A First Course in Finance

Preview, June 25, 2004

"Have I (hic) godda deal for you."

A First Course in Finance

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ISBN: no number yet.

Library of Congress: no number yet.

Book Website: http://welch.som.yale.edu/book/.

Typesetting System: pdflatex.

Cover Font	Y&Y Lucida Casual	15, 36, 43pt.
Main Body Font	Y&Y Lucida	11pt.
Other Fonts	Y&Y Lucida variations	

See http://www.yandy.com/

Most graphics were created in R, open-source and free: www.r-project.org.

The referenced spreadsheets are Excel and OpenOffice (free).

Printed: June 25, 2004.

Warning: This book is in development.
It is not error-free.

A First Course in Finance

PREVIEW, JUNE 25, 2004

Ivo Welch

Professor of Finance and Economics
Yale University

"Have I (hic) godda deal for you."

There are a large number of individuals who have helped me with this book. They will eventually be thanked here. Until then, some random collection: Rick Antle. Donna Battista. Randolph Beatty. Wolfgang Bühler. Kangbin Chua. Diego Garcia. Stan Garstka. Mary-Clare McEwing. Roger Ibbotson. Matthew Spiegel. Julie Yufe. Many victim students using earlier error-ridden drafts.

Warning: This book is in development.
It is not error-free.

Dedicated to my parents, Arthur and Charlotte.

last file change: Jun 14, 2004 (14:56h)

To the Student

Finance and Prerequisites

Finance is such an important part of modern life that almost everyone can benefit from understanding it better. What you may find surprising is that the financial problems facing **PepsiCo** or **Microsoft** are not really different from those facing an average investor, small business owner, entrepreneur, or family. On the most basic level, these problems are about how to allocate money. The choices are many: money can be borrowed or saved; money can be invested into projects, undertaken with partners or with the aid of a lender; projects can be avoided altogether if they do not appear valuable enough. Finance is about how best to decide among these alternatives—and this textbook will explain how. To do so, it has to cover a lot of theory, but trust me: when properly defanged, the theory is not difficult.

Finance as a subject of study.

What do you need to to understand this book? You do not need any specific background in finance. You do need to be thoroughly comfortable with arithmetic and generally comfortable with algebra. You do need mathematical aptitude, but no knowledge of advanced mathematical constructs, such as calculus. (Knowledge of statistics would be helpful, but I will explain the relevant concepts when the need arises.) You should own a $20 scientific calculator, but not (necessarily) a financial calculator. You should learn how to operate a spreadsheet (such as Excel in Microsoft's office or the OpenCalc spreadsheet in the excellent and free OpenOffice at www.openoffice.org). The financial world is moving rapidly away from financial calculators and toward computer spreadsheets. Because I have tried hard to keep the book self-contained and to explain everything from first principles, you do not need to go hunting for details in statistics, economics, or accounting textbooks. (This is not to say that you do not need to take courses in these disciplines: they have way more to offer than just background knowledge for a finance textbook.)

This book and the subject itself are tough, but they are not forbidding, even to an average student. The main prerequisite is mathematical aptitude, but not mathematical sophistication.

One word of caution: the biggest problem for a novice of *any* field, but especially of finance, is **jargon**: the specialized terminology known only to the initiated. Worse, in finance, much jargon is ambiguous. Different people may use different terms for the same thing, and the same term may mean different things to different people. You have been warned! This book attempts to point out such ambiguous usage. Luckily, the bark of jargon is usually worse than its bite. It is only a temporary barrier to entry into the splendid field of finance.

Jargon can trip up the reader.

Struggling with the Learning Process in a Business School

If you are taking finance in a business school, chances are that you find yourself in a classroom with considerable heterogeneity in student preparation. Unlike almost any other core course in business schools, as many as half the students regularly come from a background in which their prime function is finance-related. Usually, this work experience will not have left such students with solid enough knowledge to skip the finance core course, but it will have left them with the knowledge to help them better integrate the new information—which leaves them with a considerable headstart when it comes to test performance relative to students who come from non-quantitative and non-financial backgrounds. In universal core courses, it is thus inevitable that many

It is impossible to time a finance course in a business school core, so that both the well-prepared finance nerds and novices will be happy from the start.

students find the tempo too fast and many other students find the tempo too slow. The world is not fair—and neither is the grade competition in such a course. But there is no way around it: although the most important goal of a finance course must be to see that *all* students are solidly prepared for interviews with companies—many interviewers ask students to apply the very concepts taught in this introductory course—it makes it difficult for previously unprepared students to keep up. In its combination of unequal preparation, often splitting a class into two equal students clienteles, and in its need to cover "unskippable" material, finance is unique among core business disciplines. There are no easy shortcuts in the curriculum.

Advice to the "non-quants."

At times, students without quantitative and financial preparation inevitably feel overwhelmed by their class experience. If you are one of these students, let me advise patience, practice, and reflection: it will all eventually fall into place, kemosabe, and you can do well *in the end*. Some of my best and brightest students felt frustrated during the course, but they kept at it, studied and learned twice as hard, and ended up at the top of their class. Struggling and anxiety along the way are necessary, maybe even desirable, and in the end unavoidable.

The Book

This book is concise, focusing on the essence of arguments.

This textbook is concise. Its intent is to communicate the essential material in a straightforward (and thus compact), but also conversational (and thus more interactive) and accessible fashion. There are already many finance textbooks with well over a thousand pages. Much of the content of these textbooks is interesting and useful but not essential to an understanding of finance. (I personally find some of this extra content distracting.)

The layout of the book.

The book is organized into four parts: the Basics, Valuation, Investments, and Financing. Major sections within chapters end with questions that ask you to review the points just made with examples or questions similar to those just covered. **You should not proceed beyond a section *without* completing these questions (and in "closed book" format)!** Many, but not all, questions are easy and/or straightforward replications of problems that you will have just encountered in the chapter. Others are more challenging. Each chapter ends with answers to these review questions. Do not move on until you have mastered the material.

This is an annotation.

There are "annotations" on the left side of most paragraphs throughout the text. Suggestion: use the remaining white space in the margin to scribble your own notes, preferably in pencil so that you can revise them.

These are other notices.

Especially important concepts that you should memorize are in red boxes:

IMPORTANT: *This is an important point to remember.*

Interesting, related points that either interrupt the flow of an argument, or that are not absolutely necessary are called "Side Notes":

SIDE NOTE: This is an interesting side note.

More detailed points are called "Nerd Notes":

NERD NOTE: *If you are really interested, here is a curious fact or a derivation.*

Both side notes and nerd notes can be safely omitted from reading without compromising understanding. Sometimes, an appendix contains further advanced material.

Sense of Humor

A final warning: I have a strange sense of humor. For example, do not be turned off by the term "nerd notes." They could also have been called "technical notes"—but this would have been dry and boring.

Advice: Follow current coverage of financial topics elsewhere!

This book cannot do it all. It is important for you to keep up with current financial developments. Frequent reading of the financial section of a major newspaper (such as the **New York Times** [**N.Y.T.**]), the **Wall Street Journal** [**W.S.J.**], or the **Financial Times** [**F.T.**] can help, as can regular consumption of good business magazines, such as **The Economist** or **Business Week**. (See the website at http://welch.som.yale.edu/book/ for more useful resource links.) Although this is not a book on "how to read and understand the newspaper," you should be able to understand most of the contents of the financial pages after consuming this textbook.

Enjoy!

Ivo Welch, Yale University, 2004.

PS: And if you would like to learn more about my take on business and finance education today, feel free to read the epilogue (Chapter A).

A Quick Adoption Checklist For Instructors

This checklist will not apply after AFCIF is published (with full supplementary materials) by Addison-Wesley-Pearson. The recommended checklist for this book (AFCIF) while in beta test mode:

- ✓ Read this prologue and one or two sample chapters to determine whether you like the AFCIF approach.

  - Do not select the introduction chapter (Chapter 1), because it is not representative. If you cannot make up your mind as to which chapter to sample, you might try the chapters on taxes, inflation, and default in rates of return (Chapter 4) or in NPV (Chapter 6); the chapter on financial statements (Chapter 9) and pro formas (Chapter 10). (The following are not yet edited: you could also glance at the chapter on statistics for investments in the survey book, followed by its CAPM chapter; or you could read the chapter on valuation with corporate taxes [APV vs. WACC] in Chapter 22).) Finally, although not representative, either, you might enjoy reading the epilogue.
  - If you do like the AFCIF approach, then please continue. If you do not like AFCIF (or the chapters you read), please email ivo.welch@yale.edu why you did not like it. I promise I will not shoot the messenger: I want to learn how to improve.

- ✓ You can continue to assign whatever other finance textbook you were planning to use, but now please add AFCIF.

  - AFCIF is a full-service textbook for an introductory finance course. However, this does not mean that it cannot also work as a complement to your previous textbook. The fact that it is so different from the competition means that you and your students can benefit from a test, in which you assign both books for one year. Relative to relying on only your old textbook, AFCIF will *not* increase, but decrease your student confusion and workload. See how well AFCIF works! (Take the Pepsi challenge!) I hope that the majority of your students (and you) will prefer reading AFCIF rather than your old textbook.
  - I believe it should also be a relatively simple matter for you to plug AFCIF into your current class: The chapters are short and easy to map to your curriculum. Having the old textbook is also your insurance against using a novel textbook. It will also make students less critical of the remaining shortcomings in AFCIF, such as the limited number of exercises (and their occasionally erroneous solutions). Perhaps most importantly, AFCIF does not *yet* have full supplementary materials. It will, but until then, the older textbook may help.
  - For now, the printing cost for AFCIF adds only around $20 to student cost, so affordability should not be a concern.

  I cannot see how you can go wrong if you try out AFCIF in this manner.

- ✓ You can receive permission to post AFCIF on your class website. (The website must be secured to allow only university-internal distribution).
- ✓ Ask your copy center to print and bind the version on your website. (If need be, I can send you nicely bound copies at $50/book. Your copy center can probably do it for $20/book.)

  - Although versions on the AFCIF website at http://welch.som.yale.edu/book/ will be better than the version you download, it is good for you and your students to have one definitive reference version.

- ✓ If you are using AFCIF and you are looking for lecture notes, feel free to "steal" and adapt my lecture notes (linked at http://welch.som.yale.edu/book/) to your liking. (Please avoid the homeworks for now. Like some of the Q&A in AFCIF, the homeworks are not solid.)
- ✓ At the end of the course, please ask your students which textbook they found more helpful. Please email your conclusions and impressions to ivo.welch@yale.edu. Any suggestions for improvement are of course also very welcome.

To The Instructor: Differences and Innovations

The main concepts of finance are found in all textbooks, and this textbook is no exception. Thus, most—though not all—of the concepts and subjects in A First Course in Finance overlap with more traditional finance textbooks. This text's content is evolutionary, not revolutionary. But I do believe that it represents a dramatic departure from traditional finance textbooks in terms of its presentation. Here is my view of how this book differs from what is currently out there:

This book is intentionally different.

Conversational Tone The tone is conversational, which (I hope) makes the subject more accessible.

Conversational Tone.

Numerical Example Based I learn best by numerical example, and firmly believe that students do, too. Whenever I want to understand an idea, I try to construct numerical examples for myself (the simpler, the better). I do not particularly care for long algebra or complex formulas, precise though they may be. I do not much like many diagrams with long textual descriptions, either—I often find them too vague, and I am never sure whether I have really grasped the whole mechanism by which the concept works. Therefore, I wanted to write a textbook that relies on numerical examples as its primary tutorial method.

The method of instruction is "step-by-step numerical examples."

This approach necessitates a rearrangement of the tutorial textbook progression. Most conventional finance textbooks start with a bird's eye view and then work their way down. The fundamental difference of this book is that it starts with a worm's eye view and works its way up. In caricature form, the format of other textbooks is "institutional background, hand-waving, formulas, figures, recipé application." The format of this textbook is the posing of a critical question like "what would it be worth?," which is then explained in numerical step-by-step examples from first principles. Right under the numerical computations are the corresponding formulas. In my opinion, this structure clarifies the meaning of these formulas, and is better than either a textual exposition or an algebraic exposition. I believe that the immediate duality of numerics with formulas ultimately helps students understand the material on a higher level and with more ease. (Of course, this book also provides some overviews, and ordinary textbooks also provide some numerical examples.)

This "forward development" approach also goes well with a book that has a conversational, more interactive flavor.

Brevity Sometimes, less is more.

Brevity is important. The book focus is on explanations, not institutions.

This book is intentionally concise, even though it goes into a lot more theoretical detail than other books! Institutional descriptions are short; only concepts and models are explained in great detail.

My view is that when students are exposed to too much material, they won't read it, they won't remember it, and they won't know what is really important and what is not. Ten years after our students graduate, they should still solidly remember the fundamental ideas and be able to look up the details when they need them. Aside, many institutional details will have changed—it is the concepts that will last longer.

Self-contained means students can backtrack.

Self-Contained for Clarity Finance is a subject that *every* smart student can comprehend, regardless of background. The real problem is that many students come into class without much prerequisite knowledge, which we, the instructors, often erroneously believe they have. It is easy to mistake such "lost students" as "dumb students," especially if there is no reference source where lost students can quickly fill in the missing parts.

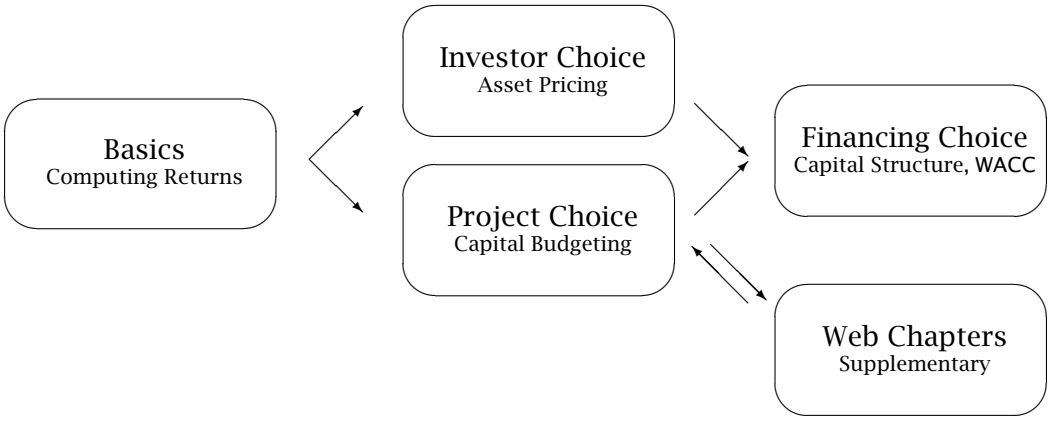
In this book, I try to make each topic's development as self-contained as possible. I try to explain everything from first principles, but in a way that every student can find interesting. For example, even though the necessary statistical background is integrated in the book for the statistics novice, the statistics-savvy student also should find value in reading it. This is because it is different in our finance context than when it is taught for its own sake in a statistics course.

Because this book tries to be as self-contained as possible, students who have failed to understand a particular lecture or topic (or who simply miss class) can now be referred back to read a self-contained chapter. My experience is that having a textbook that relates closely to the curriculum significantly reduces the need to back up and re-lecture on topics when enough students have become confused. My experience tells me that this reduces the planned time necessary to cover topics by about 10%.

Less Chapter Reordering.

Closer Correspondence with the Curriculum I believe that most finance core courses follow a curriculum that is much closer in spirit to this book—and more logical—than it is to the order in traditional finance textbooks. In the places where this book covers novel material (see below), I hope that you will find that it has merit—and if you do, that covering the topic is much easier with this book than with another book.

Every finance course must begin with some basic institutional information, some statistics (at least, expected values), and a *thorough* description of what returns are, how they are computed, and what they mean. Thereafter, capital budgeting can be taught before asset pricing, or vice versa. This book accommodates either approach by keeping these parts self-contained.



The advantage of teaching asset pricing first is that it is the more logical flow: when capital budgeting begins, students already know where the r in the NPV formula comes from. However, asset pricing relies on statistical concepts such as variance and covariance. If the finance course is taught in a first semester, contemporaneously with the statistics course, it may be better to delay covering asset pricing until the statistics course has explained some of the relevant concepts.

The capital structure part is recommended to come last: its ideas will be easier for students if they already understand NPV, payoff diagrams, and the fact that riskier securities command higher expected rates of returns.

SIDE NOTE: I have wrestled with good names for the three areas. I started with Investments (asset pricing), Valuation (capital budgeting), and Financing (capital structure); but on reflection, I now believe that all traditional designations are ambiguous.

This fellow points out when a discussion references material from chapters that have not been covered yet. This happens rarely, and the necessary knowledge is provided in self-contained fashion.

Topical Innovation The book offers a good number of specific topical and expositional innovations. Here is a selection:

Progression to Risk and Uncertainty The book starts with a risk-free world in Chapter 2, then continues into a risk-neutral world with uncertainty, and finally graduates into a risk-averse world with a risk premium. I believe it is an advantage to first teach uncertainty and default risk in terms of the much simpler concept of expected values (elaborated in my next point). The unique role of the more difficult concepts of risk measurement, risk-aversion, and risk-aversion compensation, then becomes much clearer. Equally important, the book covers the robustness of our methods—the relative importance of errors and mistakes.

First, no risk; then risk-neutral attitudes to risk; then risk-averse attitudes to risk.

Distinction Between Compensation for Default Risk and Risk Aversion I have always been shocked by how many graduating students think that a *Boston Celtics* bond quotes 400 basis points in interest above a comparable Treasury bond because of the risk-premium, which can be calculated using the CAPM formula. Learning to avoid this fundamental error is more important than fancy theories: the reason why the *Boston Celtics* bond quotes 400 extra basis points is primarily its default risk (compensation for non-payment), not a risk-premium (compensation for risk-aversion). Although many instructors mention this difference at some point, 5 minutes of default risk discussion is often lost in 5 hours worth of CAPM discussion. But if students do not understand the basic distinction, the 5 hours of CAPM discussion are not only wasted, they will have made matters worse!

Drive home "default risk."

Traditional textbooks have not helped, because they have not emphasized the distinction. In contrast, in this book, the Basics part and the Valuation part clearly break out default risk. The difference between quoted (promised) and expected returns, and default probabilities and default risk are important themes carried through.

Financials from a Finance Perspective Students need to solidly understand the relationship between financial statements and NPV, and they need to understand the basic thought process to construct pro formas. Alas, I could not find good, concise, and self-contained explanations of the important aspects and logic of accounting statements *from a finance perspective*—rather than from the minutiae-oriented accounting perspective. Consequently, AFCIF offers one chapter on financials and one chapter on pro forma construction.

Understand accounting without being an accounting textbook.

A fundamental understanding of financials is also necessary to understand comparables: for example, students must know that capital expenditures must be subtracted out if depreciation is not. Therefore, the common use of EBITDA without a capital expenditures adjustment is often wrong.

Students work out simple portfolios to understand covariances and theories.

"Hands-On" Investments The method of teaching investments is simple but revolutionary. It carries through an example of three securities with twelve actual annual rates of returns. Students can easily relate to the problem of picking the best portfolio among these three stocks. Along the way, they get to compute variances, covariances, correlations, and betas. They will internalize that the "double summation" variance formula is just a convenient computational shortcut to the variance computed from the time-series of returns on the portfolio. The correspondence between a mean-variance efficient portfolio and the risk-reward correspondence (the security markets line) similarly becomes obvious and intuitive: students can pick a better portfolio from these three stocks if one offers too little or too much return for its contribution to portfolio risk. In the end, students really understand investments, rather than just recipé formulas—and I believe they will find it clearer and easier, too.

I know this sounds incredible. This level of detail is often considered too difficult and too theoretical to be teachable in an average introductory finance course. You really have to see it to believe it.

Explain Pro-Forma, WACC, and APV better.

Better Exposition of Valuation with Corporate Taxes WACC, APV, and direct pro-forma construction all incorporate the tax advantage of debt into valuation. This is bread-and-butter for a CFO. This book offers a clear explanation of how these methods come to the same results (and when not!).

SIDE NOTE: The exposition of capital structure may be more current, but it remains similar to the treatment in traditional textbooks. The reason is that, except for the tax-related issues in WACC and APV, there is little need for formulas. Text is usually enough.

For example, corporate control considerations suggest that equity holders need to learn a lot more information about the firm than creditors ("paid or not paid") to receive their "dues." There is no advantage to understanding this in algebraic formula detail. Similarly, it does not take formulas to understand that managers like to spend money on corporate jets instead of dividends.

...and many more.

Many Other Topical Improvements For example, the yield curve gets its own chapter even before uncertainty is described in much detail, so that students understand that different horizons can come with different rates of returns. There is a self-contained chapter on comparables as a valuation technique—which many of our students will regularly have to do after they graduate. And the book tries open and honest about where our knowledge is solid and where it is shaky—and how sensitive our estimates are to the errors we inevitably commit.

Webchapters will allow a-la-carte choice.

Web Chapters and Flexible Topic Choice Although most of our curriculums are the same, covering necessary basics, there are some topics which may or may not appeal to everyone. Your preferences may differ from mine. For example, I find the comparables and pro forma chapters invaluable, because this is what most of our graduates will do when they become analysts, brokers, or consultants. However, you may instead prefer to talk more about international finance than I. It is of course impossible to satisfy everyone—and instructors have always chosen their own favorites, adding and deleting topics themselves.

Still, this book tries to help. Some chapters will not be in the printed book, but will be available only on the Web site ("Web chapters"). Chapter style and formatting are unmistakably identical to the book itself. Every instructor can therefore choose his/her own favorite selection and ask students to download it. The existing web chapters are

Resource Links A "random" selection of useful data and information resources.

Real Options Real options are briefly covered in Chapter 6 now, but not in great detail. This chapter shows how to use spreadsheets, time-series analysis, Monte Carlo simulation, and optimization to determine the value of a plant that can shut down and reopen (for a cost) as output prices fluctuate.

Option and Derivative Pricing This is a difficult subject to cover in an introductory course, because it really requires a minimum of 4-6 class sessions to do it well. This chapter tries to help you cover the basics in 2-3 class sessions. It explains option contracts, static arbitrage relations (including put-call parity), dynamic arbitrage and the Black-Scholes formula, and binomial trees.

International Finance This chapter explains the role of currency translations and international market segmentation for both investments and corporate budgeting purposes.

Ethics This chapter is experimental—and provocative. There is neither a particular set of must-cover topics, nor is there a template on how to present this material. You may disagree with my choices.

Warning: The title is optimistic.

All the material in this book has been covered in one full *semester* course for M.B.A. students at the Yale School of Management. However, it is a tight fit, even for graduate students as talented as those at Yale. It would be impossible to cover all the material in a one quarter course—although I deem all of it essential. However, a two-quarter course sequence (usually one investments and one corporate finance course) should be able to cover the material, even in an undergraduate context. For planning purposes, count on each chapter to consume *roughly* one class session. (Some chapters, such as the short chapter on background or on alternative capital budgeting methods [IRR] can obviously be covered more quickly, while others require more time.)

SIDE NOTE: An amusing concern from one instructor was that the book is *too* clear, which eliminates the need for an instructor. If you believe this to be true, then you are too familiar with the material and you underestimate how difficult finance is for new students. Neither the book alone nor your lectures alone may be enough. If we get lucky, the two together will work. Redundancy is important to the learning experience. Indeed, in my own classes, I ask the students to read the book *before class*, not after. Having a good idea of what is coming, student ask questions in the classroom that tend to become more informed.

Aside, clear and self-contained texts are common in introductory courses in other disciplines (such as mathematics, physics, and economics). These books have made it easier for students, but they have not eliminated the need for instructors. And, if *A First Course in Finance* helps to reduce the time necessary to cover the material, it is always easy for you to cover additional material. But do not get your hopes up: despite the optimistic title and the fact that students can now look up the material in the book, I still have only barely enough time to cover all necessary topics in my own 14-week class! I wish I could make the book even easier and clearer, so that I would have more spare time to entertain student questions, not less. Fortunately, even if I am forced to steamroll over a subject area somewhat too quickly, the presence of a real course text book can help alleviate student pain.

SIDE NOTE: If you use this book, please permit me to use and post *your* homework and exam questions with answers. (Of course, this is not a requirement, only a plea for help.) My intent is for the Website to become collaborative: you will be able to see what other faculty do, and they can see what you do. The copyright will of course remain with you.

Also, if you have a nice anecdote relevant for a chapter, please let me know.

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Part I

The Basics

Before we begin tonight's dream,
a word from our sponsor...

What we want to Accomplish in this Part

The goal of the first part of the book is to teach necessary background information, such as the basics of returns and rates of returns. These fundamental building blocks will form the foundation for your study of finance.

- Chapter 1 is a potpourri of important background knowledge. It describes what money is, the principal financial institutions, how taxes work, the role of transaction costs, and the importance of competitive, efficient markets, and basic statistics.

  > Typical questions: If you earn $1,000, how will you be taxed? Where and how can you invest your money? Are you likely to overpay or underpay for financial securities? If you bet on a die, what is your average outcome?

- Chapter 2 defines investment returns and *rates* of returns. (Interest rates are rates of return on loans.) It shows how you can translate rates of returns from many periods into one overall holding rate of return, and how you can "annualize" it.

  > Typical questions: If you invest $100 at an interest rate of 6% per year, how much money will you have in 30 years? If you invest $100 for a payout of $250 in 5 years, what would be your annualized rate of return?

- Chapter 3 shows how you can work with interest rates that are different for different horizons.

  > Typical questions: If you earn 30% rate of return in the first year and 60% rate of return in the second year, what is your annualized rate of return? If you can save money at a 3% annualized rate of return over one year, and at a 6% annualized rate of return over two years, what does this say about the one-year interest rate *in* one year?

- Chapter 4 shows how transaction costs, taxes, inflation, and default reduce the rates of returns that you can ultimately earn. (Default is sometimes called "credit.") The latter is our first introduction to a world of uncertainty. When there is uncertainty in loan repayments, possible future issuer default influences *quoted* interest rates. This chapter dwells at length on the difference between promised (quoted) and expected returns.

  > Typical questions: If the inflation rate is 5% per year, and the interest rate is 10% per year, how much more in goods can you actually buy if you save your money? If you have to pay 40% income taxes on interest receipts, how much more in goods can you buy now? If there is a 2% chance that your borrower will not return the money, how much extra in interest should you charge?

CHAPTER 1

INTRODUCTION

Background Stuff You Need To Know.

last file change: Jun 14, 2004 (15:01h)

last major edit: 1st edit: Feb 21

This chapter surveys some basic concepts and institutions of finance, economics, and statistics. These may seem like disparate topics, but each of them is small and self-contained, and you will use them as building blocks throughout the book. The chapter explains the major areas of finance, the meaning of "money," the role of financial institutions, the basics of taxes, the costs of transacting in financial markets, the concept of competitive low-friction markets, and some basic statistics.

ANECDOTE: The Invention of Writing
Finance may be almost as old as civilization itself, having originated in Uruk (Mesopotamia) about 6,000 years ago. Indeed, not only did the Sumerians have a financial district (like Wall Street), but writing itself may have been invented as a tool to record financial transactions! (Source: William Goetzmann, *Financing Civilization*, http://viking.som.yale.edu/.)

1·1. What is Finance?

Finance is a hybrid science of econ, stats, and accounting.

Finance is the science of capital. As such, Finance is a hybrid **Economics** (the science of choosing among economic tradeoffs), **Statistics** (the science of describing an uncertain future), and **Accounting** (the language of business). From economics, we will borrow the method of thinking about costs and benefits. From statistics, we will borrow methods of quantifying uncertainty (future risk and reward). From accounting, we will borrow methods of extracting information from firm's financial statements. However, because my goal is to write a self-contained book, I explain the concepts I borrow from other disciplines.

Individual Finance vs. Corporate Finance.

Finance is often divided into **Personal Finance** and **Corporate Finance**. As the name suggests, the former applies to a person, while the latter applies to a **Corporation** (a fancy word for a legally incorporated **Company**). The financial concepts are mostly the same, but the scale and jargon may differ. For example, the statement that "a company has issued a bond" in the context of corporate finance means the same as "an individual has borrowed money" in the context of personal finance. (Borrowing is an important concept in Finance and will be discussed at length.)

The main Finance subject areas are Valuation, Financing, and Investments.

Both individuals and corporations face three problems.

Project Choice: Capital Budgeting concerns itself with the decision to undertake or not to undertake projects: Valuable projects should be undertaken. Valueless projects should be avoided.

Investor Choice: Investments concerns itself with the decision to invest money, e.g., in a **Bank**, in the **Stock Market**, or in something else altogether (like **Real Estate** or **Stock Options**). The appropriate reward for taking on risk is the major theme in Investments.

Financing Choice: Financing concerns itself with the decisions of individual agents on how to finance projects that are to be undertaken. The two primary options are borrowing and taking on partners (called equity). Financing is also sometimes called **Capital Structure**.

These three areas rely upon one another.

All three areas are ultimately concerned with **valuation**: the process of establishing an appropriate price. Do not get hung up on precise area designations: The three areas are interlinked. When a manager decides whether to take a project, she must take into account how and at what cost it can be financed. This in turn depends on outside investors' other investment choices: if the alternatives are poor, outside investors will be more eager to finance this investment and require lower investment rates of return, which makes the project more worthwhile (from her perspective). The three areas are also not clearly separated. Valuing an internal project (a component of capital budgeting) is really similar to valuing a stock (a component of investments) and to valuing a new financing vehicle (a component of capital structure). Although these areas are explained separately, a good grasp of Finance requires an understanding of all three—but fortunately not a clear understanding of which problem fits into which area.

1·2. Money, Currency, and Inflation

Our first subject is money. So, what is money? But, of course, money is **Cash**! But that's not all.

Anecdote: The Evolution of Currencies
Before gold and silver were in general use as currency, money took many forms. In paleolithic times (9000-6000 BCE), it took the form of animals (oxen, sheep, lambs), then grains. Eventually, gold, silver, and copper came to dominate the world over. One exception was Africa, where cowrie shells were used as currency from 1200 BCE all the way to this century. Bronze and copper coins came into use in China in 1000 BCE, and elsewhere in 500 BCE. (In the west, their invention is often attributed to Lydian Kings, maybe King Croesus, around 630 BCE.) Leather-hide printed money appeared in China around 100 BCE, followed by real paper currency around 800AD. The **Gold Standard** (the backing of bills with physical gold reserves and a commitment to exchange at will) was introduced in England in 1816, and abandoned in the 1930's. (The United States was the last holdout, and only abolished the gold standard in 1971). Other scarce, small, and durable commodities—such as baseball cards, diamonds, emeralds, or cigarettes—have been used as currency at various times.

The "Money Supply" and Inflation.

Government agencies print bills and mint coins. Thus, they have some control over the **money supply**, the amount of currency circulating in the economy. If the amount of currency remains more or less constant (and money's circulation rate does not change), usually there is very little **inflation**—inflation is the process by which the same amount of money purchases less goods in the future as it does today. If the government prints a lot of new money, inflation is the inevitable result. Oversimplifying, if the economy consists of only 100 apples and 100 dollar bills, apples should cost \$1 each. If the government prints another 25 dollar bills and there are no new apples, the \$125 in cash still cover 100 apples, so apples will now cost \$1.[25] each. More money covering the same amount of goods is the most simple and common version of inflation.

ANECDOTE: History of the U.S. Dollar
In 1779, the **Office of the Secretary of the Treasury** was actually abolished in favor of a **Board of Treasury**. However, in 1781, the Continental Dollar collapsed, and so in 1789, Alexander Hamilton became the first Secretary of today's Treasury. (It was then mostly a customs authority.) The **First Bank of the U.S.** was founded in 1791, and destroyed by the British in 1816. In 1816, the **Second Bank of the U.S.** was founded. Coins began to be minted in 1793.

Silver and gold coins were used before paper currency, and eventually gave way to our non-precious metal coins today. From 1900 until 1974, the U.S. government promised to redeem money for gold. (Prior to 1900, on-and-off, the U.S., followed a mixed gold-and-silver standard.) Because coins last much longer than bills, the Fed probably nowadays regrets the change to bills.

The first U.S. notes were endorsed in 1861 and 1862. They were really private bank notes, although they were also dollar-like U.S. bills. I am waffling because they were essentially government **IOUs** ("I owe yous"), and were formally called Demand Notes, because they were payable on demand in coin at certain Treasury facilities. The color ink used to print the backs was green, thus the name **greenback**, which has survived to this day as a colloquial moniker for the dollar. From 1863 to 1929, National Bank Notes were issued by private banks, and functioned as de-facto currency.

The next step in U.S. paper currency were **silver certificates**, which were first issued in 1870. Silver certificates have not been issued since 1965, but they are still considered valid currency, because they can still be redeemed at face value from the Treasury. Of course, you can sell them for more than their face values at ebay.

Actual printed dollar bills only came into use as late as 1913 through the Federal Reserve Act, which authorized Federal Reserve Notes, now the only U.S. currency produced. This is really our dollar. From 1916 to 1933, the Federal Reserve Bank issued bills in denominations of $500, $1,000, $5,000, and even $10,000! Most of the basic design of dollar bills (except $1 and $2) were adopted in 1928. 1957–1963 saw a major face-lift and the inception of the basic $1 bill design. The design of dollar bills is evolving faster than it ever has. With color printers getting better by the month, the arms race between forgerers and the Bureau of Printing and Engraving is accelerating.

Page 32 shows some images of these currencies.

Sources: www.ustreas.gov, www.niehs.nih.gov, www.mindconnection.com, www.moneyfactory.com, www.mises.org, and www.factmonster.com. www.friesian.com.

Money Can Be Broader Than Just Cash: Consumer credit can be just like money.

Interestingly enough, you can expand or contract the U.S. money supply by creating money yourself—and, no, it is not illegal. Think about what money is: a piece of paper that has value because it is accepted elsewhere. How does a personal check that you write differ from cash? It may not be accepted *everywhere*, but if your credit is good, your check is accepted in many locations just as cash would be. In a sense, like the government, by filling out a check, you have just printed money. Eventually, you of course have to pay whoever accepted your check. Governments likewise have to accept their own printed money on demand, at least for purposes of settling your tax bill. Similarly, when you sign a credit card slip, you have just created money. In sum, your own commitments to pay can be a *substitute* for cash. The fact that the government is not the only provider of a means of payment also means that its power over controlling the money supply and thus inflation is not unlimited. If it becomes easier to use personal checks or credit cards, people use less and less government-issued cash and the government loses more and more control over the money supply.

SIDE NOTE: The many different forms of money have also necessitated different definitions of the **Money Supply**, known by such monikers as **M1** and **M2**. The government has some control over how much it allows banks to lend and/or how easy it makes it for banks to facilitate the transfer of money via checks and other currencies. Indeed, it is no longer the printing of physical bills, but bank regulation that is the Fed's prime mechanism for control of the money supply today.

SIDE NOTE: In recent years, much of the U.S. has become used to credit cards as a more convenient means of payment than cash. Japan is still mostly a cash economy. Europe is somewhere in between. Eventually, digital cash is likely to replace all paper money.

Consumers usually love non-cash methods of payments, because credit cards are often more convenient than carrying around wads of cash. Credit cards also reduce theft and other incidental losses. But many merchants have more ambivalent feelings. First, they are charged between 1 and 3 percent by banks and credit card companies. Second, the associated paper trail makes it much more difficult for merchants to (illegally) avoid declaring income. Thus, the **Underground Economy** (or **Black Market Economy**) is diminishing in importance, as economies move progressively towards cash-less means of payments with their long paper-trail.

Money is also not only U.S. dollars. It may be denominated in different **currencies**. To pay in another country, shoppers must carry the local currency. All major newspapers and many websites publish tables of major currency conversion rates. On May 31, 2002, the **Wall Street Journal** (**W.S.J.**) reported the following information on Page C1: Currencies.

Table 1.1. U.S. Dollar, on Friday May 31, 2002

Currency	Late Fri	Late Thu
Euro (in U.S. dollars)	0.9326	0.9375
Japanese Yen (per U.S. dollar)	124.30	123.33
British Pound (in U.S. dollars)	1.4559	1.4657
Canadian Dollar (per U.S. dollar)	1.5275	1.5342

Although over a hundred currencies are in circulation today, the four most important are the **U.S. Dollar**, the **British Pound**, the European **Euro**, and the **Japanese Yen** (or just **Yen**). The *Wall Street Journal* table tells its readers that one Euro purchases 0.9326 dollars. Invert the ratio (1/0.9326) to find how many euros one dollar purchases: Dollar, Pound, Euro, Yen, ...

$$1 \text{ Euro} = 0.9326 \text{ Dollars} \quad \Longleftrightarrow \quad 1.0723 \text{ Euros} = 1 \text{ Dollar} \ . \tag{1.1}$$

The Euro is said to appreciate if it purchases more dollars, i.e., if the 0.936 increases/the 1.0723 decreases. The opposite is the case for the dollar. Naturally, the agency exchanging the currency (often a bank) also charges a transaction fee: after all, to stay in business, currency exchanges need to earn money, too. They have to safely handle and account for large amounts of cash in many different currencies—not an easy task. Just think about how difficult it can be to prevent employees with access to millions of dollars of cash from stealing.

ANECDOTE: **World Currencies**
The CIA Factbook lists world currencies currently in use. As of mid 2002, there seem to be 167 official country currencies. Bryan Taylor's website http://www.globalfindata.com/ also offers an interesting description of the history of currencies.

Solve Now!

Q 1.1 *How many Yen would $500 buy (at the 124.30 exchange rate)?*

Q 1.2 *How many dollars would 500 Yen buy?*

Q 1.3 *In what sense is writing a check just like printing money? In what sense is it different?*

1·3. Financial Institutions

Now that we know what money is, we turn towards the most important institutions that handle it: banks and financial markets.

1·3.A. Banks

Banks, depositors, borrowers; financial instruments.

Banks are the most important **Financial Institutions** in most countries. Banks are institutions that pay **Interest** for the use of money lent to them by their **Depositors**. They then lend out this money to **Borrowers**, presumably at higher interest than what they are paying their depositors. Depending on the preferences of the individual depositor, depositors' money can go into different **Financial Instruments** or **Savings Vehicles**. The most important are the **Checking Account**, the **Savings Account**, and the **Certificates of Deposit** (**C.D.**). (Other financial instruments are being invented all the time.)

Savings Alternatives differ in fees, immediacy, and interest rates.

Bank savings vehicles usually differ between each other in three respects: the fixed cost of establishing and maintaining the account (including monthly fees and penalties for overdrawing accounts), the immediacy with which you can withdraw money, and the interest rate that your deposits earn. **Checking Accounts** are the most flexible in that you can withdraw money by writing checks. However, they pay only a low interest rate. **Savings Accounts** are less flexible, in that you can immediately withdraw (most of) your money, but you cannot write checks. Finally, **Certificates of Deposits** are the least liquid instruments, often requiring you to commit to keep your money with the bank for months or even years. The interest rate paid on checking accounts and saving accounts can change day by day, whereas the interest rate paid on C.D.s is usually specified in advance for a long time-period. C.D.s tend to pay the highest interest rate. In May 2002, the typical checking account paid about 1 percent interest per year, the typical savings account paid about 2 to 3 percent interest per year, and the typical C.D. paid about 3 to 4 percent interest per year. There are numerous other interest rates on other types of savings/investments, and Chapter 4 describes a large number of them.

Side Note: In many countries, banks also sell other investments, but the idea is always similar: the depositor gives cash to the bank, and receives a promise to get money in the future—perhaps. If the investment is risky, the promise may only say that the depositor receives cash under certain scenarios. Alternative investments are not usually covered by government deposit insurance (such as F.D.I.C. or F.S.L.I.C.)—if the particular investment or the bank goes under, the money may go with it. Even here in the United States, as the government has begun changing its **Banking Regulation**, banks have also begun to sell more and more non-traditional and risky investments.

With the cash that banks receive from their depositors, banks can make investments. Typically, banks extend loans to third parties. For example, they may extend a **Business Loan** to the local hardware store, or a **Mortgage Loan** to a home buyer, or a loan to you when you charge a purchase to your credit card. (The merchant receives the cash from the credit card company, while you owe it.) If the bank wants to play it safe, it can lend money to the **U.S. Treasury Department** by buying U.S. **Treasury Bonds** (**T-Bonds**, discussed in the next chapter)—but the U.S. government pays only very low interest rates. Because banks usually have to allow depositors to withdraw money at their discretion and simultaneously have to lend out money to their borrowers for a fixed period of time, they are always in danger of an imbalance. It requires a delicate balancing act to avoid a situation in which too many depositors want their money back before the bank's loans to borrowers are repaid.

Banks use deposits for loans and investments.

SIDE NOTE: Banks can liquidate many of their loans by selling them off to other banks or investors. They do not have to wait until the loan comes due in order to transform the loan into cash.

As is almost invariably the case, in order to receive a higher expected reward, you must accept more risk (and perhaps less convenience). If this were not the case, everyone (including banks) would flock toward the higher expected reward opportunity with lower risk, thus driving up the price of this opportunity, and thereby quickly reducing or eliminating it. Of course, fools can assume risk without gaining a higher reward in exchange, for example, in Las Vegas. But after eliminating all foolish behavior, if you want to earn a higher expected rate of return, you will have to assume more risk.

Reward requires Risk—but not the other way around.

IMPORTANT: *An important theme of Finance is that in order to obtain a higher reward, you usually need to accept more risk.*

SIDE NOTE: From 1926 to 2002, the annual risk and reward of some large asset-class investments were approximately as follows:

Asset Class	Reward	Risk
Short-Term U.S. Government Treasury Bills (T-Bills)	4%	3%
Long-Term U.S. Government Treasury Bonds (T-Bonds)	5.5%	10%
Long-Term Corporate Bonds	6%	9%
Large Firm Stocks	10%	20%
Small Firm Stocks	15%	30%

Inflation was about 3% per year. For the most part, it seems that higher risk and higher reward went hand-in-hand. The definition and meaning of risk will be explained in later chapters.

If the bank extends loans wisely, the borrowers who owe it money will repay these loans. If the interest rate that the bank earns on its loans is higher than the interest rate that it pays to its depositors, the bank operates profitably.

Banks live off the difference between interest paid and interest earned.

ANECDOTE: The Bank Crisis in the early 1980's
Banks and Savings&Loan Institutions usually lend money at fixed interest rates to borrowers, but pay depositors a variable interest rate that floats with economic conditions. Thus, banks do well in times of falling interest rates, and poorly in times of rising interest rates. The bank crisis of the late 1980s was partly caused, or at least aggravated by, rising interest rates.

In October 1982, depositors earned an interest rate of about 9.4% on 1-month certificates of deposits (C.D.s). At the same time, prime borrowers paid interest rates of 12.5% on loans—a good profit margin that covered the costs of lenders' operating and borrower default. Unfortunately, by August 1984, the C.D. interest rate had climbed back to 11.32%, many borrowers defaulted, and most banks and savings&loans were losing money. (Interest Rate Data Source: http://www.federalreserve.gov/releases/h15/data.htm.) This precipitated the well-publicized **Savings and Loan Crisis** (http://www.fdic.gov/bank/historical/s&l/), which required massive government bailouts and takeovers of many lenders. Less well followed is why the crisis came to end: C.D. rates dropped to 6% by mid-1986, propping up many formerly marginal banks.

When borrowers default, banks have remedies.

The exact remedies that banks have recourse to when borrowers do not repay can differ from loan to loan, from state to state, and from country to country. Usually, the lender can force a borrower who does not repay to declare **Bankruptcy**, which typically allows the lender to try to repossess and then sell some or all of the borrower's assets in order to make good on the loan. In addition, the bankruptcy will appear on the borrower's **Credit Report** (kept by private record-keeping companies but accessible to outsiders for a fee), which will make it difficult for the individual or company to borrow money in the future.

ANECDOTE: Bank Runs and the FDIC
In the movie "It's a Wonderful Life," Jimmy Stewart tries to save off depositors clamoring for their money. This is the classic **bank run**, in which depositors start to worry about the **solvency** of their bank (i.e., the ability of the bank to return they money). A bank run then becomes a self-fulfilling prophecy: depositors withdraw their money for fear of insolvency, which in turn causes the insolvency that they are afraid of.

Bank panics in the United States occurred as early as 1819 and 1837, and were fairly regular until the establishment of the **Federal Reserve Bank** as a **Lender of Last Resort** and the establishment of the **Federal Deposit Insurance Corporation (F.D.I.C.)** (which insures depositors against bank failures for deposits of up to $100,000). Knowing their deposits are safe, runs on banks have disappeared in the United States. However, there are still runs on other financial institutions.

1·3.B. Financial Markets

Financial Securities ("stocks") are traded on financial markets (the stock market). Some features of stocks explained.

Another set of important financial institutions are the **financial markets**, first and foremost the **stock market**. A stock is a piece of paper that promises its holder a share of what is left over in a **publicly traded company** (or **Public Company**) after all its debt obligations are satisfied. Stock holders are sometimes also called **residual claimants**, because they can only claim what is left over—the residue—after debt obligations have been paid off. Yet another term for such stock is **equity**. So, like a lender, an **investor** in a **stock** gives the **issuer** (i.e., the seller of the **financial securities**) cash and receives a promise to receive due consideration for future cash, if the firm can generate such cash for pay out to its investors. If the company does well, it is likely that the stock will be worth more. If the company goes bankrupt, the stock may only be worth the paper that it is printed on. Unlike a bank depositor or lender, there is really no firm promise that the stockholder will receive anything: the firm may go bankrupt and pay nothing. Still, stockholders often receive **dividend payments**, which are cash payments, similar to interest payments, but not obligatory. For example, PepsiCo (the company manufacturing and selling Pepsi Cola; stock symbol: PEP) paid its shareholders dividends of 14 cents per share in the first quarter of 2001, followed by 14.5 cents per share in each of

the following three quarters.

Upside belongs to shareholders.

Most important, only the common stockholders get all the residual (remains), after other corporate obligations are paid off. Therefore, common stock has **unlimited upside**. Common stock investors usually also receive some additional rights, such as the right to elect management. In a very real sense, stockholders are collectively the owners of the company in most normal situations—at least as long as the corporation is not repossessed by the creditors of its loans. Naturally, owning one stock among millions means that you are only a very small owner. Unlike some small business owners, common stockholders cannot lose more than their investment, which is called **Limited Liability**. This is not so for **Partners**, whose private fortune may be confiscated if their company fails.

ANECDOTE: Lloyd Names
The venerable insurance company **Lloyd's of London** is not a corporation, but a partnership. Many of its smaller "investor-partners," the so-called **Names**, found this out the hard way. After 8 billion dollars of losses in the 1980's, primarily on asbestos insurance, its average outside "external name" owed Lloyd's 263,400 pounds each. Lloyd's eventually offered a settlement. Some names, owing on average 469,000 pounds, sued for fraud instead of accepting a reduced liability of 64,000 pounds. They lost their case in court in November 2000. (Source: BBC News, Nov 3, 2000.)

Stocks can be easily bought and sold.

There are literally thousands of stocks, issued by many different companies. Many newspapers publish stock prices from major **stock exchanges** every day. In the United States, the two main stock markets are the **New York Stock Exchange** (where dealers are physically trading on a floor) and the **NASDAQ** (where trades are done by computer). Yet other financial assets, such as corporate bonds, are often traded **over-the-counter**; that is, traders call up one another to buy and sell without an organized exchange. A financial market is really a most wonderful invention. Without it, it would be difficult to finance investment projects that span decades with the money of investors who may only want to invest money for a year or two. The **stock market** allows you to invest for a while, then sell your **shares** in the **stock** (i.e., **claims** on the firm) to another investor (who typically believes these shares of stock are worth more than what they trade for). This ability to buy and sell stocks quickly for a known price and at a relatively low transaction cost distinguishes an investment in the stock market from other investments, such as the direct purchase of land or a painting or a stake in a condominium complex. The latter are more difficult to sell and the price is not known until a willing buyer is found.

SIDE NOTE: The clear delineations between banks and stock markets (or, between deposits and investments) are slowly dissolving. It is becoming possible for some firms to borrow money (i.e., at a fixed interest rate) on public exchanges (similar to the stock market), and many banks can buy stocks or financial instruments that are similar to stocks. There are also many other **Financial Intermediaries**, some of them part bank, part stock market. Others are completely different, such as financial websites and rating agencies. (They will be discussed later.) However, banks and stock markets remain the primary financial institutions today, so our book focuses largely on them.

SIDE NOTE: Some firms issue both **Common Stock** (also called **Common Equity**) and **Preferred Stock** (also called **Preferred Equity**). Preferred equity will be explained in Section 20·1.C. In brief, preferred stockholders often receive a higher dividend and better tax treatment *if they are themselves a corporation*. Therefore, most preferred stocks are not purchased by individuals but by corporations. Preferred stock is really more like a hybrid between a loan and equity (common stock) than it is like a common stock itself.

Solve Now!

Q 1.4 *What are the privileges of share ownership in a stock? What are the drawbacks?*

1·4. Taxes

Anecdote: Granola Bar and Twinkies
How sensible do you expect the tax code to be?

Not very. For example, the estate (death) tax is set to be reduced every year, until it reaches 0% by 2010. In 2011, however, it will be fully reinstated, back to today's 50%. It is unclear why the government wants us to die in 2010, but not in 2011. For another example, many states impose a sales tax on granola bars and bran muffins, but not on Twinkies and Pop Tarts. (Bon Appetit!)

1·4.A. The Basics of (Federal) Income Taxes

The Tax Code basics are simple, the details are complex.

With the exception of **tax-exempt institutions**, such as charitable institutions and pension funds (which suffer no taxes), individuals and corporations in the United States are taxed in a similar fashion. The name of the **Internal Revenue Service** (**I.R.S.**) **Tax Form** that individuals have to file is familiar to every U.S. tax filer: **1040**. The actual tax code itself is very complex, and its details change every year, but the basics have remained in place for a long time and are similar in most countries. Here we will look at the subject in broad brushstroke form.

Step 1: Compute your Taxable Income.

Earned Income (be it from **Wages** [compensation for work], from dividends [compensation paid by one's stocks], or from interest [compensation paid by one's borrowers]) is subject to both federal income taxes and state income taxes. There are some deductions that taxpayers can take that reduce their **earned income** and results in **taxable income**. Most prominently, in the United States, individuals can reduce their taxable income through mortgage interest payments. (Corporations are allowed to deduct *all* interest, not just mortgage interest.) Further, individuals may deduct other expenses, such as some educational expenses and certain retirement savings (specifically, through contribution to an **Individual Retirement Account**, such as an **I.R.A.** or a **401-K**).

Table 1.2. Sample Taxable Income Computation

	Actual Earned Income	$100,000
–	Allowed Mortgage Interest	$10,000
–	Allowed Retirement Investment	$5,000
=	Taxable Income	$85,000

Table 1.2 shows how an individual may compute his taxable income. After the taxable income has been computed, the income tax rate is applied to compute one's tax obligation. Income tax rates are **progressive**, that is, not only do you have to pay higher taxes when making more money, you have to pay *increasingly* higher taxes when making more money. **Tax rates** depend on the marital status of the individual. For example, for **Single Individuals**, the U.S. federal income tax rates for 2001 are in Table 1.3.

Tax Rates (Brackets) are "progressive."

Table 1.3. Tax Rate Table for 2001, Single Individual

Tax Rate	Description	Minimum	Maximum
15.0%	on the first $27,050	$0	$27,050
27.5%	on the next $38,500	$27,050	$65,550
30.5%	on the next $71,200	$65,550	$136,750
35.5%	on the next $160,600	$136,750	$297,350
39.1%	on the remainder	$297,350	unlimited

ANECDOTE: The Income Tax
The first federal income tax was introduced during the Civil War. It amounted to two percent per year on income above about $80,000 in 2002 dollars. Attorney Joseph H. Choate argued against the federal income tax in the Supreme Court—and won! It took a constitutional amendment to reinstate it. In his argument, Choate warned that the two percent rate might one day rise to twenty percent. (Source: Don Mathews.)

Between 1945 and 1963, the top income tax rate was around 90%; this was also the period in which the United States experienced the greatest economic boom in its history. Hopefully, this was just coincidence.

Table 1.4. Sample Income Tax Computation

15.0%	on $27,050	=	$4,057.50
27.5%	on $38,500	=	$10,587.50
30.5%	on $19,450	=	$5,932.25
Computed Total Income Tax on $85,000		=	$20,577.25

Each of the tax rates in Table 1.3 is also called a **tax bracket**, because each defines a range of income. If you say you are in the 30.5% tax bracket, you mean that you have taxable earnings between $65,550 and $136,750. Again, as with the computation of the taxable income, be warned that this particular tax rate table also contains many simplifications. For example, in 2002, there were extra refund checks that were mailed to taxpayers, and there is also an **Alternative Minimum Tax** (**A.M.T.**) that can apply. Returning to our example, ignoring all the other details of the tax code, we can see how to compute taxes on a taxable income of $85,000 in Table 1.4.

Step 2: Look up Your Tax Bracket.

SIDE NOTE: The Federal HHS Poverty Guidelines list a single individual with income below \$8,590 as "poor." Such an individual would owe Uncle Sam \$1,288.[50], or $\$1,288.^{50}/\$8,590 = 15\%$, in income taxes.

ANECDOTE: **The Use of Taxes**
Where do all the taxes go?

In 2002, there were 128 million households filing federal income taxes. About \$1 trillion (or about \$7,800 per household) went to entitlement programs (primarily **Social Security** and **Medicare**). About \$350 billion (or \$2,700 per household) each went to defense spending and to non-defense spending. In inflation-adjusted terms, defense expenditures have remained roughly constant since 1962, non-defense expenditures have doubled, and entitlements have grown eightfold. Entitlements are projected to continue growing rapidly in the future. (Source: The Heritage Foundation.)

State and Local Government expenditures in total are about half as large as federal government expenditures. In 2002, total government expenditures ran at about \$2,855 billion per year, for a **Gross Domestic Product** (**GDP**) of \$10,150 billion, almost 30% of the total. Put differently, for every two dollars spent in the private economy, one dollar is spent by government somewhere.

For entertaining reading, Martin L. Gross' book The Government Racket describes some examples of government waste that are outrageous enough to be funny.

1·4.B. Before-Tax vs. After-Tax Expenses

Taxpayers prefer before-tax expenses to (equal) after-tax expenses.

It is important for you to understand the difference between **Before-Tax Expenses** and **After-Tax Expenses**. Before-tax expenses reduce the income before taxable income is computed. After-tax expenses have to be paid after taxes have been paid. Everything else being equal, if the I.R.S. permits a payment to be designated a before-tax expense, it is more favorable to the taxpayer, because it reduces the tax burden. For example, if a taxpayer earns \$100,000 and there is only a 40% bracket, a \$50,000 before-tax expense leaves the taxpayer with

$$(\$100,000 - \$50,000) \cdot (1 - 40\%) \;=\; \$30,000 \;, \tag{1.2}$$

while the same \$50,000 expense if post-tax leaves the taxpayer only with

$$\$100,000 \cdot (1 - 40\%) \;-\; \$50,000 \;=\; \$10,000 \;. \tag{1.3}$$

Chapter 24 will show how corporations can use interest expenses to tax-shelter their income.

SIDE NOTE: A common mistake of house purchasers is that they believe that the deductability of mortgage interest means that they should keep a mortgage on the house under all circumstances. It is not rare to find a home owner with both a 6% per year mortgage and a savings account (or government bonds) paying 5% per year. Yes, the 6% mortgage payment is tax deductible, and effectively represents only an interest cost of 3% per year for a tax payer in the 50% marginal tax bracket. But, the savings bonds pay 5% per year, which are equally taxed at 50%, leaving an after-tax interest rate of 2.5% per year. Therefore, for each \$100,000 in mortgage and savings bonds, the house owner throws away \$1,000 in before-tax money (equivalent to \$500 in after-tax money).

1·4.C. Average and Marginal Tax Rates

It is also important for you to distinguish between the **Average Tax Rate** and the **Marginal Tax Rate**. In our example from Table 1.4, the former is

The Average Tax Rate is what you pay on your overall income.

$$\begin{aligned} \text{Average Tax Rate} &= \frac{\$20,577.^{25}}{\$85,000} \approx 24.21\% \\ &= \frac{\text{Total Tax Paid}}{\text{Total Taxable Income}} \end{aligned} \tag{1.4}$$

(although some people prefer computing average tax rates relative to Total Income, rather than relative to Total Taxable Income). In words, Uncle Sam receives 24.21% of this individual's taxable income.

In contrast, the marginal tax rate is the tax rate that applies to the last dollar earned (i.e., someone's tax bracket, as explained above). In our example with an income of $85,000, the table shows that

The Marginal Tax Rate is what you paid on your last dollar of income, and what you would have to pay on one more dollar of income.

$$\text{Marginal Tax Rate} = 30.5\% \ . \tag{1.5}$$

The marginal tax rate is important, because it applies to any additional activity you might undertake. For example, if you want to work one extra hour at a $40 an hour pre-tax pay rate, you only receive $(1 - 30.5\%) \cdot \$40 = \$27.^{80}$ as spendable extra cash, not $(1 - 22.2\%) = \$31.^{13}$. Thus, in the decision whether to work (which is itself a **Project**) or to go play basketball (a sort of consumption "project" that is not taxed!), it is the marginal tax rate that matters, not the average tax rate. Economists are usually more interested in the marginal tax rate than the average tax rate.

1·4.D. Capital Gains Taxes

A **capital gain** is income that is earned when something (an **asset**) that was purchased is sold for a higher price. Capital gains are peculiar in three ways:

Capital gains are taxed less than ordinary income.

1. If the asset is held for more than a year, the capital gain is not taxed at the ordinary income tax rate, but at a lower long-term capital gains tax rate. (In 2002, the long-term capital gains tax rate is 20 percent.)

2. Capital losses on the sale of one asset can be used to reduce the taxable capital gain on another sale.

3. The tax obligation occurs only at the time of the realization: if you own a painting that has appreciated by $100,000 each year, you did not have to pay $20\% \cdot \$100,000$ each year in taxes. The painting can increase in value to many times its original value, without you ever having to pay a dime in taxes, just as long as you do not sell it. In contrast, $100,000 in income per year will generate immediate tax obligations—and you even will have to pay taxes again if you invest the labor income for further gains.

ANECDOTE: Taxing the Rich, or Taxing the Wealthy?
Political debates about "taxing the rich" are plain wrong. Neither Democrats nor Republicans ever debate taxing the rich. Instead, they debate taxing high income individuals.

But there is a big difference between wealth increases and income. The lowest tax rates on wealth increases are enjoyed by the richest Americans. These are households that earn most of their wealth increases not in ordinary income, but in capital gains from existing wealth. If not realized, these wealth increases may never have been taxed, at all! For example, Warren Buffet has probably paid about 0.0% in personal income tax on his wealth increase of over $30 billion over the last 30 years.

In contrast, most ordinary households receive their annual wealth increases in ordinary income or interest receipts. These wealth increases are taxed every year, and are taxed at a much higher marginal tax rate than capital gains wealth increases. Roughly, an ordinary worker's wealth increases suffer taxes to the tune of about 25% to 50% per year. (Chapters 22–23 will describe tax sheltering in greater detail.)

Although a tax on all wealth increases would be fairer than the current income tax *and* provide better incentives to work, it would also be far more difficult to administer a tax on wealth increases than the tax on ordinary income. Nevertheless, some European countries have some forms of wealth taxes. In the United States, wealth is taxed only at death—unless you are wealthy enough to afford good estate lawyers.

1·4.E. Other Taxes

Other Income Taxes: state, county, international taxes.

In addition to federal income taxes, most states impose their own income tax. This typically adds another tax rate of between 0% and 10%, depending on state and income. Worse, each state has its own idea not only of what its tax rate and tax brackets should be, but even how taxable income should be computed. Thus, you need to learn not only the federal tax code, but also your state's tax code. If you have to file in multiple states or even in multiple countries—although there are rules that try to avoid double taxation—the details can be hair-raisingly complex. If you find yourself in such a situation, may the force be with you!

ANECDOTE: State Income Taxes
California has the highest marginal state income tax bracket that is not federal deductible: 9.3%. Montana has the highest marginal state income tax bracket that is federal tax deductible: 11%. Alaska, Nevada, South Dakota, Texas, Washington, and Wyoming levy no state income tax. Many counties pay for school education with property tax rates. In the poorer counties of Connecticut, they amount to 4% of the value of the house. In Maine (the second-highest property tax collector in the nation), residents pay 5.5% of their income in property tax. Many states also levy a sales tax. Tennessee and Louisiana have a sales tax of 8.35%; Oregon, New Hampshire, Montana, and Delaware have no sales tax.

"Social" Taxes: Social Security and Medicare.

Finally, there are **Social Security** and **Medicare** contributions. Although these are supposedly insurance premia, any money taken in today is immediately spent by the government on the elderly today. Thus, anyone young today is unlikely to receive much in return from the government in 20 to 30 years—when there will be fewer young people around to pay their retirement benefits. Thus, many financial economists consider social taxes to be as much a form of income tax as the statutory income tax.

ANECDOTE: Social Security and Intergenerational Wealth Transfers
"Trust Fund" and "Lockbox" are political jargon only. The fact is that the U.S. government takes in social tax revenues, lends itself the money (the receipts buy Treasury bonds), and then spends it. By 2030, if nothing is done, Social Security and other entitlements will have grown from one-third of the federal budget to two-thirds—and the money paid in today will be long since spent by then.

Already, current senior citizens enjoy the largest intergenerational redistribution of wealth in history, having themselves paid only modestly into Social Security and Medicare, but enjoying very high benefits. Nevertheless, the current debate is not about how to reduce senior benefits, but about how to add more benefits, primarily drug cost reimbursements. Why?

The reason is simple: 60 to 70 percent of citizens 60 years and older vote, while only 15 to 25 percent of citizens 18 to 24 years old vote. Senior citizens constitute 12 percent of the U.S. population, but fully 25 to 35 percent of the vote. With most elections decided by vote spreads of less than 5 percent, and many elderly being one-issue voters, no politician can afford not to pay more attention to the desires of the elderly than to the desires of the younger.

Sources: CNN, LWV Youthvote.

(The only thing senior citizens and those 18–24 may have in common is that 25 percent do not know who is running for president, and 75 percent do not know who is running for vice president.)

Source: Suddenly Senior.

This book also ignores many other non-income taxes. For some taxes, such as the **Sales Tax**, it is not clear how to use expertise in Finance to lower them. For other taxes, such as the **Estate Tax**, you need extremely specialized financial vehicles to avoid or reduce them. These are beyond the scope of this book.

Non Income Based Taxes: Sales Tax, Real Estate Taxes, Estate Taxes.

SIDE NOTE: Here is a small lesson in economic thinking: assuming both are in the same tax bracket, does it matter whether a tenant or the landlord pays the taxes on a rental property? *In a competitive market*, the answer is probably no: If the competitive rent is $1,000 per month, and a tax of $100 per month is imposed, if the tenant pays, the rent would probably remain the same; if the landlord pays, the rent would probably increase by $100 per month—after all, $1,000 is what a landlord has to earn on such a property in a competitive market.

1·4.F. What You Need To Know About Taxes For This Book

IMPORTANT: *You must understand*

1. *how income taxes are computed (the principles, not the details);*
2. *that expenses that can be paid from before-tax income are better than expenses that must be paid from after-tax income;*
3. *how to compute the average tax rate;*
4. *how to obtain the marginal tax rate;*
5. *that capital gains enjoy preferential tax treatment;*
6. *why the average and marginal tax rates differ, and why the marginal tax rate is usually higher than the average tax rate.*

What you will learn later about taxes.

Later, this book will explain and pay special attention to three facts: certain **Interest Expenses** can be deducted when computing **Taxable Income**; **Capital Gains** are taxed at lower tax rates than **Ordinary Income** and **Interest Income**; and some retirement account investment returns are tax-exempt. These features of the tax code offer individuals opportunities to legally reduce their tax obligations.

Solve Now!

For all questions here, assume that this investor has $1,000 in mortgage interest deductions.

Q 1.5 *What are the average and marginal federal tax rates for a single individual earning $5,000?*

Q 1.6 *What are the average and marginal federal tax rates for a single individual earning $50,000?*

Q 1.7 *What are the average and marginal federal tax rates for a single individual earning $50,000,000?*

1·5. Transaction Costs

Direct transaction costs require a money transfer.

When people engage in **Transactions**, i.e., a purchase or sale, there are costs to facilitate the transactions. For example, in the United States, if a house is sold, the seller's broker typically receives six percent of the value of the house as commission (and splits this commission with the buyer's agent). Thus, if a real estate agent manages to sell a house for $300,000, the commission is $18,000. But, there are also many other direct transaction costs, ranging from advertising, to insurance company payments, to house inspectors, to the local land registry, to postage—all of which cost money.

Anecdote: Real Estate Agents: Who works for whom?
Real estate agents are conflicted. If they sell sooner, they can spend their time focusing on other properties. Thus, the typical seller's agent will try to get the seller to reduce the price in order to make a quicker sale. Similarly, the buyer's agent will try to get the buyer to increase the offer. In a financial sense, the buyer's agent is working on behalf of the seller, and the seller's agent is working on behalf of the buyer. Interestingly, Steve Levitt found that when agents sell their own houses, their homes tend to stay on the market for about 10 days longer and sell for about 2 percent more.
Source: Steve Levitt

Indirect Transaction Costs are the loss of other opportunities.

Then there is the seller's own time required to learn as much about the value of the house as possible, and the effort involved to help the agent sell the house. These may be significant costs, even if they involve no cash outlay. After all, the seller could spend this time working or playing instead. Furthermore, not every house is suitable for every house buyer, and the seller has to find the right buyer. If the house cannot be sold immediately but stays empty for a while, the foregone rent is part of the transaction cost. The implicit cost of not having the house be put to its best alternative use is called an **opportunity cost**. Opportunity costs are just as real as direct cash costs.

SIDE NOTE: Most house buyers do not realize the extent of their transaction costs and risk. The real estate agent earns 6% of the value of the house, not of the amount of money put into the house by the purchaser. On a house purchase of $500,000, the typical loan is 80% of the purchase price, or $400,000, leaving the buyer to put in $100,000 in equity. Selling the house the day after the purchase reduces the owner's wealth of $100,000 by the commission of $30,000—for an investment rate of return of –30%. This is not the risk component; it is pure transaction cost.

If house prices decline by 10%, or the buyer overpays by 10%, the house can only be resold for $450,000, which leaves $423,000 after agent commissions. The house owner is left with $23,000 on a $100,000 investment. A 10% decline in real estate values has reduced the home owner's net worth by 77%! (PS: A 10% increase in real estate values increases the value of the house to $550,000, which means that $517,000 is left after real estate commissions. The house owner's return is thus only 17%.)

Transaction Costs for Stocks are also either direct or indirect.

Financial markets transactions also incur transaction costs. If an investor wants to buy and sell shares of a stock, the broker charges a fee, as does the stock exchange that facilitates the transaction. In addition, investors have to consider their time to communicate with the broker to initiate the purchase or sale of a stock as an (opportunity) cost.

The typical Transaction Costs for Stocks are relatively low.

Still, the transaction costs for selling financial instruments are much lower than they are for many other goods. Let's look at a few reasons why: First, even if you want to buy (or sell) $1 million worth of stock, some Internet brokers now charge as little as $10 per transaction. Your **round-trip transaction**, which is a buy and a sale, costs only $20 in broker commission. In addition, you have to pay the spread (the difference between the bid and the ask price), which is what allows the stock exchange to make money. For example, a large company's stock like PepsiCo (**Ticker Symbol** PEP) may have a publicly posted price of $50 per share. But you can neither buy nor sell at $50. Instead, the $50 is really just the average of two prices: the **bid price** of $\$49.^{92}$, at which another investor or the exchange's market-maker is currently willing to buy shares; and the **ask price** of $\$50.^{08}$, at which another investor or the exchange's market-maker is currently willing to sell shares. Therefore, you can (probably) purchase shares at $\$50.^{08}$ and sell them at $\$49.^{92}$, a loss of "only" 16 cents which amounts to round-trip transaction costs of $(\$49.^{92} - \$50.^{08})/\$50.^{08} \approx -0.32\%$. We can compute the total costs of buying and selling 20,000 shares ($1,000,000 worth) of PepsiCo stock as

Financial Roundtrip Transaction

Purchase 20,000 Shares	Pay $\$50.^{08} \cdot 20,000 = \$1,001,600$	
Add Broker Commission	$+\$10$	$= \$1,001,610$
Sell 20,000 Shares	Receive $\$49.^{92} \cdot 20,000 = \$998,400$	
Subtract Broker Commission	$-\$10$	$= \$998,390$
	Net Roundtrip Transaction Costs	–$3,220

This is not exactly correct, because the bid and ask prices that the exchange posts (e.g., on Yahoo!*Finance* or the *Wall Street Journal*) are only valid for 100 shares. Moreover, some transactions can occur inside the bid-ask spread, but for most large roundtrip orders, chances are that you may have to pay more than $\$50.^{08}$ or receive less than $\$49.^{92}$. So 0.32% is probably a bit too small. (In fact, if your trade is large enough, you may even move the publicly posted exchange price away from $50!) Your buy order may have to pay $\$50.^{20}$, and your sell may only get you $\$49.^{85}$. In real life, the true roundtrip transaction cost on a $1 million position in PEP may be more on the order of magnitude of 0.5% to 1.0%.

Opportunity costs are low, too.

Second, investors do not need to spend a lot of time to find out the latest price of the stock: it is instantly available from many sources (e.g., from the Internet such as Yahoo!*Finance*). So, the Information **Research Costs** are very low: unlike a house, the value of a stock is immediately known.

Finally, upon demand, a buyer can be found practically instantaneously, so **Search and Waiting Costs** are also very low. Recall the often multi-month waiting periods if you want to sell your house.

ANECDOTE: Paynment for Order Flow
The next financial scandal may well be **payment for order flow**. Especially (but not only) discount brokers, such as Charles Schwab, Ameritrade, and e-Trade, route their investors' orders routinely to market makers who charge high bid-ask spreads, and then pay the discount broker a rebate kickback (typically 1 to 4 cents per share). Some brokerage houses even fill customer orders with other customer orders in-house, at execution prices that are not the best prices. In any case, from the point of view of transaction costs, very wealthy investors may not necessarily be better off using discount brokers, but this is difficult for individual investors to determine conclusively. More information can be found at invest-faq.com/articles/trade-order-routing.html

Compared to other economic assets, ...

Compare the financial securities transaction costs to the transaction costs in selling a house. Broker fees alone are typically 6%: for the $100,000 equity investment in the $500,000 house, this comes to $30,000 for a round-trip transaction. Add the other fees and waiting time to this cost and you are in for other transaction costs, say, another $10,000. And houses are just one example: Many transactions of physical goods or labor services (but not all) can incur similarly high transaction costs.

Table 1.5. Comparison of Transaction Costs on Stocks and Real Estate

Cost Type	Explanation	Real Estate (House)	Financial Security (Stock)
Direct	Typical Round-trip Commission etc.	≥6%	0–1%
Search/Research	Time to Determine Fair Price	high	zero
Search/Liquidity	Time Waiting to Find Buyer	variable	zero

...financial securities have such low transaction costs that they can almost be assumed to be zero.

In contrast, financial securities markets, with their many buyers and many sellers, are comparably wonderful: here, financial transactions have comparably tiny transactions costs. For the $100,000 equity investment in a stock, the transaction costs are about $300–$500. To oversimplify, this book will make the incorrect, but convenient and reasonable assumption that financial transaction costs are zero (unless otherwise described). For individuals buying and selling financial securities only rarely (a **Buy-and-Hold** investor), the zero transaction cost assumption is quite appropriate. If you are a **Day Trader**, you better read another book!

Solve Now!

Q 1.8 *What would you guess the order of magnitude to be for a round-trip transaction in $10,000 worth of shares of Microsoft to be? Describe in percent and in absolute terms.*

Q 1.9 *List important transaction cost components, both direct and indirect.*

1·6. Competitive Markets ...or You Get What You Pay For

Have you ever heard someone say "it's only worth what people are willing to pay for it" and someone else that "it's worth much more than it's being sold for"? Who is correct? Are there any good deals?

Are there any good deals?

For the most part, the first statement is correct: most often, items are worth what buyers are willing to pay for them. For shares of stocks, it is easy to figure out what someone is willing to pay: you can find out by looking it up on the Internet. It is easy because someone else just sold shares that are exactly identical to your own shares, this price is publicly posted in the media, and there are lots of potential buyers willing to purchase the shares for the same price (or maybe just a tiny bit less). This is a situation that economists call a **competitive market**, i.e., a situation in which there are many competing buyers and sellers, so that no single buyer or seller can influence the price. In addition, to be a perfect market, a competitive market also has to have very low transaction costs, no taxes on stock trading, and no insider information and/or disagreements. A perfect market is thus a market in which there is perfect competition among many buyers and many sellers, no transaction costs, and no information differences—everyone has access to the same knowledge.

...in competitive frictionless markets?

To elaborate on this concept, finance pros often use analogies from **physics**. For example, the same way that physical movement is impeded by physical friction, economic transactions are impeded by transaction costs. Financial markets are often considered **low-friction**, or even close to **frictionless**. Competitive low-friction markets often show high **liquidity**, i.e., large flows of buys and sells. And when the amount of **trading activity** subsides, people sometimes say that "the market has *dried up*."

Physics and Finance.

Competitive low-friction markets reduce buyers' and sellers' concerns that one deal is fairer than another. For example, consider gasoline and imagine that you do not yet know when and where on your road trip you will need to pump more gas. Unlike shares of stocks, gas is not the same good everywhere: gas in one location can be more valuable than gas in another location (as anyone who has ever run out of gas can confirm). But, in populated areas, the market for gasoline is pretty competitive, and there are many buyers (drivers) and sellers (gas stations). This makes it very likely that the first gas station you see will have a reasonable, fair price. If you drive by the first gas station and it advertises a price of $1.50 per gallon, it is unlikely that you will find another gas station offering the same gas for $1 per gallon or $2 per gallon within a couple of miles. Chances are that "the price is fair," or this particular gas station would probably have disappeared by now. The same applies in many financial markets: as long as there are many, many buyers and many, many sellers (as there may be for shares of stock or certain types of mortgages), most deals are likely to be fair deals. Some shopping around may help, but an extreme amount of shopping would likely cost more in time and effort than in what it could save.

Buyers get what they pay for in a competitive low-friction market.

Nerd Note: *A competitive perfect market does not mean that you pay exactly what you value it for. Indeed, usually you get to pay less than what you value it for, and you get to receive more than what you value it for. Only the* ***marginal buyer*** *and* ***marginal seller*** *get exactly what they value it for. For example, if you are running out of gas and you are bad at pushing 2 ton vehicles, you might very well be willing to pay a lot more for gas than $2 per gallon. The difference between what you value an item for and what you pay for it is called your "surplus."*

ANECDOTE: Paying for Shorter Queues at the Pump
The typical customer has to pay about 1.5% to 2% more for a 10% reduction in congestion (wait time at the pump). On a $30 tank of gas, this is 50 cents. Wealthier neighborhoods are at the upper end of gas prices, poorer neighborhoods are at the lower end. Closer gas stations are more competitive. (Source: Png and Reitman, RJE.)

Buyers may not get what they pay for in a non-competitive high-friction market.

Unfortunately, not every good is traded in a competitive low-friction market. Return to our attempt to sell a house. What is the value of the house? What if the house is in a very remote part of the country, if potential buyers are sporadic, and if the government imposes a 50% transfer fee? Now the value of the house depends on the luck of the draw (how many potential buyers are in the vicinity), the urgency of the seller (whether the seller has the luxury to turn down a lowball first offer), and the identity of the seller (the current owner does not need to pay the government transfer fee, so he may value the house more than a potential buyer). So, it is only easy to determine the value of a good if the market is *competitive low-friction*—the value may not even be a unique number if it is not.

A "salesman" may distort the truth and claim great deals.

In sum, when someone claims that a stock or firm is really worth more than he or she is selling it for, there are only a small number of explanations: First, there can be pure kindheartedness toward any buyer or a desire by a seller to relieve himself of wealth; this rarely happens. Second, the seller may not have an easy market to unload his goods, and/or the transaction costs are very high. This makes the seller accept a lower amount of money for it—and so it would still be worth what someone is paying for it, even if this depends on the someone and the situation. Third, the seller is committing a conceptual mistake. His good is worth neither more nor less than what it is being sold for, but exactly how much it is being sold for. This can also be a convenient self-deception. Fourth, the seller is lying and is using this claim as a sales tactic.

IMPORTANT: *For liquid financial securities, such as publicly traded stocks, the assumption that the market is competitive and that the security is worth what it is trading for is pretty accurate.*

SIDE NOTE: Providing liquidity in markets is a common business. For example, think about antique stores or second-hand car dealerships. These are liquidity providers that try to buy cheap (being the only standby buyer); and try to sell expensive (being the only standby seller). Being a liquidity provider can require big risks and capital outlays. If it was easy, everyone could do it!

Solve Now!

Q 1.10 *What does the assumption of a competitive, low-friction market buy us that would not be satisfied in a different market?*

Q 1.11 *Evaluate whether supermarkets are competitive, low-friction markets.*

1·7. Preview: Efficient Markets

Finally, we call a market or the price of an investment or good to be "efficient" if the price of the investment is set using all available information. For example, say the market wanted to offer you an expected rate of return of 15% on a particular stock, and the expected value of the stock is $115. Then the price of the stock today would have to be $100 for this market to be efficient. This market would not be efficient if it set the price for this stock at $99 or $101, because the stock would then offer other than the 15% expected rate of return. Similarly, you should not be able to locate information that tells you today when/if/that the true expected value tomorrow is really $120 for the $100 stock. If you could find this information, you could on average earn more than 15%. If the market has overlooked this information, it is not efficient.

Efficient Market: Use of all information.

The application and use of the "efficient markets" concept faces a number of issues. First, where does the 15% come from? It will have to come from some model that tells us what rate of return a stock should have to offer (e.g., given its riskiness). Possible models will be discussed at length in Part III of this book. Without such a model, talking about market efficiency is meaningless. Second, what information exactly are we talking about? Insiders often have more information than the public. For example, a drug company executive may know before ordinary investors whether a drug is likely to work. Thus, the market may be efficient with respect to publicly available information, but not efficient with respect to insider information.

What is the model? What is the information set?

When markets are highly competitive and friction-free, we usually believe that they are also efficient. After all, if other buyers and sellers were to ignore a useful piece of information, you could likely earn a lot of money trading on it. This belief in efficient markets is the view of **Classical Finance**. In contrast, **Behavioral Finance** believes that markets sometimes do *not* use all information. Depending on how strong a believer in classical finance vs. behavioral finance you are, you may believe that there are no such opportunities, that there are few such opportunities, or that such opportunities are abundant. Both camps agree, however, that transaction costs and market perfection play crucial roles in determining whether a market is efficient or not. In Chapter 19, we will talk in greater length about classical vs. behavioral finance.

Competition and Efficiency

Side Note: If the terms were not so loaded, "rational vs. irrational" markets or "smart vs. stupid" markets would be better names for the classifications than "classical vs. behavioral" finance. My own personal view is that financial markets are not perfectly efficient, but exploiting market inefficiencies is somewhere between difficult and impossible. So, overall, I personally see myself as 90% in the classical camp, 10% in the behavioral camp.

We will dedicate an entire chapter (Chapter 19) to market efficiency and its consequences.

1·8. Random Variables and Expected Values

Statistics is about how to bet in an uncertain world.

The most painful of our Finance foundation sciences is **Statistics**. We need statistics if the future is uncertain. But although statistics can be a difficult subject, if you have ever placed a bet in the past, chances are that you already have a good intuitive grasp of what you will need to know about statistics in your study of finance. In fact, in the previous section, we already talked about "expected" payoffs, the statistical concepts that we now cover. We just need to tighten up your knowledge a little bit.

A "fair" bet means that both sides break even if the bet is repeated infinitely many times.

The most important statistical concept is the **Expected Value**, most often a fancy phrase for **Mean** or **Average**. The only difference is that we often use means and averages for past outcomes, whereas the expected value applies the same computation (i.e., the average) to outcomes in the future. So, the expected value of a bet is the average value if the bet is repeated many times.

> **Important**: *When it comes to forward-looking statements, **mean**, **average**, and **expected value** all mean the same thing.*

The "Expected Value" is the average outcome. An expected value can be an impossible realization.

For example, say you have an unbiased coin toss, which comes up with either heads or tails with equal probability. You receive \$1 if the coin comes up heads and \$2 if the coin comes up tails. Because you know that there is a 50% chance of a \$1 and a 50% chance of a \$2, the expected value of each coin toss is $\$1.^{50}$. Of course, exactly $\$1.^{50}$ will never come up, so the expected value need not be a possible **realization** of a single coin toss.

A random variable is a number whose realization is not yet known.

Statisticians have invented the concept of **random variables** to better formalize uncertainty. A random variable is a variable whose value (i.e., outcome) has not yet been determined. In the coin toss example, we can define a random variable named c (for "coin toss outcome") that takes the value \$1 with 50% probability and the value \$2 with 50% probability. The expected value of c is $\$1.^{50}$. To distinguish a random variable from an ordinary non-random variable, we use a tilde over the variable. To denote the expected value, we use the notation $\mathcal{E}$. So, in our bet, the actual outcome of which we call c, the uncertain outcome of which we call $\tilde{c}$, we know that

$$\begin{aligned} \mathcal{E}(\tilde{c}) &= 50\% \cdot \$1 + 50\% \cdot \$2 = \$1.^{50} \\ \mathcal{E}(\text{Value}) &= \mathcal{P}rob(\text{Head}) \cdot \$1 + \mathcal{P}rob(\text{Tail}) \cdot \$2 \quad . \end{aligned} \tag{1.6}$$

After the coin has been tossed, the actual outcome c could, e.g., be

$$c = \$2 \quad , \tag{1.7}$$

and c is no longer a random variable. Also, if you are certain about the outcome, perhaps because there is only one possible outcome, then the actual realization and the expected value are the same. The random variable is then really just a plain simple variable. Is the expected outcome of the coin toss a random variable? No: we know the expected outcome is $\$1.^{50}$ even before we toss the coin. The expected value is known, the uncertain outcome is not. The expected value is a plain variable, the outcome is a random variable. Is the outcome of the coin throw *after* it has come down heads a random variable? No: we know what it is (heads), so it is not a random variable.

Figure 1.1. A Histogram For a Random Variable With Two Equally Likely Outcomes, \$1 and \$2.

A random variable is defined by its **distribution** of possible outcomes. Statistical distributions come with probabilities. The coin throw distribution is simple: the value \$1 with 50% probability and the value \$2 with 50% probability. This is sometimes graphed in a histogram, as depicted in Figure 1.1.

Computing expected values from distributions.

For practice, let us make this a bit trickier. Assume that there is a 16.7% chance that you will get \$4, a 33.3% chance that you will get \$10, and a 50% chance that you will get \$20. You can simulate this payoff structure by throwing a die and getting \$5 if it comes up ⚀, \$10 if it comes up ⚁ or ⚂, and \$20 if it comes up ⚃, ⚄, or ⚅.

An example with 3 possible outcomes.

Now, a **Fair Bet** is a bet that costs the expected value. If repeated infinitely often, both the person offering the bet and the person taking the bet would end up even, neither taking (much) money from the other. What is a fair price for our die bet? Call the uncertain payoff $\tilde{D}$. First, you must determine $\mathcal{E}(\tilde{D})$. It is

If it costs its expected value to buy a bet, the bet is fair. An example of a fair bet.

$$\begin{aligned}
\mathcal{E}(\tilde{D}) &= 16.7\% \cdot \$4 \\
&+ 33.3\% \cdot \$10 \\
&+ 50.0\% \cdot \$20 \qquad = \$14 \\
\mathcal{E}(\tilde{D}) &= \mathcal{P}rob(\text{⚀}) \cdot (\text{Payout if ⚀}) \\
&+ \mathcal{P}rob(\text{⚁ or ⚂}) \cdot (\text{Payout if ⚁ or ⚂}) \\
&+ \mathcal{P}rob(\text{⚃ or ⚄ or ⚅}) \cdot (\text{Payout if ⚃ or ⚄ or ⚅}) \ .
\end{aligned} \tag{1.8}$$

Therefore, if you repeat this bet a million times, you would expect to earn \$14 million. If it costs \$14 to buy each bet, it would be a fair bet.

Generally, you compute expected values by multiplying each outcome by its probability and adding up all these products. If $\tilde{X}$ is a random variable with N possible outcomes, named X_1 through X_N, then you would compute

The expected value is the probability weighted sum of all possible outcomes.

$$\mathcal{E}(\tilde{X}) = \mathcal{P}rob(\tilde{X} = X_1) \cdot X_1 + \mathcal{P}rob(\tilde{X} = X_2) \cdot X_2 + \cdots + \mathcal{P}rob(\tilde{X} = X_N) \cdot X_N \ . \tag{1.9}$$

This formula is exactly the same as the one you used above:

$$\begin{aligned} \mathcal{E}(\tilde{D}) &= 16.7\% \cdot \$4 + 33.3\% \cdot \$10 + 50\% \cdot \$20 = \$14 \ . \\ &= \mathcal{P}rob(\tilde{D} = D_1) \cdot D_1 + \mathcal{P}rob(\tilde{D} = D_2) \cdot D_2 + \mathcal{P}rob(\tilde{D} = D_3) \cdot D_3 \ . \end{aligned} \tag{1.10}$$

Note that N could be a trillion possible different outcomes, and many of them could be impossible, i.e., have probabilities of zero.

Use of expected value: reward.

We will use the expected value as our measure of the (average) **reward** that we expect to receive from making an investment. Fortunately, the expected value is all that we need to know about statistics until we will get to Part III, where we will also define good measures of **risk**. (Appendix 2·4 provides more statistical background to interested nerds.)

IMPORTANT: *You must understand*

1. *the difference between an ordinary variable and a random variable;*
2. *the difference between a realization and an expectation;*
3. *how to compute an expected value, given probabilities and outcomes;*
4. *what a fair bet is.*

ANECDOTE: **Chess and Randomness**
Although **Chess** is a deterministic game, there are so many possible chess positions (maybe as many as 10^{36}) that the outcome of a game can be treated as a random (i.e., unknown) event. We have now reached the point where computers—although still unable to solve for the globally optimal move (which would make chess a truly deterministic game)—have become as good as the best human players. In 1997, a computer named **Deep Blue** beat **Gary Kasparov**, the reigning world champion, 3.5 to 2.5.

Solve Now!

Q 1.12 *A stock that has the following probability distribution (outcome P_{+1}) costs \$50. Is an investment in this stock a fair bet?*

$\mathcal{P}rob$	P_{+1}	$\mathcal{P}rob$	P_{+1}	$\mathcal{P}rob$	P_{+1}	$\mathcal{P}rob$	P_{+1}
5%	\$41	20%	\$45	20%	\$58	5%	\$75
10%	\$42	30%	\$48	10%	\$70		

Q 1.13 *An ordinary die came up with a ⚅ yesterday. What was its expected outcome before the throw? What was its realization? What is the expected outcome of yesterday's die throw today?*

1·9. Disagreements? Agency? Governance

Our book begins with the perspective of a manager who lives in a world without unique or special inside information, and who just tries to do the best with what is given to him. Like our common assumption of friction-free capital markets (no transaction costs, taxes, etc.), we first explore this simpler world to learn what we as managers *should* do. Only after we know what we should do, do we explore situations in which there is, or should be, distrust. As we do with taxes, this is not to say that distrust and/or bad behavior is unimportant, just that we take it one step at a time. Chapter 25 explores the disagreements and problems between managers and owners in great detail.

1·10. A Convention for Time

The Convention for Points in Time: "0" is now. "+1" is next period.

Finally, we must agree on a convention for time. Our most common investment scenario will be a project that begins "right here right now this moment" and pays off at some moment(s) in time in the future. We use the letter t to stand for an index in time, and we use zero (0) as the time index for "right now." We will leave open the length of each time interval: thus, time $t = 1$ could be tomorrow, next month, or next year.

Table 1.6. Sample Time Conventions

$\text{Cash}_{t=0}$	Cash Right Now (index time 0). The time index ("$t =$") is given explicitly.
$\text{Cash}_{\text{Midnight, March 3, 2025}}$	Cash on Midnight of March 3, 2025. We rely on the subscript to tell the reader that the explicit subscript t is omitted.
Cash_1	Cash in the Future (at index time 1).
$\text{Investment}_{0,\text{Midnight March 3 2025}}$	An Investment made right now to pay off on March 3, 2025.
$\text{Investment}_{0,1}$	A One Period Investment, From Right Now To Time 1.
$\text{Investment}_{t=1,2}$	A One Period Investment, From Time 1 To Time 2.
$\text{Investment}_{0,2}$	A Two Period Investment, From Right Now To Time 2.
Investment_2	A Two Period Investment, From Right Now To Time 2.

A Convention for Time Periods: "0,1" means from now to next period.

A cash payout may occur at one instant in time, and thus needs only one time index. But investments usually tie up cash over an interval of time, called a **holding period**. We will use a comma-separated pair of time indexes to describe intervals. Whenever possible, we use subscripts to indicate time. When the meaning is clear, we abbreviate phrases such as the interval "$t = 0, 1$" to simply $0, 1$, or even just as 1. This sounds more complicated than it is. Table 1.6 provides some examples.

We are now ready to proceed with Finance.

ANECDOTE: Does Time Exist?
In the August 2003 issue of the prestigious Foundations of Physics Letters, a 27 year old broadcasting school tutor from New Zealand with 6 months of university education published a paper that argues that there is no instant in time, just time intervals. The paper was reviewed by numerous physics greats, and resolves a number of physics paradoxes. However, we shall ignore this little problem, and stick to t as an index measuring an instant in time.

Source: http://www.eurekalert.org/pub_releases/2003-07/icc-gwi072703.php

1·11. SUMMARY

The chapter covered the following major points:

- Finance has three primary areas: project choice and capital budgeting, investments and asset pricing, and financing and capital structure.
- Money comes in many forms. Interestingly, anyone can create money-like claims.
- Financial Institutions, primarily banks and exchanges, facilitate saving and investing.
- Equity (Stock) is what is left after debt is paid off. Stock exchanges make it easy for investors to buy and sell equity in many publicly traded stocks.
- Investment vehicles can differ by liquidity (ease and cost of cashing), risk, and reward.
- Investors care mostly about marginal, not average tax rates: the tax rate on one extra dollar of income.
- Investors prefer tax-deductible interest expenses to non-tax deductible expenses.
- Investors prefer (long-term) capital gains to other forms of income, due to a lower marginal income tax rate.
- Financial transaction costs are comparably low.
- If there are lots of buyers and sellers and low transaction costs ("frictions"), then prices tend to be fair and efficient: neither too high nor too low.
- Expected value is a fancy term for future average outcome. Given a set of probabilities and outcome, you must know how to compute it.

A. Q&A: Answers

1. $\$500 \cdot 124.30Y/\$ = 62,150Y$.
2. $500Y/(124.30Y/\$) \approx \$4.^{02}$.
3. It is currency for a payment. It does not stay in circulation for long, though. It is usually redeemed only by the bank, rather than endorsed multiple times to other parties.
4. Privileges: Limited Liability. Voting. Residual Ownership. Drawbacks: Paid only after debt. Can lose everything.
5. Taxable Income: $4,000. Taxes of $600. Average and Marginal Tax Rates are 15%.
6. Taxable Income: $49,000. Taxes of $\$4,057.^{50} + (\$49,000 - \$27,050) \cdot 27.5\% = \$10,093.^{75}$. Average Tax Rate (relative to taxable income) is 20.6%. The Marginal Tax Rate is 27.5%.
7. Taxable Income: $49,999,000. For all practical purposes, the Average Tax Rate is just slightly below 30.5%. The Marginal Tax Rate is 30.5.
8. Microsoft is an even larger stock than PepsiCo. Therefore, a round-trip transaction would probably cost a bid-ask spread of between 0.1% and 0.3%. On a $10,000, the bid-ask cost would be around $20, and broker fees would probably be around $10 to $30 with a discount broker. Thus, $50 is a reasonable estimate.
9. Direct: Broker Costs. Market-Maker or Exchange Costs (Bid-Ask Spread). Indirect: Research Costs; Search (for Buyer/Seller) Costs; Anxiety.
10. It means stuff is worth what it is selling for.
11. In most neighborhoods, there are plenty of supermarkets, fiercely competing for business. Losing one additional or gaining one additional supermarket probably makes little difference. There are also plenty of buyers. In many ways, supermarkets are a fairly competitive business, and their products are usually priced not far away from their closest competitors. Believe it or not: supermarkets typically earn a gross spread on goods of only about 2%! This has to pay for space and personnel. *However*, in some senses, this "supermarket market" is not perfectly competitive and frictionless: there is sales tax, so third parties cannot easily "arbitrage" Arbitrage product, i.e., sell a product that is less expensive in one supermarket to the other, more expensive supermarket. Plus, once you are at the supermarket, it is often cheaper just to buy the goods there, than it is to drive to another supermarket.
12. The expected value of the stock is $52. Therefore, purchasing the stock at $50 is not a fair bet, but a good bet.
13. *If* the random variable is the number of dots on the die, then the expected outcome is 3.5. The realization was 6. The expected outcome of a known number is the known number, so the expected outcome of yesterday's die throw today is also 6. The last part of this question is really trivial nitpicking.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

A. Appendix: Historical Money: Photograph and Anecdote

These images are from the article The Color of Money by Clifford F. Thies. They are courtesy (and reproduced with permission) of www.mises.org.

1862: The 1862 greenback "IOU."

$1 Greenback of 1862 (front)

$1 Greenback of 1862 (back)

1880: The 1880 $10 U.S. Treasury Note with red seal.

$10 U.S. Treasury Note of 1880 (note the red seal)

$20 U.S. Treasury Note of 1880 (note the red seal)

1882: The 1882 $100 Gold Certificate.

$100 Gold Certificate of 1882

1899: The 1889 $1 Silver Certificate with blue seal

$1 Silver Certificate of 1899 (note the blue seal)

$1 Silver Certificate of 1923 (note the blue seal)

1923: The 1923 $1 Silver Certificate with blue seal

$100 Gold Certificate of 1928 (note the gold seal)

1928: The 1928 $100 Gold Certificate with gold seal.

$1 Silver Certificate of 1957 (note the blue seal)

1957: The 1957 $1 Silver Certificate with blue seal

$2 U.S. Treasury Note of 1928 (note the red seal)

1928: The 1928 $2 Treasury Note with red seal

$2 U.S. Treasury Note of 1963 (note the red seal)

1963: The 1923 Silver Certificate with blue seal

More interesting and unusual U.S. and state money can be found at www.money.org. www.frisian.com offers an interesting note (pun intended), and I quote: *All the small notes featured a portrait of Washington for the $1 note, Jefferson for $2, Lincoln for $5, Hamilton for $10, Jackson for $20, Grant for $50, Franklin for $100, McKinley for $500, Cleveland for $1,000, Madison for $5,000, and Chase for the $10,000. When one series of $100,000 notes was issued (1934 Gold Certificates), Wilson was put on them. Although these portraits are often called "dead presidents," three of them, Hamilton, Franklin, and*

Chase, were never Presidents. Large notes had featured many more portraits, including Martha Washington, William Tecumseh Sherman, John Marshall, James Monroe, the Sioux Indian Takokainyanka, Samuel F.B. Morse, and many others. ...

Issuing any notes larger than $100 was discontinued during World War II because of fears of German counterfeiting. With the development of other financial instruments, and now wire transfers, there is now felt to be no need for larger notes—though, when a 1995 $100 bill is only worth about $22 in 1967 dollars, it has been suggested that a $500 bill (1967 equivalent of $111) might be appropriate. Another consideration, however, is that the Police State tactics used for the War on Drugs include eliminating any money that can be used anonymously. If cash could be entirely replaced, Drug Warriors figure, the drug trade would not survive: People could not put drug purchases on their MasterCards. This strategy means that the United States government will adopt no measure to make the use of cash easier, like $500 bills.

CHAPTER 2

Rates of Returns and Interest Rates

The Basics!

last file change: Jun 14, 2004 (15:02h)

last major edit: Mar 1, 2004

The most basic concepts of finance concern returns and rates of returns. This short chapter explains compounding and annualizing of rates of returns, as well as the difference between an interest quote and an interest rate.

2·1. Basic Definitions

2·1.A. Returns, Net Returns, and Rates of Return

Defining: Return, Net Return, and Rate of Return.

The **return** of an **investment** is simply the amount of cash it returns. (**yield** is a synonym for return.) The **net return** is the difference between the return and the initial investment, which is positive if the project is profitable. The **rate of return** is the net return expressed as a percentage of the initial investment. For example, an investment project that costs \$10 today and returns \$12 in period 1 has

$$\begin{aligned}
\text{Return at Time 1} &= \$12 \\
\text{Return}_{t=1} &= C_1 \quad , \\
\text{Net Return from Time 0 to Time 1} &= \$12 - \$10 = \$2 \\
\text{Net Return}_{t=0,1} &= C_1 - C_0 \quad , \\
\text{Rate of Return from Time 0 to Time 1} &= \frac{\$12 - \$10}{\$10} = \frac{\$12}{\$10} - 1 = 20\% \\
r_1 = r_{t=0,1} &= \frac{C_1 - C_0}{C_0} = \frac{C_1}{C_0} - 1 \quad .
\end{aligned} \tag{2.1}$$

Percent (the symbol %) is a unit of 1/100. So, 20% is the same as 0.20.

Common Abbreviations: r usually means rate of return. C usually means cash.

Notice that I try to explain concepts with numerical examples. The more general algebraic formula appears right under the numerical example. This makes it easy to understand the correspondence with and use of the more formulaic algebra. Whenever possible, I choose a particular letter abbreviation that is commonly used and hopefully easy to recall. For instance, rates of returns are often abbreviated by the letter r. **Cash Flows** are typically abbreviated by the letter C. You will see this method at work throughout the rest of the text.

Capital Gains vs. Returns.

Returns can come in different forms. Many investments have intermittent payments. For example, many stocks pay cash dividends, many bonds pay cash coupons on a regular basis. Say, an investment costs \$92, pays a dividend of \$5, and then is worth \$110. What would its rate of return be?

$$\begin{aligned}
r_{0,1} &= \frac{\$110 + \$5 - \$92}{\$92} = \frac{\$110 - \$92}{\$92} + \frac{\$5}{\$92} = 25\% \\
r_{0,1} &= \frac{C_1 + \text{Div}_{0,1} - C_0}{C_0} = \underbrace{\frac{C_1 - C_0}{C_0}}_{\text{Capital Gain}} + \underbrace{\frac{\text{Div}_{0,1}}{C_0}}_{\text{Dividend Yield}}
\end{aligned} \tag{2.2}$$

The **capital gain** is the difference in the purchase price over the holding period, *not* counting interim payments. Here, the capital gain is the difference between \$110 and \$92, i.e., the \$18 change in the price of the investment. The dividend or coupon divided by the original price is called the **dividend yield** or **coupon yield**. Of course, if the dividend/coupon yield is high, you might earn a positive rate of return but experience a negative capital gain. For example, a bond that costs \$500, pays a coupon of \$50, and then sells for \$490, has a **capital loss** of \$10, but a rate of return of (\$490 + \$50 − \$500)/\$500 = 8%. We will almost always work with rates of returns, not with capital gains—though sometimes we have to draw the distinction, because the I.R.S.

treats capital gains differently from dividends (see Section 1·4).

SIDE NOTE: Most of the time, people (incorrectly but harmlessly) abbreviate a rate of return or net return by calling it just a return. For example, if you say that the return on your \$10,000 stock purchase was 10%, you obviously do not mean you received 0.1. You really mean that your rate of return was 10%. This is benign, because your listener will know what you mean.

Potentially more harmful is the use of the phrase yield, because it is often used as an shortcut for dividend yield or coupon yield. So, if you say that the yield on a bond is 5%, then some listeners may interpret this to mean that the overall rate of return is 5%, while others may interpret this to mean the coupon yield to be 5%.

If in doubt, ask!

ANECDOTE: **Yield**
In English finance, "yield" is just another standard term for rate of return. It comes from the Old High English "gieldan," which in turn comes from the German word "geltan" ("to pay"), which in turn comes from the Gothic (!) word "gildan." In German, the word root has evolved into the noun "Gelt" (money) and the verb "gelten" (as adverb "gültig"), which has a whole range of meanings: from *admitting*, to *applying*, to *counting*, to *weighing*, to *accepting*, to *be in force* (or *be in effect*), to *be classified* or to *be considered* (as something). The more sinister variation is the verb "vergelten," which means to "pay back" (or "revenge").

Formula 2.1 can be turned around to answer the question, "How much money will I receive in the future if the rate of return is 20% and I invest \$100?" The answer is

Future Payoffs Given a Rate of Return and an Initial Investment.

$$\begin{aligned} \$100 \cdot (1 + 20\%) &= \$120 \\ r_{0,1} = \frac{C_1 - C_0}{C_0} \quad \Rightarrow \quad C_0 \cdot (1 + r_{0,1}) &= C_1 \quad . \end{aligned} \tag{2.3}$$

We can now use this formula to compute all sorts of future payoffs. For example, an investment requiring \$212 today and earning a total **holding rate of return** of 60% from now (period 0) to period 12 would yield

Another example of a payoff computation.

$$\begin{aligned} \$212 \cdot (1 + 60\%) &= \$339.^{20} \\ C_{12} = \quad C_0 \cdot (1 + r_{0,12}) & \quad . \end{aligned} \tag{2.4}$$

You can call this the future value formula. If you rearrange the formula to $C_0 = C_t/(1 + r_{0,t})$. C_t is then said to be the present value of C_t, given the interest rate. The present value will be the main subject of Part II of this book.

2·1.B. Loans, Bonds, and Interest Rates

A **loan** is the commitment of a **borrower** to pay a predetermined amount of cash at a predetermined time in the future (called **maturity**), usually for cash upfront today. A loan can also be and is often called a **bond**, because it binds the borrower to pay money. Thus, "buying a bond" is the same as "extending a loan." It is the process of giving cash today and receiving a promise for money in the future. Similarly, instead of "taking a loan," one can just say that one is "giving a bond," "issuing a bond," or "selling a bond." Loans and bonds are also sometimes called **fixed income** instruments, because they promise a fixed income to the holder of the bond.

Finance Jargon: Loans, Bond, Fixed Income, Interest rate, Maturity, Coupon.

Bond: Defined by payment next year. Savings: Defined by payment this year.

Is there any difference between buying a bond for \$1,000 and putting \$1,000 into a bank account? Yes, a small one. The bond is defined by its future promised payoffs—say, \$1,100 next year—and the bond's price today is based on these future payoffs. An investment in a bank account is defined by its investment today. The interest rate can and will change every day, and next year you may end up with an amount that you do not yet know, e.g., \$1,050 (if interest rates decrease) or \$1,200 (if interest rates increase).

Side Note: You could think of a savings account as a sequence of 1-day bonds: you know the interest rate only one day in advance, and the money automatically gets reinvested tomorrow into a bond with the appropriate interest tomorrow. Incidentally, retirement plans come in two equivalent forms: **defined benefit** plans, which are defined by how much you will get when you retire, and **defined contribution** plans, which are defined by how much you are putting in today.

Interest.

The **net payoff** to a loan has a special name: it is called **interest**. The rate of return on a loan is usually called the **interest rate**. Many of our examples use the phrase "interest rate," but the examples tend also to work for other rates of returns. Not every rate of return is an interest rate: for example, an investment in a lottery ticket offers a return that is a random variable, which could be anything from zero to several million dollars.

Basis Points avoid an ambiguity in the English language: 100 basis points is 1 percent.

What does the statement "the interest rate has just increased by 5%" mean? It could mean either that the previous interest rate, say 10%, has just increased from 10% to $10\% \cdot (1 + 5\%) = 10.5\%$ or from 10% to 15%. Because this is unclear, the term **basis point** was invented. A basis point is simply 1/100 of a percent. So, if you state that your interest rate has increased by 50 basis points, you definitely mean the that the interest rate has increased from 10% to 10.5%. If you state that your interest rate has increased by 500 basis points, you mean that the interest rate has increased from 10% to 15%.

Anecdote: **Religions and Interest**
The three major Western religions have an "interesting" (pun intended) history with regard to interest, i.e., the fixed repayment of more than the principal for the purpose of profit.

- The Old Testament (Exod. xxii 25 and Deut. xxiii 19,20) seems to forbid the taking of interest among Jews. But in medieval Europe, Christian canon law prohibited Jews from many professions but allowed Jews to lend. Necessarily, money lending became a traditional occupation among Jews for many centuries. The only Christian restriction on Jewish lending was the vague prohibition of "excessive" interest.
- The New Testament is silent on the subject of interest. But the 12th canon of the First Council of Carthage (345AD) and the 36th canon of the Council of Aix (789AD) declared it reprehensible to lend money for the purpose of profit, and canonical laws of the Middle Ages absolutely prohibited it. Apparently, the prohibition had only limited success: In the Council of Vienne (1311AD), it had to be again declared that if any person obstinately maintained that there was no sin in the practice of demanding interest, he should be punished as a heretic.

  Eventually, around the 16th and 17th centuries, the lending restrictions faded away. By the 19th century, the Catholic Church admitted that borrowing even for ecclesiastical properties was useful, and nowadays itself borrows and lends at interest.

  Source: www.newadvent.org
- The Quran explicitly outlaws **Ar-Riba**, believed by interpretative consensus to mean the taking of "interest." The Quran *does* explicitly permit recovery of principal by a lender, just not interest. There is some argument as to whether inflation adjustment counts as principal recovery or interest.

  Unlike the Bible to Christians and Jews, the Quran is not just a holy book to Muslims, but literally the word of G'd. Therefore, the prohibition of lending has lasted to this day in many Muslim countries. Islam does permit equity participation, so various forms of equity flourish in Islamic countries as debt substitutes. More literal Islamic interpretations prohibit equity investment by Muslims into bank stocks (lenders) or even into stocks of firms that have debt (borrowers). Less literal interpretations doubt the meaning of the phrase Ar-Riba, which is not clearly defined as interest in the Quran itself.

Solve Now!

Q 2.1 *A project offers a return of \$1,050 for an investment of \$1,000. What is the rate of return?*

Q 2.2 *A project offers a net return of \$25 for an investment of \$1,000. What is the rate of return?*

Q 2.3 *A project has a rate of return of 30%. What is the payoff if the initial investment is \$250?*

Q 2.4 *A project has a rate of return of 30%. Rearrange Formula 2.3 to compute the initial investment if the final payoff is \$250. (This will later be called the Present Value.)*

Q 2.5 *A project has a rate of return of 30%. What is the initial investment if the final net payoff is \$50?*

Q 2.6 *The price of a zero bond offering a firm promise of \$100 in 1 year is \$95. If the bond's interest rate was 150 basis points higher, what should its price today be?*

2·2. Compounding Rates of Return

What is the total two-year holding rate of return, if you earn a one-year rate of return of 20% in each year? It is *not* 20% + 20% = 40%! Here is why: If you invest \$100 at a 20% rate of return today (year 0), you will have \$120 in year 1. Investing \$120 at a 20% rate of return from year 1 to year 2 you will end up with

Interest on Interest (or rate of return on rate of return) means rates cannot be added.

$$\begin{aligned} \$120 \cdot (1 + 20\%) &= \$144 \\ C_1 \cdot (1 + r_{1,2}) &= C_2 \quad . \end{aligned} \tag{2.5}$$

(This is based on a variant Formula 2.3. Remember that the subscripts denote time indexes, and comma-separated subscripts denote a time period.) This \$144, then, represents a total two-year rate of return of

$$\begin{aligned} \frac{\$144 - \$100}{\$100} &= 44\% \\ r_{0,2} = \frac{C_2 - C_0}{C_0} &= \frac{C_2}{C_0} - 1 \quad . \end{aligned} \tag{2.6}$$

This is more than 40%, because the original net return of \$20 in the first year earned an additional \$4 in interest in the second year. We earn interest on interest!

Compounding is different from adding.

So, how do we determine multiyear rates of returns from one-year rates of returns? The answer is the **Compounding Formula**, which could also be called the "one-plus" formula:

$$\begin{aligned} (1+20\%) \cdot (1+20\%) &= 1.20 \cdot 1.20 = 1.44 \\ (1+r_{0,2}) &= (1+r_{0,1}) \cdot (1+r_{1,2}) \quad . \end{aligned} \tag{2.7}$$

In words, one plus the two-year rate of return is one plus the first year rate of return multiplied by one plus the second year rate of return. Subtract 1 from 1.44, and the answer is $r_{0,2} = 44\%$, the rate of return (interest rate) from period 0 to period 2.

A second compounding example.

What if the interest rate next year, for whatever reason, will not be 20%, but 30%? Use the same formula

$$\begin{aligned} (1+r_{0,2}) &= (1+r_{0,1}) \cdot (1+r_{1,2}) \\ &= (1+20\%) \cdot (1+30\%) = 1.56 \quad , \end{aligned} \tag{2.8}$$

subtract 1, and the answer is the total two-year rate of return of 56%.

The general formula for compounding over many periods.

More generally, the compounding formula for obtaining a total rate of return from period i to period j is the multiplicative "one-plus" of each interest rate (subtracting one at the end).

<u>IMPORTANT</u>: *The Compounding Formula translates many individual period rates of returns into a single total holding period rate of return:*

$$(1+r_{t,t+N}) = (1+r_{t,t+1}) \cdot (1+r_{t+1,t+2}) \cdot \cdots \cdot (1+r_{t+N-1,t+N}) \quad . \tag{2.9}$$

Multi-period compounding.

The compounding formula can help answer questions such as, "If the one-year rate of return is 30% from year 1 to year 2, 40% from year 2 to year 3, and 50% from year 3 to year 4, then what is the rate of return for investing beginning next year for three years?" The answer is

$$\begin{aligned} \text{Given} \quad &: \quad r_{1,2} = 30\% \qquad r_{2,3} = 40\% \qquad r_{3,4} = 50\% \\ (1+r_{1,4}) &= (1+r_{1,2}) \cdot (1+r_{2,3}) \cdot (1+r_{3,4}) \\ &= (1+30\%) \cdot (1+40\%) \cdot (1+50\%) \\ &= (1+173\%) \quad . \end{aligned} \tag{2.10}$$

Subtracting 1, we see that the three-year rate of return for an investment that takes money next year (not today!) and returns money in four years, appropriately called $r_{1,4}$, is 173%. For example, if it were midnight of December 31, 1989 right now, each dollar invested on midnight December 31, 1990, would return \$1.[73] on midnight December 31, 1993.

ANECDOTE: Compounding Manhattan
The difference between compounding and adding seems small, at least for small interest rates and time periods, but it can add up over long periods. This is sometimes called the **power of compounding**.

The most popular anecdote for the power of compounding is the alleged purchase of the island of Manhattan for \$24 worth of goods from its native Indians in 1624. (Incidentally, despite popular belief, there is no evidence that beads were involved in the purchase!) Compounded for 375 years at an interest rate of 6% per year, each dollar invested in 1624 would have grown to over \$3 billion in 2000. This may still be small relative to the value of Manhattan today, but it is not insignificant. However, the Indians probably would not have gotten the chance to safely invest their money, anyway: the U.S. government failed to live up to most of its treaty obligations of this era. Protecting native Indian property was not high on its agenda.

NERD NOTE: *Interestingly, compounding even works over fractional time periods. So, if the interest rate is 5% per year, to find out what the interest rate over half-a-year is, compute*

$$(1 + r_{0,0.5}) \;=\; (1 + r_{0,1})^{0.5} \;=\; (1 + 5\%)^{0.5} \;=\; 1 + 2.4695\% \;. \tag{2.11}$$

Compounding 2.4695% over two (six-month) periods indeed yields 5%.

Solve Now!

Q 2.7 *If the per-year interest is \$5, what is the two-year total interest rate?*

Q 2.8 *If the per-year interest rate is 5%, what is the two-year total interest rate?*

Q 2.9 *If the per-year interest rate is 5%, what is the ten-year total interest rate?*

Q 2.10 *If the per-year interest rate is 5%, what is the hundred-year total interest rate? How does this compare to 100 times 5%?*

Q 2.11 *If the first-year interest rate is 2% and the second year interest is 3%, what is the two-year total interest rate?*

Q 2.12 *From Fibanocci's Liber Abaci, written in the year 1202: "A certain man gave one denaro at interest so that in five years he must receive double the denari, and in another five, he must have double two of the denari and thus forever. How many denari from this 1 denaro must he have in 100 years?"*

2·3. Quoting Rates of Return Per Annum

Per-Unit Measures are conceptual aids.

Is a car travelling 258,720 yards in 93 minutes fast or slow? It is difficult to say, because we are used to thinking in "miles per sixty minutes," not in "yards per ninety-three minutes." It makes sense to translate speeds into miles per hour, so that it is easy to compare speeds. We can even do this for sprinters, who cannot run a whole hour. Speeds are just a standard measure of the rate of accumulation of distance per unit of time.

A Per-Unit Standard for Rates of Returns: Annualization.

The same applies to interest rates: a rate of return of 58.6% over 8.32 years is not as easy to compare to other rates of return as a rate of return per year. So we would like to quote interest rates as something like an **average annualized interest rate**. Of course, when we compute such an average annualized interest rate, we do not mean that the investment earned the same annualized interest rate of 5.7% each year—just as the car need not have traveled at 94.8 mph (258,720 yards in 93 minutes). The average annualized interest rate is just a convenient unit of measurement for the rate at which money accumulates, a "sort-of-average measure of performance."

An Example of Annualizing a Three-Year Total Holding Return.

So, if you were earning a total three-year holding return of 173% over the three year period, what would your average *annualized* interest return be? The answer is not $173\%/3 \approx 57.7\%$, because if you earned 57.7% per year, you would have ended up with $(1 + 57.7\%) \cdot (1 + 57.7\%) \cdot (1 + 57.7\%) - 1 = 287\%$, not 173%. This incorrect answer of 57.7% ignores the compounded interest on the interest which you would earn after the first year and second year. Our goal is to find a single hypothetical interest rate which, if you received it each and every year, would give you a three-year return of 173%.

A Problem of finding a three-year annualized interest rate. Solution: Take the N-th Root of the total return (N is number of years).

Call $r_{\bar{3}}$ this hypothetical annual interest rate which we would have to earn each year for 3 years in order to end up with a total rate of return of 173%. To find r, solve the equation

$$\begin{aligned} (1 + r_{\bar{3}}) \cdot (1 + r_{\bar{3}}) \cdot (1 + r_{\bar{3}}) &= (1 + 173\%) \\ (1 + r_{\bar{3}}) \cdot (1 + r_{\bar{3}}) \cdot (1 + r_{\bar{3}}) &= (1 + r_{0,3}) \quad , \end{aligned} \tag{2.12}$$

or, for short

$$\begin{aligned} (1 + r_{\bar{3}})^3 &= (1 + 173\%) \\ (1 + r_{\bar{t}})^t &= (1 + r_{0,t}) \quad . \end{aligned} \tag{2.13}$$

Note that $r_{\bar{3}}$ is an unknown, and very different from $r_{0,3}$. Earning the same interest rate ($r_{\bar{3}}$) three years in a row should result in a total holding rate of return ($r_{0,3}$) of 173%. The correct solution for $r_{\bar{3}}$ is obtained by computing the third root of the total holding rate of return (Appendix 2·3 reviews powers, exponents and logarithms):

$$\begin{aligned} (1 + r_{\bar{3}}) &= (1 + 173\%)^{(1/3)} = \sqrt[3]{1 + 173\%} = 1.3976 \\ (1 + r_{\bar{t}}) &= (1 + r_{0,t})^{(1/t)} = \sqrt[t]{1 + r_{0,t}} \quad . \end{aligned} \tag{2.14}$$

A calculator will confirm that $r_{\bar{3}} \approx 39.76\%$,

$$(1 + 39.76\%) \cdot (1 + 39.76\%) \cdot (1 + 39.76\%) = (1 + 173\%) \quad . \tag{2.15}$$

In sum, if you invested money at an interest rate of 39.76% per annum for three years, you would end up with a total three-year return of 173%.

IMPORTANT: *The total holding rate of return over N years, called $r_{0,N}$, is translated into an annualized interest rate of return , called $r_{\overline{N}}$, by taking the N-th root:*

$$(1 + r_{\overline{N}}) = \sqrt[N]{1 + r_{0,N}} = (1 + r_{0,N})^{1/N} \quad . \tag{2.16}$$

Compounding the annualized rate of return over N years yields the total holding period rate of return.

Translating long-term net returns into annualized rates of returns.

The need to compute annualized rates of return often arises in the context of investments. For example, what kind of annualized rate of return would you expect from an investment promising a return of \$240 in 30 years for an investment of \$100 today? The first step is computing the total holding rate of return. By Formula 2.1, the total 30-year rate of return is

$$\begin{aligned} r_{0,30} &= \frac{\$240 - \$100}{\$100} = 140\% \\ r_{0,30} &= \frac{C_{30} - C_0}{C_0} \quad . \end{aligned} \tag{2.17}$$

The annualized rate of return is the interest rate r, which, if compounded for 30 years, offers a 140% rate of return,

$$\begin{aligned} (1 + r_{\overline{30}})^{30} &= (1 + 140\%) \\ (1 + r_{\overline{t}})^{t} &= (1 + r_{0,t}) \quad . \end{aligned} \tag{2.18}$$

Solve this equation by taking the 30th root,

$$\begin{aligned} (1 + r_{\overline{30}}) &= (1 + 140\%)^{1/30} = \sqrt[30]{1 + 140\%} \approx 1 + 2.96\% \\ (1 + r_{\overline{30}}) &= (1 + r_{0,30})^{1/30} = \sqrt[30]{1 + r_{0,30}} \quad . \end{aligned} \tag{2.19}$$

Thus, a return of \$240 in 30 years for \$100 investment is equivalent to about a 3% annualized rate of return.

SIDE NOTE: You may find this inuition helpful: If there was no compounding, then the holding rate of return would be the time-series sum of interest rates, and the annualized rate of return would be the time-series average of all interest rates. The effect of compounding is that, for multi-period investments, the holding rate of return is above the sum, and the annualized rate is below the average.

Solve Now!

Q 2.13 *If the total holding interest rate is 50% for a 5-year investment, what is the annualized rate of return?*

Q 2.14 *If the per-year interest rate is 10% for each of the next 5 years, what is the annualized total 5-year rate of return?*

Q 2.15 *If the annualized 5-year rate of return is 10%, what is the total 5-year holding rate of return?*

Q 2.16 *If the annualized 5-year rate of return is 10%, and if the first year's rate of return is 15%, and if the returns in all other years are equal, what are they?*

2·4. Confusion: Interest Quotes vs. Interest Rates

Adding rather than compounding can make forgivably small mistakes in certain situations—but some people are just ignorant.

Interest rates are not difficult, but they are both tedious and confusing. Some people just add interest rates instead of compounding them. When the investments, the interest rates, and the time periods are small, the difference between the correct and an incorrect computation can be minor, so this practice can be acceptable, even if it is wrong. For example, when interest rates are 1%, compounding yields $(1 + 1\%) \cdot (1 + 1\%) - 1 = 2.01\%$, which is almost the same as the simple sum coming to 2%. Making the same mistake, some people annualize interest rates by just taking the average interest rate, thus ignoring the interest on the interest. This can again be forgivable if periods and interest rates are small.

Table 2.1. Misleading Bank Interest Quotes

Bank quotes annual rate of	10%
Bank pays daily rate of	10%/365 = 0.0274%
Daily rate compounds over 365 days to	$(1 + 0.0274\%)^{365} = 10.5\%$

Banks add to the confusion, quoting interest rates in a strange but traditional way.

Even banks—who should know better—are prime culprits here. Although they do quote an annual (not an annualized!) interest rate, they will actually pay daily interest at a rate of the annual interest quote divided by 365. So, as Table 2.1 shows, in effect, if the bank quotes an annual interest rate of 10%, it is paying you 10.5% per annum ($10.^{50} for every $100) if you leave your money in the bank for a year. This may make little difference to you, but a billionairess ($1,000,000,000) would receive $274,000 instead of $261,000 if she left her money in the bank for one day. Confusion reigns supreme in the world of interest quotes!

Important: *My best advice when money is at stake: If in doubt, ask how the interest rate is computed! Even better, ask for a simple illustrative calculation.*

Nerd Note: *In a sense, it is less confusing if interest quotes bear no resemblance to interest rates. For example, in a Treasury auction, in which the government sells Treasury bills (**"T-bills"**) that surely pay $10,000 in 180 days for $9,500, the discount quote would be*

$$\textit{Quoted TB Price} = \$100 \cdot [100 - (180/360) \cdot 10] = \$9,500$$

$$\$100 \cdot [100 - (\textit{days to maturity}/360) \cdot \textit{discount rate}] \quad . \tag{2.20}$$

The Wall Street Journal (Wall Street Journal) simply prints 95, because Treasury bills are quoted in units of 100. The real interest rate at the 95 quote is $(10,000/9,500) - 1 \approx 5.26\%$. *Therefore, even ignoring the extra 5 days, the 360 day interest rate is* $1.0526^2 - 1 \approx 10.8\%$, *not 10%.*

Solve Now!

Q 2.17 *If the bank quotes an interest rate of 6% per year, what does a deposit of $100 in the bank come to after one year?*

Q 2.18 *If the bank quotes a loan rate of 8% per year, what do you have to pay back in one year if you borrow $100 from the bank?*

2·5. Summary

The chapter covered the following major points:

- Returns must be compounded, not averaged over time.
- 100 basis points are 1%.
- Bonds commit to payments in the future. An increase in interest rates decreases the price of the bond today. Bank deposits are like a sequence of 1-day bonds, where a new interest is set daily.
- Returns can be annualized for easier interpretation.
- Interest rate quotes are *not* interest rates—confusion often reigns supreme.

A. Q&A: Answers

1. $r = \dfrac{\$1,050 - \$1,000}{\$1,000} = 5\%$.
2. $r = \dfrac{\$25}{\$1,000} = 2.5\%$.
3. $r = 30\% = \dfrac{x - \$250}{\$250} \qquad \Longrightarrow x = \$325$.
4. $r = 30\% = \dfrac{\$250 - x}{x} \qquad \Longrightarrow x = \$192.^{31}$.
5. $r = 30\% = \dfrac{\$50}{x} \qquad \Longrightarrow x = \$166.^{67}$.
6. The original interest rate is $\$100/\$95 - 1 = 5.26\%$. Increasing the interest rate by 150 basis points is 6.76%. This means that the price should be $\$100/(1 + 6.76\%) = \$93.^{67}$.
7. This cannot be determined.
8. $r_{0,2} = (1 + r_{0,1}) \cdot (1 + r_{1,2}) - 1 = (1 + 5\%) \cdot (1 + 5\%) - 1 = 10.25\%$.
9. $r_{0,2} = (1 + r_{0,1})^{10} - 1 = 62.89\%$.
10. $r_{0,2} = (1 + r_{0,1})^{100} - 1 = 13,050\%$ In words, this is about 130 times the initial investment, and substantially more than 500% (5 times the initial investment).
11. $r_{0,2} = (1 + r_{0,1}) \cdot (1 + r_{1,2}) - 1 = (1 + 2\%) \cdot (1 + 3\%) - 1 = 5.06\%$.
12. First, solve for the interest rate: $1d \cdot (1 + r)^5 = 2d \rightarrow r = 14.87\%$. Therefore, in 100 years, he will have $(1 + r)^{100} = 1,048,576$ denari. Of course, you can solve this in a simpler way: you have twenty 5-year periods, in each of which the holdings double. So, the answer is 2^{20} denari.
13. $r_{0,5} = 50\% \qquad (1 + r_{\overline{5}})^5 = (1 + 50\%) \qquad \Longrightarrow \qquad r_{\overline{5}} = (1 + 50\%)^{1/5} - 1 = 8.45\%$.
14. The same.
15. $r_{\overline{5}} = 10\% \qquad (1 + r_{0,5}) = (1 + r_{\overline{5}})^5 = (1 + 61.05\%)$.
16. The basic formula is $(1 + r_{\overline{5}})^5 = (1 + r_{0,1}) \cdot (1 + r_{1,2}) \cdot (1 + r_{2,3}) \cdot (1 + r_{3,4}) \cdot (1 + r_{4,5})$. Substituting what we know, $(1 + 10\%)^5 = (1 + 15\%) \cdot (1 + x) \cdot (1 + x) \cdot (1 + x) \cdot (1 + x)$. Calculating, $1.61051 = 1.15 \cdot (1 + x)^4$, $(1 + x)^4 = 1.40$, , and $(1 + x) = 1.40^{1/4} = 1.0878$. So, the return in all other years would have to be 8.78% per year.
17. The bank quote of 6% means that it will pay an interest rate of 6%/365= 0.0164384% per day. This earns an actual interest rate of $(1 + 0.0164384\%)^{365} = 6.18\%$ per year: Each invested \$100 earns $\$6.^{18}$ over the year.
18. The bank quote of 8% means that you will have to pay an interest rate of 8%/365= 0.021918% per day. This earns an actual interest rate of $(1 + 0.021918\%)^{365} = 8.33\%$ per year: Each borrowed \$100 requires $\$108.^{33}$ in repayment.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 3

Horizon-Dependent Interest Rates

Fixed Income Basics

last file change: Jun 15, 2004 (10:32h)

last major edit: March 2004

In the real world, interest rates usually depend on the maturity of the bond. For example, a 30-year bond may offer an annualized interest rate of 5%, while a 1-year bond may offer an annualized interest rate of 3%. The graph of interest rates against time is called the yield curve. This chapter explains how to work with the yield curve.

3·1. The Yield Curve in May 2002

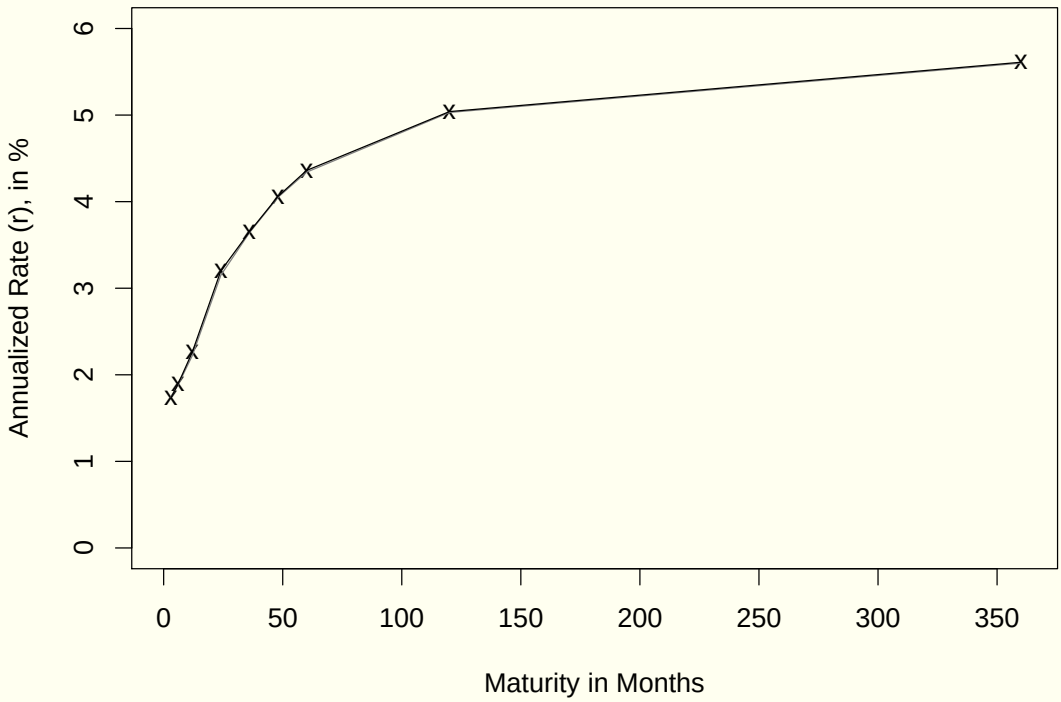
The term structure of interest tells us how the annualized rate of return is different over different horizons.

We are now ready to tackle the yield curve, which is also sometimes called the **term structure of interest rates**. The **yield curve** is today's average annualized interest (yield) that investments pay as a function of their time to maturity. If not clarified further, the yield curve usually refers to investments in **U.S. Treasuries**, although it is more precisely called the **Treasury yield curve**. These investments are obligations of the U.S. Treasury and come in three flavors: **Treasury bills** with maturities of less than one year, **Treasury notes** with maturities between one and ten years, and **Treasury bonds** with maturities greater than ten years. The 30-year bond was often called the **long bond**, at least before the Treasury suspended its issuance in October 2001.

Side Note: The U.S. Treasury will not fail to pay. It can always print more money if it runs out! Thus, there is no nominal credit risk to investing in treasuries. The Treasury bond market is also very liquid, with average *daily* trading volume of about $300 billion in 2001. Therefore, trading volume per year totals about ten times the U.S. economy's gross domestic product (**GDP**).

Figure 3.1. The Treasury Yield Curves in mid-2002

Maturity	Apr 30	May 30	May 31
1 Month	1.77%	1.72%	1.72%
3 Month	1.77%	1.74%	1.73%
6 Month	1.91%	1.88%	1.89%
1 Year	2.35%	2.22%	2.26%
2 Year	3.24%	3.15%	3.20%
3 Year	3.83%	3.64%	3.65%
4 Year	n/a	4.05%	4.06%
5 Year	4.53%	4.34%	4.36%
10 Year	5.11%	5.03%	5.04%
20 Year	5.74%	n/a	n/a
30 Year	n/a%	5.60%	5.61%



The data for May 30, 2002, and May 31, 2002 were printed in the Wall Street Journal. The data for April 30, 2002, was obtained from the U.S. Treasury website at www.ustreas.gov. As you can see, the yieldcurve changes every day—though changes are usually small. The yieldcurve for May 31, 2002, is plotted below.

For illustration, we pretend that the *Wall Street Journal* yield curve is based on zero bonds (which only have one final payment). Although this is actually not perfectly correct, the differences are usually very small. We will clarify this in Chapter 8. This is why the data on the Treasury website is slightly different—in this example, the maximum difference is for the 10-year bond, where it is 4 basis points.

Table 3.1. Relation Between Holding Returns, Annualized Returns, and Year-by-Year Returns, By Formula and on May 31, 2002

	Rates of Returns		
Maturity	Total Holding	Annualized	Individually Compounded
1 Year	$(1 + 2.26\%)$	$= (1 + 2.26\%)^1$	$= (1 + 2.26\%)$
	$(1 + r_{0,1})$	$= (1 + r_{\overline{1}})^1$	$= (1 + r_{0,1})$
2 Year	$(1 + 6.50\%)$	$= (1 + 3.20\%)^2$	$= (1 + 2.26\%) \cdot (1 + 4.15\%)$
	$(1 + r_{0,2})$	$= (1 + r_{\overline{2}})^2$	$= (1 + r_{0,1}) \cdot (1 + r_{1,2})$
3 Year	$(1 + 11.35\%)$	$= (1 + 3.65\%)^3$	$= (1 + 2.26\%) \cdot (1 + 4.15\%) \cdot (1 + 4.56\%)$
	$(1 + r_{0,3})$	$= (1 + r_{\overline{3}})^3$	$= (1 + r_{0,1}) \cdot (1 + r_{1,2}) \cdot (1 + r_{2,3})$

We will analyze the actual yield curves at the end of May 2002.

Table 3.1 shows the actual Treasury yield curves on May 30 and May 31, 2002. If you had purchased a 3-month bond at the end of the day on May 30, 2002, your annualized interest rate would have been 1.74%. The following day, a 3-month bond had a yield that was one basis point lower. (This would not be the exact same bond: the previous 90-day bond would now be an 89-day bond, and therefore would be replaced by a 90-day bond.) If you had purchased a 30-year bond at the end of the day on May 30, 2002, you would have received an annualized interest rate of 5.60% per year, which is one basis point less than it would have been the following day.

SIDE NOTE: Sometimes, it is necessary to determine an interest rate for a bond that is not traded. This is usually done by interpolation. For example, if the 90-day Treasury note had a known interest rate of 1.73% and the 92-day Treasury note had a known interest rate of 1.75%, a good interest rate for an untraded 91-day Treasury note might be 1.74%.

The annualized interest rate was higher for longer time periods.

We need notation for annualized horizon-dependent interest rates, so we shall call the the two-year annualized interest rate $r_{\overline{2}}$, the three-year annualized interest rate $r_{\overline{3}}$, and so on. The yield curve makes it clear why we need to put a subscript on the annualized yields: the annualized yield can vary with maturity. Table 3.1 relates the different types of returns. (The next section explains the iterative process by which we can extract the set of future one-year rates, also called **forward rates**, from the yield curve.)

ANECDOTE: **Life Expectancy and Credit**
Your life expectancy may be 80 years, but 30-year bonds existed even in an era when life expectancy was only 25 years—at the time of Hammurabi, around 1700 B.C.E. (**Hammurabi** established the Kingdom of Babylon, and is famous for the Hammurabi Code, the first known legal system.) A cuneiform clay tablet contains the oldest known interest rate problem for prospective students of the financial arts. The student must figure out how long it takes for 1 mina of silver, growing at 20% interest per year, to reach 64 minae. Because the interest compounds in an odd way (20% of the principal is accumulated until the interest is equal to the principal, and then it is added back to the principal), the answer to this problem is 30 years, rather than 22.81 years. This is not an easy problem to solve—and it even requires knowledge of logarithms!

3·2. Working With Yield Curves

On May 30, 2002, how much money did an investment of \$500,000 into U.S. 2-Year notes (i.e., a loan to the U.S. government of \$500,000) promise to return in two years? Refer to Table 3.1 on Page 49. Because the yield curve prints annualized rates of returns, the total two-year holding rate of return (as in Formula 2.13) is the twice compounded annualized rate of return,

Computing the holding period rate of return for 2-Year bonds.

$$\begin{aligned} r_{0,2} &= (1+3.15\%) \cdot (1+3.15\%) - 1 = 6.4\% \\ &= (1+r_{\overline{2}}) \cdot (1+r_{\overline{2}}) - 1 \ , \end{aligned} \tag{3.1}$$

so the \$500,000 would turn into

$$\begin{aligned} C_2 &= (1+6.4\%) \cdot \$500,000 = \$531,996 \\ &= (1+r_{0,2}) \cdot C_0 \ . \end{aligned} \tag{3.2}$$

(In the real world, you might have to pay a commission to arrange this transaction, so you would end up with a little less.)

What if you invested \$500,000 into 30-Year Treasuries? The 30-Year total rate of return would be

Computing the holding period rate of return for 30-Year bonds.

$$\begin{aligned} r_{0,30} &= (1+r_{\overline{30}})^{30} - 1 \\ &= (1+5.60\%)^{30} - 1 \approx 5.1276 - 1 = 412.76\% \ . \end{aligned} \tag{3.3}$$

Thus, your investment of $C_0 = \$500,000$ will turn into cash of $C_{30} \approx \$2,563,820$ in 30 years.

The yield curve is usually upward sloping. This means that longer-term interest rates are higher than shorter-term interest rates. However, this is *not* always the case. If short-term rates are higher than long-term rates, the yield curve is said to be downward sloping (or **inverted**). If medium-term rates are higher than both long-term and short-term rates, the yield curve is called humped. Inverted or humped yield curves are relatively rare.

Yield Curves are often, but not always upward sloping.

What would a flat yield curve mean? For example, if the yield curve was a horizontal line at 5%, it would mean that the interest rate, no matter over what period, was 5% per annum. Thus, borrowing money for two years, the total (non-annualized) interest rate that would have to be paid would be $(1+5\%) \cdot (1+5\%) - 1 = 10.25\%$. More generally, the interest rate over any period can then be quickly computed as

A flat yield curve means that the annualized interest rate is the same regardless of horizon.

$$(1+r_{0,t}) = (1+r_{0,1})^t \ . \tag{3.4}$$

ANECDOTE: Maximum Interest Spreads
The St. Louis Federal Reserve Bank maintains a database of historical interest rates at http://research.stlouisfed.org/fred. Since 1934, the steepest yield curve (the difference between the long-term and the short-term Treasury rate) occurred in October 1992, when the long-term interest rate was 7.3 percent and the short-term interest rate was 2.9 percent—just as the economy pulled out of the recession of 1991. In January 1940, the long-term interest rate was 2.3 percent and the short-term interest rate was practically zero. The most inverted yield curve occurred in December 1980, when long-term interest rates were 10 percent, and short-term interest rates were 12 percent, during a brief period of rapidly declining inflation rates (and expansion in between two recessions).

Another good source of recent interest rates can be found at The Treasury Management Pages at http://www.tmpages.com/.

ANECDOTE: Interest Rates over the Millenia
Historical interest rates are fascinating, perhaps because they look so similar to today's interest rates. For over 2,500 years, from about the thirtieth century B.C.E. to the sixth century B.C.E., normal interest rates in Sumer and Babylonia hovered around 10–25% per annum, though 20% was the legal maximum. In ancient Greece, interest rates in the sixth century were about 16–18%, dropping steadily to about 8% by the turn of the millenium. Interest rates in ancient Egypt tended to be about 10–12%. In ancient Rome, interest rates started at about 8% in the fifth century B.C.E., but began to increase to about 12% by the third century A.C.E. (a time of great upheaval). When lending resumed in the late Middle Ages (12th century), personal loans in England fetched about 50% per annum though they tended to hover between 10–20% in the rest of Europe. By the Renaissance, commercial loan rates had fallen to 5–15% in Italy, the Netherlands, and France. By the 17th century, even English interest rates had dropped to 6–10% in the first half, and even to 3–6% in the second half. Mortgage rates tended to be lower yet. Most of the American Revolution was financed with French and Dutch loans at interest rates of 4–5%.

Solve Now!

Q 3.1 *Using information from a current newspaper or the WWW, what does an investment of $1 in 30-year bonds yield in 30 years?*

Q 3.2 *Using information from a current newspaper or the WWW, what does an investment of $1 in 1-year bonds yield in 1 year?*

3·3. Forward Interest Rates (Optional)

An important question—not to the ordinary small investor but to large traders—is what the so-called **forward interest rate** (or, simply, **forward rate**) structure implicit in the yield curve is. A forward rate is effectively an interest rate for an investment of cash beginning not today, but in the future. We have already used forward rates: we called them, e.g., $r_{2,3}$, the one-year interest rate beginning in 2 years. The novelty will be that, by cleverly buying and selling (shorting) Treasury bonds, you can bet on future interest rates embedded in the yield curve. We now show how.

Forward interest rates are implied interest rates in the future, given by today's yield curve.

Side Note: Interest rates for investments beginning this period are called **Spot Rates** or **Spot Interest Rates**, because they are the interest that can be obtained on the spot right now. By convention, spot rates are usually quoted as annualized interest rates.

3·3.A. Extracting Forward Interest Rates From The Yield Curve

We begin by working out the future one-year interest rates that were already computed for us in Table 3.1 on Page 50. In Table 3.1, the formulas were

Working out forward rates step by step from the yield curve.

	Rates of Returns		
Maturity	Total Holding	Annualized	Individually Compounded
1 Year	$(1+r_{0,1})$	$=(1+r_{\overline{1}})^1$	$=(1+r_{0,1})$
2 Year	$(1+r_{0,2})$	$=(1+r_{\overline{2}})^2$	$=(1+r_{0,1})\cdot(1+r_{1,2})$
3 Year	$(1+r_{0,3})$	$=(1+r_{\overline{3}})^3$	$=(1+r_{0,1})\cdot(1+r_{1,2})\cdot(1+r_{2,3})$

The *Wall Street Journal* yield curve gives us the annualized interest rates, i.e., the third column. We can read them off and insert them into our table. On May 31, 2002, these interest rates were

	Rates of Returns		
Maturity	Total Holding	Annualized	Individually Compounded
1 Year	$(1+r_{0,1})$	$=(1+2.26\%)^1$	$=(1+r_{0,1})$
2 Year	$(1+r_{0,2})$	$=(1+3.20\%)^2$	$=(1+r_{0,1})\cdot(1+r_{1,2})$
3 Year	$(1+r_{0,3})$	$=(1+3.65\%)^3$	$=(1+r_{0,1})\cdot(1+r_{1,2})\cdot(1+r_{2,3})$

The first step is to compute the holding rates of returns in the second column:

	Rates of Returns		
Maturity	Total Holding	Annualized	Individually Compounded
1 Year	$(1+\ 2.26\%)$	$=(1+2.26\%)^1$	$=(1+r_{0,1})$
2 Year	$(1+\ 6.50\%)$	$=(1+3.20\%)^2$	$=(1+r_{0,1})\cdot(1+r_{1,2})$
3 Year	$(1+11.35\%)$	$=(1+3.65\%)^3$	$=(1+r_{0,1})\cdot(1+r_{1,2})\cdot(1+r_{2,3})$

Ultimately, we want to know what the implied future interest rates are. Work your way down. The first row is easy: we know that $r_{0,1}$ is 2.26%. We can also substitute this return into the other rows:

	Rates of Returns		
Maturity	Total Holding	Annualized	Individually Compounded
1 Year	$(1 + 2.26\%)$	$= (1 + 2.26\%)^1$	$= (1{+}2.26\%)$
2 Year	$(1 + 6.50\%)$	$= (1 + 3.20\%)^2$	$= (1{+}2.26\%) \cdot (1 + r_{1,2})$
3 Year	$(1 + 11.35\%)$	$= (1 + 3.65\%)^3$	$= (1{+}2.26\%) \cdot (1 + r_{1,2}) \cdot (1 + r_{2,3})$

Now we have to work on the two year row to determine $r_{1,2}$: We have one equation and one unknown in the two year row, so we can determine the interest to be

$$(1 + 6.50\%) = (1 + 2.26\%) \cdot (1 + r_{1,2}) \implies (1 + r_{1,2}) = \left(\frac{1 + 6.50\%}{1 + 2.26\%}\right) = 1 + 4.15\% \ . \tag{3.5}$$

Substitute this solution back into the table,

	Rates of Returns		
Maturity	Total Holding	Annualized	Individually Compounded
1 Year	$(1 + 2.26\%)$	$= (1 + 2.26\%)^1$	$= (1{+}2.26\%)$
2 Year	$(1 + 6.50\%)$	$= (1 + 3.20\%)^2$	$= (1{+}2.26\%) \cdot (1{+}4.15\%)$
3 Year	$(1 + 11.35\%)$	$= (1 + 3.65\%)^3$	$= (1{+}2.26\%) \cdot (1{+}4.15\%) \cdot (1 + r_{2,3})$

Now work on row 3. Again, we have one equation and one unknown in the three year row, so we can determine the interest to be

$$(1 + 11.35\%) = (1 + 2.26\%) \cdot (1 + 4.15\%) \cdot (1 + r_{2,3}) \tag{3.6}$$

$$\implies (1 + r_{2,3}) = \frac{1 + 11.35\%}{(1 + 2.26\%) \cdot (1 + 4.15\%)} \approx 1 + 4.56\% \ . \tag{3.7}$$

	Rates of Returns		
Maturity	Total Holding	Annualized	Individually Compounded
1 Year	$(1 + 2.26\%)$	$= (1 + 2.26\%)^1$	$= (1{+}2.26\%)$
2 Year	$(1 + 6.50\%)$	$= (1 + 3.20\%)^2$	$= (1{+}2.26\%) \cdot (1{+}4.15\%)$
3 Year	$(1 + 11.35\%)$	$= (1 + 3.65\%)^3$	$= (1{+}2.26\%) \cdot (1{+}4.15\%) \cdot (1{+}4.56\%)$

So, given the annualized rates of returns in the yield curve, we can determine the whole set of implied forward interest rates. For example, the implied interest rate from year 2 to year 3 is 4.56%.

Think of the annualized interest rate as the average of interest rates.

Behind this arithmetic lies a pretty simple intuition: An annualized two-year interest rate is "really sort of" an "average" interest rate over the interest rates from the first year and the second year. If you know that the average interest rate is 3.20%, and you know that the first half of this average is 2.26%, it must be that the second half of the average must be a number around 4.2% in order to average out to 3.20%. And, indeed, we worked out that the forward one-year interest rate was 4.15%. It is not exact due to compounding, but it is close enough.

SIDE NOTE: In fact, the annualized rate is called the **geometric average**.

3·3.B. Speculating on Future Interest Rates

Table 3.2. The Mechanics of an Apple Short Sale

Three Parties: Apple Lender, You, The Apple Market.

Today:

1. You borrow 1 apple from the lender in exchange for your firm promise to the lender to return this 1 apple next year. (You also pay the lender an extra 1 cent lending fee.)

2. You sell 1 apple into the apple market at the currently prevailing apple price. Say, 1 apple costs \$5 today. You now have \$5 cash, which you can invest. Say, you buy bonds that earn you a 1% interest rate.

Next year:

1. You owe the lender 1 apple. Therefore, you must purchase 1 apple from the apple market.

   - If apples now cost \$6, you must purchase 1 apple from the market at \$6. You return the apple to the lender.

     Your net return on the apple is thus −\$1, plus the \$0.05 interest on \$5, minus the 1 cent fee to the lender. You therefore lost 96 cents.
   - If apples now cost \$4, you must purchase 1 apple from the market at \$4. You return the apple to the lender.

     Your net return on the apple is thus +\$1, plus the \$0.05 interest on \$5, minus the 1 cent fee to the lender. You therefore gained \$1.04.

Net Effects:

- The apple lender has really continued to own the apple throughout, and can sell the apple in Year 1. There is no advantage for the lender to keep the apple in his own apple cellar rather than to lend it to you. In addition, the lender earns 1 cent for free by lending.

- The apple market buyer purchased an apple from you today, and will never know where it came from (i.e., from a short sale).

- The apple market seller next year will never know the use of the apple (i.e., that you will use it to return the apple loan).

- You speculated that the price of an apple would decline. Note that you did earn the interest rate along the way. Except for the fee you paid to the lender, you could sell the apple into the apple market today, just like an apple grower could have.

In the real world, short-selling is arranged so that you cannot sell the apple short, receive the \$5, and then skip town. As a short-seller, you must assure the lender that you will be able to return the apple next year.

The reason why forward interest rates are especially interesting is because firms and traders can "lock in" future interest rates today. To understand this, assume that you can buy and sell Treasury bonds, even if they do not own them. In effect, you can borrow these securities, sell them, receive the cash, buy back the bonds later, and return them to the lender. This is called a **short sale** (the opposite—buying securities—is said to be a long position). Table 3.2 explains the basic idea behind shorting. In effect, for Treasury bonds, short selling enables you to do what the government does—"issue" a security, take in money, and return it to the lender with interest. For example, you may sell short \$89,803.^{25} of a 3-year Treasury bond today with a 3.65% rate of interest and a maturity of 3 years. This will give you \$89,803.^{25} cash today, but require you to come up with \$100,000 for repayment in 3 years. (In the real world, for professional bond traders, this is easily possible and with extremely small transaction costs, perhaps 1–2 basis points. Thus, assuming transaction costs away is a reasonable assumption.) In effect, selling a bond short is a way of borrowing money.

Frictionless borrowing and lending of Treasury bonds allow investors to lock in future interest rates. How shorting works.

Being long in a security speculates that the value will go up, so selling a financial instrument (i.e., being short) speculates that the value will go down. If the price of the bond tomorrow were to go down to \$50,000 (an annualized interest rate of 26%), the trader could then purchase the government T-bill for \$50,000 to cover the \$100,000 commitment he has made for \$89,803.^{25}, a profit of \$39,803.^{25}. But if the price of the bond tomorrow were to go to \$99,000 (an annualized interest rate of 0.33%), the trader would lose \$9,196.^{75}.

Shorting is the opposite of Buying: It speculates that the value will decline.

Now assume that you are able to buy a two-year bond at an annualized interest rate of 3.20%, and able to sell (short) a three-year bond at an annualized interest rate of 3.65%, and do so without transaction costs. For the three-year bond, you would have to promise to pay back \$100 · (1 + 11.35%) = \$111.^{35} in three years (cash outflow to you) for each \$100 you are borrowing today (cash inflow to you). For the two-year bond, you would invest these \$100 (cash outflow to you) and receive \$100 · (1 + 6.50%) = \$106.^{50} in two years (cash inflow to you).

Future cash flows from the long leg and the short leg.

Table 3.3. Locking in a Future Interest Rate via the Long-Short Forward Interest Rate Spread

Time	Purchased 2-Year Bond Cash Flows		Shorted 3-Year Bond Cash Flows		Net Cash Flow	
Today	–\$100.^{00}	(outflow)	+\$100.^{00}	(inflow)	\$0.^{00}	
Year 1	\$0.^{00}		\$0.^{00}		\$0.^{00}	
Year 2	+\$106.^{50}	(inflow)	\$0.^{00}		+\$106.^{50}	(inflow)
Year 3	\$0.^{00}		–\$111.^{35}	(outflow)	–\$111.^{35}	(outflow)

Looking at your **Payout Table** 3.3, from your perspective, the simultaneous transaction in the two bonds results in an inflow of \$106.^{50} in year two followed by a cash outflow

of \$111.^{35}. Effectively, you have committed to borrowing \$106.^{50} in year 2 with payback of \$111.^{35}. The interest rate for this loan is

$$\begin{aligned} r_{2,3} &= \frac{\$111.^{35} - \$106.^{50}}{\$106.^{50}} \approx 4.56\% \\ &= \frac{C_0 \cdot (1 + r_{0,3}) - C_0 \cdot (1 + r_{0,2})}{C_0 \cdot (1 + r_{0,2})} , \end{aligned} \tag{3.8}$$

which is exactly the forward interest rate in the table.

Side Note: You can instead start with the amount that you want to borrow/lend in a future period. For example, say you want to lend \$500 in year 2 and repay whatever is necessary in year 2. Lending \$500 in year 2 requires an outflow, which you can only accomplish with an inflow today. (Therefore, the first "leg" of your transaction is that you borrow, i.e., short the 2-year bond!) Specifically, your inflow today is $\$500/(1 + 3.20\%)^2 = \$469.^{47}$. Now, invest the entire \$469.^{47} into the 3-year bond, so that you have zero net cash flow today. (The second "leg" of your transaction is that you lend, i.e., purchase the 3-year bond.) This will earn you an inflow $\$469.^{47} \cdot (1 + 3.65\%)^3 = \$522.^{78}$ in 3 years. In total, your financial transactions have committed you to an outflow of \$500 in year 2 in exchange for an inflow of \$522.^{78} in year 3—otherwise known as 1-year lending in year 2 at a precommitted interest rate of 4.56%.

Such forward interest rate swaps are so popular that there are markets that make this even simpler.

You get what you pay for: the speculation can end up for better or worse.

This particular transaction is called a **forward transaction**. Indeed, this particular type of forward transaction is so popular that an entire financial market has developed that allows speculators to easily engage in going long or short on bonds.

Should you engage in this transaction? If the one-year interest rate in 2 years will be higher than 4.56%, you will be able to borrow at a lower interest than what will be prevailing then. Of course, if the interest rate will be lower than 4.56%, you will have committed to borrow at an interest rate that is higher than what you could have gotten.

Nerd Note: ***Futures contracts*** *make shorting bonds at government interest rates easy. Futures differ from* ***forward contracts*** *in that the contract value changes are settled daily. As a result, the net value (immediate payment) of a futures contract is usually zero. This requires explanation. For example, a buyer and seller may agree that it is a fair price for a seller to deliver 100 oranges at a price of \$50 in 2 years. No money needs to change hands today between the buyer and seller: it is merely an agreement between a futures purchaser and a futures seller to do the exchange in 2 years. Now assume that tomorrow, the fair price for oranges delivered in two years goes up to \$51. In this case, the contract to receive 100 oranges for \$50 is no longer worth zero. It has become a valuable asset, the buyer of yesterday's contract has made money. In a futures contract, the seller must immediately pay this value to the buyer, and the \$50 futures will change into a \$51 future. The orange futures contract again costs zero for anyone wishing to buy or sell one.*

Details on the futures markets can be found in the ***futures*** *section of major financial newspapers. There are futures on many different commodities, ranging from financial securities, to agricultural commodities, to natural resource commodities, and even electricity! The futures contract we are interested here is called the* ***Interest Rate futures****. For example, on May 31, 2002 , the equivalent interest rate for a 2-year Treasury note from a futures contract pegged the 2-year interest rate at 3.53 percent.*

Super-Nerd Note: Bond futures are quoted in equivalent bond prices, even though initiating a contract costs zero dollars. In this case, the futures quote was "\$104-25". The dash denotes 32th, i.e., $\$104 + 25/32 = \$104.^{78125}$. *The basic contract underlying the future was a 6% Coupon Bond (which must be looked up elsewhere), due June 2003. The formula to determine the implied interest rate is*

$$\$104.^{78125} = \frac{\$3}{(1+r)^{0.5}} + \frac{\$3}{(1+r)^{1.0}} + \frac{\$3}{(1+r)^{1.5}} + \frac{\$100 + \$3}{(1+r)^{2.0}} \tag{3.9}$$

which yields the solution of $r = 3.53\%$. *Yikes!*

ANECDOTE: The Volume of Interest Rate Trading
From a Chicago Board of Trade (CBOT) press release on August 19, 2002:

> Over the past quarter-century, the CBOT's Treasury bond and 10-year Treasury note futures contracts have grown because of their transparency, liquidity and integrity. Nearly 1.4 billion Treasury bond Futures have been traded with a value of over $139 trillion, compared with the current U.S. economy's gross national product at some $10 trillion annually, and with the 10-Year note Futures volume to date at some 385 million contracts with a face value of $38.5 trillion.
>
> CBOT options on Treasury bond Futures, launched in 1982, were the first such product in the derivatives industry, and have traded over 417 million contracts at a face value of more than $41.7 trillion. Options on CBOT 10-Year Treasury notes, since their launch in 1985, have traded some 110 million contracts with a value of approximately $11 trillion.

Treasury-Bond

Daily volume record:	1,121,634 contracts on 8-27-98
Open interest record:	1,138,994 contracts on 7-6-98
Options daily volume record:	540,873 contracts on 9-10-98
Options open interest record:	1,604,136 contracts on 10-9-98

10-year Note

Futures volume record:	730,487 contracts on 7-24-02
Options volume record:	333,754 contracts on 8-14-02
Futures open interest record:	1,020,418 contracts on 8-14-02
Options open interest record:	1,933,981 contracts on 8-15-02

(PS: Each contract is for a bond with a face value of $100,000.)

These are just four interest rate related contracts from one financial exchange. Because much trading in interest related contracts is over-the-counter, no one really knows how much trading or outstanding interest (i.e., the number of open contracts at a point in time) there is. But we do know that this financial market easily dwarves the U.S. Gross Domestic Product and U.S. stock markets together.

Solve Now!

Q 3.3 *Continuing the example, compute the one-year forward interest rate $r_{3,4}$ from year 3 to year 4, if the 4-year annualized interest rate was 4.06%.*

Q 3.4 *If you want to commit to saving at an interest rate of $r_{3,4}$, what would you have to do? (Assume any amount of investment you wish, and work from there.)*

Q 3.5 *If you want to commit to saving $500,000 in 3 years at an interest rate of $r_{3,4}$, what would you have to do?*

3·4. Are 2-year or 30-year Bonds a Better Deal? Interest Rate Changes and Bond Price Changes. The Risk of Treasury Bonds.

Higher interest rate bonds do not offer a better deal.

After all this math, do you think the 2-year bond or the 30-year bond is the better deal? The answer is neither. The market for Treasury bond investments is very competitive and efficient (Section 1·6). Thus, the old tried-and-true axiom holds: **you get what you pay for**. It is just a fact of life that investments for which the interest rate is tied down for 30 years offer higher interest rates now. (The money is not tied down, because you can sell the 30-year bond tomorrow!) Why not always purchase the bond with the highest interest rate, in this instance, the 30-year bond? Because bond prices (interest rates) can change!

A bond's value changes when the current interest rate changes, and in the opposite direction.

For example, if you purchased \$500,000 of 30-year zero bonds today at the 5.60% interest rate quoted in Figure 3.1, you would earn a holding rate of return of $(1+5.60\%)^{30}-1 \approx 412.76\%$. Thus, you would receive $\$500,000 \cdot (1+5.60\%)^{30} \approx \$2,563,820$ *in 30 years*. But what if the 30-year interest rate were to increase from 5.60% to 6.00% when the financial markets will be opening tomorrow morning? You could have instead received a total rate of return of

$$r_{0,30} \;=\; (1+r_{\overline{30}})^{30}-1 \;=\; (1+6.00\%)^{30}-1 \;=\; 5.7435-1 \;=\; 474.35\% \;, \tag{3.10}$$

rather than the "puny" 412.76% at which you invested your \$500,000 today. Now, you definitely purchased the commitment to receive \$2,563,820 in 30 years. If you are willing to invest \$446,386 today at an interest rate of 6% into 30-year bonds, you would own a commitment to receive

$$C_{30} \;=\; (1+r_{\overline{30}})^{30} \cdot C_0 \;=\; (1+6.00\%)^{30} \cdot \$446,386 \;=\; \$2,563,820 \;. \tag{3.11}$$

This is exactly the same \$2,563,820 commitment that you had purchased the day before for \$500,000. So, your commitment to receive \$2,563,820 is no longer worth \$500,000, but only worth \$446,386. In one day, your investment rate of return would have been

$$r_{\text{today,tomorrow}} = \frac{\$446,386-\$500,000}{\$500,000} \;=\; -10.72\% \;. \tag{3.12}$$

In general, when interest rates go up, the prices of outstanding bonds drop and the bondholders have lost money. Put differently, when bond prices rise, bond yields decline. This relationship is more pronounced the longer-term the bond is. For example, if the bond is due tomorrow, a different interest rate tomorrow can no longer harm us. This brings us to another important insight: Treasury bonds are risk-free in the sense that they cannot default (fail to return the promised payments). But they are risky in the sense that interest changes can change their value. Only the most short-term Treasury bills (say, due overnight) can truly be considered risk-free.

IMPORTANT:

- *Bond Price and Bond Yield are inversely related. When the bond price increases, the bond yield decreases, and vice-versa.*
- *"Fixed income" does not necessarily imply a "fixed rate of return" over horizons shorter than the maturity: day to day, long-term bonds are generally riskier investments than short-term bills.*

But, if you really need cash only in 30 years, is this not just a paper loss? This is a cardinal logical error many investors commit. By committing your money one day earlier, you would have lost $53,613 of your net worth in one day! Put differently, waiting one day would have saved you $53,613. Paper money is real money. Thinking paper losses are any different from real losses is a common but capital error.

"Only" a paper loss: A cardinal error!

IMPORTANT: *"Paper losses" are real losses.*

NERD NOTE: *Between today and tomorrow, you would also earn one day's interest. On a $500,000 investment, this would be about $75. If you had invested the money in 1-day Treasury bills instead of 30-year bonds, you would have only received about $15. Strictly speaking, this $60 should be added when comparing investment strategies—but it is only about 1 basis point, and so it is often just ignored.*

Solve Now!

Q 3.6 *Using information from a current newspaper or the WWW, what is today's annualized rate of return on a 10-year bond?*

Q 3.7 *Using information from a current newspaper or the WWW, what is today's total rate of return on a 10-year bond over the 10-year holding period?*

Q 3.8 *If you invest $500,000 at today's total rate of return on a 30-day Treasury note, what will you end up with?*

3·5. Summary

The chapter covered the following major points:

- Different interest rates apply to different horizon investments. The graph of interest rates as a function of horizon is called the "term structure of interest" or "yield curve."
- The information in the set of annualized rates of returns, individual holding rates of returns, and total holding rates of returns is identical. Therefore, you can translate them into one another. For example, you can extract forward interest rates from the prevailing yield curve.
- You can lock in forward interest rates today, if you can buy and sell bonds.
- Different horizon interest rates carry different risks, which ultimately justifies why investors desire certain bonds more than others, which in turn justifies different rates of returns for different horizon bonds.
- The yield curve is usually upward sloping, but can be downward sloping (inverted), humped, or flat.

A. Appendix: Locating Current Interest Rates

The website for this book keeps some reasonably up-to-date links to bond price information on the web. As of this writing, historical yield curves can be found at SmartMoney.com, among other sources. The current yield curve, updated daily, can be found at PiperJaffray.com, among other sites

Refer to the book website for up-to-date weblinks on data sources.

Table 3.4. Locations of Interest Rates in the Wall Street Journal

Page		Content
C1		Bonds and Interest
C2		Yield Curve
C10		Individual Treasury Bonds, Notes and Bills
C12	(Top)	Credit Markets, top, Table of Yield Comparisons
C12	(Bottom)	Boxed "Money Rates"

The most common source for interest information is the *Wall Street Journal*. Unfortunately, this interest information is spread around many different locations in the C section. Although the order and content are the same from issue to issue, the exact page numbering need not be. For example, on May 20, 2002 , interest rates from Table 3.4 could be found in several different pages.

An easy source of information about bond rates is the *Wall Street Journal*.

B. Nerd Appendix: Continuous Compounding

Continuously compounded interest rates are "as if interest is paid every instant."

A subject of some interest to Wall Street traders, i.e., the people who trade bonds or options for a living, is the concept of a **continuously compounded interest rate**. This is easiest to explain by example.

Progressively more frequently paid interest payments converge to the continuously compounded interest rate.

Assume that you receive \$120 next year for an investment of \$100 today. We already know that this represents a simple rate of return of 20%. What would the interest be if it were paid twice per year, not once per year, the interest rate remained constant, and the \$100 would still come out to be \$120 at the end of the year. We have done this before:

$$(1 + r_{\text{semi-annual}}) \cdot (1 + r_{\text{semi-annual}}) = (1 + 20\%) \qquad \Longrightarrow \qquad r = 9.54\% \ . \tag{3.13}$$

Multiply this semiannual interest rate by two: 19.08%. What if you received interest twelve times a year?

$$(1 + r_{\text{monthly}})^{12} = (1 + 20\%) \qquad \Longrightarrow \qquad r = 1.53\% \ . \tag{3.14}$$

Multiply this monthly (m) interest rate by 12: 18.36%. What if you received interest 365 times a year?

$$(1 + r_{\text{daily}})^{365} = (1 + 20\%) \qquad \Longrightarrow \qquad r = 0.05\% \ . \tag{3.15}$$

Multiply this daily (d) interest rate by 365: 18.25%. Now, what would this number be if you were to receive interest every single moment in time?

The limit: Use logs and exponents to translate simple interest rates to continuously compounded interest rates.

The answer is, you guessed it, the continuously compounded interest rate and it can be computed by taking the **natural logarithm** (abbreviated ln on your calculator and below) of one plus the simple interest rate

$$\begin{aligned} r_{\text{continuously compounded}} &= \ln(1 + 20\%) = 18.23\% \\ r_{\text{continuously compounded}} &= \ln(1 + r_{\text{simple}}) \ . \end{aligned} \tag{3.16}$$

(Appendix 2·3 reviews powers, exponents and logarithms.)

Warning: Never, ever apply cc rates of return to a cash flow!

You must *never* use a continuously compounded interest rate and apply it to a cash flow to compute your return. In our example, investing \$100 would not leave you with \$118.[23] after one year. Indeed, if someone quoted you a continuously compounded interest rate, to determine how much money you will end up with, you would first have to convert the continuously compounded return into a simple interest rate

$$r_{\text{simple}} = e^{r_{\text{continuously compounded}}} - 1 = e^{18.23\%} - 1 = 20\% \ , \tag{3.17}$$

and then apply this interest rate to the cash flow.

To obtain multi-period interest returns, Continuously Compounded Interest Rates are never compounded, but added instead.

Continuously compounded interest rates have one nice feature: If the continuously compounded rate in period 1 is 10% and in period 2 is 20%, the total two-period continuously compounded rate is 30%—yes, continuously compounded interest rates are additive. No more multiplying one-pluses! This additivity is not a big advantage, and if continuously compounded interest returns were not used in other formulas (such as the Black-Scholes option formula, the subject of the Web Chapter on Options and Derivatives), this book would not have bothered describing them.

A. Q&A: Answers

1. Do it!
2. Do it!

3. The 4-year holding rate of return is $r_{0,4} = (1 + 4.06\%)^4 = 17.71\%$. Therefore, the 1-year forward rate from year 3 to year 4 is $r_{3,4} = \frac{(1 + r_{0,4})}{(1 + r_{0,3})} - 1 = \frac{(1 + 17.71\%)}{(1 + 11.35\%)} - 1 = 5.71\%$.
4. Buy \$1,000 of a 4-year zero bond and short \$1,000 of a 3-year zero bond. Today, you receive and return \$1,000, so it does not cost you anything. In 3-years, you need to pay the 3-year bond, i.e., you need to pay in \$1,113.^{50}. In 4-years, you receive from the 4-year bond \$1,177.^{10}. This is the equivalent of saving at an interest rate of 5.7%.
5. The previous example was saving \$1,113.^{50}. So, we have to this transaction at a scale that is $\$500,000/\$1113.5 \approx 449.03$ times as much. Therefore, instead of buying \$1,000 of the 4-year bond, we must buy $449.035 \cdot \$1,000 \approx \$449,035$ of the 4-year bond, and short the same amount of the 3-year bond.

6. Do it!
7. Take the above number, call it $r_{\overline{10}}$, and compute $(1 + r_{\overline{10}})^{10}$.
8. Do it.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 4

Frictions in Rates of Returns

Transaction Costs, Inflation, Taxes, Default

last file change: Jun 14, 2004 (17:54h)

last major edit: March 2004

Transaction costs, inflation, taxes, and default—these four factors eat away at the rates of return that investors earn. This chapter explains how lenders should compensate for these "frictions" in how they charge borrowers so that they can earn a fair rate of return for the use of their money.

As you will see, this chapter also makes a strong distinction between *expected* and *promised* rates of return. Although the distinction is of first-order importance, the two are often confused in practice.

Anecdote: The Ruin of the First Financial System
The earliest known example of widespread financial default occurred in the year of 1788 B.C.E., when King Rim-Sin of Uruk (Mesopotamia) repealed all loan repayments. The royal edict effectively destroyed a system of flourishing commerce and finance, which was already many thousands of years old! It is not known why he did so.

4·1. Transaction Costs

Commissions need to be taken out of meaningful rates of return.

We have already talked about transaction costs in Section 1·5, but a brief recap is in order here. As an investor, you usually care about rates of return *after* all transaction costs have been taken into account, not about quoted rates of returns from quoted prices. In most large purchases, you have to pay additional fees, such as brokerage commissions. For example, if you purchase a house for \$1,000,000 and you sell it to the next buyer at \$1,100,000 through a broker, your rate of return is not 10%. At selling time, the brokers charge you a 6% commission. There are also other costs that reduce the amount of money you receive, not to mention the many opportunity costs. Say these costs amount to \$70,000 in total. In addition, even when you purchased the house, you most likely had to pay some extra costs (such as an escrow transfer fee) above and beyond the \$1,000,000—say, \$5,000. We can illustrate how you would take these transaction costs on both sides (buy and sell) into account. Your rate of return is not 10%, but only

$$r = \frac{(\$1,100,000 - \$70,000) - (\$1,000,000 + \$5,000)}{(\$1,000,000 + \$5,000)} \approx 2.5\% \ .$$

$$\text{Rate of Return} = \frac{\begin{array}{c}\text{Dollars Returned,}\\ \text{after Transaction costs}\end{array} - \begin{array}{c}\text{Dollars Invested,}\\ \text{after Transaction costs}\end{array}}{\text{Dollars Invested, after Transaction costs}} \ . \tag{4.1}$$

Note how the \$5,000 must be added to, not subtracted from the price you paid. The price you paid was ultimately higher than \$1,000,000. The \$5,000 works against you.

Side Note: In order to make their returns look more appealing, many professional fund managers quote their investors' rates of return before taking their own fees (transaction costs) into account. Usually, a footnote at the bottom is there to satisfy the lawyers that investors can not sue for being misled.

4·2. Inflation and Real Interest Rates

Inflation is the process by which goods cost more in the future than they cost today—in which the price level is rising and money is losing its value. What effect does inflation have on returns? To answer this question, you need to know the difference between a **nominal return** and a **real return**.

Nominal Rates of Return are less than Real Rates of Return due to Inflation.

The nominal rate is what you are used to—the quoted rate that has not been adjusted for inflation. In contrast, the real rate "subtracts out" inflation to come up with a return "as if" there had been no inflation. The real rate of return reflects the fact that, in the presence of inflation, a dollar in the future will have less **purchasing power** than a dollar today.

Side Note: The *Wall Street Journal* prints one measure of the inflation rate—percent changes in the so-called **Consumer Price Index** (or **CPI**), at the end of its column *Money Rates*. On May 31, 2002, the Consumer Price Index was increasing at a rate of 1.6%.

Anecdote: **The German Hyperinflation of 1922**
The most famous episode of **hyperinflation** occurred in Germany from August 1922 to November 1923. Prices more than quadrupled every month. The price for goods was higher in the evening than in the morning! By the time Germany printed 1,000 billion Mark Bank Notes, no one trusted the currency anymore. This hyperinflation was stopped only by a drastic currency and financial system reform. But high inflation is not just a historic artifact. For example, many Latin American countries experienced annual doubling of prices in the early 1980s.

The opposite of inflation is **deflation** (negative inflation)—a process in which the price level falls. Though much rarer, it happens. In fact, in November 2002, Business Week reported that an ongoing recession and low demand continue to force an ongoing decline in Japanese prices.

Many economists now believe that a modest inflation rate between 1% and 3% per year is a healthy number.

Consider a simple example: assume that the inflation rate is 100% per year, and you can buy a bond that promises a *nominal* interest rate of 700%. What is your *real* rate of return? To find out assume that \$1 buys one apple today. With an inflation rate of 100%, you need \$2 next year to buy the same apple. Your investment return will be $\$1 \cdot (1 + 700\%) = \$8$ for today's \$1 of investment. But this \$8 now applies to apples costing \$2 each. So, your \$8 will buy 4 apples, and not 8 apples. Your real rate of return is

An Extreme 100% Inflation Rate Example: Prices Double Every Year.

$$r_{\text{real}} = \frac{4 \text{ Apples} - 1 \text{ Apples}}{1 \text{ Apples}} = 300\% \quad . \tag{4.2}$$

For each dollar invested today, you will be able to purchase only 300 percent more apples next year (not 700% more apples) than you could purchase today. This is because the purchasing power of your dollar next year will be reduced by half.

Anecdote: **Inflation Measures**
Are apples really the best measure of inflation? What if apples (the fruit) become more expensive, but Apples (the computers) become less expensive? Defining inflation itself is somewhat tricky. To solve this problem, economists have invented a *basket* or *bundle* of goods that is (trying to be) representative of what the average household consumes. Each month, economists at the **Bureau of Labor Statistics** (**BLS**) determine the average total price of this bundle, the aforementioned **Consumer Price Index**. It is generally accepted as one good generic measure of inflation—but so are other indexes, like the **Producer Price Index** (**PPI**) and the **GDP Deflator**. There are also more specialized bundles, such as computer inflation indexes (the price of equivalent computer power does not inflate, but deflate, so the rate is usually negative), or indexes for prices of goods purchased in a particular region.

SIDE NOTE: The official inflation rate can be as important as the underlying inflation rate that it tries to measure. The government must pay out more to social security recipients and purchasers of **CPI Bond**s (formally called **TIPS** for **Treasury Inflation Protected Securities**) when the inflation rate is higher. The lower the official inflation rate, the less the government has to pay.

The Conversion Formula from Nominal to Real Rates.

The correct inflation adjustment formula is a "one-plus" formula. In our example,

$$\begin{aligned} (1+200\%) &= (1+100\%) \cdot (1+50\%) \\ (1+r_{\text{nominal}}) &= (1+\text{Inflation Rate}) \cdot (1+r_{\text{real}}) \ . \end{aligned} \tag{4.3}$$

Turning this formula gives real rates of returns,

$$\begin{aligned} (1+r_{\text{real}}) &= \frac{1+200\%}{1+100\%} = 1+50\% \\ (1+r_{\text{real}}) &= \frac{(1+r_{\text{nominal}})}{(1+\text{Inflation Rate})} \ . \end{aligned} \tag{4.4}$$

IMPORTANT: *The relation between nominal rates of returns ($r_{nominal}$), real rates of returns (r_{real}), and inflation (π) is*

$$(1+r_{nominal}) = (1+r_{real}) \cdot (1+\pi) \ . \tag{4.5}$$

For small rates, adding/subtracting is ok.

As with compounding, if both inflation and the nominal interest are small, the mistake of just subtracting the inflation rate from the nominal interest rate to obtain a real interest rate is not too grave. For example, if the inflation rate is 2% and the nominal interest rate is 5%, the real interest rate is $(1+5\%)/(1+2\%) = 2.94\%$. This is close enough to $5\% - 2\% = 3\%$ not to matter much.

ANECDOTE: **Inflation-Adjusting Bonds**
What if you wanted to purchase a bond that is truly risk-free, i.e., a bond that promises a specified amount of purchasing power (a real amount, not a nominal amount)? If inflation (a **random variable**, because we do not yet know what inflation will be over the bond's holding period) were to turn out 1% higher, such a bond should pay 1% more in interest.

As it turns out, such bonds have been invented and not just recently! The world's first known inflation-indexed bonds were issued by the Commonwealth of Massachusetts in 1780 during the Revolutionary War. These bonds were invented to deal with severe wartime inflation and with angry discontent among soldiers in the U.S. Army with the decline in purchasing power of their pay. Although the bonds were successful, the concept of indexed bonds was abandoned after the immediate extreme inflationary environment passed, and largely forgotten. In 1780, the bonds were viewed as at best only an irregular expedient, since there was no formulated economic theory to justify indexation.

In 1997, the U.S. Treasury reintroduced inflation-ajdusted bonds. They are now called Treasury Inflation Protected Securities (or **TIPS**, or sometimes just **CPI Bonds**). On May 31, 2002, the *Wall Street Journal* reported on Page C10 that a 30-year CPI bond offered a real yield of about 3.375%/year. The current inflation rate was only 1.6%/year, and a normal 30-year Treasury bond offered a nominal yield of 5.6%/year. If inflation were to remain at 1.6%, the plain Treasury bond would offer a higher real rate of return because $(1+5.6\%)/(1+1.6\%) - 1 \approx 3.9\%$. But if inflation were to rise in the future, the TIPS could end up offering the higher rates of return.

Source: Robert Shiller.

IMPORTANT: *The common statement "in today's dollars" is ambiguous. Some people mean "inflation adjusted." Other people mean "compared to an investment in risk-free bonds." Ask!*

Solve Now!

Q 4.1 *From memory, write down the relationship between nominal rates of return ($r_{nominal}$), real rates of return (r_r), and the inflation rate (π).*

Q 4.2 *Using information from a current newspaper or the WWW, what is the current inflation rate?*

Q 4.3 *Using information from a current newspaper or the WWW, what is the annualized current 30-day nominal interest rate?*

Q 4.4 *Using the information from the previous two questions, what is the annualized current real interest rate?*

4·3. Taxable And Tax-Exempt Bonds

Transaction costs and inflation are not the only problems. More importantly, Uncle Sam also wants a big cut of your money. Ignoring taxes may be a good assumption for the tax-exempt Red Cross, but probably not for you. After all, in the end, all you probably care about are your after-tax returns, not your pre-tax returns.

Taxable Investors (unlike tax-exempt investors) care about post-tax inflows and outflows.

SIDE NOTE: Personal retirement accounts, such as a **401-K** or an **Individual Retirement Account** (**IRA**), are tax-advantaged, but not tax-exempt. The government exempts some contributions and returns in the 401-K from income taxes, but it does collect income taxes when the money is withdrawn.

You can purchase bonds that are legally tax-exempt. That is, you need not declare the interest on either your federal income tax statement or your state income tax statement or both. (The arrangement differs from bond to bond.) Such bonds are called **municipal bonds** or **muni bonds** or even **munis** for short. As their name suggests, they are usually issued by municipalities such as the City of Los Angeles or the City of Canton (OH).

Municipal bonds' interest payments are legally exempt from income taxes.

ANECDOTE: **Municipal Bond Tax Exemptions**
From the Morgan-Stanley website: "Since the development of the federal tax system authorized by the Sixteenth Amendment to the Constitution in 1913, the federal government has chosen not to directly tax the income from such bonds. Within the American legal and political tradition, it has been thought that taxing the interest on municipal securities would be a burden to state governments and interfere with the state's ability to borrow money. The **Tax Reform Act of 1986** continues the basic exemption from direct taxation for interest on municipal securities set forth in the Internal Revenue Code."

The May 2002 Situation.

On May 31, 2002, the *Wall Street Journal* reported on Page C12 that tax-exempt municipal 7-12 year highly rated bonds (AA) offered an annualized interest rate of 5.24%. Bonds of similar risk issued by corporations offered an interest rate of about 6.76%. Which one would be a better investment *for you*? Well, it depends.

Comparing After-Tax Returns of Tax-Exempt and Taxable Bonds.

If you invested \$1,000 into munis at a 5.24% interest rate, you would receive \$52.40 at year's end. You would get to keep all of it, because these bonds are tax-exempt. If you invested \$1,000 in taxable bonds at a 6.76% interest rate, you would receive \$67.60 at year's end. If your income tax rate is 0%, you would clearly prefer the \$67.60 to the \$52.40. However, if your marginal income tax rate is 30%, Uncle Sam would collect \$20.28 and leave you with \$47.32. The tax rate is often abbreviated with the greek letter τ (tau), so your after-tax rate of return is

$$\begin{aligned} r_{\text{post-tax}} &= (1-30\%)\cdot 6.76\% = 70\%\cdot 6.76\% = 4.73\% \\ r_{\text{post-tax}} &= (1-\tau)\cdot r_{\text{pre-tax}} \end{aligned} \tag{4.6}$$

to compute the after-tax interest rate. With a 30% tax rate, you would prefer the tax-exempt bond that pays \$52.40.

