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ENGS 91, lab 6, question 1

```
function [] = q1()

clear all; close all;
format long
```

Define Functions and Constants

```
A1      = .1;        % birthrates
A2      = .1;

B1      = 8.0e-7;    % deathrates
B2      = 8.0e-7;

C1      = 1.0e-6;    % competition rate
C2      = 1.0e-7;

N1_0    = 1e5;
N2_0    = 1e5;

t_0     = 0;        % 0 years
t_end   = 10;       % 10 years

n_max   = 9;        % solve for 1 to n_max point doublings

% Differential equations
dN1 = @(N1, N2) (N1 * (A1 - B1 * N1 - C1 * N2));
dN2 = @(N1, N2) (N2 * (A2 - B2 * N2 - C2 * N1));

% Stepsize function
StepSize = @(n) (1 ./ ( 2.^n ));

% Runge Kutta ODE next step
RK4OFunc = @MyRK4O;
```

Runge Kutta Fourth Order function defined at end of file

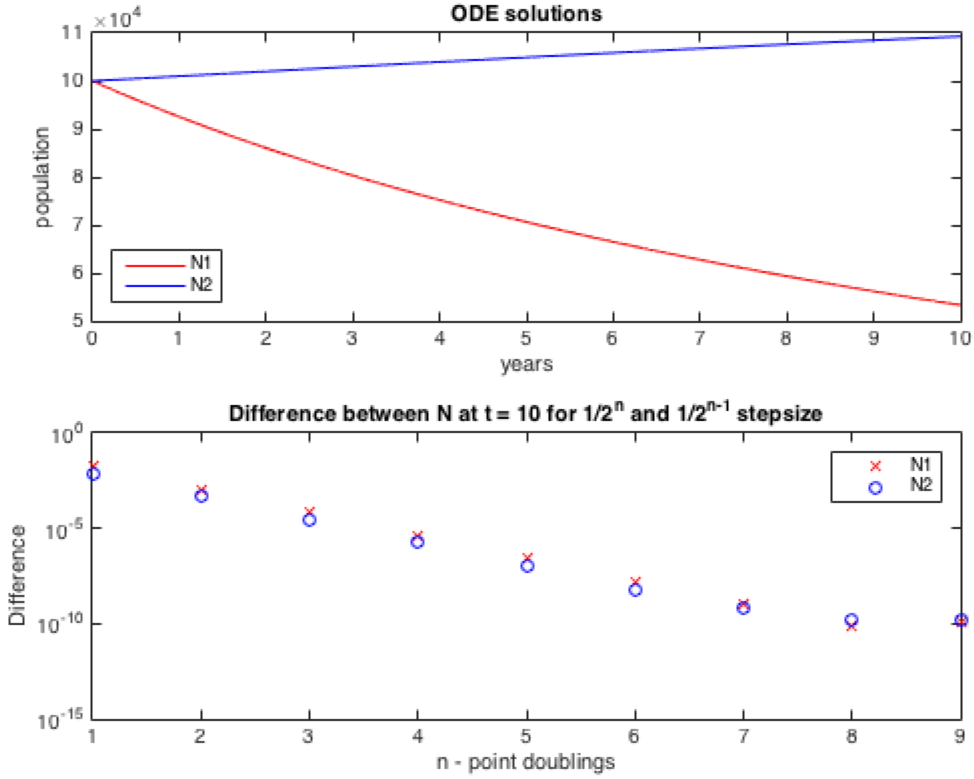
This problem's ODE solver defined at end of file

Solve ODE for different time step sizes

```
[N1, N2, t] = SolveODE(dN1, dN2, RK4OFunc, StepSize, N1_0, N2_0, t_0,
 t_end, n_max);


convergence = zeros(n_max+1,4);
prev_points = (t_end - t_0) / StepSize(0) + 1;
for i = 2:n_max+1
    points = (t_end - t_0) / StepSize(i-1) + 1;

    convergence(i,1) = points;
    convergence(i,2) = abs(N1(i,points) - N1(i-1,prev_points));

    convergence(i,3) = points;
    convergence(i,4) = abs(N2(i,points) - N2(i-1,prev_points));

    prev_points = points;
end
```

