweighs the benefit.

Hohmann Transfer #2 is 49% of the cost of the Hohmann Transfer #1 because of the inclination being performed further away from the primary; at the apogee of the transfer orbit. Maximum payload mass is increased more than tenfold from this adjustment. However, an even further improvement can be made by splitting the inclination change (5.2%/94.8%), increasing payload mass to 189 kg. This is the limit of performance for a direct Hohmann transfer maneuver which benefits from being the quickest transfer of 5 hours and 18 minutes.

A further decrease in ΔV of 91 m/s can be made by using a bi-elliptic transfer - enabling a 197 kg payload. However, it should be noted that this will increase duration of transfer by 17 hours and 52 minutes. This greater transfer time may cause complexities with communications with ground stations due to the Earth almost completing one rotation during the transfer. Perhaps a relay network such as the Deep Space Network should be considered, so that communication is maintained with the spacecraft in transit. In addition, the number of burns is increased from two to three which carries inherently greater risk and complexity.

Lastly, it is evident that satellites launched from an equatorial based launch site would be the cheapest option; allowing a payload mass of 313 kg. There is no escape from the great cost of inclination change from such a northerly position as the UK. Spaceports may be considered to be built in British Overseas Territories, nearer to the equator, in order to increase payload mass so that a wider range of customers may be interested in using the spaceport to launch a satellite in GEO. However, this would require the building of infrastructure on small islands which would likely be difficult and costly, in addition to increased transport risks and costs for launch vehicles and spacecraft to these distant places.

7. Conclusion

From this preliminary analysis, the National Space Agency have at least two options for transferring satellites (launched from the UK) to a geostationary orbit from a LEO parking orbit. The first is a direct Hohmann transfer using a split plane change (5.17/94.83); costing 4.95210 km/s. The second is a bi-elliptic transfer; costing 4.86083 km/s. It is recommended to consider the effects of increased duration and complexity of transfer for the bi-elliptic option. Alternative analysis considering electric propulsion systems with a high specific impulse may prove useful to increase payload mass, although speed of maneuver would surely be impacted. With chemical propulsion, payload mass is limited to approximately 11.5% of wet mass. Finally, to maximise payload mass, it would be necessary to launch as close to the equator as possible. British Overseas Territories such as Montserrat and Cayman Islands may be suitable places for new Spaceports in the future.