High-income tax bracket individuals should prefer tax-exempt bonds; low-income tax bracket individuals should prefer taxable bonds.

Economists sometimes like to talk about a hypothetical marginal investor. This is an investor whose marginal income tax rate is such that she would be exactly indifferent between buying the tax-exempt and the taxable bond. Using Formula 4.6, the marginal investor has a tax rate of

$$r_{\text{post-tax}} = (1-\tau_{\text{marginal}})\cdot r_{\text{pre-tax}} \quad \Leftrightarrow \quad \tau_{\text{marginal}} = 1 - \frac{r_{\text{post-tax}}}{r_{\text{pre-tax}}} \tag{4.7}$$

Any investor with a marginal income tax rate above this marginal income tax rate should prefer the tax-exempt bond. Any investor with a marginal income tax rate below this income tax rate should prefer the taxable bond.

SIDE NOTE: On May 31, 2002, the **Bond Market Data Bank** Section in the *Wall Street Journal* (Page C15) computes the ratio between the AAA municipal and Treasury securities to be around 74.6% (for short-term municipals, unlikely to default). Thus, an investor with a tax rate above 25.4% should prefer the tax-exempt municipals to the taxable Treasury. Remarkably, 30-year long-term AAA municipals paid 92% of the interest rate that comparable maturity Treasuries were paying. This is very high by historical standards, which means that the marginal investor's income tax rate of 8% is quite low.

Munis do have default (credit) risk. See next section.

Unfortunately, unlike the U.S. Treasury, municipalities can and have gone bankrupt, so that they may not fully repay. Municipal bonds are not an entirely risk-free investment. Default risk (also often called credit risk) is the subject of our next chapter.

ANECDOTE: The Orange County Bankruptcy
On December 6, 1994, Orange County became the largest municipality in U.S. history to declare bankruptcy. The county treasurer had lost \$1.7 billion in risky Wall Street investments. (He had earlier won many times this loss.) As a result, Orange County laid off 1,040 workers, and cancelled planned hiring in 563 jobs—about 10 percent of its work force. It further cancelled a number of transportation projects. After 18 months, Orange County reemerged from bankruptcy. Eventually, they also recouped \$865 million of their \$1.64 billion loss through legal proceedings. Bondholders were left unscathed, though. They received the promised payoffs.

However, bondholders do not always fare so well. In 1975, New York City defaulted and its bondholders received considerably less than originally promised. In 1983, the Washington Public Power Supply System (nicknamed **WHOOPS** by Wall Street) defaulted on \$2.25 billion of its **revenue bonds** on two of its nuclear power plants. (Unlike **general obligation bonds**, **revenue bonds** are backed only by specific assets.) Here, too, bondholders received considerably less than they had been originally promised.

4·3.A. Both Taxes and Inflation

Taxes and Inflation: Interactions?

How do inflation and income taxes interact? Let us use our simple method of "numerical example" to determine this. Consider an investment of \$100. If you earn a 10% rate of return on a \$100 investment and Uncle Sam takes $\tau = 40\%$ (or \$4 on your \$10), you get \$110 before taxes but end up with only \$106:

$$\begin{aligned} \$100 \cdot [1 + 10\% \cdot (1 - 40\%)] &= \$106 \\ C_0 \cdot [1 + r_{\text{nominal,pre-tax}} \cdot (1 - \tau)] &= C_1 \quad . \end{aligned} \tag{4.8}$$

If the inflation rate is $\pi = 5\%$ per annum, \$106 is worth in purchasing power

$$\begin{aligned} \frac{\$106}{1 + 5\%} &= \$100.^{95} \\ \frac{C_1}{1 + \pi} &= V_0 \quad . \end{aligned} \tag{4.9}$$

So, your post-tax post-inflation real rate of return is $\$100.^{95}/\$100 - 1 \approx 0.95\%$. In formula, we computed

$$\begin{aligned} r_{\text{post-tax, real}} &= \frac{\$100.^{95} - \$100}{\$100} = \frac{\frac{\$100 \cdot [1+10\% \cdot (1-40\%)]}{1+5\%} - \$100}{\$100} \\ &= \frac{10\% \cdot (1 - 40\%) - 5\%}{1 + 5\%} = 0.95\% \\ r_{\text{post-tax, real}} &= \frac{V_0 - C_0}{C_0} = \frac{\frac{C_0 \cdot [1+r_{\text{nominal,pre-tax}} \cdot (1-\tau)]}{1+\pi} - C_0}{C_0} \\ &= \frac{r_{\text{nominal,pre-tax}} \cdot (1 - \tau) - \pi}{1 + \pi} \quad . \end{aligned} \tag{4.10}$$

This is not a formula that anyone remembers, but it is both useful and a nice illustration of how you should approach and simplify complex questions—numerical example first, formula second.

If the real rate stays constant, does inflation hurt an investor? Yes, if there are taxes!

Here is an interesting question: if the real rate remains constant, does it help or hurt an investor if inflation goes up? Let's assume that the real rate of return is 20%. If inflation is 50%, then the nominal rate of return is 80% (because $(1 + 50\%) \cdot (1 + 20\%) = 1 + 80\%$): you get \$180 for a \$100 investment. Now add income taxes to the tune of 40%. The IRS sees \$80 in interest, taxes \$32, and leaves you with \$48. Your \$148 will thus be worth $\$148/(1 + 50\%) = \$98.^{67}$ in real value. Instead of a 20% increase in real purchasing power when you save money, you now suffer a $\$98.^{67}/\$100 - 1 \approx 1.3\%$ decrease in real purchasing power. Uncle Sam ends up ahead, and you end up with less purchasing power than you started with. The reason is that although Uncle Sam claims to tax only interest gains, because the interest tax is on *nominal* interest payments, you can actually lose in *real* terms. Instead, if inflation were 0% and the real rate of return were still 20%, you would have been promised \$20, Uncle Sam would have taxed you \$8, and you could have kept \$112 in real value.

IMPORTANT: *Higher inflation hurts taxable investors who earn interest income, even if real rates seem to remain constant.*

For much of the post-war U.S. history, real rates of returns on short-term government bonds have indeed been *negative* for taxed investors.

Solve Now!

Q 4.5 *If your tax rate is 20%, what interest rate do you earn in after-tax terms if the pre-tax interest rate is 6%?*

Q 4.6 *If your tax rate is 40%, what interest rate do you earn in after-tax terms if the pre-tax interest rate is 6%?*

Q 4.7 *If the private sector is a net saver, e.g., leaving the public sector as a net borrower, does Uncle Sam have an incentive to reduce or increase inflation?*

4·4. Credit Risk (Default Risk)

4·4.A. Risk-Neutral Investors Demand Higher Promised Rates

If you quote a risky borrower a particular interest rate, you must expect to earn a lower interest rate.

Unfortunately, most loans are not risk-free, because the borrower may not fully pay back what was promised. Put yourself into the position of a banker. A one-year Treasury bond offers a safe annual rate of return of 10%. You are about to lend \$1 million to a risky company. The loan maturity is one year. You must decide what interest rate to charge on the loan. If you are 100% certain that the lender will fully pay the agreed-upon amount, you can earn as much charging a 10% interest rate from the borrower as you can from buying the Treasury bond. Both give \$1,100,000 in repayment. But, there are few borrowers for whom you can be 100% certain that they will fully repay a loan. For example, assume you think there is a 50% chance that the company will pay back only \$750,000 (all it has if it goes bankrupt; this is called **default**), and a 50% chance that the company will pay back the principal plus interest (the company is called **solvent**). Now if you charge a 10% interest rate, your expected payout would be

$$\begin{array}{ccccccccc} 50\% & \cdot & \$750,000 & + & 50\% & \cdot & \$1,100,000 & = & \$925,000\ . \\ \mathcal{P}rob(\text{Default}) & \cdot & \text{Payment if Default} & + & \mathcal{P}rob(\text{Solvent}) & \cdot & \text{Payment if Solvent} & & \end{array} \tag{4.11}$$

So, your *expected* return would not be \$1,100,000, but only \$925,000. Your *expected* rate of return would not be +10%, but only $\$925,000/\$1,000,000 - 1 = -7.5\%$. Extending such a loan would not be in your interest: you can do better by putting your \$1,000,000 into government Treasury bills.

You must ask for a higher promised interest—received only in good times—in order to make up for default risk.

So, as a banker, you must demand a higher interest rate from risky borrowers, even if you just want to "break even" (i.e., earn the same \$1,100,000 that you could earn in Treasury bonds). If you solve

$$\begin{array}{ccccccc} 50\% & \cdot & \$750,000 & + & 50\% & \cdot & (\text{promised repayment}) & = & \$1,100,000 \\ \mathcal{P}rob & \cdot & \text{Payment if Default} & + & \mathcal{P}rob & \cdot & \text{Payment if Solvent} & = & \text{Treasury Payment} \end{array} \tag{4.12}$$

for the desired promised repayment, you find that you must ask the borrower for \$1,450,000. The promised interest rate is therefore $\$1,450,000/\$1,000,000 - 1 = 45\%$. Of this 45%, 10% is the time premium. Therefore, we can call the remaining 35% the **default premium**.

Common terminology

We rarely directly observe expected rates of return. Newspaper and financial documents almost always provide only the **promised interest rate**, which is therefore also called the **quoted interest rate** or the **stated interest rate**. On Wall Street, the **default risk** is often called the **credit premium**; and **default risk** is often called **credit risk**.

Side Note: If you are not risk-neutral but risk-averse, at a 45% promised interest rate, you would still prefer lending your \$1,000,000 to the government instead. You might only wish to extend the loan if you can receive more than a promised 45% rate of return, say 50%. In this case, 10% would be the time premium, 35% would be the default premium, and 5% would be the **risk premium**. If you extended such a loan, you would expect to receive

$$50\% \cdot \$750,000 + 50\% \cdot \$1,500,000 = \$1,125,000\ . \tag{4.13}$$

which is 2.5% higher than the 10% you could earn from Treasury securities.

4·4.B. A More Elaborate Example: Breakeven Interest Rates

Borrowers may sometimes not be able to repay.

Now assume that you want to lend money to your Cousin Vinny. Vinny is often not able to pay you back, despite his best intentions and promises. You believe Vinny will pay you with 98% probability what he promises, with 1% probability half of what he borrowed, and with 1% probability nothing. Vinny wants to borrow \$200 from you, and you could alternatively invest the \$200 into a government bond promising a 5% interest rate, so you would receive \$210 for certain. What interest rate would you ask of Vinny?

Table 4.1. Vinny's Payoff Table: 5% Promised Interest Rate

Receive Cash Flow C_1	How Likely (Probability)
\$210	98% of the time
\$100	1% of the time
\$0	1% of the time

If you ask Vinny to pay the risk-free interest rate, you will on average earn less than the risk-free interest rate.

Table 4.1 shows that if you ask for a 5% interest rate, your expected cash return will be

$$\begin{aligned}
\mathcal{E}(\tilde{C}_{t=1}) &= 98\% \cdot \$210 \\
&+ 1\% \cdot \$100 \\
&+ 1\% \cdot \$0 = \$206.^{80} \\
\mathcal{E}(\tilde{C}_{t=1}) &= \mathcal{P}rob\ C_{t=1} \text{ will be case 1} \cdot C_{t=1} \text{ cash flow in case 1} \\
&+ \mathcal{P}rob\ C_{t=1} \text{ will be case 2} \cdot C_{t=1} \text{ cash flow in case 2} \\
&+ \mathcal{P}rob\ C_{t=1} \text{ will be case 3} \cdot C_{t=1} \text{ cash flow in case 3} \quad .
\end{aligned} \tag{4.14}$$

This expected return of \$206.^{80} is less than the \$210 that the government promises. Put differently, with Vinny's *promised* rate of return of 5%,

$$\begin{aligned}
\text{Promised}(\tilde{r}_{0,1}) &= \frac{\$210 - \$200}{\$200} = 5.00\% \\
\text{Promised}(\tilde{r}_{0,1}) &= \frac{\text{Promised}(\tilde{C}_1) - C_0}{C_0} \quad ,
\end{aligned} \tag{4.15}$$

your expected rate of return is only

$$\begin{aligned}
\mathcal{E}(\tilde{r}_{0,1}) &= \frac{\$206.^{80} - \$200}{\$200} = 3.40\% \\
\mathcal{E}(\tilde{r}_{0,1}) &= \frac{\mathcal{E}(\tilde{C}_1) - C_0}{C_0} \quad ,
\end{aligned} \tag{4.16}$$

which is less than the 5% interest rate that Uncle Sam promises—and surely delivers.

We need to determine how much Vinny has to promise you to "break even," so that you expect to have the same \$210 that you could receive from Uncle Sam. In Table 4.1, we computed our expected payoff as the probability-weighted average payoff. We want this payoff to be not \$206.^{80}, but what we can get if we put our money in government bonds: \$210.^{00}. So, we now solve for an amount x that we want to receive if Vinny has money,

Determine how much more interest promise you need to break even.

$$\begin{aligned}
\mathcal{E}(\tilde{C}_{t=1}) &= 98\% \cdot x \\
&+ 1\% \cdot \$100 \\
&+ 1\% \cdot \$0 = \$210.^{00} \\
\mathcal{E}(\tilde{C}_{t=1}) &= \mathcal{P}rob\ C_{t=1} \text{ will be case 1} \cdot C_{t=1} \text{ cash flow in case 1} \\
&+ \mathcal{P}rob\ C_{t=1} \text{ will be case 2} \cdot C_{t=1} \text{ cash flow in case 2} \\
&+ \mathcal{P}rob\ C_{t=1} \text{ will be case 2} \cdot C_{t=1} \text{ cash flow in case 3} \quad .
\end{aligned} \tag{4.17}$$

The solution is that if Vinny promises you $x = \$213.^{27}$, you will expect to earn the same 5% interest rate that you can earn in Treasury bonds. Table 4.2 confirms that a promise of \$213.^{27} for a cash investment of \$200, which is a *promised* interest rate of

$$\begin{aligned}
\text{Promised}(\tilde{r}_{0,1}) &= \frac{\$213.^{27} - \$200}{\$200} = 6.63\% \\
\text{Promised}(\tilde{r}_{0,1}) &= \frac{\text{Promised}(\tilde{C}_1) - C_0}{C_0} \quad ,
\end{aligned} \tag{4.18}$$

provides an *expected* interest rate of

$$\mathcal{E}(\tilde{r}_{0,1}) = 98\% \cdot (6.63\%) + 1\% \cdot (-50\%) + 1\% \cdot (-100\%) = 5\% \quad . \tag{4.19}$$

Table 4.2. Vinny's Payoff Table: 6.63% Promised Interest Rate

Receive Cash Flow C_1	which is a return of	How Likely (Probability)
\$213.^{27}	+6.63%	98% of the time
\$100.^{00}	-50.00%	1% of the time
\$0.^{00}	-100.00%	1% of the time
\$200.^{00}	+5.00%	in expectation

The difference of 1.63% between the promised (or quoted) interest rate of 6.63% and the expected interest rate of 5% is the **default premium**, because it is the extra interest rate that is caused by the default risk. Of course, you only receive this 6.63% *if* everything goes perfectly. In our perfect world with risk-neutral investors,

The difference between the promised and expected interest rate is the default premium.

$$\begin{aligned}
\text{Promised Interest Rate} &= \text{Time Premium} + \text{Default Premium} \\
6.63\% &= 5\% + 1.63\% \quad .
\end{aligned} \tag{4.20}$$

IMPORTANT: *The promised (or quoted) interest rate is higher than the expected interest rate of return, except for government bonds where the two are equal. Never confuse the two! Financial securities and financial information providers usually provide only promised rates of return, not expected rates of return.*

In a risk-neutral world, all securities have the same exp. rate of return.

On average, the expected rate of return is the expected time premium plus the expected default premium. Because the *expected* default premium is zero *on average*,

$$\mathcal{E}(\text{Rate of Return}) = \mathcal{E}(\text{Time Premium}) \quad . \tag{4.21}$$

NERD NOTE: *You can compute the expected default premium as follows: you will receive (6.63% − 5% = 1.63%) in 98% of all cases; −50% − 5% = −55% in 1% of all cases (note that you lose the time-premium); and −100% − 5% = −105% in the remaining 1% of all cases (i.e., you lose not only all your money, but also the time-premium). Therefore,*

$$\mathcal{E}(\textit{Default Premium}) = 98\% \cdot (+1.63\%) + 1\% \cdot (-55\%) + 1\% \cdot (-105\%) = 0\% \quad . \tag{4.22}$$

4·4.C. Short Preview: Risk-Averse Investors Demand Higher Expected Rates

Part III will explain how much more expected rate of return riskier projects have to offer when investors are risk-averse.

In addition to the default premium, in real life, investors also demand a risk premium.

So far, we have assumed that investors are indifferent between two loans that have the same expected rate of return. Investors are what is called **risk-neutral**. But in the real world, investors would demand and expect to receive a little bit more for the risky loan. They are what is called **risk-averse**. Take your own case. Would you rather invest into a bond that is known to pay off 5% (for example, a U.S. government bond), or would you rather invest in a bond that is merely expected to pay off 5% (such as Vinny's 6.63% bond)? Like most lenders, you are likely to be better off if you know exactly how much you will receive, rather than live with the uncertainty of Vinny's situation. Thus, as a risk-averse investor, you would probably ask Vinny not only for the higher *promised* interest rate of 6.63%, which only gets you to an expected interest rate of 5%, but an even higher promise in order to get you more than 6.63%. For example, you might demand 6.75%, in which case you would expect to earn not just 5%, but a little more. The extra 12 basis points is called a **risk premium**, and it is an interest component required *above and beyond* the time premium (i.e., what the U.S. Treasury Department pays for use of money over time) plus the default premium (i.e., what the promised interest has to be for you to just expect to receive the same rate of return as what the government offers).

Recapping, we know that 5% is the time-value of money that we can receive in interest from the Treasury. We also know that 1.63% is the extra default premium that Vinny must promise a risk-neutral lender to allow the lender to expect to earn 5%, given that repayment is not guaranteed. Finally, if the lender is not risk-neutral but risk-averse, Vinny may have to pay even more than 6.63%, although we do not know exactly how much.

A more general decomposition of rates of returns.

The risk premium depends on such strange concepts as the correlation of loan default with the general economy and will be the subject of Part III of our book. For most ordinary loans by creditworthy borrowers, the risk premium is small. A reasonable guess would be 10 to 50 basis points for most U.S. corporate bonds. Most quoted loan interest rates reflect primarily the time value of money and the default risk, not the risk premium.

IMPORTANT: *When repayment is not certain, lenders demand a promised interest rate that is higher than the expected interest rate by the default premium.*

$$\begin{aligned} &\textit{Promised Interest Rate} \\ &= \textit{Time Premium} + \textit{Default Premium} + \textit{Risk Premium} \ . \end{aligned} \tag{4.23}$$

The promised default premium is positive, but it is only paid when everything goes well. The actually earned interest rate consists of the time premium, the realized risk premium, and a (positive or negative) default realization.

$$\begin{aligned} &\textit{Actual Interest Rate Earned} \\ &= \textit{Time Premium} + \textit{Default Realization} + \textit{Risk Premium} \ . \end{aligned} \tag{4.24}$$

The default realization could be more than negative enough to wipe out both the time premium and the risk premium. But it is zero on average. Therefore,

$$\begin{aligned} &\textit{Expected Interest Rate} \\ &= \textit{Time Premium} + \textit{Expected Risk Premium} \ . \end{aligned} \tag{4.25}$$

NERD NOTE: *There also seems to be a premium for investors willing to purchase financial securities that have higher transaction costs. This is sometimes called a* ***liquidity premium****. We do not yet fully understand it, but the current finance research that tries to measure and understand the liquidity premium better, is exciting. In our introductory text, we have to ignore it.*

Behavioral finance argues that some securities could also offer an irrational premium—but such premiums are even more difficult to find and to empirically confirm than liquidity premiums.

SIDE NOTE: What if the lender and borrower have different information and/or disagree about the default risk? For example, the lender may believe that the appropriate default risk is 50% and the appropriate default premium is 5%—but the borrower knows that he will have the money for sure. In this case, borrower and lender would determine different expected interest rates and costs of capital. The borrower would perceive the cost of capital (the expected interest rate) to be much higher than the lender. This will be the subject of Section 5·3.B.

4·4.D. Bond Rating Agencies

Bond rating agencies indicate probability of default.

In general, it is in the interest of the borrower to reduce the lender's doubt about future repayment. This ordinarily allows the corporation to borrow at a lower interest rate. But gauging default risk is not an easy task. Because judging credit risk costs time and money, borrowers usually wish to communicate their quality—their probability of default—to potential lenders, who might otherwise just pass up lending altogether. In the real world, credit rating agencies serve this function. The two biggest rating agencies are **Moody's** and **Standard&Poors**. For a fee, they rate the issuer's probability that the bonds will default. This fee depends on a number of factors, such as the identity of the issuer, the desired detail in the agencies' investigations and descriptions, and the features of the bond (e.g., a bond that will pay off within one year is usually less likely to default before maturity than a bond that will pay off in thirty years; thus, the former is easier to grade). The credit rating agencies ultimately do not provide a whole set of default probabilities (e.g., 1% chance of 100% loss, 1.2% chance of 99% loss, etc.), but just an overall rating. It is up to the ratings' reader to translate the rating into an appropriate compensation for default risk. The top rating grades are called **investment grade**, the bottom **speculative grade** (or **junk**). There is often a sharp difference in interest rates between the worst investment grade bond and the best speculative grade bond, because many investing institutions are allowed to hold only investment grade bonds.

Table 4.3. Ratings by Agencies and Default Rates

			Cumulative Default Rate, in %, over number of years, Y.		
Investment Grade:	Moody's	Standard & Poors	5 Y	10 Y	20 Y
Exceptional	Aaa, Aaa1, Aaa2, Aaa3	AAA, AAA-, AA+	0.1%	0.8	3.0
Excellent	Aa, Aa1, Aa2, Aa3	AA, AA-, A+	0.4	1.0	2.5
Good	A, A1, A2, A3	A, A-, BBB+	0.7	2.1	5.5
Adequate	Baa, Baa1, Baa2, Baa3	BBB, BBB-, BB+	2.1	5.0	12.2
Speculative Grade:	Moody's	Standard & Poors	5 Y	10 Y	20 Y
Questionable	Ba, Ba1, Ba2, Ba3	BB, BB-, B+	11.5	18.2	27.0
Poor	B, B1, B2, B3	B, B-, CCC+	26.0	34.7	39.6
Very Poor	Caa, Caa1, Caa2, Caa3	CCC, CCC-, CC+	?	?	?
Extremely Poor	Ca, Ca1, Ca2, Ca3	CC,CC-, C+	?	?	?
Lowest	C	C	?	?	?

Source: Moody's 1992 Corporate Bond Defaults, Moody's Investors Service. The report itself, with much interesting additional information, is available at riskcalc.moodysrms.com/us/research/defrate/0085.pdf.

Default by Rating

Table 4.3 gives a sketch of how likely default is, given creditrating, albeit a dated one. Few bonds default right after issuance, but many lower rated bonds eventually will miss at least one coupon payment, which is considered default. Note that in the real world, as in our Vinny example, default thus does not necessarily mean zero payment; it can mean as little as just one missed payment. In the text, Moody's special 1974–1992 report also indicated that a defaulted debt issue would be worth between 30% and 70% of its stated value, with 40% as the average.

Recent Reports.

In addition, Moody's publishes an interesting monthly report on credit risk, including corporate bond default forecasts (which change with the business cycle). Older Moody's reports can be found at riskcalc.moodysrms.com/us/research/mdr.asp. In August 2002, the report described that no bonds rated Aaa–A2 bonds had defaulted, 0.3–0.6% of bonds rated A3–Ba3 had defaulted, and 1.8%, 7.6%, 9.9%, and 21.0% of bonds rated B1, B2, B3, and Caa–C had defaulted.

Ratings are useless for making money.

Unfortunately, although bond rating agencies will update their rating if the condition of the firm changes, the empirical evidence suggests that these bond ratings are not very good in helping an investor earn superior rates of returns. In fact, the ratings seem to respond more to drops in the value of the underlying bonds than vice-versa. The bond rating agencies seem to be more reactive than proactive. A longer discussion of bond ratings appears on Page 490f.

4·4.E. Historical Interest Rates With Different Default Risks

Table 4.4. Promised Interest Rates For Some Loans in May 2002.

Security (Bond)	Annual Yield	Similar U.S. Treasury
FNMA May 2003	2.36%	2.22%
FNMA March 2031	6.29%	5.60%
Boston Celtics 6s38	9.40%	5.60%
United Airlines 11.21s14	14.40%	4.82%
Corporate High-Quality 1–10 years	4.89%	≈ 3%–4%
Corporate Medium-Quality 1–10 years	6.24%	≈ 3%–4%
Corporate High-Quality 10+ years	6.76%	≈ 4%–5%
Corporate Medium-Quality 10+ years	7.65%	≈ 4%–5%
High-Yield (Junk Bond) Corporates	11.36%	?

Source: *Wall Street Journal*, Page C13. FNMA is a quasi-governmental agency that underwrites home mortgages. See also Page 85.

Sample Rates of Returns

Table 4.4 lists the borrowing rates of various issuers in May 2002. (Many other current interest rates can be found at www.bloomberg.com/markets/rates/index.html and bonds.yahoo.com/rates.html.) Most of the differences between these borrowers' promised interest rates and Treasury interest rates are due to default risk, which compensates lenders for differential default probabilities.

Warning: I have cheated a little: not all of the differences in promised rates of return in Table 4.4 are due to default risk. Some of these differences reflect transaction costs and liquidity premia, which we ignore due to lack of space; some of these differences reflect bond maturity and duration differences, which I did not know for these bonds; some of these differences reflect risk aversion premia, which we will discuss in detail only in Part III; and some of these differences reflect additional bond features (such as embedded bond options), which we will discuss only in Part IV (Financing). For all these reasons, the quoted yields in Table 4.4 are not exactly comparable to treasuries. Nevertheless, the differences are mostly due to default risk.

ANECDOTE: Subprime and Minority Lending
Is it ethical to ask poorer people for higher interest rates? Are these borrowers worse off having received offers of loans at interest rates that are **usurious** (exorbitant)? What would happen if usurious interest rates were legally prohibited? Would poorer borrowers be worse off or better off? This is not just an academic question.

In late 1998, many lenders were accused of discrimination against minority borrowers. Subprime mortgages typically have interest rates between 100 and 400 basis points higher than ordinary mortgages. About 18% of white borrowers suffered subprime mortgages, while 50% of African-Americans did. The lenders defended themselves with the argument that ethnicity-neutral credit scoring models suggested lower credit-worthiness of the average minority applicant and his/her home purchase.

Although chances are that both views are correct (loan default data are not easy to come by, and the existing data are difficult to properly interpret), there is some evidence of residual discrimination even after credit-worthiness has been accounted for. If this is true, there should be excellent opportunities for third party mortgage lenders to earn higher profits by studying the mortgage market and offering more competitive mortgages in minority neighborhoods. Regardless of your views on the ethics of higher interest rates, we all hope that more and lower-interest rate lending will come to minority neighborhoods soon! For an example of a minority lender, and to read some more interesting information, please see www.sbk.com, website of the Shorebank Community Development Group.

Solve Now!

Q 4.8 *For what kind of bonds are expected and promised interest rates the same?*

Q 4.9 *Recompute the Vinny example assuming that the probability of receiving full payment is only 95%, and the probability of absolutely no payment is 4%. At the promised interest rate of 5%, what is the expected interest rate?*

Q 4.10 *Recompute the Vinny example assuming that the probability of receiving full payment is only 95%, and the probability of absolutely no payment is 4%. What interest rate is required as a promise to ensure an expected interest rate of 5%?*

Q 4.11 *Using information from a current newspaper or the WWW, what is the annualized yield on corporate bonds (high-quality, medium-quality, high-yield) today?*

Q 4.12 *Advanced question: How would a 40% income tax rate on interest receipts (both from government and Vinny loans) change the interest rate you would ask from Vinny?*

Q 4.13 *Advanced question: Return to the apples example from Section 4·2, in which the inflation rate was 100% and the nominal rate of interest was 700%. Now, assume that there is also a 25% default rate. That is, 25% of all apples are returned with worms inside, and will therefore not be sellable (and be worth \$0). What is your real rate of return? What is the formula?*

Q 4.14 *Really Advanced question: Return to the Taxes-and-Inflation example from Section 4·3.A: A 10% nominal rate of return, a tax rate of 40%, and an inflation rate of 5%. (We worked out that the post-inflation, post-tax rate of return was 0.95%.) Now, add a default rate, d, of 2%, where all money is lost (-100% return). What is the real, post-inflation, post-tax, post-default rate of return? (Hint: Losses are tax-deductible, too. Assume that the default rate reduces the nominal rate of return (on which taxes are charged), because we do not just take 1 such loan, but 1 million, which practically assures us of the exact default rate without any sampling variation.)*

4·5. Summary

The chapter covered the following major points:

- There are four types of "frictions," which reduce earned rates of returns: transaction costs, inflation, taxes, and default.
- Nominal interest rates can be translated into real interest rates, given the inflation rate.
- Taxable interest rates can be converted into equivalent tax-exempt interest rates, given the appropriate marginal tax-rate.
- Default causes promised interest rates to be higher than expected interest rates; default risk is often called credit risk.
- Quoted interest rates are almost always promised interest rates.
- Most of the difference between promised and expected interest rates is due to default, and not to higher expected rates of return (i.e., extra compensation) for bearing more risk.

The appendix covers a plethora of different interest rates.

A. Appendix: A List of Commonly Quoted Bonds

In the real world, there are many different interest rates. Every borrower and every lender may pay a slightly different interest rate, depending on the bond's default risk, risk premium, liquidity, maturity, identity, convenience, etc. It is impossible to describe every common bond or rate. The C-section of the *Wall Street Journal* describes daily interest rates on many common and important interest instruments: the C-1 Page **Markets Diary**, the C-2 Page **Interest Rates and Bonds** section, some boxes on a later page describing the interest rates paid on individual government and government agency bonds (headlines **Treasury Bonds, Notes and Bills** and **Government Agency & Similar Issues**, the **Credit Markets** page thereafter, which includes the important **Yield Comparisons** and **Money Rates** boxes, as well as some Corporate Bonds, and what is often the final page, which contains the **Bond Market Data Bank**. In addition, futures on interest rates (similar to forward rates) are listed in the B-section.

Other interest rate information sources.

Here are short descriptions of some of the interest rates printed in the *Wall Street Journal* on a daily basis, as well as some other Bond subclasses.

Some real-world interest rates explained.

Agency Bonds Issued by quasi-governmental agencies, typically backed by explicit or implicit guarantees by the U.S. government. **FNMA** (pronounced **Fannie-Mae**) is a good example of such an agency. It holds mortgages as assets (indeed, if you have a mortgage, chances are that FNMA owns it), and finances the purchases of these mortgages with bonds it issues (the rates of which are reported in the *Wall Street Journal*). The FNMA bonds are themselves collateralized (backed) by the mortgages.

ARM Rate or Adjustable Rate Mortgage A mortgage with an interest rate that is usually reset once per year according to a then prevailing interest rate, prespecified by a formula, but subject to some upper limit (CAP). Repayable by the borrower.

Bankers Acceptances Loans by banks to importers, used to pay the exporting firm. Backed by the issuing bank if the importer defaults. Usual maturities are 30 to 180 days.

Certificate of Deposit (CD) Rate paid by banks to bank retail customers willing to commit funds for a short-term or medium-term period. Unlike ordinary savings accounts, CDs are not insured by the government if the bank fails.

Callable Bonds Bonds that the issuer can redeem. Will be discussed in Chapter 20.

Collateralized Mortgage Obligations or CMO A security backed by a pool of real estate mortgages, with specified claims to interest and principal payments. For example, there are **Interest Only** (**IO**) bonds and **Principal Only** (**PO**) bonds, which only entitle bond holders to either the principal or the interest that the pool of mortgages receives.

Collateralized Trust Bonds Often issued by corporations, these bonds pledge as collateral the securities owned by a subsidiary.

Commercial Paper Short-term bonds issued by corporations to the public markets. Often backed by bank guarantees. Being short-term and often backed, commercial paper is usually very low risk.

Consumer Credit Rates For example, the *Wall Street Journal* lists the going **Credit-Card Rate**, and **Car Loan Rate**.

Convertible Bonds Bonds that the holder can convert into common equity. Will be discussed in Chapter 20.

Debenture Unsecured general obligation bond.

Discount Rate The interest rate that the Federal Reserve charges banks for short-term loans of reserves.

Equipment Obligations Unlike Debentures, these corporate bonds usually pledge specific equipment as collateral.

Eurobond Bonds issued by U.S. government outside the domain of the S.E.C. (e.g., Europe) and purchased by foreign investors. Eurobonds need not be denominated in Dollars.

Federal Funds Rate Banks must hold financial reserves at the Federal Reserve Bank. If they have more reserves than they legally need, they can lend them to other banks. The rate at which they lend to one another overnight is the federal funds rate.

GICs or Guaranteed Investment Contracts Usually issued by insurance companies and purchased by retirement plans. The interest rate is guaranteed, but the principal is not.

G.O. Bonds or General Obligation Bonds Bonds whose repayment is not guaranteed by a specific revenue stream. See also **Revenue Bonds**.

High-Yield Bonds Sometimes also called **Junk Bonds**, high-yield bonds are bonds (usually of corporations) that have credit ratings of BB and lower.

Home Equity Loan Rate The rate for loans secured by a house. Usually second mortgages, i.e., taken after another mortgage is already in place.

Investment Grade Bonds Bonds that have a rating higher than BBB. This is a common classification for Corporate Bonds.

Jumbo Mortgage Rate Like the N-year Mortgage Rate (see below), but for loans which exceed the FNMA limit on mortgage size.

LIBOR London Interbank Offer Rate Typical rate at which large London banks lend dollars to one another.

Money-Market Rate Rate paid to cash sitting in a brokerage account and not invested in other assets.

Mortgage Bonds are secured by a particular real-estate property. In case of default, the creditor can foreclose the secured property. If still not satisfied, the remainder of the creditor's claim becomes a general obligation.

Municipal Bond Bonds issued by a municipality. Often tax-exempt.

N-year Mortgage Rate A fixed-rate loan, secured by a house, with standard coupon payments. The rate is that paid by the borrower. Usually limited to an amount determined by FNMA.

Prime Rate An interest rate charged by the average bank to their best customers for short-term loans.

REPO Rate A Repo is a repurchase agreement, in which a seller of a bond agrees to repurchase the bond, usually within 30 to 90 days, but also sometimes overnight. This allows the bond holder to obtain physical cash to make additional purchases, while still being fully exposed to (speculate on) the bond.

Revenue Bond A bond secured by a specific revenue stream. See also G.O. Bond.

Savings Bonds Issued by the U.S. Treasury, Savings Bonds can only be purchased from or sold to agents authorized by the Treasury Department. They must be registered in the name of the holder. **Series E Bonds** are Zero Bonds; **Series H Bonds** are semi-annual coupon payers and often have a variable interest feature. In contrast to Savings Bonds, other bonds are typically **Bearer Bonds**, which do not record the name of the owner and are therefore easy to resell (or steal).

Tax-Exempt Bonds Typically bonds issued by municipalities. Their interest is usually exempt from some or all income taxes. The designation **G.O. Bond** means **General Obligation Bond**, i.e., a Bond that was not issued to finance a particular obligation. In contrast, a **Revenue Bond** is a Bond backed by specific municipal revenues—but they may or may not be tax-exempt.

Yankee Bonds U.S. Dollar denominated and SEC-registered bonds by foreign issuers.

Note: mortgage (and many other) bonds can be paid off by the borrower before maturity. Repayment is common especially if interest rates are dropping. Prepayment.

ANECDOTE: Sumerian Debt Contracts
Among the earliest known collateralized debt contracts is a tablet from Sumeria (Mesopotamia), which promises delivery of silver and gives as security the son of the borrower. (The tablet can be viewed at www.museumofmoney.org/babylon/index.html.) Such contracts are illegal today, but de-facto "debt slavery" for debts not repaid is common in many countries according to the September 2003 issue of National Geographic Magazine.

A. Q&A: Answers

1. $(1 + r_{\text{nominal}}) = (1 + r_{\text{real}}) \cdot (1 + \pi)$.
2. Do it! (As of 2002, it should be between 1% and 2% per year.) This rate can be found at the end of the **Money Rates** box in the WSJ.
3. Do it! (This changes too often to give a useful figure here.) It can also found in the Money Rates box.
4. Do it!

5. For every \$100, you receive \$6. Uncle Sam takes 20% of \$6, or $\$1.^{20}$. So, your after tax rate of return is $\$4.^{80}/\$100 = 4.8\%$. You could have also computed $(1 - 20\%) \cdot 6\% = 4.8\%$ directly.
6. For every \$100, you receive \$6. Uncle Sam takes 40% of \$6, or $\$2.^{40}$. So, your after tax rate of return is $\$3.^{60}/\$100 = 3.6\%$. You could have also computed $(1 - 40\%) \cdot 6\% = 3.6\%$ directly.
7. Increase. In the real world, interest rates may have to rise to compensate private savers for this extra "tax" on money.

8. Only for government bonds. Most other bonds have some kind of default risk.
9. The expected payoff is now $95\% \cdot \$210 + 1\% \cdot \$100 + 4\% \cdot \$0 = \$200.^{50}$. Therefore, the expected rate of return is $\$200.^{50}/\$200 = 0.25\%$.
10. You require an expected payoff of \$210. Therefore, you must solve for a promised payment $95\% \cdot P + 1\% \cdot \$100 + 4\% \cdot \$0 = \$210 \rightarrow P = \$209/0.95 = \$220$. On a loan of \$200, this is a 10% promised interest rate.
11. Do it! This information can be found in the **Yield Comparisons** exhibit in the **Credit Markets** section in the WSJ.
12. It would not matter.
13. Our numeraire is one apple ($1a$) that costs \$1. We will get \$8 in nominal terms, next year ($a \cdot (1 + r_{\text{nominal,pre-tax}}) = a \cdot (1 + 700\%) = 8 \cdot a$). This will purchase apples that cost \$2 each ($(1 + \pi) = (1 + 100\%) = \$2$), i.e., 4 apples ($a \cdot (1 + r_{\text{nominal,pre-tax}})/(1 + \pi) = 1 \cdot (1 + 700\%)/(1 + 100\%) = 4$). However, one of the apples ($d = 25\%$) is bad, so we will get only 3 apples ($a_1 = a_0 \cdot (1 + r_{\text{nominal,pre-tax}})/(1 + \pi) \cdot (1 - d) = 1a_0 \cdot (1 + 700\%)/(1 + 100\%) \cdot 75\% = 3 \cdot a_0$, where d is the 25% default rate) . Therefore, the real rate of return is $(a_1 - a_0)/a_0$ or

$$\begin{aligned} r_{\text{real,post-tax,post-default}} &= \frac{(1a \cdot \frac{1+700\%}{1+100\%} \cdot 75\%) - 1a}{1a} = 300\% - 1 = 200\% \\ r_{\text{real,post-tax,post-default}} &= \frac{[1a \cdot \frac{1+r_{\text{nominal,pre-tax}}}{1+\pi} \cdot (1-d)] - 1a}{1a} . \end{aligned} \tag{4.26}$$

The "1a" of course cancels, because the formula applies to any number of apples or other goods.

14. Instead of 10%, we earn only $98\% \cdot 10\% + 2\% \cdot (-100\%) = 7.8\%$. Translated into a formula, this is $(1 - d) \cdot r_{\text{nominal,pre-tax}} + d \cdot (-100\%) = r_{\text{nominal,pre-tax}} - d \cdot (1 + r_{\text{nominal,pre-tax}}) = 10\% - 2\% \cdot (1 + 10\%) = 7.8\%$. Now, in Formula 4.10,

$$\begin{aligned} r_{\text{post-tax, real, post-default}} &= \frac{V_0 - C_0}{C_0} = \frac{\frac{C_0 \cdot [1 + r_{\text{nominal,pre-tax}} \cdot (1-\tau)]}{1+\pi} - C_0}{C_0} \\ &= \frac{r_{\text{nominal,pre-tax}} \cdot (1 - \tau) - \pi}{1 + \pi} , \end{aligned} \tag{4.27}$$

replace the nominal interest rate $r_{\text{nominal,pre-tax}}$ with the default reduced nominal rate $r_{\text{nominal,pre-tax}} - d \cdot (1 + r_{\text{nominal,pre-tax}})$, so the new formula is

$$\begin{aligned} r_{\text{post-default, post-tax, real}} &= \frac{V_0 - C_0}{C_0} \\ &= \frac{\frac{C_0 \cdot \left[1 + (r_{\text{nominal,pre-tax}} - d \cdot (1 + r_{\text{nominal,pre-tax}})) \cdot (1 - \tau)\right]}{1 + \pi} - C_0}{C_0} \\ &= \frac{(r_{\text{nominal,pre-tax}} - d \cdot (1 + r_{\text{nominal,pre-tax}})) \cdot (1 - \tau) - \pi}{1 + \pi} \\ &= \frac{7.8\% \cdot (1 - 40\%) - 5\%}{1 + 5\%} = -0.3\% \quad . \end{aligned} \tag{4.28}$$

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

Part II

Project Choices / Capital Budgeting

"It's not all bad. We still have enough cash for bribing the prison guards."

Transition

You now have a solid understanding of what rates of return really mean and how to work with them. Rates of returns differ by horizon and their quotes usually do not adjust for inflation, taxes, risk, and liquidity.

You now understand the factors why the interest rate for the 10-year Boston Celtics bond quotes 400 basis points over a 10-year Treasury bond and 600 basis points over a 90-day tax-exempt municipal bond. With additional information—such as good estimates of the inflation rate, your own tax rate, the Celtics' default probability, transaction costs, and the prevailing yield curve—you can even evaluate the relative attractiveness of one versus the other. Getting these estimates is not an easy task—which is the primary reason why it is not at all easy to work with rates of returns, even though we called the previous part the "basics" of finance.

Perhaps even more important, you should now have the ability to think of solving problems in terms of numerical examples that you can then translate into more general formulas. So, if I throw a new problem at you, even if you do not remember any formulas, given time, you should now be able to "invent" them. Try it: can you construct an example with both inflation and taxes to determine an equivalent risk-free rate of return? (The solution is the formula you should not have remembered from Page 73.) Could you add an x% chance of total default risk? (Think back to the apple example, and assume there is a risk that one in twenty apples will have a worm inside!)

We now begin to look at the problem from the perspective of someone choosing to undertake a project, which could be a family or a company. For instance, we may take the perspective of the CFO of the Boston Celtics, who needs to raise money by issuing a bond. Then, as manager of the Celtics, should you invest your investor's money into Treasury bonds, the European NBA, Shaquille O' Neal, or an advertising campaign? It turns out that this problem of "capital budgeting" (where to invest your capital) is an exact analogue of the rate of return problem (where to save capital).

To work the capital allocation process, we will also rely on our knowledge of rates of returns: if the cost of raising capital (the expected interest rate that we will have to offer to attract investors) is higher than the expected rate of return that our project investment opportunities provide, then we would be better off not to undertake such projects in the first place. This is why expected rates of returns will be a direct input into the most important capital budgeting formula, the famous "Net Present Value" (NPV), that we shall cover now.

In sum, the goal of this part of the book is to understand company behavior—specifically, the capital allocation decision process.

What we want to Accomplish in this Part

The goal of this part of the book is to teach you how to make good project investment decisions, how to value projects and firms, and how to use financial statements. It takes the perspective of the manager.

- Chapter 5 shows how to discount money that you will receive in the future into equivalent dollars today. It explains how to do this even if markets are imperfect.

  Typical question: What is today's value of a mortgage that pays $1,000 each month for 30 years?

- Chapter 6 shows how to discount money when future payment is not assured (similar to the credit risk Chapter 4). It also explains the basics of debt and equity.

  Typical question: If you purchase a building and finance it with a mortgage, what is a fair investment rate of return for the mortgage company and for you?

- Chapter 7 details a whole range of factors that you should think about when you want to value a project.

  Typical questions: If the interest rate is 10% per year, inflation is 5% per year, and the income tax rate is 40% per year, what is today's value of a particular project? How does this value change if there are associated negative consequences—for example, if the project pollutes the air? How does your assessment of the value change if someone else makes up the cash flow estimates?

- Chapter 8 illustrates how you can compute the (internal) rate of return and duration for projects with many future cash flows.

  Typical question: If your project costs $100, and returns $50 next year and $100 in ten years, what is your project's internal rate of return?

- Chapter 9 demonstrates how to extract economic cash flow estimates from corporate financial statements.

  Typical question: What are the economic cash flows in PepsiCo's financial statements that we would use to estimate the net present value of PepsiCo?

- Chapter 10 discusses how to construct "pro formas," which are financial projections to value the firms and projects.

  Typical questions: How would you create forecasts of PepsiCo's future cash flow statements? How long into the future should you forecast?

- Chapter 11 shows how to assemble information about your own firm, using publicly available information from comparable firms.

  Typical questions: When is the P/E (price earnings) ratio a good number to look at? What should be the P/E ratio of our project?

CHAPTER 5

Present Value, Future Value, and Net Present Value (NPV)

Capital Budgeting At Its Best

last file change: Jun 15, 2004 (10:34h)

last major edit: March 24, 2004

Firms must allocate cash to projects within their budgets. Therefore, the process of deciding which projects to undertake and which projects to pass up is called **capital budgeting**. Capital budgeting is at the heart of corporate decision-making.

If the interest rate is positive, then money in the future will necessarily be worth less than the same amount of money today—after all, you could always put today's money into the bank in order to have more of it in the future. This is called the **time value of money**—a dollar today is worth more than a dollar tomorrow.

This chapter explains how to determine the value *today* of a project that will yield cash flows *in the future*. To obtain a project's value today, you must compute the **net present value** (NPV). NPV translates all *future* cash flows—both inflows and outflows—into their equivalent *present* values today, and then adds them up to find the "net": the "net present value" of the project.

This chapter also discusses **Perpetuities** and **Annuities**, which are projects that have peculiar cash flow patterns. Their valuation formulas have quick and intuitive NPV solutions, and are thus often useful tools.

ANECDOTE: Fibonacci and the Invention of Net Present Value

William Goetzmann argues that Leonardo of Pisa, commonly called Fibonacci, may have invented not only the famous "Fibonacci series," but also the concept of net present value, which is the focus of our chapter.

Fibonacci's family were merchants in the Mediterranean in the 13th century, with trade relations to Arab merchants in Northern Africa. This must have exposed Fibonacci to algebra, invented by Al Khwārizmī. But unlike Al Khwārizmī, Fibonacci wrote about mathematics primarily as a tool to solve merchants' problems—in effect, to understand the pricing of goods and currencies relative to one another. Think about how rich you could get if you could determine faster than your competition which goods were worth more in relation to others! In fact, you should think of Fibonacci and other Pisan merchants as the "financial engineers" of the 13th century.

Fibonacci's most famous treatise, *Liber Abaci*, was written at age 30. In our chapter, you will get to solve his "On a Soldier Receiving 300 Bezants for his Fief" problem:

> A soldier is granted an annuity by the king of 300 bezants per year, paid in quarterly installments of 75 bezants. The king alters the payment schedule to an annual year-end payment of 300. The soldier is able to earn 2 bezants on 100 per month (over each quarter) on his investment. How much is his effective compensation after the terms of the annuity changed?

To solve this problem, you must know how to value payments at different points in the future—you must understand the time value of money. What is the value of 75 bezants in one quarter, two quarters, etc.? What is the value of 300 bezants in one year, two years, etc.? Yes, money sooner is usually worth more than money later—but you need to determine by exactly how much in order to determine how good or bad the change is for the king and the soldier. To answer, you must use the interest rate Fibonacci gives, and then compare the two different cash flow streams—the original payment schedule and the revised payment schedule—in terms of a common denominator. This common denominator will be the two streams' present values.

5·1. Projects, Firms, Families, and Markets

As far as finance is concerned, every project is a set of flows of money (**cash flows**). Most projects require an upfront **cash outflow** (an **investment** or **expense** or **cost**) and are followed by a series of later **cash inflows** (**payoffs** or **revenues** or **returns**). It does not matter whether the cash flows come from garbage hauling or diamond sales. Cash is cash. However, it is important that all costs and benefits are included as cash values. If you would have to spend more time or have more distaste to haul trash, you would have to translate this into an equivalent cash negative. Similarly, if you want to do a project "for the fun of it," you must translate this into a cash positive. The discipline of finance takes over after all positives and negatives (inflows and outflows) from the project "black box" have been translated into their monetary cash value.

To value projects, make sure to use all costs and benefits, including, e.g., opportunity costs and pleasure benefits.

Anecdote: Restaurant Failures
In New York City, two out of every five new restaurants close within one year. Nationwide, the best estimates suggest that about 90% of all restaurants close within two years. If successful, the average restaurant earns a return of about 10% per year. One explanation for why so many entrepreneurs are continuing to open up restaurants, despite seemingly low financial rates of returns, is that restauranteurs so much enjoy owning a restaurant that they are willing to buy the prestige of owning a restaurant. If this is the case, then to value the restaurant, you must factor in how much the restauranteur is willing to pay for the prestige of owning a restaurant, just as you would factor in the revenues that restaurant patrons generate. (But see also Page 155.)

Projects need not be physical. For example, a company may have a project called "customer relations," with real cash outflows today and nebulous future inflows. You (a student) are a project: you pay for education and will earn a salary in the future. In addition, some of the payoffs from education are metaphysical rather than physical. If knowledge provides you with pleasure, either today or in the future, education yields a value that should be regarded as a positive cash flow. Of course, for some students, the distaste of learning should be factored in as a cost (equivalent cash outflow). All such non-financial cash flows must be appropriately valued and considered if you want to arrive at a good project valuation!

These examples show that cash flows must be universal, not just direct cash flows.

A firm can be seen as nothing more than a collection of projects. Similarly, so can a family. Your family may own a house, a car, tuition payments, education investments, etc.,—a collection of projects. This book assumes that the value of a firm is the value of all its projects' net cash flows, and nothing else. It is now our goal to learn how to determine overall project value, given cash flows.

In finance, firms are basically collections of projects.

Solve Now!

Q 5.1 *In computing the cost of your M.B.A., should you take into account the loss of salary while going to school? Cite a few non-monetary benefits, too, and try to attach monetary value to them.*

Q 5.2 *If you purchase a house and live in it, what are your inflows and outflows?*

5·2. The Present Value of Future Cash Flows

5·2.A. Discount Factors and The Present Value (PV)

The "Present Value Formula" is the rate of return definition, inverted to translate future cash flows into (equivalent) today's dollars.

Recall the basic **Rate of Return** Formula 2.1,

$$r_{0,1} = \frac{C_1 - C_0}{C_0} = \frac{C_1}{C_0} - 1 \quad , \tag{5.1}$$

which states that the rate of return from period 0 to period 1 is the difference in cash flows divided by the original cash flow. Turn this formula around to answer the question: how much would you have to save today if you want to receive a given amount of money next year (at $t = 1$)? Or, if you have to borrow money to undertake a project that will return C_1 next year, what is the value of these future cash flows to you *today*?

A concrete example.

For example, if the interest rate is 10%, how much would you have to save to receive \$100 next year? The answer is

$$\begin{aligned} \frac{\$100}{1+10\%} &= \$90.^{91} \\ C_0 = \frac{C_1}{1+r_{0,1}} &= \mathsf{PV}_0(C_1) \ . \end{aligned} \tag{5.2}$$

Check this calculation: investing \$90.^{91} at an interest rate of 10% will indeed return \$100 next period. Thus, we can also state that the **present value** (PV) of \$100 next year is \$90.^{91} today—the value today of tomorrow's cash flows. If you can borrow or lend at the interest rate of 10%, you will be indifferent between receiving \$100 next year and receiving \$90.^{91} today. In contrast, if the standard rate of return in the economy were 12% and you could invest \$90.^{91} in a project to receive \$100 next year, this project would not be a good deal. The project's present value would be

$$\begin{aligned} \frac{\$100}{1+12\%} &= \$89.^{29} \\ C_0 = \frac{C_1}{1+r_{0,1}} & \quad , \end{aligned} \tag{5.3}$$

which would be less than its cost of \$90.^{91}. If the standard economy-wide rate of return were 8% and you could invest \$90.^{91} to receive \$100 next year, the project would be a great deal. Today's present value of the project's future payoffs would be

$$\begin{aligned} \frac{\$100}{1+8\%} &= \$92.^{59} \\ C_0 = \frac{C_1}{1+r_{0,1}} & \quad , \end{aligned} \tag{5.4}$$

which would exceed the project cost of \$90.^{91}.

Side Note: In the valuation part of the book, the rate of return is given. The investments part of the book covers where the appropriate cost of capital comes from.

The present value formula translates future money into today's money. For example, if the interest rate were 10% per period, \$100 in two periods would be worth

The PV formula.

$$\begin{aligned} \frac{\$100}{(1+10\%)^2} &= \$82.^{64} \\ C_0 = \frac{C_2}{(1+r_{0,1})\cdot(1+r_{1,2})} &= \frac{C_2}{1+r_{0,2}} \ . \end{aligned} \tag{5.5}$$

The rate of return, r, with which the project can be financed, is often called the **cost of capital**. It is the rate of return at which you (the project) can borrow (or invest) money. The quantity

The interest rate can now be called the "cost of capital;" the discount factor is closely related.

$$\frac{1}{1+r_{0,t}} \tag{5.6}$$

is called the **discount factor** (or sometimes, less correctly, the **discount rate**, though we use it as a name for $r_{0,t}$ in this context). Discounting—the conversion of a future cash flow into its equivalent present value—is the reverse of compounding. In other words, the discount factor translates one dollar in the future into the equivalent amount of dollars today. Because nominal interest rates are usually positive, discount factors are usually less than 1. Consequently, a dollar in the future is usually worth less than a dollar today. This is called the **time value of money**.

5·2.B. Net Present Value (NPV)

An important feature of present value is that it is expressed in the same unit: cash today. To see this, say that a project generates \$10 in 1 year and \$8 in 5 years. You cannot add them to come up with \$18—it would be like adding apples and oranges. However, if you translate both future cash flows into their present values, you *can* then add them. For example, if the interest rate was 5% per annum over 1 year and 6% per annum over 5 years (or $(1+6\%)^5 = (1+33.8\%)$ in total), then

Present Values are alike and thus can be added, subtracted, compared, etc.

$$\begin{aligned} \mathsf{PV}_0(\$10 \text{ in one year}) &= \frac{\$10}{1+5\%} = \$9.^{52} \\ \mathsf{PV}_0(\$8 \text{ in five years}) &= \frac{\$8}{(1+6\%)^5} = \$5.^{98} \ . \end{aligned} \tag{5.7}$$

It follows that the project's total value *today* (time 0) would be \$15.^{50}.

Net present value (NPV) is really the same as present value, except that the word "net" upfront reminds us to add and subtract *all* cash flows, including any upfront net investment today. The NPV calculation is always the same. First, translate all future cashflows into today's dollars and second, add them all up. So, for example, if obtaining our project costs \$12 today, then

The definition and use of NPV.

$$\mathsf{NPV}_0 = -\$12 + \frac{\$10}{1+5\%} + \frac{\$8}{(1+6\%)^5} = \$3.^{50} \ . \tag{5.8}$$

With its cost of \$12, you can think of the NPV of \$3.^{50} as the difference between the (market) value of the future cash flows and the project's cost—a sort of value added. If the NPV is positive, as it is here, you should take the project. If it is negative, you should reject the project. NPV is definitively the best method for capital budgeting—the process by which you should accept or reject projects.

Table 5.1. Hypothetical Project Cash Flow Table

Time t	Project Cash Flow	Annualized Interest Rate	t-year Total Interest Rate
Today	-\$900	0.0%	0.0%
Year +1	+\$200	5.0%	5.0%
Year +2	+\$200	5.5%	11.3%
Year +3	+\$400	6.0%	19.1%
Year +4	+\$400	6.5%	28.6%
Year +5	-\$100	7.0%	40.2%

Here is a typical NPV Example.

Because the NPV concept is so important, we will work another example. Consider the project described in Table 5.1. This project is a bit unusual in that it not only requires an upfront investment but also a final "cleanup" expense at its end (e.g., a reserve to cover future legal claims against us). The table's last two columns provide the appropriate interest rates. In our perfect world, the interest rate is the same regardless of whether we borrow or lend.

Table 5.2. Hypothetical Project Cash Flow Table, Augmented

Time	Project Cash Flow	Per Annum Interest Rate	t-year Total Interest Rate	Discount Factor	Present Value
Today	-\$900	0.0%	0.0%	1.000	-\$900.^{00}
Year +1	+\$200	5.0%	5.0%	0.952	\$190.^{48}
Year +2	+\$200	5.5%	11.3%	0.898	\$179.^{69}
Year +3	+\$400	6.0%	19.1%	0.840	\$335.^{85}
Year +4	+\$400	6.5%	28.6%	0.778	\$311.^{04}
Year +5	-\$100	7.0%	40.2%	0.713	-\$71.^{33}
			Net Present Value (Sum):	=	+\$45.^{73}

Discount factors multiply future cash flows to determine today's value.

Table 5.2 augments Table 5.1 with discount factors and present values: The fifth column is the discount factor, i.e., the amount that we need to set aside today to receive \$1 in the relevant year. As we saw above, compute it by dividing 1 by one plus the appropriate total interest rate, or $1/(1 + r_{0,t})$. For example, to receive \$1 in year 3, we must invest $\$1 \cdot 1/(1 + 19.1\%) = \$0.^{840}$ today.

How much does it cost to receive this project's total payoff pattern? Simply multiply the promised payoffs by the appropriate discount factor to get their present value equivalent. As we already know, each of these numbers is a present value (PV). Because present values are additive, you then sum up all the terms to compute the overall net present value. The sixth column shows the present value of each year's payoffs, their sum being the NPV of \$45.[73].

You can think of the project as a sequence of discounted cash flows.

One way that NPV is often justified is to think of a sequence of bonds (promises) that have identical cash flows as the project. For example, we could instead buy one bond that promises \$200 next year, and it would cost \$190.[48] today (\$200 multiplied by the discount factor). We could then buy another bond that promises \$200 in two years, which would cost \$179.[69]. And so on. Because these bonds would have exactly the same payoffs as the project, *i.e., because the bonds* ***replicate*** *the project,* their net present values should be the exactly the same, too.

NPV can be justified because it is the cost of a set of loans (bonds) that yield exactly the same benefits as the project.

In our example, the net present value of the project is positive. Thus, the project would be a good use of our money. However, if the upfront expense was \$1,000 instead of \$900, the NPV would be –\$54.[27], and we would be better off investing the money into the appropriate sequence of bonds from which the discount factors were computed.

The NPV Decision Rule: accept only positive NPV projects.

IMPORTANT: *The* ***Net Present Value*** *Formula is*

$$\begin{aligned} NPV &= CF_0 + \frac{CF_1}{1+r_{0,1}} + \frac{CF_2}{1+r_{0,2}} + \frac{CF_3}{1+r_{0,3}} + \frac{CF_4}{1+r_{0,4}} + \ldots \\ &= \sum_{t=0}^{\infty} \left(\frac{CF_t}{1+r_{0,t}} \right) \quad , \end{aligned} \tag{5.9}$$

where the subscripts are time indexes, CF_t is the net cash flow at time t (positive for inflows, negative for outflows), and $r_{0,t}$ is the relevant interest rate for investments from today to time t. (The Greek sigma notation is a shorter way of writing the longer summation above. You will see it again, so if you are not familiar with it, it is explained in Appendix Section 2·3.)

The ***Net Present Value Capital Budgeting Rule*** *states that you should accept projects with a positive NPV and reject projects with a negative NPV.*

Present value, and its variant net present value, are the most essential concepts in finance. NPV is the method by which cash flows are translated into the same units—dollars today—which can then be compared or added. NPV always works the same way: compute appropriate discount factors to translate future money back into today's equivalent monetary value. Then, add up all present values. *Before moving on, you must be comfortable with the mechanics of computing the net present value of projects.* Unfortunately, despite its conceptual simplicity, the application of NPV in the real world is usually very difficult, because you must estimate cash flows and discount factors. This will be the subject of later chapters.

Recap: NPV may be the most important building block in finance.

5·2.C. The Link Between Cash Flow (Earnings) Growth, Dividend Growth, and Value

Should we invest in a fast grower or a slow grower?

Why would anyone invest in a company that grows very slowly? Would it not always be better to invest in a company that grows faster with higher future cash flows? For example, consider company "Grow" (G) that will produce

$$G_1 = \$100 \quad G_2 = \$150 \quad G_3 = \$250 \ , \tag{5.10}$$

and company "Shrink" (S) that will produce

$$S_1 = \$100 \quad S_2 = \$90 \quad S_3 = \$80 \ . \tag{5.11}$$

Why would anyone ever consider buying company S? Isn't G a better company to buy than S?

Let's find out: Compute the values.

For argument's sake, assume that both company G and company S face annualized interest rates—costs of capital—of 10%. Then the price of G today is

$$\mathsf{NPV}_0(G) = \frac{\$100}{(1+10\%)^1} + \frac{\$150}{(1+10\%)^2} + \frac{\$250}{(1+10\%)^3} \approx \$402.^{71} \ , \tag{5.12}$$

and the price of S today is

$$\mathsf{NPV}_0(S) = \frac{\$100}{(1+10\%)^1} + \frac{\$90}{(1+10\%)^2} + \frac{\$80}{(1+10\%)^3} \approx \$225.^{39} \ . \tag{5.13}$$

Your investment dollar grows at the same rate, disconnected from the cash flow rate.

If you invest in G, then next year you will have \$100 cash, and own a company with \$150 and \$250 cash flows coming up. G's value at time 1 (so NPV now has subscript 1) will thus be

$$\mathsf{NPV}_1(G) = \$100 + \frac{\$150}{(1+10\%)^1} + \frac{\$250}{(1+10\%)^2} \approx \$442.^{98} \ . \tag{5.14}$$

Your investment will have earned a rate of return of $\$442.^{98}/\$402.^{71} - 1 = 10\%$. If you instead invest in S, then next year you will receive \$100 cash, and own a company with "only" \$90 and \$80 cash flows coming up. S's value will thus be

$$\mathsf{NPV}_1(S) = \$100 + \frac{\$90}{(1+10\%)^1} + \frac{\$80}{(1+10\%)^2} \approx \$247.^{93} \ . \tag{5.15}$$

Your investment will have earned a rate of return of $\$247.^{39}/\$225.^{39} - 1 = 10\%$. In either case, you will earn the fair rate of return of 10%. So—whether cash flows are growing at a rate of +50%, −10%, +237.5%, or −92% is irrelevant: *the market price today already reflects the future growth rate.* In an efficient market, there is no necessary connection between the growth rate of the underlying project cash flows or earnings, and the growth rate of your investment money (i.e., your expected rate of return).

SIDE NOTE: If the fast grower G were also riskier, it might have to offer a higher expected rate of return. (The relation between risk and reward will be covered in Part III.) In this case, your investment dollar would be expected to earn more in G than in S. However, this is because of the extra risk that you would be taking on, not because of G's faster growth rate.

The same argument applies to dividends: in the end, all earnings must be paid out (i.e., as dividends). This does not need to occur at the same time: your earnings can grow today, and your dividends can shrink today. But the net present value of your total earnings and your total dividends must both be equal to the price of the firm in our idealized world—or you would get something for nothing. For example, firm G could be a slow dividend payer or a fast dividend payer. It could pay \$100 now, \$150 next year and \$250 in two years. Or, it could reinvest the money, effectively on your behalf, (at the same 10%, of course), and then pay one big lump sum dividend of $\$100 \cdot (1+10\%)^2 + \$150 \cdot (1+10\%) + \$250 = \$536$ at the end of period 2. The dividend payout policy does not matter to what G's value is today.

Dividend Payout Timing can shift around, too.

> **IMPORTANT**: *In an idealized efficient world, the price and value of the firm are determined by the net present value of the firm's underlying projects. The time patterns of earnings or dividends only matter indirectly, because they are a reflection of these project cash flows. A fast-earnings grower, a slow-earnings grower, a fast-dividend payer, or a slow-dividend payer are all fair investments.*

If you want to write this relationship in a formal way,

$$\begin{aligned}\text{NPV} &= \sum_{t=0}^{\infty} \frac{\text{Cash Flow}_t}{1+r_{0,t}} &&= \sum_{t=0}^{\infty} \frac{\text{Dividend}_t}{1+r_{0,t}} \\ &= \text{Cash Flow}_0 + \frac{\text{Cash Flow}_1}{1+r_{0,1}} + \frac{\text{Cash Flow}_2}{1+r_{0,2}} + \ldots &&= \text{Dividend}_0 + \frac{\text{Dividend}_1}{1+r_{0,1}} + \frac{\text{Dividend}_2}{1+r_{0,2}} + \ldots \ .\end{aligned} \tag{5.16}$$

For our growing firm G,

$$\begin{aligned}\$442.^{98} &= \$100 + \frac{\$150}{1+10\%} + \frac{\$250}{(1+10\%)^2} &&= \$100 + \frac{\$150}{1+10\%} + \frac{\$250}{(1+10\%)^2} \\ &&&= \$0 + \frac{\$0}{1+10\%} + \frac{\$536}{(1+10\%)^2} \\ \text{NPV} &= \sum_{t=0}^{\infty} \frac{\text{Cash Flow}_t}{1+r_{0,t}} &&= \sum_{t=0}^{\infty} \frac{\text{Dividend}_t}{1+r_{0,t}}\end{aligned} \tag{5.17}$$

5·2.D. Valuing Coupon Bonds and The Effects of Yield Curve Changes

Bond Naming

To illustrate more NPV applications, we now look at bond pricing. Bonds come in many different flavors, but one useful distinction classifies them into **coupon bonds** and **zero bonds** (**default bonds**). A coupon bond pays its holder cash at many different points in time, whereas a zero bond only pays a single lump sum at the maturity of the bond. Many coupon bonds promise to pay a regular coupon similar to the interest rate prevailing at the time of the bond's original sale, and then return the principal plus a final coupon at the end of the bond.

Coupon bonds pay not only at the final time.

Bonds are specified by their promised payout patterns.

Here is an example: Assume that a default-free coupon bond will pay \$1,500 each half-year for five years, plus an additional \$100,000 in 5 years. This payment pattern is so common that it has acquired some common terminology: A bond with coupon payments that remain the same for the life of the bond is called a **level-coupon bond**. The \$100,000 would be called the "principal," in contrast to the \$1,500 semi-annual coupon. Level bonds are named by just adding up all the coupon payments during the year (here, \$3,000), and dividing these total annual coupon payments by the principal. So this particular bond would be called a "3% coupon bond" (\$3,000 coupon per year, divided by the principal of \$100,000). This is just a naming convention for the bond. In Section 2·4, we called this an interest *quote*, as distinct from an interest *rate*. Even if the bond were to cost \$100,000 today (and we shall see below that it does not), the interest rate would of course not be 3% per annum, but $(1 + 1.5\%)^2 - 1 \approx 3.02\%$ per annum.

Today's Price of the Bond

Step 1: Write down the project's payment pattern.

What is the value of this "3% coupon bond" today? First, write down the payment structure for our 3% semi-annual coupon bond:

Year	Due Date	Bond Payment	Year	Due Date	Bond Payment
0.5	Nov 2002	\$1,500	3.0	May 2005	\$1,500
1.0	May 2003	\$1,500	3.5	Nov 2005	\$1,500
1.5	Nov 2003	\$1,500	4.0	May 2006	\$1,500
2.0	May 2004	\$1,500	4.5	Nov 2006	\$1,500
2.5	Nov 2004	\$1,500	5.0	May 2007	\$101,500

Step 2: find the appropriate costs of capital.

Second, find the appropriate expected rates of return to use for discounting. Because we assumed that our bond is default-free, it is just as good as a government bond (in our perfect world), which allows us to use the government yield curve to extract appropriate discount factors. Assume it is May 30, 2002, so we can return to our yield curve from Figure 3.1 on Page 49.

Maturity	Yield	Maturity	Yield	Maturity	Yield
3 Month	1.74%	2 Year	3.15%	5 Year	4.34%
6 Month	1.88%	3 Year	3.64%	10 Year	5.03%
1 Year	2.22%	4 Year	4.05%	30 Year	5.60%

To value our bond, we want to use the NPV formula, so we need to find the discount factors. We begin by computing the holding rates of returns from the yield curve, using the methods from Section 2·2. For example, the 6-month and 2-year holding rates of return are computed as

$$\begin{aligned} 1 + r_{0,0.5} &= (1 + 1.88\%)^{0.5} = (1 + 0.94\%) \\ 1 + r_{0,2} &= (1 + 3.15\%)^2 = (1 + 6.40\%) \\ 1 + r_{0,t} &= (1 + r_{\bar{t}})^t \quad . \end{aligned} \tag{5.18}$$

Eventually, you find the table of holding rates of return that corresponds to the yield curve,

Maturity	Yield	Maturity	Yield	Maturity	Yield
3 Month	not needed	2 Year	6.40%	5 Year	23.67%
6 Month	0.94%	3 Year	11.32%	10 Year	not needed
1 Year	2.22%	4 Year	17.21%	30 Year	not needed

But how do we obtain a holding rate of return for the coupon that will be paid in 18 months? We do not know the annualized 18-month interest rate, but we do know that the 1-year annualized interest rate is 2.22% and the 2-year annualized interest rate is 3.15%. So it is reasonable to guess that the 1.5 year annualized interest is roughly the average interest rate of the 1-year and 2-year annualized interest rates—about 2.7%. Therefore, we would estimate the 1.5 year holding rate of return to be

$$\begin{aligned} 1 + r_{0,1.5} &\approx (1 + 2.7\%)^{1.5} = (1 + 4.08\%) \\ 1 + r_{0,t} &= (1 + r_{\bar{t}})^t \quad . \end{aligned} \tag{5.19}$$

You have to do similar interpolations for the coupon payments in 2.5, 3.5 and 4.5 years. Collect this information—our payments and discount holding rates—into one table:

Year	Due Date	Bond Payment	Annual. Interest	Holding Interest
0.5	Nov 2002	\$1,500	1.88%	0.94%
1.0	May 2003	\$1,500	2.22%	2.22%
1.5	Nov 2003	\$1,500	≈2.7%	4.08%
2.0	May 2004	\$1,500	3.15%	6.40%
2.5	Nov 2004	\$1,500	≈3.4%	8.72%
3.0	May 2005	\$1,500	3.64%	11.32%
3.5	Nov 2005	\$1,500	≈3.8%	13.94%
4.0	May 2006	\$1,500	4.05%	17.21%
4.5	Nov 2006	\$1,500	≈4.2%	20.34%
5.0	May 2007	\$101,500	4.34%	23.67%

Our third step is to compute the discount factors, which are just $1/(1 + r_{0,t})$, and to multiply each future payment by its discount factor. This will give us the present value (PV) of each bond payment:

Step 3: Compute the discount factor is $1/(1 + r_{0,t})$.

Year	Due Date	Bond Payment	Annual. Interest	Holding Interest	Discount Factor	Present Value
0.5	Nov 2002	\$1,500	1.88%	0.94%	0.991	\$1,486.^{03}
1.0	May 2003	\$1,500	2.22%	2.22%	0.978	\$1,467.^{42}
1.5	Nov 2003	\$1,500	≈2.7%	4.08%	0.961	\$1,441.^{20}
2.0	May 2004	\$1,500	3.15%	6.40%	0.940	\$1,409.^{77}
2.5	Nov 2004	\$1,500	≈3.4%	8.72%	0.920	\$1,379.^{69}
3.0	May 2005	\$1,500	3.64%	11.32%	0.898	\$1,347.^{47}
3.5	Nov 2005	\$1,500	≈3.8%	13.94%	0.878	\$1,316.^{48}
4.0	May 2006	\$1,500	4.05%	17.21%	0.853	\$1,279.^{75}
4.5	Nov 2006	\$1,500	≈4.2%	20.34%	0.831	\$1,246.^{47}
5.0	May 2007	\$101,500	4.34%	23.67%	0.809	\$82,073.^{26}
					Sum	\$94,447.^{55}

Therefore, in an efficient market, we would expect this 3% semi-annual level-coupon bond to be trading for \$94,447.^{55} today. Because the current price of the bond is below the so-named final principal payment of \$100,000, our bond would be said to trade at a **discount**. (The opposite would be a bond trading at a **premium**.) Note again that \$1,500 is definitely not 1.5% of the \$94,447.^{55} purchase price of the bond today! The "3% coupon bond" is only a naming convention for the bond's payment pattern, not a statement about its interest rate.

Common naming conventions for this type of bond: coupon rate is not interest rate!

SIDE NOTE: In the real world, many corporations that issue bonds attempt to choose coupon rates similar to the prevailing interest rate, so that at inception, the bond will be trading at neither premium nor discount. So, at least for this one brief at-issue instant, the coupon rate and the interest rate may actually be close.

Bond Price Changes

Bond values change every day due to two causes.

But the bond price will change every day, both to reflect accumulated earned interest and to reflect changes in the yield curve. For a precise estimate, we must recompute the entire bond price again tomorrow, using tomorrow's yield curve. However, we already know that our bond's market price will follow a seesaw pattern: on ordinary non-coupon payment days, we would expect the bond price to appreciate by the amount of interest earned. That is, if I purchased the bond from you today and sold it back to you tomorrow, I would expect to receive a little bit more than I paid for it to compensate me for the time value of my money. However, on the instant that the $1,500 coupon is paid, in our perfect market, the price of the bond will have to drop by the amount of the coupon—if it were expected to drop by less than the $1,500 in this instant, you could purchase the bond just before the coupon is paid, pocket the $1,500, and then sell the bond again. Similarly, noone would be willing to hold the bond if it were expected to drop by more than $1,500.

Second, yield curve changes influence the bond price value.

The effects of unexpected changes in the yield curve are more interesting. Assume that the economy-wide yield curve has suddenly dropped across the board by an (enormous) 100 basis points each. Our revised yield curve is

Maturity	Yield	Maturity	Yield	Maturity	Yield
3 Month	0.74%	2 Year	2.15%	5 Year	3.34%
6 Month	0.88%	3 Year	2.64%	10 Year	4.03%
1 Year	1.22%	4 Year	3.05%	30 Year	4.60%

If you recall Section 3·4, your intuition should tell you that the value of our bond should go up when the interest rate goes down. Does it for our level-coupon bond? What will its new price be? The revised bond pricing table is

Year	Due Date	Bond Payment	Annual. Interest	Holding Interest	Discount Factor	Present Value
0.5	Nov 2002	$1,500	0.88%	0.44%	0.996	$1,493.^{44}
1.0	May 2003	$1,500	1.22%	1.22%	0.988	$1,481.^{92}
1.5	Nov 2003	$1,500	1.69%	2.54%	0.975	$1,462.^{87}
2.0	May 2004	$1,500	2.15%	4.35%	0.958	$1,437.^{52}
2.5	Nov 2004	$1,500	2.40%	6.10%	0.943	$1,413.^{82}
3.0	May 2005	$1,500	2.64%	8.13%	0.924	$1,387.^{21}
3.5	Nov 2005	$1,500	2.85%	10.32%	0.906	$1,359.^{72}
4.0	May 2006	$1,500	3.05%	12.77%	0.887	$1,330.^{15}
4.5	Nov 2006	$1,500	3.20%	15.20%	0.868	$1,302.^{05}
5.0	May 2007	$101,500	3.34%	17.85%	0.849	$86,123.^{91}
					Sum	$98,792.^{60}

Again, the bond price increase is equivalent to a reduction in the implied interest rate, and vice-versa.

As we had surmised, the reduction in the government interest rate has increased the value of our bond from $94,447.^{55} to $98,792.^{60}, which comes to a (one-instant) holding rate of return of $\$98,792.^{60}/\$94,447.^{55} - 1 \approx 4.6\%$. The value increase makes economic sense. Think of investors as deciding whether to put money into our bond or into government bonds. Previously, investing $100 into the alternative 1-year government bond would have yielded $2.^{22}, but it yields only $1.^{22} now. Our bond's unchanging, fixed promise of $1,500 is thus more valuable in competition with this government bond's promise. This is a general implication and worthwhile noting:

IMPORTANT: *The price and implied rate of return on a bond with fixed payments move in oppositive directions. When the price of the bond goes up, its implied rate of return goes down.*

In Chapter 8, we will discuss the yield-to-maturity of bonds, which is sort of an average interest rate implicit in the bond's payment; as well as duration, which is sort of an average payment date. You already have enough background to proceed to read this chapter. The next chapters will just cover some different applications of NPV first.

Solve Now!

Q 5.3 *A bond promises to pay \$160 in 12 months. The annual applicable interest rate is 5%. What is the bond's price today?*

Q 5.4 *Yale's school of management charges \$31,500 tuition per year. Assuming that the tuition stays the same, and that the one-year relevant interest rate (cost of capital) is 4%, what is the present value of the tuition payment occuring in one year?*

Q 5.5 *Yale's school of management charges \$31,500 tuition per year. Assuming that the tuition stays the same, and that the two-year relevant total holding interest rate (cost of capital; $r_{0,2}$) is 10%, what is the present value of the tuition payment occuring in two years?*

Q 5.6 *Yale's school of management charges \$31,500 tuition per year. What is the present value of a 2-year education, paid for in one year and in two years?*

Q 5.7 *Repeat these examples if Yale's tuition rises with inflation at 1.5% per year.*

Q 5.8 *Write down the NPV formula from memory.*

Q 5.9 *What is the NPV capital budgeting rule?*

Q 5.10 *If the cost of capital is 5%/yr in year 1, and 6%/yr in year 2, what is the discount factor for a year 2 cash flow?*

Q 5.11 *Interpret the meaning of the discount factor.*

Q 5.12 *Determine the NPV of the project in Table 5.1, if the annualized interest rate is 200 basis points higher in each year than stated.*

Q 5.13 *The discount rate is 15%/annum over all periods. Firm F's cash flows start with \$500 and grow at 20% per annum for 3 years. Firm S's cash flows also start with \$500 but shrink at 20% per annum for 3 years. What are the prices of these two firms, and what is the expected growth rate of your money that you would invest into these two companies?*

Q 5.14 *In the 3% coupon bond example, what would happen if each government interest rate on the yield curve would have gone up by 100 basis points, instead of down by 100 basis points?*

5·3. The Role of Perfect Capital Markets

Who owns a project is not important in a perfectly competitive zero-friction capital market.

In competitive zero-friction capital markets, the value of a project is its NPV and therefore does not depend on who owns it, and/or on when the owner needs cash.

5·3.A. Does *When* You Need Cash Influence Decisions and Value?

The Time Value of Money.

We already know about the time value of money, the fact that cash today is worth more than cash tomorrow. (A positive time value of money means nothing more than that the interest rate is positive.) If you do not agree—that is, if you do value money tomorrow more than you value money today—then just give it to me until you need it back. I will deposit it in my bank to earn the interest rate. In our ideal, perfect capital markets, of course, you can do better: you can shift money between periods at an "exchange rate" that reflects the time value of money.

Side Note: The fact that money tomorrow is worth more than money today is only true for nominal interest rates, because nominal interest rates are never negative. If there is inflation (Section 4·2), real interest rates can be negative. Thus, even if you value purchasing power tomorrow more than purchasing power today, you may not have a mechanism by which you can increase your purchasing power by saving instead of consuming. The time value of *real* money may thus be negative.

An "eager" consumer will take the project.

The shifting-at-will is worth illustrating. Assume that you have \$150 cash on hand and that you have exclusive access to a project that costs \$100, and returns \$200 next year. The appropriate interest rate (cost of capital) is 10%—but you *really*, *really*, *really* want to live it up today, because you know you will die on December 31. So, how much can you consume? And, would you take the project? Here is the NPV prescription in a perfect market:

An "eager beaver" consumer still takes any positive NPV project.

- Sell the project in the competitive market for its NPV:

$$-\$100 + \left(\frac{\$200}{1+10\%}\right) = \$82 \quad . \tag{5.20}$$

- Spend the \$150 + (\$182 − \$100) = \$232 today. You will be better off taking the project than consuming just your \$150 cash at hand.

A "sleeper" consumer also takes the project.

Now, let us assume that you are Austin Powers, the frozen detective, who cannot consume this year. So, how much will you be able to consume next year? And, would you take the project? The NPV answer is

- Sell the project in the competitive market for

$$-\$100 + \frac{\$200}{1+10\%} = \$82 \ . \tag{5.21}$$

- Put the \$82 into the bank for 10% today. Get \$90.^{20} next year.
- Also put your \$150 into the bank at 10% interest.
- Next year, consume \$90.^{20} + \$165 = \$255.^{20}.

The point of this argument is simple: regardless of when you need cash, you are better off taking all positive NPV projects, and then using the capital markets to shift consumption into your preferred time for consumption. It makes no sense to let *consumption decisions* influence *investment decisions*. This is called the **separation of decisions**: you can make investment decisions without concern for your consumption preferences. (This separation of investment and consumption decisions does not hold in imperfect markets, in which you can face different borrowing and lending interest rates, as we shall see next.)

The moral of the story: consumption and investment decisions can be separated in a perfect environment.

ANECDOTE: Neuroeconomics

Does economics make sense to humans? A new field called **neuroeconomics** runs experiments on individuals while their brain activity is being scanned. For example, subjects may be asked to evaluate a bet while they lie in a magnetic-resonance imager (MRI).

These tests can investigate if there is anything wrong with our argument that you should be indifferent between \$100 today and \$110 next year, given an interest rate of 10%. It turns out that if I promise you the former (\$100 today), a different section of your brain will spring into action than if I offer you the latter (\$110 next year). On our most basic brain level, we perceive the two differently. In our book, however, we will ignore this difference in perception—and perhaps try to retrain your brain to use the same regions for money today and money tomorrow, if appropriately translated using the prevailing interest rate.

Aside, neuroeconomics offers other fascinating results. We apparently get irrationally attached to what we consider to be "ours." When measuring happiness resulting from two different but identical bets,

> **Bet 1:** Receive \$10 first for free. A few minutes later, enter a bet in which you can lose \$10 or lose nothing;
> **Bet 2:** Receive \$10 with 50% probability and \$0 with 50% probability;

economists found that subjects are happier if they are asked to participate in bet 2 instead of bet 1. It turns out that risky gains activate a brain area in the ventromedial region and risky losses stimulate a brain area in the dorsomedial region. The ventromedial area is connected with emotional response, whereas the dorsomedial area is connected more with higher cognitive function.

(Incidentally, drug addicts, in particular, have especially high discount rates, not only for the addiction substance, but also for other substances.)

SIDE NOTE: Now, after they have lost their clients' money, many brokers like to confuse this basic truth by claiming that they invested their clients' money for the long-term, and not for the short-term. The fact is that no matter whether an investor needs money sooner or later, the broker should always purchase the highest NPV investments. This gives clients the most wealth today; if need be, investors could have saved the extra cash today for future consumption themselves.

5·3.B. Different Borrowing and Lending Interest Rates

Unfortunately, real-world markets are not always perfect, frictionless, and fully competitive. What can cause different expected borrowing and lending rates? For example, if there is only one bank that you can borrow from, this bank may have market power over you, and exploit it by charging a very high borrowing-lending interest spread. Similarly, if it is very difficult and costly to administer loans, banks may have no choice but to charge much higher lending rates. But perhaps most commonly, if you have no credit history and if your lender is afraid of not receiving the promised repayments—from the perspective of a bank, individual borrowers are often extremely high-risk—the lender will demand a high default premium (see Section 4·4). *You* may know that this rate of return is not justified, because *you* know that you will indeed return the lender's money—but you may not be able to convince the lender (it is in your interest to try to

Investors may face different savings and investment interest rates, especially if the lender does not believe he will be repaid, but the borrower knows he will repay.

do so!). From your perspective, the expected rate of return that the lender is asking, i.e., your cost of capital, is very high—borrowing money at an interest rate that includes a substantial default premium when you are certainly paying back the money is not an especially good deal for you. In this scenario, you cannot borrow at the same "low default" interest rate that you could earn if you invested into equivalently risky projects elsewhere. As in the two previous cases, your borrowing and lending rates are different.

Do not confuse different *promised* borrowing/lending rates (not discussed here) with different *expected* borrowing/lending rates (discussed here).

It is important that you do not confuse this scenario with the fact that you usually have to *promise* to pay the bank a higher interest rate than what you can obtain when you deposit money at the bank—as long as this interest rate differential merely reflects the appropriate premium for default and risk. After all, instead of putting your money into 100% safe government insured bank account, you could also invest the money you borrow from the bank into securities that are as risky as you are, which would also offer you a higher promised rate of return. So, the fact that your risky borrowing interest rate is higher than your 100% insured bank savings interest rate is not the issue discussed in this section. Instead, at issue are situations in which you have to pay a much higher *expected* rate of return than what you could earn by investing in equally risky securities.

If savings and investment interest rates differ, the project's value (NPV) depends on the owner's position

So what is the value of a simple project if you can lend money (invest cash) at 3%, and borrow money (receive cash) at 7%? For example, what is the value of a project that invests $1,000 today and returns $1,050 next period?

- If you have $1,000 and your alternative is to invest your money in the bank, you would only get $1,030 from the bank. You should take this project to earn $20 more than you could earn from the bank.
- If you do not have the $1,000, you would have to borrow $1,000 from the bank to receive $1,050 from the project. But because you will have to pay the bank $1,070, you would lose $20 net. You should not take the project.

The proper project decision (and value) now depends on how much cash you have. Taking current holdings into account when making investment choices can be much more difficult.

NERD NOTE: *The project's appropriate cost of capital includes both borrowed money and self-invested equity capital. In this example, we assumed that the project would be fully debt financed, so we just called the cost of capital the borrowing/lending rate.*

For large companies, a perfect market assumption with reasonably equal borrowing and lending rates is reasonable; for entrepreneurs, it is not.

For many large companies, the interest rate spread between similarly risky borrowing and lending rates is not excessive, so they can pretend they live in a perfect market in which they can separate the project choice from their financial situation.

Small businesses.

For many individuals and entrepreneurs, however, borrowing interest rates seem higher than saving interest rates. It is probably difficult for them to convey credibly their intent and ability to pay back their loans. (They also tend to have irrationally optimistic views of their prospects, which argues against this view.) Many entrepreneurs even resort to financing projects with credit cards, which can cost many hundred basis points above Treasury bonds. The high borrowing costs thus can prevent rational entrepreneurs from taking many projects that they would have undertaken if they had the money on hand. Put differently, richer entrepreneurs should optimally take more projects than poorer entrepreneurs. But be careful: as we noted above, the quoted spread between borrowing and lending rates has a large component that is due not to information disagreements, or loan administration frictions, or market-power by the bank, but that is simply due to **credit risk**, which was the subject of Section 4·4 and which will again be the subject in our next chapter.

In the next chapter, we will explain why an entrepreneur may have to promise to pay the bank 10%/year, although banks offer only a 3% rate of return on investments. The 10% *promised* rate of return may only be a 3% *expected* rate of return.

SIDE NOTE: Valuation services are an important revenue business for many finance professors and consulting firms. When asked to value small, non-public businesses—for example, for purposes of assessing the inheritance tax or in disputes among former business partners—it is customary and legally acceptable to first compute the value of an equivalent publicly traded business or company, and then to apply a "private discount" of around 10% to 30% in order to reflect the limited access to capital (because lenders tend not to believe in high default rates for young companies).

ANECDOTE: The Sopranos and Usury Laws
The official state of the Sopranos, New Jersey, has both a civil usury law (with a 16% limit) and a criminal usury law (with a 30% maximum for licensed lenders). But so-called payday lenders can and do legally charge interest rates that work out to about 400% per year. This is not far off from the going organized-crime Las Vegas loan rate of 5% *per week*.

Solve Now!

Q 5.15 *You have $500 and terminal cancer. You cannot wait until the following project completes: The project costs $400 and offers a rate of return of 15%, although equivalent interest rates are only 10%. What should you do?*

Q 5.16 *Your borrowing rate is 10%/year. Your lending rate is 4%/year. The indivisible project costs $1,000 and returns a rate of return of 8%. If you have $900 to invest, should you take the project?*

Q 5.17 *(Cont'd). You can think of the $900 as the amount of money that you are not consuming. Say, your wealth is $2,000, but in the previous question, you wanted to consume $1,100. How much could you consume and still want to take the project?*

5·4. Perpetuities and Annuities

"Perpetuities" and "Annuities" are projects with special kinds of cash flows, which permit the use of short-cut formulas.

Perpetuities and **Annuities** are projects that have quick NPV solutions, which are especially useful when you need to come up with quick rule-of-thumb estimates.

5·4.A. The Perpetuity Formula

Table 5.3. Perpetuity Stream of \$2 With Interest Rate $r = 10\%$

Time	Cash Flow	Discount Factor	Present Value
0	Nothing! You have no cash flow here!		
1	\$2	$1/(1+10\%)^1$	$\$1.^{82}$
2	\$2	$1/(1+10\%)^2$	$\$1.^{65}$
3	\$2	$1/(1+10\%)^3$	$\$1.^{50}$
.	...	...	...
50	\$2	$1/(1+10\%)^{50}$	$\$0.^{02}$
.	...	...	...
t	\$2	$1/(1+10\%)^t$	$\$2/(1+10\%)^t$
.	...	...	...
Net Present Value (Sum):			$= \$20.^{00}$

An Example Perpetuity that pays \$2 forever.

A simple perpetuity is a stream of cash flows that are the same for each period and continue *forever*. If the interest rate is also constant, there is a nice shortcut to computing the net present value of the perpetuity. For example, Table 5.3 describes a perpetuity paying \$2 forever if the interest rate is 10% per annum.

The Shortcut Perpetuity Formula.

To confirm the table's last row, which gives the perpetuity's net present value as \$20, you can spend from here to eternity to add up the infinite number of terms. But if you use a spreadsheet to add up the first 50 terms, you will get an NPV of $\$19.^{83}$. If you add up the first 100 terms, you will get an NPV of $\$19.^{9986}$. Trust me that the sum will converge to the formula

$$\begin{aligned} \text{Perpetuity NPV} &= \frac{\$2}{10\%} = \$20 \\ \text{NPV}_t &= \frac{\text{CF}_{t+1}}{r} \quad . \end{aligned} \tag{5.22}$$

The "t+1" in the formula indicates that the first cash flow begins the following period, not this period. (A proof of the perpetuity formula is in one of the exercises.)

IMPORTANT: *A perpetuity project that pays the same C dollars forever, beginning next period, and which is discounted at the same annual cost of capital forever, has a net present value today of*

$$PV_0 = \frac{C_1}{r} \quad . \tag{5.23}$$

ANECDOTE: The Oldest Institutions and Perpetuities
Perpetuities assume that projects last forever. But nothing really lasts forever. The oldest Western institution today may well be the Roman Catholic Church, which is of course about 2,000 years old. The oldest existing corporation in the United States is The Collegiate Reformed Protestant Dutch Church of the City of New York, formed in 1628 and granted a corporate charter by King William in 1696. The Canadian Hudson's Bay Company was founded in 1670, and claims to be the oldest continuously incorporated company in the world.

Guantanamo Naval Base was leased from Cuba in 1903 as a perpetuity by the United States in exchange for 2,000 pesos per annum in U.S. gold, equivalent to $4,085. In a speech, Fidel Castro has redefined time as "whatever is indefinite lasts 100 years." In any case, the Cuban government no longer recognizes the agreement, and does not accept the annual payments—but also dares not to try to throw out the Gringos.

5·4.B. The Growing Perpetuity Formula

Table 5.4. Perpetuity Stream With $C_{+1} = \$2$, Growth Rate $g = 5\%$, and Interest Rate $r = 10\%$

Time	Cash Flow	Discount Factor	Present Value
0	Nothing. You have no cash flows here.		
1	$(1+5\%)^0 \cdot \$2 = \$2.^{000}$	$(1+10\%)^1$	$\$1.^{818}$
2	$(1+5\%)^1 \cdot \$2 = \$2.^{100}$	$(1+10\%)^2$	$\$1.^{736}$
3	$(1+5\%)^2 \cdot \$2 = \$2.^{205}$	$(1+10\%)^3$	$\$1.^{657}$
.	... $\cdot$ ... = ...	...	...
30	$(1+5\%)^{29} \cdot \$2 = \$8.^{232}$	$(1+10\%)^{30}$	$\$0.^{236}$
.	... $\cdot$ ... = ...	...	...
t	$(1+5\%)^{t-1} \cdot \$2 =$	$(1+10\%)^t$	...
.	... $\cdot$ ... = ...	...	...
Net Present Value (Sum):	$\$2/(10\% - 5\%) =$		$= \$40.^{00}$

A growing perpetuity assumes that cash flows grow by a constant rate forever.

A generalization of the perpetuity formula is the **growing perpetuity** formula, in which the cash flows grow by a constant rate g each period. The cash flows of a sample growing perpetuity—which pays \$2 next year, grows at a rate of 5%, and faces a cost of capital of 10%—are in Table 5.4. The present value of the first 50 terms adds up to \$36.[28]. The first 100 terms add up to \$39.[64]. The first 200 terms add up to \$39.[98]. Eventually, the sum approaches the formula

$$\text{Growing Perpetuity} \qquad \begin{aligned} \text{NPV of Perpetuity}_0 &= \frac{\$2}{10\% - 5\%} = \$40 \\ \text{NPV of Perpetuity}_t &= \frac{\text{CF}_{t+1}}{r - g} \quad . \end{aligned} \tag{5.24}$$

As before, the "t+1" indicates that cash flows begin next period, not this period. Note that the growth timing occurs one period after the discount factor timing. For example, the time 30 discount factor is power-30, while the growth factor is power-29. The most common use of the growing perpetuity formula is when nominal project cash flows are assumed to grow by the rate of inflation. This will become clearer when we use this formula—and heavily so in the section on pro-formas (10·2).

IMPORTANT: *Cash flows that are growing at a rate of g each period and discounted at a constant interest rate r (which must be higher than g), are worth*

$$NPV = \frac{C}{r - g} \quad . \tag{5.25}$$

The first cash flow of C occurs next period, the second cash flow of $C \cdot (1 + g)$ occurs in two periods, and so forth, forever.

NERD NOTE: *What would happen if the cash flows grew faster than the interest rate ($g \geq r$)? The NPV in the perpetuities formulas is only less than infinity, because in today's dollars, each term in the sum is a little less than the term in the previous period. If g were greater than r, however, the cash flow one period later would be worth more even in today's dollars—and taking a sum over an infinite number of increasing terms would yield infinity as the value. A value of infinity is clearly not sensible, as nothing in this world is worth an infinite amount of money today. And, therefore, the growing perpetuity formula yields a non-sensical negative value if $g \geq r$, as it should.*

5·4.C. A Growing Perpetuity Application: Stock Valuation with the Gordon Dividend Growth Model

Perpetuities are imperfect approximations, but often give a useful upper bound.

With their fixed interest and growth rates and eternal payment requirements, perpetuities are rarely exactly correct. But they can be very helpful for quick estimations. For example, a stable business with profits of \$1 million next year, which is likely to grow at the inflation rate of, say, 2.0% and which has a nominal cost of capital of 8% should probably be worth no more than

$$\begin{aligned} \text{Business Value} &= \frac{\$1,000,000}{8\% - 2\%} \approx \$16.7 \text{ million} \\ \text{Business Value} &= \frac{C}{r - g} \quad , \end{aligned} \tag{5.26}$$

because the actual project will not exist forever. Of course, in real life, there are often even more significant uncertainties: next year's profit may be different, the firm may grow at a different rate (or may grow at a different rate for a while) or face a different cost of capital for 1-year loans than it does for 30-year loans. Thus, \$16.7 million should be considered a quick-and-dirty useful approximation, perhaps for an upper bound, and not an exact number.

The growing perpetuity model is sometimes directly applied to the stock market. For example, if you believe that a stock's dividends or earnings will grow by 5% forever, and if you believe that the appropriate rate of return is 10%, and you expect the stock to earn and/or pay dividends of \$10 next year, then you would feel that a stock price of

The "Gordon Growth Model": constant eternal dividend growth.

$$P_0 = \frac{C_0}{r-g} = \frac{\$10}{10\% - 5\%} = \$200 \tag{5.27}$$

would be appropriate. In this context, the growing perpetuity model is often called the **Gordon growth model**.

5·4.D. The Annuity Formula

An **annuity** is a stream of cash flows for a given number of periods. Unlike a perpetuity, payments stop after n periods.

An Annuity pays the same amount for T years.

For example, what is the value of an annuity that lasts for $n = 3$ periods, starting to pay \$5 next period? Let us first do this the slow way: we can simply hand-compute the net present value to be

Solving an example Annuity problem the slow way.

$$\begin{aligned} \text{NPV}_0 &= \frac{\$5}{1+10\%} + \frac{\$5}{(1+10\%)^2} + \frac{\$5}{(1+10\%)^3} = \$12.^{4343} \\ \text{NPV}_0 &= \frac{C_1}{(1+r_{0,1})} + \frac{C_2}{(1+r_{0,2})} + \frac{C_3}{(1+r_{0,3})} \quad . \end{aligned} \tag{5.28}$$

This is easy for three periods, but it does become tedious when the number of periods (n) is large. Just try valuing a 30 period annuity, or a 360 month annuity by hand.

So, what is the shortcut to compute the net present value of an annuity? It is the annuity formula

Using The Shortcut Annuity Formula

$$\text{NPV} = \text{CF}_{t+1} \cdot \left\{ \frac{1 - [1/(1+r)]^n}{r} \right\} \quad . \tag{5.29}$$

In our example in Formula 5.28, we had a cash flow of \$5 for 3 periods, so

$$\text{NPV} = \$5 \cdot \left\{ \frac{1 - [1/(1+10\%)]^3}{10\%} \right\} = \$12.^{4343} \quad , \tag{5.30}$$

which shows that Formula 5.29 works!

NERD NOTE: *If you are interested in the proof of the annuity formula, the NPV of an annuity is the NPV of one perpetuity starting now, minus the NPV of a perpetuity starting at time $T+1$. Incidentally, there is also a growing annuity formula, which nobody remembers. It is*

$$NPV = \frac{CF_{t+1}}{r-g} \cdot \left[1 - \frac{(1+g)^T}{(1+r)^T} \right] \quad . \tag{5.31}$$

Look it up when you need it.

ANECDOTE: Lottery Cheats
Lotteries and university fund-raisers are not so honest. If you promise to donate \$1,000 to a university for 100 years, they will announce this as a gift of \$100,000, instead of as a gift of about \$16,617 (at an interest rate of 6%). The \$40 million payout that the lottery advertises is also not paid out immediately, but over 20 years: so, it is really an annuity of \$2 million per year and thus worth only about \$23 million.

There are, however, a number of companies which, for a fee, are willing to accept the lottery payouts in exchange for a lump sum payment to the winner. Can you guess which formula they are using to compute how much cash to offer you upfront in exchange for the lottery payments?

5·4.E. An Annuity Application: Fixed-Rate Mortgage Payments

Mortgages are annuities, so the annuity formula is quite useful.

Fixed rate mortgage loans are basically annuities. They promise a specified stream of equal cash payments to a lender. A 30-year mortgage with monthly payments is really a 360-payments annuity. (The "annu-ity" formula should really be called a "month-ity" formula in this case.) So, assume that you want to take out a 30-Year Mortgage Loan for \$500,000 at an interest rate of 7.5% per annum. What would be your monthly payment?

Lenders quote interest rates using the same convention that banks use.

Before we can proceed further, you need to know one more bit of institutional knowledge here: Mortgage providers, like banks (see Page 44), quote interest by just dividing the mortgage quote by 12 and not by compounding, so the true monthly interest rate is $7.5\%/12 = 0.625\%$, and not $(1 + 7.5\%)^{1/12} - 1 = 0.605\%$.

The mortgage payment can be determined by solving the Annuity formula.

Because the 30-year mortgage is an annuity with 360 equal payments, you need to determine the fixed cash flow that has an NPV of \$500,000, equal to the amount of money you are borrowing:

$$\begin{aligned} \$500,000 &= \mathrm{CF}_{t+1} \cdot \left\{ \frac{1 - [1/(1 + 0.625\%)]^{360}}{0.625\%} \right\} \\ \mathsf{NPV}_t &= \mathrm{CF}_{t+1} \cdot \left\{ \frac{1 - [1/(1 + r)]^{n}}{r} \right\} \quad . \end{aligned} \tag{5.32}$$

Solving this for the cash flow tells you that the monthly payment on your \$500,000 mortgage will be \$3,496.^{07} for 360 months, beginning next month.

SIDE NOTE: Uncle Sam allows mortgage borrowers to deduct the interest, but not the principal, from their tax bills. Interest on the above mortgage is imputed as follows: In the first month, Uncle Sam proclaims $0.625\% \cdot \$500,000 = \$3,125$ to be the mortgage interest payment. Therefore, the principal repayment is $\$3,496.^{07} - \$3,125 = \$371.^{07}$ and remaining principal is \$499,628.^{93}. The following month, Uncle Sam proclaims $0.625\% \cdot \$499,628.^{93} = \$3,122.^{68}$ as the interest payment, $\$3,496.^{07} - \$3,122.^{68} = \$373.^{39}$ as the principal repayment, and \$499,255.^{54} as the remaining principal. And so on.

NERD NOTE: *In applications, the growing perpetuity and annuity formulas are flexible standards, but they should not be deemed immutable. There is no magical reason why cash flows in firms have to follow the exact time patterns assumed. One can think of other useful cash flow scenarios. For example, for other cash flow patterns, the formulas*

$$NPV = \frac{CF_{t+1}}{r - g + g/T} \quad \text{or} \quad NPV = \frac{CF_{t+1}}{r - g + g/\sqrt{T}}, \tag{5.33}$$

could be useful. They have an extra parameter T that measures the time over which growth is likely to last. If the user sets $T = 1$, growth ebbs instantly, and the formula is the perpetuity formula. If the user sets $T = \infty$, the formula is the growing perpetuity formula. This formula would be a compromise between eternal stagnation and eternal growth. Such choice may be what you want when valuing your company.

Supernerdnote: Both formulas are equivalent to working with lower growth rates in the plain perpetuity formula, i.e., replacing g by $g \cdot c$ where c is a constant that varies with the user-selected T ($c = 1 - 1/T$ and $c = 1 - 1/\sqrt{T}$).

Solve Now!

Q 5.18 *From memory, write down the perpetuity formula. Be explicit on when the first cash flow occurs.*

Q 5.19 *From memory, write down the growing perpetuity formula.*

Q 5.20 *If you can, from memory, write down the annuity formula.*

Q 5.21 *What is the NPV of a perpetuity paying \$5 each month, beginning next month, if the monthly interest rate is a constant 0.5%/month (6.2%/year)?*

Q 5.22 *A perpetuity will still be a perpetuity next period. But, given that one payment will be received, how can it still be worth the same? Take a 10% perpetuity for \$20, which is thus worth $\$20/10\% = \$200$. If you hold this for one year, what payments will you receive, and what will your remaining perpetuity (after the payment) be worth? Show that you get the same value today (i.e. \$200) if you discount the sum of these two to today. (This argument can be used to prove the perpetuity formula itself.)*

Q 5.23 *What is the NPV of a perpetuity paying \$5 each month, beginning this month (in 1 minute), if the monthly interest rate is a constant 0.5%/month (6.2%/year), and the cash flows will grow at a rate of 0.1%/month (1.2%/year)?*

Q 5.24 *What is the NPV of a 360 month annuity paying \$5 per month, beginning at \$5 next month, if the monthly interest rate is a constant 0.5%/month (6.2%/year)?*

Q 5.25 *What is the monthly payment on a 15-year mortgage for every \$1,000 of mortgage at a quoted interest rate of 6.2% per year (here, 0.5% per month)?*

Q 5.26 *Solve Fibonacci's annuity problem: Compare the NPV of a stream of quarterly cash flows of 75 bezants vs. the NPV of a stream of annual cash flows of 300 bezants. Payments are always at period-end. The interest rate is 2 bezants per month. What is the relative value of the two streams? Compute the difference for a 1-year investment first.*

Q 5.27 *In L'Arithmetique, written in 1558, Jean Trenchant posed the following question: "In the year 1555, King Henry, to conduct the war, took money from bankers at the rate of 4% per fair [quarter]. That is better terms for them than 16% per year. In this same year before the fair of Toussaints, he received by the hands of certain bankers the sum of 3,945,941 ecus and more, which they called 'Le Grand Party' on the condition that he will pay interest at 5% per fair for 41 fairs after which he will be finished. Which of these conditions is better for the bankers?" Translated, the question is whether a perpetuity at 4% per quarter is better or worse than a 41-month annuity at 5%.*

5·5. Summary

The chapter covered the following major points:

- The discounted present value (PV) of future cash flows is the most important concept in finance. The net present value (NPV) is the sum of all present values of a project.
- The NPV formula can be written as

$$\text{NPV} = C_0 + \frac{C_1}{1+r_{0,1}} + \frac{C_2}{1+r_{0,2}} + \ldots \quad . \tag{5.34}$$

  In this context, r is called the discount rate, $1/(1+r)$ is called the discount factor.
- The Net Present Value Capital Budgeting Rule states that you should accept projects with a positive NPV and reject projects with a negative NPV.
- Firms are worth the NPV of their assets. In a perfect market, the time pattern of cash flows and dividends can be costlessly rearranged.
- In a perfect market, consumption and investment decisions can be made independently. This breaks down if markets are not perfect: capital budgeting decisions can then depend on the cash position of the project owner.
- NPV can still be used if the market is not perfect, although it then requires special care in selecting the appropriate interest rate.
- The NPV of a growing perpetuity—with constant-growth (g) cash flows C beginning next year and constant annualized interest rate r—is

$$\text{NPV} = \frac{C}{r-g} \quad . \tag{5.35}$$

- The application of the growing perpetuity formula to stocks is called the Gordon dividend growth model.
- The NPV of an annuity—T periods of constant C cash flows (beginning next year) and constant annualized interest rate r—is

$$\mathsf{NPV} = \mathrm{CF}_{t+1} \cdot \left\{ \frac{1 - [1/(1+r)]^n}{r} \right\} \quad . \tag{5.36}$$

- Fixed-rate mortgage payments can be considered annuity payments.

A. Q&A: Answers

1. Definitely yes. Foregone salary is an opportunity cost that you are bearing. This can be reasonably estimated, and many economic consulting firms regularly opine thereon. As to non-monetary benefits, there is the reputation that the degree offers you, the education that betters you, and the consumption pleasure from the beer that fattens you.
2. Inflows: Value of Implicit Rent. Capital Gain if house appreciates. Outflows: Maintenance Costs. Transaction Costs. Mortgage Costs. Real Estate Tax. Theft. Capital Loss if house depreciates. And so on.
3. $\$150/(1+5\%) \approx \$142.^{86}$.
4. $\$31,500/(1+4\%) = \$30,288$.
5. $\$31,500/(1+10\%) = \$28,636$.
6. Add the previous two answers: $\$30,288 + \$28,636 = \$58,924$.
7. Assuming a 1.5% inflation rate, the tuition will be $\$31{,}972.^{50}$ next year; $\$32{,}452.^{09}$ the year after. This means that the present values are $\$30{,}742.^{32}$ and $\$29{,}501.^{90}$.
8. If you cannot do this by heart, do not go on until you have it memorized.
9. Accept if NPV is positive. Reject if NPV is negative.
10. $1/[(1+5\%)\cdot(1+6\%)] = 0.8985$.
11. It is today's value of one future dollar, i.e., at a specific point in time in the future.
12. $-\$900 + \$200/(1+7.0\%)^1 + \$200/(1+7.5\%)^2 + \$400/(1+8.0\%)^3 + \$400/(1+8.5\%)^4 - \$100/(1+9\%)^5 = 1.152$. The NPV is positive, therefore this is a worthwhile project.
13. F's cash flows are \$500, \$600, and \$720. Its value is therefore $\$1{,}361.^{88}$. Firm S's cash flows are \$500, \$400, and \$320. Its value is therefore $\$947.^{65}$. Both firms offer your investment dollar a 15% rate of return.
14. Our revised yield curve is

Maturity	Yield	Maturity	Yield	Maturity	Yield
3 Month	2.74%	2 Year	4.15%	5 Year	5.34%
6 Month	2.88%	3 Year	4.64%	10 Year	6.03%
1 Year	3.22%	4 Year	5.05%	30 Year	6.60%

Therefore,

Year	Due Date	Bond Payment	Annual. Interest	Holding Interest	Discount Factor	Present Value
0.5	Nov 2002	\$1,500	2.88%	1.43%	0.986	$\$1{,}478.^{86}$
1.0	May 2003	\$1,500	3.22%	3.22%	0.969	$\$1{,}453.^{21}$
1.5	Nov 2003	\$1,500	3.69%	5.59%	0.947	$\$1{,}420.^{65}$
2.0	May 2004	\$1,500	4.15%	8.47%	0.922	$\$1{,}382.^{84}$
2.5	Nov 2004	\$1,500	4.40%	11.37%	0.898	$\$1{,}346.^{91}$
3.0	May 2005	\$1,500	4.64%	14.58%	0.873	$\$1{,}309.^{18}$
3.5	Nov 2005	\$1,500	4.85%	18.03%	0.847	$\$1{,}270.^{87}$
4.0	May 2006	\$1,500	5.05%	21.78%	0.821	$\$1{,}231.^{71}$
4.5	Nov 2006	\$1,500	5.20%	25.62%	0.796	$\$1{,}1194.^{04}$
5.0	May 2007	\$101,500	5.34%	29.71%	0.771	$\$78{,}252.^{73}$
					Sum	$\$90{,}340.^{99}$

Therefore, the interest rate increase has reduced the value of our bond from $\$94{,}447.^{55}$ (see text) to $\$90{,}340.^{99}$, which is an instantaneous value loss of 4.35%.

15. Take the project. If you invest \$400, the project will give $\$400\cdot(1+15\%) = \$460$ next period. The capital markets will value the project at $\$418.^{18}$. Sell it at this amount. Thereby, you will end up being able to consume $\$500 - \$400 + \$418.^{18} = \$518.^{18}$.

16. You would have to borrow \$100 at an interest rate of 10% in order to take the project. If you take the project, you will therefore have $\$1,000 \cdot 1.08 - \$110 = \$970$ next period. If instead you invest \$900 at the 4% savings rate, you will receive only \$936. So, definitely take the project.

17. Say you invest I. If you put it into the bank, you receive $I \cdot (1 + 4\%)$. If you put I into the project, you receive $\$1,000 \cdot (1 + 8\%)$ from the project, borrow $(\$1,000 - I)$ at an interest rate of $(1 + 10\%)$. Therefore, you must solve

$$I \cdot (1 + 4\%) \;=\; \$1,000 \cdot (1 + 8\%) - (\$1,000 - I) \cdot (1 + 10\%) \tag{5.37}$$

The solution is $I = \$333.^{33}$, which means that if you want to consume more than \$1,666.^{66}, you should not take the project. Check: [1] If you consume \$1,700, you have a remaining \$300 to invest. The bank would pay \$312 next year. The project would pay off \$1,080, but you would have to borrow \$700 and pay back \$770, for a net of \$310. You should not take the project [2] If you consume \$1,600, you have a remaining \$400 to invest. The bank would pay \$416 next year. The project would pay off \$1,080, but you would have to borrow \$600 and pay back \$660, for a net of \$420. You should take the project.

18. CF_1/r. The first cash flow occurs next period, not this period.

19. $\mathrm{CF}_1/(r - g)$. The first cash flow occurs next period, not this period.

20. Remembering this formula is not as important as remembering the other growing perpetuity formula. The annuity formula is $\mathrm{CF}_1 \cdot \left\{ \frac{1 - [1/(1+r)]^n}{r} \right\}$.

21. $\mathsf{NPV} = \frac{\mathrm{CF}_1}{r} = \$5/0.005 = \$1,000$.

22. Next period, you will receive \$20, plus you will own a perpetuity *then* worth \$200. Your total is therefore \$220, and discounted at 10%, it is exactly worth the $\$200/(1 + 10\%) = \$200$ that the perpetuity formula gives today. So, the perpetuity formula is time-consistent. This argument can also be used to prove the perpetuity formula (by induction). The formula is

$$\frac{C}{1+r} + \frac{C}{(1+r)^2} + \cdots + \frac{C}{(1+r)^t} + \cdots = \frac{C}{r} \;. \tag{5.38}$$

We want to show that this is a true statement. Divide by C,

$$\frac{1}{1+r} + \frac{1}{(1+r)^2} + \cdots + \frac{1}{(1+r)^t} + \cdots = \frac{1}{r} \;. \tag{5.39}$$

Multiply (5.39) by $(1 + r)$

$$1 + \frac{1}{(1+r)} + \cdots + \frac{1}{(1+r)^{t-1}} + \cdots = \frac{(1+r)}{r} \;. \tag{5.40}$$

Subtract (5.40) from (5.39),

$$1 = \frac{(1+r)}{r} - \frac{1}{r} \tag{5.41}$$

which simplifies to be a true statement.

23. You get \$5 today, and next month you will receive a payment of $(1 + \pi) \cdot \mathrm{CF} = 1.001 \cdot \$5 = \$5.^{005}$. The growing perpetuity is worth $\mathsf{NPV} = \frac{\mathrm{CF}_1}{r - g} = \$5.^{005}/(0.5\% - 0.1\%) = \$1,251.^{25}$. So, the total value is \$1,256.^{25}.

24.
$$\mathrm{CF}_1 \cdot \left\{ \frac{1 - [1/(1+r)]^n}{r} \right\} = \$5 \cdot \left\{ \frac{1 - [1/(1+0.005)]^{360}}{0.005} \right\}$$
$$= \$5 \cdot \left\{ \frac{1 - 0.166}{0.005} \right\} \approx \$833.^{96}$$

25. For \$1,000 of mortgage, solve for CF_1 in

$$\mathsf{NPV} = \mathrm{CF}_1 \cdot \left\{ \frac{1 - [1/(1+r)]^n}{r} \right\}$$

$$\$1,000 = \mathrm{CF}_1 \cdot \left\{ \frac{1 - [1/(1+0.005)]^{15 \cdot 12 = 180}}{0.005} \right\}$$

$$\$1,000 = CF_1 \cdot 118.504 \iff CF_1 = \$8.^{43}$$

In other words, for every \$1,000 of loan, you have to pay \$8.[44] per month. For other loan amounts, just rescale the amounts.

26. For 1 year, the 300 bezants are worth $300/1.02^12 = 236.55$ bezants today. The quarterly interest rate is $1.02^3 - 1 = 6.12\%$. Therefore, the 4-"quartity" is worth $75/.0612 \cdot [1 - 1/1.0612^4] = 300/1.0612^1 + 300/1.0612^2 + 300/1.0612^3 + 300/1.0612^4 = 259.17$ bezants. The soldier would have lost 22.62 bezants, which is 8.7% of what he was promised. (The same 8.7% loss applies to longer periods.)

27. For each ecu (e), the perpetuity is worth $1e/0.04 = 25e$. The annuity is worth $1e/0.05 \cdot (1 - 1/1.05^{41}) = 17.29e$. Therefore, the perpetuity is better.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 6

Uncertainty and Errors in Capital Budgeting (NPV)

State Contingent Payoffs

last file change: Jun 14, 2004 (12:16h)

last major edit: March 30, 2004

Although net present value is conceptually easy, its inputs are not easily determined. The prime problem is how to deal with uncertain future outcomes when valuing projects *today*. This chapter is the second in exploring discounted cash flow valuation. It explains how NPV applies when future cash flows are uncertain, using an example that works out how borrowers and lenders must be compensated.

In Section 6·1, the goal is to learn to compute net present values with imperfect knowledge about future outcomes. The principal tool is the **payoff table** (or **state table**), which assigns probabilities to each possible future outcome. It is itself a major tool of finance. Section 6·2 introduces the **contingent claim**, i.e., a financial claim whose payoff depends on the not-yet-known future state. It values two contingent claims: a **loan** (**debt**) and **levered ownership** (also called **levered equity**). It shows that owning only the levered equity is riskier than owning the project outright. Finally, Section 6·3 shows how robust NPV is when you make a mistake.

Anecdote: Limited Liability in the 13th Century
In the 13th century, many trade ventures in Northern Italy were financed via so-called **commenda** contracts. An investor (**commendator**) financed most of the required capital, while an agent (**tractator**) contributed both effort and usually a little capital. The commenda choices were between a bilateral contract, in which the tractator and commedator were both similar investors, sharing profits and losses; and a unilateral contract, in which the tractator enjoyed some **limited liability**—that is, the investor's remaining wealth could not be touched—but smaller profits. (Limited liability is desirable if the outcome will be disaster, and not valuable if the outcome will be great.) To value claims with limited liability is the subject of this chapter.

6·1. Project Ownership and Payoff Tables

Most projects are financed with a mix of debt and equity.

Virtually all companies are financed with both debt and levered equity. The latter is simply what accrues to the business owner *after* the debt is paid off. But companies are not alone: more than likely, the house you are living in is financed with a mortgage. And if you have student loans, you *yourself* are only the levered owner of your future income stream. That is, you only get to consume the residual income *after* your loans are paid back. But what will the levered owner and the lender get if the company's projects fail, if the house collapses, or if your career takes a turn towards Rikers Island? How should they demand appropriate compensation? The split of net present value streams into loans and levered equity—to facilitate the asset purchase (for you, education!)—lies at the heart of finance.

The example of this section: A house in Tornado Alley can have one of two possible future values.

We will illustrate this division through the hypothetical purchase of a house for which the future value is uncertain. This house is peculiar, though: it is situated in Tornado Alley, and there is a 20% chance that a tornado will destroy it by next year. In this case, its value will only be the land—say, $20,000. Otherwise, with 80% probability, the house will be worth $100,000.

Table 6.1. House Payoff Table

Event	Probability	Value
Tornado	20%	$20,000
Sunshine	80%	$100,000

Side Note: Even though our neighborhood is hard-hit by tornadoes, at least they are timely! Fortunately, we can ignore that tornadoes could strike after the first year, too—the reason is that the house could be sold for $100,000 next year if no tornado has struck by then. Naturally, the $100,000 market value next year would itself be the result of many factors, including the possibility that a tornado might strike in subsequent years. (See also Page 481.)

To obtain the expected cash value of the house, multiply each (possible) outcome by its probability.

Table 6.1 shows the payoff table for full house ownership. The expected house value *next year* is therefore

$$\begin{aligned}\mathcal{E}(\text{Value}_{t=1}) &= 20\% \cdot \$20,000 \\ &+ 80\% \cdot \$100,000 = \$84,000 \\ &= \mathcal{P}rob(\text{Tornado}) \cdot (\text{Value if Tornado}_{t=1}) \\ &+ \mathcal{P}rob(\text{Sunshine}) \cdot (\text{Value if Sunshine}_{t=1}) \quad .\end{aligned} \tag{6.1}$$

Then discount back the expected cash value using the appropriate cost of capital.

Now, assume that the appropriate expected rate of return for a project of type "house" with this type of riskiness and with one-year maturity is 10%. Our goal is to determine the correct price for the house *today*.

SIDE NOTE: The 10% discount rate is provided by demand and supply in the financial markets. In a risk-neutral world, the average expected rate of return on each and every project is the same. In a risk-averse world, it depends on how the value of the house covaries with the value of other assets. This covariance and the consequent appropriate discount rate will be explained in Part III of this book. For now, asume the discount rate is known.

IMPORTANT: *In the presence of uncertain cash flows, in the NPV formula, actual future cash flows are replaced by expected discounted cash flows, and actual appropriate rates of return are replaced by appropriate expected rates of return.*

There are two methods to arrive at the correct result. The first method to compute today's present value is to take the expected value of the house next period and to discount it at 10 percent:

Under uncertainty, use NPV on expected (rather than actual, known) cash flows, and use the appropriate expected (rather than actual, known) rates of returns. The NPV principles remain untouched.

$$\begin{aligned} \mathsf{PV}_{t=0} &= \frac{\$84,000}{1+10\%} \approx \$76,363.^{64} \\ &= \frac{\mathcal{E}(\text{Value}_{t=1})}{1+\mathcal{E}(r_{t=0,1})} \quad . \end{aligned} \tag{6.2}$$

Table 6.2. House Payoff Table, Augmented

Event	Probability	Value	Discount Factor	⇒	PV
Tornado	20%	\$20,000	$^1/_{(1+10\%)}$	⇒	\$18,181.^{82}
Sunshine	80%	\$100,000	$^1/_{(1+10\%)}$	⇒	\$90,909.^{09}

The second method is to compute the discounted state-contingent value of the house, and then take expected values. To do this, augment Table 6.1. Table 6.2 shows that *if* the tornado hits, the present value is \$18,181.^{82}. *If* the sun shines, the present value is \$90,909.^{10}. Thus, the expected value of the house can also be computed

Taking expectations and discounting can be done in any order.

$$\begin{aligned} \mathsf{PV}_{t=0} &= 20\% \cdot \$18,181.^{82} \\ &+ 80\% \cdot \$90,909.^{09} \approx \$76,363.^{64} \\ &= \mathcal{P}rob(\text{Tornado}) \cdot (\text{PV of House if Tornado}) \\ &+ \mathcal{P}rob(\text{Sunshine}) \cdot (\text{PV of House if Sunshine}) \quad . \end{aligned} \tag{6.3}$$

Both methods lead to the same result: you can either first compute the expected value next year ($20\% \cdot \$20,000 + 80\% \cdot \$100,000 = \$84,000$), and then discount this expected value of \$84,000 to \$76,363.^{34}; or you can first discount all possible future outcomes (\$20,000 to \$18,181.^{82}; and \$100,000 to \$90,909.^{09}), and then compute the expected value of the discounted values ($20\% \cdot \$18,181.^{82} + 80\% \cdot \$90,909.^{09} = \$76,363.^{34}$.)

The state-contingent rates of returns can also be probability-weighted to arrive at the average (expected) rate of return.

What would the rates of returns be in both states, and what would be the overall expected rate of return? If you have bought the house for \$76,363.^{64}, and no tornado strikes, your actual rate of return (abbreviated $r_{t=0,1}$) will be

$$\text{if Sunshine:} \qquad r_{t=0,1} = \frac{\$100,000 - \$76,363.^{64}}{\$76,363.^{64}} \approx +30.95\% \ . \tag{6.4}$$

If the tornado does strike, your rate of return will be

$$\text{if Tornado:} \qquad r_{t=0,1} = \frac{\$20,000 - \$76,363.^{64}}{\$76,363.^{64}} \approx -73.81\% \ . \tag{6.5}$$

Therefore, the expected rate of return is

$$\begin{aligned} \mathcal{E}(\tilde{r}_{t=0,1}) &= 20\% \cdot (-73.81\%) \\ &+ 80\% \cdot (+30.95\%) = 10.00\% \\ \mathcal{E}(\tilde{r}_{t=0,1}) &= \mathcal{P}rob(\text{Tornado}) \cdot (r_{t=0,1} \text{ if Tornado}) \\ &+ \mathcal{P}rob(\text{Sunshine}) \cdot (r_{t=0,1} \text{ if Sunshine}) \ . \end{aligned} \tag{6.6}$$

The probability state-weighted rates of return add up to the expected overall rate of return. This is as it should be: after all, we derived the proper price of the house today using a 10% expected rate of return.

Solve Now!

Q 6.1 *What changes have to be made to the NPV formula to handle an uncertain future?*

Q 6.2 *Under risk-neutrality, a factory can be worth \$500,000 or \$1,000,000 in two years, depending on product demand, each with equal probability. The appropriate cost of capital is 6% per year. What is the net present value of the factory?*

Q 6.3 *A new product may be a dud (20% probability), an average seller (70% probability) or dynamite (10% probability). If it is a dud, the payoff will be \$20,000; if it is an average seller, the payoff will be \$40,000; if it is dynamite, the payoff will be \$80,000. What is the expected payoff of the project?*

Q 6.4 *(Continued.) The appropriate expected rate of return for such payoffs is 8%. What is the NPV of the payoff?*

Q 6.5 *(Continued.) If the project is purchased for the appropriate net present value, what will be the rates of return in each of the three outcomes?*

Q 6.6 *(Continued.) Confirm the expected rate of return when computed from the individual outcome specific rates of returns.*

6·2. Splitting Project Payoffs into Debt and Equity

6·2.A. The Loan

We now know how to compute the NPV of state-contingent payoffs—our house paid off differently in the two states of nature. We now generalize this slightly to more arbitrary state-contingent claims. Our house was a state-contingent claim—its payoff dependend on the outcome—but so is an asset that promises to pay $1 if the sun shines and $2 if a tornado hits. Debt and equity are the most important contingent claims.

State-contingent claims have payoffs that depend on future states of nature.

So we now asume you want to finance the house purchase of $76,363.[64] with a mortgage of $25,000. In effect, the single project "house" is being turned into two different projects, each of which can be owned by a different party. The first project is the project "Mortgage Lending." The second project is the project "Residual House Ownership," i.e., ownership of the house but bundled with the obligation to repay the mortgage. This "residual house ownership" investor will not receive a dime until *after* the debt has been satisfied. Such residual ownership is called the **levered equity**, or just the **equity**, or even the **stock** in the house, in order to avoid calling it "what's-left-over-after-the-loans-have-been-paid-off."

Asume the house is funded by a mortgagor and a residual, levered house owner.

Side Note: More commonly, house owners finance 80% of the purchase price with a mortgage. In this case, the house with a value of $76,363.[63] could be financed with a mortgage worth $61,090.[91] today.

What sort of interest rate would the creditor demand? To answer this question, we need to know what will happen if the house were to be condemned, because the mortgage value ($25,000 today) will be larger than the value of the house if the tornado hits ($20,000 next year). We are assuming that the owner could walk away from it and the creditor could repossess the house, but not any of the borrower's other assets. This arrangement is called **limited liability**. The house owner cannot lose more than the money that he originally puts in. Limited liability is a mainstay of many financial securities: for example, if you purchase stock in a company in the stock market, you cannot be held liable for more than your investment, regardless of how badly the company performs.

The first goal is to determine the appropriate promised interest rate on a "$25,000 value today" mortgage loan on the house.

Side Note: Such mortgages are called **no-recourse loans**. In many states, the law forces all initial mortgages to be no-recourse, but not refinanced mortgages. Nevertheless, in the real world, even in **full-recourse loan** situations, the lender will often permit the house owner to walk away from a house, if only because the legal process to obtain other assets of the borrower is so expensive that it is usually not worthwhile for mortgage providers to press such claims.

Table 6.3. Payoff to Mortgage Creditor, Providing $25,000 Today

Event	*Prob*	Value	Discount Factor
Tornado	20%	$20,000	$1/(1+10\%)$
Sunshine	80%	Promised	$1/(1+10\%)$

Start with the Payoff Table, and write down payoffs to project "Mortgage Lending."

To compute the NPV for the project "Mortgage Lending," we return to the problem of setting an appropriate interest rate, given credit risk (from Section 4·4). Start with the payoff table in Table 6.3. The creditor receives the property worth \$20,000 if the tornado strikes, or the full promised amount (to be determined) if the sun shines. To break even, the creditor must solve for the payoff to be received if the sun will shine in exchange for lending \$25,000 today. This is the "quoted" or "promised" payoff.

$$\begin{aligned} \$25,000 &= 20\% \cdot \left(\frac{\$20,000}{1+10\%}\right) \\ &+ 80\% \cdot \left(\frac{\text{Promise}}{1+10\%}\right) \\ \text{Loan Value}_{t=0} &= \mathcal{P}rob(\text{Tornado}) \cdot (\text{Loan PV if Tornado}) \\ &+ \mathcal{P}rob(\text{Sunshine}) \cdot (\text{Loan PV if Sunshine}) \ . \end{aligned} \tag{6.7}$$

The solution is a promise of

$$\begin{aligned} \text{Promise} &= \frac{(1+10\%)\cdot\$25,000 - 20\%\cdot\$20,000}{80\%} = \$29,375 \\ &= \frac{(1+\mathcal{E}(r))\cdot\text{Loan Value} - \mathcal{P}rob(\text{Tornado})\cdot\text{Value if Tornado}}{\mathcal{P}rob(\text{Sunshine})} \end{aligned} \tag{6.8}$$

in repayment, paid by the borrower only if the sun will shine.

The state-contingent rates of returns in the tornado ("default") state and in the sunshine state can be probability weighted to arrive at the expected rate of return.

With this promised payoff of \$29,375, if the sun will shine, the lender's rate of return will be the **promised rate of return**:

$$\text{if Sunshine:} \qquad r_{t=0,1} = \frac{\$29,375 - \$25,000}{\$25,000} = +17.50\% \ , \tag{6.9}$$

The lender will not provide the mortgage at a lower promised interest rate. If the tornado strikes, the owner walks away, and the lender's rate of return will be

$$\text{if Tornado:} \qquad r_{t=0,1} = \frac{\$20,000 - \$25,000}{\$25,000} = -20.00\% \ . \tag{6.10}$$

Therefore, the lender's expected rate of return is

$$\begin{aligned} \mathcal{E}(\tilde{r}_{t=0,1}) &= 20\% \cdot (-20.00\%) \\ &+ 80\% \cdot (+17.50\%) = 10.00\% \\ \mathcal{E}(\tilde{r}_{t=0,1}) &= \mathcal{P}rob(\text{Tornado}) \cdot (r_{t=0,1} \text{ if Tornado}) \\ &+ \mathcal{P}rob(\text{Sunshine}) \cdot (r_{t=0,1} \text{ if Sunshine}) \ . \end{aligned} \tag{6.11}$$

After all, in our risk-neutral environment, anyone investing for one year expects to earn an expected rate of return of 10%.

6·2.B. The Levered Equity

We now know how the lender must be compensated: we must promise \$29,375 next year for a mortgage that allows us to raise \$25,000 today. Our interest now turns towards proper compensation for ourselves—the expected payoffs and expected rate of return to the levered house owner.

Now compute the payoffs of the 60% post-mortgage (i.e., levered) ownership of the house.

Table 6.4. Payoff To Levered House (Equity) Owner

Event	$\mathcal{P}rob$	Value	Discount Factor
Tornado	20%	\$0.^{00}	$^{1}/_{(1+10\%)}$
Sunshine	80%	\$70,624.^{80}	$^{1}/_{(1+10\%)}$

The house is worth \$76,363.^{64} today. The lender provides \$25,000. Thus, as residual house owner, you need to pay \$51,363.^{64}—presumably from personal savings. If the tornado strikes, these savings will be lost and the lender will repossess the house. However, if the sun shines, the house will be worth \$100,000 minus the promised \$29,375, or \$70,625. The owner's payoffs in Table 6.4 help to determine that the *expected* future levered house ownership payoff is $20\% \cdot \$0 + 80\% \cdot \$70,625 = \$56,500$. Therefore, the present value of levered house ownership is

The method is exactly the same.

$$\begin{aligned} \mathsf{PV}_{t=0} &= 20\% \cdot \left(\frac{\$0.^{00}}{1+10\%}\right) + 80\% \cdot \left(\frac{\$70,625}{1+10\%}\right) \\ &= \mathcal{P}rob(\text{Tornado}) \cdot (\mathsf{PV}\text{ if Tornado}) + \mathcal{P}rob(\text{Sunshine}) \cdot (\mathsf{PV}\text{ if Sunshine}) \\ &= \$51,363.^{64} \quad . \end{aligned} \tag{6.12}$$

If the sun shines, the rate of return will be

Again, knowing the state-contingent cash flows permits computing state-contingent rates of returns and the expected rate of return.

$$\text{if Sunshine:} \qquad r_{t=0,1} = \frac{\$70,624.^{80} - \$51,363.^{63}}{\$51,363.^{63}} = +37.50\% \quad . \tag{6.13}$$

If the tornado strikes, the rate of return will be

$$\text{if Tornado:} \qquad r_{t=0,1} = \frac{\$0 - \$51,363.^{63}}{\$51,363.^{63}} = -100.00\% \quad . \tag{6.14}$$

The expected rate of return of levered equity ownership, i.e., the house with the bundled mortgage obligation, is

$$\begin{aligned} \mathcal{E}(\tilde{r}_{t=0,1}) &= 20\% \cdot (-100.00\%) \\ &+ 80\% \cdot (+37.50\%) = 10.00\% \\ \mathcal{E}(\tilde{r}_{t=0,1}) &= \mathcal{P}rob(\text{Tornado}) \cdot (r_{t=0,1}\text{ if Tornado}) \\ &+ \mathcal{P}rob(\text{Sunshine}) \cdot (r_{t=0,1}\text{ if Sunshine}) \quad . \end{aligned} \tag{6.15}$$

SIDE NOTE: A promised rate of return, as we define it, is simply the best ("promised") outcome rate of return. For bonds, we can immediately find the promised rate of return: it is the quoted rate of return. For levered equity, the promised rate of return, i.e., the best outcome rate of return, is neither obvious, nor commonly used. You can think of a promised equity rate of return as that which the seller of the levered equity would try to hype with. So, for levered equity, a promised rate of return is an unusual and solely theoretical concept only; for bonds, it is a practical concept—a number that is quoted as an interest rate in the real world.

6·2.C. Reflections On The Example: Debt and Equity Risk

Table 6.5. Payoff Splits

Event	*Prob*	House Value	Mortgage Value	Levered Ownership
Tornado	20%	\$20,000	\$20,000	\$0
Sunshine	80%	\$100,000	\$29,375	\$70,625
Expected Value$_{t=1}$		\$84,000	\$27,500	\$56,500
	NPV$_{t=0}$	\$76,364	\$25,000	\$51,364
	$\mathcal{E}(r_{t=0,1})$	10%	10%	10%

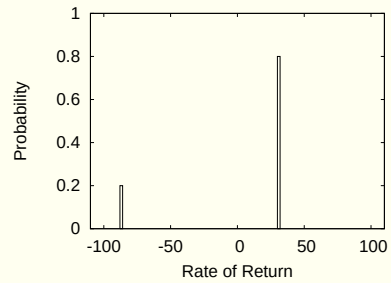
IMPORTANT: *Whenever possible, in the presence of uncertainty, write down a payoff table to describe the probabilities of each possible event ("state") with its state-contingent payoffs.*

There are three possible investment opportunities here. The bank is just another investor, with particular payoff patterns.

Table 6.5 summarizes the state-contingent payoffs, expected payoffs, net present value, and expected rates of return for the house scenario. Because owning the mortgage and the levered equity is the same as owning the full house, the last two columns must add to the values in the "House Value" column. You could decide to be any kind of investor: a creditor (bank), that is someone who is loaning money in exchange for promised payment; a levered house owner, that is, someone who is taking a "piece left over after a loan"; or an unlevered house owner, that is, someone who is investing money into an unlevered project. You might take the whole piece (that is, 100% of the claim; all three investments are just claims) or you might just invest, say \$5, at the appropriate fair rates of return that are due to investors in our perfect world, where everything can be purchased at or sold at a fair price. (Further remaining funds can be raised elsewhere.)

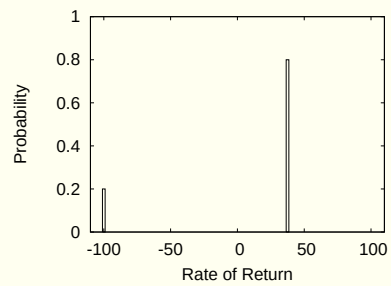
Figure 6.1. Probability Histograms of Project Returns

Histogram of Rate of Return for Project of Type "Full Ownership"



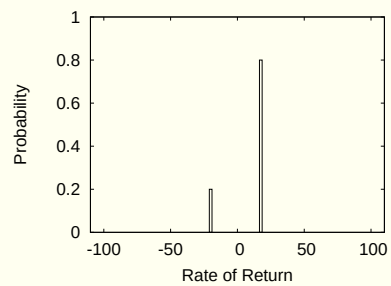
Histogram of Rate of Return for Project of Type "Levered Ownership"

(Most Risky)



Histogram of Rate of Return for Project of Type "Loan Ownership"

(Least Risky)



	20% probability Tornado	80% probability Sunshine	Expected Average
Loan (Ownership)	$\frac{\$20,000}{\$25,000} - 1 =$ -20.00%	$\frac{\$29,375}{\$25,000} - 1 =$ $+17.50\%$	$\frac{\$27,500}{\$25,000} - 1 =$ 10.00%
Levered Ownership	$\frac{\$0}{\$51,364} - 1 =$ -100.00%	$\frac{\$70,625}{\$51,364} - 1 =$ $+37.50\%$	$\frac{\$56,500}{\$51,364} - 1 =$ 10.00%
Full Ownership	$\frac{\$20,000}{\$76,364} - 1 =$ -73.81%	$\frac{\$100,000}{\$76,364} - 1 =$ $+30.95\%$	$\frac{\$84,000}{\$76,364} - 1 =$ 10.00%

NERD NOTE: *Chapter 14 introduces a measure of risk, the standard deviation (Sdv). The standard deviations of the rates of return for these three investments are:*

$$\begin{aligned}
\textit{Loan Ownership} \quad Sdv &= \sqrt{20\% \cdot (-20.00\% - 10\%)^2 + 80\% \cdot (17.50\% - 10\%)^2} &= 19.94\% \\
\textit{Levered Ownership} \quad Sdv &= \sqrt{20\% \cdot (-100.00\% - 10\%)^2 + 80\% \cdot (37.50\% - 10\%)^2} &= 59.04\% \\
\textit{Full Ownership} \quad Sdv &= \sqrt{20\% \cdot (-73.81\% - 10\%)^2 + 80\% \cdot (30.95\% - 10\%)^2} &= 46.07\%
\end{aligned} \tag{6.16}$$

Loan ownership is safest, levered ownership is riskiest, and full ownership is in between.

Leveraging (mortgaging) a project splits it into a safer loan and a riskier levered ownership, although everyone expects to receive 10% on average.

So, which investment is most risky: full ownership, loan ownership, or levered ownership? Figure 6.1 plots the histograms of the rates of returns to each investment type. As you can see, the loan is least risky, followed by the full ownership, followed by the levered ownership. Intuitively, by taking the mortgage, the medium risky project "house" has been split into a more risky project "levered house" and into a less risky project "mortgage." The combined "full house ownership" project has an average risk.

If everyone is risk-neutral, everyone should expect to earn 10%.

It should not come as a surprise to learn that all investment projects expect to earn a 10% rate of return. After all, 10% is the time-premium for investing money. Recall from Page 79 that the expected rate of return (the cost of capital) consists only of a time-premium and a risk premium. (The **default premium** is a component only of **promised interest rates**, not of **expected interest rates**; see Section 4·4). By assuming that investors are risk-neutral, we have assumed that the risk premium is zero. Investors are willing to take any investment that offers an expected rate of return of 10%, regardless of risk.

Unrealistic, maybe! But ultimately, maybe not.

Although this example is a little sterile because it has assumed away risk preferences, it is nevertheless very useful. Almost all projects in the real world are financed with loans extended by one party and levered ownership held by another party. Understanding debt and equity is as important to corporations as it is to house owners. After all, stocks in corporations are basically levered ownership claims that provide money only *after* the corporation has paid back its loans. The foregoing example gives you the skills to compute state-contingent, promised, and expected payoffs, and state-contingent, promised, and expected rates of returns—the necessary tools to work with debt, equity, or any other state-contingent claim.

Indeed, not one of these essential concepts and skills will change when we will introduce higher expected rates of returns for claims with more risk (i.e., when equity will command a higher *expected* rate of return than debt) in Section 21·5.A! We will again very much rely on payoff tables.

Solve Now!

Q 6.7 *In our example, the house was worth \$76,364, the mortgage was worth \$25,000 and the equity was worth \$51,364. The mortgage thus financed 32.7% of the cost of the house, and the equity financed \$67.3%. Is the arrangement identical to one in which two partners purchase the house together—one puts in \$25,000 and owns 32.7%, and the other puts in \$51,364 and owns 67.3%?*

Q 6.8 *For ilustration, we assumed that our sample house was not lived in. It consisted purely of capital amounts. But, in the real world, part of the return earned by a house owner is rent. Now assume that rent of \$11,000 is paid strictly at year-end, and that both the state of nature (tornado or sun) and the mortgage loan payment happens only after the rent has been safely collected. The new house has a resale value of \$120,000 if the sun shines, and a resale value of \$20,000 if the tornado strikes.*

1. *What is the value of the house today?*
2. *What is the promised interest rate for a lender providing $25,000 in capital today?*
3. *What is the value of residual ownership today?*
4. *Conceptual Question: What is the value of the house if the owner chooses to live in the house?*

Q 6.9 *Repeat the example if the loan does not provide $25,000, but promises to pay off $25,000. How much money do you get for this promise? What is the promised rate of return. How does the riskiness of the project "full house ownership" compare to the riskiness of the project "levered house ownership"?*

Q 6.10 *Repeat the example if the loan promises to pay off $20,000. Such a loan is risk-free. How does the riskiness of the project "full house ownership" compare to the riskiness of the project "levered house ownership"?*

Q 6.11 *Under risk-neutrality, a factory can be worth $500,000 or $1,000,000 in two years, depending on product demand, each with equal probability. The appropriate cost of capital is 6% per year. The factory can be financed with proceeds of $500,000 from loans today. What are the promised and expected cash flows and rates of return for the factory (without loan), for the loan, and for a hypothetical factory owner who has to first repay the loan?*

6·2.D. More Than Two Possible Outcomes

How does our example generalize to multiple possible outcomes? For example, assume that the house could be worth $20,000, $40,000, $60,000, $80,000, or $100,000 with equal probability, and the appropriate expected interest rate were 10%—so the house has an NPV of $\$60,000/(1+10\%) \approx \$54,545$.[45]. If a loan promised $20,000, how much would you expect to receive? But, of course, $20,000!

Multiple outcomes will cause multiple breakpoints.

$$\begin{aligned} \mathcal{E}(\text{ Payoff(Loan=\$20,000) }) &= \$20,000 \ . \\ \mathcal{E}\left(\begin{array}{c}\text{Payoff of Loan}\\ \text{if } \$0 \le \text{Loan} \le \$20,000\end{array}\right) &= \text{Loan} \ . \end{aligned} \tag{6.17}$$

If a loan promised $20,001, how much would you expect to receive? $20,000 for sure, plus the extra "marginal" $1 with 80% probability. In fact, you would expect only 80 cents for each dollar promised between $20,000 and $40,000. So, if a loan promised $40,000, you would expect to receive

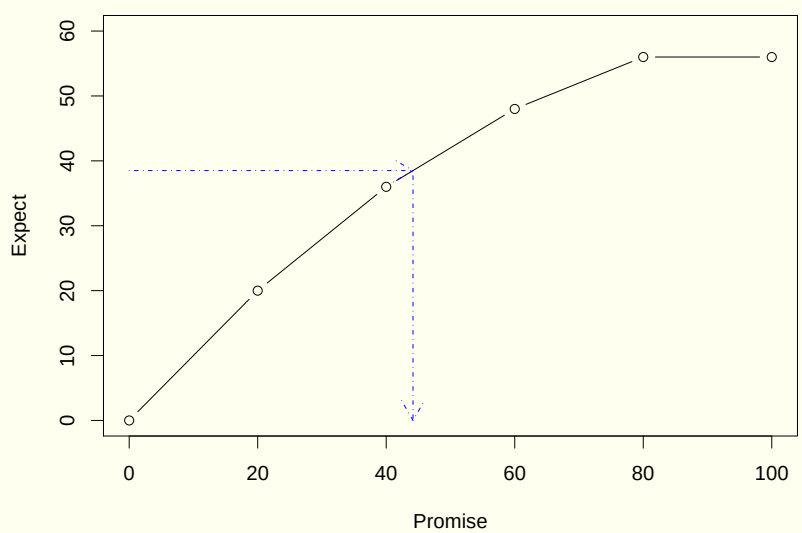
$$\begin{aligned} \mathcal{E}(\text{ Payoff(Loan = \$40,000) }) &= \$20,000 + 80\% \cdot (\$40,000 - \$20,000) = \$36,000 \ . \\ \mathcal{E}\left(\begin{array}{c}\text{Payoff of Loan}\\ \text{if } \$20,000 \le \text{Loan} \le \$40,000\end{array}\right) &= \$20,000 + 80\% \cdot (\text{Loan} - \$20,000) \ . \end{aligned} \tag{6.18}$$

If a loan promised you \$40,001, how much would you expect to receive? You would get \$20,000 for sure, plus another \$20,000 with 80% probability (which is an expected \$16,000), plus the marginal \$1 with only 60% probability. Thus,

$$\begin{aligned}
\mathcal{E}(\text{ Payoff(Loan = \$40,001) }) &= \$20,000 + 80\% \cdot (\$40,000 - \$20,000) \\
&\quad + 60\% \cdot \$1 = \$36,000.^{60} \ . \\
\mathcal{E}\left(\begin{array}{c}\text{Payoff of Loan} \\ \text{if } \$40,000 \le \text{Loan} \le \$60,000\end{array}\right) &= \$20,000 + 80\% \cdot \$20,000 \\
&\quad + 60\% \cdot (\text{Loan} - \$40,000) \ .
\end{aligned} \tag{6.19}$$

And so on. Figure 6.2 plots the expected payoffs as a function of the promised payoffs. With this figure, mortgage valuation becomes easy. For example, how much would the loan have to promise to provide \$35,000 today? The expected payoff would have to be $(1 + 10\%) \cdot \$35,000 = \$38,500$. Figure 6.2 shows that an expected payoff of \$38,500 corresponds to around \$44,000 in promise. (The exact number can be worked out to be \$44,167.)

Figure 6.2. Promised vs. Expected Payoffs



SIDE NOTE: We cannot borrow more than \$54,545, the project's NPV. So, we can eliminate the idea of obtaining a \$55,000 mortgage.

NERD NOTE: *In the real world, outcomes are rarely "one of five possible outcomes," although discrete simplifications of continuous outcomes will usually come fairly close to the right solution.*

The most common continuous distribution—although hard to apply here, because outcomes should never be negative—is the so-called gaussian-normal distribution, better known as the bell-shaped curve. For example, a project outcome distribution may be gaussian-normal with mean μ and variance σ^2. Then, the expected value of a debt contract promising D is

$$\mathcal{E}(D) = D - (D - \mu) \cdot CDF\left(\frac{D-\mu}{\sigma}\right) - \sigma \cdot PDF\left(\frac{D-\mu}{\sigma}\right) \tag{6.20}$$

*where CDF is the so-called cumulative normal distribution (*NORMDIST *in Excel with last argument true) and PDF is the so-called marginal normal distribution (*NORMDIST *in Excel with last argument false). For a project with an expected value of \$100 million and a standard deviation of* $\pm\$20$ *million, debt that promises to pay \$90 million expects to pay around* $\mathcal{E}(D) = \$90 - (\$90 - \$100) \cdot CDF(-0.5) - \$20 \cdot PDF(-0.5) = \$90 - (\$90 - \$100) \cdot 0.308 - \$20 \cdot 0.352 = \$86.04$ *million—which then needs to be discounted to today.*

How bad would we be if we approximated the bell shape with nine discrete outcomes, $\$70, \$80, \$80, \$100, \$100, \$100, \$120, \$120, \$140$*? (The mean is \$100 million, the standard deviation is* $\pm\$19.4$ *million.) We would get \$70 million 1/9 of the time, \$80 million 2/9 of the time, and \$90 million 6/9 of the time. The net sum would be \$85.6 million. The difference of \$0.5 million is modest, and swamped by the real-world uncertainty about μ and σ.*

The general method to handle any continuous distributions is exactly the same as it is for discrete distributions—creation of a graph mapping promised to expected payoffs, only we use integrals instead of summations. If $f(v)$ is the distribution of possible value outcomes v, and D is the promised debt payoff, then the expected debt payoff is

$$\mathcal{E}(\textit{Debt Payoff}) = \int_v \min(v, D) \cdot f(v)\, dv \quad . \tag{6.21}$$

The min *function parametrizes the debt payoff: if value v is less than D, you get v; if v is greater than D you get the (smaller) D. The levered equity's expected payoffs would be*

$$\mathcal{E}(\textit{Levered Equity Payoff}) = \int_v [v - \min(v, D)] \cdot f(v)\, dv \quad . \tag{6.22}$$

No normal business person would ever work with integrals for possible future business outcomes, though. Such aberrations are reserved for egghead finance professors and Wall Street "quants."

Solve Now!

Q 6.12 *What is the expected payoff if the promised payoff is \$45,000?*

Q 6.13 *What is the promised payoff if the promised payoff is \$45,000?*

Q 6.14 *What is the formula equivalent to (6.19) for promised payoffs between \$60,000 and \$80,000?*

Q 6.15 *Assume that the probabilities are not equal: \$20,000 with probability 12.5%, \$40,000 with probability 37.5%, \$60,000 with probability 37.5%, and \$80,000 with probability 12.5%.*

1. *Draw a graph equivalent to Figure 6.2.*
2. *If the promised payoff of a loan is \$50,000, what is the expected payoff?*
3. *If the prevailing interest rate is 5% before loan payoff, then how much repayment does a loan providing \$25,000 today have to promise? What is the interest rate?*

You do not need to calculate these values, if you can read them off your graph.

Q 6.16 *(More Difficult:) Can you work out the exact \$44,167 promise for the \$35,000 (today!) loan?*

Q 6.17 *A new product may be a dud (20% probability), an average seller (70% probability) or dynamite (10% probability). If it is a dud, the payoff will be \$20,000; if it is an average seller, the payoff will be \$40,000; if it is dynamite, the payoff will be \$80,000. The appropriate expected rate of return is 6% per year. If a loan promises to pay off \$40,000, what are the promised and expected rates of return?*

6·3. Robustness: How Bad are Mistakes?

How bad are mistakes?

Although it would be better to get everything perfect, it is often impossible to come up with perfect cash flow forecasts and appropriate interest rate estimates. Everyone makes errors. So, how bad are mistakes? How robust is the NPV formula? Is it worse to commit an error in estimating cash flows or in estimating the cost of capital?

6·3.A. Short-Term Projects

The benchmark case: A short-term project, correctly valued.

Assume that your project will pay off \$200 next year, and the proper interest rate for such projects is 8%. Thus, the correct project present value is

$$\mathsf{PV}_{\text{correct}} = \frac{\$200}{1+8\%} \approx \$185.^{19} \quad . \tag{6.23}$$

Committing an error in cash flow estimation.

If you make a 10% error in your cash flow, e.g., mistakenly believing it to return \$220, you will compute the present value to be

$$\mathsf{PV}_{\text{CF error}} = \frac{\$220}{1+8\%} \approx \$203.^{70} \quad . \tag{6.24}$$

The difference between \$203.70 and \$185.19 is a 10% error in your NPV.

In contrast, if you make a 10% error in your cost of capital (interest rate), mistakenly believing it to require a cost of capital (expected interest rate) of 8.8% rather than 8%, you will compute the present value to be

Committing an error in interest rate estimation.

$$\mathsf{PV}_{r\ \text{error}} = \frac{\$200}{1+8.8\%} \approx \$183.^{82} \quad . \tag{6.25}$$

The difference between \$183.82 and \$185.19 is less than a \$2 or 1% error.

IMPORTANT: *For short-term projects, errors in estimating correct interest rates are less problematic in computing NPV than are errors in estimating future cash flows.*

6·3.B. Long-Term Projects

Now take the same example, but assume the cash flow will occur in 30 years. The correct present value is now

A long-term project, correctly valued and incorrectly valued.

$$\mathsf{PV}_{\text{correct}} = \frac{\$200}{(1+8\%)^{30}} \approx \$19.^{875} \quad . \tag{6.26}$$

The 10% "cash flow error" present value is

$$\mathsf{PV}_{\text{CF error}} = \frac{\$220}{(1+8\%)^{30}} \approx \$21.^{86} \quad , \tag{6.27}$$

and the 10% "interest rate error" present value is

$$\mathsf{PV}_{r\ \text{error}} = \frac{\$220}{(1+8.8\%)^{30}} \approx \$17.^{52} \quad . \tag{6.28}$$

This calculation shows that cash flow estimation errors and interest rate estimation errors are now about equally important. So, for longer-term projects, estimating the correct interest rate becomes relatively more important. However, in fairness, estimating cash flows (and maybe even measuring appropriate interest rates) thirty years into the future is as just about as difficult as reading a crystal ball. Of course, as difficult as it may be, we have no alternative. We must simply try to do our best at forecasting.

Both cash flow and cost of capital errors are now important.

IMPORTANT: *For long-term projects, errors in estimating correct interest rates and errors in estimating future cash flows are both problematic in computing NPV.*

Solve Now!

Q 6.18 *What is the relative importance of cash flow and interest rate errors for a 10-year project?*

Q 6.19 *What is the relative importance of cash flow and interest rate errors for a 100-year project?*

6·4. Summary

The chapter covered the following major points:

- The key tool for thinking about uncertainty is the payoff table. Each row represents one possible state outcome, which contains the probability that the state will come about, the total project value that can be distributed, and the allocation of this total project value to different state-contingent claims. The state-contingent claims "carve up" the possible project payoffs.
- Most real-world projects are financed with the two most common state-contingent claims, debt and equity. The conceptual basis of debt and equity is firmly grounded in payoff tables. Debt financing is the safer investment. Equity financing is the riskier investment.
- If debt promises to pay more than the project can deliver in the worst state of nature, the debt is risky and requires a promised interest rate in excess of its expected interest rate.
- NPV is robust to uncertainty about the expected interest rate (the discount rate) for short-term projects. However, NPV is not robust with respect to either expected cash flows or discount rates for long-run projects.

A. Q&A: Answers

1. The actual cash flow is replaced by the expected cash flow, and the actual rate of return is replaced by the expected rate of return.
2. $\$750,000/(1+6\%)^2 \approx \$667,497.^{33}$.
3. $\mathcal{E}(P) = 20\% \cdot \$20,000 + 70\% \cdot \$40,000 + 10\% \cdot \$80,000 = \$40,000$.
4. $\$37,037$.
5. $\$20,000/\$37,037 - 1 = -46\%$, $\$40,000/\$37,037 - 1 = +8\%$, $\$80,0000/\$37037 - 1 = +116\%$.
6. $20\% \cdot (-46\%) + 70\% \cdot (+8\%) + 10\% \cdot (+116\%) = 8\%$.

7. No! Partners would share payoffs proportionally, not according to "debt comes first." For example, in the tornado state, the 32.7% partner would receive only $6,547.^{50}, not the entire $20,000 that the debt owner receives.
8.
   (a) In the sun state, the value is $120,000+$11,000= $131,000. In the tornado state, the value is $11,000+$20,000= $31,000. Therefore, the expected house value is $111,000. The discounted house value today is $100,909.^{09}.
   (b) Still the same as in the text: the lender's $25,000 loan can still only get $20,000, so it is a promise for $29,375. So the quoted interest rate is still 17.50%.
   (c) $\$100,909.^{09} - \$25,000 = \$24,899.^{09}$.
   (d) Still $100,909.^{09}, assuming that the owner values living in the house as much as a tenant would.

   Owner-consumed rent is the equivalent of corporate dividends paid out to levered equity. Note: you can repeat this example assuming that the rent is an annuity of $1,000 each month, and tornadoes strike mid-year.
9. In the tornado state, the creditor gets all ($20,000). In the sunshine state, the creditor receives the promise of $\$25,000$. Therefore, the creditor's expected payoff is $20\% \cdot \$20,000 + 80\% \cdot \$25,000 = \$24,000$. To offer an expected rate of return of 10%, you can get $\$24,000/1.1 = \$21,818$ from the creditor today. The promised rate of return is therefore $\$25,000/\$21,818 - 1 = 14.58\%$.
10. The loan pays off $20,000 for certain. The levered ownership pays either $0 or $80,000, and costs $\$64,000/(1+10\%) = \$58,182$. Therefore, the rate of return is either -100% or $+37.5\%$. We have already worked out full ownership. It pays either $20,000 or $100,000, costs $84,000, and offers either -73.81% or $+30.95\%$. By inspection, the levered equity project is riskier. In effect, house ownership has become riskier, because the owner has chosen to sell off the risk-free component, and retain only the risky component.
11. The expected factory value is $750,000. The discounted (today's) value is $\$750,000/1.06^2 = \$667,497.^{33}$. The promised value is $1,000,000. The loan must have an expected payoff of $1.06 \cdot \$500,000 = \$561,800$ (6% expected rate of return, two years). Because the loan can pay $500,000 with probability 1/2, it must pay $623,600 with probability 1/2 to reach $561,800 as an average. Therefore, the promised loan rate of return is $\$623,600/\$500,000 - 1 = 24.72\%$ over two years (11.68% per annum). The levered equity must therefore pay for/be worth $\$667,497.^{33} - \$500,000.^{00} = \$167,497.^{33}$ (alternatively, levered equity will receive $\$1,000,000 - \$623,600 = \$376,400$ with probability 1/2 and $0 with probability 1/2), for an expected payoff of $188,200. (The expected two-year holding rate of return is $\$188,200/\$167,497 - 1 = 12.36\%$ [6% per annum, expected].) The promised rate of return is $(\$1,000,000 - \$623,600)/\$167,497.^{33} - 1 = 124.72\%$ (50% promised per annum).
12. From the graph, it is around $40,000. The correct value can be obtained by plugging into Formula (6.19): $39,000.
13. From the graph, it is around $55,000. The correct value can be obtained by setting Formula (6.19) equal to $55,000 and solving for "Loan." The answer is indeed $55,000.

14.
$$\begin{aligned}\mathcal{E}\left(\begin{array}{c}\text{Payoff of Loan}\\ \text{if } \$60,000 \le \text{Loan} \le \$80,000\end{array}\right) &= \$20,000\\ &+ 80\% \cdot \$20,000\\ &+ 60\% \cdot \$20,000\\ &+ 40\% \cdot (\text{Loan} - \$60,000)\ .\end{aligned} \tag{6.29}$$

15.

(a)

(b) The exact expected payoff is $1/8 \cdot \$20,000 + 3/8 \cdot \$40,000 + 1/2 \cdot \$50,000 = \$42,500$.

(c) The loan must expect to pay off $(1 + 5\%) \cdot \$25,000 = \$26,250$. Therefore, solve $1/8 \cdot \$20,000 + 7/8 \cdot x = \$26,250$, so the exact promised payoff must be $x = \$27,142.^{90}$.

16. The loan must yield an expected value of \$38,500. Set formula (6.19) equal to \$38,500 and solve for "Loan." The answer is indeed \$44,166.^{67}.

17. With 20% probability, the loan will pay off \$20,000; with 80% probability, the loan will pay off \$40,000. Therefore, its expected payoff is \$36,000. Its price is $\$36,000/(1 + 6\%) = \$33,962$. Therefore, the promised rate of return is $\$40,000/\$33,962 - 1 \approx 17.8\%$. The expected rate of return was given: 6%.

18. Consider a project that earns \$100 in 10 years, and where the correct interest rate is 10%.

- The correct PV is $\$100/(1 + 10\%)^{10} = \$38.^{55}$.
- If the cash flow is incorrectly estimated to be 10% higher, the incorrect PV is $\$110/(1 + 10\%)^{10} = \$42.^{41}$.
- If the interest rate is incorrectly estimate to be 10% lower, the incorrect PV is $\$100/(1 + 9\%)^{10} = \$42.^{24}$.

So, the misvaluation effects are reasonably similar at 10% interest rates. Naturally, percent valuation mistakes in interest rates are higher if the interest rate is higher; and lower if the interest rate is lower.

19. Although this, too, depends on the interest rate, interest rate errors almost surely matter for any reasonable interest rates now.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 7

Capital Budgeting (NPV) Applications and Advice

Tips and Tricks!

last file change: Jun 14, 2004 (17:53h)

last major edit: April 5, 2004

It is worthwhile repeating that the NPV formula itself is easy, but its application is hard. The correct inputs are difficult to estimate and mistakes can lurk in many corners.

This chapter tries to help the NPV user avoid common mistakes. To this end, the chapter covers a variety of topics relevant to the practical application of NPV. The first two sections discuss taxes and inflation. The remaining sections cover issues of project economics and common conceptual mistakes.

The NPV checklist in Appendix 2·1 will summarize most advice given in this chapter, as well as advice given later in the book.

7·1. Handling Taxes and Inflation

The most fundamental rule is to never mix apples and oranges.

The basic rule is: never mix apples and oranges. The beauty of NPV is that every project, every action is translated into the same units: today's dollars. Keep everything in the same units in the presence of taxes and inflation so that the NPV advantages are not negated.

7·1.A. Inflation

NPV under inflation: nominal cash flows must be discounted with nominal interest rates.

When you use the NPV formula, always discount nominal cash flows with nominal costs of capital, and real (inflation-adjusted) cash flows with real (inflation-adjusted) costs of capital. The nominal interest rates already reflect the fact that nominal cash flows next year are worth less than they are this year.

> **Important**: *Discount nominal cash flows with nominal interest rates. Discount real cash flows with real interest rates. Never discount nominal cash flows with real interest rates, or vice-versa.*

A Previous Example Revisited.

Return to our earlier "apple" example from Section 4·2. With 200% nominal interest rates and 100% inflation, the real interest rate is $(1 + 200\%)/(1 + 100\%) - 1 = 50\%$. What is the value of a project that gives 12 apples next year, given that apples cost \$1 each today and \$2 each next year?

There are two methods you can use.

1. Discount real cash flows (i.e., 12A) with the real interest rate. Thus, the 12 future apples are worth
$$\frac{\text{Real Cash Flow}_1}{1 + \text{real rate}_{0,1}} = \frac{12A}{1 + 50\%} = 8A \ , \tag{7.1}$$
in today's apples. Because an apple costs \$1 today, the eight apples are worth \$8.

2. Discount the nominal value of 12 apples next year ($\$2 \cdot 12 = \$24$) with the nominal interest rate. Thus, the 12 future apples are worth
$$\frac{\text{Nominal Cash Flow}_1}{1 + \text{nominal rate}_{0,1}} = \frac{\$24}{1 + 200\%} = \$8 \ . \tag{7.2}$$

Both methods arrive at the same result.

> **Nerd Note**: *The reason is that*
> $$\begin{aligned} NPV &= \frac{\$24}{1 + 200\%} = \frac{12A}{1 + 50\%} = \frac{12A \cdot (1 + 100\%)}{(1 + 50\%) \cdot (1 + 100\%)} \\ &= \frac{N}{1 + n} = \frac{R}{1 + r} = \frac{R \cdot (1 + \pi)}{(1 + r) \cdot (1 + \pi)} \ , \end{aligned} \tag{7.3}$$
> *where N is the nominal cash flow, n the nominal interest rate, R the real cash flow, r the real interest rate, and π the inflation rate. The inflation rate cancels out.*

Usually, use nominal interest rates.

Most of the time, it is easier to work in nominal quantities. Nominal interest rates are far more common than real interest rates, and you can simply use published inflation rates to adjust the future price of goods to obtain future expected nominal cash flows.

Do not reduce expected cash flows by inflation when discounting with nominal interest rates.

A common mistake occurs when the project cash flows grow with inflation. More than one analyst has erroneously discounted nominally growing cash flows with real interest rates. For example, if inflation is 2% per year, the interest rate is 8% per year (for a real interest rate of 5.88%), and a given project has cash flows that grow at 1% faster than inflation forever, starting with \$20 next year, the *correct* NPV is

$$\begin{aligned} \text{NPV} &= \frac{\$20}{1+8\%} + \frac{\$20\cdot(1+3\%)}{1+8\%} + \frac{\$20\cdot(1+3\%)^2}{(1+8\%)^2} + \cdots \\ &= \frac{\$20}{8\%-3\%} = \$400 \quad . \end{aligned} \tag{7.4}$$

The project value is *neither* \$20 growing at the real rate of 1%, discounted by the nominal interest 8% (i.e., \$20/(8% − 1%)); *nor* \$20 growing at the nominal rate of 3% discounted by the real interest rate of 5.88% (i.e., \$20/(5.88% − 3%)).

Side Note: There is also a more mundane common error in the timing of the inflation rate in the first term of a perpetuity. For example, if the inflation rate is 2% per period, and the first cash flow *next period* were given as \$100 *in today's real dollars*, then the *nominal* cash flow next period will be \$102. To determine the NPV *in today's real dollars*, using only nominal quantities throughout (the nominal cash flow next year, the nominal interest rate, and as nominal growth rate the inflation rate), you would therefore use next year's nominal cash flow—a C of \$102—in the formula,

$$\text{NPV} = \frac{\$102}{8\%-2\%} = \$1,700 \quad . \tag{7.5}$$

and not \$100/6% ≈ \$1,666.[67].

7·1.B. Taxes

Compute everything in After-Tax Dollars!

You should care about cash that you can use for consumption. It should not matter whether you receive \$100 that has to be taxed at 50% or whether you receive \$50 that does not have to be taxed. As far as NPV is concerned, everything should be computed in after-tax dollars. This includes all cash flows, whether today or tomorrow, whether cash inflows or outflows.

Important: *Do all NPV calculations in after-tax money.*

You must compute the after-tax opportunity cost of capital.

Unfortunately, adjustments here are not as easy as they are for inflation, where you could simply work either in all-real or all-nominal inflation, and still get the same conclusion. For example, consider taking a project that costs \$10,000 and returns \$13,000 next year. Your tax rate is 40%, and 1-year equivalently risky bonds return 25% if their income is taxable, and 10% if their income is not taxable. First, you must decide what your opportunity cost of capital is. Section 4·3 tells you that if you put \$100 into taxables, you will own \$125, but the IRS will confiscate (\$125 − \$100) · 40% = \$10. You will thus own \$115 in after-tax income. Tax-exempts grow only to \$110, so you prefer the taxable bond—it is the taxable bond that determines your opportunity cost of capital. Your equivalent after-tax rate of return is therefore 15%. This 15% is our after-tax "opportunity" cost of capital.

You must apply it to the after-tax expected cash flows.

Return to your \$10,000 project now. Intuitively, your taxable project returns 30% taxable (\$3,000), while taxable bonds return 25% (\$2,500), so NPV should tell us to take this project. Uncle Sam will take $40\% \cdot \$3,000 = \$1,200$, leaving you with \$11,800. Therefore, the NPV of your project is

$$\begin{aligned} \text{NPV} &= -\$10,000 + \frac{\$11,800}{1+15\%} = \$260.^{87} \\ \text{NPV} &= \text{CF}_0 + \frac{\mathcal{E}(\text{CF}_1)}{1+\mathcal{E}(r_{0,1})} . \end{aligned} \tag{7.6}$$

It makes intuitive sense: if you had invested money into the bonds, you would have ended up with \$11,500. Instead, you will end up with \$11,800, the \$300 difference occurring next year. Discounted, the \$261 seems intuitively correct. Of course, there is an infinite number of ways of getting incorrect solutions, but recognize that none of the following calculations which use the pre-tax expected cash flows (and try different discount rates) give the same correct result:

$$\begin{aligned} \text{NPV} &\neq -\$10,000 + \frac{\$13,000}{1+25\%} = \$400 \\ \text{NPV} &\neq -\$10,000 + \frac{\$13,000}{1+15\%} = \$1,304.^{35} \\ \text{NPV} &\neq -\$10,000 + \frac{\$13,000}{1+10\%} = \$1,818.^{18} \end{aligned} \tag{7.7}$$

You have no choice: you cannot work with pre-tax expected cash flows. Instead, you need to go through the exercise of carefully computing after-tax cash flows and discounting with your after-tax opportunity cost of capital.

Do not forget that even when inflows require after-tax dollars, sometimes outflows are taxed again.

A common mistake is to ignore taxes or tax timing. An extreme personal finance case is a lottery ticket; you pay for the lottery ticket with post-tax money, but then are taxed on winnings (certainly, big winnings). Ignoring income tax can vastly overstate the NPV of a lottery ticket. Outside the personal realm, this situation often occurs when a long-term investment is compared to multiple short-term investments. For example, consider a farmer in the 40% tax bracket who purchases grain that costs \$300, and which triples its value every year.

- If the IRS considers this farm to be one long-term two-year project, the farmer can use the first harvest to reseed, so \$300 seed turns into \$900 in one year and then into a \$2,700 harvest in two years. Uncle Sam considers the profit to be \$2,400 and so collects taxes of \$960. Our farmer is left with post-tax profits of \$1,440.
- If the IRS considers this production to be two consecutive one-year projects, then the farmer ends up with \$900 at the end of the first year. Uncle Sam collects $40\% \cdot \$600 = \$240$, leaving the farmer with \$660. Replanted, the \$660 grows to \$1,980, of which the IRS collects another $40\% \cdot \$1,980 = \$792$. Our farmer is left with post-tax profits of $60\% \cdot \$1,980 = \$1,188$.

The discrepancy between \$1,440 and \$1,188 is due to the fact that the long-term project can avoid the interim taxation. Similar issues arise whenever an expense can be reclassified from "reinvested profits" (taxed, though often with some credit at reinvestment) into "necessary maintenance."

SIDE NOTE: When zero bonds were first introduced, the I.R.S. had no rules to guard against this problem, and zero bonds could defacto avoid interim taxation. Since then, the I.R.S. has created elaborate rules to prevent this tax arbitrage.

SIDE NOTE: You should particularly pay attention to getting taxes right if you are planning to undertake real estate transactions. These have special tax exemptions and tax depreciation writeoffs that are essential to getting the project valuation right.

Solve Now!

Q 7.1 *The inflation rate is 1.5% per year. The real rate of return is 2% per year. A perpetuity project that payed $100 this year will provide income that grows by the inflation rate. Show what this project is truly worth. Do this in both nominal and real terms. (Be clear on what never to do.)*

Q 7.2 *You are in the 33.33% tax bracket. A project will return $14,000 for a $12,000 investment—a $2,000 net return. The equivalent tax-exempt bond yields 15%, and the equivalent taxable bond yields 20%. What is the NPV of this project?*

Q 7.3 *Compare a 10-year zero bond and a 10% coupon bond, both paying 10%, with an appropriate (economy-wide) interest rate of 10%. If the I.R.S. does not collect interim interest on the zero bond, and the marginal tax rate is 25%, then what is the relative NPV of the two bonds?*

Q 7.4 *Assume you have both taxes and inflation. You are in the 20% tax bracket, and the inflation rate is 5%/year. A 1-year project offers you $3,000 return for a $20,000 investment. Taxable bonds offer a rate of return of 10%/year. What is the NPV of this project? Extra-credit if you can derive the formula yourself!*

7·2. Economics and Project Interactions

7·2.A. The Ultimate Project Selection Rule

Important: ***The Ultimate Project Selection Rule:*** *Consider all possible project combinations, and select the combination of projects that gives the highest NPV.*

There are too many possible action choices in the real world to evaluate (to compute NPV for). You need rules and heuristics!

This is easier said than done. It *is* easy for two projects: there are only four options to consider: take neither, take one, take the other, or take both. But the complexity quickly explodes when there are more projects. For three projects, there are eight options. For four projects, there are sixteen options. For ten projects, there are about a thousand options. For twenty projects, there are over a million options...and even the simplest corporate projects can easily involve hundreds of decisions that have to be made. (Nerds: the formula is 2^N, where N is the number of projects.) These choices do not even consider the fact that some projects may allow other projects to be added in the future, and that many projects are not just "accept" or "reject," but "how much project to take." The goal is therefore to find suitable **heuristics**, i.e., rules that are often (but not always) correct, and that simplify decisions.

A Greedy Algorithm?

One common algorithm is to consider project combinations, one at a time. Start with the project combination that, if you were only allowed to take two projects (one pair from a set of many different projects), would give you the highest NPV. Then take this pair as fixed, i.e., treat it as a single project. Now see which project adds the most value to your existing pair. Continue until adding the best remaining project no longer increases value. Computer scientists call this the **greedy algorithm**. It is a good heuristic, because it drastically cuts down the possible project combinations to consider, and usually gives a pretty good set of projects.

Nerd Note: *There are many possible enhancements to this algorithm, such as forward and backward iterations, in which one considers replacing one project at a time with every other option. Combinatorial enhancements are really the domain of* ***computer science*** *and* ***operations research****, not of finance.*

7·2.B. Project Pairs — A Classification

Project combinations can be classified into positive, zero, and negative interaction combinations.

By necessity, the most common and reasonable heuristic is to consider NPV projects or project combinations one by one, or at least category by category, and consider adding an additional project or a small set of projects, one at a time. Considering pairs of projects is common practice in many corporate situations. Even better, in many cases, one of these projects is already present, given, and unalterable, and just the addition of a single new project has to be considered. This simplifies the problem even further: it permits testing whether the new project adds or subtracts value from the total. In this case, economists use the **margin** concept—holding the existing project as it is, what is the *additional* contribution of the new project? The rest of this section focuses on tools to evaluate pairs of projects.

Economics can provide helpful insights and heuristics. We can decompose the overall net present value of two projects into three terms.

A basic taxonomy: zero, positive, and negative interaction projects.

$$\text{Overall NPV} = \text{NPV Project One} + \text{NPV Project Two} + \text{NPV Interactions} . \tag{7.8}$$

The final term suggests that we should classify project combinations into one of three different categories:

1. Projects with zero interactions.
2. Projects with positive interactions.
3. Projects with negative interactions.

Interactions are also sometimes called **externalities** in economics, because they are concerned with how one project has external influences on other projects—sometimes imposing external costs and sometimes providing external benefits. We now discuss each of these cases.

7·2.C. Zero Project Interactions

Most project payoffs in this world are **independent**, that is they have no mutual interactions. For example, the project payoffs of a mall in Connecticut have nothing to do with the project payoffs of a consumer's purchase of a computer in California. Independent project payoffs permit compartmentalization. This makes decision-making very easy:

Project independence is the most common case, and allows simple decision making.

- Taking each Positive NPV project increases firm value.
- Taking each Zero NPV project leaves firm value unchanged.
- Taking each Negative NPV project decreases firm value.

(These rules do not hold if projects have non-zero externalities, as we shall explain below.) Project NPV's are additive, because all cash flows have been translated into the same units, today's dollars, and the project interaction term is zero. Project independence makes decisions a lot easier: for twenty projects, only twenty independent decisions (accept or reject) have to be made, not a million.

A common mistake occurs when management incorrectly mingles the cost of capital of a potential acquisition target with their own cost of capital.

The most common confusion arises in complex projects and under risk aversion, e.g., in M&A, when acquirors use their own cost of capital, rather than project-specific costs of capital.

Project specific costs of capital can only be discussed in Part III, because in our risk-neutral world, all project costs are the same. This means that we have to delay illustrating this mistake until Page 419.

IMPORTANT: *You can evaluate zero interaction projects independently. Add NPV's.*

7·2.D. Positive Project Interactions

Positive interactions exist when taking one project increases the value of another project. Projects with positive interactions are often considered as "bundles." Indeed, in many cases, what makes a project a project in the firm's mind is the indivisibility of its components.

If one project has a positive influence on the net present value of another project, you cannot consider it without considering this positive influence. For example, consider creating a product as one project and an advertising campaign as another project. The advertising campaign project is of lesser use without a product, and the product is of lesser use without the advertising campaign. You must consider creating a product and an advertising campaign together. Such positive externalities are even more plentiful in smaller decisions: A secretary with word processing skills is less useful without a word processor, and a word processor is less useful without a secretary who can use it. A computer keyboard is less useful without a computer, and a computer is less useful without a keyboard.

SIDE NOTE: Some projects or products only make sense if purchased together. In this case, producers may bundle them together and/or purchasers may only buy them as bundles.

The ability to spread Fixed Costs is a common positive externality.

A common source of positive externalities in corporations arises from what economists call **economies of scale**: the more product the firm produces, the lower the per-unit production cost. For example, the fixed cost of software is the R&D to develop it. The variable cost is almost zero: burning another CD is almost free. As a consequence, R&D provides positive externalities for each project "customer." Software developers often consider producing either a large number of programs, or none at all. "Unused capacity" utilization is a closely related source of positive externalities: For example, a garage may be only used for half its space. Adding the project "another car" that could also park in the garage would reduce this car's depreciation. The garage has positive externalities on project "corporate cars."

IMPORTANT: *When deciding whether to take a project, credit all positive interactions to the project.*

Agency issues often prevent proper crediting.

Internal conflict and cost allocation procedures issues often hinder large corporations from taking advantage of many positive externalities. For example, laying a fast Internet connection to the factory could enhance more than just the division wanting to install it. The firm should factor in the increase in value to all divisions when deciding on the project "Internet Connection." However, in real life, other division managers might argue that they should not be charged for the Internet connection, because they did not request and therefore do not really need it (even if it were to increase their divisions' values): they prefer getting Internet for free instead of paying for it.

SIDE NOTE: Nowadays, managers who want to acquire other companies usually claim the presence of large positive externalities. **Synergies**, the managerial term for positive externalities between a potential acquisition target and their own firm, is an important managerial buzzword.

ANECDOTE: Synergies?
A google search of "unrealized synergies" yielded about 7,000 pages, while a search of "realized synergies" yielded about 38,000. Among the latter is a press release from September 4, 2001, in which Hewlett-Packard touts the potential benefits of a merger with Compaq:

> Cost synergies of approximately $2.0 billion are expected in fiscal 2003, the first full year of combined operations. Fully realized synergies are expected to reach a run rate of approximately $2.5 billion by mid-fiscal 2004. These anticipated synergies result from product rationalization; efficiencies in administration, procurement, manufacturing and marketing; and savings from improved direct distribution of PCs and servers.