When the difference between the solutions to the ODEs at t = 10 becomes too small for machine precision, the difference will cease to decrease even though the step size continues to shrink.

```
figure()
subplot(2,1,1)
plot(t(end,1:end),N1(end,1:end),'r', ...
    t(end,1:end),N2(end,1:end),'b')
xlabel('years')
ylabel('population')
title('ODE solutions')
legend('N1','N2','Location','southwest')

subplot(2,1,2)
semilogy(0:n_max, convergence(:,2), 'rx',...
    0:n_max, convergence(:,4), 'bo');
xlabel('n - point doublings')
ylabel('Difference')
title('Difference between N at t = 10 for 1/2^n and 1/2^{n-1}
 stepsize')
legend('N1','N2')
```



Record the "True" values (limited by our machine precision)

Step Size needed is StepSize(n = 9) (1/2^9)

```
N1_true = N1(end,end)
N2_true = N2(end,end)

best_n = 8;
limit_step_size = StepSize(best_n)
```

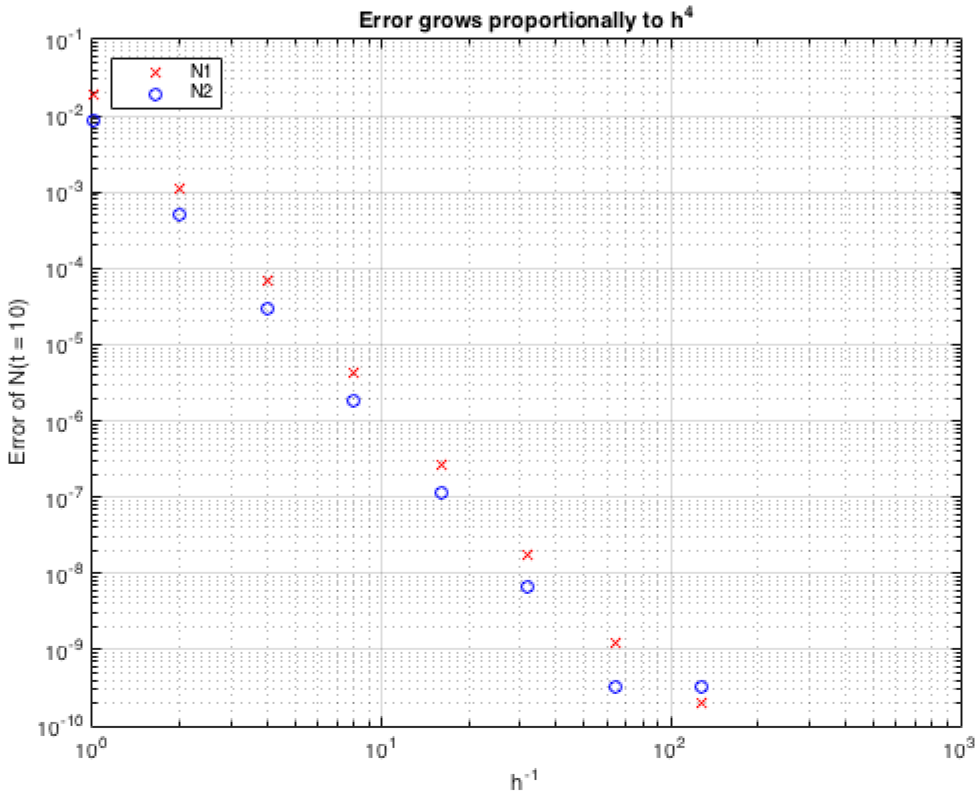
```
N1_true =

     5.331779361724573e+04


N2_true =

     1.092840108399848e+05


limit_step_size =

   0.003906250000000
```