Incidentally, most of the synergistic cost cutting will come from fewer employees—like yourself.

7·2.E. Negative Project Interactions

Projects can also have negative influences on one another, and thereby decrease one another's value. Economists sometimes call negative externalities **diseconomies of scale**. For example,

Negative interactions exist when taking one project decreases the value of another project. Pollution, cannibalization, and limited attention span are examples thereof.

Pollution and Congestion If there is only one major road to two divisions, and the traffic of one division clogs up the traffic to the other division, it can cause a loss of cash flow in the other division. A division that wants to expand and thereby clog up more of the existing infrastructure will not want to pay for the congestion cost that its own expansion will impose on the other divisions. (Of course, it is the overall firm's headquarters that should step in and allow the expansion only if the NPV is positive after taking into account the negative externalities imposed on other divisions.)

Cannibalization If a new Apple computer can produce more revenues than an older Windows machine, the Apple would eliminate the positive cash flows produced by the existing Windows machine. The cash flow of the project "replace Windows with Apple" is thus not just the $100,000 produced by the Apple, but only the $100,000 minus the $50,000 that the now unused Windows machine produced.

Bureaucratization and Internal Conflict If more projects are adopted, project management may find it increasingly difficult to make good decisions in a reasonable time frame. This may require more cumbersome bureaucracy and reduce cash flows for all other divisions.

Resource Exhaustion Perhaps the most common source of negative externalities—and often underestimated—is **limited attention span**. Management can only pay so much attention to so many different issues. An extra project distracts from the attention previously received by existing projects.

IMPORTANT: *When deciding whether to take a project, charge all negative interactions to the project.*

Agency issues often prevent proper costing.

Again, as in the case of positive externalities, agency issues and cost allocation systems often prevent proper accounting for negative externalities in the real world. Clearly, companies that are better at overcoming these issues will end up being more profitable.

7·2.F. One More Project: Marginal Rather Than Average Contribution

The Capital Budgeting Rule for one extra project requires taking all project interactions into account.

Usually, managers do not make the decision for all projects at the same time. Instead, many projects are already in place and—although existing projects should also constantly be evaluated in an ideal world—the manager has to make a decision about a single new project only. This new project may have positive or negative externalities on other existing projects.

IMPORTANT: *The decision on whether to take one additional project can be made by the rule*

Take New Project if

Total Firm NPV> with New Project	*Total Firm NPV without New Project*

This means that the single new project should be credited with any value increase or value decrease that it confers on other projects.

This is sometimes called "making decisions on the margin": the new project's costs and revenues are considered, including all extra value increases and decreases that it confers on other projects.

An example.

For example, consider a firm that can produce goods for an average price of

$$\text{Average Price Per Good} = \$4 + \frac{\$10}{x+1}, \tag{7.9}$$

so producing one good costs $\$4+\$10/(1+1) = \$9$/good, and one-hundred goods costs $\$4+\$10/(100+1) = \$4.^{10}$/good. The company is currently selling 5 goods domestically, each for a price of $\$8.^{00}$. It earns a

$$\text{Total Profit @ 5 items} = 5\cdot\$8 - 5\cdot\left[\$4 + \frac{\$10}{(5+1)}\right] = \$11.^{67}. \tag{7.10}$$

The company is considering a new foreign sales division that would cost \$16 to open, and that could sell another 5 units at \$8. The average price for the company producing 10 units would be $\$4+\$10/11= \$4.^{91}$/unit. Therefore, 5 units cost $\$24.^{55}$ to produce. The total cost of $\$16 + \$24.^{55} = \$40.^{55}$ exceeds the total profit of $5 \cdot \$8 = \$40$. If considered by itself, opening a foreign sales division is not a positive NPV project.

Now compute the total firm profit if the firm were to open the foreign sales division. 10 units would sell for a profit of \$80. Subtracting the opening costs of \$16 and production costs of $10 \cdot \$4.^{91} = \$49.^{19}$ would earn a

The foreign sales division also lowers the cost of domestic production!

$$\text{Total Profit @ 10 items} = 10 \cdot \$8 - 10 \cdot \left[\$4 + \frac{\$10}{(10+1)}\right] - \$16 \approx \$14.^{91} . \tag{7.11}$$

This is more than the $\$11.^{67}$ that the firm earned without the foreign sales division. The reason is that the foreign office has an additional marginal benefit: it reduces the average production cost experienced by the domestic office. This cost improvement is a positive externality that must be credited to the project—or the firm will make the bad decision of not opening the foreign division.

IMPORTANT: *When considering a project **on the margin** (i.e., extra), credit/charge to this project all externalities that this project conveys onto the existing firm.*

7·2.G. Sunk Costs

A **sunk cost** is a cost that the firm must incur (or has already incurred), whether or not the project is undertaken. Such a cost should not enter the decision process: it is no longer relevant. All that should matter is how much a project costs and contributes *on the margin*. For a sunk cost, there is no cost contribution on the margin.

A sunk cost is incurred whether the project is taken or not. It should have no influence.

For example, asume that a firm has spent \$100 million on a drug to cure ulcers. It will cost only \$1 million more to bring the drug to market. However, one researcher has just discovered that ulcers are caused by bacteria and thus can be cured with the firm's antibiotics. Should the firm spend the \$1 million to bring the original ulcer drug to market? The answer is no. The \$100 million has already been spent and it is sunk. The decision is now only whether to invest \$1 million, and it looks as if spending it would be a waste of money.

An example of a sunk cost that should be ignored.

ANECDOTE: Sinking Ulcer Medicines and Helicobactor Pylori
In the past, ulcers were a major problem in developing countries. Expensive drugs, such as bismuth subcitrate, could cure them, but 80% of patients would suffer a relapse within a year. In the early 1980s, Barry Marshall and Robin Warren, two researchers from the Australian hinterland, isolated *Helicobacter Pylori*—the bacterium that causes ulcers. It took years for the medical establishment to accept that bacteria could survive in the acidic environment of the stomach. Nowadays, a good dose of antibiotics can cure ulcers and expensive ulcer medications have become exotics. Even the best new ulcer medication, having cost billions of dollars in development, may no longer be worth even the \$1 million cost of drug introduction. The colloquial description for the sunk cost problem is "do not throw good money after bad (already lost) money."

A common mistake is to forget that "overhead" is often a sunk cost. For example, assume that a firm has spent \$500,000 on a computer that is currently idle half the time. It serves only one division. Assume that another division can take an additional project that produces \$60,000 in net present value, but that will consume twenty percent of the computer's time. Should the firm take this project? If twenty percent of the cost of the computer is allocated to this new project (i.e., $20\% \cdot \$500,000 = \$100,000$), the net present value of the new project would appear to be a negative $-\$40,000$. But the correct valuation is not to allocate the existing overhead to divisions. The \$500,000 on

Allocating already existing overhead to a project valuation is a common example of bad project decision making.

overhead has already been spent. The computer is a sunk cost—assuming that it really would sit idle otherwise and find no better purpose. It may seem unfair to charge only one division for the computer. But taking this additional project will produce $60,000 in profits without cost—clearly, a good thing.

Solve Now!

Q 7.5 *A company must decide if it should move division A to a new location. If division A moves, it will be housed in a new building that reduces its operating costs by $10,000 per year forever. The new building costs $120,000. Moving division A allows division B to expand within the old factory. This enables B to increase its profitability by $3,000 per year forever. If the discount rate is 10%, should division A move?*

Q 7.6 *A firm can purchase a new punch press for $10,000. The new press will allow the firm to enter the widget industry, and thereby earn $2,000 in profits per year forever. However, the punch press will displace several screw machines that produce $1,500 in profits per year. If the interest rate is 10%, should the new punch press be purchased?*

7·3. Other Errors

7·3.A. Expected, Typical, and Most Likely Scenarios

The NPV formula requires expected cash flows, not typical cash flows. The difference is low-probability events.

Under uncertainty, the NPV formula requires the **expected cash flows** in the numerator. A common decision error is to think of the *typical cash flow* or the *most likely cash flow*, instead. This can lead to failure to consider low-probability events: a plane crash, a legal suit, an especially severe recession, or a terrific new client.

An Example.

For example, a business may have the following payoffs:

Event	Probability	Value
Good Business	50%	$1,200,000
Normal Business	45%	$1,000,000
Lawyers Sue For Punitive Damages	5%	$-\$10,000,000$

The most likely payoff is $1,200,000. The typical payoff is $1,000,000. The expected payoff, however, is only

$$\begin{aligned} \mathcal{E}(\text{Payoff}) &= 50\% \cdot \$1,200,000 + 45\% \cdot \$1,000,000 + 5\% \cdot (-\$10,000,000) \\ &= \$550,000 \quad . \end{aligned} \tag{7.12}$$

It is the latter that is required in an NPV analysis. If you run this business 100 times, you would receive $1.2 million 50 times, $1 million forty-five times, and lose $10 million 5 times.

Nerd Note: *Statisticians call the most likely outcome the **Mode**, and the typical outcome the **Median**, and the expected (or average) outcome the **Mean**. If the statistical distribution is symmetric, e.g., as it is in the case of the so-called normal distribution, then the three statistics are all the same.*

7·3.B. Future Contingencies and Real Options

A **real option** (sometimes called **strategic option**) is the ability to change course in the future, depending on the economic environment at the time. For example, a business may expand its size, accelerate its production, and venture out into related or spin-off businesses, if the demand for its products increases, or if the costs of its inputs fall. Similarly, the firm may reduce, delay, or stop production if its economic environment deteriorates.

A real option is the value of flexibility to change course in the future.

For example, a factory may cost $1 million to build in year 1. In year 2, it can produce $2 million worth of inputs into $3 million worth of outputs. The expected profits next year of $1 million are no better than the actual cost today. From an NPV perspective, this factory does not appear worthwhile. Or does it?

An Example.

Table 7.1. State Contingent Factory Payoff Table

		Ignore Real Option	Recognize Real Option
	Prob	Dumb NPV	Smart NPV
Demand is low	1/2	$1,000,000	$0
		– $2,000,000	
		= –$1,000,000	$0
Demand is high	1/2	$5,000,000	$5,000,000
		– $2,000,000	– $2,000,000
		$3,000,000	$3,000,000
	Expected Value	$1,000,000	$1,500,000

If the product can be either in high demand (yielding $5 million) or in low demand (yielding $1 million)—both equally likely—the expected value of output can indeed be $3 million, as stated. This is what the firm expects to earn if managers always operate the factory, regardless of demand. But if managers can shut down the factory if demand is low, then owning the factory is similar to owning a contingent equity claim with limited liability (Chapter 6): you can get the upside with less downside. The payoff table in Table 7.1 shows that if the factory always runs, its expected net cash flows are less than if managers shut down the factory when demand is low. It is the managerial flexibility that increases the expected factory cash flow from $1 million to $1.5 million, which means that the factory is well worth building under reasonable cost-of-capital scenarios.

It is not the expected value that matters.

Strategic Options are everywhere.

Strategic options become both more important and more difficult to handle when there are many periods and many possible economic scenarios. Indeed, many projects are nothing but strategic options: For example, the value of unused land around cities is essentially the option that the city might expand enough to make building on the land economically worthwhile. Research and development often has no immediate usefulness, or even usefulness in the most likely scenario—but there is a chance that it might yield a highly profitable discovery. This strategic option value has to be properly considered in the expected cash flow computation, or the project value will be underestimated.

Strategic options are tough to value: use scenario analysis or Monte-Carlo simulations.

The proper valuation of strategic options is as important as it is difficult. The correct method is **scenario analysis**, conducted in a computer spreadsheet, in which the manager specifies different scenarios, one at a time, each resulting in its own cash flows. The appropriate project NPV is the average over many different scenarios—each with different probabilities and possibly different managerial responses. This is not easy. Think about how difficult it was just to figure out how many different project combinations you could choose *today*. Now, figure out what you could do in the future under an infinite variety of possible *future* scenarios—which you have to do when valuing strategic options. Out of necessity, most businesses just consider the most obvious strategic options.

NERD NOTE: *The solution process becomes very complex if your decisions in each time period cannot be made independently. For example, if it costs money to close and reopen a plant, then your decision to close the plant must also depend on your assessment of how quickly the product price will recover. If there is a good chance of recovery soon, you may choose to operate the plant even at a small loss right now.*

Monte-Carlo simulation is an automated form of scenario analysis, in which managers can specify a whole range of possible future scenarios and let the spreadsheet work out the expected outcome under many different scenarios. The Monte-Carlo simulation computes the probability-weighted average over all scenarios. This method is beyond the scope of this chapter.

Real Options are the subject of a full web chapter.

ANECDOTE: **Monte-Carlo and Nuclear Bombs**
Although previously known, the Monte Carlo method itself was not thoroughly developed until about 1944. The real use of Monte Carlo methods as a research tool stems from work on the atomic bomb during World War II. This work involved a direct simulation of the probabilistic problems concerned with random neutron diffusion in fissile material. But even at an early stage of these investigations, von Neumann and Ulam improved the methods used in this particular form of Russian Roulette.

7·3.C. Human Estimation Biases

Inflows are usually not what they should be.

Most cash flow and cost-of-capital estimates rely on judgments. Unfortunately, it is often difficult to obtain accurate judgments. There are at least two problem areas, both of which have spawned entire research areas:

1. Our brains tend to commit systematic decision errors.
2. People often have incentives not to tell the truth.

Managers who do not recognize these biases will systematically make poor decisions.

Human Biases: There are literally dozens of well-known behavioral errors, but limited space allows us to highlight just three: **overconfidence**, **relativism**, and **compartmentalization**.

Innate Human Decision Biases cause predictable valuation mistakes.

Overconfidence is the tendency of people to believe that their own assessments are more accurate than they really are. In lab experiments, ordinary people are found to be dramatically overconfident. When asked to provide a confidence interval so that their estimates are contained in their confidence interval nine out of ten times, the typical accuracy ends up being only around five out of ten correct.

To test your own susceptibility to these problems, you can take a self-test at the book website, http://welch.som.yale.edu/book/.

ANECDOTE: Who is not overconfident?
Empirical studies show that the only population segments who are not overconfident are weather forecasters and clinically depressed patients.

ANECDOTE: Small Business Failures
In New York City, two of every five new restaurants close within one year. Nationwide, the best estimates suggest that about 90% of all restaurants close within two years. If successful, the average restaurant earns a return of about 10% per year. So, owners seem to lose money on average. So, why open yet another restaurant? We mentioned earlier (Page 97) that restauranteurs may just enjoy owning restaurants. But a more likely explanation is that restauranteurs are over-optimistic, and just do not realize how tough it is to profitably run a restaurant.

More generally, a **Small Business Administration** study of small business failures from 1989 to 1992 found that 33% of businesses failed within 2 years, 50% within 4 years, and 66% within 6 years. Yet in a survey of about 3,000 entrepreneurs, 81% of entrepreneurs believed that their chances of success were at least 70%, and 33% believed that they had zero chance of failure!

Relativism is the tendency of people to consider issues in scale when they should not. For example, most people are willing to drive 15 minutes to a farther store to save $20 on the purchase of groceries, but they would not be willing to drive 15 minutes to a farther car dealer to save $100 on the purchase of a new car. The money appears to be less in the context of a car purchase than in the context of a grocery purchase. This is flawed logic, equivalent to ignoring scale in comparing project IRR's (see Chapter 8). The project is the same in both cases: the cost is "driving 15 minutes extra," and its benefit is $50.

SIDE NOTE: When is the last time you drove 5 minutes to a farther gas station or waited 5 minutes extra in line for gas that was 5 cents a gallon less than the closer gas station? For a gas tank of 15 gallons, this is 75 cents. With 10 minutes round-trip distance, this comes to an hourly pay rate of $4.50. After reading this book, your hourly rate should be higher than that!

Compartmentalization Compartmentalization is the tendency of people to categorize into budgets. For example, most people are more inclined to spend on an area when the area has produced an unexpected windfall earlier. For example, winning a lottery at a baseball game makes winners more likely to purchase more baseball tickets, even though the project "baseball game" has not changed in profitability. Similarly, an unexpected loss may stop people from otherwise profitable investment. For example, if an individual loses his baseball game ticket, he is less likely to purchase a replacement, even though the cost and benefit of purchasing the ticket is the same as it was when the original ticket was purchased.

7·3.D. Incentive (Agency) Biases

For more detail, read the webchapter on "corporate governance," which is all about agency conflict.

So-called incentive problems arise when the information provider has incentives that are different from those of the project owner.

Although everyone has incentives to lie—or at least color the truth—corporations are especially rife with such distortions. Again, lack of space forces us to highlight just a few issues:

Competition for Capital For example, division managers want to obtain capital for their projects. A less optimistic but more accurate estimate of the project cash flows may induce headquarters to allocate capital to another division instead. Thus, division managers often end up in a race to make their potential projects appear most profitable.

Employment Concerns For example, scientists tend to highlight the potential and downplay the drawbacks of their areas of research. After all, not doing so may cost them their jobs.

Perks For example, division managers may enjoy perks, such as having their own secretaries or private planes. This may lead them to overstate the usefulness of the project "administrative assistance" or "private plane transportation."

Slack For example, division managers may want to hide the profitability of their divisions, fearing that excess profitability would be siphoned off into other divisions.

Reluctance to Take Risk For example, managers may not want to take a profitable NPV project, because they can only get fired if it fails—and may not be rewarded enough if it succeeds. A popular saying used to be "noone was ever fired for buying IBM," although these days Microsoft has replaced IBM.

Naturally, human biases can reinforce incentive problems: "wishful thinking" is a disease from which we all suffer. It comes about from self-rationalization of what is in our interest.

The best "solution" is ample skepticism and common sense.

There is no obvious solution to these decision bias problems. You must be skeptical of all estimates and judgments and take the biases and incentives of each information provider into account. Rewarding division managers for telling the truth may help—but such mechanisms are very difficult to implement and commit to in the real world. (Will you reward subordinates who tell you that they screwed up? Will they believe you will?) An independent assessment or audit often helps, too—but many consultants suffer from the same disease: they know that they are most likely to be rehired if they tell you what you want to hear.

Side Note: Long-term projects may pay off decades after their conception. It would be difficult to reward or punish managers decades after their decisions for bad decisions.

Solve Now!

Q 7.7 *A company rents 40,000 square feet of space and is using 30,000 square feet for its present operations. It wishes to add a new division that will use the remaining 10,000 square feet. If it adds the division, equipment will cost $210,000 once, and the operations will generate $50,000 in profits every year. Presently, the office staff costs $160,000 per year. However, the expansion requires a larger staff, bringing costs up to $180,000 per year. If the cost of capital* $r = 10\%$*, should the firm expand?*

Q 7.8 *The CEO projects earnings of $100 million next year. List three reasons why this is not a good input into an NPV valuation.*

Q 7.9 *List "strategic options" that a firm needs to incorporate in its project valuation.*

Q 7.10 *A business produces 100,000 gadgets, costing $1 each to produce and sellable for $1.80 each (last year and just now). To produce another 100,000 gadgets requires running the machine at night, which increases production costs from $1 to $2. The business can last for up to 2 years (but think about how you would solve this for 5 years). In every year, with 10% probability, the output price doubles; with 10% probability, the output price halves; with 80% probability, the price stays the same as in the previous year. Shutting down the factory for 1 year costs $9,000. Reopening it costs $10,000. The cost of capital is a constant 5% per year. What is the value of this factory? (This is a difficult problem, but unfortunately not an unrealistic one.)*

Q 7.11 *Describe common human decision biases, and how they are likely to bias NPV calculations.*

Q 7.12 *Describe common agency biases, and how they are likely to bias NPV calculations.*

7·4. Conclusion

The problem is tough! We can only offer some help.

No doubt about it: good capital budgeting is a difficult problem. Each subsection covered in this chapter can easily be expanded into a full chapter—some even into entire books. (One web chapter works out the valuation of real options in more detail.) There are pitfalls everywhere. To make the task a little easier, Appendix 2·1 contains an NPV checklist. In the end, though, capital budgeting is an art as much as a science and has to rely as much on common sense and intuition as it can rely on rules. The best analysis combines both.

7·5. Summary

The chapter covered the following major points:

- Always discount nominal cash flows with nominal interest rates, and real cash flows with real interest rates. The former is usually easier. Never mix real and nominal quantities.
- Compute NPV in after-tax dollars, using both after-tax cash flows and after-tax cost of capital. Further, realize that projects may suffer interim taxation.
- Attribute to each project's NPV its influence on other projects, both positive or negative. Unfortunately, political reasons inside corporations often make this difficult.
- Think in terms of marginal costs and benefits of a new project. Ignore sunk costs when making project choices.
- Use the *expected* cash flows, not the *most likely* cash flows in the NPV numerator.
- Do not forget about valuing "strategic options" (or "real options"), which is the value of your ability to change course depending on future conditions. It includes your flexibility to delay or accelerate projects, and to expand or shut down projects.
- Realize that common human and agency biases usually distort expected cash flow estimates.

A. Q&A: Answers

1. In nominal terms, the rate of return is $n_{0,1} = (1 + 2\%) \cdot (1 + 1.5\%) - 1 = 3.53\%$, the cash flow will be $\$101.^{50}$. Therefore, NPV $= \$101.^{50}/(3.53\% - 1.5\%) = \$5,000$. In real, inflation-adjusted terms, the rate of return is 2%, the $\$101.^{50}$ next year are still worth \$100 in today's dollars, so NPV $= \$100/2\% = \$5,000$. Never discount \$100 by 3.53%, or $\$101.^{50}$ by 2%.

2. Your opportunity cost of capital is determined by the tax-exempt bond, because $66.7\% \cdot 20\% < 15\%$. Your project's \$2,000 will turn into $66.7\% \cdot \$2,000 = \$1,334$ after-tax earnings, or \$13,334 after-tax cash flow. Therefore, your NPV is $-\$12,000 + \$13,334/(1 + 15\%) = -\$405.22$. Check: The after-tax rate of return of the project's cash flow are $\$13,334/\$12,000 - 1 \approx 11\%$. This is less than 15%. You are better off investing in tax-exempt bonds.

3. The coupon bond has an after-tax rate of return of 7.5%. Start with \$1,000 of money. Reinvestment yields an after-tax rate of return of 7.5% (\$75 in the first year on \$1,000). So, after 10 years, you are left with $\$1,000 \cdot 1.075^{10} = \$2,061$. In contrast, the zero bond has a single pre-tax payout of $\$1,000 \cdot (1 + 10\%)^{10} = \$2,593.^{74}$, for which the I.R.S. would collect $\$1,593.^{74} \cdot 25\% = \$398.^{43}$ in year 30, for a post-tax zero-bond payout of \$2,195. The tax savings on the zero bond are therefore \$134 in 10 years, or \$52 in present value.

4. What is our after-tax rate of return on taxable bonds? \$100 will grow to \$110 ($(1 + 10\%) \cdot \$100 = \$110$) pre-tax, minus the 20% what Uncle Sam collects. Uncle Sam takes $(1 + 10\%) \cdot \$100 = \$110$, subtracts \$100, and then leaves us with only 80% thereof:

$$\begin{aligned} r_{\text{after-tax}} &= \frac{80\% \cdot (\$110 - \$100)}{\$100} = 8\% \\ &= \frac{(1-\tau) \cdot (\mathrm{CF}_1 - \mathrm{CF}_0)}{\mathrm{CF}_0} , \end{aligned} \tag{7.13}$$

where τ is your tax-rate of 20%. $(\mathrm{CF}_1 - \mathrm{CF}_0)/\mathrm{CF}_0$ is the pre-tax rate of return, so this is just

$$r_{\text{after-tax}} = 80\% \cdot 10\% = (1-\tau) \cdot r_{\text{pre-tax}} . \tag{7.14}$$

Now, in pre-tax terms, our project offers a 15% rate of return. In after-tax terms, the project offers $80\% \cdot \$3,000 = \$2,400$ net return, which on our investment of \$20,000 is a 12% after-tax rate of return. (On the same \$20,000, the taxable bond would offer only $80\% \cdot (\$22,000 - \$20,000) = \$1,600$ net return (8%). So, we know that the NPV should be positive.) Therefore, the project NPV is

$$\begin{aligned} \mathsf{NPV} &= -\$20,000 + \frac{\$20,000 + 80\% \cdot (\$22,400 - \$20,000)}{1 + 8\%} \approx \$740.^{74} \\ &= \mathrm{CF}_0 + \frac{\mathrm{CF}_0 + (1-\tau) \cdot (\mathrm{CF}_1 - \mathrm{CF}_0)}{1 + r_{\text{after-tax}}} . \end{aligned} \tag{7.15}$$

I can now give you any other cash flows or interest rates, and you can easily substitute into these formulas to obtain the NPV.

5. The answer is yes, because it will cost the company \$120,000 to move division A. Moving saves \$10,000/10%= \$100,000 in division A costs and \$3,000/10%=\$30,000 in division B costs. The total savings are \$130,000 which is \$10,000 greater than the cost of the building.

6. The answer is no, because the press earns \$2,000/0.10=\$20,000. But the press costs \$10,000 to purchase and eliminates \$1,500/0.10=\$15,000 of profits from the screw machines. So the total cost of the press, including the \$15,000 in opportunity costs, is \$25,000. The project's value is $\$20,000 - \$25,000 = -\$5,000$.

7. Yes. The PV of the division's profits will be \$50,000/0.10=\$500,000. The division costs are \$210,000 for new equipment and \$20,000 per year in increased overhead. The PV of the increased overhead is \$20,000/0.10=\$200,000. So the total PV cost of the new division is $\$210,000 + \$200,000 = \$410,000$, and the PV of the benefits equal \$500,000.

8. First, it is probably the most likely outcome, not the expected outcome. It is probably more likely that the firm goes bankrupt due to totally unforeseen circumstances than it is likely that the firm will have a windfall. Second, the CEO has an incentive to distort the truth, and report better projections than are most likely. This is an agency problem. And, third, the CEO is probably subject to human biases, too.

9. See Page 153.

10. Problems like this need to be solved "backwards." You can start in period 2 with a prevailing price of $\$0.^{45}$, $\$0.^{90}$, $\$1.^{80}$, $\$3.^{60}$, or $\$7.^{20}$; and your factory can be either open or closed. In this final period,

    - If the price is $\$0.^{90}$ or lower, you definitely want to close the factory, because \$9,000 loss is better than \$10,000 loss. If the factory is already closed, lucky you.
    - If the price is $\$1.^{80}$ or higher, you definitely want the factory to be open, because \$80,000 profit fortunately outweighs all opening and closing costs. If the factory is already open, lucky you.

    Now consider what to do in year 1. If the price drops to $\$0.^{90}$, you have a decision to make: operate the factory for a year, hoping that the future will be better, or close the factory. Operating losses would be \$10,000. Closing immediately would cost only \$9,000. So, if you operate today, you incur an extra \$1,000 loss. In exchange, there is a 10% chance that the price will go back up, in which case you got lucky. In this case, you will have saved \$10,000 reopening costs. Thus, you are exactly indifferent between closing and operating if the price has dropped. (Of course, if the price is higher today, operating today is the correct choice.)
    The problem of determining optimal choices as a function of environmental variables can get incredibly complex very easily. Scenario analysis (or just plain real-world experience and intuition) is really the only analysis method. This goes beyond the scope of an introductory textbook.

11. See Page 155.

12. See Page 156.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 8

Other Important Capital Budgeting Topics

The Internal Rate of Return (IRR), Duration, and Other Capital Budgeting Rules

last file change: Jun 14, 2004 (13:29h)

last major edit: April 5, 2004

This chapter explains commonly used capital budgeting rules (methods to decide among projects) other than NPV. But the best prescription is simple: when correctly applied, "net present value" is *the* correct measure. Stick to it. Any valuation rule that comes up with a different solution is incorrect.

However, it is possible that another rule can also come up with the correct recommendation. (After all, even a stopped clock shows the right time twice a day!) *Only if* another rule gives the same answer as NPV are you permitted to use this other rule. In particular, there is one common method, which often—but not always—yields the same correct answer: the internal rate of return, abbreviated IRR. And IRR has its own advantages. It is used in other contexts, too, because it can be interpreted as a "sort of average rate of return" when a project has different cash flows at different times. In the context of bonds, it is called the **yield to maturity**.

8·1. The Internal Rate of Return (IRR)

Why study IRR if we already know the correct answer?

The one important alternative to net present value that typically gives the same answer as NPV is the **Internal Rate of Return**, often abbreviated as **IRR**. It is important to know what IRR is, if only because it is in common use.

Important: *The Internal Rate of Return is the quantity IRR, which, given a complete set of project cash flows, solves the NPV equation set to zero,*

$$0 = C_0 + \frac{C_1}{1+IRR} + \frac{C_2}{(1+IRR)^2} + \frac{C_3}{(1+IRR)^3} + \ldots \tag{8.1}$$

The IRR capital budgeting rule states that if a project's IRR is above its appropriate interest rate (cost of capital), it should be taken.

Computing an Example IRR.

For example, for a project with cash flows −\$100, +\$55, and +\$60.^{50} in successive years, you must solve the equation

$$-\$100 + \frac{\$55}{1+\text{IRR}} + \frac{\$60.^{50}}{(1+\text{IRR})^2} = 0 \tag{8.2}$$

for IRR. For this particular set of cash flows, the solution is an internal rate of return of 10%, because

$$-\$100 + \frac{\$55}{1+10\%} + \frac{\$60.^{50}}{(1+10\%)^2} = 0 \ . \tag{8.3}$$

How does one usually find IRR's (the 10%)? For now, trial and error—although spreadsheets can make this relatively easy. Try different values for IRR's and slide towards numbers that make the left-hand side of the IRR formula closer to zero.

Nerd Note: *IRR is determined by solving a polynomial equation, where the order is the number of time periods. For a two-period project, a quadratic equation needs to be solved. As you may know from algebra, quadratic equations have closed form solutions and can have 0, 1, or 2 solutions. For a ten-period project, you have an equation with up to a power 10 term, which typically has no closed form solution and which can have between 0 and 10 different valid solutions.*

The IRR Rule: Accept if the cost of capital is below the IRR.

As to the IRR capital budgeting rule, in our example, if the project's appropriate cost of capital is 9%, the project ought to be accepted; if the cost of capital is 11%, the project ought to be rejected.

IRR is a characteristic of a project's cash flows. (It is not an interest rate.)

Even though the internal rate of return is also quoted in percent and compared against an interest rate (the cost of capital or hurdle rate), in itself, IRR is *not* an interest rate. Instead, it is a characteristic of a project cash flow stream. A given cash flow stream directly implies an IRR. In fact, you can compute the IRR for a project, never having looked at financial markets, interest rates, or costs of capital. This is IRR's most important advantage over NPV: *t can be calculated before you know what the appropriate interest rate (cost of capital) is.* Armed with the IRR, you can then contact various capital providers to see if the project is worthwhile. Also, you can first investigate different project scenarios, delaying the determination of the appropriate cost of capital r for the project.

IMPORTANT: *The IRR is a characteristic of project cash flows. It is not an interest rate, even though the IRR is compared to an interest rate in the IRR capital budgeting rule.*

Typically, applying the IRR capital budgeting rule yields the same (correct) decision as the NPV capital budgeting rule. IRR is guaranteed to work if the first, up-front cash flow is a single negative number (an investment) followed only by positive cash flows (paybacks), or vice-versa. This is why IRR has survived as a common method for "capital budgeting": most projects have precisely such cash flow patterns. But, because you cannot do any better than doing right, always using NPV is safer. It is just that if you use IRR *correctly* and in the right circumstances, it can often work equally well and sometimes give you nice intuition.

IRR is safe to use when there is only one positive or only one negative cash flow.

8·2. Problems Of IRR

8·2.A. Computation Problems

Unfortunately, IRR has some problems. First, IRR is difficult to use when projects have both future positive and negative cash flows (there are often multiple and sometimes no solutions). For example, return to our earlier project on Page 100. There are at least two internal rates of return: $r = 8\%$ and $r = -79.6\%$ (round to -80%). Confirm this:

IRR can have multiple solutions.

$$\begin{aligned} -\$900 + \frac{\$200}{1+8\%} + \frac{\$200}{(1+8\%)^2} + \frac{\$400}{(1+8\%)^3} + \frac{\$400}{(1+8\%)^4} + \frac{-\$100}{(1+8\%)^5} &\approx 0 \ , \\ -\$900 + \frac{\$200}{1-80\%} + \frac{\$200}{(1-80\%)^2} + \frac{\$400}{(1-80\%)^3} + \frac{\$400}{(1-80\%)^4} + \frac{-\$100}{(1-80\%)^5} &\approx 0 \ . \end{aligned} \tag{8.4}$$

So, does this project yield an internal rate of return 8% or an internal rate of return of -80%? The fact is that both IRR's are valid according to the definition. Should you accept the project if the prevailing interest rate is 5%? The answer is not obvious, and we need to go back to the NPV rule to learn that the correct answer is yes.

And, what is the proper IRR of a project that yields \$10 today and \$20 tomorrow, and never demands an investment? There is no IRR that makes it zero. Or a project that costs \$10 today and \$20 tomorrow, and never yields a positive cashflow? These projects have no IRR solutions, though they are admittedly far-fetched. But neither does a project with cash flows -10, $+\$28$, and $-\$20$ in consecutive years, where the non-existence is not so obvious.

Some great (and some awful) projects have absolutely no IRR.

Table 8.1. IRR Calculation in Excel

	A
1	-1000
2	600
3	600
4	=IRR(A1:A3)

IRR can be difficult to compute.

A relatively minor problem is that finding the IRR of project cash flows is not easy. Excel's built-in equation-solver helps computing the IRR in a relatively painless manner. For example, Table 8.1 shows how to obtain an IRR in a simple example (cell **A3** will become 0.13). Again, if there are multiple or no solutions, the answer cannot be trusted—and Excel will definitely not warn you!

8·2.B. Comparison Problems

In capital budgeting, if the yield curve is not flat, it is not clear what the appropriate cost of capital r is (to which IRR should be compared to).

Second, when the term structure of interest rates is not flat (e.g., when the annualized two-year interest rate is different from the one-year interest rate), and the IRR lies between them, there is no rule as to which one to compare the IRR to. For example, the yield curve may have the 1-year interest rate at 10%, the 2-year interest rate at 9%, the 3-year interest rate at 8%, the 4-year interest rate at 7%, and the 5-year interest rate at 6%. Should you accept a 5-year project with an 8% IRR? After all, its IRR is above its 1-year cost of capital (interest rate) but below its 5-year cost of capital. There is no clear answer.

IRR has no concept of project scale.

Third, when projects are exclusive alternatives, the IRR rule can be misleading: would you prefer a project with a 100% IRR, or a project with a 10% IRR? What if the first project is an investment opportunity of \$5 (returning \$10), and the second project is an investment opportunity of \$1,000 (returning \$100)? The latter is the better project, even though its IRR is worse.

8·3. So Many Returns: The Internal Rate of Return, the Cost of Capital, the Hurdle Rate, and the Expected Rate of Return

It is time to recap four rates that are commonly used in finance: the **Internal Rate of Return**, the **Cost of Capital**, the **Hurdle Rate**, and the **Expected Rates of Return**. The differences are sometimes subtle, and finance professors tend to use these terms interchangeably—which is o.k. in many, but not all situations. So, here is a summary:

Finance professors like to use terms interchangeably

Internal Rate Of Return The internal rate of return is a feature of cash flows, and has nothing to do with capital markets. It can be calculated before the appropriate cost of capital is known. It is most different from the three rates below.

Cost of Capital The cost of capital is determined by the capital markets. It is the expected rate of return that investors demand in order to willingly give us money. Therefore, the cost of capital is sometimes called the "required expected rate of return."

If your project's actual expected rate of return is above the required expected rate of return, it is a positive NPV project. In competitive capital markets, loans are usually zero net present value (or the borrower or lender is giving away free money), which means that the terms "cost of capital," "required expected rate of return," and "the project's actual expected rate of return" can be used interchangeably.

> **Side Note**: The cost of capital is determined by competition among investors in the financial markets. However, if investors are risk-averse (see Part III), they demand higher expected rates of return for riskier projects, so the cost of capital depends on the project characteristics. (This is not the case in a world of risk-neutral investors, where the cost of capital is independent of project characteristics.)

Expected Rate of Return See description above.

Hurdle Rate The appropriate project hurdle rate is the expected rate of return above which management decides to accept and go forward with the project. It is set neither by the financial markets, nor by the project, but by management. Bad management could choose any arbitrary and even idiotic hurdle rates. Good management should accept all projects that have positive net present value.

Usually, this means that good managers should set a project's hurdle rate to be equal to the project's cost of capital, and management should then determine whether the project's IRR exceeds this hurdle rate.

If management makes smart decisions, one can use the phrases "hurdle rate," "cost of capital," and "required expected rate of return" interchangeably.

Solve Now!

Q 8.1 *A project has cash flows of -$100, $55, and $70 in consecutive years. What is the IRR?*

Q 8.2 *From memory, write down the equation that defines IRR.*

Q 8.3 *Give an example of a problem that has multiple IRR solutions.*

Q 8.4 *Give an example of a problem that has no IRR solution.*

8·4. The Bond Context

8·4.A. Yield-to-Maturity

The "Yield to Maturity" is the IRR for a bond.

You can think of bonds as investment projects with well-specified payments. As with ordinary projects, the cash flows of bonds can differ across periods. For example, a bond that costs \$100 today may promise coupon payoffs of \$10 next year, and \$35 in two years with a repayment of \$100. Is its rate of return 10%, 35%, or the average? The internal rate of return allows us to compute an interest rate that intuitively summarizes all cash payments into one convenient interest-like quantity. In this example,

$$-\$100 + \frac{\$10}{1+\mathsf{IRR}} + \frac{\$135}{(1+\mathsf{IRR})^2} = 0 \quad \Rightarrow \quad \mathsf{IRR} = 21.3\% \ . \tag{8.5}$$

In the context of bonds, the IRR is called the **Yield To Maturity** (**Y.T.M.**). The YTM is therefore a sort of average interest rate implicit in the bond's promised cash flows.

YTM is usually unique.

Fortunately, the "multiple solution" and "no solution" IRR problems described in Section 8·2 rarely matter in the bond context, because bonds usually have one big outflow upfront, followed by many future inflows. This guarantees that the IRR of a bond exists and is unique.

8·4.B. Duration

Maturity ignores interim payment structure.

A related problem is how we measure the "term length" of bonds. The final payment, i.e., the maturity, is flawed: zero bonds and coupon bonds may have the same maturity, but a high coupon bond could pay out a good amount of money early on. For example, a coupon bond could pay 99% in coupon in the first month, and leave 1% for a payment in 30 years. It would count as a 30-year bond, the same as a zero-bond that pays 100% in 30 years.

Duration is an "average" payout date.

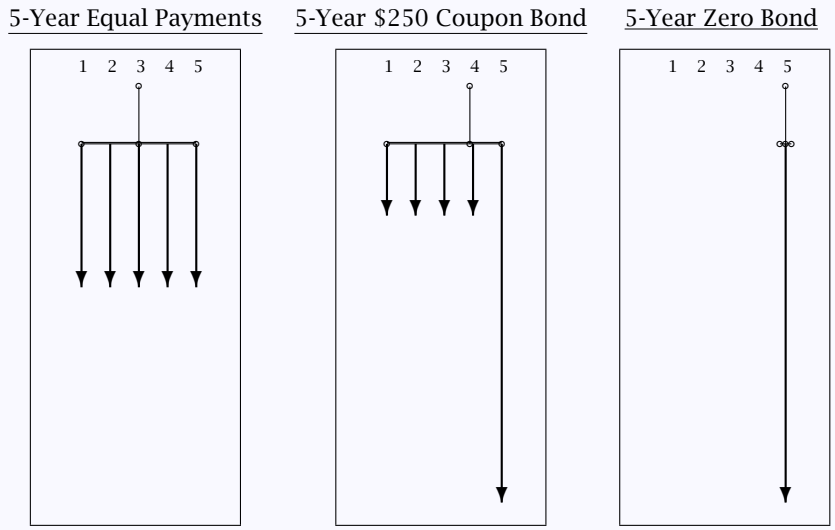
To measure the payout pattern of a bond, investors often rely on both maturity and **duration**—a measure of the *effective* time-length of a project. The simplest duration measure computes the time-weighted average of bond payouts, divided by the price. For example, a 5 Year Coupon Bond that pays \$250 for 4 years and \$1,250 in the fifth year, has a duration of 3.89 years:

$$\text{Plain Duration} = \frac{\$250\cdot 1 + \$250\cdot 2 + \$250\cdot 3 + \$250\cdot 4 + \$1,250\cdot 5}{\$250 + \$250 + \$250 + \$250 + \$1,250} = 3.89$$

$$\frac{\text{Payment at Time 1}\cdot 1 + \text{Payment at Time 2}\cdot 2 + \ldots + \text{Payment at Time T}\cdot T}{\text{Payment at Time 1} + \text{Payment at Time 2} + \ldots + \text{Payment at Time T}} \ . \tag{8.6}$$

The idea is that we now consider this 5-year coupon bond to be shorter-term than a 5-year zero bond (which has a 5-year duration)—and perhaps more similar to a 3.9-year zero bond.

SIDE NOTE: Duration is sometimes explained through a physical analog: If all payments were weights hanging from a (time) line, the duration is the point where the weights balance out, so that the line tilts neither right nor left.



Macauley Duration improves on plain duration by using the present value of payouts, not just nominal payouts. Thus, it also depends on the prevailing yield curve. If the interest rate on all horizons is 5%, the Macauley duration for our coupon bond is

Macauley duration uses PV, and is usually a little bit less than plain duration.

$$\text{Macauley Duration} = \frac{\$238\cdot 1 + \$227\cdot 2 + \$216\cdot 3 + \$206\cdot 4 + \$979\cdot 5}{\$238 + \$227 + \$216 + \$206 + \$979} \approx 3.78$$

$$\frac{\text{PV(Payment at Time 1)}\cdot 1 + \text{PV(Payment at Time 2)}\cdot 2 + \ldots + \text{PV(Payment at Time T)}\cdot T}{\text{PV(Payment at Time 1)} + \text{PV(Payment at Time 2)} + \ldots + \text{PV(Payment at Time T)}} . \tag{8.7}$$

Dividing by $(1+r)$ gives yet another duration measure, the **Modified Macauley Duration**, here $3.78/(1+5\%) = 3.6$.

Duration is an important and widely used variable in judging the exposure (risk) of a bond portfolio to changes in economy-wide interest rates. For example, if our bond portfolio has an average duration of 6.9 years, then it is probably more exposed to the 7-year Treasury bond than the 5-year or 10-year Treasury bonds.

Duration is used as an exposure measure.

NERD NOTE: *The interest-rate sensitivity of a bond is roughly its duration divided by one-plus the bond's yield. Therefore, a bond's price change with respect to a change in interest rate is roughly*

$$\textit{Bond Price Change} \approx \left(\frac{\textit{Duration}}{1+\textit{YTM}}\right)\cdot \textit{Interest Rate Change} . \tag{8.8}$$

This ignores complex changes in the term structure, but it is often a useful quick-and-dirty sensitivity measure.

Note that you could use duration as a measure of term length for projects other than bonds, too. However, it is important to this measure that all incoming cash flows are positive. Otherwise, these duration measures can give you pure nonsense.

Duration can be used for any positive flow projects.

NERD NOTE: *In Chapter 3, we pretended that the Wall Street Journal yieldcurve was based on zero bonds. But this description was not exactly correct: The yields graphed by the Wall Street Journal are not really those on **zero bonds**, which have only one final big payoff, but those on **coupon bonds**, which have interim coupon payments. The duration of coupon bonds is a complicated function, but it is always lower than it is on the equivalent zero bond. Therefore, the graphed slope in the Wall Street Journal is flatter than it would be if the Wall Street Journal used zero bonds. For short-term bonds, the yield on a **coupon bond** and a **zero bond** is so close that this is not a problem. However, for a long-term coupon bond, e.g., a 30-year bond, a good sum of money is paid out before the final maturity. Thus, if the yield curve is upward sloping, the annualized yield of a 30-year "zero-bond" would be a little higher. For example, on May 31, 2002, the Wall Street Journal quoted a government bond yield of 4.98% for a coupon bond maturing in February 2011. The yield on the equivalent **Treasury strip** bond, which is a pure zero bond, was 5.09%, or about 11 basis points higher. (This was a rather large difference, because the yield curve was very steep.) For bond traders, even 5 basis points can make an important difference, so bond traders usually graph their own yield curves from Treasury strips. But, for ordinary investors, 11 basis points difference is often swamped by other uncertainties.*

Solve Now!

Q 8.5 *Compute the yield-to-maturity of a two-year bond that costs \$25,000 today, pays \$1,000 at the end of the first year and at the end of the second year. At the end of the second year, it also repays \$25,000. What is the bond's IRR?*

Q 8.6 *Compute the duration of this two-year bond.*

Q 8.7 *If the yield curve is a flat 3%, compute the Macauley duration for this two-year bond.*

Q 8.8 *If the yield curve is a flat 10%, compute the Macauley duration for this two-year bond.*

Q 8.9 *If the yield curve is a flat 10%, compute the Modified Macauley duration for this two-year bond.*

Q 8.10 *Compute the yield-to-maturity of a 25-year bond that costs \$25,000 today, and pays \$1,000 at year-end for the following 25 years. In the final year ($t = 25$), it also pays \$25,000. What is the IRR?*

Q 8.11 *Compute the plain duration of this 25-year bond.*

Q 8.12 *If the yield curve is a flat 3%, compute the Macauley duration for this 25-year bond.*

Q 8.13 *If the yield curve is a flat 10%, compute the Macauley duration for this 25-year bond.*

Q 8.14 *If the yield curve is a flat 10%, compute the Modified Macauley duration for this 25-year bond.*

8·5. Other Capital Budgeting Rules

Alternative Capital Budgeting Rules: Power!

Although most corporations in the real world rely on NPV and IRR, some follow a whole variety of different capital budgeting rules. In some corporations, power rules: the most influential employees get most new funding. In other corporations, even stranger historical methods for deciding whether to take projects have taken hold.

Alternative Capital Budgeting Rules: Payback!

One commonly used alternative rule is the **payback rule**. Projects are assumed to be better if they recover the original investment faster. For the most part, this is a stupid idea. Consider the following three projects:

	Year 1	Year 2	Year 3	Year 4
Project A	−\$5	+\$8		
Project B	−\$5	+\$4	\$1,000	
Project C	−\$5	+\$4	\$0	+\$1 million

Project A has the shortest (best) payback period, but it is the worst of the three projects. Project B has the next shortest payback period, but it is the second-worst of the three projects (assuming reasonable interest rates). Project C has the longest (worst) payback period, but is the best project. At best, the payback period can be an interesting number. In many ordinary situations, the results of the payback rule do not lead to such severe mistakes, but just to mild mistakes (as would, for example, a rule that ignores all time-value of money). At worst, payback will lead you to take the wrong projects and ruin your company. But why take a chance when you know better capital budgeting methods?

Important: *Simple Advice: Stick to net present value and, if need be, to the internal rate of return. Do not use alternative capital budgeting rules for investment decisions.*

Anecdote: **Real-World Use of NPV, IRR, and Payback**
Among large Fortune-500 type firms, about 75% of CFOs use NPV and IRR in their capital budgeting decision (and more frequently if they have an MBA). Yet, small firms' CFOs tend to *also* use the payback criterion (and more frequently if they do *not* have an MBA and if they are "mature").
(Source: Graham and Harvey, Duke, 2001.)

8·6. Summary

The chapter covered the following major points:

- The IRR (internal rate of return) is computed from a project's cash flows—and without the use of any cost-of-capital information. It solves the NPV formula to equal zero.
- IRR can be interpreted as a "sort of" average rate of return implicit in these cash flows.
- Projects with an IRR above the cost of capital often, but not always, have positive net present value (NPV), and vice-versa.
- IRR can suffer from serious computation problems, having multiple or no solutions. IRR suffers from comparison problems, because it does not adjust for project scale.
- In the context of bonds, IRR is called the "yield to maturity."
- Duration is a measure of the effective time-length of a project.
- Other capital budgeting rules regularly provide non-sensical results and therefore should generally be avoided. At best, the information they provide can sometimes be "interesting."

A. Q&A: Answers

1. Using Excel, the answer pops out as 16%. Check: $-\$100 + \$55/1.16 + \$70/1.16^2 = 0$.
2. See equation 8.1 on Page 162.
3. For example, $-\$100, +\$120, -\$140, +\$160, -\$1$. (The solutions are −99.3716% and 16.796%. The important aspect is that your example has multiple inflows and multiple outflows.)
4. $-\$100, -\$50$. There are no rates of return that can possibly make this a zero net present value.
5. You are seeking the solution to

$$-\$25,000 + \sum_{t=1}^{2} \frac{\$1,000}{(1+r)^t} + \frac{\$25,000}{(1+r)^2} = 0 \ . \tag{8.9}$$

The correct solution is 4%.

6.

$$\text{Plain Duration} = \frac{\sum_{t=1}^{2} \$1,000 \cdot t + \$25,000 \cdot 2}{\sum_{t=1}^{2} \$1,000 + \$25,000} \approx 1.96 \ . \tag{8.10}$$

7.

$$\begin{aligned}\text{Macauley Duration at 3\%} &= \frac{\sum_{t=1}^{2} \frac{\$1,000 \cdot t}{(1+3\%)^t} + \frac{\$25,000 \cdot 2}{(1+3\%)^2}}{\sum_{t=1}^{2} \frac{\$1,000}{(1+3\%)^t} + \frac{\$25,000}{(1+3\%)^2}} \\ &= \frac{\frac{\$1,000 \cdot 1}{(1+3\%)^1} + \frac{\$1,000 \cdot 2}{(1+3\%)^2} + \frac{\$25,000 \cdot 2}{(1+3\%)^2}}{\frac{\$1,000}{(1+3\%)^1} + \frac{\$1,000}{(1+3\%)^2} + \frac{\$25,000}{(1+3\%)^2}} \approx 1.22 \ .\end{aligned} \tag{8.11}$$

The units here are years, because we quoted the multiplication factors "1" and "2" are in years.

8.

$$\begin{aligned}\text{Macauley Duration at 10\%} &= \frac{\sum_{t=1}^{2} \frac{\$1,000 \cdot t}{(1+10\%)^t} + \frac{\$25,000 \cdot 2}{(1+10\%)^2}}{\sum_{t=1}^{2} \frac{\$1,000}{(1+10\%)^t} + \frac{\$25,000}{(1+10\%)^2}} \\ &= \frac{\frac{\$1,000 \cdot 1}{(1+10\%)^1} + \frac{\$1,000 \cdot 2}{(1+10\%)^2} + \frac{\$25,000 \cdot 2}{(1+10\%)^2}}{\frac{\$1,000}{(1+10\%)^1} + \frac{\$1,000}{(1+10\%)^2} + \frac{\$25,000}{(1+10\%)^2}} \approx 1.48 \ .\end{aligned} \tag{8.12}$$

9.

$$\text{Modified Macauley Duration} = 1.48/(1+10\%) \approx 1.34 \ . \tag{8.13}$$

10. You are seeking the solution to

$$-\$25,000 + \sum_{t=1}^{25} \frac{\$1,000}{(1+r)^t} + \frac{\$25,000}{(1+r)^{25}} = 0 \ . \tag{8.14}$$

The middle piece is an annuity, so $\sum_{t=1}^{25} \$1,000/(1+r)^t = \$1,000 \cdot \frac{1-[1/(1+r)^{25}]}{r}$. The correct solution is 4%.

11.

$$\text{Plain Duration} = \frac{\sum_{t=1}^{25} \$1,000 \cdot t + \$25,000 \cdot 25}{\sum_{t=1}^{25} \$1,000 + \$25,000} \approx 19.62 \tag{8.15}$$

12.
$$\text{Macauley Duration at 3\%} = \frac{\sum_{t=1}^{25} \frac{\$1{,}000 \cdot t}{(1+3\%)^t} + \frac{\$25{,}000 \cdot 25}{(1+3\%)^{25}}}{\sum_{t=1}^{25} \frac{\$1{,}000}{(1+3\%)^t} + \frac{\$25{,}000}{(1+3\%)^{25}}} \approx 16.98 \ . \tag{8.16}$$

13.
$$\text{Macauley Duration at 10\%} = \frac{\sum_{t=1}^{25} \frac{\$1{,}000 \cdot t}{(1+10\%)^t} + \frac{\$25{,}000 \cdot 25}{(1+10\%)^{25}}}{\sum_{t=1}^{25} \frac{\$1{,}000}{(1+10\%)^t} + \frac{\$25{,}000}{(1+10\%)^{25}}} \approx 11.81 \ . \tag{8.17}$$

14.
$$\text{Modified Macauley Duration} = 11.81/(1+10\%) \approx 10.74 \ . \tag{8.18}$$

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 9

Financial Statement Analysis

Translating Accounting into Finance (Economic Cash Flows).

last file change: Jun 14, 2004 (14:02h)

last major edit: April 14, 2004

Financial Accounting is the "language of business." Although this book is not about financial statements, you must understand both their logic and their fundamentals. They contain information about the cash flows we need for an NPV analysis, as well as a lot of other useful information. Without understanding accounting, you also cannot understand corporate income taxes, a necessary NPV input.

This chapter begins with a simple hypothetical project. Its economics make computing NPV easy. It then explains how accountants would describe the project in a financial statement. We then translate between the finance and the accounting descriptions. Finally, the chapter applies the same analysis to the financial statement of a real corporation, PepsiCo (PEP).

This chapter also gently introduces some basics on corporate income taxes and capital structure, which will be explained in greater detail in Chapter 22.

9·1. Financial Statement Basics

Companies communicate their internals through standardized financial reports.

Publicly traded companies report their **financial results** (or **financials**) in **financial reports** to their shareholders and to the public. The most important financial report is the **Annual Report**, which is filed with the S.E.C. in Form **10-K**. (There is also a **Quarterly Report**, called **10-Q**.) Almost all Annual Reports begin with a general description of the business and business developments, followed by the more formal presentation of the firm's financials. This book is mostly interested in the financials: after all, financiers are not so much interested in *how* the firm makes its money as in *how much* money it makes.

Side Note: If you have never seen a company Annual Report (which includes the financial statements), please spend some time reading one. Any business library should have a selection. Most large corporations also publish their financials on their websites. If you own shares of stock in a publicly traded company, the Annual Report will automatically be mailed to you once a year.

Financial reports follow accounting conventions.

The purpose and therefore the logic of accounting (and specifically, net income) is different from the logic of finance (and specifically, cash flows). This chapter tries to explain how to read, interpret, and transform financial statements into the cash flows that an NPV analysis demands. It necessarily must omit many nuances of accounting, which would be useful to analysts in other contexts.

The difference between income and cash flows are accruals.

The principal difference between the concepts of income and cash flows are **accruals**: economic transactions that have delayed cash implications. For example, if I owe your firm $10,000 and have committed to paying you tomorrow, the accountant would record your current firm value to be $10,000 (perhaps time- and credit-risk discounted). In contrast, the financier would consider this to be a zero cash-flow—until tomorrow, when the payment actually occurs. The contrast is that the accountant wants the financial statements to be a good representation of the economic value of the firm *today* (i.e., you already own the payment commitment), instead of a representation of the exact timing of inflows and outflows. But timing of cash flows is exactly what we need for an NPV analysis—which is what the financier wants.

Financiers see actual cash flows: an expense spike, followed by years of no expenses.

Accruals can be classified into long-term accruals and short-term accruals. The primary **long-term accrual** is **depreciation**. When a financier purchases a maintenance-free machine, he sees a machine that costs a lot of cash today, and produces cash flows in the future. If the machine needs to be replaced every 20 years, then the financier sees a sharp spike in cash outflows every 20 years, followed by no further expenditures (but hopefully many cash inflows).

Accountants smooth asset values over time.

The accountant, however, sees the machine as an asset that uses up a fraction of its value each year. So, an accountant would try to determine an amount by which the machine deteriorates in each year, and would only consider this prorated deterioration to be the annual outflow (called an **expense**). A $1 million machine would not have an outflow of $1 million in the first year, followed by $0 in the remaining 19 years. Instead, it would be an expense of $50,000 in each of 20 years. (This is the most common method of depreciation and is called **straight-line depreciation**.)

Accountants use impairment schedules.

To complicate matters further, accountants often use standardized fixed depreciation schedules (impairment rules) over which particular assets are depreciated. Houses, for example, are depreciated over 30 years—often regardless of whether the house is constructed of wood or brick. The predetermined value schedule is usually not accurate: For example, if investors develop a taste for old buildings, it could be that a building's value has doubled since its construction, even though the financial statements might

record this building to be worth nothing.

SIDE NOTE: Even this is oversimplified. On occasion, accountants invoke procedures which allow them them to reduce the value of an asset midway through its accounting life—but more often downward than upward.

If the machine happens to continue working after 20 years, although the financials will have treated it as an expense in Year 20, they will treat it as a zero expense in Year 21. It remains worth $0 because it cannot depreciate any further. The financier sees no difference between Year 20 and Year 21, just as long as the machine continues to work.

There is some inconsistency when the machine has been fully depreciated.

Short-term accruals come in a variety of guises. To a financier, what matters is the timing of cash coming in and cash going out. A sale for credit is not cash *until* the company has collected the cash. To the accountant, if the firm sells $100 worth of goods on credit, the $100 is booked as **revenue** (which flows immediately into **net income**), even though no money has yet arrived. In the accounting view, the sale has been made. To reflect the delay in payment, accountants increase the **accounts receivables** by $100. (Sometimes, firms simultaneously establish an allowance for estimated non-payments [bad debts].)

For short-term accruals, such as receivables, accounting logic relies on predicted cash flows.

Another short-term accrual is **corporate income tax**, which a financier considers to be an outflow only when it has to be paid—at least not until (the corporate equivalent of) April 15 of the following year. However, on the income statement, when a firm in the 40% corporate tax bracket makes $100 in profits, the income statement immediately subtracts the corporate income tax of $40 (which will eventually have to be paid due to the $100 in profits) and therefore records net income of only $60. To reflect the fact that the full $100 cash is still around, $40 is recorded as **tax payables**.

The logic of finance relies on actual cash flows (or immediate values), only.

In sum, for a financier, the machine costs a lot of cash today (so it is an immediate negative), the accounts receivables are not yet cash inflows (so they are not yet positives), and the corporate income tax is not yet a cash outflow (so it is not yet a negative). There is definite sense in the approaches of both accounting and finance: the accounting approach is better in giving a snapshot impression of the firm value; the finance approach is better in measuring the timing of the cash inflows and cash outflows for valuation purposes—but it leans much more heavily on the requirement that all future cash flows are fully considered. Today's cash flows alone do *not* usually make for a good snapshot of the firm's situation.

Both approaches have relevant advantages and disadvantages.

ANECDOTE: **Trashy Accounting at Waste Management**
On December 14, 1998, Waste Management (WMX) settled a class action lawsuit by shareholders for $220 million, then the largest such settlement. (This will soon be dwarfed by the Enron, MCI WorldCom, and Arthur Andersen debacles.) The suit alleged that WMX had overstated its income by $1.32 billion over an 8-year period. From 1994 through 1997, about 47% of the company's reported income was fictitious.

One of WMX's dubious practices was that it had changed the accounting life of its waste containers from 12 years to 18 years. Therefore, each year, it subtracted less depreciation, which increased its reported earnings by $1.7 billion. Of course, managers were handsomely rewarded for their superior earnings performance.

There are at least two reasons why you need to understand financial statements: First and fortunately, companies report financial statements, not just the pure cash flows for discounting. Second and unfortunately, the I.R.S. levies corporate income tax based on a tax-rule variant of the income statements. Because taxes are real costs, finance must be able to understand and construct financial statements that properly subtract taxes from the cash flows projected from projects.

Accounting also matters to finance, because the I.R.S. computes taxes using accounting principles.

Solve Now!

Q 9.1 *What are the main differences between how accounting (net income) and finance (NPV cash flows) see projects?*

Q 9.2 *Why is a firm not just a firm and accounting numbers not just "funny money"? That is, what is the most important direct cash flow influence of Accounting in most corporations?*

9·2. The Hypothetical Bottom-Up Example — Long-Term Accruals (Depreciation)

9·2.A. Doing Accounting

The hypothetical project to illustrate the accounting perspective and the finance perspective.

To understand how depreciation drives a wedge between income and cash flows, let us start with a simple project for which the cash flows are known. Our firm is basically just one machine, described in Table 9.1.

Table 9.1. A Hypothetical Project

Project	
Real Physical Life	6 Years
Capital Expenditure	\$75, year 1
	\$75, year 2
Raw Output	\$70/year
− Input Costs (cash)	\$5/year
− Selling Costs (cash)	\$5/year
= Net Output	\$60/year
Overall Cost of Capital	12%/year
Corporate Tax Rate (τ)	40%/year

Available Financing — Executed	
Debt Capacity	\$50
Debt Interest Rate	10%/year (=\$5/year)

Accounting Treatment	
Project Life	3 Years

Note: This debt contract provides cash necessary in Year 1, and requires a first interest payment in year 2. Both principal and interest are repaid in Year 6.

Elaborate on the example.

With \$50 of debt at 10% interest, the firm's annual interest payments are \$5. The machine (and its tax depreciation schedule) is rather unusual: it has no maintenance costs; it requires capital expenditures in the second year; it produces fully even in year 1; and divine intelligence tells us that the machine will really last 6 years. But the IRS has decreed that depreciation of this particular machine occurs over 3 years. So, \$75 generates \$25 in depreciation, three years in a row, beginning in the year of the capital expenditure.

By assuming a lower interest rate of 10% on the debt than on the overall firm's cost of capital of 12%, we are in effect assuming that financial markets are risk-averse—as they truly are in the real world. Risk aversion is the subject of Part III.

Table 9.2. Income Statement of Hypothetical Machine

	Year	1	2	3	4	5	6
	Sales (Revenues)	\$70	\$70	\$70	\$70	\$70	\$70
–	Cost of Goods Sold (COGS)	\$5	\$5	\$5	\$5	\$5	\$5
–	Selling, General & Administrative Expenses (SG&A)	\$5	\$5	\$5	\$5	\$5	\$5
=	EBITDA	\$60	\$60	\$60	\$60	\$60	\$60
–	Depreciation	\$25	\$50	\$50	\$25	\$0	\$0
=	EBIT (operating income)	\$35	\$10	\$10	\$35	\$60	\$60
–	Interest Expense	\$0	\$5	\$5	\$5	\$5	\$5
=	EAIBT (or EBT)	\$35	\$5	\$5	\$30	\$55	\$55
–	Corporate Income Tax (at 40%)	\$14	\$2	\$2	\$12	\$22	\$22
=	Net Income	\$21	\$3	\$3	\$18	\$33	\$33

Excerpts From the Cash Flow Statement

Year	1	2	3	4	5	6
Capital Expenditures	–\$75	–\$75	-	-	-	-
Net Debt Issue	+\$50	-	-	-	-	–\$50

Note: Though broken out in this sample income statement, parts of depreciation are often rolled into cogs or sg&a. It is always fully broken out (and thus can be obtained from) the cash flow statement.

Accountants produce four financial statements: the **balance sheet** and the **statement of owners' equity** (both of which we do not use in this chapter); and the **income statement**, and the **cash flow statement**. The cash flow statement contains the firm's **capital expenditures**, which in this case are \$75 in Year 0 and \$75 in Year 1, followed by \$0 in all subsequent years. The income statement for this machine is shown in Table 9.2.

The accounting view: balance sheet, income statement, and cash flow statement.

Side Note: Sign Warning: You might think that a machine purchase of \$75 would be recorded as a Capital Expenditure of +\$75. Alas, the accounting convention is to record this as a negative number, i.e., −\$75. You are assumed to know that a negative expenditure is not a cash inflow, but a cash outflow. (But beware: capital expenditures is recorded as a positive asset on the balance sheet.)

Some common accounting abbreviations (jargon) explained.

In going down the left-most column, you will notice that—like finance—accounting has its own jargon. **COGS** abbreviates **cost of goods sold**; **SG&A** abbreviates **selling, general & administrative expenses**. These are expenditures that have to be subtracted from **sales** (or **revenues**) to arrive at **EBITDA**: **earnings before interest and taxes, depreciation, and amortization**. Next we subtract out depreciation, which is a subject that deserves the long sidenote below and that we will return to in a moment. Thus, we arrive at **operating income**, also called **EBIT** (**earnings before interest and taxes**). Finally, we take out interest expense at a rate of 10% per year and corporate income tax (which we can compute from the firm's tax rate of 40%) and arrive at plain **earnings**, also called **Net Income**. Net income is often called the "bottom line," because of where it appears.

SIDE NOTE:

- **Depreciation**, **depletion**, and **amortization** are all "allocated expenses" and not real cash outflows. The difference in names derives from the asset types to which they apply. depreciation applies to **Tangible Assets**, such as plants. depletion applies to **Natural Resources**. amortization applies to **Intangible Assets**. Because depreciation, depletion, and amortization are really all the same thing, they are often just lumped together under the catch-all phrase "depreciation," a convention which we follow in this chapter.
- For tutorial purposes, our example commits an accounting error. Corporations have no consistent treatment of depreciation on the income statement: some depreciation is typically rolled into cost of goods sold (such as depreciaton on manufacturing equipment; thus our machine's depreciation would usually be reported as part of cost of goods sold). Other depreciation may be rolled into selling, general & administrative expenses, and yet other depreciation may be broken out. Therefore, the only complete picture of depreciation can be found on the cash flow statement, where the sum-total of all forms of depreciation is added back to net income.
- The rules for publicly reported financial statements are called **GAAP** (Generally Accepted Accounting Principles) and change rarely. They are set by a number of policy makers, most prominently **FASB** (the Financial Accounting Standards Board). For public reporting purposes, firms are supposed to seek to match true depreciation to reported depreciation.

- Although the I.R.S. financial statements follow the same logic as the public financial statements, the statements themselves are not the same. The I.R.S. rules change often and are set by congress. (Even states can have their own rules.) The difference is particularly pronounced when it comes to depreciation schedules. Tax depreciation rules are set by the I.R.S., and they often apply strict mechanical schedules, regardless of the true asset life. Consequently, there are many companies that report positive earnings to their shareholders, and negative earnings to the I.R.S..
- In some countries, the financial and tax statements of public companies are identical. In the author's opinion, this is not a bad idea. In the United States, the public debate is how to get companies not to be aggressive in reporting financial earnings (on which executive compensation is often based!). If companies had to release their tax statements to the public, aggressive earnings manipulation would become not only more difficult, but also more costly.
- To compute corporate income tax, our chapter should have used not the depreciation on the firm's reported financial statements, but the depreciation on the firm's unreported *tax* statements. We commit this error to facilitate the exposition.
- There is a small timing difference between interest expense and when interest is actually paid. This is usually about 1 month in timing. The value impact of this difference is small, so we shall ignore it, and just use interest expense as if it were immediately paid.

(If these accounting details excite you—important as they may be—please see a shrink.)

In Table 9.2, note how capital expenditures reduce net income (in the income statement) only slowly through depreciation. In the first year, the first \$25 depreciation from the \$75 capital expenditures are accounted for; in the second year, the second \$25 depreciation from the first \$75 capital expenditures are accounted for, plus the first \$25 depreciation from the second \$75 capital expenditures are accounted for; and so on.

Depreciation smoothes capital expenditures.

9·2.B. Doing Finance

Now, forget accounting for a moment, and instead let us value the machine from an underlying finance perspective. The firm consists of three components: the machine itself, the tax obligation, and the loan.

The finance perspective focuses only on adding actual cash inflows and cash outflows. Note difference between full and levered ownership.

$$\begin{aligned} \text{NPV Project} &= \text{NPV Machine} - \text{NPV Taxes} \quad , \\ \text{NPV Levered Ownership} &= \text{NPV Machine} - \text{NPV Taxes} + \text{NPV Loan} \quad . \end{aligned} \tag{9.1}$$

Full project ownership is equivalent to supplying both the debt and equity, and earning the cash flows due to both creditors and shareholders. Levered equity ownership adds the project "loan" to the package. As full project owner (debt plus equity), in the first year, you must originally supply \$50 more in capital than if you are just a levered equity owner, but in subsequent years, the interest and principal payments are not negatives for you. (For convenience, we ignore intra-year payment timing complications.)

We first work out the real cash flows of the first component, the machine itself. Without taxes and loan, the machine produces

Let us look at the first component of the firm—the machine's real cash flows, without taxes and loan.

$$\begin{aligned} \text{NPV}_{\text{Machine}} &= \frac{\$60-\$75}{(1+12\%)^1} + \frac{\$60-\$75}{(1+12\%)^2} + \frac{\$60}{(1+12\%)^3} \\ &+ \frac{\$60}{(1+12\%)^4} + \frac{\$60}{(1+12\%)^5} + \frac{\$60}{(1+12\%)^6} = \$119.^{93} \\ \text{NPV}_{\text{Machine}} &= \text{CF}_0 + \frac{\text{CF}_1}{1+r_{0,1}} + \frac{\text{CF}_2}{1+r_{0,2}} + \frac{\text{CF}_3}{1+r_{0,3}} \\ &+ \frac{\text{CF}_4}{1+r_{0,4}} + \frac{\text{CF}_5}{1+r_{0,5}} + \frac{\text{CF}_6}{1+r_{0,6}} \quad . \end{aligned} \tag{9.2}$$

Unfortunately, corporate income tax—the second component—is an actual cost which cannot be ignored. Looking at Table 9.2, we see that Uncle Sam collects \$14 in the first year, then \$2 twice, then \$12, and finally \$22 twice.

The tax obligation is a negative NPV project, which must be calculated and then valued.

To value the future tax obligations, we need to know the appropriate discount factor. Unfortunately, we need to delay this issue until Section 22·3.B (Page 522). It is both convenient and customary (if not exactly correct) to use the firm's overall cost of capital as the discount rate for its tax obligations.

Assume that the stream of tax obligations has the same discount rate (12%) as that for the overall firm. With this cost-of-capital assumption, the net present cost of the tax liability is

Value the tax liability, and determine the project NPV.

$$\begin{aligned} \text{NPV}_{\text{Tax Liability}} &= \frac{\$14}{(1+12\%)^1} + \frac{\$2}{(1+12\%)^2} + \frac{\$2}{(1+12\%)^3} \\ &+ \frac{\$12}{(1+12\%)^4} + \frac{\$22}{(1+12\%)^5} + \frac{\$22}{(1+12\%)^6} = \$46.^{77} \quad . \end{aligned} \tag{9.3}$$

Put together,

$$\text{NPV Project} = \text{NPV Machine} - \text{NPV Taxes} = \$119.^{93} - \$46.^{77} = \$73.^{16} \ . \tag{9.4}$$

Table 9.3. Cash Flows and Net Income Summary

	Year 1	2	3	4	5	6	Discount Rate	NPV
CF Machine w/o Tax	–\$15	–\$15	+\$60	+\$60	+\$60	+\$60	12%	$\$119.^{93}$
CF Uncle Sam	–\$14	–\$2	–\$2	–\$12	–\$22	–\$22	12%	$-\$46.^{77}$
CF **Project**	–\$29	–\$17	+\$58	+\$48	+\$38	+\$38	12%	$\$73.^{16}$
CF Loan	+\$45	–\$5	–\$5	–\$5	–\$5	–\$55	10%	$\$0.^{00}$
CF **Levered Ownership**	+\$18	–\$22	+\$53	+\$43	+\$33	–\$17	?	$\$73.^{16}$
For comparison, Net Income	\$21	\$3	\$3	\$18	\$33	\$33	n/a	n/a

Note: The cost of capital (expected rate of return) is higher in this example for the machine than it is for the loan. This will be explained when we discuss the role of risk-aversion, when the safer loan will command a lower cost of capital than the riskier machine.

The loan usually is a "Zero NPV" project, unless you can get an unusually great deal or unusually bad deal on the loan.

Now consider the third component—the loan. Assume that we play not the "full project owner," but only the "residual levered equity owner," so we do not extend the loan ourselves. Instead, a perfectly competitive capital market extends us a loan. We can assume that our company "got what it paid for," a fair deal: the loan, which provides \$50 and pays interest at a rate of 10%, should be zero NPV. (This saves us the effort to compute the NPV of the loan!)

$$\text{NPV}_{\text{Loan}} = \$0 \ . \tag{9.5}$$

Be my guest, though, and do the effort:

$$\text{NPV}_{\text{Loan}} = \frac{+\$50}{1.1^1} + \frac{-\$5}{1.1^2} + \frac{-\$5}{1.1^3} + \frac{-\$5}{1.1^4} + \frac{-\$5}{1.1^5} + \frac{-\$50}{1.1^6} = \$0 \ . \tag{9.6}$$

Therefore, the project NPV with the loan, i.e., levered equity ownership, is the same as the project NPV without the loan. This makes sense: we are not generating or destroying any value by walking over to the bank. Therefore

$$\begin{aligned} \text{NPV Levered Ownership} &= \text{NPV Machine} - \text{NPV Taxes} + \text{NPV Loan} \\ &= \$119.^{93} - \$46.^{77} + \$0 = \$73.^{16} \ . \end{aligned} \tag{9.7}$$

Although the NPV remains the same, the cash flows to levered equity ownership are different from the cash flows to the project. The cash flows (and net income) are shown in Table 9.3. Note how different the cash flows and Net Income are. Net Income is highest in Years 5 and 6, but levered cash flow in Year 6 is negative. In contrast, in Year 3, the year with the highest levered cash flow, Net Income is lowest.

9·2.C. Translating Accounting into Finance

If we did not know about the details of this machine but saw only the financial statements, could we compute the correct firm value by discounting the Net Income? Discounting Net Income with a cost of capital of 12% would yield

Discounting the Net Income does not give the true project NPV.

$$\begin{aligned}\text{Incorrect NPV}_{\text{via Net Income}} &= \frac{\$21}{(1+12\%)^1} + \frac{\$3}{(1+12\%)^2} + \frac{\$3}{(1+12\%)^3} \\ &+ \frac{\$18}{(1+12\%)^4} + \frac{\$33}{(1+12\%)^5} + \frac{\$33}{(1+12\%)^6} = \$70.^{16}\ ,\end{aligned} \tag{9.8}$$

which is definitely not the correct answer of $\$73.^{16}$. Neither would it be correct to discount the Net Income with a cost of capital of 10%,

$$\begin{aligned}\text{Incorrect NPV}_{\text{via Net Income}} &= \frac{\$21}{(1+10\%)^1} + \frac{\$3}{(1+10\%)^2} + \frac{\$3}{(1+10\%)^3} \\ &+ \frac{\$18}{(1+10\%)^4} + \frac{\$33}{(1+10\%)^5} + \frac{\$33}{(1+10\%)^6} = \$75.^{24}\ .\end{aligned} \tag{9.9}$$

So, how do we reverse-engineer the correct cash flows from the financials that we can then discount? We first need to translate the financials back into the cash flows that NPV analysis can use. We just need to retrace our steps. We have both the income statement and cash flow statement at our disposal. First, to obtain the machine cash flow, we can apply the formula

Cash flows can be reverse engineered from the corporate financial statements.

		Year 1	Year 2
	EBIT	+\$35	+\$10
+	Depreciation	+\$25	+\$50
"+"	(−)Capital Expenditures	+(−\$75)	+(−\$75)
=	CF Project, Pre-Tax	−\$15	−\$15

(9.10)

to the numbers from Table 9.2. We add back the depreciation, because it *was not* a real cash outflow; and we subtract the capital expenditures, because it *was* a real cash flow. I find the formula most intuitive if I think of the depreciation + capital expenditures terms as undoing the accountants' smoothing of the cost of machines over multiple periods.

Side Note: The numbers to the right are from the first two years, and clarify the meaning of the formulas. See, the formula signs themselves are ambiguous, because accountants use different sign conventions in different spots. For example, because capital expenditures is usually quoted as a negative term on the cash flow statement, in order to subtract out capital expenditures, we just add it. In the formula below, we want to subtract corporate income tax, which appears on the income statement (Table 9.2) as a positive. Therefore, we have to subtract the positive. Sigh... I try to clarify the meaning (and to warn you) with the quotes around the +, and the cumbersome " +'' (−) in the formulas themselves.

Now we need to subtract corporate income taxes (and, again, look at the numbers them-

Finish the reverse-engineering.

selves to clarify the signs in your mind; income-tax is sometimes quoted as a negative, sometimes as a positive). This gives us the after-tax project cash flow,

		Year 1	Year 2
	EBIT	+\$35	+\$10
+	Depreciation	+\$25	+\$50
"+"	(−)Capital Expenditures	+(−\$75)	+(−\$75)
−	(+)Corporate Income Tax	−(+\$14)	−(+\$2)
=	CF Project, After-tax	−\$29	−\$17

(9.11)

Net Income already has corporate income tax subtracted out, but it also has interest expense subtracted out. So, the same cash flow results if we start with Net Income instead of EBIT, but add back the interest expense,

		Year 1	Year 2
	Net Income	+\$21	+\$3
+	Depreciation	+\$25	+\$50
"+"	(−)Capital Expenditures	+(−\$75)	+(−\$75)
+	Interest Expense (*add* back to Net Income)	\$0	+(+\$5)
=	CF Project, After-tax	−\$29	−\$17

(9.12)

Cash flow to levered equity holders.

Investors (equity and debt together) must thus come up with \$29 in the first year and \$17 in the second year. What part of this is provided by creditors? In the first year, creditors provide \$50; in the second year, creditors get back \$5. Therefore, levered equity actually receives a positive net cash flow of \$21 in the first year, and a negative cash flow of \$22 in the second year. Therefore, with the loan financed from the outside, we must add all loan inflows (principal proceeds) and subtract all loan outflows (both principal and interest). Therefore, the cash flow for levered equity shareholders is

		Year 1	Year 2
	EBIT	+\$35	+\$10
+	Depreciation	+\$25	+\$50
"+"	(−)Capital Expenditures	+(−\$75)	+(−\$75)
−	Corporate Income Tax	−(+\$14)	−(+\$2)
=	CF Project	−\$29	−\$17
+	Net Debt Issue	+\$50	+\$0
−	Interest Expense	−\$0	−\$5
=	CF Levered Equity Ownership	+\$21	−\$22

(9.13)

Again, Net Income already has both corporate income tax and interest expense subtracted out, so again the same result obtains if we instead use the formula

		Year 1	Year 2
	Net Income	+\$21	+\$3
+	Depreciation	+\$25	+\$50
"+"	(−)Capital Expenditures	+(−\$75)	+(−\$75)
+	Net Debt Issue	+\$50	\$0
=	CF Levered Equity Ownership	+\$21	−\$22

(9.14)

ANECDOTE: Solid Financial Analysis
EBITDA is all the rage among consultants and Wall Street, because it seems both closer to cash flows than EBIT and more impervious to managerial earnings manipulation through accruals. Sadly, discounting EBITDA can be worse than discounting EBIT *if* capital expenditures are not netted out—and they usually are not netted out. (Forgetting about capital expenditures when depreciation is not netted out is equivalent to assuming that product falls like manna from heaven. EBIT may spread capital expenditures over time periods in a strange way, but at least it does not totally forget it!) Sometimes, a little bit of knowledge is more dangerous than no knowledge.

In June 2003, a Bear Sterns analyst valued American Italian Pasta, a small N.Y.S.E.-listed pasta maker. Unfortunately, Herb Greenberg from TheStreet.com discovered that he forgot to subtract capital expenditures. This mistake increased the value of AIP from \$19 to \$58.[49] (then trading at \$43.[65]). Bear-Sterns admitted the mistake, and came up with a new valuation, in which Bear-Stern's boosted AIP's estimate of the company's operating cash flows and dropped its estimate of the cost of capital. Presto! The NPV of AIP was suddenly \$68 per share. How fortunate that Bear-Sterns' estimates are robust to drastic errors.

Solve Now!

Q 9.3 *Show that the formulas 9.10–9.14 yield the cash flows in Table 9.3*

Q 9.4 *Using the same cash flows as in the NPV analysis in Table 9.3, how would the project NPV change if you used a 10% cost of capital (instead of 12%) on the tax liability?*

Q 9.5 *Rework the example (income statement, cash flow statement excerpts, cash flows, and NPV) with the following parameters.*

Project	
Real Life	*5 Years*
Cost	*\$120, year 1*
Raw Output	*\$80/year*
− Input Costs	*\$6/year*
− Selling Expense	*\$8/year*
= Net Output	*\$66/year*
Overall Cost of Capital	*8%/year*
Corporate Tax Rate (τ)	*50%*

Available Financing — Executed	
Debt Capacity	*\$100*
Debt Interest Rate	*8%/year*

Accounting Treatment	
Accounting Life	*4 Years*

Debt does not require interest payment in Year 1. The world is risk-neutral, because debt and project require the same expected rate of return (cost of capital).

Q 9.6 *For the example in the text, do both the financial statements and the cash flow analysis using monthly discounting. Assume that the loan is taken at year start, and most expenses and income occur pro-rata. (Warning: Time-Intensive Question. Use Excel. Do not do by hand!)*

9·3. The Hypothetical Bottom-Up Example — Short-Term Accruals

Here are other items that require cash (inflows or outflows), which have not appeared in this simple example.

We now know how to translate some of the financials into more NPV-relevant cash flows. But we can still improve the accuracy of our cash flow formula by showing how short-term accruals can be extracted from the financials.

9·3.A. Working Capital

The definition of working capital.

To run a business day-to-day requires cash. Firms must put money into cash registers (to make change), into inventories (to have something to sell), and into extending credit to buyers (to get them to bite). This is called **working capital**. Accountants define working capital as **current assets** minus **current liabilities**. Current Assets are **cash**, **accounts receivables**, and **inventories**. current liabilities are **accounts payables**, **bank overdrafts**, **tax payables**, and other soon-due bills.

$$\begin{aligned} \text{working capital} &= (\text{current assets}) - (\text{current liabilities}) \\ &= (\text{cash} + \text{accounts receivables} + \text{inventories}) - (\text{accounts payables}) \end{aligned} \tag{9.15}$$

Net Income books cash before it comes in, so accounts receivables need to be taken out.

The cash flow effects of working capital changes are best explained with an example. Say that a firm sells $100 of goods on credit. The firm books $100 as Net Income. But because the $100 is not yet available, the firm also books $100 into accounts receivables. To compute actual cash flows, recognize that the cash has not yet materialized: we need to subtract out the $100 accounts receivables from the $100 Net Income.

Table 9.4. Multi-Year Working Capital

	Year	0	1	2	3
Finance	1. Sales and Net Income	$0	$100	$300	$0
	2. Actual Cash Receipts (for NPV CF)	$0	$0	$100	$300
Accntng	3. Reported Net Income	$0	$100	$300	$0
	4. Reported accounts receivables	$0	$100	$300	$0

Table 9.5. Multi-Year Working Capital

	Year	0	1	2	3
Finance	1. Sales and Net Income	$0	$100	$300	$0
	2. Actual Cash Receipts (for NPV CF)	$0	$0	$100	$300
Accntng	3. Reported Net Income	$0	$100	$300	$0
	4. Reported accounts receivables	$0	$100	$300	$0
	Your Computations				
	5. Change in accounts receivables	*$0*	*+$100*	*+$200*	*-$300*
	6. Net Income (Line 3) - Change in accounts receivables (Line 5)	*$0*	*$0*	*+$100*	*+$300*

Line 6 recovers Line 2 from the financials.

This becomes more interesting if we consider multiple years. For example, the firm in Table 9.4 always sells on credit and is always paid by its customers the following year. An NPV analysis requires the firm's actual cash receipts in Line 2, but accountants have provided only the information in lines 3 and 4. How do we get back the information in Line 2? Year 1 has already been discussed: we subtracted accounts receivables from Net Income to obtain the actual cash inflows of $0. Year 2 is more interesting: The firm previously had accounts receivables of $100, but now has accounts receivables of $300. It is the +$200 *change* in accounts receivables (= $200 − $100) that needs to be subtracted from the $300 in Net Income in order to infer the actual cash receipts of $100. In Year 3, the firm no longer grows and is liquidated, so the remaining receivables turn into cash that can be recaptured from the business. Again, the formula to obtain the NPV cash flow (Line 2) subtracts the change in working capital (accounts receivables) of $0 − $300 = −$300 from the $0 Net Income to conclude that we got +$300 cash inflow. Table 9.5 shows these calculations.

The difference between cash flows and Net Income are year-to-year changes in working capital.

Other short-term accruals that are components of working capital work similarly. For example, although corporate income tax is deducted on the income statement for the year in which the earnings have occurred, firms do not have to immediately pay these taxes. Instead, they can often defer them—at least until (the corporate equivalent of) April 15 of the following year. To the extent that more taxes can be delayed, more cash is available than is suggested by Net Income. Therefore, delayed taxes must be added back to Net Income when computing finance cash flows. Of course, at some point in the future, these tax payables will have to be paid, and they will then have to be counted as a cash outflow of the firm. But, for now, the permitted delay in payment is like a government loan at zero interest—and one that the accounting item Net Income ignores.

Working Capital already contains other delayed payments, making our life easier.

Side Note: We started this subsection with a Year 1 computation: we subtracted $100 in accounts receivables from the $100 Net Income worked only because the accounts receivables were the same as the *change* in accounts receivables, because the original accounts receivables were zero.

SIDE NOTE: Alas, as with capital expenditures (see Page 177), the cash flow statement has its sign conventions. The change in cash, accounts receivables, and inventories is recorded as a negative. But accounts payables do not have the opposite sign from accounts receivables, though they are already an outflow (negative); they are left as is. As a result, to compute the firm's working capital from its line items (accounts receivables, accounts payables, etc.), you do not subtract current liabilities (e.g., accounts payables) from current assets (e.g., accounts receivables), but add them together.

Here is an example of the accounting sign conventions. Table 9.7 on Page 193 lists PepsiCo's changes in working capital as 84, 416, and 79 for the years 2001, 2000, and 1999. The more detailed financials are

Cash Flow Statement	December		
	2001	2000	1999
Current Assets			
Accounts Receivables	+7	−52	−141
Inventories	−75	−51	−202
Prepaid Expenses, etc.	−6	−35	−209
Current Liabilities			
Accounts Payables, etc.	−236	+219	+357
Corporate Income Tax, Payable	+394	+335	+274
Net Change in Operating working capital	+84	+416	+79

Excludes effects of acquisitions and dispositions.

Because these figures come from the cash flow statement, to obtain the net change in operating working capital, all figures are simply added up, not netted out! (The sign of current liabilities has already been reversed for you.) Incidentally, you may note that these numbers cannot be inferred from other parts of the financial statements, because they exclude the effects of acquisitions and dispositions, as well as non-operating working capital.

ANECDOTE: Working Capital Management
Entrepreneurs usually fail for one of two reasons, and both are common: The first is that the business is just not a good idea to begin with. There is not much you can do about this. The second is that the business is too good of an idea, and the entrepreneur is not equipped to handle the success. The growth in sales consumes so much cash for increases in working capital that the firm fails to pay back its own loans: The cash is tied up in production, or in inventory, or in credit extended to customers (payment to be received), when instead it is needed to flow back to the bank. For growing firms, proper working capital management is an issue of first-order importance.

Expand our valuation formula for another source of cash.

We can now expand our formulas to include changes in working capital. For example, Formula 9.11 transmutates into

$$\begin{aligned}\text{NPV Project} = \quad & \text{EBIT} + \text{Depreciation} \quad \text{“+”} \quad \text{Capital Expenditures} \\ & - \text{Corporate Income Tax} - \text{Increase in Working Capital} \ .\end{aligned} \tag{9.16}$$

9·3.B. Other Sources and Uses of Cash

The logic of the remaining items below is similar: what matters is how a cash flow variable changes over the years. What follows are only brief descriptions of the sources of these cash flow providers/consumers.

Other, so-far neglected sources of cash hidden in the financial statements.

Changes in Deferred Taxes arise when firms use different depreciation schedules on their tax financials than on their public financials. Many firms (are allowed to) use accelerated depreciation for tax purposes. The resulting discrepancy in tax timing is recorded here.

> **SIDE NOTE**: Corporate Income Tax is an obligation that the company incurs (which therefore needs to be subtracted from EBIT). However, because taxes often need not be paid immediately, they must be added back to cash flows (at least for a while, until they are really paid). This is done via a different item, changes in Income Tax Payable, which is a component of changes in working capital. Therefore, the Changes in Deferred Taxes is a totally different animal.

Investment in Goodwill is an odd item whose name is a total misnomer. It has to do with cash laid out when firms acquire other firms.

Increase in Net Other Assets is a catch-all for a variety of other operating activities.

Non-Operating Cash Flows is a catch-all for a variety of non-operating activities. For example, firms may earn cash in financial transactions, such as currency hedging.

9·3.C. A Semi-Complete Formula To Compute Cash Flows

Putting short-term and long-term accruals and other sources/uses of cash together yields a second and more complete formula for estimating cash flows for an NPV analysis from financial statements:

A semi-complete cash flow formula:

IMPORTANT:

Cash Flows for NPV analysis can be estimated using the formula

		Earnings before Interest and Taxes (EBIT)
	–	*Corporate Income Tax*
	+	*Changes in Deferred Taxes*
=		*Net Operating Profit*
	+	*Depreciation*
=		*Gross Cash Flow*
	–	*Increase in Working Capital* (incl. tax payables, etc.)
	–	*Capital Expenditures*
	–	*Investment in Goodwill*
	–	*Increase in Net Other Assets*
=		*Free Cash Flow from Operations*
	+	*Non-Operating Cash Flows*
=		*Total Project Firm Cash Flow (to Debt and Equity)*
	+	*Net Issuance of Debt*
	–	*Interest Expense*
=		*Total Cash Flow to Levered Equity*

*The corporate **cash flow statement** was introduced to help investors construct this information. However, accountants consider interest an operating expense, while financiers consider it a distribution to owners.*

Apply our formula to PepsiCo.

We will illustrate this formula with PepsiCo's numbers in the following section. Do not consider the formula to be the perfect, end-all formula to compute NPV cash flows. Every business operates and reports differently. For example, PepsiCo has foreign exchange effects, which our formula here does not take into account. Still, the formula is a good start for estimating realized cash flows for an NPV analysis for most firms in the real world.

NERD NOTE: *Because accountants keep score of both assets and liabilities, the exact same cash flows can also be obtained from the financing side of the business:*

		Change in Marketable Securities
	+	*Decrease in Debt*
	+	*After-Tax Interest Expense*
	+	*Dividends*
	+	*Share Repurchases*
	–	*After-Tax Interest Income*
=		*Total Financial Flow*

(Actually, accountants charge after-tax interest as an operating cost, so care must be taken to recognize it as a distribution.) For complete references, see Corporate Valuation *by Bradford Cornell (Irwin), or* Valuation *by Copeland, Koller, Murrin (McKinsey).*

ANECDOTE: **Predicting the Enron debacle?**
Companies can manage reported earnings, often legally. But short of lying outright (which is not unheard of, either), firms cannot easily manipulate reported cash flows. Therefore, if a firm has unusually high accruals (depreciation, working capital) relative to its industry competitors and to its firm growth, it is often a hint that something fishy could be going on. For example, such a firm may engage in aggressive and premature booking of sales as income, even when customers may return and/or never pay for product.

So, was the Enron Accounting debacle of 2001 predictable? In 2000, I was involved with a hedge fund that shorted the stocks of firms that had unusually high short-term accruals. (Short-term accruals are often easier to manipulate on short notice than long-term accruals, and tend to come back sooner to haunt the manager.) Our fund identified Enron as being in the top 10% of most aggressive earnings reporters, so we shorted Enron. But our upstart fund had too little money under management and was unlikely to receive new money inflow soon, so we decided to close the fund—6 months before the Enron scandal hit, and before we would have become famous.

Solve Now!

Q 9.7 *From memory, can you recall the main components of cash flow used in an NPV analysis? Do you understand the logic?*

Q 9.8 *A firm reports the following financials.*

Year	*0*	*1*	*2*	*3*	*4*	*5*	*6*
Reported Sales= Net Income	*$0*	*$100*	*$100*	*$300*	*$300*	*$100*	*$0*
Reported Accounts Receivables	*$0*	*$100*	*$120*	*$340*	*$320*	*$120*	*$0*

Can you describe the firm's customer payment patterns? Extract the cash flows.

Q 9.9 *Is the firm's lifetime sum of Net Income (about) equal to the firm's lifetime sum of cash flows?*

Q 9.10 *Construct the financials for a firm that has quarterly sales and net income of $100, $200, $300, $200, $100, and half of all customers pay immediately, the other half always pay* ***two*** *quarters after purchase.*

Q 9.11 *(Difficult:) Amazonia can pay suppliers after it has sold to customers. Amazonia has 25% margins and is reporting*

Month	*Jan*	*Feb*	*Mar*	*Apr*	*May*
Reported Sales	*$0*	*$100*	*$100*	*$400*	*$0*
Reported Net Income	*$0*	*$25*	*$25*	*$100*	*$0*
Reported Accounts Payables	*$0*	*$75*	*$75*	*$300*	*$0*

What are Amazonia's actual cash flows?

Q 9.12 *A new firm reports the following financials:*

	Income Statement	*December*		
		2001	*2000*	*1999*
=	*Revenue*	*200*	*162*	*150*
	COGS	*60*	*58*	*57*
	+ SG&A	*20*	*19*	*18*
=	**Operating Income**	*120*	*85*	*75*
-	*Net Interest Income (Gains&Losses)*	*35*	*35*	*35*
=	**Income Before Tax**	*85*	*50*	*40*
–	*Corporate Income Tax at 40%*	*34*	*20*	*16*
=	**Income After Tax**	*51*	*39*	*24*
–	*Extraordinary Items*	*0*	*0*	*0*
=	**Net Income**	*51*	*39*	*24*

The firm also reports

Source	*Item*	*2001*	*2000*	*1999*
Cash Flow Statement	*Capital Expenditures*	*0*	*30*	*200*
Cash Flow Statement	*Depreciation*	*25*	*23*	*20*
Balance Sheet	*Deferred Taxes*	*20*	*16*	*0*
Balance Sheet	*Accum Depreciation*	*68*	*43*	*20*
Balance Sheet	*Working Capital*	*35*	*25*	*20*

(You will need to compute changes in deferred taxes, which are $\$20 - \$16 = \$4$ *in 2001, as well as changes in working capital.) Can you compute an estimate of cash flows produced by this firm?*

9·4. A Top-Down Example: PepsiCo's Past Cash Flows

We are now ready to conduct a cash flow estimation on a real-world example. Our goal is to understand the financial situation of PepsiCo (ticker symbol PEP). It is messier than the nice theoretical world that we have worked with so far, and there are still some details that we need to explain as we go. Tables 9.6 and 9.7 reproduce the official 2001 financial statements of PepsiCo.

Ready to value PepsiCo?

You should recognize the basic format of PepsiCo's income statement: it looks very similar to that of our hypothetical machine—although there are many small deviations from our hypothetical machine's easier accounting (and every company reports its financial statements in slightly different formats). PepsiCo's accountants prefer to put later years in columns to the left. In 2001, PepsiCo had $26,935 million in sales. COGS and SG&A added up to $\$10,754 + \$11,608 = \$22,362$ million. Therefore, EBITDA was $\$26,935 - \$22,362 = \$4,573$ million. Amortization subtracted $165 million. Other expenses amounted to $387 million, leaving us with EBIT of $\$4,573 - \$165 - \$356 - \$31 = \$4,021$ million. In PepsiCo's case, the combination of bottling equity income and transaction gains, interest expenses, and interest income was determined to be its net interest income of $8 million. Uncle Sam demanded $1,367 million, leaving shareholders with Net Income of $2,662.

PepsiCo's income statement looks just like the hypothetical example.

Side Note: The $165 depreciation that we need to add back for our NPV analysis is the $1,082 million on the cash flow statement, not the $165 million amortization that PepsiCo reports on its income statement. The reason is that PepsiCo lumped most but not all depreciation into sg&a. Amortization omits depreciation of non-physical plant assets. For clarity, I have added one line in Table 9.6, which breaks out the depreciation from selling, general & administrative expenses. Edgarscan also reported net interest income/expense of $8, which we therefore added to the table.

Non-Cash Items contain M&A costs of $356 from PepsiCo's acquisition of Quaker Oats in August 2001. Note that financials are often **restated**, i.e., changed ex-post to reflect the acquisition of other businesses. This particular procedure is called **Pooling**. The idea is to report financials *as if* the two companies had always been conjoined.

Table 9.6. PepsiCo Income Statement, Restated for Acquisitions.

	Consolidated Income Statement	December		
	(in millions)	2001	2000	1999
=	**Net Sales**			
	New PepsiCo	$26,935	$25,479	$22,970
	Bottling operations	–	–	2,123
=	Total Net Sales	26,935	21,661	25,093
	Costs and Expenses			
	Cost of Goods Sold	10,754	10,226	10,326
	Selling, General & Administrative Expenses	11,608	11,104	11,018
	Note: sg&a incl. depreciation of	*917*	*946*	*963*
	Amortization of Intangible Assets	165	147	193
	Merger-related Costs (Unusual Expenses)	356	–	–
	Other Impairment and Restructuring (Unusual Expenses)	31	184	73
–	= **Total Costs and Expenses**	22,914	21,661	21,610
=	**Operating Income**			
	New PepsiCo	4,021	3,818	3,430
	Bottling operations and equity investments	–	–	53
	Total Operating Project	4,021	3,818	3,483
	Bottling equity income and transaction gains/(losses), net	160	130	1,083
	Interest Expense	(219)	(272)	(421)
	Interest Income	67	85	130
	= Net Interest Income (Gains&Losses)	*=8*	*=-57*	*=792*
=	**Income Before Income Taxes**	4,029	3,761	4,275
–	**Provision for corporate income tax**	1,367	1,218	1,770
=	**Income After Income Taxes**	2,662	2,543	2,505
–	Extraordinary Items	0	0	0
=	**Net Income**	$ 2,662	$ 2,543	$ 2,505

Source: http://www.pepsico.com/investors/filings/200110k.shtml.

SIDE NOTE: There are many other locations for financial accounting information. PepsiCo provides it also at http://www.pepsico.com/investors/annual-reports/2001/statement_income.shtml. Financial data aggregators are also popular sources. See, e.g., www.marketguide.com). or PriceWaterhouse-Coopers' EdgarScan.

Table 9.7. PepsiCo Abbreviated Cash Flow Statement, Restated for Acquisitions.

	Cash Flow Statement	December		
	(in millions)	2001	2000	1999
	Net Income	2,662	2,543	2,505
+	Depreciation, Depletion, Amortization	1,082	1,093	1,156
+	Deferred Taxes (changes in)	162	33	573
+	Non-Cash Items	211	355	-708
+	Changes in Working Capital	84	416	79
=	**Total Operating Activity**	4,201	4,440	3,605
	Capital Expenditures	-1,324	-1,352	-1,341
+	Other Investing	-1,313	-644	169
=	**Total Investing Activity**	-2,637	-1,996	-1,172
	Financing Cash Flow Items	-5	-254	-382
+	Dividends	-994	-949	-935
+	Net Issuance of Stock	-579	-740	-902
+	Net Issuance of Debt	-341	-705	391
=	**Total Financing Activity**	-1,919	-2,648	-1,828
-	Foreign Exchange Effects	0	-4	3
=	**Net Change In Cash**	-355	-208	608

Source: http://www.pepsico.com/investors/filings/200110k.shtml. A more complete—and complex—version of PepsiCo's cash flow statement appears in Table 9.11 on Page 201.

Side Note: Although PepsiCo's Other Investing showed a dramatic downward trend, the complete version shows that it has no clear source. It contains acquisitions and investments in unconsolidated affiliates (whatever they may be), sales of businesses and PP&E, and other investments. The financing cash flow items are the Quaker Oats share purchases in the acquisition; PepsiCo's own share repurchases were lumped into net issuance of stock.

PepsiCo's cash flow statement does most of the work necessary to determine the cash flows.

PepsiCo's cash flow statement, in Table 9.7, reproduces much of our efforts to produce cash flows for an NPV analysis. It starts with the Net Income and adds back depreciation—unlike in the income statement, this includes all depreciation of all sorts of assets. The **deferred taxes** of $162 (i.e., tax depreciation schedule differences) and **non-cash items** of $211 are two more opaque items that provided PepsiCo with extra cash during the year—more than was indicated by its Net Income of $2,662. The **change in working capital**, explained in Section 9·3.A, is cash that PepsiCo had to add to its cash registers, inventory, and loans made to its retailers. In this case, PepsiCo actually could reduce and thereby generate $84 in cash from its working capital. In total, PepsiCo's business generated cash flows of $4,201 million. Of this cash, PepsiCo invested $\$1,324 + \$1,313 = \$2,637$ million into things like plant and equipment, advertising and sponsorships, etc. Again, these were actual cash flows consumed in the course of running PepsiCo.

PepsiCo's cash flow is not given directly, but can be easily computed.

Adding **total operating activity** of $+\$4,201$ and **total investing activity** of $-\$2,637$ gives \$1,564 in **operating activity net of investing activity**. Finally, we need to add back any interest expense that was taken out from Net Income. (After all, the project generated these funds and they were paid out, just like dividends are paid out.) In PepsiCo's case, it is not an interest expense, but net interest income, so the cash flow that we would use in an NPV analysis of the business of PepsiCo for 2001 is

$$\begin{aligned} \mathrm{CF}_{\mathrm{Project}} &= \text{Cash Flow from Operating Activity} \\ &+ \text{Cash Flow from Investing Activity} \\ &+ \text{Interest Expense} \ \text{(from the income statement)} \\ &= \$4,201 + (-\$2,637) + (-\$8) = \$1,564 \quad . \end{aligned} \tag{9.17}$$

(PepsiCo is the rare company that did not pay interest income, but earned interest income in 2001!) We are done with the project overall. We are, however, still interested in the cash flow that is earned by PepsiCo's levered equity (without the creditors). We need to add cash obtained from **net issuance of debt** (the difference of debt principal that we raised and debt principal that we repaid, which we can read from the cash flow statement), and we need to subtract interest that we paid.

$$\begin{aligned} \mathrm{CF}_{\mathrm{Equity}} &= \mathrm{CF}_{\mathrm{Project}} + \text{Net Issuance of Debt} - \text{Interest Expense} \\ &= \$1,564 - \$341 + -\$8 = \$1,215 \ . \end{aligned} \tag{9.18}$$

Sometimes, you might want to know the individual components, because, as we noted earlier, the proper discount rate on the \$341+\$8 debt cash flows might be lower (say 10%) than the (say 12%) discount rate on the project cash flows. We will see more of this later.

IMPORTANT:

Project Cash Flows (CF), *which pay out to owners collectively (both debt holders and equity holders in the corporation) are*

$$\begin{aligned} CF_{Project} = & \text{ Cash Flow from Operating Activity} \\ & + \text{ Cash Flow from Investing Activity} \\ & + \text{ Interest Expense} \quad . \end{aligned} \tag{9.19}$$

Net Income, a component of cash flow from operating activity, has had interest expense subtracted out. But interest expense is cash that is being returned to (debt) investors. Thus, to obtain the total amount generated by the project and available (paid out to) the sum-total of both creditors and shareholders, the interest expense (from the income statement) must be added back.

Equity Cash Flows (CF), *which are available only to levered equity shareholders—that is, they have subtracted out all inflows and outflows to creditors (debtholders)—are*

$$\begin{aligned} CF_{Equity} = & \text{ Cash Flow from Operating Activity} \\ & + \text{ Cash Flow from Investing Activity} \\ & + \text{ Net Issuance of Debt} \\ = & \; CF_{Project} + \text{ Net Issuance of Debt} - \text{ Interest Expense} \quad . \end{aligned} \tag{9.20}$$

Note: If you project forward, you may want to discount the debt components embedded in these cashflows at lower costs of capital.

SIDE NOTE: PepsiCo had an increase in Net Income from 1999 to 2001. Did it also have an increase in cash flows? The answer is no. In 1999, PepsiCo had NPV cash flows of $\$3,605 - \$1,172 - \$792 = \$1,641$; in 2000, it had cash flows of $\$4,440 - \$1,996 + \$57 = \$2,501$; in 2001, it had NPV cash flows of $\$4,201 - \$2,637 - \$8 = \$1,556$. Yet, even in 2000, managers used **changes in working capital** to prevent PepsiCo's cash flows from dropping even further. It may be that PepsiCo did not show stellar three year improvement, after all. On the other hand, the cash was not discarded but used. Naturally, judging whether these were profitable investment uses is a difficult matter.

The cash flow statement in Table 9.7 then proceeds to gives us information on what PepsiCo did with its projects' (post interest) cash flows: It used $994 million to pay dividends, $579 million to repurchase its own equity shares, and $341 million to repurchase its own debt, for total capital market activities of $1,919 million. In fact, it paid out more than it made in 2001 to the tune of $355 million. (Presumably, PepsiCo still had cash lying around, e.g., from December 1999.)

PepsiCo's use of cash from the business is also in the cash flow statement.

NERD NOTE: *Did PepsiCo pay the $1,367 in corporate income tax, noted on its income statement? No! Note that it is only called "provision for corporate income taxes," not corporate income tax. The balance sheet in Table 9.10 shows that PepsiCo's taxes payable increased by* $\$183 - \$64 = \$119$. *In addition, tax accounting allowed some other contortions, which the balance sheet identifies as an increased in deferred taxes of* $\$1,496 - \$1,367 = \$129$. *Thus, PepsiCo's actual income tax payments was probably only* $\$1,367 - \$119 - \$129 = \$1,119$.

Solve Now!

Q 9.13 *The 2002 10-K Annual Statement of Coca Cola is available from Edgar. (If need be, you can use the financials provided on Page 198. However, it would be good for you to look this up on the Web, instead.) Approximate the cash flows that you would use in valuing Coca Cola. Then, use the cash flow statement. How different are the numbers that Coca Cola reports from those that you would infer from the income statement combined with capital expenditures and depreciation (i.e., from our first formula, 9.11 on Page 182)? What if you had used our second formula 9.16 on Page 186, which also subtracts out changes in working capital?*

Q 9.14 *What are the cash flows produced by PepsiCo's projects in 1999, 2000, and 2001? What are the cash flows available to residual equity shareholders in 1999, 2000, and 2001?*

9·5. Summary

The chapter covered the following major points:

- Although every financial is a little different, the major elements are typically the same in all financial statements. Financial statements also serve more purposes than just NPV calculations, and are well worth studying in more detail (elsewhere).
- Earnings (Net Income) are *not* the cash flow inputs required in an NPV analysis.
- Accountants use "accruals" in their net income (earnings) computation, which we need to undo in order to extract actual cash flows.
- The primary long-term accrual is "depreciation," an allocation of capital expenditures.
- The primary short-term accrual is "changes in working capital," an allocation of soon-expected but not-yet-executed cash inflows and cash outflows. Examples are accounts payables, accounts receivables, and tax payables.
- If a cash flow statement is available, it conveniently handles most of the difficulties in undoing accruals for the NPV analysis. However, accountants believe interest expense to be a cost of operations, while financiers believe it to be a payout to owners. Thus, interest expense requires some special handling.
- Formula 9.19 shows how to compute cash flows that accrue to project owners (debt plus equity). Formula 9.20 shows how to compute cash flows that accrue to levered equity owners (equity only).

A. Appendix: Supplementary Financials

The following tables provide further financial statements for Coca Cola and PepsiCo.

file=financials.tex: LP

a. Coca Cola's Financials From EdgarScan

Table 9.8. Coca Cola's Financials from EdgarScan, Restated.

	Income Statement	December 2001	2000	1999
=	Revenues	20,092	19,889	19,284
	COGS	6,044	6,204	6,009
	+ SG&A (incl. Depreciation)	8,696	8,551	8,480
	+ Other Expenses	0	1,443	813
–	= **Total Operating Expenses**	14,740	16,198	15,302
=	**Operating Income**	5,352	3,691	3,982
	+ Other Net Income	607	155	174
=	**EBIT**	5,959	3,846	4,156
+	Interest Expense	289	447	337
=	**Income Before Tax**	5,670	3,399	3,819
–	Income Tax	1,691	1,222	1,388
=	**Income After Tax**	3,979	2,177	2,431
–	Extraordinary Items	-10,000	0	0
=	**Net Income**	3,969	2,177	2,431

	Cash Flow Statement	December 2001	2000	1999
	Net Income	3,969	2,177	2,431
+	Depreciation and Depletion	803	773	792
+	Deferred Taxes	56	3	97
+	Non-Cash Items	–256	1,484	1,120
+	Changes In Working Capital	–462	–852	–557
=	**Total Operating Activity**	4,110	3,585	3,883
	Capital Expenditures	–769	–733	–1,069
+	Investments	–1	–218	–342
+	Other Investing	–418	–214	–2,010
=	**Total Investing Activity**	–1,188	–1,165	–3,421
	Dividends	–1,791	–1,685	–1,580
+	Net Issuance of Stock	–113	–198	–153
+	Net Issuance of Debt	–926	–585	+956
=	**Total Financing Activity**	–2,830	–2,072	–471
–	Foreign Exchange Effects	–45	–140	–28
=	**Net Change In Cash**	47	208	–37

b. Coca Cola's Financials From Yahoo!*Finance*

Table 9.9. Coca Cola financial statements from Yahoo!*Finance*, Not Restated

	Income Statement	December 2001	2000	1999
=	Revenues	20,092	20,458	19,805
	COGS	6,044	6,204	6,009
	+ SG&A	8,696	10,563	9,814
	+ Depreciation and Amortization			
	+ Unusual Expenses			
–	= **Total Operating Expenses**			
=	**Operating Income**	5,352	3,691	3,982
	+ Other Net Income	607	155	174
=	**EBIT**	5,959	3,846	4,156
+	Interest Expense	289	447	337
=	**Income Before Tax**	5,670	3,399	3,819
–	Income Tax	1,691	1,222	1,388
=	**Income After Tax**	3,979	2,177	2,431
–	Extraordinary Items	-10,000	0	0
=	**Net Income**	3,969	2,177	2,431

	Cash Flow Statement	December 2001	2000	1999
	Net Income	3,969	773	792
+	Depreciation and Depletion	803	773	792
+	Deferred Taxes			
+	Non-Cash Items			
+	Changes In working capital			
=	**Total Operating Activity**	4,110	3,585	3,883
	Capital Expenditures	–769	–733	–1,069
+	Investments	–1	–218	–518
+	Other Investing	–418	–214	–1,834
=	**Total Investing Activity**	–1,188	–1,165	–3,421
	Financing Cash Flow Items			
+	Dividends	–1,791	–1,685	–1,580
+	Net Issuance of Stock	–113	–198	–153
+	Net Issuance of Debt	–926	–585	+956
=	**Total Financing Activity**	–2,830	–2,072	–471
–	Foreign Exchange Effects	–45	–140	–28
=	**Net Change In Cash**	47	208	–37

c. PepsiCo's Unabbreviated Balance Sheet

Table 9.10. PepsiCo's Balance Sheet

(in millions except per share amounts)	12/29/01	12/30/00
ASSETS		
Current Assets		
Cash and cash equivalents	$ 683	$ 1,038
Short-term investments, at cost	966	467
	1,649	1,505
Accounts and notes receivable, net	2,142	2,129
Inventories	1,310	1,192
Prepaid expenses and other current assets	752	791
Total Current Assets	5,853	5,617
Property, Plant and Equipment, net	6,876	6,558
Intangible Assets, net	4,841	4,714
Investments in Unconsolidated Affiliates	2,871	2,979
Other Assets	1,254	889
Total Assets	$21,695	$20,757
LIABILITIES AND SHAREHOLDERS' EQUITY		
Current Liabilities		
Short-term borrowings	$ 354	$ 202
Accounts payable and other current liabilities	4,461	4,529
Income taxes payable	183	64
Total Current Liabilities	4,644	4,593
Long-Term Debt	2,651	3,009
Other Liabilities	3,876	3,960
Deferred Income Taxes	1,496	1,367
Preferred Stock, no par value	26	49
Deferred Compensation — preferred	-	(27)
Common Shareholders' Equity		
Common stock, par value 1 2/3 c per share	30	34
(issued 1,782 and 2,029 shares, respectively)		
Capital in excess of par value	13	375
Deferred compensation	-	(-21)
Retained earnings	11,519	16,510
Accumulated other comprehensive loss	(1,646)	(1,374)
Less: repurchased common stock, at cost	(1,268)	(7,920)
(26 and 280 shares, respectively)		
Total Common Shareholders' Equity	8,648	7,604
Total Liabilities and Shareholders' Equity	$21,695	$20,757

The original is available at
http://www.pepsico.com/investors/annual-reports/2001/pepsico_financials2001.pdf.

d. PepsiCo's Unabbreviated Cash Flow Statement

Table 9.11. PepsiCo's Unabbreviated Cash Flow Statement

in millions

	52 Weeks Ending 12/29/01	53 Weeks Ending 12/30/00
Cash Flows - Operating Activities		
Net income	$ 2,662	$ 2,543
Adjustments to reconcile net income to net cash provided by operating activities		
Bottling equity income, net	(160)	(130)
Depreciation and amortization	1,082	1,093
Merger-related costs	356	–
Other impairment and restructuring charges	31	184
Cash payments for merger-related costs and restructuring charges	(273)	(38)
Deferred income taxes	162	33
Deferred compensation - ESOP	48	36
Other noncash charges and credits, net	209	303
Changes in operating working capital, excluding effects of acquisitions and dispositions		
Accounts and Notes Receivables	7	(52)
Inventories	(75)	(51)
Prepaid expenses and other current assets	(6)	(35)
Accounts payable and other current liabilities	(236)	219
Income taxes payable	394	335
Net change in operating working capital	84	416
Net Cash Provided by Operating Activities	4,201	4,440
Cash Flows - Investing Activities		
Capital spending	(1,324)	(1,352)
Acquisitions and investments in unconsolidated affiliates	(432)	(98)
Sales of businesses	–	33
Sales of property, plant & equipment	–	57
Short-term investments. by original maturity		
More than three months – purchases	(2,537)	(4,950)
More than three months – payments	2,078	4,585
Three months or less, net	(41)	(9)
Other, net	(381)	(262)
Net Cash Used for Investing Activities	(2,637)	(1,996)

(Continued on the following page.)

(Continued from previous page.)

in millions

	52 Weeks Ending 12/29/01	53 Weeks Ending 12/30/00
Cash Flows - Financing Activities		
Proceeds from issuances of long-term debt	324	130
Payments of long-term debt	(573)	(879)
Short-term borrowings, by original maturity		
More than three months — proceeds	788	198
More than three months — payments	(483)	(155)
Three months or less, net — payments	(397)	1
Cash dividends paid	(994)	(949)
Share repurchases - common	(1,716)	(1,430)
Share repurchases - preferred	(10)	–
Quaker share repurchases	(5)	(254)
Proceeds from issuance of shares in connection with the Quaker merger	524	–
Proceeds from exercises of stock options	623	690
Net Cash Used for Financing Activities	(1,919)	(2,648)
Effect of Exchange Rate Changes on Cash and Cash Equivalents	–	(4)
Net (Decrease)/Increase in Cash and Cash Equivalents	(355)	(208)
Cash and Cash Equivalents - Beginning of year	1,038	1,246
Cash and Cash Equivalents - End of period	$ 683	$ 1,038
Supplemental Cash Flow Information		
Interest Paid	$ 159	$ 226
Income taxes paid	$ 857	$ 876
Acquisitions		
Fair value of assets acquired	$ 604	$ 80
Cash paid and debt issued	(432)	(98)
Liabilities Assumed	$ 172	$ (18)

The original is available at
http://www.pepsico.com/investors/annual-reports/2001/statement_cash_flows.shtml

A. Q&A: Answers

1. Accruals, specifically depreciation and delayed payments/receipts.
2. Uncle Sam uses Accounting methods to compute corporate income taxes. Secondary influences, not discussed in the text, come from the fact that many contracts are contingent on accounting numbers (e.g., debt covenants).

3. Do it!
4. A 12% instead of a 10% interest rate would increase the NPV of the tax obligation from $46.77 to $50.16. Therefore, the project value would increase by $3.39.
5. The income statementis now

	Year	1	2	3	4	5
	Sales (Revenues)	$80	$80	$80	$80	$80
–	Cost of Goods Sold (COGS)	$6	$6	$6	$6	$6
–	Selling, General & Administrative Expenses (SG&A)	$8	$8	$8	$8	$8
=	EBITDA	$66	$66	$66	$66	$66
–	Depreciation	$30	$30	$30	$30	$0
=	EBIT (Operating Income)	$36	$36	$36	$36	$66
–	Interest Expense	-	$8	$8	$8	$8
=	EAIBT (or EBT)	$36	$28	$28	$28	$58
–	Corporate Income Tax	$18	$14	$14	$14	$29
=	Net Income	$18	$14	$14	$14	$29

Cash Flow Statement Excerpt

Year	1	2	3	4	5
Capital Expenditures	–$120	-	-	-	-
Net Debt Issue	+$100	-	-	-	–$100

The cash flow formula is EBIT plus depreciation (or use EBITDA instead) minus capital expenditures, minus corporate income tax: $36 + $30 – $120 – $18 = –$72. The first levered equity cash flows are –$72 + $100 = +$28.

Cash Flow	Discount Rate	1	2	3	4	5	NPV
Machine	8%	–$54	$66	$66	$66	$66	$152.41
Uncle Sam	8%	–$18	–$14	–$14	–$14	–$29	–$69.81
Project	8%	–$72	+$52	+$52	+$52	+$37	$82.60
Loan	8%	+$100	–$8	–$8	–$8	–$108	$0
Levered Ownership	8%	+$28	+$44	+$44	+$44	–$71	$82.60

6. T: ***The answer will eventually be posted on my website. (It is not there yet.)***

7. See Page 188.
8.

Year	0	1	2	3	4	5	6
Reported Net Income	\$0	\$100	\$100	\$300	\$300	\$100	\$0
Reported accounts receivables	\$0	\$100	\$120	\$340	\$320	\$120	\$0
Change in accounts receivables	\$0	\$100	\$20	\$220	-\$20	-\$200	-\$120
Cash Flow	\$0	\$0	\$80	\$80	+\$320	+\$300	+\$120

The firm's customers did not all pay the next period. Therefore, the cash flows were delayed.

9. Yes. Cash flows just have different timing. For example, firm's capital expenditures are not booked immediately, but the sum of all lifetime depreciation adds up to the sum of all lifetime capital expenditures. (This abstracts away from some pathological accounting cases that we have not covered.).
10. The cash flows are

Quarter	0	1	2	3	4	5	6	7
Reported Net Income	\$0	\$100	\$200	\$300	\$200	\$100	\$0	\$0
Immediate Cash Flows	\$0	\$50	\$100	\$150+	\$100+	\$50+	\$0	\$0
+ Delayed Cash Flows				+\$50	+\$100	+\$150	+\$100	+\$50
⇒ = Cash Flows	=\$0	=\$50	=\$100	=\$200	=\$200	=\$200	=\$100	=\$50
⇒ Change in A/R	-	\$50	\$100	\$100	\$0	-\$100	-\$100	-\$50
⇒ Accounts Receivables	\$0	\$50	\$150	\$250	\$250	\$150	\$50	\$0

It is easier to obtain the change in A/R first: we know that Net Income plus Change in A/R must add up to cash flows. So, $ChangeinA/R = NetIncome - cashflows$. And, knowing Change in A/R, accounts receivables itself requires simply adding up.

11. In Year 1, Amazonia has cash inflows of \$100 (\$25 net income plus \$75 change in accounts payables). In Year 2, Amazonia has another \$100 in sales, but payables stay the same. (It has to pay its old suppliers \$75, even though it gets to keep \$75 from its new suppliers.) So, Amazonia gets cash inflows of \$25 only. In Year 3, Amazonia gets net income cash inflows of \$100, plus the \$225 change in payables, for cash inflows of \$325. Finally, in Year 4, Amazonia has cash outflows of \$300. The pattern is thus

Month	Jan	Feb	Mar	Apr	May
Cash Flows	\$0	\$100	\$25	\$325	−\$300

Note that Amazonia has total 5-month cash flows of \$150, just as it has total 5-month Net Income of \$150. The working capital has only influenced the timing attribution.

12. Use the Formula on Page 188:

				2001	2000	1999
		Earnings before Interest and Taxes (EBIT)		120	85	75
	-	Corporate Income Tax	-	34	20	16
	+	Changes in deferred taxes	+	4*	16*	?
=		Net Operating Profit	=	90	81	?
	+	Depreciation	+	25	23	20†
=		Gross Cash Flow	=	113	104	?
	-	Increase in Working Capital	-	10*	5*	?
	-	Capital Expenditures	-	0	30	200
=		Free Cash Flow from Operations	=	105	69	?

* Note that the balance sheet gave the level of deferred taxes and the level of working capital, not the *changes* in these variables. You had to compute the differences yourself. † Depreciation is only available from the cash flow statement, not from the balance sheet.

13. For reference, in 2001, the cash flow statement reports depreciation of +$803, and capital expenditures of −$769. Our first formula 9.11 was Net Income plus depreciation minus corporate income tax, or $3,969+$803-$769=$4,003. Looking at the actual Coca Cola 2001 cash flow statement on Page 198, our calculation omits deferred taxes (+$56), non-cash items (-$256), changes in working capital (-$462), and "investments" of −$418 − $1. In total, our first formula 9.11 therefore would have omitted $1,081. Our second formula 9.16 would have captured at least changes in working capital, for an error reduction of $462, and a total error of $619.

14. Use Formulas 9.19 and 9.20. PepsiCo's project cash flows, available for satisfaction of both creditors and shareholders, are

	Cash Flow from Operating Activity	4,201	4,440	3,605
+	Cash Flow from Investing Activity	-2,637	-1,996	-1,172
+	Interest Expense	8	-57	792
=	Cash Flow From Projects	1,572	2,387	3,225

PepsiCo's shareholder cash flows are

	Cash Flow from Operating Activity	4,201	4,440	3,605
+	Cash Flow from Investing Activity	-2,637	-1,996	-1,172
+	Net Issuance of Debt	-341	-705	391
=	Cash Flow To Equity	1,223	1,739	2,824

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 10

Pro Forma Financial Statements

Valuing Projects From Projected financial statements.

last file change: Jun 14, 2004 (14:03h)

last major edit: Apr 21, 2004

Do you want to propose a new project? When you propose a new project to your boss, to the board of directors, or to an external venture capitalist, you will almost surely be asked to produce a business plan. A critical part of that business plan will have to be your "pro forma" financials. These financials will be used as the baseline for discussion and valuation of your proposed project.

Managers and entrepreneurs are not the only producers of pro formas. Analysts for major investment banks or for firms seeking acquisitions or mergers also have to produce pro formas to back up their analyses of corporate value. Their task is both easier and harder than that of the entrepreneur: analysts can rely on historical financial statistics and often a stable history upon which to base their pro formas, but they also often lack the detailed knowledge of the business internals and of the corporate intentions that the internal managers and entrepreneurs would have.

Every business is different, and thus every pro forma is different. Still, this chapter tries to give some guidance to the process of creating pro formas.

Anecdote: Pro Forma
According to Merriam-Webster, *pro forma* is a Latin term meaning "for form" and dated ca. 1580. Pro forma has two definitions: "provided in advance to prescribe form or describe items"; and "made or carried out in a perfunctory manner or as a formality." In many (rejected) business plans, the latter may be a better description than the former!

10·1. The Goal

Detailed pro formas help us think about the business.

A **pro forma** is a model of a hypothetical future scenario. In our context, this usually means a model of the financial performance in this hypothetical scenario. Creating a pro forma is essentially the same challenge that we encountered in Chapter 6, where we had to forecast expected cash flows. The only novelty in this chapter is that we now need to forecast the future of the firm in the context of financial accounting, rather than just in the context of the NPV expected cash flows. But creating a full pro forma—a more comprehensive projection of the financials in an accounting context—is not work for nothing: it helps impose some discipline and structure on our thinking about the value of our proposed project. It forces us to think about "details" such as what we believe sales and costs will be, how we will manage working capital, how quickly our contribution to corporate earnings can turn positive, whether taxes will be an important factor, and so on.

But forecasting for pro formas is hard and different from business to business.

But there are also reasons why we finance professors often prefer to avoid the topic. We would rather relegate the task of creating pro formas into the "black box" which we leave to other disciplines to teach for two reasons. First, constructing a pro forma is difficult, and we naturally prefer the "easy" tasks! NPV may be challenging, but in comparison to the difficulties you will encounter in making good financial projections, NPV will seem almost trivial. Second, in contrast to such topics as NPV (which is a universal concept), financials and pro formas are unique for each business. Forecasting the financials for a new cancer drug is obviously different from forecasting the financials for a new toy fad, which is obviously different from forecasting the financials for a retail store, which is obviously different from forecasting the financials for aluminum mining, and so on. So, many of our guidelines for creating good pro formas are necessarily less universal and more ad hoc than what we have learned in finance so far. At best, I can give you some general advice. Unfortunately for you, there is no way around pro formas: to be an effective entrepreneur, manager, or analyst, you must learn both how to produce and how to critically consume financial pro formas.

Side Note: For learning how to create pro formas, the Harvard method of case study is well suited. "Learning by doing" works well in this context.

Decide on a detailed projection period and a terminal value period.

The standard method for creating a pro forma separates the future into a "projection" time period, for which you forecast the financials in great detail, and a **terminal value**, which you can think of as the "then market value" of the business—a going-concern value of the business if you were to sell it at this point in the future. You have to decide for how many years you want to project in detail before capping the value analysis with your terminal value. We will work our way backwards, discussing first the terminal value.

Here is the template of what we need to do.

As our guinea pig, we will use PepsiCo. Our goal is to produce a good pro forma to determine its market value today. The template for our discussion is Table 10.1. It shows the three big areas to work on: the horizon choice T, the terminal value at time T, and the financials during the initial projection phase, from time 1 to time $T-1$.

SIDE NOTE: PepsiCo is an established company, so our example follows the mindset of an external analyst, rather than the mindset of an entrepreneur suggesting a new project without a history. PepsiCo's performance will *not* begin with a sharp initial growth curve, which is only later followed by a more stable period. PepsiCo is already in its mature, stable state, and we have no detailed knowledge of how the next year will be different from what will happen in ten years, so we could have just relied on a terminal value and dispensed with the initial phase altogether. We do create the detailed projections (rather than just the terminal value) only because we want to illustrate the process.

Table 10.1. The Pro Forma Problem for PepsiCo

To be determined

Pro Forma Net Income Statement									
Year	−3	−2	−1	0	+1	+2	...	+T	Terminal Value
	1999	2000	2001	2002	2003	2004	...		
Net Sales	22,970	25,479	26,935	to be projected in detail					"wholesale" projection needed
– COGS	10,326	10,226	10754						
...									
= Net Income	2,505	2,543	2,662						
Pro Forma Cash Flow Statement									
Net Income	2,505	2,543	2,662	to be projected in detail					"wholesale" projection needed
+ Depreciation	1,156	1,093	1,082						
...	...	...	...						
= Operating Cash Flow	3,605	4,440	4,201						

10·2. Pro Forma Terminal Values

10·2.A. The Economics of Horizon Choice

How many years of detail?

Our first goal is to determine a suitable value for the horizon choice T in Table 10.1. Remember that the horizon is the span of time up to which we project detailed financials and beyond which we substitute a "wholesale" terminal value estimate

In relative terms, the very-long run may not be more daunting than the intermediate run. Uncertainty may not grow dramatically with horizon.

As an initial step, we take a brief detour into forecasting. There is one surprising and key insight: we may be able to project present value better in the very long term than in the intermediate term! This is best explained by analogy. If I ask you to forecast the temperature in 2 hours, your (short-term) forecast will be pretty good. If I ask you to forecast the temperature on August 1, 2055, you will not be as precise, but most likely still reasonably accurate. (Here in New York, the temperature will probably be 85 degrees, plus or minus 15 degrees.) But if I ask you to forecast the temperature at an intermediate horizon, say August 1 of next year, your 1-year forecast will likely be no different and no better than your 50-year forecast.

In NPV terms, long-term uncertainty can often be less problematic.

Now, say you want to set up an ice cream store, and estimate its NPV. The effect of the temperature uncertainty in August of next year is less discounted and thus more important than the effect of the temperature uncertainty in August of 2055. For example, say your store expects to earn \$100,000, and a 15 degree temperature difference can cause you to earn anything between \$75,000 and \$125,000. At a 15% discount rate, the temperature uncertainty for August of next year can cause a value difference of about $\$50,000/(1+15\%)^1 \approx \$43,478$ in present value today. But the same temperature uncertainty in 50 years causes only a value difference of about $\$50,000/(1+15\%)^{50} \approx \$46$ in present value. If you want to get the value *today* right, the intermediate-term uncertainty is of more concern to you than the long-term uncertainty.

Economics and Strategy: Scarce resources make rents!

The role of intermediate term vs. long term uncertainty generalizes beyond ice cream stores, because knowledge of economics and strategy allows us to put reasonable bounds on long-term future profitability (in 10, 20, or 30 years). At such far-out horizons, we do not expect businesses to still have unusually large growth rates and to earn **economic rents**—where economic rents are defined as investment rates of returns that are much in excess of the cost of capital. Economic rents can only be achieved when a firm has assets and capabilities that are scarce, valuable, and difficult to imitate. Examples of such scarce resources are the presence of a unique and excellent manager-owner (e.g., Bill Gates), scale or other natural economies (e.g., operating system software), unduplicable corporate reputation (e.g., General Electric's name), patents (e.g., Glaxo's retroviral drugs), or consumer switching costs (e.g., cable television). In the very long run, over decades, scarce resources tend to become less scarce, as new technologies and consumers make old advantages obsolete. Walmart may seem like a juggernaut today, but in 50 years, it will almost surely not have the scarce resources that will allow its owners to continue earning rates of return much above their investment cost of capital. (If Walmart did maintain its growth rates, it would probably have to start colonizing other planets!)

Side Note: Who benefits from Walmart's scarce resources? It is not new shareholders that purchase shares today: they receive only appropriate compensation for their willingness to bear risk. Instead, it is the existing owners and providers of the scarce resources: rents are captured by Walmart's original shareholders and managers, not by the anonymous stock market investor putting up money today.

To determine how long it might take before a product becomes a commodity (and thus produce only very low economic profits), we need more general economics and specific business knowledge. If there are few scarce resources and entry barriers, then it may only take a couple of years before unusually high corporate growth rates slow down and there are no more economic rents. For example, there are few entry barriers to flat-screen television technology today. Consequently, you can count on the industry that produces flat-screen televisions to earn few excess rents within 10 years. (If you do not believe this, think back to the days of $500 DVD players; today, all entry barriers have disappeared, and you can purchase a DVD player for $30.) Other products, however, can enjoy more scarcity and entry barriers for longer periods of time. For example, if you can get a patent on a cure for cancer, you will be able to earn economic rents for 15 to 25 years—although competitors' drugs will eventually come onto the scene and your patent will eventually run out.

The force (of economics) has worked on products historically, too.

The economic simplifications that help you decide on when a firm is likely to settle into a lower economic growth rate are taught in great detail in a business strategy course (e.g., Porter's five forces). To determine when your business' economic rents are likely to erode, you should ask questions such as

The strategy model: what delays erosion of economic rents?

1. How long before your entry barriers will erode?
2. How long before your success will be mimicked by the competition?
3. How long before you can be squeezed by suppliers or customers?

Side Note: If you still do not believe me, think back. Can you name the companies from the 1990s that still earn large economic rents? If you had picked a company and a similar one in 1990, are both companies still around? For example, Dell may still be doing well, but Gateway looked just as good in 1990—and there are literally dozens of now bankrupt companies that looked no different then, either. You should not have expected to earn large economic rents if you had bet on a computer hardware vendor then.

So, to choose T (our detail horizon), we can consider the "underlying firm economics." If we set T at a point where long-run economic forces will have eroded most of the economic edge of the company, we can then assume that the company will henceforth earn only "ordinary profits." This suggests that your goal should be to capture the initial rapid and possibly unstable growth phase with detailed financial forecasts, and the stable period with the terminal value. But there is also a second consideration for choice of T. It relates to the discount rates. You want to pick a horizon such that the discount factor is high enough so that the precise choice of T does not matter *too* much. Even at a 10% discount rate, each dollar in 20 years is worth only about 15 cents in present value today. The high discount rate can help plaster over the errors that our crude terminal value will inevitably commit. Whether you choose 20 years or 25 years as the detail horizon, then, does not matter too much in terms of present value. When it comes to horizons that are so far away, the best we can hope for is a halfway reasonable estimate of market value, anyway.

The two considerations for setting T: business economics and discount factors.

For most businesses, you would pick a terminal value in between 3 and 20 years, with 5 to 10 years being most common. Let us apply our economic intuition about choice of T to PepsiCo. PepsiCo is a very stable company, so we do not need to project 20 years of financials in great detail. We can instead "lump" the value created in all future years into one terminal market (sale) value fairly soon. This is a relief—it saves us from guessing too many numbers about which we really have no clue (not being PepsiCo managers, but simple outside analysts). Thus, for convenience, in our example, we choose a horizon $T = 5$ years.

Typical values for T: 5-10 years.

10·2.B. Projecting A Terminal (Market) Value

The standard method of terminal valuation is fairly easy: use a growing perpetuity.

Our second goal is to determine the firm's terminal market value. The most common method to obtain this value is to use the growing perpetuity formula 5.24 from Page 114. Following my directive—because you trust me—you assume that in year 5 (2005), PepsiCo's NPV-appropriate economic cash flow will be $2,532 million (which will be derived below); that PepsiCo's nominal cost of capital is 8%/annum; and that PepsiCo's eternal growth rate of cash flows (not sales!) will be 3%/annum. (This is just a little above the [current] inflation rate of 2% per annum, and just a little below PepsiCo's most recent sales growth rate, also computed below). Using the growing perpetuity formula, you would then estimate PepsiCo's terminal market value as of 2004 to be

$$\begin{aligned} \text{Terminal Value}_{2004} &= \frac{\$2,532}{8\% - 3\%} \approx \$50,640 \\ \text{Terminal Value}_{+4} &= \frac{\mathcal{E}(\text{CF}_{+5})}{r - g} \quad . \end{aligned} \tag{10.1}$$

This $50,640 million value in Year 4 represents the value of all future cash flows that PepsiCo will create from Year 5 to eternity. Think of it as the presumed PepsiCo market value if we had to sell it, as of 2004. Of course, because it is the value in Year 4, it has to be discounted back. PepsiCo's long-term future, based on the cash flows from Year 5 to eternity, contributes

$$\begin{aligned} \text{PV}\ (\text{Terminal Value}_{+4}) &= \frac{\$50,640}{(1+8\%)^4} \approx \$37,222 \\ \text{PV}\ (\text{Terminal Value}_{+4}) &= \frac{\text{Terminal Value}_{+4}}{(1+r_{\bar{4}})^4} \end{aligned} \tag{10.2}$$

in terms of today's value.

Side Note: You should be economically consistent in your choice of expected cash flows and the expected growth rate of cash flows. You have an investment rate that is part of your expected cash flows (see the cash flow formula on Page 188), and an implied growth rate of cash flows g (in Formula 10.1). Do not assume an investment rate of 10%/year forever and a cash flow growth rate of 1%/year forever, or vice-versa!

Side Note: A common way to obtain g, your growth rate of cash flows, is from growth rates of already mature, publicly traded comparable firms in the same industry, taking into account reinvestment behavior. In PepsiCo's case, we might use Coca Cola's implied cash flow growth rate, adjusted for different cost of capital and different reinvestment rates. This, of course, would not work if Coca Cola were itself an unstable growth company, and/or would not work if we did not believe that financial markets can get value about right. This method of valuing from comparables will be the subject of Chapter 11.

The terminal value can be obtained by alternative methods, too

Your ultimate goal is to get the best estimate of the future market value, not to produce an exercise in NPV analysis. There are no laws that say that you have to use the growing perpetuity formula on cash flows to obtain your terminal market value. Instead of using the assumption that growth will remain eternally the same (say, 3%/year), you could develop another formula that assumes high growth rates for a few years (say, 5% next year), followed by growth rate declines until the growth rate reaches the inflation rate (say, 2% per year). You are also allowed to use any other information. For example, you could decide to use Net Income instead of economic cash flows. After all, accounting does try to smooth out sharp spikes, and maybe today's Net Income is a better representation of long-term cash flows than today's cash flows. (It better recognizes that recent investments may have only temporarily depressed cash flows.) Use your head and your

imagination! Finance is, after all, not just a science, but also an art.

SIDE NOTE: What would happen if you had followed my suggestion to estimate a terminal value based on Net Income in 2001? In our case, Net Income was higher than economic cash flow and grew positively.

	Cash Flow			Net Income
1999	3,605-1,172	=	2,433	2,505
2000	4,440-1,996	=	2,444	2,543
2001	4,201-2,637	=	1,564	2,662

Thus, basing the terminal value on Net Income instead of cash flows would likely lead to a higher terminal value for PepsiCo.

10·3. The Detailed Projection Phase

10·3.A. The Economics in the Initial Growth Phase

Initial growth projections are highly product specific, and indirectly determine terminal values, too.

Our third goal is to determine the project value during the beginning growth period, from next year up to year $T-1$. In general, what can we say about it? Probably not much. Pharmaceutical drug research, aluminum mining, fad toys, and a new stamping machine each have their own financial and accounting patterns. In contrast to the terminal value, long-run economic forces are unlikely to apply in the short run. Unfortunately, we cannot simply brush over the need to obtain accurate forecasts of the initial growth phase. These projections will have a significant impact on project wealth, and not just because they have a direct contribution to the present value. The terminal value itself is usually relative to a baseline expected cash flow in year T that is established from the initial projections. For example, in our PepsiCo valuation, we used \$2,532 as the expected cash flow in year 5. If instead we had believed that PepsiCo's cash flow in Year 5 had been 20% higher (\$3,000), PepsiCo's terminal value would have been 20% higher (\$60,000 instead of \$50,640), too, as would its present value (\$49,362 instead of \$37,222). Thus, a lot of weight rests on these baseline building blocks.

In general, we have little to say. If we are external analysts, we may be able to use historical financials.

If you are an analyst asked to produce a pro forma for an existing company, you can use historical cash flows for some guidance about future cash flows. Still, even then, to do this well, you have to understand a lot about the underlying economics of the business, and you have to make many assumptions. In this process, much additional information that we have so far mostly ignored—such as the the specific industry economics or the corporate balance sheet (which we have basically ignored)—could come in very handy. Although this book can give you little guidance for the general case of coming up with initial projections, it can try to illustrate some of the process with an attempt to do a naïve financial projection for PepsiCo. We know very little about PepsiCo's business or the plans of its managers, so accuracy is not our goal—illustration is.

Projecting economic cash flows directly or indirectly (via detailed financials).

The two primary methods of projecting financials are: direct extrapolation of the accounting component that we are interested in (i.e., the economic NPV cash flows for the project), which you may not even count as a "real" pro forma; and complex financial modeling (explained in Section 10·3.C). The former is a drastic shortcut, used by analysts when time and knowledge is limited, whereas the latter is more common.

10·3.B. Method 1: Direct Extrapolation of Historical Cash Flows

Directly project the final cash flows themselves forward. Here, it gives bad results. Recognize such problems!

The first method is really a cheat, which tries to avoid having to do the full-blown financial pro forma analysis. It directly projects the historical cash flows forward, e.g., by assuming a constant growth rate forever. For example, using Formula 9.19, PepsiCo's cash flows for 1999 to 2001, you can see that

$$\begin{aligned}
\text{Cash Flow}_{1999} &= \$3,605 - \$1,172 - (+\$792) = \$1,641 \\
\text{Cash Flow}_{2000} &= \$4,440 - \$1,996 - (-\$57) = \$2,501 \\
\text{Cash Flow}_{2001} &= \$4,201 - \$2,637 - (+\$8) = \$1,556 \\
&= \underset{\text{Operating Cash Flow}}{} + \underset{\text{Investing Cash Flow}}{} - \underset{\text{Net Interest Income}}{} .
\end{aligned} \tag{10.3}$$

(Note that Formula 9.19 calls for adding back interest expense, or equivalently, subtracting out interest income.) Over the three years, PepsiCo showed a cash flow decline of about $\$1,556/\$1,641 - 1 \approx 5\%$. This comes to an annual decline of about $(\$1,556/\$1,641)^{1/2} - 1 \approx 2.6\%$. Over the most recent 12 months, cash flows even dropped by one-third! You could assume that PepsiCo's cash flows will continue to decline at this rate forever. But do you really believe that PepsiCo's cash flow decline will continue, or do you believe that it will reverse? If you investigate PepsiCo's cash flow statement in Table 9.7 further, you will find that much of PepsiCo's decline was due to a heavy increase in (other) investing activity, not to a decline in its business (sales). Hopefully, such investing activity will eventually pay off in *more* cash, not less cash. This demonstrates how hazardous simplistic extrapolation of financials can be: You really need to know more about the business itself, rather than just project the past financial trends. Only if you make good projections about future performance will you get good estimates of the value of the firm. Purely mechanical models usually just don't work well. Again, always remember that valuation requires much common sense, and is as much an art as it is a science.

Settle on growth assumptions: fast first, slow for eternity.

Now let's say that after much investigation, you have come to the conclusion that the recent investment activity will cause PepsiCo to experience annual 10%/year increases in economic cash flows over the next 5 years, beyond which PepsiCo will revert back to a 3%/year long-term growth. In this case, using the terminal value of $50,640 from Formula 10.1, we arrive at our very abbreviated pro forma in Table 10.2.

Table 10.2. Direct Economic Cash Flow Projections

	Known	Pro Forma Cash Flow Statement: "Detailed" Model Growth at 10%				Terminal Value Growth at 3%	
	2000	2001	2002	2003	2004	2005	2005 to ∞
	Year 0	Year +1	Year +2	Year +3	Year +4	Year +5	Year +5 to ∞
Projected Annual CF[1]	1,572	1,729	1,902	2,092	2,302	2,532	see next row
Terminal Value Equivalent for 2005 to eternity[2]					$\frac{2,532}{8\% - 3\%}$		
Total CF	1,572	1,729	1,902	2,092	52,942		
Discount Factor[3]	1	0.93	0.86	0.79	0.74		
Total Present Value		1,601	1,761	1,937	38,914		

Explanations (Notes):
[1]: Projecting 10% due to investments, until (incl.) 2005.
[2]: Projecting 3% eternal growth, beginning 2005 to 2006.
[3]: Discount factor based on 8% cost of capital.

The total market value today that this pro forma indicates for PepsiCo is only about $45 billion. A peek at PepsiCo's stock market-value of around $90 billion (plus the value of debt) suggests that our pro forma value estimate is way low. Why the difference? Basically, either the financials markets are off—or we are. Given our level of knowledge and the sophistication of our pro forma, I would bet on the latter. PepsiCo may grow at a rate higher than 10% for 4 years and/or for longer; PepsiCo's long-term growth rate may be higher than 3%; or PepsiCo's cost of capital may be lower than 8% (all of which would mean that we underestimated PepsiCo's sale value in 4 years).

Reasons why our pro forma value estimate for PepsiCo is too low.

10·3.C. Method 2: Pro Forma Projections With Detailed Modeling of Financials

The second and more common method to project economic cash flows during the initial period is to project the entire financials, which provide the individual components for the economic cash flows we seek. Doing so is often (but not necessarily) better than direct economic cash flow projections for two reasons. First, the cash flows tend to be volatile (e.g., due to capital expenditures' lumpiness!), and are thus difficult to directly project out. (This can sometimes be alleviated by projecting from Net Income rather than from cash flows.) Second, this method sometimes makes it easier to intelligently incorporate our knowledge of the business model into our economic cash flow estimates. For example, we may happen to know that unusual expenses will be zero next year. By forecasting the full financials, such knowledge will automatically flow into our cash flow estimates.

A more sophisticated method attempts to model the entire financials, not just the "end product," the economic cash flows.

The Income Statement

Table 10.3. A Possible PepsiCo Pro Forma Income Statement Model for 2002

	Income Statement	December			(est)	(est)
		1999	2000	2001	2002	2003 ...
=	Sales[1]	25,093	25,479	26,935	27,906	...
	COGS[2]	10,326	10,226	10,754	10,760	...
	+ SG&A[3]	11,018	11,104	11,608	12,279	...
	+ Deprec/Amort[4]	193	147	165	168	...
	+ Unusual Expenses[5]	73	184	387	279	...
–	= **Operating Expenses**[6]	21,610	21,661	22,914	23,486	...
=	**Operating Income**[7]	3,483	3,818	4,021	4,420	...
+	Net Interest Income[8]	792	–57	8	0	...
=	**Income Before Tax**[9]	4,275	3,761	4,029	4,420	...
–	Corporate Income Tax[10]	1,770	1,218	1,367	1,591	...
=	**Income After Tax**[11]	2,505	2,543	2,662	2,828	...
–	Extraordinary Items[12]	0	0	0	0	...
=	**Net Income** [13]	2,505	2,543	2,662	2,828	...

Explanations (Notes):

[1]: grows by historical 3.6%.
[2]: 3,506+26% of sales.
[3]: 44% of sales.
[4]: 3-year historical average.
[5]: 1% of sales.
[6]: sum the above.
[7]: subtract the above.
[8]: too ignorant and lazy.
[9]: subtract the above.
[10]: 36% of IBT.
[11]: subtract the above.
[12]: too ignorant and lazy.
[13]: subtract the above.

The base for detailed pro formas is sales prediction.

This method usually starts by forecasting future sales in the income statement. For example, in Table 10.3, we can compute that PepsiCo sales grew at an annualized $(\$26,935/\$25,093)^{1/2} - 1 \approx 3.6\%$ from 1999 to 2001. We can assume that PepsiCo sales will continue in 2002 at the same growth rate. The pro forma line item has a footnote that explains our assumption. Indeed, every good pro forma must have detailed footnotes explaining the assumptions behind each and every line item projection. (Admittedly, our own notes are too perfunctory and do not even explain where the 3.6% came from.) Our projected PepsiCo sales in 2002 would be $\$26,935 \cdot (1+3.6\%) \approx \$27,906$. Of course, as a real-world analyst or PepsiCo manager, we might know whether PepsiCo was about to launch many exciting new products or whether it had little in the pipeline. Such information would help us to adjust the sales estimate for a more accurate projection.

Direct extrapolation of COGS is possible.

We could repeat the same exercise for COGS: a pure growth model would be that COGS' historical growth rate of $(\$10,754/\$10,326)^{1/2} - 1 \approx 2.05\%$ will continue in 2002. If applied to the year 2001 COGS of \$10,754, our 2002 COGS forecast would thus be $\$10,754 \cdot (1 + 2.05\%) \approx \$10,975$.

But, armed with our sales scenario of \$27,906 in 2002, we can now consider a wider set of models: we can forecast COGS not only relative to its own history, but also relative to projected sales for 2002, which we already have. We also know the historical relationship between COGS and sales, which we can use to predict a relationship between 2002 sales and 2002 COGS. For example, PepsiCo's history tells us that COGS was $\$10,326/\$25,093 \approx 41.15\%$ of sales in 1999, 40.135% of sales in 2000, and 39.93% of sales in 2001. The simplest sales-based model might just project that COGS would be a slowly declining fraction of sales in 2002. For example, COGS might be

Other income statement items can now also be projected in relation to (as a fraction of) sales.

$$\begin{aligned} \mathcal{E}(\text{COGS}_{2002}) &\approx 0 + 39.5\% \cdot \mathcal{E}(\text{sales}_{2002}) \\ &= 39.5\% \cdot 27,906 \approx \$11,023 \ . \end{aligned} \tag{10.4}$$

A more sophisticated model might pose that there are **economies of scale**. In this case, COGS would not go up one-to-one with sales. Instead, COGS would have both a "fixed component," whose cost would not change with sales (e.g., the factories), and a "variable component," whose costs would increase with sales (e.g., the cola syrup). We might try to plot COGS against sales for 1999–2001, and determine visually that a good line fit would be

$$\begin{aligned} \mathcal{E}(\text{COGS}_{2002}) &= \$3,500 + 25\% \cdot \mathcal{E}(\text{sales}_{2002}) \\ &= a + b \cdot \mathcal{E}(\text{sales}_{2002}) \ . \end{aligned} \tag{10.5}$$

This says that \$3.5 billion is unalterable factory costs, but for each extra dollar of sales, we have to purchase only 25 cents of syrup. Substituting in our estimated 2002 sales of \$27,906 million, we would project COGS for 2002 to be

$$\mathcal{E}(\text{COGS}_{2002}) \approx \$3,500 + 25\% \cdot (\$27,906) \approx \$10,500 \ . \tag{10.6}$$

Or, we could use heavier statistical artillery and run an **OLS** regression on PepsiCo's last three years of data. (Don't worry if you do not know what this is.) Such a regression suggests that a better line fit would be

$$\mathcal{E}(\text{COGS}_{2002}) \approx \$3,506 + 26\% \cdot \mathcal{E}(\text{sales}_{2002}) \ , \tag{10.7}$$

so our prediction would change to

$$\mathcal{E}(\text{COGS}_{2002}) \approx \$3,506 + 26\% \cdot 27,906 \approx 10,760 \ . \tag{10.8}$$

We could also instead pose that PepsiCo's future will be more like its industry's or Coca Cola's past. After adding to our COGS/sales historical PepsiCo relation COGS/sales points from our soda industry or comparables, we might prefer that line. If we were even more sophisticated, we could recognize that COGS contains some depreciation. Thus, PepsiCo's past known capital expenditures could also influence the accounting COGS, which we could throw into our statistical regression, too. The sky is the limit. For arguments sake, in Table 10.3, we now adopt \$10,760 as our reference predicted COGS.

Other items in the table may follow other models.

We can repeat these processes to predict other income statement items. Like COGS, SG&A contains both fixed and variable expenses, as well as depreciation that relates to past investments. SG&A might thus best be modeled as a combination of a fixed component, plus a sales-variable component, plus a past capital expenditure based component. But given that no money (only book space) is at stake, for the rest of the income statement, we will play it simple. The footnotes describe the method of projection for each item. Clearly, if our money was at stake, we would want to know as much about the business as possible and use this knowledge to come up with a better model of PepsiCo business relations. Again, the limit is your imagination, and your accounting and economic knowledge. Different items could receive totally different treatments, too. For example, we could relate net interest income to how much debt PepsiCo currently has, what we know current and what we believe future interest rates to be. Given our inadequate knowledge of the business, and with no real money on the line, we leave our rudimentary model "as is" in Table 10.3.

The Cash Flow Statement: Depreciation.

The cash flow statement model would rely on the income statement model.

Next, we model the cash flow statement. Our PepsiCo attempt is in Table 10.4. We start by transfering the projected Net Income from our pro forma income statement model into the cash flow statement model. For the remaining cash flow items, we remain perfunctory—after all, this is only an illustration. Although we really have no idea what causes depreciation and depletion, a number on the order of 1,100 looks "reasonably reasonable" to me, given that we know nothing about PepsiCo's physical plant, and given the stabilities of PepsiCo's prior history of depreciation and capital expenditures. (We also ignore the fact that we already modeled some parts of depreciation in the income statement, although we really should check consistency.)

Table 10.4. A Possible PepsiCo Pro Forma Cash Flow Statement Model

	Cash Flow Statement	December			(est)
		1999	2000	2001	2002
	Net Income [1]	2,505	2,543	2,662	2,828
+	Depreciation and Depletion[2]	1,156	1,093	1,082	1,100
+	Deferred Taxes[3]	573	33	162	300
+	Non-Cash Items[4]	-708	355	211	0
+	Changes in Working Capital [5]	79	416	84	-200
=	**Total Operating Activity**[6]	3,605	4,440	4,201	≈3,700
	Capital Expenditures[7]	-1,341	-1,352	-1,324	-1,300
+	Other Investing[8]	169	-644	-1,313	0
=	**Total Investing Activity**[9]	-1,172	-1,996	-2,637	-1,300
	Operating and Investing				2,400

Explanations (Notes):

[1]: transfer $2,828 from IS.
[2]: cat in the hat.
[3]: 15%-20% of Income Tax.
[4]: too ignorant and lazy.
[5]: 27% of revenue *Increase*.
[6]: sum of above, rounded.
[7]: $-1,200 + 4\% \cdot$ Earnings.
[8]: too ignorant and lazy.
[9]: sum the above, rounded.

The Cash Flow Statement: Other Components.

As a model for deferred taxes, we adopt around 18% of PepsiCo's income taxes, which fits the history reasonably well. We know nothing about non-cash items, and PepsiCo's history does not suggest a clear pattern, so we choose zero. Changes in working capital are more interesting. Our projected sales increase for 2002 is around $1 billion, which would suggest that we will need more working capital. Yet, PepsiCo also grew in prior years, and still managed to pull working capital out of the business, rather than put it in! This is rather unusual. Without further knowledge, and after much (pretend) analysis of the underlying business, we presume that PepsiCo will need to put $200 million into the business to finance sales growth. The result of all these assumptions is a projected operating cash flow of $3.7 billion. Finally, after equally long consideration of PepsiCo's business, and equally long interviews with PepsiCo management, we determine that PepsiCo is planning to invest $1.3 billion into capital expenditures, and nothing into other activities.

Economic Project Cash Flows

The projected cash flow is now much higher, due to our Other Investing assumptions.

Having guestimated the cash flow statement components for 2002, we can now compute the economic cash flow for our NPV analysis, using the basic cash flow formula 9.19 from Page 195: economic project cash flow for PepsiCo is the sum of operating cash flows and investing cash flows plus interest expense (projected to be zero), which comes to around $2.4 billion. Note that this is nearly 50% larger than the $1.6 billion that we estimated in Table 10.2. This is not because our forecasting technique is different, but primarily because we are now projecting other investing activity to be zero. (It accounted for around $1 billion in cash consumption in 2001.) If you wish, you can repeat similar forecasting exercises for PepsiCo in 2002–2005, then rework a terminal market value for 2004 off your 2005 cash flow projection, and discount it all back.

The Emperor's Clothes

Do not automatically trust pro formas! They often look very professional even if they are not credible.

Do our projections seem arbitrary to you? They should, *because they are arbitrary*—and I have made a point of telling you so throughout. But look at the financials in Tables 10.3 and 10.4. If we did not round but quoted a few more digits (for pseudo-accuracy), if we expanded our footnotes with some more mumbo-jumbo, and if we added a few more columns for 2003 to 2005, a naïve reader might be fooled into thinking that we were sophisticated analysts who knew what we were doing! It is important that you not end up being such a naïve consumer of pro formas. A well-written pro forma can easily convey an image of professional knowledge where there is none. (Form over content may work here!) But in the case of pro formas, even the best emporer wears only a bathing suit.

Do not lose the forest and discuss mini details.

Another danger for the unwary pro forma reader is falling into the trap of looking at the trees, rather than the forest. You can easily get involved in endless discussions of a particular projected item in someone else's pro forma. In real life, every pro forma relies on plenty of heroic assumptions. You must look at the big picture as well as at the minor assumptions. There is devil in both detail and in the sum-total.

What a good pro forma is and is not.

The universal use of heroic assumptions does not mean that there are no good pro formas. A less naïve reader indeed can distinguish a good one from a bad one. A good pro forma pitched to a sophisticated boss must use solid economics and have detailed footnotes explaining and justifying just about every important line item. It is a starting point for a good discussion, not an end in itself.

Scenario Analysis

The biggest problem. This is a scenario, not an expected value! Overall failure is often not considered.

The biggest problem in most pro formas, however, is not even in the details. It is the fact that a pro forma is just one particular scenario, and usually an optimistic one. I think of most pro formas as seeking to model an average outcome *but conditional on the project not aborting altogether*. For example, if someone pitches you a new magazine, most of the time, the pro forma will project a medium optimistic scenario—*on condition that the magazine succeeds*. It probably won't take into account the fact that 50% of all magazines fold within a year. It is your task as the consumer of the pro forma to determine for yourself the probability of overall magazine failure, or you will end up misled. (Immediate death does not matter for our PepsiCo pro forma. PepsiCo is likely to stay around for a few more years.)

More analysis helps to determine expected (rather than just most likely) cash flows.

Is there a better alternative? Many of the best business plans will provide **scenario analyses** (also called **sensitivity analyses**)—a rundown of different possible outcomes. For example, what would happen if the new product were to be wildly successful, or if it were to fall on hard times (though few pro forma I have seen in the real world consider complete failure)? What would happen in a recession? What has happened in past recessions? What would happen if sales were to decline by 5% next year, rather than grow by 3.6% per year? What would happen if sales were to decline for a number of years, not just for one year? We really should create modified scenario analyses for our PepsiCo pro forma. Computer spreadsheets were invented precisely to help with such analyses. An even more sophisticated method of analysis is so-called **Monte-Carlo Simulation**, which is like automated sensitivity analysis for a whole range of different scenarios. But this is beyond the scope of a first textbook in Finance.

Monte-Carlo Analysis is explained in the web chapter on real options.

In-a-Pinch Advice: Fixed vs. Variable Components

What is fixed, what is variable? Some advice.

Is it possible to predict *in general* how firms' income statements and cash flow statements are likely to develop in the future? Is depreciation better modeled as consisting of fixed+variable components, or is it better modeled as a fixed component only, or as a variable component only? Is COGS more sales-variable or more stable, or are dividends? Of course, every business is different, so there are no uniform answers here. Some firms rely more on fixed-cost technologies, others on variable cost technologies. However, rather than not provide any guidance, I will now try to describe how corporate financials have evolved *on average* in publicly traded companies. Our specific interest is whether particular accounting items have been better explained by their own history or by sales growth. Although such knowledge of how the average publicly traded firm has evolved can sometimes help you in a pinch (when you need something quickly and without much thought), it is better if you regard this section as a "jumpstart" to get you to do more economic thinking, exploration, and business modeling of your particular company.

Our basic public company financial item prediction model will be

Our projections consist only of a fixed component and a variable (sales-related) component.

$$\mathcal{E}\left(X_{t+1}\right) \approx \gamma_{\text{fixed}} \cdot X_t + \gamma_{\text{variable}} \cdot \left\{X_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.9}$$

where X is a financial statement number, such as COGS or SG&A, and t is a year index. For example, statistical history suggests that

$$\begin{aligned}\mathcal{E}\left(\text{SG\&A}_{t+1}\right) &\approx 36\% \cdot \text{SG\&A}_t + 68\% \cdot \left\{\text{SG\&A}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} \\ &= \gamma_{\text{fixed}} \cdot \text{SG\&A}_t + \gamma_{\text{variable}} \cdot \left\{\text{SG\&A}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} .\end{aligned} \tag{10.10}$$

This says that the typical firm's SG&A was about one-third related to its own past SG&A value, and two-thirds related to SG&A adjusted for sales growth. How would you use this prediction in our PepsiCo pro forma? In 2001, PepsiCo had SG&A of \$11,608, and sales of \$26,935. We had already projected 2002 sales to be \$27,906 for a 3.6% increase. Thus, Formula 10.10 suggests

$$\begin{aligned}\mathcal{E}\left(\text{SG\&A}_{2002}\right) &\approx 36\% \cdot \$11,608 + 68\% \cdot \left[\$11,608 \cdot \left(\frac{\$27,906}{\$26,935}\right)\right] . \\ &\approx 36\% \cdot \$11,608 + 68\% \cdot \left[\$11,608 \cdot (1 + 3.6\%)\right] \\ &\approx 36\% \cdot \$11,608 + 68\% \cdot \$12,025 \approx \$12,356 .\end{aligned} \tag{10.11}$$

The left part in the formula measures the "fixed effect," i.e., the degree to which SG&A remains the same as last year's SG&A, independent of PepsiCo's 2002 sales growth. The right part in the formula measures the "variable effect," i.e., how SG&A has to increase with sales growth in 2002.

Side Note: The reason why the coefficients in formula 10.11 do not add up to 1 is that SG&A increased on average in the sample—perhaps due to inflation. If γ_{fixed} is 1 and γ_{variable} is 0, then the best prediction of X next year is the same as X this year. If γ_{fixed} is 0 and γ_{variable} is 1, then the best prediction of X next year is obtained by multiplying last year's X by the observed or predicted sales increase from this year to next year.

It is important that you do not believe that the precise coefficient estimates of 36% and 68% are applicable to *your* company. They are based on mechanical statistical models, which rely only on historical information for publicly trading companies that may be totally unrelated to your own, and on a time period that is ancient history. The coefficient estimates can serve only as "quick-and-dirty" stand-ins until you use your skills and smarts to produce something better. They are here only to help give you some initial guidance in your own economic exploration of whether a particular financial item in your firm tends to be more fixed or more variable.

Again, use the estimates for guidance, and—if need be—as stand-ins, but do not believe they fit your project well.

Moreover, keep in mind that most of the time, you will be asked to create a pro forma when the company contemplates a change in policy, or when you want to propose a new project. The historical behavior of large publicly traded companies is unlikely to be a good representation of what will happen in such circumstances. Instead, your pro forma forecasts must be specific in addressing contemplated policy changes. So, please do better than the formulas below.

Projection formulas can definitely be hazardous to your wealth. Watch it.

Enough words of caution. Here are some nuggets of forecasting advice:

Sales This is the most important variable. You must forecast this number as diligently as you possibly can. Other variables below can depend on this critical estimate.

COGS In our average publicly traded companies,

$$\mathcal{E}\left(\text{COGS}_{t+1}\right) \approx 6\% \cdot \text{COGS}_t + 95\% \cdot \left\{\text{COGS}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.12}$$

Coefficients so close to 0 and 1, respectively, suggest that cost of goods sold is best explained as a constant ratio of sales (unless the firm deliberately shifts production into different [fixed cost] production). Like all other formulas below, this formula is based on the history of reasonably large publicly traded U.S. firms (and thus is neither necessarily applicable to smaller firms nor to the future).

SG&A Selling, general & administrative expenses tend to have both a fixed and a variable component. A typical firm may be modeled by assuming that two-thirds is related to the sales increase, and one-third is related to historical SG&A. A formula estimated on reasonably large publicly traded U.S. firms suggests that

$$\mathcal{E}\left(\text{SG\&A}_{t+1}\right) \approx 36\% \cdot \text{SG\&A}_t + 68\% \cdot \left\{\text{SG\&A}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.13}$$

Unusual Expenses No particular advice.

Operating Income Either construct from the items above (i.e., use the accounting identities), or forecast as

$$\mathcal{E}\left(\text{Oper.Inc.}_{t+1}\right) \approx -41\% \cdot \text{Oper.Inc.}_t + 120\% \cdot \left\{\text{Oper.Inc.}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.14}$$

Note that operating income is extremely sensitive to sales growth: any extra sales on the margin has more than a one-to-one effect on operating income. This is why the first coefficient is negative and the second is above 1. It makes economic sense: operating income goes positive only above some break-even sales point. (A strong sensitivity to sales growth also appears in some other variables below.)

Interest Income/Payments Either construct from debt and/or previous year's interest payments, or forecast as

$$\mathcal{E}\left(\text{Intst Inc.}_{t+1}\right) \approx 22\% \cdot \text{Intst Inc.}_t + 67\% \cdot \left\{\text{Intst Inc.}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.15}$$

Remember: If a change in capital structure policy is contemplated, this item needs to reflect it.

Income Before Tax Either construct from items above, or forecast as

$$\mathcal{E}\left(\text{Inc.bef.Tax}_{t+1}\right) \approx -32\% \cdot \text{Inc.bef.Tax}_t + 116\% \cdot \left\{\text{Inc.bef.Tax}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.16}$$

Income Tax Either construct from items above, or forecast as

$$\mathcal{E}\left(\text{Income Tax}_{t+1}\right) \approx -55\% \cdot \text{Income Tax}_t + 123\% \cdot \left\{\text{Income Tax}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.17}$$

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Income After Tax Either construct from items above, or forecast as

$$\mathcal{E}\left(\text{Inc.aft.Tax}_{t+1}\right) \approx -30\% \cdot \text{Inc.aft.Tax}_t + 113\% \cdot \left\{\text{Inc.aft.Tax}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.18}$$

Extraordinary Items No specific advice.

Net Income Either construct from items above, or forecast as

$$\mathcal{E}\left(\text{Net Inc.}_{t+1}\right) \approx -42\% \cdot \text{Net Inc.}_t + 114\% \cdot \left\{\text{Net Inc.}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.19}$$

Depreciation, Depletion, Amortization Either construct from items above, or forecast as

$$\mathcal{E}\left(\text{DDA}_{t+1}\right) \approx 42\% \cdot \text{DDA}_t + 62\% \cdot \left\{\text{DDA}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.20}$$

Deferred Taxes Very strongly related to sales growth.

Non-Cash Items Very sticky, but negatively related to sales growth.

Changes in Working Capital Changes in working capital are cash consumers, especially when the firm is growing fast!

$$\mathcal{E}\left(\Delta WC_{t+1}\right) \approx 46\% \cdot \Delta WC_t + (-43\%) \cdot \left\{\Delta WC_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.21}$$

Capital Expenditures Capital expenditures seem to be strongly related to sales growth.

$$\mathcal{E}\left(\text{CapExp}_{t+1}\right) \approx 0\% \cdot \text{CapExp}_t + 100\% \cdot \left\{\text{CapExp}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.22}$$

Note: If a change in capital expenditures policy is contemplated, this item needs to reflect it.

Other Investing Very sticky, but negatively related to sales growth.

Total Cash Flows From Investing Activity

$$\mathcal{E}\left(\text{CF-Inv}_{t+1}\right) \approx (-320\%) \cdot \text{CF-Inv}_t + 340\% \cdot \left\{\text{CF-Inv}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.23}$$

Very strongly related to sales growth.

Capital Expenditures Best construct from the items above, or forecast as

$$\mathcal{E}\left(\text{CapExp}_{t+1}\right) \approx (-300\%) \cdot \text{CapExp}_t + 300\% \cdot \left\{\text{CapExp}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.24}$$

Financing Cash Flow Items No useful relationship.

Dividends Very sticky, but negatively related to sales growth.

$$\mathcal{E}\left(\text{Dividends}_{t+1}\right) \approx 159\% \cdot \text{Dividends}_t + (-82\%) \cdot \left\{\text{Dividends}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.25}$$

Net Stock Issuing No useful relationship. Strongly related to sales growth.

Net Debt Issuing Strongly related to sales growth.

$$\mathcal{E}\left(\text{Debt-Issue}_{t+1}\right) \approx (-192\%) \cdot \text{Debt-Issue}_t + 195\% \cdot \left\{\text{Debt-Issue}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.26}$$

Total Cash Flows From Financing Activity Mildly related to sales growth.

$$\mathcal{E}\left(\text{CF-Fin}_{t+1}\right) \approx (-0.07) \cdot \text{CF-Fin}_t + 0.25 \cdot \left\{\text{CF-Fin}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.27}$$

Foreign Exchange Effects Sticky.

$$\mathcal{E}\left(\text{FX}_{t+1}\right) \approx 0.75 \cdot \text{FX}_t + (-0.52) \cdot \left\{\text{FX}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.28}$$

Total Net Cash Flows

$$\mathcal{E}\left(\text{Net CF}_{t+1}\right) \approx 272\% \cdot \text{Net CF}_t + (-267\%) \cdot \left\{\text{Net CF}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{10.29}$$

NERD NOTE: *The formulas are estimated using a statistical technique called "regression analysis." For super-nerds, to normalize firms, all variables were normalized by sales, regressions were run firm-by-firm, and the coefficients were then averaged over firms. Even more sophisticated modeling assumptions and techniques did no better than the simple regression approach adopted here.*

In conclusion, do not trust these formulas. They are merely tools you can use for constructing a first draft of your pro forma—they are not good blueprints. Forecasting the performance of any business, but especially a new business, remains an art that relies on the underlying sciences of economics, statistics, accounting and finance. Don't just rely on statistics alone. Use common sense. Use good knowledge of the economics of the business and the industry. Document your reasoning in informed and detailed footnotes. And then—pray!

Solve Now!

Q 10.1 *Complete the 2002 forecast in the cash flow statement model in Table 10.4. Create a forecast for 2003. (Iterate on depreciation and investing to determine sensible inputs into both.)*

10·4. Summary

The chapter covered the following major points:

- The purpose of pro formas is to project financials, which we then often use to compute a project's NPV today.
- Pro formas are often not very reliable and idiosyncratic—but there is no alternative.
- Pro formas are usually split into a detailed forecast period and a terminal value.
- A good horizon choice for the detailed forecast period depends on the economics of the business and the prevailing discount rate.
- A quick-and-dirty pro forma analysis may just project the line items of direct use. A more complete pro forma analysis can try to project many intermediate components.
- A useful distinction is to think of fixed vs. variable (sales-contingent) forecasts.
- Scenario analysis helps to better understand pro forma uncertainty.
- Use caution in constructing and interpreting pro formas.

A. Q&A: Answers

1. There is no clear and unique answer to this question. Here is a reasonable attempt.

	Income Statement	December 2001	2002	2003	Model Used
=	Sales	26,935	27,906	28,911	grows by 3.6%
	COGS	10,754	10,761	11,023	3,506+26% of revenue
	+ SG&A	11,608	12,279	12,721	44% of revenue
	+ Deprec/Amort	165	168	168	3-year average
	+ Unusual Expenses	387	279	289	1% of revenue
–	= **Operating Expenses**	22,914	23,486	24,201	Sum The Above
=	**Operating Income**	4,021	4,420	4,710	Subtract The Above
+	Net Interest Income	8	0	0	Too Ignorant and Lazy
=	**Income before Tax**	4,029	4,420	4,710	Subtract The Above
–	Corporate Income Tax	1,367	1,591	1,696	36% of IBT
=	**Income After Tax**	2,662	2,828	3,014	Subtract The Above
–	Extraordinary Items	0	0	0	Too Ignorant and Lazy
=	**Net Income**	2,662	2,828	3,014	

	Cash Flow Statement	December 2002	2003	Model Used
=	Net Income	$2,828	$3,014	transferred
+	Depreciation and Depletion	1,149	1,216	formula
+	Deferred Taxes	286	305	18% of Income Tax
+	Non-Cash Items	-46	-46	Average Historical
+	Changes in Working Capital	43	71	27% of Revenue *Increase*
=	**Total Operating Activity**	4,260	4,560	Sum Above
=	Capital Expenditures	-1,313	-1,321	$-1,200 - 4\% \cdot$ Earnings
+	Other Investing	-1,000	-1,000	Arbitrary. Sticky.
=	**Total Investing Activity**	-2,313	-2,321	Sum The Above

Depreciation: 42% times prior year depreciation plus 62% times sales-grossed-up prior depreciation.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 11

Valuing From Comparables

A Practical Approach

last file change: Jun 14, 2004 (13:33h)

last major edit: April 26, 2004

You should now be well aware that constructing pro formas and computing NPV are difficult—they require good knowledge of *all* future expected cash flows (which includes all future cash flow *contingencies*). There is an alternative approach: practitioners often resort to "valuation by comparables," or "comps" for short.

The basic idea behind valuation by comparables is simple and best understood by analogy: assume that you want to determine the value of 5 red marbles. If black marbles cost $2 a piece, and if you are willing to make the assumption that red marbles are valued like black marbles, then you can compute that the value of your 5 red marbles should be $10. It is not necessary to forecast what value marbles will have in the future or what discount factor to apply: the price of black marbles has already taken this information into account.

Of course, the more similar black marbles are to red marbles, the better this method will work. (If black marbles are made from coal and red marbles are made from rubies, you will undervalue your red marbles!) Furthermore, the method of comparables assumes that public markets already value comparable firms appropriately, so that the value of your firm or your new project can be assessed in terms of its similarity to comparable firms.

11·1. The Price-Earnings (PE) Ratio

11·1.A. Definition

The price-earnings ratio is price divided by earnings, either per-share or overall.

The **price-earnings ratio**, commonly abbreviated as **P-E ratio**, **P/E ratio**, or **PE Ratio**, is the most popular and important comparables ratio. The P/E ratio divides the overall firm market value—its market price—by the income flow (earnings) the firm generates. In the real world, price-earnings ratios are often quoted as the current market price divided by the consensus of analysts about the *expected* earnings for the *next* year, rather than by the most recently reported earnings—in other words, the ratio focuses more on the future than on the history. This suits us well: In much of this chapter, we use the growing perpetuity formula 5.24 (on Page 114), which already calls for next year's earnings anyways.

Side Note: The intuition would remain the same if we used the most recently reported earnings. Therefore, this chapter keeps the language a bit loose. Also, later in our chapter, we shall discuss some other ratios. It will then become clear why the P/E ratio is the most popular comparables measure.

Firm-value and price-per-share based ratios are the same.

Let us start by exploring the meaning of the P/E ratio with an example. A firm with a market value of $\mathbf{P} = \$200$ million and expected earnings of $\mathbf{E} = \$10$ million next year would have a price-earnings ratio of 20. Both inputs could be expressed in per-share terms, rather than in aggregate value. So, if this firm has 40 million shares outstanding, each share would be worth $\mathbf{P} = \$5$ and produce earnings of $\mathbf{E} = \$0.^{25}$. The price-earnings ratio would still be 20.

Other interpretations.

One way to look at the price-earnings ratio is that it attaches an implicit overall value to each dollar of earnings. In this case, each extra dollar of earnings translates into an extra \$20 worth of valuation—the shares sell for twenty times earnings. Sometimes it is convenient to use the reciprocal of the P/E ratio, the so-called **earnings yield**. Intuitively, it measures what percentage of price is due to current earnings. In our example, the earnings yield would be $10/200 = 5\%$.

Side Note: If the earnings are zero or negative, the price-earnings ratio is meaningless, and often indicated as NA or N/A. In contrast, the earnings-price ratio (earnings yield) can reasonably be negative and be meaningful. If the earnings are positive, then a higher price-earnings ratio implies a lower earnings-price yield and vice-versa.

Side Note: After an entire chapter on how to compute cash flows in order to avoid net income, why compute a price-earnings ratio, rather than a price-cash flow ratio? The reason is that cash flows are more "spiky" than earnings. When a firm makes a large capital expenditure or acquisition, it may have a large negative cash flow one year, followed by positive cash flows in the following years. This is not a problem in an NPV analysis, because the higher future cash flows will also enter in the future terms. But, for computing a representative ratio with just one year's information, the current accounting earnings are probably more representative than a current cash flow would be. After all, earnings try to smooth inflows and outflows of large expenditures over many periods.

11·1.B. The Theoretical Relation between P/E Ratio and Firm Growth

Our first goal in constructing a valuation by comparables is to determine where price-earnings ratios come from, and what they say about the firm. Assume that a firm—call it A—is expected to earn cash of \$100 next year, and its appropriate cost of capital is 15%. This firm is a perpetuity whose income will grow by 5% per annum forever. The growing perpetuity formula 5.24 on Page 114 states that the value of this firm is

Determining a sensible price-earnings ratio for a hypothetical firm, which is a simple growing perpetuity.

$$\begin{aligned} V_A &= \frac{\$100}{15\% - 5\%} = \$1,000 \\ \text{Value}_A \approx \mathbf{P}_A &= \frac{\text{Cash Flow}_A \approx \mathbf{E}_A}{\text{Interest Rate}_A - \text{Growth Rate}_A} . \end{aligned} \tag{11.1}$$

With a price of \$1,000 and expected earnings of \$100, A's price divided by its expected earnings is

$$\begin{aligned} \frac{\mathbf{P}_A}{\mathbf{E}_A} &= \frac{\$1,000}{\$100} = 10 \\ \frac{\mathbf{P}_A}{\mathbf{E}_A} &= \frac{\left[\frac{\mathbf{E}_A}{\mathcal{E}(r_A)-\mathcal{E}(g_A)}\right]}{\mathbf{E}_A} = \frac{1}{\mathcal{E}(r_A) - \mathcal{E}(g_A)} . \end{aligned} \tag{11.2}$$

What if the firm grew not by 5% but by 10% per year (forever)? Then the price earnings ratio would be

Faster growing firms have higher price-earnings ratios.

$$\begin{aligned} \frac{\mathbf{P}_A}{\mathbf{E}_A} &= \frac{1}{15\% - 10\%} = 20 . \\ &= \frac{1}{\mathcal{E}(r_A) - \mathcal{E}(g_A)} . \end{aligned} \tag{11.3}$$

The P/E ratio is higher. So, high price-earnings ratios are a reflection of the market's expectation about how fast the firm will grow relative to its cost of capital.

What if the market expected this firm to shrink by 5% each year? Such a firm would have a price-earnings ratio of only

Slower growing firms have lower price-earnings ratios.

$$\frac{\mathbf{P}_A}{\mathbf{E}_A} = \frac{1}{15\% - (-5\%)} = 5 . \tag{11.4}$$

Cigarette producers, for example, may suffer from negative annual growth rates and as a result have low price-earnings ratios. In May 2002, RJR Nabisco and Philip Morris (now Altria) had P/E ratios of about 12. Contrast this with high-growth firms, such as AMGEN (a high-tech pharmaceutical), which had a P/E ratio of about 40 and Microsoft, which had a P/E ratio of about 45.

Do you find it confusing that the earnings can grow by 5%, but investors expect to receive 15% rate of return? Shouldn't investors' expected rate of return be the growth rate of earnings? No—not at all. (Indeed, the expected rate of return ($\mathcal{E}(r)$) cannot be equal to the growth rate of earnings (g), or the NPV would be infinite.) The reason is that the price today already capitalizes all future earnings. Although we have already discussed why there is no direct link between earnings and rates of return (in Section 5·2.C), this is so important it deserves another example: say, the appropriate cost of capital for a firm is 10%, and it will produce \$100 next year, \$50 the next year, and \$0 thereafter. There is no uncertainty. Clearly, the cashflows/earnings of the firm are shrinking dramatically. But the value of the firm today is $\$100/1.1 + \$50/1.1^2 \approx \$132.^{23}$. Next year, the investor will receive \$100 and hold a remaining project of $\$50/1.1^1 \approx \$45.^{45}$, for a total wealth of $\$145.^{45}$. The (expected) rate of r is $\$145.^{45}/\$132.^{23} - 1 = 10\%$, even though the growth rate of earnings is negative.

The growth rate of earnings is not the expected rate of return to investors.

IMPORTANT: *The price-earnings ratio is higher when the firm has more growth opportunities.*

NERD NOTE: *The perpetuity formula provides most of the necessary intuition, i.e., that price-earnings ratios reflect the market's underlying expectation of growth. However, the same basic effect (that higher growth firms have higher* P/E ratio*s) could also be shown with other assumptions about the growth and death of firms.*

11·1.C. PVGO (The Present Value of Growth Opportunities)

PVGO (Present Value of Growth Opportunities).

Another common perspective about the meaning of P/E ratios comes from decomposing a firm into two components: the ratio of one hypothetical firm that has the projected earnings of the company, but has stopped growing; and the ratio of another hypothetical firm that has zero earnings right now and consists just of the projected growth opportunities. The latter part is called the **Present Value of Growth Opportunities** (**PVGO**). We can split the market value of any company—regardless of its actual earnings—into these two components.

An example split of firms' earnings into "steady" and "PVGO."

For example, consider three firms, all priced at \$100 and all with an appropriate cost of capital of 10%. The first firm has expected earnings of \$10, the second firm has expected earnings of \$8, and the third firm has expected earnings of \$15. We will decompose each firm's value into the two components.

No Growth: The first firm is worth

$$\begin{aligned} \$100 &= \frac{\$10}{10\%} + \quad ? \quad = \$100 + ? \\ \mathbf{P} &= \frac{\mathbf{E}}{r} + \text{PVGO} \quad . \end{aligned} \tag{11.5}$$

To be an equality, the question mark must stand for \$0. The market has priced this first firm exactly as if it had no expectation of any future growth. Thus, 100% of this firm's value comes from the "steady component," 0% from the "growth component."

Growth: In contrast, if the second firm, also trading at \$100, earned only a constant \$8 forever, its constant growth component would only be worth \$80,

$$\begin{aligned} \$100 &= \frac{\$8}{10\%} + \quad ? \quad = \$80 + ? \\ \mathbf{P} &= \frac{\mathbf{E}}{r} + \text{PVGO} \quad . \end{aligned} \tag{11.6}$$

Therefore, this firm's "steady component" is worth \$80, and its growth opportunities must be worth PVGO = \$20.

Negative-Growth: Finally, if the third firm were expected to earn a constant \$15 forever, it should have been worth \$150 today. To justify its market value of \$100, we must believe that it will have negative growth in the future,

$$\begin{aligned} \$100 &= \frac{\$15}{10\%} + \quad ? \quad = \$150 + ? \\ \mathbf{P} &= \frac{\mathbf{E}}{r} + \text{PVGO} \quad , \end{aligned} \tag{11.7}$$

specifically, a subtractive part worth PVGO = −$50.

Is there a relationship between a firm's P/E ratio and its cost of capital? To find out, rearrange Formula 11.5:

The E/P yield is the interest rate plus the normalized present value of growth opportunities

$$\mathbf{P} = \frac{\mathbf{E}}{r} + \text{PVGO} \quad \Leftrightarrow \quad \mathbf{E} = r\cdot\mathbf{P} - r\cdot\text{PVGO} \quad \Leftrightarrow \quad \frac{\mathbf{E}}{\mathbf{P}} = r\cdot\left(1 - \frac{\text{PVGO}}{\mathbf{P}}\right) . \tag{11.8}$$

The formula states that a stable company without any growth opportunities ($g = 0 \Rightarrow$ PVGO $= 0$) has an earnings-price yield equal to its cost of capital, r. A growing firm ($g > 0 \Rightarrow$ PVGO > 0) has an earnings-price yield lower than its cost of capital. And a dying firm (PVGO < 0) has an earnings-price yield higher than its cost of capital.

11·1.D. Empirical Evidence: P/E Ratios and Growth Rates

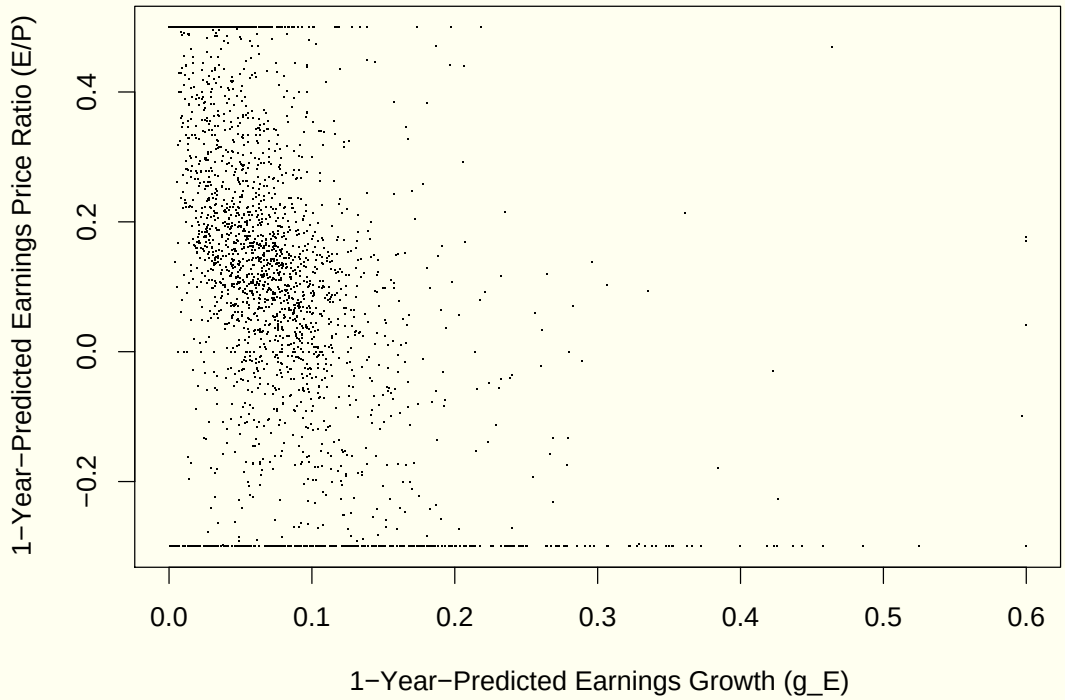
The P/E ratio theory works nicely on paper, but does it hold water in the real world? The implication of the theory is that if we plot long-term expected earnings growth ($g_{\mathbf{E}}$) against $\mathbf{E}/\mathbf{P}$, we should get a negative relation.

Do high growth firms in the real-world have higher P/E ratios?

$$\mathbf{P} = \frac{\mathbf{E}}{r - g_{\mathbf{E}}} \quad \Longleftrightarrow \quad \frac{\mathbf{E}}{\mathbf{P}} = r - g_{\mathbf{E}} . \tag{11.9}$$

Unfortunately, to make the theory operationally useful, we have to make two more assumptions. First, we do not know the eternal growth rate: we only know the most recent earnings ($\mathbf{E}_0$) and the earnings predicted by analysts for next year (call them $\hat{\mathbf{E}}_{+1}$), so we can only compute the expected growth rate for one year: $\mathcal{E}(g_{\mathbf{E}}) = (\hat{\mathbf{E}}_{+1} - \mathbf{E}_0)/\mathbf{E}_0$. We have to make the leap that firms with higher short-term growth rates also have higher long long-term growth rates, so we can use the former as a standin for the latter. Second, each firm may have its own costs of capital (unequal r); if firms with high growth rates $g_{\mathbf{E}}$'s also have sufficienly high costs of capital r's, then we might not even be able to see any relationship between earnings yield and earnings growth. Both of these problems could conceivably scramble any negative expected relationship between the earnings yield and the growth rate of earnings. Thus, we need to look at the empirical evidence to determine how practically useful the theory is.

Figure 11.1. Relation between 1-Year Predicted Earnings-Growth Rates and 1-Year Predicted Earnings-Price Yields, as of December 2000.



Analysts' consensus earnings forecasts were obtained from **I/B/E/S**.

Some detail information about the figure.

Figure 11.1 plots the predicted next-year earnings-growth rate against the earnings yield (the ratio of predicted earnings over today's stock price), for firms with market capitalization of \$50 million or more, *as of December 2000*. Each dot is one firm.

The evidence supports the theory: high-growth firms have lower E/P (and thus higher P/E) ratios.

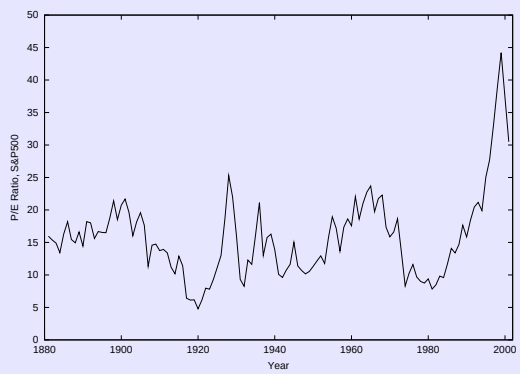
The figure shows how firms with higher earnings growth rates had lower earnings-yields (higher price-earnings ratios), just as the theory had predicted. Eyeballing the figure, we can see that firms that are neither growing nor contracting tend to have an earnings price ratio of about 8% ($\mathbf{P}/\mathbf{E} \approx 12$), firms growing by 20% tend to have an earnings price ratio of about 5% ($\mathbf{P}/\mathbf{E} \approx 20$), and firms growing by about 40% tend to have an earnings price ratio of about 2.5% ($\mathbf{P}/\mathbf{E} \approx 40$).

NERD NOTE: *To reduce the influence of some extremely unusual firms, a few firms with* E/P yields *in excess of 100% were truncated at 100%. More importantly, earnings growth (g_E, the independent variable on the x-axis) was truncated at -30% and $+50\%$, again to reduce the influence of extreme observations. Firms with such negative growth rates and such high positive growth rates are sufficiently unusual that pretending that they had just -30% or $+50\%$ growth is reasonable. Indeed, it is particularly unreasonable to pretend that negative earnings growth can last forever, so we would not expect a good relationship between price-earnings ratios and earnings growth for contracting firms. Finally, firms with lagged negative earnings (not earnings growth rates!) had to be ignored, too, because it is impossible to compute a meaningful earnings growth rate when the denominator is negative.*

If you had been hired in December 2000 to assess the value of a privately held firm for which you only knew the earnings, Figure 11.1 would have been very useful. For example, if this firm had earnings of \$10 million, and was expected to grow them to \$12 million by December 2001, the figure would have indicated that this 20% earnings growth rate on average would have translated into likely E/P yields between about 2% and 10%, with 5% being perhaps the best number. Therefore, reasonable value estimates for this company might have been somewhere between $50 \cdot \$12$ million $\approx \$600$ million and $10 \cdot \$12$ million $\approx \$120$ million, with $20 \cdot \$12$ million $\approx \$240$ million being a decent average estimate.

Using the figure to estimate a comparables-based firm value, as of December 2000.

ANECDOTE: Shiller's Irrational Exuberance



This figure shows the history of the price-earnings ratio for the S&P500. (The S&P500 index contains the 500 largest publicly traded firms in the United States.)

We can relate these findings back to Formula 11.9, which states that the earnings-price ratio is $(r - g_E)$. Many experts believe that the expected rate of return on the stock market has not changed dramatically in the last 100 years; and in 1999, stock market bulls also believed that the expected rate of return r in the stock market would be at least 10% per year *in real terms (inflation-adjusted)* forever. Therefore, when the stock market's P/E ratio reached 44 at the end of 1999, such optimists must have believed in earnings growth $g_E = r - {}^{E}/_{P} = 10\% - 1/44 \approx 7.5\%$ per year, *in real terms* and *forever*. This is truly an astronomical figure, and should have been tough to swallow. A forecast of real GDP growth based on its historical long-run average puts its real growth rate at only about 2.5% per year. Put differently, at the historical 2.5% real growth rate, the P/E ratio of 44 could have only implied an expected real rate of return in the stock market of $2.5\% + 1/44 \approx 5.0\%$ per year.

This argument—that either the stock market's ${}^{P}/_{E}$ multiple or popular stock return expectations were out of line with reasonable earnings growth estimates (and thus that the stock market was overvalued)—was most forcefully advanced in Robert Shiller's Bestseller **Irrational Exuberance**. It was published just before the stock market peaked in 2000. (The plotted data were obtained from Shiller's website.)

SIDE NOTE: During economic booms, earnings growth is high, and, although P/E ratios are high, too, they do not seem high enough; after all, such earnings growth is unsustainable. During recessions, earnings growth can be negative. But P/E ratios remain relatively too high, because investors expect that earnings-growth will eventually improve again.

For example, in December 2000, corporate earnings growth was running at an average rate of +40%, and therefore unsustainable. Naturally, if such an earnings growth rate could be sustained forever, the price over today's earnings ratio would have to be truly astronomical! By December 2001, i.e., post 9/11, the opposite had happened: the typical current earnings growth rate had dropped to −40%, and yet firms were still worth something!

An important point is that the relation between earnings growth and earnings-price yields, using only one-year ahead earnings forecasts, does not hold over time. Do not use Figure 11.1 to estimate $^{\mathbf{P}}/_{\mathbf{E}}$ ratios from earnings growth *today*! Instead, if you need to value a firm based on its current growth rates *today*, recreate the graph and implied E/P yield with data as of today.

11·1.E. P/E Ratio Application Example: Valuing Beverage Companies

Table 11.1. Financial Newspaper Printed Financials, from May 31, 2002

YTD	52-Week				YLD		VOL		NET
%CHG	HI	LO	STOCK (SYM)	DIV	%	$^{\mathbf{P}}/_{\mathbf{E}}$	100s	CLOSE	CHG
13.5	31.91	23.55	Cadbury Schweppes (CSG)	.70g	2.4	21	475	29.20	-0.20
15.4	57.91	42.59	Coca Cola (KO)	.80	1.5	35	47,565	54.39	0.24
4.6	53.50	43.08	PepsiCo (PEP)	.60f	1.2	34	26,539	50.93	0.00

The description of the table states that the P/E ratio is based on the closing price and on diluted per-share earnings ignoring extraordinary items, as available, for the most recent four quarters. Fully diluted earnings means that all common stock equivalents (convertible bonds, preferred stock, warrants, and rights) have been included.

We now apply the P/E ratio valuation method to PepsiCo. We use common newspaper information.

Table 11.1 reproduces the stock price report from May 31, 2002, in the same format as that of a prominent financial newspaper. The price-earnings ratio for Coca Cola was 35, for PepsiCo 34, and for Cadbury Schweppes 21. The (day's closing) price-per-share for Coca Cola was $\$54.^{39}$, for PepsiCo $\$50.^{93}$, and for Cadbury Schweppes $\$29.^{20}$. Using this information, we can back out Coca Cola's earnings-per-share as

$$\frac{\$54.^{39}}{\mathbf{E}_{\text{KO}}} = 35 \qquad \Longrightarrow \qquad \mathbf{E}_{\text{KO}} = \left(\frac{\$54.^{39}}{35}\right) \approx \$1.^{55} \ . \tag{11.10}$$

SIDE NOTE: The most convenient source of financial information on individual stocks may no longer be the newspaper. The World-Wide-Web, such as Yahoo!*Finance* makes it even easier to find more comprehensive financial information.

Task: Value PepsiCo now.

We now proceed with the valuation-by-comparables method, once again using PepsiCo as our guinea pig. Pretend that we do not know PepsiCo's value, but we do know PepsiCo's internal financials (earnings). Our task is to value the shares of PepsiCo in light of the value of shares of Coca Cola. If the $^{\mathbf{P}}/_{\mathbf{E}}$ comparables valuation method works, we can then check whether estimated value roughly fits the true value of PepsiCo.

So, can we really use Coca Cola as a comparable company for PepsiCo? This requires making the heroic assumption that Coca Cola is a company very similar to PepsiCo in terms of ratios and earnings. (The choice of comparable is discussed in general below.) We are indeed heroes, so we assume that Coca Cola's P/E ratio of 35 can be applied to PepsiCo earnings of $\$50.^{93}/34 \approx \$1.^{50}$ per share,

Applying Coca Cola's P/E ratio to value PepsiCo, given PepsiCo's earnings.

$$\begin{aligned} \frac{\mathbf{P}_{\text{PEP}}}{\$1.^{50}} &= 35 \quad \Rightarrow \quad \mathbf{P}_{\text{PEP}} = 35 \cdot \$1.^{50} = \$52.^{50} \\ \left(\frac{\mathbf{P}_{\text{PEP}}}{\mathbf{E}_{\text{PEP}}}\right) &= \left(\frac{\mathbf{P}_{\text{KO}}}{\mathbf{E}_{\text{KO}}}\right) . \end{aligned} \tag{11.11}$$

The valuation-by-comparables method suggests that PepsiCo should be worth $\$52.^{50}$. This is higher than the $\$50.^{93}$ that PepsiCo shares are currently trading for, but a difference of \$2 (about 5%) is well within the range of typical valuation uncertainty. So, here the method of comparables works quite well in predicting a correct market value for PepsiCo.

In PepsiCo's case, valuation-by-comparables against Coca Cola seems to work well.

Now, let us assume that we instead owned Cadbury Schweppes (CSG), that it was not yet publicly traded, and that it had just earned $\$1.^{39}$ per share. (This can be inferred from CSG's P/E ratio of 21 and closing price of $\$29.^{20}$.) Applying the Coca Cola P/E ratio of 35 to Cadbury Schweppes' earnings, we would expect CSG to trade for

In Cadbury Schweppes's case, valuation-by-comparables against either PepsiCo or Coca Cola does not work well.

$$\begin{aligned} \frac{\mathbf{P}_{\text{CSG}}}{\$1.^{39}} &= 35 \quad \Rightarrow \quad \mathbf{P}_{\text{CSG}} = 35 \cdot \$1.^{39} = \$48.^{67} \\ \left(\frac{\mathbf{P}_{\text{CSG}}}{\mathbf{E}_{\text{CSG}}}\right) &= \left(\frac{\mathbf{P}_{\text{KO}}}{\mathbf{E}_{\text{KO}}}\right) , \end{aligned} \tag{11.12}$$

—and we would be far off! The value of Cadbury Schweppes shares in the public markets is $\$29.^{20}$ per share, not $\$48.^{67}$ per share. In any case, the method of comparables has not worked well in predicting the correct market value for shares in Cadbury Schweppes.

SIDE NOTE: Incidentally, we could also have seen this directly by comparing the price-earnings ratios of CSG and KO: Coca Cola has a much higher price-earnings ratio than Cadbury Schweppes. Coca Cola and PepsiCo earnings seem to be "worth more" than Cadbury Schweppesearnings. We still went through the exercise because I wanted to pointed out the pitfall in the P/E method.

Solve Now!

Q 11.1 *Which is likely to have a higher price-earnings ratio: Microsoft or Conagra?*

Q 11.2 *A firm has earnings of \$230 this year, grows by about 6% each year, and has a price-earnings ratio of 40. What would its price-earnings ratio be if it could grow by 7% each year instead? How much would its value increase?*

Q 11.3 *A firm has earnings of \$200, and a price-earnings ratio of 20. What is its implied growth rate, if its cost of capital is about 10%?*

11·2. Problems With P/E Ratios

If comparables are dissimilar, either the market is wrong or the comparable is wrong.

So, what went wrong in the Cadbury Schweppes valuation? There are basically two possible explanations. The first explanation is that the stock market valuations—either of CSG or KO, or both—are just plain wrong. In this case, it makes little sense to use the methods of comparables. But this is unlikely. If the market values were systematically wrong, we could presumably easily get rich if we purchased undervalued firms. If it is not obvious, Chapter 19 will explain why getting rich is not easy—and which is why only about half of all investors beat the market—so we will assume that misvaluation is not the principal reason. The second explanation is that our assumption that the two firms were basically alike is incorrect. This is the more likely cause. There is a long litany of reasons why comparables are not really comparable—reasons why the technique failed us in valuing Cadbury Schweppes. Here is an outline of possible problems, on which the remainder of this chapter focuses:

Project composition differences.

- **Selection of Comparable Firms**. Comparing businesses is almost always problematic. Every firm is a unique combination of many different projects. Cadbury Schweppes owns Dr. Pepper, 7-Up, A&W Root Beer, Canada Dry, Hawaiian Punch, Snapple, Mott's Apple products, Clamato juice, plus some confectionary brands. This may not be comparable to Coca Cola, which owns Coca Cola Bottling, Minute Maid, Odwalla, and some other drink companies. Each of these businesses has its own profitability and each may deserve its own P/E ratio.

  Even for the main business, as any connoisseur knows, the Pepsi and Coca Cola sodas are not substitutes. Different consumer tastes may mean different growth rates, and especially in different countries.

> **Anecdote: Dr. Pepper's Bubble Gum**
> Dr. Pepper is not widely sold in Europe. To unfamiliar palates, Dr. Pepper tastes like liquid bubble gum. However, its caffeine content can keep the American students from falling asleep while reading my book!

  Selection of comparable firms will be discussed in Subsection 11·2.A. P/E ratio aggregation issues with multi-division firms will be covered in Subsection 11·2.B. The worst kind of averaging P/E ratios will be examined in Subsection 11·2.C.

"Meaning of numbers" differences.

- Comparing the accounting numbers may be problematic. Here are a few of the "maybe problem issues" in regard to the Cadbury Schweppes valuation:
  - Maybe as a British firm, Cadbury Schweppes uses altogether different accounting methods.
  - Maybe Cadbury Schweppes has just had an unusual year, or a year in which it plowed most of its cash into advertising, causing temporarily unusually lower earnings and much higher earnings in the future.
  - Maybe, if we did not use earnings from the most recent four quarters, but instead forecast earnings over the next four quarters, the numbers would be more comparable. How to adjust better for differences in the timing of reports will be the subject of Subsection 11·2.D.
  - Maybe Cadbury Schweppes and Coca Cola have different debt ratios. The influence of debt on P/E ratios will be explained in Subsection 11·2.E.
  - Maybe extraordinary items (excluded from this measure of earnings) should be included to make these firms more comparable. The use of other financial ratios will be discussed in Section 11·3.

11·2.A. Selection of Comparison Firms

The single biggest problem with comparables may be the selection of appropriately comparable firms. Say, you own a little soda producer, the *Your Beverage Corporation* (YBC), with earnings of \$10 million. You want to select public firms to use as your comparables from the universe of firms. Usually, this means publicly traded companies. So, which of the 10,000 or so publicly traded companies are most comparable? Are firms more similar if they are similar in assets, similar in their business products and services, similar in their geographical coverage, similar in their age, similar in their size and scale, etc.? Do they have to be similar in all respects? If so, chances are that not a single of the 10,000 firms will qualify!

Finding comparables: general criteria for evaluating similarity.

Let us assume that after extensive research and much agonizing, you have identified the (same) three companies: KO, PEP, and CSG. Which one is most similar? We know that depending on which firm you select, your valuation could be \$250 million (if Cadbury Schweppes, unlevered, was most similar), \$410 million (if PepsiCo was most similar), or \$500 million (if Coca Cola was most similar). What now?

Which firm is the single best comparable?

Selecting comparables depends both on the judgment and on the motives of the analyst. In the YBC case, one analyst may consider all three firms (KO, PEP, and CSG) to be similar, but CSG to be most similar because it is the smallest comparison firm. She may determine a good P/E ratio would be 30. Another analyst might consider Coca Cola and Pepsi-Co to be better comparables, because they tend to serve the same market as YBC. He may determine a good P/E ratio would be 45. The owner of YBC may want to sell out and try to find a buyer willing to pay as much as possible, so she might claim Coca Cola to be the only true comparable, leading to a P/E ratio of 50. The potential buyer of YBC may instead claim Cadbury Schweppes to be the only comparable, and in fact attribute an extra discount to CSG: after all, YBC is a lot smaller than CSG, and the buyer may feel that YBC deserves only a P/E ratio of 10. There is no definitive right or wrong choice.

Different conclusions about the value of the same firm: Analyst errors and biases can create wide variations in valuations.

Another choice may be to not select either the P/E ratio of 10 or the P/E ratio of 45, but to "split the difference." A reasonable P/E ratio that is better than either 10 or 45 may be 30. This might mean valuations of around \$300 to \$400 million. Unfortunately, though this may be the best solution, it is not a good solution. The next two section shows you why averaging P/E ratios is really not a good procedure.

Incorrect, but practical averaging. Let's pray!

11·2.B. (Non-) Aggregation of Comparables

Companies are collections of many projects. Is the P/E ratio of the company the same as the weighted average P/E ratio of its subsidiaries, so that we can seamlessly work with either individual subsidiary P/E ratios or with overall company P/E ratios? Unfortunately, the answer is no.

Can we aggregate (take averages of) P/E ratios? No!

Consider two firms. A has a $g_{\mathbf{E}} = 5\%$ growth rate and earnings of \$100 (next year); and B has a $g_{\mathbf{E}} = 14\%$ growth rate and earnings of \$50 (next year). Both have a 15% cost of capital. Their respective values should be

The average of individual P/E ratios is not the overall P/E ratio

$$\begin{aligned} \mathbf{P}_{\mathsf{A}} \equiv V_{\mathsf{A}} &= \frac{\$100}{15\% - 5\%} = \$1,000 \quad \Longrightarrow \quad \mathbf{P}/\mathbf{E} = 10 \ , \\ \mathbf{P}_{\mathsf{B}} \equiv V_{\mathsf{B}} &= \frac{\$50}{15\% - 14\%} = \$5,000 \quad \Longrightarrow \quad \mathbf{P}/\mathbf{E} = 100 \ . \end{aligned} \tag{11.13}$$

What would happen if these two firms merged into a single conglomerate, called AB? Assume AB does not operate any differently—the two firms would just report their financials jointly. AB must be worth \$6,000—after all, nothing has changed, and we know that NPV's are additive. It would have earnings of \$150. Thus, its P/E ratio would be $\$6,000/\$150 = 40$.

$$\text{Correct But Unknown AB P/E ratio:} \qquad \frac{\mathbf{P}_{\mathsf{AB}}}{\mathbf{E}_{\mathsf{AB}}} = 40 \quad \Longrightarrow \quad \mathbf{P}_{\mathsf{AB}} = 40 \cdot \mathbf{E}_{\mathsf{AB}} \ . \tag{11.14}$$

Your goal is to value AB. Fortunately, you just happen to know a perfectly comparable firm for division A (trading at about $\mathbf{P}/\mathbf{E} = 10$), and a perfectly comparable firm for division B (trading at about $\mathbf{P}/\mathbf{E} = 100$). You even have a good idea of the relative size of the divisions inside AB (5:1). Knowing the combined earnings of AB of \$150, you want to estimate a value for AB, based on your two comparables. Unfortunately, neither the unweighted average P/E ratio nor the weighted average P/E ratio gives you the correct desired P/E ratio of 40:

$$\begin{aligned} &\text{Unweighted P/E ratio Average of A and B} \quad \left(\frac{1}{2}\right) \cdot \left(\frac{\mathbf{P}_{\mathsf{A}}}{\mathbf{E}_{\mathsf{A}}}\right) + \left(\frac{1}{2}\right) \cdot \left(\frac{\mathbf{P}_{\mathsf{B}}}{\mathbf{E}_{\mathsf{B}}}\right) = 55 \ , \\ &\text{Weighted P/E ratio -Average of A and B} \quad \left(\frac{1}{6}\right) \cdot \left(\frac{\mathbf{P}_{\mathsf{A}}}{\mathbf{E}_{\mathsf{A}}}\right) + \left(\frac{5}{6}\right) \cdot \left(\frac{\mathbf{P}_{\mathsf{B}}}{\mathbf{E}_{\mathsf{B}}}\right) = 85 \ . \end{aligned} \tag{11.15}$$

Applying either of these two P/E ratios to your \$150 in earnings would result in price assessment for AB that would be too high.

IMPORTANT: *It is difficult to interpret the price-earnings ratio for merged firms, because price-earnings ratios cannot be averaged.*
Mergers change the P/E ratio, *even if they do not create value.*

Lack of easy aggregation makes it difficult to value even well-defined firms, if the comparables are divisions inside larger firms.

The inability to aggregate divisions is not only an issue for the firm that is to be valued, but it also makes it difficult to extract a single comparable ratio from a division from inside conglomerates. In our case, let's assume that we only wanted to value the U.S. Dr. Pepper division, and that the U.S. Coca-Cola soda division was a perfect comparable for the U.S. Dr. Pepper. But how do we extract a P/E ratio for the Coca-Cola division, if all we know is the P/E ratio of the overall Coca Cola company with all its international subsidiaries, Minute Maid, Odwalla, etc.?

The consequences of the aggregation failure mean, strictly speaking, that only the most basic single-product firms should be compared.

We have no good methods to aggregate and disaggregate P/E ratios. Therefore, strictly speaking, we can only compare full firms that are similar, which means that P/E ratios are likely to work well only for simple and well-defined companies, and not so well for complex conglomerates. In retrospect, it would have been a coincidence if our naïve attempt to apply the overall P/E ratio of Coca Cola or PepsiCo to Cadbury Schweppes's overall earnings would have worked. Indeed, in retrospect, it was an amazing coincidence that PepsiCo and Coca Cola had such similar $\mathbf{P}/\mathbf{E}$ ratios. We lived in blissful ignorance.

In Part III, we will introduce "beta" as a valuation measure. Unlike P/E ratios, beta nicely aggregates and disaggregates. This makes it relatively easy to compute betas for conglomerates from their divisions, and to extract a division beta (given the conglomerate beta and comparable betas for other divisions).

11·2.C. A Major Blunder: Never Average P/E ratios

On top of these unavoidable P/E ratio problems, many an analyst has mistakenly created a much worse and avoidable problem by averaging P/E ratios. Averaging overlooks the fact that earnings can be (temporarily) zero or negative, which can totally mess up any P/E ratio analysis. For example, consider the example where the choice of industry comparables for X consists of A and B.

Using a firm with negative earnings as the comparable makes absolutely no sense.

	Value (**P**)	Earnings (**E**)		P/E ratio	E/P yield
Firm A	\$1,000	+\$10	$\Longrightarrow$	100	1.000%
Firm B	\$20	−\$5	$\Longrightarrow$	−4	−0.250%
		Industry Average:		48	0.375%
Firm X	?	\$2			

If Firm B were the only comparable, it would imply a negative value for Firm X,

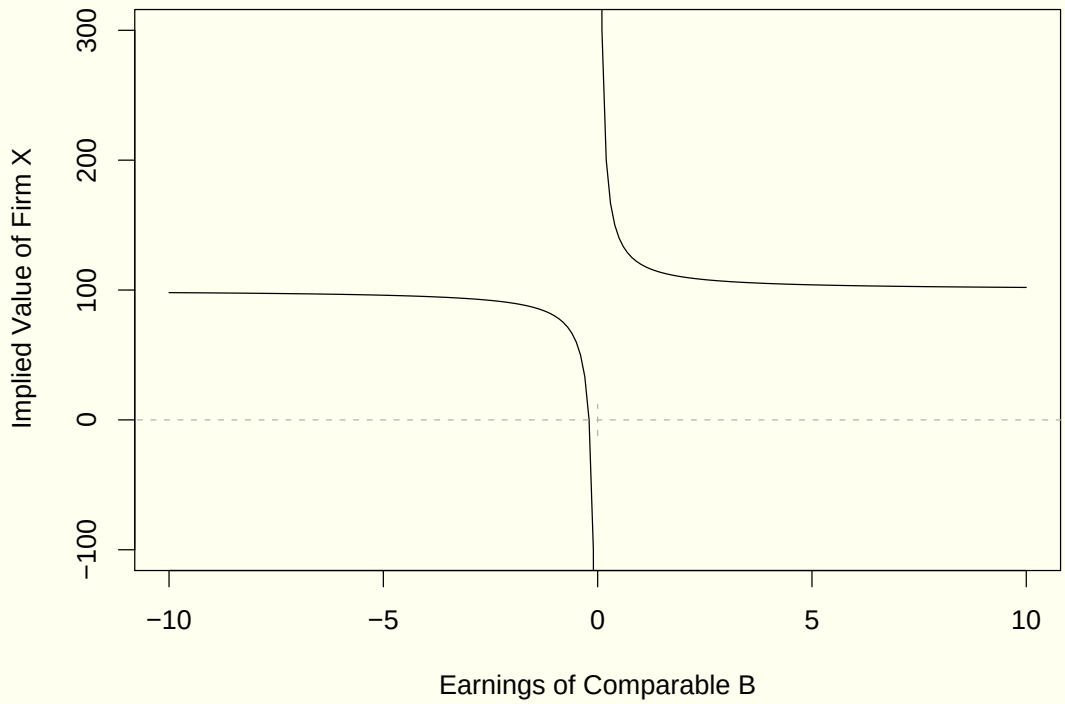
$$V_{\mathsf{X}} = \mathbf{E}_{\mathsf{X}} \cdot (P_{\mathsf{B}}/\mathbf{E}_{\mathsf{B}}) = \$2 \cdot (-4) = -\$8 \ . \tag{11.16}$$

A value of minus eight dollars for a firm with positive earnings and limited liability is not sensible. Luckily, this comparables-derived valuation is so nonsensical that no analyst would not notice it.

But the problem is often overlooked when an analyst uses a $^{\mathbf{P}}/_{\mathbf{E}}$ industry average. For example, assume our analyst uses an average of both comparables: Firm A has a P/E ratio of 100, Firm B has a P/E ratio of −4. Thus, the average P/E ratio would be 48 (= $[100 + (-4)]/2$), which is a reasonable looking average that would not raise a red flag. A thoughtless analyst could conclude that Firm X should be worth $V_{\mathsf{X}} = \mathbf{E}_{\mathsf{X}} \cdot (P_{\mathsf{A,B}}/\mathbf{E}_{\mathsf{A,B}}) = \$2 \cdot 48 = \$96$.

Averaging P/E ratios can look reasonable at first glance...

Figure 11.2. Implied Value vs. Earnings Changes of One Comparable



If the earnings of the comparable B are \$1, we get a sensible value for our firm X. If the earnings are a little bit lower, we get a non-sensically high number; if the earnings are a little bit lower, we get a non-sensically low number; and if the earnings are yet a little bit lower, we again get a non-sensical number—but one that can appear at first glance to be of reasonable magnitude.

...but it is not.

Figure 11.2 makes the absurdity of this method even clearer. What happens to the implied value of Firm X if Firm B's earnings change? As Firm B improves its performance from about −\$5 to about −\$1, the implied value remains a seemingly reasonable \$90 to \$100. Beyond −\$1, earnings improvements in the comparable B create non-sensically huge negative implied firm values for X. Then, further improvements suddenly create non-sensically huge positive firm values. Finally, finally once the earnings of B are above \$1 or so, you again get seemingly reasonable values (of about \$100) for X. So, small changes in earnings can produce either seemingly sensible or non-sensible valuations. In other examples, even one comparable with earnings close to zero among a dozen comparables can totally screw up an average of many comparable P/E ratios.

Excluding firms does not help.

In an effort to deal with this problem, a common industry practice is to drop out firms with non-positive earnings from $^{P}/_{E}$ averages. Unfortunately, this is not a good solution, either. First, we want an accurate valuation, and the stock market did value Firm B at \$20. We have no good reason to ignore firms with low earnings. Second, dropping out some firms does not solve the problem: the firm would enter the $^{P}/_{E}$ average if its earnings are +5 cents (leading to a very high industry $^{P}/_{E}$ average), but be dropped out if its earnings are −5 cents (potentially leading to a much lower industry average). A small change in the P/E ratio of one comparable among the industry would have a disproportionately large impact on comparables valuation due to arbitrary inclusion/exclusion of comparables, rather than due to closeness of earnings to zero.

NERD NOTE: *The reason for all these problems with price-earnings ratios is that earnings are in the denominator. The function* $^1/_{\mathbf{E}}$ *is both discontinuous and very steep when earnings are close to zero. In contrast, the price (value) is guaranteed to be positive.*

The two better alternatives.

Fortunately, there are two easy overall alternatives to obtain good "pseudo averaged P/E ratios," even if some firms' earnings in the industry are low:

1. Work with earnings yields (E/P yields) instead of P/E ratios.

   In the example, the E/P yield of Firm A is $\$10/\$1,000 = 1\%$; the E/P yield of Firm B if it earned -1 cent is $-\$0.^{01}/\$20 = -0.05\%$. The average E/P yield is thus $[1\% + (-0.05\%)]/2 = 0.475\%$. Inverting this back into a P/E ratio provides a halfway sensible value for the P/E ratio ($1/0.475\% \approx 211$).

2. Add up all market capitalizations in the industry and all earnings in the industry, and then divide the two.

   In the example where B earned 1 cent, the total industry earnings would be $\$10.^{00} + \$0.^{01} = \$10.^{01}$, the entire industry market value would be $\$1,000 + \$20 = \$1,020$, and the average P/E ratio would be $\$1,020/\$10.^{01} = 101.9$. Note that in this method, firms are not equally weighted, but weighted by their relative market valuation. This may or may not be desirable: In our example, firm A would become the dominant determinant of our comparable valuation ratio.

Neither of these methods will give a very appealing comparable if the *total* industry average earnings are very small or negative. Our averaging alternatives can only avoid the problem of excessive influence of a small number of negative (or small) earnings firm in the average.

IMPORTANT: *Although neither* P/E ratio*s nor* E/P yield*s, strictly speaking, can be averaged, for lack of a better alternative and in order to avoid relying on a single comparable, averaging is still often done. Two pieces of advice: First, do not take averages literally. The goal is to produce an "intuitive average" derived from multiple comparables, not an exact number. Second, never average* P/E ratio*s: instead, either average* E/P yield*s, or divide total* **P** *sums by total* **E** *sums.*

11·2.D. Computing Trailing Twelve Month (TTM) Figures

When comparable firms report annual statements in different months, the time change in economic climate introduces yet another problem.

There is one "small" mechanical detail left: Timing. First, is it meaningful to use annual earnings for a firm if the last annual report was from eleven months ago? Or should you use just the last quarter's numbers? Second, some firms report earnings in June, others in December. Should you compare financials that are timed so differently, especially if the economy has changed during this time lag? For example, consider the following reports:

	2001				2002			
	mar	jun	sep	dec	mar	jun	sep	dec
Firm 1	Q1:\$1	Q2:\$2	Q3:\$3	Ann:\$15	Q1:\$5	Q2:\$6	Q3:\$7	Ann:\$31

Your own firm closed its financial year in September 2002 with annual earnings of \$12 and Q4 earnings of \$2. What are the relevant comparable earnings from Firms 1 and 2? Should you work with your annual earnings and choose the comparable but dated annual earnings from December 2001 for firm 1? If you use the dated annual earnings number, to obtain firm 1's P/E ratio, should you divide by the current price or by the December 2001 price? Or should you just work with your quarterly earnings of \$2, and compare it to the 4th quarter earnings of Firm 1? Or to the calendar-similar \$7 in earnings? Here is some advice.

This time difference can be reduced, even though quarterly accounting statements themselves should be avoided. Instead, use quarterlies and annuals to compute "trailing twelve month" (TTM) figures.

Generally, the best method to adjust flows (such as earnings) into a "most recent annualized equivalent" is to use a so-called **trailing twelve months** (**TTM**) adjustment. In our example, this means adding the earnings from Q4-2001 through Q3-2002,

$$\begin{aligned} \text{As If Annual} &= \$9 + \$5 + \$6 + \$7 = \$27 \\ \text{TTM Earnings} &= \text{Q4-01} + \text{Q1-02} + \text{Q2-02} + \text{Q3-02} \quad . \end{aligned} \tag{11.17}$$

Using the reported earnings, you can also compute this

$$\begin{aligned} \text{As If Annual} &= \$15 + (\$5 - \$1) + (\$6 - \$2) + (\$7 - \$3) = \$27 \\ \text{TTM Earnings} &= \text{Ann-01} + (\text{Q1-02} - \text{Q1-01}) + (\text{Q2-02} - \text{Q2-01}) + (\text{Q3-02} - \text{Q3-01}) \quad . \end{aligned} \tag{11.18}$$

SIDE NOTE: In most cases, quarterlies should *not* be used. Most firms do more business in December, and December can be the first month in a quarter or the last month in a quarter. Not only are different quarters difficult to compare across firms, the fourth quarter may be difficult to compare to other quarters from the same firm.

"Trailing twelve months" only works for "flow" numbers (such as income), not for stock numbers (such as assets).

There are two caveats: first, TTM-adjust only "flow" numbers (such as earnings or sales), never "stock" numbers (such as corporate assets). Stock numbers are whatever they have been reported as most recently. Second, firms sometimes change their fiscal year, often to make it intentionally more difficult to compare numbers. In this case, extra care must be exercised.

11·2.E. Debt Adjustments?

This section assumes that riskier projects have to offer higher *expected* rates of return. Why this is the case will only fully become clear in Part III. Trust me for now.

Does leverage influence P/E ratios?

Companies can be financed through a mix of debt and equity. We want to know if the P/E ratio of a firm is changed by its **leverage**. If the same firm with more debt in its capital structure has a different P/E ratio, then we cannot compare two otherwise identical companies with different debt ratios without adjustment.

Recapitalizing a 100-0 firm into a 50-50 firm.

Return to a simple example. With \$100 in annual earnings, a growth rate of 0% now, and an appropriate cost of capital of 10%, a hypothetical firm A would be worth \$1,000. A has no debt (leverage). We now work out what happens to the price-earnings ratio of A if it were to recapitalize itself. That is, A takes on a loan of \$500, and pays the cash proceeds from the loan to the equity share owner. This is called a **debt-for-equity** (or **debt-for-stock**) exchange. Such transactions are fairly common.

From Chapter 4, we already know that the cost of capital (interest rate) consists of multiple components: the time-premium, the default-premium (on average, zero), and the risk-premium. From Chapter 6·2, we know that levered equity is riskier than debt. Also, trust me now when I claim that if investors are not risk-neutral, the cost of capital is generally higher for riskier securities. The *expected* rate of return on risky investments must be higher than the *expected* rate of return on safer investments; it must be higher for equity than for debt; and it must be higher for riskier debt than for safer debt. In the example in this section, we shall assume that the appropriate expected rate of return for a firm financed with 50% debt is 7.5% for the debt, and 12.5% for the remaining 50% "levered equity" (residual ownership).

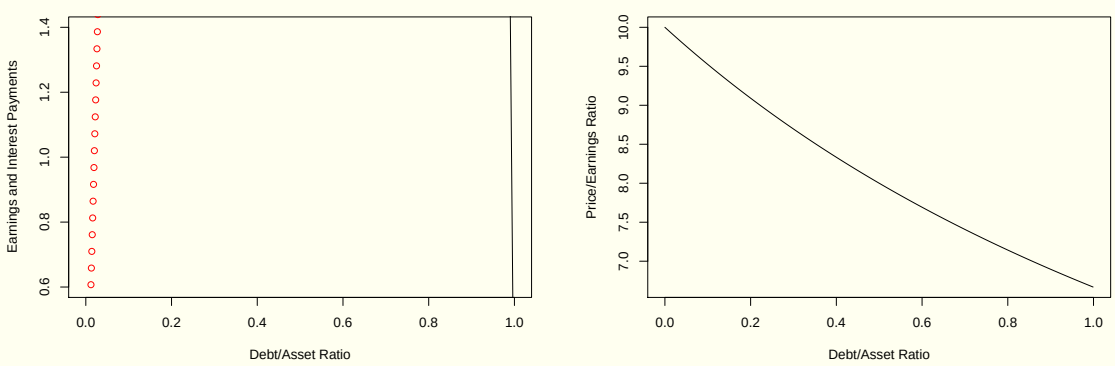
...with different costs of capital for bonds and stocks.

SIDE NOTE: The cost of capital for the firm overall remains at 10% after the firm is financed differently, because

$$\begin{array}{ccccccccc} 50\% & \cdot & 7.5\% & + & 50\% & \cdot & 12.5\% & = & 10\% \\ \text{proportion of debt} & \cdot & \text{cost of captal for debt} & + & \text{proportion of equity} & \cdot & \text{cost of capital for equity} & = & \text{cost of capital for the firm} \end{array} . \tag{11.19}$$

The cost of capital for the overall company has not changed. Section 21 will discuss this in more detail.

Figure 11.3. Earnings, Interest Payments, and Price-Earnings ratios as a Function of Leverage



As the firm's debt ratio goes up, its interest payments increase, its earnings decrease—and so does its price-earnings ratio. The same underlying projects can thus have a price-earnings ratio of

How much total interest payment does the debt have to promise? The loan value is \$500, the interest rate is 7.5%, and payments are forever. Therefore, the perpetuity formula can be solved as follows:

The changed price-earnings ratio under the 50-50 capital structure.

$$\$500 = \frac{\text{Annual Payment}}{7.5\%} \implies \text{Annual Payment} = \$37.^{50}/\text{year} \tag{11.20}$$

$$\text{PV} = \frac{\text{CF}}{\mathcal{E}(r)} .$$

Having issued debt, the equity no longer receives \$100 per year, but only what is left after interest payments, i.e., $\$100 - \$37.^{50} = \$62.^{50}$ per year. Consequently, the price-earnings ratio of this firm's equity shares will be the value of the equity (now \$500) divided by the earnings available to equity,

$$\mathbf{P}/\mathbf{E} = \$500/\$62.^{50} = 8.0 \text{ (times)} \quad . \tag{11.21}$$

In sum, when the same firm is financed with a 50/50 debt equity ratio, rather than a 0/100 debt equity ratio, its price-earnings ratio is 8 rather than 10. Figure 11.3 shows this graphically for a whole range of different debt-asset ratios: the higher the firm's debt load, the higher the interest payments, the lower the equity earnings, and the lower the price-earnings ratio. (The figure assumes that the expected [not just the promised!] debt interest rate increases in the debt-asset ratio according to the formula $5\% + 5\% \cdot D/A$.)

IMPORTANT: *The price-earnings ratio is higher when the firm has lower leverage (debt-equity ratio).*

Revisiting CocaCola, PepsiCo, and Cadbury: Using leverage information to improve valuation.

To compare firms that we deem to be identical in operations, but different for their capital structures, we need to adjust them to equivalent, unleveraged values. Let us put this insight to practical use. Assume that the companies of Coca Cola, PepsiCo, and Cadbury Schweppes are identical, even if their equities may not be because of their different leverage ratios. Some gathering of information from Yahoo!*Finance* allows us to put together Table 11.2:

Table 11.2. Financial Information from Yahoo

		Leverage		Financials, in billion-\$			
STOCK (SYM)	$\mathbf{P}/\mathbf{E}$	D/E	$\frac{D}{D+E} = D/V$	Interest Expense	EBITDA	Earn-ings	Equity Value
Coca Cola (KO)	35	56%	36%	$\$0.^{244}$	$\$6.^{14}$	$\$3.^{91}$	$\$136.^{85}$
PepsiCo (PEP)	34	33%	25%	$\$0.^{207}$	$\$4.^{26}$	$\$2.^{74}$	$\$93.^{16}$
Cadbury Schweppes (CSG)	21	49%	33%	$\$0.^{155}$	$\$1.^{28}$	$\$0.^{72}$	$\$15.^{12}$

Note: The firm's asset value A is the sum of its equity value E and debt value D. The formula to translate debt-equity (D/E) ratios into debt-asset (D/A) ratios is

$$\frac{1}{D/A} = \frac{1}{D/(D+E)} = \left(\frac{D+E}{D}\right) = 1 + \left(\frac{E}{D}\right) \quad \Rightarrow \quad \frac{D}{E} = \frac{1}{\left(\frac{1}{D/A} - 1\right)} \quad . \tag{11.22}$$

We are now ready to ask what would happen if the three companies were not leveraged, but were financed only by equity. First, the earnings would not be diminished by the interest expense that the firms are currently paying. We must add back the interest expense to the earnings to get "as if unlevered" earnings. Second, the market capitalization of equity would increase. After all, the firm's projects would no longer be financed by debt and equity, but by equity alone. If a firm has a debt/equity ratio of 33% (1/3), for every dollar worth of debt it currently has 3 dollars worth of equity. Thus, an unlevered equivalent firm would be financed with \$4 in equity. To convert a debt/value ratio to an unlevered value, divide the (levered) equity value by one minus the debt/value ratio. Putting this together, Table 11.3 constructs "as if unlevered" $^{\mathbf{P}}/_{\mathbf{E}}$ ratios by dividing the unlevered firm capitalization by the unlevered equity earnings (all quoted in billion dollars):

Translating P/E ratios of levered firms into P/E ratios as if unlevered.

Table 11.3. Computing Unlevered $^{\mathbf{P}}/_{\mathbf{E}}$ Ratios

STOCK (SYM)	$(^{\mathbf{P}}/_{\mathbf{E}})_{\text{Lv}}$	Unlevered Earnings	Unlevered Value	$(^{\mathbf{P}}/_{\mathbf{E}})_{\text{UnLv}}$
Coca Cola (KO)	35	$\$3.^{91} + \$0.^{244} = \$4.^{15}$	$\$136.^{85}/(1-36\%) \approx \$214$	52
PepsiCo (PEP)	34	$\$2.^{74} + \$0.^{207} = \$2.^{95}$	$\$93.^{16}/(1-25\%) \approx \$124$	42
Cadbury (CSG)	21	$\$0.^{72} + \$0.^{155} = \$0.^{88}$	$\$15.^{12}/(1-33\%) \approx \$23$	26

Side Note: These computations are only approximations and wrong for at least two reasons. First, we assume no taxes. If by unlevering the firm incurred more taxes, these taxes should be subtracted from firm value. Chapter 22 will discuss the role of corporate taxes in more detail. Fortunately, in this particular case (for these three companies), not much would change. Second, the unlevering depends on assuming that the firm continues forever, so that the perpetuity formula is applicable. This could also be wrong. Again, fortunately, in this particular case (for these three companies), not much would change for other growth assumptions.

Does it appear as if Cadbury Schweppes (the underlying unlevered company) now is a lot more like PepsiCo (the company) than levered Cadbury Schweppes shares were to levered PepsiCo shares? Unfortunately, the answer is the opposite of what we had hoped for. The P/E ratios of Cadbury Schweppes are even more different from those of Coca Cola and PepsiCo than they were before. We also have some more information to evaluate our earlier remarkable finding that PepsiCo could be accurately valued with the comparable of Coca Cola. We chose Coca Cola because we believed that the firm of Coca Cola would be similar to PepsiCo, not that the equity shares of Coca Cola would be similar to those of PepsiCo. But, in this case, the firms of Coca Cola and PepsiCo are less similar than the equity shares of Coca Cola and PepsiCo: their unlevered P/E ratiosare farther apart than their levered P/E ratios. So, if we had properly applied the valuation ratio of one firm to the other firm, we would have concluded that PepsiCo and Coca Cola are not so similar, after all, and that we should not have compared the two. We just got lucky. We lived in ignorant bliss.

Unfortunately, in our case, after proper adjustment, the P/E ratios have become more different, not more similar.

Solve Now!

Q 11.4 *List the main problems with using comparables.*

Q 11.5 *A firm with a* P/E ratio *of 20 wants to take over a firm half its size with a* P/E ratio *of 50. What will be the* P/E ratio *of the merged firm?*

Q 11.6 *The following are quarterly earnings and assets for Coca Cola and PepsiCo:*

	KO				PEP			
	Earnings (Qua/Ann)		Assets (Qua/Ann)		Earnings (Qua/Ann)		Assets (Qua/Ann)	
6/2002	1,290		25,287		888		24,200	
3/2002	801		23,689		651		22,611	
12/2001	914	3,979	22,417	22,417	667	2,662	21,695	21,695
9/2001	1,074		22,665		627		23,036	
6/2001	1,118		22,387		798		19,503	
3/2001	873		22,248		570		18,660	
12/2000	242	2,177	20,834	20,834	698	2,183	18,339	18,339
9/2000	1,067		23,141		755		17,659	
6/2000	926		23,258		668		17,492	

If it is now in July 2002, what would be good comparable earnings and comparable assets for these two firms?

Q 11.7 *A firm that is 100% equity financed is considering a recapitalization to a 50-50 debt-equity ratio. What will happen to its* P/E ratio*?*

Q 11.8 *A firm has a* P/E ratio *of 12 and a debt-equity ratio of 66%. What would its unlevered* P/E ratio *(i.e., the* P/E ratio *of its underlying business) approximately be?*

Q 11.9 *On October 9, 2002, the seven auto manufacturers publicly traded in the U.S. were*

Manufacturer	Market Cap	Earnings
Volvo (ADR)	\$5.7	-\$0.18
Ford	\$14.1	-\$5.30
GM	\$18.8	\$1.83
Nissan (ADR)	\$27.0	\$2.55
DaimlerChrysler	\$32.3	\$4.63
Honda (ADR)	\$37.7	\$3.09
Toyota (ADR)	\$87.3	\$4.51

(All quoted dollars are in billions. Ignore leverage. ***ADR*** *means* ***American Depositary Receipt****, a method by which foreign companies can list on the New York Stock Exchange.) On the same day, Yahoo! Deutschland reported that Volkswagen AG had earnings of 3.8 billion euro. In terms of sales, Volkswagen was most similar to Volvo and Ford. What would you expect Volkswagen to be worth?*

Q 11.10 *What are the basic assumptions in applying the* P/E ratio *of one company to the earnings of another company?*

11·3. Other Financial Ratios

There are a large number of other ratios in use, and this section presents a potpourri of some popular ones. The list is not exhaustive, as indeed only the imagination limits the quantities that can be used in the denominator. Two ratios that are relatively similar to the P/E ratio discussed in the previous section are

Price-earnings ratios are not the only measure. The goal is to find a measure that is proportional to value.

Alternative Price-Earnings Ratios Earnings can be defined in a variety of ways: with or without extraordinary items, diluted, etc. There is no right or wrong way: the goal is to find a ratio that makes the comparables firm appear to be as similar as possible.

> **Side Note**: One measure of earnings that we discussed in Chapter 9 is ebitda (earnings before interest and taxes, depreciation, and amortization). The rationale is that accounting depreciation is so fictional that it should not be subtracted out. But ebitda has problems with leverage (interest expense) and capital expenditures.

Price/Book-Value-of-Equity Ratios This ratio is commonly used, and often abbreviated as the market/book ratio. It should not be recommended for valuation purposes. The reason is that the book value of equity is close to meaningless. Accountants use the book value of equity to balance assets and liabilities, in accordance with all sorts of accounting conventions (such as depreciation). As a result, this measure is especially different across firms with different ages of fixed assets (such as buildings).

However, sometimes neither earnings nor book-value of equity are meaningful. For example, biotech firms may need to be valued even before they have meaningful, positive earnings. The idea is to substitute another, more meaningful quantity for earnings. In biotech firms, a better (but still very bad) measure than earnings may be the number of scientists. An analyst might find that biotech firms are worth $1,000,000 per employed scientist. In principle, the comparables valuation method remains the same as it was when used with price earnings ratios. The alternative ratio typically still has price (either equity value or overall firm value) in the numerator, but a quantity other than earnings (e.g., sales or number of scientists) in the denominator. The analyst chooses comparable firm(s), determines an appropriate typical comparables ratio, and finally multiplies this comparables ratio by the firm's own quantity to determine its value. This may work well only if firms are comparable enough among the chosen dimension that application of the ratio of some firms to the ratio of others offers a meaningful price.

Many biotech firms have neither earnings nor sales. So, what to use?

Price/Sales Ratios This ratio is especially popular for industries that do not have positive earnings. Therefore, it was commonly used during the Internet bubble, when

few Internet firms had positive earnings. Presumably, firms with higher sales should be worth more.

One problem was that firms, such as Amazon during the bubble, were known to sell merchandise at a loss. Naturally, it is relatively easy to sell $100 bills for $99! So, the more Amazon sold, the more money it lost—but the more valuable Amazon appeared to be. After all, with higher sales, the Price/Sales ratio suggested that Amazon would be worth more.

Price/Employees Ratio This ratio is popular when financials are deemed not trustworthy. Of course, it assumes that the employees at the comparable firm are as productive as the employees in the company to be valued.

Price/Scientists Ratio This ratio is popular for upstart technology firms, which have neither earnings nor sales. One problem is that it induces firms to hire incompetent scientists on the cheap in order to increase their valuations. After all, firms with more scientists are presumed to be worth more.

None of these ratios even adjusts moderately for differences in debt ratios, and, if they become common, all are easily subject to manipulation by the firm to be valued.

SIDE NOTE: The latter ratios are not just mildly sensitive to leverage changes (like the price-earnings ratio), but extremely sensitive. They fail to adjust *at all* for leverage changes. For example, consider a firm worth $100 with sales of $100. If this firm is financed with 100% equity, it would appear that each dollar of sales is worth one dollar in the firm. But, if the firm now issues $90 worth of debt and uses it to repurchase the equity, it would have a capital structure with only $10 equity remaining. Now, it would appear that it takes $10 worth of sales to support $1 worth of equity value. This is obviously incorrect. If you want to use the price/sales ratio, you must compare all firms "as if fully unlevered."

Solve Now!

Q 11.11 *On July 28, 2003 (all quoted dollars are in billions):*

Firm	*Cash*	*Sales*	*Dividends*	*Value*	*D/E*
CSG	*n/a*	*$9.2*	*$0.4*	*$12.2*	*153%*
KO	*$3.6*	*$20.3*	*$2.2*	*$110.8*	*43%*
PEP	*$1.8*	*$25.9*	*$1.1*	*$81.0*	*22%*

Hansen Natural had $210,000 in cash, $9.22 million in sales, zero dividends, and a debt/equity ratio of 10%. What would a price/cash ratio predict its value to be? A price/sales ratio? A price/dividend ratio? Elaborate on some shortcomings.

11·4. Closing Thoughts: Comparables or NPV?

Use both valuation methods, and use common sense to decide what you believe.

Should you use comparables or net present value? Both methods rely on inputs that you will almost surely not know perfectly: You will never have the perfect comparable, and you will never know the expected future cash flow perfectly. Fortunately, the cause of errors is different for these two methods. Therefore, if you use both, you can get a better idea of where the true value is. This does not mean that you should average the valuation estimates obtained from NPV and comparables; instead, you should perform both analyses, and then take a step back and make up your mind as to which combination of methods seems to make most sense in your particular situation. Yes, valuation is as much an art as it is a science.

11·5. Summary

The chapter covered the following major points:

- The P/E (price earnings) ratio reflects the firm's growth opportunities and cost of capital.
- P/E ratios and other methods based on comparables can provide an alternative valuation of firms and projects.
- Comparables suffer from a variety of problems, some of which can be corrected, and some of which are intrinsic and uncorrectable.
- The comparables valuation techniques and NPV have different weaknesses, which therefore often makes it worthwhile to consider and judge both.

A. Appendix: Other Ratios Used in Corporate Analysis

Not all ratios are used to estimate firm value. Other ratios can help to assess financial health and profitability—or are merely interesting. For example, a number of ratios are often used to judge financial health (proximity to bankruptcy) and profitability. We sort the following ratios so that ratios at the top tend to reflect financial health, while ratios at the bottom tend to reflect profitability. Most ratios vary systematically by industry and over the business cycle. Thus, they should only be compared to similar firms at the same time.

Side Note: I would also advise special caution when ratios involve quantities from the balance sheet, such as the book value of equity. Book value of equity is often so far from the true market value, that ratios based on the book value are almost meaningless.

Here are some of the more interesting and common ratios:

Debt Ratio The ratio of debt as a fraction of assets. Many variations exist: debt can be either all debt, or just long-term debt. Assets can be measured either in terms of book value or in terms of market value. Because market value of debt is rarely available, a common variant adds the book value of debt and the market value of equity. (If interest rates have not changed dramatically since issue, this is a good approximation of overall market value.)

Debt/Equity Ratio Like the debt ratio, but eliminates debt from the denominator.

Interest Coverage The ratio of debt payments due as a fraction of cash flows. Many variations exist: debt payments can be only interest due, or include both principal and interest. Cash flows can by any of a number of choices: all cash flows, operating cash flows, net income plus depreciation minus capital expenditures, and net income plus depreciation are all popular.

Current Ratio The ratio of **current assets** over **current liabilities** is a measure of liquidity. Often interpreted to be "healthy" if greater than 1.5. The **quick ratio** (**acid-test**) deletes inventories from current assets (and is often considered healthy if it is greater than 1.0—I am not sure why). The **cash ratio** further eliminates receivables from current assets.

Turnover The ratio of a current asset (or sometimes liability) component divided by sales. **Receivables turnover** indicates how quickly customers are paying. **Inventory turnover** indicates how quickly we are using up our inventory. **Payables turnover** measures how quickly we are paying our own bills.

These measures are sometimes inverted (one divided by the ratio) and multiplied by 365 to obtain a "number of days" measure. For example, if the receivables turnover is 3, then our average receivables collection period is $1/3 \cdot 365 \approx 120$.

Profit Margin The net or gross profit divided by sales.

Return on Assets (ROA) The EBIT divided by net operating assets. Often compared to the firm's interest rate. A variant is **return on sales**, which divides by sales instead of assets.

Book/Market Ratio The book-value of the equity divided by the market value of the equity. If the book value of equity is representative of how much the assets would cost to replace, then this is a measure of how much the market values the (special) growth opportunities that the company has created from replaceable assets.

Retained Earnings Ratios The retained earnings, divided either by sales or assets. A firms that decides to start retaining more earnings should pay out more in the future. Because the retained earnings should be reinvested, such firms should have higher expected earnings growth.

Dividend Payout Ratio The paid out dividends divided by earnings (any flavor thereof). Dividends can include share repurchases. The idea is that a firms that pays out more earnings today should pay out less in the future, because—in contrast to a firm that retains earnings—it lacks the paid out cash for reinvestment.

Dividend Yield The amount of dividends divided by the share price. Dividends can include share repurchases (in which case, this is called the **payout ratio**). Dividends are a flow measure, while the stock price is a stock measure. Thus, dividends can be measured at the beginning of the period or at the end of the period (in which case, this is called the **dividend-price ratio**).

> NERD NOTE: ***Economics:*** *Firms that have a low payout yield today either have to see smaller stock price increases in the future (which means a lower expected rate of return), or they have to increase their payout yield sometime in the future. Otherwise, noone would want to hold them today.*

B. Nerd Appendix: A Formula For Unlevering P/E ratios

Start with a special case in which firms have comparable leverage.

Return to the example in Section 11·1. Our comparison benchmark firm was

Firm B Financing	E	P	P/E
100-0 Equity-Debt Financed	$100.00	$1,000	10
50-50 Equity-Debt Financed	$62.50	$500	8

So, here is our problem: How do we value firm A, if the underlying projects of firm A are similar to those of firm B? Should we use 8 or 10 as our price-earnings ratio? The answer is easy if A has the same debt equity ratio as B. Then we can just use the comparison firm's price equity ratio as a proxy for our own price equity ratio. That is, if Firm A produced $200 in income *for equity holders* each year, after financing with a 50/50 debt equity ratio, then we would expect its equity value to be about

$$\begin{aligned}\text{Implied A Value} &= \left(\frac{\text{Price}}{\text{Earnings}}\right)_A \cdot \text{Earnings}_A \\ &\approx \left(\frac{\text{Price}}{\text{Earnings}}\right)_B \cdot \text{Earnings}_A \\ &= \qquad 8 \cdot \$200 \qquad = \$1,600 \ ,\end{aligned}$$

where we write out Earnings to avoid confusion with equity. As just stated, this equity valuation depends on the assumption that Firm A had borrowed an equal amount in outstanding debt ($1,600), i.e., that it had a 50/50 debt equity ratio to which we could apply our B multiple.

Problem is: leverage may be different. Try to unlever both.

But what do we do if our A, with its reported income of \$200 for equity holders, had borrowed \$2,500 instead of \$1,600? Or, what if A had not borrowed anything? We need to determine an appropriate comparable that we can apply to A and that takes into account the actual leverage of A and of B. The easiest method is to convert all price-earnings ratios to a 0/100 debt equity ratio. In this case, we already know the result that we wish to obtain: a formula should tell us that unlevering B should translate the previous ratio of 8 into a price-earnings ratio of 10. Next, we need to look up the interest rate that A is paying for borrowing. Let us assume that A's records inform us that the interest rate is 8%. This means that A is paying \$200 in interest each year, before it paid off the \$200 to its equity holders. Consequently, we know that a 0/100 debt equity A would have produced earnings of \$200 + \$200 = \$400. Thus,

$$\begin{aligned}
\text{Implied A Value} &= \left(\frac{\text{Price}}{\text{Earnings}}\right)_{\text{A}} \cdot \text{Earnings}_{\text{A}} \\
&\approx \left(\frac{\text{Price}}{\text{Earnings}}\right)_{\text{B}} \cdot \text{Earnings}_{\text{A}} \\
&= \qquad 10 \cdot \$400 \qquad = \$4,000
\end{aligned}$$

This would be the value of an all equity firm. Because A has borrowed \$2,500, its equity must be worth \$1,500.

An attempt at a delevering formula.

So the trick is to convert all debt equity ratios into "all equity" ratios first. Unfortunately, there is no general formula to do so: the translation formula depends on the time when the cash flows are likely to occur. The following formula, based on a plain perpetuity assumption on the firm's earnings, often works reasonably well:

$$\begin{aligned}
\left(\frac{\text{Price}}{\text{Earnings}}\right)_{\text{Relevered}} &= \frac{1 - (r_{\text{Firm}} - r_{\text{Debt}}) \cdot \left(\frac{\text{Debt}}{\text{Earnings}}\right)}{r_{\text{Firm}}} \\
&= \left[1 - (r_{\text{Firm}} - r_{\text{Debt}}) \cdot \left(\frac{\text{Debt}}{\text{Earnings}}\right)\right] \cdot \left(\frac{\text{Price}_{\text{Firm}}}{\text{Earnings}_{\text{Firm}}}\right)
\end{aligned}$$

where "firm" means an all-equity no-debt financed firm. Let us apply this formula to firm B. Let us assume that we know that B has a value of \$1,000 and a price-earnings ratio of 10 when it has no debt, but it is now considering a move to a 50/50 debt equity ratio. We want to determine this capital structure change on its new price-earnings ratio. Use the formula:

$$\begin{aligned}
\left(\frac{\text{Price}}{\text{Earnings}}\right)_{\text{Relevered}} &= \left[1 - (r_{\text{Firm}} - r_{\text{Debt}}) \cdot \left(\frac{\text{Debt}}{\text{Earnings}}\right)\right] \cdot \left(\frac{\text{Price}_{\text{Firm}}}{\text{Earnings}_{\text{Firm}}}\right) \\
&= \left[1 - (10\% - 7.5\%) \cdot \left(\frac{\$500}{\$62.^{50}}\right)\right] \cdot 10 \\
&= [1 - 0.025\% \cdot 8] \cdot 10 \\
&= 0.8 \cdot 10 = 8 \quad .
\end{aligned}$$

This formula makes it easy to figure out how an all equity firm's price earnings ratio changes when it takes on debt. If we know the debt equity ratio of a firm and want to convert it into a debt-free equivalent price-earnings ratio, then

IMPORTANT: *To convert the observed leveraged* P/E ratio *into a hypothetical* P/E ratio *for an unlevered firm, the following approximation formula can be helpful:*

$$\left(\frac{\text{Price}}{\text{Earnings}}\right)_{Firm} = \frac{1}{\left[1-(r_{Firm}-r_{Debt})\cdot\left(\frac{Debt}{\text{Earnings}}\right)\right]}\cdot\left(\frac{\text{Price}}{\text{Earnings}}\right)_{Equity}. \tag{11.23}$$

This formula makes very specific assumptions on the type of firm, specifically that earnings are a plain perpetuity. Nevertheless, it often is good enough approximation in other cases, too, though.

A. Q&A: Answers

1. Microsoft is growing faster, so it would have a higher P/E ratio.
2. $E/P = r - g \Rightarrow r = P/E + g = 1/40 + 6\% = 8.5\%$. Therefore, $E/P = 8.5\% - 7\% = 1.5\%$ and its P/E ratio would shoot from 40 to 66.7. The percent change in value would therefore be $66.6/40 - 1 \approx 66\%$.
3. $E/P = r - g \Rightarrow g = r - E/P = 10\% - 1/20 = 5\%$.
4. See the text.
5. Do an example. Acquiror has value of \$100, so it needs to have earnings of \$5. Acquiree has value of \$50, so it needs to have earnings of \$1. This means that the combined firm will have earnings of \$6 and value of \$150. Its P/E ratio will thus be 25.
6. Earnings: The TTM Earnings for KO is $3,979 + (801 - 873) + (1,290 - 1,118) = 4,079$. The TTM Earnings for PEP is $2,662 + (651 - 570) + (888 - 798) = 2,833$. Assets: You would not compute a TTM, but use the most recent assets: \$25,287 for Coca-Cola and \$24,200 for Pepsico.
7. We know from the text that the P/E ratio would go down.
8. This question cannot be answered if we do not know the different costs of capital. For example, if the firm's cost of capital is equal to the debt cost of capital, the P/E ratio would not change at all!
9. Yahoo reported an actual market value of \$10.52 billion euros, and an earnings yield of 36.9% ($^{P}/_{E}$ of 2.7). The easy part is supplementing the table:

Manufacturer	Market Cap	Earnings	P/E ratio	E/P yield
Volvo (ADR)	\$5.7	-\$0.^{18}	-31.7	-3.2%
Ford	\$14.1	-\$5.^{30}	-2.7	-37.6%
GM	\$18.8	\$1.^{83}	10.3	9.7%
Nissan (ADR)	\$27.0	\$2.^{55}	10.6	9.4%
DaimlerChrysler	\$32.3	\$4.^{63}	7.0	14.3%
Honda (ADR)	\$37.7	\$3.^{09}	12.2	8.2%
Toyota (ADR)	\$87.3	\$4.^{51}	19.4	5.2%
Sum	\$222.9	\$11.^{13}	25.1	6.0%
Average	\$31.8	\$1.^{59}	3.6	0.9%

The hard part is deciding on a suitable $^{P}/_{E}$ comparable. The first method (average E/P yield, then invert) suggests adopting the astronomical ratio of $1/0.9\% = 111$, due to Ford's enormous loss in terms of market capitalization (Ford had \$85 billion in sales, and a positive EBITDA of \$4.8 billion. But Ford also has ongoing depreciation on the order of \$15 billion per year, but capital and other expenditures on the order of \$18 (2001) to \$37 billion (2000 and 1999).) The second method (sum up E's and P's first) suggests \$222.9/\$11.1 = 20, but it weighs the larger [and Japanese] firms more highly. Nevertheless, in this case, the second method came closer to the actual Volkswagen $^{P}/_{E}$ multiple of 27.
Incidentally, by mid-2003, VW had introduced a couple of flops, and its earnings had sagged to \$2.5 billion, though its market capitalization had increased to \$15 billion. This meant that Volkswagen's $^{P}/_{E}$ multiple had shrunk from 27 to 6 in just nine months!
10. The firms are alike substitutes in all respects, including product, product lines, and debt ratio.
11. These ratios are usually performed without debt adjustment—the equivalent of surgery without anesthesia. This is a huge problem, but it also makes this exercise relatively easy.

Firm	Value/Cash	Value/Sales	Value/Dividends
CSG	n/a	1.3	30
KO	185	5.5	50
PEP	45	3.1	74

- The cash-based ratio suggests a value between $10 million and $39 million. The cash-based ratio values all firms as if only current cash has any meaning, and the ongoing operations are irrelevant (except to the extent that they have influenced current cash).
- The sales-based ratio suggests a value between $12 million, $29 million, and $50.6 million. Because the smaller comparables have lower ratios, one might settle on a lower value. The sales-based ratio ignores that CSG's equity value is relatively low because more of its value is capitalized with debt than with equity.
- The dividend-based ratio suggests a zero value. Obviously, this is not a perfect estimate. Firms can choose different payout policies.

Hansen's actual value on this day was $51.4 million.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

Part III

Investor Choices

Investments

"This next song's about spreading risk in a volatile market by diversification."

This introduction appears in the Survey text only.

Transition

We are still plagued by one problem: We do not yet know where the cost of capital, the $\mathcal{E}(\tilde{r})$, in the present value formula comes from. We just assumed we knew it.

In the real world, the cost of capital is determined by our investors—and our investors have many choices. So we have to understand their mindset. They can put money in bonds, stocks, projects, etc.—or into our firm's projects. If investors very much like the alternatives, then our firm's cost of capital will be high. Think of our corporate cost of capital as our investors' *Opportunity Cost of Capital*.

To learn more about where the cost of capital comes from, we have to study not just our own project, but more generally how investors think—what they like and dislike.

Of course, it is nice that we are often ourselves investors, so learning how to best invest money in the financial markets has some nice side benefits.

SIDE NOTE: In the Investments part of the book, we will use the terms "stock" and "security" very loosely. Our concern is really the choice among many investment opportunities, which includes bonds, options, futures, real estate, etc. It is just more convenient to use the phrase stock, rather than "any possible investment opportunity."

What we want to Accomplish in this Part

The goal of this part of the book is to teach you how public investors think. This is important not only when you are an investor who wants to decide how to allocate money among the thousands of possible financial investments, but also for the corporate manager who wants to get you and other investors to part with their money in order to finance new corporate projects.

- Chapter 12 gives a short tour of historical rates of returns to whet your apetite.
- Chapter 13 describes how you can trade equity securities (a.k.a. "stocks") , and how securities can be combined into portfolios.

  Typical questions: what was the rate of return on a portfolio that invested money into the 30 stocks that form the **Dow-Jones Industrial Average** (**DJIA**) index?

- Chapter 14 shows how to measure the risk and reward of securities and portfolios.

  Typical questions: if you held a portfolio of the 30 stocks that form the DJIA, what would your risk be? How would it change if you rearrange how much money you put into each of the 30 stocks?

- Chapter 15 shows how holding many stocks in your portfolio can reduce your risk.

  Typical questions: How much lower would your risk be if you held all 30 stocks in the DJIA instead of just a single stock in the DJIA?

- Chapter 16 shows how you can determine the *best* portfolio.

  Typical questions: Can you invest better than just into the 30 stocks in the DJIA? How much risk does PepsiCo contribute to your portfolio? Would it be better if you shifted more money into PepsiCo stock, or should you stick to your current portfolio?

- Chapter 17 shows what expected rates of returns securities have to offer *if* investors make smart investment decisions. It gives the formula (cookbook) version of the so-called CAPM model, which answers this question.

  Typical questions: What is a fair reward for PepsiCo's stock investors, given how much risk it contributes to the overall market portfolio?

- Chapter 18 explains more of the theory behind the CAPM.
- Chapter 19 expands on the efficient markets concept, first mentioned in the introduction. It also explains the difference between arbitrage and good bets.

  Typical questions: what kind of information can you use to beat the market?

CHAPTER 12

A First Look at Investments

Historical Rates of Returns and Investment Opportunities

last file change: Jun 24, 2004 (11:13h)

last major edit: June 24, 2004

The subject of Investments is so interesting that we are going to break our own rules. Instead of laying all the foundations before we look at the evidence, we shall first look at the evidence: the world of returns on stocks, bonds, and money-markets. Don't worry if you cannot follow everything in this chapter: it will all be explained in due course—and you can always come back later.

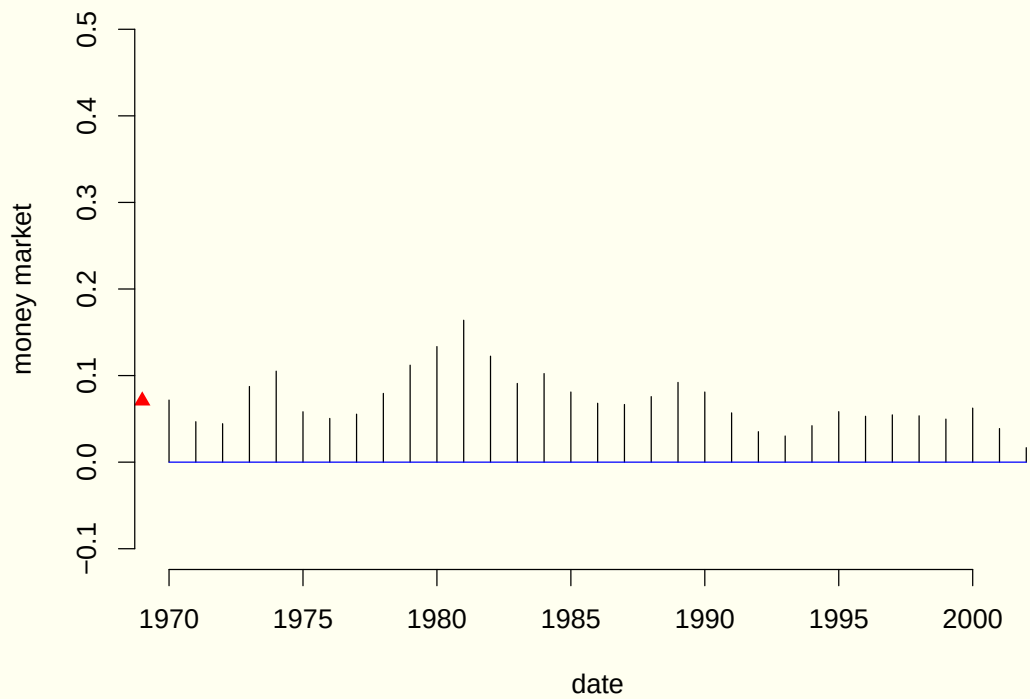
Ready for our expedition into the world of historical rates of returns?

12·1. Stocks, Bonds, and Money-Market, 1970–2002

12·1.A. Graphical Representation of Historical Money-Market Returns

Figure 12.1. The Time Series of Money-Market Returns, 1970–2002

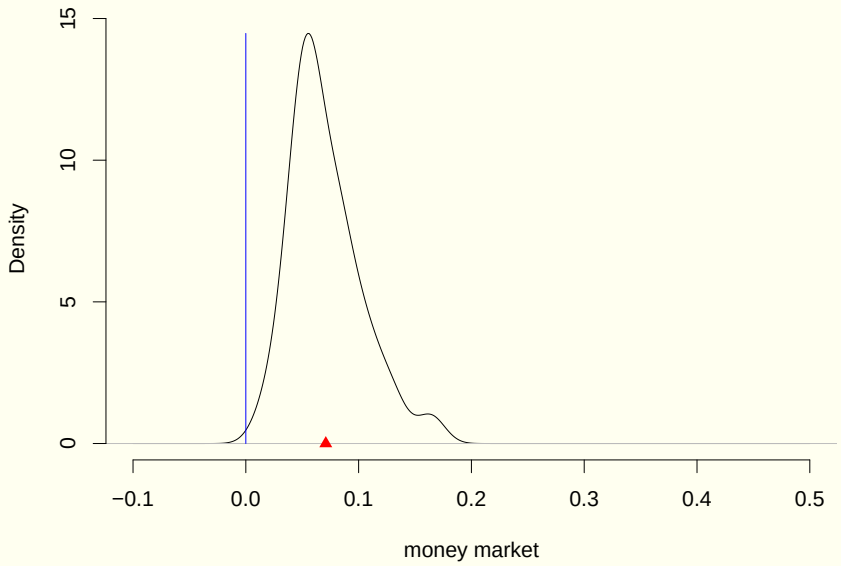
1970	7.17%	1980	13.35%	1990	8.10%	2000	6.24%
1971	4.67%	1981	16.39%	1991	5.69%	2001	3.88%
1972	4.44%	1982	12.24%	1992	3.52%	2002	1.67%
1973	8.74%	1983	9.09%	1993	3.02%		
1974	10.51%	1984	10.23%	1994	4.21%		
1975	5.82%	1985	8.10%	1995	5.83%	Average	7.1%
1976	5.05%	1986	6.80%	1996	5.30%	Std Dev	± 3.2%
1977	5.54%	1987	6.66%	1997	5.46%		
1978	7.94%	1988	7.57%	1998	5.35%		
1979	11.20%	1989	9.21%	1999	4.97%		



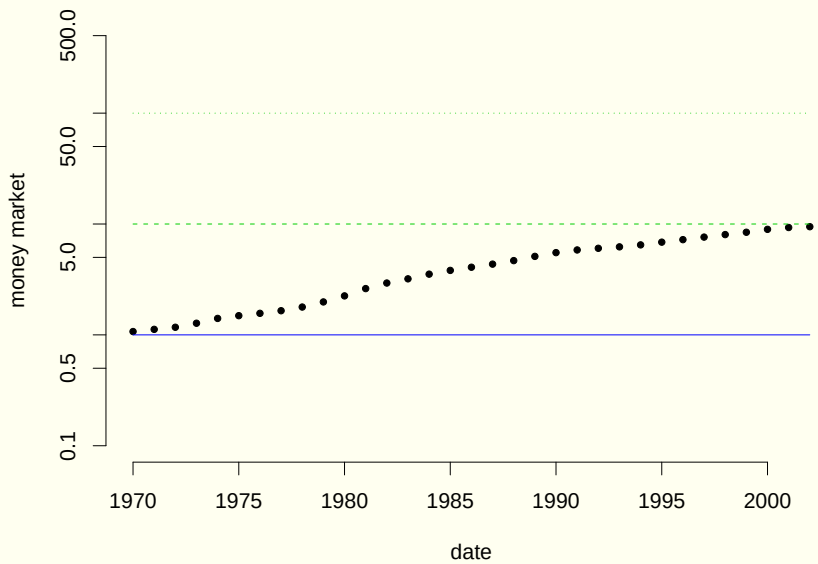
The time series diagram.

We begin with Figure 12.1, which shows the year-by-year rates of returns of a money-market like financial investment. (A money-market investment is basically a very short-term, almost risk-free bond, often called "cash." This particular series is the so-called **Federal Funds rate**.) You would have earned 7.17% in 1970, 4.67% in 1971, and so on. The table in Figure 12.1 allows you to compute the average rate of return over all 33 years as 7.1% per annum—also marked by the red triangle in the graph.

Figure 12.2. Other Representation of Money-Market Returns, 1970–2002



Return Starts with	1	2	3	4	5	6	7	8	9.	10.	11.	>12.
Number of Years	1	0	3	4	8	3	3	3	2	2	0	3



1970	$1.^{07}$	1980	$2.^{25}$	1990	$5.^{52}$	2000	$8.^{95}$
1971	$1.^{12}$	1981	$2.^{62}$	1991	$5.^{83}$	2001	$9.^{30}$
1972	$1.^{17}$	1982	$2.^{94}$	1992	$6.^{04}$	2002	$9.^{46}$
1973	$1.^{27}$	1983	$3.^{20}$	1993	$6.^{22}$		
1974	$1.^{41}$	1984	$3.^{53}$	1994	$6.^{48}$		
1975	$1.^{49}$	1985	$3.^{82}$	1995	$6.^{86}$		
1976	$1.^{57}$	1986	$4.^{08}$	1996	$7.^{23}$		
1977	$1.^{65}$	1987	$4.^{35}$	1997	$7.^{62}$		
1978	$1.^{78}$	1988	$4.^{68}$	1998	$8.^{03}$		
1979	$1.^{98}$	1989	$5.^{11}$	1999	$8.^{43}$		

The histogram shows how spread out returns are.

Figure 12.2 takes the same data as Figure 12.1, but presents it differently. The density histogram (on the top) is based on the number of returns that fall within a range. For example, the table in Figure 12.1 shows that only 2002 had a return in the 1-2% range. Most returns were in the 4% to 7% range, but there were also 3 returns of at least 12%. Therefore, the histogram has its main peak at around the mean (7%), and a smaller peak at around 13%. The histogram makes it easy to see how spread out returns are.

The cumulative rate of return graph shows how long-run investments would have fared.

The cumulative return graph in Figure 12.1 plots the cumulative returns on a logarithmic scale. For example, by the end of 1973, the compound return of \$1 invested in 1970 would have been

$$\begin{aligned} &\$1 \cdot (1+7.17\%) \cdot (1+4.67\%) \cdot (1+4.44\%) \cdot (1+8.74\%) \approx \$1.^{27} \\ &I_0 \cdot (1+r_{1970}) \cdot (1+r_{1971}) \cdot (1+r_{1972}) \cdot (1+r_{1973}) \quad . \end{aligned} \tag{12.1}$$

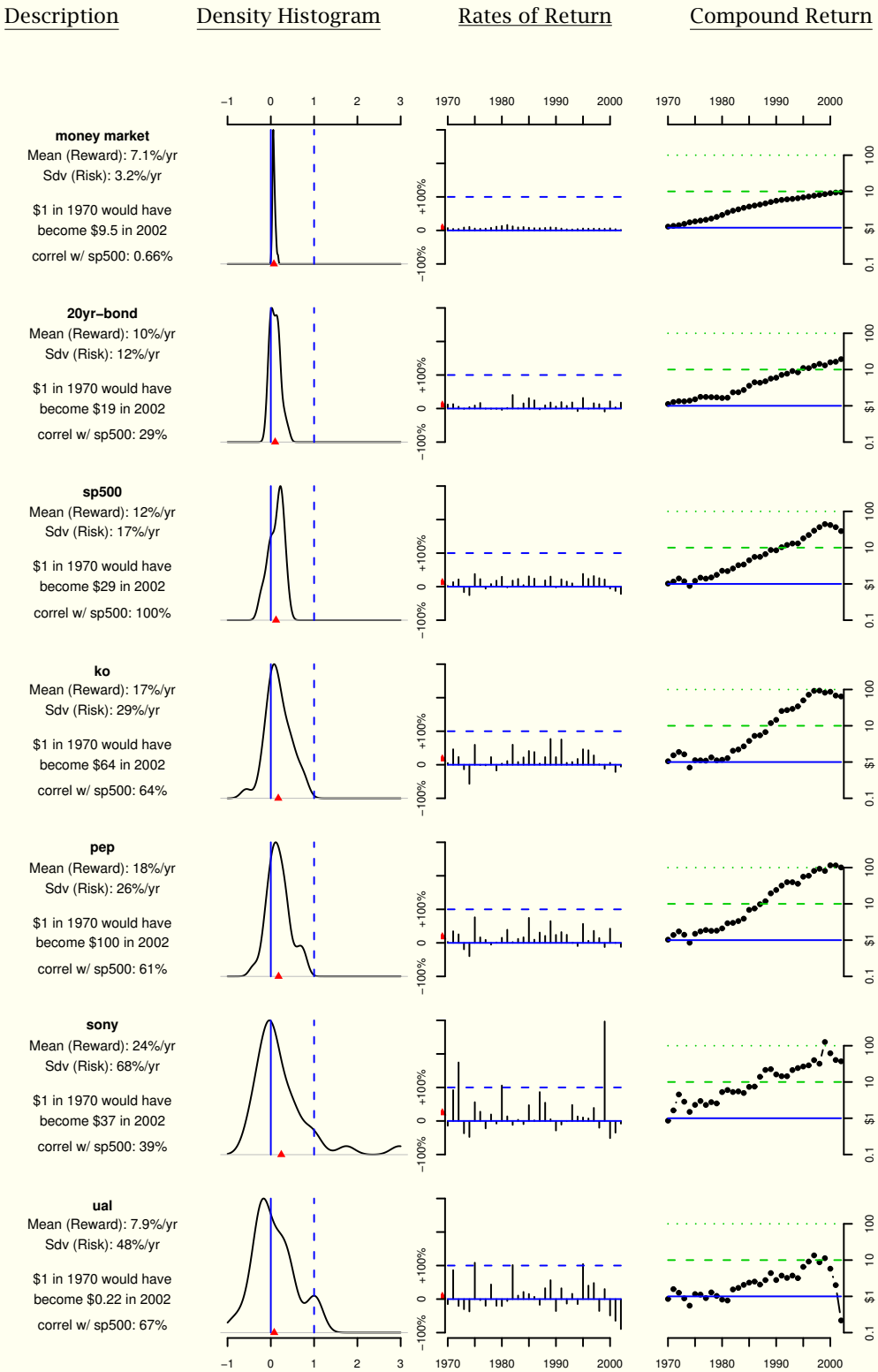
The cumulative return perspective corrects for the fact that a return of –50% followed by +100% is a net zero return, even though the average rate of these two returns would be a positive +25%. The graph shows that a money-market investment of \$1 at the start of 1970 would have ended up as \$9.[46] at the end of 2002, of course ignoring all taxes.

12·1.B. Comparative Investment Performance

Explore the complex figure first. Stare at it.

We can now compare the historical rates of returns on different investment opportunities, using exactly the same graphs as those in Figure 12.1 and Figure 12.2. Figure 12.3 repeats them for a set of historical investment choices *all on the same scale*. It displays a lot of information about the performance of some of the investments that investors could have chosen. Do not expect to understand everything at first glance: you need to stare at the elements of Figure 12.3 for a while to comprehend them.

Figure 12.3. Investment Performance, 1970–2002



The first row is again money-market.

We have already seen the first row—investments in **money-market** securities, such as very short-term bonds issued by the Treasury (the **federal funds rate**). We only changed the scale for direct comparison to the other investments in the graphs below. Note how tight the distribution of money-market returns was around its 7% mean.

The second row, long-term bonds, offered more reward, but was more risky, too.

The second row describes investments in 20-year Treasury bonds. The graph in column 3 shows that the bars are now sometimes negative (years in which you would have earned a negative rate of return), but there are also years in which you would have done much better. This is why the histogram is much wider for 20-year bonds, and this is why the risk was 12% per year—although the average rate of return was a much higher 10% per year. By 2002, your \$1 in 1970 would have become \$19.^{00}.

The third row, stocks overall, offered even more reward, but was even more risky.

The third row describes an investment in a so-called index-fund replicating the S&P500 index (described later). Sometimes, this is colloquially called "stocks" or "the stock market." Stocks would have been even more risky (mean rate of return 12% per year; risk 17% per year), but your \$1 in 1970 would have been worth \$29 in 2002.

SIDE NOTE: The compound inflation rate from 1970 to 2002 was 4.8. (Put differently, \$1 in 1970 purchased as much as \$4.^{80} did in 2003.) Therefore, the \$9.^{50} end result in money-market would have been worth \$9.^{50}/4.8 ≈ \$1.^{98} in 1970-inflation-adjusted dollars. Over 30 years, you would have only doubled your real purchasing power. You can easily compute equivalent real returns for the other investment opportunities.

Furthermore, the difference between \$29 in stocks and \$9.^{50} in cash or \$19 in bonds is an understatement *for you*. Interest was taxable each year, while the capital gain in stocks was not (the dividend gain would have been taxable). Very roughly, a highly taxed investor would have ended up with about \$5 in cash, \$13 in bonds, and \$26 in bonds. Therefore, in real *and* after-tax terms, from 1970 to 2002, a highly taxed investor would have ended up just about even if invested in cash, doubled or tripled if invested in bonds, and quintupled if invested in stocks. This was a good 30 years for stocks!

Individual stocks can offer more reward, or be even more risky.

The remaining four rows are individual stock investments. Their rates of returns and histograms are all over the place: investing in a single stock is a rather risky venture, even if the stock is a stalwart (like Coca Cola [KO], PepsiCo [PEP], Sony [SNE], or United Airlines [UAL]).

Figure 12.4. Rate of Returns on The S&P500 and Coca-Cola (KO)

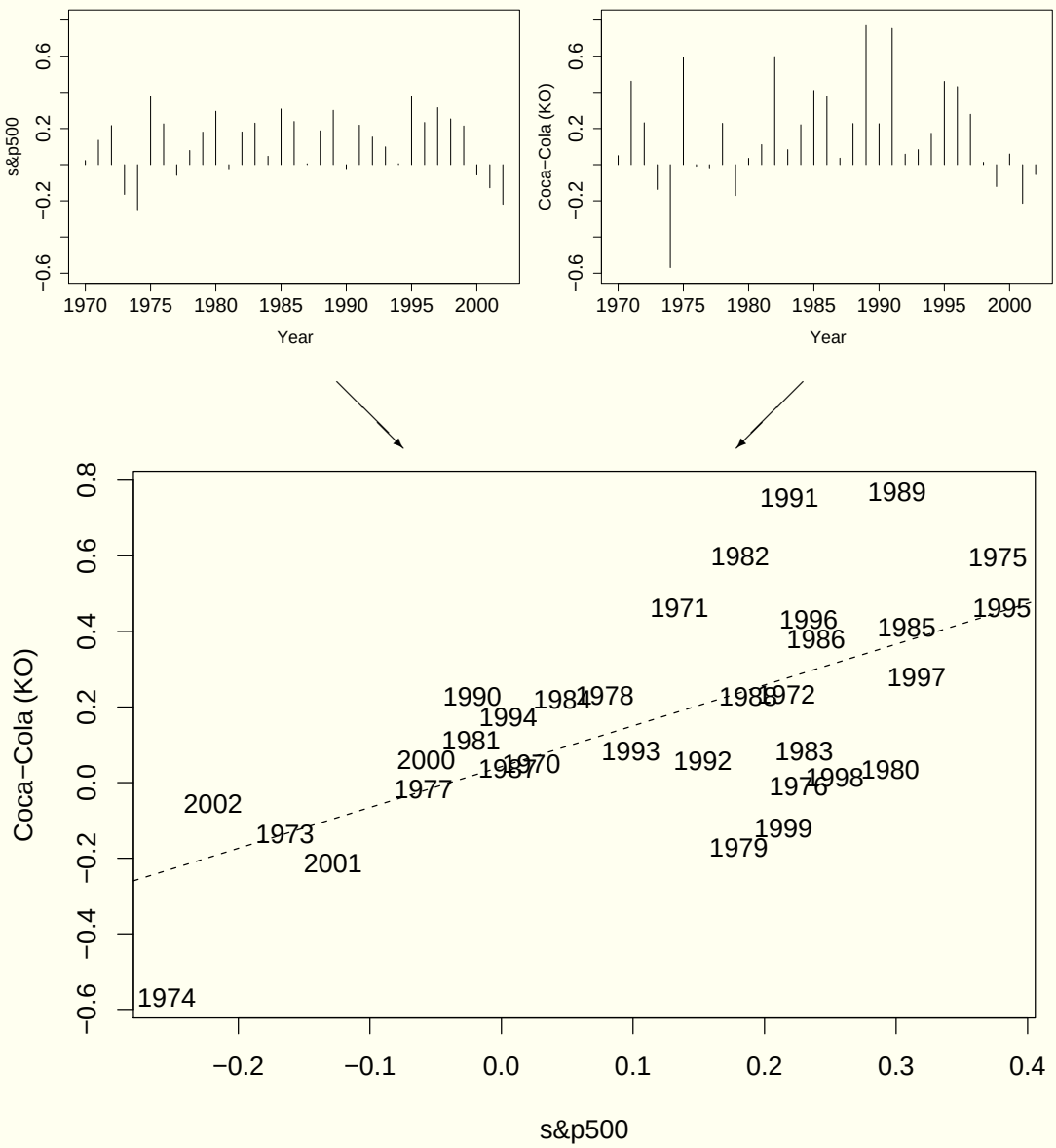


Figure 12.4 highlights the rates of returns on the S&P500 and one specific stock, Coca-Cola (KO). The top row redraws the third column in Figure 12.3. Do you notice a correlation between these two rates of returns? Are the years in which one is positive (or above its mean) more likely to also see the other be positive (or above its mean), and vice-versa? It does seem that way. For example, the worst rates of return for both are 1974. Similarly, 1973, 2000, and 2001 were bad years for investors in either the S&P500 or Coca-Cola. In contrast, 1989 and 1975 were good years for both. The correlation is not perfect: in 1979, the S&P500 had a good year, but Coca-Cola had a bad one. The lower plot in Figure 12.4 plots the returns against one another, and tries to fit a line between the two. (The slope of this line is called the market-beta of Coca-Cola, and will be explained in much more detail later.) The figure shows that there is clearly a positive correlation between these rates of return. Figure 12.3 quantifies this correlation as

What is the correlation mentioned in the figure?

64%. (A correlation can never exceed +100% or –100%.) Put differently, Coca-Cola had a positive market-beta. It is very common for investments to correlate positively with the stock market: in years of malaise, almost everything tends to be in malaise. In years of exuberance, almost everything tends to be exuberant. Keep in mind that this is not always, but often, the case.

12·2. Visible and General Historical Stock Regularities

The main empirical regularities.

What can you learn from these graphs? Actually, almost everything that there is to learn about Investments—although it will need to be explained in more detail below.

- The history indicates that stocks offered higher average rates of return than bonds, which in turn offered higher average rates of return than money-market investments. However, keep in mind that this was only *on average*. In any given year, the relationship might have been reversed.

- Although stocks did well (on average), you could have lost your shirt investing in them, especially if you had bet on just one individual stock. For example, if you had invested $1 into United Airlines in 1970, you would have only 22 cents left in 2002 (and $0 shortly thereafter).

- Money-markets were the safest investment. Bonds were riskier. Stocks were riskier, yet.

  (Sometimes, stocks are called "noisy," because it is really difficult to predict what they will turn out to offer.)

- There was some sort of relationship between risk and reward: the riskiest investments tended to have higher mean rates of return.

  (The risk has to be looked at "in context." Thus, please do not overread the simple relationship between the mean and the standard deviation here.)

- Large portfolios consisting of many stocks tended to have less risk than individual stocks. The S&P500 fund had a risk of 17%, much less than the risk of any of the individual stocks.

  (This is due to **diversification**, explained at length later.)

- A positive average rate of return usually, but not always translates into a positive compound holding rate of return. United Airlines had a positive average rate of return, despite having lost money.

  (You already know why: A stock that doubles and then halves has rates of return of +100% and –50%. It would have earned you a 0% total compound rate of return. But the average rate of return would have been positive, $[100\% + (-50\%)]/2 = +25\%$.)

- Stocks tend to move together. For example, if you look at 2001–2002, not only did the S&P500 go down, but the individual stocks also tended to go down. In 1998, on the other hand, most tended to go up (or at least not down much). The mid 1990s were good to all stocks. And so on. In contrast, money market returns had little to do with the stock market. Long-term bonds were in between.

  (For all practical purposes, a correlation of 100% indicates that two variables always move together 1-to-1; a correlation of 0% indicates that two variables move about

independently.) On annual frequency, the correlation between money-market and stock market (the S&P500) was about zero; between long-term bond returns and stock market around 30%; and between our individual stocks and the stock market around 40% to 70%. The fact that investment rates of returns tend to move together will be important. It will be the foundation for the **market-beta**, a measure of risk that we shall propose in Chapter **??**.

12·3. History or Opportunities?

History is only useful over longer horizons, not over just a few years.

Ultimately, we are not interested in history per se. We want to know about the future, and history is useful only because it is our indicator for the future. But which history? One year? Thirty years? One hundred years? Trust me when I state that if we had drawn the graphs beginning in 1926 instead of 1970, our conclusions would have all remained the same. However, if we had started in 2001, what would we have seen? Two awful years for stock investors. We know intuitively that this would not have been a representative sample period. To make any sensible inferences about what is going on in the financial markets, we need many years of history, not just one, two, or three—and certainly not the 6-week investment performance touted by some funds or friends (who also often display remarkable selective memory!). The flip side of this argument is that we cannot reliably say what the rate of return will be over one year. We will be better in forecasting the average annual rate of return over 5 to 10 years than over 1 year. Any single year will be very noisy.

Still, history can be rather misleading. The Nikkei is a good example.

Instead of relying on just one year, relying on statistics computed over many years is much better. However, although 20 to 30 years of performance is the minimum number necessary to learn something about return patterns, this is still not sufficient. Again, we are really interested in what will happen in the next 5 to 10 years, not what did happen in the last 5 to 10 years. Yes, the historical performance can help us judge, but it should not be blindly trusted. For example, an investor in UAL in 2000 might have guessed that the average rate of return for UAL would have been positive—and would have been sorely disappointed. Investors in the Japanese stock market in 1990 had seen the Nikkei-225 stock market index rise from 10,000 to 40,000—a four fold increase in just four years—a 40% rate of return every year. If they had believed history, they would have expected $40,000 \cdot (1 + 40\%)^{13} \approx 3.2$million. Instead, the Nikkei had fallen below 8,000 in April 2003, and has only recently recovered to 12,000. History would have been an awful guide.

But, we do not have much choice other than to use some history.

Nevertheless, despite the intrinsic hazards in using historical information in forecasting stock market returns, we do not have much else to rely on. So, we will use historical statistics, but we will also try to be careful in not overinterpreting it. Of course, if we see crazily high or crazily low past historical rates of returns, we should try to exercise additional caution.

Historical risk and betas are better predictors of the future risk and betas.

Fortunately for us, though, it turns out that correlations and risk (explained in the following chapters) tend to be more stable than historical means. So, even though we do not believe that a \$1 investment in PepsiCo will return \$100 in 30 years, or that PepsiCo will return 18% next year, or that PepsiCo has an expected rate of return of 18% over the long run, we can believe that PepsiCo has a risk of around 25% to 30% per year, and that its correlation with the S&P500 is around 60%.

Presume we know expected statistics, although we do now future realized statistics.

To understand Investments better, for almost the entire rest of the book, we shall pretend that we know the expected future statistical outcomes. For example, we may work with expected rates of returns as if we really knew these *expected* rates of returns perfectly. This is not the same as assuming that we know the *actual* rates of returns. For example, we may know that the lotto combination "5,10,12,33,34,38" has an expected investment rate of return of –5 cents, but we do not know whether it will win the jackpot.

SIDE NOTE: For the most part, we will work with historical statistics as if they were our expected statistics. So, we will try to slip in "unnoticed" that the historical statistical outcomes tell us something about the future expected statistical outcomes. After the sermon about how historical statistics need not be perfectly indicative of the future statistics, you should realize that equating the two will be a big and not necessarily innocuous jump. We do not have much choice. Fortunately, as we have already mentioned, historical betas and risks are pretty indicative of future betas and risks. Unfortunately, historical expected rates of return are not too indicative of future expected rates of returns. Know what you can and what you cannot trust.

12·4. SUMMARY

The chapter covered the following major points:

- Historical rate of return patterns of stocks, bonds, and money-market investments were graphed in Figure 12.3
- Stocks, on average, had higher expected rates of returns than bonds, which in turn had higher expected rates of return than money-market investments.
- Individual stocks were most risky.
- Large stock market portfolios had lower risk than individual stock holdings.
- Stocks tended to correlate: when one stock had a good year, the others tended to have a good year, too.

CHAPTER 13

Securities and Portfolios

Applications

last file change: Jun 15, 2004 (11:50h)

last major edit: n/a

This chapter appears in the Survey text only.

This chapter first explains where stocks come from and where they are traded. It then explains the process of going long (speculating that an asset will go up) and going short (speculating that an asset will go down). Finally, it explains portfolios and indexes.

13·1. Market Microstructure

13·1.A. Exchanges and Non-Exchanges

The standard process.

Most small investors use a broker to buy or sell equity shares. They can place either **market orders**, which ask for execution at the current price; or **limit orders**, which ask for execution if the price is above or below a limit that the investor can specify. The broker then routes the order to a centralized trading location. For example, an order for a stock like IBM can be traded on the **New York Stock Exchange** (**NYSE**), where a human being, called the designated **specialist**, handles all IBM orders. The other major exchange is **Nasdaq**, which is a purely electronic exchange without a specialist. (There are Nasdaq marketmakers, who, unlike ordinary investors, can see the unexecuted **limit order book**.)

New Alternative Trading Institutions: ECNs

But your broker can route your trade also to other locales. Recently, a set of **electronic crossing networks** (**ECN**) have made big inroads into the business, replacing exchanges. The largest are **Archipelago** and **Island**. ECN's compete in terms of cost and speed of execution. These ECNs can trade the same stocks that exchanges are trading.

Auctions have lower execution costs, but also lower execution speed.

Continuous trading, that is trading at any moment an investor wants to execute, relies on the presence of a standby intermediary, who is willing to absorb shares when noone else is available. This is risky business, and thus this intermediary must earn a good rate of return to do so. In contrast, some countries have exchanges that do not have continuous trading, but **auctions** (say, twice a day). In an auction, execution does not happen immediately, but a whole range of buy orders and sell orders have usually accumulated, eliminating the expensive intermediary. Thus, auctions generally offer lower trading costs but slower execution. Even in the United States, there are institutions, such as **ITG**, which focus on matching large institutional trades with one another in auction-like crossing systems. (If no match on the other side is found, the order is simply not executed. If a match is made, by cutting out the middle-man [specialist], the execution will have cost a lot less.)

Side Note: There are many more financial markets, too: The **NASD** (National Association of Securities Dealers) also operates an **OTC over-the-counter** market for the stocks of smaller firms. Foreign securities trade on their local national exchanges. And many bonds are only or primarily traded OTC. For the most part, the transaction costs in OTC or foreign financial markets for retail investors are prohibitively high.

13·1.B. Brokers and Prime Brokers

Brokers execute and keep track; Prime brokers leave execution to others.

Most individuals place their order with brokers at a brokerage firm, such as **Ameritrade**, **Charles Schwab**, or **Merrill Lynch**. Brokers do two things: they route and thereby facilitate the execution of trades; and they keep track of the individual's money and securities. (They also facilitate purchasing **on margin**, whereby an investor can borrow money to purchase stock.) Many larger investors, such as **funds** (described in Section 13·3.B), break these two functions: the investor can employ its own traders, and the broker takes care only of the book-keeping of the investor's portfolio (as well as margin provision and the finding of securities to short, see below.) Such limited brokers are called **prime brokers**.

13·1.C. How Securities Appear and Disappear

Firms first sell shares in IPOs.

Most publicly traded equities appear on public exchanges through an **Initial Public Offering** (**IPO**). This is an involved process, whereby a privately traded company first sells shares to ordinary investors. When sold by a reputable **underwriter**, shares are usually sold at a fixed price, and typically about 10% below the first day after-market open trading price. (Such IPO shares are allocated to the brokerage firm's favorite customers, and can be an important source of profit.)

Money flows into the financial markets through IPOs and SEOs.

Usually, about a third of the company is sold in the IPO, and the typical IPO offers shares between $20 million and $100 million, though some are much larger (e.g., privatizations, like British Telekom). About two thirds of all such IPO companies never amount to anything and die quickly, but the remaining third soon thereafter offer more shares in **Seasoned Equity Offerings** (**SEO**). These days, much expansion in the number of shares in publicly traded companies, however, comes not from seasoned equity offerings, but from employee stock option plans, which eventually become unrestricted publicly traded companies.

Publicly traded companies must report financials, and restrict insider trading.

Publicly traded companies must regularly report their financials and other information to the SEC, and their executives have fiduciary obligations to their shareholders. Insider may not trade on brand new information, although some insider trading is allowed (and seems to be done fairly profitably by these insiders).

Money flows out from the financial markets in dividends and share repurchases.

Many companies pay some of their earnings in **dividends** to investors. Dividends of course do not fall like manna from heaven. For example, a firm worth $100 million may pay $1 million, and would therefore be worth $99 million after the dividend distribution. A $100 shareholder would therefore own (roughly) $99 in stock and $1 in dividends. (Shareholders usually must pay some taxes on dividend receipts.) Alternatively, firms may pay out earnings in **share repurchases**. For example, the firm may dedicate the $1 million to share repurchases, and the $100 investor may then either have received $100 for his shares, or, holding onto them, would own $(100/900,000,000)$ of a $900,000,000 firm—still $100 worth.

Shares can also shrink out of the financial markets in bankruptcies, liquidations, and delistings.

Firms can shrink more drastically, too: some firms voluntarily liquidate, determining that they can pay their shareholders more if they sell their assets and return the money to shareholders. More commonly, firms make bad investments, and fall in value to the point where they are delisted from the exchange and/or go into bankruptcy. Fortunately, investors enjoy **limited liability**, which means that they can at most lose their investment and not have to pay for any further sins of the company.

13·2. Equities Transaction Costs

13·2.A. Going Long

The costs of buying shares depend on the stock, but typically are less than 0.5 percent (round-trip) for a large firm's stock.

The process of buying stocks is familiar to almost everyone: you call up your broker to purchase 100 shares of a stock (say PepsiCo) with cash in your account, and the shares appear in your account and the cash disappears from your account. When you want to sell your shares, you call your broker again to sell the shares and the appropriate cash value returns to your account. There are some transaction costs in the process: the broker collects a commission (typically ranging from about $8 at a discount broker to $100 at a full-service broker); and you are most likely to buy your shares at the **Ask Price**, which is higher than the **Bid Price**, at which you can sell the shares. For a stock like PepsiCo, trading around $50, the so-called bid-ask spread may be 10 to 20 cents or about 0.2 percent. So, buying and then immediately selling 1,000 shares of PepsiCo ($50,000), a **Round-trip Transaction**, might cost you **Transaction Costs** of around $100 to $200 (lost to the bid-ask spread) plus $16 to $200 (lost to your broker). You would thus have around $49,600 to $49,900 left.

Side Note: Discount brokers often receive payment from their market-makers in exchange for routing their clients' transactions to them. The market-maker recoups this payment to the broker with execution that is more likely to be at the unfavorable side of the spread, so that the market-maker can earn more spread. The discount brokerage client therefore receives an inferior bid-ask spread. Net in net, however, for all but the largest transactions, discount brokers still tend to be cheaper than full service brokers.

13·2.B. Going Short: The Academic Fiction

Idealized shorting, as used in academia, gives shorters interest on the stock that is being sold.

But, what if you want to speculate that a stock is going down rather than speculate that a stock is going up? This is called **shorting** a stock. (In optional Section 3·3.B, we had already discussed shorting in the context of treasury securities and apples.) Optimally, you would want to do the same thing that the PepsiCo company does: give an investor who wants to buy shares in PEP the exact same payoffs (incl. dividends!) that PEP will promise in exchange for his giving you $50. If the share price declines to $30, upon termination of the short, you would have received his $50 and only repay him $30—you would have earned $20. In addition, having his $50, you could earn the interest on $50. This is the idealized world of academic finance and this book, in which borrowing and lending can be done without friction. The upper half of Table 13.1 shows an example of a particular portfolio that involves a shorted security. Note that the portfolio return Formula 13.1 works without modification.

Anecdote: **Eternal Shorts?**
The short must make good on all promises that the underlying firms make. There are some rare instances in which this can cause unexpected problems. For example, when Heartland Industrial Partners acquired Mascotech for about $2 billion in 2000, the latter promised to dispose of some non-operating assets and distribute the proceeds to the original shareholders. As of 2002, that has not yet happened. Anyone having written a short still has an escrowed obligation as of 2002, that cannot be closed out.

13·2.C. Going Short: The Real World

In the real world, shorting is not so easy. First, you need to credibly guarantee that you can give the share purchaser all the cash flows that PepsiCo shares offer. (What have you committed to if the share price triples? Remember that you have unlimited liability as a short!) Second, a real investor in PepsiCo also receives the accounting statements of PepsiCo in the mail and can vote at the annual meetings. Presumably, you cannot offer this service. Third, you need to find an investor willing to be on the other side. Fourth, there are rules and regulations that the S.E.C. imposes on short-selling that you have to follow.

Real-world shorting requires more than just offering the same cash flows as the stock

NERD NOTE: *The most important SEC regulation concerning shorting is that the broker must borrow the shares from a willing owner and then resell them to a third party. It is not enough for another investor to be willing to take the other side of the short trade: instead, the shares have to be actually physically found from an investor holding them. For some smaller stocks, it can occasionally be difficult to find someone willing to lend the shares, which can make shorting difficult or impossible. In addition, there is a second SEC rule that states that shares can only be shorted on an up-tick. The intent is to reduce further short-selling during a stock market crash, when up-ticks are rare.*

All of the details necessary to execute a short can be arranged by your broker. Unlike buying shares long, execution of a short is often not instantaneous. But more importantly, the broker's service comes at a price. The broker usually does not return to you the $100 paid by the person buying the shares, so that you can invest the proceeds in bonds. That is, if the stock price declines to $90, you still made $10, but the interest on the $100 is earned by your broker, not by you. In addition, as with a purchase of shares, the broker earns commissions and the bid-ask spread goes against you. The lower panel in Table 13.1 contrasts the pure academic version of shorting (used in this book) to the more real-world version of shorting.

Real-world shorting loses the use of (some) proceeds. This is a "friction."

SIDE NOTE: Large clients can usually negotiate to receive at least some of the interest earned on the $100, at least for large, liquid stocks. Hypothetically, if such a large investor were both short one share of a firm and long one share of the same firm loses about 100 to 300 basis points per year. So, on a $100 share, the cost of being long one share and being short one share would typically be $1 to $3 per share per year. This money is shared between the brokerage firm and the investor willing to loan out shares to you for shorting (so that you can sell them to someone else). Nowadays in the real-world, large stock index funds earn most of their profits through lending out shares to shortsellers.

Table 13.1. Shorting

Academic Shorting Example

Investor Wealth: \$200.

You can sell \$100 worth of KO shares (or an equivalent promise) to another investor, who wants to hold KO shares. This gives you \$200 + \$100 = \$300 of cash, which you can invest into Pepsico.

Portfolio P: $w_{\text{KO}} = -\$100,\ w_{\text{PEP}} = \$300, \qquad \Longrightarrow w_{\text{KO}} = -50\%,\ w_{\text{PEP}} = 150\%.$

Hypothetical Rates of Return: KO $= -10\%$, PEP $= +15\%$.

$\Longrightarrow$ Portfolio Rate of Return: $r_P = -50\% \cdot (-10\%) + 150\% \cdot (+15\%) = +27.5\%.$

\$100 KO shares borrowed became a liability of \$90, for a gain of \$10;

\$300 PEP shares invested became an asset of \$345, for a gain of \$45.

$\Longrightarrow$ The net portfolio gain is \$55 on an original investment of assets worth \$200, which comes to a +27.5% rate of return.

Real World Retail Investor Shorting Example

Investor Wealth: \$200.

The broker finds another investor to borrow shares from and sells the shares (on your behalf) for \$100 to another investor, who wants to hold KO shares. Brokers keeps \$100. You still have only \$200 in cash, which you can invest into Pepsico.

Portfolio P: $w_{\text{KO}} = -\$100,\ w_{\text{PEP}} = \$200, \qquad \Longrightarrow w_{\text{KO}} = -50\%,\ w_{\text{PEP}} = 100\%.$

Hypothetical Rates of Return: KO $= -10\%$, PEP $= +15\%$.

$\Longrightarrow$ Portfolio Rate of Return: $r_P = -50\% \cdot (-10\%) + 100\% \cdot (+15\%) = +20.0\%.$

\$100 KO shares borrowed became a liability of \$90, for a gain of \$10;

\$200 PEP shares invested became an asset of \$230, for a gain of \$30.

$\Longrightarrow$ The net portfolio gain is \$40 on an original investment of assets worth \$200, which comes to a +20% rate of return.

Both examples ignore trading costs for buying and selling securities. The retail investor is assumed to receive absolutely no shorting proceeds. Institutional investors can receive some, but not all of the shorting proceeds.

Solve Now!

Q 13.1 *What are the main differences between academic theoretical perfect shorting and real-world practical shorting?*

Q 13.2 *If you simultaneously buy and short \$5,000 of IBM at the beginning of the year, and you terminate these two positions at the end of the year, how much would it cost you in the real world?*

Q 13.3 *Assume you believe that stock in KO will go up by 12% and stock in PEP will go up by 15% over the next year. The current risk-free interest rate is 2% per year. You have \$300,000 to invest, and your broker allows you to go short up to \$100,000.*

1. *How much could you go long in PEP?*
2. *If your forecast comes true, how much money would you earn in an academic fiction world? What would your rate of return be?*
3. *If your forecast comes true, how much money would you earn in the real world?*

13·3. Portfolios and Indexes

13·3.A. Portfolio Returns

Investors do not purchase just one security. They usually form entire **portfolios**. The ultimate goal of Investments is to select the best portfolio.

Important: *A portfolio is a set of investment weights.*

When an investor chooses multiple investments, the overall investment rate of return is the investment-weighted rate of return on each investment, with the weights being the relative investment proportions. For example, consider an investor who owns three stocks: PEP, KO, and CSG. If the investor has \$500 in PEP, \$300 in KO, and \$200 in CSG, then her total investment is \$1,000, and her portfolio investment weights are 50% in PEP, 30% in KO, and 20% in CSG. If the rate of return on PEP was 5%, the rate of return on KO was 2%, and the rate of return on CSG was –4%, the rate of return on her overall portfolio (P) was

Overall portfolio returns are the investment-weighted average returns of its constituents.

$$\begin{aligned} \tilde{r}_P &= \left(\frac{\$500}{\$1,000}\right)\cdot(+5\%) + \left(\frac{\$300}{\$1,000}\right)\cdot(+2\%) + \left(\frac{\$200}{\$1,000}\right)\cdot(-4\%) = +2.3\% \\ &= w_{\text{PEP}}\cdot r_{\text{PEP}} + w_{\text{KO}}\cdot r_{\text{KO}} + w_{\text{CSG}}\cdot r_{\text{CSG}} . \end{aligned} \tag{13.1}$$

Is this really true? A \$500 investment in PEP at a 5% rate of return gave \$25. A \$300 investment in KO at a 2% rate of return gave \$6. A \$200 investment in CSG at a –4% rate of return gave –\$8. The net dollar return on the \$1,000 was therefore \$25 + \$6 – \$8 = \$23. The rate of return was $(r_{P,t=1} - r_{P,t=0})/r_{P,t=0} = \$23/\$1,000 - 1 = 2.3\%$. This also

clarifies that the portfolio formula also works with absolute investments instead of relative investments:

$$\begin{aligned} \tilde{r}_P &= (\$500)\cdot(+5\%) + (\$300)\cdot(+2\%) + (\$200)\cdot(-4\%) = +\$23 \\ &= w_{\text{PEP}}\cdot r_{\text{PEP}} + w_{\text{KO}}\cdot r_{\text{KO}} + w_{\text{CSG}}\cdot r_{\text{CSG}} \quad . \end{aligned} \tag{13.2}$$

SIDE NOTE: When a security has additional payouts (such as dividends) over the measurement period, its rate of return should really be written as

$$r_{t-1,t} = \frac{(P_t + \text{Dividends}_{t-1,t}) - P_{t-1}}{P_{t-1}} \quad . \tag{13.3}$$

Alternatively, one could quote a net price at the end of the period, which includes dividends. Our discussion mostly just ignore dividends and stock splits. That is, when we talk about returns and prices, we mean returns that take into account all payments to an investor.

Notation: number investments. A portfolio is a set of known weights.

The nature of the subject of Investments is to evaluate all possible investment choices in order to determine the best portfolio. So, we need to come up with good notation that does not make discussing this task too cumbersome. We shall continue to use R and r as the designated names for "rate of return." But with thousands of possible investment choices, it is rather inconvenient to work with ticker symbols (or even full stock names). It would also be tedious to write "average the returns over all possible stocks (ticker symbols) and other securities." Therefore, we change the names of our securities to the numbers $1, 2, 3, \ldots, N$. We also usually use the letter P to name a portfolio (or, if we work with multiple portfolios, with a capital letter close by, such as Q or O). We call the investment weight in security i by the moniker w_i, where i is a number between 1 and N. Finally, we rely on "summation notation": $\sum_{i=1}^{N} f(i)$ is the algebraic way of stating that we compute the sum $f(1) + f(2) + \ldots f(N)$. For example, $\sum_{i=2}^{4} \sqrt{i}$ is notation for $\sqrt{2} + \sqrt{3} + \sqrt{4} \approx 5.15$. (Appendix Chapter 2·3 reviews summations.)

IMPORTANT: *The rate of return R on a portfolio P that consists of securities 1 through N is*

$$r_P = \sum_{i=1}^{N} w_i \cdot r_i = w_1 \cdot r_1 + w_2 \cdot r_2 + \ldots + w_N \cdot r_N \quad , \tag{13.4}$$

where w_i is the investment weight in the i-th security (from N choices). If weights are quoted as a fraction of the overall investment portfolio, their sum must add up to 100%,

$$\sum_{i=1}^{N} w_i = 100\% \quad . \tag{13.5}$$

OK, we cheated on notation: each rate of return R should have three subscripts: one to name the financial security (e.g., PEP or i), one for the beginning of the period (e.g., t), and one for the end of the period (e.g., $t+1$)—too many for our tastes. When there is no danger of confusion—or the formula works no matter what periods we choose (as long as we choose the same period for all securities), we omit the time subscripts.

Notation: Abbreviations

13·3.B. Funds and Net Holdings

One can think of portfolios, consisting of stocks, the same ways as one can think of stocks themselves. Indeed, **Funds** can be bought and sold just like firms, only their underlying assets are in turn more financial assets. Investors often like buying shares in funds because a professional manager may pick securities better than they can, plus funds allow individual investors to in effect purchase thousands of stocks, even if they only have $1,000 to invest. Depending on their legal arrangements, they may be called **Exchange-Traded Funds** (**ETF**) (bought and sold on a financial market), **Mutual Funds** (bought and sold by the general public, but not on an exchange), or **Hedge Funds** (not marketed to the broad public, and therefore not subject to S.E.C. restrictions). Many mutual funds have prices that are listed daily in the *Wall Street Journal*. Like exchange funds, they can be purchased easily through most stock brokers.

Funds are themselves portfolios of other financial assets.

> **ANECDOTE: More funds or more stocks?**
> In 1999, U.S. equity funds managed roughly 3 trillion dollars of assets, or about one-third of U.S. stock market capitalization. More surprisingly, there were more U.S. equity funds than there were U.S. stocks: In 1999, there were 8,435 equities, but 11,882 equity funds.
>
> Source: Harry Mamaysky and Matthew Spiegel.

Mutual funds come in two flavors. In an **open-end** mutual fund, an investor can tender her shares and receive the underlying assets in due proportion. For example, if a fund has sold 50 shares to its investors, and used the money to purchase 200 shares of PepsiCo and 300 shares of Coca Cola, then each mutual fund share represents 4 PepsiCo shares and 6 Coca Cola shares. In an open-ended fund, the owner of the mutual fund share can, at her will, redeem her share in the fund into the 4+6 underlying securities. This forces an **Arbitrage** link between the price of the fund and the value of its assets: if the price of the fund drops too much relative to the underlying assets, then investors would redeem their mutual fund shares. In a **closed-end** mutual fund, redemption is not permitted. If the underlying fund assets are very illiquid (e.g., real-estate in emerging countries), an open-ended like redemption request would be very expensive or even impossible to satisfy. Closed-end funds often trade for substantially less than their underlying assets, and for significant periods of time. Among the explanations for this "closed end fund discount" are the significant fees collected by the fund managers.

Open-ended vs. closed-ended Mutual Funds.

Mutual funds have to disclose their holdings on a quarterly basis to the S.E.C., which makes it easy for investors to compute their net exposures (at least on the reporting day). Assume Fund A holds $500,000 of PepsiCo and $1,500,000 of Coca Cola. Assume Fund B holds $300,000 of Coca Cola and $700,000 of Cadbury Schweppes. What is the net portfolio of an investor with 60% of her wealth in Fund A and 40% of her wealth in Fund B? Fund A has holdings of

How to compute net underlying holdings.

$$\begin{aligned} &\text{Fund A Holdings: } w_{A,\text{PEP}} = 0.25 \ , \ w_{A,\text{KO}} = 0.75 \ , \ w_{A,\text{CSG}} = 0 \ . \\ &\text{Fund B Holdings: } w_{B,\text{PEP}} = 0 \ , \ w_{B,\text{KO}} = 0.30 \ , \ w_{B,\text{CSG}} = 0.70 \ . \end{aligned} \tag{13.6}$$

The portfolio weights are multiplicative, so the fund investor's net exposures are

$$\begin{aligned} w_{\text{PEP}} &= 60\% \cdot 0.25 + 40\% \cdot 0.00 = 15\% \ , \\ w_{\text{KO}} &= 60\% \cdot 0.75 + 40\% \cdot 0.30 = 57\% \ , \\ w_{\text{CSG}} &= 60\% \cdot 0.00 + 40\% \cdot 0.70 = 28\% \ , \\ w_i &= w_A \cdot w_{A,i} + w_B \cdot w_{B,i} \ . \end{aligned} \tag{13.7}$$

So, for each \$2,000, the investor defacto owns \$300 of shares in PepsiCo, \$1,140 of shares in Coca Cola, and \$570 of shares in Cadbury Schweppes. In sum, investors can think of funds and portfolios the same way they think of stocks: they are investment opportunities representing combinations of assets. The investor can always compute the underlying stock holdings represented by the funds.

SIDE NOTE: If this investor wanted to hold purchase these two funds, but neutralize her exposure to PepsiCo, she should short some PepsiCo and use the proceeds to purchase more of the two funds. Shorting is discussed in more detail later in this section.

ANECDOTE: **High Losses Plus High Taxes**
In the second half of 2001, many mutual funds lost significant amounts of money in the collapse of the Tech bubble. Although their values (and with them, the assets of their clients) had declined, many mutual funds had realized capital gains early in the year. The I.R.S. requires these to be declared by investors as capital gains, which were therefore taxed. Thus, many unlucky investors experienced losses, and still had to pay taxes.

(To prevent tax arbitrage, there are special tax rules on mutual funds. The precise treatment of the taxes owed by mutual funds [and often passed through to their investors] is complex and beyond the scope of this book.)

ANECDOTE: **Finance Professors' Portfolios**
A large number of finance professors invest personal funds with low-cost mutual funds, which buy-and-hold a wide cross-section of assets and avoid active trading. Vanguard funds are particularly popular.

ANECDOTE: **Risk and Conglomeration**
In the 1970's, a lot of firms diversified to become conglomerates. Management argued that conglomerates tended to have lower risk, which created value for shareholders. This argument was of course total non-sense: Investors could diversify for themselves. It was the managers who valued lower risk, with the lower chance to lose their jobs and the higher compensation due to running a bigger company. Worse, because conglomerates often operate less efficiently than individual stand-alone, focused companies, diversification actually often destroyed firm value. In the 1980s, there were many so-called bust-up buyouts, which created value by purchasing conglomerates to sell off the pieces.

T: ***This is a very important point. Check whether it is elsewhere again.***

13·3.C. Some Common Indexes

An index is almost like a fund or portfolio, but it is not something that one can invest in because an index is just a number. (There are however funds that try to mimick indexes.) Most commonly, an index is a weighted sum of the prices of a predetermined basket of securities. For example, the **Dow-Jones 30** index is a weighted average of the prices of 30 pre-selected big stocks. (Table b on Page 319 lists them.) If an investor purchased a portfolio holding the same 30 stocks (or a fund doing this for her), her investment rate of return would be fairly close to the percentage change in the index—except for one big difference. When stocks pay dividends, their stock prices decline by the amounts of dividends paid. (Otherwise, you could earn an arbitrage by purchasing the stocks, collecting the dividends, and then reselling them for the same price.) The dividends are earned by investors mimicking the index, even though the index would decline by the percent paid out in dividends. Therefore, an investor should be able to outperform an index. Unfortunately, in the real world, most portfolio managers fail to outperform the underlying indexes. This is primarily due to transaction costs and excessive trading.

Although an Index is a weighted average of stock prices, it is just a number, not a portfolio.

Side Note: Most indexes are not adjusted for dividends, but they are adjusted for **Stock Splits**. In a 3:1 stock split, a firm trading at $120 per old share would henceforth trade at $40 per new share. Each investor who held one old share would receive three new shares. The "guardians of the index" would adjust the index formulas by tripling the weight on the stock that split. A few indexes are not price indexes, but so-called **Total Return Indexes**. For example, the German **Dax Performance Index** reinvests dividends (i.e., adjusts the index value as if dividends were reinvested).

There are literally hundreds of indexes, created and published by hundreds of companies. The **Money&Investing** Section of the *Wall Street Journal* lists just a sampling. In the United States, the most prominent stock market indexes are the **S&P 500** (holding 500 large stocks), the **Dow-Jones 30** (holding 30 stocks, selected to cover different industries), and the **Nasdaq** index (holding the largest Nasdaq companies). The **Russell 2000** covers 2,000 small-firm stocks. There are also other asset class indexes. For example, **Lehman Brothers** publishes the **MBS** (Mortgage Bond Securities) index; **Dow Jones** also publishes a corporate bond index; and **Morgan Stanley** publishes a whole slew of country stock prices indexes (**MSCI EAFE**). Furthermore, each country with a stock market has its own domestic index. Some foreign stock market indexes are familiar even to casual investors: the **Financial Times Stock Exchange** Index, spelled **FTSE** and pronounced "foot-sy" for Great Britain; the **Nikkei**-225 Index for Japan; and the **DAX** index for Germany.

There are more indexes than one can count, and new ones get invented all the time.

Anecdote: **The Presidential Election Market**
The University of Iowa runs the Iowa Electronic Market, which are indexes measuring the likelihood for each presidential candidate to win the next presidential election. Investors and students can trade futures based on these indexes. This market tends to be a better forecaster of who the next president will be than the press.

13·3.D. Equal and Value Weighted Portfolios

An Example of An Equal-weighted and A Value-weighted portfolio.

Two kinds of portfolios deserve special attention, the equal-weighted and the value-weighted portfolio. Assume that there are only three securities in the market. The first is worth \$100 million, the second \$300 million, and the third \$600 million. An **Equal-Weighted** portfolio purchases an equal amount in each security. If you had \$30 million, you would invest \$10 million into each security. Table 13.2 shows what happens when one stock's price changes: security $i = 1$ quadruples in value. If you do not trade, your portfolio holdings would be too much in security 1 relative to securities 2 and 3. To maintain an equal-weighted portfolio, you would have to rebalance. Indeed, you would have to trade \$40 worth of stock.

Table 13.2. Maintaining an Equal-Weighted Portfolio

	Time 0			Time 1			
Security i	Market Value	Investor Pfio	Rate of Return	Market Value	Investor, No Trade	Investor, Desired	Necessary Trading
1	\$100	\$10	+300%	\$400	\$40	\$20	−\$20
2	\$300	\$10	0%	\$300	\$10	\$20	+\$10
3	\$600	\$10	0%	\$600	\$10	\$20	+\$10
Sum	\$1,000	\$30		\$1,300	\$60	\$40	\$0

IMPORTANT: *To maintain an equal-weighted portfolio, continuous rebalancing is necessary.*

Table 13.3. Maintaining a Value-Weighted Portfolio

	Time 0			Time 1			
Security i	Market Value	Investor Pfio	Rate of Return	Market Value	Investor, No Trade	Investor, Desired	Necessary Trading
1	\$100	\$3	+100%	\$200	\$6	\$6	\$0
2	\$300	\$9	0%	\$300	\$9	\$9	\$0
3	\$600	\$18	0%	\$600	\$18	\$18	\$0
Total	\$1,000	\$30		\$1,100	\$33	\$33	\$0

In contrast, a **Value-Weighted** portfolio purchases an amount proportional to the availability of each security. In this case, with \$30 million, a portfolio that invests \$3 million in the first security (weight: \$2/\$20 = 10%), \$9 million in the second security (weight: 30%), and \$18 million in the third security (weight: 60%) is value-weighted. Even though it is easier at the beginning to select an equal-weighted portfolio (one does not need to know how much of each security is available), over time, it is easier to maintain a value-weighted portfolio than an equal-weighted portfolio. In Table 13.3, the first security has again doubled in value. It now has a market capitalization of \$200 million. Without trading, the value-weighted portfolio increases its holdings in this security from \$3 million to \$6 million. The portfolio weight in the first security would therefore increase to \$6/\$33 ≈ 18%, the second security would drop to \$9/\$33 ≈ 27%, and the third security would drop to \$18/\$22 ≈ 55%. But these are exactly the weights that a value weighted portfolio of \$33 million, if initiated at time 1, would ask for! This is because any changes in the market values of securities are reflected both in the overall market capitalizations and the weight of the security in the portfolio, so a value-weighted portfolio requires no trading.

Value-weighted portfolios are easier to maintain.

SIDE NOTE: There is one real-world exception: it could be that new firms come to the market. The investor may not have owned the not-yet-publicly trading firm in the proper proportion, and thus needs to initiate it into her portfolio. So, rebalancing could be required to include the new security.

IMPORTANT: *To maintain a value-weighted portfolio, no rebalancing is usually necessary.*

There is a second important feature of value-weighted portfolios: it is possible for everyone in the economy to hold a value-weighted portfolio, but not possible for everyone in the economy to hold an equal-weighted portfolio. Return to our example: with \$1 billion in overall market capitalization, presume there are only two investors: the first has \$100 million in wealth, the second has \$900 million in wealth. Equal-weighted portfolios would have the first investor allocate \$33 million to the first security and have the second investor allocate \$300 million to the first security. In sum, they would want to purchase \$333 million in the first security—but there is only \$100 million worth of the first security to go around. In contrast, holding value-weighted portfolios, both investors could be fully satisfied. For example, for the first security, the first investor would allocate \$10 million, the second investor would allocate \$90 million, and the sum-total would equal the \$100 million available in the economy.

It is possible for everyone in the economy to hold a value-weighted portfolio, but not an equal-weighted portfolio.

IMPORTANT: *It is possible for all investors in the economy to hold value-weighted portfolios. It is impossible for all investors in the economy to hold equal-weighted portfolios.*

Time Convergence? Two more trivia: First, over time, if an investor does not trade, even non-value weighted portfolios tend to become more value-weighted. Stocks that increase in value tend to constitute larger and larger fractions of the portfolio. This is also why most indexes end up behaving more like value-weighted portfolios than equal-weighted portfolios. Second, over short time frames (say a month or even a year), broad stock market indexes within a country tend to be very highly correlated (say, above 95%), no matter whether they are equal-weighted, value-weighted, or arbitrary (e.g., the Dow-Jones 30 or the S&P500). Therefore, if the newspaper reports the return of the S&P500 yesterday, it is a pretty good guideline either for broader portfolios (like a value-weighted overall stock market portfolio) or for narrower portfolios (like the Dow-Jones 30).

Solve Now!

Q 13.4 *An investor's portfolio consists of 40% of stock A and 60% of stock B. The first stock has a rate of return of +4%, the second has a rate of return of +6%. What is the overall portfolio rate of return?*

Q 13.5 *An investor owns \$40 in stock A and \$60 in stock B. The first stock has a return of* $\$1.^{60}$*, the second has a return of* $+\$3.^{60}$*. What is the overall portfolio return?*

Q 13.6 *An investor owns \$40 in stock A and \$60 in stock B. The first stock has a return of +4%, the second has a return of +6%. What is the overall portfolio rate of return?*

Q 13.7 *Write down the formula for the return of a portfolio, given individual security returns and their weights. First use summation notation, then write it out.*

Q 13.8 *A portfolio consists of \$200 invested in PEP, and \$600 invested in CSG. If the stock price per share on PEP increased from \$30 to \$33, and the stock price per share in CSG declined from \$40 to \$38 but CSG paid a dividend of \$1 per share, then what was the portfolio's return and rate of return?*

Q 13.9 *What is* $\sum_{j=1}^{5} j^2$*?*

Q 13.10 *What is* $\sum_{i=1}^{5} 2 \cdot i$*?*

Q 13.11 *What is* $\sum_{j=1}^{5} (j-5)$*?*

Q 13.12 *What is the difference between a Hedge Fund and a Mutual Fund?*

Q 13.13 *What is the difference between an open-ended mutual fund and a closed-ended mutual fund?*

Q 13.14 *An investor holds two funds. Fund A has holdings in stocks 1, 2, and 3 of 0.15, 0.5, and 0.35, respectively. Fund B has holdings in stocks 1, 2, and 3, of 0.4, 0.2, and 0.4, respectively. The investor would like to have a portfolio that has a net investment weight of 30% on the first stock. If the investor has decided only to hold funds and not individual stocks, what would his exposure be on stocks 2 and 3?*

Q 13.15 *What is the difference between an index and a mutual fund?*

Q 13.16 *List a few prominent financial indexes.*

Q 13.17 *How does an index differ from a portfolio?*

Q 13.18 *Compute the value-weighted dollar investments of the two investors (with wealths $100 million and $900 million, respectively) for the second and third securities in the example on Page 283.*

Q 13.19 *Continuing with this example, what would be the dollar investments and relative investments if the first security were to double in value? Does this portfolio require rebalancing to remain value-weighted?*

Q 13.20 *There are two stocks: stock 1 has a market capitalization of $100 million, stock 2 has a market capitalization of $300 million.*

1. *What are the investment weights of the equal-weighted portfolio?*
2. *What are the investment weights of the value-weighted portfolio?*
3. *There are 5 equally wealthy investors in this economy. How much of stock 1 would they demand if they all held the equal-weighted portfolio? If they held the value weighted portfolio?*
4. *If the first stock appreciates by 10% and the second stock depreciates by 30%, how much trading would such an investor have to do to continue holding an equal-weighted portfolio?*
5. *Repeat the previous question with a value-weighted portfolio.*

Q 13.21 *To maintain an equal-weighted portfolio, does an investor have to sell recent winner stocks or recent loser stocks? (Is this a bad or a good idea?)*

Q 13.22 *How different would the one-month performance of an investment in an S&P500 mimicking portfolio be, relative to the performance of the value-weighted market portfolio?*

13·3.E. Quo Vadis? Random Returns on Portfolios

A Roadmap. Most of our attention will be devoted to the case where returns are not yet known: they are still random variables, often denoted with a tilde above the unknown quantity (e.g., $\tilde{r}$). The goal of Investments is to select a portfolio P (that is a set of N investment weights $w_1, w_2, \ldots, w_N$) which is likely to offer the highest future performance with the least risk. Normally, rates of returns are unknown, and therefore written with tildes

$$\tilde{r}_p \;=\; \sum_{i=1}^{N} w_i \cdot \tilde{r}_i \;=\; w_1 \cdot \tilde{r}_1 + w_2 \cdot \tilde{r}_2 + \ldots w_N \cdot \tilde{r}_N \;\; . \tag{13.8}$$

We now need to determine

1. a good measure for the reward (likely performance) of a portfolio;
2. and a good measure for the risk of a portfolio.

For this, we shall need statistics, our next chapter.

13·4. Summary

The chapter covered the following major points:

- Securities appear through initial public offerings. They disappear through delistings and bankruptcies.
- Going long (purchasing) a publicly traded liquid stock requires paying broker fees and half of the bid-ask spread. Going short (selling without owning) does not provide full use of proceeds, and thus adds some holding costs that depend on the prevailing interest rate.
- Portfolio returns are a weighted average of individual returns.
- Fund holdings can be deconstructed into individual exposures.
- An index is computed as the weighted averages of its component prices. It is just a number. Funds are collections of underlying assets. The fund value is also the weighted average of prices. Some funds attempt to mimick indexes.
- Unlike most other portfolios, the value-weighted market portfolio can be owned by each and every investor in equilibrium.

A. Q&A: Answers

1. For academic shorting, you just promise the same cash that the shares themselves are paying. For real-world shorting, you ask your broker to find a holder of the shares, borrow them (i.e., and promise him the same payoffs), and then sell them to someone else. Most importantly, the broker who arranges this will not give you the cash obtained from selling the borrowed shares, that is the broker will earn the interest on the cash, rather than you. Other important differences have to do with the fact that you can be called upon to terminate your short if the lender of shares wants to sell the shares (you have to return the borrowed shorts), and with the fact that you have to find someone willing to lend you the shares.
2. The bid-ask transaction round-trip costs (bid-ask spread and broker commissions) for either the long or the short would be around 30-60 basis points. In addition, you would have to provide the cash for the share purchase; the cash from the share short is most likely kept by the broker. The loss of proceeds would cost you another 300 to 500 basis points per year in lost interest proceeds, depending on who you are and whether you already have the money or whether you have to borrow the money. So, if the shares cost \$5,000, you would have buy transaction costs of around \$25, sell transaction costs of around \$25, for total transaction costs of \$50; plus interest opportunity costs of around \$100 to \$250.
3.
   1. \$400,000.
   2. If you use the KO short proceeds to purchase stock in PEP, then $\$400,000 \cdot (+15\%) - \$100,000 \cdot (+12\%) = \$48,000$. On \$300,000 net investment, so your rate of return would be 16%.
   3. $\$300,000 \cdot (+15\%) - \$100,000 \cdot (+12\%) = \$33,000$. On \$300,000 net investment, so your rate of return would be 11%. You would be better off forgetting about the shorting and earn the 15% on PEP.
4. $r_p = w_1 \cdot r_1 + w_2 \cdot r_2 = 40\% \cdot 4\% + 60\% \cdot 6\% = 5.2\%$.
5. $r_p = r_1 + r_2 = \$5.^{20}$.
6. $r_p = w_1 \cdot r_1 + w_2 \cdot r_2 = \$40 \cdot 4\% + \$60 \cdot 6\% = \$5.^{20}$. Note that the formula works with dollar investments, too.
7. See Formula 13.4 on Page 278.
8. $r_{\text{PEP}} = 10\%.r_{\text{CSG}} = (\$38 + \$1 - \$40)/\$40 = -2.5\%$. Therefore, the absolute return and the rate of return on the portfolio was

$$\begin{aligned} r_P &= \$200 \cdot (10\%) + \$600 \cdot (-2.5\%) &&= +\$5 \quad . \\ r_P &= \left(\frac{\$200}{\$800}\right) \cdot (10\%) + \left(\frac{\$600}{\$800}\right) \cdot (-2.5\%) &&= +0.625\% \quad . \end{aligned} \tag{13.9}$$

9. $\sum_{i=1}^{5} i^2 = 1 + 4 + 9 + 16 + 25 = 55$. (Note that i is just a counter name, which can be replaced by any letter, so this answer is correct.)
10. $\sum_{j=1}^{5} (2 \cdot j) = 2 + 4 + 6 + 8 + 10 = 30$.
11. $\sum_{j=1}^{5} (j - 5) = -4 - 3 - 2 - 1 - 0 = -10$.
12. See Page 279.
13. See Page 279.

14.

$$\begin{aligned} w_A \cdot 15\% + (1 - w_A) \cdot 40\% &= 30\% \quad w_A = 40\% \\ w_A \cdot w_{A,1} + w_B \cdot w_{B,1} &= w_1 \end{aligned} \tag{13.10}$$

If the investor purchases 40% in fund A, her net holdings in each stock are

$$\begin{aligned} \text{Stock 1:} \quad 40\% \cdot 15\% + 60\% \cdot 40\% &= 30\% \ ; \\ \text{Stock 2:} \quad 40\% \cdot 50\% + 60\% \cdot 20\% &= 32\% \ ; \\ \text{Stock 3:} \quad 40\% \cdot 35\% + 60\% \cdot 40\% &= 38\% \ . \\ w_A \cdot w_{A,i} + w_B \cdot w_{B,i} &= w_i \ . \end{aligned} \tag{13.11}$$

15. An index is a series of numbers; a mutual fund holds stocks. A mutual fund can hold stocks to mimick the return on an index.
16. See text for possible indexes to mention.
17. Indexes are numbers, not investments. Their percent changes differs from a mimicking portfolio rate of return in that dividends are ignored.
18. In the second security: \$30 and \$270, for the first and second investor, respectively. In the third security: \$60 million and \$540 million, respectively.
19. As already computed in the text, the first security would increase from \$100 million to \$200 million, and thus total market capitalization would increase from \$1 billion to \$1.1 billion. Therefore, the weight of the three securities would be 18%, 27%, and 55%. (Moreover, the portfolio increased by 10% in value, which means that the first investor now has holdings worth \$110 million, the second investor has holdings worth \$990 million.) The dollar investments are even simpler: the first investor started with \$10, \$30, and \$60 million, respectively, and now holds \$20, \$30, and \$60 million. The second investor now holds \$180 \$270, and \$540 million, respectively.
20.
    1. $w_1 = 50\%, w_2 = 50\%$.
    2. $w_1 = 25\%, w_2 = 75\%$.
    3. Each investor would own \$80 million worth of securities. If each investor held the equal-weighted portfolio, the total demand for stock 1 would be $\$40 \cdot 5 = \$200$. This is impossible. If each investor held the value-weighted portfolio, the total demand for stock 1 would be $\$20 \cdot 5 = \$100$. This is definitely possible.
    4. The equal-weighted portfolio would have started out at $w_1 = \$40m, w_2 = \$40m$. The returns would have made the portfolio $w_1 = \$44m, w_2 = \$28m$ for a total value of $\$72m$. The revised portfolio would have to be $w_1 = \$36m, w_2 = \$36m$. Therefore, the investor would have to sell $\$8m$ in security 1 in order to purchase $\$8m$ in security 2.
    5. The value-weighted investor portfolio started out at $w_1 = \$20m, w_2 = \$60m$, and without trading would have become $w_1 = \$22m, w_2 = 42m$ for a portfolio worth $\$64m$. The investor's weights would be $w_1 = 34.375\%$, $w_2 = 65.625\%$.
    In terms of the market, the first stock would have appreciated in value from \$100 million to \$110 million, the second would have depreciated from \$300 million to \$210 million. The value-weighted market portfolio would therefore invest $w_1 = \$110/\$320 \approx 34.375\%$ and $w_2 = \$280/\$320 \approx 65.625\%$. Therefore, no trading is necessary.
21. Recent winners have to be sold, recent losers have to be bought. Although covered in more detail only in Chapter 19, the answer to the question in parenthesis is that if the stock market is competitive, past returns should have little predictive power for future returns, so this trading strategy is not necessarily a good or a bad idea.
22. The correlation between these two is high. Therefore, it would make little difference.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 14

STATISTICS

last file change: Jun 15, 2004 (11:50h)

last major edit: n/a

This chapter appears in the Survey text only.

This chapter attempts to distill the essential concepts of a full course in statistics into 30 pages. Thus, it is not an easy chapter, but it is also not as painful as you may imagine.

The chapter is written to allow you to understand the formulas for risk and reward of portfolios under uncertainty. These formulas will be used extensively in subsequent chapters. You cannot understand Investments without understanding this chapter.

14·1. Historical and Future Rates of Return

T: ***Some of this is now a repeat from the motivating chapter.***

We have little else but history to guide us.

As an investor, our goal is to find the best possible investment portfolio. Easier said than done. What do we know about how a stock, say, IBM, will perform in the future? Not much. Our prime source of information about how IBM *will* perform is how it *did* perform. If it returned 10% per year over the last 10 years *on average*, maybe it is a good guess that it will return 10% over the next year, too. If it had a risk of plus or minus 20% per year, maybe it is a good guess that it will have a risk of plus or minus 20% over the next year, too. But, is historical performance really representative of the future?

Don't take history too literally.

Clearly, it makes no sense to assume that the future returns will be *exactly* the same as the past returns. An investment in a particular 6 lotto numbers paid off big times last year, but this does not mean that the exact same investment gamble will pay off again. The same 6 lotto number investment are not especially likely to appear again. More sensibly, we should look at the risk/reward characteristics of the *average* 6 lotto number investment, from which we would probably conclude that the average 6 number lotto investment is not a great idea.

For stocks, we use general historical characteristics to indicate future general characteristics.

Similarly, for stocks, it makes more sense to assume that future returns will be *only on average* like past returns in terms of risk and reward. It is not that we believe this to be exactly true, but it is usually the best guess we can make *today*. Of course, we also know that the future will turn out different from the past—some firms will do better, others worse—but we generally have no better information than history. (Any investor who can systematically estimate future risk and reward better than others is bound to become rich.)

Sometimes, even general historical characteristics are obviously wrong.

Be warned, though. History is sometimes outright implausible as a predictor of the future. For example, Microsoft is almost surely not going to repeat its spectacular historical performance over the last 20 years. There are many examples when investors, believing history too much, made spectacularly wrong investment decisions. For example, in 1998-2000, Internet stocks had increased in value by more than 50% per year, and many investors believed that it was almost impossible to lose money on Internet stocks. Of course, these investors, who believed historical Internet returns were indicative of future Internet returns, lost *all* their money over the following 2 years. Despite these problems, we can only repeat: in many cases, stock's average historical risk-reward behavior is often the best guidance we have. We can use them, even though we should retain a certain skepticism—and perhaps even use common sense to adjust historical averages into more sensible estimates for the future.

Side Note: The theory of Investments, which is the subject of this part of our book, makes it easy on itself. It just *assumes* that you already know a stock's general risk/reward characteristics, and then proceeds to give you advice what portfolio to choose, given your expectations.

Anecdote: **Forecasting in New York State**
Persons pretending to forecast the future shall be considered disorderly under subdivision 3, section 901 of the criminal code and liable to a fine of $ 250 and/or six months in prison.

(Section 889, New York State Code of Criminal Procedure.)

14·2. The Data: Twelve Annual Rates of Returns

The goal of this chapter is to explain portfolios and stock returns under uncertainty. This is best done with a concrete example. Table 14.1 contains the actual twelve annual rates of returns from 1991 to 2002 for three possible investments: an S&P500 mutual fund, IBM stock, and Sony (ADR) shares.

The data example that is used throughout the Investments part of the book.

Side Note: An **ADR** (**American Depositary Receipt**) is a relatively easy way by which large foreign companies can trade shares on the New York Stock Exchange. Their shares are put into an escrow, which operates like an open-end fund holding only shares in this one company.

Table 14.1. Historical Annual Rates of Returns for S&P500, IBM, and Sony

Year	$\tilde{r}_{\text{S\&P500}}$	$\tilde{r}_{\text{IBM}}$	$\tilde{r}_{\text{Sony}}$	Year	$\tilde{r}_{\text{S\&P500}}$	$\tilde{r}_{\text{IBM}}$	$\tilde{r}_{\text{Sony}}$
1991	+0.2631	−0.2124	−0.1027	1997	+0.3101	+0.3811	+0.3905
1992	+0.0446	−0.4336	−0.0037	1998	+0.2700	+0.7624	−0.2028
1993	+0.0706	+0.1208	+0.4785	1999	+0.1953	+0.1701	+2.9681
1994	−0.0154	+0.3012	+0.1348	2000	−0.1014	−0.2120	−0.5109
1995	+0.3411	+0.2430	+0.1046	2001	−0.1304	+0.4231	−0.3484
1996	+0.2026	+0.6584	+0.0772	2002	−0.2337	−0.3570	−0.0808

Source: Yahoo!*Finance*. (Sidenote: Sony is the SNE ADR.) .

We want to understand risk and reward. Although we are really interested in the returns of 2003 (and beyond), unfortunately all we have are these historical rates of return. So, we shall make the common presumption that historical returns are good indicators of future returns. Further, we also make the common presumption that each year was an equally likely outcome drawn from an underlying statistical process, so that each year is equally informative to us. Formally, future returns are **random variables**, because their outcomes are not yet known. This is denoted with a tilde over the unknown variable, e.g.,

Pretend that historical rates of returns are representative of future returns. Introduce tilde notation.

$$\tilde{r}_{\text{S\&P500}} \ , \ \tilde{r}_{\text{IBM}} \ , \ \tilde{r}_{\text{Sony}} \ , \tag{14.1}$$

but because we only know historical rates of returns, we shall just use the historical data series in place of the tilde-d future variables.

Nerd Note: *The tilde stands not just one month, but for a whole range of possible outcomes, estimated based on the entire historical series; after all, standing today, next month's rate of return can be anything. (We do not yet have one number for it.) Conceptually, there is a difference between historical realizations and next month's rate of return. Practically, there is almost no difference, because we have no choice but to pretend that the historical return series represent the distribution of future returns.*

In statistics books, the historical average is often denoted with a bar, rather than with the $\mathcal{E}$ function, in order to distinguish the two. For example, $\overline{R}$ would be the historical average rate of return (e.g., 4%, known); and $\mathcal{E}\tilde{r}$ would be the unknown future rate of return. Then, the statistics books usually explain that we use the historical $\overline{R}$ as a proxy for the unknown true $\mathcal{E}\tilde{r}$. (The same applies to variance, standard deviations, covariances, etc.) In our finance book, we will keep the distinction rather vague.

14·3. Univariate Statistics

14·3.A. The Mean

Everyone knows how to compute an average.

If you had invested in a random year, what you would have expected to earn? The reward is measured by the single most important statistic, the **Mean** (also called **Average** or **Expected Value**). Everyone has computed this before, so let's just state that the means are

$$\mathcal{E}(\tilde{r}_{\text{S\&P500}}) = 0.101 \ , \ \mathcal{E}(\tilde{r}_{\text{IBM}}) = 0.154 \ , \ \mathcal{E}(\tilde{r}_{\text{Sony}}) = 0.242 \ . \tag{14.2}$$

Sony was clearly the best investment over these 12 years (mostly due to its spectacular performance in 1999), but IBM and the S&P500 did pretty well, too.

Nerd Note: *The average rate of return is not the compound rate of return. An investment in Sony beginning in 1999 for three years would have had a compound three-year rate of return of* $(1 + 297\%) \cdot (1 - 51\%) \cdot (1 - 35\%) \approx 26.5\%$ *(8.1% annualized). Yet, its average annual rate of return is* $(297\% + (-51\%) + (-35\%))/3 \approx 70.3\%$. *This is an extreme example: usually the difference between average and compound returns is much smaller. The reason for the difference is volatility across years, which negatively affects the compound holding returns.*

For our purposes of forecasting a single year's return, assuming that each historical outcome was equally likely, we want to work with average rates of returns. For computing long-term holding period performance, we want to work with compound rates of return.

14·3.B. The Variance and Standard Deviation

A Tale of three investments.

How can we measure portfolio risk? Intuitively, how does the risk of the following three investment choices compare?

1. An investment in a bond that yields 10% per year for sure.
2. An investment that yields −15% half the time, and +35% half the time.
3. An investment in the S&P500.

All three investments have a mean of about +10% per annum. But the mean tells us nothing about the risk. We need a statistic that tells us how variable the outcomes are around the mean. Contestant #1 has no variability, so it is clearly safest. What about the other two?

A naïve attempt at measuring spread fails.

For contestant #2, half the time, the investment outcome is 25% below its mean (of +10%); the other half, it is 25% above its mean. Measuring outcomes relative to their means (as we have just done) is so common, that it has its own name, **Deviation from the Mean**. So, perhaps we can we use the average deviation from the mean to measure typical variability? Try it. With two outcomes, each outcome is equally likely. Thus,

$$\text{Bad Variability Measure} = {}^1\!/_2 \cdot (-25\%) + {}^1\!/_2 \cdot (+25\%) = 0 \ . \tag{14.3}$$

The average deviation is zero, because the minus and plus cancel. Therefore, the simple average of the deviations is not a good measure of spread. We need a measure that tells us the typical variability is plus or minus 25%, not plus or minus 0%. A better measure must recognize that a negative deviation from the mean is the same as a positive deviation from the mean. Squaring each deviation from the mean eliminates the "opposite sign problem." The average of the squared deviations is called the **Variance**:

The root cause of the problem, and better alternatives: variance and standard deviation.

$$\mathcal{V}ar \;=\; {}^1\!/_2 \cdot (-25\%)^2 + {}^1\!/_2 \cdot (+25\%)^2 \;=\; {}^1\!/_2 \cdot 0.0625 + {}^1\!/_2 \cdot 0.0625 \;=\; 0.0625 \;. \tag{14.4}$$

NERD NOTE: *There are other measures of risk, but they are obscure enough to deserve our ignorance (at least until an advanced Investments course). One such measure is the mean absolute deviation (MAD) from the mean,*

$$MAD \;=\; {}^1\!/_2 \cdot |(-25\%)| + {}^1\!/_2 \cdot |(+25\%)| \;=\; {}^1\!/_2 \cdot 25\% + {}^1\!/_2 \cdot 25\% \;=\; 25\% \;. \tag{14.5}$$

In our case, the outcome happens to be the same as the standard deviation, but this is not generally the case. The MAD gives less weight than the standard deviation to observations far from the mean. Another such measure is the semivariance ($S\mathcal{V}$), which computes only observations below zero or below the mean. That is, all positive returns (or deviations from the mean) are simply ignored. For example,

$$S\mathcal{V} \;=\; {}^1\!/_2 \cdot (-25\%)^2 + {}^1\!/_2 \cdot (0) \;=\; {}^1\!/_2 \cdot 0.0625 \;=\; 0.03125 \;. \tag{14.6}$$

The idea is that as an investor, you only fear realizations that are negative (or below the mean).

You can think of the variance as the "average squared deviation from the mean." Alas, we want a measure that tells us that our typical variability is about plus or minus 25%, not plus or minus 6.25% (the variance). So, we now take the square root of the variance. This gives us the **Standard Deviation**:

The variance and standard deviation measure the expected spread of a random variable.

$$Sdv \;=\; \sqrt{\mathcal{V}ar} \;=\; \sqrt{0.0625} \;=\; 25\% \;. \tag{14.7}$$

This has the intuitively pleasing correct order of magnitude of 25%. Although it is not really correct, it is often convenient to think of the standard deviation as the "average deviation from the mean." (It would be more correct to call it "the square root of the average squared deviation from the mean," but this is unwieldy.)

SIDE NOTE: Aside from its odd value of 0.0625, there is a more important interpretation problem with the variance, that did not show in our example, because rates of returns are unitless. If you are interested in the variability of a variable that has units, like dollars, the units are often uninterpretable. For example, if you receive either \$10 or \$20, the deviation from the mean is either −\$5 or +\$5, and the variance is $(\$5)^2 = \$^2 25$, not \$25—the same way by which multiplying 2 meters by 2 meters becomes 4 square-meters, not 4 meters. Square-meters has a good interpretation (area); dollars-squared does not. The standard deviation takes the square root of the variance, and therefore returns to the same units (dollars) as the original series, $\sqrt{\$^2 25} = \$5$ in the example.

Because the standard deviation is just the square root of the variance, if the variance of one variable is higher than the variance of another variable, so is its standard deviation.

Standard deviation and variance have the same ordering.

Table 14.2. Deviations From the Mean for S&P500, IBM, and Sony

Year	$\tilde{r}_{\text{S\&P500}}$	$\tilde{r}_{\text{IBM}}$	$\tilde{r}_{\text{Sony}}$	Year	$\tilde{r}_{\text{S\&P500}}$	$\tilde{r}_{\text{IBM}}$	$\tilde{r}_{\text{Sony}}$
1991	+0.1620	−0.3661	−0.3448	1997	+0.2090	+0.2273	+0.1485
1992	−0.0565	−0.5874	−0.2458	1998	+0.1656	+0.6086	−0.4448
1993	−0.0305	−0.0330	+0.2364	1999	+0.0942	+0.0163	+2.7261
1994	−0.1165	+0.1474	−0.1073	2000	−0.2025	−0.3658	−0.7529
1995	+0.2400	+0.0892	−0.1374	2001	−0.2315	+0.2693	−0.5904
1996	+0.1015	+0.5046	−0.1648	2002	−0.3348	−0.5105	−0.3228
		Mean (of Deviations) over all 12 years			0.0	0.0	0.0

Important Nuisance: For historical data, do not divide the squared deviations by N, but by N-1.

For contestant #3, we must compute variability measures from the historical data series. Recall that to compute the variance, we subtract the mean from each outcome, square the deviations, and then average them. Table 14.2 computes the deviations from the mean for the S&P500, IBM, and Sony. Computing variances from these deviations is now straightforward: square and average. Alas, there is one nuisance complication: because there is a difference between historical realizations and expected future outcomes, statisticians divide by $N-1$, not by N. Therefore, the estimated variance (divides by $N-1$) is a little bit higher than the average squared deviation from the mean (divides by N).

SIDE NOTE: The reason for this $N-1$ adjustment is that the future standard deviation is not actually known, but only estimated given the historical realizations. This "extra uncertainty" is reflected by the smaller divisor.

The best intuition comes from a sample of only one historical data point: What would you believe the variability would be if you only know one realization, say 10%? In this case, you know nothing about variability. If you divided the average squared deviation (0) by N, the variance formula would indicate a zero variability. This is clearly wrong. If anything, you should be especially worried about variability, for you now know nothing about it. Dividing by $N-1=0$, i.e., $\mathcal{V}ar = 0/0$, indicates that estimating variability from one sample point makes no sense.

The division by N rather than $N-1$ is not important when there are many historical sample data points, which is usually the case in finance. Thus, most of the time, you could use either method, although you should remain consistent. This book uses the $N-1$ statistical convention, if only because it allows checking computations against the built-in formulas in Excel and other statistical packages.

Executing the formulas on historical data and dividing by N-1 yields variances and standard deviations.

The variance of each series is obtained by squaring each deviation from the mean, adding these squares, and dividing by 11 ($N-1$).

$$\begin{aligned}
\mathcal{V}ar(\tilde{r}_{\text{S\&P500}}) &= \frac{(+0.1620)^2+(-0.0565)^2+\ldots+(-0.3348)^2}{11} = 0.0362 \;; \\
\mathcal{V}ar(\tilde{r}_{\text{IBM}}) &= \frac{(-0.3661)^2+(-0.5874)^2+\ldots+(-0.5105)^2}{11} = 0.1503 \;; \\
\mathcal{V}ar(\tilde{r}_{\text{Sony}}) &= \frac{(-0.3448)^2+(-0.2458)^2+\ldots+(-0.3228)^2}{11} = 0.8149 \;; \\
\mathcal{V}ar(\tilde{r}) &= \frac{[\tilde{r}_{t=0}-\mathcal{E}(\tilde{r})]^2+[\tilde{r}_{t=1}-\mathcal{E}(\tilde{r})]^2+\ldots+[\tilde{r}_{t=T}-\mathcal{E}(\tilde{r})]^2}{T-1} \;.
\end{aligned} \tag{14.8}$$

The square roots of the variances are the standard deviations:

$$\begin{aligned} Sdv(\tilde{r}_{\text{S\&P500}}) &= \sqrt{3.62\%} = 19.0\% \quad ; \\ Sdv(\tilde{r}_{\text{IBM}}) &= \sqrt{15.03\%} = 38.8\% \quad ; \\ Sdv(\tilde{r}_{\text{Sony}}) &= \sqrt{81.49\%} = 90.3\% \quad . \end{aligned} \tag{14.9}$$

Wow! Sony was not only the best performer (with its mean rate of return of 24.2%/year), but it also was by far the riskiest investment. It had a whopping 90.3%/year standard deviation. (Like the mean, the large standard deviation is primarily caused by the single large rate of return outlier [of +297%] in 1999.) Incidentally, it appears that the S&P500 contestant #3 with its 19% risk is a safer investment than contestant #2 with its 25% risk.

For a preview, the goal of Investments is to determine the portfolio that offers the highest expected reward for the lowest amount of risk. This is why we need both the mean and the standard deviation.

Preview: Use of risk measures in finance.

SIDE NOTE: As mentioned in the introduction, from 1926 to 2002, the annual risk and reward of some large asset-class investments were approximately

Asset Class	$\mathcal{E}(\tilde{r})$	$Sdv(\tilde{r})$
Short-Term U.S. Government Treasury Bills	4%	3%
Long-Term U.S. Government Treasury Bonds	5.5%	10%
Long-Term Corporate Bonds	6%	9%
Large Firm Stocks	10%	20%
Small Firm Stocks	15%	30%

Inflation was about 3% per year. For the most part, it seems that higher risk and higher reward went hand-in-hand.

Solve Now!

Q 14.1 *Use Excel to confirm all numbers computed in this section.*

Q 14.2 *The annual rates of return on the German DAX index were*

1991	+0.1286	*1994*	−0.0706	*1997*	+0.4624	*2000*	−0.0754
1992	−0.0209	*1995*	+0.0699	*1998*	+0.1842	*2001*	−0.1979
1993	+0.4670	*1996*	+0.2816	*1999*	+0.3910	*2002*	−0.4394

Compute the mean, variance, and standard deviation of the DAX.

14·4. Bivariate Statistics: Covariation Measures

14·4.A. Intuitive Covariation

Table 14.3. Covariation Examples

Negative	Zero (or Low)	Positive
Agility vs. Weight	IQ vs. Gender or Race	Height vs. Basketball Scoring
Wealth vs. Disease	Wealth vs. Tail when flipping coin	Wealth vs. Longevity
Age vs. Flexibility	Age vs. Blood Type	Age vs. Being CEO
Mood vs. Temperature	Sunspots vs. Temperature	Grasshoppers vs. Temperature
Net Returns vs. Commissions Paid	IBM Returns in 1999 vs. Exxon Returns in 1986	Returns on S&P500 vs. Returns on IBM

Personal statistics (such as Weight) apply only to adults. Returns are rates of returns on stock investments, net of commissions.

A measure of dependence should be positive when two variables tend to move together, negative if they tend to move in opposite directions.

Our goal is now to find measures of covariation that are positive when two variables tend to move together; that are zero when two variables have nothing to do with one another; and that are negative when one variable tends to be lower when the other variable tends to be higher. Table 14.3 offers some examples of covariation in real life. We will consider three possible measures: covariance, correlation, and beta.

14·4.B. Covariation: Covariance, Correlation, and Beta

Illustrative Data Series

Covariance is best to understand with an extreme, albeit absurd example.

Table 14.4 shows nine data series I made up. Figure 14.1 shows that B and C are easily related to A: B is A plus 3; C is negative one-half times B. D and E seem to bear no relationship to A: there is no pattern connecting A and D; and E is 5 regardless of what A is. Figure 14.2 shows that the relationships between A on the one hand and Series F through I on the other hand are more interesting: F and G tend to be higher when A is higher, but in different ways: F is more reliably higher, but even a large increase in A predicts an F that is only a little higher. In contrast, even a small increase in A predicts a much higher G, but very unreliably so. Figures H and I repeat this, but for negative relations.

Side Note: Later, we will ask such questions as "if A were the rate of return on the S&P500, and B were the rate of return on IBM, then how do the two return series covary?" This is important, because it can help us determine investment opportunities that have lower risk.

We need a suitable numerical statistic for how these series covary. Actually, we need more than just one statistic: we need one statistic to tell us how accurately two variables move together (e.g., A predicts F better than it predicts G), and another statistic to tell us how sensitive two variables are relative to one another (e.g., A has more effect on G than on F). Put differently, we want one measure of how well the line predicts the points, and another measure of how steep the line is.

We need (at least) two measures of covariation, not just one.

Table 14.4. Illustrative Data Series

Observation	A	B	C	D	E	F	G	H	I
Year 1	−5	−2	+1.0	−10	+5	2	−10	+5	+12
Year 2	+6	+9	−4.5	−9	+5	4	−9	+3	+14
Year 3	+3	+6	−3.0	+2	+5	3	+10	+4	−10
Year 4	−1	+2	−1.0	+12	+5	2	−8	+5	+9
Year 5	+7	+10	−5.0	0	+5	4	+12	+3	−10
Mean	+2	+5	−2.5	−1	+5	+3	−1	+4	+3
Var	25	25	6.25	81	0	1	121	1	144
Sdv	5	5	2.5	9	0	1	11	1	12

Figure 14.1. B Through E as functions of A

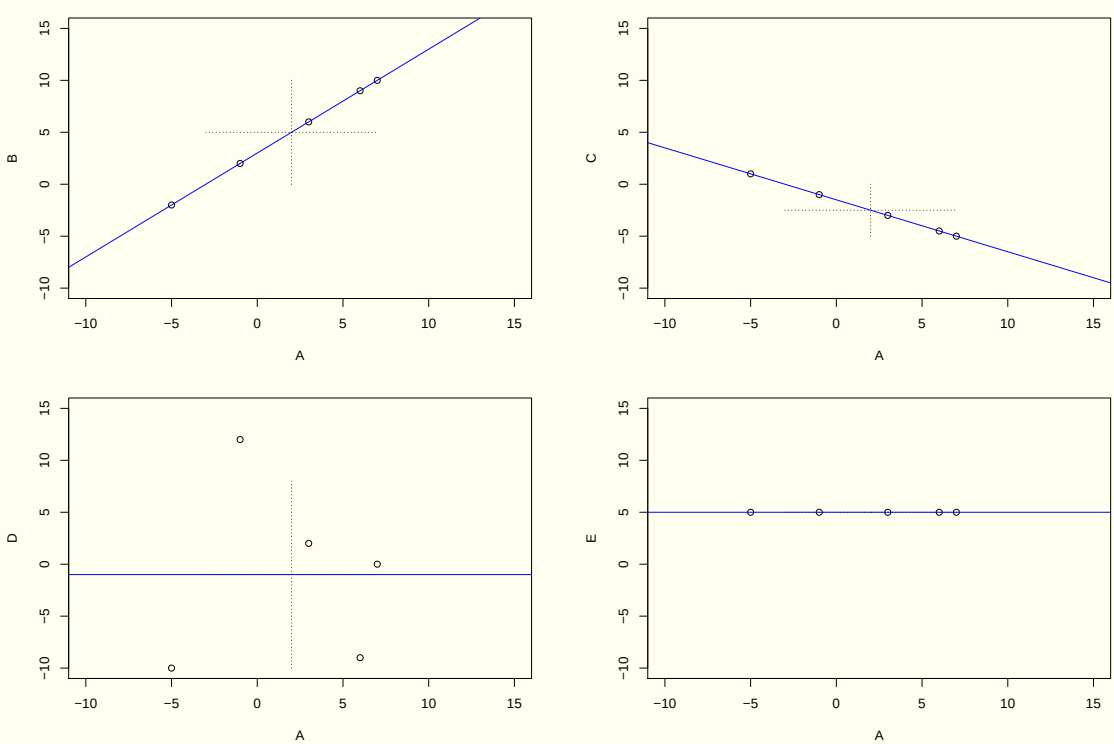
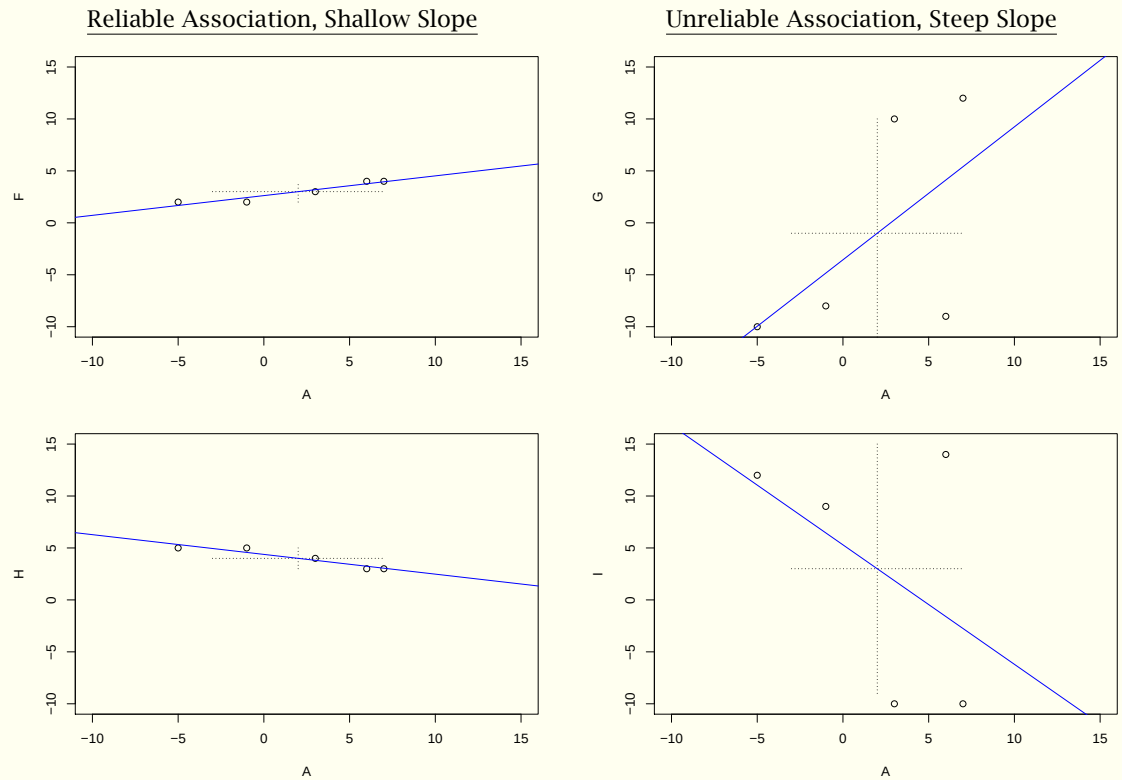


Figure 14.2. F through I as functions of A



The main idea is to think about where points lie, within the quadrants in the figure.

To do so, we rely on the covariation measures. The idea is that two series, call them X and Y, are positively correlated when X tends to be above its mean while Y is also above its mean, and when X tends to be below its mean while Y is also below its mean. And that X and Y are negatively correlated when X tends to be above its mean while Y is below its mean, and X tends to be below its mean while Y is above its mean. The crosses in Figure 14.1 and 14.2 show the means graphically. Any point that lies in the upper-right or lower-left quadrants—where both coordinates of a data point are either below or above the series mean—indicates positive covariation. Any point that lies in the lower-right or upper-left quadrants indicates negative covariation. So, we first compute for each x-y data point the deviation from its x-mean and y-mean:

Observation	A	B	C	D	E	F	G	H	I
Year 1	−7	−7	+3.5	−9	0	−1	−9	+1	+9
Year 2	+4	+4	−2.0	−8	0	+1	−8	−1	+11
Year 3	+1	+1	−0.5	+3	0	0	+11	0	−13
Year 4	−3	−3	+1.5	+13	0	−1	−7	+1	+6
Year 5	+5	+5	−2.5	+1	0	+1	+13	−1	−13
Mean	0	0	0.0	0	0	0	0	0	0
Var	25	25	6.25	81	0	1	121	1	144
Sdv	5	5	2.5	9	0	1	11	1	12

Covariance

How do we make a positive number for every point that is either above both means or below both means? The product of either two positive or two negative numbers is positive, and the product of one positive and one negative number is negative. So if we multiply deviations from the mean, we will get positive signs in the upper-right and lower-left quadrants (the deviations are either both positive or both negative) and negative signs in the upper-left and lower-right quadrants (only one deviation is positive, the other is negative).

Multiply the deviations from the mean to get the right sign for each data point relative to each quadrant.

The **covariance** is the average of these products of two variables' deviations from their own means. Try it (again, following the standard method of dividing by $N-1$ rather than N):

The covariance is the average of the multiplied deviations from their means.

$$Cov(A,B) = \frac{(-7)\cdot(-7)+(+4)\cdot(+4)+(+1)\cdot(+1)+(-3)\cdot((-3))+(+5)\cdot((+5))}{4} = 25$$

$$Cov(A,B) = \frac{(a_1-\bar{a})\cdot(b_1-\bar{b})+\ldots+(a_5-\bar{a})\cdot(b_5-\bar{b})}{N-1} \quad . \tag{14.10}$$

Note how each term in the sum is non-negative, and therefore pulls the average (the covariance) towards being a positive number. This positive number tells us that A and B covary positively. Repeat this for the remaining series:

$$\begin{aligned}
Cov(A,C) &= \frac{(-7)\cdot(+3.5)+(+4)\cdot(-2)+(+1)\cdot(-0.5)+(-3)\cdot(+1.5)+(+5)\cdot(-2.5)}{4} = -12.5\\
Cov(A,D) &= \frac{(-7)\cdot(-9)+(+4)\cdot(-8)+(+1)\cdot(+3)+(-3)\cdot(+13)+(+5)\cdot(+1)}{4} = 0\\
Cov(A,E) &= \frac{(-7)\cdot(0)+(+4)\cdot(0)+(+1)\cdot(0)+(-3)\cdot(0)+(+5)\cdot(0)}{4} = 0\\
Cov(A,F) &= \frac{(-7)\cdot(-1)+(+4)\cdot(+1)+(+1)\cdot(0)+(-3)\cdot(-1)+(+5)\cdot(+1)}{4} = 4.75\\
Cov(A,G) &= \frac{(-7)\cdot(-9)+(+4)\cdot(-8)+(+1)\cdot(+11)+(-3)\cdot(-7)+(+5)\cdot(+13)}{4} = 32\\
Cov(A,H) &= \frac{(-7)\cdot(+1)+(+4)\cdot(-1)+(+1)\cdot(0)+(-3)\cdot(+1)+(+5)\cdot(-1)}{4} = -4.75\\
Cov(A,I) &= \frac{(-7)\cdot(+9)+(+4)\cdot(+11)+(+1)\cdot(-13)+(-3)\cdot(+6)+(+5)\cdot(-13)}{4} = -28.75\\
Cov(X,Y) &= \frac{(x_1-\bar{x})\cdot(y_1-\bar{y})+\ldots+(x_5-\bar{x})\cdot(y_5-\bar{y})}{N-1} \quad .
\end{aligned} \tag{14.11}$$

When we compute our covariance computations, we see that the signs work great: A and C covary negatively on average; A and D (or E) have zero covariation; A and F (or G) have positive covariation; and A and H (or I) have negative covariation. In the A vs. G figure, there was one lonely point in the lower-right quadrant: in the A vs. G covariance computation, it is the $(+4)\cdot(-1)$ term, which is the only negative component in the sum. Without this one data point, the association between A and G would be more strongly positive than 4.75.

Just like the variance, the covariance is difficult to interpret. Indeed, it not only shares the weird squared-units problem, but the covariance of a variable with itself is the variance! This can be seen from the definitions: Both multiply each historical outcome by

Covariance gives us the right sign, but not much more. It is often abbreviated as a sigma with two subscripts.

itself, and then divide by the same number. The covariance statistic is so important and used so often that the Greek letter sigma (σ) has become the standard abbreviation:

$$\begin{aligned} \text{Covariance between } X \text{ and } Y\text{:} \quad & Cov(X,Y) \qquad \sigma_{X,Y} \\ \text{Variance of } X\text{:} \quad & Var(X) \qquad \sigma_{X,X} = \sigma_X^2 \ . \end{aligned} \tag{14.12}$$

These are sigmas with two subscripts. If we use only one subscript, we mean the standard deviation:

$$\text{Standard Deviation of } X\text{:} \quad Sdv(X) \qquad \sigma_X \ . \tag{14.13}$$

This is easy to remember if you think of two subscripts as the equivalent of multiplication (squaring).