Increase the step size by factors of two

Compare the solution to the ODEs at t = 10 for these larger step sizes. The SolveODE function that I invoked earlier actually solves the ODE's for step sizes in factors of 1/2, 1/4, 1/8, ... So we can just use the data from those solutions to see what happens as we increase the step size (instead of resolving for step sizes we've already computed).

```
error1 = zeros(1,best_n);
error2 = zeros(1,best_n);

for i = (best_n-1):-1:0
    points = (t_end - t_0) / StepSize(i) + 1;

    % calculate error from "true" value and value at t = 10 for step
 size
    error1(best_n - i) = abs(N1_true - N1(i+1,points));
    error2(best_n - i) = abs(N2_true - N2(i+1,points));
end

% Plot log of the error for each increased step size
figure()
n_val = (best_n-1):-1: 0;
stepsizes = StepSize( n_val );

loglog(1./stepsizes, error1(1:end),'rx',...
        1./stepsizes, error2(1:end),'bo');
grid on
xlabel('h^{-1}')
ylabel('Error of N(t = 10)')
title('Error grows proportionally to h^{4}')
legend('N1','N2','Location','northwest')

inverse_step_sizes = 1 ./ stepsizes;

% Get Slopes
l1 = polyfit(log10(inverse_step_sizes(3:end)), log10(error1(3:end)),
 1)
l2 = polyfit(log10(inverse_step_sizes(3:end)), log10(error1(3:end)),
 1)
```

```
l1 =

  -4.009724696780905  -1.736852490283321


l2 =

  -4.009724696780905  -1.736852490283321
```




```
    end
```

Solve this ODE function -- specific to this problem (modified from q4 on lab5)

```
                Outputs a n X 2^n matrix of solution values
                where for values(row, column)
                Row is the n in the step size calculation
                Column is the ith step

                Can be applied for Single Step solution functions

    function [N1, N2, t] =
     SolveODE(N1RateFunc,N2RateFunc,SolutionFunc,StepFunc,N1_0,N2_0,t_0,t_end,n_max)

        % Find out how many steps will be needed for the n_max method
        max_len = (t_end - t_0) / StepFunc(n_max);
        t = zeros(n_max + 1, max_len);
        N1 = zeros(n_max + 1, max_len);
        N2 = zeros(n_max + 1, max_len);

        t(:,1)  = t_0;
        N1(:,1) = N1_0;
        N2(:,1) = N2_0;
```

```
    n = 0:n_max;

    % Calculate Solutions iterations for each n value
    for i = 1:length(n)
        delta_t = StepFunc(n(i));
        steps = (t_end - t_0) / delta_t;
        for j = 1:steps
            t(i,j+1) = t(i,1) + j * delta_t;
            [N1(i,j+1),N2(i,j+1)] =
 SolutionFunc(N1RateFunc,N2RateFunc,N1(i,j),N2(i,j),delta_t);
        end
    end

end
```

Runge-Kutta 4th Order Method for solving ODE given rate function, current (y,t) and step size Modified for this problem

```
function [N1_next, N2_next] =
 MyRK4O(N1RateFunc,N2RateFunc,N1_0,N2_0,step)
    % R# - rate of change for N1
    % V# - rate of change for N2

    % First evaluation
    R1      = N1RateFunc(N1_0, N2_0);
    V1      = N2RateFunc(N1_0, N2_0);

    % Second Evaluation
    R2      = N1RateFunc(N1_0 + step/2 * R1, N2_0 + step/2 * V1);
    V2      = N2RateFunc(N1_0 + step/2 * R1, N2_0 + step/2 * V1);

    % Third Evaluation
    R3      = N1RateFunc(N1_0 + step/2 * R2, N2_0 + step/2 * V2);
    V3      = N2RateFunc(N1_0 + step/2 * R2, N2_0 + step/2 * V2);

    % Fourth Evaluation
    R4      = N1RateFunc(N1_0 + step * R3, N2_0 + step * V3);
    V4      = N2RateFunc(N1_0 + step * R3, N2_0 + step * V3);

    % Get Next Values
    N1_next = N1_0 + step/6 * (R1 + 2*(R2 + R3) + R4);
    N2_next = N2_0 + step/6 * (V1 + 2*(V2 + V3) + V4);
end
```

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