Correlation

Correlation is closely related to, but easier to interpret than Covariance.

To better interpret the covariance, we need to somehow normalize it. Our first normalization of the covariance gives us the **Correlation**. It divides the covariance by the standard deviation of both variables.

$$\begin{aligned}
Correlation(A,B) &= \frac{Cov(A,B)}{Sdv(A)\cdot Sdv(B)} = \frac{+25}{5\cdot 5} = +1.00 \\
Correlation(A,C) &= \frac{Cov(A,C)}{Sdv(A)\cdot Sdv(C)} = \frac{-12.5}{5\cdot 2.5} = -1.00 \\
Correlation(A,D) &= \frac{Cov(A,D)}{Sdv(A)\cdot Sdv(D)} = \frac{0}{5\cdot 1} = \pm 0.00 \\
Correlation(A,E) &= \frac{Cov(A,E)}{Sdv(A)\cdot Sdv(E)} = \frac{0}{5\cdot 0} = \text{not defined} \\
Correlation(A,F) &= \frac{Cov(A,F)}{Sdv(A)\cdot Sdv(F)} = \frac{4.75}{5\cdot 1} = +0.95 \\
Correlation(A,G) &= \frac{Cov(A,G)}{Sdv(A)\cdot Sdv(G)} = \frac{32}{5\cdot 11} = +0.58 \\
Correlation(A,H) &= \frac{Cov(A,H)}{Sdv(A)\cdot Sdv(H)} = \frac{32}{5\cdot 1} = -0.95 \\
Correlation(A,I) &= \frac{Cov(A,I)}{Sdv(A)\cdot Sdv(I)} = \frac{32}{5\cdot 12} = -0.48 \\
Correlation(X,Y) &= \frac{Cov(X,Y)}{Sdv(X)\cdot Sdv(Y)}
\end{aligned} \tag{14.14}$$

The correlation measures the reliability of the relationship between two variables. It is always between -100% and $+100\%$, which makes it easy to interpret: a higher absolute correlation means more predictability. The correlation is unit-less, regardless of the units of the original variables themselves, and is often abbreviated with the Greek letter rho (ρ). The perfect correlation between A and B, C, or D tells us that a straight line can be drawn between A and the B, C, or D data points. The correlation between A and F (95%) and the correlation between A and H (-95%) are almost as strong: the points almost lie on a line. The correlation between A and G, and the correlation between A

and I are weaker: the points do not seem to lie on a straight line, and knowing A will not permit us to *perfectly* predict G or I.

SIDE NOTE: If two variable are always acting identically, they have a correlation of 1. Therefore, we can determine the maximum covariance between two variables:

$$1 = \frac{Cov(\tilde{X},\tilde{Y})}{Sdv(\tilde{X})\cdot Sdv(\tilde{Y})} \iff Cov(\tilde{X},\tilde{Y}) = Sdv(\tilde{X})\cdot Sdv(\tilde{Y}) \ . \tag{14.15}$$

It is mathematically impossible for the absolute value of the covariance to exceed the product of the two standard deviations.

NERD NOTE: *The correlation squared is the R^2 in a bivariate OLS regression with a constant.*

Beta

The correlation cannot tell us that A has more pronounced influence on B than on C: if A is higher by 1, our prediction of B is higher by 1, too; but if A is higher by 1, our prediction of C is lower by only −0.5. We need a measure for the slope of the best-fitting line that you could draw through the points. Our second normalization of the covariance does this: it gives us this slope, the so-called **beta**. It divides the covariance by the variance of the X variable (here, A).

Beta is yet another covariation measure, and has an interesting graphical interpretation.

$$\begin{aligned}
\beta_{B,A} &= \frac{Cov(A,B)}{Sdv(A)\cdot Sdv(A)} = \frac{+25}{5\cdot 5} = +1.00 \\
\beta_{C,A} &= \frac{Cov(A,C)}{Sdv(A)\cdot Sdv(A)} = \frac{-12.5}{5\cdot 5} = -0.50 \\
\beta_{D,A} &= \frac{Cov(A,D)}{Sdv(A)\cdot Sdv(A)} = \frac{0}{5\cdot 5} = \pm 0.00 \\
\beta_{E,A} &= \frac{Cov(A,E)}{Sdv(A)\cdot Sdv(A)} = \frac{0}{5\cdot 5} = \pm 0.00 \\
\beta_{F,A} &= \frac{Cov(A,F)}{Sdv(A)\cdot Sdv(A)} = \frac{4.75}{5\cdot 5} = +0.19 \\
\beta_{G,A} &= \frac{Cov(A,G)}{Sdv(A)\cdot Sdv(A)} = \frac{32}{5\cdot 5} = +1.28 \\
\beta_{H,A} &= \frac{Cov(A,H)}{Sdv(A)\cdot Sdv(A)} = \frac{32}{5\cdot 5} = -0.19 \\
\beta_{I,A} &= \frac{Cov(A,I)}{Sdv(A)\cdot Sdv(A)} = \frac{32}{5\cdot 5} = -1.15 \\
\beta_{Y,X} &= \frac{Cov(X,Y)}{Sdv(X)\cdot Sdv(X)} = \frac{Cov(X,Y)}{\mathcal{V}ar(X)}
\end{aligned} \tag{14.16}$$

The first subscript on beta denotes the variable on the Y axis, the second subscript on beta denotes the variable on the X axis. It is the latter that provides the variance (the divisor). Betas are not limited to any range. Beta values of +1 and −1 denote the diagonal lines, beta values of 0 and ∞ denote the horizontal and vertical line. Inspection of Figure 14.1 reveals that the slope of *A* vs. *B* is *1 to 1*, while the slope of *A* vs. *C* is shallower *1 to -2*. This is exactly what beta tells us: $\beta_{B,A}$ is 1.0, while $\beta_{C,A}$ is −0.5. Unlike

the correlation, beta cannot tell us whether our line fits perfectly or imperfectly. But, unlike the correlation, beta can tell us how much you should change your prediction of Y if the X values change.

SIDE NOTE: The most common way to write down the formula for a line is as $y = \alpha + \beta \cdot x$. Thus, the line slope got the name Beta. A slope of 0 is a horizontal line. Slopes of $+1$ and -1 are the two diagonal lines. Slopes can be bigger than $+1$ and smaller than -1. A slope of infinity is a vertical line.

SIDE NOTE: Unlike correlation and covariance, the order of the two variables matters in computing beta. For example, $\beta_{A,E} = 2.0$ is not $\beta_{E,A}$:

$$\beta_{A,E} = \frac{Cov(A,E)}{Sdv(E) \cdot Sdv(E)} = \frac{5.00}{13.7} = +0.365 \ . \tag{14.17}$$

NERD NOTE: *In a statistical package, beta can be obtained either by computing covariances and variances and then dividing the two; or by running a so-called* ***Linear Regression****, in which the dependent variable is the Y variable and the independent variable is the X variable.*

$$\tilde{r}_i = \alpha + \beta \cdot \tilde{r}_{\mathcal{M}} + \epsilon \ . \tag{14.18}$$

Both methods yield the same answer.

Beta has another interesting use. For example, if you believe IBM will outperform the stock market, but you do not know whether the stock market will do well or do poorly, you may want to ***hedge*** *out the market risk. If β is 2, for example, it would mean that for each 10% rate of return in the stock market, IBM would move 20%. If you purchase \$100 in IBM and go short $\beta \cdot \$100 = \$200$ in the stock market, then you have neutralized the systematic stock market risk and therefore are left only with the idiosyncratic risk of IBM.*

NERD NOTE: *If portfolio P consists of two stocks, called 1 and 2 with weights w_1 and w_2, then the beta of this portfolio with respect to some other portfolio M is*

$$\beta_{P,\mathcal{M}} = w_1 \cdot \beta_{1,\mathcal{M}} + w_2 \cdot \beta_{2,\mathcal{M}} \ , \tag{14.19}$$

i.e., the value-weighted average of the betas of its components.

Supernerdnote Proof:

$$\begin{aligned} \beta_{w_1 \cdot \tilde{x}_1 + w_2 \cdot \tilde{x}_2, \tilde{x}_{\mathcal{M}}} &= \frac{Cov(w_1 \cdot \tilde{x}_1 + w_2 \cdot \tilde{x}_2, \ \tilde{x}_{\mathcal{M}})}{\mathcal{V}ar(\tilde{x}_{\mathcal{M}})} \\ = \frac{Cov(w_1 \cdot \tilde{x}_1, \tilde{x}_{\mathcal{M}})}{\mathcal{V}ar(\tilde{x}_{\mathcal{M}})} + \frac{Cov(w_2 \cdot \tilde{x}_2, \tilde{x}_{\mathcal{M}})}{\mathcal{V}ar(\tilde{x}_{\mathcal{M}})} &= w_1 \cdot \frac{Cov(\tilde{x}_1, \tilde{x}_{\mathcal{M}})}{\mathcal{V}ar(\tilde{x}_{\mathcal{M}})} + w_2 \cdot \frac{Cov(\tilde{x}_2, \tilde{x}_{\mathcal{M}})}{\mathcal{V}ar(\tilde{x}_{\mathcal{M}})} \\ = w_1 \cdot \beta_{\tilde{x}_1, \tilde{x}_{\mathcal{M}}} + w_2 \cdot \beta_{\tilde{x}_2, \tilde{x}_{\mathcal{M}}} & \ . \end{aligned} \tag{14.20}$$

An exercise below will confirm that I have applied the covariance laws correctly. (Covariance is a linear operator.)

When it comes to stock returns, we will really want to know the future slope, although we will only have the historical slope.

And, as with all other statistical measures, please keep in mind that we are computing a historical beta (line slope), although we are really interested in the future beta (line slope)!

Summary of Covariation Measures

Table 14.5 summarizes the covariation measures.

Table 14.5. Comparison of Covariation Measures

	Units	Magnitude	Order of Variables	computed as	Measures
Covariance	squared	practically meaningless	irrelevant	$\sigma_{X,Y}$	?
Correlation	unit-less	between -1 and $+1$	irrelevant	$\sigma_{X,Y} \,/\, (\sigma_X \cdot \sigma_Y)$	Reliability
beta (Y,X)	unit-less	meaningful (slope)	relevant	$\sigma_{X,Y} \,/\, \sigma_{X,X}$	Slope
beta (X,Y)	unit-less	meaningful (slope)	relevant	$\sigma_{X,Y} \,/\, \sigma_{Y,Y}$	Slope

All covariation measures share the same sign. If one is positive (negative), so are all others. Recall that $\sigma_{X,X} = \sigma_X^2$, which must be positive.

ANECDOTE: Unbiased Coin Tossing
If we want to get a yes/no random variable that is unbiased, tossing a coin is a good idea—or is it? The correlation between a random variable's outcome and its own outcome is called the "auto-correlation." (Auto means self in Latin, not car. Automobile is a vehicle that moves by itself.)

In 2004, Diaconis, Holmes and Montgomery, statisticians at Stanford and UC Santa-Cruz, discovered that human coin-tossing is not random, at all: They found that around 51% of all real-world coin-tosses ended up on the side on which they started out. Human coin tosses are *not* independent random 50-50 draws, but display auto-correlation.

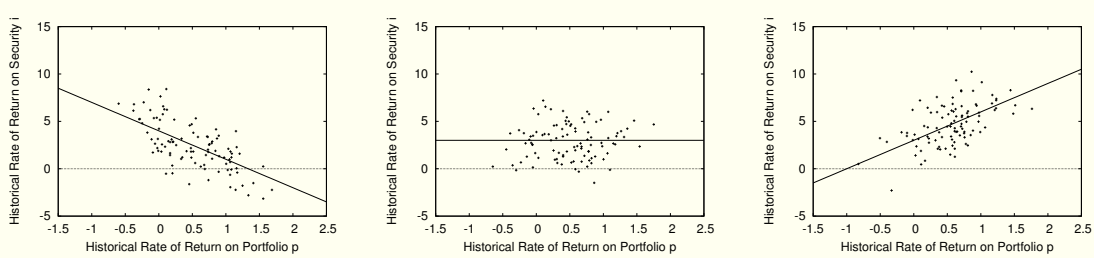
Source: Science News.

14·4.C. Computing Covariation Statistics For the Annual Returns Data

Figure 14.3 graphs some more realistic examples of rates of returns among financial securities. The correlations are negative, zero, and positive, respectively. The figures also draw the best fitting line.

Some drawings of more reasonable correlations.

Figure 14.3. Three Different Beta Measures of Risk for Security i.



The betas of security i with respect to portfolio p are -3, 0, and $+3$ in the three figures, respectively. The standard deviations of x in these graphs are 1, so the betas and covariances are identical. The correlation between x and y is -83% in the left figure, 0 in the middle figure, and $+83\%$ in the right figure.

SIDE NOTE: Although we should really write $\beta_{\tilde{r}_{\text{IBM}},\tilde{r}_{\text{S\&P500}}}$, we often omit the $\tilde{r}$ for convenience, and just elevate the subscripts when there is no risk of confusion. Thus, we would just write $\beta_{\text{IBM,S\&P500}}$, instead.

Applying the covariance formula to our historical data.

Now back to work! Compute the covariance, correlation, and beta for our three investment choices, S&P500, IBM, and Sony. Return to the deviations from the means in Table 14.2. The covariances are the sums of their products, divided by $(N-1)$—tedious, but not difficult to compute:

$$\begin{aligned}
Cov(\tilde{r}_{\text{S\&P500}},\tilde{r}_{\text{IBM}}) &= \frac{(+0.162)\cdot(-0.366)+\ldots+(-0.335)\cdot(-0.511)}{11} = 0.0330 \;; \\
Cov(\tilde{r}_{\text{S\&P500}},\tilde{r}_{\text{Sony}}) &= \frac{(+0.162)\cdot(-0.345)+\ldots+(-0.335)\cdot(-0.323)}{11} = 0.0477 \;; \\
Cov(\tilde{r}_{\text{IBM}},\tilde{r}_{\text{Sony}}) &= \frac{(-0.366)\cdot(-0.345)+\ldots+(-0.511)\cdot(-0.323)}{11} = 0.0218 \;. \\
Cov(\tilde{x},\tilde{y}) &= \frac{(x_1-\overline{x})\cdot(y_1-\overline{y})+\ldots+(x_N-\overline{x})\cdot(y_N-\overline{y})}{N-1}
\end{aligned} \tag{14.21}$$

All three covariance measures are positive. We know from Page 299 that, aside from their signs, the covariances are almost impossible to interpret.

Applying the correlation formula to our historical data.

Therefore, we compute the correlations, our measure of how well the best-fitting line fits the data. The correlations are the covariances divided by the two standard deviations:

$$\begin{aligned}
Correlation(\tilde{r}_{\text{S\&P500}},\tilde{r}_{\text{IBM}}) &= \frac{3.30\%}{19.0\%\cdot 38.8\%} = 44.7\% \;; \\
Correlation(\tilde{r}_{\text{S\&P500}},\tilde{r}_{\text{Sony}}) &= \frac{4.77\%}{19.0\%\cdot 90.3\%} = 27.8\% \;; \\
Correlation(\tilde{r}_{\text{IBM}},\tilde{r}_{\text{Sony}}) &= \frac{2.18\%}{38.8\%\cdot 90.3\%} = 6.2\% \;. \\
Correlation(\tilde{x},\tilde{y}) &= \frac{Cov(\tilde{x},\tilde{y})}{Sdv(\tilde{x})\cdot Sdv(\tilde{y})} \;.
\end{aligned} \tag{14.22}$$

So, the S&P500 has correlated much more with IBM, than the S&P500 has correlated with Sony (or IBM with Sony). This makes intuitive sense. Both S&P500 and IBM are U.S. investments, while Sony is a stock that is trading in an entirely different market.

Applying the beta formula to our historical data.

Finally, we want to compute the beta of $\tilde{r}_{\text{Sony}}$ with respect to the $\tilde{r}_{\text{S\&P500}}$; and the beta of $\tilde{r}_{\text{IBM}}$ with respect to the $\tilde{r}_{\text{S\&P500}}$:

$$\begin{aligned}
\beta_{\text{IBM,S\&P500}} &= \frac{3.30\%}{(19.0\%)^2} = 0.91 \;; \\
\beta_{\text{Sony,S\&P500}} &= \frac{4.77\%}{(19.0\%)^2} = 1.31 \;. \\
\beta_{y,x} &= \frac{Cov(\tilde{x},\tilde{y})}{Sdv(\tilde{x})\cdot Sdv(\tilde{x})} \;.
\end{aligned} \tag{14.23}$$

This is the slope of the best-fitting line when the S&P500 is on the X-axis and IBM (Sony) is on the Y axis. The next chapters will elaborate more on the importance of beta in finance.

We have now computed all the statistics that this book will use: means, variances, standard deviations, covariances, correlations, and betas. Only modestly painful, I hope.

The Marquis De Sade would not have been happy.

Solve Now!

Q 14.3 *What is the correlation of a random variable with itself?*

Q 14.4 *What is the slope (beta) of a random variable with itself?*

Q 14.5 *Return to the historical rates of returns on the DAX from Question 14.2 (Page 295). Compute the covariances, correlations and betas for the DAX with respect to each of the other three investment securities.*

Q 14.6 *Supernerd Question: Compute the annual rates of return on a portfolio of 1/3 IBM and 2/3 Sony. Then compute the beta of this portfolio with respect to the S&P500. How does this compare to the beta of IBM with respect to the S&P500, and the beta of Sony with respect to the S&P500?*

14·5. Risk and Reward of Portfolios

14·5.A. The Formulas

The previous chapter explained that the return of a portfolio is a weighted sum of random variables (constituent security returns). We now want to determine the mean and standard deviation of portfolio returns. This can become a little complicated. By starting with formulas, and then showing how to use them, this will be less painful than it first seems. Bear with me.

A Roadmap: We need to learn more about the risk and reward of weighted sums.

As in Chapter 13, i is a counter to enumerate each and every possible investment, from N choices. $\tilde{r}_i$ are the (random) returns. w_i are the investment weights that define the portfolio. The return of portfolio P in each and every month is

The Formal Definitions.

$$r_P = \sum_{i=1}^{N} w_i \cdot r_i \quad , \tag{14.24}$$

but because you do not yet know the returns,

$$\tilde{r}_P = \sum_{i=1}^{N} w_i \cdot \tilde{r}_i \quad . \tag{14.25}$$

The first formula is for the mean return of the portfolio,

$$\mathcal{E}(\tilde{r}_P) = \mathcal{E}\left(\sum_{i=1}^{N} w_i \cdot \tilde{r}_i\right) = \sum_{i=1}^{N} w_i \cdot \mathcal{E}(\tilde{r}_i) \quad . \tag{14.26}$$

Our second formula is for the variance of the return on the portfolio,

$$\mathcal{V}ar(\tilde{r}_P) \;=\; \mathcal{V}ar\left(\sum_{i=1}^{N} w_i \cdot \tilde{r}_i\right) \;=\; \sum_{j=1}^{N}\left\{\sum_{k=1}^{N}\left[w_j \cdot w_k \cdot Cov(\tilde{r}_j, \tilde{r}_k)\right]\right\} \tag{14.27}$$

We can write out and rearrange the terms in the formula. This, too, looks rather intimidating, but it will become clearer in the next section. So, for now, just register that these formulas are here.

$$\begin{aligned}
\mathcal{V}ar(\tilde{r}_P) &= \sum_{j=1}^{N}\left\{\sum_{k=1}^{N}\left[w_j \cdot w_k \cdot Cov(\tilde{r}_j, \tilde{r}_k)\right]\right\} \\
&= \sum_{j=1}^{N}\left\{w_j \cdot w_1 \cdot Cov(\tilde{r}_j, \tilde{r}_1) + w_j \cdot w_2 \cdot Cov(\tilde{r}_j, \tilde{r}_2) + \ldots + w_j \cdot w_N \cdot Cov(\tilde{r}_j, \tilde{r}_N)\right\}
\end{aligned} \tag{14.28}$$

$$\begin{aligned}
= \quad & w_1 \cdot w_1 \cdot Cov(\tilde{r}_1, \tilde{r}_1) + w_1 \cdot w_2 \cdot Cov(\tilde{r}_1, \tilde{r}_2) + \ldots + w_1 \cdot w_N \cdot Cov(\tilde{r}_1, \tilde{r}_N) \\
+ \quad & w_2 \cdot w_1 \cdot Cov(\tilde{r}_2, \tilde{r}_1) + w_2 \cdot w_2 \cdot Cov(\tilde{r}_2, \tilde{r}_2) + \ldots + w_2 \cdot w_N \cdot Cov(\tilde{r}_2, \tilde{r}_N) \\
+ \quad & w_3 \cdot w_1 \cdot Cov(\tilde{r}_3, \tilde{r}_1) + w_3 \cdot w_2 \cdot Cov(\tilde{r}_3, \tilde{r}_2) + \ldots + w_3 \cdot w_N \cdot Cov(\tilde{r}_3, \tilde{r}_N) \\
+ \quad & \ldots + \ldots + \ldots + \ldots \\
+ \quad & w_N \cdot w_1 \cdot Cov(\tilde{r}_N, \tilde{r}_1) + w_N \cdot w_2 \cdot Cov(\tilde{r}_N, \tilde{r}_2) + \ldots + w_N \cdot w_N \cdot Cov(\tilde{r}_N, \tilde{r}_N) \\
= \quad & w_1^2 \cdot \mathcal{V}ar(\tilde{r}_1) + 2 \cdot w_1 \cdot w_2 \cdot Cov(\tilde{r}_1, \tilde{r}_2) + \ldots + 2 \cdot w_1 \cdot w_N \cdot Cov(\tilde{r}_1, \tilde{r}_N) \\
+ \quad & w_2^2 \cdot \mathcal{V}ar(\tilde{r}_2) + \ldots + 2 \cdot w_2 \cdot w_N \cdot Cov(\tilde{r}_2, \tilde{r}_N) \\
+ \quad & \ldots + \ldots \\
+ \quad & w_N^2 \cdot \mathcal{V}ar(\tilde{r}_N)
\end{aligned} \tag{14.29}$$

$$\begin{aligned}
= \quad & \left[\text{sum of } w_i^2 \text{ times the variance of return } i \text{ (on the diagonals above)}\right] \\
+ \quad & \left[\text{sum of 2 times } w_i \text{ times } w_j \text{ times each possible covariance pair}\right] \quad .
\end{aligned}$$

(For the supernerd, the variance formula is derived in Appendix c.)

IMPORTANT: *The mean and variance formulas for portfolios are important enough to deserve memorizing.*

SIDE NOTE: Note that there are usually many more covariance terms than variance terms in Formula 14.29. For 2 stocks inside a portfolio, there are 2 variance terms and 1 covariance term. (Of course, covariance terms are always multiplied by a factor of 2.) For 3 stocks, there are 3 variance terms and 3 mutual covariance terms (each security with each other security). For 4 stocks, there are 4 variance terms and 6 covariance terms. For $N = 100$ stocks, there are 100 variance terms and $N \cdot (N-1)/2 = 4,950$ covariance terms. With more and more securities in the portfolio, there are fewer and fewer own return variances and more and more return covariances. Thus, on first glance, it seems that the overall portfolio variance could be driven more by the many covariance terms than the few variance terms. This will play a major role in later chapters.

The first form in Formula 14.29 writes the variance in compact mathematical notation. The second form states that to compute an overall variance, sum up all the constituent variances ($\mathcal{V}ar(\tilde{r}_i)$), each multiplied by its squared weight (w_i^2); and then add each pairwise covariance ($Cov(\tilde{r}_i, \tilde{r}_j)$), multiplied by two times its two weights ($2 \cdot w_i \cdot w_j$). The meaning of the formulas will become clearer when we use them. Trust me: the formulas are easier to use than they are to write. So, let's try it.

The formulas look nasty, but are not so bad.

14·5.B. Usage Example 1: The Annual Returns Data

We now compute the risk and reward of a portfolio that consists of the S&P500 ($w_{\text{S\&P500}} = 70\%$), IBM ($w_{\text{IBM}} = 20\%$) and Sony ($w_{\text{Sony}} = 10\%$). This portfolio returns

Application of the Formulas will make them a lot easier to understand.

$$\tilde{r}_P \;=\; 70\% \cdot \tilde{r}_{\text{S\&P500}} \;+\; 20\% \cdot \tilde{r}_{\text{IBM}} \;+\; 10\% \cdot \tilde{r}_{\text{Sony}} \quad . \tag{14.30}$$

What are the portfolio's risk and reward? Using the expected rates of returns computed in Formula 14.2 on Page 292, the portfolio means Formula 14.26 states

$$\begin{aligned}
\mathcal{E}(\tilde{r}_P) \;&=\; \mathcal{E}\left(\sum_{i \in \substack{\text{S\&P500}\\ \text{IBM}\\ \text{Sony}}} w_i \cdot \tilde{r}_i\right) \;=\; \sum_{i \in \substack{\text{S\&P500}\\ \text{IBM}\\ \text{Sony}}} w_i \cdot \mathcal{E}(\tilde{r}_i) \\
&=\; 70\% \cdot \mathcal{E}(\tilde{r}_{\text{S\&P500}}) \;+\; 20\% \cdot \mathcal{E}(\tilde{r}_{\text{IBM}}) \;+\; 10\% \cdot \mathcal{E}(\tilde{r}_{\text{Sony}}) \\
&=\; 70\% \cdot 0.101 \;+\; 20\% \cdot 0.154 \;+\; 10\% \cdot 0.242 \\
&=\; 12.57\% \quad .
\end{aligned} \tag{14.31}$$

The portfolio variance formulas are more complex, and certainly tedious; but they will only be difficult if you are not sure about how to use summation signs. The variance can be computed via the first form (Formula 14.27) or the second form (Formula 14.29), and we will show both. Using the variances for our three stocks computed in Formula 14.8 (Page 294) and the covariances computed in Formula 14.21 (Page 304), Formula 14.27 states that

The variance formula looks hairier.

$$\mathcal{V}ar(\tilde{r}_P) \;=\; \sum_{i \in \substack{\text{S\&P500}\\ \text{IBM}\\ \text{Sony}}} \left\{ \sum_{j \in \substack{\text{S\&P500}\\ \text{IBM}\\ \text{Sony}}} \left[w_i \cdot w_j \cdot Cov(\tilde{r}_i, \tilde{r}_j) \right] \right\}$$

Write out the inner summation, that is, expand all the j's.

$$\begin{aligned}
=\; \sum_{i \in \substack{\text{S\&P500}\\ \text{IBM}\\ \text{Sony}}} \Big\{ & w_i \cdot w_{\text{S\&P500}} \cdot Cov(\tilde{r}_i, \tilde{r}_{\text{S\&P500}}) \;+\; w_i \cdot w_{\text{IBM}} \cdot Cov(\tilde{r}_i, \tilde{r}_{\text{IBM}}) \\
& + w_i \cdot w_{\text{Sony}} \cdot Cov(\tilde{r}_i, \tilde{r}_{\text{Sony}}) \Big\}
\end{aligned}$$

Now write out the outer and only remaining summation, that is, expand all the i's.

$$
\begin{aligned}
= & \Big[w_{\text{S\&P500}} \cdot w_{\text{S\&P500}} \cdot Cov(\tilde{r}_{\text{S\&P500}}, \tilde{r}_{\text{S\&P500}}) + w_{\text{IBM}} \cdot w_{\text{S\&P500}} \cdot Cov(\tilde{r}_{\text{IBM}}, \tilde{r}_{\text{S\&P500}}) \\
& \quad + w_{\text{Sony}} \cdot w_{\text{S\&P500}} \cdot Cov(\tilde{r}_{\text{Sony}}, \tilde{r}_{\text{S\&P500}}) \Big] \qquad \leftarrow \text{(the 3 terms where } i \text{ is S\&P500)} \\
+ & \Big[w_{\text{S\&P500}} \cdot w_{\text{IBM}} \cdot Cov(\tilde{r}_{\text{S\&P500}}, \tilde{r}_{\text{IBM}}) + w_{\text{IBM}} \cdot w_{\text{IBM}} \cdot Cov(\tilde{r}_{\text{IBM}}, \tilde{r}_{\text{IBM}}) \\
& \quad + w_{\text{Sony}} \cdot w_{\text{IBM}} \cdot Cov(\tilde{r}_{\text{Sony}}, \tilde{r}_{\text{IBM}}) \Big] \qquad \leftarrow \text{(the 3 terms where } i \text{ is IBM)} \\
+ & \Big[w_{\text{S\&P500}} \cdot w_{\text{Sony}} \cdot Cov(\tilde{r}_{\text{S\&P500}}, \tilde{r}_{\text{Sony}}) + w_{\text{IBM}} \cdot w_{\text{Sony}} \cdot Cov(\tilde{r}_{\text{IBM}}, \tilde{r}_{\text{Sony}}) \\
& \quad + w_{\text{Sony}} \cdot w_{\text{Sony}} \cdot Cov(\tilde{r}_{\text{Sony}}, \tilde{r}_{\text{Sony}}) \Big] \qquad \leftarrow \text{(the 3 terms where } i \text{ is Sony)}
\end{aligned}
\tag{14.32}
$$

Substitute all the investment weights.

$$
\begin{aligned}
= & \Big[70\% \cdot 70\% \cdot Cov(\tilde{r}_{\text{S\&P500}}, \tilde{r}_{\text{S\&P500}}) + 20\% \cdot 70\% \cdot Cov(\tilde{r}_{\text{IBM}}, \tilde{r}_{\text{S\&P500}}) \\
& \quad + 10\% \cdot 70\% \cdot Cov(\tilde{r}_{\text{Sony}}, \tilde{r}_{\text{S\&P500}}) \Big] \\
+ & \Big[70\% \cdot 20\% \cdot Cov(\tilde{r}_{\text{S\&P500}}, \tilde{r}_{\text{IBM}}) + 20\% \cdot 20\% \cdot Cov(\tilde{r}_{\text{IBM}}, \tilde{r}_{\text{IBM}}) \\
& \quad + 10\% \cdot 20\% \cdot Cov(\tilde{r}_{\text{Sony}}, \tilde{r}_{\text{IBM}}) \Big] \\
+ & \Big[70\% \cdot 10\% \cdot Cov(\tilde{r}_{\text{S\&P500}}, \tilde{r}_{\text{Sony}}) + 20\% \cdot 10\% \cdot Cov(\tilde{r}_{\text{IBM}}, \tilde{r}_{\text{Sony}}) \\
& \quad + 10\% \cdot 10\% \cdot Cov(\tilde{r}_{\text{Sony}}, \tilde{r}_{\text{Sony}}) \Big]
\end{aligned}
$$

Substitute all the covariances, and use your calculator.

$$
\begin{aligned}
= & \Big[70\% \cdot 70\% \cdot 0.0362 + 20\% \cdot 70\% \cdot 0.0330 + 10\% \cdot 70\% \cdot 0.0218 \Big] \\
+ & \Big[70\% \cdot 20\% \cdot 0.0330 + 20\% \cdot 20\% \cdot 0.1503 + 10\% \cdot 20\% \cdot 0.0218 \Big] \\
+ & \Big[70\% \cdot 10\% \cdot 0.0477 + 20\% \cdot 10\% \cdot 0.0218 + 10\% \cdot 10\% \cdot 0.8149 \Big] \\
= & \quad 0.04869 \quad .
\end{aligned}
$$

Alternatively, the second form in Formula 14.29 is

$$
\begin{aligned}
\mathcal{V}ar(\tilde{r}_P) = & \; w_{\text{S\&P500}}^2 \cdot \mathcal{V}ar(\tilde{r}_{\text{S\&P500}}) + w_{\text{IBM}}^2 \cdot \mathcal{V}ar(\tilde{r}_{\text{IBM}}) + w_{\text{Sony}}^2 \cdot \mathcal{V}ar(\tilde{r}_{\text{Sony}}) \\
& + 2 \cdot w_{\text{S\&P500}} \cdot w_{\text{IBM}} \cdot Cov(\tilde{r}_{\text{S\&P500}}, \tilde{r}_{\text{IBM}}) \\
& + 2 \cdot w_{\text{S\&P500}} \cdot w_{\text{Sony}} \cdot Cov(\tilde{r}_{\text{S\&P500}}, \tilde{r}_{\text{Sony}}) \\
& + 2 \cdot w_{\text{IBM}} \cdot w_{\text{Sony}} \cdot Cov(\tilde{r}_{\text{IBM}}, \tilde{r}_{\text{Sony}}) \\
= & \; (70\%)^2 \cdot 3.62\% + (20\%)^2 \cdot 15.03\% + (10\%)^2 \cdot 81.49\% \\
& + 2 \cdot 70\% \cdot 20\% \cdot 3.30\% + 2 \cdot 70\% \cdot 10\% \cdot 4.77\% + 2 \cdot 20\% \cdot 10\% \cdot 2.18\% \\
= & \; 3.19\% + 1.679\% = 4.869\% \quad .
\end{aligned}
\tag{14.33}
$$

Computing variances for portfolios is tedious, but not difficult. With some practice, it will become second-nature.

The standard deviation is just the square root.

The portfolio's standard deviation (risk) is

$$
Sdv(\tilde{r}_P) = \sqrt{\mathcal{V}ar(\tilde{r}_P)} = \sqrt{4.869\%} = 22.1\% \quad . \tag{14.34}
$$

Table 14.6. Historical Annual Rates of Returns for Portfolio P

Year	$\tilde{r}_{S\&P500}$	$\tilde{r}_{IBM}$	$\tilde{r}_{Sony}$	$\tilde{r}_P$	Year	$\tilde{r}_{S\&P500}$	$\tilde{r}_{IBM}$	$\tilde{r}_{Sony}$	$\tilde{r}_P$
1991	+0.263	−0.212	−0.103	+0.131	1997	+0.310	+0.381	+0.391	+0.332
1992	+0.045	−0.434	−0.004	−0.056	1998	+0.270	+0.762	−0.203	+0.319
1993	+0.071	+0.121	+0.479	+0.121	1999	+0.195	+0.170	+2.968	+0.468
1994	−0.015	+0.301	+0.135	+0.063	2000	−0.101	−0.212	−0.511	−0.165
1995	+0.341	+0.243	+0.105	+0.298	2001	−0.130	+0.423	−0.348	−0.042
1996	+0.203	+0.658	+0.077	+0.281	2002	−0.234	−0.357	−0.081	−0.243

The return of portfolio P is $\tilde{r}_P = 70\% \cdot \tilde{r}_{S\&P500} + 20\% \cdot \tilde{r}_{IBM} + 10\% \cdot \tilde{r}_{Sony}$.

But what have we really computed? Return to first principles: if we had really purchased this portfolio, each annual return of P would have been 70% of the return on the S&P500, plus 20% of the return on IBM, plus 10% of the return on Sony. Table 14.6 shows the rates of return of portfolio P, next to its constituents. From these actual rates of returns on portfolio P, we can compute the expected rate of return on portfolio P as

Do not lose the forest in the trees: know what you really have computed.

$$\begin{aligned} \mathcal{E}(\tilde{r}_P) &= \frac{(+0.131) + (-0.056) + (+0.121) + \ldots + (-0.243)}{12} \\ &= 0.1257 \quad = \quad 12.57\% \quad . \end{aligned} \tag{14.35}$$

This is exactly what we computed in Formula 14.31. We can also compute the variance from these actual rates of returns on portfolio P,

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_P) &= \frac{(0.131 - 0.1257)^2 + (-0.056 - 0.1257)^2 + \ldots + (-0.2430 - 0.1257)^2}{11} \\ &= 0.04869 \quad = \quad 4.869\% \quad . \end{aligned} \tag{14.36}$$

This is exactly what we computed in Formula 14.33. So, the seemingly complex Formula 14.26 and Formula 14.29 are just methods to compute the overall portfolio risk and reward, that can be applied if you know the means, variances, and pairwise covariances for each individual security. The complex formulas are time-savers if the historical rate of return series are long, and if we want to determine the overall portfolio means and variances for many different portfolios (with many different portfolio weights). Knowing the formulas will allow us to more quickly answer such questions as "how does the risk/reward change if we increase one weight a little while reducing another?"

NERD NOTE: *The portfolio P keeps the weights constant. Therefore, if you wanted to replicate P, you would have to rebalance every period.*

14·5.C. Usage Example 2: PepsiCo, CocaCola, and Cadbury

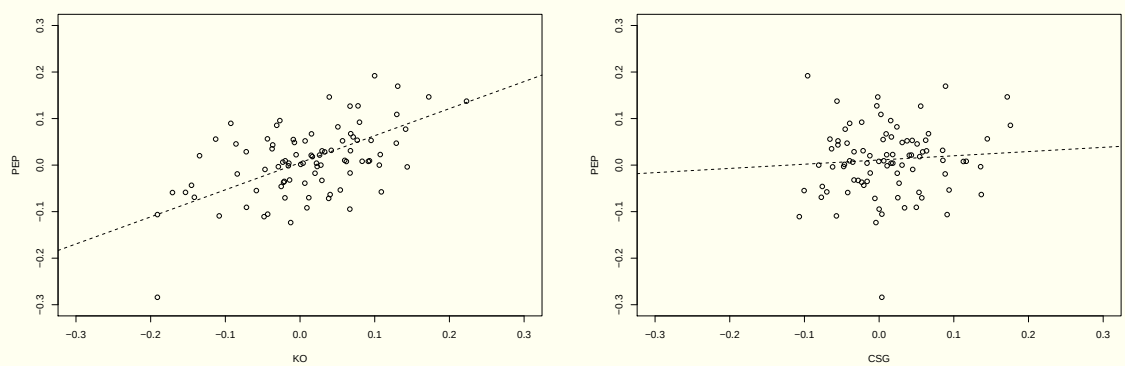
Table 14.7. PepsiCo, Coca Cola, and Cadbury Schweppes Monthly Return Statistics, from September 1995 to August 2002.

Investment		Means	Standard Deviations	Correlations PEP	KO	CSG
1	PEP	0.83%	7.47%	100.0%		
2	KO	0.90%	8.35%	53.2%	100.0%	
3	CSG	1.19%	6.29%	10.8%	9.9%	100.0%

The table provides only statistics for the rates of return, not the series themselves.

We now use portfolio formulas on a second example with monthly data, based on the historical means, standard deviations, and correlations of Coca Cola, PepsiCo, and Cadbury Schweppes, over the 1995-2002 period. This example is deliberately reminiscent of the example from the previous section, but we are now given only the correlations, not the detailed historical return series themselves. Table 14.7 shows that the average return of each of these three stocks was about 1% per month, or 10%–12% per year. The monthly means were significantly lower than the monthly standard deviations. Over the sample period, Cadbury Schweppes had higher performance and lower risk than either PepsiCo or Coca Cola. Note also how high the 53% correlation between PepsiCo and Coca Cola is, especially relative to the 10.8% and 9.9% Cadbury Schweppes correlations. Figure 14.4 plots the data points. Coca Cola stock seems to behave more like PepsiCo stock than like Cadbury Schweppes stock.

Figure 14.4. 1,765 Daily Stock Returns of PepsiCo vs. Coca Cola, and PepsiCo vs. Cadbury Schweppes from August 1995 to August 2002



As always, we assume that historical means, standard deviations, and correlations are indicative of future means, standard deviations, and correlations. We want to determine the risk and reward of a portfolio that invests 20% in PEP, 30% in KO, and 50% in CSG. Thus, portfolio P is defined by

A real-world example that computes the risk and reward of a portfolio of three stocks.

$$\tilde{r}_P = 20\% \cdot \tilde{r}_{\text{PEP}} + 30\% \cdot \tilde{r}_{\text{KO}} + 50\% \cdot \tilde{r}_{\text{CSG}} \ . \tag{14.37}$$

The reward is easy. It is

$$\begin{aligned} \mathcal{E}(\tilde{r}_P) &= 20\% \cdot 0.83\% + 30\% \cdot 0.90\% + 50\% \cdot 1.19\% = 1.03\% \\ \mathcal{E}(\tilde{r}_P) &= 20\% \cdot \mathcal{E}(\tilde{r}_{\text{PEP}}) + 30\% \cdot \mathcal{E}(\tilde{r}_{\text{KO}}) + 50\% \cdot \mathcal{E}(\tilde{r}_{\text{CSG}}) \ . \end{aligned} \tag{14.38}$$

The portfolio risk is more difficult. As inputs, we need covariances, not standard deviations or correlations. The covariances of variables with themselves (i.e., the variances) of the three stocks can be computed from the standard deviations, by squaring:

$$\begin{aligned} \sigma_{\tilde{r}_{\text{PEP}},\tilde{r}_{\text{PEP}}} &= (0.0747)^2 = 0.005585 \ ; \\ \sigma_{\tilde{r}_{\text{KO}},\tilde{r}_{\text{KO}}} &= (0.0835)^2 = 0.006967 \ ; \\ \sigma_{\tilde{r}_{\text{CSG}},\tilde{r}_{\text{CSG}}} &= (0.0629)^2 = 0.003956 \ . \end{aligned} \tag{14.39}$$

The covariances have to be computed from the correlations. Recall Formula 14.14,

$$\rho_{1,2} = \frac{\sigma_{1,2}}{\sigma_1 \cdot \sigma_2} \iff \sigma_{1,2} = \rho_{1,2} \cdot \sigma_1 \cdot \sigma_2 \ . \tag{14.40}$$

With this formula, we can compute the covariances,

$$\begin{aligned} \sigma_{\tilde{r}_{\text{PEP}},\tilde{r}_{\text{KO}}} &= \rho_{\tilde{r}_{\text{PEP}},\tilde{r}_{\text{KO}}} \cdot \sigma_{\tilde{r}_{\text{PEP}}} \cdot \sigma_{\tilde{r}_{\text{KO}}} = 0.532 \cdot 0.0747 \cdot 0.0835 = 0.003318 \ ; \\ \sigma_{\tilde{r}_{\text{PEP}},\tilde{r}_{\text{CSG}}} &= \rho_{\tilde{r}_{\text{PEP}},\tilde{r}_{\text{CSG}}} \cdot \sigma_{\tilde{r}_{\text{PEP}}} \cdot \sigma_{\tilde{r}_{\text{CSG}}} = 0.108 \cdot 0.0747 \cdot 0.0629 = 0.000507 \ ; \\ \sigma_{\tilde{r}_{\text{KO}},\tilde{r}_{\text{CSG}}} &= \rho_{\tilde{r}_{\text{KO}},\tilde{r}_{\text{CSG}}} \cdot \sigma_{\tilde{r}_{\text{KO}}} \cdot \sigma_{\tilde{r}_{\text{CSG}}} = 0.099 \cdot 0.0835 \cdot 0.0629 = 0.000522 \ . \end{aligned} \tag{14.41}$$

We now have all inputs that we need to compute the portfolio return variance:

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_P) &= \mathcal{V}ar\Big(20\% \cdot \tilde{r}_{\text{PEP}} + 30\% \cdot \tilde{r}_{\text{KO}} + 50\% \cdot \tilde{r}_{\text{CSG}}\Big) \\ &= (20\%)^2 \cdot \sigma_{\tilde{r}_{\text{PEP}},\tilde{r}_{\text{PEP}}} + (30\%)^2 \cdot \sigma_{\tilde{r}_{\text{KO}},\tilde{r}_{\text{KO}}} + (50\%)^2 \cdot \sigma_{\tilde{r}_{\text{CSG}},\tilde{r}_{\text{CSG}}} \\ &\quad + 2 \cdot 20\% \cdot 30\% \cdot \sigma_{\tilde{r}_{\text{PEP}},\tilde{r}_{\text{KO}}} + 2 \cdot 30\% \cdot 50\% \cdot \sigma_{\tilde{r}_{\text{PEP}},\tilde{r}_{\text{CSG}}} + 2 \cdot 20\% \cdot 50\% \cdot \sigma_{\tilde{r}_{\text{KO}},\tilde{r}_{\text{CSG}}} \\ &= (20\%)^2 \cdot 0.005585 + (30\%)^2 \cdot 0.006967 + (50\%)^2 \cdot 0.003956 \\ &\quad + 2 \cdot 20\% \cdot 30\% \cdot 0.003318 + 2 \cdot 30\% \cdot 50\% \cdot 0.000507 + 2 \cdot 20\% \cdot 50\% \cdot 0.000522 \\ &= 0.002496 \ . \end{aligned} \tag{14.42}$$

Therefore, the risk of portfolio P is

$$\mathcal{S}dv(\tilde{r}_P) = \sqrt{\mathcal{V}ar(\tilde{r}_P)} = \sqrt{0.002496} = 5.00\% \ . \tag{14.43}$$

Note that this standard deviation is lower than the standard deviation of each of the three stocks by themselves (Table 14.7). This is caused by **Diversification**, which is

explored in great detail in the next section.

Solve Now!

Q 14.7 *Compute the standard deviation (risk) of portfolio P from Table 14.6 to 4 digits. Is it the same as what we computed in the text?*

Q 14.8 *Compute the expected rate of return (reward) and standard deviation (risk) for another portfolio, called B, that invests 10.7% in the S&P500, 64.5% in IBM and 24.8% in Sony. Compute this both from the formulas and from a historical rate of return series of this portfolio.*

Q 14.9 *Compute the risk and reward for portfolio C:*

$$w_{S\&P500} = 0.5, \quad w_{IBM} = 0.5, \quad w_{Sony} = 0 \ . \tag{14.44}$$

Q 14.10 *Compute the risk and reward for portfolio D:*

$$w_{S\&P500} = 0.614, \quad w_{IBM} = 0.288, \quad w_{Sony} = 0.98 \ . \tag{14.45}$$

Q 14.11 *What is the risk and reward of a portfolio that invests 10% in PEP, 10% in KO, and 80% in CSG?*

Q 14.12 *What is the slope (beta) of the lines in Figure 14.4?*

14·6. Historical Statistics For Some Asset-Class Index Portfolios

This section describes the 1997-2002 investment performance of some Vanguard funds.

Enough with statistical torture! Let's look at the historical performances of some realistic investment portfolios. Table 14.8 describes historical rates of return from portfolios managed by **Vanguard** over the January 1997 to October 2002 period. (**Vanguard** is a prominent low-cost provider of **Index Funds**.) Each Vanguard fund purchases a large number of securities, often simply everything that qualifies in an asset category (e.g., all suitable bonds, all European stocks, all real estate investment trusts, etc.), and without much attempt of picking winners within each class. Naturally, although we really are interested in forward-looking statistics, standing here today, all we have are historical statistics. So, we describe the properties of historical rates of returns of these portfolios.

Side Note: Experience shows that historical means are not good predictors of future means, but historical standard deviations and betas are good predictors of their future equivalents. See the local Nerd Appendix a.

Bonds offered only low rates of returns.

The first statistic that this chapter described was the mean. The **Historical Mean** (also called **Sample Mean**) of the monthly rates of return describes how an investor would have fared on average. The second column in Table 14.8 shows that over the sample period, the short-term government bond fund earned a rate of return of about 60 basis points (*per year*). The intermediate government bond fund earned 100 basis points; municipal bonds earned 80 basis points; corporate junk bonds earned 180 basis points; corporate convertible bonds earned about 2.4%.

Part IV explains corporate bonds in more detail.

Equities offered higher rates of return.

Continuing on to pure equity (stock) investments, the 500 large stocks in Vanguard's S&P500 fund earned about 4.9% per year. The tax-managed version of the same investment strategy minimizes trading (to minimize capital gains). It did even better than the unmanaged version, earning a rate of return of 6.0% per year. **Value Firms** are large and unexciting companies. **Growth Firms** are small, fast-growing, exciting companies. Yet, following a long-standing historical trend, Value Firms earned higher rates of returns than Growth Firms. (During this sample period, Small Growth Firms did best, though.) Neither could outperform the S&P500 over the sample period. Among industries, **Healthcare Firms** earned the highest rates of returns, and **Utilities** firms earned the lowest rates of return. International investors fared especially poorly in this sample period: Japanese stocks in particular lost 7.4% *per annum* over the sample period.

Side Note: The Vanguard S&P500 index portfolio outperformed the S&P500 index itself (second to last line in the table). This is presumably [a] because the fund also received dividends, which are not counted in the S&P index itself; and [b] because the fund could lend out securities to short-seller and thereby earn some extra return. Incidentally, to minimize transaction costs, Vanguard does not perfectly mimick the index, which causes further so-called **tracking error**.

Equities were riskier than bonds.

The second set of statistics that this chapter described included the standard deviation. The third column in Table 14.8 shows that short-term and medium-term government bonds and municipal bonds were exceptionally safe. Their monthly rates of return varied only a little over the sample period. A large cluster of investment strategies had risks of about 15% to 25% per year, including the overall S&P500 stock market index investment strategy. The riskiest stock market investment strategy in the sample

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Table 14.8. Asset Class Portfolios, Based on Monthly Rates of Returns, Annualized, January 1997 to October 2002.

	Annualized Returns		Monthly Returns							Ann.Market-Model	
Asset	Mean	StdDev	%Neg	%Pos	Worst	Q2	Median	Q3	Best	α_i	$\beta_{i,\text{S\&P}}$
govbonds: short-term	0.6%	2.0%	43%	51%	−1.5%	−0.3%	0.1%	0.4%	1.2%	0.7%	−0.0
govbonds: intermediate	1.0%	4.4%	44%	53%	−3.2%	−0.7%	0.2%	0.8%	2.9%	1.2%	-0.0
bond: long-term munis	0.8%	4.5%	43%	54%	−3.1%	−1.1%	0.4%	0.9%	2.7%	1.0%	-0.0
bond: high-yield (junk) bonds	1.8%	16.2%	46%	54%	−15.0%	−2.3%	0.4%	2.9%	9.9%	−1.5%	0.7
bond: corporate convertibles	2.4%	15.4%	50%	49%	−12.8%	−2.7%	−0.2%	3.2%	10.6%	−0.5%	0.6
u.s. s&p500	4.9%	18.5%	46%	54%	−14.5%	−3.0%	0.7%	4.7%	9.5%	0.1%	1.0
u.s. tax managed	6.0%	21.3%	46%	54%	−17.6%	−3.4%	0.4%	5.2%	9.6%	0.6%	1.1
u.s. value firms	1.6%	18.1%	46%	53%	−16.1%	−3.2%	0.3%	3.8%	8.6%	−2.7%	0.9
u.s. growth firm	−2.1%	25.7%	49%	51%	−21.7%	−5.2%	0.8%	5.6%	15.2%	−8.2%	1.3
u.s. small growth firms	7.7%	25.5%	46%	54%	−19.3%	−4.6%	0.3%	6.3%	20.1%	3.1%	1.0
u.s. small-cap firms	2.7%	22.6%	49%	51%	−19.3%	−4.3%	0.3%	5.2%	16.5%	−1.3%	0.8
u.s. energy firms	5.2%	23.7%	53%	47%	−18.0%	−4.1%	−0.5%	2.8%	19.0%	2.3%	0.6
u.s. gold and metals	3.6%	35.1%	51%	47%	−19.9%	−6.1%	−0.8%	6.0%	39.3%	1.1%	0.5
u.s. healthcare firms	13.6%	14.0%	36%	64%	−10.5%	−1.0%	1.8%	3.3%	10.3%	11.3%	0.5
u.s. real estate inv. trusts	−0.4%	12.6%	50%	49%	−9.2%	−2.2%	−0.2%	2.0%	9.6%	−1.2%	0.2
u.s. utilities firms	−2.3%	13.9%	57%	43%	−11.9%	−2.5%	−0.7%	2.4%	7.7%	−4.2%	0.4
intl emerging market firms	−4.4%	28.0%	50%	50%	−26.1%	−6.2%	−0.4%	4.5%	15.4%	−9.7%	1.1
intl european firms	1.9%	17.6%	41%	56%	−13.1%	−3.4%	0.3%	3.2%	9.7%	−1.7%	0.7
intl pacific firms	−7.4%	21.3%	59%	40%	−12.1%	−5.3%	−2.0%	4.0%	17.6%	−10.6%	0.7
intl growth firms	−1.6%	17.5%	46%	53%	−13.2%	−3.2%	0.5%	3.1%	9.6%	−5.1%	0.7
intl value firms	−4.2%	18.7%	57%	43%	−15.0%	−3.1%	−0.7%	2.9%	13.0%	−7.5%	0.7
s&p 500 index	4.8%	18.5%	46%	54%	−14.6%	−3.1%	0.5%	5.0%	9.7%	0.0%	1.0
dow jones 30 index	6.3%	18.7%	43%	57%	−15.1%	−3.6%	0.9%	4.0%	10.6%	1.8%	0.9

period would have been U.S. Gold and Metals, the safest would have been **Real Estate Investment Trust** (**REITs**) and **Utilities** firms.

The third set of statistics that this chapter described was covariation, which included beta. The last column in the Table 14.8 shows the beta of the rate of return of each investment portfolio with the rate of return on the S&P500 index,

An important statistic, used later again, is the covariation of investments with the stock market.

$$\beta_{P,\text{S\&P500}} = \frac{Cov(\tilde{r}_P, \tilde{r}_{\text{S\&P500}})}{Var(\tilde{r}_{\text{S\&P500}})} \quad . \tag{14.46}$$

These so-called market-betas tell us how much a particular investment portfolio's rate of return covaried with the rate of return on the S&P500. A beta of 1 tells us that the rate of return of a portfolio tended to covary one-to-one with the rate of return in the U.S. stock market. A beta of 0 tells us that the rate of return of a portfolio tended to be unrelated to what happened to the U.S. stock market.

SIDE NOTE: The linear regression by which the beta measure can be obtained is so common, it is called the **Market Model**, and this beta is called the **Market Beta** of a portfolio. It can be obtained from the time-series regression

$$\tilde{r}_P = \alpha + \beta \cdot \tilde{r}_{\text{S\&P500}} \quad . \tag{14.47}$$

Again, we use a historical beta as estimate for the future beta.

The last column shows that government bonds had practically no covariation with the S&P500. Corporate bonds, energy stocks, precious metals, healthcare stocks, real estate investment trusts (REITs) and utilities had very mild covariation, indicated by betas around 0.5. The next-most correlated segment are international stocks, having betas of around 0.7. But many other portfolios varied about 1-to-1 with the overall stock market. Note that U.S. growth firms swung even more than 1-to-1 with changes in the stock market: The beta of 1.3 tells us that a 10% increase in the stock market tended to be associated with a 13% increase in the growth firm portfolio. This is typical.

Equities covaried more with the s&p than bonds.

SIDE NOTE: For curiosity, Table 14.8 also shows some other statistics, such as the percent of all months which earned a positive rate of return. (Naturally, one minus this percent are the months in which the portfolio had a negative return.) The Table further shows the single worst month, the single best month, the median month (half of all return months were better, half were worse). Finally, although this is beyond what we have covered so far, the alpha (α) in the Table is sometimes interpreted as a risk-adjusted reward measure, the higher the better.

Some other random observations: The table shows no systematic relationship between risk and rate of return over the sample period. However, it is the case that the least risky and least covarying investment strategies (government bonds) provided a very modest, but positive average rate of return. With hindsight, it would have been terrific to invest in U.S. healthcare stocks: they had the most spectacular return, plus a very modest risk. Naturally, with hindsight, you could have selected the right six numbers for the lottery. So, which numbers can we trust to be indicative of their future equivalent? First and foremost, covariation measures. They tend to be very stable. Next, standard deviations are reasonable stable. Means, however, are very untrustworthy: it is not especially likely that Healthcare firms will continue to outperform other stocks, and that Japanese firms will continue to underperform other stocks.

Solve Now!

All questions refer to Table 14.8.

Q 14.13 *Which investment class portfolio would have done best over the sample period? Do you believe this will continue?*

Q 14.14 *Which investment class portfolio would have done worst over the sample period? Do you believe this will continue?*

Q 14.15 *Assuming an investor held only one asset class, which investment class portfolio was safest during the sample period? Do you believe this will continue?*

Q 14.16 *Assuming an investor held only one asset class, which investment class portfolio was riskiest during the sample period? Do you believe this will continue?*

Q 14.17 *Which asset class portfolio had the lowest covariation with the S&P500 index? Do you believe this will continue?*

Q 14.18 *Which asset class portfolio had the highest covariation with the S&P500 index? Do you believe this will continue?*

A. Appendix: More Historical Statistics

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a. Country Fund Rates of Return

Table 14.9. AMEX Country Funds, Based on Monthly Rates of Returns, Annualized, January 1997 to October 2002.

	Annualized Returns		Monthly Returns							Ann.Market-Model	
Asset	Mean	StdDev	%Neg	%Pos	Worst	Q2	Median	Q3	Best	α_i	$\beta_{i,\text{S\&P}}$
australia index fund amex	−0.4%	23.1%	47%	49%	−17.5%	−4.2%	0.0%	3.4%	14.5%	−4.1%	0.8
canada index fund amex	5.0%	23.5%	40%	57%	−22.4%	−4.0%	0.7%	5.5%	11.2%	−0.2%	1.1
sweden index fund amex	1.2%	30.3%	46%	53%	−21.2%	−4.9%	0.6%	5.3%	21.7%	−4.4%	1.2
germany index fund amex	3.9%	28.6%	46%	53%	−24.0%	−4.6%	0.9%	5.1%	24.4%	−1.2%	1.1
hong kong index fund amex	−5.7%	36.0%	56%	41%	−28.1%	−7.4%	−1.3%	5.7%	38.4%	−11.7%	1.2
japan index fund amex	−8.0%	24.2%	59%	41%	−16.5%	−5.9%	−1.2%	3.7%	21.1%	−11.4%	0.7
belgium index fund amex	5.9%	26.6%	44%	56%	−19.0%	−2.7%	0.4%	4.0%	37.9%	2.3%	0.8
netherlands index fund amex	0.2%	22.2%	46%	53%	−17.3%	−3.3%	0.3%	3.6%	14.4%	−3.9%	0.9
austria index fund amex	1.2%	21.0%	47%	49%	−19.7%	−4.2%	0.0%	4.4%	11.9%	−1.0%	0.5
spain index fund amex	6.5%	25.0%	44%	53%	−22.6%	−4.1%	0.3%	4.8%	15.5%	2.0%	0.9
france index fund amex	6.2%	23.5%	43%	57%	−15.0%	−2.5%	0.8%	3.9%	16.8%	2.0%	0.9
singapore index fund amex	−7.5%	37.7%	49%	46%	−27.0%	−6.5%	0.0%	3.6%	40.3%	−13.8%	1.3
uk index fund amex	0.2%	15.4%	43%	54%	−11.9%	−3.2%	0.6%	3.1%	8.6%	−3.1%	0.7
mexico index fund amex	11.3%	36.1%	44%	53%	−35.3%	−7.0%	3.0%	9.0%	20.6%	4.7%	1.4
s&p 500 index	4.8%	18.5%	46%	54%	−14.6%	−3.1%	0.5%	5.0%	9.7%	0.0%	1.0

b. Dow-Jones Constituents

Table 14.10. Dow Jones Constituents, Based on Monthly Rates of Returns, Annualized, January 1997 to October 2002.

	Annualized Returns				Monthly Returns					Ann.Market-Model	
Asset	Mean	StdDev	%Neg	%Pos	Worst	Q2	Median	Q3	Best	α_i	$\beta_{i,\text{S\&P}}$
alcoa	15.1%	41.3%	49%	51%	−23.9%	−6.2%	0.4%	8.2%	51.1%	8.8%	1.3
american express	17.3%	31.4%	36%	63%	−29.3%	−3.0%	1.7%	8.4%	16.9%	11.3%	1.2
boeing	−2.9%	34.6%	46%	53%	−34.6%	−7.1%	0.6%	6.6%	20.2%	−6.0%	0.7
citigroup	24.9%	37.4%	43%	57%	−34.0%	−5.8%	2.4%	9.0%	25.8%	17.6%	1.5
caterpillar	9.6%	34.0%	46%	54%	−17.4%	−5.9%	0.8%	6.7%	40.8%	5.7%	0.8
du pont	4.4%	29.0%	50%	50%	−17.0%	−6.1%	−0.2%	6.5%	21.7%	0.6%	0.8
disney	0.3%	32.2%	46%	54%	−26.8%	−6.4%	0.4%	5.0%	24.2%	−4.0%	0.9
eastman kodak	−5.5%	34.6%	53%	47%	−34.4%	−6.6%	−0.5%	5.5%	24.2%	−8.4%	0.6
general electric	12.8%	28.4%	50%	49%	−17.7%	−4.1%	−0.2%	5.9%	19.2%	7.6%	1.1
general motors	5.1%	36.7%	50%	49%	−24.1%	−5.0%	−0.5%	5.8%	25.4%	0.1%	1.0
home depot	22.8%	35.1%	41%	59%	−20.6%	−4.1%	2.8%	8.8%	30.2%	16.8%	1.2
honeywell	6.0%	44.5%	44%	56%	−38.4%	−4.1%	0.7%	6.5%	51.0%	0.1%	1.2
hewlett packard	4.0%	48.2%	51%	49%	−32.0%	−9.1%	−1.4%	8.7%	35.4%	−3.1%	1.5
ibm	20.2%	39.6%	50%	50%	−22.6%	−5.8%	−0.6%	7.8%	35.4%	13.2%	1.5
intel	15.8%	53.1%	46%	54%	−44.5%	−9.6%	0.8%	11.4%	33.9%	7.5%	1.7
international paper	6.0%	36.0%	47%	53%	−22.3%	−6.3%	0.5%	5.3%	27.0%	1.6%	0.9
johnson and johnson	19.5%	26.4%	43%	57%	−16.0%	−2.7%	1.5%	6.2%	17.4%	17.0%	0.5
jp morgan	5.9%	42.5%	46%	54%	−30.6%	−6.3%	0.7%	8.2%	32.9%	−1.9%	1.6
coca-cola	3.7%	30.7%	49%	51%	−19.1%	−4.7%	0.1%	6.7%	22.3%	0.9%	0.6
mcdonalds	0.6%	28.8%	44%	56%	−25.7%	−5.7%	1.0%	5.9%	17.7%	−3.1%	0.8
3m	12.8%	26.0%	44%	56%	−15.8%	−3.9%	1.1%	6.0%	25.8%	10.4%	0.5
philip morris	6.9%	33.0%	41%	57%	−23.7%	−4.6%	0.8%	5.3%	24.4%	5.1%	0.4
merck	12.1%	32.0%	49%	51%	−21.7%	−5.6%	0.1%	6.5%	22.8%	9.3%	0.6
microsoft	28.4%	50.0%	47%	51%	−34.4%	−8.7%	0.1%	11.9%	40.8%	20.0%	1.7
proctor and gamble	13.7%	28.4%	41%	59%	−35.4%	−1.8%	1.4%	5.3%	24.7%	12.6%	0.2
sbc communications	8.1%	34.4%	50%	50%	−18.8%	−7.0%	−0.2%	5.7%	29.3%	4.5%	0.8
att	−5.7%	40.3%	54%	46%	−23.8%	−8.5%	−2.6%	6.1%	39.1%	−10.3%	1.0
united technologies	18.1%	34.1%	40%	60%	−32.0%	−3.4%	1.9%	7.0%	24.6%	12.6%	1.1
walmart	31.5%	31.1%	33%	66%	−20.8%	−3.2%	3.0%	8.2%	26.4%	27.2%	0.9
exxon	7.1%	18.3%	50%	50%	−10.2%	−3.5%	−0.1%	4.7%	17.7%	4.9%	0.5
dow jones 30 index	6.3%	18.7%	43%	57%	−15.1%	−3.6%	0.9%	4.0%	10.6%	1.8%	0.9
s&p 500 index	4.8%	18.5%	46%	54%	−14.6%	−3.1%	0.5%	5.0%	9.7%	0.0%	1.0

B. Appendix: Summary

The chapter covered the following major points:

- We often use historical rate of return statistics to predict future rate of return statistics. This is a leap of faith—often, but not always correct.
- The chapter gave the formulas to compute
  - means, variances, and standard deviations.
  - covariances, correlations, and betas.
  - risk and reward of portfolios.
- The chapter gave statistics on the historical performance of some typical asset-class portfolios and stocks.

C. Nerd Appendix: More Statistical Theory

a. Historical and Future Statistics

The theory usually assumes that although you do not know the *outcome* of a random variable, you do know the statistics (such as mean and standard deviation) for the outcomes. That is, you can estimate or judge a random variable's unknown mean, standard deviation, covariance (beta), etc. Alas, while this is easy for the throw of a coin or a die, where we know the physical properties of what determines the random outcome, this is not so easy for stock returns. For example, what is the standard deviation of next month's rate of return on PepsiCo?

Physical processes often have known properties. Stock returns do not.

We just do not have a better alternative than to assume that PepsiCo's returns are manifestations of the same statistical process over time. So, if we want to know the standard deviation of PepsiCo's next month's rate of return, we assume that each historical monthly rate of return—at least over the last couple of years—was a draw from the same distribution. Therefore, we can use the historical rates of return, assuming each one was an equally likely outcome, to estimate the future standard deviation. The mechanics of the computation for obtaining the estimated future standard deviation are then exactly the same as those we used to obtain an actual standard deviation, where we know the outcome.

Use historical statistics as estimators of future statistics.

But, using historical statistics and then arguing that they are representative of the future is a bit of a leap. Empirical observation has taught us that doing so works well for standard deviations and covariation measures: that is, the historical statistics obtained from monthly rates of returns over the last 3 to 5 years appear to be fairly decent predictors of future betas and standard deviations. Unfortunately, the historical mean rates of returns are fairly unreliable predictors of the future rates of returns.

This works well for standard deviations and covariation statistics, but not for means.

Q 14.19 *When predicting the outcome of a die, why do we not use historical statistics on die throws as predictors of future die throws?*

Q 14.20 *Are historical financial securities' mean rates of returns good predictors of future mean rates of returns?*

Q 14.21 *Are historical financial securities' standard deviations and correlations of rates of returns good predictors of their future equivalents?*

b. Improving Future Estimates From Historical Estimates

The main remaining problem in the reliability of historical estimates of covariances is what statisticians call "regression to the mean." That is, the most extreme historical estimates are likely caused not only by the underlying true estimates, but even more so by chance. For example, if all securities had a true standard deviation of 30% per annum, over a particular year, some might show a standard deviation of 40%, others might show a standard deviation of 20%. Those with the high 40% historical standard deviations are most likely to have lower than their historical standard deviations (dropping back to 30%). Those with the low 20% historical standard deviations are most likely to have higher than their historical standard deviations (increasing back to 30%).

Extreme outcomes have more of two components: higher expected outcome and a higher error term (which will not repeat).

Shrinkage just reduces the estimate, hoping to adjust for extremes' errors.

Statisticians handle such problems with a technique called "shrinkage." The historical estimates are reduced ("shrunk") towards a more common mean. Naturally, the exact amount by which historical estimates should be shrunk and what number they should be shrunk towards is a very complex technical problem—and doing it well can make millions of dollars. This book is definitely not able to cover this subject appropriately.

Advice: take the average of the market historical statistic and your individual stock historical statistic. It probably predicts better.

Still, a student reading this book might wonder if there is something reasonable that can be done to obtain better estimates of future mean returns, better estimates of future standard deviations, and better estimates of future betas. So, here is a two minute non-formal heuristic estimation job: To predict a portfolio statistic, average the historical statistic on your particular portfolio with the historical statistic on the overall stock market. This may not work wonders, but it is likely to work better than the historical statistic of the particular portfolio by itself. With more time and statistical expertise, you could use other information, such as beta, the industry historical rate of return, or the average P/E ratio of the portfolio, to produce even better guestimates of future portfolio behavior.

An example of shrinking.

For example, look at Table 14.10 on Page 319. If we were asked to guestimate an expected annual rate of return for Walmart over the next year, we would not quote Walmarts historical 31.5% as our estimate of Walmart's future rate of return. Instead, we could quote an average of 31.5% and 6.3% (the historical rate of return on the market from 1997 to 2002), or about 20% per annum. (This assumes that I am not permitted to use more sophisticated models, such as the CAPM.) We would also guestimate Walmart's risk to be the average of 31.1% and 18.7%, or about 25% per year. Finally, we would guestimate Walmart's beta to be about 0.95. The specific market index to which we shrink matters little (the Dow-Jones 30 or the S&P500)—but it does matter that we do shrink! An even better target to shrink towards are the industry average statistics. (Some researchers go as far as to just estimate industry betas, and forego computing the individual firm beta altogether! This is like shrinking to an extreme degree.) However, good shrinking targets are far beyond the scope of this book. Would you like to bet that the historical statistics are better guestimates than the shrunk statistics? (If so, feel free to invest your money into Walmart and expect 31.5%!)

SIDE NOTE: It may be more than coincidental that the word "shrink" has a second meaning.

What works reasonably well, what does not.

Here is a summary of some recommendations. Based on regressions using 5 years of historical monthly data, to predict one year ahead statistics, reasonable shrinkage methods for reasonably large stocks (members of the S&P500 stocks), statistics for rates of return can be obtained as follows:

Mean	Nothing works well.
Market-Model Alpha	Nothing works well.
Market-Model Beta	Average the historical beta with the number "1."
Standard Deviation	Average the historical standard deviation of the stock and the historical standard deviation of the S&P500. Then increase by 30%, because, historically, for unknown reasons, volatility has been increasing.

The market model is the linear regression in which the x variable is the rate of return on the S&P500, and the y variable is the rate of return on the stock in which we are

interested.

c. The Portfolio Variance Formula

This section requires more knowledge of the algebra of expectations than we have covered, so you may not be able to follow the manipulations. It is also not necessary to follow this derivation: it is here only for completeness, and to reiterate how many more covariance terms than variance terms are in the portfolio variance formula.

Definitely Not Necessary!

The covariance can be defined as

Here comes algebra!

$$Cov(\tilde{x}, \tilde{y}) \equiv \mathcal{E}\{[\tilde{x} - \mathcal{E}(\tilde{x})] \cdot [\tilde{y} - \mathcal{E}(\tilde{y})]\} \quad . \tag{14.48}$$

Therefore,

$$\mathcal{V}ar(\tilde{r}_P) \equiv \mathcal{E}\{[\tilde{r}_P - \mathcal{E}(\tilde{r}_P)]^2\} \quad . \tag{14.49}$$

We can write out the return on the portfolio, $\tilde{r}_P$,

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_P) &= \mathcal{E}\{[\sum_{i=1}^{N} w_i \cdot \tilde{r}_i - \mathcal{E}(\sum_{i=1}^{N} w_i \cdot \tilde{r}_i)]^2\} = \mathcal{E}\{[\sum_{i=1}^{N} w_i \cdot \tilde{r}_i - \sum_{i=1}^{N} w_i \mathcal{E}(\tilde{r}_i)]^2\} \\ &= \mathcal{E}\{[\sum_{i=1}^{N} w_i \cdot [\tilde{r}_i - \mathcal{E}(\tilde{r}_i)]]^2\} = \mathcal{E}\{[w_1 \cdot \tilde{r}_1 + w_2 \cdot \tilde{r}_2 + \ldots w_N \cdot \tilde{r}_N]^2\} \quad . \end{aligned} \tag{14.50}$$

where lowercase r_i are the demeaned returns.

replace each return with its demeaned returns

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_P) &= \mathcal{E}\{[w_1 \cdot \tilde{r}_1 + w_2 \cdot \tilde{r}_2 + \ldots w_N \cdot \tilde{r}_N]^2\} \\ &= \mathcal{E}\{[w_1 \cdot \tilde{r}_1 + w_2 \cdot \tilde{r}_2 + \ldots w_N \cdot \tilde{r}_N] \cdot [w_1 \cdot \tilde{r}_1 + w_2 \cdot \tilde{r}_2 + \ldots w_N \cdot \tilde{r}_N]\} \end{aligned} \tag{14.51}$$

start multiplying in each term from the left sum

$$\begin{aligned} &= \mathcal{E}\{w_1 \cdot \tilde{r}_1 \cdot [w_1 \cdot \tilde{r}_1 + w_2 \cdot \tilde{r}_2 + \ldots w_N \cdot \tilde{r}_N]\} \\ &+ \mathcal{E}\{w_2 \cdot \tilde{r}_2 \cdot [w_1 \cdot \tilde{r}_1 + w_2 \cdot \tilde{r}_2 + \ldots w_N \cdot \tilde{r}_N]\} \\ &+ \ldots \\ &+ \mathcal{E}\{w_N \cdot \tilde{r}_N \cdot [w_1 \cdot \tilde{r}_1 + w_2 \cdot \tilde{r}_2 + \ldots w_N \cdot \tilde{r}_N]\} \end{aligned} \tag{14.52}$$

further multiply out

$$\begin{aligned} &= \mathcal{E}\{w_1 \cdot w_1 \cdot \tilde{r}_1 \cdot \tilde{r}_1 + w_1 \cdot w_2 \cdot \tilde{r}_1 \cdot \tilde{r}_2 + \ldots + w_1 \cdot w_N \cdot \tilde{r}_1 \cdot \tilde{r}_N]\} \\ &+ \mathcal{E}\{w_2 \cdot w_1 \cdot \tilde{r}_2 \cdot \tilde{r}_1 + w_2 \cdot w_2 \cdot \tilde{r}_2 \cdot \tilde{r}_2 + \ldots + w_2 \cdot w_N \cdot \tilde{r}_2 \cdot \tilde{r}_N]\} \\ &+ \ldots \\ &+ \mathcal{E}\{w_N \cdot w_1 \cdot \tilde{r}_n \cdot \tilde{r}_1 + w_N \cdot w_2 \cdot \tilde{r}_N \cdot \tilde{r}_2 + \ldots + w_N \cdot w_N \cdot \tilde{r}_N \cdot \tilde{r}_N]\} \end{aligned} \tag{14.53}$$

distribute the expectation operator

$$
\begin{aligned}
&= \mathcal{E}\{w_1 \cdot w_1 \cdot \tilde{r}_1 \tilde{r}_1\} + \mathcal{E}\{w_1 \cdot w_2 \cdot \tilde{r}_1 \cdot \tilde{r}_2\} + \ldots + \mathcal{E}\{w_1 \cdot w_N \cdot \tilde{r}_1 \cdot \tilde{r}_N]\} \\
&+ \mathcal{E}\{w_2 \cdot w_1 \cdot \tilde{r}_2 \cdot \tilde{r}_1\} + \mathcal{E}\{w_2 \cdot w_2 \cdot \tilde{r}_2 \cdot \tilde{r}_2\} + \ldots + \mathcal{E}\{w_2 \cdot w_N \cdot \tilde{r}_2 \cdot \tilde{r}_N]\} \\
&+ \qquad \ldots \\
&+ \mathcal{E}\{w_N \cdot w_1 \cdot \tilde{r}_N \tilde{r}_1\} + \mathcal{E}\{w_N \cdot w_2 \cdot \tilde{r}_N \cdot \tilde{r}_2\} + \ldots + \mathcal{E}\{w_N \cdot w_N \cdot \tilde{r}_N \cdot \tilde{r}_N]\}
\end{aligned}
\tag{14.54}
$$

now, recognize that each expectation term is just the covariance

$$
\begin{aligned}
&= Cov(w_1 \cdot \tilde{r}_1, w_1 \cdot \tilde{r}_1) + Cov\{w_1 \cdot \tilde{r}_1, w_2 \cdot \tilde{r}_2\} + \ldots + Cov\{w_1 \cdot \tilde{r}_1, w_N \cdot \tilde{r}_N\} \\
&+ Cov(w_2 \cdot \tilde{r}_2, w_1 \cdot \tilde{r}_1) + Cov\{w_2 \cdot \tilde{r}_2, w_2 \cdot \tilde{r}_2\} + \ldots + Cov\{w_2 \cdot \tilde{r}_2, w_N \cdot \tilde{r}_N\} \\
&+ \qquad \ldots \\
&+ Cov(w_N \cdot \tilde{r}_N, w_1 \cdot \tilde{r}_1) + Cov\{w_N \cdot \tilde{r}_N, w_2 \cdot \tilde{r}_2\} + \ldots + Cov\{w_N \cdot \tilde{r}_N, w_N \cdot \tilde{r}_N\} \quad .
\end{aligned}
\tag{14.55}
$$

and simplify covariances

$$
\begin{aligned}
&= w_1 \cdot w_1 \cdot Cov(\tilde{r}_1, \tilde{r}_1) + w_1 \cdot w_2 \cdot Cov\{\tilde{r}_1, \tilde{r}_2\} + \ldots + w_1 \cdot w_N \cdot Cov\{\tilde{r}_1, \tilde{r}_N\} \\
&+ w_2 \cdot w_1 \cdot Cov(\tilde{r}_2, \tilde{r}_1) + w_2 \cdot w_2 \cdot Cov\{\tilde{r}_2, \tilde{r}_2\} + \ldots + w_2 \cdot w_N \cdot Cov\{\tilde{r}_2, \tilde{r}_N\} \\
&\qquad \ldots \\
&+ w_N \cdot w_1 \cdot Cov(\tilde{r}_N, \tilde{r}_1) + w_N \cdot w_2 \cdot Cov\{\tilde{r}_N, \tilde{r}_2\} + \ldots + w_N \cdot w_N \cdot Cov\{\tilde{r}_N, \tilde{r}_N\} \quad .
\end{aligned}
\tag{14.56}
$$

The covariance terms where the return indices are identical are the N variance terms. Because $Cov(x, y) = Cov(y, x)$, there are two covariance terms for each mutual combination of securities. If you think of this sum as being arranged on a square grid, the N variance terms are on the diagonal, the $N \cdot N - 1$ off-diagonal terms are the mutual covariance terms.

d. Translating Mean and Variance Statistics Into Probabilities

Translating standard deviations into more intuitive risk assessments

Although we now know enough to compute a measure of risk, we have not bothered to explore how likely outcomes are. For example, if a portfolio's expected rate of return is 12.6% per year, and its standard deviation is 22% per year, what is the probability that we will lose money (earn below 0%)? What is the probability that we will earn 15% or more? 20% or more?

We assume a normal (bell-shaped) distribution.

It turns out that if the underlying distribution looks like a bell curve—and many common portfolio return distributions have this shape—there is an easy procedure to translate mean and standard deviation into the probability that the return will end being less than x. In fact, this probability correspondence is the only advantage that bell shaped distributions provide! Everything else works regardless of the actual shape of the distribution.

For concreteness sake, assume we want to determine the probability that the rate of return on this portfolio is less than +5%:

An example: Z-score and probability.

Step 1 Subtract the mean from 5%. In our case, with the expected rate of return of 12.6%, the result is 5% − 12.6% = −7.6%.

Step 2 Divide this number by the standard deviation. In our case, this is −7.6% divided by 22%, which comes to be −0.35. This number is called the **Score** or **Z-score**.

Step 3 Look up the probability for this Score in the **Cumulative Normal Distribution Table** in Table B.1 (Page 638). For the score of −0.35, this probability is about 0.36.

In sum, we have determined that if returns are drawn from a distribution with a mean of 12.6% and a standard deviation of 22%, then the probability of observing a single rate of return of +5% or less is about 36%. It also follows that the probability that the return will be greater than +5% must be 100% − 36% = 64%.

SIDE NOTE: In the real world, this works well enough—but not perfectly. So, do not get fooled by theoretical pseudo-accuracy. Anything between 30% and 40% is a reasonable prediction here.

How well does the Z-score fit in our application?Now recall portfolio P in Table 14.6. P had a mean of 12.6% and a standard deviation of 22%. We have just computed that about one third of the 12 annual portfolio returns should be below +5%. 1991, 1993, 1994, 1995, 1996, 1997, 1998, and 1999 performed better than +5%; 1992, 2000, 2001, and 2002 performed worse. So, as predicted by applying the normal distribution table, about one third of our annual returns returned 5% or less.

How likely is it that you will lose money?A common question is "what is the probability that the return will be negative?" Use the same technique,

Step 1 Subtracting the mean of 0% yields 0.0% − 12.6% = −12.6%.

Step 2 Dividing −12.6% by the standard deviation of 22% yields the score of −0.57.

Step 3 For this score of −0.57, the probability is about 28%.

In words, the probability that the rate of return will be negative is around 25% to 30%. And, therefore, the probability that the return will be positive is around 70% to 75%. The table shows that 4 out of the 12 annual rates of returns are negative. This is most likely **sampling error**: with only 12 annual rates of returns, it was impossible for the distribution of data to accurately follow a bell shape.

NERD NOTE: *Many portfolio returns have what is called "fat fails." This means that the probability of extreme outcomes—especially extreme negative outcomes—is often higher than suggested by the normal distribution table. For example, if the mean return were 30% (e.g. for a multi-year return) and the standard deviation were 10%, the score for a value of* 0 *is* −3*. The table therefore suggests that the probability of drawing a negative return should be 0.135%, or about once in a thousand periods. Long experience with financial data suggests that this is often much too over-confident for the real-world. In some contexts, the true probability of even the most negative possible outcome (*−100%*) may be as high as 1%, even if the Z-score suggests 0.0001%!*

ANECDOTE: Long Term Capital Management
Long-Term Capital Management (LTCM), a prominent hedge fund run by top finance professors and Wall Street traders, collapsed in 1999 in what their quantitative models believed to be a "10 sigma" (i.e., a 10 standard deviation) event. According to the normal distribution probability table, such an event that has a score of -10 should occur with a probability of less than 0.0001%, or 1 in 1,000,000 periods of trading.

We can conclude that either their models were wildly over-optimistic, or their assumption of a normal distribution was incorrect, or the 1 in 1,000,000 actually occurred. Chances are that it was a little bit of all three.

(In essence, LTCM's model believed that it would be exceedingly unlikely that all their bets would go sour at the same time. Of course, they did all go sour together, so the LTCM principals lost most of their wealth.)

e. Correlation and Causation

Correlation does not imply Causation.

A warning: covariation is related to, but not the same as **Causation**. If one variable "causes" another, then the two variables will be correlated. But the opposite does not hold. For example, snow and depression are positively correlated, but neither causes the other. Instead, there is another variable (winter) that has an influence on both snow and depression.

Solve Now!

Q 14.22 *If the mean is 20 and the standard deviation is 15, what is the probability that the value will turn out to be less than 0?*

Q 14.23 *If the mean is 10 and the standard deviation is 20, what is the probability that the value will turn out to be positive?*

Q 14.24 *If the mean is 50 and the standard deviation is 20, what is the probability that the value will turn out to be greater than 80?*

Q 14.25 *If the mean is 50 and the standard deviation is 20, what is the probability that the value will turn out to be greater than 30?*

A. Q&A: Answers

1. Do it!
2. The mean was 9.837%. The variance was 7.7%. standard deviation was 27.74%.
3. $$Correlation(\tilde{x},\tilde{x}) = \frac{Cov(\tilde{X},\tilde{X})}{Sdv(\tilde{X})\cdot Sdv(\tilde{X})} = \frac{Var(\tilde{X})}{Var(\tilde{X})} = 1 \ . \tag{14.57}$$
4. $$\beta_{\tilde{x},\tilde{x}} = \frac{Cov(\tilde{X},\tilde{X})}{Var(\tilde{X})} = \frac{Var(\tilde{X})}{Var(\tilde{X})} = 1 \ . \tag{14.58}$$
5. $$\begin{aligned} Cov(\tilde{r}_{\text{DAX}},\tilde{r}_{\text{S\&P500}}) &= 0.0394 \ , \\ Cov(\tilde{r}_{\text{DAX}},\tilde{r}_{\text{IBM}}) &= 0.0464 \ , \\ Cov(\tilde{r}_{\text{DAX}},\tilde{r}_{\text{Sony}}) &= 0.1264 \ . \end{aligned} \tag{14.59}$$

   We have already computed the standard deviation of S&P500, IBM, and Sony as 19.0%, 38.8%, and 90.3%; and for the DAX, as 27.74%. Therefore,

   $$\begin{aligned} Correlation(\tilde{r}_{\text{DAX}},\tilde{r}_{\text{S\&P500}}) &= 74.6\% \ , \\ Correlation(\tilde{r}_{\text{DAX}},\tilde{r}_{\text{IBM}}) &= 43.1\% \ , \\ Correlation(\tilde{r}_{\text{DAX}},\tilde{r}_{\text{Sony}}) &= 50.5\% \ . \end{aligned} \tag{14.60}$$

   The beta of the DAX with respect to the S&P500 is

   $$\begin{aligned} \beta_{\tilde{r}_{\text{DAX}},\tilde{r}_{\text{S\&P500}}} &= \frac{0.0394}{0.0362} = 1.088 \ . \\ \beta_{\tilde{r}_{\text{DAX}},\tilde{r}_{\text{IBM}}} &= \frac{0.0464}{0.388^2} = 0.308 \ . \\ \beta_{\tilde{r}_{\text{DAX}},\tilde{r}_{\text{Sony}}} &= \frac{0.1264}{0.903^2} = 0.155 \ . \end{aligned} \tag{14.61}$$
6. This is to confirm the supernerdnote on Page 302.
7. The variance is 0.0487, the standard deviation is 0.2207. (Remember to divide by $N-1$, not by N.)
8. $$\begin{aligned} \mathcal{E}(\tilde{r}_B) &= 17.00\% \\ Sdv(\tilde{r}_B) &= 35.65\% \ . \end{aligned} \tag{14.62}$$

9.
$$\begin{aligned} \mathcal{E}(\tilde{r}_C) &= 12.74\% \\ \mathcal{S}dv(\tilde{r}_C) &= 25.12\% \ . \end{aligned} \tag{14.63}$$

10.
$$\begin{aligned} \mathcal{E}(\tilde{r}_C) &= 13.00\% \\ \mathcal{S}dv(\tilde{r}_C) &= 22.90\% \ . \end{aligned} \tag{14.64}$$

11.
$$\mathcal{E}(\tilde{r}_P) = 10\% \cdot 0.83\% + 10\% \cdot 0.90\% + 80\% \cdot 1.19\% = 1.125\% \tag{14.65}$$

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_P) &= 10\% \cdot (7.47\%)^2 + 10\% \cdot (8.35\%)^2 + 80\% \cdot (6.29\%)^2 \\ &\quad +2\cdot 10\%\cdot 10\%\cdot 53.2\% + 2\cdot 10\%\cdot 80\%\cdot 10.8\% + 2\cdot 10\%\cdot 80\%\cdot 9.9\% \\ &= 0.0004858 \quad , \\ \rightarrow \ \mathcal{S}dv(\tilde{r}_P) &= 6.97\% \quad . \end{aligned} \tag{14.66}$$

12. We want the slope of a line where KO is the X variable. Therefore, the slope is

$$\begin{aligned} \beta_{\text{PEP,KO}} &= \frac{\mathcal{C}ov(\tilde{r}_{\text{PEP}}, \tilde{r}_{\text{KO}})}{\mathcal{V}ar(\tilde{r}_{\text{KO}})} = \frac{0.003318}{0.006967} \approx 0.48 \ . \\ \beta_{\text{PEP,CSG}} &= \frac{\mathcal{C}ov(\tilde{r}_{\text{PEP}}, \tilde{r}_{\text{CSG}})}{\mathcal{V}ar(\tilde{r}_{\text{CSG}})} = \frac{0.0005}{0.004} \approx 0.13 \ . \end{aligned} \tag{14.67}$$

13. Healthcare Firms. Unlikely: We know that historical expected rates of returns are not reliable predictors of future expected rates of returns.
14. Pacific (Japanese) Firms. Unlikely: We know that historical expected rates of returns are not reliable predictors of future expected rates of returns.
15. Government Bonds, short term. Likely: Historical standard deviations tend to be good predictors of future standard deviations.
16. Gold and Metals, then Emerging Stock Market Investments. (These are stock markets from developing countries.) Likely: Historical standard deviations tend to be good predictors of future standard deviations.
17. Look at the final column, $\beta_{i,\text{S\&P500}}$. Government bonds of all kinds had almost no correlation with the S&P500. Among more risky securities, REITs were almost uncorrelated, too. Likely: historical covariations tend to be good predictors of future covariations.
18. Growth firms. These contain many technology firms. Likely: historical covariations tend to be good predictors of future covariations.

19. Because we know the true distribution of future die throws. The historical values are measured with errors.
20. No.
21. Yes, reasonably so.
22. The score is $(0-20)/15 = -1.3$. Therefore, the probability is 9.68%, i.e., roughly 10%.
23. The score is $(0-10)/20 = -0.5$. Therefore, the probability is around 30% that the value will be negative, or 70% that it will be positive.
24. The score is $(80-50)/20 = +1.5$. Therefore, the probability is around 93% that the value will be below 80, or 7% that it will be above 80.

25. The score is $(30 - 50)/20 = -1.0$. Therefore, the probability is around 16% that the value will be 30, and 84% that it will be above 30.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 15

The Principle of Diversification

Eggs and Baskets

last file change: Jun 15, 2004 (11:48h)

last major edit: n/a

This chapter appears in the Survey text only.

Having the statistical artillery now to describe risk (i.e., the standard deviation), we are ready to abandon the previously maintained assumption of investor risk-neutrality. Henceforth, our investors will no longer be indifferent among investments with the same expected rates of return. Instead, investors will now prefer the less risky investment *if* two investment options have the same expected rate of return.

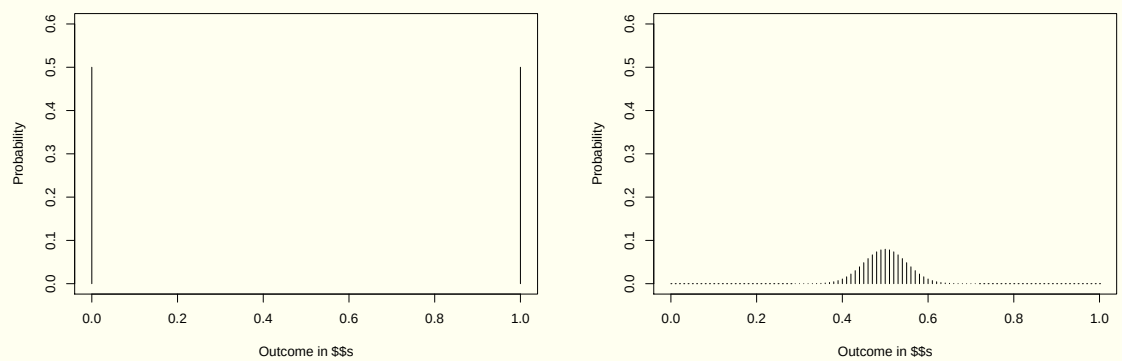
Important: *The remainder of this book assumes that investors prefer portfolios that offer higher expected rates of return and lower standard deviation. Investors care about their portfolio performances at one particular point in the future. Investors are assumed not to care about any other characteristics of their portfolios (e.g., whether a firm in their portfolios produces cigarettes or cancer cures.)*

15·1. Diversification: The Informal Way

Don't put all your eggs into one basket (on one bet), if you dislike variance!

The first and most important principle of Investments is that of **Diversification**. This book—and finance—expounds on it in great detail, but it can be explained with a simple example. Compare two bets. The first bet depends on the outcome of a single coin toss. If heads, the bet pays off \$1 (if tails, the bet pays off nothing). The second bet depends on the outcome of 100 coin tosses. Each coin toss, if heads, pays off $^1/_{100}$ of a dollar, zero otherwise. The expected outcome of either bet is 50 cents. But, as Figure 15.1 shows, the standard deviation of the payoff—the risk—of the latter bet is much lower.

Figure 15.1. Payoffs Under Two Bets With Equal Means.



The left figure is the distribution of payoff where you bet \$1 on head for 1 coin throw. The right figure is the distribution of payoff where you bet \$0.^{01} on head for each of 100 coin throws.

Anecdote: Risk: Aversion or Las Vegas?
We are assuming that investors dislike risk. Putting money into many different bets, rather than one big bet accomplishes this goal. But, is this a good strategy on the roulette table, too? Should you bet your entire money on red (one big bet only), or should you bet it one dollar at a time on red? From a purely financial perspective, the answer is that the single bet is better: if you bet one dollar at a time, you are indeed likely to have a lower variance of payoffs around your expected rate of return. Unfortunately, in roulette, your expected rate of return is *negative*. The casino would be perfectly happy to have you pay up your (negative) expected rate of return *without any risk* for each role of the ball.

Indeed, if your strategy is to gamble as long until you either are bankrupt or double your money, it is easy to show that you are most likely not to go bankrupt if you bet everything into relatively few bets.

15·2. Diversification: The Formal Way

15·2.A. Uncorrelated Securities

Recall from Section 13·3 (Page 277) that a portfolio's return, r_P, is

The base case: independent security returns.

$$r_P \equiv \sum_{i=1}^{N} w_i \cdot r_i \ , \tag{15.1}$$

where P is the overall portfolio, w_i are the investment weights (proportions), and i is a counter that enumerates all investments from 1 to N. If we do not yet know the return outcomes, then our returns are random variables, so

$$\tilde{r}_P \equiv \sum_{i=1}^{N} w_i \cdot \tilde{r}_i \ . \tag{15.2}$$

It is now time to put the laws of expectations and standard deviations (from Section 14·5) to good use. To illustrate how diversification works, make the following, admittedly unrealistic assumptions;

1. that all securities offer the same expected rate of return (mean)

$$\mathcal{E}(\tilde{r}_i) = 5\% \quad \text{for all } i \ ; \tag{15.3}$$

2. that all securities have the same risk (standard deviation of return) of

$$Sdv(\tilde{r}_i) \equiv \sigma(\tilde{r}_i) \equiv \sigma_i = 40\% \quad \text{for all } i \ , \tag{15.4}$$

   which is roughly the annual rate of return standard deviation for a typical U.S. stock;

3. and that all securities have rates of return that are independent from one another. Independence implies that security returns have zero covariation with one another, so $Cov(\tilde{r}_i, \tilde{r}_j) = 0$ for any two securities i and j—just as long as i is not j.

$$Cov(\tilde{r}_i, \tilde{r}_j) = 0 \quad \text{for all different } i \text{ and } j \ . \tag{15.5}$$

   (This last assumption is the most unrealistic of the three.)

What are the risk and return characteristics of the portfolio, $\tilde{r}_P$, if it contains N securities? For an equal-weighted portfolio with N securities, each investment weight is $1/N$, so the portfolio rate of return is

The portfolio definition.

$$\tilde{r}_P = \sum_{i=1}^{N} w_i \cdot \tilde{r}_i = \sum_{i=1}^{N} (1/\mathrm{N}) \cdot \tilde{r}_i = 1/\mathrm{N} \cdot \tilde{r}_1 + 1/\mathrm{N} \cdot \tilde{r}_2 + \ldots + 1/\mathrm{N} \cdot \tilde{r}_N \ . \tag{15.6}$$

The Expected Rate of Return.

Start with one security: In this case, $r_P \equiv r_1$, so

$$\mathcal{E}(\tilde{r}_P) = \mathcal{E}(\tilde{r}_1) = 5\% \ , \quad \mathcal{S}dv(\tilde{r}_P) = \mathcal{S}dv(\tilde{r}_1) = \sqrt{\mathcal{V}ar(\tilde{r}_1)} = 40\% \quad . \tag{15.7}$$

Two securities now. The portfolio consists of a 50-50 investment in securities 1 and 2:

$$\tilde{r}_P = 1/2 \cdot \tilde{r}_1 + 1/2 \cdot \tilde{r}_2 \ . \tag{15.8}$$

The average expected rate of return on a portfolio is the average of the expected rates of returns on its components:

$$\begin{aligned} \mathcal{E}(\tilde{r}_P) &= \mathcal{E}(1/2 \cdot \tilde{r}_1 + 1/2 \cdot \tilde{r}_2) \\ &= 1/2 \cdot \mathcal{E}(\tilde{r}_1) + 1/2 \cdot \mathcal{E}(\tilde{r}_2) \\ &= 1/2 \cdot 5\% + 1/2 \cdot 5\% = 5\% \ . \end{aligned} \tag{15.9}$$

More generally, the rate of return is 5%, no matter how many securities enter the portfolio:

$$\begin{aligned} \mathcal{E}(\tilde{r}_P) &= \sum_{i=1}^{N} \mathcal{E}\left(1/\mathrm{N} \cdot \tilde{r}_i\right) = \sum_{i=1}^{N} 1/\mathrm{N} \cdot \mathcal{E}(\tilde{r}_i) \\ &= \sum_{i=1}^{N} 1/\mathrm{N} \cdot 5\% = 5\% \quad . \end{aligned} \tag{15.10}$$

It is not a surprise that the average return on N securities, each averaging 5%, is also 5%.

The base case: Variance and Standard Deviation are lower for 2 securities.

The variance and standard deviation of the rate of return on the portfolio P are more interesting. Begin with two securities.

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_P) &= \mathcal{V}ar(1/2 \cdot \tilde{r}_1 + 1/2 \cdot \tilde{r}_2) \\ &= (1/2)^2 \cdot \mathcal{V}ar(\tilde{r}_1) + (1/2)^2 \cdot \mathcal{V}ar(\tilde{r}_2) + 2 \cdot (1/2) \cdot (1/2) \cdot \mathcal{C}ov(\tilde{r}_1, \tilde{r}_2) \\ &= 1/4 \cdot 0.16 + 1/4 \cdot 0.16 + 2 \cdot 1/2 \cdot 1/2 \cdot 0 \\ &= 1/4 \cdot 0.16 + 1/4 \cdot 0.16 \\ &= 1/2 \cdot 0.16 \\ \iff \mathcal{S}dv(\tilde{r}_P) &= \sqrt{\mathcal{V}ar(\tilde{r}_P)} = \sqrt{1/2 \cdot 0.01} = 70.7\% \cdot 40\% = 28.3\% \quad . \end{aligned} \tag{15.11}$$

We were able to drop out the covariance term, because we assumed security returns to be independent. Pay close attention to the final line: the portfolio of one security had a risk of 40%. The portfolio of two securities has the lower risk of 28.3%.

SIDE NOTE: If you find it easier to understand the formula if you see some data, below are two sample series that are consistent with our assumptions: each has 5% mean and 40% standard deviation, and they have zero mutual covariance. The final column is the rate of return on the portfolio that invests half in each security. You can confirm that the standard deviation of this portfolio is indeed 28.284%, as we have just computed via our Formula.

Year	$\tilde{r}_1$	$\tilde{r}_2$	$\tilde{r}_P$	Year	$\tilde{r}_1$	$\tilde{r}_2$	$\tilde{r}_P$
1980	25.420	-11.476	6.972	1985	21.268	-35.560	-7.146
1981	39.376	62.064	50.720	1986	-45.524	30.620	-7.452
1982	41.464	-19.356	11.054	1987	42.976	59.616	51.296
1983	30.500	-41.256	-5.378	1988	-10.324	30.360	10.018
1984	-67.552	15.772	-25.890	1989	-27.604	-40.784	-34.194
				$\mathcal{E}$	5.000	5.000	5.000
				Sdv	40.000	40.000	28.284

Now, compute the standard deviation for an arbitrary number of securities in the portfolio. Recall the variance formula,

Variance and Standard Deviation for N securities.

$$\begin{aligned}
\mathcal{V}ar(\tilde{r}_P) &= \mathcal{V}ar\left(\sum_{i=1}^{N} c_i \cdot \tilde{r}_i\right) = \sum_{j=1}^{N}\left\{\sum_{k=1}^{N}\left[c_j \cdot c_k \cdot Cov(\tilde{r}_j, \tilde{r}_k)\right]\right\} \\
w_1^2 \cdot \mathcal{V}ar(\tilde{r}_1) &+ w_2^2 \cdot \mathcal{V}ar(\tilde{r}_2) + \ldots + w_N^2 \cdot \mathcal{V}ar(\tilde{r}_N) \\
&+ 2 \cdot w_1 \cdot w_2 \cdot Cov(\tilde{r}_1, \tilde{r}_2) + 2 \cdot w_1 \cdot w_3 \cdot Cov(\tilde{r}_1, \tilde{r}_3) + \ldots + 2 \cdot w_1 \cdot w_N \cdot Cov(\tilde{r}_1, \tilde{r}_N) \\
&+ 2 \cdot w_2 \cdot w_1 \cdot Cov(\tilde{r}_2, \tilde{r}_1) + 2 \cdot w_2 \cdot w_3 \cdot Cov(\tilde{r}_2, \tilde{r}_3) + \ldots + 2 \cdot w_2 \cdot w_N \cdot Cov(\tilde{r}_2, \tilde{r}_N) \\
&+ \ldots + \ldots + \ldots + \ldots \\
&+ 2 \cdot w_N \cdot w_1 \cdot Cov(\tilde{r}_N, \tilde{r}_1) + 2 \cdot w_N \cdot w_3 \cdot Cov(\tilde{r}_N, \tilde{r}_3) + \ldots + 2 \cdot w_N \cdot w_{N-1} \cdot Cov(\tilde{r}_N, \tilde{r}_{N-1}) \quad .
\end{aligned} \tag{15.12}$$

In our example, all the covariance terms are zero, all the weights are $1/N$, and all individual security variances are the same, so this is

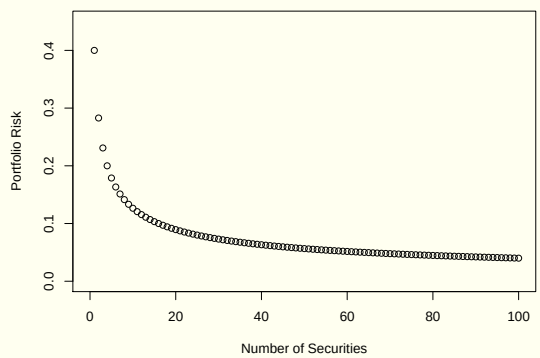
$$\begin{aligned}
\mathcal{V}ar(\tilde{r}_P) &= (1/N)^2 \cdot \mathcal{V}ar(\tilde{r}_1) + (1/N)^2 \cdot \mathcal{V}ar(\tilde{r}_2) + \ldots + (1/N)^2 \cdot \mathcal{V}ar(\tilde{r}_N) \\
&= (1/N)^2 \cdot \mathcal{V}ar(\tilde{r}_i) + (1/N)^2 \cdot \mathcal{V}ar(\tilde{r}_i) + \ldots + (1/N)^2 \cdot \mathcal{V}ar(\tilde{r}_i) \\
&= N \cdot (1/N)^2 \cdot \mathcal{V}ar(\tilde{r}_i) = (1/N) \cdot \mathcal{V}ar(\tilde{r}_i) \quad . \\
\iff Sdv(\tilde{r}_P) &= \sqrt{1/N \cdot \mathcal{V}ar(\tilde{r}_i)} = \sqrt{1/N} \cdot Sdv(\tilde{r}_i) = \sqrt{1/N} \cdot 40\% \quad .
\end{aligned} \tag{15.13}$$

This formula states that for 1 security, the risk of the portfolio is 40%; for 2 securities, it is 28.3%; for 4 securities, it is $\sqrt{1/4} \cdot 40\% = 20\%$; for 16 securities, it is 10%; for 100 securities, it is 4%, and for 10,000 securities, is 0.4%. In a portfolio of infinitely many securities, the risk of the portfolio disappears. In other words, the investor would practically be certain to earn the expected rate of return (here 5%). The decline in risk is also interesting. The square root function on N declines steeply only for the first few securities, but then progressively less so for subsequent securities. Going from 1 to 4 securities reduces the risk by 50%. The next 5 securities (going from 4 to 9 securities) only reduce the risk by 17% (from $\sqrt{1/4} = 50\%$ to $\sqrt{1/9} = 33\%$). To drop the risk from 50% to 25% requires 12 extra securities; to drop the risk from 50% to 10% requires 96 extra securities. In other words, *if* security returns are independent, diversification works

A reflection on the effectiveness of diversification.

really well in the beginning, but less and less as more securities are added. It is important to have, say, a dozen independent securities in the portfolio, which drops the portfolio risk by two-thirds; additional diversification through purchasing more securities is nice, but it is not as important, in relative terms, as these first dozen securities.

Figure 15.2. Diversification if security returns were independent.



15·2.B. Correlated Securities

Diversification fails when security returns are perfectly positively correlated.

When does diversification fail? Recall from Page 301 that the maximum possible correlation of 1 implies that

$$Correlation(\tilde{X}, \tilde{Y}) \;=\; +1 \quad \Longleftrightarrow \quad Cov(\tilde{X}, \tilde{Y}) \;=\; Sdv(\tilde{X}) \cdot Sdv(\tilde{Y}) \quad . \tag{15.14}$$

Therefore, the covariance of two perfectly correlated securities' rates of returns (in our example) is

$$\begin{aligned} Cov(\tilde{r}_i, \tilde{r}_j) &= 40\% \cdot 40\% \;=\; 0.16 \quad . \\ Cov(\tilde{r}_i, \tilde{r}_j) &= Sdv(\tilde{r}_i) \cdot Sdv(\tilde{r}_j) \quad . \end{aligned} \tag{15.15}$$

The variance of a portfolio of two such stocks is

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_P) &= \mathcal{V}ar(1/2 \cdot \tilde{r}_1 + 1/2 \cdot \tilde{r}_2) \\ &= 1/2^2 \cdot \mathcal{V}ar(\tilde{r}_1) + 1/2^2 \cdot \mathcal{V}ar(\tilde{r}_2) + 2 \cdot 1/2 \cdot 1/2 \cdot Cov(\tilde{r}_1, \tilde{r}_2) \\ &= 1/4 \cdot 0.16 + 1/4 \cdot 0.16 + 2 \cdot 1/2 \cdot 1/2 \cdot 0.16 \\ &= 1/2 \cdot 0.16 + 1/2 \cdot 0.16 \;=\; 0.16 \\ Sdv(\tilde{r}_P) &= \sqrt{\mathcal{V}ar(\tilde{r}_P)} = 40\% \quad . \end{aligned} \tag{15.16}$$

In other words, when two securities are perfectly correlated, diversification does not reduce portfolio risk. It should come as no surprise that you cannot reduce the risk of a portfolio of one PepsiCo share by buying another PepsiCo share. (The returns of PepsiCo shares are perfectly correlated.) In general, the smaller or even more negative the covariance term, the better diversification works.

IMPORTANT:

- *Diversification fails when underlying securities are perfectly positively correlated.*
- *Diversification reduces portfolio risk better when its underlying securities are less correlated.*
- *Diversification works perfectly (reducing portfolio risk to zero) when underlying securities are perfectly negatively correlated.*

Question 15.3 below asks you to prove the last point.

15·2.C. Measures of Diversification: Covariance, Correlation, or Beta?

Covariance, Correlation, and Beta are multiple possible measures of covariation.

It seems that diversification works better when the covariation between investment securities is smaller. The correct measure of overall risk remains, of course, the standard deviation of the portfolio's rate of return. But, the question we are now interested in is "what is the best measure of the contribution of an individual security to the risk of a portfolio?." We need a measure of the risk contribution of just one security inside the portfolio to the overall portfolio risk.

Candidates to measure how a new security helps portfolio diversification.

Consider an investor who already holds a portfolio p, and who is now considering to add "a little bit" of security i. How much does this new security help or hurt diversification? Because this investor is adding fairly little, it is a reasonable approximation to assume that the rest of the portfolio remains as it was (even though the new security really becomes part of portfolio p and thereby changes p). The three candidates to describe how correlated the new investment opportunity is with the rest of the portfolio are

$$\begin{array}{lcl} \text{The Covariance:} & Cov(\tilde{r}_i, \tilde{r}_p) & \text{(Uninterpretable)} \\ \text{The Correlation:} & \dfrac{Cov(\tilde{r}_i, \tilde{r}_p)}{Sdv(\tilde{r}_i) \cdot Sdv(\tilde{r}_p)} & \text{(Interpretable)} \\ \text{The } \textbf{Portfolio Beta}\text{:} & \dfrac{Cov(\tilde{r}_i, \tilde{r}_p)}{\mathcal{V}ar(\tilde{r}_p)} & \text{(Interpretable)} \end{array} \tag{15.17}$$

Although all three candidates share the same sign, each measure has its advantages: The covariance is used directly in the portfolio formula (e.g., Formula 15.11, Page 334), but its value is difficult to interpret. The correlation is easiest to interpret because it lies between -1 and $+1$, but its real problem as a measure of risk for a new security (which we want to add to our portfolio) is that ignores its relative variability.

Debunking correlation: it is not a good risk measure.

This merits an explanation. The following two securities both have perfect correlation with your portfolio (this makes our illustration easy, but it is not necessary). In our example, there are only two equally likely possible outcomes:

	Your Portfolio	Security A	Security B
Case 1	+24%	+12%	+200%
Case 2	-12%	-6%	-100%
$\mathcal{E}(\tilde{r})$	+6%	+3%	50%
$Sdv(\tilde{r})$	+18%	+9%	150%

Now assume you had \$75 in your portfolio, but you are adding \$25 of either A or B. Therefore, your new combined portfolio rate of return would be

	Your Portfolio Plus Security A	Your Portfolio Plus Security B
Case 1	$75\% \cdot (+24\%) + 5\% \cdot (+12\%) = 21.0\%$	$75\% \cdot (+10\%) + 25\% \cdot (+200\%) = 68\%$
Case 2	$75\% \cdot (-12\%) + 5\% \cdot (-6\%) = -10.5\%$	$95\% \cdot (+10\%) + 5\% \cdot (+100\%) = -34\%$
$\mathcal{E}(\tilde{r}_P)$	5.25%	51.00%
$Sdv(\tilde{r}_P)$	15.75%	153.00%

Adding stock B causes your portfolio risk to go up more. Yes, B adds more benefit in terms of expected rate of return, but B's cost is higher portfolio risk. Yet, the correlation between your portfolio and either security A or security B was the same. The correlation would not have told you that B is the riskier add-on.

But the beta of a stock with your portfolio still works as a good risk measure!

In contrast, the beta of security A with respect to your portfolio P tells you the right thing: the beta of security A with respect to your portfolio P is 0.5 (which you can compute either with the formula $Cov(A,P)/\mathcal{V}ar(P)$, or by recognizing that A is always one-half of P); the beta of security B with respect to your portfolio P is 8.33. Therefore, beta tells us that adding security B would increase your overall portfolio risk more than adding security A. Unlike correlation, beta takes into account the scale of investments.

A concrete example that shows that correlation is not a great diversification measure.

We can demonstrate this scale problem within the concrete context of our earlier example: the annual returns of the S&P500, IBM, and Sony. On Page 304, we determined that, compared to IBM, Sony had a higher beta with the market, but a lower correlation. So, does a portfolio consisting of one-half S&P500 and one-half IBM portfolio I have more or less risk than a portfolio consisting of one-half S&P500 and one-half Sony portfolio S?

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_I) &= (1/2)^2 \cdot 3.62\% + (1/2)^2 \cdot 15.03\% + 2 \cdot 1/2 \cdot 1/2 \cdot 3.30\% = 5.075\% \ , \\ \mathcal{V}ar(\tilde{r}_S) &= (1/2)^2 \cdot 3.62\% + (1/2)^2 \cdot 81.49\% + 2 \cdot 1/2 \cdot 1/2 \cdot 4.77\% = 21.874\% \ . \end{aligned} \tag{15.18}$$

Even though Sony has lower correlation with the S&P500 than IBM, Sony's higher variance negates this advantage: the S portfolio is riskier than the I portfolio. The covariances reflect this accurately: the covariance of the S&P500 with Sony is higher than the covariance of the S&P500 with IBM. Beta preserves this ordering, too, because both covariances are divided by the same variance (the variance of S&P500). It is only the correlation that would have misleadingly indicated that Sony would have been the better diversifier. This is our main insight:

IMPORTANT: *Beta is a measure of diversification that takes scale into account. The lower the beta, the better the diversification.*

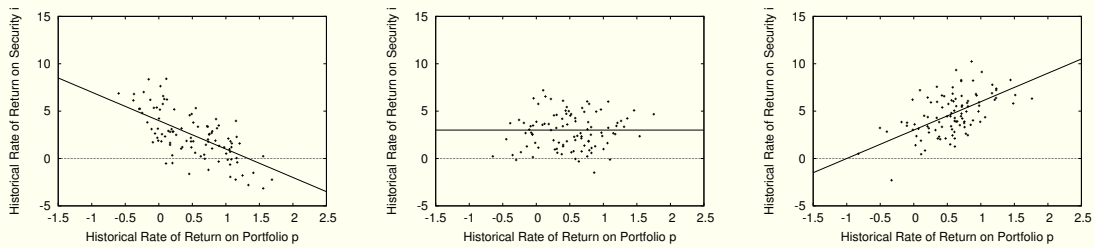
The covariance would have worked equally well, but it is more difficult to intuitively interpret. Correlation is not suitable as a quantitative measure, because it fails to recognize investment scales.

The other two measures.

The remainder of this section just provides some additional intuition: Beta has a nice slope interpretation. If you graph the rate of return on the portfolio P (e.g., the S&P500) on the x axis, and the rate of return on the new security i (e.g., Sony) on the y axis, then beta represents the slope of a line that helps you predict how the rate of return of security i will turn out *if* you know how the rate of return on your portfolio turned out.

Beta has a nice slope interpretation, too.

Figure 15.3. Three Different Beta Measures of Risk for Security i.



The betas of security i with respect to portfolio P are -3, 0, and $+3$ in the three figures, respectively.

Figure 15.3 shows three different betas: the right-most figure shows a positive beta. The investor would expect i to not help her much in diversification: when her portfolio P does better, so does i. In the figure, beta has a value of 3. This indicates that if P were to offer an additional 5% rate of return, the rate of return on the security i would be expected to change by an additional 15%. More importantly, if P hits hard times, i hits even harder times. An investor would be better off purchasing only a very, very small amount of such a security; otherwise, she could quickly end up with a portfolio that looks more like i than like P. In contrast, in the left-most figure, a purchaser of security i would expect i to help considerably in diversification: when P does worse, i does better! With a beta of -3, the security i tends to go up by an additional 15% when the portfolio P goes down by an additional 5%. Therefore, i provides excellent "insurance" against downturns in P. Finally, the middle figure shows a situation where a security has a zero beta, which is the case when i's rate of return is independent of P's rate of return.

Beta can be visually judged by graphing the rate of return on a security against the rate of return on the overall portfolio. It is the slope of the line.

Side Note: The intercept in Figure 15.3 is sometimes called the "alpha," and it can measure how much expected rate of return the security is likely to offer, holding its extra risk constant. For example, if stocks i and j have lines as follows

$$\begin{aligned}
\mathcal{E}(\tilde{r}_i) &\approx +15\% + 1.5 \cdot \mathcal{E}(\tilde{r}_p) \quad , \\
\mathcal{E}(\tilde{r}_j) &\approx -10\% + 1.5 \cdot \mathcal{E}(\tilde{r}_p) \quad , \\
\mathcal{E}(\tilde{r}_k) &\approx \alpha_k + \beta_{k,p} \cdot \mathcal{E}(\tilde{r}_p) \quad ,
\end{aligned} \tag{15.20}$$

the α of stock i was 15%, while the α of stock j was -10%. It appears that stock i offers a holder of portfolio p a lot of positive return compared to j, holding its exposure to portfolio p constant. Naturally, investors like high alphas and low betas—but such opportunities are difficult to find, especially for already reasonably good (well-diversified) portfolios, p.

ANECDOTE: Value-At-Risk (VAR)
The latest in risk measurement techniques among banks and other financial institutions—and a great step forward if executed correctly—is **VaR** (**Value-at-Risk**). It often replaces older risk-scoring systems, in which (for example) all commercial loans received one score, government loans another, etc. The goal of Value at Risk is to compute the risk (standard deviation) when all investment (loans) are evaluated in a portfolio framework. Value at Risk can come to very different conclusions than these older systems, especially if payoffs to loans are very negatively or very positively correlated.

Solve Now!

Q 15.1 *Assume that every security has a mean of 12%, and a standard deviation of 30%. Further, assume that each security has no covariation with any security. What are the risk and reward of a portfolio of N stocks that invests equal amount in each security?*

Q 15.2 *Assume that every security has a mean of 12%, and a standard deviation of 30%. Further, assume that each security has 0.0025 covariation with any security. What are the risk and reward of a portfolio of N stocks that invests equal amount in each security? If you find this difficult, solve this for 2 stocks. Can you guess what the risk is for a portfolio of infinitely many such stocks?*

Q 15.3 *Compute the Variance of a two stock portfolios if the two securities are not perfectly positively, but perfectly negatively correlated.*

Q 15.4 *You own a \$1,000 portfolio, p, whose expected rate of return has a mean of 10% and a standard deviation of 20%. You are considering buying a security q that has a mean of 15% and a standard deviation of 50%. The correlation between the rates of return on p and q is 20%.*

1. *What is the covariance between the rate of return of p and q?*
2. *What is the beta between the rate of return on p and q?*
3. *Consider purchasing \$1,000 in q. What would the portfolio risk be?*

A new security, named n has appeared. It has a mean of 150% and a standard deviation of 500%, and the same 20% correlation with p. (Sidenote: Such a security could be created by a fund that borrows in order to purchase more than 100% in q.)

1. *What is the covariance between the rate of return of p and n?*
2. *What is the beta between the rate of return on p and n?*
3. *Consider purchasing \$1,000 in n. What would the portfolio risk be?*

q and n have equal correlation with portfolio p. Does it follow that they would both be equal risk-contributors, if added to the portfolio?

15·3. Does Diversification Work in the Real World?

15·3.A. Diversification Among The Dow-Jones 30 Stocks

Diversification depends on the specific portfolio.

To answer whether portfolio diversification works in the real world, one would have to ask for exactly which two (or more securities) the effect of diversification is to be examined. The answer necessarily depends on the specific portfolio chosen (the weights and the securities).

The risk of investing in a Dow-Jones 30 Stock.

An intuitive feel for the effectiveness of diversification in the U.S. stock markets can be gathered by looking at the 30 stocks constituting the Dow-Jones 30. The Dow-Jones company has chosen 30 stocks in its Dow-Jones 30 Index (see also Section 13·3.C, Page 281) to avoid industry and firm-type concentration. However, the Dow-Jones contains only very large firms, both in market capitalization and sales.

The average stock in the Dow-Jones has a risk of about 30% to 40%.

Table 15.1 shows the risks (standard deviations of the rate of returns) for the 30 Dow-Jones stocks, measured either from January 1997 to October 2002 or from January 1994 to October 2002. Their risks ranged from about 18% to 50% (annualized), averaging about 30% to 35% for a typical stock. Each individual stock is a fairly risky investment.

The risk of investing in all Dow-Jones 30 Stocks.

However, even though the Dow-Jones 30 Index is not an equal-weighted portfolio, when its risk is computed over the same sample periods, its risk was only 16.5% or 18.7% per year. Naturally, this is still pretty risky. But it is only about half the risk of the average Dow-Jones stock.

Inferring correlations.

Are the returns of these 30 stocks positively correlated? Note that 18% is only about half of the 35% risk that would obtain if all the securities' returns were perfectly positively correlated. So, diversification works: the correlation among the Dow-Jones 30 stock returns is not close to one. However, the correlation among the Dow-30 stocks is also not zero. In this case, we would have expected the equal-weighted portfolio to have a risk of about $\sqrt{1/30}\cdot 35\% \approx 6.4\%$ per year. Instead, the risk of the Dow-Jones portfolio, at 18% per year, is 2.8 times as high. In sum, this evidence suggests that these 30 stocks move together, i.e. that they tend to have mutual positive correlations. More broadly, when the U.S. stock market does well, most stocks do well at the same time (and vice-versa).

Important: *The Dow-Jones 30 Market Index Portfolio has a risk of about 15% to 20% per year. (Broader U.S. stock market indexes, like the S&P500 Index, tend to have slightly lower risks.)*

The 30 component stocks in the Dow-Jones 30 Index are mutually positively correlated, which limits the effectiveness of diversification—but not so much as to render diversification useless.

Table 15.1. Risk for Dow Jones Constituents, Based on Monthly Rates of Returns, Then Annualized.

	about 10 years 1994/01-2002/10		about 5 years 1997/01-2002/10	
Asset	Mean	StdDev	Mean	StdDev
alcoa	18.4%	36.0%	15.1%	41.3%
american express	21.4%	27.9%	17.3%	31.4%
boeing	8.9%	30.1%	−2.9%	34.6%
citigroup	27.7%	33.6%	24.9%	37.4%
caterpillar	14.0%	31.2%	9.6%	34.0%
du pont	12.2%	26.2%	4.4%	29.0%
disney	6.7%	28.8%	0.3%	32.2%
eastman kodak	4.5%	30.0%	−5.5%	34.6%
general electric	17.1%	25.1%	12.8%	28.4%
general motors	5.2%	32.6%	5.1%	36.7%
home depot	18.5%	31.3%	22.8%	35.1%
honeywell	11.2%	37.9%	6.0%	44.5%
hewlett packard	14.8%	42.9%	4.0%	48.2%
ibm	25.8%	36.0%	20.2%	39.6%
intel	28.7%	46.7%	15.8%	53.1%
international paper	7.5%	31.6%	6.0%	36.0%
johnson and johnson	23.2%	23.9%	19.5%	26.4%
jp morgan	15.0%	36.8%	5.9%	42.5%
coca-cola	13.1%	26.5%	3.7%	30.7%
mcdonalds	6.2%	25.6%	0.6%	28.8%
3m	14.8%	23.2%	12.8%	26.0%
philip morris	13.5%	29.9%	6.9%	33.0%
merck	19.2%	28.6%	12.1%	32.0%
microsoft	35.8%	42.6%	28.4%	50.0%
proctor and gamble	16.7%	24.7%	13.7%	28.4%
sbc communications	9.4%	28.9%	8.1%	34.4%
att	−4.4%	36.5%	−5.7%	40.3%
united technologies	21.9%	29.2%	18.1%	34.1%
walmart	20.6%	28.8%	31.5%	31.1%
exxon	10.0%	16.8%	7.1%	18.3%
Average	15.3%	31.0%	10.6%	35.0%
Median	14.8%	29.9%	8.1%	34.1%
dow jones 30 index	10.5%	16.5%	6.3%	18.7%
s&p 500 index	8.6%	16.1%	4.8%	18.5%

SIDE NOTE: In Chapter a (Page 321), we asserted that historical standard deviations of rates of returns tend to be relatively stable. This table shows that, although the riskiness of firms does change over time (it is different over 5 years and 10 years), it does change only slowly, even for the individual Dow-30 stocks. (There would be even more stability when asset class portfolios are considered.) This stability gives us confidence in using historical risk measures (e.g., standard deviations) as estimates of future risk.

Make sure not to buy just tech stocks, or just growth stocks, or ...

The square root in the portfolio standard deviation formula suggests that most of the diversification typically comes from the first 10 to 50 stocks. Therefore, it is more important to be suitably diversified across different types of stocks (to avoid mutual positive covariances) than it is to add every single possible stock to a portfolio. A good choice for an investor who holds the Dow-Jones 30 stocks and who wants to further diversify using U.S. stocks would be to add small or high-growth firm stocks to her

portfolio (or, better, a mutual fund that focuses on small firms).

SIDE NOTE: To compute the overall portfolio risk, Formula 15.12 (Page 335) states that for ten stocks, there are 45 unique covariance terms, but only 10 variances. For 50 stocks, there are over 1,200 unique covariance terms, but only 50 variances...no surprise that covariance terms can quickly dominate the variance terms.

SIDE NOTE: Research shows that when one looks at the average stock on the N.Y.S.E., about 75% of its risk can be diversified away, about 25% of its risk cannot be diversified away. Put differently, undiversifiable co-movements among all stocks in the stock market are responsible for about one-quarter of the typical stocks' return variance; three-quarters are idiosyncratic day-to-day fluctuations, which average themselves away for investors who hold highly diversified stock market portfolios.

15·3.B. Mutual Funds

There are too many stocks for ordinary investors to buy them all.

Diversification clearly reduces risk, but it can also be expensive to accomplish. How can an investor with $50,000 to invest purchase 500 securities? The trading transaction costs when purchasing $100 of a security are prohibitive. With about 10,000 publicly traded equities in the U.S. stock market, even purchasing just $1,000 in every stock traded would require $10 million, well beyond the financial capabilities of ordinary investors.

Mutual funds are investment vehicles to accomplish diversification.

Mutual Funds, already mentioned in Section 13·3.B, come to the rescue. As already described in Chapter 13, a mutual fund is a firm that consists of nothing but holdings in other assets, usually financial assets. In a sense, a mutual fund is a large portfolio that can be purchased as a bundle. However, there is a catch. On the one hand, investing in a mutual fund rather than in its individual underlying assets can reduce an investor's costs, ranging from the time necessary to research stocks and initiate transactions, to direct transaction costs (the bid-ask spread). But, on the other hand, mutual funds often charge a variety of fees to their investors and may force investors to realize taxable gains when they would rather not.

ANECDOTE: **Portfolios of Finance Professors**
Many finance professors invest their own money into passively managed, low-cost mutual funds, often **Index Funds**. Index funds are buy-and-hold portfolios which require minimal investment selection abilities by their managers, and which usually incur minimal trading costs.

15·3.C. Alternative Assets

Other Assets Are Equally or More Important.

A common error committed by investors is that they focus only on the diversification among stocks traded on the major U.S. stock exchanges. But there are many other financial and non-financial instruments that can aid investors in diversifying their risk. Because these instruments are often less correlated with an investor's portfolio than domestic U.S. stocks, these assets can be especially valuable in reducing the portfolio risk. Among possible investment assets are:

- Savings Accounts.
- Bonds.
- Commodities (such as **Gold**).
- Other Futures (such as agricultural commodities).
- Art.

- Real Estate.
- Mortgage and Corporate Bonds.
- Labor Income.
- International Stocks.
- Hedge Funds.
- Venture and Private Equity Funds.
- Vulture and Bankruptcy Funds.

In addition, if you are a smart investor, you would not only consider the diversification within your stock portfolio, but across your entire wealth. Your wealth would include your house, your education, your job, etc. Many of these alternative investments could also have low covariation with your overall wealth.

SIDE NOTE: Some of the above mentioned asset categories may be better held in modest amounts. Furthermore, some of these areas do not resemble the highly liquid, fair, and efficient financial markets that U.S. stock investors are used to. Instead, some are rife with outright scams. Therefore, it might be wise to hold such assets through sophisticated and dedicated professional investors (mutual funds), who have the appropriate expertise.

Solve Now!

Q 15.5 *In Table 15.1, Exxon had the lowest standard deviation among the Dow-30 stocks. Why not just purchase Exxon by itself?*

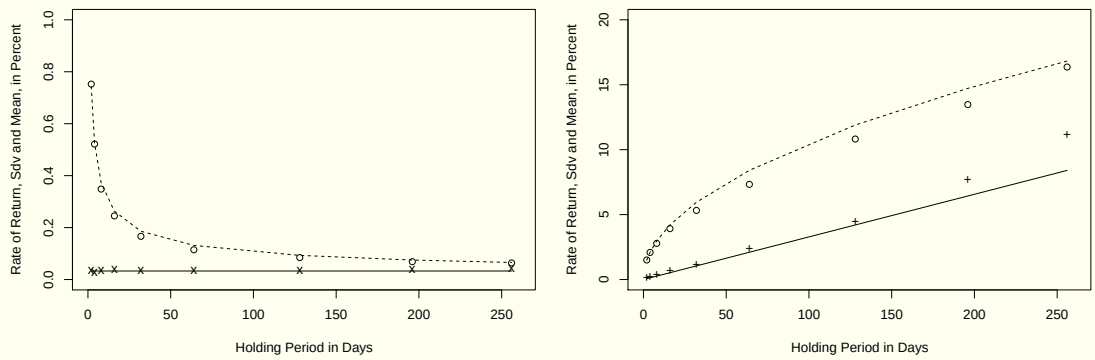
Q 15.6 *How much does diversifying over all 500 stocks in the S&P500 help in terms of risk reduction relative to investing in the 30 stocks of the Dow Jones-30?*

Q 15.7 *Why do mutual funds exist?*

Q 15.8 *Should an investor just own U.S. stocks? Does the value-weighted market portfolio just contain stocks?*

15·4. Diversification Over Time

Figure 15.4. Average Risk and Return over Time in the S&P500, 1990–2002



The x-axis is the investment horizon, i.e., the number of consecutive investment days. The two top series (lines and dots) are the standard deviation, the two bottom series are the means. Lines are the theoretical values computed with the formulas below; circles and pluses are from simulated actual investment strategies.

Most investors think of diversification across securities within a portfolio, but diversification also works over time. Figure 15.4 shows the risk and reward if you had invested in the S&P 500 from 1990–2002 for x consecutive trading days. The left graph shows your average daily rate of return; the right side your total compounded rate of return. For example, the right figure shows that if you had held an S&P500 portfolio during a random 25 day period (about a month), you would have earned a little less than 1%, but with a risk (standard deviation) of about 5%. The left graph quoted this in average daily terms: over 25 days, a 1% mean and 5% risk was an average reward of about 0.04%/day with a risk of about 0.2%/day.

Explaining the Figure Illustraing Time Diversification.

Note from the right graph how the risk-reward tradeoff changes with time. Over an investment horizon of a full year (255 trading days), you would have expected to earn about 10% with a risk of about 15%. The risk would have been 1.5 times the reward. In contrast, over an investment horizon of one day, you would have expected to earn about 0.04% with a risk of about 1.1%. Your risk would have been about 30 times your reward! If you stare at the right-side figure, you should notice that the reward goes up a little more than linearly (the compounding effect!), while the risk goes up like a parabola, like a square root function. Indeed, this is the case, and the rest of this section shows why.

Time Diversification at Work!

Recall that a portfolio that earns $\tilde{r}_{t=1}$ in period 1, $\tilde{r}_{t=2}$ in period 2, and so on until period T ($\tilde{r}_{t=T}$), will earn an overall rate of return of

Over periods shorter than a few years, the ordinary portfolio returns is roughly the sum of time period returns.

$$\begin{aligned}\tilde{r}_{t=0,t=T} &= (1+\tilde{r}_{t=0,1})\cdot(1+\tilde{r}_{t=1,2})\cdot\ldots\cdot(1+\tilde{r}_{t=T-1,T}) \\ &\approx \tilde{r}_{t=0,1}+\tilde{r}_{t=1,2}+\ldots+\tilde{r}_{t=T-1,T}+\text{Multiplicative } \tilde{r} \text{ terms}\end{aligned} \tag{15.21}$$

The multiplicative terms reflect the power of compounding—which clearly matters over many years—but perhaps less so over periods of just a few months or years. After all,

for a monthly return that may typically be on the order of 1%, a multiplicative term is on the order of $1\% \cdot 1\% = 0.01\%$.

Stock returns have to be about independent across time-periods to avoid great money-making opportunities. This determines the riskiness of portfolios over multiple periods.

Now, if you forget about the small multiplicative terms, you already know that if you expect to earn 1% per period, then over x periods you expect to earn x%:

$$\mathcal{E}(\tilde{r}_{t=0,t=T}) \;=\; T \cdot \mathcal{E}(\tilde{r}_t) \quad . \tag{15.22}$$

For the variance, we need some intuition. Assume that the return variance in each time period is about the same and can just be called $\tilde{r}_t$. Further assume that the covariance terms among returns in different times (e.g., $\tilde{r}_{t=0,1}$ and $\tilde{r}_{t=1,2}$) are about zero. After all, If they were not, this would mean that one could predict future returns with past returns. Suffice it to say that this sort of prediction is not an easy task—if it were, I would be rich! Put this all together and use the variance Formula 15.23:

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_{t=0,t=T}) \;=\; & \mathcal{V}ar\left(\tilde{r}_{t=0,1} + \tilde{r}_{t=1,2} + \ldots + \tilde{r}_{t=T-1,T}\right) \\ & + \text{ many covariance terms, all about zero} \\ \approx\; & T \cdot \mathcal{V}ar(\tilde{r}_t) \quad , \end{aligned} \tag{15.23}$$

It follows that

$$\mathcal{S}dv(\tilde{r}_{t=0,t=T}) \;\approx\; \sqrt{T} \cdot \mathcal{S}dv(\tilde{r}_t) \quad . \tag{15.24}$$

But this is just the relationship in the graph: The risk increases with the square root of time!

Annualization

The last formula is commonly used to "annualize" portfolio risk. For example, if an investment strategy has a monthly risk of 5% (i.e., the standard deviation of it's rate of return), and the question is what kind of risk such an investment strategy would have per annum, you can compute this annual standard deviation to be about $\sqrt{12} \cdot 5\% \approx 17\%$. Conversely, if the five-year variance is 39%, then the annualized variance is 17%, because $\sqrt{5} \cdot 17\% \approx 39\%$.

IMPORTANT: *A quick and dirty (and common) method to annualize portfolio risk (the standard deviation of the rate of return) is to multiply the single-period rate of return by the square root of the number of periods. For example, the annual portfolio risk is about $\sqrt{12}$ or 3.5 times as high the monthly risk.*

The most common method, on Wall Street and in academia, is to compute risk and reward from monthly rates of return, but to report their annualized values.

SIDE NOTE: T: ***Elevate this?!*** A common measure of portfolio performance is the **Sharpe Ratio**, named after Nobel Prize Winner William Sharpe. It is the expected rate of return of a portfolio *above and beyond* the risk-free rate of return ($r_{\mathcal{F}}$), divided by the standard deviation of the rate of the return:

$$\text{Sharpe Ratio} = \frac{\mathcal{E}(\tilde{r}_P - r_{\mathcal{F}})}{Sdv(\tilde{r}_P - r_{\mathcal{F}})} = \frac{\mathcal{E}(\tilde{r}_P) - r_{\mathcal{F}}}{Sdv(\tilde{r}_P)} \tag{15.25}$$

Although the Sharpe ratio is intuitively appealing and in widespread use, it has many problems. A relatively minor issue among them is the fact that the Sharpe ratio depends on the time-interval that is used to measure returns. In the example with 1% monthly mean and 5% monthly standard deviation, if the risk-free rate were 6% per year, the Sharpe-Ratio of the portfolio would be about $^{(1\%-0.5\%)}/_{5\%} = 0.1$ if measured over a one month time interval; $^{(12\%-6\%)}/_{17\%} = 0.35$ if measured over a year; and $^{(60\%-30\%)}/_{39\%} = 0.77$ if measured over five years.

SIDE NOTE: **Asset allocation** is the relative division of one's assets between stocks and bonds.

Many practitioners suggest that you should put more of your money into risky stocks when you are young. Over the very long run—and young people have naturally longer investment horizons—the expected rate of return vs. risk relationship looks more favorable than it does over shorter investment horizons. After all, the mean goes up with time, while the risk goes up only with the square-root of time. This means that the *average* rate of return is less risky when you are young than when you are old.

But academics are divided on this advice. They point out that the portfolio rate of return is the product of the individual returns:

$$\tilde{r}_{0,T} = (1 + \tilde{r}_{0,1}) \cdot (1 + \tilde{r}_{1,2}) \cdot \ldots \cdot (1 + \tilde{r}_{T-1,T}) \ . \tag{15.26}$$

It should not matter whether you choose risky stocks over safer bonds at time $t = 0$ (when you are young) or at time $t = T$ (when you are old). That is, you should be interested in the *total* rate of return, not the *average* rate of return.

Even more sophisticated arguments take into account that you can adjust better (e.g., working harder) when you are young if they experience a bad portfolio return; or that the stock market rate of return may be **mean reverting**. That is, the market rate of return may be negatively correlated with itself over very long time periods, in which case, the long-run risk could be a little lower than the short-run risk.

Solve Now!

Q 15.9 *What is a reasonable assumption for stock return correlations across different time periods?*

Q 15.10 *Table 14.8 on Page 314 showed that the S&P500 has an annual standard deviation of about 20% per year. As of November 2002, the S&P500 stands at a level of about 900. What would you expect the daily standard deviation of the S&P500 index to be? Assume that there are about 255 trading days in a year.*

Q 15.11 *Assume you have a portfolio that seems to have a monthly standard deviation of 5%. What would you expect its annual standard deviation to be?*

Q 15.12 *If the risk-free rate over the 1997-2002 sample period was about 3% per annum, what was the monthly and what was the annual Sharpe ratio of the Dow-Jones 30 index?*

Q 15.13 *Assume you have a portfolio that seems to have had a daily standard deviation of 1%. What would you expect its annual standard deviation to be? Assume there are 255 trading days in a year.*

Q 15.14 *The S&P500 was quoted in the first two weeks of June 2003 as*

06/02/2003	*967.00*	*06/06/2003*	*987.76*	*06/12/2003*	*998.51*
06/03/2003	*971.56*	*06/09/2003*	*975.93*	*06/13/2003*	*988.61*
06/04/2003	*986.24*	*06/10/2003*	*984.84*		
06/05/2003	*990.14*	*06/11/2003*	*997.48*		

Compute the mean and standard deviation of annual returns of a portfolio that would have mimicked the S&P500. Based on these returns, what would you expect to be the risk and reward if you held the S&P500 for one year? Assuming a risk-free rate of return of 3%/annum, what would be the proper estimate for a Sharpe ratio of daily, monthly, and annual returns? Is this consistent with the statistics in Table 14.8? Can you speculate why?

15·5. Summary

The chapter covered the following major points:

- Diversification—investment in different assets—reduces the overall risk (standard deviation).
- Diversification works better when assets are uncorrelated.
- The beta of a new asset (with respect to the existing portfolio) is a good measure of the marginal contribution of the new asset to the risk of the overall portfolio.
- Diversification works over time, too. The portfolio reward (expected rate of return) grows roughly linearly over time, but the portfolio risk (standard deviation) grows roughly with the square-root of time. That is, if the expected rate of return is 1% per month, then the T month expected rate of return is approximately $T \cdot 1\%$. If the standard deviation of the rate of rate of return is 10% per month, then the T month standard deviation is approximately $\sqrt{T} \cdot 10\%$. The latter formula is often used to "annualize" risk.

Appendix

A. Nerd Appendix: Investor Preferences Assumed Irrelevant

The start of this chapter stated that this book assumes that investors care only about the expected rate of return and the risk of their portfolios. They prefer more expected rate of return and less risk at a predetermined point in time. They care about no other characteristics of their portfolios, or the performance of their portfolio before or after this point in time. How innocuous are these assumptions?

What we have assumed about investor preferences.

First, we have assumed that investors do not care about anything other than returns. But, an investor could care about whether his portfolio companies invest ethically. Or, more basically, an investor may prefer higher rates of returns if he were to lose his job. He may prefer a portfolio with a lower mean (given the same standard deviation), just as long as the better outcomes occur in recessions.

What non-return preferences we have assumed away.

Second, it is also conceivable that investors might care about other portfolio characteristics. For example, the following two portfolios both have a mean return of 20% and a standard deviation of 20%.

What return preferences we have assumed away.

Pfio "Symmetric"	with 50% probability, a return of 0%	with 50% probability, a return of +40%
Pfio "Skewed"	with 33% probability, a return of –8%	with 67% probability, a return of +34%

However, the symmetric portfolio cannot lose money, while the skewed portfolio can. On the other hand, the skewed portfolio has the better return more frequently. By focusing only on mean and standard deviation, we have assumed away any preference between these two portfolios.

Third, we have assumed concern for the portfolio over more than just one time period. For example, if in the symmetric portfolio case, it were true that a return of 0% were always later followed by a return of 100%, while a return of +40% were always followed by an unavoidable return of –100% (nuclear war!), then an investor may not care about the return at the end of the first measurement period.

What return processes we have assumed away.

A. Q&A: Answers

1. The mean is 12%, the standard deviation is $30\%/\sqrt{N}$.
2. The mean is 12%. The variance now still has N variance terms, but $N \cdot (N-1)$ terms that are each $1/N \cdot 1/N \cdot 0.001$. Thus, the variance is now

$$\mathcal{V}ar(\tilde{r}_P) = N \cdot (1/N^2 \cdot (30\%)^2) + N \cdot (N-1) \cdot 1/N \cdot 1/N \cdot (0.0025) \quad . \tag{15.27}$$

Therefore, for 2 stocks, the variance is $1/2 \cdot .09 + 2 \cdot 1 \cdot (1/4 \cdot 0.0025) = 0.045 + 1/2 \cdot 0.0025 = 0.04625$. $Sdv = 21.5\%$. This is a little higher than the 21.2% from the previous question. For many stocks, $N \to \infty$ is

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_P) &= N \cdot (1/N^2 \cdot (30\%)^2) + N \cdot (N-1) \cdot 1/N \cdot 1/N \cdot (0.0025) \quad . \\ &\approx N \cdot (1/N^2 \cdot (30\%)^2) + \cdot (0.0025) \quad . \\ \lim_{N \to \infty} Sdv(\tilde{r}_P) &= \sqrt{0.0025} = 5\% \quad . \end{aligned} \tag{15.28}$$

3.

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_P) &= \mathcal{V}ar(1/2 \cdot \tilde{r}_1 + 1/2 \cdot \tilde{r}_2) \\ &= 1/2^2 \cdot \mathcal{V}ar(\tilde{r}_1) + 1/2^2 \cdot \mathcal{V}ar(\tilde{r}_2) + 2 \cdot 1/2 \cdot 1/2 \cdot Cov(\tilde{r}_1, \tilde{r}_2) \\ &= 1/4 \cdot 0.01 + 1/4 \cdot 0.01 + 2 \cdot 1/2 \cdot 1/2 \cdot (-0.01) \\ &= 1/2 \cdot 0.01 - 1/2 \cdot 0.01 = 0.00 \\ Sdv(\tilde{r}_P) &= \sqrt{\mathcal{V}ar(\tilde{r}_P)} = 0\% \quad . \end{aligned} \tag{15.29}$$

Therefore, with perfect negative correlation and equal investment weights, diversification works perfect!

4. The covariance is the correlation multiplied by the two standard deviations. For the q portfolio,

$$Cov(\tilde{r}_q, \tilde{r}_p) = Correlation(\tilde{r}_q, \tilde{r}_p) \cdot Sdv(\tilde{r}_q) \cdot Sdv(\tilde{r}_p) = 20\% \cdot 20\% \cdot 50\% = 0.02 \quad . \tag{15.30}$$

$$\beta_{\tilde{r}_q, \tilde{r}_p} = \frac{Cov(\tilde{r}_q, \tilde{r}_p)}{\mathcal{V}ar(\tilde{r}_p)} = \frac{0.02}{0.04} = 0.5 \quad . \tag{15.31}$$

Therefore, the new portfolio with weights of 0.5 each is

$$\begin{aligned} \tilde{r} &= 0.5 \cdot \tilde{r}_p + 0.5 \cdot \tilde{r}_q \quad . \\ \mathcal{V}ar(\tilde{r}) &= w_p \cdot \mathcal{V}ar(\tilde{r}_p) + w_q \cdot \mathcal{V}ar(\tilde{r}_q) + 2 \cdot w_q \cdot w_p Cov(\tilde{r}_q, \tilde{r}_p) \\ &= 0.5^2 \cdot (20\%)^2 + 0.5^2 \cdot (50\%)^2 + 2 \cdot 0.5 \cdot 0.5 \cdot 0.02 \\ &= 0.01 + 0.0625 + 0.01 = 0.0825 \quad . \\ Sdv(\tilde{r}) &= 28.7\% \quad . \end{aligned} \tag{15.32}$$

For the n portfolio,

$$Cov(\tilde{r}_n, \tilde{r}_p) = Correlation(\tilde{r}_n, \tilde{r}_p) \cdot Sdv(\tilde{r}_n) \cdot Sdv(\tilde{r}_p) = 20\% \cdot 20\% \cdot 500\% = 0.2 \ . \tag{15.33}$$

$$\beta_{\tilde{r}_n, \tilde{r}_p} = \frac{Cov(\tilde{r}_n, \tilde{r}_p)}{\mathcal{V}ar(\tilde{r}_p)} = \frac{0.2}{0.04} = 5 \ . \tag{15.34}$$

Therefore, the new portfolio with weights of 0.5 each is

$$\begin{aligned} \tilde{r} &= 0.5 \cdot \tilde{r}_p + 0.5 \cdot \tilde{r}_n \ . \\ \mathcal{V}ar(\tilde{r}) &= w_p \cdot \mathcal{V}ar(\tilde{r}_p) + w_q \cdot \mathcal{V}ar(\tilde{r}_q) + 2 \cdot w_q \cdot w_p Cov(\tilde{r}_q, \tilde{r}_p) \\ &= 0.5^2 \cdot (20\%)^2 + 0.5^2 \cdot (500\%)^2 + 2 \cdot 0.5 \cdot 0.5 \cdot 0.2 \\ &= 0.01 + 6.25 + 0.1 = 6.36 \ . \\ Sdv(\tilde{r}) &= 252.2\% \ . \end{aligned} \tag{15.35}$$

Even though n has an equal correlation with portfolio p, its diversification aid is overwhelmed by its scale: n is just so much riskier than q. Thus, the portfolio with n is definitely more risky than the portfolio with q. This lesser diversification effect is reflected in the (higher) covariance and the (higher) beta with our portfolio p, but not in the (equal) correlation.

5. This is not a question the reader might necessarily know how to answer. However, it should get the reader to think. Exxon is indeed a low-risk stock. However, it is also a low mean stock. An investor would have done a little better purchasing the diversified Dow-30 index (remember: indexes do not count dividends!). However, it is reasonably likely that in the future, the diversified Dow-Jones 30 index will have lower risk than Exxon. More than likely, Exxon was just lucky over the sample period. For example, oil prices were relatively stable.
6. There is almost no difference in risk between these two indexes. To reduce risk further, one should instead invest in asset classes that are different from large corporate equities.
7. They allow an investor with a limited amount of money to own a large number of securities, without incurring large transaction costs.
8. No. There are many alternative asset classes, such as commodities, art, and real estate, etc..
9. That they are zero. If they were not zero, it would mean that an investor could use past returns to predict future returns. Presumably, this would aid the investor to earn extra returns.
10. The standard deviation is likely to be

$$\begin{aligned} Sdv_{\text{annual}}(\tilde{r}) &\approx \sqrt{T} \cdot Sdv_{\text{daily}}(\tilde{r}) \\ 20\% &\approx \sqrt{255} \cdot Sdv_{\text{daily}}(\tilde{r}) \\ \Rightarrow \quad Sdv_{\text{daily}}(\tilde{r}) &\approx 1.25\% \ . \end{aligned} \tag{15.36}$$

This corresponds to a typical daily movement of about 11 points. In statistical terms, if the S&P500 is about normally distributed, about $^2/_3$ of all days, it should move up or down no more than 11 points. In about $^9/_{10}$ of all days, it should move up or down no more than 22 points.

11. $\sqrt{12} \cdot 5\% \approx 17.3\%$.
12. For the annual Sharpe ratio, we can use the numbers in the Table 15.1:

$$\text{Annual Sharpe Ratio} = \frac{6.3\% - 3.0\%}{18.7\%} \approx 0.18 \ . \tag{15.37}$$

For the monthly Sharpe ratio, we need to de-annualize the mean and standard deviation. The excess mean is $3.3\%/12 \approx 0.275\%$. The monthly standard deviation is about $18.7\%/\sqrt{12} \approx 5.4\%$. Therefore, the monthly

$$\text{Monthly Sharpe Ratio} = \frac{0.275\%}{5.4\%} \approx 0.05 \quad . \tag{15.38}$$

13. $\sqrt{255} \cdot 1\% \approx 16.0\%$.

14. Over 10 real days, you would have earned a compound rate of return of $988.61/967 - 1 = 2.2\%$. With 36.5 such 10-day periods over the year, the compound annual return would have been 124%. As to the daily rates of return, they were 0.47%, 1.51%, 0.40%, –0.24%, –1.20%, 0.92%, 1.28%, 1.03%, –1.00%. The simple arithmetic average rate of return was an even higher 0.35%/day (3.5% over the 10 days). Clearly, this was a great ten days, not likely to repeat. So, I would not trust these 10 day means. The standard deviation is 0.97%/day. This indicates an annualized standard deviation of about $\sqrt{365} \cdot 0.97\% \approx 18.5\%$. (Over 10 days, the estimated standard deviation is 3.07%/(**10 days**).) As is fairly common, the annualized standard deviation is fairly reasonable. Finally a risk-free rate of return of 3% per annum is less than 0.01%./day. The Sharpe-ratio would therefore be approximately $(0.35\% - 0.01\%)/0.97\% \approx 0.35$. Quoted in annual terms, the Sharpe ratio would be approximately $(0.35\% - 0.01\%) \cdot 365/(0.97\% \cdot \sqrt{365}) = \sqrt{365} \cdot 0.35 \approx 6.7$. Quoted in monthly terms, it would be $\sqrt{30} \cdot 0.35 \approx 1.9$.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 16

The Optimally Diversified Portfolio

How much of each security?

last file change: Jun 15, 2004 (11:49h)

last major edit: n/a

This chapter appears in the Survey text only.

We have already established that

1. Diversification reduces risk.
2. The covariance between investment asset returns determines the effectiveness of diversification. Beta is an equally good measure.
3. Because risk typically increases "with the square root," it is especially important to have at least a handful and better a dozen of very different types of assets.
4. Publicly traded stocks are mutually positively correlated, but not perfectly so.

But we have not yet determined the optimal portfolio weights that you should assign to individual assets in their portfolios. For example, should you purchase equal amounts in every security? Should you purchase relatively more of securities with higher expected rates of returns, lower variances, or lower covariances? This chapter answers these questions.

16·1. The Mean-Variance Efficient Frontier

We know that diversification reduces risk, which investors like—but this does not tell us how much our investors should purchase in each security. It may be better to purchase 30% in A and 70% in B, than 50% in each. So how do we determine our best investment weights?

16·1.A. Two Risky Securities

For two securities, compute the expected rate of return and risk computations for possible portfolios.

Start with only two possible investment securities. You have only one decision to make: how much to invest in the first security. (The remainder is automatically allocated to the second security). For example, assume that each security has a risk (standard deviation) of 10%. The first security, call it A, has an expected rate of return of 10%, and the second security, call it B, has an expected rate of return of 5%. Should you ever choose to purchase the second security, which is so obviously inferior to the first?

Recap the formulas we will use.

The formulas for the portfolio risk and reward are

$$\begin{aligned} \mathcal{E}(\tilde{r}_p) &= w_A \cdot 10\% + (1 - w_A) \cdot 5\% \\ \mathcal{E}(\tilde{r}_p) &= w_A \cdot \mathcal{E}(\tilde{r}_A) + (1 - w_A) \cdot \mathcal{E}(\tilde{r}_B) \quad , \end{aligned} \tag{16.1}$$

$$\begin{aligned} Sdv(\tilde{r}_p) &= \sqrt{w_A^2 \cdot 0.01 + (1 - w_A)^2 \cdot 0.01 + 2 \cdot w_A \cdot (1 - w_A) \cdot Cov(\tilde{r}_A, \tilde{r}_B)} \\ Sdv(\tilde{r}_p) &= \sqrt{w_A^2 \cdot \mathcal{V}ar(\tilde{r}_A) + (1 - w_A)^2 \cdot \mathcal{V}ar(\tilde{r}_B) + 2 \cdot w_A \cdot (1 - w_A) \cdot Cov(\tilde{r}_A, \tilde{r}_B)} \quad . \end{aligned} \tag{16.2}$$

where w_A is any investment weight you may choose to place in the first security.

Table 16.1. Portfolio Characteristics For Various Portfolios, w_A. (Under Various Covariance Scenarios.)

			Correlation is				
			−1.00	−0.75	0.00	+0.75	+1.00
			Risk (*Sdv*) if Covariance is				
w_A	$1 - w_A$	$\mathcal{E}(\tilde{r}_p)$	−0.01	−0.0075	0	+0.0075	+0.01
1	0	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
0	1	5.0%	10.0%	10.0%	10.0%	10.0%	10.0%
1/5	4/5	**6.0%**	6.0%	6.6%	8.2%	**9.6%**	10.0%
1/3	2/3	6.8%	3.3%	4.7%	7.5%	9.4%	10.0%
1/2	1/2	7.5%	0.0%	3.5%	7.1%	9.3%	10.0%
2/3	1/3	8.3%	3.3%	4.7%	7.5%	9.4%	10.0%
4/5	1/5	9.0%	6.0%	6.6%	8.2%	9.6%	10.0%
Portfolios Involving Shorting							
−1	+2	0.0%	30.0%	28.3%	22.4%	14.1%	10.0%
+2	−1	15.0%	30.0%	28.3%	22.4%	14.1%	10.0%

Now, assume a particular covariance, say $Cov(\tilde{r}_A, \tilde{r}_B)$ of 0.0075. (Below, we try other covariance scenarios, too.) What are the achievable risks and rewards? There is only one decision variable, the weight w_A, but you can choose any weight you desire. For example, the third row (columns 3 and 7) in Table 16.1 shows that the portfolio p that has $w_A = 1/5$ (and thus $w_B = 4/5$) has an expected rate of return of 6.0% and a standard deviation of 9.6%. Table 16.1 shows this risk/reward "performance pair." Other columns show portfolio risk under some other covariance assumptions.

We can compute the possible portfolio choices.

Figure 16.1 on Page 356 plots the data in Table 16.1 in a coordinate systems, in which the overall portfolio standard deviation is on the x-Axis, and the overall portfolio expected rate of return on the y-Axis. Each coordinate (point) represents a single portfolio, with its particular risk and reward characteristic. Verify visually that in the lower left graph, which uses the assumed covariance of 0.0075, there is indeed a point at $x = 9.6\%$ and $y = 6.0\%$. Tracing out all possible portfolios (i.e., weights w_A) produces a curve. In this example, the smallest risk always obtains when the two securities are equally weighted. Above and below $w_A = 1/2$, the risk steadily increases. Also, regardless of covariance, when you choose only one or the other security ($w_A = 0$ or $w_A = 1$), the portfolio risk is 10% (as was assumed).

We can also draw each portfolio in a coordinate system of risk vs. reward. Each portfolio is one point in x-y space.

SIDE NOTE: The traditional graph draws the standard deviation on the "risk" or x axis. Still, it is common to call this the "mean-variance graph" instead of the more correct "mean-standard deviation graph."

Repeating the exercise for different covariance scenarios, our earlier insight that lower covariance helps diversification also shows up in both Table 16.1 and Figure 16.1. For example, if the two securities are perfectly negatively correlated, which implies a covariance of -0.01, then an equal-weighted portfolio of the two securities has zero risk. (When one security's value increases, the other security's value decreases by an equal amount, thereby eradicating any risk.) If the two securities are perfectly positively correlated, diversification does nothing, and the portfolio standard deviation remains at 10% no matter what weights are chosen.

Higher correlations "compress" the function.

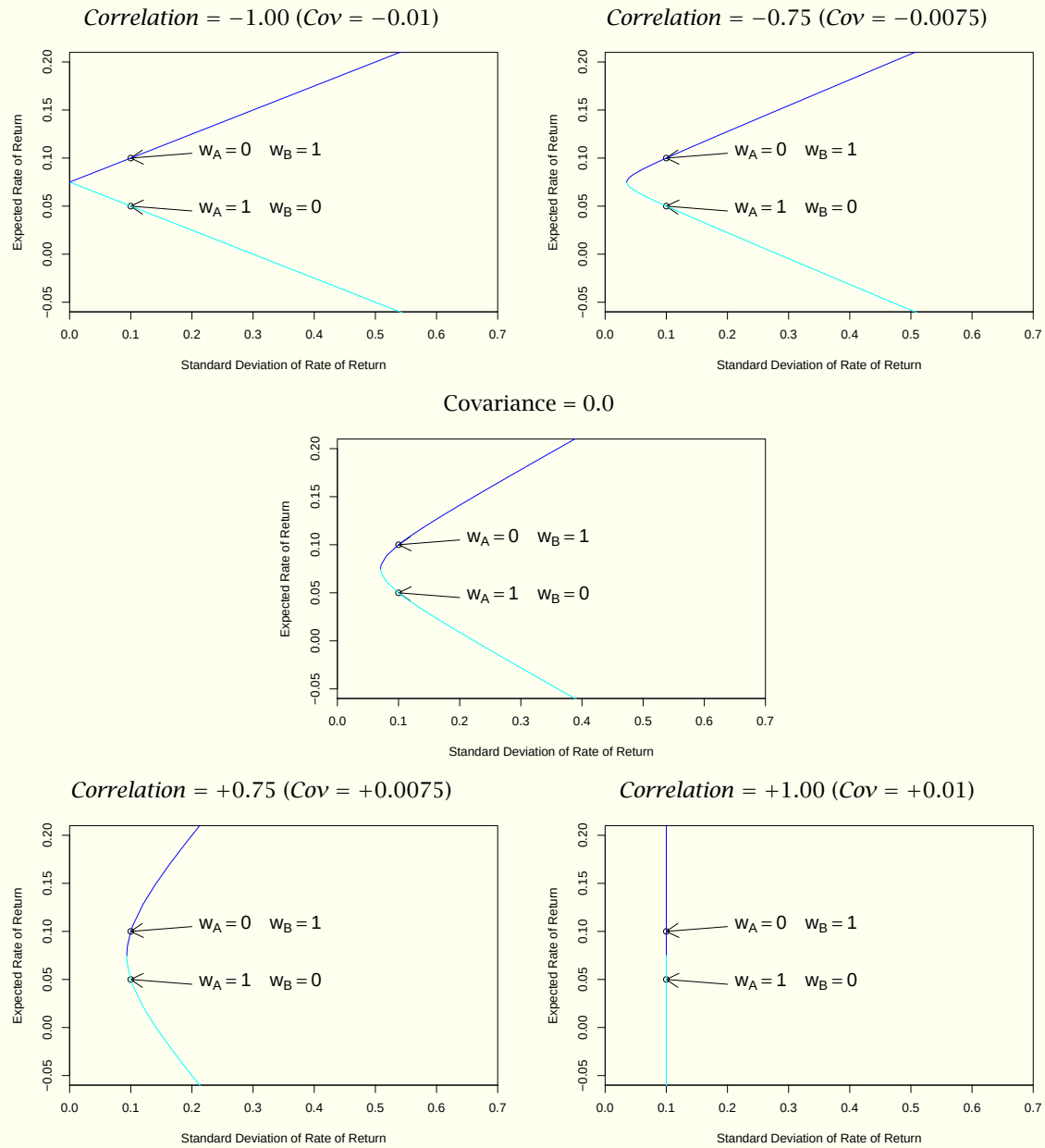
We can say more about where portfolios lie on the curves. Portfolios with similar weights lie close to one another in mean-variance space: after all, their expected rates of return and standard deviations are similar. You can even visually judge where a given portfolio combination must lie. For example, a portfolio with 95% weight on the first security and 5% weight on the second security would have to lie 95% of the way between the two on any of the curves, i.e., rather close to the $w_A = 1$ portfolio.

Similar portfolios are close to one another.

All portfolios that have positive weights in both securities lie between the two portfolios marked by arrows ($w_A = 1, w_B = 0$ and $w_A = 0, w_B = 1$). But Table 16.1 and Figure 16.1 also show portfolios that involve shorting (Section 13·2.B). For example, say you have \$200, but you want to invest \$300 in security 1. This means that you have to short \$100 in security 2, in order to obtain the money to have the full \$300 for security 1. Your investment weights are now $w_A = +150\%$ and $w_B = -50\%$. You can verify that this portfolio, and any other portfolio that involves shorting security 2, lie above the upper arrows in Figure 16.1.

Where are portfolios with both long and short positions?

Figure 16.1. Achievable Mean-Variance Combinations



The highest achievable expected rates of returns for each choice of risk is called the "Mean-Variance Efficient Frontier."

Depending on your risk tolerance, you may choose a more risky or a less risky portfolio. However, if you are sane, you will only choose from among the portfolios on the upper half of the hyperbola. They give the highest reward for a given amount of risk. This set of points/portfolios is called the **Mean-Variance Efficient Frontier**, or, in brief, the **Efficient Frontier**. This will be used so often that we shall abbreviate "Mean-Variance Efficient" as MVE.

SIDE NOTE: The phrase **Efficient Frontier** (more precisely, **Mean-Variance Efficient Frontier**) here has nothing to do with the concept of **Efficient Markets** in Chapter 19. The word *Efficient* may be the most overused phrase in Economics.

NERD NOTE: *We can solve for the functional form of all weighted combinations of two portfolios by substituting out w_A from Formula 16.1 (portfolio mean) and Formula 16.2 (portfolio standard deviation). The result is*

$$Sdv[\ \mathcal{E}(\tilde{r}_P)\ ]^2 = a \cdot \mathcal{E}(\tilde{r}_P)^2 + b \cdot \mathcal{E}(\tilde{r}_P) + c \quad , \tag{16.3}$$

where

$$\begin{aligned}
a &\equiv d^{-1} \cdot \mathcal{V}ar(\tilde{r}_A) - 2 \cdot Cov(\tilde{r}_A, \tilde{r}_B) + \mathcal{V}ar(\tilde{r}_B) \quad , \\
b &\equiv d^{-1} \cdot 2 \cdot \mathcal{E}(\tilde{r}_A) \cdot [Cov(\tilde{r}_A, \tilde{r}_B) - \mathcal{V}ar(\tilde{r}_B)] \\
&\quad + d^{-1} \cdot 2 \cdot \mathcal{E}(\tilde{r}_B) \cdot [Cov(\tilde{r}_A, \tilde{r}_B) - \mathcal{V}ar(\tilde{r}_A)] \quad , \\
c &\equiv d^{-1} \cdot \mathcal{E}(\tilde{r}_B)^2 \cdot \mathcal{V}ar(\tilde{r}_A) + \mathcal{E}(\tilde{r}_A)^2 \cdot \mathcal{V}ar(\tilde{r}_B) \\
&\quad - d^{-1} \cdot 2 \cdot \mathcal{E}(\tilde{r}_B) \cdot \mathcal{E}(\tilde{r}_A) \cdot Cov(\tilde{r}_A, \tilde{r}_B) \quad , \\
d &\equiv [\mathcal{E}(\tilde{r}_A) - \mathcal{E}(\tilde{r}_B)]^2 \quad .
\end{aligned} \tag{16.4}$$

*This formula states that the variance of the rate of return ($Sdv(\tilde{r}_P)^2$) on an arbitrary weighted portfolio P is a quadratic formula in its expected rate of return ($\mathcal{E}(\tilde{r}_P)$): a **Parabola**. Therefore, the MVE Frontier function is a **Hyperbola** in a graph of the expected rate of return against their standard deviations. Although this is by no means obvious, it turns out that this is the case even if there are more than 2 securities: the MVE Frontier is always a hyperbola. This only breaks down when there are additional constraints, such as short-sales constraints.*

Solve Now!

Q 16.1 *In the example in Table 16.1, if the covariance is -0.01, is there an arbitrage opportunity?*

Q 16.2 *Assuming no transaction and shorting costs, does going short a security and going long a security produce a risk-free investment? If so, and if there is another risk-free security, can it be used to construct an arbitrage opportunity?*

Q 16.3 *In the example, with risk of 10% for each security, compute the standard deviations of various portfolios' returns if the covariance between the two securities is +0.005. Graph the mean against the standard deviation.*

Q 16.4 *Security 1 has a risk (standard deviation) of 20% and an expected rate of return of 6%; Security 2 has a risk of 30% and an expected rate of return of 10%. Assume the two securities have +0.80 correlation. Draw the MVE Frontier.*

16·1.B. Many Risky Securities

With more securities, the computations become cumbersome...

The MVE Frontier is easy to compute when you can only choose one investment weight, which then determines your relative allocation between the two assets. If you can choose from three securities, you must consider combinations of two portfolios weights (the third weight is one minus the other two). This is a two-dimensional optimization problem. With four securities, you have three portfolio weights to determine, and so on.

...but the principle remains the same.

The portfolio selection principle remains the same: each portfolio is a point in the graph of portfolio means (expected rates of return) vs. portfolio standard deviation. After plotting all possible portfolios, you should choose from those points (portfolios) that lie on the MVE Frontier, the upper half of the hyperbola.

Return to our three stocks.

We can illustrate the portfolio choice by trying many random investment portfolios from three stocks from Page 291, S&P500, IBM, Sony, and selecting the best ones. Assume that the historical annual rates of return from 1991–2002 are representative of the future. They were

		Covariance between $\tilde{r}_i$ and $\tilde{r}_j$		
Security	$\mathcal{E}(\tilde{r}_i)$	1=S&P500	2=IBM	3=Sony
1=S&P500	10.110%	3.6224%	3.2980%	4.7716%
2= IBM	15.379%	3.2980%	15.0345%	2.1842%
3= Sony	24.203%	4.7716%	2.1842%	81.4886%

What would the risk-reward tradeoffs of these portfolio combinations have looked like? Recall the necessary Formula 14.31 and Formula 14.33 (Page 308). With three securities, the formulas to compute the overall portfolio mean and standard deviation are

$$\begin{aligned}
\mathcal{E}(\tilde{r}_p) &= w_1 \cdot \mathcal{E}(\tilde{r}_1) + w_2 \cdot \mathcal{E}(\tilde{r}_2) + w_3 \cdot \mathcal{E}(\tilde{r}_3) \quad , \\
\mathcal{V}ar(\tilde{r}_p) &= w_1^2 \cdot \mathcal{V}ar(\tilde{r}_1) + w_2^2 \cdot \mathcal{V}ar(\tilde{r}_2) + w_3^2 \cdot \mathcal{V}ar(\tilde{r}_3) \\
&\quad +2 \cdot w_1 \cdot w_2 \cdot Cov(\tilde{r}_1, \tilde{r}_2) + 2 \cdot w_1 \cdot w_3 \cdot Cov(\tilde{r}_1, \tilde{r}_3) \quad , \\
&\quad +2 \cdot w_2 \cdot w_3 \cdot Cov(\tilde{r}_2, \tilde{r}_3) \quad , \\
\text{and } w_3 &\equiv 1 - w_1 - w_2 \quad .
\end{aligned} \tag{16.5}$$

NERD NOTE: *If you use Excel to check the historical calculations, your numbers will be a little smaller. Excel computes so-called population covariances, rather than the so-called sample covariances that we are using. (If Excel were consistent, it would name its population covariance function "covp" instead of "cov.") The difference is that the former divides by N, while the latter divides by $N-1$. For large data sets, it does not matter whether you divide by N or $N-1$; but in our small demonstration data set, it does.*

Pick some portfolios and use the formulas.

Armed with formulas and statistics, we can now determine portfolio means and standard deviations for some portfolios ($w_{\text{S\&P500}}, w_{\text{IBM}}, w_{\text{Sony}}$). For example, for the portfolio that invests 20% in S&P500, 40% in IBM, and 40% in Sony,

$$\begin{aligned}
\mathcal{E}(\tilde{r}_p) &\approx 20\% \cdot 10.110\% + 40\% \cdot 15.379\% + 40\% \cdot 24.203\% \approx 17.85\% \quad , \\
\mathcal{E}(\tilde{r}_p) &= w_1 \cdot \mathcal{E}(\tilde{r}_1) + w_2 \cdot \mathcal{E}(\tilde{r}_2) + w_3 \cdot \mathcal{E}(\tilde{r}_3) \quad ,
\end{aligned} \tag{16.6}$$

and

$$\begin{aligned}
\mathcal{V}ar(\tilde{r}_p) &\approx (20\%)^2 \cdot 3.6224\% + (40\%)^2 \cdot 15.0345\% + (40\%)^2 \cdot 81.4886\% \\
&\quad +2 \cdot (20\%) \cdot (40\%) \cdot 3.2980\% + 2 \cdot (20\%) \cdot (40\%) \cdot 4.7716\%, \\
&\quad +2 \cdot (40\%) \cdot (40\%) \cdot 2.1842\% \quad , \\
\mathcal{V}ar(\tilde{r}_p) &= w_1^2 \cdot \mathcal{V}ar(\tilde{r}_1) + w_2^2 \cdot \mathcal{V}ar(\tilde{r}_2) + w_3^2 \cdot \mathcal{V}ar(\tilde{r}_3) \\
&\quad +2 \cdot w_1 \cdot w_2 \cdot Cov(\tilde{r}_1, \tilde{r}_2) + 2 \cdot w_1 \cdot w_3 \cdot Cov(\tilde{r}_1, \tilde{r}_3) \quad , \\
&\quad +2 \cdot w_2 \cdot w_3 \cdot Cov(\tilde{r}_2, \tilde{r}_3) \quad .
\end{aligned} \tag{16.7}$$

which comes to $(41.93\%)^2$. Table 16.2 lists some more portfolio combinations.

Table 16.2. Risk and Reward of Some Portfolio Involving Only S&P500, IBM, and Sony

$(w_{\text{S\&P500}}, w_{\text{IBM}}, w_{\text{Sony}})$	Mean	*Sdv*	$(w_{\text{S\&P500}}, w_{\text{IBM}}, w_{\text{Sony}})$	Mean	*Sdv*
(1,0,0)	0.1010	0.1903	(-1,1,1)	0.2947	0.9401
(0,1,0)	0.1537	0.3877	(1,-1,1)	0.1893	0.9936
(0,0,1)	0.2420	0.9027	(1,1,-1)	0.0128	0.9635
(0.5,0.5,0)	0.1274	0.2513	(1,0.5,-0.5)	0.0569	0.5018
(0.5,0,0.5)	0.1715	0.4865	(0.5,1,-0.5)	0.0833	0.5919
(0,0.5,0.5)	0.1979	0.5022	(0.5,-0.5,1)	0.2156	0.9332
(0.2,0.4,0.4)	0.1785	0.4193	(-0.5,0.5,1)	0.2683	0.9051
(0.4,0.2,0.4)	0.1680	0.4077	(0.4,0.4,0.2)	0.1503	0.2901

Mean and *Sdv* are quoted in percent per annum—and based on historical data, although we presume them to be representative of the future.

Figure 16.2 plots the data from Table 16.2. Looking at the set of choices, if you were extremely risk averse, you might have chosen just to invest in (a portfolio that was almost all) S&P500. If you are more risk tolerant, you might have chosen a portfolio that invested 20% in the S&P500, 40% in IBM, and 40% in Sony. However, regardless of your risk aversion, the (0.5,0,0.5) portfolio, which invested 50% in S&P500, zero in IBM, and 50% in Sony, would have been a poor choice: it would have had a lower portfolio mean *and* higher standard deviation than alternatives, such as the (0.2,0.4,0.4) portfolio.

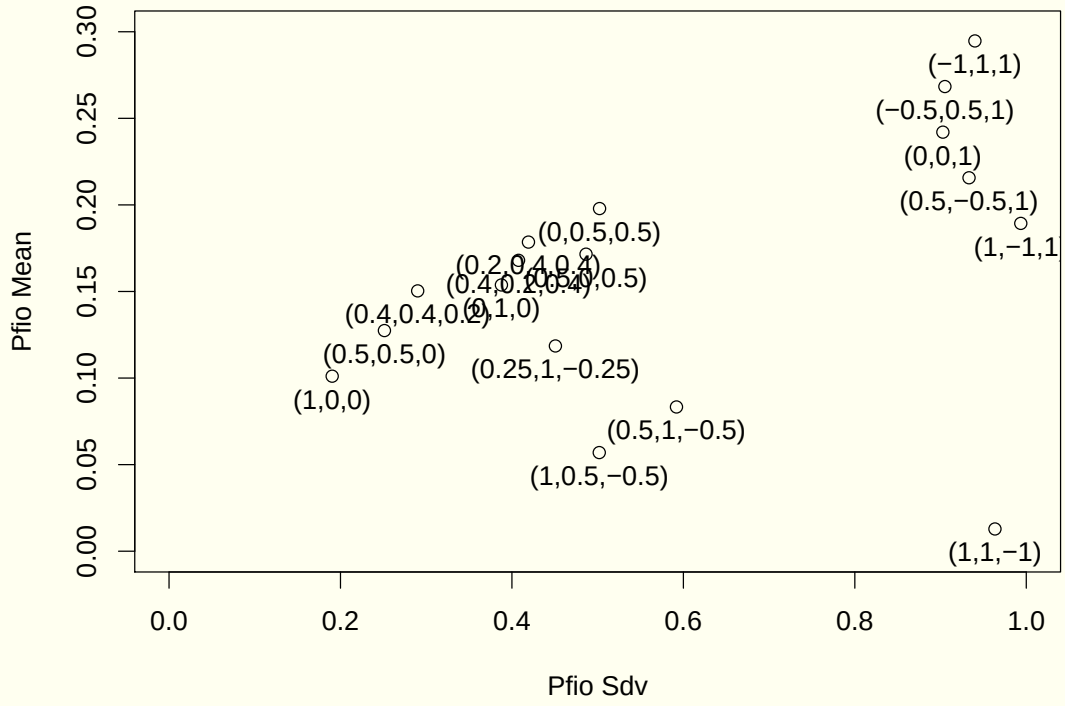
Plot the mean and standard deviations of these portfolios.

Note also that again, portfolios with very similar portfolio weights have similar means and standard deviations, and therefore lie close to one another. For example, the portfolio that invests (0.5,0.5,0) is relatively close to the portfolio (0.4,0.4,0.2). The so-called **Minimum Variance Portfolio** among these 16 portfolios seems to be the pure play in the S&P500.

Similarity in portfolio weights means proximity in the Figure.

SIDE NOTE: The technique for obtaining the MVE Frontier can remain the same for more than three securities: select many possible portfolio combinations and plot the outcomes. However, the number of possible portfolio weights becomes quickly overwhelming. If the portfolio optimization is done through such trial-and-error, sometimes called **Monte-Carlo Simulation**, it would be better to draw not just totally random portfolio weights, but to draw portfolio weights that lie closer to the best portfolio combinations already obtained. In this case, you can use a more convenient way to obtain the MVE Frontier, using matrix algebra. This is done in Nerd Appendix a.

Figure 16.2. The S&P500, IBM, and Sony Portfolio Combinations From Table 16.2

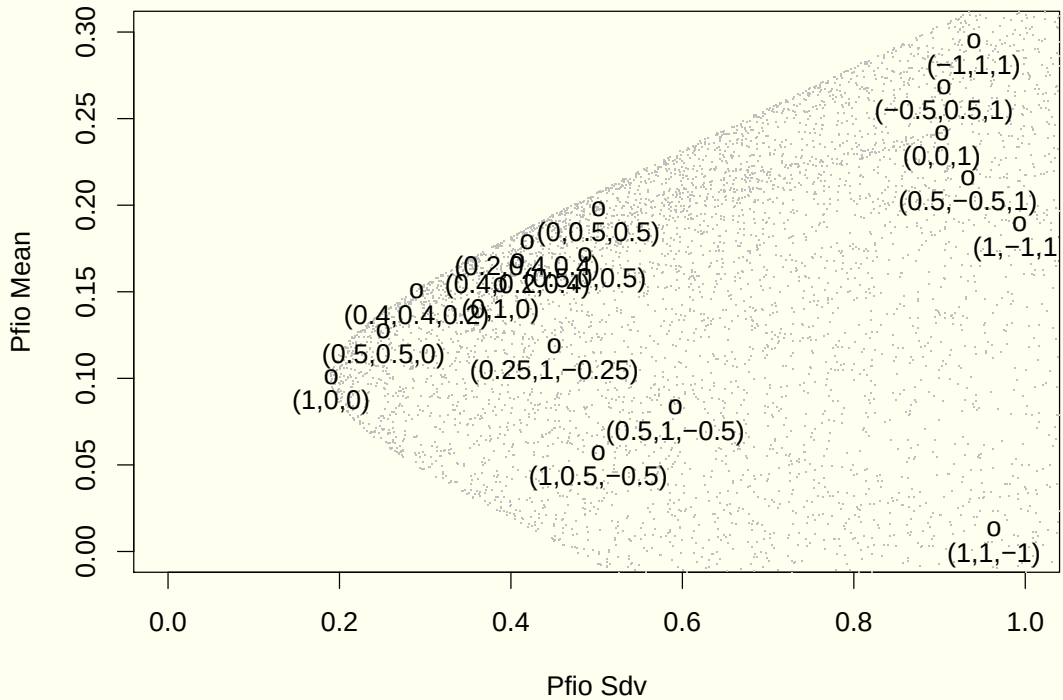


Numbers in parentheses are the investment weights in the three portfolio assets (S&P500, IBM, Sony). For example, a \$200 portfolio that invests \$200 in S&P500, \$200 in IBM, and -\$200 in Sony has a mean of about 2% and a standard deviation of about 90%, which places it in the lower right corner.

Plotting the mean and standard deviations of many random portfolios shows that the set of feasible portfolios produces a cloud of points, with a distinct border: the MVE Frontier.

Figure 16.3 plots 10,000 further portfolios, randomly chosen. The possible investment choices are no longer the line that they were with two assets, but a cloud of points. Nevertheless, the MVE Frontier looks remarkably similar to the shape it had with just two assets. You should never purchase a portfolio that is a point inside the cloud but not on the frontier: there are better portfolios with higher rewards and lower risks towards its upper left. The minimum variance portfolio is still a portfolio that is mostly invested in the S&P500, though a magnifying glass reveals that adding a little of IBM and shorting a little of Sony is better—specifically, 99% of S&P500, 2.4% of IBM, and −1.4% of Sony would reach as low a risk as 18.96%.

Figure 16.3. 10,000 Randomly Chosen Portfolios Involving Only S&P500, IBM, and Sony



Numbers in parentheses are the investment weights in the three portfolio assets (S&P500, IBM, Sony). For example, a \$200 portfolio that invests \$100 in S&P500, \$200 in IBM, and -\$100 in Sony has a mean of about 8% and a standard deviation of about 60%, which places it about in the middle of the grey area. If we randomly drew another trillion portfolios, the entire region inside the hyperbola would be grey. That is, for any mean/standard deviation combination that is inside the hyperbola, you can find a set of portfolio weights that achieve it.

Please keep in mind that this plot is *ex-post*, i.e., based on historical data. Our real interest is of course not the past, but the future. Unfortunately, we do not have a much better choice than to rely on history. In the real world, where we would use many more securities, the historical MVE Frontier would tend to be *indicative* of the future MVE Frontier, but not a perfect predictor thereof. Use common sense!

The historical MVE frontier, usually used, is not the same as what we really want to know: the future MVE frontier.

SIDE NOTE: The historical portfolio mean estimates tend to be less reliable than the portfolio standard deviation estimates. Perhaps not surprisingly, academic research has found that the minimum variance portfolio (among broadly diversified portfolios) tends to outperform other portfolio choices. That is, not only is its risk lower than that of other portfolios (as might have been expected), but it seems as if its actual expected rate of return has beaten its predicted rate of return out-of-sample.

Solve Now!

Q 16.5 *For the three stocks, in Excel, randomly draw 1,000 random investment weights into the first security; 1,000 weights into the second security; and compute 1 minus these two weights to be the investment weight into the third security. Use the formulas and the following table to compute the risk and reward of each of the 1,000 portfolios in two separate columns.*

		Covariance between $\tilde{r}_i$ and $\tilde{r}_j$		
Security	$\mathcal{E}(\tilde{r}_i)$	1	2	3
1	10.0%	0.010	0.001	0.003
2	5.0%	0.001	0.010	0.006
3	7.5%	0.003	0.006	0.010

Now plot the means and standard deviations of each portfolio in an X-Y graph.

Q 16.6 *Continued: What is the* ***Minimum Variance Portfolio****?*

Q 16.7 *Continued: What is the best portfolio with an expected rate of return of about 11%?*

Q 16.8 *Continued: Would any investor purchase the portfolio (-12%,32%,81%) ?*

16·2. Real-World MVE Frontier Implementation Problems

Figure 16.3 also shows the practical drawback of MVE portfolios. Many points on the MVE Frontier require shorting of securities (the weight is negative), but this may or may not be possible in the real world. Even if modest shorting is possible, the portfolio optimization often recommends strange portfolios that suggest shorting huge amounts in one security in order to go long a huge amount in another security.

Unfortunately, many portfolios on the MVE Frontier require shorting securities, and often massive amounts so.

Chances are that this is not because these are great portfolios, but because the historical covariance and mean estimates are not perfect predictors of future covariances and means. The historical covariance estimates cannot be easily relied upon, not so much because they are bad in themselves in an absolute sense, but because the optimization technique is very sensitive to *any* covariance estimation errors. This can be explained with a hypothetical bet optimizer: assume that you throw a coin 100 times, and observed 51 heads. A naïve portfolio optimizer relying on historical realizations would determine that the "heads bet" is much better than the "tails bet," and might recommend betting a million dollars on heads and against tails. This is not because the bet optimizer has made a mistake—after all, if it were truly 51%, this would be a pretty good bet. It is also not because 51% is a bad estimate of the true probability (of 50% if the coin is fair). Instead, it is the interaction: the bet optimizer is very sensitive to historical sample variation.

Here is why.

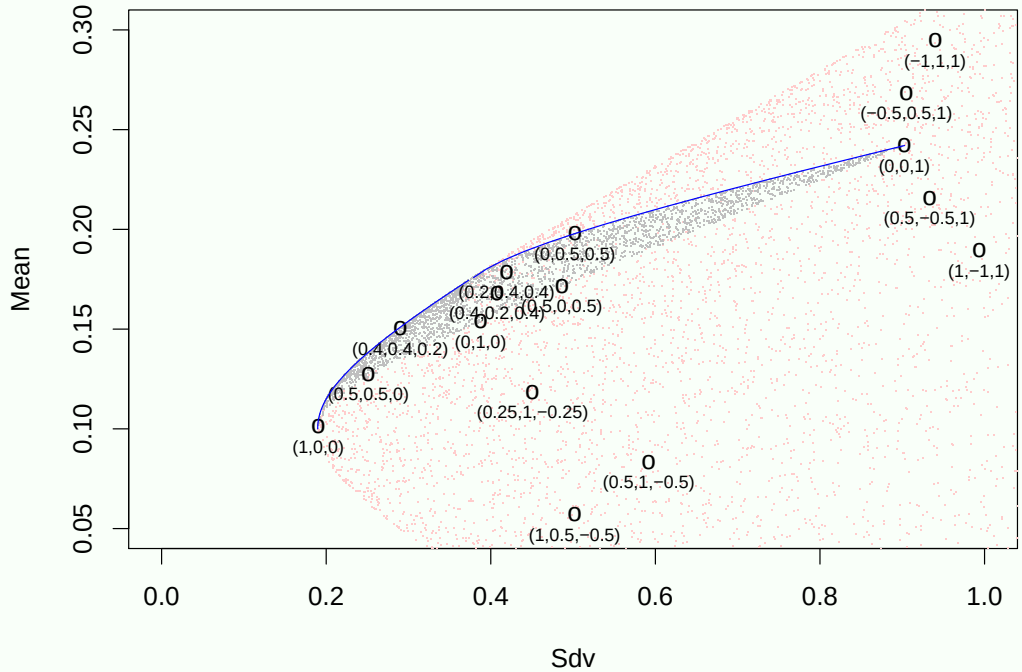
This suggests two methods to address this problem in order to make portfolio optimization a useful tool. Both rely on complicated statistical analysis, but simple ideas:

There are two solutions to the problem.

1. You can improve the estimates of future covariances and means, and not just rely blindly on their historical equivalents. In essence, these improvements rely on techniques that try to "pull in" extreme outlier returns. In the end, these techniques usually yield decent variance and covariance estimates. However, these techniques often still fail in producing reliable *expected* rate of return estimates. So, definitely, do not believe the historical mean rates of return to be representative of future mean rates of return. Noone really knows how to estimate future expected rates of return well.

2. You can use a portfolio optimizer that restricts the amount that can be shorted. This can be done by assuming a (high) cost of shorting, or by disallowing short-sales altogether.

NERD NOTE:



The portfolios with only positive investment weights (no shorting) are highlighted in grey in the graph on the right. It shows that an investor who is not permitted to short assets has a different MVE frontier. Although this particular graph fails to show this, the short-free frontier with many assets is usually entirely below (inside) the unrestricted MVE Frontier, because all MVE portfolios want to short at least one stock.

There is not enough data to estimate a good covariance matrix for 10,000 securities.

There is a closely related other problem with optimization. To obtain meaningful results, you need to have at least as many time periods (return observations) as there are terms in the covariance formula, just as you would need at least as many equations as you have unknowns to pin down a system of equations. Alas, with 10,000 securities, there are 50 million terms in the covariance formula. If you use daily returns, you would have to wait 196 thousand years to have just one data point per estimated covariance. (Mathematically, you could do the estimation with 10,000 daily data points—still 40 years. These would be fairly unreliable, of course.) Therefore, you cannot reliably estimate a good variance-covariance matrix with historical data for too many assets.

Restrict yourself to larger classes.

And, therefore, mean-variance optimization can only be used when there are only a few portfolios, preferably broad asset-class portfolios, to choose from. Fortunately, such broad asset-class portfolios also tend to have low *and more reliable* historical variances and covariances estimates. Unfortunately, you must narrow down your investment choices into a small number of asset class portfolios *before* you can use the MVE Frontier toolbox.

Where to use this technique!

In sum, portfolio optimization is a very usable technique, as long as you restrict yourself to just a few big asset classes for which you have good historical data. You should not use mean-variance optimization with poor variance-covariance estimates. If you try to use it for individual stocks, and/or when you do not have long historical returns data, chances are that your results will not be very satisfying.

ANECDOTE: Mean-Variance Optimization in Practice
Mean-variance optimization techniques, such as those discussed in this section, are in common use among professional investors. For example, a **Fund of Funds** itself invests in other **Hedge Funds**. After determining their hedge fund investment candidates, fund of funds usually compute the mean-variance frontier among their candidates, and allocate money accordingly.

T: ***This could be a sidenote instead***

16·3. COMBINATIONS OF PORTFOLIOS ON THE MVE FRONTIER

If your portfolio invests 50% each into two portfolios on the MVE Frontier, is the resulting portfolio still on the MVE Frontier? Put differently, when two MVE investors marry, is their total portfolio still MVE? The answer is yes.

Any combination of MVE portfolios is itself also MVE.

IMPORTANT: *If two investors hold portfolios that are Mean-Variance Efficient, then the merged portfolio is also Mean-Variance Efficient.*

Unfortunately, it requires Nerd Appendices b and c to prove that the combination of two MVE Frontier portfolios is also MVE (as was shown in Figure 16.5). However, if you either believe my equation 16.4 or if you trust your eyes that the MVE frontier is indeed a nice hyperbola without kinks, then we can understand this graphically. For convenience, we use the same frontiers as we did in Figure 16.1. Assume that the MVE-Frontier is the solid line, and contains the two MVE portfolios A and B. The question is whether combinations of A and B have to also lie along the solid line, or whether they can produce portfolios that lie on the dashed lines. In the left graph, portfolios that combine A and B are better than MVE. Therefore, the solid line could not have been the MVE-Frontier to begin with. In the left graph, if portfolio B is shorted to buy more of portfolio A, then the resulting portfolio is again better than what is available on the MVE. Therefore, the solid line could not have been the MVE-Frontier, either. The only combinations of A and B that do not break the MVE Frontier are the portfolios that lie on the MVE Frontier. This is what we stated: the combination of two MVE portfolios must itself be MVE.

Sorry, there is no easy way to prove this in general.

Figure 16.4. Two MVE Portfolios

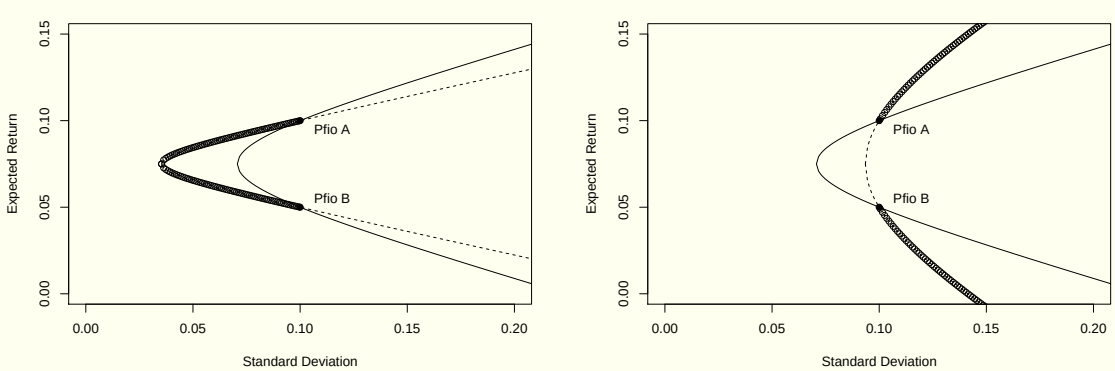


Figure 16.5 gives a concrete example.

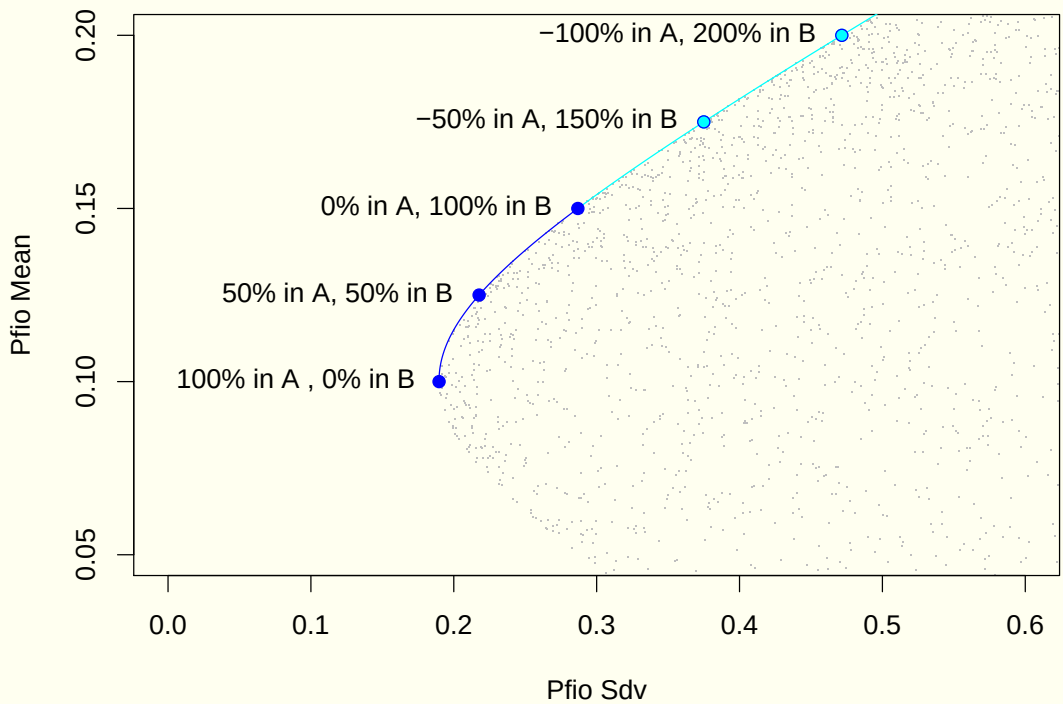
We can also show how combinations of MVE portfolios must be MVE in our three stock data example. Figure 16.5 plots 100 portfolio combinations from two portfolios on the MVE Frontier. The minimum variance portfolio A is mostly the S&P500. A second MVE portfolio B requires shorting a large amount in the S&P500, but in our ideal world of no transaction costs, this is not a problem. Two hand-computed combination MVE portfolios are in Table 16.3.

Table 16.3. Two Hand-Computed Combinations of MVE Portfolios

	Portfolio:	A	B	"Hand 1"	"Hand 2"
	Weight in A:	+100%	0%	+50%	−25%
	Weight in B:	0%	+100%	+50%	+125%
(net	S&P500	+99.448%	−90.620%	+4.414%	−138.138%
hol-	IBM	+2.124%	+135.770%	+68.910%	+169.088%
dings)	Sony	−1.573%	+54.925%	+26.676%	+69.049%
Reward	$\mathcal{E}(\tilde{r}_P)$	10.000%	25.000%	17.500%	28.750%
Risk	$Sdv(\tilde{r}_P)$	18.969%	67.555%	37.498%	83.271%

Figure 16.5 plots many portfolio combinations of the MVE portfolios A and B. Visually, it appears as if these combinations "trace out" all other MVE portfolios. (Points to the right of the "100% in B" portfolio require shorting portfolio A in order to obtain the money to purchase more of portfolio B.)

Figure 16.5. S&P500, IBM, and Sony Portfolio Combinations



Portfolio A invests 99.448% in S&P500, 2.124% in IBM, and -1.573% in Sony. Portfolio B invests -90.62% in S&P500, +135.77% in IBM, and +54.925% in Sony. Other portfolios along the efficient frontier are combinations of A and B.

Consequently, once you have identified two MVE portfolios, you can determine the best portfolio for any desired expected rate of return. For example, if you want to know what the best portfolio C is that offers an expected rate of return of 15%, determine how much of portfolio A and how much of portfolio B you have to purchase in order to expect a rate of return of 15%. The answer is

It is now easy to determine other portfolios on the MVE Frontier.

$$\begin{aligned} w \cdot 10\% + (1-w) \cdot 25\% &= 15\% \quad \Leftrightarrow \quad w = {}^{2}/_{3} \ . \\ w \cdot \mathcal{E}(\tilde{r}_A) + (1-w) \cdot \mathcal{E}(\tilde{r}_B) &= \mathcal{E}(\tilde{r}_C) \end{aligned} \tag{16.8}$$

So, portfolio C should consist of ${}^{2}/_{3}$ of portfolio A and therefore ${}^{1}/_{3}$ of portfolio B. If you wish, you can think of portfolios A and B as two funds, so you can use the method from Section 13·3.B to compute the underlying net investment weights of portfolio C:

Portfolio:	A	B	C
Weight in A	100%	0%	$2/3$
Weight in B	0%	100%	$1/3$
S&P500	+99.448%	-90.620%	+36.092%
IBM	+2.124%	+135.770%	+46.648%
Sony	-1.573%	+54.925%	+17.260%
$\mathcal{E}(\tilde{r})$	10.000%	25.000%	15.000%
$Sdv(\tilde{r})$	18.969%	67.555%	28.684%

C is the best portfolio that offers an expected rate of return of 15% per year.

Solve Now!

Q 16.9 *Among the three stocks, what is the portfolio that yields an expected rate of return of 12.5% per year?*

Q 16.10 *Among the three stocks, what is the portfolio that yields an expected rate of return of 20% per year?*

16·4. A Risk-Free Security

Risk-free securities do exist.

From a practical perspective, we can always find a Treasury bond that is essentially risk-free. So, we can proceed under the assumption that we have access to one such security. This also means that we know already one MVE portfolio—after all, you should not be able to get a higher interest rate for a risk-free investment—or you could become rich. Fortunately, the presence of the risk-free security also makes the portfolio optimization problem a lot easier.

SIDE NOTE: The special role applies *only* to the risk-free security, not to any other kind of bond. Although we did discuss these *risky* bonds in the preceding parts of the book (Chapters 4 and 6), you should look at them as just another possible risky investment asset, the same way you should look at any equity stock or fund you might purchase. They are just another risky component inside your larger investment portfolio.

16·4.A. Risk-Reward Combinations of Any Portfolio Plus the Risk-Free Asset

It is very easy to compute the expected rate of return and risk for a portfolio in which one of the two securities is risk-free.

So, what are the achievable risk-reward combinations if there is a risk-free security? For example, consider portfolios from the combination of a risk-free security (called $\mathcal{F}$) which pays 6% for sure, with only one risky security (called R) which pays a mean rate of return of 10.56% and has a standard deviation of 12%. The formulas for portfolio mean and standard deviation are

$$\begin{aligned} \mathcal{E}(\tilde{r}_P) &= (1-w_R)\cdot 6\% &+& \quad w_R \cdot 10.56\% \\ \mathcal{E}(\tilde{r}_P) &= (1-w_R)\cdot \mathcal{E}(r_{\mathcal{F}}) &+& \quad w_R \cdot \mathcal{E}(\tilde{r}_R) \\ &= \quad r_{\mathcal{F}} &+& \quad w_R \cdot \left[\mathcal{E}(\tilde{r}_R) - r_{\mathcal{F}}\right] \end{aligned} \tag{16.9}$$

Easy. More interestingly,

$$Sdv(\tilde{r}_P) = \sqrt{w_{\mathcal{F}}^2 \cdot \mathcal{V}ar(r_{\mathcal{F}}) + w_R^2 \cdot \mathcal{V}ar(\tilde{r}_R) + 2 \cdot w_{\mathcal{F}} \cdot w_R \cdot Cov(\tilde{r}_R, r_{\mathcal{F}})} \ . \tag{16.10}$$

But a risk-free rate of return is just a constant rate of return, so it has no variance and no covariance with anything else—it is what it is. Now combine our portfolio R with the risk-free rate, purchasing $w_{\mathcal{F}}$ in the risk-free asset and $w_R = (1 - w_{\mathcal{F}})$ in R.

$$\begin{aligned} Sdv(\tilde{r}_P) &= \sqrt{\cancel{w_{\mathcal{F}}^2 \cdot Var(r_{\mathcal{F}})} + w_R^2 \cdot Var(\tilde{r}_R) + \cancel{2 \cdot w_{\mathcal{F}} \cdot w_R \cdot Cov(\tilde{r}_R, r_{\mathcal{F}})}} \\ &= \sqrt{w_R^2 \cdot Var(\tilde{r}_R)} \\ &= w_R \cdot Sdv(\tilde{r}_R) \quad . \\ Sdv(\tilde{r}_P) &= w_R \cdot 12\% \quad . \end{aligned} \tag{16.11}$$

This is nice: the standard deviation of the rate of return on any portfolio is simply the weight times the standard deviation of the risky asset (here, 13%). Now substitute Formula 16.11 into Formula 16.9 to eliminate w_R $(= Sdv(\tilde{r}_P)/Sdv(\tilde{r}_R))$, and you obtain

$$\begin{aligned} \mathcal{E}(\tilde{r}_P) &= r_{\mathcal{F}} + w_R \cdot \left[\mathcal{E}(\tilde{r}_R) - r_{\mathcal{F}}\right] \\ &= r_{\mathcal{F}} + \left[\frac{\mathcal{E}(\tilde{r}_R) - r_{\mathcal{F}}}{Sdv(\tilde{r}_R)}\right] \cdot Sdv(\tilde{r}_P) \\ &= a + b \cdot Sdv(\tilde{r}_P) \quad . \end{aligned} \tag{16.12}$$

This is the equation for a line, where a (the risk-free rate) is the intercept and b $(= [\mathcal{E}(\tilde{r}_R) - r_{\mathcal{F}}]/Sdv(\tilde{r}_R))$ is the slope. In our case,

$$\mathcal{E}(\tilde{r}_P) = 6\% + \left[\frac{10.56\% - 6\%}{12\%}\right] \cdot Sdv(\tilde{r}_P) = 6\% + 38\% \cdot Sdv(\tilde{r}_P) \quad . \tag{16.13}$$

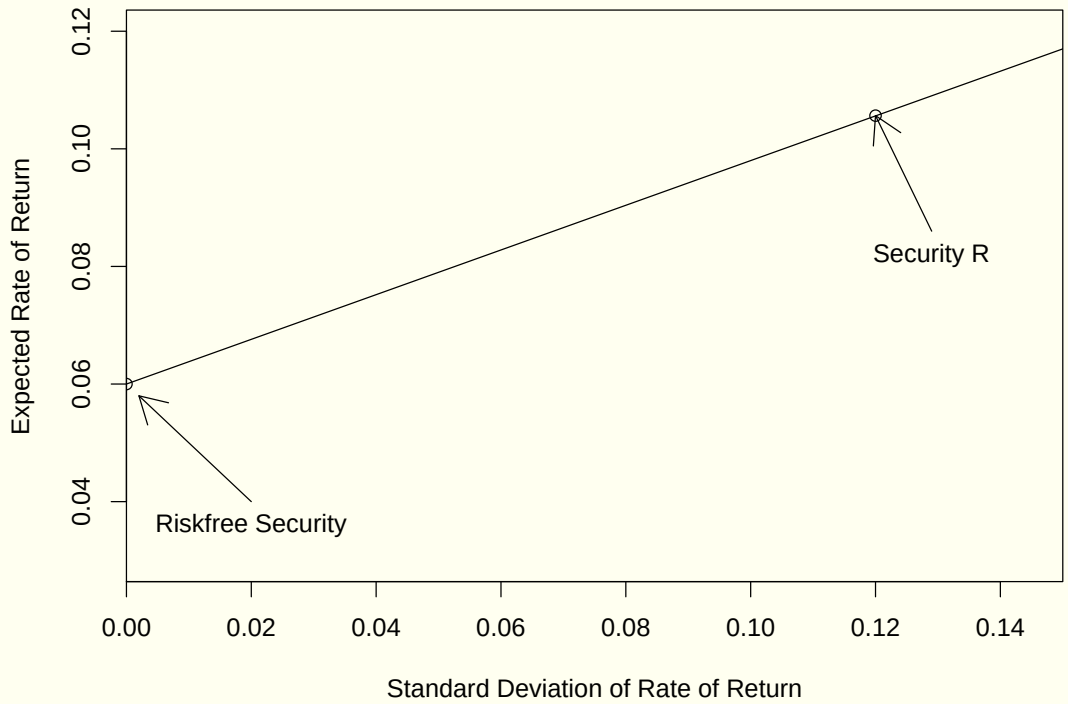
Side Note: The slope in these formulas is called the **Sharpe Ratio**,

$$\text{Sharpe Ratio} \equiv \frac{\mathcal{E}(\tilde{r}_R) - r_{\mathcal{F}}}{Sdv(\tilde{r}_R)} \quad . \tag{16.14}$$

For a given portfolio P, $[\mathcal{E}(\tilde{r}_P) - r_{\mathcal{F}}]/Sdv(\tilde{r}_P)$, it is the ratio of the expected reward vs. the absorbed risk ratio. (Strictly speaking, the ratio is quoted in terms of excess-over-the-riskfree-rate.) It is a common measure of how good the reward is relative to its risk. A portfolio has a higher Sharpe ratio if it has a higher expected rate of return and a lower standard deviation of return.

Figure 16.6. Combinations of The Risk-Free Security ($\mathcal{F}$) And a Risky Asset (R)

Risky Weight	0	$^1/_4$	$^1/_2$	$^3/_4$	1	$^3/_2$
Mean (Expected Rate of Return)	6%	7.14%	8.28%	9.42%	10.56%	12.84%
Standard Deviation	0%	3%	6%	9%	12%	18%

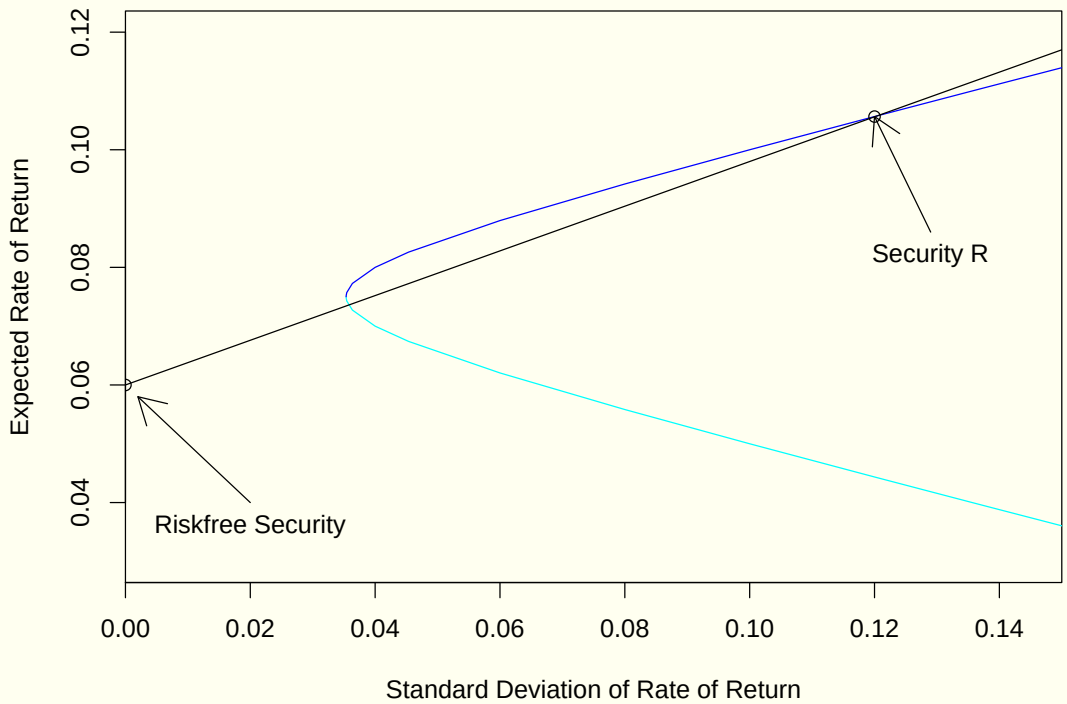


The combinations are a straight line in the mean-standard deviation graph.

Figure 16.6 uses Formulas 16.9 and 16.11 to plot the mean and standard deviation of portfolio pairs for our particular security R. All portfolio combinations lie on a straight line, which starts at the risk-free rate (at Sdv=0%, $\mathcal{E}$=6%) and goes through the risky portfolio R (at Sdv=12%, $\mathcal{E}$=10.56%). As we know from Formula 16.12, this is general: The portfolios that consist of the risk-free security and a risky security have risk/reward tradeoffs that lie on a straight line.

16·4.B. The Best Risk-Reward Combinations With A Risk-Free Asset

Figure 16.7. Combinations of The Risk-Free Security ($\mathcal{F}$) And a Risky Asset (R)

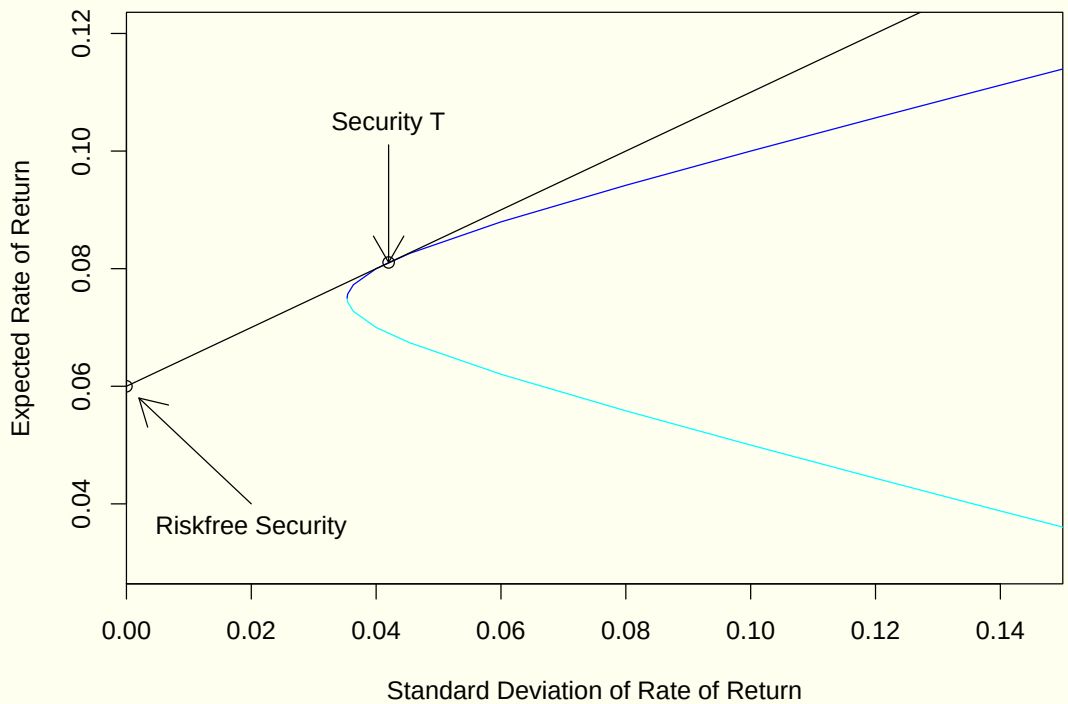


The MVE among risky assets is based on the example from Table 16.1 with a covariance of -0.0075. (This means the first security has a reward of 5% and a risk of 10%; the second has a reward of 10% and a risk of 10%.) Incidentally, Portfolio R has weights of $(-0.113, 1.113)$, which means it has an expected rate of return of 10.57% and a risk of 12%.

Would you still purchase a combination of R, if you could instead purchase any other combination of A and B?

Now return to the efficient frontier from Table 16.1 and Figure 16.1, and assume the covariance is -0.0075. R happens to be among the portfolios that you can purchase, combining A and B. Figure 16.7 repeats the last graph, but also plots the other securities you could purchase, putting together two base securities, A and B. This raises an interesting question: Presume that we have drawn the MVE Frontier using all risky assets. (It will always looks like a sideways hyperbola, so the drawn figure is accurate even for the more general case.) Now a risk-free asset becomes available, so you can draw a line between the risk-free rate and whatever portfolio you want to purchase. How does your new achievable MVE Frontier look like? Would you ever purchase portfolio R in Figure 16.7, which is on the previous MVE Frontier (or any combination of the risk-free security and our portfolio R)?

Figure 16.8. The Mean-Variance Efficient Frontier With a Risk-Free Asset



Incidentally, portfolio $\mathcal{T}$ has weights of $(0.38, 0.62)$, which means it has an expected rate of return of 8.1% and a risk of 4.18%.

The new MVE Frontier is a line from the risk-free rate to the "tangency" portfolio.

No! You can do better. Figure 16.8 draws the so-called tangency portfolio, $\mathcal{T}$. By combining portfolio $\mathcal{T}$ and the risk-free asset, you can do better than purchasing even a tiny bit of R. In fact, the figure shows that you would draw a line from the risk-free asset to this **Tangency Portfolio** ($\mathcal{T}$), and you would only purchase combinations of these two. No risky portfolio other than $\mathcal{T}$ would ever be purchased by *any* rational investor.

IMPORTANT: *If there is a risk-free security, every investor would purchase a combination of the risk-free security and the Tangency Portfolio. No combination of risky assets other than the Tangency Portfolio $\mathcal{T}$ would ever be purchased by any investor.*

16·4.C. The Formula to Determine the Tangency Portfolio

We can actually figure out the tangency portfolio. It is the portfolio that has the highest slope, i.e., the highest Sharpe ratio (see also Page 347). We also know that the tangency portfolio is the portfolio on the risky MVE Frontier, which has the maximum slope. This makes it easy to determine the tangency portfolio. Let us find the tangency portfolio for our three well-familiar stocks. Recall the first three MVE portfolios for S&P500, IBM, and Sony from Table 16.3 on Page 366. The toughest part actually turns out to be determining the covariance between two efficient frontier portfolios, for us between A and B. Recall that

Portfolio:	A	B	"Any"
Weight in A:	+100%	0%	+50%
Weight in B:	0%	+100%	+50%
S&P500	+99.448%	−90.620%	+4.414%
IBM	+2.124%	+135.770%	+68.910%
Sony	−1.573%	+54.925%	+26.676%
$\mathcal{E}(\tilde{r}_P)$	10.000%	25.000%	17.500%
$\mathcal{S}dv(\tilde{r}_P)$	18.969%	67.555%	37.498%
Sharpe Ratio @ $r_{\mathcal{F}} = 6\%$	$\frac{10\% - 6\%}{19\%} = 0.21$	$\frac{25\% - 6\%}{67.6\%} = 0.28$	$\frac{17.5\% - 6\%}{37.5\%} = 0.31$

You can trust me that the $\mathcal{C}ov(\tilde{r}_A, \tilde{r}_B) = 0.03505$, or you can read the nerdnote box below. We also know that we can trace out the MVE Frontier by purchasing any combinations of these three portfolios. Our goal is to find, for a given risk-free interest rate, the tangency portfolio. First, write down again the risk and reward of portfolios of two portfolios on the prior risky efficient frontier. Call them A and B. For any portfolio itself comprised of A and B,

The Agenda. Work out the Sharpe ratio, so mean first, covariance next, and standard deviation last.

$$\begin{aligned} \mathcal{E}(\tilde{r}_p) &= w_A \cdot 10\% + (1 - w_A) \cdot 25\% = 25\% - 15\% \cdot w_A \\ &= w_A \cdot \mathcal{E}(\tilde{r}_A) + (1 - w_A) \cdot \mathcal{E}(\tilde{r}_B) \ . \end{aligned} \tag{16.15}$$

and

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_p) &= w_A^2 \cdot (18.969\%)^2 + w_B^2 \cdot (67.555\%)^2 + 2 \cdot w_A \cdot w_B \cdot 3.505\% \ . \\ \mathcal{V}ar(\tilde{r}_p) &= w_A^2 \cdot \mathcal{V}ar(\tilde{r}_A) + w_B^2 \cdot \mathcal{V}ar(\tilde{r}_A) + 2 \cdot w_A \cdot w_B \cdot \mathcal{C}ov(\tilde{r}_A, \tilde{r}_B) \ . \end{aligned} \tag{16.16}$$

With a risk-free rate of 6%, the Sharpe Ratio for p is therefore

$$\mathrm{SR}(w_A) = \frac{\mathcal{E}(\tilde{r}_p) - r_{\mathcal{F}}}{\mathcal{S}dv(\tilde{r}_p)} = \frac{25\% - 15\% \cdot w_A - 6\%}{\sqrt{w_A^2 \cdot (18.969\%)^2 + (1 - w_A)^2 \cdot (67.555\%)^2 + 2 \cdot w_A \cdot (1 - w_A) \cdot 3.505\%}} \ . \tag{16.17}$$

You can now use Excel or Calculus to determine the w_A that maximizes this fraction. The solution is $w_A = 68.087\%$, which means that the net portfolio weights in this tangency portfolio are

$$\begin{aligned} w_{\text{S\&P500}} &= 0.68 \cdot (+99.448\%) + 0.32 \cdot (-90.620\%) = 38.791\% \ , \\ w_{\text{IBM}} &= 0.68 \cdot (+2.124\%) + 0.32 \cdot (135.770\%) = 44.751\% \ , \\ w_{\text{Sony}} &= 0.68 \cdot (-1.573\%) + 0.32 \cdot (54.925\%) = 16.458\% \ . \end{aligned} \tag{16.18}$$

The mean rate of return of this tangency portfolio is 14.787%, the standard deviation is 27.998%, and the Sharpe ratio is 0.3138. So, you should only purchase a combination of the risk-free security and this tangency portfolio, with weights determined by your peculiar taste for risk.

NERD NOTE: *There are two methods to extract the covariance: one is brute force to manipulate the component security covariances. The more clever approach recognizes that we already know*

$$\begin{aligned}
(18.969\%)^2 &= 1^2\cdot\mathcal{V}ar(\tilde{r}_A) + 0^2\cdot\mathcal{V}ar(\tilde{r}_B) + 2\cdot 1\cdot 0\cdot Cov(\tilde{r}_A,\tilde{r}_B)\\
(67.555\%)^2 &= 0^2\cdot\mathcal{V}ar(\tilde{r}_A) + 1^2\cdot\mathcal{V}ar(\tilde{r}_B) + 2\cdot 0\cdot 1\cdot Cov(\tilde{r}_A,\tilde{r}_B)\\
(37.498\%)^2 &= 0.5^2\cdot\mathcal{V}ar(\tilde{r}_A) + 0.5^2\cdot\mathcal{V}ar(\tilde{r}_B) + 2\cdot 0.5\cdot 0.5\cdot Cov(\tilde{r}_A,\tilde{r}_B)\\
\mathcal{V}ar(\tilde{r}_p) &= w_A^2\cdot\mathcal{V}ar(\tilde{r}_A) + w_B^2\cdot\mathcal{V}ar(\tilde{r}_B) + 2\cdot w_A\cdot w_B\cdot Cov(\tilde{r}_A,\tilde{r}_B)\ ,
\end{aligned} \tag{16.19}$$

which simplifies into

$$\begin{aligned}
(18.969\%)^2 &= \mathcal{V}ar(\tilde{r}_A)\\
(67.555\%)^2 &= \mathcal{V}ar(\tilde{r}_B)\\
(37.498\%)^2 &= 0.25\cdot\mathcal{V}ar(\tilde{r}_A) + 0.25\cdot\mathcal{V}ar(\tilde{r}_B) + 2\cdot 0.5\cdot 0.5\cdot Cov(\tilde{r}_A,\tilde{r}_B)\\
&= 0.25\cdot(18.969\%)^2 + 0.25\cdot(67.555\%)^2 + 0.5\cdot Cov(\tilde{r}_A,\tilde{r}_B)\ .
\end{aligned} \tag{16.20}$$

Solving, we obtain $Cov(\tilde{r}_A,\tilde{r}_B) = 0.03505$.

16·4.D. Combining The Risk-Free Security And the Tangency Portfolio

The special case with a risk-free security.

If a risk-free security exists, so that the "true" MVE Frontier is the straight line tracing out all combinations of the risk-free security and the tangency portfolio $\mathcal{T}$, then the fact that the combination of two MVE portfolios is still MVE is easy to understand. Say, we want to marry two portfolios. The groom owns \$100 in the tangency portfolio $\mathcal{T}$, and \$900 in the risk-free asset. The bride owns \$2,000 in $\mathcal{T}$, purchased partly with \$500 worth of debt. The couple then owns $\$100 + \$2,000 = \$2,100$ in $\mathcal{T}$ and $\$900 - \$500 = \$400$ in the risk-free asset. This is a MVE portfolio, because it consists only of the risk-free asset and the risky tangency portfolio $\mathcal{T}$.

NERD NOTE: *However, the opposite statement—if the merged portfolio is mean-variance efficient, then the individual components are mean-variance efficient—is not true. It is possible that A and B do not invest in $\mathcal{T}$, but that their merged investment-weighted stock market portfolio is $\mathcal{T}$. For example, allow the tangency portfolio to consist itself of two securities, X and Y, say in proportions 1/3 and 2/3, respectively. If the groom holds \$300 in X and the bride holds \$600 in Y, then the couple's portfolio is MVE. But each individual's portfolio was not. Who says marriage does not pay?*

Solve Now!

Q 16.11 *For our three securities, what is the Sharpe ratio of a portfolio that invests 50% in the risk-free security and 50% in the tangency portfolio $\mathcal{T}$.*

Q 16.12 *What is the tangency portfolio if the risk-free rate were not 6%, but 3%? Before solving it, think where should lie geometrically.*

Q 16.13 *What is the tangency portfolio if the risk-free rate were not 6%, but 9%? Before solving it, think where it should lie geometrically.*

16·5. What does a Security need to offer to be in a MVE Frontier Portfolio?

16·5.A. What if the Risk-Reward Relationship is Non-Linear?

Return to Figure 16.3 on Page 361, which graphs the risk and reward for many portfolios. Among them are

Some Portfolios.

Portfolio	$w_{\text{S\&P500}}$	w_{IBM}	w_{Sony}	$\mathcal{E}(\tilde{r}_P)$	$Sdv(\tilde{r}_P)$
A	0.400	0.200	0.400	16.80%	40.77%
B	0.107	0.645	0.248	17.00%	35.65%
C	0.500	0.500	0.000	12.74%	25.12%
D	0.614	0.288	0.097	13.00%	22.90%
E	0.990	0.024	-0.014	10.03%	18.96%
B	1.000	0.000	0.000	19.03%	10.10%
C	0.361	0.466	0.173	15.00%	28.68%
E	−0.146	0.734	0.286	18.00%	39.38%
F	−0.906	1.357	0.549	25.00%	67.55%

The A portfolio is clearly inefficient: it has a lower expected rate of return and a higher standard deviation than the B portfolio. Comparing the two portfolios, it appears as if the A portfolio has too much of the S&P500, too little of IBM, and too much of Sony. Put differently, if you were holding portfolio A, then S&P500 and Sony would really be too expensive for you, while IBM would be a bargain. You would be better off rebalancing away from the S&P500 and Sony securities, and towards IBM.

B is better than A. For an A investor, some expensive stocks should be reduced, some cheap stocks should be increased.

Whether a stock is too expensive depends on your current portfolio: if you were holding portfolio C, portfolio D would be a better choice—but D asks for less holdings in IBM than C, not more. So, from your perspective of an owner of portfolio C, the S&P500 and Sony would look like bargains, because they would help you to reduce your portfolio risk, while IBM would look too expensive.

Expensive vs. cheap depends on the portfolio, not just on the stocks themselves.

Important: *The risk contribution of a security depends on the particular portfolio. For some portfolios, a security may appear like a bargain, for others like a gouge. An investor for whom the security appears like a bargain should invest more in it; an investor for whom the security appears like a gouge should divest more of it.*

The annual portfolio returns, including portfolio A:

It turns out that this insight is the basis for an important method to measure the risk/reward characteristics of an individual security, i.e., whether this individual security is a bargain, given your specific overall portfolio. Assume you are holding portfolio A. Its annual rates of return would have been

Year	S&P500	IBM	Sony	Pfio A	Year	S&P500	IBM	Sony	Pfio A
1991	0.263	−0.212	−0.103	0.022	1997	0.310	0.381	0.391	0.356
1992	0.045	−0.435	−0.004	−0.070	1998	0.267	0.762	−0.203	0.178
1993	0.071	0.121	0.478	0.244	1999	0.195	0.170	2.968	1.299
1994	−0.015	0.301	0.135	0.108	2000	−0.101	−0.212	−0.511	−0.287
1995	0.341	0.243	0.105	0.227	2001	−0.130	0.423	−0.348	−0.107
1996	0.203	0.658	0.077	0.244	2002	−0.234	−0.357	−0.081	−0.197
					$\mathcal{E}$	0.101	0.154	0.242	0.168
					$\mathcal{V}ar$	0.037	0.150	0.815	0.166
					$\mathcal{S}dv$	0.190	0.388	0.903	0.408

Compute the beta of each stock with respect to portfolio A, and graph each beta against its expected rate of return.

Although we could work directly with covariances, betas are easier to interpret. So, as the measure for how each stock contributes to the risk of portfolio A, we adopt the beta of each stock *with respect to* portfolio A. The covariances may be tedious to compute, but they are not difficult:

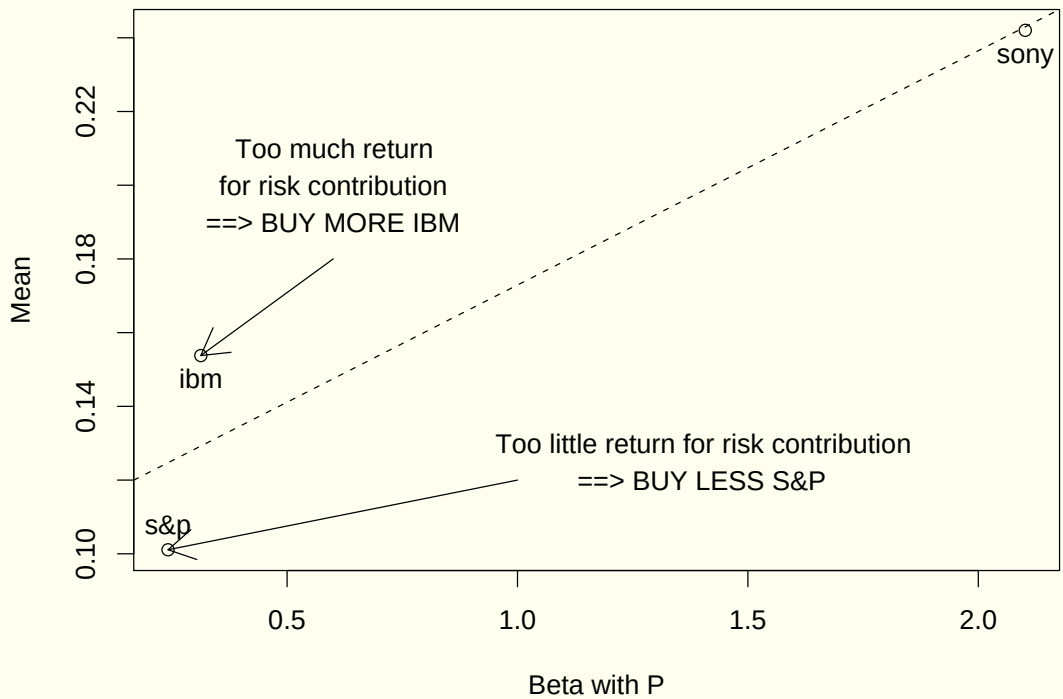
$$\begin{aligned}
\mathcal{C}ov(\tilde{r}_{\text{S\&P500}}, \tilde{r}_A) &= \frac{(+0.2631 - 0.101)\cdot(0.0217 - 0.168) + \ldots + (-0.2337 - 0.101)\cdot(-0.1971 - 0.168)}{11} = 0.0402 \\
\mathcal{C}ov(\text{IBM}, \tilde{r}_A) &= \frac{(-0.2124 - 0.154)\cdot(0.0217 - 0.168) + \ldots + (-0.3570 - 0.154)\cdot(-0.1971 - 0.168)}{11} = 0.0520 \\
\mathcal{C}ov(\text{Sony}, \tilde{r}_A) &= \frac{(-0.1027 - 0.242)\cdot(0.0217 - 0.168) + \ldots + (-0.0808 - 0.242)\cdot(-0.1971 - 0.168)}{11} = 0.3494
\end{aligned} \tag{16.21}$$

To obtain beta, divide each of these securities' covariances by the same number, the variance of A:

$$\begin{aligned}
\beta(\tilde{r}_{\text{S\&P500}}, \tilde{r}_A) &= \frac{\mathcal{C}ov(\tilde{r}_{\text{S\&P500}}, \tilde{r}_A)}{\mathcal{V}ar(\tilde{r}_A)} = \frac{0.0402}{0.1662} = 0.2417 \\
\beta(\tilde{r}_{\text{IBM}}, \tilde{r}_A) &= \frac{\mathcal{C}ov(\tilde{r}_{\text{IBM}}, \tilde{r}_A)}{\mathcal{V}ar(\tilde{r}_A)} = \frac{0.0520}{0.1662} = 0.3129 \\
\beta(\tilde{r}_{\text{Sony}}, \tilde{r}_A) &= \frac{\mathcal{C}ov(\tilde{r}_{\text{Sony}}, \tilde{r}_A)}{\mathcal{V}ar(\tilde{r}_A)} = \frac{0.349}{0.1662} = 2.1023
\end{aligned} \tag{16.22}$$

Now, plot the expected rates of return against the betas (for portfolio A, of course), and try to draw a line through these three points. Figure 16.9 shows that the three points do not lie on one line. IBM seems to have too high an expected rate of return for its beta (it lies above the line), while the S&P500 has too low an expected rate of return. Therefore, just as we knew from comparing portfolio A and B, it appears that good advice to portfolio A owners would be to rebalance, away from the expensive S&P500, and towards the cheaper IBM. (Thereafter, you should recheck the relative expected pricing of these securities relative to your new portfolio.)

Figure 16.9. Means of Stocks vs. Betas with respect to non-MVE A portfolio



A is a lousy portfolio, because it bought too little of IBM, given its risk contribution to A; and too much of S&P500, given its risk contribution to A. Portfolios that have either higher mean and the same risk, or the same mean and lower risk, could have beeen achieved by buying more IBM and less of S&P500.

Indeed, as it turns out, the fact that the three securities do not perfectly lie on one line is proof-positive that portfolio A can be improved upon, i.e., that portfolio A is not on the MVE Frontier.

MVE of a pfio means its securities must like on a line in beta-exp.ret. space.

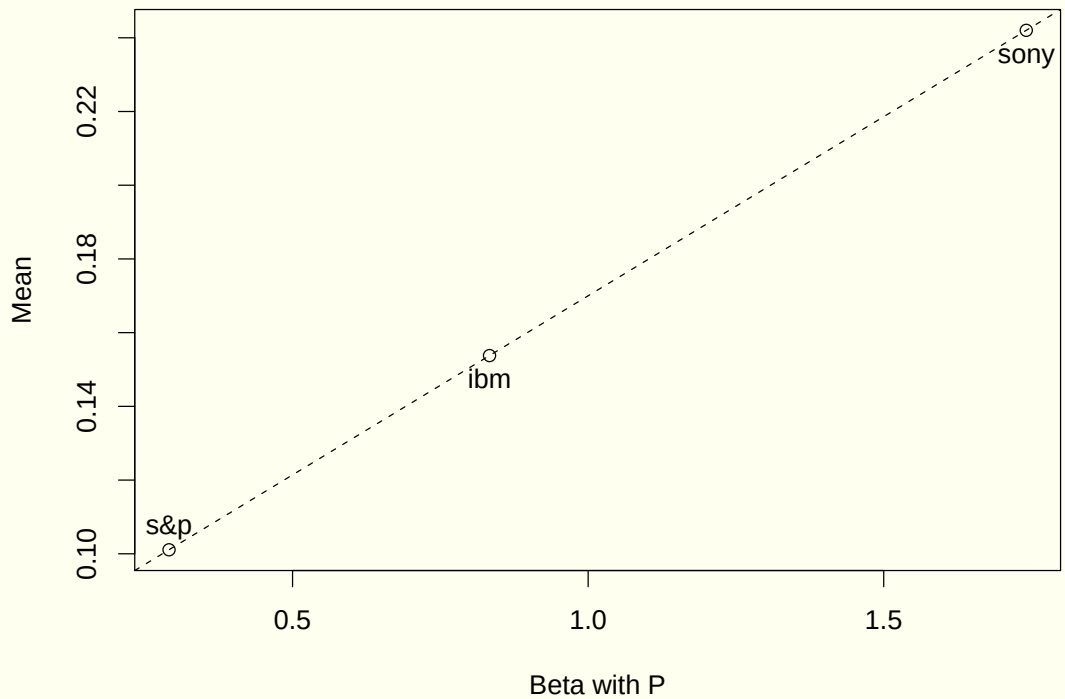
16·5.B. What if the Risk-Reward Relationships is Linear?

If we repeat the same exercise for portfolio B, we find that

$$\begin{aligned}
\beta(\tilde{r}_{\text{S\&P500}}, \tilde{r}_B) &= \frac{Cov(\tilde{r}_{\text{S\&P500}}, \tilde{r}_B)}{Var(\tilde{r}_B)} = \frac{0.0370}{0.1275} = 0.2909 \ , \\
\beta(\tilde{r}_{\text{IBM}}, \tilde{r}_B) &= \frac{Cov(\tilde{r}_{\text{IBM}}, \tilde{r}_B)}{Var(\tilde{r}_B)} = \frac{0.1059}{0.1275} = 0.8330 \ , \\
\beta(\tilde{r}_{\text{Sony}}, \tilde{r}_B) &= \frac{Cov(\tilde{r}_{\text{Sony}}, \tilde{r}_B)}{Var(\tilde{r}_B)} = \frac{0.2213}{0.1275} = 1.7403 \ .
\end{aligned} \tag{16.23}$$

For the MVE portfolio B, when graphing expected rate of return against beta (with respect to portfolio B), all stocks lie exactly on one straight line.

Figure 16.10. Means of Stocks vs. Betas with respect to MVE B portfolio



B is a good portfolio. You cannot do better by purchasing more or less of S&P500, IBM, or Sony in terms of your mean/variance tradeoff. (This also means that to obtain more mean, you would also have to accept more risk.)

An example of an MVE portfolio: each stock offers the right exp. return for its beta.

Figure 16.10 plots the beta of each stock with respect to portfolio B against the expected rate of return of each stock. All securities lie on a single line. This actually means that if you hold portfolio B, each security offers an expected rate of return (reward) that is appropriate for its beta (risk). There is no way to improve upon portfolio B by changing these portfolio weights. Therefore, portfolio B is MVE.

Preview: This will be the CAPM equation.

This linear relationship between the expected rate of return of each security with its beta with a portfolio, *if the portfolio is Mean-Variance Efficient*, will play a profound role in the CAPM, which will be discussed in the next chapters.

NERD NOTE: *If we had worked with covariances instead of betas, the slope of the line would change, but not the fact that all points either lie on a line or do not lie on a line.*

IMPORTANT: *If all securities' expected rates of returns lie on a line when plotted against the beta with respect to some portfolio, then this portfolio is MVE. If they do not lie on a line, then this portfolio is not MVE.*

Let us describe our example more generally. We know that the security holdings are optimized for each security i in a Mean-Variance Efficient portfolio $\mathcal{E}$—or else $\mathcal{E}$ would not be MVE! The exact relationship if all the components in Portfolio $\mathcal{E}$ are optimized is

To enter the tangency portfolio, each stock must follow a particular relationship

$$\begin{array}{c}\text{Portfolio } \mathcal{E} \text{ is} \\ \text{MVE}\end{array} \iff \mathcal{E}(\tilde{r}_i) = a + b \cdot \beta_{\tilde{r}_i,\tilde{r}_\mathcal{E}}. \tag{16.24}$$

where i is an index naming each and every stock. The formula contains two constants, a and b, which are numbers that depend on the portfolio $\mathcal{E}$. Formula 16.24 relates how the expected rate of return for each and every single available security must be related to its beta with respect to $\mathcal{E}$, i.e., lie along a straight line.

NERD NOTE: *Nerd Appendix d proves Formula 16.24.*

Here is a different way to think of the relationship. If you purchase an MVE portfolio, the relationship between each security's expected rate of return and its beta with respect to your own portfolio comes about automatically. It could even be stated that you purchase a best possible (MVE) portfolio, *if and only* if you always add more of stocks which seem relatively cheap for your portfolio (securities with too high an expected rate of return for their betas, i.e., which lie above your portfolio's line); and always reduce stocks which seem relatively expensive (securities with too low an expected rate of return for their betas, i.e., which lie below your portfolio's line).

A different way to think about portfolio optimization: given a portfolio, rebalance from more expensive towards cheaper stocks.

16·5.C. The Line Parameters

We can yet learn more about the Line Formula 16.24: we can determine the two constant numbers a and b. If $\beta_{i,\mathcal{E}}$ (the covariance) is zero for a particular investment i, then a is the expected rate of return of this investment. If a risk-free security is available it would have such zero covariance, so

We can determine a and b.

$$\begin{array}{c}\text{Portfolio } \mathcal{E} \text{ is} \\ \text{MVE}\end{array} \iff \mathcal{E}(\tilde{r}_i) = a + b \cdot 0 = r_\mathcal{F} \ . \tag{16.25}$$

So you can replace a with $r_\mathcal{F}$,

$$\begin{array}{c}\text{Portfolio } \mathcal{E} \text{ is} \\ \text{MVE}\end{array} \iff \mathcal{E}(\tilde{r}_i) = r_\mathcal{F} + b \cdot \beta_{\tilde{r}_i,\tilde{r}_\mathcal{E}} \ . \tag{16.26}$$

We can also consider an investment in portfolio $\mathcal{E}$ itself. (Naturally, buying more or less of it does not improve the portfolio performance.) The beta of portfolio $\mathcal{E}$ with respect to itself is 1. Therefore

$$\begin{array}{c}\text{Portfolio } \mathcal{E} \text{ is} \\ \text{MVE}\end{array} \iff \mathcal{E}(\tilde{r}_\mathcal{E}) = r_\mathcal{F} + b \cdot 1 = r_\mathcal{F} + b \ . \tag{16.27}$$

You can solve Formula 16.27 for b and find $b = \mathcal{E}(\tilde{r}_\mathcal{E} - r_\mathcal{F})$. Putting this all together, the line equation must be

$$\begin{array}{c}\text{Portfolio } \mathcal{E} \text{ is} \\ \text{MVE}\end{array} \iff \mathcal{E}(\tilde{r}_\mathcal{E}) = r_\mathcal{F} + \left[\mathcal{E}(\tilde{r}_\mathcal{E}) - r_\mathcal{F}\right] \cdot \beta_{i,\mathcal{E}} \tag{16.28}$$

NERD NOTE: *If there is no risk-free security, then $r_{\mathcal{F}}$ is simply replaced by the expected rate of return of a zero-beta stock. For our Portfolio B from Figure 16.10, this means that we can only state that the line is*

$$\mathcal{E}(\tilde{r}_{\mathcal{E}}) \;=\; a \;+\; [\mathcal{E}(\tilde{r}_{\mathcal{E}}) - a] \cdot \beta_{i,\mathcal{E}} \quad . \tag{16.29}$$

The formula states that inside any MVE portfolio, each stock must offer an appropriate expected rate of return for its risk.

Now, for any portfolio on the (upper half) of the MVE frontier, b is positive. This gives the line formula in 16.28 a nice intuitive interpretation. In order to find its way into your MVE portfolio, each security has to offer a reward (expected rate of return) that is appropriate for its contribution to your portfolio's risk:

- A stock that fluctuates a lot with your $\mathcal{E}$ portfolio itself (having high rates of returns when your portfolio $\mathcal{E}$ has high rates of return) has a high beta with respect to your portfolio. Such a stock does not help much in diversifying away the risk inside $\mathcal{E}$. Therefore, this stock must offer you a relatively high expected rate of return to enter your $\mathcal{E}$ portfolio.
- A stock whose returns tend to move in the opposite direction from $\mathcal{E}$'s returns has a negative beta. Therefore, this stock can offer a relatively low expected rate of return and it would still be sufficiently desirable to enter into your MVE portfolio.

This is not necessarily the case for non-MVE portfolios.

This "appropriateness" relationship between risk and reward is a peculiar property of MVE portfolios. If you hold a portfolio that is not MVE, some of your stocks offer too low or too high an expected rate of return for their risk contributions to your portfolio.

IMPORTANT: *All securities inside a MVE portfolio have to offer a fair reward for their risk-contribution. If even a single security offered either too much or too little reward for its risk, there would be a better portfolio, and the original portfolio would not have been MVE to begin with.*

Offering a fair risk/reward means that when the expected rates of return are graphed against the beta (with respect to the MVE portfolio $\mathcal{E}$) for each and every security, the relationship must be a straight line,

$$\mathcal{E}(\tilde{r}_i) \;=\; a \;+\; b \;\cdot\; \beta_{\tilde{r}_i,\tilde{r}_{\mathcal{E}}} \quad . \tag{16.30}$$

where a is the risk-free rate $r_{\mathcal{F}}$ and b is $\mathcal{E}(\tilde{r}_{\mathcal{E}}) - r_{\mathcal{F}}$.

Keep The Forest in Mind: The goal is to find the best overall portfolio.

Be warned: we have covered a lot of variances and covariances in many different aspects now. Do not confuse them. The mean-variance graphs that we drew in Figures 16.1–16.3 had standard deviation on the x-Axis. The lines that we drew in Figures 16.9 and 16.10 had β on the x-Axes. It is important to remember that you are only interested in minimizing your overall portfolio variance, given your desired overall portfolio expected rate of return. You are not interested in the expected rate of return of an individual stock per sé. You are not interested in the standard deviation of an individual stock per sé. You are not interested in the beta of an individual stock per sé. The individual stock statistics are only useful information only insofar as they help you in determining the best overall portfolio.

16·5.D. An Example of Using The Linear Risk-Reward Relationship

An Example to Illustrate This Formula.

Say, you have a Mean-Variance Efficient Portfolio $\mathcal{E}$ that has a mean $\mathcal{E}(\tilde{r}_{\mathcal{E}})$ of 8%, a standard deviation of 4% (variance of 0.0016). Further, the risk-free rate of return is 6%. What is the relationship between the expected rate of return of each security and the beta of each security? The answer is that Formula 16.24 states that every security should have an expected rate of return of

$$\mathcal{E}(\tilde{r}_i) \;=\; 6\% + [8\% - 6\%] \cdot \beta_{\tilde{r}_i,\tilde{r}_{\mathcal{E}}} \tag{16.31}$$

We did not need to use the information about the 4% risk!

SIDE NOTE: Beta is easier to write without extra confusion if we eliminate one level of subscripts, writing $\mathcal{E}(\tilde{r}_i) = 6\% + [8\% - 6\%] \cdot \beta_{i,\mathcal{E}}$.

Solve Now!

Q 16.14 *Assume that you are told that portfolio $\mathcal{E}$ is MVE. Further, you are told that $\mathcal{E}$ has an expected rate of return of 14%. Finally, you are told that security 1 has an expected rate of return of 8% and a beta with respect to $\mathcal{E}$ of 0.5. What expected rate of return would a security i with a beta with respect to $\mathcal{E}$ of $\beta_{i,\mathcal{E}} = 2$ have to have, in order for portfolio $\mathcal{E}$ to be MVE (i.e., not able to be improved upon)?*

Q 16.15 *A new stock has appeared. Its variance-covariance is*

	$\tilde{r}_{S\&P500}$	$\tilde{r}_{IBM}$	$\tilde{r}_{Sony}$	$\tilde{r}_N$	$\tilde{r}_B$
$\tilde{r}_{S\&P500}$	3.622%	3.298%	4.772%	4.362%	3.698%
$\tilde{r}_{IBM}$	3.298%	15.034%	2.184%	4.866%	10.587%
$\tilde{r}_{Sony}$	4.772%	2.184%	81.489%	57.956%	22.124%
$\tilde{r}_N$	4.362%	4.866%	57.956%	41.978%	17.974%
$\tilde{r}_B$	3.698%	10.587%	22.124%	17.974%	12.707%

The new stock has a mean rate of return of 21.029%. Recall that the mean rate of return of the MVE portfolio B was 17%.

1. *Compute the beta of $\tilde{r}_N$ with respect to portfolio $\tilde{r}_B$.*
2. *Plot this new stock into Figure 16.10. Does it lie on the line? What does this mean?*
3. *Add a little bit of stock N to portfolio B. That is, buy 99% of B and 1% of N. Compute the risk and reward of this combination portfolio. Does this combination portfolio offer superior risk/reward characteristics than portfolio B alone?*
4. *Subtract a little bit of stock N to portfolio B. That is, buy 101% of B and -1% of N. Compute the risk and reward of this combination portfolio. Does this combination portfolio offer superior risk/reward characteristics than portfolio B alone?*

Q 16.16 *Repeat this question, but assume that the expected rate of return on stock N is only 10%.*

16·6. Summary

This was a long and complex chapter, so this summary is more detailed than our usual chapter summaries:

1. Each portfolio is a point in a graph of overall portfolio expected rate of return (reward) against overall portfolio standard deviation (risk).

   Portfolios with similar compositions are close to one another.

   The formulas to compute means and standard deviations from a set of portfolio weights are on Page 305.

2. For two securities, the possible risk vs. reward choices are a curve.

   For more than two securities, the possible risk vs. reward choices are a cloud of points.

3. The Mean-Variance Efficient Frontier is the upper left envelope of this cloud of points. No one should purchase a portfolio that is not on the MVE frontier.

4. In real life, computing the MVE Frontier is useful especially when choosing among a small set of asset classes. Otherwise, the historical standard deviations and expected rates of returns are too unreliable to produce good forecasts of future rates of return.

5. If there is a risk-free security, the combination of the risk-free security and any other portfolio is a straight line.

6. If there is a risk-free security, the MVE Frontier is the line between the risk-free rate and the Tangency portfolio. No one should purchase a portfolio that is not on this line.

7. The combination of two MVE portfolios is also a MVE.

8. If two MVE portfolios are known, finding other MVE portfolios is a matter of taking weighted combinations of these two MVE portfolios.

9. MVE portfolios have an important property: Each security *inside* the MVE portfolio offers a fair reward for its risk contribution.

   - If any single security were too cheap, you could do better by buying more of it—which means that the portfolio would not have been MVE to begin with.
   - If any single security were too expensive, you could do better by buying less of it—which means that the portfolio would not have been MVE to begin with.

10. The reward of each security is suitably measured by its expected rate of return. The risk of each security inside the MVE portfolio is suitably measured by its beta *with respect to* your specific MVE portfolio.

    To be fairly priced, every security in the portfolio must lie on a straight line when the expected rates of return are plotted against these betas. Algebraically, this means that the relation is

$$\mathcal{E}(\tilde{r}_i) \;=\; a + [\mathcal{E}(\tilde{r}_{\mathcal{E}}) - a] \cdot \beta_{i,\mathcal{E}} \quad , \tag{16.32}$$

for every security i inside the MVE portfolio $\mathcal{E}$. If there is a risk-free security offering an expected (actual) rate of return $r_{\mathcal{F}}$, then this line becomes

$$\mathcal{E}(\tilde{r}_i) \;=\; r_{\mathcal{F}} + \Big[\mathcal{E}(\tilde{r}_{\mathcal{E}}) - r_{\mathcal{F}}\Big] \cdot \beta_{i,\mathcal{E}} \quad . \tag{16.33}$$

This relationship will play a prominent role in the subsequent CAPM chapters.

A. Nerd Appendix: Proofs For The Supernerd

a. The Optimal Portfolio Weights Formula

Warning: Linear Algebra.

This section relies on algebra (matrices and vectors). This is necessary, because it is simply too messy to deal with 10,000 individually named securities, with their requisite 10,000 expected rates of returns and the requisite 50 million covariance terms. Unless you already know linear algebra inside out, and/or you want to write a computer program to determine the MVE Frontier, please ignore this section. It is not necessary to an understanding of this book.

Variable Definitions and the Problem Setup.

Using matrix notation, the portfolio mean and variance can be expressed as

$$\begin{aligned} \mathcal{E}(\tilde{r}_P) &\equiv \vec{w}'\vec{m} \equiv \sum_{i=1}^{N} w_i \cdot Exp(\tilde{r}_i) \quad , \\ \mathcal{V}ar(\tilde{r}_P) &\equiv \vec{w}'\mathbf{S}\vec{w} \equiv \sum_{i=1}^{N}\sum_{j=1}^{N} w_i \cdot w_j \cdot \sigma_{i,j} \quad , \end{aligned} \tag{16.34}$$

where $\mathbf{S}$ is the matrix of variance-covariances, $\vec{m}$ is the vector of expected rates of return on each security (which is easier to write than $\overline{\mathcal{E}(\tilde{r})}$), and $\vec{1}$ is a vector containing only the scalar 1 in each of N positions. A prime denotes a row rather than a column vector. The problem to solve is therefore

$$\begin{array}{llcll} \min\limits_{\vec{w}} & \mathcal{V}ar(\tilde{r}_P) & \equiv & \vec{w}'\mathbf{S}\vec{w} & \text{Minimize Portfolio Variance,} \\ \text{subject to} & \vec{1}'\vec{w} & = & 1 & \text{Portfolio Weights add to 1,} \\ & \mathcal{E}(\tilde{r}_P) & \equiv & \vec{m}'\vec{w} & \text{Given a user specified mean return } \mathcal{E}(\tilde{r}_P). \end{array} \tag{16.35}$$

This problem is usually solved via Lagrangian Optimization techniques. Introduce λ and γ as two new Lagrangian multiplier parameters, and optimize

$$\min_{\vec{w},\lambda,\gamma} L(\vec{w},\lambda,\gamma) \equiv \vec{w}'\mathbf{S}\vec{w} - \lambda \cdot \left(1 - \vec{1}'\vec{w}\right) - \gamma \cdot \left[\mathcal{E}(\tilde{r}_P) - \vec{m}'\vec{w}\right] \quad . \tag{16.36}$$

The MVE set is obtained by differentiating with respect to the weights $\vec{w}$ and the two Lagrangian constants.

Nerd Note: *If you are not familiar with Lagrangian techniques, differentiating with respect to the two Lagrangian multipliers produces the two original constraints, thereby ensuring that the optimization really works across the constrained set only. For example, $\partial L/\partial\lambda = 1 - \vec{1}'\vec{w}$, which if set to zero (as it is in the first-order condition) reimposes the constraint $1 = \vec{1}'\vec{w}$.*

The solution are the optimal portfolio weights, $\vec{w}^*$.

After some manipulation, the solution is

$$\begin{aligned} \vec{w}^* &= \lambda^* \cdot \mathbf{S}^{-1}\vec{1} + \gamma^* \cdot \mathbf{S}^{-1}\vec{m} \\ \lambda^* &= \frac{C - \mathcal{E}(\tilde{r}_P)\cdot B}{D} \\ \gamma^* &= \frac{\mathcal{E}(\tilde{r}_P)\cdot A - B}{D} \end{aligned} \tag{16.37}$$

where A, B, C, and D are simple numerical constants (scalars):

$$\begin{aligned} A &\equiv \vec{1}'\mathbf{S}^{-1}\vec{1} \qquad B \equiv \vec{1}'\mathbf{S}^{-1}\vec{m} \\ C &\equiv \vec{m}'\mathbf{S}^{-1}\vec{m} \qquad D \equiv A\cdot C - B^2 \end{aligned} \tag{16.38}$$

Using this formula, we can obtain the formula relating the mean and variance, the parabola

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_P) &= \vec{w}'^*\mathbf{S}\vec{w}^* = \vec{w}'^*\mathbf{S}\left(\lambda^*\mathbf{S}^{-1}\vec{1} + \gamma^*\mathbf{S}^{-1}\vec{m}\right) \\ &= \lambda^* + \gamma^* \cdot \mathcal{E}(\tilde{r}_P) = \frac{A\cdot\mathcal{E}(\tilde{r}_P)^2 + 2\cdot B\cdot\mathcal{E}(\tilde{r}_P) + C}{D} \quad . \end{aligned} \tag{16.39}$$

This formula generalizes Formula 16.4 on Page 357.

The minimum variance portfolio.

The weights of the minimum variance portfolio are worth writing down. They are the solution when the mean constraint $\vec{m}'\vec{w} = \mathcal{E}(\tilde{r}_P)$ is not imposed. The minimum variance portfolio is then

$$\vec{w}^*_{\substack{\text{min}\\ \text{var}}} = \frac{\mathbf{S}^{-1}\vec{1}}{A} \quad . \tag{16.40}$$

Note that this portfolio does not depend on any security's expected rate of return.

b. The Combination of MVE Portfolios is MVE — With Risk-Free Security.

Combining two MVE portfolios is MVE again.

It is difficult to prove that the combination of MVE portfolios is MVE—except in the special case in which a risk-free security exists. Call the MVE tangency portfolio $\mathcal{T}$. Because $\mathcal{T}$ is MVE,

$$\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + \left[\mathcal{E}(\tilde{r}_{\mathcal{T}}) - r_{\mathcal{F}}\right] \cdot \beta_{i,T} \qquad \forall i \quad . \tag{16.41}$$

Now invest w_f in the risk-free security ($\mathcal{F}$) and $(1 - w_f)$ in $\mathcal{T}$. Call this portfolio Z.

$$\tilde{r}_Z = w_f \cdot r_{\mathcal{F}} + (1 - w_f)\cdot\tilde{r}_{\mathcal{T}} \quad . \tag{16.42}$$

Both $\mathcal{F}$ and $\mathcal{T}$ are MVE portfolios. The question is whether Z is also MVE. The expected rate of return on Z is

$$\mathcal{E}(\tilde{r}_Z) = w_f \cdot r_{\mathcal{F}} + (1 - w_f)\cdot\mathcal{E}(\tilde{r}_{\mathcal{T}}) \quad . \tag{16.43}$$

The beta of any security i with respect to this portfolio Z is

$$\begin{aligned}
\beta_{i,Z} &= \frac{Cov(\tilde{r}_i, \tilde{r}_Z)}{\mathcal{V}ar(\tilde{r}_Z)} = \frac{Cov[\tilde{r}_i,\ w_f \cdot r_{\mathcal{F}} + (1-w_f)\cdot \tilde{r}_{\mathcal{T}}]}{\mathcal{V}ar[w_f \cdot r_{\mathcal{F}} + (1-w_f)\cdot \tilde{r}_t ngnt]} \\
&= \frac{(1-w_f)\cdot Cov(\tilde{r}_i, \tilde{r}_{\mathcal{T}})}{(1-w_f)^2 \cdot \mathcal{V}ar(\tilde{r}_{\mathcal{T}})} = \left(\frac{1}{1-w_f}\right)\cdot\left[\frac{Cov(\tilde{r}_i, \tilde{r}_{\mathcal{T}})}{\mathcal{V}ar(\tilde{r}_{\mathcal{T}})}\right] \\
&= \frac{\beta_{i,T}}{1-w_f} \quad ,
\end{aligned} \tag{16.44}$$

because the risk-free rate has neither a variance nor a covariance with any other return series. The new portfolio Z is also MVE, if and only if

$$\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + \left[\mathcal{E}(\tilde{r}_Z) - r_{\mathcal{F}}\right]\cdot \beta_{i,Z} \quad \forall i \ . \tag{16.45}$$

Substitute Formula 16.43 for $\mathcal{E}(\tilde{r}_Z)$, and Formula 16.44 for $\beta_{i,Z}$:

$$\begin{aligned}
\mathcal{E}(\tilde{r}_i) &= r_{\mathcal{F}} + \left\{\left[w_f \cdot r_{\mathcal{F}} + (1-w_f)\cdot \mathcal{E}(\tilde{r}_Z)\right] - r_{\mathcal{F}}\right\}\cdot\left(\frac{\beta_{i,T}}{1-w_f}\right) \\
&= r_{\mathcal{F}} + \left[(1-w_f)\cdot \mathcal{E}(\tilde{r}_Z) - (1-w_f)\cdot r_{\mathcal{F}}\right]\cdot\left(\frac{\beta_{i,T}}{1-w_f}\right) \\
&= r_{\mathcal{F}} + \left[\mathcal{E}(\tilde{r}_Z) - r_{\mathcal{F}}\right]\cdot \beta_{i,T} \qquad \forall i \ .
\end{aligned} \tag{16.46}$$

This is true if and only if a portfolio is MVE. Therefore, the new portfolio Z is also MVE.

c. The Combination of Mean-Variance Efficient Portfolios is Mean-Variance Efficient — Without Risk-Free Security.

The combination of any two risky portfolios is a parabola in mean-variance space.

There are many ways to prove that the combination of two MVE portfolios is also MVE. The brute-force method is the linear algebra solution from the previous section. In Formula 16.37, $\mathbf{S}^{-1}\vec{1}$ is one portfolio (a vector of [unnormalized] portfolio weights), $\mathbf{S}^{-1}\vec{m}$ is another portfolio (another vector of portfolio weights). λ and μ determine the relative investment proportions in each.

We start with an "intuitive" proof.

The rest of this subsection is an alternative conceptual argument which may or may not help you to get some more intuition. Ignore it if you wish. Return to Formula 16.4, which shows that the relationship between two risky investment assets is

$$\mathcal{V}ar[\ \mathcal{E}(\tilde{r}_P)\ ] = a\cdot \mathcal{E}(\tilde{r}_P)^2 + b\cdot \mathcal{E}(\tilde{r}_P) + c \ , \tag{16.47}$$

where a, b, and c were constants that depend on the particular expected rate of return between two securities, their individual variances, and their mutual covariance.

Being only a parabola means that the combination portfolio of any two MVE Frontier portfolios has to curve at the same rate as the MVE Frontier.

Now, imagine two portfolios on the MVE frontier, call them P and Q. P is on the upper half of the parabola, Q is its equal-variance counterpart on the lower part of the parabola. Call M the actual minimum variance portfolio. The question is whether the combination of the P and Q portfolios traces out the MVE Frontier. We already know that the combination of P and Q is a quadratic equation. The previous section showed that the MVE Frontier is a quadratic equation, too. The question is can the quadratic frontier between P and Q define a set other than the MVE Frontier? In a quadratic equation, three points define the parabola. Thus, if P and Q are already chosen as two portfolios with equal risk, then all possible combinations of these portfolios would have to lie on a parabola, and the minimum variance portfolio based only on P and Q would

have to lie on the same horizontal line that the actual minimum variance portfolio M lies on. What would happen if the combination of P and Q produced a minimum variance portfolio with lower variance than M? Then our original statement that M is the minimum variance portfolio would be false. What would happen if the combination of P and Q produced a minimum variance portfolio with higher variance than M? Then, the parabola would overshoot the MVE Frontier when either P and Q is shorted. However, then the real MVE Frontier would not be a simple quadratic equation, but one quadratic equation to the left of P and Q and another quadratic equation to the right of P and Q. We already know that this cannot be the case, either.

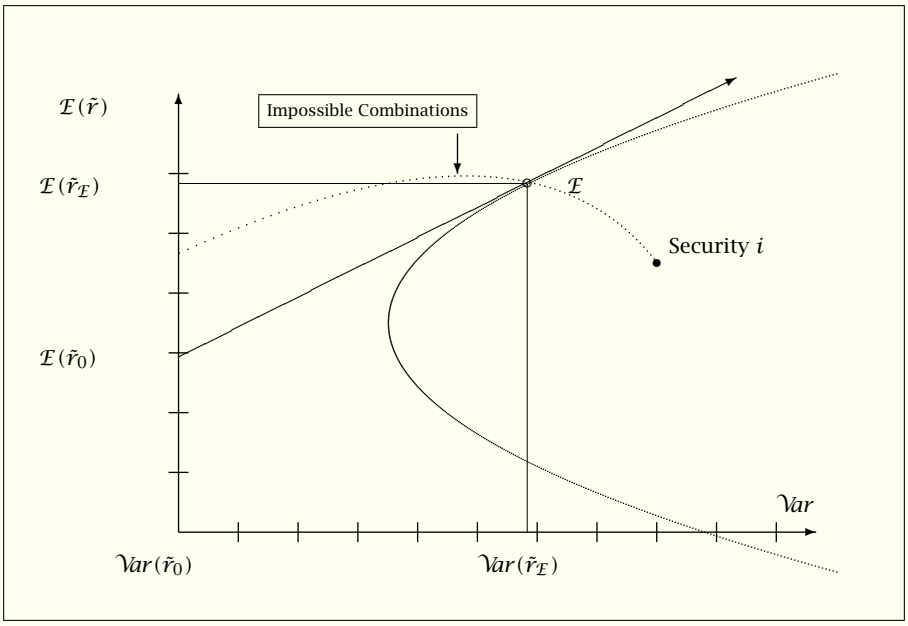
Therefore, any combination of any two risky MVE portfolios traces out the MVE Frontier.

Therefore, the combinations of P and Q trace out the MVE frontier. In other words, the combination of the two MVE risky portfolios P and Q is also MVE. Any other portfolio, say R, that is a weighted combination of P and Q can be represented by a unique weight on P vs. Q. Combining many arbitrary of these unique-weight portfolios on P and Q is still a weighted combination of P and Q, and therefore MVE.

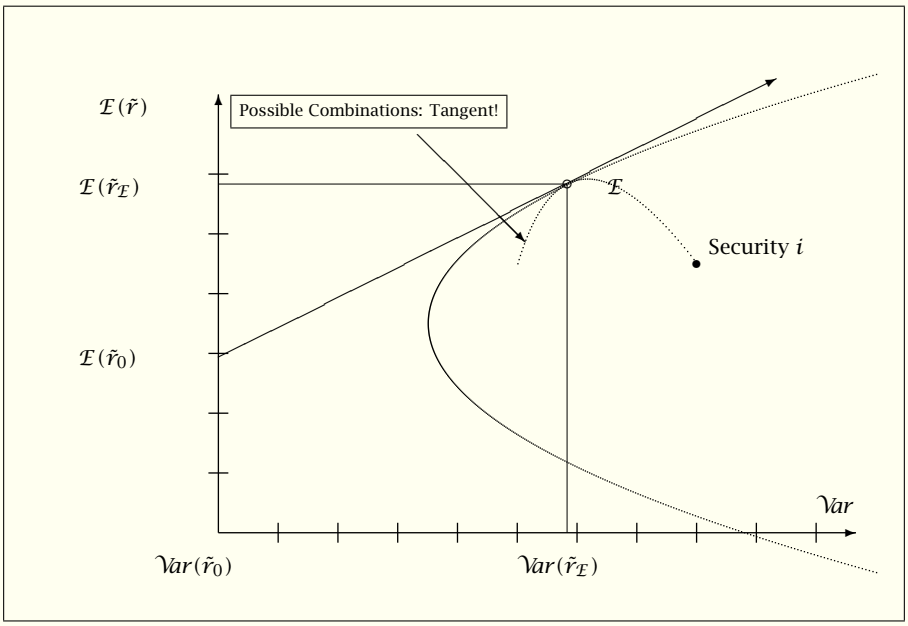
d. Proof of the Linear Beta vs. Expected Rate of Return Relationship for MVE Frontier Portfolios

Figure 16.11. Possible and Impossible Combinations of a new Security with a Mean-Variance Efficient Portfolio

Impossible Combinations of a new Security with a Mean-Variance Efficient Portfolio:



Possible Combinations of a new Security with a Mean-Variance Efficient Portfolio:



Our intent is to prove that all securities in a MVE portfolio must follow a linear beta-expected return relationship. It is the mathematical relationship that is required for every stock to just be at its optimum level, so that you are indifferent between purchasing one more or one less cent of this security. The derivation relies on the so-called envelope theorem, which states that adding a little, little bit or subtracting a little, little bit at the optimal investment choice $w_i^\star$ should make only a minimal difference *for the optimal portfolio*. If the expected rate of return were too high or too low for a security given its beta, you could do better than $\mathcal{E}$, either buying a little bit more or a little bit less of this security. But then, $\mathcal{E}$ would not be MVE.

Consider the combination of the MVE Portfolio and a new security.

Figures 16.11 shows a MVE portfolio called $\mathcal{E}$. You are considering adding w_i of a security i (security i *alone* does not have to be MVE). Could it be that the combination portfolio were not tangent to the frontier, i.e. where $w_i \approx 0$ and $w_\mathcal{E} \approx 1$? The answer must be no—or it would be possible to purchase a combination portfolio that outperforms the MVE Frontier. Perturbing the weight in the $\mathcal{E}$ by adding security i (where $w_i = 0$) has to result in a mean-variance tradeoff that is tangent to the MVE Frontier, as shown in the graph in Figure 16.11. We can use this insight to prove the linear relationship between the expected rate of return on security i and its beta with respect to $\mathcal{E}$.

If we combine an MVEportfolio with a little bit of another portfolio, we should not do better.

1. By inspection, the slope of the MVE Frontier at $\mathcal{E}$ is

$$\text{Slope} \;=\; \frac{\mathcal{E}(\tilde{r}_\mathcal{E}) - \mathcal{E}(\tilde{r}_0)}{Sdv(\tilde{r}_\mathcal{E}) - Sdv(\tilde{r}_0)} \;=\; \frac{\mathcal{E}(\tilde{r}_\mathcal{E}) - \mathcal{E}(\tilde{r}_0)}{Sdv(\tilde{r}_\mathcal{E})} \quad , \tag{16.48}$$

   because $Sdv(\tilde{r}_0)$ is 0.

2. Consider an arbitrary combination portfolio (call it P). It is defined by

$$\tilde{r}_P \;=\; w_i \cdot \tilde{r}_i + (1 - w_i) \cdot \tilde{r}_\mathcal{E} \quad . \tag{16.49}$$

   The characteristics of this portfolio P are

$$\begin{aligned} \mathcal{E}(\tilde{r}_P) &= w_i \cdot \mathcal{E}(\tilde{r}_i) + (1 - w_i) \cdot \mathcal{E}(\tilde{r}_\mathcal{E}) \\ Sdv(\tilde{r}_P) &= \sqrt{w_i^2 \cdot \mathcal{V}ar(\tilde{r}_i) + (1 - w_i)^2 \cdot \mathcal{V}ar(\tilde{r}_\mathcal{E}) + 2 \cdot w_i \cdot (1 - w_i) \cdot Cov(\tilde{r}_i, \tilde{r}_\mathcal{E})} \quad . \end{aligned} \tag{16.50}$$

   How do the mean and standard deviation of P change with w_i?

$$\begin{aligned} \frac{\partial \mathcal{E}(\tilde{r}_P)}{\partial w_i} &= \mathcal{E}(\tilde{r}_i) - \mathcal{E}(\tilde{r}_\mathcal{E}) \quad , \\ \frac{\partial Sdv(\tilde{r}_P)}{\partial w_i} &= \left\{ \frac{Cov(\tilde{r}_i, \tilde{r}_\mathcal{E}) - \mathcal{V}ar(\tilde{r}_\mathcal{E}) + w_i \cdot [\mathcal{V}ar(\tilde{r}_i) - 2 \cdot Cov(\tilde{r}_i, \tilde{r}_\mathcal{E}) + \mathcal{V}ar(\tilde{r}_\mathcal{E})]}{\sqrt{w_i^2 \cdot \mathcal{V}ar(\tilde{r}_i) + (1 - w_i)^2 \cdot \mathcal{V}ar(\tilde{r}_\mathcal{E}) + 2 \cdot w_i \cdot (1 - w_i) \cdot Cov(\tilde{r}_i, \tilde{r}_\mathcal{E})}} \right\} \quad . \end{aligned} \tag{16.51}$$

   Now, compute the slope of the curve that defines the sets of combination portfolios P, i.e., how its expected rate of return changes as its standard deviation is changed. Because we can only change w_i, we need to compute this slope by dividing the two derivatives in Formula 16.51:

$$\frac{\partial \mathcal{E}(\tilde{r}_P)}{\partial Sdv(\tilde{r}_P)} \;=\; \frac{\frac{\partial \mathcal{E}(\tilde{r}_P)}{\partial w_i}}{\frac{\partial Sdv(\tilde{r}_P)}{\partial w_i}} \quad . \tag{16.52}$$

What we really need to know is the slope when $w_i = 0$, i.e., when the weight is solely on the MVE Frontier portfolio $\mathcal{E}$. So, substitute $w_i = 0$ into this messy expression, and you get

$$\begin{aligned} \lim_{w_i \to 0} \left[\frac{\partial \mathcal{E}(\tilde{r}_P)}{\partial Sdv(\tilde{r}_P)} \right] &= \frac{\lim_{w_i \to 0} \left[\frac{\partial \mathcal{E}(\tilde{r}_P)}{\partial w_i} \right]}{\lim_{w_i \to 0} \left[\frac{\partial Sdv(\tilde{r}_P)}{\partial w_i} \right]} \\ &= \frac{\mathcal{E}(\tilde{r}_i) - \mathcal{E}(\tilde{r}_{\mathcal{E}})}{\left[\frac{Cov(\tilde{r}_i, \tilde{r}_{\mathcal{E}}) - \mathcal{V}ar(\tilde{r}_{\mathcal{E}})}{\mathcal{V}ar(\tilde{r}_{\mathcal{E}})} \right]} \quad . \end{aligned} \tag{16.53}$$

3. We now use the fact that the slope of portfolio P at $w_i = 0$ has to be the same slope as the MVE Frontier. That is, the line defined in equation 16.48 and the curve defined in equation 16.53 must be tangent.

$$\begin{aligned} \text{Slope of Tangent Line} &= \text{Slope of Curve at } w_i = 0 \\ \frac{\mathcal{E}(\tilde{r}_{\mathcal{E}}) - \mathcal{E}(\tilde{r}_0)}{Sdv(\tilde{r}_{\mathcal{E}})} &= \frac{\mathcal{E}(\tilde{r}_i) - \mathcal{E}(\tilde{r}_{\mathcal{E}})}{\left[\frac{Cov(\tilde{r}_i, \tilde{r}_{\mathcal{E}}) - \mathcal{V}ar(\tilde{r}_{\mathcal{E}})}{\mathcal{V}ar(\tilde{r}_{\mathcal{E}})} \right]} \quad . \end{aligned} \tag{16.54}$$

Use $\mathcal{V}ar(\tilde{r}_{\mathcal{E}}) = Sdv(\tilde{r}_{\mathcal{E}}) \cdot Sdv(\tilde{r}_{\mathcal{E}})$ to simplify this equation to

$$\begin{aligned} \mathcal{E}(\tilde{r}_i) &= \mathcal{E}(\tilde{r}_0) + \left[\frac{Cov(\tilde{r}_i, \tilde{r}_{\mathcal{E}})}{\mathcal{V}ar(\tilde{r}_{\mathcal{E}})} \right] \cdot [\mathcal{E}(\tilde{r}_{\mathcal{E}}) - \mathcal{E}(\tilde{r}_0)] \\ &= \mathcal{E}(\tilde{r}_0) + \beta_{im} \cdot [\mathcal{E}(\tilde{r}_{\mathcal{E}}) - \mathcal{E}(\tilde{r}_0)] \quad , \end{aligned} \tag{16.55}$$

which is Formula 16.24 that we wanted to prove! This formula has to hold for every available security i. Indeed, a portfolio is MVE *if and only if* this equation holds for all available investment opportunities i.

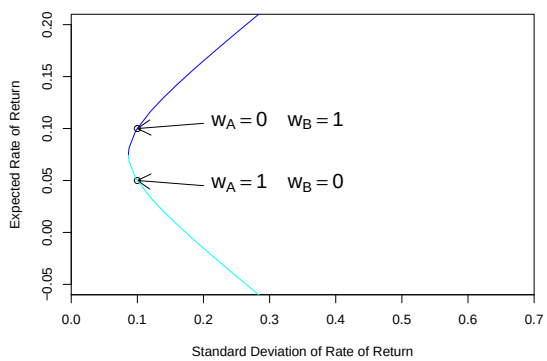
NERD NOTE: *Figure 16.11 is drawn for better illustration, not for accuracy. To avoid eventually intersecting the MVE Frontier, the possible combinations would necessarily have to intersect $\mathcal{E}$ from below, not from above (as drawn).*

A. Q&A: Answers

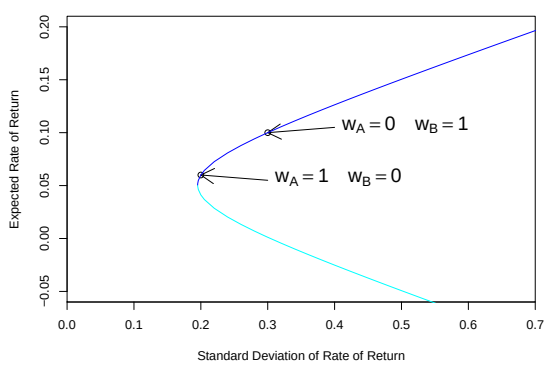
1. Formally, no. The risk is always 10%. However, by shorting the lower expected rate of return security and longing the higher expected rate of return security, you can invest in an opportunity with an infinite expected rate of return. Unfortunately, even though this is expected, it is possible that in one period, the higher expected rate of return security actually earns a negative rate of return, in which case the portfolio would actually lose money. Remember: expectations apply to infinitely or very often repeated gambles, not to one gamble.
2. Indeed, going long and going short is risk-free. However, it is also a zero-investment and zero-payoff strategy (except for frictions, of course). Therefore, it is not useful in constructing an arbitrage opportunity.
3. The portfolios have the following risks:

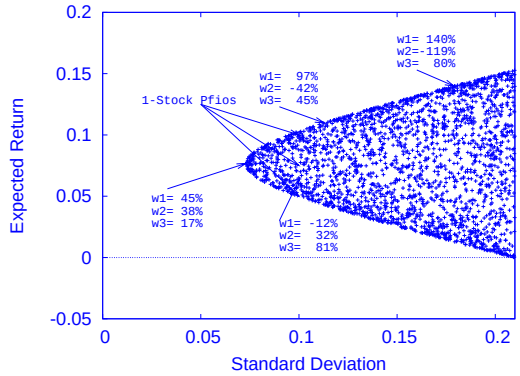
Weight w_A	-1	0	$1/3$	$1/2$	$2/3$	1
Risk (σ_P)	17.3%	10%	8.8%	8.7%	8.8%	10%

A graph of these portfolios (and weights in between) is



4. First, compute the covariance: $Correlation = Cov/(Sdv_A \cdot Sdv_B) \Rightarrow Cov = 20\% \cdot 30\% \cdot 80\% = 0.048$. The MVE frontier is





5.

6. The **Minimum Variance Portfolio** in this world is a portfolio that invests about 45% into security 1, about 38% into security 2, and the rest (about 17%) into security 3.

7. An investor with more taste for risk would choose a portfolio higher up along the MVE Frontier. The least risky portfolio with an expected rate of return of 11% invests about 97% into security 1, shorts 42% in security 2, and invests 45% into security 3.

8. No, because it is inside the main "cloud" of points. There are better portfolios that offer higher expected rates of return for the same amount of risk. The three "pure" portfolios, which invest in only one stock are not on the MVE Frontier, either. However, the portfolio that invests 100% in security 1 and nothing in the other two securities comes fairly close *by accident*.

9. You can save time if you work directly off combinations of portfolio A and portfolio C.

	A	C	$1/2 \cdot A + 1/2 \cdot C$
S&P500	99.5%	36.1%	67.770%
IBM	2.1%	46.6%	24.386%
Sony	-1.6%	17.2%	7.844%
$\mathcal{E}(\tilde{r})$	10.0%	15.0%	12.500%
$Sdv(\tilde{r})$	19.0%	67.6%	30.949%

10. You can save time if you work directly off combinations of portfolio B and portfolio C.

	B	C	$1/2 \cdot A + 1/2 \cdot C$
S&P500	-90.6%	36.1%	-27.264%
IBM	135.7%	46.6%	91.172%
Sony	54.9%	17.2%	36.092%
$\mathcal{E}(\tilde{r})$	25.0%	15.0%	20.000%
$Sdv(\tilde{r})$	19.0%	67.6%	59.266%

11. Still the same: 0.35. Indeed, this is the case for any MVE portfolio now.

12. If you sketch it, the intuition is that when the risk-free rate is lower, the tangency portfolio should have less risk and less reward. With $r_{\mathcal{F}} = 3\%$, the weight on portfolio A is now 81.606%. The tangency portfolio has a mean rate of return of 12.759% (excess return, 9.759%), and a standard deviation of 22.344%. The Sharpe ratio is about 0.4368.

13. If you sketch it, the intuition is that when the risk-free rate is higher, the tangency portfolio should have more risk and more reward. With $r_{\mathcal{F}} = 3\%$, the weight on portfolio A is now -23.949%. The tangency portfolio has a mean rate of return of 28.592% (excess return, 19.592%), and a standard deviation of 82.607%. The Sharpe ratio is about 2371.

14.

$$8\% \;=\; \alpha + (14\% - \alpha) \cdot 0.5 \qquad \Longleftrightarrow \qquad \alpha = 2\% \;. \tag{16.56}$$

Therefore,

$$\mathcal{E}(\tilde{r}_i) \;=\; 2\% + (14\% - 2\%) \cdot 2 \;=\; 26\% \;. \tag{16.57}$$

15.

(a) $\beta_{\mathsf{N,B}} = Cov(\tilde{r}_N, \tilde{r}_B)/\mathcal{V}ar(\tilde{r}_B) = 17.974\%/12.707\% = 1.414$.

(b) Yes, stock N does lie exactly on the line. This means that portfolio B is still MVE: you cannot do better.

(c) The mean rate of return of this combination portfolio is $\mathcal{E}(\tilde{r}) = 99\%\cdot 17\% + 1\%\cdot 21.029\% = 17.040\%$. For B to remain MVE, the combination portfolio must not be more risky. It is $\mathcal{V}ar(\tilde{r}) = (99\%)^2\cdot 12.707\% + (1\%)^2\cdot 41.978\% + 2\cdot 1\%\cdot 99\% = 12.814\%$. This is higher than the variance of $\tilde{r}_B$ alone. Therefore, this combination portfolio is not superior to $\tilde{r}_B$.

(d) The mean rate of return of this $\mathcal{E}(\tilde{r}) = 101\%\cdot 17\% + (-1\%)\cdot 21.029\% = 16.960\%$. This portfolio already has lower mean, so it cannot dominate portfolio B. We do not even have to compute the risk. (The *Sdv* happens to be $\sqrt{0.126}$.)

16.

(a) Beta has not changed. $\beta_{\mathsf{N,B}} = Cov(\tilde{r}_N, \tilde{r}_B)/\mathcal{V}ar(\tilde{r}_B) = 17.974\%/12.707\% = 1.414$.

(b) No, stock N does not lie on the line. This means that portfolio B is no longer MVE: you can do better.

(c) The mean rate of return of this combination portfolio is $\mathcal{E}(\tilde{r}) = 99\%\cdot 17\% + 1\%\cdot 10.0\% = 16.93\%$. This portfolio already has lower mean, so it cannot dominate portfolio B.

(d) The mean rate of return of this combination portfolio is $\mathcal{E}(\tilde{r}) = 101\%\cdot 17\% + (-1\%)\cdot 10.0\% = 17.07\%$. The variance of the combination portfolio is $\mathcal{V}ar(\tilde{r}) = (101\%)^2\cdot 12.707\% + (-1\%)^2\cdot 41.978\% + 2\cdot(-1\%)\cdot 99\% = 12.603\%$. Thus, this combination portfolio has both higher mean and lower risk than portfolio B. Therefore, portfolio B is no longer MVE!.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 17

The CAPM: A Cookbook Recipé Approach

How much expected return should investments offer?

last file change: Jun 15, 2004 (11:47h)

last major edit: n/a

This chapter appears in the Survey text only.

The previous chapters explored *your* best investment choices. This chapter explores the consequences if *all* important investors in the economy were to engage in such good investment choices.

This chapter offers the "recipé version" of the CAPM. Chapter 18 will explain and critically evaluate the theory behind the CAPM. Chapters 21 and 22 will show how to use the CAPM in a capital budgeting context.

17·1. Conceptual Basics

Knowing what (all) other investors do is less important to an investor, than it is to a firm that wants to sell to investors.

Why is it important if all other investors are purchasing MVE portfolios? It is not important if you want to *buy* investments. Regardless of what other investors are purchasing, you can make your own investment decisions. But it is very important for corporations who want to *sell* investments.

The firm must know the opportunities available to investors to know what they like and dislike.

Think about a firm that acts on behalf of its owner/investors. Investors have an opportunity cost when they give the firm their money—the firm can invest their money, or it can let investors put their money into alternative projects elsewhere (by returning it to investors instead of putting it into a new project). A manager acting on behalf of the investors needs to determine what projects investors find worthwhile and what projects investors do not find worthwhile; which projects investors would want to buy and which projects to pass up on.

This is why the appropriate expected rate of return is called the cost of capital.

This is the process by which managers need to determine the price at which the important investors in the economy are willing to purchase projects, so that they themselves can make the decisions of what projects to take and what projects to avoid. This task of "finding a fair price" is the process of determining an appropriate "costs of capital" (expected rate of return) for a project. (See also Section 8·3.) The so-called **Capital Asset Pricing Model** (or CAPM) provides this number.

Important: *As a corporate manager, the* CAPM *stipulates an* ***opportunity cost of capital*** *for our corporation's investors. This helps us determine whether our projects are beneficial or detrimental for our owners; whether we should take them or instead return money to our investors for better uses elsewhere.*

17·2. Required Returns: The Informal Way

After diversification, the second most important principle of investments is that it takes extra compensation to get investors to accept extra risk. This does not mean that you usually can expect to receive extra compensation for risks that you do not need to take. You can easily avoid betting in Las Vegas, so you are unlikely to receive extra compensation—a high expected rate of return—when betting there. But it does mean that when a firm wants you to accept risks that you cannot easily diversify away, it must offer you a higher expected rate of return in order to get you to accept this risk.

Higher Risk can mean higher reward...but not always.

17·3. The CAPM

17·3.A. The Premise and Formula

The basic premise of the Capital Asset Pricing Model is that investors—to whom the firm wants to sell its projects—hold a well-diversified market portfolio. Because, in the CAPM, this market portfolio is also mean-variance efficient, the project must offer investors a fair risk/reward tradeoff in order to join it. (This was explained in Section 16·5.)

The CAPM Formula is similar to the Formula from Section 16·5, except the efficient portfolio is M.

The project's risk contribution is measured by beta with respect to the investors' portfolio, stock market $\mathcal{M}$,

Beta measures project risk contribution.

$$\beta_{i,\mathcal{M}} \equiv \frac{Cov(\tilde{r}_i, \tilde{r}_{\mathcal{M}})}{Var(\tilde{r}_{\mathcal{M}})} , \tag{17.1}$$

which you are assumed to know. (In real life, you can estimate it.) Stocks that move a lot together with the market (high beta) are relatively risky, because they do not help investors already holding the market portfolio to diversify. Reward is measured by the expected rate of return, $\mathcal{E}(\tilde{r}_i)$. Securities contributing more risk have to offer a higher expected rate of return.

Important: *The* CAPM *states that the relationship between risk contribution ($\beta_{i,\mathcal{M}}$) and reward ($\mathcal{E}(\tilde{r}_i)$) is*

$$\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + \left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right] \cdot \beta_{i,\mathcal{M}} , \tag{17.2}$$

where $r_{\mathcal{F}}$ is the risk-free rate, and $\mathcal{E}(\tilde{r}_{\mathcal{M}})$ is the expected rate of return on the stock market.
*$\left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right]$ is also called the **Equity Premium** or **Market Risk Premium**.*

The CAPM specifically places little importance on the standard deviation of an individual stock. A stock's own standard deviation is only a measure of how risky it is *by itself*, which would only be of relevance to an investor who holds just this one stock and nothing else. But our investors are smarter. They care about their portfolio risk, and keep it low by holding the widely diversified overall stock market portfolio. To them, the standard deviation of a stock only matters to the (usually small) extent to which it can influence the market beta.

The CAPM Formula is about the suitable risk/reward tradeoff for an investor holding the market portfolio.

Our Agenda: Cover all the CAPM Inputs.

In sum, to estimate an appropriate CAPM expected rate of return for a project or firm, i.e., the cost of capital, you need three inputs:

1. The risk-free rate of return, $r_{\mathcal{F}}$.
2. The expected rate of return on the market, $\mathcal{E}(\tilde{r}_{\mathcal{M}})$.
3. A firm's or project's beta with respect to the market, $\beta_{i,\mathcal{M}}$.

We are really interested in the future expected rate of return on the market, and the future beta of a firm/project with respect to the market, and not in the past average rates of return or beta. However, we usually must estimate the latter two inputs, and often we have to rely primarily on historical data. Sections 17·4 and 17·5 will talk about how we estimate each input in more detail.

17·3.B. A Brief Usage Example and the Security Markets Line (SML)

An example of what rate of returns individual securities should offer.

Let us assume that the risk-free rate is 3% per year, and that the stock market offers an expected rate of return of 8% per year. The CAPM formula states that a stock with a beta of 1 should offer an expected rate of return of $3\% + (8\% - 3\%)\cdot 1 = 8\%$ per year; that a stock with a beta of 0 should offer an expected rate of return of $3\% + (8\% - 3\%)\cdot 0 = 3\%$ per year; that a stock with a beta of $1/2$ should offer an expected rate of return of $3\% + (8\% - 3\%)\cdot 0.5 = 5.5\%$ per year; that a stock with a beta of 2 should offer an expected rate of return of $3\% + (8\% - 3\%)\cdot 2 = 13\%$ per year; and so on:

Stock	$\beta_{i,\mathcal{M}}$	$\Rightarrow$	$\mathcal{E}(\tilde{r}_i)$
A	-1.0	$\Rightarrow$	-2.0%
B	-0.5	$\Rightarrow$	0.5%
C	0.0	$\Rightarrow$	3.0%

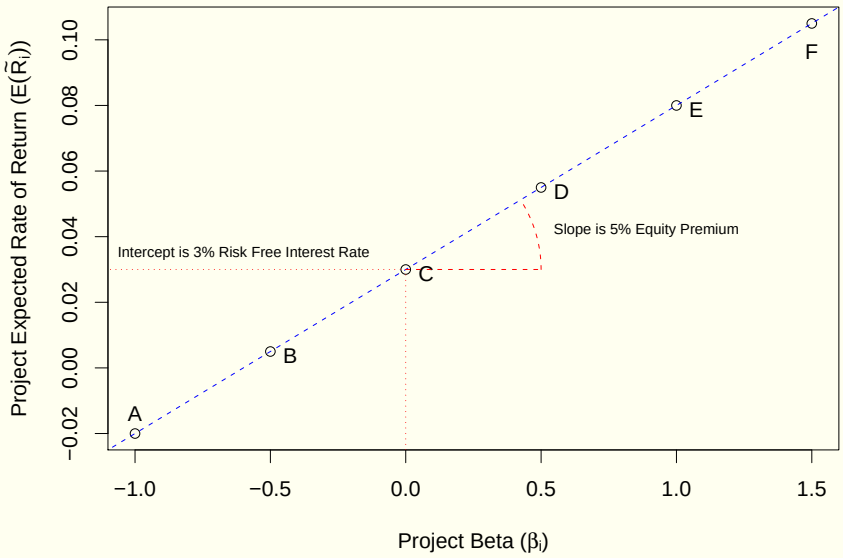
Stock	$\beta_{i,\mathcal{M}}$	$\Rightarrow$	$\mathcal{E}(\tilde{r}_i)$
D	0.5	$\Rightarrow$	5.5%
E	1.0	$\Rightarrow$	8.0%
F	1.5	$\Rightarrow$	10.5%

What happens if a stock offers too much or too little expected rate of return?

If a stock offered more than its due expected rate of return, investors would want to buy more of the stock than would be available: its price would be too low. It would be too good a deal. Investors would immediately flock to it, and because there would not be enough of this stock, investors would bid up its price and thereby lower its expected rate of return. Conversely, if a stock offered less than its due expected rate of return, investors would not be willing to hold enough of the stock: the stock's price would be too high. Neither should happen for very long in the real world.

NERD NOTE: *The formal* CAPM *model really does not allow for any such dynamics, i.e., how we get back to the* CAPM *if securities are mispriced. In the* CAPM, *everything is always perfect and right from start. So, the above is really only an intuitive explanation of how the* CAPM *process works. Also, realize that there is no arbitrage if stocks do not follow the* CAPM *relation: stocks that offer too high or too low a rate of return can be a good or a bad deal and thus would attract too many or too few investors. But there is no risk-free profit to be made.*

Figure 17.1. The Security Market Line For Stocks A–F



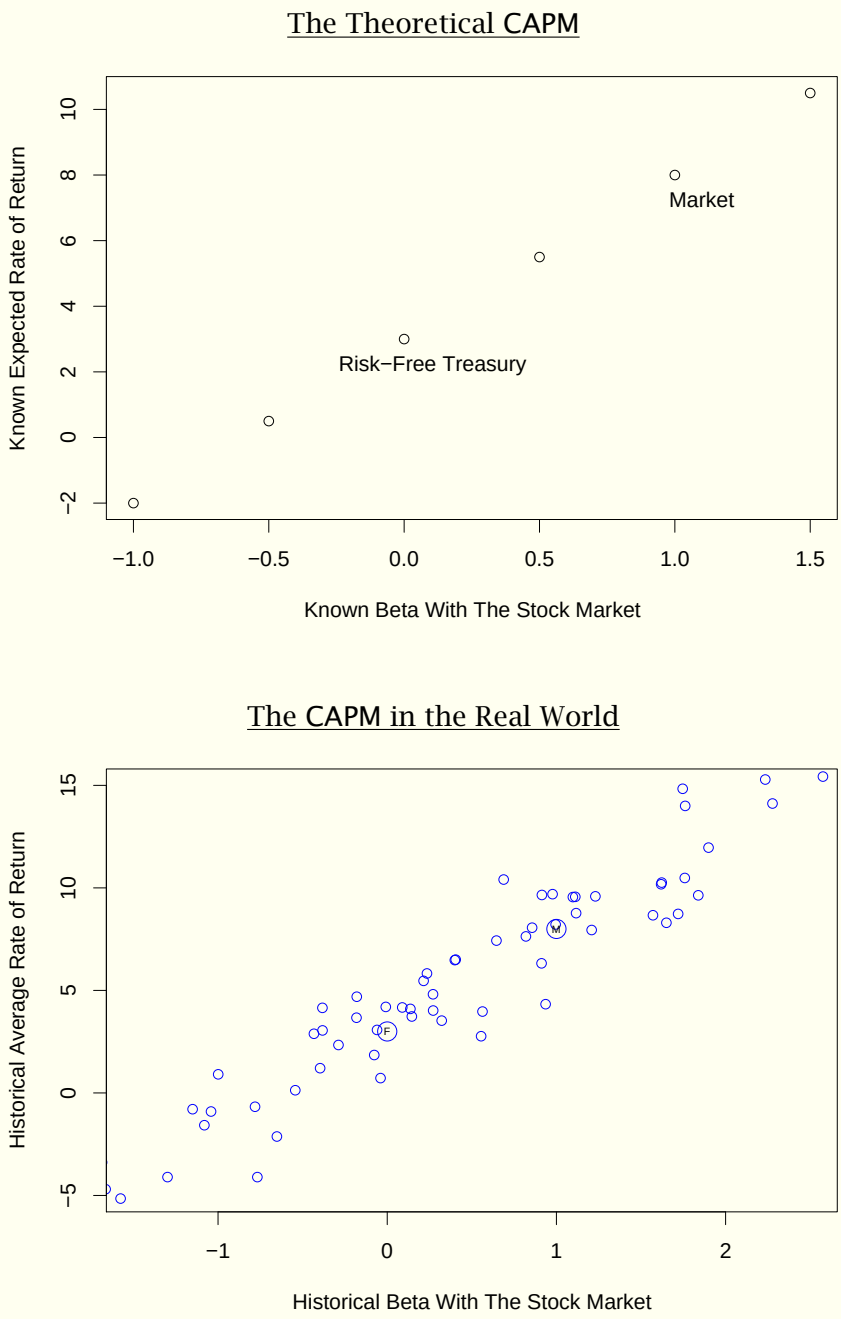
Stock	$\beta_{i,\mathcal{M}}$	$\mathcal{E}(\tilde{r}_i)$
A	−1.0	−2.0%
B	−0.5	0.5%
C	0.0	3.0%

Stock	$\beta_{i,\mathcal{M}}$	$\mathcal{E}(\tilde{r}_i)$
D	0.5	5.5%
E	1.0	8.0%
F	1.5	10.5%

This graph plots the CAPM relation $\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{M}} = 3\% + (8\% - 3\%) \cdot \beta_{i,\mathcal{M}}$.

The CAPM equation is often graphed in the so-called **security markets line**, which shows the relationship between the expected rate of return of a project and its beta. Figure 17.1 draws our first security markets line, using stocks A through F. As the name suggests, all stocks must lie on a straight line. In other words, the SML line is just a graphical statement of the formulaic one for the CAPM.

Figure 17.2. The Security Market Line in a CAPM World



Historical average returns and historical betas are estimated from the data—and hopefully representative of the true underlying mean returns and true betas, which in turn means that they are also indicative of the future mean returns and betas.

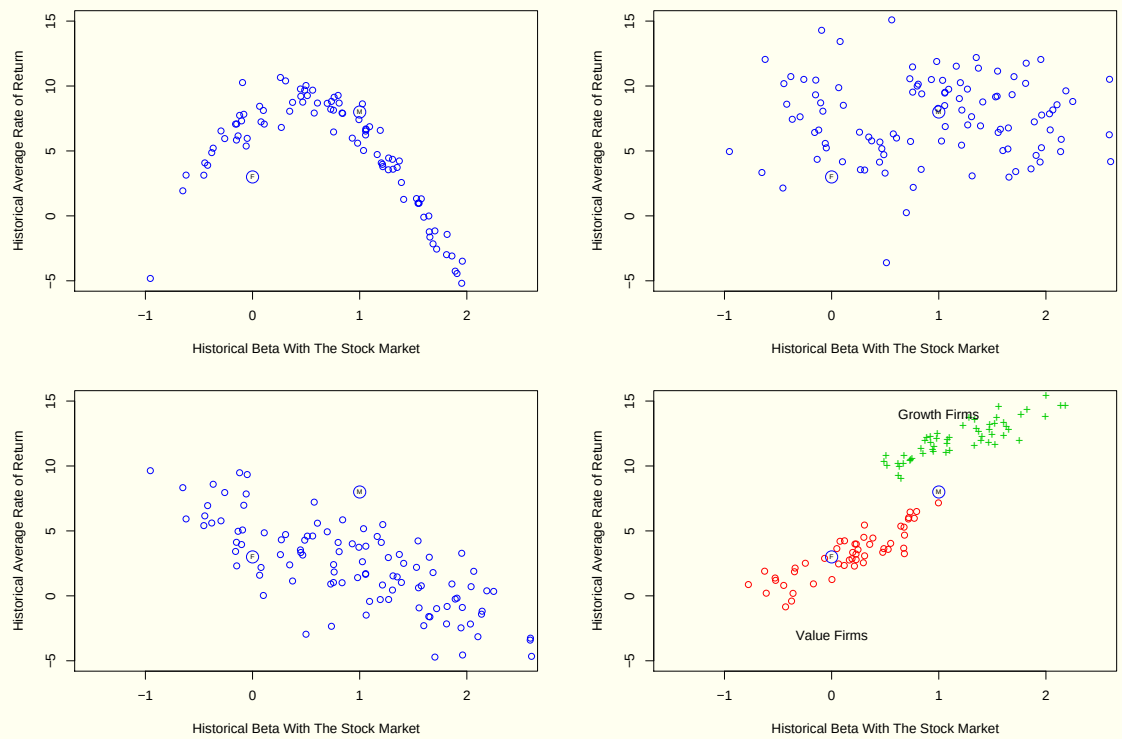
The "Security Market Line" in an Ideal CAPM World

Figure 17.2 plots two graphs to contrast the difference between the perfect world in which we know all the CAPM inputs, and a "wonderful" world in which the CAPM holds, but we do not know all the inputs. As in our previous example, we assume the risk-free rate is 3% per year, and the expected rate of return on the stock market is 8% per year. Again, each stock is a point in this coordinate system. In the left graph, as before, we know all inputs, including the *expected* rates of return on each stock and the betas of each stock with the market, so we can plot our straight line.

The "Security Market Line" in a Realistic CAPM World

Unfortunately, neither true betas nor true expected rates of return are published. So, more realistically, we have no choice but to *estimate* both the expected rates of return and betas from historical data. (For convenience, we call these "historical" estimates in our figures, even though they could incorporate other knowledge, too.) We can only plot an "estimated security market line," not the "true security market line," using historical data—of course, hoping that these are good unbiased estimates of beta and expected rates of returns (and this is a big if). (Section 14·1 already discussed some of the pitfalls.) Of course, for ordinary stocks, the expected rate of return cannot always be the actual realized rate of return (or the stock would be risk-free). The right graph in Figure **??** shows a graph of historical actual rates of returns plotted against historical betas. Our workable version of the CAPM states that there should roughly be a linear relationship between data-estimated beta and the data-estimated expected rate of return, just as we have drawn it here.

Figure 17.3. The Security Market Line in non-CAPM Worlds



Each point is one stock (or fund)—its historical beta and its historical average rate of return. (The market and risk-free rate are noted.) In these figures, the security market line does not appear linear, as the CAPM suggests. Therefore, if these patterns are not just statistical mirages, you can usually do better than investing in the market. In the top figures and left figures, you would want to overemphasize low market-beta stocks and underemphasize high-beta stocks in your portfolio. In the bottom right figure, you would disproportionally purchase growth-firms.

The "Security Market Line" if the CAPM is the wrong model.

Under what circumstances would we lose faith in the CAPM? Figure 17.3 plots how security market relations could look like if the CAPM did not work. In the top left figure, the rate of return does not seem to linearly increase with beta if beta is greater than about 0.5. In the top right figure, the rate of return seems unrelated to beta. In the bottom left figure, higher beta securities offer *lower* expected rates of returns. And in the bottom right figure, even though we would still draw a straight positive line between the two clusters of firm, it is not really beta that matters, but whether a firm is a growth firm or a value firm. After taking into account what the firm is, beta would not matter. You could earn higher expected rates of return buying stocks based on firm type, than based on beta. Naturally, in the short-run or if our procedures to extract beta or expected rates of return are poor, we might observe such relationships even if the CAPM really held.

Solve Now!

Q 17.1 *The risk-free rate is 4%. The expected rate of return on the stock market is 7%. What is the appropriate cost of capital for a project that has a beta of 3?*

Q 17.2 *The risk-free rate is 4%. The expected rate of return on the stock market is 12%. What is the appropriate cost of capital for a project that has a beta of 3?*

Q 17.3 *The risk-free rate is 4%. The expected rate of return on the stock market is 7%. A corporation intends to issue publicly traded bonds which promise a rate of return of 6%, and offer an expected rate of return of 5%. What is the implicit beta of the bonds?*

Q 17.4 *Draw the security market line if the risk-free rate is 5% and the equity premium is 4%.*

Q 17.5 *What is the equity premium, both mathematically and intuitively?*

Q 17.6 *Draw some possible security markets relations which would not be consistent with the* CAPM.

17·4. Obtaining CAPM Constant Inputs: The Risk-Free Rate and the Equity Premium

The Risk-Free Rate is relatively easy to obtain.

The risk-free rate is easily obtained from Treasury bonds. But which one? From a practical perspective, it makes sense to use the yield on a Treasury bond that is of similar length as a project's approximate lifespan. For example, to value a machine that produces for three years, it makes sense to use an average of the 1-year, 2-year, and 3-year risk-free interest rates.

Nerd Note: *The* CAPM *is a one-period model, and has no concept of multiple periods. Thus, it gives us no real guidance for what interest to use when interest rates differ over different horizons (i.e., when the yield curve is not flat). The advice to use "similar-time-span" interest rates is only a good heuristic.*

The equity premium is not only more interesting than the risk-free rate, it is *the most interesting* single number in finance.

Important: *The* ***equity premium****,* $\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}$*, may be the single most important number in finance. It measures the extra expected rate of return that risky projects have to offer, above and beyond what risk-free projects have to offer.*

The equity premium must be provided as a CAPM input. Estimates are all over the map; reasonable ones can range from 2% to 8% per year.

The equity premium is not only interesting in itself, it is also a CAPM input of great importance. The value you choose can have a tremendous influence on your estimated costs of capital. Of course, the CAPM assumes that the *expected* rate of return on the market is known. Yet, in real life, the equity premium is not posted anywhere, and no one really knows the correct number. There are a number of methods to guestimate it, which unfortunately do not tend to agree with one another. Rather than just throw you one estimate and ask you to use it (in which case you would be startled to find that someone else has used another number and came up with another cost of capital estimate), let's discuss the intuition behind each method and the number the intuition would suggest. In this way, you can make up your own mind as to what you deem to be an appropriate equity premium estimate.

Method 1: Historical Averages.

The first estimation method simply relies on historical average equity premia as good indicators of future risk premia. As of 2003, the arithmetic average equity premium since 1926 was about **8.4%** per annum. However, if you start computing the average in 1869, the equity premium estimate drops to around **6.0%**. Maybe you should start in 1771? Or 1971? Which is the best estimation period? Noone really knows what the right start date should be. If you choose too few years, your sample average could be unreliable. If you choose too many years, your earlier sample period may be so different from today that it is no longer relevant.

Method 2: Inverse Historical Averages.

The second estimation method argues that high past equity premia are not indicative of high future equity premia, but rather indicative of **bubbles** in the stock market. The proponents of this view usually cannot quantify the appropriate equity premium, except to argue that it is lower after recent market runups—exactly the opposite of what proponents of the historical perspective argue. It need not be bubbles, however—if the expected rate of return declines, the PV formula tells us that the price of an asset today goes up. So, if some factor in the economy had made investors compete for and accept lower expected rates of return, it would have shown up in increased stock prices.

SIDE NOTE: A bubble is a run-away market, in which rationality has temporarily disappeared. There is a lot of debate as to whether bubbles in the stock market ever occurred. A strong case can be made that Technology stocks experienced a bubble from around 1998 to 2000. Noone has yet come up with a rational story based on fundamentals that can explain *both* why the Nasdaq Index had climbed from 2,280 in March 1999 to 5,000 on March 27, 2000, *and* why it then dropped back to 1,640 on April 4, 2001.

Method 3: Dividend or Earnings Yields.

The third method uses historical dividend yields (i.e., the dividend payments received by stockholders) to predict rates of returns. Higher dividend yields make stocks more attractive and therefore predict higher future equity premia. (This is usually done in two steps: first, one estimates a statistical regression that predicts next year's equity premium with this year's dividend yield; second, one substitutes in the currently prevailing dividend yield.) Unfortunately, as of 2003, current dividend yields are so low that the predicted equity premia are just about **0%** or even negative. Variations of this method use the earnings yield, with similar results.

Method 4: What is reasonable reward for risk?

The fourth method wonders how much rate of return is required to entice reasonable investors to switch from bonds into stocks. Even with an equity premium as low as 3%, over 25 years an investor would end up with more than twice the money when investing in equity rather than bonds. Naturally, in an efficient market, nothing comes for free, and the reward for risk-taking is just about fair. So, equity premia of 8% just seem too high for the amount of risk observed in the stock market. This method generally suggests equity premia of about **1%** to **3%**.

ANECDOTE: The Power of Compounding
Assume you invested $1 in 1925. How much would you have in December 2001? If you had invested in large-firm stocks, you would have ended up with $2,279 (10.7% compound average return). If you had invested in long-term government bonds, you would have ended up with $51 (5.3%). If you had invested in short-term Treasury bills, you would have ended up with $17 (3.8%). Of course, inflation was 3.1%, so $1 in 1925 was more like $0.[10] in real terms. Source: Ibbotson Associates, Chicago.

Method 5: Just ask.

The fifth method just asks people or experts what they deem reasonable. The ranges can vary widely, and seem to correlate with very recent stock market returns. For example, in late 2000, many investors polled by Fortune or Gallup/Paine-Webber expected equity premia as high as 15%/year. (They were acutely disappointed: the stock market dropped by as much as 30% over the following two years. Maybe they just got the sign wrong!) The consulting firm, McKinsey, uses a standard of around 5% to 6%, and the social security administration uses a standard of around 4%. In a survey of finance professors in August 2001, the common equity premium estimate ranged between **3.5%** for a 1-year estimate to **5.5%** for a 30-year estimate.

Pick a good estimate, and use it for all similar-horizon projects.

What to do? Welcome to the club! No one knows the correct answer. Here is my personal take: First, I have my doubts that equity premia will return to the historical levels of 8% anytime soon. (The twentieth century was the American Century for a good reason: there were a lot of positive surprises for American investors.) So, I personally prefer equity premia estimates between 2% and 4%. But reasonable individuals can prefer equity premia estimates as low as 1% or as high as 8%. Anything outside this range is tough to swallow. Second, whatever equity premium you choose, be consistent. Do not use 3% for investing in one asset (say, factories), and 8% for investing in another (say, stocks or different factories). At least, being consistent will minimize relative mistakes in choosing one project over another.

ANECDOTE: The American Century?
Was this really the "American Century?" The inflation-adjusted compound rate of return in the United States was about 6% per year from 1920 to 1995.

In contrast, an investor who would have invested in Romania in 1937 would have experienced not only the German invasion and Soviet domination, but also a real annual capital appreciation of about −27% per annum over the 4 years of Hungarian stock market existence (1937–1941). Similar fates befell many other East European countries. Many other countries not experiencing political disasters also proved to be less stellar investments. For example, Argentina had a stock market from 1947 to 1965, even though its only function seems to have been to wipe out its investors. Peru tried three times: from 1941 to 1953, its stock market investors lost all their money. From 1957 to 1977, its stock market investors again lost all their money. But three times is a charm: From 1988 to 1995, its investors earned a whopping 63% real rate of return. India's stock market started in 1940, and offered its investors a real rate of return of just about −1% per annum. Pakistan started in 1960, and offered about −0.1% per annum.

Even European countries with long stock market histories and no political trouble did not perform as well as the United States. For example, Switzerland and Denmark earned nominal rates of return of about 5% per annum from 1921 to 1995, while the United States earned about 8% per annum.

The United States stock market was indeed an unusual above-average performer in the twentieth century. Will the twenty-first century be the Chinese century?

Source: Goetzmann and Jorion.

NERD NOTE: *There are also many measurement details here. For example, the equity premium is traditionally measured against shorter-term interest rates. However, for capital budgeting for a longer-term project, you should subtract out a longer-term interest rate. The long-term rate is usually higher than the short-term rate, which means that the appropriate equity premium should be lower, perhaps by as much as 2%. This is one reason why I prefer a lower equity premium estimate over longer horizons. Furthermore, over spans of many years, fluctuations in market returns together with the compounding formula imply that you should further reduce your appropriate equity premium estimate. For example, if the rate of return on the market were 50% followed by* -50%*, the average and thus expected rate of return per would be zero, but the holding rate of return would be* $(1+50\%)\cdot(1-50\%) = -25\%$*. Over long periods, these so-called cross terms reduce the expected rate of return on the market, relative to the average short-term rate of return on the market. The so-called geometric average equity premium was only 6.2%, instead of 8.4%: although you would have earned 8.4% in an average from the last 80 years, an investment would have only earned you a total rate of return of* $(1+6.2\%)^{80}$ *compounded.*

No way around it: the equity premium is the most important number in finance, and we need to pull it out of our hats.

Unfortunately, we cannot allow our poor knowledge of the equity premium to stop us from using the CAPM. Yes, the equity premium may be difficult to guestimate, but there is really no way around taking a stance. We must judge the appropriate reward for risky projects relative to risk-free projects. The need to do so is not exclusive to the CAPM and corporations. It matters also for your personal investments: if you believe that the equity premium is higher, you should allocate relatively more of your personal assets to purchasing stocks rather than bonds. It is not only because of the CAPM formula that the equity premium may be the single most important number in finance.

The CAPM tells us about relative pricing, not absolute pricing.

In the end, you can think of the CAPM as telling us the *relative* expected rate of return for projects, not their *absolute* expected rate of return. Given as input an estimate of how much risky projects should earn relative to non-risky projects, it can determine the right costs of capital for projects of any chosen riskiness. But the basic judgment of the appropriate spread between risky and non-risky projects is left up to you.

Solve Now!

Q 17.7 *What is today's risk-free rate for a 1-year project? For a 10-year project?*

Q 17.8 *What are appropriate equity premium estimates? What are not?*

17·5. Obtaining CAPM Inputs: The Investment Project's Market Beta

Finally, the project beta needs to be estimated. **Market Beta** measures how project rates of return fluctuate with the market. Unlike the previous two inputs, which are the same for every project/stock in the economy, the beta input depends on the specific project characteristics: different investments have different betas. For example, the following five projects, each with its own beta, would be expected to fluctuate with the overall market as follows:

Unlike the risk-free rate and the equity premium, beta is specific to each project.

	if beta ($\beta_{i,\mathcal{M}} = Cov(\tilde{r}_i, \tilde{r}_{\mathcal{M}})/\mathcal{V}ar(\tilde{r}_{\mathcal{M}})$) is				
	-1.0	0.0	$+0.5$	$+1.0$	$+2.0$
If market increases by 10% ($\tilde{r}_{\mathcal{M}} = +10\%$), then $\tilde{r}_i$ is expected to be	-10%	$\pm 0\%$	$+5\%$	$+10\%$	$+20\%$
If market decreases by 10% ($\tilde{r}_{\mathcal{M}} = -10\%$), then $\tilde{r}_i$ is expected to be	$+10\%$	$\pm 0\%$	-5%	-10%	-20%

Nerd Note: *A slightly better calculation would be to show how the excess rate of return on a project above the risk-free rate covaries with the excess rate of return on the market above the risk-free rate. This can be seen by rearranging the* CAPM *equation,*

$$\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + \left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right] \cdot \beta_{i,\mathcal{M}} \quad \Leftrightarrow \beta_{i,\mathcal{M}} = \frac{\mathcal{E}(\tilde{r}_i) - r_{\mathcal{F}}}{\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}} \quad . \tag{17.3}$$

The table shows how the absolute magnitude of beta determines the dampening or amplifying effect of the stock market on each project. The sign determines whether the investment tends to move with or against the stock market. We have also been deliberately vague about the "market." In most practical applications, the S&P500 is used in the United States.

Beta can be thought of as an amplifier of market movements.

Depending on the project for which you need a beta, the estimate can be easy or difficult to obtain. We now discuss the three most common sources for beta estimates.

Our plan.

17·5.A. Betas For Publicly Traded Firms

We have already mentioned that obtaining a beta has become relatively easy for stocks that are already publicly trading: indeed, there are many services (e.g., Yahoo!*Finance*) that publish betas. The average beta in the stock market is 1, and most stocks have betas somewhere between about 0 and 3. Large, low-tech firms tend to have lower betas than small, high-tech betas, but this is not always the case. Some sample market betas on asset classes, country portfolios, and Dow-Jones 30 stocks can be found in Tables 14.8–14.10 (Page 314, Page 318, and Page 319).

Beta is easy to get for publicly traded stocks.

Side Note: The published betas themselves are estimated from historical time-series regressions, a statistical technique that attempts to fit the best α and β in $\tilde{r}_i = \alpha_i + \beta_i \cdot \tilde{r}_{\mathcal{M}} + \epsilon$ to historical data. The regression technique does what we did earlier by hand: to get beta, it computes the covariance divided by variance computation for us.

NERD NOTE: *In the real world, predicting future betas by running a regression with historical rate of return data is too naïve. The reason is that a stock that happened to have a really high return on one day will show too high a beta if the overall stock-market happened to have gone up this day and too low a beta if the overall stock market happened to have gone down on this day. Such extreme observations tend not to repeat under the same market conditions in the future.*

Statisticians suggest "shrinking" the simple naïve historical time-series beta estimates to improve the forecast . The simplest shrinker might suggest halving the distance between the time-series regression and the number "1." More sophisticated shrinkers take the regression uncertainties into account.

If you can, estimate betas with weekly returns over a 3–5 year window.

17·5.B. Betas From Comparables with Leverage Adjustments: Equity Beta vs. Asset Beta

Using comparable publicly traded stocks with unlevered Betas.

It is more difficult to estimate betas for projects for which no historical rate of return experience is available. However, many projects are deemed sufficiently similar to publicly traded firms (for which betas are available) so that managers can adopt the methods of comparables (Chapter 11). For example, if a new soda company is deemed similar to PepsiCo, managers would adopt the beta of PepsiCo in computing the CAPM expected rate of return.

Leverage Adjustments: An Intuitive Example.

However, you must draw a clear distinction between equity betas and asset betas. Usually, you have an intuition that your **project beta** (also called **asset beta**) is the same as that of a publicly traded company, but all that you get to see is the comparable's equity beta. You must adjust the **equity beta** for the comparable's leverage, because stocks that are more levered have higher equity market-betas—they are riskier. Recall the example from Chapter 6: when a project was split into debt and equity, the debt became less risky, while the equity became more risky. This turns out to matter for betas, too.

Table 17.1. The Effect of Leverage on Beta

	Market-Beta	Value Today	Exp. Value Next Year	If Market Underperforms	If Market Overperforms	Spread
Stock Market	$\beta_{i,\mathcal{M}} = 1$	\$10 trillion	\$11 trillion	\$9 trillion	\$ 13 trillion	
	Performance relative to Previous Year, \$10t:		10%	–10%	+30%	40%
	Unlevered Capital Structure					
Project	$\beta_{\text{project},\mathcal{M}} = 1/2$	\$100	\$105	\$95	\$115	
	Performance relative to Previous Year, \$100:		5%	–5%	+15%	20%
	Alternative Capital Structure					
\$80 in Debt	$\beta_{\text{debt},\mathcal{M}} = 0$	\$80	\$80	\$80	\$80	
	Performance relative to Previous Year, \$80:		0%	–0%	+0%	
Levered Equity	$\beta_{\text{equity},\mathcal{M}} = ?$	\$20	\$25	\$15	\$35	
	Performance relative to Previous Year, \$20:		25%	–25%	+75%	100%

To determine how leverage changes beta, consider Table 17.1. In this example, the stock market, worth \$10 trillion today, is expected to increase by 10% to \$11 trillion next year. However, relative to this expected value, the market can either underperform or overperform (by 20%). For convenience, we assume that the risk-free rate is zero. Now, our own project is worth \$100 today and has a beta of $1/2$. Therefore, it is expected to return \$105, but either 10% above or 10% below its mean, depending on the stock market rate of return. This makes sense: for a 40% difference in the rate of return on the stock market (the final column in the Table), our project has a 20% difference in its rate of return. This confirms that the project's market-beta is indeed $1/2$: for a 40% swing in the overall stock market, our project value experiences a 20% swing.

Our project has a beta of 0.5, consistent with Table

We can think of our project in an alternative capital structure that consists of debt and equity, i.e.,

The beta of levered equity scales with leverage.

$$\begin{aligned} \text{Value}_{\text{Project}} &= \text{Value}_{\text{Debt}} + \text{Value}_{\text{Equity}} \quad , \\ 100\% &= \frac{\text{Value}_{\text{Debt}}}{\text{Value}_{\text{Project}}} + \frac{\text{Value}_{\text{Equity}}}{\text{Value}_{\text{Project}}} \\ 100\% &= w_{\text{Debt}} + w_{\text{Equity}} \quad , \end{aligned} \tag{17.4}$$

where we have taken the liberty to call the weight of each security in the capital structure w.

For convenience, we assume that the debt is risk-free, so it pays off \$80 no matter what. This also means that the debt beta is 0, and, in reality, debt betas are indeed often close to zero. This makes it easier to determine the value of the now levered equity, given the value of the unlevered equity: it must be expected to be \$105 − \$80 = \$25. The \$25 expected payoff for a \$20 investment represents an expected rate of return of 25%. Note how our leverage increase from 0 to 80% has increased the expected rate of equity by a factor of 5 (from 5% to 25%). This is because our firm's weighted-average cost of capital (called WACC, and discussed in detail later) is the same 5% in both capital structures:

The WACC remains the same regardless of capital structure.

$$\begin{aligned} 5\% &= 80\% \cdot 0\% + 20\% \cdot 25\% \\ \mathcal{E}(\tilde{r}_{\text{Project}}) &= w_{\text{Debt}} \cdot \mathcal{E}(\tilde{r}_{\text{Debt}}) + w_{\text{Equity}} \cdot \mathcal{E}(\tilde{r}_{\text{Equity}}) \quad . \end{aligned} \tag{17.5}$$

Our goal is to determine what the beta of levered equity would be for our project. That is, we want to know the behavior of the levered equity if the market goes up vs. if the market goes down. If the stock market underperforms, the levered equity value becomes \$95 − \$80 = \$15, which is a –25% rate of return. If the stock market overperforms, the levered equity value becomes \$115 − \$80 = \$35, which is a +75% rate of return. So, our levered equity covers a range of 100% (either –25% or +75%) when the market covers a range of 40% (from \$10 trillion to \$9 trillian, or –10%, or from \$10 trillion to \$13 trillion, or +30%). The market beta of our levered equity is therefore 5 times the beta of our unlevered project. The asset beta is 0.5, the equity beta is $5 \cdot 0.5 = 2.5$.

The beta of levered equity scales with leverage.

This is actually just a special version of a general linear property of betas: as we learned earlier, you can take weighted averages of betas. (On Page 302, we already discussed that the beta of a portfolio is the value-weighted beta of its component securities.) Therefore,

The weighted average beta is the overall beta.

$$\begin{aligned} 0.5 &= 80\% \cdot 0 + 20\% \cdot 2.5 \\ \beta_{\text{Project},mkt} &= w_{\text{Debt}} \cdot \beta_{\text{Debt},\mathcal{M}} + w_{\text{Equity}} \cdot \beta_{\text{Equity},\mathcal{M}} \quad . \end{aligned} \tag{17.6}$$

IMPORTANT: *If project A consists of part B and part C,*

$$A = B + C \tag{17.7}$$

then the overall market beta of the combined project A is the weighted average market beta of its components,

$$\beta_{A,\mathcal{M}} = w_B \cdot \beta_{B,\mathcal{M}} + w_C \cdot \beta_{C,\mathcal{M}} , \tag{17.8}$$

where w are weights according to value today, and add up to 1. The components could be any type of investments, and in particular be the debt and equity of the same firm. Therefore,

$$\begin{aligned} \textit{Project} &= \textit{Debt} + \textit{Equity} \\ \Longrightarrow \beta_{Project,\mathcal{M}} &= w_{Debt} \cdot \beta_{Debt,\mathcal{M}} + w_{Equity} \cdot \beta_{Equity,\mathcal{M}} . \end{aligned} \tag{17.9}$$

Leverage Adjustments: The Formula.

Yahoo!*Finance* always quotes the beta of the levered equity, not the beta of the firm's underlying projects. If we believe our own project is similar to the projects of a publicly traded company, we need to determine the public firm's underlying project beta from its quoted equity beta. For example,

$$\begin{aligned} \$100 &= \$80 + \$20 \\ \text{Firm} &= \text{Debt} + \text{Equity} . \end{aligned} \tag{17.10}$$

Therefore,

$$\begin{aligned} 0.5 &= 80\% \cdot \beta_{\text{Debt},\mathcal{M}} + 20\% \cdot \beta_{\text{Equity},\mathcal{M}} \\ \beta_{\text{Firm},\mathcal{M}} &= w_{\text{Debt}} \cdot \beta_{\text{Debt},\mathcal{M}} + w_{\text{Equity}} \cdot \beta_{\text{Equity},\mathcal{M}} . \end{aligned} \tag{17.11}$$

If the debt level of the firm is relatively low, the debt will be almost risk-free—which means that the β of the debt will be about zero (as it was in our example where debt value did not move with the market). In this case, we can drop the first term.

$$\begin{aligned} 0.5 &\approx 20\% \cdot \beta_{\text{Equity},\mathcal{M}} \quad \Rightarrow \quad \beta_{\text{Equity},\mathcal{M}} \approx \frac{0.5}{20\%} = 2.5 \\ \beta_{\text{Firm},\mathcal{M}} &\approx w_{\text{Equity}} \cdot \beta_{\text{Equity},\mathcal{M}} \quad \Rightarrow \quad \beta_{\text{Equity},\mathcal{M}} \approx \frac{\beta_{\text{Firm},\mathcal{M}}}{w_{\text{Equity}}} . \end{aligned} \tag{17.12}$$

Thus, as long as corporate debt is small enough not to be much influenced by stock market fluctuations, the beta of the underlying firm is approximately the reported beta of the equity times the fraction of equity in the capital structure.

17·5.C. Betas Based on Economic Intuition—And a Common Blunder

Intuitive Betas Guestimating.

Sometimes, there are projects for which there are no good publicly traded firms from which we can extract a beta estimate. In such cases, you need to make a judgment: how will the rate of return of your project covary with the stock market? First, rearrange the CAPM Formula:

$$\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + \left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right] \cdot \beta_{i,\mathcal{M}} \quad \Longleftrightarrow \quad \beta_{i,\mathcal{M}} = \frac{\mathcal{E}(\tilde{r}_i) - r_{\mathcal{F}}}{\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}} \quad . \tag{17.13}$$

The right side of this formula helps us translate our intuition into a beta guestimate. We can ask such questions as "What rate of return (above the risk-free rate) will our project have if the stock market were to have +10% or –10% rate of return (above the risk-free rate)?" Clearly, such guess work is difficult and error-prone.

Betas are rarely zero, even if it appears at first sight that they should be zero.

Unfortunately, intuitively judging covariation is even *more* difficult than it appears. The reason is a source of covariation in the real world that is often forgotten. For example, take a fair coin flip in ten years. If heads, you will receive +\$2,000; if tails, you will receive nothing. Also, assume the stock market will be worth either \$10 trillion or \$20 trillion. It would seem that a fair coin flip—which has nothing to do with the stock market—should have a beta of zero. But this intuition is wrong. It is correct that there is no covariation that comes from the expected cash flows. *But there is covariation that comes from interim discount rate fluctuation.* For example, assume that the appropriate discount rates are 5%/year and 6%/year, respectively, so that the two investment opportunities would have values of

$$\begin{aligned} \mathcal{E}(\text{Coin Payoff}) &= \frac{50\%\cdot\$2,000 + 50\%\cdot\$0}{(1+5\%)^{10}} \approx \$614 \;; \\ \mathcal{E}(\text{Market Value}) &= \frac{50\%\cdot\$10t + 50\%\cdot\$20t}{(1+6\%)^{10}} \approx \$8.3t \;. \end{aligned} \tag{17.14}$$

Now, assume that, for some reason, the appropriate time premium on *all* investments in the economy increases by 1%. This changes the value of both investments.

$$\begin{aligned} \mathcal{E}(\text{Coin Payoff}) &= \frac{50\%\cdot\$2,000 + 50\%\cdot\$0}{(1+6\%)^{10}} \approx \$558 \;; \\ \mathcal{E}(\text{Market Value}) &= \frac{50\%\cdot\$10t + 50\%\cdot\$20t}{(1+7\%)^{10}} \approx \$7.6t \;. \end{aligned} \tag{17.15}$$

Note how both prices have moved in the same direction. Their rates of return covary positively together, which means that the seemingly totally idiosyncratic coin flip bet has a positive (though small) stock market beta! In fact, even a project that returns \$1,000 for sure in 10 years would end up having a positive stock market beta today!

In the real world, many seemingly zero-beta projects have positive betas.

In sum, because of covariation in discount rates, long-term investments can have positive market betas. Even long-term government bonds sometimes have betas that are positive, when their rates of return (prior to expiration) are measured against the rate of return of the stock market.

NERD NOTE: *Actually, the* CAPM *is a single-period model that cannot easily handle covariation in discount rates. In other words, we are trying to apply a square model into a round real-world fact.*

SIDE NOTE: There is another reason why project values may increase when the expected rate of return on the stock market drops: Debt financing may become cheaper. This can influence the expected project cash flows. As a consequence, even many long-term bonds end up having positive betas.

NERD NOTE: *Here is some more economics to help guestimating market betas: Firms with more fixed costs rather than variable costs are riskier, therefore tend to have to higher betas, therefore have to offer a higher expected rate of return.*

The supernerd may enjoy the following, otherwise irrelevant proof. Assume a firm A might consist of sales revenues (R), minus its fixed costs (F) and variable costs (V),

$$A = R - F - V \iff R = A + F + V \quad . \tag{17.16}$$

You can think of this as three firms, and R as a portfolio. Therefore, the beta on R must be the weighted betas of A, F, and V,

$$\beta_R = \beta_A \cdot \left(\frac{A}{A+F+V}\right) + \beta_F \cdot \left(\frac{F}{A+F+V}\right) + \beta_V \cdot \left(\frac{V}{A+F+V}\right) \quad . \tag{17.17}$$

Fixed costs are fixed: they do not change. Therefore, β_F is about zero.

$$\beta_R = \beta_A \cdot \left(\frac{A}{R}\right) + \beta_V \cdot \left(\frac{A}{R}\right) \quad . \tag{17.18}$$

If costs and revenues have about the same beta, then

$$\beta_R = \beta_A \cdot \left(\frac{A}{R}\right) + \beta_R \cdot \left(\frac{A}{R}\right) \iff \beta_A = \beta_R \cdot \left(\frac{R-V}{A}\right) \quad . \tag{17.19}$$

Because $R - V = A + F$,

$$\beta_A = \beta_R \cdot \left(1 + \frac{F}{A}\right) \quad . \tag{17.20}$$

This states that if everything else is equal, projects with higher fixed costs have higher asset betas.

Solve Now!

Q 17.9 *If you had representative historical project returns, how would you obtain the stock market beta?*

Q 17.10 *Look up the beta for IBM at Yahoo!Finance.*

Q 17.11 *A comparable firm (in a comparable business) has an equity beta of 2.5 and a debt/asset ratio of 2/3. The debt is almost risk-free. Estimate the beta for our firm if projects have alike betas, but our firm will carry a debt/asset ratio of 1/3.*

Q 17.12 *(Continued.) If the risk-free rate is 3% and the equity premium is 2%, what is the expected rate of return on the comparable firm's equity and on our own equity?*

Q 17.13 *A comparable firm (in a comparable business) has an equity beta of 2.5 and a debt/equity ratio of 2. The debt is almost risk-free. Estimate the beta for our firm if projects have alike betas, but our firm will carry a debt/equity ratio of 1/2.*

Q 17.14 *(Continued.) If the risk-free rate is 3% and the equity premium is 2%, what is the expected rate of return on the comparable firm's equity and on our own equity?*

Q 17.15 *You own a stock market portfolio that has a market beta of 2.4, but you are getting married to someone who has a portfolio with 0.4. You are three times as wealthy as your future significant other. What is the beta of your joint portfolio?*

Q 17.16 *Assume that you can short. If your portfolio has a market beta of 0.6 and you can short a fund with a market beta of 1, what portfolio do you have to purchase to eliminate all market risk?*

Q 17.17 *Why can even a long-term risk-free bond that has seemingly no covariation with the stock market have a positive beta?*

17·6. Robustness: How Bad are Mistakes?

We know that we don't know the inputs for the CAPM perfectly. After all, expected rates of return are never actually observed but can only be guestimated from historical rates of return. At best, we can make educated guesses. So, how robust is the CAPM with respect to errors in inputs?

Where will we inevitably go wrong?

Errors in the risk-free rate ($r_{\mathcal{F}}$) are likely to be modest. The risk-free rate can be considered to be known for all practical reasons. This leaves us with having to judge the influence of errors in estimating the expected market rate of return ($\mathcal{E}(\tilde{r}_{\mathcal{M}})$) and errors in estimating betas ($\beta_{i,\mathcal{M}}$).

Errors in the risk-free rate tend to be very small.

Reasonable beta estimates typically have relatively low uncertainty. That is, good comparables can often be found in the public market. After some care is exercised, a typical range of uncertainty about beta might be about plus or minus 0.2. For example, say the equity premium is truly 4%. Now, if you believe your beta is 2, but it is really 1.8 instead, then you would overestimate the appropriate expected rate of return by $2 \cdot 4\% - 1.8 \cdot 4\% = 0.8\%$. Although this level of uncertainty is not insignificant, it is relatively modest.

Errors in beta estimates tend to be modest.

Reasonable equity premium estimates can range from about 1% per year to about 8% per year—a large range. *To date, there is no universally accepted method to estimate the expected rate of return on the market, so this disagreement cannot be easily settled with data and academic studies.* Some academics believe that the historic appreciation of prices in the stock market is the best indicator for the typical appreciation of stock prices in the long run. These individuals predict high expected rates of return on the market. Other academics believe that the historic appreciation of prices in the stock market occurred because prices rose *in order to* lower the expected rates of return for the future. These individuals predict low expected rates of return on the market.

Disagreement on the equity premium tends to be large...

...and these differences in equity premium estimates often have enormous influence.

The problem is that reasonable differences of opinion in estimating the expected rate of return on the market often have a large influence on expected rate of return estimates. For example, take a stock with a beta of 2. The CAPM might advise this corporation that potential investors demand either an expected rate of return of 5% per year or an expected rate of return of 20% per year (or anything in between). This is—to put it diplomatically—a miserably large range of possible cost of capital estimates. (And this range does not even consider the fact that *actual* future rates of returns will necessarily differ from *expected rates of return*!) Of course, in the real world, managers who want to take a project will argue that the expected rate of return on the market is low. This means that their own project looks relatively more attractive. Potential buyers of projects will argue that the expected rate of return on the market is high. This means that they can justify a lower price offer for the project.

Use the CAPM as guidance, not as gospel!

So, where does this leave the CAPM? First, you need to realize that there are really no better alternatives. No matter how poor or imprecise the CAPM estimates are, without a better alternative, you do not have any choice but to use it. Second, users of the CAPM need to be aware of its limitations. The CAPM is a model that can often provide a "*reasonable* expected rate of return," but not an "*accurate* expected rate of return." Anyone who believes that CAPM expected rates of return should be calculated with digits after the decimal point is deluded. At best, the CAPM can offer expected rates of returns that are of the "right order of magnitude."

ANECDOTE: Cost of Capital Expert Witnessing

When Congress tried to force the so-called Baby Bells (the split up parts of the original AT&T) to open up their local telephone lines to competition, it decreed that the Baby Bells were entitled to a fair return on their infrastructure investment—with fair return to be measured by the CAPM. (The CAPM is either the de facto or legislated standard for measuring the cost of capital in many other regulated industries, too.) The estimated value of the telecommunication infrastructure in the United States is about $10 to $15 billion. A difference in the estimated equity premium of 1% may sound small, but even in as small an industry as local telecommunications, it meant about $1,000 to $1,500 million a year—enough to hire hordes of lawyers and valuation consultants opining in court on the appropriate equity premium. Some of my colleagues bought nice houses with the legal fees.

I did not get the call. I lack the ability to keep a straight face while stating that "the equity premium is exactly x%," which was an important qualification for being such an expert.

Put together NPV and CAPM robustness considerations.

In sum, in Chapter 6, we saw the relative importance of getting the inputs into the NPV formula correct. The basic conclusion was that for short-term projects, getting the cash flows right is more important than getting the expected rate of return right; for long-term projects, getting both right is important. We just discussed the relative importance of getting the equity premium and the project beta right. Recall that our basic conclusion was that the CAPM formula is first and foremost exposed to errors in the market risk premium (equity premium), though it is also somewhat exposed to beta estimates. Putting these two conclusions together suggests that for short-term projects, worrying about exact beta estimates is less important than worrying about estimating cash flows first and the appropriate equity premium second. For long-term projects, the order of importance remains the same, but the difference in the relative importance of good estimates of expected cash flows and good estimates of the equity premium estimates shrinks. In contrast, honest mistakes in beta, *given reasonable care*, should not matter too much.

Solve Now!

Q 17.18 *To value an ordinarily risky project, that is, a project with a beta in the vicinity of about 1, what is the relative contribution of your personal uncertainty (lack of knowledge) in: the risk-free rate, the equity premium, the beta, and the expected cash flows? Consider both long-term and short-term investments.*

17·7. The Finer Points of CAPM Application

This section covers a number of important nuances and pitfalls in the use of the CAPM. As always, the concept is straightforward, but the devil is in the details.

17·7.A. Using the CAPM Cost of Capital in Multiperiod Contexts

Which risk-free rate?

Assume that the expected rate of return on the market is 12%. The risk-free rate of return is 2% over 1 year, 4% over 10 years, and 5% over 30 years. Your own investment has a market-beta of 2.5, and is expected to return money in 10 years. So, how do we use the CAPM? Which interest rate do we pick in a multi-year context?

Advice: Pick the closest-term interest rate.

The answer is actually "ad-hoc." See, the CAPM has no concept of more than one single time-period. It therefore does not understand why there is a term-structure of interest (different expected rates of returns over different horizons). The sensible heuristic method is to pick the risk-free rate closest in some economic sense (maturity or duration) to our project. Here, we have a 10-year project, so we should probably use 4% rate of return.

Nerd Note: *Is the expected per-annum rate of return on the market, $\mathcal{E}(\tilde{r}_{\mathcal{M}})$, different over 1 year than it is over 10-years? After all, if this is the case for bonds, why not for stocks? We do not know, because our estimates of the expected rate of return on the market are too unreliable to allow us to answer this question. Put differently, this question is swamped by our uncertainty about $\mathcal{E}(\tilde{r}_{\mathcal{M}})$ in the first place.*

For long-term projects, it can matter whether you first compound and then use the CAPM, or vice-versa.

We also have a computational issue. Should we first compute the CAPM expected rate of return, and then compound, or vice-versa? This can "seem to" matter: If we first compound, we get 10-year rates of return of

$$\begin{aligned} r_{\mathcal{F},0,10} &= (1+r_{\mathcal{F},0,1})^{10}-1 = (1+4\%)^{10} \approx 48.0\% \\ \tilde{r}_{\mathcal{M},0,10} &= (1+\tilde{r}_{\mathcal{M},0,1})^{10}-1 = (1+12\%)^{10} \approx 210.6\% \ , \end{aligned} \tag{17.21}$$

so the CAPM formula offers

$$\begin{aligned} \mathcal{E}(\tilde{r}_{0,10}) &= 48\% + (210.6\% - 48\%)\cdot 2.5 \approx 454\% \\ &= r_{\mathcal{F},0,10} + [\mathcal{E}(\tilde{r}_{\mathcal{M},0,10}) - r_{\mathcal{F},0,10}]\cdot \beta_{i,\mathcal{M}} \ , \end{aligned} \tag{17.22}$$

or $\mathcal{E}(r_{\overline{10}}) = (1+\tilde{r}_{0,10})^{1/10} = (1+454\%)^{1/10} \approx 18.7\%$ per year. On the other hand, if we first compute the CAPM expected rate of return for one year, we find

$$\begin{aligned} \mathcal{E}(\tilde{r}_{0,1}) &= 4\% + (12\% - 4\%)\cdot 2.5 = 24\% \\ \mathcal{E}(\tilde{r}_{0,1}) &= r_{\mathcal{F},0,1} + [\mathcal{E}(\tilde{r}_{\mathcal{M},0,1}) - r_{\mathcal{F},0,1}]\cdot \beta_{i,\mathcal{M}} \ . \end{aligned} \tag{17.23}$$

Over 10 years, this comes to

$$\mathcal{E}(\tilde{r}_{0,10}) \;=\; [1+\mathcal{E}(\tilde{r}_{0,1})]^{10}-1 \;=\; (1+24\%)^{10} \approx 759.4\% \;. \tag{17.24}$$

Is today's value of our project's expected payoff \$100 in 10 years then \$100/(1+454%) ≈ \$18.^{05} or \$100/(1+759%) ≈ \$11.^{64}? Noone knows (and the CAPM again does not tell us)—but few people wonder. The reason is the importance of intrinsic uncertainty about the project vs. the computational uncertainty: over 10 years, how accurate do you believe your market-beta estimate is? Or your estimate of the expected rate of return on the market? More importantly, in an NPV context, how accurate is your estimate of the your \$100 expected cash flow? These uncertainties usually swamp our computational calculation method differences. So, computationally, either method can be used, but you should be aware that their uses can lead to different results.

SIDE NOTE: The numerical differences are only dramatic for projects with market-betas that are considerably above 1 or below 0. If beta is exactly 0 or 1, there is no difference.

17·7.B. Using the CAPM Cost of Capital in the NPV Context: Revisiting Risk Premium and Default Premium

We usually use the CAPM expected rate of return in the NPV denominator.

The reason why we worked through the CAPM in the first place was to obtain the quantities that we need in the denominator of the NPV formula,

$$\text{NPV} \;=\; C_0 \;+\; \frac{\mathcal{E}(\tilde{C}_1)}{1+\mathcal{E}(\tilde{r}_{0,1})} \;+\; \frac{\mathcal{E}(\tilde{C}_2)}{1+\mathcal{E}(\tilde{r}_{0,2})} \;+\; \ldots \tag{17.25}$$

The CAPM tells us that cash flows that correlate more with the overall market are of less value to our investors, and therefore require a higher expected rate of return ($\mathcal{E}(\tilde{r})$) in order to pass muster (well, the hurdle rate).

Do not lose the forest: the CAPM has nothing to do with default risk.

Although we have mentioned this before, it is important to reiterate that beta and the expected rate of return from the CAPM do not directly take default risk into account. In the NPV formula, the default risk enters the valuation in the expected cash flow numerator, not the expected rate of return denominator. So, recall our super-important box on Page 79, which decomposed rates of returns into three parts:

$$\begin{aligned}
\text{Promised Rate of Return} &= \text{Time Premium} + \text{Default Premium} + \text{Risk Premium} \;. \\
\text{Actual Earned Rate} &= \text{Time Premium} + \text{Default Realization} + \text{Risk Premium} \;. \\
\text{Expected Rate of Return} &= \text{Time Premium} + \text{Expected Risk Premium} \;.
\end{aligned} \tag{17.26}$$

The CAPM gives you the expected rate of return, which consists of the time premium and the expected risk premium. It does not give you any default premium.

A specific example.

For example, if you want to determine the NPV of a corporate bond, absolutely do not believe that just because you are using expected rates of return obtained from the CAPM in the denominator, you have already taken default risk into account. You must still adjust the numerator (promised payments) for the probability of default. Say that you are considering buying a zero bond that has a beta of 0.25, and promises to deliver \$200 next year. This bond pays off 95% of the time, and 5% of the time it totally defaults. Assume that the risk-free rate of return is 6% per annum, and the expected rate of return

on the market is 10%. The correct way to value this bond today is to first compute the expected payoff;

$$\begin{aligned} \mathcal{E}(\tilde{C}) &= 95\% \cdot \$200 + 5\% \cdot 0 = \$190. \\ &= \mathcal{P}rob(\text{No Default}) \cdot \text{Promise} + \mathcal{P}rob(\text{Default}) \cdot \text{Nothing} \quad ; \end{aligned} \tag{17.27}$$

then to compute the appropriate expected rate of return,

$$\begin{aligned} \mathcal{E}(\tilde{r}) &= 6\% + (10\% - 6\%) \cdot 0.25 = 7\% \\ &= r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{\text{Bond},\mathcal{M}} \quad ; \end{aligned} \tag{17.28}$$

and finally, the value of the bond is

$$\text{NPV} = \frac{\mathcal{E}(\tilde{C}_1)}{1 + \mathcal{E}(\tilde{r}_{0,1})} = \frac{\$190}{1 + 7\%} \approx \$177.^{57} \quad . \tag{17.29}$$

The promised (or quoted) rate of return on this bond is

$$\begin{aligned} \text{Promised } r_{0,1} &= \frac{\$200 - \$177.^{57}}{\$177.^{57}} \approx 12.6\% \\ &= \frac{\text{Promised } C_1 - C_0}{C_0} \quad . \end{aligned} \tag{17.30}$$

Although we rarely need to decompose quoted interest rates in practice, quantifying the three components in our example helps us to better conceptualize where the quoted rates come from. For our bond, the time-premium of money is 6% per annum—it is the rate of return an equivalent-term Treasury bond offers. The time-premium plus the risk-premium is provided by the CAPM, and it is 7% per annum. Therefore, 1% per annum is our "average" compensation for our willingness to hold this risky bond, rather than the risk-free Treasury bond. The remaining 12.6% − 7% ≈ 5.6% per annum is the default premium: we do not expect to earn money from this part "on average;" we only earn it if the bond does not default.

$$\begin{aligned} 12.6\% &= 6\% + 5.6\% + 1\% \quad . \\ \text{Promised Interest Rate} &= \text{Time Premium} + \text{Default Premium} + \text{Risk Premium} \quad . \end{aligned} \tag{17.31}$$

Most bonds have fairly small market betas and thus risk premia. Most of the premium that risky bonds quote above the equivalent risk-free Treasury rates is due to default risk.

SIDE NOTE: In the real world, corporate bonds often have a liquidity premium built-in, which compensates investors for not being able to easily buy/sell these securities. The broker/market-makers tend to earn this premium. The liquidity premium differs across investors: retail investors are charged higher liquidity premia than bond funds. As a retail investor, it is best not to purchase individual bonds.

17·7.C. Does Risk-Reducing Diversification Create Value?

Diversification reduces risk, but does not create value.

Let us now reopen an issue we first visited in Chapter 7. In the 1960s through 1970s, many firms became **conglomerates**, that is, companies with widely diversified and often unrelated holdings. Can we add value purely by such diversification? The answer is "usually no." Diversification indeed reduces the standard deviation of the rate of return of the company, but our investors can diversify risk for themselves. For example, if our \$900 million firm ABC (e.g., with a beta of 2, and a risk of 20%) is planning to take over the \$100 million firm DEF (e.g., with a beta of 1, and also risk of 20%), the resulting firm is worth \$1 billion dollars. ABC+DEF has indeed an idiosyncratic risk lower than 20% if the two firms are not perfectly correlated, but our investors (or a mutual fund) could just purchase 90% of ABC and 10% of DEF and thereby achieve the very same diversification benefits. If anything, we have robbed investors of a degree of freedom here: they no longer have the ability to purchase, say, 50% in ABC and 50% in DEF. (In a CAPM world, this does not matter.) The CAPM makes it explicit that the cost of capital does not change unduly. Say, both firms follow the CAPM equation, and say that the risk-free rate is 3% and the equity premium is 5%,

$$\begin{aligned}
\mathcal{E}(\tilde{r}_{\text{ABC}}) &= 3\% + 5\% \cdot 2 = 13\% \ , \\
\mathcal{E}(\tilde{r}_{\text{ABC}}) &= r_{\mathcal{F}} + \left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right] \cdot \beta_{\text{ABC},\mathcal{M}} \ , \\
\mathcal{E}(\tilde{r}_{\text{DEF}}) &= 3\% + 5\% \cdot 1 = 8\% \ , \\
\mathcal{E}(\tilde{r}_{\text{DEF}}) &= r_{\mathcal{F}} + \left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right] \cdot \beta_{\text{DEF},\mathcal{M}} \ .
\end{aligned} \tag{17.32}$$

The newly formed company will have an expected rate of return—cost of capital—of

$$\begin{aligned}
\mathcal{E}(\tilde{r}_{\text{ABC+DEF}}) &= 90\% \cdot 13\% + 10\% \cdot 8\% = 12.5\% \ , \\
\mathcal{E}(\tilde{r}_{\text{ABC+DEF}}) &= w_{\text{ABC}} \cdot \mathcal{E}(\tilde{r}_{\text{ABC}}) + w_{\text{DEF}} \cdot \mathcal{E}(\tilde{r}_{\text{DEF}}) \ ,
\end{aligned} \tag{17.33}$$

and a market-beta of

$$\begin{aligned}
\beta_{\text{ABC+DEF},\mathcal{M}} &= 90\% \cdot 2 + 10\% \cdot 1 = 1.9 \\
\beta_{\text{ABC+DEF},\mathcal{M}} &= w_{\text{ABC}} \cdot \beta_{\text{ABC},\mathcal{M}} + w_{\text{DEF}} \cdot \beta_{\text{DEF},\mathcal{M}} \ .
\end{aligned} \tag{17.34}$$

The merged company will still follow the CAPM,

$$\begin{aligned}
\mathcal{E}(\tilde{r}_{\text{ABC+DEF}}) &= 3\% + 5\% \cdot 1.9 = 12.5\% \\
\mathcal{E}(\tilde{r}_{\text{ABC+DEF}}) &= r_{\mathcal{F}} + \left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right] \cdot \beta_{\text{ABC+DEF},\mathcal{M}} \ .
\end{aligned} \tag{17.35}$$

Its cost of capital has not unduly increased or declined. In our ideal CAPM world, no value has been added or destroyed—even though ABC+DEF has a risk lower than the 20% per annum that its two constituents had.

Synergies or Dis-synergies drive M&A value, not diversification. Managers also have agency conflicts in M&A activity.

In the real world, diversified firms often do not operate as efficiently as stand-alone firms, e.g., due to limited attention span of management or due to more bureaucratization. Such mergers destroy firm value. Of course, other mergers can add value due to synergies, as we discussed in Chapter 7. More often, however, the unspoken rationales for mergers are that managers prefer the reduced idiosyncratic uncertainty and higher salaries guaranteed by larger firms to the higher risk and lower salaries in sharply

focused, smaller firms. This is another example of an agency conflict.

17·7.D. Avoiding Cost-of-Capital Mixup Blunders That Destroy Value

When Acquiring Another Company

In Section 7·2.C, we also dropped a cryptic hint that practitioners sometimes forget that NPVs are additive. We are now ready to explain what we really meant. The most common abuse of the CAPM arises from the use of the firm's overall cost of capital for individual projects, and here is why.

Common misuse of CAPM: a uniform cost of capital.

Assume the risk-free rate of return is 3% and the equity premium is 4%. Our old firm, cleverly named O, is worth \$100 and has a market beta of $1/2$. An acquisition target (or just a new project), cleverly named N, costs \$10 and is expected to pay off \$11 next year. (Its rate of return is therefore 10%. We covered IRR more generally in more detail in Chapter 8.) The beta of this new project is 3.

What happens if the firm uses its overall cost of capital for projects, rather than a project specific cost of capital?

The simplest method to compute the value of project N relies on NPV, already discussed on Page 147. We value the new project using its own expected cash flows and own cost of capital. *Who* owns it should matter little: the project is worth what it is worth. So, N should offer an expected rate of return of

The Solution: Compute the NPV of the project.

$$\begin{aligned} \underline{\text{Correct Cost of Capital:}} \quad \mathcal{E}(\tilde{r}_N) &= 3\% + 4\% \cdot 3 = 15\% \\ \mathcal{E}(\tilde{r}_N) &= r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{N,\mathcal{M}} \ , \end{aligned} \tag{17.36}$$

and the true NPV of the new project is

$$\text{NPV}_N = -\$10 + \frac{\$11}{1+15\%} \approx -\$0.^{44} \ . \tag{17.37}$$

Therefore, if firm O adopts project N, N's owners would be -\$0.^{44} poorer than they would be if their managers did not adopt it (i.e., \$100 vs. \$99.^{56}).

Unfortunately, this is not obvious to some practitioners. In many firms, it is standard policy to evaluate *all* projects by the firm's overall cost of capital. Would such a firm take the N project? Evaluated with a market beta of 0.5, the hurdle rate for the project would be

Bad Company Policy.

$$\begin{aligned} \underline{\text{Incorrect Cost of Capital:}} \quad \mathcal{E}(\tilde{r}) &= 3\% + 4\% \cdot 0.5 = 5\% \ . \\ &= r_{\mathcal{F}} + \left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right] \cdot \beta_{O,\mathcal{M}} \ . \end{aligned} \tag{17.38}$$

With its internal rate of return of $\$11/\$10 - 1 = 10\%$, a poor manager would indeed take this project.

If the O firm did take project N, how would its value change? With a beta of $1/2$, the old firm had an expected rate of return of $3\% + 4\% \cdot 1/2 = 5\%$. Its expected value next year would be \$105. Using NPV, we see that the present value of the combined firm would be

The loss if the firm makes the mistake.

$$\begin{aligned} \text{PV}_{\text{combined}} &= \frac{\$105}{1+5\%} + \frac{\$11}{1+15\%} \approx \$109.^{56} \ . \\ &= \text{PV}_O + \text{PV}_N \ . \end{aligned} \tag{17.39}$$

This is \$0.^{44} less than the original value of \$100 plus the \$10 acquisition cost of the new project. Taking the project has made the N owners 44 cents poorer.

NERD NOTE: *Some companies use the blended cost of capital post-acquisition as the appropriate cost of capital. However, this also leads to incorrect decisions. We already know that \$109.^{48} is the combined firm value. The new project would therefore represent a weight in the new firm of*

$$w_N = \frac{PV_N}{PV_{combined}} = \frac{\frac{\$11}{1+15\%}}{\$109.^{48}} \approx 8.73\% \ . \tag{17.40}$$

The beta of the new firm would be

$$\beta_{combined} = w_O \cdot \beta_O + w_N \cdot \beta_N \approx 91.26\% \cdot 0.5 + 8.73\% \cdot 3 = 0.718 \ . \tag{17.41}$$

The combined cost of capital would therefore be

$$\mathcal{E}(\tilde{r}_{combined}) = 3\% + 4\% \cdot 0.718 = 5.872\% \ , \tag{17.42}$$

and the 10% IRR of N is still above the blended cost of capital: the firm would take it. Again, firm value would be

$$\begin{aligned} PV &= \frac{\$105 + \$11}{1 + 5.872\%} = \$109.^{56} \\ &= \frac{\mathcal{E}(CF_N) + \mathcal{E}(CF_O)}{1 + \mathcal{E}(\tilde{r}_{combined})} \ . \end{aligned} \tag{17.43}$$

SIDE NOTE: In the real world, when capital markets are not perfect, it is possible for acquirers to create value. For example, if an acquiree previously had no access to capital markets, as explained in Section 5·3.B (Page 109), then the cost of capital to the acquiree can change when it is acquired. The correct acquiree cost of capital, however, is *neither* the cost of capital of the acquirer, *nor* the blended post-acquisition cost of capital of the firm. Instead, the correct cost of capital is that appropriate for the acquiree's projects, given the "now ordinary" access to capital markets. For example, if an inventor of holographic displays previously had faced a cost of capital of, say, 303%, primarily due to credit card and credit shark financing, and if this inventor's business is purchased by IBM with its beta of 6.5% (beta of 1.5), the proper cost of capital is neither IBM's beta cost, nor a blended average between 303% and 6.5%. Instead, the holographic project division should be evaluated at a cost of capital that is appropriate for projects of the beta risk class "holographic display projects."

When Acquiring Another Project

Projects must be discounted by their own beta.

It is important to realize that not only firms-to-be-acquired, but also smaller projects themselves consist of components with different betas, which therefore have different costs of capital. For example, when firms keep cash on hand in Treasury bonds, such investments have a zero beta, which is lower than the beta for the firms' other projects. These bonds need not earn the same expected rate of return as investments in the firm's risky projects. (The presence of this cash in the firm lowers the average beta of the firm and thus the average cost of capital for the firm by the just-appropriate amounts.)

Another Example Problem.

Here is another application example: Assume that we consider purchasing a rocket to launch a Telecomm satellite next year. It would take us 1 year to obtain the rocket, at which point we would have to pay \$100 million. Then we launch it. If the rocket fails (25% chance), then our investment will be lost. If the rocket succeeds, the satellite will produce a revenue stream with an appropriate beta of 2. (Telecomm revenues tend to have a high covariance with the market.) Telecomm's expected cash flows will be \$20 million *forever*. Assume that the risk-free rate is 3% per year and the market equity-premium is 4%.

The Example Solution.

The correct solution is to think of the rocket as one project and of the Telecomm revenues as another project. The rocket project has only idiosyncratic risk, therefore its beta is close to zero, and its discount factor is the same as the risk-free rate of return, 3%. The rocket value (in millions of dollars today is)

$$\mathsf{NPV}_{\text{rocket}} \approx \frac{-\$100}{1+3\%} \approx -\$97 \ . \tag{17.44}$$

The Telecomm revenues would be a perpetuity. With a beta of 2, their cash flows would be discounted at about 11%. However, the cash flows will only occur with a probability of 75%. Therefore,

$$\mathsf{NPV}_{\text{Telecomm}} \approx \frac{\mathcal{E}(\text{Telecomm Profits})}{\mathcal{E}(\tilde{r}_{\text{Telecomm Revenues}})} = \frac{75\% \cdot \$20}{3\% + 4\% \cdot 2 = 11\%} = \frac{\$15}{11\%} = \$136 \ . \tag{17.45}$$

Therefore, this project has a net present value of about \$39 million dollars.

ANECDOTE: Real-life CFO Sophistication
There is no doubt that projects must be discounted by their project-specific cost of capital. Yet, just about half of surveyed CFOs always—and *incorrectly*—use the firm's overall cost of capital, rather than the project-specific cost of capital. And even fewer CFOs correctly discount cash flows of different riskiness within projects. Do you really want to be as sophisticated as your average CFO today?

(Source: Graham and Harvey, Duke, 2001.)

Keeping Sane!

You will never get this perfectly right. Get it right where it matters!

We now know that components must be discounted at their own discount rates if we want to get it right. However, if you want to value each paperclip by its own cost of capital, you will never come up with a reasonable value. You need to keep a perspective as to what reasonable errors are and what unreasonable errors are. The question is one of magnitude: if you are acquiring a totally different company or project, with a vastly different cost of capital, then the choice of cost of capital matters. However, if you are valuing a project that is uncertain, and the project is relatively small, and its cost of capital is reasonably similar to your overall cost of capital, you can probably live with some error. Your mileage may vary!

17·7.E. Valuing Gifts — Certainty Equivalence

How to value a project if the efficient price today is not known?

Here is an important brainteaser: What is today's value of a future expected windfall of \$100 next year?

At a price of zero, is the appropriate cost of capital in the CAPM formula infinite?

How do you even compute the beta of the windfall's rate of return with the rate of return on the stock market? After all, a windfall is a gift, so its price is \$0 today, and your actual rate of return will be infinite! But we clearly should be able to put a value on this gift. Indeed, our intuition tells us that this cash flow is most likely worth a little less than \$100, the specifics depending on how the cash flow covaries with the stock market. But, how do we compute this value? The solution to this puzzle is that the *price* of the gift may be \$0 today, but its *value* today (PV_0) is not—and it is the latter, i.e., the fair value, that is used in the CAPM, not the former.

IMPORTANT: *The CAPM works with expected rates of return that are computed from the true asset value (PV_0) today, not from what you are buying the asset for (P_0). In an efficient market, what you get is what you pay for, so $P_0 = PV_0$. But, if you manage to obtain a better or worse deal ($P_0 < PV_0$ or $P_0 > PV_0$), e.g., buying the asset from a benevolent or malevolent friend, then you cannot use P_0 to compute the expected rate of return when you want to value the asset. (The same applies to $\tilde{P}_1$: the expected value tomorrow must be true expected value, not the sweetheart deal value at which you may let go of the asset.)*

We need to rearrange the CAPM formula into so-called Certainty Equivalence: We work out an expected value that we can discount with the risk-free rate.

So, our problem is that we could compute a rate of return for the cash flow if we knew PV_0, then from the rate of return we could compute the project beta, which we could use to find the discount rate to translate the expected cash flow back into the price PV_0 today. Alas, we do not know PV_0, so we cannot compute a rate of return. To solve this dilemma, we must use an alternative form of the CAPM formula, called its **certainty equivalence** form. It is

$$\begin{aligned} PV_0 &= \frac{\mathcal{E}(P_1) - [\mathcal{E}(\tilde{r}_{\mathcal{M}} - r_{\mathcal{F}})/\mathcal{V}ar(\tilde{r}_{\mathcal{M}})] \cdot Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1 + r_{\mathcal{F}}} \\ &= \frac{\mathcal{E}(P_1) - \lambda \cdot Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1 + r_{\mathcal{F}}} \quad , \end{aligned} \tag{17.46}$$

where λ is $\{[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}]/\mathcal{V}ar(\tilde{r}_{\mathcal{M}})\}$. If you believe that the expected annual equity premium is 5%, and that the variance of the rate of return on the market is around 0.04 (a standard deviation of 20%), you would choose a lambda of around 1.25. If the risk-free rate today is 3%, you would value projects as

$$PV_0 = \frac{\mathcal{E}(\tilde{P}_1) - 1.25 \cdot Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1 + 3\%} = \underbrace{\frac{\mathcal{E}(P_1)}{1 + 3\%}}_{\text{as-if-riskfree}} - \underbrace{\frac{1.25 \cdot Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1 + 3\%}}_{\text{risk discount}} . \tag{17.47}$$

The name "certainty-equivalence" is apt. The first form shows that, after we have reduced the expected value of the future cash flow ($\mathcal{E}(\tilde{P}_1)$) by some number that relates to the cash flow's covariance with the market, we can then treat this reduced value as if it were a perfectly certain future cash flow and discount it with the risk-free rate. The second form shows that we can decompose the price today into an "as-if-risk-neutral" value discounted only for the time-premium (with the risk-free rate) and an additional discount for covariance risk with the stock market.

Watch out: the covariance is different in this form!

The covariance between the future value $\tilde{P}_1$ and the rate of return on the market is related, but not identical to the project's market-beta. It is *not* the covariance of the project's *rate* of return with the market rate of return, either. It is the covariance of the project's *cash flow* with the market rate of return, instead.

Our problem solved with zero covariance.

With the certainty equivalence formula, we can now value the \$100 expected windfall. Assuming that the risk-free rate is 3% per annum, and that the lambda is the aforementioned 1.25,

$$\begin{aligned} PV_0 &= \frac{\$100 - 1.25 \cdot Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1 + 3\%} \\ PV_0 &= \frac{\mathcal{E}(P_1) - \lambda \cdot Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1 + r_{\mathcal{F}}} \quad . \end{aligned} \tag{17.48}$$

If you believe that the windfall's payout does not covary with the rate of return on the market, so $Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}}) = 0$, then

$$\begin{aligned} \mathsf{PV}_0 &= \frac{\$100 - 1.25 \cdot 0}{1 + 3\%} = \frac{\$100}{1 + 3\%} = \$97.^{09} \\ \mathsf{PV}_0 &= \frac{\mathcal{E}(P_1) - \lambda \cdot Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1 + r_{\mathcal{F}}} \quad . \end{aligned} \tag{17.49}$$

But what if you believe that your windfall does covary with the market? How can you guestimate the cash flow's covariance with the rate of return on the stock market? The answer is that we need to write down some scenarios, and then use our covariance computation formula (from from Chapter 14) This is easiest to understand in an example. Let us assume that we believe that if the market goes up by 28%, our windfall will be \$200; if the market goes down by 12%, our windfall will be \$0. We also believe these two outcomes to be equally likely.

Our problem solved with positive covariance.

	Prob:	1/2	1/2			
		Bad	Good	Mean	*Var*	*Sdv*
Stock Market		-12%	+28%	8%	4%	20%
Our Windfall		\$0	\$200	\$100	$\$^2$ 10,000	\$100

Now use the covariance formula to compute the average product of deviations from the mean. This is

$$\begin{aligned} Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}}) &= \frac{(\$200 - \$100) \cdot (28\% - 8\%) \ + \ (\$0 - \$100) \cdot (-12\% - 8\%)}{2} = \$20 \\ &= \frac{\sum_{j=1}^{N} [P_{1,\text{outcome j}} - \mathcal{E}(P_1)] \cdot [\tilde{r}_{\mathcal{M},\text{outcome j}} - \mathcal{E}(\tilde{r}_{\mathcal{M}})]}{N} \end{aligned} \tag{17.50}$$

Lambda is still 1.25, because it is the equity premium (8% − 3% = 5%) divided by the variance of the rate of return on the stock market ($[(28\%-8\%)^2+(-12\%-8\%)^2]/2 = 0.04$. We can now use the certainty equivalence formula for our expected windfall of \$100 next year. It is worth

$$\begin{aligned} \mathsf{PV}_0 &= \frac{\$100 - 1.25 \cdot \$20}{1 + 3\%} = \frac{\$75}{1 + 3\%} = \$72.^{82} \\ \mathsf{PV}_0 &= \frac{\mathcal{E}(P_1) - \lambda \cdot Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1 + r_{\mathcal{F}}} \quad . \end{aligned} \tag{17.51}$$

NERD NOTE: *Knowing the fair price of \$72.^{82}, we can now easily check that we have really just recomputed the* CAPM *formula. The project will either provide a rate of return of* $\$200/\$72.^{82} - 1 = 174\%$*, or a rate of return of* -100%*, for an average rate of return of 37%. The beta computed with rates of return is*

$$\beta_{i,\mathcal{M}} = \frac{Cov(\tilde{r}_i, \tilde{r}_\mathcal{M})}{\mathcal{V}ar(\tilde{r}_\mathcal{M})} = \frac{\frac{(+174\%-37\%)\cdot(+28\%-8\%)+(-100\%-37\%)\cdot(-12\%-8\%)}{2}}{\frac{(+28\%-8\%)^2+(-12\%-8\%)^2}{2}} = \frac{0.274}{0.04} = 6.85 \ . \tag{17.52}$$

The ordinary CAPM *formula states that the expected rate of return, given this beta of 6.85, should be be*

$$\mathcal{E}(\tilde{r}_i) = r_\mathcal{F} + [\mathcal{E}(\tilde{r}_\mathcal{M}) - r_\mathcal{F}] \cdot \beta_{i,\mathcal{M}} = 0.03 + (0.08 - 0.03) \cdot 6.85 \approx 0.37 \ , \tag{17.53}$$

which is indeed what we had computed above.

NERD NOTE: *Here is a proof of the certainty equivalence form. Start with the* CAPM *formula:*

$$\mathcal{E}(\tilde{r}_i) = r_\mathcal{F} + [\mathcal{E}(\tilde{r}_\mathcal{M}) - r_\mathcal{F}] \cdot \beta_{i,m} \ . \tag{17.54}$$

Rewrite beta

$$\mathcal{E}(\tilde{r}_i) = r_\mathcal{F} + [\mathcal{E}(\tilde{r}_\mathcal{M}) - r_\mathcal{F}] \cdot \frac{Cov(\tilde{r}_i, \tilde{r}_\mathcal{M})}{\mathcal{V}ar(\tilde{r}_\mathcal{M})} \ . \tag{17.55}$$

Rewrite the rate of return, $\tilde{r}_i = \tilde{P}_1/PV_0 - 1$,

$$\mathcal{E}(\tilde{P}_1/PV_0 - 1) = r_\mathcal{F} + [\mathcal{E}(\tilde{r}_\mathcal{M}) - r_\mathcal{F}] \cdot \frac{Cov(\tilde{P}_1/PV_0 - 1, \tilde{r}_\mathcal{M})}{\mathcal{V}ar(\tilde{r}_\mathcal{M})} \ . \tag{17.56}$$

We want to simplify $Cov(\tilde{P}_1/PV_0 - 1, \tilde{r}_\mathcal{M})$*. You do not know this yet, but trust me when I claim that covariances are easy to manipulate:* $Cov(a \cdot \tilde{x} + b) = a \cdot Cov(\tilde{x})$*, if* a *and* b *are known constants.* $1/PV_0$ *plays the role of the constant* a*,* -1 *plays the role of the constant* b*. So we can write* $Cov(\tilde{P}_1/PV_0 - 1, \tilde{r}_\mathcal{M}) = Cov(\tilde{P}_1, \tilde{r}_\mathcal{M})/PV_0$*. We already know how to manipulate expectations, so* $\mathcal{E}(\tilde{P}_1/PV_0 - 1) = \mathcal{E}(\tilde{P}_1)/PV_0 - 1$*. Substitute back in these unrolled expectation and covariances, and you get*

$$\mathcal{E}(\tilde{P}_1)/PV_0 - 1 = r_\mathcal{F} + [\mathcal{E}(\tilde{r}_\mathcal{M}) - r_\mathcal{F}] \cdot \frac{Cov(\tilde{P}_1, \tilde{r}_\mathcal{M})}{PV_0 \cdot \mathcal{V}ar(\tilde{r}_\mathcal{M})} \ . \tag{17.57}$$

If you solve this equation for PV_0*, you will arrive at Formula 17.46.*

Solve Now!

Q 17.19 *Which risk-free rate should you be using for a project that will yield \$5 million each year for 10 years?*

Q 17.20 *Give an example that the numbers can differ when you first compound and then use the* CAPM*, or vice-versa. For your example to show the difference, what parameters should you choose?*

Q 17.21 *A corporate bond with a beta of 0.2 will pay off next year with 99% probability. The risk-free rate is 3% per annum, the risk-premium is 5% per annum. What is the price of this bond, and its promised rate of return?*

Q 17.22 *Continue: Decompose the bond's quoted rate of return into its components.*

Q 17.23 *A $300 million firm has a beta of 2. The risk-free rate is 4%, the equity premium is 3%. A supplier has approached the firm for a 1-year loan of $100 million that has a beta of 0. The supplier is willing to pay 6% interest, and there is no default risk. The firm has a policy of only accepting projects with a hurdle rate of 10%.*

1. *If the firm changes its policy and extends the loan, how would its value change?*
2. *If the firm changes its policy and extends the loan, approximately how would its beta change?*
3. *If the firm changes its policy and extends the loan, approximately how would its cost of capital change?*
4. *If the firm changes its policy and extends the loan, approximately what would its cash flows be expected to be?*
5. *If the firm changes its policy and extends the loan, can you compute the combined firm's NPV by dividing its expected cash flow by its combined cost of capital?*
6. *Should the firm change its policy?*

Q 17.24 *Going to your school has total additional and opportunity costs of $30,000 this year and up-front. With 90% probability, you are likely to graduate from your school. If you do not graduate, you have lost the entire sum. Graduating from the school will increase your 40-year lifetime annual salary by roughly $5,000 per year, but more so if the stock market rate of return is high than when it is low. For argument's sake, assume that your extra-income beta is 1.5. Assume the risk-free rate is 3%, the equity premium is 5%. What is the value of your education?*

Q 17.25 *Although you are a millionaire, keeping all your money in the market, you have managed to secure a great deal: if you give your even richer Uncle Vinny $10,000 today, he will help you buy a Ferrari, expected to be worth $200,000, if his business can afford it. He is an undertaker by profession, so his business will have the money if the stock market drops, but not if it increases. For simplicity, assume that the stock market drops 1 in 4 years and by –10% when it does and increases by 18% per annum if it does not drop. (Write it out as four separate possible state outcomes to make your life simpler.) The risk-free rate is 6%. What is your Uncle's promise worth at market value?*

17·7.F. For Further Capital Budgeting Applications: See Later Chapters.

For further capital budgeting application of the CAPM, please refer to Chapters 21 and 22. Chapters 21 explains how to use the CAPM in a perfect world without taxes. Chapter 22 explains how to use the CAPM in the presence of (corporate) income taxes. Because the primary use of the CAPM is to determine appropriate costs of capital in corporations, it is only in these later chapters that this book offers enough examples to familiarize you with CAPM applications.

We delay applications until later.

Solve Now!

Q 17.26 *If the stock market has gone up by 16%, and the risk-free rate of return is 4%, then what would you expect a firm i with a stock-market beta of 1.5 to have returned?*

Q 17.27 *A project i is likely to go up by 20% if the stock market goes up by 10%. It is also likely to go down by 20% if the stock market goes down by 5%. If the risk-free rate of return is 4%, what would you expect the beta to be?*

17·8. Summary

The chapter covered the following major points:

- The CAPM provides an "opportunity cost of capital" for investors, which corporations can use as the hurdle rate (or cost of capital) in the NPV formula. The CAPM formula is
$$\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + \left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right] \cdot \beta_{i,\mathcal{M}} \quad . \tag{17.58}$$
Thus, there are three inputs: the risk-free rate of return, the expected rate of return on the stock market, and the project market-beta. Only the latter is project specific.
- The line plotting expected rates of return against market-beta is called the security-markets line (SML).
- Use a risk-free rate that is similar to the approximate duration or maturity of the project.
- There are a number of methods to estimate market-beta. For publicly traded firms, it can be computed from stock return data or obtained from commercial data vendors. For private firms or projects, a similar publicly traded firm can often be found. Finally, managerial scenarios can be used to estimate market-betas.
- The expected rate of return on the market is often a critical input, especially if market-beta is high—but it is difficult to guess.
- The CAPM provides an expected rate of return, consisting of the time-premium and the risk-premium. In the NPV formula, the default-risk and default-premium works through the expected cash flow numerator, not the cost of capital denominator.
- Corporations can reduce their risk by diversification—but if investors can do so as easily, diversification per sé does not create value.
- To value a project, corporations should not use the cost of capital (market-beta) applicable to the entire firm, but the cost of capital (market-beta) applicable to the project. However, because the effort involved can be enormous, you should use individual, project-specific costs of capital primarily when it makes a difference.
- Use the certainty-equivalence form of the CAPM when projects are purchased for a price other than their fair present market-value.

A. Q&A: Answers

1. $\mathcal{E}(\tilde{r}) = 4\% + (7\% - 4\%) \cdot 3 = 13\%$.
2. $\mathcal{E}(\tilde{r}) = 4\% + (12\% - 4\%) \cdot 3 = 28\%$.
3. Solve $\mathcal{E}(\tilde{r}) = 4\% + (7\% - 4\%) \cdot \beta_{i,\mathcal{M}} = 5\%$. Therefore, $\beta_{i,\mathcal{M}} = 1/3$.
4. Do it!
5. $\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}$ is the premium that the stock market expects to offer, above and beyond the rate that risk-free investments offer.
6. See Figure 17.3.

7. Use the Treasury rate for the 1-year project, e.g., from the *Wall Street Journal*. Because the 10-year project could have a duration of flows anywhere from 5 to 10 years, depending on use, you might choose a risk-free Treasury rate that is between 5 and 10 years.
8. An estimate between 2% and 8% per year is reasonable. Anything below 0% and above 10% would be unreasonable.

9. You could run a time-series regression of project rates of returns on the stock market.

$$\tilde{r}_p = \alpha + \beta \cdot \tilde{r}_{\mathcal{M}} \quad . \tag{17.59}$$

(PS: Although not discussed, a slightly better regression would be $\left(\tilde{r}_p - r_{\mathcal{F}}\right) = \alpha + \beta \cdot (\tilde{r}_{\mathcal{M}} - r_{\mathcal{F}})$.) The statistical package (or Excel) would spit out the project beta. The nerdnote explains that if you want to improve the forward-looking estimate, you might want to shrink this beta towards 1.

10. Beta can be found in Yahoo!*Finance*'s "Profile." In June 2003, IBM's beta was 1.48.
11. You can compute an unlevered beta.

$$\begin{aligned} \beta_{P,\mathcal{M}} &= \left(\frac{2}{3}\right) \cdot 0 + \left(\frac{1}{3}\right) \cdot 2.5 \\ \beta_{P,\mathcal{M}} &= w_{\mathsf{D}} \cdot \beta_{\mathsf{D},\mathcal{M}} + w_{\mathsf{E}} \cdot \beta_{\mathsf{E},\mathcal{M}} \quad . \end{aligned} \tag{17.60}$$

Therefore, $\beta_{P,\mathcal{M}} = 0.83$. We assume our project has the same beta, but a smaller debt ratio:

$$\begin{aligned} 0.833 &= \left(\frac{1}{3}\right) \cdot 0 + \left(\frac{2}{3}\right) \cdot \beta_{\mathsf{E},\mathcal{M}} \Rightarrow \beta_{\mathsf{E},\mathcal{M}} = 1.25. \\ \beta_{P,\mathcal{M}} &= w_{\mathsf{D}} \cdot \beta_{\mathsf{D},\mathcal{M}} + w_{\mathsf{E}} \cdot \beta_{\mathsf{E},\mathcal{M}} \quad . \end{aligned} \tag{17.61}$$

12. The comparable project's equity expected rate of return would be $3\% + 2\% \cdot 2.5 = 8\%$. Our own equity's expected rate of return would be $3\% + 2\% \cdot 1.25 = 5.5\%$

13. A debt/equity ratio of 2 is the same as the debt asset ratio of 2/3: two parts debt, one part equity. A debt/equity ratio of 1/2 is the same as the debt asset ratio of 1/3: one part debt, two parts equity. To convert a debt-equity ratio into a debt-asset ratio, recognize that

$$\begin{aligned} \frac{1}{D/A} &= \frac{1}{D/(D+E)} = \left(\frac{D+E}{D}\right) = 1 + \left(\frac{E}{D}\right) \\ \Longrightarrow \quad \frac{D}{E} &= \frac{1}{\left(\frac{1}{D/A} - 1\right)} = \frac{1}{\left(\frac{1}{2/3} - 1\right)} \\ &= \frac{1}{\left(\frac{3}{2} - 1\right)} = \frac{1}{\left(\frac{1}{2}\right)} = 2 \quad . \end{aligned} \tag{17.62}$$

14. This is the same as above, too.
15. $\beta_{\text{combined},\mathcal{M}} = (3/4) \cdot 2.4 + (1/4) \cdot 0.4 = 1.9$.
16. $$\beta_{i,\mathcal{M}} = w_p \cdot 0.2 + (1 - w_p) \cdot 1 = 0 \quad \Rightarrow \quad w_p = 1.25 \quad . \tag{17.63}$$

    So, if your wealth is \$100, you want to short \$25 in the fund to increase your own portfolio holdings to \$125. The net portfolio has zero market beta.
17. Because both the bond and the stock market are exposed to common variation in the time premium.
18. For short investments, the expected cash flows are most critical to estimate well (see Section 6·3). For long-term projects, cost of capital becomes more important to get right. Betas and risk-free rates are usually relatively trouble free, having only modest degrees of uncertainty. The equity premium will be the most important problem factor.
19. A 5-year interest rate is a reasonably good guess. You should not be using a 30-day Treasury bill, or a 30-year Treasury bond.
20. Yes, it matters. You should choose a long horizon payoff, and a very high or very low market-beta.
21. It does not matter what you choose as the per-unit payoff of the bond. \$100 is expected to return \$99. So, the price of the bond is

    $$\text{NPV} = \frac{\$99}{1 + (3\% + 5\% \cdot 0.2)} \approx \$95.^{19} \tag{17.64}$$

    Therefore, the promised rate of return on the bond is $\$100/\$95.^{19} - 1 \approx 5.05\%$.
22. The risk-free rate is 3%, so this is the time premium. The expected risk premium is 1%. The remaining 1.05% is the default premium.
23. The CAPM cost of capital is 10%. Its current projects are expected to provide \$30, the new project would provide \$6, \$2 above the risk-free rate. Therefore, the value of the firm would go up by \$2 next year, which has to be discounted to today. The new project "loan" would be about $^1/_4$ of the new firm. Therefore, the new beta of the firm will be

    $$\beta_{\text{W},\mathcal{M}} = {}^3/_4 \cdot 2 + {}^1/_4 \cdot 0 = 1.5 \quad . \tag{17.65}$$

    The firm's cost of capital would therefore decline from 10% to 8.5%. At this cost of capital, the extra \$2 would add about $\$1.^{84}$ to the firm value. The firm's cash flows would change from \$430 to $\$330 + \$106 = \$436$. Discounted at the 8.5% interest rate, this comes to about $\$401.^{84}$. Subtracting off the \$100 cost of the loan confirms the NPV. The firm should change its policy.
24. The cost needs to be discounted with the current interest rate. Since payment is upfront, this cost is \$50,000 now! The appropriate expected rate of return for cash flows (of your earnings) is $3\% + 5\% \cdot 1.5 = 10.5\%$. You can now use the annuity formula, to determine the NPV if you graduate.

    $$\frac{\$5,000}{10.5\%} \cdot \left[1 - \left(\frac{1}{1 + 10.5\%}\right)^{40}\right] = \$47,619 \cdot 98.2\% \approx \$46,741.^{46} \quad . \tag{17.66}$$

With 90% probability, you will do so, which means that the appropriate risk-adjusted and discounted cash flow is about \$42,067.[32]. The NPV of your education is therefore about \$12,067.

25. This is a certainty equivalence question. Although it is not a gift per se, you cannot assume that \$10,000 is a fair market value, so that you can compute a rate of return of 1,900%—after all, it is your Uncle trying to do something nice for you. There are four outcomes:

Prob:	1/4	1/4	1/4	1/4	
	Crash	No-Crash	No-Crash	No-Crash	Mean
Stock Market	-10%	+18%	+18%	+18%	11%
Ferrari	\$200	\$0	\$0	\$50	

Plug this into the formula

$$\begin{aligned} Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}}) &= 1/4 \cdot [\$150,000 \cdot (-21\%) + (-\$50,000) \cdot (+7\%) + \\ &\quad (-\$50,000) \cdot (+7\%) + (-\$50,000) \cdot (+7\%)] = -\$21,000 \quad . \end{aligned} \tag{17.67}$$

We also need to determine the variance of the market. It is

$$Cov(\tilde{r}_{\mathcal{M}}, \tilde{r}_{\mathcal{M}}) = \frac{(-21\%)^2 + (+7\%)^2 + (+7\%)^2 + (+7\%)^4}{4} = 147\%^2 \tag{17.68}$$

(which incidentally comes to a standard deviation of 12% per annum, a bit low.) With the risk-free rate of 6%, lambda (λ) is $(11\% - 6\%)/147\%^2 \approx 3.4$.

So, we can now use the certainty equivalence formula: the expected value of the Ferrari is \$50,000. If it were a safe payoff, it would be worth \$47,170. Because you get more if the rest of your portfolio goes down, it is actually great insurance for you. So, you value it $3.4 \cdot (-\$21,000)/(1 + 6\%) \approx \$67,358$ higher than \$47,170: the Ferrari is worth \$114,528. You have to pay \$10,000 today, of course, so you have managed to secure a deal for \$104,528.

26. You can use the CAPM formula here: $4\% + (16\% - 4\%) \cdot 1.5 = 22\%$.

27. Rearrange the CAPM formula to $\beta_{i,\mathcal{M}} = \frac{\mathcal{E}(\tilde{r}_i) - r_{\mathcal{F}}}{\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}}$. So, the first beta estimate would be $\frac{20\% - 4\%}{10\% - 4\%} \approx 2.7$. The second beta estimate would be $\beta_{i,\mathcal{M}} = \frac{-20\% - 4\%}{-5\% - 4\%} = -2.7$. Therefore, a reasonable estimate of beta would be the average, here 2.7.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 18

THE CAPM: THE THEORY AND CAPM BENCHMARKING

Where does the CAPM equation come from?

last file change: Jun 15, 2004 (11:50h)

last major edit: n/a

This chapter appears in the Survey text only.

This chapter explains the CAPM theory in detail *and* its shortcomings. Actually, because we have already discussed all the necessary ingredients, mostly in Chapter 16 which is absolutely necessary background for this chapter, this chapter is surprisingly simple.

ANECDOTE: The CAPM Rules!
Among large Fortune-500 type firms, about 75% of CFOs use the CAPM as their primary method to estimate their cost of capital. CAPM use is more common among large firms and firms in which the CFO has an M.B.A. Unfortunately, it is neither clear whether the CAPM is the right model to use, nor whether CFOs use the CAPM correctly. (Source: Graham and Harvey, Duke, 2001.)

18·1. The Theory

18·1.A. The Logic and Formula

Here are the underlying mathematical facts, which were already explained.

We have already covered every necessary aspect of the CAPM theory in earlier chapters:

- In Section 13·3.D, we learned that the portfolio held by investors in the aggregate is the (market-capitalization) value-weighted portfolio.
- In Section 16·3, we learned that the combination of Mean-Variance Efficient portfolios is itself MVE.
- In Section 16·5, we learned that each stock i in a MVE-portfolio must offer a fair expected rate of return for its risk contribution to $\mathcal{E}$. Formula 16.29 states that for each and every security i in the MVE portfolio $\mathcal{E}$, this means

$$\mathcal{E}(\tilde{r}_{\mathcal{E}}) \;=\; r_{\mathcal{F}} \;+\; \Big[\mathcal{E}(\tilde{r}_{\mathcal{E}}) - r_{\mathcal{F}}\Big] \cdot \beta_{i,\mathcal{E}} \;\;. \tag{18.1}$$

Here is the CAPM Theory.

The CAPM is a theory that has only one hypothesis and only one implication.

CAPM **Hypothesis** The CAPM hypothesis is that each and every investor chooses a MVE portfolio.

Investors may not hold MVE portfolios.

Unlike the method to choose weights for a MVE portfolio—which are just mathematical optimization and which have to be correct—the CAPM is a real theory that may not hold in practice: investors might have different investment objectives and therefore not hold MVE portfolios.

CAPM **Implication** If the CAPM hypothesis is correct, then the mathematical implication is that the aggregate value-weighted market portfolio is also MVE.

The CAPM could be wrong in practice.

If the CAPM hypothesis is not correct, there is no particular reason to expect the value-weighted portfolio to lie on the MVE Frontier. Instead, the value-weighted market portfolio could have risk-reward characteristics that place it far inside the MVE Frontier.

This is it: the Capital-Asset Pricing Model, which won the 1990 Nobel Prize in Economics.

IMPORTANT: *The* CAPM *conjectures that all investors purchase MVE portfolios. As a necessary mathematical consequence, the value-weighted market portfolio is also MVE.*

Put altogether again, the logic of the CAPM is:

OK, let's do with algebra and logic.

Mathematical Fact To enter a MVE Frontier portfolio $\mathcal{E}$, each stock in $\mathcal{E}$ has to offer an appropriate reward for its risk. This formula, which relates its expected rate of return to its covariation with $\mathcal{E}$, is

$$\mathcal{E}(\tilde{r}_i) \;=\; r_{\mathcal{F}} + \Big[\mathcal{E}(\tilde{r}_{\mathcal{E}}) - r_{\mathcal{F}}\Big] \cdot \beta_{i,\mathcal{E}} \;. \tag{18.2}$$

Stocks that offer expected rates of return higher than suggested by this formula would generate too much aggregate investor demand; stocks that offer lower expected rates of return would generate too little aggregate investor demand.

CAPM **Theory Assumption** Every investor purchases a MVE portfolio.

$\Longrightarrow$ **Implication** The value-weighted market portfolio is MVE.

$\Longrightarrow$ **Implication**

$$\mathcal{E}(\tilde{r}_i) \;=\; r_{\mathcal{F}} + \Big[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\Big] \cdot \beta_{i,\mathcal{M}} \;. \tag{18.3}$$

In this case, the market portfolio consisting only of risky securities must be the tangency portfolio T.

NERD NOTE: *This is called the Sharpe-Lintner* CAPM. *There is also a so-called "Fischer Black"* CAPM, *in which there is no risk-free rate of return. Everything works the same, except that the risk-free rate is replaced by the constant a. All stocks in the economy follow the* CAPM *formula*

$$\mathcal{E}(\tilde{r}_i) \;=\; a + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - a] \cdot \beta_{i,\mathcal{M}} \;, \tag{18.4}$$

with respect to the value-weighted market portfolio M.

18·1.B. Some Odds and Ends

- The theoretically correct market portfolio in the CAPM is the market-capitalization value-weighted portfolio, consisting of all possible investment assets in the economy, not just stocks in the U.S. stock market.

  The market portfolio is theoretically clear, but practically difficult to identify.

  However, in traditional Investments use, the assumed goal is to pick the best portfolio among publicly traded stocks. It is as if the CAPM theory was restated to conjecture that investors seek to optimize the risk/reward relationship only within their portfolios of publicly traded domestic stocks. In this case, the proper market portfolio would be the market-capitalization value-weighted U.S. stock market portfolio. However, even this portfolio is difficult to obtain every day. Fortunately, the value-weighted U.S. stock market portfolio has high correlation with other broad stock market indexes, such as the S&P500. Thus, the S&P500 is often used as a reasonably good substitute.

SIDE NOTE: Although this is reasonable use for CAPM testing and CAPM application, it is not good investment advice: you can definitely diversify across more assets than just domestic stocks. International stocks, commodities, real-estate, and even education represent other investment classes that are readily available to join smart investors' portfolios.

NERD NOTE: *In the real world, different investors have different investment opportunities. You can invest in your education and reap the rewards, but I cannot invest in your education. This problem is ignored. The* CAPM *totally ignores this problem.*

Mutual Funds facilitate holding the market portfolio.

- Transaction costs would be high enough to prevent ordinary small investors from themselves purchasing the widely diversified portfolio prescribed by the CAPM. Mutual Funds (Section 13·3.B), however, have made this relatively easy.

If the market portfolio is not MVE, then the CAPM does not hold and the CAPM formula cannot be used.

- What if the market portfolio does not sit on the MVE Frontier? Then there will be stocks for which the CAPM Formula 18.4 does not hold. And then it should not be assumed that the CAPM formula is the appropriate relation predicting stocks' expected rates of return.

SIDE NOTE: Indeed, then anything else could conceivably explain higher or lower required rates of returns. For example, instead of firms with higher betas, it may be firms with higher P/E ratios or firms with older CEOs or firms containing the letter "Z" that may end up having to offer higher expected rates of returns in order to induce investors to willingly purchase and hold them in equilibrium. Naturally, such a situation would make it very difficult for corporations to determine the appropriate cost of capital that their projects should offer.

Higher beta implies higher expected rate of return.

- Because $\mathcal{E}(\tilde{r}_{\mathcal{M}}) > r_{\mathcal{F}}$, the intuition of the CAPM formula is that stocks with higher stock-market betas have to offer higher expected rates of return. Stocks with high betas are less helpful in reducing the risk of an investor who already holds the market portfolio.

From the perspective of a widely diversified investor, a tiny bit of zero-beta stock is just as good as investment in the risk-free security.

- A risky project that has a beta of zero need only offer an expected rate of return no higher than the risk-free rate itself. This is because each investor would only hold a tiny amount of this security. Having one cent in the risk-free rate or in a security whose return is expected to be the same and with no correlation to the rest of the portfolio is practically the same. In the real and naturally not perfectly CAPM world, this is not exactly correct—but it is often still a reasonably good approximation.

Solve Now!

Q 18.1 *What is the main* CAPM *hypothesis, what is the main* CAPM *implication?*

Q 18.2 *Write down the* CAPM *formula without looking at the text. You must memorize it!*

Q 18.3 *Under what circumstances is the market-beta a good measure of risk?*

Q 18.4 *What can plotting both the stock-market portfolio into the MVE Frontier graph tell you?*

Q 18.5 *What can checking whether every single security follows the* CAPM *formula (i.e., that every single possible investment is on the security markets line in a graph of expected rate of return against beta) tell you about the MVE Frontier (a graph of expected rate of return against standard deviation)?*

Q 18.6 *If all but one security are right on the security-markets line, is the market portfolio MVE?*

Q 18.7 *Is the* CAPM *market portfolio the value-weighted or the equal-weighted portfolio?*

Q 18.8 *Can the S&P500 be used as a proxy for the market?*

Q 18.9 *In the formula, why do higher beta stocks offer higher expected rates of return?*

Q 18.10 *If there is a risk-free security, does the stock market portfolio have to be the tangency portfolio for the* CAPM *to hold, or can it be any portfolio on the mean-variance frontier of risky assets?*

Q 18.11 *Is it possible for a risky stock to offer an expected rate of return that is less then the risk-free rate?*

18·2. Does the CAPM Hold?

18·2.A. Listing All the CAPM Assumptions

Let's judge the CAPM Model Assumptions.

We have sneaked in the CAPM assumptions one at a time, perhaps to make them appear more palatable. It is excusable if they have slipped the reader's mind by now. So, to help the reader judge how likely it is that the CAPM assumptions are reasonably satisfied in the real world, it is worthwhile to repeat them all at once. These are all the assumptions that we have used to conjecture that each and every investor buys an MVE portfolio:

- Market Assumptions:
  - The market is perfectly competitive.
  - There are no transaction costs.
  - There are no taxes.
  - The investment opportunity choices are identical for each investor. Therefore, the value-weighted market portfolio is identical for all investors. It consists of all assets that can be invested in. It includes such assets as real estate, bonds, international markets, etc.
  - The previous assumption means that are no untraded assets that only some, but not other investors can hold. That is, you cannot have your own house, or your own children, or your own education, or your own labor income, or your own executive stock options.

NERD NOTE: *If you had such assets, you might not want stocks to minimize your portfolio variance only. Instead, you might want to purchase relatively more stocks that go up when your other untraded assets go down, and vice-versa.*

- Informational Assumptions:
  - Financial Markets *are* efficient in the sense of Chapter 19: as will be described, this means that the market does not ignore information in the setting of financial prices, which an investor could use to outperform the market securities line. Investors cannot use public information to pick stocks better than the market.
  - There is no actual inside information. That is, investors are not able to better pick stocks than the market.
  - There is no perceived inside information. That is, investors do not *believe* that they are able to better pick stocks than the market.
  - All investors share the same "opinion" about security expected rates of return and security variance/covariances.
  - All model parameters are perfectly known:
    - The expected rate of return on each and every stock (including the market portfolio) is known.
    - All covariances (and thus betas) are known.
    - If you as a researcher want to test the CAPM, i.e., whether other variables matter, you must also know possible alternative return factors (e.g., firm-size) that investors can use.
  - There are no managerial agency problems (i.e., managers enriching themselves) that can be changed if a large investor holds more of the particular stock with agency problems.

- Preference Assumptions:
  - Investors care only about their portfolio performance. They do not care about other characteristics of their holdings (e.g., socially responsible corporate behavior).
  - Investors like mean and dislike variance. This fully describes their portfolio preferences. (They do not care, e.g., about skewness or kurtosis of their future investment rates of return.)
  - Agents maximize their portfolio performance independently each period. (Although this may appear innocuous, it implies that investors do not choose their portfolios to insure themselves against changes in future investment opportunities. You cannot buy stocks that you believe to pay off more in a recession, because you believe then will be a good time to double up investment.)
  - Investors do not care about how their portfolios perform relative to other variables, including variables related to their own personal characteristics (except for risk-tolerance). For example, investors must not care about earning a different rate of return in states in which they find themselves ill or states in which inflation is high.

If these assumptions hold, the CAPM will hold. Of course, it could also happen that the value-weighted market portfolio just happens to lies on the MVE frontier, in which case the CAPM security markets line would just happen to be a line by accident. Fortunately, neither must the market lie *perfectly* on the MVE frontier, nor must the assumptions hold *perfectly* in order for the CAPM to be a useful model. After all, the CAPM is only a model, not reality. Only mathematics works the same in theory and in practice. The real question is whether the CAPM assumptions are sufficiently badly violated to render the CAPM a useless model.

The CAPM is just a helpful model, not perfect reality.

SIDE NOTE: Here is the author's personal opinion about why the CAPM fails:

Most investors do not invest scientifically. They buy stocks that they believe to be undervalued and sell stocks that they believe to be overvalued. Why do even small and relatively unsophisticated investors—often relying on investment advice that qualifies as the modern equivalent of **hocus-pocus**—believe they are smarter than the financial markets, whilst even professional investors have difficulties beating the market? This is perhaps the biggest puzzle in finance.

Although people's tendencies to categorize choices might have given us some hope that people at least maximize MVE within the domain of their own pure market investment portfolios, perhaps subject to their idiosyncratic estimates of expected rates of returns, the evidence suggest that investors are not even internally consistent. Even among the stocks for which they hold opinions about high or low expected rates of returns, investors tend not to choose securities to reach their own best MVE Frontiers.

Unfortunately, if every investor behaves differently, it is also not clear what the expected rate of return on a particular investment really should be or has to be—other than that it need not necessarily be that suggested by the CAPM. It could be high, it could be low, it could be anything.

This phenomenon is the domain of behavioral finance, eventually to be further expounded in a web chapter.

18·2.B. Is the CAPM a good representation of reality?

In real life, we know that some investors purchase portfolios other than MVE portfolios. This should not come as a big surprise: chances are that you yourself are one of these investors, even if you are only a small investor. Therefore, in the strictest sense, we already know that the CAPM does not hold. But, again, the CAPM is only a model. The question is whether the CAPM simplification of reality is helpful in understanding the world, i.e., whether the model is sufficiently close to reality to be useful. Perhaps, most of the real big investors "who matter" do invest in portfolios sufficiently similar to MVE portfolios, so that the value-weighted market portfolio is on the MVE Frontier, meaning that the CAPM "roughly" works.

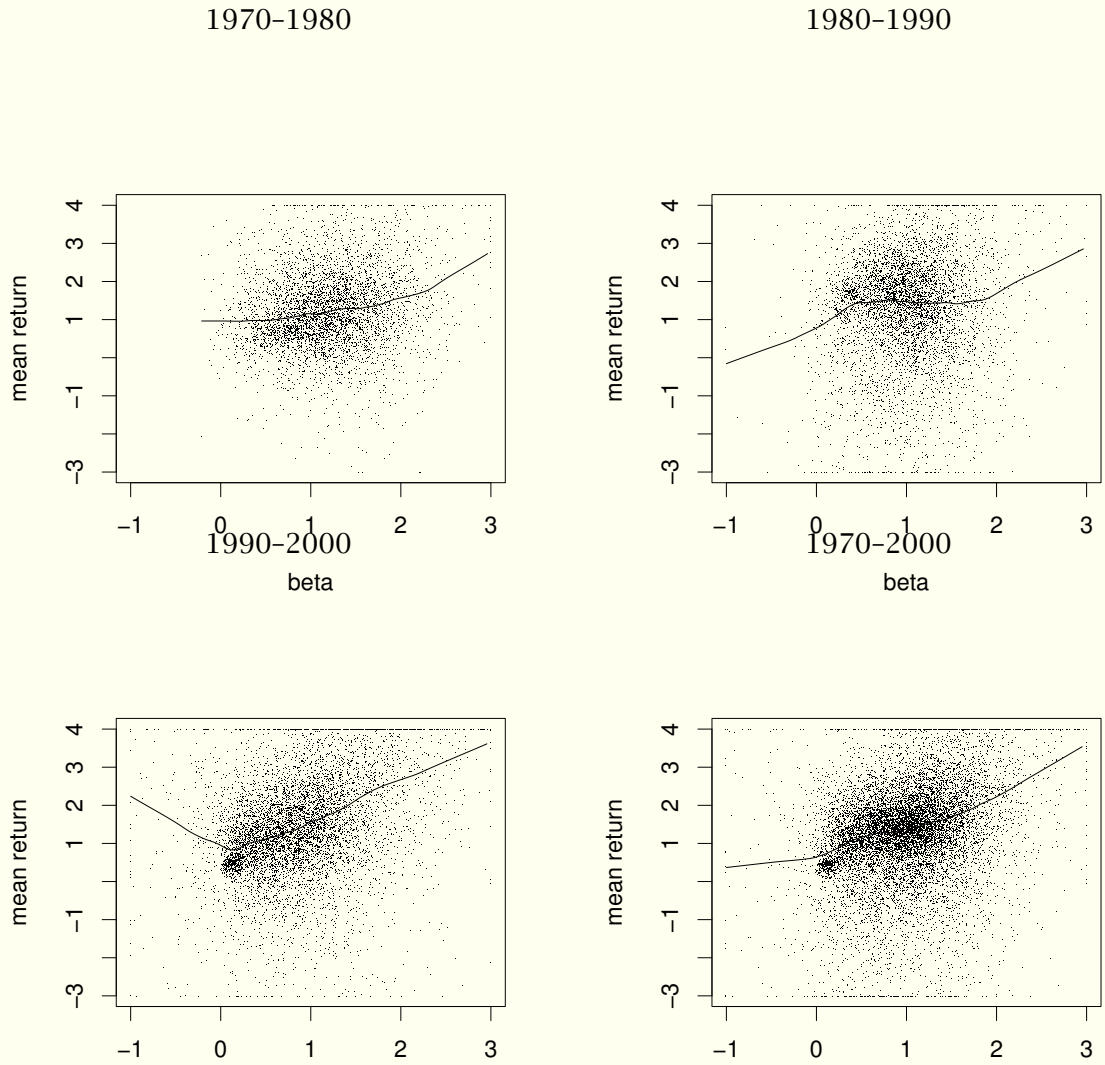
I do not hold the stock market portfolio, therefore the CAPM is incorrect. True, but...

Unfortunately, the empirical evidence suggests that the market portfolio does not lie too closely on the MVE Frontier, that the security markets line is not perfectly linear, and that expected security returns relate to other characteristics in addition to beta. Figure 18.1 plots average monthly returns against stock betas. Care must be exercised when viewing these figures, because there are few stocks with betas below 0 and above 2. Therefore, the smoothed lines are not too reliable beyond this range. In the 1970s, the relationship between beta and average (monthly) return was generally upward-sloping. In the eighties, although stocks with betas below 0.5 and 1.5 had similar average rates of returns, stocks outside the 0.5 to 1.5 beta range (not too many!) had a clear positive relationship. In the 1990s, firms with positive betas showed a nice positive relationship, though the (few) stocks below $\beta = 0$ did not. Finally, over the whole sample period, the relationship between beta and average monthly return was reasonably upward sloping. Stocks with a beta of about 0 paid an average rate of return of about 0.8% per month,

Why use the CAPM when there is evidence against it?

while stocks with a beta of about 2 paid an average rate of return of about 2% per month. This, and other evidence, suggests that, although the CAPM is reasonable enough an approximation for the cost of capital required in capital budgeting, it is not a great tool for determining the best investment strategy.

Figure 18.1. Average Historical Rates of Return Against Historical Market Beta.



The web chapter on empirical asset pricing examines, among other issues, noise and non-stationarity problems when testing investments models.

Why the CAPM continues to be and should be used.

Although the CAPM formula is mistaken, it continues to be the dominant model. Why?

1. Although the market portfolio does not seem to lie directly on the MVE Frontier, it does not appear to be *too* far away.

NERD NOTE: *Not all academics agree with this assessment.*

2. The CAPM provides good intuition for the characteristics which should determine the expected rates of returns that have to be offered by stocks: stocks that help diversified investors achieve lower risk should be sellable even if they offer lower expected rates of return.

3. For many purposes and many stocks, the CAPM provides reasonable expected rates of returns, not too far off from those gleamed from more complex, difficult, and/or arbitrary models.

4. There is no good alternative model that is either simpler to use or convincingly better in providing appropriate expected rates of return for investments.

5. The CAPM is *the* gold standard for cost-of-capital estimations in the real world. It is in wide use. Understanding it (and its shortcomings) is therefore crucial for any student with an interest in finance.

18·2.C. Why do we use the CAPM?

What are our alternatives to the CAPM? Investors have good, albeit tedious alternatives...

So, why do we need a model like the CAPM ? Could we not just rely on optimal portfolio theory (Chapter 16) instead? From the perspective of an investor, this is a feasible choice. Instead of believing that the market portfolio is MVE, i.e., that a part of the investor's portfolio should be allocated to it, the investor has to solve the more general problem of where the MVE Frontier is. This is tedious, but possible. Even so, in many situations, i.e., when not too much money is at stake, even though the stock market portfolio is not exactly MVE, it is probably still close enough for most investors to just choose it anyway in order to avoid the pain of computation. Nevertheless, the CAPM should not be used for investment purposes where more than $100 million is at stake. There may be better investment strategies. Again, for smaller investment portfolios, the advice of purchasing a combination of the value-weighted market portfolio (including international stocks and other assets) and a risk-free rate is probably reasonably close to an MVE portfolio.

...while corporations really have few or no alternatives.

From the perspective of a firm, however, relying on optimal portfolio theory is not a feasible choice. Corporate managers need to be able to compute what expected rate of return investors would demand for particular project. If they do not know what investors want in equilibrium, managers are in a guessing game. So, take the CAPM for what it is useful for: it is a model which provides decent guestimates of appropriate cost of capital for non-financial projects, in which the exact discount rate is not too critical. Do not use it if a lot of money can be lost if the discount rate is misestimated by a percent or two.

Solve Now!

Q 18.12 *List as many assumptions about the* CAPM *that you can recall. Which ones are most problematic?*

18·3. PORTFOLIO BENCHMARKING WITH THE CAPM?

Can the CAPM Formula be beaten in a CAPM world?

The performance evaluation of investment managers is an example of how problematic the application of the CAPM formula can be. The typical CAPM use in **Portfolio Benchmarking** has investors compute the beta of portfolios held by their investment managers, and check if the manager's portfolio return over the year beat the securities market line. Unfortunately, if the CAPM holds, the security markets-line cannot be beaten. After all, the definition of the MVE Frontier is that it is the set of portfolios which offers the best possible combination of risk and return. If the CAPM does not hold, then the security markets line has no reason for living. It would be totally ad-hoc. So, strictly speaking, the CAPM cannot be used for portfolio benchmarking.

There is a good intuitive reason to use the CAPM anyway.

Nevertheless, the CAPM formula gives good intuition on what constitutes good performance contribution to a widely diversified investor. It also works if the fund manager with the presumably better information and stock picking ability is too small to move prices. This explains why the use of the CAPM formula for performance evaluation continues—despite all its problems.

18·4. Summary

The chapter covered the following major points:

- The logic of the CAPM is that if every investor holds a mean-variance efficient portfolio, then the (value-weighted) market portfolio is mean-variance efficient. This in turn means that stocks follow the linear relationship between the expected rate of reward and market-beta (the security markets line).
- The CAPM relies on many assumptions. It is only a model.
- The CAPM is a useful simplification in certain contexts (such as capital budgeting), because it is intuitive, easy-to-use, and often gives a reasonable enough cost-of-capital estimate.
- The CAPM is not a reasonable description of reality in many other contexts. It should not be used for stock investing purposes.
- There are no good alternatives to the CAPM. This is why the CAPM, with all its fault, is still the predominant model in most corporate contexts.

A. Nerd Appendix: The Arbitrage Pricing Theory (APT) Alternative

The **Arbitrage Pricing Theory** (**APT**) is an alternative to the CAPM. This Chapter provides a brief treatment. For more detail, consider an advanced course/text.

Assume that the returns of stocks—here S1, S2 and S3—are driven by some (say two) time-series factors:

The price-generation process: a mean plus random factors multiplied.

$$\begin{aligned}
\tilde{r}_{S1} &= 24\% + (+4)\cdot\tilde{f}_A + (+1)\cdot\tilde{f}_B + \tilde{\epsilon}_{S1} \\
\tilde{r}_{S2} &= 5\% + (-1)\cdot\tilde{f}_A + (+3)\cdot\tilde{f}_B + \tilde{\epsilon}_{S2} \\
\tilde{r}_{S3} &= 8\% + (+2)\cdot\tilde{f}_A + (-1)\cdot\tilde{f}_B + \tilde{\epsilon}_{S2} \\
\tilde{r}_{Si} &= \mathcal{E}(\tilde{r}_{Si}) + a_{Si}\cdot\tilde{f}_A + b_{Si}\cdot\tilde{f}_B + \tilde{\epsilon}_{Si} \ .
\end{aligned} \tag{18.5}$$

What exactly is a **Factor**? A factor is just an expected economy wide outcome, de-facto some time-series, after it has been de-meaned. So, factor $\tilde{f}_A$ could be the actual rate of return on the S&P500 net of the typical (or expected) rate of return on the S&P500. Factor $\tilde{f}_B$ could be the economy's growth rate, or the percent change in the price of a barrel of oil, or the rate of return on IBM—again, quoted as a "surprise" by first subtracting out its expected value:

$$\mathcal{E}(\tilde{f}_A) = 0 \qquad \mathcal{E}(\tilde{f}_B) = 0 \ . \tag{18.6}$$

This is just a normalization, and can always be done. To give a feeling for returns that are consistent with our specific APT model, Table 18.1 shows some historical rates of returns that would have lead to this APT model.

Table 18.1. A Set of Rates of Returns Well Described By Our APT Model

	Factors				Rates of Return		
Year	raw $\tilde{f}_A$	raw $\tilde{f}_B$	$\tilde{f}_A$	$\tilde{f}_B$	$\tilde{r}_{S1}$	$\tilde{r}_{S2}$	$\tilde{r}_{S3}$
1980	0.30	0.31	0.58	0.09	2.65	−0.26	1.15
1981	−0.37	−0.82	−0.09	−1.04	−1.14	−2.98	0.95
1982	1.20	0.85	1.48	0.63	6.79	0.47	2.41
1983	−0.54	0.62	−0.26	0.40	−0.38	1.51	−0.83
1984	−1.81	0.12	−1.53	−0.10	−5.97	1.29	−2.88
1985	−0.93	0.56	−0.65	0.34	−2.01	1.71	−1.55
1986	1.39	−0.07	1.67	−0.29	6.63	−2.50	3.72
1987	−1.50	0.19	−1.21	−0.03	−4.64	1.17	−2.32
Mean	−0.28	+0.22	0.00	0.00	+0.24	+0.05	+0.08

ANECDOTE: CFO Concerns for other Risks
Surveyed CFOs seem to also be concerned with interest rate risk (especially small firms), exchange rate risk (especially large Fortune-500 type firms), business cycle risk, and inflation risk. (Source: Graham and Harvey, Duke, 2001.)

Epsilon's should be uncorrelated. Common variation should be in factors.

Common stock variation when all/many stocks are formed into a portfolio should be only in the factors, not the pricing error, so the pricing error ($\tilde{\epsilon}_{Si}$) for each stock should not only be zero on average, but should outright become zero if the portfolio has a very large number of stocks. If the errors are still not idiosyncratic to each firm, but continue to have common variation with other errors, you need to introduce a new factor that captures this common variation.

Our return generation process intercept is indeed the expected rate of return.

With the assumptions of de-meaned factors ($\mathcal{E}(\tilde{f}) = 0$) and mean-zero errors ($\mathcal{E}(\tilde{\epsilon}_{Si}) = 0$), the intercept is indeed the expected rate of return on the stock. To see this, take the expectation of both sides of Formula 18.5:

$$\begin{aligned} \mathcal{E}(\tilde{r}_{Si}) &= \mathcal{E}\left[\text{Intercept} + a_{Si}\cdot\tilde{f}_A + b_{Si}\cdot\tilde{f}_B + \tilde{\epsilon}_{Si}\right] \\ &= \text{Intercept} + a_{Si}\cdot\mathcal{E}(\tilde{f}_A) + b_{Si}\cdot\mathcal{E}(\tilde{f}_B) + \mathcal{E}(\tilde{\epsilon}_{Si}) \\ &= \mathcal{E}(\tilde{r}_{Si}) \qquad . \end{aligned} \tag{18.7}$$

The a_{Si} and b_{Si} coefficients are called **Factor Loadings** or **Factor Exposures**. A stock's factor loadings describe how the stock responds to factor surprises. In our example, stock S1 responds to surprises in the first factor more and in an opposite way as stock S2: if $\tilde{f}_A$ where to turn out as 5%, it would change the actual rate of return on stock S1 by $+4\cdot 5\% = 20\%$ and change the actual rate of return on stock S2 by $-1\cdot 5\% = -5\%$. Each stock can be described by how it responds to the two factors—aside from the epsilon, which is just some error or noise that is not supposed to correlate with anything else. Again, this is important: the factors are supposed to capture everything that makes two stocks covary with one another.

The APT question is almost like the CAPM question.

The question that the APT is trying to answer is the same question that the CAPM is trying to answer: given how a stock moves together with something, what should its expected rate of return be? In the CAPM context, this translates into: given the beta of a stock with respect to the stock market, what should its expected rate of return be? In the APT context, this translates into: given the factor loadings (which describe how a stock comoves with the factors), what should its expected rate of return be?

The APT trick is to diversify away all risk.

The APT trick is to form a portfolio P that has zero exposure with the factors. For example, if we were to invest \$100 into S1 and \$100 into S2, a 10% factor realization by $\tilde{f}_A$ would increase our S1 investment by $\$100\cdot(+4)\cdot 10\% = \$40$, and our S2 investment by $\$100\cdot(-1)\cdot 10\% = -\$10$. Net, we would gain \$30. The rate of return on such a portfolio definitely still depends on the factor $\tilde{f}_A$. But, if we were to invest \$100 into S1 and \$400 into S2, a 10% factor realization by $\tilde{f}_A$ would increase our S1 investment by $\$100\cdot(+4)\cdot 10\% = \$40$, and our S2 investment by $\$400\cdot(-1)\cdot 10\% = -\$40$. For this particular portfolio, no matter what the $\tilde{f}_A$ factor will turn out to be in the future, it will not affect its value. The portfolio has zero net exposure to factor $\tilde{f}_A$.

A portfolio if one stock fails to follow the APT pricing equation.

Now assume that there is also a fourth stock available for investment. This fourth stock has factor exposures of $(+6)$ and (-2), respectively, but we do not yet know its expected rate of return.

$$\begin{aligned} \tilde{r}_{S4} &= \mathcal{E}(\tilde{r}_{S4}) + (+6)\cdot\tilde{f}_A + (-2)\cdot\tilde{f}_B + \tilde{\epsilon}_{S3} \\ \tilde{r}_{Si} &= \mathcal{E}(\tilde{r}_{Si}) + a_{Si}\cdot\tilde{f}_A + b_{Si}\cdot\tilde{f}_B + \tilde{\epsilon}_{Si} \ . \end{aligned} \tag{18.8}$$

Our ultimate goal is to determine what expected rate of return it should offer. Our first step is to determine a portfolio of four stocks (defined by w_{S1}, w_{S2}, w_{S3}, and w_{S4}) which has zero net exposure to the factors. What is the *actual* rate of return of such a portfolio?

$$\begin{aligned}
\tilde{r}_p &= w_{S1}\cdot\tilde{r}_{S1} + w_{S2}\cdot\tilde{r}_{S2} + w_{S3}\cdot\tilde{r}_{S3} + w_{S4}\cdot\tilde{r}_{S4} \\
&= w_{S1}\cdot[\mathcal{E}(\tilde{r}_{S1}) + (+4)\cdot\tilde{f}_A + (+1)\cdot\tilde{f}_B + \tilde{\epsilon}_{S1}] \\
&\quad + w_{S2}\cdot[\mathcal{E}(\tilde{r}_{S2}) + (-1)\cdot\tilde{f}_A + (+3)\cdot\tilde{f}_B + \tilde{\epsilon}_{S2}] \\
&\quad + w_{S3}\cdot[\mathcal{E}(\tilde{r}_{S3}) + (+2)\cdot\tilde{f}_A + (-1)\cdot\tilde{f}_B + \tilde{\epsilon}_{S3}] \ . \\
&\quad + w_{S4}\cdot[\mathcal{E}(\tilde{r}_{S4}) + (+6)\cdot\tilde{f}_A + (-2)\cdot\tilde{f}_B + \tilde{\epsilon}_{S4}] \ .
\end{aligned} \tag{18.9}$$

Collect the factor-related terms,

$$\begin{aligned}
\tilde{r}_p = \ & w_{S1}\cdot\mathcal{E}(\tilde{r}_{S1}) + w_{S2}\cdot\mathcal{E}(\tilde{r}_{S2}) + w_{S3}\cdot\mathcal{E}(\tilde{r}_{S3}) + w_{S4}\cdot\mathcal{E}(\tilde{r}_{S4}) \\
& + [\ w_{S1}\cdot(+4) + w_{S2}\cdot(-1) + w_{S3}\cdot(+2) + w_{S4}\cdot(+6)\ ]\cdot\tilde{f}_A \\
& + [\ w_{S1}\cdot(+1) + w_{S2}\cdot(+3) + w_{S3}\cdot(-1) + w_{S4}\cdot(-2)\ ]\cdot\tilde{f}_B \\
& + w_{S1}\cdot\tilde{\epsilon}_{S1} + w_{S2}\cdot\tilde{\epsilon}_{S2} + w_{S3}\cdot\tilde{\epsilon}_{S3} + w_{S4}\cdot\tilde{\epsilon}_{S4} \ .
\end{aligned} \tag{18.10}$$

To find a portfolio whose return does not depend on the factors, the net exposure on each factor must be zero.

$$\begin{aligned}
[\ w_{S1}\cdot(+4) + w_{S2}\cdot(-1) + w_{S3}\cdot(+2) + w_{S4}\cdot(+6)\ ] &= 0 \ , \\
[\ w_{S1}\cdot(+1) + w_{S2}\cdot(+3) + w_{S3}\cdot(-1) + w_{S4}\cdot(-2)\ ] &= 0 \ .
\end{aligned} \tag{18.11}$$

We also do not want to spend any money on this portfolio today, so quote our investment in dollars (rather than in percent), and choose a portfolio so that it costs no money:

$$[\ w_{S1} + w_{S2} + w_{S3} + w_{S4}\ ] = \$0 \ . \tag{18.12}$$

There is not just one such portfolio, but many. For example, the portfolio

$$w_{S1} = -\$92.^{86} \ , \quad w_{S2} = +\$71.^{43} \ , \quad w_{S3} = -\$78.^{57} \ , \quad w_{S4} = +\$100 \tag{18.13}$$

happens to be one such a portfolio. It has zero net exposure to each factor and it costs nothing today. Check this:

$$\begin{aligned}
&[\ (-\$92.^{86})\cdot(+4) + (+\$71.^{43})\cdot(-1) + (-\$78.^{57})\cdot(+2) + (+\$100)\cdot(+6)\ ] = \$0 \ , \\
&[\ (-\$92.^{86})\cdot(+1) + (+\$71.^{43})\cdot(+3) + (-\$78.^{57})\cdot(-1) + (+\$100)\cdot(-2)\ ] = \$0 \ , \\
&[\ (-\$92.^{86}) + (+\$71.^{43}) + (-\$78.^{57}) + (+\$100)\ ] = \$0 \ .
\end{aligned} \tag{18.14}$$

Other portfolios that have zero net exposure and zero costs are multiplies of this portfolio. For example, the following portfolio works equally well:

$$w_{S1} = -\$92.^{86}/2 \ , \quad w_{S2} = +\$71.^{43}/2 \ , \quad w_{S3} = -\text{-}\$78.^{57}/2 \ , \quad w_{S4} = +\$100/2 \ . \tag{18.15}$$

To recap, we have determined a portfolio that costs nothing today and has no factor exposure. Regardless of what the factors will turn out to be, the portfolio will be unaffected.

If so, our portfolio allows us to earn an arbitrage!

Now, what is the *actual* rate of return on this portfolio?

$$\begin{aligned}
\tilde{r}_p = \; & w_{S1} \cdot \mathcal{E}(\tilde{r}_{S1}) + w_{S2} \cdot \mathcal{E}(\tilde{r}_{S2}) \\
& + w_{S3} \cdot \mathcal{E}(\tilde{r}_{S3}) + w_{S4} \cdot \mathcal{E}(\tilde{r}_{S4}) \\
& + w_{S1} \cdot \tilde{\epsilon}_{S1} + w_{S2} \cdot \tilde{\epsilon}_{S2} \\
& + w_{S3} \cdot \tilde{\epsilon}_{S3} + w_{S4} \cdot \tilde{\epsilon}_{S4} \quad . \\
= \; & (-\$92.^{86}) \cdot \mathcal{E}(\tilde{r}_{S1}) + (+\$71.^{43}) \cdot \mathcal{E}(\tilde{r}_{S2}) \\
& + (-\$78.^{57}) \cdot \mathcal{E}(\tilde{r}_{S3}) + (+\$100) \cdot \mathcal{E}(\tilde{r}_{S4}) \\
& + w_{S1} \cdot \tilde{\epsilon}_{S1} + w_{S2} \cdot \tilde{\epsilon}_{S2} + w_{S3} \cdot \tilde{\epsilon}_{S3} + w_{S4} \cdot \tilde{\epsilon}_{S4} \, .
\end{aligned} \tag{18.16}$$

We now need to make the big leap of faith that the APT requires: Assume that the epsilons are not just expected to be zero, but that they are *actually* always zero. In a large portfolio of thousands of stocks, the epsilons should be noise that should diversify away. (PS: In reality, even the 5,000 stocks traded in the U.S. are not enough to make this come true. This APT leap of faith is definitely an approximation.)

$$\begin{aligned}
\tilde{r}_p \approx \; & (-\$92.^{86}) \cdot \mathcal{E}(\tilde{r}_{S1}) + (+\$71.^{43}) \cdot \mathcal{E}(\tilde{r}_{S2}) + (-\$78.^{57}) \cdot \mathcal{E}(\tilde{r}_{S3}) + (+\$100) \cdot \mathcal{E}(\tilde{r}_{S4}) \\
= \; & (-\$92.^{86}) \cdot 24\% + (+\$71.^{43}) \cdot 5\% + (-\$78.^{57}) \cdot 8\% + (+\$100) \cdot \mathcal{E}(\tilde{r}_{S4}) \\
= \; & (-\$22.^{29}) + (+\$3.^{57}) + (-\$6.^{29}) + (+\$100) \cdot \mathcal{E}(\tilde{r}_{S4}) \\
= \; & -\$25.^{00} + (+\$100) \cdot \mathcal{E}(\tilde{r}_{S4}) \, , \\
\approx \; & w_{S1} \cdot \mathcal{E}(\tilde{r}_{S1}) + w_{S2} \cdot \mathcal{E}(\tilde{r}_{S2}) + w_{S3} \cdot \mathcal{E}(\tilde{r}_{S3}) + w_{S4} \cdot \mathcal{E}(\tilde{r}_{S4}) \quad .
\end{aligned} \tag{18.17}$$

Note that $\tilde{r}_p$ is an *actual* rate of return, while $\mathcal{E}(\tilde{r}_{S4})$ is just a number, known in advance. For this special portfolio, however, we have managed to eliminate all risk! The factor risk was eliminated with clever portfolio choice, the epsilon risk was assumed to be zero (diversified away). Therefore, we can eliminate the tilde over the rate of return, and write

$$r_p = -\$25.^{00} + \$100 \cdot \mathcal{E}(\tilde{r}_{S4}) \quad . \tag{18.18}$$

Now, what would you do if the expected rate of return on the fourth stock were 30% instead of 25%? If you purchased this portfolio, you would be guaranteed a rate of return of

$$r_p = -\$30.^{00} + \$100 \cdot 25\% = +\$5 \quad . \tag{18.19}$$

Remember that this portfolio costs nothing and has no risk. (It is an **Arbitrage Portfolio**.) With a positive payoff, such a portfolio would constitute an arbitrage! Indeed, the only expected rate of return on the fourth stock that does not permit arbitrage is 25%. It can be shown that the expected rates of return on each and every stock that prevents the presence of arbitrage must follow the Formula

$$\begin{aligned}
\mathcal{E}(\tilde{r}_{Si}) &= 1\% + 5\% \cdot a_{Si} + 3\% \cdot b_{Si} \\
\mathcal{E}(\tilde{r}_{Si}) &= r_{\mathcal{F}} + \lambda_a \cdot a_{Si} + \lambda_b \cdot b_{Si} \quad .
\end{aligned} \tag{18.20}$$

If even one stock does not follow this formula, you can construct a profitable arbitrage portfolio. The intercept in this formula must be the risk-free rate, because the risk-free rate does not covary with either factor, and therefore has zero a_{Si} and zero b_{Si}. (It has no exposure to either factor.)

SIDE NOTE: The Arbitrage Pricing Theory is a misnomer. It should be called the Non-Arbitrage Pricing Theory, because it determines the expected rates of returns if there are no arbitrage opportunities.

Note the similarity of Formula 18.20 to the CAPM. If λ_b is zero (meaning that a factor exposure to the second factor does not influence expected rates of return), and if λ_a is the equity premium $\mathcal{E}(\tilde{r}_{\mathcal{M}} - r_{\mathcal{F}})$, then the APT formula is the CAPM formula.

The APT equation looks like a generalized form of the CAPM.

SIDE NOTE: The CAPM can almost be considered to be a special case of the APT, in which there is only factor, which is the excess rate of return on the stock market. (Almost because the two models differ by some technical conditions.) But APT flexibility and generality comes at a price: you need to specify what the important pricing factors are—and the APT can offer no guidance here.

In the real world, application of the arbitrage pricing theory requires determining what the right factors are, so that the errors are small and diversify away. Sometimes, this is done by specifying macroeconomic factors, or industry portfolios; sometimes, this is done by using a statistical technique called principal component analysis (or factor extraction).

The APT's main problem: what are the factors?

SIDE NOTE: Many academic papers use four factors as predictors of expected rates of returns. (There is a debate about whether these are risk-factors, inefficient stock market behavior, or just too many academic researchers finding random past coincidences.) At a minimum, controlling for such factors helps to control for expected rate of return differences that are already known.

The three factors, additional to the stock market, are **book-to-market ratios**, **firm capitalizations**, and **momentum**. It appears as if **value firms** (with high book-to-market ratios) command a premium over **growth firms**, as if small firms command a premium over large firms, and as if firms having recently appreciated command a premium over stocks recently depreciated.

Solve Now!

Q 18.13 *Add the rates of return from security 4 (factor exposures of* $+6$ *and* -2*) with an expected rate of return of 25% to Table 18.1. Then write down the annual rates of return of the arbitrage portfolio.*

Q 18.14 *Assume that you have determined that three stocks are always behaving as follows:*

$$\begin{aligned}
\tilde{r}_{S1} &= 9\% + (+1)\cdot\tilde{f}_A + (-1)\cdot\tilde{f}_B \\
\tilde{r}_{S2} &= -4\% + (+2)\cdot\tilde{f}_A + (+3)\cdot\tilde{f}_B \\
\tilde{r}_{S3} &= 15\% + (+3)\cdot\tilde{f}_A + (-1)\cdot\tilde{f}_B
\end{aligned} \tag{18.21}$$

A new stock has just arrived on the scene, and it has a factor exposure to each of the two factors of $+4$*. What is its expected rate of return?*

Q 18.15 *If the new stock has an expected rate of return of +2%, how could you make money?*

A. Q&A: Answers

1. Because each and every investor purchases an MVE portfolio, the value-weighted market portfolio is MVE.
2. See Formula 18.4
3. It is a measure of risk contribution to an investor's portfolio, if this portfolio is the market portfolio.
4. If the market portfolio lies on the MVE Frontier, then the CAPM holds and all securities must lie on the security markets line.
5. Whether the stock market portfolio is on the MVE Frontier.
6. No.
7. The value-weighted portfolio.
8. Maybe yes, maybe no—but it certainly is *commonly* used as a proxy for the value-weighted domestic stock market portfolio.
9. Because $\mathcal{E}(\tilde{r}_{\mathcal{M}}) > r_{\mathcal{F}}$: the stock market is assumed to have a higher expected rate of return than risk-free Treasury bonds.
10. It has to be the tangency portfolio.
11. Yes, if this stock helps exceptionally well to diversify the market portfolio (beta is negative).
12. See Subsection 18·2.A.
13. The revised table is

	Factors				Rates of Return				
Year	raw $\tilde{f}_A$	raw $\tilde{f}_B$	$\tilde{f}_A$	$\tilde{f}_B$	$\tilde{r}_{S1}$	$\tilde{r}_{S2}$	$\tilde{r}_{S3}$	$\tilde{r}_{S4}$	Pfio
1980	0.30	0.31	+0.580	+0.089	2.65	−0.26	1.15	3.60	0.05
1981	−0.37	−0.82	−0.085	−1.038	−1.14	−2.98	0.95	1.87	0.05
1982	1.20	0.85	+1.480	+0.632	6.79	0.47	2.41	7.92	0.05
1983	−0.54	0.62	−0.255	+0.401	−0.38	1.51	−0.83	−2.03	0.05
1984	−1.81	0.12	−1.529	−0.096	−5.97	1.29	−2.88	−8.68	0.05
1985	−0.93	0.56	−0.648	+0.337	−2.01	1.71	−1.55	−4.26	0.05
1986	1.39	−0.07	+1.671	−0.294	6.63	−2.50	3.72	10.91	0.05
1987	−1.50	0.19	−1.213	−0.030	−4.64	1.17	−2.32	−6.92	0.05
Mean	−0.28	+0.22	0.000	0.000	+0.24	+0.05	+0.08	+0.25	0.05

14. The formula for the expected rates of return is

$$\begin{aligned} \mathcal{E}(\tilde{r}_i) &= 2\% + 3\% \cdot 4\% - 4\% \cdot 4\% = -2\% \ , \\ \mathcal{E}(\tilde{r}_i) &= 2\% + 3\% \cdot a_i - 4\% \cdot b_i \ . \end{aligned} \tag{18.22}$$

15. A sample arbitrage portfolio is

$$w_{S1} = \$112.^{50} \ , \quad w_{S2} = -\$125 \ , \quad w_{S3} = -\$87.^{50} \ , \quad w_{S4} = +\$100 \ . \tag{18.23}$$

Any portfolio that consists of multiples of these weights also works. The known rate of return on this portfolio is

$$r_p \;=\; \$2 \;+\; \$100 \cdot \mathcal{E}(\tilde{r}_{S4}) \;. \tag{18.24}$$

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 19

EFFICIENT MARKETS

Or You Get What You Pay For!

last file change: Jun 21, 2004 (16:53h)

last major edit: n/a

T: ***quantify possible inefficiencies***

This chapter revisits the concept of competitive, perfect, efficient markets, first mentioned in Chapter 1 (Introduction), in more detail. It develops three basic concepts of finance in more detail: **Arbitrage** and **No-Arbitrage**, **Good Bets**, and **Efficient Market** (**E-M**). No study of finance is complete without these.

This chapter also discusses the consequences of the E-M concept: what efficient markets mean for predicting stock performance; how to interpret the success of famous investors; and how to use the efficient markets concept to run an **Event Study** to help assess the valuation impact of some corporate events. The idea of an event study is that if the public market is valuing projects appropriately, and if the stock price increases by $1,000,000 on the minute when the firm first announces the event, then the value of the new project is likely to be worth about $1,000,000.

19·1. Arbitrage and Great Bets

Important: *An **arbitrage** is a business transaction without any net cash outflow, either today or in the future, and under any circumstance.*

arbitrage is the "Perpetuum Mobile" of economics. It is defined in terms of cash outlays and risk.

Naturally, even small arbitrage transactions are very difficult to find. Arbitrage is not the same as earning money without risk: after all, we know that investments in Treasury securities earn a positive risk-free rate of return. But this requires you to lay out cash today. Arbitrage is also not the same as "money in" today: if you are willing to accept risk, it is possible to receive cash today: For example, insurance companies take money from you in exchange for the possibility that they may have to pay you in the future.

Anecdote: **Arbitrage Departments, or Wishful Thinking Departments?**
In a fit of wishful thinking, many Wall Street firms have established so-called Arbitrage Departments, implying that they can find and exploit arbitrage opportunities in the financial markets. It would be less sexy to call them "Wishful Thinking Departments."

In a sense, positive NPV projects under certainty are arbitrage.

Naturally, arbitrage opportunities are difficult to find in practice, especially in very competitive financial markets. But in theory, what could a hypothetical arbitrage opportunity look like? For example, if you can purchase an item for \$1, borrow at an interest rate of 9%, and sell the item tomorrow for \$1.[10] for sure, you earn 1 cent for certain today without any possible negative outflows in the future. Although this is not a lot, if you can repeat this transaction one billion times, you can earn \$10 million. Now, it might be possible to find and execute a risk-free arbitrage for 10,000 items (earning you \$100). If you ever stumble upon such an opportunity, please execute it—it is a positive NPV project! But, it is highly unlikely that you can find such an arbitrage opportunity that works for one billion items to earn you \$10 million.

Side Note: "Lack of arbitrage" is one justification for NPV: under **perfect certainty**, positive NPV projects should be difficult to come by, at least in very large or infinite amounts. Anything else is an arbitrage.

Arbitrage could conceivably occur between different financial markets.

Another hypothetical example of arbitrage: PEP shares are quoted for \$51 on the Frankfurt Stock Exchange, and for \$50 on the **New York Stock Exchange**. The arbitrage is executed by shorting a share at a price of \$51 in Frankfurt, taking the \$51 and investing them in one share of PEP on the N.Y.S.E.. You pocket \$1 today. Whatever you have promised to the Frankfurt buyer will be covered by your ownership of the N.Y.S.E. PEP share. If you can do this with \$1 million of PEP shares, you earn \$20,000 without effort or risk.

Side Note: In the real world, you have to make sure that you can really sell and purchase 20,000 shares for this price. The share price can move when you want to transact a significant amount of shares. You would have to account for the cost of executing this transaction, both in direct financial terms and in what it costs you to set up your own operation to do this. And, there is risk if time passes between the purchase and sale of the shares.

In the real world, small arbitrage opportunities can occur from time to time, but large financial firms are constantly running automated computer trading programs that search for even tiny arbitrage opportunities in order to exploit them as soon as they appear—and thereby make them disappear.

The difference between arbitrage and a good bet.

The concept of arbitrage is usually stronger than the concept of a **good bet**. A good bet would be a chance to win $1,000,000 with 99% probability and to lose $1 with 1% probability. But because there is a chance of losing money in some circumstances, this is "just" a great bet. It is not arbitrage. The difference is important: everyone would want to take advantage of arbitrage opportunities, but someone sufficiently risk-averse may not like a good bet, even if it is an absolutely wonderful bet. Conversely, you might consider a single 1 cent arbitrage to be not a particularly great bet.

There are probably neither great bets nor arbitrage in very competitive financial markets.

Still, unless financial markets were exceedingly strange, we would not expect to find either arbitrage opportunities or great bets. If you agree with this assessment—basically that money does not grow on trees—we can draw some surprisingly strong conclusions about how financial markets work. If you disagree, you should not be sitting in class, but somewhere on a beach, ranking among the richest people in the world. There is little this book can teach you.

Solve Now!

Q 19.1 *Explain when and why you would prefer a good bet to an arbitrage opportunity.*

19·2. Market Efficiency and Behavioral Finance

Important: *Warning: Market Efficiency is a different concept than Mean-Variance Efficiency. The reuse of the word "efficiency" is unfortunate.*

19·2.A. Basic Definition and Requirements

Market Efficiency means markets use all information.

Formally, financial economists call a market efficient when it uses all available information in its price setting. This means that you cannot use available information to predict future returns better than the market. Unfortunately, this leaves the question vague of where the market wants to set returns. For example, the CAPM might state that the expected rate of return on PepsiCo should be 10% (setting a price of $50 for an expected payoff of $55), but you as an investor could determine when the current price of PepsiCo really offered a rate of return of 20% (an expected payoff of $60). You could now draw one of two conclusions: first, the CAPM is not the correct model, and the market wanted to set the expected rate of return for PepsiCo to be 20%; second, the stock market is not efficient. In a sense, the problem with market efficiency is that it is almost a matter of faith: if you wish to proclaim a belief in market efficiency, and if you then find that "prices are off," you can still always claim that your model for the appropriate expected returns in financial markets was wrong, in order to deny that the market was inefficient.

Useful?

Even though stock market efficiency is a fairly modest claim—at least as long as we remain vague on what the correct model of appropriate expected rates of return is—it is still a surprisingly useful concept. For example, it is pretty safe to say that no model of financial markets is likely to claim that investors can find great bets "+$1 million with 99% probability" and "−$1 with 1% probability." Such an expected return would be way out of line. Even expected rates of return of 100% per year are surely unreasonable for stocks such as PepsiCo. Few people doubt that the stock market is, to such a first

approximation, efficient. Still, there is a large gray zone: we do not know the correct model of expected stock returns well enough to know if the stock market set the price of PepsiCo stock so as to offer an expected rate of return on PepsiCo of 10% a year or 12% a year.

Efficient Markets means that the announcement stock reaction estimates change in NPV.

The fact that stock markets are pretty efficient means that, by-and-large, you can trust financial markets to get asset values about right and to do so immediately. One advantage is that if you hold a portfolio of many large and liquid stocks, you do not have to perform costly due diligence to determine whether stocks are priced relatively fairly. All you need to do is to make sure you are appropriately diversified, which you can probably accomplish by purchasing just a few large index-mimicking portfolios.

ANECDOTE: Trading Places and Citrus Futures
The 1983 hit comedy **Trading Places**, starring Dan Akroyd and Eddie Murphy, centers around the trading of Orange Juice Frozen Concentrate **Futures Contracts** (a security that promises delivery of oranges) on the **New York Futures Exchange**. If it is going to rain or there is a frost, oranges will be scarcer and the futures price will rise. You can learn more about Futures contracts at the website of the **New York Mercantile Exchange** at www.nyce.com.

In a 1984 paper in the American Economic Review, Richard Roll found that these Citrus Futures contracts predict whether the U.S. Weather Service's forecast for Central Florida temperatures is too high or too low. It is a great example how financial markets help aggregate information better than the best non-financial institution. This should not be a surprise. After all, there is a lot of money at stake!

Market Perfection and Market Efficiency.

In an inefficient market, you could find opportunities that earn unusually good rates of returns, but you could not necessarily find arbitrage opportunities. You could also not rely on market prices—they could be inappropriately too high or too low. Still, if a market is perfectly competitive (i.e., if there are gazillions of buyers and sellers), and there are no trading and/or information frictions, it is difficult to believe that you can outsmart such a market. After all, thousands of other traders would try to flock to good bargains and try to avoid bad bargains. Therefore, the question of market efficiency is intimately related to how friction-free markets are: it is easier to believe that markets are not perfectly efficient if information and transaction costs are so high that you cannot expect to go out, study, and become rich.

SIDE NOTE: If it costs nothing to trade stocks, it would be easy for an investor to trade on any information that the market has not yet incorporated in the stock price—and thereby to earn an unusually good expected rate of return. On the other hand, if it is very expensive to trade, the market may not respond to news immediately, and no one can do much about it.

19·2.B. A (New) Classification Of Market Efficiency Beliefs

Financial markets are probably close to efficient.

Nevertheless, almost all financial economists believe in basic market efficiency for large markets and liquid securities. No respectable economist believes that it is easy to get super-rich trading on easily available information. The disagreement is, loosely, about whether stock markets are "99% efficient" or "97% efficient." The area that proposes the latter view is often called **Behavioral Finance**. Still, given that the transaction costs in financial markets are so low, even tiny violations of market efficiency may offer highly profitable investment strategies.

Financial economists can be classified into camps based on their degree of belief in market efficiency: A taxonomy.

True Believer Financial prices always reflect the best estimate of net present value of all future cash flows.

Firm Believer Financial prices sometimes deviate from the appropriate best estimate of future cash flows. If there were no transaction costs, investors might be able to obtain unusually good bets.

Mild Believer Financial prices sometimes deviate from the appropriate best estimate of future cash flows (and the financial price next period). Despite transaction costs, investors are able to obtain unusually good bets, at least sometimes.

Non Believer Financial prices sometimes or even frequently deviate from the appropriate estimate, and thereby allow investors to obtain arbitrage opportunities.

A firm believer need not be a true believer: financial prices may follow random walks, but these random walks in prices could be barely related to the random walk in the values of the true underlying firms. A mild believer need not be a firm believer: transaction costs may be low enough to permit great strategies. A non-believer need not be a mild believer: financial markets may just beg to be exploited.

In this classification of market efficiency, most finance professors are either in the "mild believer" or "firm believer" camp, or somewhere in between. That is, most financial economists believe that deviations from market efficiency are mild and usually of second order importance, but possibly present. The debates between the two more extreme side of these camps, the "rationalists" and "behavioralists," is nothing short of exciting. Virtually no academic is a non believer, and only a very few remain in the true believer camp. Our best estimate is that we are somewhere between the mild and firm believer camps.

I personally am in the firm-to-mild camp. In my view, most investors believe that they have more knowledge and control than they actually have. This is why trading volume in the stock market is so high. Investors believe that they can predict when stocks are going to go up or down. Some pundits like to call this **investor psychology**. However, I also believe that an individual investor is unlikely to be able to find rate of return patterns in the stock market to earn high excess returns. (Some sophisticated funds may be able to earn a few extra basis points per year, but even after decades of academic research to identify better performing funds, academics usually find that about half of all funds outperform and half underperform.) My opinion.

ANECDOTE: The Limits of Arbitrage in the Internet Bubble
Even in cases where it is probable that the market mispriced stocks, such as technology stocks during the famous "Internet bubble" at the turn of the millenium, it was almost impossible for an individual investor to take advantage of the market inefficiency. Trust me, I know. In 1999, I believed *Yahoo!* (YHOO) was worth less than what it was trading for. So, I speculated that its stock would go down. After I had lost more than three times my original investment, I realized that I had to either close my bet or risk personal bankruptcy. So, I terminated my bet, having lost a lot of money. Yes, I would have been right in the end if I had held on longer, but I could not afford taking the risk of personal bankruptcy.n

ANECDOTE: A Conversation with Eugene Fama
I have posted an impromptu email conversation between myself and Eugene Fama (perhaps the most famous finance professor alive and a strong defender of market efficiency) at welch.som.yale.edu/academics/famaconversation.html. This will give you an authentic impression of the ongoing dialogue among Finance professors.

The low signal-to-noise ratio causes the dispute.

Why is this debate so tough to settle? The reason is that the **signal-to-noise ratio** in financial returns is low. A typical stock may have an expected rate of return of 0.05% per trading day (14% per year), but a variance of 2-3% per trading day, which is about 50 times as high. (The signal-to-noise description draws on an analogy from physics, the signal [the expected increase one is looking for] is small compared to the noise [the **Volatility**].) This means it is difficult for a researcher to determine whether a particular trading strategy has earned high returns [a] because it took on risk, and the researcher has just not recognized the risk appropriately; [b] because it had a lucky outcome, which will not repeat; or [c] because the market was/is inefficient. On the plus side, this allows finance professors to continue to write papers to argue one side or the other. Still, most finance professors agree that, when most individuals earn an unusual amount of money in a day or a week, it is more likely due to luck than due to ability.

IMPORTANT: *On a typical day, the typical stock moves up or down by about 10 to 100 times as much as it offers in expected rate of return. Therefore, it is not easy to attribute past observed stock price performance to investor ability or inability.*

SIDE NOTE: Historically, financial academics used to distinguish between weak-form, semi-strong form, and strong-form market efficiency.

- The weak form presumes that financial markets are efficient enough not to allow the use of historical stock price information to earn inappropriately high rates of return. This means that **technical trading** (trading based solely on historical price patterns) would not earn excess returns.
- The semi-strong form presumes that financial markets are efficient enough not to allow the use of any publicly available information to earn inappropriately high rates of return. This means that **fundamental trading** (trading based on price and underlying firm information) would not earn excess returns.
- The strong form presumes that the market incorporates even the most private information held by the deepest insiders in corporations. This means that no trading would earn excess returns.

In this classification of market efficiency, most finance professors believe that large and liquid financial markets (such as the N.Y.S.E., Nasdaq, or the CBOE) are weak-form efficient—and maybe even semi-strong form efficient. However, there is very good evidence against strong form efficiency: insider trading may be illegal, but it works.

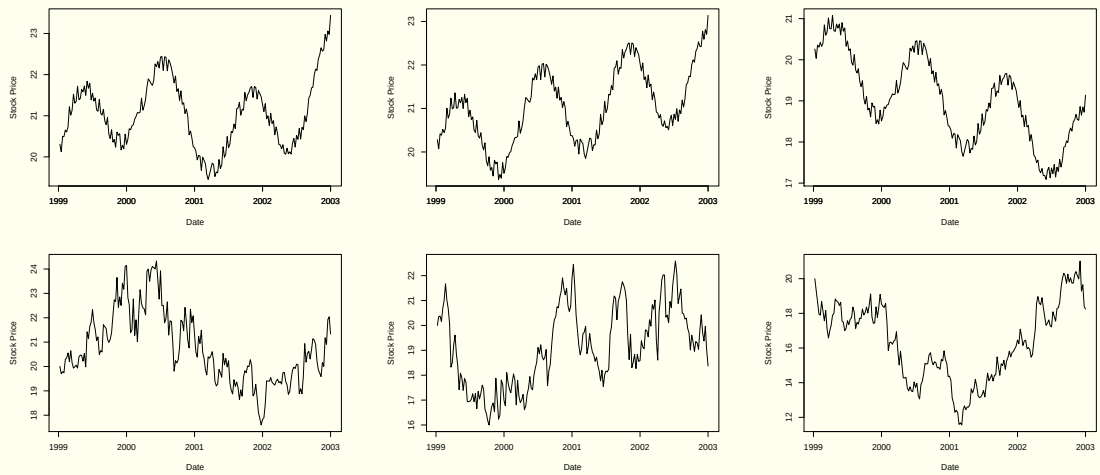
Solve Now!

Q 19.2 *What does it mean for a stock market to be efficient?*

Q 19.3 *Is it more or less likely for a financial market to be efficient when transaction costs are low?*

19·3. Consequence I: Random Walks

Figure 19.1. Potential Stock Price Patterns



If these patterns were systematic, some of them should make you rich. Which ones?

Cycles in the market?

Here are some trick questions: Look at Figure 19.1. Looks like how stock market patterns look like, does it not? Perhaps. Make sense to think that these are representative for the future? Absolutely not! See, the figures in the top row display a strong regular cycling pattern. If this were representative for the future, you will easily become rich! You should purchase the stock only when it has "bottomed out," which you can reasonably detect if you have seen a multi-month period of losses, followed by about a quarter of stable returns. It need not be the kind of cycles I drew here: any good predictable patterns (such as "everytime the price hits $22, it drops by $2") would allow us to get rich. The figures in the bottom row, on the other hand, could be representative. Yes, each rate of return to the next month is just a tiny little higher than the previous (which is because the expected rate of return on stocks is positive); but the important thing is that there is a lot of *noise*, up or down. (Noise is by definition unpredictable.) It must be, or you could outsmart the stock market. In fact, whenever we look stocks in the markets, they have looked very much like the bottom figures and very unlike the top figures.

Can we predict Stock Prices?

Let us look at some. May 31, 2002 was a decidedly uneventful day for the stock market. The Dow Jones rose 13.56 from 9,911.69 to 9,925.25, a change of 0.14%. On this day, the most actively traded stocks (but not biggest price movers) were MCI WorldCom (rate of return of −1%), Nasdaq100 (−2%), Palm (−30%), Sun (0%), and Oracle (+6%). So, let us put our statistical and financial expertise to good use and ask a fundamental question of Finance: *In a perfect market, if the shares of a company cost $50 today, what do we expect them to cost tomorrow?* Could we reasonably have predicted stock price movements of 1%, i.e., the return of MCI?

The order of magnitude of expected stock returns on a typical day. Can the price tomorrow be much higher, on average? No!

What would you do if you knew that this stock price would increase on average by 1% to \$50.^{50} in a day? This may not sound impressive, but this would be the world's most amazing stock to buy. See, 1% per day represents an annual return of

$$1 + r_{0,365} = (1 + r_{0,1})^{365} \approx (1 + 1\%)^{365} = (1 + 3,678.34\%) \quad . \tag{19.1}$$

Instead, in the real world, you can only expect to earn rates between 2% and 30% per year, depending on what risk you are willing to take. After all, any fund manager who can consistently outperform by a "lousy" 200 basis points per year would be considered a star! In sum, you must not be able to predict returns as high as 1%/day, at least without divine guidance or inside information.

Can the price tomorrow be much lower, on average? No!

Still, MCI did not increase but decrease by 1%. But, if the expectation were for shares to trade for \$49.^{50} tomorrow, would you not want to sell the shares for \$50.^{00} today, instead? After one year, with such daily rates of return, you would be left with only 2.5% of your original investment. If you could cheaply short the stock, you could even make money from the 1% decline. After all, by selling shares today at \$50.^{00} and repurchasing them tomorrow for \$49.^{50} (i.e., selling short; see Chapter 13), you could reap a positive 1% return. In any case, every owner would rush to the market to sell, no one would want to buy, and the price would immediately be lower.

So, what can the price tomorrow be, on average?

These arguments suggest that we would expect daily rates of returns averaging somewhere about 0% to 0.07%/day. Intuitively, this is what an efficient stock market is: we do not believe anyone can get rich so easily, so it must be mostly impossible to predict where stocks are going. The best expectation of the price tomorrow must be roughly the price today. Formally,

$$\begin{aligned} \mathcal{E}(\tilde{P}_{\text{tomorrow}}) \approx P_{\text{today}} \quad &\Leftrightarrow \quad \mathcal{E}(\tilde{P}_{\text{tomorrow}}) - P_{\text{today}} \approx 0 \\ \Leftrightarrow \quad \frac{\mathcal{E}(\tilde{P}_{\text{tomorrow}}) - P_{\text{today}}}{P_{\text{today}}} \approx 0 \quad &\Leftrightarrow \quad \mathcal{E}(\tilde{r}_{0,\text{tomorrow}}) \approx 0 \quad . \end{aligned} \tag{19.2}$$

where P is the common notation for "price." This particular process is often called a **Random Walk**. Thus, we now know that, in the absence of easy ways to get rich, stock prices follow a random walk.

IMPORTANT: *A Random Walk is a process in which the expected value tomorrow is the same as the value today. Naturally, actual values tomorrow will most likely be different from the value today. Empirical evidence shows that stock prices tend to follow roughly a random walk. This means that it is not easy to get rich.*

NERD NOTE: *This is actually a slightly inaccurate description. Mathematicians call the process described above a* ***Martingale****, i.e., a process for which the next outcome is expected to be the same as the previous outcome. Martingales are examples of Markov processes, named after the Russian mathematician Andrei Markov (1856-1922).*

In contrast, real random walk processes are allowed to have drifts, e.g., as in

$$\mathcal{E}(\tilde{r}) \;=\; \frac{\mathcal{E}(P_t) - P_{t-1}}{P_{t-1}} \;\approx\; \alpha \quad \Leftrightarrow \quad \mathcal{E}(P_t) \;\approx\; (1+\alpha)\cdot P_{t-1} \;. \tag{19.3}$$

Random walks can thus have a non-zero expected change, and stock prices indeed tend to have a tiny, usually positive, drift. This is because they have to offer compensation for time and risk. This "alpha" is very small, say 0.03%*/day, but it eventually accumulates to offer an investor a reasonable rate of return per annum.*

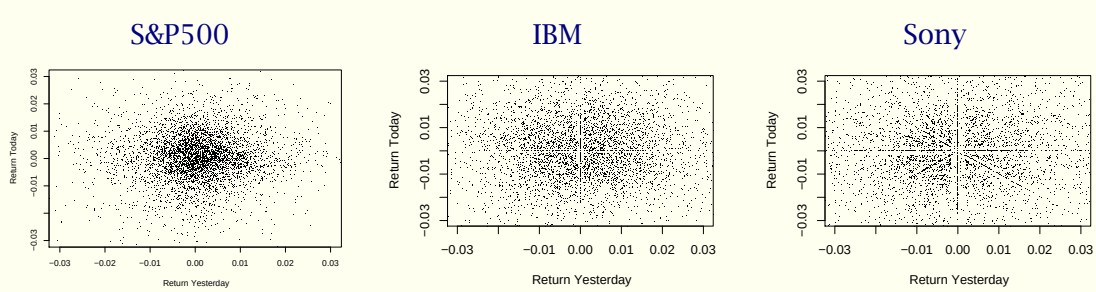
ANECDOTE: **Great Mathematicians and Gambling: The Origin of the Random Walk**
In the 1700s, it was not beneath mathematicians to study how to gamble in order to gamble better. In the 1700s, Jacob Bernoulli (1654–1705) and Abraham DeMoivre (1667–1754) studied the random walk of a gambler's stake in fair games.

Later reinventions and applications of the Random Walk concept abound: Jan Ingenhausz (1730-1799), a physician and plant physiologist, placed charcoal powder on an alcohol film and observed that the grains moved randomly. The botanist Robert Brown (1773-1858) reported erratic dancing of small particles in fluids at rest. Albert Einstein (1879-1955) considered such fluids to be composed of discrete molecules, whose many collisions with a Brown-ian particle caused the particle to jump in random directions—a random walk. Einstein's analysis not only explained **Brownian motion**, but also bolstered the case for the existence of atoms, which was not yet universally accepted. The first recorded use of the phrase "random walk" was by Lord Raleigh (1842-1919) in 1899. (Raleigh made a connection between diffusive heat flow and random scattering and showed that a one-dimensional random walk could provide an approximate solution to a parabolic differential equation.) The name is believed to have originated with the description of a drunk who stands on a ladder. The drunk can walk up or down and does so in a random fashion—just like stocks.

Fortunately, in 1900, Louis Bachelier introduced the random walk theory of financial market fluctuations (although Pearson introduced the term "random walk" only later, in 1905), finding that bond prices could diffuse in the same manner as heat. Unfortunately, this has only pointed out the obvious: it is not easy for an investor to outperform the market. The first rigorous and published investigation of the random walk hypothesis were done by Cowles, an eclectic investor and economist at Yale in the 1930s and 1940s.

Source: Mostly Michael F. Schlesinger, Office of Naval Research, Scienceweek.com, 2001.

Figure 19.2. The Relation between Lagged and Current Rates of Return for S&P500, IBM, and Sony.



Note: The figures chop off some outliers, especially the crash of 1987 and mini-crash of 1989, but even if they are included, there is no apparent predictability.

Predicting with past rates of return.

Traders have tried all sorts of strategies, trying to become rich. For example, one popular misperception is that stocks that rose yesterday are more likely to fall back today (or continue to rise). Figure 19.2 shows how the daily rate of return on our three stocks (S&P500, IBM, and Sony) related to the previous day's rate of return from 1985 to 2003. The graphs show no pattern that would allow you to get rich quickly. There is definitely not much juice in trying to predict how a stock will perform tomorrow, given how it performed today (or yesterday, or any earlier day). Similar conclusions apply if we extended our use of historical price information beyond yesterday.

It is also unlikely that one stock is expected to go up today, and another stock to go up tomorrow, and so on.

Another variation on attempts to find market efficiency violations tries to predict not which stocks systematically go up every day, but just for the next day: maybe it is possible to predict that one stock should go up today, and another tomorrow. But, if you reflect on this statement, you realize that you could shift your money from one stock to another to take advantage of different stocks on different days. Again, unless the expected daily returns are tiny, it would be too easy to get rich. And, again, with average rates of return being tiny compared to the ups and downs, a good stock market pick is more likely to come from a lucky or unlucky draw than from a systematic ability.

Transaction costs destroy most hope for high turnover strategies.

Of course, in the real-world, there are financial transaction costs that would also prevent you from really earning the $0.50 (1%). You would have to pay money to the broker to buy the shares, and again to sell shares. (This is why financial markets are not exactly perfectly competitive, only approximately perfectly competitive.) Even small transaction costs can render trading strategies with very high turnover unprofitable. After all, even if the bid-ask spread is only 10 basis points, if incurred 255 trading days a year, you would only be left with $(1-0.1\%)^{255} = 77.5\%$ of your original investment. So, for a daily trading strategy to earn money, it needs to earn at least an annual rate of return of 25% before it can overcome the trading frictions.

ANECDOTE: Women are better Investors than Men

Analyzing 35,000 households from 1991 to 1997, Terry Odean and Brad Barber found that men trade 45 percent more than women. Apparently, men are too overconfident in their trading prowess: On average, men's investment rates of returns are lower than women's, by a little less than one percent per year. This can be almost entirely attributed to the higher transaction costs that men incur for transactions which do not gain them higher returns.

Despite strong evidence to the contrary, many investors still believe that stock prices do not follow random walks, as evidenced by the plethora of financial talk shows and investment news letters. It would be better for audiences to watch more sports and cooking shows and fewer investment shows—especially for men!

Solve Now!

Q 19.4 *From memory, write down the formula for a random walk.*

Q 19.5 *If stocks follow a random walk, can the price tomorrow be different from the price today?*

Q 19.6 *What is the typical movement of a stock on an average day?*

Q 19.7 *What is the typical expected rate of return on a stock on an average day?*

19·4. Consequence II: Monkeys and Typewriters

So, what about all the stock analysts on **television** shows that explain which stocks are undervalued and which stocks are overvalued? And what about **Technical Analysis**, the art of seeing shoulders, price barriers, etc., in historical prices and using them to forecast future oprices? And what about famous investors, such as **Warren Buffett**, **George Soros**, and many others?

What about celebrity investors?

First, realize that even an ability to forecast by 0.01% (yes, 0.0001) better per day yields an annual rate of return that is 3.7% higher than it would otherwise be. 370 basis points excess return per year by a fund would be widely considered indeed stellar performance. And even if the stock market is not a perfect random walk, we would still conclude that by-and-large, it is a random walk. We are talking about tiny deviations from the random walk.

Even top investors can have at most mild predictive ability. None can be expected to have even the ability to foresee systematically a 0.1% movement per day.

Second, there are about 10,000 **Mutual Funds** today, which invest money on their investors' behalf. How many of them are likely to outperform the stock market overall next year (at least before they collect fees)? If they have no ability, about 5,000. How many of these will outperform the year thereafter? About 2,500. Even if there is absolutely no ability, pure randomness will mean that about 10 funds will outperform the market every year for ten years in a row. What will happen to the funds that underperformed several years in a row? They disappear quietly. What will happen to the funds that outperformed several years in a row? They proudly announce their performances, advertise, boast, and collect more investments from outside investors. They are even better dressed, supported by larger "research teams," appear more "professional," and fly in executive jets. They are the ones that will be most visible. Indeed, if you made money ten years in a row in the stock market, would you not believe yourself that you have the ability to pick stocks?

And, pure chance means some investors succeed many years in a row.

In truth, maybe there are some individuals that can pick stocks, but the evidence suggests that luck is far, far more important than ability. But whenever academics (or the Wall Street Journal) have searched for better forward-looking performance among professional fund managers who outperformed in the past, they have found little or no exceptional performance. If managers were truly capable of systematically earning better rates of returns by picking stocks, you would expect those managers who have picked better in the past to also pick better in the future. The evidence here is mixed: about 54% of mutual funds that have outperformed their benchmark over the last 1–3 years tend to outperform their benchmark over the following 1–3 years. (This is better than 50%, but not by much.) But, as fund prospectuses aptly note, and as the evidence suggests, for the most part, **Past Performance is no Predictor of Future Performance**.

If you look for future performance, past performance may be your best guide, even if it is a very, very poor guide.

Important: *In an efficient market, in which no one can pick stocks better than anybody else, a large number of investors will beat the market. A small number of investors will beat the market again and again. In the real world, there is little evidence that investors who did well picking stocks in the past are better picking stocks in the future when compared to investors who did poorly.*

Old-fashioned work and liquidity provision works better than stock picking.

There are of course other ways to make money: Warren Buffett's fund, **Berkshire-Hathaway**, for example, runs an insurance and aircraft business. These businesses make money. But, it is money earned the old fashioned way: through hard work and risk taking. So, next time someone has a great stock tip, regard it with some skepticism: it probably has a fifty-fifty chance of being right.

SIDE NOTE: Here is a great business idea. I give you stock tips, and I only ask for money if you make money. In fact, I only want 10 percent of your winnings. "So you have nothing to lose." I only get something if I help you make money.

Now, if I pick a stock randomly, I have a fifty-fifty chance of making money. If you gain, I get something. If you lose, I pay nothing. In a sense, this is how many mutual and hedge funds operate: fund managers participate in the upside, but not the downside.

ANECDOTE: **The Three Top Investment Books of 1996**
The three best-selling investment books of spring 1996 were David and Tom Gardner's **Motley Fool Investment Guide**, based on a popular investment web site; Matt Seto's **The Whiz Kid of Wall Street's Investment Guide**, Seto being 17 years of age then; and the **Beardstown Ladies' Common-Sense Investment Guide**, septuagenarians whose first book mixed cooking recipés with investment advice. All touted "common sense methods" to beat the market, earning 30 percent per year or more. Not a week went by without dozens of prominent radio and TV shows featuring their sound advice. What did I need my Ph.D. in finance for? It is difficult to argue with performance!

Naturally, best-selling books are a great business. However, the stock performance of these three experts was not.

1. From 1996 to 2002, the *Motley Fool* investment recommendations of a number of hypothetical portfolios have been discontinued. In 1997, they launched a real-money portfolio, called DRIP. From 7/28/1997 to 7/31/2002, it lost about 10%, while the S&P500 lost 2.5% and Nasdaq lost 15%. One should not judge a fund by just 5 years of performance (and certainly not without risk adjustment), but it does appear that the Motley Fool has not exactly found the Holy Grail of investment opportunities.
2. Matt Seto has stopped publishing his investment performance and has decided to pursue a career as a student.
3. And the Beardstown Ladies, five books richer, were found to have miscalculated their returns: their returns were not 30 percent, but 9 percent—significantly lower than the 15 percent turned in by the S&P500 stock market index during their investment period.

How disappointing: I would have thought that about one of them would have continued beating the market, one would have done about the same as the market, and one would have underperformed it. Now, where are my five minutes of fame?

Solve Now!

Q 19.8 *If a firm employees 10,000 analysts, how many of them are likely to issue forecasts that beat the market ten years in a row if none of them has any ability and there are no transaction costs?*

19·5. Consequence III: Event Studies

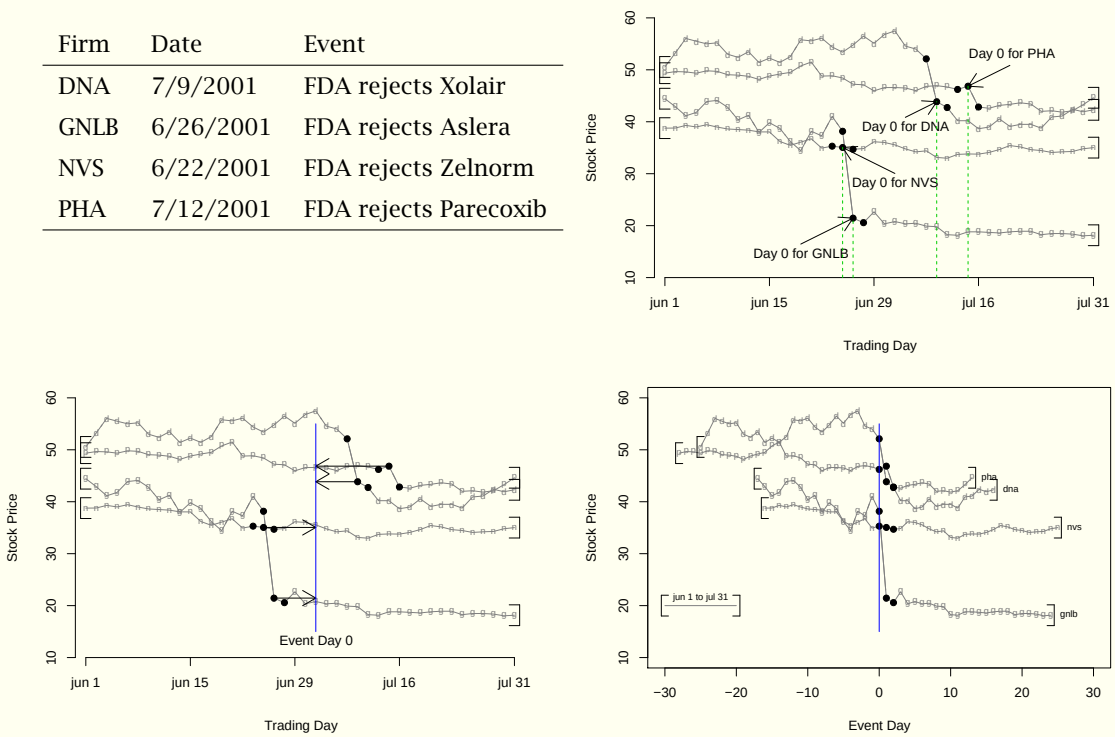
19·5.A. An Exogenous Event: The Value Impact of FDA Drug Rejections

An Example: Estimating the value loss when the FDA rejects a drug application.

The immediacy of reaction in an efficient market has a surprising advantage: price reactions can help to estimate value consequences, using a technique called **Event Study**. For example, say you wanted to know what the value implication of the rejection of a novel drug application by the **Food and Drug Administration** (**F.D.A.**) is. You could compute the sudden decline in future expected cash flows, discount it properly, and come up with an appropriate valuation estimate. This is not an easy task. But, if stock markets value pharmaceutical stocks appropriately, the stock price reaction to the announcement of the F.D.A. drug application rejection would be a good indicator of the value loss. After all, if the stock market did not react *immediately*, you could on average make enormous profits by shorting the relevant stock.

Figure 19.3. Event Study: Lining up Event Date Information

Firm	Date	Event
DNA	7/9/2001	FDA rejects Xolair
GNLB	6/26/2001	FDA rejects Aslera
NVS	6/22/2001	FDA rejects Zelnorm
PHA	7/12/2001	FDA rejects Parecoxib



(GNLB's stock price was scaled by a factor of 11 to fit better into this graph.)

Figure 19.3 starts with 4 events that we have identified: the FDA rejected Xolair, a Genentech (ticker DNA) drug, on July 9, 2001; Aslera, a Genelabs (ticker GNLB) drug, on June 26; Zelnorm, a Novartis (NVS) drug, on June 22; and Parecoxib, a Pharmacia (PHA) drug on July 12. The right top figure plots the price history for these 4 stocks during June and July 2001. The event day itself is marked in this graph. To find out what happens when the FDA rejects a drug, we need to line up all the returns in event time, as illustrated in the bottom two figures. When we compute the rate of return over the three days around our event date (for we are not sure how accurate our identification of

the announcement day is, and whether announcements happen before opening or after closing of the stock market), we find that

Firm	Date	Event	"Couple of Days" Return
DNA	7/9/2001	FDA rejects Xolair	dropped about 18%
GNLB	6/26/2001	FDA rejects Aslera	dropped about 46%
NVS	6/22/2001	FDA rejects Zelnorm	staid put
PHA	7/12/2001	FDA rejects Parecoxib	dropped about 7%

Having lined up everything in event time, we can do more analysis. For example, we can subtract the rate of return on the market on each event date to eliminate noise induced by the general movement on the event date. We can compute the average rate of return on the event, which seems to be around –18%—getting rejected by the FDA is not a good thing for a firm's market value. If you know more statistics, you can compute if this value drop is "statistically significant." (It is.) You can investigate if bigger firms experience a bigger or a smaller drop. For this, you need to locate the market value at the time:

Firm	Event Return	Equity Market Value
GNLB	−46%	\$0.1 billion
NVS	±0%	\$2.7 billion
DNA	−18%	\$26 billion
PHA	−7%	\$58 billion

Therefore, the evidence suggests that smaller firms are harder hit, but that the relationship is not necessarily perfectly monotonic. It would be even more interesting if we knew what fraction of the drug development portfolio the particular rejected drug would be—we could then determine whether drug developers whose main portfolio drug was rejected suffer more. We do not have this data, so no such test! We could test whether it has become worse or better to have one's drug rejected by the FDA by sorting our events by the event day.

Firm	Event Return	Event Day
NVS	±0%	6/22/2001
GNLB	−46%	6/26/2001
DNA	−18%	7/9/2001
PHA	−7%	7/12/2001

There does not seem to be a clear relationship here—as sane analysts would have suspected—so the evidence does not suggest that the market looked any more favorably or less favorably upon rejections in June 2001 relative to July 2001. We could investigate whether information seems to have leaked prior to the FDA announcement: if so, we should have seen a drop even before event day 0. Just eyeballing the bottom bottom right graph in Figure 19.3, it appears as if very little information leaked over the month prior to the announcement. (An alternative reason for such a pattern could occur if the FDA were more likely to reject a drug if the stock price had recently gone down. There is no such evidence, either.) We could investigate whether we can earn profits buying or selling after the event—under market efficiency, this should not be the case. Just looking at the bottom right graph in Figure 19.3, this appears not to be the case.

NERD NOTE: *If we had more statistical background, instead of just sorting our events as we did in the in-text tables above, we should run a regression to predict the announcement rate of return with variables we deem to be important determinants of the value change upon FDA rejections.*

Table 19.1. Sample Event Study of FDA Drug Rejections

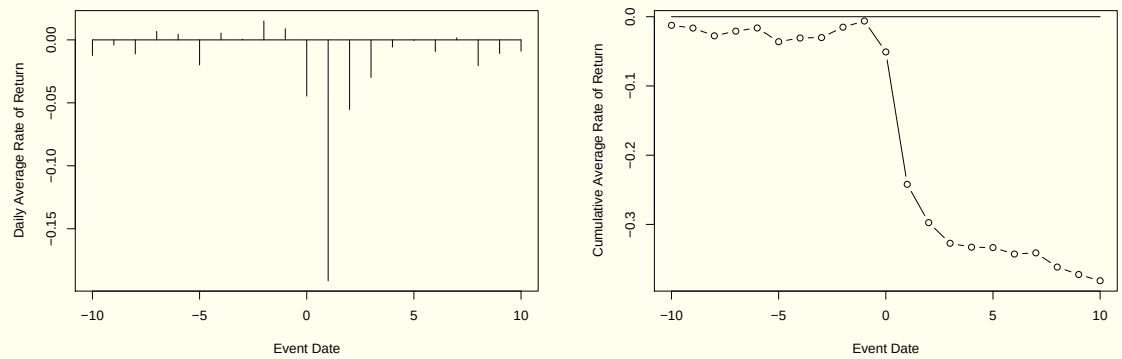
	Event		Announcement	Rate of Return		
Symbol	Company	Drug	Day-Day	Stock	Market	Net
AVE	Aventis	Refludan	5/17/00-5/19/00	2.87	−2.82	**+5.69**
AVN	Avanir	Docosanol	12/22/98-12/24/98	−65.56	0.19	**−65.75**
CEPH	Cephalon	Myotrophin	5/7/97-5/9/97	−35.00	1.12	**−36.12**
CRXA	Corixa	Bexxar	3/12/02-3/14/02	−38.32	1.08	**−39.40**
DNA	Genentech	Xolair	7/9/01-7/11/01	−17.96	1.55	**−19.51**
FRX	Forest Labs	Synapton	11/17/98-11/19/98	−3.79	1.17	**−4.96**
GILD	Gilead Sciences	Adefovir	11/01/99-11/3/99	−21.08	0.01	**−21.09**
GNLB	Genelabs	Aslera	6/26/01-6/28/01	−46.11	0.78	**−46.89**
GSK	Glaxo-Smith-Kline	Augmentin	12/14/01-12/18/01	−0.15	1.77	**−1.92**
IMCL	imClone	Erbitux	12/27/01-12/31/01	−20.31	0.08	**−20.39**
JNJ	Johnson& Johnson	Risperdal Consta	6/28/02-7/2/02	−1.24	4.20	**−5.44**
MAXM	Maxim	Maxamine	12/12/00-12/14/00	−44.78	2.20	**−46.98**
NVS	Novartis	Zelnorm	6/22/01-6/26/01	0.35	−0.70	**+1.05**
PFE	Pfizer	Zeldox	6/18/98-6/22/98	−2.51	0.29	**−2.80**
PHA	Pharmacia	Parecoxib	7/12/01-7/16/01	−7.32	0.47	**−7.79**
PRCS	Praecis	Plenaxis	6/11/01-6/13/01	−31.74	1.02	**−32.76**
SCIO	Scios	Natrecor	4/27/99-4/29/99	−57.67	1.47	**−59.14**
SEPR	Sepracor	Soltara	3/6/02-3/8/02	−58.73	0.13	**−58.86**
VPHM	ViroPharma	Picovir	5/9/02-5/13/02	−28.94	0.14	**−29.08**
WPI	Watson	Estradiol/ Progestin	10/30/00-11/1/00	−5.52	1.61	**−7.13**
					Average	**−24.96%**
					Standard Deviation	**±22.31%**

To eliminate noise, average many events from many firms.

Unfortunately, as we already know, individual stocks do move around day by day. So, four firms may not be representative enough, because stock returns are noisy. Instead, we should average the event returns for many FDA rejections, hoping that the noise averages away and the signal becomes more apparent. (Having more firms is like tuning a radio station with a more powerful antenna.) Table 19.1 presents the same event study—to estimate the value consequences of FDA rejections—for 20 pharmaceutical firms. It is usually a good idea to reduce the influence of overall market returns, so we subtract the overall stock market return on the same day from each firm's stock return. After all, we do not want to attribute the fact that the stock market declined by, e.g., 1% on the announcement day, to the drug announcement. (You also have to hope that there are no other major corporate events that *always* occur on the same day as the FDA drug application rejection.) The event study suggests that when firms announce

that their drug applications are rejected by the FDA, they lose on average about 24%, plus or minus 22%.

Figure 19.4. Average Returns and Cumulative Average Returns in Event Time for 20 FDA Rejections



Event Study Graphs—perhaps we can make money?!

Whereas Table 19.1 gives only the average rate of return on the three days of the announcement at the bottom of the table, Figure 19.4 gives the average rate of return on each event day. The left graph indicates that we most likely misidentified the timing of the announcement day: if the announcement happened after market closing, then we only see a market reaction on day +1. On days +2 and +3, however, there appears to be a slight market imperfection: the stocks seem to go down further, by a total of about 5%—if this pattern is systematic, it is not only a violation of market efficiency, but it would be enough to earn us a lot of money! The right graph plots the cumulative return—it adds the return to a running sum. It shows that not much happens before the announcement, so there is no evidence of widespread news leakage or insider trading prior to our event, but there is some negative trend post announcement. It appears as if the market "underreacted." This would be worth further investigation, e.g., to see if it holds up in bigger samples and if it is more pronounced for certain, identifiable firms—except that I have told both you and the world about it now. If slow market reaction was a real effect, you can be almost sure that it will now disappear, as traders try to exploit it.

Event study are not a panacea.

Event studies are not without drawbacks. First, they work well only if the event is significant enough to significantly influence the overall stock market valuation: if a \$1 billion stock fluctuates on average by \$10 million a day, it is impossible to use an event study to determine the value of a project worth \$100,000. Using a physics analogy, the noise would drown out the signal. A reasonable rule of thumb is to take the ratio of the typical daily stock market value fluctuation (here, \$10 million) divided by the order of magnitude of the value consequence over (here, \$100,000, so the ratio is $\$10,000,000/\$100,000 = 100$), and then require 50 times as many event observations as this ratio. For the example, this would require 5,000 event observations—too many to make this study feasible. Second, event studies rely on the fact that stock markets react only to news, i.e., the unanticipated component of an information release. For example, if a company was expected with 80% probability to win a contract worth \$1,000,000, the stock price would have already reflected \$800,000. The news that the company actually won the contract would raise the stock price only by \$200,000, not by \$1,000,000. The

news that the company would not have won the contract would drop the stock price by $800,000, however. Isolating market expectations can be very difficult. More than likely, the analyst would not know after-the-fact how expected the event was by the market at-the-time. (And insider trading before the event may have already moved the stock price to the $1,000,000 before the public announcement.) Therefore, in many cases, the event study technique is better at helping to determine whether an event is good or bad for a company (e.g., the announcement of a new law), than it is in helping to compute an exact value gain. Third, the event study technique relies on the fact that the event can be precisely isolated in time. If other events occur in the same time window, any value consequence may be from these other events, not from the event that is examined. Unfortunately, many value consequences occur at the same time, e.g., at the annual meetings.

19·5.B. Endogenous Regular Reports: Earnings Announcement Responses

T: ***TO BE WRITTEN.***

How does the market respond to earnings? Yes, if earnings are reported to be surprisingly above expectations, then we would expect the stock price to increase; if they are reported to be surprisingly below expectations, then we would expect the stock price to decrease. But by how much? Should an extra dollar in earnings per share correspond to an extra dollar in share price? It depends—if management is likely to waste shareholders' money, the stock price may not even budge; if management is likely to return and/or wisely invest the money, it would go up. Perhaps more importantly, does the extra dollar signal more unexpected extra dollars in future quarters—i.e., we underestimated the firm—or is the earnings increase temporary? And, what part of earnings matter? Does the market respond to all of earnings, or is it just operating cash flows, or is it just pure cash? And, are there any patterns in how the market responds to earnings? For example, does the market—if it is inefficient—underreact or overreact to unexpected earnings? If so, how quickly does it correct itself, and can you get rich exploiting it? These are all fascinating questions, and there have literally been many books and hundreds of articles written on each of these questions.

19·5.C. Short Preview: Other Events

Event studies are a very powerful tool to measure the value effects of many changes. The most important criterion is that we can identify a clear announcement release date, so that the information has not leaked out earlier. Whether the stock market follows the CAPM or not is fairly irrelevant on a 1–3 day event, because the average CAPM return is around 5 basis points for a stock per day and easily dwarfed by daily noise.

Event studies work even if the CAPM does not.

There have been event studies on all sorts of events, ranging from new legislation, to corporate name changes, to analysts' opinions, to corporate earnings, to stock splits, to corporate dividends, to corporate debt and equity issuance and retirement, to deaths of the founder, etc.

Event studies have been used on many different events. We will rely on event studies in later sections.

We shall take a closer look at the capital structure event study evidence (dividends, and debt and equity issuing effects) in Chapter ??.

ANECDOTE: A Rote Finance Ph.D.?
In the late 1960s, a main paradigm shift in finance occurred: market efficiency became the accepted wisdom. Shortly thereafter, in 1968 and 1969, two papers appeared that practically invented the event study technique (Ball and Brown, 1968; and Fama, Fisher, Jensen, and Roll, 1969) as we know it today. These papers were so successful, simple, and informative, that in the late 1970s and early 1980s, finance and accounting academics became outright crazed in wanting to run event studies. They had a great hammer and were looking for nails everywhere. And there were literally hundreds of nails to be hammered.

At this time, it was relatively easy to get a Ph.D. in finance or accounting, and to publish in a top journal. One had to identify some new event with enough observations, and voi-la—a Ph.D. (Some additional work may have been required.) Running an event studies required very little thinking. It was the closest thing to a "safe Ph.D." that academic finance and accounting ever had to offer.

This is not to say that many of these event studies were uninteresting—just that it had become "too easy" to execute one. (Incidentally, when a founder dies, the stock price usually goes up, not down!) The technique was too powerful for its own good. Nowadays, the pendulum has swung back—and too far: event studies often elicit an automatic disdain from "serious academics" (replace serious with snobby, if you wish) even though they remain one of the most powerful empirical techniques in finance.

SIDE NOTE: The event study technique is very powerful with daily stock returns, at least as long as the signal (the average event day reaction) has a value consequence of at least 1% and better 3% to 5% and there are more than just a couple of dozen observations. Using this technique, we now know that values increase on average, e.g., • when firms announce increases in dividends and share repurchases, stock splits, • when firms being taken over by another firm, and • when the founding CEO dies. And values decrease, e.g., • when firms announce new stock sales, • when they overpay for other firms in acquisitions, • when they announce bad earnings, and • when they fend off an acquiror having made a bid. And values do not change, e.g., when firms divested from South Africa during the **South African boycott** (it was essentially a sideshow).

Certain legislation can systematically have a positive or negative impact on firms, and this value impact can be measured. For example, we can determine which firms were helped and which firms were hurt when telecommunication and airline markets were deregulated.

Solve Now!

Q 19.9 *Which of the following are good candidates for ascertaining the value effects with an event study, and why:*

1. *An acquiror wants to buy the firm.*
2. *The CEO dies.*
3. *The CEO ages.*
4. *Positive earnings surprise at the annual meetings.*
5. *Purchase of a new machine.*
6. *A law is passed to force the company to reduce its emissions.*
7. *An ad campaign.*

Q 19.10 *What kind of response ("unusual" stock price change and "unusual" rate of return) would you expect when the company announces that it has struck oil and plans to pay it out next month? What reaction do you expect over this month? What reaction do you expect on the day when it pays the dividends?*

Q 19.11 *What are the factors that make an event study more likely to be more or less informative?*

19·6. Summary

The chapter covered the following major points:

- Arbitrage is a riskless bet with no negative outflows under any circumstances. In contrast, a great bet can be risky. (A great bet can be profitable to a small arbitrage opportunity.)
- Market efficiency simply means that the stock market uses all available information in setting the stock price to offer the fair rate of return.
- It is rarely clear what this "fair rate of return" is. Noise in the stock market makes it difficult to measure the fair rate of return, which in turn makes testing for or against market efficiency a slippery exercise.
- Market efficiency prescribes that stocks roughly follow random walks in the short run.
- Beliefs in efficient markets come in different flavors.
- The overall evidence suggests that it is not easy to become rich—a belief shared by most finance professors. The strength of their beliefs in market efficiency—the extent to which professors believe that market prices always reflect underlying value—separates finance professors into "rationalists" and "behavioralists."
- Market efficiency does not mean that there are not some investors that will beat the stock market ten years in a row *ex-post*; only that any one investor is unlikely to beat the stock market *ex-ante* ten years in a row.
- Event studies allow us to ascertain the corporate value impact of sharp events, such as legislative action (FDA rulings) or corporate events (such as dividend increases).
- The event-study evidence suggests that firm value increases *on average* when firms pay out more (dividends, repurchases), and decreases when firms raise more capital (equity issuances)—but there are many firms and events for which this is not the case.

A. Q&A: Answers

1. If the arbitrage opportunity can only be done once and gains \$10, it is probably worse than a good bet that loses 1 cent one percent of the time, and gains \$1,000,000 ninety-nice percent of the time.

2. The market uses all available information in the setting of its price.
3. More likely.

4. See Formula 19.2.
5. The price can definitely and most likely will be different. Only the *expected* price is the same as the price today.
6. The typical movement (variation) is around plus or minus 1% to 3% a day. The average rate of return is much lower.
7. If a stock has an expected rate of return of 20% per year—which is definitely on the high side for most firms—the daily rate of return would be

$$(1 + 20\%)^{1/365} - 1 \approx 0.05\%.$$

8. If each of them has a chance of 50-50 in any given year, then the answer is $10,000/2^{10} \approx 10$.

9.
   (a) An acquiror wants to buy the firm: Super. Usually unannounced and big event.
   (b) The CEO dies: Maybe. Depends on suddenness (anticipation) and replace-ability of CEO.
   (c) The CEO ages: Bad. No sudden information release. Value effect not big enough.
   (d) Positive earnings surprise at the annual meetings: Maybe. Problem is many other things may happen at the same time.
   (e) Purchase of a new machine: Probably Bad. Problem is that one machine is usually too small to make a big value difference.
   (f) A law is passed to force the company to reduce its emissions: Maybe. The value consequences could be large enough, but by the time the law passes, it has long since been anticipated.
   (g) An ad campaign: Bad. First, there is no unique date on which to pin down the information release, and the value effects are often not too overwhelming, either.

10. The share price response would immediately be positive. Over the following month, you would not expect any unusual upward or downward drift: it should be about zero. Finally, when the firm pays out the special dividend, the rate of return should be zero on average, too, although its share price will have to drop by the amount of dividend paid to keep the return around zero.
11. The effect should be big, unanticipated, and there should be many other companies that have already experienced similar events in the past.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

Part IV

Financing Choices
Capital Structure

"It's not begging if you've been to college. It's attempting to arrange interim financing while restructuring your debt load."

Transition

We are still plagued by one problem: Although we know how firms should value projects, and although we know the determinants of our cost of capital, we do not yet know how we can best get new investors to part with their cash. We just assumed that if our firm had the right project (with a positive NPV), then the funds would come by themselves.

In the real world, this may not be the case, but our firm does have many venues for raising funds. We now discuss both the types of securities that our firm can sell to potential investors and the selling process itself. Although this topic is less mathematical, the issues themselves are no less important. What use is a project if it cannot be financed?

Of course, when we personally need cash, it is also good to know what the pluses and minuses of different financing options are.

What we want to Accomplish in this Part

The goal of this part of the book is to teach how firms should finance projects with debt and equity, and how this influences the firm's cost of capital.

- Chapter 20 explains how you should think of securities that firms sell (issue), and how these securities are sold into the financial markets.

  Typical questions: What rights do stock investors have, e.g., if the underlying firm goes bankrupt? What is preferred stock?

- Chapter 21 shows that *if everything is perfect*, then the value of the firm is the value of its underlying assets and not dependent on whether it is financed with debt or equity.

  Typical questions: How does the value and expected rate of return of the firm's equities change if the firm issues a large bond in order to buy back stock?

- Chapter 22 shows how firms should make capital structure and capital budgeting decisions if they have to pay corporate income taxes.

  Typical questions: What is the firm's cost of capital and value if it finances itself with 50% debt and 50% owner's equity, instead of with 100% owner's equity.

- Chapter 23 shows how firms should make capital structure decisions if both they and their owners have to pay corporate income taxes.

  Typical questions: Should a fast growing firm finance itself with more or less debt than a profitable value firm?

- Chapter 24 explains other considerations that can have important influences on capital structure.

  Typical questions: If investors are concerned that managers like to raise funds primarily when they know that their current projects are not very good, then what can managers do to calm investor fears in order to be able to raise more money?

- Chapter 25 focuses in more detail on corporate governance.

  Typical questions: If investors are concerned that managers like to steal or waste money, what can managers do (and what do they do) to calm investor fears?

CHAPTER 20

Financial Securities and Investment Banking

What Can A Company Sell To Get Money?

last file change: Jun 24, 2004 (20:37h)

last major edit: n/a

After the firm has determined what projects to take, the question is how to finance them. This is called **Financing**, **Capital Structure**, or simply **Investment Banking**. (The difference between **I-banking** and ordinary banking is that the I-banker handles corporations, not individuals.)

In principle, firms have two basic choices: They can accept new partners, which is called issuing equity (stock). Or they can borrow money, which is called issuing leverage (bonds). Most other securities are hybrids of these two basic choices.

This chapter describes some of the choices that corporations have, and how to think of them. The next chapters will explain how to choose among them.

20·1. The Basic Building Blocks

Side Note: The term **Security** originally meant a financial instrument (piece of paper promising payoffs) that is registered with the **Securities Exchange Commission** (**S.E.C.**). However, the term is now used much more liberally, e.g., for foreign securities and privately placed securities, which really are not securities in the original sense of the term.

Bonds and Stocks are the basic building blocks. Homes and Corporations are really the same.

The two basic building blocks with which corporations are financing themselves, i.e. raising external money, are **Bonds** (**Leverage**) and **Stocks** (**Equity**). In reading this chapter, you will be surprised that you already know most of what you need to know. Firms issuing bonds and stocks are really no different from house owners borrowing with mortgages and keeping the rest (levered equity), as described in Chapter 6.

Firms can choose any security features they wish.

Over time, many variations and hybrids of the two basic claims have developed. The features of securities are not written in stone. Firms can and do experiment. Naturally, if a security offers more features or protections that are of value to investors, then these investors will be willing to pay more for the security upfront. In an efficient market, both companies and investors get the appropriate fair shake (price), regardless of the features chosen by corporations offering securities for sale. The securities features described in this chapter are among those that have survived, evolved, and thrived over the years. Of course, corporations could issue securities that do not maximize firm value, even if they are fairly priced. For example, a security might offer its owner the right (or obligation!) to become CFO if it were to rain in Los Angeles next April 21. This security would fetch an appropriate price, but it would significantly lower the value of the firm.

Owning the firm really requires owning all securities without further obligations.

By definition, you own the firm only if you own all **claims** (**securities**) that a firm has issued. It is not enough for you to own only all of the stock. Think about the levered equity homeowner: she does not really own the house, because she cannot dispose of the house and keep the money without repaying the mortgage. It is she plus the mortgage company that together own the house. Similarly, the levered equity holders in a corporation do not fully own it. The value of the firm is by definition the sum-total value of all its outstanding securities.

20·1.A. Bonds

Bonds are Loans, with specified obligations by the company to its creditors.

Chapter 2 introduced bonds, which are just loans that promise specific payoffs at specific times in the future. The borrower (or issuer) receives cash upfront and contractually promises to return cash (usually with interest) in the future. The returned cash can be classified into interest payments and repayment of principal. Corporate interest payments typically occur every 3 or 6 months and are tax-deductible by the corporation. At **Maturity**, corporate bonds usually pay back the principal.

Bondholders have no control rights, unless the firm goes bust.

Bondholders do not usually have the right to participate in the selection of management. *But*, if the firm misses a payment, the bondholders have the right to force a bankruptcy proceeding, and presumably the right to force a liquidation to receive what they were contractually promised. In theory this allows bondholders to take over and then fully own the company. The **Absolute Priority Rule**, commonly abbreviated **APR**, means that bondholders first receive what they were promised, before more junior claimants (such as equity) receive anything. But in practice, forcing liquidation is not too easy. Still, few managers survive bankruptcy, so they generally try to avoid missing bond payments.

ANECDOTE: Judge Lifland and Eastern Airlines' Creditors
APR, at least, is the theory. In practice, bankruptcy courts can and often do violate the pre-agreed priority rules in the bankruptcy process. In turn, because corporate managers can choose where to file for bankruptcy, they usually do file in the court where they expect to fare best.

The **Southern District of New York** Bankruptcy Judge Burton Lifland was so notorious for violating creditors' rights, that he attracted not only **Eastern Airlines'** bankruptcy, but also those of **Manville**, **Orion Pictures**, and **LTV**. But it was Eastern Airlines that was Judge Lifland's crowning achievement: When it went bankrupt in March 1989, it was fully solvent. Unsecured creditors would have likely been satisfied in full. Instead, Judge Lifland allowed Eastern to continue operating for two more years, partially on the basis that closing it would have disrupted Christmas travel. Eastern's ongoing operation evaporated about $1.5 billion through operating losses and another $100 million through legal fees. In the end, unsecured creditors received practically nothing of their $2.3 billion claim.

Despite frequent modest APR violations and despite such occasional spectacular examples of drastic APR violations, APR violations are usually mild. (They may even be necessary. After all, society would not want our lawyers to end up starving—or, worse, mugging our senior citizens, instead!) These days, creditors are aware of expected violations and legal fees, and therefore take them into account when they purchase bonds and stocks in the first place. Thus, the presence of lawyers increases corporations' costs of financing.

Bonds come in a thousand flavors, and then some.

Bonds come in many different flavors. The issuer can choose what specific rights to offer to buyers and what rights to reserve for himself. Naturally, the more rights a bond buyer receives, the higher is the value of the security when the issuer sells them. Some bond flavors are unique to corporations, other flavors are shares with other bonds. The following are common bond specifications:

Bond Seniority specifies which security receives first dip in case of bankruptcy and liquidation. A **Senior Bond** will have to be satisfied in full before a **Subordinated Bond** (or **Junior Bond**) may receive any money. In turn, equity receives its funds only after even the most junior bonds have been fully satisfied.

Bond Covenants promise that the firm will keep certain promises, or else it will be forced to repurchase (**redeem**) the bond. Among the more common covenants are restrictions on what the firm can do with its assets, how much dividends it may pay, how many and what kind of other securities it may issue, what kind of financial ratios (e.g., debt/equity ratio) it needs to maintain, who the auditor is, what happens if the corporation defaults on any other bond, and how much of its own bonds it will repurchase each year. This last feature is called a **Sinking Fund** commitment and is very common.

Collateral or **Security** are specific corporate assets pledged to a specific bond in case of default. For example, mortgage bonds are collateralized by the value of the underlying real estate. If the issuer fails to pay, the bondholders may repossess the underlying real estate, and use it to satisfy their claim. If the real estate is not enough to satisfy the claim of the **Secured Bond**, the remaining claim becomes an ordinary bond, waiting in line with other creditors for payment.

Convertible Bonds give the bond holders the right to convert their bonds into another security at prespecified terms, usually equity. Convertible Bonds are further described below.

Callable Bonds allow the issuer (the firm) to "call in" outstanding bonds at a prespecified price. Usually, this right is not given for the first 5 years of a bond and is often granted in connection with convertibility. (Almost all bonds issued prior to the mid 1980s included this feature.) Issuers tend to call bonds when interest rates drop. Callable Bonds are further described below.

Puttability: Puttable Bonds allow bondholders to return the bonds to the issuer, in exchange for pre-agreed-upon payment.

CFOs must also make decisions on the following corporate bond features. Because these features are shared by non-corporate bonds, we have already described them in Chapter 2.

Bond Maturity is the time to final payback. Indeed, borrowing may be very short-term (as little as overnight!), or very long-term (as long as forever). Bonds of different maturities may have different names. For example, **Commercial Paper** is short-term debt, often guaranteed by a bank's credit line (see below), and therefore is almost risk-free to the lender. On the corporate balance sheet, **Funded Debt** is the term for debt that has a maturity of less than 1 year. **Unfunded Debt** has a maturity of more than 1 year.

SIDE NOTE: Maturity can sometimes be like "super-seniority." That is, a subordinated bond may be repaid long before a more senior bond comes due; and the money paid to the subordinated bond can usually not be repatriated to satisfy the senior creditor's first claim.

Bond Duration is a measure of how soon payments are made. It was described on Page 166.

Coupon Bonds vs. Zero Bonds: Zero bonds pay a fixed amount of money only at a final date. Coupon bonds make (interest) payments on a regular schedule, typically (but not always) twice a year, and the principal is repaid as **Balloon Payment** at the end.

SIDE NOTE: In a practice called **Bond Stripping**, investment banks sometimes separate the coupon obligations from the final balloon payment. That is, they purchase bonds, put them into an escrow account, and offer two types of new securities collateralized by the escrowed bonds: one pays all the coupon, the other pays all the principal. The reason for doing this is to exploit tax loopholes in Japan and other countries.

Fixed Interest-Rate Debt vs. Floating Interest-Rate Debt: Bonds can promise to pay a predetermined interest rate over the life, or a spread relative to some other interest rate. Floating debt is often issued at a spread relative to the **Prime Rate** (an average interest rate that banks usually offer their best customers) or relative to **L.I.B.O.R.** (**London Interbank Offer Rate**). The interest rate on floating rate debt is often **Capped** or **Collared**; that is, the interest rate will never exceed a pre-determined ceiling.

SIDE NOTE: Highly reputable companies can borrow at interest rates that are about LIBOR. More risky companies typically pay interest rates that are about 100-300 basis points (or 1-3%) above LIBOR.

There is an endless array of possible bond features.

There is no limit to the imagination as far as bond features are concerned. For example, the Russian carmaker **Avtovaz** issued **Lada Bonds** in 1994, which allowed the holders to convert their bonds into Lada cars. Other bonds' payoffs have been linked to commodities, such as the price of oil, financial securities, or exchange rates.

SIDE NOTE: **Par Value** is a vacuous concept, sometimes used to compute coupon payout schedules. Principal and Par Value, and/or Interest and Coupon Payment need not be identical, not even at the time of issue, much less later. For the most part, par value is best ignored.

A bank loan is like a bond, but usually with only one bondholder—the bank. The advantage is that the bank may know the firm better, and thereby grant better terms. The disadvantage is that there is less competition among banks for extending loans than there is among public bondholders. Bank loans can also take the form of a **Credit Line**. Credit lines are like instant debt, permitting borrowers to draw down money (and pay higher interest) only upon need. (Borrowers typically agree to pay a low interest rate even on the unused part of credit line.) The opposite of a credit line is **Negotiated Debt**, where both the bank and the firm commit to a fixed loan.

Bank Loans may be Credit Lines.

SIDE NOTE: These are the deconstructed bond features. In the real world, bond features usually come in bundles.

For a mid to large cap company, there would typically be a syndicate lead. J.P. Morgan Chase and Citibank are two large players in the market. The bank credit facility would be structured into a number of segments or **Tranches**. The credit facility would include a **Revolver** (revolving credit line) and one or more tranches of **Term Debt**. The revolver is a line of credit which the company can borrow and repay and borrow again until a commitment termination date/maturity. The term debt would be structured in one or more tranches. The principal payment schedule and maturity date would be different for each of the tranches. Tranche A would begin to amortize right away and would have the shortest term to maturity. The Tranche B term loan would amortize and mature before the Tranche C term loan, etc. The revolver and term loans would share in the same security package. In terms of pricing, the pricing on the Revolver and Term A loan (a spread over LIBOR) would be the same. The revolving credit banks would also receive a fee for the unused/undrawn portion of the revolver. The Tranche B and C lenders would receive a wider spread over LIBOR to compensate them for the added credit risk of having a longer term loan to the company. The Revolver and Term A loan is marketed as a package. Because institutions and mutual fund are not set up to provide revolving credit, this "pro rata" piece is purchased by commercial banks. The market for Tranches B, C, D, etc., term debt is becoming more liquid and this is purchased by mutual funds, commercial banks, hedge funds, etc. The deal is brought to the capital markets with proposed pricing by the syndicate lead and will be priced at the market clearing price.

Smaller companies would have a relationship with either a smaller syndicate of commercial banks, or perhaps a regional bank in the case of a very small company. The structure would in all likelihood be less complex—a revolver and only one Tranche of term debt or just a revolver. In terms of pricing, there would be some type of premium pricing to compensate the lenders for the added credit risk of lending to a small company and for holding a less liquid security. The price would be negotiated and agreed to by the borrower and lender.

20·1.B. Ordinary Equity (Common Stock)

Stock, a common abbreviation for **Ordinary Equity** or **Common Equity**, is like ownership: **stock shareholder** (or just **stockholders**) receive whatever is left over *after* the promises to bondholders have been honored. Equity typically has the lowest APR priority. But shareholders enjoy unlimited upside. Stockholders receive dividend payments, and—if they are lucky—capital appreciation. Unlike contractually committed coupon interest payments, dividend payments can be omitted at the firm's discretion. In a very real sense, equity holders are like the owners (or partners) of the firm, with one exception: equity holders enjoy **Limited Liability** (see also Subsection 1·3.B on Page 12). They have **Control Rights**, appointing the **Board of Directors**, which in turn appoints and oversees **Management**.

Stock ownership is ownership post-debt, earning dividends and capital gains, and having control rights.

At least, this is what the theory is. The Web Chapter on corporate governance thoroughly dispells this illusion.

Most shares are issued by the corporation. Large corporations' shares typically trade on stock exchanges.

Although shares are usually issued and sold by corporations to investors in exchange for cash, sometimes shares are granted (for example to management, or employees). Over 10,000 large U.S. firms now have their common stock traded on a major public stock exchange, like **Nasdaq** or the **New York Stock Exchange**. There, any investor can easily purchase and sell shares, and closing prices for the previous day can easily be found in most newspapers.

SIDE NOTE: Firms with fewer than 100 investors and which do not try to sell their securities to the public are not (or at least less) regulated by the S.E.C. Many hedge funds and other corporation consider this a god-sent exclusion. The necessary S.E.C. paperwork, both initial and ongoing, can be strangling for small firms.

Dividend payments come from after-tax income.

Unlike coupon payments to bondholder, which may be paid with before-tax income, dividend payments have to come from after-tax income. In other words, corporate profits are first taxed before they can be used to pay dividends. (See Subsection 1·4.B for the consequences.) Then, even though the corporation has already paid corporate income tax on its earnings, individuals have to pay personal income tax on their dividend receipts. This is sometimes called the **Double Taxation of Dividends**. The tax code in many non-U.S. countries, such as Great Britain, tries to remedy this somewhat.

SIDE NOTE: The Bush Tax Cuts of 2003 similarly try to remedy the double taxation.

20·1.C. Preferred Equity (Stock)

Preferred Equity gets better dividend treatment by the corporation.

Many companies also issue **Preferred Equity**. Preferred equity is an instrument somewhere between debt and equity. Preferred equity holders usually receive higher dividends than ordinary equity holders, hence the description "preferred." Unlike ordinary equity, where dividends are declared annually at the discretion of management, preferred equity dividends are usually specified at issuance (for example, $2.^{25}$ per calender quarter per share). Unlike bondholders who fail to receive their promised coupon payment, preferred equity holders cannot force the firm into bankruptcy if dividends are not paid—but their contract usually states that they have priority over common equity holders: the preferred equity receive dividends before ordinary equity receives dividends (and often with interest!).

Preferred Dividends receive better tax treatment when held by other corporations.

The real advantage of preferred equity over ordinary equity is that, although not tax-deductible to the issuing corporation, the preferred dividend payments are not fully taxable when received by other firms. The I.R.S. considers only 15–30% of preferred dividends taxable for the recipient. Preferred equity is therefore usually not held by individuals, but by corporations investing in other corporations. (Individuals can earn higher post-tax yields in other securities.)

More preferred equity features.

Preferred Equity is junior (subordinated) to the company's bonds, which means that in case of bankruptcy, preferred equity holders will only be paid after bondholders are paid. Preferred Equity is often retired on a fixed schedule—even though many preferred equities have no formal maturity. Like common stock, some preferred stock is traded on public stock exchanges. Though preferred equity usually enjoys covenant restrictions, it has neither precedence to bonds, nor control rights, nor unlimited upside (rights to the residual value of the firm). Naturally, any such features can be explicitly contractually added. For example, in many small firms, it is not uncommon for preferred equity to be both convertible into common equity ("convertible preferred stock") and to explicitly provide for voting rights. The holders of such claims are usually themselves corpora-

tions, often **Venture Capitalists**, that can write off the securities if the firm fails; and convert them into common equity if the firm succeeds.

Solve Now!

Q 20.1 *In what sense is preferred equity like bonds? In what sense is preferred equity like stocks?*

Q 20.2 *Write down all bond features (variations) that you remember.*

Q 20.3 *What is a control right?*

Q 20.4 *What is limited liability?*

20·2. Viewing Securities as State-Contingent Claims

20·2.A. Stocks and Bonds

Financial Instruments are (mostly) state contingent claims

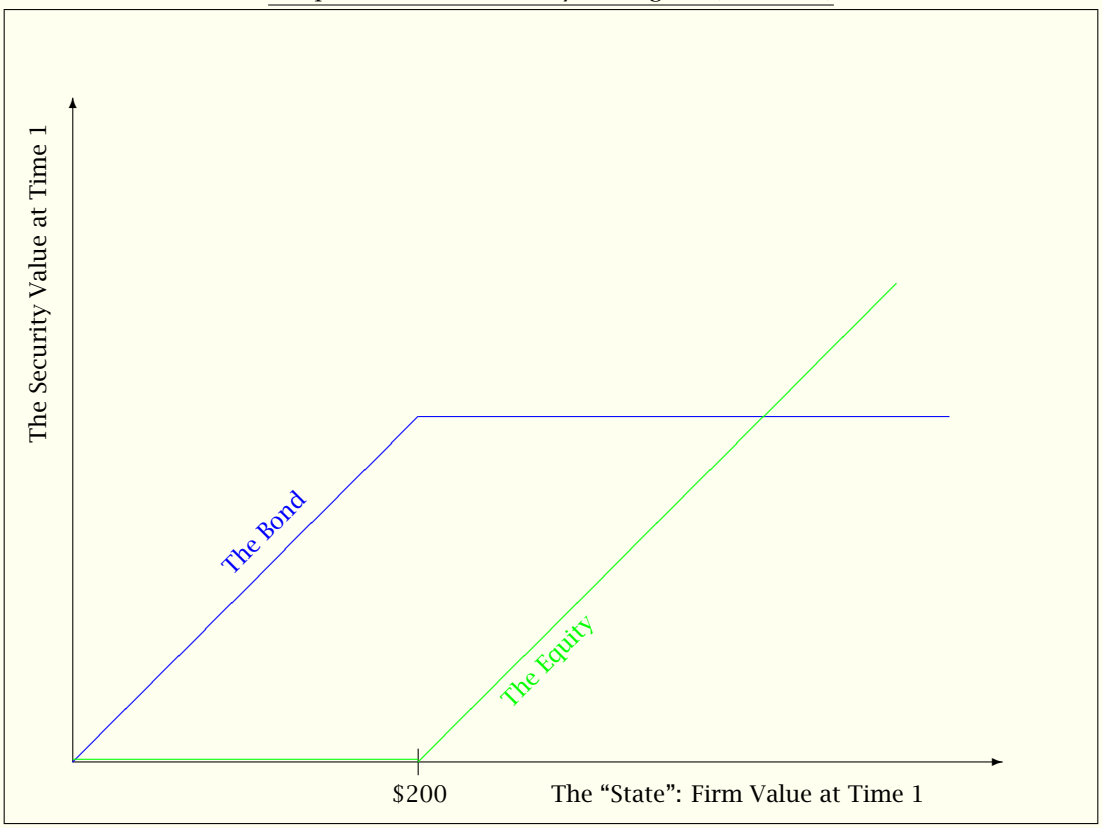
Your main problem as CFO is to decide which capital structure maximizes the value of your firm. In our world of academic fiction—in a perfect market in which there are no transaction or other costs—it matters little whether you first purchase all securities and then resell securities to new owners, or whether you just maximize the existing securities values. The goal is to obtain maximum value for your firm.

Side Note: In a perfect market, it is not important whether the firm continues to exist after the bond comes due: what matters is the value of the firm at that time. After all, the firm could be liquidated (sold) for its fair value, and stockholders and bondholders would both receive their due. This fair corporate value would also be the total value of the securities at this time. (On Page 124, in our tornado/sunshine example, we explained that it does not matter whether the house is sold at the end of period 1.)

Table 20.1. Sample Bond and Stock Payoff Table (at Time 1)

Firm Value	Bond Value	Equity Value
$0	$0	$0
$50	$50	$0
$100	$100	$0
$150	$150	$0
$200	$200	$0
$250	$200	$50
$300	$200	$100
$350	$200	$150
...	...	...

Sample Bond and Stock Payoff Diagram (at Time 1)



Payoff diagrams, again.

Again, payoff tables and payoff diagrams (discussed in Chapter 6 (and also in the Web Chapter on Options and Derivatives) make it easy to understand the basic promises of many securities. For example. consider a firm with a capital structure that consists of equity and a single bond that promises to pay $200 next year. Next year (and thereafter), the value of the corporation is the total value promised to bondholders' and shareholders. How much each will receive will depend on the value of the firm. Table 20.1 shows that if the firm is worth $100, bondholders will receive $100 and shareholders will receive nothing. If the firm is worth $200, bondholders will receive $200 and shareholders will receive nothing. If the firm is worth $300, bondholders will receive $200

and shareholders will receive $100. If the firm is worth $400, bondholders will receive $200 and shareholders will receive $200. And so on. This is the best way to think of the contractual obligations of bonds, stocks, and any other securities. (There are other important considerations, though, which are covered in subsequent chapters.) If you call the future value of the firm (the base asset) the underlying **State**, financial securities issued by the corporation are **State-Contingent Claims**. Bondholders, equity holders, or any other security holders have rights to specific payments which depend on the future state of the firm.

NERD NOTE: *Payoff diagrams are useful as conceptual aids at a given point in time. They fail to illustrate features that are themselves a function of time. For example, payoff diagrams are less useful to illustrate the value of a security that consists of many differently timed or even randomly timed future payoffs.*

Many financial instruments are combinations of stocks and bonds.

Over time, firms have become more complex. As already discussed, modern corporations can use a variety of financial instruments that promise different future payoffs to various buyers under various scenarios. These are usually **Hybrid Financial Instruments**, which often contain some aspects of both bonds and stocks, such as **Convertible Debt**, **Callable Debt**, **Warrants**, **Derivatives**, **Trade Credit** (borrowing from product suppliers), **Leasing** Financing (instead of purchasing an asset, leasing it), and so on.

20·2.B. Convertible Bonds

Convertible bonds allow the bondholder to exchange the bond into something else, usually equity. An example.

A **Convertible Bond** permits the bondholder to convert his bonds into equity at a predetermined price at predetermined dates. Here is an idealized example, in which the bond can only be converted at one point in time: a firm with 400 outstanding shares of equity has also sold 200 individual bonds that promise $10 each in January 2005. However, each bond can be converted, at the bondholder's discretion, into 3 shares of stock.

Understanding the specific example: at the time the bond comes due.

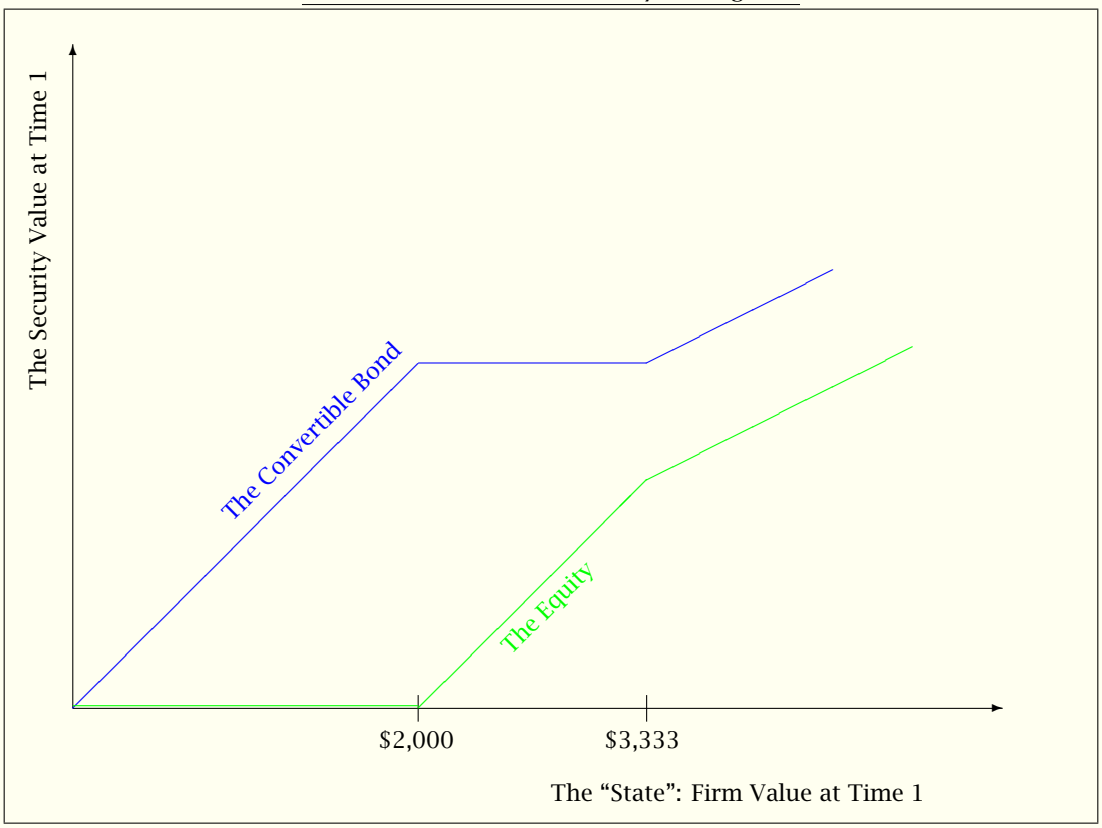
If the value of the firm in January 2005 will be $2,000 or less, then bondholders will own the entire firm. If the value of the firm will be $100,000, bondholders will make the following calculation: If they take advantage of the convertibility feature and exchanged their 200 bonds for 600 shares, there would be 1,000 share in total. These 1,000 shares will own 60% of the firm, which would be worth $60,000—a whole lot more than the $2,000 that they would receive if they did not request conversion. Therefore, the bondholders would convert.

NERD NOTE: *Each bondholder would be even better off if he could convince his fellow bond holders not to convert, but this is not realistic. After all, these other bondholders would be making themselves worse off.*

Table 20.2. Sample Convertible Bond and Stock Payoff Diagram at Time 1

Firm Value	Convertible Bond Value	Common Equity Value
$0	$0	$0
$1,000	$1,000	$0
$1,500	$1,500	$0
$2,000	$2,000	$0
$2,500	$2,000	$500
$3,000	$2,000	$1,000
$3,333	$2,000	$1,333
$3,500	$2,100	$1,400
$4,000	$2,400	$1,600
$4,500	$2,700	$1,800
...	...	...

Convertible Bond and Stock Payoff Diagrams



Determining the cutoff at which convertible bondholders will prefer converting.

It is possible to determine at what firm value the bondholders would be indifferent between converting and not converting: it is where 60% of the firm would be equal to $2,000. This is the case if firm value is equal to $3,333. Summarizing the situation, below $2,000, the bondholders get everything. Between $2,000 and $3,333, the bondholders get $2,000 and the shareholders get the residual above $2,000. And above $3,333, the shareholders and bondholders both benefit from higher values, with bond-

holders receiving 60% of the firm's value and shareholders receiving 40% of the firm's value. The payoff diagram helps to understand this security. Exhibits 20.2 shows the convertible debt payoff diagram.

One final question: why would shareholders be willing to give bondholders this right, which in effect deprives them of much upside? The answer must be that by doing so, bondholders are willing to pay more for the bond upfront. And, indeed, we know that if financial markets are competitive, bondholders will get what they pay for.

Owners are willing to give the convertible right, because it increases the cash they receive upfront.

20·2.C. Call Options, Warrants, and Units

Firms sometime issue **Call Options** (or just **Calls**), **Warrants**, or **Units**. Also described in the Web Chapter on Options and Derivatives, a call option is a right to purchase the shares from the call seller for a given price at or by a given point in time. For example, a call might be the right to purchase 300 shares of the firm for $50/share on January 15, 2005. If shares will then be worth more than $50/share, then the call option will have value.

Call Options are rights to purchase equity at a predetermined price in the future.

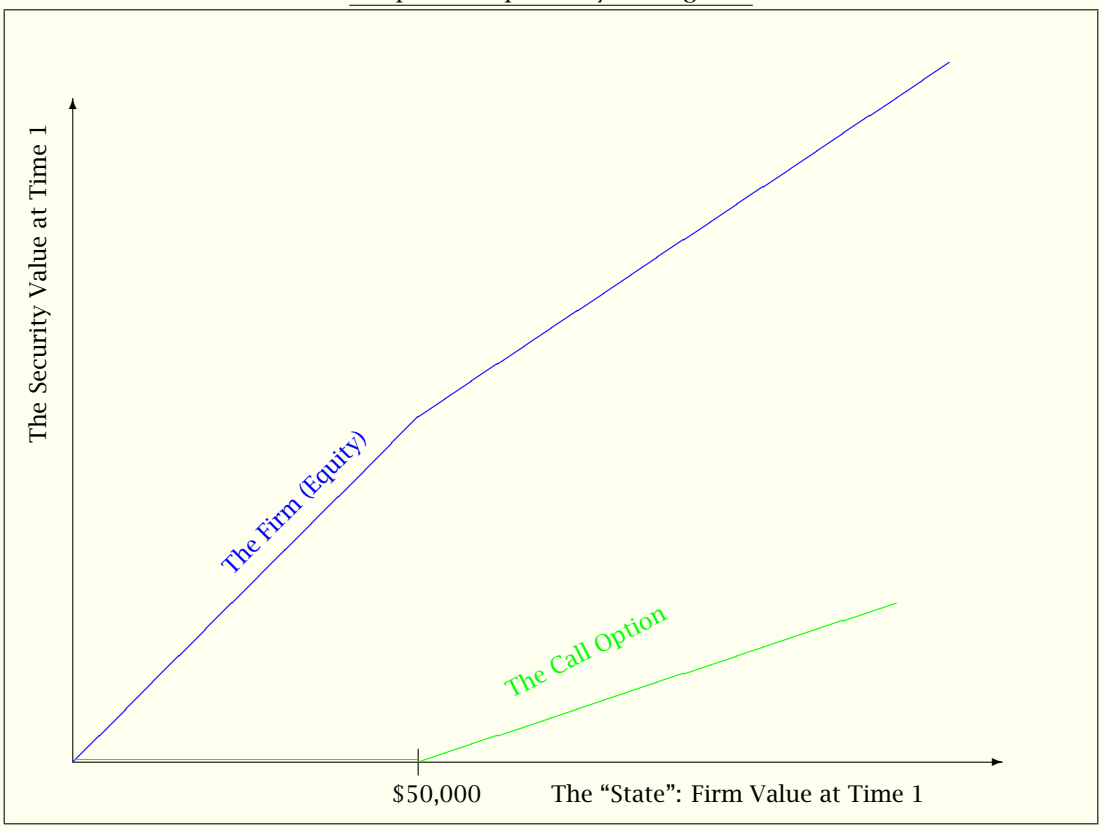
NERD NOTE: *In the Web Chapter on Options and Derivatives, we show how put-call parity implies that call option holders usually (but not always!) do not want to exercise their call options early, even if their contract permits them to. Therefore, focusing on a call option that expires on a particular date rather than a call option that allows conversion anytime up to January 15, 2005, is not harmful.*

Table 20.3. Sample Call Option Payoff Diagram at Time 1

Firm Value	Option Value	Equity Value
$0	$0	$0
$10,000	$0	$10,000
$20,000	$0	$20,000
$30,000	$0	$30,000
$40,000	$0	$40,000
$50,000	$0	$50,000
$60,000	$3,000	$57,000
$70,000	$6,000	$64,000
...	...	...

The call option strike price is $50 for 300 shares. With 1,000 shares in total, the option is "payment of $15,000 in exchange for 30% of the firm." At a firm value of $60,000, this means $18,000 minus $15,000 payment. At a firm value of $70,000, this means $21,000 minus $15,000 payment.

Sample Call Option Payoff Diagrams



Working the Example.

In Exhibit 20.3, the firm has 1,000 outstanding equity shares. If call option owners will choose to exercise, the call option for 300 shares will therefore be equivalent to owning 30% of the firm. Now, if the per-share price will be \$40 (total firm value \$40,000) at option expiration, the call option will be worthless. If the per-share price will be \$60, call option holders will demand exercise in exchange for payment of \$50. The firm will repurchase its own shares at \$60/share but receive only \$50/share. This will cost old shareholders net \$10/share for 300 shares, or \$3,000 in total. This net benefit goes to new shareholders. Old equity holders will own 70% of the \$60,000 firm, plus the \$15,000. In sum, their wealth will be \$57,000.

SIDE NOTE: If the old shareholders wish, they can use the \$15,000 to repurchase 250 shares at \$60/share from the new shareholders. Equivalently, instead of the market repurchase, old shareholders could just hand the new shareholders 5% of the firm (50/1,000) and retain 95%.

Companies usually issue Warrants, not Call Options.

Companies rarely issue call options, because it is usually easier for them to just issue new shares (to satisfy the call option) than it is for them to repurchase existing shares. Such call option-like securities are called **Warrants**. They give warrant holders exactly the same rights that call option holders would have, except that the money that the warrant holder pays flows into the corporation, and the corporation issues new shares to satisfy the warrants. If the above security (300 shares at \$50/share) is a warrant rather than an option, if the firm ends up being worth \$60,000, the warrant holders would exercise their rights, the firm would then be worth \$75,000, and warrant holders would own $300/(1,000+300) \approx 23\%$ of this firm, the equivalent of \$17,307.[69] in exchange for their \$15,000 payment. Note that this is slightly less than the \$18,000 in stock that a call option holder would have received in exchange for \$15,000 payment. Therefore, a warrant is worth less than an equivalent call option. Exhibits 20.4 shows the warrant payoff diagram.

Units are usually a Share and a Warrant.

A **Unit** is just a bundle of multiple types of securities that are sold together, usually involving a warrant and another security (a stock or a bond). The purchaser can keep both types of securities, or unbundle them and sell them separately.

NERD NOTE: *One can show that a convertible bond is the same as a hypothetical Unit consisting of a plain bond and a warrant.*

Large firms usually have multiple claims outstanding.

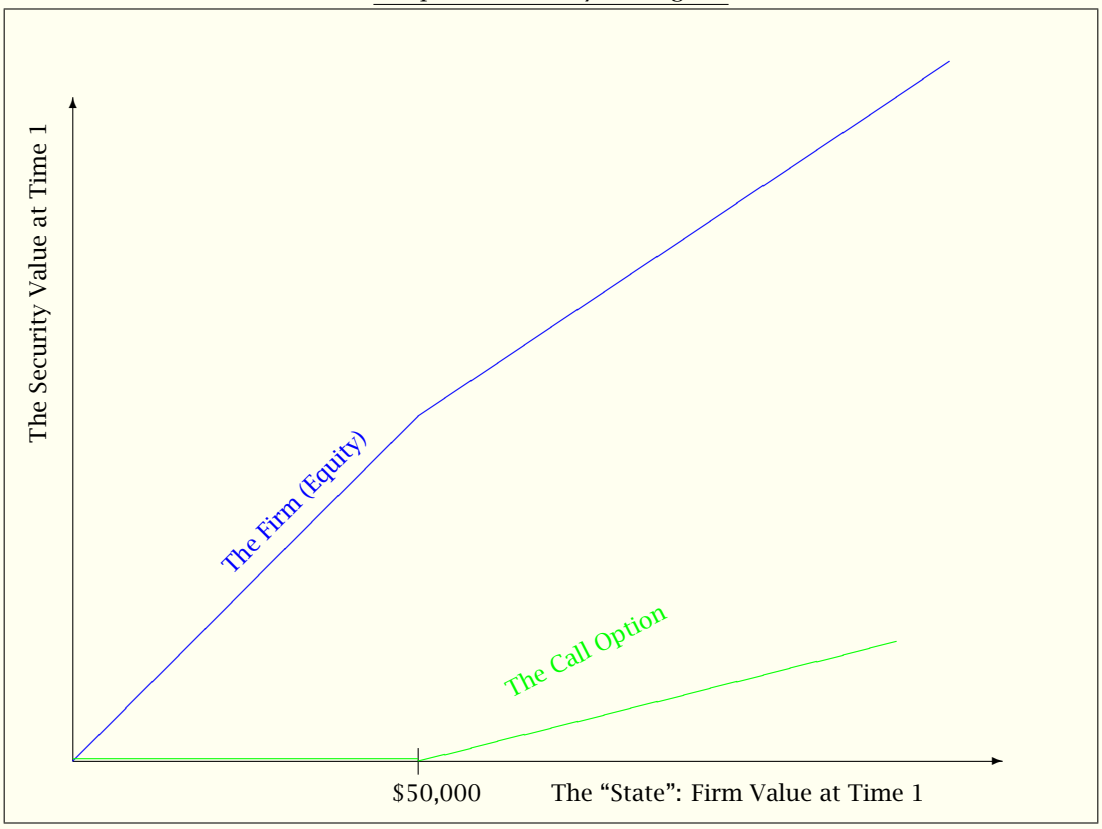
In the real world, firms are financed by a whole set of different securities. The same firm may have senior debt, junior debt (perhaps with a conversion feature), equity, and a warrant. The right way to think about all these securities still involves the "magic" payoff table (and payoff diagram): if the firm ends up worth very little, only the senior debt is paid. If the firm is worth a little more, both the senior and the junior debt are paid. If the firm is worth even more, the equity will become worth something; and finally, the warrant and/or the conversion feature will be valuable.

Table 20.4. Sample Warrant Payoff Diagram at Time 1

Pre-Warrant Firm Value	Post-Warrant Firm Value	Warrant Value	Old Equity Value
\$0	\$0	\$0	\$0
\$10,000	\$0	\$0	\$10,000
\$20,000	\$0	\$0	\$20,000
\$30,000	\$0	\$0	\$30,000
\$40,000	\$0	\$0	\$40,000
\$50,000	\$0	\$0	\$50,000
\$60,000	\$75,000	\$2,308	\$57,692
\$70,000	\$85,000	\$4,615	\$65,385
...	...	...	

The warrant strike price is \$50 for 300 shares. With 1,000 shares in total, the option is "payment of \$15,000 in exchange for $300/1,300 = 23\%$ of the firm." At a pre-warrant firm value of \$60,000, this means $23\% \cdot \$75,000 \approx \$17,308$ minus \$15,000 payment. At a pre-warrant firm value of \$70,000, this means $23\% \cdot \$85,000 \approx \$19,615$ minus \$15,000 payment. (Old Equity owners now own 77% of a \$85,000 firm, which is \$65,385.)

Sample Warrant Payoff Diagram



20·2.D. Callable Bonds — Or Where Payoff Diagrams and Tables Are Not Useful

While a convertible bond gives bondholders extra rights, callable bonds give the firm extra rights: They permit the firm to call the bonds for a predetermined price. For example, a callable bond contract may state that shareholders can redeem the bond by paying back principal plus 10% rate of interest in May 2002. Naturally, when a bond contains a call feature, it is less valuable than an otherwise identical bond. Therefore, issuers of bonds with call features are likely to receive less money when they sell these bonds. Put differently, they must pay a higher interest rate upfront. In effect, every mortgage is a callable bond, because the seller of the bond (the homeowner, that is you) can just pay back the remaining loan balance (the **Principal**) and be absolved of all further obligations.

Callable Bonds reserve the right to repay the loan.

Unlike the conversion feature discussed above, a bond's call feature has an element that cannot be easily captured by a payoff diagram: corporations may call bonds when the interest rate has declined so replacement bonds have become much cheaper. (Similarly, homeowners tend to repay their mortgages and refinance when the mortgage interest rate has dropped.) But, the interest rate is not a one-to-one function of the firm value in the future, and so the payoff diagram for a fixed point in time would usually be misleading.

Actual calling may depend not just on the value of the firm, but on such things as interest rate.

SIDE NOTE: Many bonds have both call and conversion features. The call is often used to force conversion. That is, the corporation calls the bonds, and the holder of the bond finds that it is in his interest to convert the bond into equity, rather than to accept repayment.

Solve Now!

Q 20.5 *Assume a medical insurance pays 90% of medical expenses, subject to a $500 deductible, and subject to a limit of $10,000 payout by the insurance company per year. Write down an insurance payoff table and graph an insurance payoff diagram, as a function of your medical expenses. What is the slope of the line at each segment?*

Q 20.6 *A firm is financed with a senior bond that promises to pay $100, a junior bond that promises to pay $200 (of lower seniority but equal maturity to the senior bond), and equity. Write down the payoff table and then draw the payoff diagrams when the two bonds are due.*

20·3. The Selling Process For Corporate Securities

Underwriters help sell corporate securities.

The process of selling securities is neither easy nor frictionless. Firms can raise between 50-90% (60-80%) of their funding needs internally (retained earnings plus depreciation, i.e. cash flow not paid out), but the remainder must be raised from investors. The financial intermediary who brings securities to the public markets is the **Underwriter** (i.e., one of the functions of an investment banker). Among the top underwriters in the United States are the likes of **Merrill Lynch**, **Lehman Brothers**, **Salomon Smith Barney**, and **Goldman Sachs**. Virtually all offerings of securities for exchange-traded firms are underwritten these days. Formerly, the term "underwriter" designated the fact that an underwriter guaranteed the issuing proceeds. These days, this guarantee is given on the morning of the offering, when the underwriter already knows at what price investors are willing to purchase shares. If it looks like the issue cannot be sold, the underwriter will demand a lower price and/or refuse to bring the issue to market altogether. So, the actual underwriting guarantee itself is no longer important.

Side Note: Until November 1999, the **Glass-Steagall Act of 1933** prohibited U.S. banks to mix commercial and investment banking. This was one reason why large commercial banks, such as **Citibank** or **Chase Manhattan Bank** could not play in this space. In other countries, there was typically no distinction between commercial and investment banks. (Glass-Steagall also prevented interstate banking for a long time, but this restriction has also been repealed.)

Underwriters help sell corporate securities.

Instead, the main role of the underwriter today is to provide the legal expertise of the underwriter in carrying through the process (**Issue Origination**), to smoothly place shares with specific investors, often with large institutional investors (**Issue Placement**), and to signal confidence in the firm by putting the underwriter's reputation on the line. Underwriters also help in other ways. For example, they are known to spread **Hype** through optimistic analyst reports, which increases the sellability of shares.

Anecdote: **Analysts Recommend: Buy!**
The number of analysts' **Buy Recommendations** outnumbers the number of **Sell Recommendations** by a ratio of about 5:1; when limited to **Strong Buy** and **Strong Sell** recommendations, this ratio changes to over 10:1. Investment banks are well aware that a sell recommendation by one of their analysts is likely to induce such firms to not only exclude the analyst from further information about the firm, but also to induce the firm select a different underwriter—and so the investment banks discourage sell recommendations. Although this analyst bias was always widely known by professional investors, it had received scant attention in the press and thus recognition by small investors—until 2001, when it suddenly became a public scandal. It is still somewhat of a mystery why then, but not before.

Table 20.5. Typical U.S. Fees, 1990-1994, in Percent.

Proceeds	IPOs			SEOs			Cnvtbl			Bonds		
($ millions)	*N*	UWC	TC	*N*	UWC	TC	*N*	UWC	TC	*N*	UWC	TC
2-10	337	9.1	17.0	167	7.7	13.3	4	6.1	8.8	32	2.1	4.4
10-20	389	7.2	11.6	310	6.2	8.7	14	5.5	8.7	78	1.4	2.8
20-40	533	7.0	9.7	425	5.6	6.9	18	4.2	6.1	89	1.5	2.4
40-60	215	7.0	8.7	261	5.1	5.9	28	3.3	4.3	90	0.7	1.3
60-80	79	6.7	8.2	143	4.6	5.2	47	2.6	3.2	92	1.8	2.3
80-100	51	6.5	7.9	71	4.3	4.7	13	2.4	3.0	112	1.6	2.2
100-200	106	6.0	7.1	152	3.9	4.2	57	2.3	2.8	409	1.8	2.3
200-500	47	5.7	6.5	55	3.3	3.5	27	2.0	2.2	170	1.8	2.2
500-	10	5.2	5.7	9	3.0	3.2	3	2.0	2.1	20	1.4	1.6
All	1,767	7.3	11.0	1,593	5.4	7.1	211	2.9	3.8	1,092	.62	2.2

Source: Lee-Lochhead-Ritter, 1996. IPOs are initial public offerings. SEOs are seasoned equity offerings (i.e., post-IPO). Cnvtbl are convertible bonds. Bonds are straight bonds. *N* are the number of observations. UWC is underwriter compensation (in percent of proceeds). TC are total costs, including registration fees, printing fees, legal, and auditing costs (in percent of proceeds).

Typical real-world fees.

Table 20.5 shows that selling equity shares is more expensive than selling bond. For example, placing a $100 million bond offering may cost the firm $2.3 million, while placing a $100 million equity offering may cost $3.9 million. This difference is because investors often incur more risk in buying a $1 million equity offering than they incur in buying a $10 million bond offering. Risky capital is more difficult to engage, which is reflected in higher underwriting fees. Similarly, when companies first sell shares in an **Initial Public Offering** (**IPO**), which is a relatively risky proposition, costs are usually higher than when companies later issue more shares.

Firms select underwriters and method of placement of the security.

Many if not most firms select their underwriters based on one-to-one negotiation, or even based on personal friendships. About 70% of securities issues in the United States are sold to the public, 30% are sold to private parties (without S.E.C. involvement). But investment banking has truly gone global, and many foreign companies issue securities in the United States and vice-versa.

SIDE NOTE: There is some evidence to suggest that underwritten security issuing is cheaper, but less frequent (except for utilities where it is often legislated). The cynical view (in the Web Chapter on Corporate Governance) is that this is because underwriters "wine and dine" corporate executives, and form a placement network for executives wanting to move. Investment bankers are not likely to recommend executives who minimize the investment bank's profits to bigger and better companies.

SIDE NOTE: **Shelf Registration** (called **Rule-415**) is another form of issue, which allows companies to create shares, and not sell them all at once to the market in an offering, but to slowly sell shares to investors that are buying shares on an exchange over a two year period.

In Europe, it is common for firms to conduct **Rights Offerings**, rather to simply sell shares to the public. Interestingly, by appropriately setting the price of the new shares, issuers can force their old shareholders to participate. For example, if there are 100 shares each trading for \$1, and the corporation wants to raise \$10, it can offer each shareholder the right to purchase one additional share at 10 cents. A shareholder who does not participate will find himself with $1/199 \approx 0.5\%$ of a firm worth \$110. This $\$0.^{55}$ is less than the 1% (or \$1) she held before the offering. Participating in the rights offering, she would own $2/200 = 1\%$ of the \$110 firm, worth $\$1.^{10}$, for a cost of $\$0.^{10}$. Thus her best choice is to buy into the offering. Such offerings are called **Forced Rights Offerings**. In some cases, where participation is not forced, underwriters are paid to **Stand By** in order to sell shares not sought after by existing investors to new investors.

Primary vs. Secondary Shares. IPOs vs. Seasoned Offerings.

A poor choice of jargon are the phrases **Primary Shares** and **Secondary Shares**. The latter seems to suggest that an offering is not an **Initial Public Offering** (**I.P.O.**), but an offering made by an already publicly listed company. Unfortunately, those are called **Seasoned Offerings**. Instead, a secondary share is a share that is sold by an investor in the firm (e.g., by the founder), in which case the company does not enjoy the issue proceeds. A primary share is a share that is newly minted and sold by the firm itself, in which case the proceeds go to the firm itself.

Dilution means that equity issues tend to be very costly.

There are not only these direct costs, but also indirect costs. Most important is the fact that outstanding stock usually declines by about 2% on average upon the announcement of a new seasoned equity offerings. (This is not the case for debt issues. The reasons will be discussed in later chapters.) Because the size of the new equity offering tends to be smaller than the already outstanding equity, this 2% drop translates into about 30% **dilution**; that is, upon announcement of a \$100 million SEO, the outstanding equity tends to drop by about \$30 million. This dilution is definitely a cost that the CFO needs to weigh when issuing equity.

Bonds usually are illiquid—and, when they do trade, trade "over-the-counter."

After the offering, equity securities are usually bought and sold on stock exchanges. The two most important exchanges in the United States are **Nasdaq** and the **N.Y.S.E.**. Bonds, on the other hand, often do not trade on any exchange (such as the **New York Bond Exchange**). And when they do trade, the markets tend to be not very liquid. (The trading volume is low.) Instead, most bonds are traded **Over-The-Counter**, that is by large investors who call up individual investment banks' desks. The transaction price is usually not disclosed in such cases, and trading is fairly rare.

SIDE NOTE: Because the vast majority bond transactions take place between brokers rather than on an exchange, accurate prices are difficult to come by. Thus, individual investors are typically better off staying away from purchasing individual bonds, lest they wish to be taken advantage of. A better alternative would be to purchase a good mutual funds that just holds bonds.

Bond Rating Agencies.

We already mentioned bond rating agencies on Page 80. Issuers can pay **Moody's**, **Standard&Poors**, or **Fitch** to rate their bonds. This typically costs \$5,000 to \$25,000 per bond issue. Having a public bond rating, helps potential investors gauge the risk. Indeed, many institutions are prohibited from buying any unrated bonds, making ratings a necessity for many large bond offerings. Only the very largest and most stable firms can issue **Investment Grade Bonds**. All other firms can only issue **High-Yield Bonds**, that is, bonds rated BB or worse (see also Table 4.3 on Page 81).

ANECDOTE: High-Yield or Junk?
High-Yield Bonds are also sometimes called **Junk Bonds**, a term that sounds worse than it is. This term was originally coined by **Michael Milken**, who reinvigorated this market in the 1980s. He later regretted his choice. Compared to **Investment Grade** Bonds, Junk Bonds are indeed riskier, but they also offer higher expected rates of return. Of course, when compared to equity (common stock), Junk Bonds are safer and offer lower expected rates of return.

Michael Milken later pleaded guilty to securities fraud—although even a decade later, professional views are still divided on whether his behavior or that of his prosecutors was less ethical.

ANECDOTE: Legal Monopolies: Bond Ratings
Federal securities laws recognize just three raters: Moody's, Standard&Poor's, and Fitch. The three raters have not always enjoyed such privileged status. At the beginning of the twentieth century, they were simply investment service agencies that provided investors with research for a fee. But in the second half of the century, the S.E.C. began to rely on ratings to determine what sort of securities certain financial institutions could own. In the 1970s, Moody's and S&P began to charge issuers instead of investors.

In 1994, the **Jefferson County School District** No R-1 of Colorado decided not to obtain a Moody's ranking. To their surprise, Moody's decided to publish an unsolicited and unusually detailed "Special Comment," anyway. It was a negative rating that downgraded the school district, and interestingly it occurred on the day of the pricing of the bond. Although Jefferson County sued, a judge later ruled that Moody's was protected by the First Amendment's *Freedom of Speech* clause.

This legal protection also helped the three major credit rating agencies in **Enron**'s case. They received substantial fees from Enron. Interestingly, even when Enron was already trading at $3 per share and the market was aware of its trouble, all three agencies still gave investment grade ratings to Enron's debt.

SIDE NOTE: Please browse the Issuing Calendar in the Wall Street Journal, and a Moody's or S&P Bond Manual in your local library. The Moody's descriptions are now published by a sister company of Moody's, called **Mergent**.

Solve Now!

Q 20.7 *What are the main functions of underwriters today?*

Q 20.8 *What is a rights offering? Is there anything unique about it?*

Q 20.9 *Roughly, how expensive is selling $100 million worth of equity? $100 million worth of bonds?*

Q 20.10 *Who is Standard&Poor's?*

20·4. Other Sources of Funds

There are alternative money providers.

Firms can obtain financing not only from public markets (investment banking), but also from plain old commercial banks—and even most large publicly traded corporations do. (Small firms rely on banks almost exclusively as their loan providers.) But, **Insurance Companies**, **Pension Funds**, **Mutual Funds**, **Foundations**, **Venture Capital Funds** and **Private Equity Funds**, and even any one of a multitude of government support programs have also jumped into the fray, and may help provide certain companies with needed capital.

There are ways to avoid getting money altogether.

Other methods to implicitly obtain financing is not to pay in the first place. **Trade Credit** by a supplier, the issuing of **Hedging Contracts** (which may promise future delivery of a good in exchange for cash today), **Leasing** instead of **Buying**, **Securetization** (in which the firm sells off assets such as **Accounts Payables**) instead of retaining its assets, etc. are all plausible and common methods to finance operations.

20·5. Summary

The chapter covered the following major points:

- Corporate securities, such as debt and equity (preferred and common), have different cash flow and control rights.
- Corporate borrowing can come in many different flavors.
- Corporate securities can be considered state-contingent claims, which can be described with payoff tables and payoff diagrams.
- Firms raise most of their capital needs through internal retained earnings. For the remainder, underwriters help firms issue corporate securities through a variety of mechanism. Typical fees appear in Table 20.5.

A. Q&A: Answers

1. Preferred equity is like a bond in that it does not participate in the upside, and in that common shareholders do not get their dividends until preferred shareholders have received their dividends. Preferred equity is like a stock in that its payments are not tax deductible, and in that preferred shareholders have no ability to force the firm into bankruptcy if their dividends are not paid.
2. See text.
3. A control right is the right to influence decisions, specifically by changing management and/or the board.
4. The fact that all that the owner can lose is his investment. He cannot forfeit his house or other possessions because the corporation does bad stuff.
5.

Medical Costs	Insurance Payout
$0	$0
$250	$0
$500	$0
$750	$225
$1,000	$450
$2,000	$1,350
…	…
$11,500	$9,900
$11,600	$9,990
$11,611	$10,000
$11,700	$10,000
$12,000	$10,000
$13,000	$10,000
…	…

The "slope" is zero until $500 is reached, then 90% until $11,611.[11] is reached, then zero again.

6.

Firm	Senior	Junior	Equity
$0	$0	$0	$0
$50	$50	$0	$0
$100	$100	$0	$0
$150	$100	$50	$0
$200	$100	$100	$0
$250	$100	$150	$0
$300	$100	$200	$0
$350	$100	$200	$50
$400	$100	$200	$100
$450	$100	$200	$150
…	…	…	…

7. Formal process management, selling, certification.
8. It gives existing shareholders the right to purchase more shares at a given price. The unique aspect is that participation by existing investors can be almost forced.

9. About $4 million and $1.5 million.
10. A large financial services corporation that sells credit ratings, among other products.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 21

Idealized Capital Structure and Capital Budgeting

Should A Company Issue Stocks or Bonds?

last file change: Jun 14, 2004 (13:38h)

last major edit: n/a

This chapter explains how to look at the basics of financing: in a "perfect" world (no transaction costs, no taxes, etc.) the value of the firm is determined by the value of the projects (the NPV), not by whether the firm is financed with equity or leverage. This chapter also explains the basics of WACC.

Subsequent chapters will explain financing in the real world, not in this perfect world.

21·1. Conceptual Basics

21·1.A. The Firm, The Charter, and The Capital Structure

The firm's charter defines the firm structure. The agreement of who receives what part of future corporate income is the Capital Structure

The best way to think about an optimal **Firm Structure** is that the current owners of a firm want to sell their firm today for the highest possible price. Their goal is to design a **Corporate Charter** that maximizes the total market-value of their firm today; that is, the price that new investors are willing to pay to acquire the firm from the old investors. The corporate charter must not only specify the **Voting Rules** (the procedure to replace incumbent managers and how the charter can be changed in the future), but also how future earnings are to be split among possibly different owners and stakeholders (such as customers, workers, and suppliers). This agreement on how to split up future earnings—either explicitly outlined or implicitly allowed to be changed in the future—is the firm's **Financial Structure** or **Capital Structure**: it is the rules that specify who receives the proceeds of (possibly uncertain) future cash-flows.

Anecdote: German Stakeholders
In Germany, stakeholders have an explicit role in corporations. In the case of joint stock companies, limited liability companies and cooperatives with more than 500 employees, one third of the members of the **Supervisory Board** must be employees. In the case of companies in the iron, coal and steel industry, provision is made for equal representation on the supervisory board by shareholders and representatives of the work force. A director representing the employees with responsibility for social affairs is also appointed (qualified co-determination).

21·1.B. Maximization of Equity Value or Firm Value?

Should management maximize shareholder value?

Management is usually appointed by the shareholders. An important question is whether managers should be directed to see themselves as representative of the shareholders or as representatives of the firm. The common view is that it is the task of management to maximize shareholder wealth. But, equity is only one part of the firm's capital structure, but there is also debt—and many financial instruments with both debt and equity features. Does it even make a difference whom management is representing?

When is there a potential problem?

The legal situation in the United States is that management has a legal fiduciary duty to shareholders—except if decisions made by the firm can threaten its insolvency. In this case, management's legal fiduciary responsibility extends to creditors, too. In general, managers see themselves more as representatives of shareholders than as representatives of creditors. When both bondholders and shareholders benefit from a manager's actions, there is no problem. But what if there are situations in which optimizing shareholder value (i.e., the value of the equity) is not the same as optimizing the overall firm's value? For example, assume there will be a transaction by which managers can increase the value of equity by \$1, but at a cost to the value of debt by \$3. (We later show exactly how to do this. It is not difficult to find such opportunities.) This "expropriative" transaction would destroy \$2 in the net value of the firm. This is the type of situation that confronts management with a dilemma: should it maximize the firm value, or should it maximize shareholder value? Recall that it is shareholders who vote managers into office and allow them to stay in office. Whether it is right or wrong, managers will find it in their interest to execute this transaction when the time comes—even though it destroys firm value.

But put yourself into the shoes of the old shareholders who own the entire firm and are trying to set up the corporate charter that maximizes the value of their firm, i.e., for sale to new investors at the highest price. You want to find the best capital structure *today*, i.e., before you have found investors. Clearly, today's purchasers of your debt will take into consideration what you may do in the future, and they will rationally demand compensation. If you cannot commit today *not* to undertake the $3-for-$1 transaction in the future, prospective bond buyers realize today (before the fact, short-hand **ex-ante**) that you (management) will have the incentive to execute it later (after the fact, short-hand **ex-post**), even if you tell them that you will not.

Bond buyers understand future conflict of interest. If managers ignored bondholder concerns, bondholders demand a higher interest rate upfront.

> **IMPORTANT**: *In estimating net present value and deciding on the appropriate price to pay, the buyers of financial securities take into account what the firm (with its shareholders and managers) is likely to do in the future.*

If the firm were to undertake this transaction in the future, what would it be worth today? It would be worth less than a firm that will not destroy $2 of value in the future. Therefore, management today has a choice: it can either find a way to commit today not to exploit bondholders in the future; or it can sell the firm today for its net present value, assuming value destruction tomorrow—because everyone realizes that managers will be trapped tomorrow into destroying $2 of firm value. To avoid this, managers should want to do everything in their power to constrain themselves from expropriating bondholders in the future. This will maximize the value of the firm in the future, which in turn maximizes the value of the firm today.

Therefore, managers need to pay attention to the needs of bondholders—even though they are voted into power only by equity holders and are always tempted to focus only on improving equity value.

> **IMPORTANT**: *The optimal capital structure is the structure that maximizes the value of the firm, not the value of the equity. Management should maximize firm owner value, not shareholder value.*
>
> *In order to obtain a better price for its bonds (a lower interest rate), the firm has the incentive to commit itself today (**Ex-Ante**) to treating bondholders well in the future (**Ex-Post**). The ex-ante capital structure that results in the highest firm value today is the optimal capital structure.*

This is the most important insight with respect to capital structure, and worth repeating: the cost of *ex-post* actions against bondholders is not only borne by bondholders tomorrow, but it is also borne by the owners today. Indeed, *caveat emptor* ("buyer beware") applies: bond and stock purchasers can only be hurt to the extent that future opportunistic actions by management are unforeseen surprises. Thus, it is in management's (owners') own interest today to commit not to exploit future owners and bondholders tomorrow—especially if everyone knows that when the time comes, management would like to change its mind.

The basics of capital structure theory is to realize that future events have impact on corporate value today. The goal is to use capital structure to maximize firm value today.

The web chapter on corporate governance will return to this idea.

Competition among management teams pressures firms to improve Capital Structures.

What would happen if the current management team cannot commit to avoid such bad future 3-for-1 exchanges? In this case, another management team with such a self-restraint capability would value the firm more highly than the current management team. It could purchase the firm and make an immediate profit. Therefore, competition among management teams can push firms towards the best capital structure. Again, the general thread emphasized throughout this chapter is that firms that can commit to do "the right thing" tomorrow (*ex-post*) are worth more today (*ex-ante*). It is a direct consequence that firms which maximize firm value are worth more than firms which maximize just shareholder value.

Solve Now!

Q 21.1 *Explain the difference between ex-ante and ex-post, especially in the capital structure context. Give an example where the two differ.*

Q 21.2 *Can an ex-post maximizing choice be ex-ante bad?*

21·2. Modigliani and Miller (M&M), The Informal Way

Anecdote: M&M: Milk, Cream, and Pizza
(The following is an excerpt from a speech by Merton Miller, accepting an honorary doctorate at Louvain, Belgium, in 1986. His coauthor, Franco Modigliani, had just won the first Nobel Prize; Merton Miller would receive his own Nobel Prize a few years later.)

How difficult it is to summarize briefly the contribution of these papers was brought home to me very clearly last October after Franco Modigliani was awarded the Nobel Prize in Economics in part–but, of course, only in part–for the work in finance. The television camera crews from our local stations in Chicago immediately descended upon me. "We understand," they said, "that you worked with Modigliani some years back in developing these M&M theorems and we wonder if you could explain them briefly to our television viewers." "How briefly?", I asked. "Oh, take 10 seconds," was the reply.

Ten seconds to explain the work of a lifetime! Ten seconds to describe two carefully reasoned articles each running to more than 30 printed pages and each with 60 or so long footnotes! When they saw the look of dismay on my face, they said: "You don't have to go into details. Just give us the main points in simple, common sense terms."

The main point of the first or cost-of-capital article was, in principle at least, simple enough to make. It said that in an economist's ideal world of complete and perfect capital markets, and with full and symmetric information among all market participants, the total market value of all the securities issued by a firm would be governed by the earning power and risk of its underlying real assets and would be independent of how the mix of securities issued to finance it was divided between debt instruments and equity capital. Some corporate treasurers might well think that they could enhance total value by increasing the proportion of debt instruments because yields on debt instruments, given their lower risk, are, by and large, substantially below those on equity capital. But, under the ideal conditions assumed, the added risk to the shareholders from issuing more debt will raise required yields on the equity by just enough to offset the seeming gain from use of low cost debt.

Such a summary would not only have been too long, but it relied on shorthand terms and concepts, like perfect capital markets, that are rich in connotations to economists, but hardly so to the general public. I thought, instead, of an analogy that we ourselves had invoked in the original paper. "Think of the firm," I said, "as a gigantic tub of whole milk. The farmer can sell the whole milk as is. Or he can separate out the cream and sell it at a considerably higher price than the whole milk would bring. (Selling cream is the analog of a firm selling low yield and hence high-priced debt securities.) But, of course, what the farmer would have left would be skim milk, with low butter-fat content and that would sell for much less than whole milk. Skim milk corresponds to the levered equity. The M&M proposition says that if there were no costs of separation (and, of course, no government dairy support programs), the cream plus the skim milk would bring the same price as the whole milk."

(continues.)

ANECDOTE: (continued.)

The television people conferred among themselves for a while. They informed me that it was still too long, too complicated and too academic. "Have you anything simpler?", they asked. I thought of another way that the M&M proposition is presented which emphasizes the notion of market completeness and stresses the role of securities as devices for "partitioning" a firm's payoffs in each possible state of the world among the group of its capital suppliers. "Think of the firm," I said, "as a gigantic pizza, divided into quarters. If now, you cut each quarter in half into eights, the M&M proposition says that you will have more pieces, but not more pizza."

Again there was a whispered conference among the camera crew and the director came back and said: "Professor, we understand from the press release that there were two M&M propositions. Maybe we should try the other one."

He was referring, of course, to the dividend invariance proposition and I know from long experience that attempts at brief statements of that one always cause problems. The term "dividend" has acquired too great a halo of pleasant connotations for people to accept the notion that the more dividends the better might not always be true. Dividends, however, as we pointed out in our article, do not fall like manna from heaven. The funds to pay them have to come from somewhere–either from cutting back on real investment or from further sales (or reduced purchases) of financial instruments. The M&M dividend proposition offered no advice as to which source or how much to tap. It claimed, rather, that once the firm had made its real operating and investment decisions, its dividend policy would have no effect on shareholder value. Any seeming gain in wealth from raising the dividend and giving the shareholders more cash would be offset by the subtraction of that part of their interest in the firm sold off to provide the necessary funds. To convey that notion within my allotted 10 seconds I said: "The M&M dividend proposition amounts to saying that if you take money from your left-hand pocket and put it in your right-hand pocket, you are no better off."

Once again whispered conversation. This time, they shut the lights off. They folded up their equipment. They thanked me for my cooperation. They said they would get back to me. But I knew that I had somehow lost my chance to start a new career as a packager of economic wisdom for TV viewers in convenient 10-second sound bites. Some have the talent for it; and some just don't.

These simple, common sense analogies certainly do less than full justice to the M&M propositions; crude caricatures or cartoons they may be but they do have some resemblance. So much, in fact, that looking back now after more than 25 years it is hard to understand why they were so strongly resisted at first. One writer–David Durand, the same critic who had so strongly attacked the Markowitz model–even checked out the prices for whole milk, skim milk and cream in his neighborhood supermarket. He found, of course, that the M&M propositions didn't hold exactly; but, of course, empirical relations never do.

21·3. Modigliani and Miller (M&M), The Formal Way In Perfect Markets

Consider an idealized, perfect world.

The famous **Modigliani-Miller** (**M&M**) propositions (honored with two **Nobel Prizes**) provide a good start to understanding firms' capital structure decisions. Modigliani and Miller argued that under perfect conditions, the total value of all financial securities is the same regardless of whether the firm is financed by equity or debt or anything in between. They proved their argument by showing that there would be arbitrage opportunities if the value of the firm depended on how it is financed. Because there should be no arbitrage in real life, it follows that firms should be able to choose any mix of securities without impact on the firm's value. This perfect world which M&M describe relies on the following assumptions:

- There are no transaction costs.
- Capital markets are perfectly competitive, with a large number of investors competing for many securities.
- There are no taxes.
- There are no bankruptcy costs (including costs of financial distress before bankruptcy).
- There are no differences in opinion and information;

These are typically referred to as "perfect capital markets." They are also enough to imply two more statements about the world:

- Borrowing and lending interest rates are equal.
- The firm's investment plan has been determined; it does not depend on its choice of financing.

Now, these M&M assumptions do not hold in reality. However, once you understand how the M&M argument works, it becomes easier to understand what happens when these assumptions are violated, and to understand how important such violations can be. Indeed, the next two chapters cover these cases.

The Modigliani-Miller Proposition state that in this ideal world, capital structure does not matter.

Here is how Modigliani and Miller's proof works: Take as given that the firm has already decided on what projects to take. It now wishes to consider how to finance these projects. Because we have also assumed that we all agree on all current and future projects' expected cash flows and proper discount rates, we must agree on the present value (PV) of these projects today. The M&M claim is that the PV of the firms' projects must equal to the PV of the firms' issued claims today. In other words, the claim is that if the firm issues 100% equity, the equity must sell for the PV of the projects. If the firm issues 50% debt and 50% equity, the two together must sell for the same PV. If the firm issues x% debt and (1-x%) equity, the two together must sell for PV.

If capital structure had a value influence, there would be arbitrage opportunities.

Why? Call the value of the projects under the best capital structure PV. (This is [almost by definition] the present value that the firm's projects can fetch in our perfect capital market, of course.) Assume that the managers could find—and actually did choose—a capital structure that makes the firm worth $1 less than PV. For clarity sakes, assume that the the firm is worth PV=$100 under the optimal capital structure of 80% equity and 20% debt; and assume that the firm is worth only $99 under the capital structure of 50% equity and 50% debt that the firm has actually chosen. Then, all one needs to do to get rich is to purchase all old equity and all old debt, i.e., the entire firm, for $99. Now

issue claims duplicating the optimal capital structure (assumed to be 80% equity/20% debt). These claims will sell for \$100, and we pocket \$1.

NERD NOTE: *This argument even works when you buy not 100% of the firm, but only 1% of the firm. That is, if you buy 1% of all the firm's securities, you will receive 1% of the projects' payoffs. You can then sell your securities repacked to imitate the new (and better) capital structure for 1% of the firm's value, and receive an arbitrage profit of 1%· \$1.*

Competition: Others would want to arbitrage, too—until the M&M proposition works (firm value is as if it was financed optimally).

Unfortunately, we would not be the only ones to notice this opportunity. After all, all information is assumed to be common knowledge (both by the firm and other investors). So, the old managers would simply ask for bids from other investors. The only price at which no one will overbid us for the right to purchase the firm's current securities is \$100. But notice that this means that the value of the old securities are equal to the price that the firm is worth under the optimal capital structure. So, regardless of the financial structure that managers choose, they can sell their claims for \$100, i.e. the present value of their projects.

ANECDOTE: Zero-Bond Tax Arbitrage in the 1980's

Has it ever happened that some managers chose an inferior capital structure, and outside investors bid up the prices of firms' securities in order to sell repackaged securities? Maybe. Up to the early eighties, the I.R.S. taxed bond coupons, the cash interest payment that bondholders received each year. In 1982, PepsiCo invented the first long-term zero-coupon bond, which allowed bondholders to in effect reinvest coupon without being taxed. To see this, consider an extreme situation in which the tax rate is 50% and the interest rate is 10%. A \$10,000 coupon bond could pay \$1,000 for two years, and then return the principal, in which case the government receives \$500 in tax payments each year. The investor could thus reinvest only \$500 to earn interest after the first year. In contrast, in a zero-coupon bond, the firm essentially promises to reinvest the full \$1,000 dollar on behalf of the investor.

Once it became clear that zero bonds were valuable, investment banks began to bid up the prices of already outstanding coupon bonds, and repackaged their payoff streams into the payoff streams of equivalent zero-coupon bonds. "Unfortunately," this process did not continue until the prices of all outstanding coupon bonds reflected the repackaging zero-bond value. Instead, the I.R.S. stepped in and changed the tax rules, taxing accrued interest instead of coupon payments.

Linking M&M under uncertainty to state-contingent payoffs (under risk-neutral pricing).

The M&M argument should really not come as a surprise to the reader. In Section 6·2 (Page 127), without calling it the M&M argument, we had already made use of it (that the cost of capital is not influenced by the chosen capital structure): a house was worth what the house was worth, not whether it was financed with debt or equity. This was M&M precisely. At the risk of boring the reader, we shall do one more quick example now. Assume that the value of the firm's projects will be either \$100 or \$200 with equal probability next year. The firm's value today will be $\$150/(1+r_{\mathsf{W}})$ where r_{W} is the appropriately risk-adjusted interest rate. The firm's projects are thus worth \$136.[36] today.

SIDE NOTE: We are running out of subscripts. We really should write $r_{\mathsf{W},0,1}$ to describe that we mean the rate of return on the overall firm W from 0 to 1. However, because the examples in this chapter are all one-period examples, we will drop the time subscripts. Instead, we use subscripts W (for the firm), E (for equity), and D (for debt). The present values (PV) in this chapter are always as of time 0, and the expected cash flows are always as of time 1. Sometimes, we use a subscript to spell out "pre-tax" or "post-tax."

NERD NOTE: *In real life, the firm's cost of capital (interest rate) depends on the correlation of its project payoffs with market-wide risk factors. Consequently, the cost of capital for equity is higher than the cost of capital for debt. However, to illustrate our arguments expediently, we here assume that investors are risk-neutral, so that regardless of the risk, the interest rate is 10%. In Section 6·2, we worked an example with differing costs of capital.*

Firm value is independent of capital structure.

Under hypothetical capital structure 1, the firm issues debt with face value (FV) \$50. Consequently, bondholders face no uncertainty, and will pay \$50/(1 + 10%) = \$45.^{45}. Equity holders will receive either \$50 or \$150, and are thus prepared to pay \$100/(1 + 10%) = \$90.^{91} (in a real-life example, the interest rate would have to be higher to compensate investors for risk). Simply adding the value of the firm's claims adds up to the same \$136.^{36}. Under hypothetical capital structure 2, the firm issues debt with face value \$150. Consequently, bondholders will now receive either \$100 or \$150, and are willing to pay \$113.^{64} today. Equity holders will receive \$0 or \$50, and are willing to pay \$22.^{73} for the privilege. Again, the value of all claims adds to \$136.^{36}.

Table 21.1. Illustration of the M&M Proposition with Risk-Neutral Investors

		Bad Luck	Good Luck	Today's Expected Value
Prob:		1/2	1/2	
Project	W	\$100	\$200	\$136.^{37}
Capital Structure 1: Bond with Face Value \$50				
Bond(FV=\$50)	D	\$50	\$50	\$45.^{45}
Equity	E	\$50	\$150	\$90.^{91}
Capital Structure 2: Bond with Face Value \$150				
Bond(FV=\$150)	D	\$100	\$150	\$113.^{64}
Equity	E	\$0	\$50	\$22.^{73}

The cost of capital in this example is 10% for all securities. (All insights would hold if the cost of capital varied with security risk. See next chapter.)

Arbitraging does not require purchase and sale of the entire firm; it also works with a fraction of the firm.

There are many ways to skin a cat. Yet another way to prove the Modigliani-Miller proposition relies on the ability of investors to effectively lever or unlever the firm *themselves*. For example, in the previous example, assume that the firm has chosen capital structure 1, but you prefer capital structure 2, because you prefer the equity payoffs of \$50 in the good state, \$0 in the bad state instead of the \$150/\$50 payoffs. How can you get the payoffs *you* prefer? All you need to do is borrow the present value of \$25 (i.e., \$22.^{73}), and purchase only half a \$90.^{91} share at \$45.^{45}. In the bad state, you now receive $1/2 \cdot \$50 = \$25$ from your equity holdings and you must repay \$25, so you end up with zero. In the good state, you now receive $1/2 \cdot \$150 = \$75$, repay \$25, and thus end up with \$50—but these are precisely the payoffs you wanted! How much did it cost you to "replicate" the payoffs from capital structure 2? You had outflows of \$45.^{45}, and inflows of \$22.^{73}, so your net capital outflow was \$22.^{73}—exactly the same cost as it would have been if the firm had chosen capital structure 2 for you.

IMPORTANT: *In Modigliani and Miller's (M&M) perfect world, arbitrage restrictions force the value of the firm to be the same, regardless of the firm's mix of debt and equity. When managers make investment decisions, they can ignore the financing options. Financing and Investment are two entirely separate issues. (Advance Warning: This total separation will not hold in an imperfect world!)*

NERD NOTE: *Although M&M did not explicitly state it, if investment decisions are reversible, the perfect markets assumptions would also allow us to prove that the firm's investment decisions do not influence the firm's value, either. By the same logic that was used to prove the M&M claim, investors would simply bid up the price of the company for the right to control the firm, to reverse the firm's sub-optimal investment decision, and to implement the optimal investment decision. If the firm did not choose the best investments, someone could buy the firm, redesign the charter to force the firm to proceed to the best real investment, and sell the firm for an arbitrage profit.*

Solve Now!

Q 21.3 *Under what assumptions does capital structure not matter?*

Q 21.4 *Explain the M&M argument to your 10-year old brother.*

Q 21.5 *What does risk-neutrality "buy" us in the M&M argument?*

21·4. DIVIDENDS

In a perfect world, dividends do not create value.

The second Modigliani-Miller proposition is even simpler than the first: it states that corporate dividend policy should not matter in their perfect market setting, either. From the corporate perspective, if managers pay $1 in dividends, this money has to come from somewhere. Dividends do not fall like manna from heaven, so no value (NPV) is created or destroyed when firms pay dividends. Money that was previously owned by investors but held inside the corporate shell is just being moved to the same investors, so that it is outside the corporate shell. The owners do not have any more or any less wealth because of the dividend payment. From the financial market perspective, if managers undertook a dividend policy that destroyed value, then any investor could step in to purchase the firm, fire the management, institute the better dividend policy, and resell the firm for the difference. With many investors vieing to do this if even just a penny can be earned, the only firm value and dividend policy that does not allow anyone to arbitrage (get rich without effort) is the value of the firm under an optimal dividend policy.

In a perfect world, firms always undertake the best projects.

Though not an M&M proposition, the perfect markets assumption also implies that management's project policy should not matter. If the firm were not to undertake all positive NPV projects and reject all negative NPV projects, you could buy all the shares, fire the old management, institute the best underlying firm investment project policy, resell all the shares, and earn the difference in firm value. Alas, you are not the only one: everyone can do this. Therefore, in this perfect world, firms always do the right thing, maximizing firm NPV.

Like the M&M capital structure proposition, the point of the M&M dividend proposition is not to argue that dividends do not matter. It is to point out what perferct market violations must be the case for dividend policy to matter, and how much these violations can matter. For example, if it costs a roundtrip premium of $10 million to purchase and then resell a firm, then it cannot be that the wrong dividend policy destroys more than $10 million. If it did, you could make money even in our imperfect world.

The M&M logic helps us think in our imperfect world.

Side Note: These days, the average dividend yield of large firms is around 1% of firm value a year. This is so little that the real-world market frictions are larger than what you could earn by correcting this policy if it were incorrect. That is, if the optimal payout would be 0% or 2% instead, the maximum 1% value increase is too little to make it worthwhile in the real world for anyone to step in and correct it.

When firms announce higher dividends, its value usually goes up. The reason is that it sends a credible signal from management about future firm prospects and managerial behavior that investors would otherwise not have. This can be the case, because in the real world, managers and investors do not have the same information. Only in the perfect market world is there no disagreement and equal information.

21·5. The Weighted Cost of Capital (WACC) in a Perfect M&M World

This chapter shows how to determine the values of debt and equity in the perfect scenario of the M&M world. To do so, it draws on Net Present Value, the Capital Asset Pricing Model, and Capital Structure concepts.

21·5.A. The Numerical Example

We now return to the example from Section 6·1 (Page 124): the house with the mortgage. You must remember that Chapter to fully understand this Chapter. Briefly, in the example, the *expected* rate of return was the same for projects of all risk classes. In contrast, in this chapter, we take into account that investors are risk averse, so that riskier investments have to offer higher expected rates of returns. Our goal is to compute the **Weighted Average Cost of Capital** (or WACC), which is the average cost of capital when a firm is financed with both debt and equity. The basic tools will be exactly the same as those in Section 6·1: payoff tables, promised rates of return, and expected rates of return.

All tools learned in Section 6·2 still apply under risk aversion.

From Chapter 20, we also know now that we can call debt and equity to be contingent claims on the underlying project. Although we continue calling this project a house (as we did in Section 6·1), you can consider the corporation to be the same as an unlevered house, the mortgage the same as corporate debt, and levered house equity ownership the same as corporate equity. There is no conceptual difference.

This example applies to more than just houses.

Recall the information from Section 6·1: The probability of sunshine is 80%, the probability of a tornado is 20%. If the sun shines, the house (firm) is worth $100,000; if the tornado strikes, the house is worth only $20,000. The appropriate cost of capital (at which investors are willing to borrow or save) is 10% for the overall house. We had computed that this house must be worth $76,363.[64].

Risk Aversion means expected debt interest rates of return are lower than project expected rates of return.

Table 21.2. All Provided Inputs For Valuing The House

		Financing Scheme 1	Financing Scheme 2	
		100% Equity	Bond promises \$28,125	Levered Equity after \$28,125 obligation
prob(Sunshine)= 80%	\$100,000.^{00}			
prob(Tornado)= 20%	\$20,000.^{00}			
$\mathcal{E}$ Payoff				
$\mathcal{E}$ Rate of Return (r)		10%	6%	
Price P_0 Today				

Here we introduce different costs of capital.

The novelty is that we now assume that Treasury bonds pay only about 5% *expected* rate of return. Through differences in expected rates of returns, we have defacto assumed that investors are risk-averse! The debt on the house is not exactly risk-free, though. Therefore, we assume that we take on a particular, risky bond that promises to pay \$28,125 and which requires only a 6% *expected* rate of return. (This 6% expected rate of return is appropriately higher than risk-free debt and appropriately lower than the 10% required expected rate of return for projects that are of the riskiness of "unlevered house" ownership.) Table 21.2 summarizes our given inputs.

Compute the state-contingent payoffs.

Our goal is to determine now what the appropriate cost of capital for the levered equity is. **Step 1:** Determine how much the house owners receive if they own the entire house (scheme 1) vs. if they promise \$28,125 to bond holders and only retain the levered equity (scheme 2). Naturally, in each state, the bond and the levered equity together must own the entire house, so:

		Financing Scheme 1	Financing Scheme 2	
		100% Equity	Bond promises \$28,125	Levered Equity after \$28,125 obligation
prob(Sunshine)= 80%	\$100,000.^{00}	\$100,000.^{00}	\$28,125.^{00}	\$71,875.^{00}
prob(Tornado)= 20%	\$20,000.^{00}	\$20,000.^{00}	\$20,000.^{00}	\$0.^{00}
$\mathcal{E}$ Payoff				
$\mathcal{E}$ Rate of Return (r)		10%	6%	
Price P_0 Today				

Compute the expected payoffs.

Step 2: Compute the expected value of each security, using the probabilities of sunshine vs. tornado. Note that the expected payoffs of the bond and the levered stock together must add up to the expected payoff on the house (i.e., as if the house were 100% equity financed).

		Financing Scheme 1	Financing Scheme 2	
		100% Equity	Bond promises \$28,125	Levered Equity after \$28,125 obligation
prob(Sunshine)= 80%	\$100,000.00	\$100,000.00	\$28,125.00	\$71,875.00
prob(Tornado)= 20%	\$20,000.00	\$20,000.00	\$20,000.00	\$0.00
$\mathcal{E}$ Payoff		\$84,000.00	\$26,500.00	\$57,500.00
$\mathcal{E}$ Rate of Return (r)		10%	6%	
Price P_0 Today				

Step 3: Discount the expected cash flows by the appropriate cost of capital demanded by the capital providers:

Discount the expected payoffs on the overall project and on the debt.

		Financing Scheme 1	Financing Scheme 2	
		100% Equity	Bond promises \$28,125	Levered Equity after \$28,125 obligation
prob(Sunshine)=80%	\$100,000.00	\$100,000.00	\$28,125.00	\$71,875.00
prob(Tornado)=20%	\$20,000.00	\$20,000.00	\$20,000.00	\$0.00
$\mathcal{E}$ Payoff		\$84,000.00	\$26,500.00	\$57,500.00
$\mathcal{E}$ Rate of Return (r)		10%	6%	
Price P_0 Today		\$76,363.64	\$25,000.00	

Step 4: Invoke our "perfect market" or M&M assumptions. Everyone can buy or sell without transaction costs, taxes, or any other impediments. By "absence of arbitrage," the value of the house if financed by a bond plus levered equity must be the same as the value of the house if it is 100% equity financed. Put differently, if you own all of the bond and all of the levered equity ownership, you own the same thing as the house—and vice-versa.

Determine the value of the levered equity.

SIDE NOTE: If the contingent claims were selling for less than the unlevered ownership, someone could purchase all bonds and stocks for a low price, and resell it as 100% equity for a higher price. If they were selling for more, someone could purchase the 100% equity of the house, and promise to pay the same payoffs as if the house was debt plus levered-equity financed. Naturally, this arbitrage need not be true if there are taxes or other transaction costs. After all, the fact that the total value remains the same depends on the **Perfect Market** assumption. Therefore, in the real world, this relationship holds only up to transaction costs (which are reasonably low for large corporations).

So, use the fact that the value of the levered equity plus the value of the bond should equal the total house value.

Use the "arbitrage" condition.

		Financing Scheme 1	Financing Scheme 2	
		100% Equity	Bond promises \$28,125	Levered Equity after \$28,125 obligation
prob(Sunshine)=80%	\$100,000.00	\$100,000.00	\$28,125.00	\$71,875.00
prob(Tornado)=20%	\$20,000.00	\$20,000.00	\$20,000.00	\$0.00
$\mathcal{E}$ Payoff		\$84,000.00	\$26,500.00	\$57,500.00
$\mathcal{E}$ Rate of Return (r)		10%	6%	
Price P_0 Today		\$76,363.64	\$25,000.00	\$51,363.64

Compute the appropriate expected rate of return on the levered equity.

Step 5: Levered equity ownership, which sells for \$51,363.^{64} and expects to pay off \$57,500.^{00}, offers an expected rate of return of $\$57,500.^{00}/\$51,363.^{64} - 1 = +11.95\%$. So, knowing only the cost of capital on a bond and on 100% stock, we could determine the cost of capital on levered equity!

		Scheme 1	Scheme 2	
		100% Equity	Bond promises \$28,125	Levered Equity after \$28,125 obligation
prob(Sunshine)=80%	\$100,000.^{00}	\$100,000.^{00}	\$28,125.^{00}	\$71,875.^{00}
prob(Tornado)=20%	\$20,000.^{00}	\$20,000.^{00}	\$20,000.^{00}	\$0.^{00}
$\mathcal{E}$ Payoff		\$84,000.^{00}	\$26,500.^{00}	\$57,500.^{00}
$\mathcal{E}$ Rate of Return (r)		10%	6%	11.95%
Price P_0 Today		\$76,363.^{64}	\$25,000.^{00}	\$51,363.^{64}

Small Detour: Compute the riskiness of a dollar investment in each financial instrument.

Summarize the rates of return in the two possible states on each financial claim:

	Contingent		Expected
	Tornado	Sunshine	Appropriate
Unlevered Ownership	$\frac{\$20,000}{\$76,364} - 1 =$	$\frac{\$100,000}{\$76,364} - 1 =$	$\frac{\$84,000}{\$76,364} - 1 =$
	-73.81%	$+30.95\%$	$+10.00\%$
Loan Ownership	$\frac{\$20,000}{\$25,000} - 1 =$	$\frac{\$28,125}{\$25,000} - 1 =$	$\frac{\$26,500}{\$25,000} - 1 =$
	-20.00%	$+12.50\%$	$+6.00\%$
Levered (post-Loan) Ownership	$\frac{\$0}{\$51,364} - 1 =$	$\frac{\$71,875}{\$51,364} - 1 =$	$\frac{\$57,500}{\$51,364} - 1 =$
	-100.00%	$+39.93\%$	$+11.95\%$

Debt is less risky than unlevered ownership which is less risky than levered ownership.

As was also the case in the example with risk-neutral investors (Figure 6.1, Page 131), the levered equity ownership is more risky than unlevered ownership, which in turn is more risky than the loan. But, unlike in the example with risk-neutral investors, the **Cost of Capital** (the expected rate of return at which an owner can obtain financing) is higher for levered equity ownership than it is for unlevered ownership, which in turn is higher than it is for loan ownership. Indeed, we could work out *exactly* how high this expected rate of return on levered equity ownership is by resorting to the "absence of arbitrage" argument in the perfect M&M world: Given the expected rate of return on the house and on the bond, we could determine the expected rate of return on levered equity ownership. (Alternatively, if we had known the appropriate expected rate of return on levered equity ownership and the rate of return on the bond, we could have worked out the appropriate expected rate of return on unlevered ownership.)

Side Note: In the real world, house owners would naturally like to pay the lowest rate of return on loans possible. Similarly, anyone selling a firm would like to receive the highest price (lowest cost of capital) possible. What the seller of the loan can get away with will depend on the demand and supply of capital. However, this does not tell us *why* demand and supply met at 6% for the debt and 11.95% for the levered equity in our example (although one implies the other).

So, what can tell us the appropriate expected rate of return for a risky security? But, of course, the CAPM from Chapter 17!

21·5.B. The WACC Formula (Without Taxes)

We can now translate the numerical example into a formula for the "Weighted Average Cost of Capital," or WACC. No matter which state will come about, the debt and equity together own the firm, which we shall call W:

A Line-by-line derivation of the WACC formula.

$$\begin{array}{lccccc} \text{Sunshine:} & \$28,125 & + & \$71,875 & = & \$100,000 \\ \text{Tornado:} & \$20,000 & + & \$0 & = & \$20,000 \\ \text{Any:} & \mathsf{D}_{t=1} & + & \mathsf{E}_{t=1} & = & \mathsf{W}_{t=1} \quad . \end{array} \tag{21.1}$$

Therefore, the expected value of debt and equity together must be equal to the expected value of the firm:

$$\begin{array}{ccccc} \$57,500 & + & \$26,500 & = & \$84,000 \\ \mathcal{E}(\mathsf{E}_{t=1}) & + & \mathcal{E}(\mathsf{D}_{t=1}) & = & \mathcal{E}(\mathsf{W}_{t=1}) \end{array} \tag{21.2}$$

Each expected value in the future is one plus the expected rate of return multiplied by the value today. (The value today is known, the rate of return is expected.) Thus,

$$\begin{array}{ccccc} \$51,363 \cdot (1+11.95\%) & + & \$26,500 \cdot (1+6\%) & = & \$76,363.^{64} \cdot (1+10\%) \\ \mathcal{E}\left[\mathsf{E}_{t=0} \cdot (1+r_{\mathsf{E}})\right] & + & \mathcal{E}\left[\mathsf{D}_{t=0} \cdot (1+r_{\mathsf{D}})\right] & = & \mathcal{E}\left[\mathsf{W}_{t=0} \cdot (1+r_{\mathsf{W}})\right] \\ \mathsf{E}_{t=0} \cdot [1+\mathcal{E}(\tilde{r}_{\mathsf{E}})] & + & \mathsf{D}_{t=0} \cdot [1+\mathcal{E}(\tilde{r}_{\mathsf{D}})] & = & \mathsf{W}_{t=0} \cdot [1+\mathcal{E}(\tilde{r}_{\mathsf{W}})] \quad . \end{array} \tag{21.3}$$

Divide by $\mathsf{W}_{t=0}$ to express this in terms of percentages of firm value:

$$\begin{array}{ccccc} \left(\dfrac{\$51,363.^{00}}{\$76,363.^{64}}\right) \cdot (1+11.95\%) & + & \left(\dfrac{\$26,500.^{00}}{\$76,363.^{64}}\right) \cdot (1+6\%) & = & 1+10\% \\ \left(\dfrac{\mathsf{E}_{t=0}}{\mathsf{W}_{t=0}}\right) \cdot [1+\mathcal{E}(\tilde{r}_{\mathsf{E}})] & + & \left(\dfrac{\mathsf{D}_{t=0}}{\mathsf{W}_{t=0}}\right) \cdot [1+\mathcal{E}(\tilde{r}_{\mathsf{D}})] & = & [1+\mathcal{E}(\tilde{r}_{\mathsf{W}})] \quad . \end{array} \tag{21.4}$$

$\mathsf{E}_{t=0}/\mathsf{W}_{t=0}$ is the weight of equity in the firm's value, so we can call it w_{E}. Similarly, $\mathsf{D}_{t=0}/\mathsf{W}_{t=0}$ is better written as w_{D}.

SIDE NOTE: We really should write these as $w_{\mathsf{E},t=0}$ and $w_{\mathsf{D},t=0}$, just as we should write $r_{\mathsf{E},t=0,1}$. But no one is so explicit, so neither will we. When no time subscript is present, we mean "now."

$$\begin{array}{ccccc} 67.26\% \cdot (1+11.95\%) & + & 32.74\% \cdot (1+6\%) & = & 1+10\% \\ w_{\mathsf{E}} \cdot [1+\mathcal{E}(\tilde{r}_{\mathsf{E}})] & + & w_{\mathsf{D}} \cdot [1+\mathcal{E}(\tilde{r}_{\mathsf{D}})] & = & [1+\mathcal{E}(\tilde{r}_{\mathsf{W}})] \quad . \end{array} \tag{21.5}$$

Multiply the weight percentages into the brackets,

$$\begin{array}{ccccccccc} 67.26\% & + & 67.26\% \cdot 11.95\% & + & 32.74\% & + & 32.74\% \cdot 6\% & = & 1+10\% \\ w_{\mathsf{E}} & + & w_{\mathsf{E}} \cdot \mathcal{E}(\tilde{r}_{\mathsf{E}}) & + & w_{\mathsf{D}} & + & w_{\mathsf{D}} \cdot \mathcal{E}(\tilde{r}_{\mathsf{D}}) & = & 1+\mathcal{E}(\tilde{r}_{\mathsf{W}}) \quad . \end{array} \tag{21.6}$$

Because debt and equity own the firm, $w_{\mathsf{D}} + w_{\mathsf{E}} = 1$, and the "+1" terms on both sides cancel. This is the so called **Weighted Average Cost of Capital** (WACC) Formula:

$$\begin{array}{ccccc} 67.26\% \cdot 11.95\% & + & 32.74\% \cdot 6\% & = & 10\% \\ w_{\mathsf{E}} \cdot \mathcal{E}(\tilde{r}_{\mathsf{E}}) & + & w_{\mathsf{D}} \cdot \mathcal{E}(\tilde{r}_{\mathsf{D}}) & = & \mathcal{E}(\tilde{r}_{\mathsf{W}}) \quad . \end{array} \tag{21.7}$$

Chapter 22 will explain how WACC must be modified in the presence of corporate income taxes.

The promised debt interest rate is not used in the WACC formula; the expected rate is.

One item to remember: the cost of debt in the WACC formula is not the interest rate that the bank is charging. The bank's quoted interest rate is the promised rate of return to debt, and therefore higher than the expected interest rate (by the default premium). So, how do you get the expected rate of return on debt? Pretty much the same way as you get the expected rate of return on equity: if you believe the CAPM. then you try to estimate the beta from the debt's historical monthly rates of return, and then you use the CAPM equation. Fortunately, debt betas are often fairly small, especially for short-term and default-free debt, in which case you will end up with an r_D reasonably close to the risk-free rate. As we need it, the CAPM only provides expected rates of return, due to the time-premium and the systematic risk premium, and pretty much ignores the debt's idiosyncratic risk.

21·5.C. The Big Picture: How to Think of Debt and Equity

IMPORTANT: *Think of debt and equity, contingent claims, and contingent claims valuation is as summarized in the following bullet points.*

- Debt and equity claims partition the firm's payoffs.
- As explained in Chapter 6, levered equity (i.e., the residual claim after the debt is satisfied) is riskier than outright ownership (i.e., the firm itself). Debt is safer than outright ownership (i.e., the firm itself).
- In a world of uncertainty, riskier investments have to offer higher expected rates of return.
- Levered equity has to offer a higher expected rate of return than outright ownership. Debt can offer a lower expected rate of return than outright ownership.
- In the M&M world, an "absence of arbitrage" relationship ensures that the total value of a project equals the sum-total of the values of all its financing instruments.

  This implies that the appropriate expected rate of return on outright ownership has a specific relationship with the expected rates of return on the bond and stock:

$$\mathcal{E}(\tilde{r}_{\text{Firm}}) = w_{\text{Equity}} \cdot \mathcal{E}(\tilde{r}_{\text{Equity}}) + w_{\text{Debt}} \cdot \mathcal{E}(\tilde{r}_{\text{Debt}}) \quad , \tag{21.8}$$

  where the weights w_{Equity} and w_{Debt} are the values of equity and debt when quoted as a fraction of the overall firm value. This overall cost of capital is also called WACC.
- In the perfect M&M world, the project's WACC remains the same, no matter how the firm is financed. It is determined by the underlying payoffs of the project.

21·6. A Major Blunder: If all securities are more risky, is the firm more risky?

Many practitioners commit a serious logical mistake. They argue that

More debt does not increase the firm's cost of capital, because it increases both debt and equity cost of capital.

1. If the firm takes on more debt, the debt becomes riskier and the cost of capital for the debt (r_D) rises;
2. If the firm takes on more debt, the equity becomes riskier and the cost of capital for the equity (r_E) rises.
3. Because the firm consists of only debt and equity, the firm also becomes riskier when the firm takes on more debt.

The fact that both debt and equity become riskier as the firm takes on more leverage does not mean that the firm becomes riskier.

The first two statements are correct. With higher leverage, the cost of capital on debt increases, because the debt becomes riskier: in default, the debt is less likely to receive what it was promised. The equity also becomes riskier: the cost of capital on equity rises, because in financial default, which is now also more likely to occur, more cash goes to the creditors before equity holders receive anything. It is the final statement—therefore, the firm becomes riskier—that is wrong. The reason is that when the firm takes on more debt, the weight on the debt (w_D) increases and the weight on the equity ($w_E = 1 - w_D$) decreases. Because the cost of capital for debt (r_D) is lower than the cost of capital for equity (r_E), the weighted sum remains the same. Here is an example, where I have made up appropriate costs of capital to illustrate the point:

$$\begin{array}{lrcrcr} \text{Low Debt} & 30\% \cdot 5.0\% & + & 70\% \cdot 12.2\% & = & 10\% \\ \text{High Debt} & 60\% \cdot 6.0\% & + & 40\% \cdot 16.0\% & = & 10\% \\ & w_D \cdot \mathcal{E}(\tilde{r}_D) & + & w_E \cdot \mathcal{E}(\tilde{r}_E) & = & \mathcal{E}(\tilde{r}_W) \ . \end{array} \tag{21.9}$$

This example shows that statements 1 and 2 are correct and statement 3 is incorrect: the costs of capital for both debt and equity are higher when the firm has more leverage, but the overall cost of capital for the firm has not changed. In the perfect M&M world, the overall cost of capital is independent of the mix between debt and equity.

Solve Now!

Q 21.6 *In a perfect market, if the cost of equity for a company is 15% and the cost of debt is 10%, and if the company is financed by 80% debt and by 20% equity, what would be the company's cost of equity if it reduced its debt from 80% to 50%, so that it could qualify as a AAA rated firm with a debt interest rate of 8%?*

Q 21.7 *Compared to Firm B, Firm A has both a higher cost of capital for its debt and a higher cost of capital for its equity. Does this necessarily imply that firm A has a higher cost of capital for the overall firm than Firm B?*

21·7. Using the CAPM and WACC Cost of Capital in the NPV Formula

NPV, WACC, and CAPM are often all used together.

We have yet to combine NPV, WACC, and the CAPM. This is easy: we use the CAPM to provide appropriate expected rates of returns on debt and equity, compute the weighted average to obtain a WACC, and then use this WACC as the denominator in the NPV formula. For example, consider a project that can be financed with low-risk debt with a market-beta of 0.1, worth \$400 today; and high-risk equity with a market-beta of 2.5, worth \$250 today. The risk-free rate of return is 4%, the equity premium is 3%. What is the cost of capital of this project?

The standard method is to first compute the appropriate expected rates of return for the debt and the equity,

$$\begin{aligned}
\mathcal{E}(\tilde{r}_{\text{debt},\mathcal{M}}) &= 4\% + 3\% \cdot 0.1 = 4.3\% \ , \\
\mathcal{E}(\tilde{r}_{\text{equity},\mathcal{M}}) &= 4\% + 3\% \cdot 2.5 = 11.5\% \ , \\
\mathcal{E}(\tilde{r}_{i,\mathcal{M}}) &= r_{\mathcal{F}} + \left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right] \cdot \beta_{i,\mathcal{M}} \ .
\end{aligned} \tag{21.10}$$

Second, compute the WACC:

$$\begin{aligned}
\text{WACC} &= \left(\frac{\$400}{\$400+\$250}\right) \cdot 4.3\% + \left(\frac{\$250}{\$400+\$250}\right) \cdot 11.5\% \\
&= 61.5\% \cdot 4.3\% + 38.5\% \cdot 11.5\% \\
&= 7.1\% \ .
\end{aligned} \tag{21.11}$$

An alternative method relies on the weighted average project beta,

$$\begin{aligned}
\beta_{\text{firm},\mathcal{M}} &= \left(\frac{\$400}{\$400+\$250}\right) \cdot 0.1 + \left(\frac{\$250}{\$400+\$250}\right) \cdot 2.5 = 1.025 \ . \\
\beta_{\text{firm},\mathcal{M}} &= w_{\text{debt}} \cdot \beta_{\text{debt},\mathcal{M}} + w_{\text{equity}} \cdot \beta_{\text{firm},\mathcal{M}} \ .
\end{aligned} \tag{21.12}$$

This means that the project's cost of capital is

$$\begin{aligned}
\mathcal{E}(\tilde{r}_{\text{firm},\mathcal{M}}) &= 4\% + 3\% \cdot 1.025 = 7.1\% \\
\mathcal{E}(\tilde{r}_{i,\mathcal{M}}) &= r_{\mathcal{F}} + \left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right] \cdot \beta_{i,\mathcal{M}} \ ,
\end{aligned} \tag{21.13}$$

which is the same as the cost of capital estimate in Formula 21.11. We could now use this cost of capital estimate to discount the project's expected cash flows to obtain a present value estimate. For example, if the project earns \$800 with probability 48% and \$600 with probability 52%, then

$$\text{PV} = \frac{48\% \cdot \$800 + 52\% \cdot \$600}{1 + 7.1\%} \approx \$650. \tag{21.14}$$

Solve Now!

Q 21.8 *Assume the risk-free rate of return is 3% and the equity premium is 4%. A firm worth \$100 (all numbers in million dollars) has a market beta of 3. A new project costing \$10 appears, which is expected to pay off \$11 next year. The beta of this new project is 0.5. However, the firm evaluates all projects by its overall cost of capital. Would this firm*

take the project? How does the beta and the value of the firm change if it takes vs. if it does not take this project?

21·8. Summary

The chapter covered the following major points:

- Entrepreneurs have an incentive to set up a capital structure that maximizes firm value, not equity value, even if—or because—managers later would want to behave opportunistically.
- In the perfect market scenario of M&M, the value of all securities is equal to the value of the firm's underlying projects, and thus unrelated to either the split between debt and equity, or the dividend policy of the firm.
- Arbitrage in this perfect world forces the firm's cost of capital to be invariant to the split between debt and equity. It is equal to the weighted average cost of capital (WACC) of debt and equity.
- Higher leverage does not imply that the overall cost of capital increases, even though both debt and equity become riskier.
- The CAPM is compatible with this perfect-world point of view.

A. Nerd Appendix: Compatibility of Beta, the WACC, and the CAPM Formulas in a Perfect World.

Reconciling Beta, WACC, and the CAPM.

For the nerd, we now show that the "leverage adjustment for beta" Formula 17.9 (Page 410), the WACC Formula 21.8 (Page 510), and the CAPM Formula 18.4 (Page 433) are mutually compatible, at least in the perfect markets scenario.

Recap WACC.

In this chapter, we developed the basic WACC formula (the cost of capital for the overall firm W—not to be confused with $\mathcal{F}$, the subscript for the risk-free security):

$$\mathcal{E}(\tilde{r}_{\mathsf{W}}) \;=\; w_{\mathsf{D}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{D}}) + w_{\mathsf{E}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{E}}) \quad .$$

where W is the firm, D is the total debt, and E is the total equity of the firm.

Recap CAPM.

Substitute the CAPM Formula 18.4 into the three expected rates of return in the WACC Formula:

$$\begin{aligned} r_{\mathcal{F}} + \Big[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\Big]\cdot\beta_{\mathsf{W},\mathcal{M}} \;&=\; w_{\mathsf{D}}\cdot\Big\{r_{\mathcal{F}} + \Big[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\Big]\cdot\beta_{\mathsf{D},\mathcal{M}}\Big\} \\ &+\; w_{\mathsf{E}}\cdot\Big\{r_{\mathcal{F}} + \Big[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\Big]\cdot\beta_{\mathsf{E},\mathcal{M}}\Big] \quad . \end{aligned}$$

And show the compatibility.

Now pull out the risk-free rates of returns,

$$\begin{aligned} & r_{\mathcal{F}} + \Big[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\Big]\cdot\beta_{\mathsf{W},\mathcal{M}} \\ = {} & w_{\mathsf{D}}\cdot r_{\mathcal{F}} + w_{\mathsf{E}}\cdot r_{\mathcal{F}} + w_{\mathsf{D}}\cdot\Big\{\Big[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\Big]\cdot\beta_{\mathsf{D},\mathcal{M}}\Big\} + w_{\mathsf{E}}\cdot\Big\{\Big[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\Big]\cdot\beta_{\mathsf{E},\mathcal{M}}\Big\} \quad . \end{aligned}$$

Recognize that $(w_{\mathsf{E}} + w_{\mathsf{D}}) = 1$, so $(w_{\mathsf{E}} + w_{\mathsf{D}})\cdot r_{\mathcal{F}} = r_{\mathcal{F}}$, so

$$\Big[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\Big]\cdot\beta_{\mathsf{W},\mathcal{M}} = w_{\mathsf{D}}\cdot\Big[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\Big]\cdot\beta_{\mathsf{D},\mathcal{M}} \;+\; w_{\mathsf{E}}\cdot\Big[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\Big]\cdot\beta_{\mathsf{E},\mathcal{M}} \quad .$$

Divide by $\Big[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\Big]$ to obtain

$$\beta_{\mathsf{W},\mathcal{M}} \;=\; w_{\mathsf{D}}\cdot\beta_{\mathsf{D},\mathcal{M}} + w_{\mathsf{E}}\cdot\beta_{\mathsf{E},\mathcal{M}} \quad ,$$

which is exactly our relationship in Formula 17.9, which relates betas to one another! Indeed, all three formulas share the same intuition: firms and securities with higher betas are riskier and thus have to offer higher expected rates of returns.

Solve Now!

Q 21.9 *A firm consists of 60% equity with a market-beta of 2, and 40% debt with a market-beta of 0.25. The equity premium is 5%, the risk-free rate is 3%. Compute the firm's cost of capital from the overall-firm beta. Then compute the equity's cost of equity capital and debt's cost of debt capital, and compute the firm's cost of capital from these costs of capital.*

A. Q&A: Answers

1. Ex-ante means "before the fact"; Ex-post means "after the fact." To the extent that owners can set up a situation (charter) that encourages best (i.e., from a perspective of the firm) ex-post behavior, the ex-ante value (for which the firm can be sold right now) is maximized. However, if the situation (charter) is such that owners will later try to expropriate others or such that managers make bad decisions in the future, the ex-ante value today is less.
2. Yes. See the example of the \$3 for \$1 transaction in the text.
3. A perfect market: no transaction costs, perfect competition, no taxes and bankruptcy costs, and no differences in information.
4. The idea is to explain it really simply.
5. Nothing really. We do not need it. We only use it because it makes the Tables simpler to compute.
6. First, compute the WACC without taxes: $0.8 \cdot 10\% + 0.2 \cdot 15\% = 11\%$. We now want to compute the cost of debt that satisfies $0.5 \cdot 8\% + 0.5 \cdot x = 11\%$, so $x = 14\%$.
7. No. The example in the "Important Error" Section illustrates this fallacy.
8. The solution proceeds the same way as in the text on Page 419. The project should have an appropriate rate of return of

$$\mathcal{E}(\tilde{r}_i) = 3\% + 4\% \cdot 0.5 = 5\% \ . \tag{21.15}$$

With a 10% expected rate of return, the project would not be taken if the firm used a beta of 3 (implying a cost of capital of 15%, which is higher than the expected rate of return of 10%). Nevertheless, the project has a net present value of

$$\text{NPV} = -\$10 + \frac{\$10 \cdot (1 + 10\%)}{1 + 5\%} = \$0.^{48} \ , \tag{21.16}$$

which is lost if it is not taken. Please confirm that this is also the outcome if the combined firm value is computed.

9. Using the first method, the overall beta is $60\% \cdot 2 + 40\% \cdot 0.25 = 1.3$. Using the CAPM, the cost of capital is $\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{M}} = 3\% + 5\% \cdot 1.3 = 9.5\%$. Using the second method, the equity's cost of capital is $\mathcal{E}(\tilde{r}_{\text{E}}) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{M}} = 3\% + 5\% \cdot 2 = 13\%$, the debt's cost of capital is $\mathcal{E}(\tilde{r}_{\text{E}}) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{M}} = 3\% + 5\% \cdot 0.25 = 4.25$. Therefore, the firm's cost of capital is $\mathcal{E}(\tilde{r}_i) = 60\% \cdot 13\% + 40\% \cdot 4.25\% = 9.5\%$.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 22

The Corporate Tax Advantage of Debt

The Tax-Adjusted Weighted Average Cost of Capital (WACC)

last file change: Jun 14, 2004 (13:38h)

last major edit: n/a

The presence of taxes in the real world may be the most glaring violation of the M&M assumptions. If the firm has to pay corporate income taxes, managers can use capital structure policy to create value. To do so, they rely either on the APV formula or on the tax-adjusted WACC formula. This chapter explains it all.

Note that the relevant tax that this chapter is concerned with is only the *corporate* income tax, not the *personal* income tax. (You can assume our shareholders—or yourself—to be a tax-exempt pension fund, if this helps you.) The next chapter will consider both corporate and personal income taxes.

22·1. Capital Budgeting If Equity and Debt Were Equally Taxed

Table 22.1. Hypothetical Firm

Investment Cost in Year 0	\$200
Before-Tax Return in Year 1	\$280
Before-Tax Net Return From Year 0 to Year 1	\$80
Corporate Income Tax Rate (τ)	30%
Appropriate Cost of Capital From 0 to 1	12%

Side Note: Again, we are running out of subscripts. We really should write $r_{\mathsf{W},0,1}$ to describe that we mean the rate of return on the overall firm W from 0 to 1. However, because most of the examples in this chapter are one-period, we will drop the time subscripts if there is no risk of confusion. We use subscripts W (for the firm), E (for equity), and D (for debt). The present values (PV) in this chapter are always as of time 0, and the expected cash flows are always as of time 1. Sometimes, we use a subscript to spell out "before-tax" or "after-tax."

A Basic Corporate Example

In this chapter, we will follow the simple hypothetical firm that is described in Table 22.1. As in Section 4·3, we follow the standard practice to denote the tax rate with the Greek letter τ [**Tau**]. We start with a brief discussion of an entirely unrealistic case, in which the firm faces one tax rate, regardless of how it is financed. This helps to illustrate that investors care about after-tax returns, not before-tex returns.

Taxes mean that the after-tax rate of return is lower than the before-tax rate of return.

Consider financing our firm entirely with equity. With \$280 in before-tax earnings on the \$200 investment, our firm has a "before corporate income tax" internal rate of return of $(\$280 - \$200)/\$200 = 40\%$. But, with taxes to the tune of 30% on its net return of \$80, Uncle Sam collects \$24. The firm's "post corporate income tax" net rate of return is therefore only $(\$256 - \$200)/\$200 = 28\%$.

Investors demand a certain rate of return, regardless of how the firm gets there.

Now assume that financial markets are such that investors are willing to provide capital for projects with this riskiness at a rate of 12%. Naturally, they do not care what happens inside the firm, only what the firm will pay them in the end. It is all the same to investors whether the firm earns 12% before-tax and manages to avoid all internal income taxes; or whether the firm earns 24% and pays half of it in corporate income taxes; or whether the firm earns 600% of which 98% is confiscated by the government ($1 + 600\% \cdot (1 - 98\%) = 1 + 12\%$). From the perspective of our small firm, we are price-takers, and our investors only care about the cash that we will ultimately return to them—and this (after-corporate income tax) rate of return has to be 12%. Now, if the specific firm in our example faces a 30% corporate tax rate, it must earn 17.14% in before-tax rate of return in order to be able to offer investors 12% in actual rate of return. Check this: an investment of \$100 that turns into \$117.^{14} has to pay Uncle Sam 30% in taxes on income of \$17.^{14} for a total income tax of \$5.^{14}, which leaves the firm \$112 to return to its investors after the corporate income tax is paid. Returning to our "\$280 before-corporate-income-tax" firm, with its 12% required after-corporate-income tax cost of capital, the firm's actual PV is

$$\mathsf{PV} = \frac{\mathcal{E}(\mathrm{CF}_{\text{after-tax}})}{1 + \mathcal{E}(\tilde{r}_{\text{after-tax}})} = \frac{\$256}{1 + 12\%} = \$228.^{57} \quad . \tag{22.1}$$

22·2. Differential Taxation in The U.S. Tax Code

We now move on to a more realistic tax code. In many countries, the United States included, individuals and corporations face similar tax treatments, tax schedules, and tax rates. Although tax code details vary year to year, country to country, state to state, county to county, and even city to city, most tax codes are pretty similar in spirit (if not in effective tax rates). Thus, the concepts of this chapter (and the concepts of the next chapter) apply pretty universally.

Tax codes worldwide violate the M&M no-tax assumption.

When corporations earn money, the form of payout matters. Firms pay taxes on their earnings *net of interest payments*. That is, unlike dividend distributions or money used to repurchase shares or money reinvested, the I.R.S. considers interest payments to be a cost of production. Therefore, it allows the payment of interest to be treated as a before-tax expense rather than as an after-tax distribution of earnings. As we explained on Page 19, the result is that a corporation saves on taxes when it distributes its earnings in the form of interest payments. For example, if PepsiCo's operations really produced $100, and if $100 of interest were owed, then PepsiCo could pay the full $100 to its creditors and Uncle Sam would get nothing. However, if not paid out in interest, Uncle Sam would first collect corporate income taxes, say 30%. PepsiCo could only keep (or distribute) the $70 that would be left over. The point of this chapter is to show how to exploit this difference in the relative tax treatment between payments to debt and all other uses of money. It allows the astute CFO to add value by choosing a clever capital structure.

Tax codes subsidizes borrowing: firms can pay interest from before-tax income, but pay dividends from after-tax income.

Side Note: Like corporations paying income taxes on earnings net of interest payments, individuals pay income taxes on their incomes net of **mortgage interest** payments. Unfortunately, when it comes to non-mortgage interest payments (e.g., car loan payments), the IRS treats individuals differently from corporations, in that individuals are not allowed to deduct such other interest payments.

Personal advice: in Section 4·3, we compared tax-exempt and taxable savings, but we can do the same analysis for tax-exempt and taxable borrowings. Given the choice of two, say equal, interest rate loans, choose the mortgage! To illustrate why it is better to pay interest with before-tax dollars rather than after-tax dollars, assume you earn $100,000 and are in the 30% tax bracket. If you have mortgage interest payments of $70,000, because you are permitted to pay the $70,000 with before-tax income, you would be left with $30,000 worth of before-tax income, a tax obligation of $10,000, and after-interest after-tax income of $20,000. In contrast, if you have car loan payments of $70,000, you would first have to pay $30,000 in tax. You would be left with $70,000 in after-tax before-interest income, all of which would have to pay the interest, so that you would be left with $0 in after-tax after-interest income.

ANECDOTE: Special Tax Breaks and Corporate Welfare
"Special Income Tax Provisions" are tax breaks enacted by Congress for specific activities, often on behalf of a single corporation. The special income tax provisions amount are commonly estimated to be about $1 trillion a year—more than the total amount of federal discretionary spending! These provisions are a main cause why corporations—large corporations, really—have paid less and less in income taxes relative to the rest of the population and relative to other OECD countries. In 1965, corporate income taxes were 4.1% of U.S. GDP; in 2000, about 2.5%; in 2002, about 1.5%. And, for comparison to the 1.5% U.S. tax rate, in 2000, Germany's rate was 1.8%, Canada's rate 4.0%.

It would be wonderful if the low U.S. corporate income tax rate would attract businesses to locate into the U.S. and to create jobs. Alas, because the low effective corporate income tax rates comes about through strange corporate tax shelters (often through relocation of headquarters into foreign countries), the U.S. often ends up with the worst of both worlds: both incentives for companies to move out of the United States and low corporate income tax receipts. The only president in recent history to buck the trend may have been Ronald Reagan, who slashed both the corporate income tax and the ability of companies to circumvent it.

Source: "Testimony of Robert S. McIntyre" (www.ctj.org).

Solve Now!

Q 22.1 *A debt-equity hybrid security is making a payout of $500 to its holders. If the firm is in the 33% tax bracket, how much does the firm have to earn if the I.R.S. designates the payment an interest payment? How much does the firm have to earn if the I.R.S. designates the payment a dividend distribution?*

22·3. Firm Value Under Different Capital Structures

Introducing an interest tax subsidy leads to a corporate preference for debt.

We know that firms are indifferent between debt and equity in a perfect world. We also know that Uncle Sam subsidizes firms which pay interest, relative to firms which either retain earnings or pay dividends. Therefore, we know that, *on tax grounds*, firms should have a preference for debt. Our goal now is to determine the exact value of the firm in the presence of this tax subsidy of debt interest payments, afforded by the I.R.S. treatment.

In the next few chapters, we will describe why too much debt can create other costs, which raise the firm's overall cost of capital. Fortunately, the formulas in this chapter will still hold, because these other costs will be reflected in different cost of capital (r) components.

Table 22.2. Two Financing Scenarios for a 1-Year Firm

Investment Cost in Year 0	$\$200.^{00}$
Before-Tax Return in Year 1	$\$280.^{00}$
Before-Tax Net Return From Year 0 to Year 1	$\$80.^{00}$
Corporate Income Tax Rate (τ)	30%
Appropriate Average Cost of Capital From 0 to 1	12%

Scenario 1: 100% Equity Financing.

Taxable Profits Next Year	$\$80.^{00}$
Corporate Income Taxes Next Year	$\$24.^{00}$
Owners Will Keep Next Year	$\$56.^{00}$

Scenario 2: \$200 Debt Financing at 11%. Rest is Levered Equity.

Interest Payments	$\$22.^{00}$
Taxable Profits Next Year	$\$58.^{00}$
Corporate Income Taxes Next Year	$\$17.^{40}$
Equity Owners Will Keep Next Year	$\$40.^{60}$
Equity+Debt Owners Will Keep Next Year	$\$22.^{00} + \$40.^{60} = \$62.^{60}$

22·3.A. Future Corporate Income Taxes and Owner Returns

The benchmark: value without any tax shelters.

First work out the value of the firm if it is 100% equity financed, i.e., if the firm does not exploit the I.R.S.' help. The all-equity Scenario 1 in Table 22.2 shows that our firm then earns \$280 on an investment of \$200. At a 30% corporate income tax rate, it will pay corporate income taxes of $30\% \cdot \$80 = \$24$, and then could pay out \$56 in dividends.

If debt financed, Uncle Sam gets less; therefore, the owners get more.

Table 22.2 also shows a second scenario, in which the firm borrows \$200 today at an interest rate of 11% for interest payments next year of \$22. Therefore, its corporate profits will be $\$80 - \$22 = \$58$, on which it would have to pay Uncle Sam $\$17.^{40}$, permitting owners (creditors and shareholders—and you may be both) to keep $\$62.^{60}$, the sum of \$22 for its creditors and $\$40.^{60}$ for its equity holders. Relative to the 100% equity financed case, financing via debt allows the owners to shelter $\$62.^{60} - \$56.^{00} = \$6.^{60}$ from corporate income taxes. A quicker way to compute the tax savings is to multiply the tax rate by the interest payments: If the I.R.S. allows the firm to deduct \$22 in interest payments, the firm will save $\$22 \cdot 30\% = \$6.^{60}$ in corporate income taxes. These $\$6.^{60}$ in tax savings will occur next year, and will therefore have to be discounted back.

ANECDOTE: The RJR Buyout Tax Loophole
In a **Leveraged Buyout** (**L.B.O.** for short) leverage can increase dramatically—and this can significantly reduce corporate income taxes. In 1988, First Boston's plan to take over R.J.R. Nabisco relied on an esoteric tax loophole just about to be closed. By "monetizing" its food operations, a fancy way to increase leverage, the deferring of taxes would have saved an estimated $3-$4 billion dollars of R.J.R.'s corporate income taxes—which would have increased the annual federal U.S. deficit by 2 percent!

22·3.B. The Discount Factor on Tax Obligations and Tax Shelters

We know the future tax-related cash flows. How do we discount them?

Table 22.2 showed that if the firm is 100% equity financed, corporate income taxes will be $24 next year. If the project is financed with $200 in debt at an 11% interest rate, corporate income taxes will be $17.40 next year. The interest payment designation of $22 has create a *future* tax savings of $6.60. So, what should you use as the appropriate discount rate (cost of capital) for the future tax obligation (either $24 or $17.40) or for the relative tax shelter ($6.60)?

A simple example with risky payoffs.

The easiest way to gain intuition about proper discount rates for taxes and tax shelters is an example, in which the debt is risk-free. For the moment, assume that the value of the firm will be either $250 (bad) or $310 (good) with equal probability. Therefore, the $200 debt at 11% interest is risk-free. Now, if a cash flow does not covary with the firm's outcome, we know that we can use the debt's (risk-free) cost of capital of 11%; if a cash flow does covary with the firm's outcome, it requires a discount rate above the firm's debt cost of capital.

Table 22.3. Two Financing Scenarios for a 1-Year Firm

Scenario 1: All Equity Financing.

	$\mathcal{E}$(Value)	Bad	Good
Before-Tax Return Next Year	$280.00	$250.00	$310.00
Taxable Profits Next Year	$80.00	$50.00	$110.00
Corporate Income Taxes Next Year	$24.00	$15.00	$33.00
Owners Will Keep Next Year	$56.00	$35.00	$77.00

Scenario 2: $200 Debt Financing at 11%. Rest is Levered Equity.

	$\mathcal{E}$(Value)	Bad	Good
Before-Tax Return Next Year	$280.00	$250.00	$310.00
Interest Payments	$22.00	$22.00	$22.00
Taxable Profits Next Year	$58.00	$28.00	$88.00
Corporate Income Taxes Next Year	$17.40	$8.40	$26.40
Equity Owners Will Keep Next Year	$40.60	$19.60	$61.60
Equity+Debt Owners Will Keep Next Year	$62.60	$41.60	$83.60

Scenario 2 vs. Scenario 1 Tax Savings

	$\mathcal{E}$(Value)	Bad	Good
Before-Tax Return Next Year	$280.00	$250.00	$310.00
Scenario 1 corporate income taxes	$24.00	$15.00	$33.00
Scenario 2 corporate income taxes	$17.40	$8.40	$26.40
Relative Net Tax Savings Next Year	$6.60	$6.60	$6.60

Table 22.3 shows that under either financing scenario the income tax obligation is risky and covaries with the firm's return. Uncle Sam is basically a co-owner, partaking in the good and the bad times. At this point, we could try to work out an exact cost of capital for the tax obligation (using the algebra of betas), but most everyone just uses the same cost of capital for both the firm and its tax obligation. Doing so is not exactly correct, but it is a good working assumption. The real reason why we punt is that it would be very difficult to derive the correct discount rate, and the added valuation precision would be modest. Aside, if we wanted to get discount rates on individual component cash flows 100% right, why stop with the corporate income tax payments? Why not also determine individual discount rates for every other component of the company (taxes,

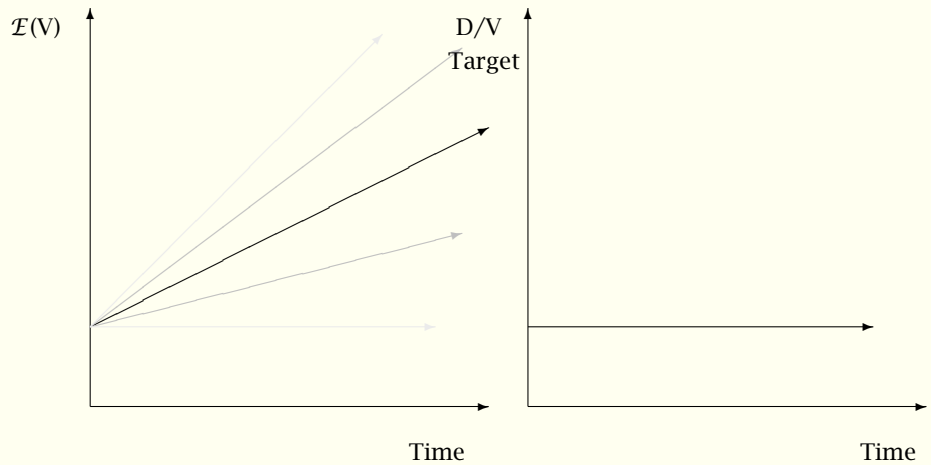
Use the overall firm cost of capital on income taxes.

depreciation, SG&A, marketing, etc.), and/or for the comparables from which we may obtain comparable beta estimates? This is not only impractical, but also beyond my capabilities.

The tax shelter is safer than the firm, and thus may (or may not) warrant a lower discount rate.

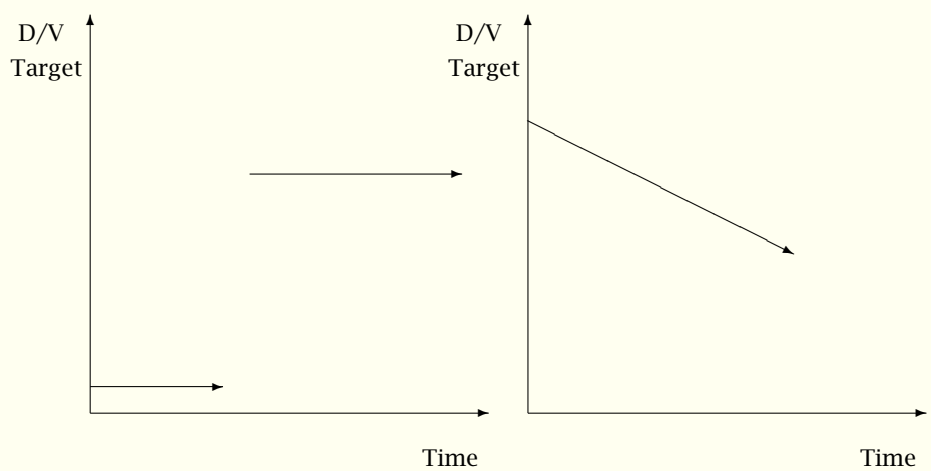
The bottom panel in Table 22.3 illustrates a second interesting fact: the tax savings due to debt remain the same \$6.[60], regardless of firm performance. Thus, in this one-year investment, if we had to pick a discount rate for the tax savings, we should probably choose the debt's cost of capital, here 11%. But although our example gives us good basic intuition that the tax shelter is not risky (the tax saving does not vary with firm value, accruing to owners no matter what the outcome will be), and therefore that the tax shelter should be discounted at the firm's cost of debt capital, this only works in this simple one-period scenario. The reason is that if your firm value doubles by next year, you can probably borrow twice as much next year, and thus enjoy higher tax savings henceforth. If your firm follows such an intelligent dynamic borrowing strategy, the tax shelter obtained by debt financing will not remain constant, but will increase with the firm value, too. To compute the lifetime tax shelter afforded to our firm by its ability to take on debt, we must therefore realize that intelligent capital structure policies will induce the dollar amount of debt (and thus the tax shelter) to also covary positively with firm value. For this reason, it is often sensible to discount the tax shalter not with debt's cost of capital, but with the firm's cost of capital (or a discount rate somewhere in between).

Figure 22.1. Thinking About Proper Discount Rates For The Tax Shelter



(Background: The typical firm value grows over time.)

A firm with constant debt policy. Use $\mathcal{E}(\tilde{r}_W)$ to discount the tax shelter.



A typical R&D Project. Use $\mathcal{E}(\tilde{r}_W)$ to discount the tax shelter (which will only matter in later years).

An overlevered firm, e.g., an LBO, which reduces its debt ratio over time, keeping debt dollars constant. Use $\mathcal{E}(\tilde{r}_D)$ to discount the tax shelter.

Different dynamic debt policies, and intuition about reasonable shelter discount rates.

Figure 22.1 may help you to think about reasonable choices for the discount rate on the tax shelter. We assume that we are dealing with the typical firm, which tends to grow over time (upper left). If the firm policy is to keep a constant debt target (upper right), firm growth will translate into higher and higher dollar debt and interest payments. Therefore, the higher and higher tax shelter will be exposed to about the same risk as the firm overall, which means that you should use the firm's overall cost of capital, $\mathcal{E}(\tilde{r}_W)$ to discount the tax shelter. The lower-left graph shows a typical R&D project, which will initially provide no debt capacity and thus no debt tax shelter—so, this is a period

during which the chosen discount rate does not matter one way or the other. Thereafter, the R&D may pay off, so here is where the R&D will provide a positive expected payoff. The value of the tax shelter and the value of the firm will likely be highly correlated, so the tax shelter should again be discounted by the overall firm cost of capital, $\mathcal{E}(\tilde{r}_{\mathsf{W}})$. The lower right graphs shows a firm that plans to reduce its (proportional) debt level over time, perhaps keeping the same absolute dollar interest payments. Such a firm would expect to save about the same dollar amount in taxes each year, regardless of firm performance. In this case, you should use the debt cost of capital ($\mathcal{E}(\tilde{r}_{\mathsf{D}})$).

Taxes are important; we are only fudging the divisor, not the numerator!

Of course, this entire discussion is only about the discount factor: it is not about the idiosyncratic risk in the expected tax shelter, a quantity which figures into the numerator, not the denominator. For example, the R&D project may not generate any tax shelter half the time—in which case, the expected tax shelter (in the NPV numerator) to be discounted would be something like

$$\text{Expected Tax Shelter} \;=\; 50\% \cdot \begin{pmatrix}\text{Tax Shelter If R\&D is Successful}\\ \text{= Tax-Rate Times Interest Paid}\end{pmatrix} + 50\% \cdot \$0 \quad . \tag{22.2}$$

Think about, but do not lose sleep over the precise discount rate on the tax shelter.

At this point, you probably wonder why I advise you to get the discount rate wrong. The reason is that it will make your life a lot easier, without sacrificing meaningful real-world accuracy. Let's put our error into proper perspective: The debt in a typical large firm may be about 25% of the firm value. The interest rate paid in one year is therefore about 5% on this 25% of the firm value. The tax shelter itself is therefore only about 30%–40% (the corporate income tax rate) times 5% times 25% of the firm value. And now you are wondering if you should divide this modest number by, say, 1.10 (a reasonable firm cost of capital) or by 1.05 (a reasonable debt cost of capital)?! For tax shelters far in the future, the tax shelter will be heavily discounted to be an even smaller number. Hopefully, this over-the-envelope estimate has convinced you that, in the real-world, your overall project valuation is robust with respect to moderate variations in the choice of discount rate. (I typically use whatever is most convenient, although I try to keep track of whether I think my assumptions overestimate or underestimate the true firm value.) In our Scenario 2 in Table 22.3, how much does the choice of discount rate on the tax sheltermatter? If you use an 11% discount factor on the $\$6.^{60}$ tax shelter, you would conclude that debt will save you the equivalent of $\$5.^{95}$. If you use a 12% discount factor, you would conclude that debt will save you $\$5.^{89}$. The difference of $\$0.^{05}$ on an expected before-tax return of \$280 is obviously inconsequential. Even if you commited the most "extreme" possible error here, which is using a zero discount rate on the tax shelter, you would misestimate only 60 cents on \$280 in expected return. In almost all real-world corporate decisions, your uncertainty about your \$280 estimate will dwarf this 60 cents. So, please "gimme a break"—and yourself, too!

NERD NOTE: *We are also ignoring many other real-world tax issues for the same reason: they are too hard to work out correctly, and the gained accuracy improvement would often be trivial. For example, delayed tax payments, tax-loss carryforwards, different marginal corporate income tax rates at different income levels, and default-ability on income tax payments throw more wrenches into our machine (well, into our tax shelter valuation estimate). Worrying about these issues is not too worthwhile, except to some egghead academics.*

IMPORTANT:

- *The discount rates on the tax obligations and on the tax shelter can usually not be exactly estimated. Therefore we usually use reasonable and convenient approximations. You should remain aware of the debt policy that your firm follows, and therefore how your choice of discount rate introduces a (modest) valuation bias one way or the other.*
- *It is reasonable and common to value tax liabilities at a discount rate equal to the firm's overall cost of capital ($\mathcal{E}(\tilde{r}_{\mathsf{W}})$).*
- *It is reasonable and common to value the tax shelter afforded by the debt at a discount rate that lies anywhere between the debt's cost of capital ($\mathcal{E}(\tilde{r}_{\mathsf{D}})$) and the firm's overall cost of capital ($\mathcal{E}(\tilde{r}_{\mathsf{W}})$), , the choice of which should depend on the firm's multi-period debt policy.*

Solve Now!

Q 22.2 *A \$1 million construction project is expected to return \$1.2 million. You are in a 45% combined federal and state marginal income tax bracket. Your annual income is \$200,000 per year. If you finance the project with an \$800,000 mortgage at an interest rate of 5%, how much will Uncle Sam receive? If you finance the project with cash, how much will Uncle Sam receive?*

Q 22.3 *Continue. If the appropriate project interest rate is 8%, what is the NPV of the tax savings from financing the project with a mortgage?*

22·4. Formulaic Valuation Methods: APV and WACC

But we need a formula that works for intermediate leverage. The choices are WACC and APV.

We now know how to compute the tax cash flows and the tax shelter. We have also thought about reasonable discount rates. Our next goal is to find a valuation formula that allows us to compute the firm value today not only for the current financing arrangement, but also for other financing arrangements that we might contemplate for our firm. There are essentially three methods: The first method is to construct the firm's pro-forma financials under the new hypothetical capital structure, and then to value the cash flows directly. Without stating it such, we have already done this in chapter 9 (and we shall repeat it in the next section). The second method is called **Adjusted Present Value** or **A.P.V.** for short. The third method is a tax-modified version of the **Weighted Average Cost of Capital** or **W.A.C.C.** Formula. Properly applied, all methods should provide the same (or at least very similar) answers. This section explains the latter two methods (APV and WACC) in some detail; the next section works a multi-year example with all three methods.

22·4.A. Adjusted Present Value (APV): Theory

The main idea of APV: value the firm, then add the tax subsidy.

APV decomposes the value of the firm into two components: the value of the firm if it were all equity-financed and fully taxed, plus a tax subsidy for each dollar that can be declared as "interest" rather than as "dividend." In our example from Table 22.2, \$256 is the expected cash flow of our firm if it were 100% equity financed. The APV method then adds the tax subsidy. For example,

- If the firm is all equity financed, the tax subsidy is zero.
- If the firm could have the (utopian) interest payments of \$80, the I.R.S. would believe that the firm had not earned a penny. Therefore, the owners could keep an extra \$24 above the \$256 all-taxed scenario *next year*.
- Iff the firm will have interest payments of \$19, the I.R.S. would see \$280 − \$19 = \$261 in return minus \$200 investment cost for a net return of \$61. The I.R.S. would therefore collect 30%·\$61 = \$18.^{30}, which is \$5.^{70} less than the \$24 that the I.R.S. would have collected if the firm had been 100% equity financed. The expected tax savings could also have been directly calculated as $\tau \cdot (\mathcal{E}(\tilde{r}_D) \cdot D_0) = 30\% \cdot (\$19) = \$5.^{70}$.

Computing tax savings, given the debt level.

Let's make a formula out of this method. Our first step to a more general valuation formula in the presence of corporate income taxes is to relate the amount of debt today to the interest payments next year. Let's return to our example, in which we borrow \$200 at an interest rate of 11%. The expected interest payments are now

$$\begin{aligned} \text{Expected Interest Payment} &= 11\% \cdot \$200 = \$22 \\ \text{Expected Interest Payment} &= \mathcal{E}(\tilde{r}_D) \cdot D \quad . \end{aligned} \tag{22.3}$$

The future tax savings *relative to an all-equity financed firm* is the amount of corporate income tax that the firm will *not* have to pay on the interest.

$$\begin{aligned} \text{Expected Tax Saving} &= 30\% \cdot [11\% \cdot \$200] = \$6.^{60} \\ \text{Expected Tax Saving} &= \tau \cdot [\mathcal{E}(\tilde{r}_D) \cdot D] \quad . \end{aligned} \tag{22.4}$$

In words, Uncle Sam will expect to receive \$6.60 less from the owners of the project, because \$22 in profit repatriation was designated as "interest."

SIDE NOTE: Many managers mistakenly use the *quoted* debt interest instead of the *expected* debt interest. Although this is incorrect, fortunately if the firm is reasonably large and not highly levered, the difference between the expected and promised interest payments is often small.

The \$6.60 in tax savings will occur next year, and will therefore have to be discounted. So the APV Formula computes the discounted value of an all-equity financed firm (with after-tax cash flows of \$256 next year), and then adds back the *discounted* tax savings:

APV discounts tax savings and adds them to an all-equity type hypothetical firm.

\$200 debt at 11% interest, i.e.,\$22 interest payment discounted at 11%:

$$\text{APV} = \frac{\$256}{1+12\%} + \frac{30\%\cdot\$22}{1+11\%} = \$234.52$$

$$\text{APV} = \frac{\mathcal{E}(\text{CF})}{1+\mathcal{E}(\tilde{r}_{\text{W}})} + \frac{\tau\cdot[\mathcal{E}(\tilde{r}_{\text{D}})\cdot \text{D}]}{1+\mathcal{E}(\tilde{r}_{\text{D}})} \tag{22.5}$$

$$\text{APV} = \begin{matrix}\text{Value as}\\ \text{if 100\% equity}\\ \text{financed}\end{matrix} + \begin{matrix}\text{Tax Subsidy}\\ \text{from Interest}\\ \text{Payments}\end{matrix}.$$

As described at length in Section 22·3.B, you could also reasonably use the firm's cost of capital to discount the tax savings,

\$200 debt at 11% interest, i.e.,\$22 interest payment discounted at 12%:

$$\text{APV} = \frac{\$256}{1+12\%} + \frac{30\%\cdot\$22}{1+12\%} = \$234.46$$

$$\text{APV} = \frac{\mathcal{E}(\text{CF})}{1+\mathcal{E}(\tilde{r}_{\text{W}})} + \frac{\tau\cdot[\mathcal{E}(\tilde{r}_{\text{D}})\cdot \text{D}]}{1+\mathcal{E}(\tilde{r}_{\text{W}})} \tag{22.6}$$

$$\text{APV} = \begin{matrix}\text{Value as}\\ \text{if 100\% equity}\\ \text{financed}\end{matrix} + \begin{matrix}\text{Tax Subsidy}\\ \text{from Interest}\\ \text{Payments}\end{matrix}.$$

APV generalizes easily to multiple years: just compute the tax savings for each year and add them up, the same way that we add up present values.

IMPORTANT: *The Adjusted Present Value (*APV*) Formula computes an "as if all equity financed" NPV (i.e., after corporate income tax) and then adds back the tax subsidy:*

$$\text{APV}_0 = \begin{matrix}\textit{Value as if Firm is 100\%}\\ \textit{equity financed and fully}\\ \textit{taxed.}\end{matrix} + \begin{matrix}\textit{Tax Subsidies}\\ \textit{from Interest}\\ \textit{Payments}\end{matrix} \tag{22.7}$$

$$\text{APV}_0 = \sum_{t=0}^{\infty}\left[\frac{\mathcal{E}(CF_t)}{1+\mathcal{E}(\tilde{r}_{\text{W},t})}\right] + \sum_{t=0}^{\infty}\left[\frac{\tau\cdot\mathcal{E}(IP_t)}{1+\mathcal{E}(\tilde{r}_{?,t})}\right].$$

$\mathcal{E}(CF_t)$ are the firm's expected corporate cash flows if the firm were 100% equity financed and fully taxed; $\mathcal{E}(IP_t)$ are the expected interest payments (often the interest rate times current debt, $\mathcal{E}(\tilde{r}_{\text{D}})\cdot\text{D}$); τ is the marginal corporate income tax rate; and r is the discount rate. Subscript W *denotes the overall cost of capital; subscript '?' denotes that the discount rate on the expected interest rate depends on corporate debt policy, and that the exact discount rate choice (often either $\mathcal{E}(\tilde{r}_{\text{W}})$ or $\mathcal{E}(\tilde{r}_{\text{D}})$) is usually not too important.*

22·4.B. APV: Application to a 60/40 Debt Financing Case

An example of how to value a firm financed with 60% debt.

In the example above, the firm with \$200 debt was worth $\$234.^{46}$ today, which comes to a debt ratio of $\$200/\$234.^{46} \approx 85\%$. Now assume that the firm instead considers a new capital structure, in which it would borrow only $\$139.^{16}$. The firm has determined that this lower-debt capital structure would reduce its debt cost of capital to 9% per annum. What would the firm's value be?

Problem solved.

According to the APV Formula, we begin with the value of a 100%-equity firm, which is $\$256/(1+12\%)$, and add back the tax subsidy. Interest payments on $\$139.^{16}$ of debt will be $9\% \cdot \$139.^{16} = \$12.^{52}$ *next year*. Taxes saved will be $30\% \cdot \$12.^{52} = \$3.^{76}$ *next year*. This is worth $\$3.^{45}$ *today*. Therefore

$$\begin{aligned}
\text{APV} &= \frac{\$256.^{00}}{1+12\%} + \frac{30\% \cdot 9\% \cdot \$139.^{16}}{1+9\%} \\
&= \$228.^{57} + \$3.^{45} = \$232.^{02} \\
\text{APV} &= \frac{\mathcal{E}(\text{CF})}{1+\mathcal{E}(\tilde{r}_{\text{W}})} + \frac{\tau \cdot \mathcal{E}(\tilde{r}_{\text{D}}) \cdot \text{D}}{1+\mathcal{E}(\tilde{r}_{\text{D}})} \\
&= \text{“as if all equity financed” firm} + \text{tax subsidy} \quad .
\end{aligned} \tag{22.8}$$

If you prefer discounting the expected tax shelter with the firm's cost of capital, use

$$\begin{aligned}
\text{APV} &= \frac{\$256.^{00}}{1+12\%} + \frac{30\% \cdot 9\% \cdot \$139.^{16}}{1+12\%} \\
&= \$228.^{57} + \$3.^{36} = \$231.^{93} \\
\text{APV} &= \frac{\mathcal{E}(\text{CF})}{1+\mathcal{E}(\tilde{r}_{\text{W}})} + \frac{\tau \cdot \mathcal{E}(\tilde{r}_{\text{D}}) \cdot \text{D}}{1+\mathcal{E}(\tilde{r}_{\text{D}})} \\
&= \text{“as if all equity financed” firm} + \text{tax subsidy} \quad .
\end{aligned} \tag{22.9}$$

This is the APV answer: In the presence of corporate income taxes, a firm financed with $\$134.^{16}$ in debt would be worth about \$232. Incidentally, at \$232 in value, our firm's debt ratio today is $\$139.^{16}/\$231.^{93} \approx 60\%$. If we want the firm's overall cost of capital $\mathcal{E}(\tilde{r}_{\text{W}})$ to remain at 12%, then we can solve $\mathcal{E}(\tilde{r}_{\text{W}}) = w_{\text{D}} \cdot \mathcal{E}(\tilde{r}_{\text{D}}) + w_{\text{E}} \cdot \mathcal{E}(\tilde{r}_{\text{E}}) = 60\% \cdot 9\% + 40\% \cdot 16.5\% \approx 12\%$ to obtain a cost of equity capital of $\mathcal{E}(\tilde{r}_{\text{W}}) \approx 16.5\%$.

22·4.C. Tax-Adjusted Weighted Average Cost of Capital (WACC) Valuation: Theory

To show that WACC and APV are compatible, we derive the tax-adjusted WACC Formula from the APV Formula.

The second method to compute the value of the firm uses a **Tax-Adjusted W.A.C.C. (Weighted Average Cost of Capital)**. If we start with the APV Formula and manipulate it, it will be most apparent that the two methods yield the same value. We will stick with our 60/40 debt/equity financed example, with a 30% corporate income tax rate, a 9% cost of debt capital, and \$280 in before-tax return (\$256 after-tax return). The firm

borrowed \$139.^{16} at a 9% interest rate for net interest payments of \$12.^{52}. The corporate income tax shield was \$3.^{76}; and the APV Formula 22.9 valued the firm at

$$\begin{aligned} \mathsf{PV} &= \frac{\$256}{1+12\%} + \overbrace{\frac{30\% \cdot \overbrace{(9\% \cdot \$139.^{16})}^{\$12.^{52}}}{1+12\%}}^{=\$3.^{76}} = \$231.^{93} \\ \mathsf{PV} &= \frac{\mathcal{E}(\mathrm{CF})}{1+\mathcal{E}(\tilde{r}_{\mathsf{W}})} + \frac{\tau \cdot [\mathcal{E}(\tilde{r}_{\mathsf{D}}) \cdot \mathsf{D}]}{1+\mathcal{E}(\tilde{r}_{\mathsf{W}})} \quad . \end{aligned} \tag{22.10}$$

The main difference between APV and WACC is that WACC expresses the debt as a ratio of firm value. Therefore,

$$\begin{aligned} 60\% &= \$139.^{16}/\$231.^{93} \qquad \$139.^{16} = 60\% \cdot \$231.^{93} \\ w_D &= \mathsf{D}_0/\mathsf{PV} \qquad \Longrightarrow \qquad \mathsf{D}_0 = w_D \cdot \mathsf{PV} \quad . \end{aligned} \tag{22.11}$$

Substitute this into the APV formula,

$$\begin{aligned} \mathsf{PV} &= \frac{\$256}{1+12\%} + \frac{30\% \cdot [9\% \cdot (60\% \cdot \$231.^{93})]}{1+12\%} = \$231.^{93} \\ \mathsf{PV} &= \frac{\mathcal{E}(\mathrm{CF})}{1+\mathcal{E}(\tilde{r}_{\mathsf{W}})} + \frac{\tau \cdot [\mathcal{E}(\tilde{r}_{\mathsf{D}}) \cdot (w_{\mathsf{D}} \cdot \mathsf{PV})]}{1+\mathcal{E}(\tilde{r}_{\mathsf{W}})} \quad . \end{aligned} \tag{22.12}$$

The trouble is that we now have PV on both sides of the equation. We need to solve for PV. Multiply both sides by $[1+\mathcal{E}(\tilde{r}_{\mathsf{W}})]$,

$$\begin{aligned} (1+12\%) \cdot \$231.^{93} &= \$256 + 30\% \cdot [9\% \cdot (60\% \cdot \$231.^{93})] \\ [1+\mathcal{E}(\tilde{r}_{\mathsf{W}})] \cdot \mathsf{PV} &= \mathcal{E}(\mathrm{CF}) + \tau \cdot [\mathcal{E}(\tilde{r}_{\mathsf{D}}) \cdot (w_D \cdot \mathsf{PV})] \quad . \end{aligned} \tag{22.13}$$

Move the second term to the left side,

$$\begin{aligned} (1+12\%) \cdot \$231.^{93} - 30\% \cdot [9\% \cdot (60\% \cdot \$231.^{93})] &= \$256 \\ [1+\mathcal{E}(\tilde{r}_{\mathsf{W}})] \cdot \mathsf{PV} - \tau \cdot [\mathcal{E}(\tilde{r}_{\mathsf{D}}) \cdot (w_D \cdot \mathsf{PV})] &= \mathcal{E}(\mathrm{CF}) \quad . \end{aligned} \tag{22.14}$$

Pull out the PV,

$$\begin{aligned} \$231.^{93} \cdot [1+12\% - 30\% \cdot 9\% \cdot 60\%] &= \$256 \\ \mathsf{PV} \cdot [1+\mathcal{E}(\tilde{r}_{\mathsf{W}}) - \tau \cdot \mathcal{E}(\tilde{r}_{\mathsf{D}}) \cdot w_{\mathsf{D}}] &= \mathcal{E}(\mathrm{CF}) \quad . \end{aligned} \tag{22.15}$$

Divide both sides by the PV multiplier,

$$\begin{aligned} \$231.^{93} &= \frac{\$256}{1+12\% - 30\% \cdot 60\% \cdot 9\%} \\ \mathsf{PV} &= \frac{\mathcal{E}(\mathrm{CF})}{1+\mathcal{E}(\tilde{r}_{\mathsf{W}}) - \tau \cdot [\mathcal{E}(\tilde{r}_{\mathsf{D}}) \cdot w_{\mathsf{D}}]} \quad , \end{aligned} \tag{22.16}$$

and you have the basic WACC formula. The WACC idea is to discount the "as-if-100%-equity financed and fully taxed" cash flows not with the overall firm's cost of capital $\mathcal{E}(\tilde{r}_{\mathsf{W}}) = 12\%$, but with a cost of capital that is 12% minus a tax shelter subsidy of $\tau \cdot w_{\mathsf{D}} \cdot \mathcal{E}(\tilde{r}_{\mathsf{D}}) = 30\% \cdot 60\% \cdot 9\% \approx 1.62\%$. The revised discount rate—the expression in

brackets—is $1+12\%-30\%\cdot 9\%\cdot 60\% \approx 1+10.38\%$. 10.38% is often called the tax-adjusted WACC.

The tax-adjusted WACC generalizes ordinary WACC.

This tax-modified weighted cost of capital is used to discount future cash flows in the NPV formula—just like the plain non-tax adjusted WACC Formula 21.8 was used in the case without corporate income taxes. The innovation relative to the non tax-adjusted WACC is that the cost of capital for debt is lowered by $(1-\tau)$ to reflect the corporate income tax subsidy. If the corporate tax rate τ is zero, then the tax subsidy is useless, $1-\tau = 1$, and the tax-adjusted and the plain WACC formulas are the same. Indeed, when analysts refer to WACC, they usually mean the tax-adjusted WACC, not the zero-tax perfect-M&M-world WACC.

The more common form of WACC also breaks out equity cost of capital.

The WACC formula is often slightly rearranged. Split $\mathcal{E}(\tilde{r}_{\mathsf{W}})$ into its cost of equity and cost of debt components, $\mathcal{E}(\tilde{r}_{\mathsf{W}}) = w_{\mathsf{D}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{D}}) + w_{\mathsf{D}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{D}})$. In our example, to keep the weighted average firm cost of capital at the constant $\mathcal{E}(\tilde{r}_{\mathsf{W}}) = 12\%$, we solve $\mathcal{E}(\tilde{r}_{\mathsf{W}}) = w_{\mathsf{D}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{D}}) + w_{\mathsf{E}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{E}}) = 60\%\cdot 9\% + 40\%\cdot\mathcal{E}(\tilde{r}_{\mathsf{E}}) = 12\%$, and find $\mathcal{E}(\tilde{r}_{\mathsf{E}}) = 16.5\%$. Substitute this, and you get the more common form of the WACC formula,

$$\begin{aligned}
\mathsf{PV} &= \frac{\$256}{1+10.38\%} = \frac{\$256}{1+40\%\cdot 16.5\% + (1-30\%)\cdot(60\%\cdot 9\%)} \\
\mathsf{PV} &= \frac{\mathcal{E}(\mathrm{CF})}{1+\mathsf{WACC}} = \frac{\mathcal{E}(\mathrm{CF})}{1+w_{\mathsf{E}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{E}}) + (1-\tau)\cdot(w_{\mathsf{D}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{D}}))}
\end{aligned} \tag{22.17}$$

IMPORTANT: *The Tax-Adjusted Weighted Average Cost of Capital Formula (WACC) formula computes an "as if all equity financed" future cash flow," (i.e., after corporate income tax) and then discounts it back with a lower cost of capital that itself reflects the tax shelter:*

$$PV_0 = \frac{\mathcal{E}(CF_t)}{1+\mathsf{WACC}} \quad \text{, where} \quad .$$

$$\mathsf{WACC} = \mathcal{E}(\tilde{r}_{\mathsf{W}}) - \tau\cdot\mathcal{E}(\tilde{r}_{\mathsf{D}})\cdot w_{\mathsf{D}} = w_{\mathsf{E}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{E}}) + w_{\mathsf{D}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{D}})\cdot(1-\tau) \quad . \tag{22.18}$$

$\mathcal{E}(CF)$ are the firm's expected corporate cash flows as if the firm were 100% equity financed and fully taxed; w_{E} is the fraction of equity in the capital structure, so $w_{\mathsf{D}} = (1-w_{\mathsf{E}})$ is the fraction of debt in the capital structure; $\mathcal{E}(\tilde{r}_{\mathsf{E}})$ is the firm's equity cost of capital; $\mathcal{E}(\tilde{r}_{\mathsf{D}})$ is the debt cost of capital (i.e., interest payments); and τ is the marginal corporate income tax rate.

If the cost of capital and capital structure (debt ratio) stay constant, the WACC *formula generalizes to multiple periods,*

$$PV_0 = \sum_{t=0}^{\infty} \frac{\mathcal{E}(CF_t)}{\{1+\ [\ w_{\mathsf{E}}\cdot r_{\mathsf{E}} + w_{\mathsf{D}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{D}})\cdot(1-\tau)\ ]\}^t} \quad . \tag{22.19}$$

WACC *cannot handle changing debt ratios over time.*

SIDE NOTE: About half of all publicly traded firms in the United states have a marginal tax rate of zero (!), e.g., due to tax-loss carryforwards or due to clever tax shelters. For these companies, the corporate tax rate is 0, which means that they cannot use debt to reduce their cost of capital.

NERD NOTE: *Our derivation in this section was made simpler by using the same discount rate on the tax liability as on the firm value, and by committing minor algebraic rounding-type errors elsewhere. Still, it captured the basic correspondence between* APV *and* WACC *quite well:* WACC *expresses the firm's debt as a fraction of current firm value, and then collects the current firm value terms on the left side.*

Solve Now!

Q 22.4 *Consider a 25/75 debt/equity financing case for our firm: the before-tax return is $280, the tax rate is 30%, the overall cost of capital is 12%, and debt when the firm is 25% debt financed must offer an expected rate of return of 8%. First compute the* WACC*, then compute the debt as 25% the* WACC *value, and show how the* APV *yields the same result.*

Q 22.5 *Consider financing our firm with $100 debt: the before-tax return is $280, the tax rate is 30%, the overall cost of capital is 12%, and this debt must offer an expected rate of return of 8.7%. First compute the* APV*, then compute the capital structure in ratios, and finally show that the* WACC *yields the same result.*

Q 22.6 *If you are thinking of debt in terms of a (constant) fraction of firm value, would you prefer* WACC *or* APV*? If you are thinking of debt in terms of a (constant) dollar amount, would you prefer* WACC *or* APV*?*

22·4.D. A Major Blunder: Applying APV and WACC to the Current Cash Flows

Another Common Mistake: Make sure you use the correct Cash Flow to Discount.

Unfortunately, both WACC and APV are often used incorrectly. Analysts frequently forget that the expected cash flows in the numerator are the "as if fully equity financed and fully after-tax" cash flows. The correct expected cash flow in the numerator of both WACC and APV is $256. It is neither the before-tax project cash flows ($280 in our example), nor the after-tax cash flows under the current financing scheme (e.g., $231.[93] if the firm were currently financed 60/40; or $234.[79] if the firm were currently financed 90/10). A reader who has worked through the examples in this chapter should understand why this would provide the wrong answer.

IMPORTANT: WACC *and* APV *operate with expected "as if 100% equity-financed and after corporate income tax" cash flows, not the firm's current cash flows (which depend on the current debt/equity financing).*

Solve Now!

Q 22.7 *A firm in the 20% marginal tax bracket is currently financed with $500 debt and $1,000 equity. The debt carries an interest rate of 6%, the equity's cost of capital is 12%. The risk-free rate is 4%, the equity premium is 3%. What is the firm's beta? The firm is pondering a recapitalization to $1,000 debt, which would increase the debt's interest rate to 8%. The firm exists only for 1 more year. What would the new equity be worth?*

Solve Now!

Q 22.8 *A firm in the 40% income tax bracket has an investment that costs \$300 in year 0, offers a pretax return in year 1 of \$500. Assume that the firm's cost of capital, as provided by the external capital markets, is approximately* $\mathcal{E}(\tilde{r}_D) = 15\% + w_D \cdot 5\%$. *Compute the APV, WACC and a WACC based value if the firm borrows \$50 to finance it. Repeat if the firm borrows \$100.*

22·5. A Sample Application of Tax-Adjusting Valuation Techniques

Table 22.4. Income Statement of Hypothetical Machine

	Year	1	2	3	4	5	6
	gross sales (revenues)	$70	$70	$70	$70	$70	$70
–	cost of goods sold (cogs)	$5	$5	$5	$5	$5	$5
–	selling, general & administrative expenses (sg&a)	$5	$5	$5	$5	$5	$5
=	ebitda (net sales)	$60	$60	$60	$60	$60	$60
–	depreciation	$25	$50	$50	$25	$0	$0
=	ebit (operating income)	$35	$10	$10	$35	$60	$60
–	interest expense	$0	$5	$5	$5	$5	$5
=	eaibt (or EBT)	$35	$5	$5	$30	$55	$55
–	corporate income tax (at 40%)	$14	$2	$2	$12	$22	$22
=	net income	$21	$3	$3	$18	$33	$33

	Excerpts From the Cash Flow Statement						
	Year	1	2	3	4	5	6
	net income	$21	$3	$3	$18	$33	$33
+	depreciation	$25	$50	$50	$25	$0	$0
=	**Total Operating Activity**	$46	$53	$53	$43	$33	$33
	capital expenditures	–$75	–$75	-	-	-	-
=	Total Investing Activity	–$75	–$75	-	-	-	-
	Financing Cash Flow	-	-	-	-	-	-
	Net Equity Issue	+$26	-	-	-	-	-
	Dividends	-	-	–$53	–$43	–$33	–$8
	net debt issue	+$25	-	-	-	-	–$25
=	**Total Financing Activity**	+$51	-	–$53	–$43	–$33	–$33
=	**Net Change In Cash**	+$22	–$22	$0	$0	$0	$0

Let's value a pro-forma firm.

We now apply our corporate income-tax related valuation techniques on a multi-year, pro-forma firm. In Table 22.4, we return to our basic hypothetical machine from Chapter 9, Table 9.2 on Page 177. To make the example more useful, we assume an appropriate debt interest rate is assumed to be 20%, so a loan of \$25 must offer an expected \$5 in interest per annum. The appropriate overall cost of capital for our corporation is assumed to be 30%. Table 22.4 shows that, equity holders pay in \$26 and receive a total of \$112 in dividends. Debtholders invest \$25 and receive \$25 in total interest payments. (Our firm follows an odd capital distribution policy, but so be it.) What is our firm worth?

22·5.A. Direct Valuations from Pro-Forma Financials

Method 1: Direct Flows from the Financials.

Our first method to value the firm uses the pro-forma cash flows directly. First, return to the project cash flow formula 9.19 from Page 195. It tells us that the project cash flows that we need for our valuation are

	Total Operating Activity	\$46	\$53	\$53	\$43	\$33	\$33
+	**Total Investing Activity**	–\$75	–\$75	-	-	-	-
+	interest expense	-	\$5	\$5	\$5	\$5	\$5
=	**Project Cash Flows**	–\$29	–\$17	+\$58	+\$48	+\$38	+\$38

We discount these cash flows, using the assumed 30% firm cost of capital:

$$\begin{aligned} \mathsf{NPV}_{t=0} &= \frac{-\$29}{1+30\%} + \frac{-\$17}{(1+30\%)^2} + \frac{+\$58}{(1+30\%)^3} + \frac{+\$48}{(1+30\%)^4} \\ &+ \frac{+\$38}{(1+30\%)^5} + \frac{+\$38}{(1+30\%)^6} = \$28.^{95} \quad . \end{aligned} \tag{22.20}$$

So, you would be willing to pay \$28.95 *today* for the right to buy (and finance) the firm, which will initiate *next year* with this exact capital structure. But wait: did we not forget about the tax-shelter that came with the debt? No, we did not! The pro-forma itself had already incorporated the correct interest expense, which had percolated into a corporate income tax and thus net project value.

22·5.B. APV

Method 2: APV.

Our second method to value our firm is APV. But careful: the cash flows in Formula (22.20) are *not* the cash flows that we need for the APV analysis, because these are not the cash flows *as if 100% equity financed*. APV states that we want to use the *as if 100% equity financed* cash flows, and then add back the tax shield. (If we used the cash flows in Formula 22.20 and then added the tax-shield [due to the interest payment designation], we would be counting the tax-shield twice.)

Detour: Construct as-if-100%-equity financials.

We know that if the firm were fully equity financed, the tax obligation would go up. By how much? In years 2–6, the taxable net income would be \$5 more, so at our 40% corporate income tax rate, we would have to pay not \$2, but \$4 in taxes. So, we would have to pay an extra \$2 in taxes each year. To make sure we have this right, let's see how the financials of a 100% equity financed firm would look like:

Excerpts From the Income Statement, As If 100% Equity Financed							
	Year	1	2	3	4	5	6
=	ebit (operating income)	$35	$10	$10	$35	$60	$60
–	interest expense	$0	$0	$0	$0	$0	$0
=	eaibt (or EBT)	$35	$10	$10	$35	$60	$60
–	corporate income tax (at 40%)	$14	$4	$4	$14	$24	$24
=	net income	$21	$6	$6	$21	$36	$36

Excerpts From the Cash Flow Statement, As If 100% Equity Financed							
	Year	1	2	3	4	5	6
	net income	$21	$6	$6	$21	$36	$36
+	depreciation	$25	$50	$50	$25	$0	$0
=	**Total Operating Activity**	$46	$56	$56	$46	$36	$36
	capital expenditures	–$75	–$75	-	-	-	-
=	Total Investing Activity	–$75	–$75	-	-	-	-

We can now reuse our formula on the 100% equity financed version of our firm:

	(As If 100% Equity Financed.) Year:	1	2	3	4	5	6
	Total Operating Activity	$46	$56	$56	$46	$36	$36
+	**Total Investing Activity**	–$75	–$75	-	-	-	-
+	interest expense	$0	$0	$0	$0	$0	$0
	Project Cash Flows	–$29	–$19	+$56	+$46	+$36	+$36

Comparing this to the equivalent table on Page 536, the project cash flows in our 100% equity financed firm have indeed lost the tax shelter of $2 in each of years 2–6.

We can now discount these "as if 100% equity financed" cash flows with the firm's appropriate cost of capital, which we have assumed to be 30%. Standing at time 0, this gives us

Return to the main task: APV valuation

$$\begin{aligned}\mathsf{NPV}_{\text{Project, 100\% Equity Financed}} &= \frac{-\$29}{(1+30\%)} + \frac{-\$19}{(1+30\%)^2} + \frac{+\$56}{(1+30\%)^3} + \frac{+\$46}{(1+30\%)^4} \\ &+ \frac{+\$36}{(1+30\%)^5} + \frac{+\$36}{(1+30\%)^6} = \$25.^{20} \quad .\end{aligned} \tag{22.21}$$

The APV formula tells us that we now need to add back the expected tax shield from the debt. The interest tax shields in years 2–6 are the interest payments ($5 per year) multiplied by the corporate tax rate (40%), or $2 per year. What is the value of this tax shelter?

$$\begin{aligned}\mathsf{NPV}_{\text{Tax Shelter}} &= \frac{\$0}{(1+30\%)} + \frac{+\$2}{(1+30\%)^2} + \frac{+\$2}{(1+30\%)^3} + \frac{+\$2}{(1+30\%)^4} \\ &+ \frac{+\$2}{(1+30\%)^5} + \frac{+\$2}{(1+30\%)^6} = \$3.^{75} \quad .\end{aligned} \tag{22.22}$$

Therefore, our APV method tells us that the firm value is

$$\mathsf{APV}_{t=0} = \$25.^{20} + \$3.^{75} = \$28.^{95} \quad . \tag{22.23}$$

This is the same answer that we found in Formula 22.20.

22·5.C. WACC

Method 3: WACC

Our third method to value the firm is WACC. We again start with the firm's cash flows, as if 100% equity financed.

Year, As If 100% Equity Financed	1	2	3	4	5	6
Project Cash Flows	–\$29	–\$19	+\$56	+\$46	+\$36	+\$36

But we shall now use the appropriate tax-adjusted cost of capital for discounting. There is one tricky issue now: what is the firm's capital structure? That is, we need to determine $w_{\mathsf{D}} = (1 - w_{\mathsf{E}})$ first, before we can use the WACC method in our example. In the real world, we could just look up the current values, so trust me (and the nerdnote below) that the debt is about 31 percent of the firm's value today. The remaining inputs we need are the overall corporate cost of capital is 30%, and the debt cost of capital of 20%

Return to the main task: WACC valuation

So, the firm's weighted average cost of capital is

$$\begin{aligned} \mathsf{WACC} &\approx 30\% - 40\% \cdot 31\% \cdot 20\% \approx 27.5\% \\ \mathsf{WACC} &\approx r_{\mathsf{W}} - \tau \cdot w_{\mathsf{D}} \cdot r_{\mathsf{D}} \quad . \end{aligned} \tag{22.24}$$

Therefore,

$$\begin{aligned} \mathsf{NPV}_0 = \frac{-\$29}{(1+27.5\%)} + \frac{-\$19}{(1+27.5\%)^2} + \frac{+\$56}{(1+27.5\%)^3} + \frac{+\$46}{(1+27.5\%)^4} \\ + \frac{+\$36}{(1+27.5\%)^5} + \frac{+\$36}{(1+27.5\%)^6} \approx \$29.^{05} \quad , \end{aligned} \tag{22.25}$$

which is a tiny 10 cents off the value of the APV formula. A part of this difference is rounding error. Another part of this difference is the fact that the fraction of debt in the capital structure does not remain a constant 31% over the years! Thus, we are mildly misapplying WACC in our case. Little harm done.

NERD NOTE: *Our equity will have payments of \$53, \$43, \$33, and \$8 in the final four years. Let's assume for a moment a 35% cost of capital on equity. (We will verify this later.) With a 35% cost-of-equity-capital assumption, the market value of the equity in year 1 (not year 0!) will be*

$$PV_{Equity,t=1} \approx \frac{\$53}{(1+35\%)^2} + \frac{\$43}{(1+35\%)^3} + \frac{\$33}{(1+35\%)^4} + \frac{\$8}{(1+35\%)^5} \approx \$58.^{28} \ . \tag{22.26}$$

The market value of the equity in year 1 will not be the \$26 that the equity holders have to put in! The equity can be sold for more and is therefore worth the higher \$58.^{28}. The debt however is priced right: when discounted appropriately, it is worth \$25. So, as a fraction of financing in the capital structure, equity will constitute

$$w_E = \frac{\$58.^{28}}{\$58.^{28} + \$25.^{00}} \approx 69\% \ . \tag{22.27}$$

Now, we know that the debt's cost of capital is 20%. To have a consistent example in which the firm's cost of capital is 30% requires an appropriate rate of return on the equity of around 35% without a tax subsidy:

$$\begin{aligned} 69\% \cdot x \ + \ (1-69\%)\cdot 20\% \ &\approx \ 30\% \quad \Rightarrow \quad x \approx 35\% \ . \\ w_E \cdot \mathcal{E}(\tilde{r}_E) \ + \ (1-w_D)\cdot \mathcal{E}(\tilde{r}_D) \ &= \ \mathcal{E}(\tilde{r}_W) \ . \end{aligned} \tag{22.28}$$

so our assumption of a 35% cost-of-equity capital above was ok.

22·5.D. Some Concluding Thoughts

Warning

It is important that you do not use the wrong expected cash flows. In the first method (from pro-formas) you already have both the projected debt cash flows and the projected equity cash flows, so you can make your life even simpler, and just use those instead—pro-forma cash flows already take the debt tax shield into account. In contrast, in both the APV and WACC method, you must use not the expected cash flows of the firm under the current capital structure (much less the expected cash flows to the current equity), but the cash flows that would accrue if the firm was fully equity financed.

A textbook world? Usually not.

In a perfect textbook world, and with constant debt ratios, the three methods should come out with the same result. In the real world, this is rarely the case, although the differences are often rounding error, especially when compared to such very uncertain inputs as the expected cash flows.

Preview: With too much debt, other forces may increase the cost of capital.

At this point, you should wonder: if debt always lowers the cost of capital, why not choose 99% debt? The answer is that we have so far ignored all non-tax consequences of debt. These will be covered in our next chapters. The short answer is that, yes, if we were in a perfect M&M world with only this one distortion, corporate incomes taxes, then having as much corporate leverage as possible would be ideal. However, the world is not M&M, and by taking on more corporate debt, although the $(1-\tau)$ factor reduces the cost of debt, at some point, these other forces will raise $\mathcal{E}(\tilde{r}_E)$ and $\mathcal{E}(\tilde{r}_D)$ to the point that further increases in debt are no longer value-increasing. From the perspective of a corporate manager, these changes are almost transparent: the $\mathcal{E}(\tilde{r}_E)$ and $\mathcal{E}(\tilde{r}_D)$ that our investors demand will go up, and the WACC formula will still be the right formula to use.

Solve Now!

Q 22.9 *Construct a pro-forma for the following firm: A 3-year project costs \$150 (year 1), and produces \$70 in year 1, \$60 in year 2, and \$55 in year 3. Depreciation, both real and financial, is 3 years. Projects of this riskiness (and with this term structure of project payoffs) have an 18% cost of capital. The marginal corporate income tax rate is 40%.*

1. *Assume that the firm is 100% equity financed. Construct the pro-forma, and compute expected project cash flows.*
2. *Compute the Project IRR.*
3. *Compute the project NPV.*
4. *Assume that this firm expects to receive an extra bonus of \$2 in years 2 and 3 from a benevolent donor. What would be the project's cash flows and IRR now?*

For the remaining question, assume that the firm instead has a capital structure financing \$50 in debt raised in year 1 at a 10% (expected) interest rate. There is no interest paid in year 1, just in years 2 and 3. The principal is repaid in year 3.

5. *Construct the pro-forma now. What is the IRR of this project?*
6. *From the pro-forma, what is the NPV of the debt-financed project?*
7. *Compute the NPV via the APV method.*
8. *Via the APV method, how much would firm value be if the firm would have taken on \$40 in debt (assume same interest rate of 10%)?*
9. *How much money must the equity provide in year 1? What is the debt ratio of the firm? Does it stay constant over time? Is this a good candidate firm for the WACC method?*
10. *What is the debt/value ratio in year 1?*
11. *Assume the equity capital must also be raised in year 0, and the firm can earn a 46% rate of return on this equity capital in year 0. For example, if the firm raises \$10 in equity in year 0, then the firm will have \$14.60 to finance the project in year 1. How much equity will the firm have to raise in year 0 if it wants to cover both interest and project costs?*
12. *What is the debt/value ratio in year 0?*
13. *What is the expected debt/expected value ratio in year 1? Why is it different from year 0?*
14. *Is this a zero NPV project?*
15. *What is the value of the tax shelter that is part of the project NPV?*
16. *What is the value of this project if it were purely equity financed and fully taxed?*
17. *How does firm value change if the firm borrows not \$50, but \$25 at an interest rate of not 10%, but 8%?*
18. *Tough question: is it legitimate to compute the weighted average cost of capital of this firm in year 0 from debt/equity ratios in year 0, in order to check whether the firm cost of capital is equal to the weighted average cost of capital?*

22·6. The Tax Subsidy on PepsiCo's Financial Statement

Table 22.5. PepsiCo's Income Statement (Revisited).

	Income Statement	December 2000
=	Revenue	25,479
	COGS	10,226
	+ SG&A	11,104
	+ Depreciation and Amortization	147
	+ Unusual Expenses	184
–	= **Total Operating Expenses**	21,661
=	**Operating Income**	3,818
+	Net Interest Income	–57
=	**Income Before Tax**	3,761
–	Income Tax	1,218
=	**Income After Tax**	2,543
–	Extraordinary Items	0
=	**Net Income**	2,543

Chapter 9 covered financial statement analysis. Can we infer the tax subsidy from the Income Statement? The answer is yes. Recall PepsiCo's Income Statement. In 2000, PepsiCo had \$3.818 billion in operating income, but only had to pay income taxes on \$3.761 billion. With income taxes of \$1.218 billion, PepsiCo suffered an average corporate income tax rate of about 32.4%. If PepsiCo had been purely equity financed, it would have had to pay taxes on its operating income of \$3.818 billion, or about \$1.237 billion. Thus, by having \$57 million in interest, relative to a hypothetical dividend payout of \$57 million, PepsiCo enjoyed a tax shield in 2000 from its interest payments of

One can infer the tax subsidy from corporate financial statements.

$$\begin{aligned} \text{Debt Tax Shield} &= \tau \cdot [\mathcal{E}(\tilde{r}) \cdot \mathsf{D}] \\ &= 32.4\% \cdot \$57 \text{ million} = \$19 \text{ million} \ . \end{aligned} \tag{22.29}$$

Side Note: For companies like PepsiCo with income as high, the marginal and the average tax rates are practically the same, so we can assume that PepsiCo would have had to pay its average tax rate if it had paid out its interest in dividends instead. (We are also ignoring deferred taxes here.)

Solve Now!

Q 22.10 *Compute the Tax Shield for a Company of Your Choice (or Coca Cola if you have no favorite!).*

22·7. Odds and Ends

22·7.A. When to use APV? When to use WACC?

Although the APV and WACC methods are the same,

So, should you use APV or WACC? We already know that both APV (Formula 22.7) and WACC (Formula. 22.19) come up with the same result, but through different routes. APV adds back a dollar quantity, the tax savings; WACC adjusts the discount factor.

Nerd Note: *On a more technical note,* WACC *leans more heavily on some extra assumptions about borrowing rates being competitive and zero NPV. Therefore,* WACC *works only in "normal" situation in which creditors are paid the appropriate cost of capital on the debt (as low as possible, given the appropriate risk).*

Each can be more useful than the other, depending on the situation.

But there is an advantage to having both formulas.

- The APV formula makes it relatively easier to determine how an extra dollar of debt capacity increases firm value. When thinking of a specific addition or project with a specific cost, this may be the easier formula to use.
- The WACC formula makes it relatively easier to determine how an extra percentage in debt capacity increases firm value. When thinking of a target ratio change in capital structure policy, this may be the easier formula to use.

Side Note: What drives project debt capacity? The simple answer is that more tangible (collateralizable) projects tend to add more debt capacity. The easy answer may be to just ask your banker what expected rate the market will demand. Of course, if you are the banker, you get the difficult part: you must guess, too—and this is easier if you have have read this chapter. In general, determining appropriate expected rates of return is a difficult issue. Such factors as the correlation of project cash flows should be taken into account.

Personal Author Advice:

In many cases, I personally find APV easier to work with than WACC. For example, projects may add debt capacity only at some stage in their life, which is easy to handle with APV. An **Research and Development** (**R&D**) project may require equity investment upfront, followed by the construction of a laboratory that can be debt-financed. The laboratory adds debt capacity, the R&D does not. APV makes it easy to add in the debt capacity only in later stages. APV also makes it easier to assign different discount factors to the firm's projects and to the firm's tax shields, if you desire my advice in Section 22·3.B. In contrast, WACC requires assumptions about the capital structure policy in future years—and not just an assumption of corporate neglect. If the firm does nothing, the value of the firm and thus the value of equity next year will almost certainly be different. To keep a constant debt ratio, a corporation must issue more debt when its equity value increases, and repurchase more debt when its equity value decreases. But even on the expected path, often used in pro-forma analysis, the equity is expected to increase in value more than the debt each year. This means that without corporation action, the debt ratio is expected to go down over time. If the debt ratio is not constant over time, our bets are off because multi-year compounding of WACC is tricky business.

22·7.B. A Heuristic Tax-Savings Rule

Why bother with such puny tax savings?

Are the tax savings not too small to bother? Before you draw this conclusion, realize that the firm need not invent anything new or work extra hard to obtain the tax savings. And, tax savings materialize year after year after year. In fact, this provides a nice over-the-thumb heuristic of what the firm can gain in value from one dollar extra in debt.

A rule of thumb: Each perpetual dollar of debt increases firm value by the corporate income tax-rate.

Start with the APV formula. If a large firm today takes on and maintains an extra \$1 billion in debt rather than an extra \$1 billion in equity, the interest is on the order of about 6%, or \$60 million per year. The tax rate for many corporation is about 40%, leading to a savings of \$24 million per year—this can pay for a nice executive bonus. But this is only the first year. If the \$24 million per year saving is a perpetuity, at an appropriate cost of capital of 6%, we can compute the total value increase to the firm today to be \$24/6% = \$400 million.

$$\begin{aligned} \text{Value Increase} &= \frac{40\% \cdot 6\% \cdot \$1 \text{ billion}}{6\%} = \$400 \text{ million} \\ \text{Value Increase} &= \frac{\tau \cdot \mathcal{E}(\tilde{r}_{\mathsf{D}}) \cdot \mathsf{D}}{\mathcal{E}(\tilde{r}_{\mathsf{D}})} = \tau \cdot \mathsf{D} \quad . \end{aligned} \tag{22.30}$$

This is a nice shortcut: for every dollar extra in eternal debt, the value of the firm increases by the tax rate of the firm. And Formula 22.30 is so easy, you can often compute it in your head. For example, compare financing a \$1 million project with 50% debt (Rather than all-equity), in which a firm in the 40% marginal tax bracket plans not to repay any of the debt principal or to take on new debt. The tax savings would be $40\% \cdot \$500,000 = \$200,000$.

22·7.C. Getting Expected Rates of Return, the CAPM, and Taxes

Formally, it is wrong to use the CAPM in a world of taxes.

Formally speaking, in the presence of taxes (or other imperfections), the CAPM does not hold and should not be used. But this does not help us in the real world, where we have no choice but to compute appropriate discount factors in the presence of corporate taxes.

Informally, no one has a better alternative.

So, it has become common practice to combine the tax-adjusted WACC Formula (see Chapter 22) with a cost of capital derived from a CAPM model.

$$\mathcal{E}(\tilde{r}_{\mathsf{W}}) = w_{\mathsf{E}} \cdot \mathcal{E}(\tilde{r}_{\mathsf{E}}) + (1-\tau) \cdot w_{\mathsf{D}} \cdot \mathcal{E}(\tilde{r}_{\mathsf{D}}) \tag{22.31}$$

Most often, the CAPM provides the cost of capital for the equity ($\mathcal{E}(\tilde{r}_{\mathsf{E}})$), while the borrowing interest rate provides the cost of capital for the debt ($\mathcal{E}(\tilde{r}_{\mathsf{D}})$). Of course, this application is also usually incorrect, because corporate debt has a default premium, too. You should not use the *quoted* interest rate but the *expected* interest rate (cost of capital)—and here $\mathcal{E}(\tilde{r}_{\mathsf{D}})$ is a component of the NPV numerator, not denominator. Fortunately, for large companies with low default probabilities, this error often does not make much difference.

Expected costs of capital reflect investors' tax rates.

From Section 7·1.B, we know that we should work only with expected after-tax cash flows and expected after-tax costs of capital. Do we need to adjust the expected debt cost of capital and expected equity cost of capital (from the CAPM) for income taxes? From our corporate managerial perspective, are the two costs of capital pre-tax or after-tax rates of return (which we need)? They are after-tax returns! The reason is in the nature of net present value: the expected cash flows at the top is what we, as a corporate intermediary, provide to our investors; the appropriate cost of capital is what our investors have for their cost of capital, given all their other opportunities. We, the corporation, worry about the numerator, the investors worry about the denominator. They will be giving us capital at an appropriate expected rate of return—and they will have worried about their own income taxes (if even applicable; they may be tax-exempt!) at the time that they were willing to give us (or other investment opportunities in the economy) the money to invest at these expected rates of returns.

NERD NOTE: *Of course, if we suddenly imposed or eliminated personal income taxes, the* CAPM *relationship itself should change. Investors would presumably be willing to give us and every other investment opportunity capital at a lower expected rate of return. But, this is transparent to us corporate users of the* CAPM.

22·7.D. Can Investment and Financing Decisions Be Separate?

If the world is not perfect, projects with different financing options can offer different values. Thus, financing and investment decisions must be considered together, not separately.

In the perfect M&M world, investment and financing decisions could be made independently: Managers could focus on production choices and leave the financing to the nerds in the finance department. Unfortunately, if debt is tax advantaged, this will not be the case outside the M&M world. For example, consider two projects with equal costs, equal payoffs, and equal cost of capitals. (Alternatively, just consider their NPV's to be the same.) But the first project is a **Research and Development** (**R&D**) project, the second is the purchase of a building. In the real world, it is difficult to find a bank to lend money for R&D: after all, if the firm fails to pay its interest payments, there is often little that the bank can collect and resell. Buildings, on the other hand, are easy to repossess. Therefore, the building offers more **Debt Capacity** than the R&D project. This can make it more valuable than the otherwise equally promising R&D project. Managers cannot choose among projects without taking into consideration how each project aids the debt capacity of the firm.

IMPORTANT: *In an imperfect world, unlike the M&M world, managers cannot ignore or delay financing decisions when making real investment decisions. The two decisions are intertwined.*

SIDE NOTE: A second complication derives from the fact that the value of the debt capacity can depend on who the owner is. Although most profitable and older firms are in the same highest tax bracket, some younger, growing, and unprofitable firms are in lower tax brackets. To these younger firms, the debt capacity is worth a lot less than it is to a large firm like PepsiCo (which can immediately use the tax deduction).

Solve Now!

Q 22.11 *A firm has expected before-tax earnings of $20 per year forever, starting next year. It is financed with half debt (Risk-free, at 5% per year) and half equity (at 10% per year), and this is eternally maintained, and if the firm is in the 25% tax bracket, then what is its NPV?*

Q 22.12 *(Continued.) If this firm took on $50 in debt and maintained the debt load at $50 forever, rather than maintain a 50/50 debt-equity ratio, then what would this firm's value be?*

A. Nerd Appendix: Inflation and Taxes

When inflation increases, even real interest rates must also increase.

In addition to the government's direct benefits of printing money, inflation has an interesting indirect effect. Holding the agreed-upon interest fixed, inflation benefits borrowers and hurts lenders. Lenders who receive interest must pay taxes on the nominal amount of interest, not the real amount of interest. The reverse holds for borrowers. For example, assume interest rates are 3% and there is no inflation. A savings account holder with $100 in the 33% tax bracket has to pay 1% to Uncle Sam ($1), and gets to keep 2% ($2). Now assume that interest rates are 12% and inflation is 9%. The savings account holder would now have to pay 4% ($4) in taxes, and own $108 the coming year. However, because money has lost 9% of its value, the $108 is worth less than $100 the following year. In effect, although real rates are identical in the no-inflation and inflation scenarios, a lender who pays taxes on nominal interest receipts gets to keep less in real terms if p there is inflation. (It is straightforward to check that the opposite is true for borrowers.) The implication of this argument is simple: to compensate lenders for their additional tax burdens (on nominal interest), *real* interest rates must rise with inflation.

A. Q&A: Answers

1. \$500 if designated an interest payment. \$750 if designated a dividend distribution, because only \$500 is left after corporate income taxes have been paid.

2. With an internal rate of return of 20%, Uncle Sam would see \$90,000 if you pay cash. If you finance with 80% debt, you will have \$40,000 in interest to deduct from \$200,000 in return, and thus pay taxes only on \$160,000. This lowers your tax bill to \$72,000. (Side Advice: If you borrow \$800,000, you may have to invest your \$800,000 elsewhere. If you do not choose tax-exempts, Uncle Sam may receive more taxes therefrom.)

3. The net subsidy is $\$90,000 - \$72,000 = \$18,000$ next year. At an appropriate cost of capital of 8%, this is an NPV of $\$16,667$.

4. The WACC valuation is

$$\mathsf{PV}_0 = \frac{\$256}{1 + 12\% - 25\% \cdot 30\% \cdot 8\%} = \$229.^{80} \ . \tag{22.32}$$

The firm has $\$229.^{80} \cdot 25\% = \$57.^{45}$ of debt and $\$172.^{35}$ in equity value today. Its APV is

$$\begin{aligned} \mathsf{APV}_0 &= \frac{\mathcal{E}(\mathrm{CF}_1)}{1 + \mathcal{E}(\tilde{r}_\mathsf{W})} + \frac{\tau \cdot r_\mathsf{D} \cdot \mathsf{D}_0}{1 + \mathcal{E}(\tilde{r}_\mathsf{W})} \\ &= \frac{\$256}{1 + 12\%} + \frac{30\% \cdot 8\% \cdot \$57.^{45}}{1 + 12\%} = \$229.^{80} \ . \end{aligned} \tag{22.33}$$

5. The APV valuation is

$$\begin{aligned} \mathsf{APV}_0 &= \frac{\mathcal{E}(\mathrm{CF}_1)}{1 + \mathcal{E}(\tilde{r}_\mathsf{W})} + \frac{\tau \cdot r_\mathsf{D} \cdot \mathsf{D}_0}{1 + \mathcal{E}(\tilde{r}_\mathsf{W})} \\ &= \frac{\$256}{1 + 12\%} + \frac{30\% \cdot 8.7\% \cdot \$100}{1 + 12\%} = \$230.^{90} \ . \end{aligned} \tag{22.34}$$

Therefore, the \$100 debt is 43.3% of the firm's value today, and

$$\mathsf{PV}_0 = \frac{\$256}{1 + 12\% - 43.3\% \cdot 30\% \cdot 8.7\%} = \$230.^{90} \ . \tag{22.35}$$

6. WACC for ratio, APV for dollar amounts. Look at the previous two questions. You cannot figure out the APV in the first question before you determine the WACC, and the opposite in the second question.

7. • The firm's overall cost of capital today is $6\% \cdot {}^1/_3 + 12\% \cdot {}^2/_3 = 10\%$. (Because $4\% + 3\% \cdot 2 = 10\%$, the beta is 2.) • The easy way is to recognize that the firm is sheltering $\$500 \cdot 6\% = \$30$ through interest payments. If it refinanced with \$1,000, it could now shelter $\$1,000 \cdot 8\% = \$80$. Uncle Sam would get to see an additional \$50 less in income, which means that the firm would pay $\$50 \cdot 20\% = \$10$ less in income tax *next year*. • Now we need to determine the appropriate discount rate for \$10 in tax savings. For convenience, use the debt cost of capital: 8%. This means that our recapitalization increases firm value by $\$10/1.08 \approx \$9.^{26}$. (If you prefer to use the overall firm cost of capital, you would obtain $\$9.^{09}$.) • The question intentionally gave additional irrelevant information.

8. Because we know that the cost of capital if all financed by debt has to be the cost of capital for the firm, we know that the firm's overall cost of capital is $\mathcal{E}(\tilde{r}_\mathsf{D}) = 15\% + 100\% \cdot 5\% = 20\%$. Now, this project will offer \$200 pretax profit in year 1. Discounted back at the firm's cost of capital,

the NPV without taxes is $-\$300 + \$500/(1 + 20\%) = \$116.^{67}$. But, if equity financed, the I.R.S. will declare taxes due on \$200 of profit, or \$80. So, the NPV with taxes and all equity financed is $-\$300 + \$420/(1 + 20\%) = \$50$.

Now, right after the investment, the firm has a value of $\$420/1.2 = \$350$. With debt of \$50 (\$100), the firm carries a debt load of around 15% (30%). The cost of debt capital formula given in the question suggests that $\mathcal{E}(\tilde{r}_D) = 15\% + 15\% \cdot 5\% = 15.75\%$ (16.5%). (Note: the question is a bit ambiguous in that it does not tell us what to use as firm value is a bit ambiguous. The 15% and 30% debt ratios are reasonable values, though.)

Interest payments on \$50 (\$100) at a cost of capital of 15.75% (16.5%) are $\$7.^{88}$ ($\$16.^{50}$) *next year*. Facing a tax rate of about 40%, Uncle Sam would thereby subsidize the project to the tune of $40\% \cdot \$7.^{88} = \$3.^{15}$ ($\$6.^{60}$), which in today's value would be worth around $\$3.^{15}/(1 + 20\%) \approx \$2.^{63}$ ($\$6.6/1.2 \approx \$5.^{50}$). Therefore, under APV, if financed with \$50 in debt (\$100 in debt), the project is worth $\$50 + \$2.^{63} = \$52.^{63}$ ($\$50 + \$5.^{50}$).

The cost of capital, if 15% of the firm is financed by debt at an interest rate of 15.75% is the solution to $15\% \cdot 15.75 + 85\% \cdot \mathcal{E}(\tilde{r}_E) = 20\% \rightarrow \mathcal{E}(\tilde{r}_E) = 20.75\%$. So, the WACC is given by the formula, $w_E \cdot \mathcal{E}(\tilde{r}_E) + w_D \cdot \mathcal{E}(\tilde{r}_D) \cdot (1 - \tau) = 85\% \cdot 20.75\% + 15\% \cdot 15.75\% \cdot (1 - 40\%) \approx 19.06\%$. Similarly, if \$100 is borrowed, $\mathcal{E}(\tilde{r}_E) = 21.5\%$, and $\text{WACC} = w_E \cdot \mathcal{E}(\tilde{r}_E) + w_D \cdot \mathcal{E}(\tilde{r}_D) \cdot (1 - \tau) = 70\% \cdot 21.5\% + 30\% \cdot 16.5\% \cdot (1 - 40\%) \approx 18.02\%$. The WACC based value is thus $-\$300 + \$420/(1 + 19.06\%) \approx \$52.^{76}$. Note that we have made enough little assumptions and approximations to allow us to be off by a little in the APV and WACC computations ($\$52.^{76}$ and $\$52.^{63}$).

9.

(a) The pro-forma for a 100% equity financed firm is

	income statement			
		Year 1	Year 2	Year 3
	ebitda (=Net Sales)	\$70	\$60	\$55
–	depreciation	\$50	\$50	\$50
=	ebit (=operating income)	\$20	\$10	\$5
–	interest expense	\$0	\$0	\$0
=	eaibt	\$20	\$10	\$5
–	corporate income tax	\$8	\$4	\$2
=	net income	\$12	\$6	\$3
	cash flow statement			
	net income	\$12	\$6	\$3
+	depreciation	\$50	\$50	\$50
=	Operating Cash Flow	**\$62**	**\$56**	**\$53**
	capital expenditures	–\$150	\$0	\$0
=	Investing Cash Flow	**–\$150**	**0**	**0**
Economic Project Cash Flows: Operating CF+ Investing CF+ Interest				
Project Cash Flows		**–\$88**	**+\$56**	**+\$53**

(b) The IRR of our project solves

$$\frac{-\$88}{1 + \text{IRR}} + \frac{+\$56}{(1 + \text{IRR})^2} + \frac{+\$53}{(1 + \text{IRR})^2} = 0 \ . \tag{22.36}$$

Thus, the IRR of a purely equity financed project is 15.7%.

(c) The NPV of the purely equity finaned project is

$$\text{NPV} = \frac{-\$88}{1 + 18\%} + \frac{+\$56}{(1 + 18\%)^2} + \frac{+\$53}{(1 + 18\%)^3} = -\$2.^{10} \ . \tag{22.37}$$

This is in line with the fact that the project IRR of 15.7% is less than the 18% cost of capital.

(d) The cash flows would increase to −\$88, +\$58, and +\$55. The IRR would increase to 18.6%.

(e) The debt-financed pro-forma would now be

	income statement			
		Year 1	Year 2	Year 3
	ebitda (=Net Sales)	\$70	\$60	\$55
–	depreciation	\$50	\$50	\$50
=	ebit(=operating income)	\$20	\$10	\$5
–	interest expense	**\$0**	**\$5**	**\$5**
=	eaibt	\$20	\$5	\$0
–	corporate income tax	\$8	\$2	\$0
=	net income	\$12	\$3	\$0
	cash flow statement			
	net income	\$12	\$3	\$0
+	depreciation	\$50	\$50	\$50
=	Operating Cash Flow	**\$62**	**\$53**	**\$50**
	capital expenditures	–\$150	\$0	\$0
=	Investing Cash Flow	**–\$150**	**0**	**0**
Economic Project Cash Flows: Operating CF + Investing CF + Interest				
Project Cash Flows		–\$150+\$62	+\$53+\$5	+\$50+\$5
	=	**–\$88**	**+\$58**	**+\$55**
	Economics of Financing			
Debt		+\$50	+\$5	+\$55
Equity		+\$38	+\$53	+\$0

Not surprisingly, these are the same as our aforementioned cash flows, with a \$2 income-tax subsidy in years 2 and 3. So, the IRR is again 18.6%.

(f) The NPV of the debt-financed firm is

$$\text{NPV} = \frac{-\$88}{1+18\%} + \frac{+\$58}{(1+18\%)^2} + \frac{+\$55}{(1+18\%)^3} = +\$0.^{55} \quad . \tag{22.38}$$

So, with the tax subsidy, this project becomes worthwhile.

(g) The APV of this project would be the value as-if-100%-equity-financed, which is –\$2.^{10} (Formula 22.37), plus the discounted tax subsidies in years 2 and 3. These have a value of

$$\text{Tax Subsidy} = \frac{\$2}{(1+18\%)^2} + \frac{\$2}{(1+18\%)^3} = \$1.^{44} + \$1.^{22} = \$2.^{66} \quad . \tag{22.39}$$

Therefore, the APV would be $-\$2.^{10} + \$2.^{66} = \$0.^{56}$.

(h) By APV, the expected tax subsidy would shrink from $\tau \cdot \mathcal{E}(IP) = 40\% \cdot \$5 = \$2$ per year to $\tau \cdot \mathcal{E}(IP) = 40\% \cdot \$4 = \$1.^{60}$ per year. The expected value of the tax subsidy would therefore be

$$\text{Tax Subsidy} = \frac{\$1.^{60}}{(1+18\%)^2} + \frac{\$1.^{60}}{(1+18\%)^3} = \$2.^{12} \quad . \tag{22.40}$$

The net project value would be about \$0.^{02}.

(i) In year 0, the weight of the debt is $w_{D,0} = \$50/\$88 \approx 57\%$. But after year 2 and before year 3, the debt is expected to be 100% of the capital structure, so its weight in the capital structure is drastically changing each year. So, this firm is not at all a good candidate for WACC application.

NERD NOTE: *Do not try to compute a weighted average cost of capital from the debt and equity internal rates of return (10% and 40%, respectively). If the debt would remain at 57% of the firm's capital structure, then the appropriate rate of return of equity would have to be around 30% so that the weighted cost of capital would come out to* $\mathcal{E}(\tilde{r}_{\mathsf{W}}) = w_{\mathsf{D}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{D}}) + w_{\mathsf{E}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{E}}) = 18.6\%$. *This is much lower than the equity IRR of 40% (which is the same as its expected rate of return from year 1 to year 2), because from year 2 to 3, the equity becomes a much smaller part of the firm. What bites us in this case is the fact that we have a strong term structure of investment weights.*

10. Do it!

11. The weighted average cost of capital (WACC) is

$$\begin{aligned}\mathsf{WACC} &= w_{\mathsf{D}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{D}})\cdot(1-\tau) + w_{\mathsf{E}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{E}}) \\ &= 50\%\cdot 5\%\cdot(1-25\%) + 50\%\cdot 10\% = 6.875\% \ .\end{aligned} \tag{22.41}$$

The numerator has to be post corporate income tax; therefore, it is $(1-\tau)\cdot\mathrm{CF} = \$15$. This is an annuity, therefore the NPV is

$$\mathsf{NPV} = \frac{\$15}{6.875\%} = \$218.^{18} \ . \tag{22.42}$$

12. The cost of capital for a fully equity financed firm without a tax subsidy would be 7.5%, because it had 50% debt at 5% and 50% debt at 10%. Therefore, the "as if fully equity financed" value is

$$\mathsf{NPV} = \frac{\$15}{7.5\%} = \$200.^{00} \ . \tag{22.43}$$

Now, we need to add back the tax subsidy. With \$50 in debt, risk-free and therefore with an interest rate of 5%, the interest payments would be $\mathcal{E}(\tilde{r}_{\mathsf{D}})\cdot\mathsf{D} = \$2.^{50}$ per year. The taxes saved would be $\tau\cdot\$2.^{50} = \$0.^{625}$, which is an eternal cash flow. At the interest rate of 5%, the value of the tax subsidy today is $\$12.^{50}$. Therefore, the value of this firm is $\$200+\$12.^{50}= \$212.^{50}$.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

B. Q&A: Summary

The chapter covered the following major points:

- In the imperfect real world, the U.S. tax code favors debt (borrowing) over equity (partnered-financing). Managers should take this corporate income tax advantage into account.
- The calculation of the income tax advantage can be done either through a direct pro-forma (financing scenario) or through an adjusted WACC method or through the APV method.
- In the imperfect real world, financing and investment decisions can no longer be separated: projects that add more debt capacity may add value through the financing channel.

CHAPTER 23

Personal Income Tax Considerations

Clientele Effects

last file change: Jun 10, 2004 (17:30h)

last major edit: n/a

This chapter explains the role of personal income taxes. After all, corporate income taxes are just one side of what Uncle Sam receives: he also wants his share from investors' income. This chapter discusses how personal income taxes that investors have to pay can change the optimal corporate capital structure for firms and create clienteles: small growth firms should have more equity in their capital structure than large cash firms.

This chapter also explains dividends.

23·1. The Basics of Personal Income Taxes

23·1.A. The Tax Code For Security Owners

The type of income matters: capital gains income is better for taxable investors than interest income.

When investors receive money, the form of income in which they get money matters to them, too. The description on Page 17 explained how income taxes are computed: interest and dividend incomes are taxed at ordinary income tax rates, which is usually above the tax rate applicable to (long-term) capital gains. Moreover, capital gains are only taxed when they are realized (the investment is sold) and capital gains can be offset with capital losses from other investments. (Interest income and dividend income cannot be reduced by capital losses.) With capital gains' lower statutory tax rates, the ability of investors to decide when to realize capital gains, and the ability to use other capital losses to reduce the taxable capital gains, investors prefer the tax treatment of capital gains income over the tax treatment of dividend and interest income.

Side Note: There are some very intricate tax rules on how capital gain income and interest income on bonds must be computed. This is to prevent corporations from paying out interest income in a form that makes them capital gains for their investors. (The loophole that these regulations had to close were **Zero Bonds**.) The tax treatment of bonds is an ongoing cat-and-mouse game between the I.R.S. and corporate tax lawyers.

Nerd Note: *In some countries, such as the United Kingdom, dividend recipients receive a credit for taxes paid by the corporation. Relative to the United States, dividends are thus more attractive in the United Kingdom.*

23·1.B. The Principle is "Joint Tax Avoidance"

The owners care not about where taxes are paid (corporation or personal), just that as little as possible is paid in total.

Corporate income taxes are not the end of the story. If you were to own a corner shop, you would care how much you could ultimately spend, not whether the IRS taxes you right at the cash register of your business, or taxes you personally when you move the cash from the register into your pocket. It is the money in your pocket, which you have left over after Uncle Sam has had his dip, that matters.

Important: *Both corporate and personal taxes that can be avoided is cash that the owners can keep. Reducing the total taxes ultimately collected by Uncle Sam (in the future) increases the value of the firm (today).*

Investors are the corporate owners.

Similarly, an investor who receives interest payments and who must pay personal income taxes cares only about after-tax receipts. To the corporate manager, these taxes are not directly visible: the firm sees them "only" in its cost of capital. When a firm offers securities that cost the investors a lot of personal tax liabilities, these investors will demand a higher compensation (expected rate of return) than they would if they did not have to pay taxes. This manifests itself in higher expected rates of returns demanded by the capital markets when the firm wants to obtain financing.

IMPORTANT: *The owners' personal tax burden is visible to corporate managers in the cost of capital that they have to pay in order to obtain financing. Thus, the* WACC *or* APV *formulas from Chapter 22 remain correct. It is in the cost of capital inputs to the* WACC *formula (the expected rates of returns demanded by equity holders and bond holders) through which the personal income taxes paid by investors manifests itself. In the real world, the financial markets provide managers costs of capital that reflect personal tax and a host of other investor concerns.*

SIDE NOTE: The Bush Tax Cuts of 2003 are intended to reduce the double taxation of dividends. The details are not worked out yet, although it appears that investors will receive a credit for dividend taxes paid by corporations. The tax cuts will not change anything in the WACC or APV formulas. However, it does change the appropriate expected rate of return that equityholders demand. For example, assume that the taxes on equity distributions dropped from 50% to 25%. If shareholders previously demanded an expected rate of return of 12%, of which they could keep 9%, they may now only demand 9% to begin with. Thus, the corporation would now plug the lower $r_E = 9\%$ into its WACC formula, but the formula itself would stay the same.

Distributions in interest save firms money, but not investors. Distributions in capital gains save investors money, but not firms.

Owners (and managers on their behalf) should try to arrange their firms' capital structures in order to minimize the total taxes paid after cash has been distributed to and personally taxed by their owners, not just minimize their own corporate income taxes. Firms know that investors cannot shelter interest income and dividend income, but that they can easily shelter capital gains income (delaying realization, lower long-term capital gains rate, offsetting capital losses).

Companies can shift tax burdens from them to their investors and vice-versa.

From the perspective of the company, if it can transform its cash into interest income for its investors, it saves on its own corporate income tax, but its investors will face the full brunt of Uncle Sam (and thus demand a high expected rate of return). If the company transforms its cash into capital gains for its investors, it does not enjoy the tax subsidy from dividends, but its investors may pay much lower taxes—and thus demand only a lower expected rate of return.

Investors should care only about value today!

It is worth repeating this argument in terms often argued by the less educated: "But, would changing the capital structure, e.g., taking on more debt, not dilute earnings? Or increase the risk?" (The latter statement actually contains two errors, and they unfortunately do not cancel: the risk of the firm is the risk of the projects, not the risk of the residual equity. We already know that the blended risk stays the same.) The correct view is that earnings are tertiary concerns. Put yourself into the shoes of the owner investors today. You want only one thing: more value, more money, more wealth. To the extent that Uncle Sam receives less money, what you own is worth more. It is these tax savings that matter to investors, not any ephemeral concerns about equity or debt or risk split-up or earnings dilution, or any of a hundred other reasons sometimes given by management.

23·1.C. Tax Clienteles

Complication: Different investors face different personal tax rates.

To make matters more complicated, managers have to be concerned that, in real life, not every investor faces the same tax rate. There are low-tax investors, like pension funds, pay low personal taxes either on dividend or interest distributions. And there are high-tax investors, like most individual investors, who pay relatively lower taxes when they receive income as capital gains instead of as interest.

"Clientele" means different firms attract different investors

The net effect of putting personal tax and corporate tax considerations together causes "clienteles" to form:

High-tax, profitable firms should pay via interest (thus, have debt).

- Firms in the highest corporate income tax bracket can readily use the tax shelter provided by debt. Because individual investors cannot easily shelter interest income from personal income taxes, it is not as much in their interest to hold them, as it is for pension funds and other untaxed entities. These tax-exempt institutions should flock to purchase the bonds issued by high-tax corporations.

Low-tax, unprofitable firms should pay via share repurchases (thus, have equity).

- Firms in the lowest corporate income tax bracket have less use for the tax shelter provided by debt. Because individual investors could not tax-profitably invest in bonds, they prefer that these low-corporate tax firms issue securities that tax them less: with capital gains being the best medium for individual taxable investors, low tax bracket corporations should finance themselves with equity, not with debt, and use earnings to repurchase securities or to reinvest in the corporation.

SIDE NOTE: There are two categories of firms in low tax brackets: First, firms can be in a low tax bracket if they have undergone financial distress in the past with current carry-forward **Net Operating Losses** (**N.O.L.s**). Second, small firms, often in high growth industries, have usually not yet earned enough net income to exceed the expenses put up by their owners.

NERD NOTE: *Current tax-laws allow firms to either receive a refund equal to the excess deductions in the current year for taxes paid up to three years ago, or to carry forward deductions and apply them to future taxes. Further, if a company cannot use its tax deductions, it can sometimes sell itself to a company that can use the tax deductions.*

Carried to extremes, Uncle Sam might not get anything.

In an "ideal" world, Uncle Sam may not receive anything, at all: if there are enough tax-exempt investors to absorb all debt from high-tax corporations, the corporations can avoid all corporate income taxes by paying out their net income as interest payments. Neither corporation nor investors pay tax. Low tax corporations pay no taxes on any income either, and their investors could just hold the equity and not realize the capital gains.

The extreme tax avoidance is illustrative, but not realistic.

Admittedly, this is a caricature: there are reasons why tax-exempt investors do not hold only or all of high-corporate-tax firm's debt; reasons why high-tax corporate tax firm's have equity; reasons why taxable investors hold debt; and reasons why low-tax bracket firms have debt. But, the tax rationale for this sorting argument should be clear. Furthermore, in the real world, clientele sorted ownership effects are important. For example, corporate bonds are indeed overwhelmingly owned by tax-exempt institutions

NERD NOTE: *In equilibrium, security yields and investor holdings are jointly determined, not sequentially as indicated by this discussion. But, profit seeking means that in equilibrium, proper sorting will take place. If tax-exempt investors were to sort themselves into the wrong stocks, the yields on corporate interest-bearing securities would increase, and tax-exempt investors should find it irresistible to pass up on corporate bonds (which, in order to be purchased by taxable retail investors, would really have to offer high rates of returns). Tax-exempts would find it in their interest to sell their equities and move into these bonds. In equilibrium, dividend and interest bearing securities should have lower pre-tax rates of returns and higher post-tax rates of returns than zero-dividend stocks.*

23·1.D. An Illustration of Clienteles and Joint Tax Burdens

Figure 23.1. Clientele Tax Avoidance

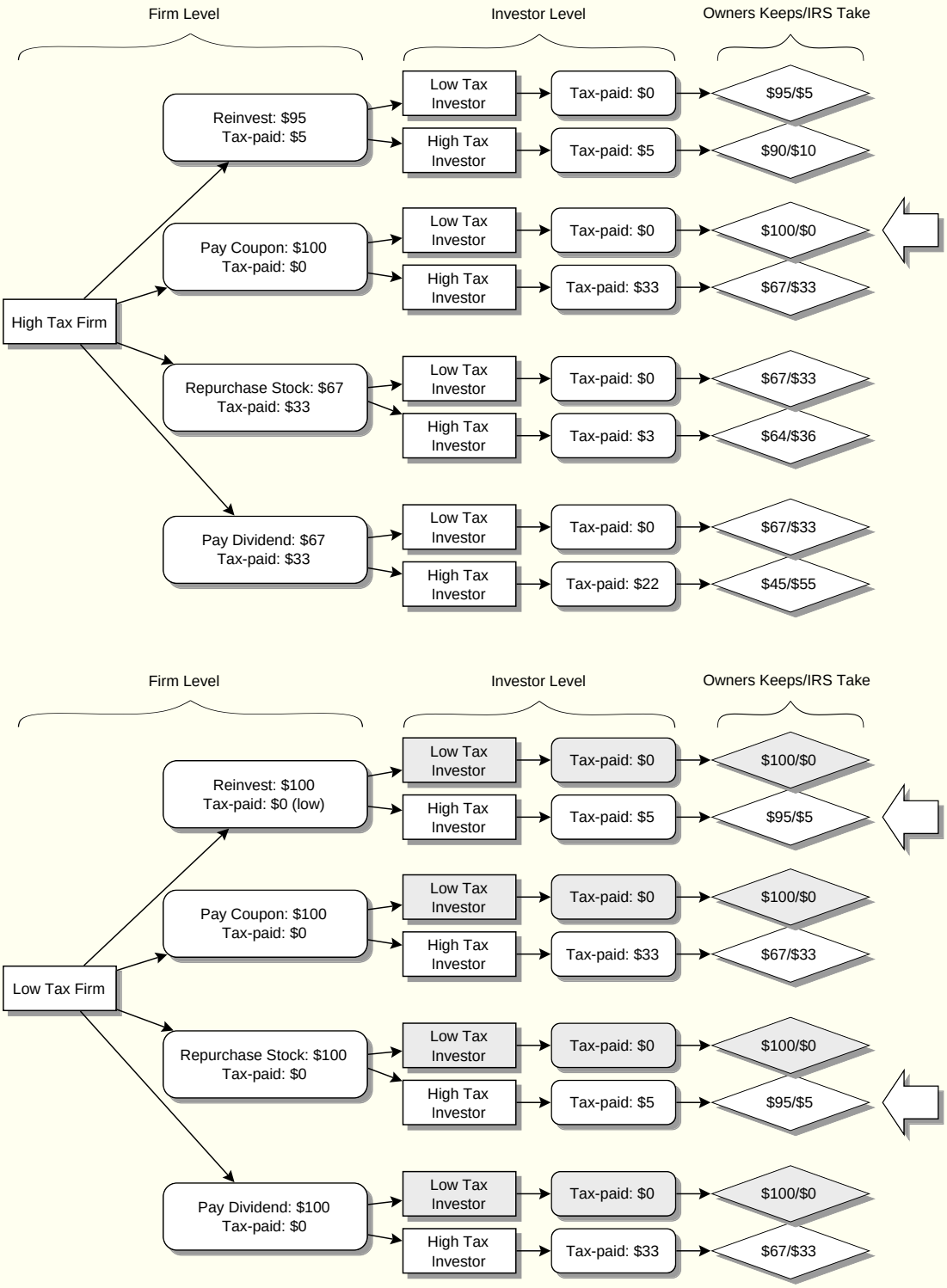


Figure 23.1 illustrates the choices available to firms and investors—and Uncle Sam's take—numerically. The figures assume that when firms reinvest, they pay a small tax penalty (5%). Firms can only reinvest post-corporate-tax income, but they do get credit for their reinvestments. In addition, some reinvestments can be called "maintenance" and never get classified as taxable net income in the first place. Analogously, when investors receive capital gains, we assume that they cannot delay taxes forever and/or cannot always find appropriate capital losses to offset these gains, so we assume a 5% income tax cost penalty in today's terms.

The figures show where the taxes flow.

The game to be played is to think of a corporate economy in which there is one high-tax firm and one low-tax firm; and one high-tax investor and one low-tax investor. (The important assumption here is that the tax-exempt investors are not so abundant that they can purchase everything. Over-abundance would eliminate the influence of taxed investors; personal taxes would not matter at all; and all firms could pay out their earnings in interest.) The joint goal of this corporate economy is to minimize the dead-weight losses that accrue to the I.R.S. In the real world, there are of course many investors and many firms, and it is the price system that will adjust the prices of different securities (and with it the costs of capital) to make sure that there is no money left on the table for the I.R.S. that could be avoided with a better clientele sorting policy.

Think of a limited number of tax-exempt firms.

Figure 23.1 shows that when one adds up total taxes-paid, high-tax firms are best off issuing debt to pay interest to low-tax investors. Figure 23.1 shows that low-tax firms are comparatively better off than high-tax firms if they use earnings to generate capital gains, likely held by low-tax investors. (Figure 23.1 has grayed out the boxes for low tax investors, because high-tax firms have "used up" low-tax investors in their competition for tax-sheltering services.) In total, in this corporate economy, with one high-tax firm and one low-tax firm and one high-tax and one tax-exempt investor, Uncle Sam gets to keep only $5 for every $200 earned in the corporate sector.

The best clienteles.

NERD NOTE: *Even in the U.S., with its relatively simple tax code and its armies of academic finance researchers, taxes remain a complex and empirically unresolved issue. Academics are still debating who benefits most from tax arbitrage, dividend-paying issuers or tax-exempt investors. If there are many tax-exempt institutions to compete with one another for the privilege of conducting tax arbitrage, firms could borrow at rates as if all their investors were tax-exempt. Yet, if there are too many firms paying dividends, they will compete to pay a premium to find tax-exempt institutions, up to and including the cost of paying taxes—and it would be the tax-exempt investors who would benefit most from their tax-exempt status. As always, the truth is probably in the middle, where the tax-arbitrage benefits are shared by tax-exempt investors and taxed firms.*

Solve Now!

Q 23.1 *Explain the (personal and corporate) tax treatments if the company pays its operating cash flow in interest, repurchase, or dividends.*

Q 23.2 *Would you expect large, stable firms to be predominantly held by pension funds or by individuals? Would you expect young, growing firms to be predominantly held by pension funds or by individuals?*

23·2. Dividends and Share Repurchases

Share repurchases and dividends are perfect substitutes in a perfect world.

We had already discussed payout policies in Chapter 19. Share repurchases are a direct substitute for dividends. A naïve misconception is that share repurchases are different because they benefit only shareholders tendering their shares into the repurchase—unlike dividends, which benefit all shareholders. This argument is false, which is easiest to demonstrate in a perfect M&M world. A firm with 100 shareholder, each owning \$10 worth of shares, could pay \$50 worth of dividends (50 cents to each shareholder), and the firm would be worth \$950. Each shareholder would have a share worth \$9.50 and \$0.50 in dividends. If the firm repurchased \$50 worth of shares, the firm would now have 95 shareholders, each owning \$10 worth of shares. In the perfect world, without taxes, all shareholders are equally well off with either a repurchase or a dividend payment.

In the real world, dividends are tax-disadvantaged on the investor side, relative to share repurchases.

Leaving the M&M world, introducing taxes, if there is less money for dividend payments than there is for a share repurchase, it must follow that share repurchases should be preferred. Dividends are difficult to shelter, while in share repurchases, the tendering investors will be those who find it easy to shelter the capital gains. Net in net, personal taxes on dividends are high and personal taxes on share repurchases are low.

Side Note: This is set to change in the future, however, with the enactment of the Bush tax reforms: investors are likely to receive a partial tax credit for taxes paid by the corporation, although the details have not yet been fully worked out. This tax credit would drastically reduce the advantage of share repurchases over dividends.

The existence of dividends is a mystery.

The tax clientele argument explains why some investors prefer holding equity over debt (to receive capital gains instead of taxable interest payments) and why dividend taxes are not as important as one might think (low-tax pension fund investors can avoid personal income taxes on dividends). But, why do firms pay dividends, instead of repurchasing their equity or paying interest? After all, academics have pointed out for a long time that every investor has to pay the same personal taxes on interest and dividends, and every firm is at least as well off paying interest coupons as it is paying dividends.

Share repurchases are becoming relatively more important over time.

No one really knows the answer why companies have paid out so much in dividends rather than in repurchases, although managers no longer appear to know either. Bagwell and Shoven (1989) find that big firms paid out about seven times more through dividends than they paid out through share repurchases in 1977. By 2000, dividends and share repurchases were about equally important. The presence of dividends is already less of a mystery than they once were.

Because dividend payout ratios are small these days, they hurt less.

Moreover, in 2002, typical **Dividend Yields** (the amount of dividends divided by the share price) have declined to 2 to 3 percent per year. Consequently, taxes for an investor in the 30% to 50% tax bracket are on the order of 50 to 100 basis points per year. In addition, some tax-exempt institutions may hold these stocks and not pay any income taxes on the dividend income. Putting this all together, it appears that dividends may not be the smartest method for managers to pay out, but the wasted money is too small to offer large arbitrage opportunities (buying the firm, firing the CFO and changing the payout policy, and selling the firm for a higher price).

Side Note: Because dividends are heavily taxed, like interest payments, they can attract some tax-exempt investors. Indeed, the desire to attract this clientele may be one reason for some companies to pay dividends in the first place.

ANECDOTE: Ralph Nader and Microsoft
On January 4, 2002, Ralph Nader wrote an open letter to William H. Gates, III, Chairman of Microsoft, that begins as follows:

> We are writing to ask Microsoft to change its practice of not paying dividends to shareholders. Our reasons are as follows.
>
> 1. The quantitative failure to pay dividends year after year is an inappropriate and we believe unlawful device to shelter Microsoft earnings from federal income taxes.
>
> By not paying dividends, wealthy Microsoft shareholders such as yourself avoid paying the top marginal tax rate of 39.6 percent that would apply to income distributed as dividends. By taking earnings entirely through stock sales, wealthy shareholders lower their tax rate to the maximum 20 percent that applies to capital gains. According to the most recent SEC reports on insider trades, you personally sold more than $2.9 billion in Microsoft stock last year, benefiting enormously from the lower tax rate that applies to stock sales.

And this letter does not even point out that 20% is an overstatement: Bill is only taxed on *realized* capital gains! If Bill does not sell his shares, he suffers zero taxes on increases in his wealth over the year. And, with the **G.O.P.** elimination of the **Estate Tax**, neither may his heirs. The Bush tax reforms of 2003 have further significantly reduced the taxes on dividend payments. And Microsoft promptly started paying dividends.

Here is an interesting question: Is it the fault of William Gates (incidentally, a prolific political campaign donor) or is it the fault of the U.S. government that he has minimal legal taxes to pay?

Solve Now!

Q 23.3 *If a normal investor cannot participate in a share repurchase program, is she better off with a dividend payout than with a share repurchase?*

Q 23.4 *What companies should pay dividends?*

23·3. Other Capital Structure Related Tax Avoidance Schemes (NOLs, Leases, Dividend Capture, Units, Transfer Pricing, etc.)

There are too many tax schemes in existence to list in just one book. They are also changing all the time. Here are some examples.

There is an arms race between the I.R.S. and investors: investors keep looking for new tax avoidance schemes and the I.R.S. tries to close these new loopholes. Wall Street employs armies of tax experts to help its clients avoid taxes. There are a large number of past (now closed) and current tax avoidance schemes. Some of the more noteworthy schemes are:

- Sometimes, high-tax firms may be able to purchase low-tax firms, and thereby immediately use the acquired firm's existing **Net Operating Losses N.O.L.s**.

  For example, the Financial Times (Feb 10, 1994) reports that the £2.5B GKN corporation made a hostile bid for the £300M Westland corporation, solely to exploit its own pent-up advance corporation tax.

- Compared to purchasing on credit, **Leasing** can be a tax advantageous arrangement. If the borrower does not have enough income to efficiently use the interest deduction, someone else should be the official owner of the asset and "lease" it to the borrower.

- If personal investors want to hold dividend-paying stocks without paying dividend taxes, they can do so for most of the year—as long as they sell them to institutions prior to the **Ex-Dividend Day**, which is the day on which the owner of the shares is determined for the distribution of the dividend.

  In real life, this "tax-arbitrage" indeed happens; tax-exempt funds compete to purchase these shares, driving up the share prices before the ex-dividend date. The Financial Times reports that such transactions are known as "bed-and-breakfast" deals for equity, "bond-washing" for bonds—even though both the I.R.S. and the **Bank of England** have specifically prohibited such tax arbitrage. The latter has imposed a 1-week holding period for tax-free institutions purchasing around dividend dates.

  Of course, there are limits to such dividend tax arbitrage. Some high-tax investors would have to pay capital gains taxes when they sell their shares, and thus prefer paying taxes on the dividends instead of on their capital gains. In addition, the round-trip transaction costs limit the possible profits from the avoidance of dividend taxes.

- **Units** are combination securities that consist of a debt security and an equity security. If the firm pays interest, it shifts its tax burden to the unit owners. If the firm pays dividend, it shifts its unit owners' tax burdens to itself.

- Multinational corporation can shift difficult-to-value assets producing income from a high-tax country into a low-tax country. For example, corporate income taxes in **Switzerland** (federal and canton) can be as low as 7.8% (for holding companies) and as high as 25%. This contrasts with corporate income tax rates usually as high as 45% in the United States.

  Now consider a company that has just developed a patent worth \$10 million per year. If the U.S. branch owned the patent, the firm would retain only $(1 - 45\%)\cdot\$10 = \$4.5$ million per year. If the Swiss branch owned the patent, the firm would retain up to $(1 - 7.8\%)\cdot\$10 = \$4.5\$9.2$ million per year. Why stop at \$10 million? If the Swiss branch charged the U.S. branch \$20 million per year, the firms

U.S. tax obligations (resulting from profits from other businesses) would decline by $9 million per year (45% times $20 million), but Swiss tax obligations would increase by $1.56 million. Still, this is a healthy $7.4 million net gain.

This tax-efficient capital transfer can also be accomplished with capital structure. For example, if the Swiss branch lent funds to the U.S. branch at an interest rate of 36% per year, rather than 6% per year, the effect would be a reduction of the firm's tax liabilities. For every $1,000 in excess interest paid (at the 36% instead of the 6% rate), the company would retain an extra (45% − 7.8% = 37.2%) $372 in profits. Companies can play similar, but less drastic tax games by choosing the U.S. state and municipality in which they are headquartered.

The I.R.S. is very much aware of these issues. For example, on June 24, 2002, the *Wall Street Journal* reports that the I.R.S. is trying to prevent firms from shifting intellectual property, such as patents, to other countries in which corporations would have less taxes to pay.

ANECDOTE: Stanley Toolworks and Foreign Domiciles
We may even be lucky if Wall Street succeeds in escaping the I.R.S.: some corporations may just leave the United States altogether if taxes are too high. **Stanley Toolworks**, a hundred year old prominent Connecticut based global manufacturer of tools, was in the process of locating its headquarters to the Bermudas in mid-2002. This would allow its foreign subsidiaries to escape U.S. taxes. In the end, only unusually strong media attention and a public outcry prevented this departure.

ANECDOTE: Tax Reductions for the Needy? For-Profit Corporations with No Tax Obligations
The Washington Post reported a study by Institute on Taxation and Economic Policy which showed that 41 companies not only owed no taxes, but received money back in at least one of the three years studies—1996–1998—although they reported a total of $25.8 billion in pretax profits. 24 companies—nearly one in 10 studied—received tax rebates in 1998 alone, including such household names as Texaco Inc., Chevron Corp., PepsiCo Inc., MCI WorldCom Inc., Goodyear Tire & Rubber Co. and General Motors Corp. Texaco, for example, received a tax rebate of $67.7 million, which meant that it paid taxes at a rate of negative 37.2 percent on the $182 million in profit it reported in 1998. In dollar terms, the study found that tax breaks enabled the companies to reduce their taxes by $98 billion over the three years, with 25 companies receiving almost half of that amount. General Electric Co. topped the list, with $6.9 billion in breaks, which cut its tax bill by 77 percent over the three years. A G.E. spokesman also questioned the report's methodology, noting that of the $6.9 billion in breaks cited, $2.4 billion was deferred taxes "that we will pay."

The twenty-four companies that payed less than zero in federal income taxes in 1998 were

Company	98 Profit	98 Tax	98 Rate
Lyondell Chemical	$80.0	–$44.0	–55.0%
Texaco	$182.0	–$67.7	–37.2%
Chevron	$708.0	–$186.8	–26.4%
CSX	$386.6	–$102.1	–26.4%
Tosco	$227.4	–$46.7	–20.6%
PepsiCo	$1,583.0	–$302.0	–19.1%
Owens&Minor	$46.1	–$7.9	–17.1%
Pfizer	$1,197.6	–$197.2	–16.5%
J.P. Morgan	$481.1	–$62.3	–12.9%
Saks	$83.0	–$7.9	–9.5%
Goodyear	$400.7	–$33.2	–8.3%
Ryder	$227.5	–$16.4	–7.2%
Enron	$189.0	–$12.5	–6.6%
Colgate-Palmolive	$348.5	–$19.6	–5.6%
MCI WorldCom	$2,724.2	–$112.6	–4.1%
Eaton	$478.8	–$18.0	–3.8%
Weyerhaeuser	$405.0	–$9.5	–2.3%
General Motors	$952.0	–$19.0	–2.0%
El Paso Energy	$383.7	–$3.0	–0.8%
WestPoint Stevens	$142.6	–$1.2	–0.8%
MedPartners	$49.6	–$0.4	–0.7%
Phillips Petroleum	$145.0	–$1.1	–0.7%
McKesson	$234.0	–$1.0	–0.4%
Northrop Grumman	$297.7	–$1.0	–0.3%
Total	$11,953.0	–$1,272.9	–10.6%

23·4. Summary

The chapter covered the following major points:

- The managerial objective should be to minimize the overall tax burden—the sum of taxes paid by the corporation and its investors.
- Investor clientele effects arise, because they reduce overall tax costs.
- Overall tax reduction considerations suggest the following choices:

Choice	Low Tax Investors (e.g., Pension Funds)	High Tax Investors (e.g., Wealthy Individuals)
Better	Hold bonds or high-dividend stocks	Hold low-dividend stocks with high capital gains
Worse	Hold low-dividend stocks with high capital gains	Hold bonds or high-dividend stocks

(Note: the Bush tax cuts may reduce the tax disadvantage of dividends, which means that low-dividend and high-dividend stocks will be as good as low-capital gain and high-capital gain stocks. In this case, low-tax investors should focus more on bonds than on high dividend stocks, and high-tax investors can focus on stocks of any kind.)

Choice	High Tax Corporations (e.g., "cash cows")	Low Tax Corporations (e.g., "growth firms")
Better	Finance With Bonds	Finance With Stocks
Worse	Finance With Stocks	Finance With Bonds

- Despite the introduction of other effects—the presence of personal income taxes—the corporate income tax-augmented WACC formula from Chapter 22 remains valid. This is because investors' personal income taxes flow into the firm's cost of capital through the costs of capital quoted by the financial markets.

A. Q&A: Answers

1. The firm first pays taxes on money used for repurchase and dividends, but can use pre-tax money for interest payments. Investors can easily shelter repurchases (as capital gains), but face the full brunt of Uncle Sam on interest or dividend payments.
2. Stable and old: Pension Funds. Young and growing: Individuals.
3. No! Even this investor is better off with a share repurchase, but his increase in wealth now comes from in increase in unrealized (and therefore still untaxed) capital gains.
4. None. If absolutely necessary, it should be firms with many tax-exempt investors.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 24

Non-Tax Capital Structure Considerations

Bankruptcy Costs, Inside Information, Behavior

last file change: Jun 21, 2004 (16:41h)

last major edit: n/a

Taxes are important, but they are not the sole determinant of capital structure. This chapter explains many factors not tax related that should influence capital structure. Unlike taxes, the effects of these influences are often not easy to quantify in the real world. This does not mean that these influences are not important.

24·1. Operating Policy Distortions: Behavior in Bad Times

If some capital structure policies can lead to inferior investment policies, then capital structure choice can matter to firm value.

Think of a firm operating in a value-maximizing fashion, set up as a machine that is and always will be making optimal decisions. This firm will have the highest possible NPV today. Now imagine that a particular capital structure induces managers to deviate from this optimal behavior in the future. The resulting reduction in future cash flows reduces the firm's NPV, and should be considered a cost attributable to the inferior capital structure.

WACC still works for managers. These factors impact changes in the cost of capital, r, that managers face.

As with personal taxes, managers need not compute what expected value consequences their decisions will have: if they choose a capital structure leading to inferior future payoffs, they have to give away a larger percentage of the firm today, which manifests itself in the WACC that they experience. Of course, a smart manager could also compute the value of the firm "as if operated in a purely value-maximizing fashion," and then subtract out the expected costs imposed by the future suboptimal behavior caused by a particular capital structure. However, this would be so difficult that no manager does it.

Important: *Corporate income taxes require the use of* WACC *or* APV *formulas. All other distortions, ranging from personal investor taxes to bankruptcy distortions, manifest themselves in the cost of capital (the appropriate interest rate) that investors demand. The optimal capital structure allows managers to raise financing at the lowest cost of capital, and thereby maximizes the wealth of current owners.*

24·1.A. Direct and Indirect Bankruptcy Costs

Bankruptcy/default per se is not the problem.

A firm that has debt in its capital structure can go bankrupt. The important insight with respect to bankruptcy is that it is not bankruptcy *per se* that is the problem, but only the deadweight losses in and around bankruptcy that matters. To see this, go back to Table 21.1 and reconsider the second capital structure. In this perfect M&M world, the bad state is "bankruptcy," because the bond payments of \$150 cannot be met and the bond officially defaults. Still, the firm value today remains at \$136.[37]. Everyone (including bondholders) knows that the firm is transferred to bondholders, their payoffs in the bad state are \$100, and this is priced appropriately when the bond is sold.

Side Note: Creditors may also face uncertainties if bankruptcy courts do not uphold the agreed-upon **Absolute Priority Rule** (that bond holders are to be paid in full before equity holders receive anything). Known deviations from promised **absolute priority** simply change the contingent payoffs and thus the effective values of the securities. Such known violations do not reduce the total value of the firm. Relative to strict **APR**, the value of the bonds is just lower by the amount that the value of the equity is higher.

It is the additional costs arising from bankruptcy/default that are the problem.

But the world is not perfect, and there can be direct and indirect bankruptcy costs. **Bankruptcy Costs** are those which occur in the financial distress state. In the example in Table 21.1, if from the remaining \$100 in the bad state, \$20 were lost as deadweight costs in bankruptcy, then firm value today under the second capital structure would no longer be \$136.[64], but \$9.[09] less. Thus, the second capital structure would be worse than first capital structure, in which the firm could never go bankrupt. **Indirect Bankruptcy**

Costs are those which occur even before any financial distress state happens. In the example in Table 21.1, they would be costs that can occur today, even if we do not yet know which of the two future states will come about.

The most obvious direct costs are the resources (legal fees and management time) that the bankruptcy process consumes. There can also be inefficient production of goods in financial distress, liquidation costs, inefficiently low fire-auction prices of the firm's assets, customer departures due to confidence loss, etc. All these costs directly reduce the claims of the firm in bankruptcy, and are the real welfare losses of bankruptcy.

Direct legal and administrative direct bankruptcy costs are easily visible.

SIDE NOTE: In the United States, there are two primary legal forms of financial distress: **Chapter 7 Liquidation** and **Chapter 11 Bankruptcy**. Chapter 11 can be a long drawn out process, in which a firm is reorganized under the existing management, supervised by a court trustee. Although the history of bankruptcy, both in the U.S. and worldwide, are fascinating, the legal details of bankruptcy are beyond the scope of this book.

However, similar to the argument in the M&M proof, direct bankruptcy costs must have limits: if they are too high, someone can purchase all the securities, which eliminates the financial distress and with it all direct bankruptcy costs. In this sense, direct distress costs can be considered a special form of transaction costs: the costs of purchasing all securities to eliminate deadweight costs.

The limit to the costs' importance is the cost of combining securities.

SIDE NOTE: One problem is that when all other creditors agree to a bailout, you as the final bondholder would not want to agree. You would insist on your claim, and in order to execute the bailout, you would often have to be bought out at much better terms than the bailout creditors. The problem is that every creditor realizes this, and would prefer to hold out. Given such bargaining considerations, it may be cheaper for a potential acquiror to wait until the firm is sold in a fire-sale than to try to acquire securities before deadweight costs appear.

Some firms may even be able to reduce the rebundling costs by issuing **units** (unit securities), which bundle debt and equity. If the firm fails to pay interest in the future, creditors will be eager to compromise in order avoid financial distress—after all, there is little reason to force bankruptcy in order to collect assets from oneself.

Disagreement and bargaining among creditors makes efficient reorganization difficult.

The indirect costs of bankruptcy can be even more detrimental, because they can arise even before formal distress. They originate from the fact that firms can shed promised claims in bankruptcy, even if they would like to commit today (ex-ante) not to shed them in the future. This causes a loss of value when future distress is possible.

Loss of (or less) trust is an example of indirect bankruptcy costs.

- When products require customer investments, customers may be reluctant to purchase the product and invest, knowing that their investments could turn out to be wasted if the firm were to disappear.

  For example, the value of a computer is not only determined by its hardware, but also by the manufacturer's continued provision of hardware and software support and development. An "obsolete" computer or piece of software, no matter how good, is practically worthless. Even if the firm promised to continued development of faster hardware to preserve their customers' software investments, if liquidated, a bankrupt firm may not be able to keep this promise. The inability of the firm to commit to honoring its promises in the future hurts its business today.

ANECDOTE: Lotus Woes Leave Customers Skittish
On May 1, 1995, the computer trade magazine *PCWeek*'s cover story read "Besieged by ongoing financial turmoil...Latest Lotus woes leave some customers skittish." Customers aware of the possibility of bankruptcy may choose an (inferior) product from a competing vendor with a lower possibility of bankruptcy, which in turn reduces the value of the first firm today.

- When product sales require promises of future contact, customers may be reluctant to purchase the product, given that the future promised rebate may fail to materialize.

  For example, airlines depend on frequent flier plans to attract business travellers. When the promise of future free flights loses its credibility, an airline becomes severely handicapped. In effect, any firm whose products require warranties should weigh whether issuing debt might not alarm its customers. Such products may require future service, and customers may be reluctant to purchase the product, knowing that the service may become unobtainable in the future.

ANECDOTE: Chrysler's Viability
On July 23, 1981, the *Wall Street Journal* reported on Chrysler's first positive quarterly earnings after a long hiatus:

> Telegraphing even this tiny profit via the nightly news to dealers and potential customers is extremely important for Chrysler which is trying to shore up its image as a viable automaker. In the past, every time Chrysler's losses mounted, customers fled its showrooms, fearing the company would collapse.

- When product quality is difficult to judge, customers become afraid that companies may cut corners in order to avoid financial distress.

Without trustworthy warranty programs, competing in some businesses is very difficult.

  Have you ever wondered whether an airline in financial distress cuts corner on airplane maintenance? (You should!) Consequently, the price at which such an airline can sell tickets may be below that of a financially solid airline. Similarly, wholesalers will not deliver their goods to near-bankrupt retailers unless they are assured of payment. Because bankrupt retailers may no longer be able to purchase credit, the costs of their goods may increase—and their competitive advantage may erode.

- Some business rely on **Trade Credit**, in which suppliers extend credit to buyers. If the supplier fears that the buyer can go bankrupt, he may not extend trade credit and thereby even cause the onset of bankruptcy.

SIDE NOTE: Bankruptcy concerns will also hurt product resale values, for the same reasons that they hurt sales today. This can further amplify the real value impact of bankruptcy costs.

IMPORTANT: *Bankruptcy costs favor equity over debt as a cheaper financing vehicle.*

The Empirical Evidence on Bankruptcy Costs

What firms are likely to suffer high deadweight losses in bankruptcy? We know that many U.S. railroads have declared bankruptcy dozens of times, without interruption in service. Even large retailers, like **Federated Department Stores** (Macy's), have been in and out of bankruptcy several times. Airlines have at least some easily transferable and collaterizable assets (airplanes), and thus may have fewer deadweight losses. In contrast, firms with large intangible assets (warrants, name recognition, etc.) need to be more concerned to reduce the probability of future bankruptcy. They should choose lower-leverage capital structure.

The fact that some firms go bankrupt "regularly" speaks for their relatively low bankruptcy costs.

Still, the importance of bankruptcy costs as an important determinant of capital structure remains an empirical issue. The current academic consensus is that bankruptcy costs matter for some firms and industries, particularly during recessions; but their expected costs are small for most healthy Fortune 500 companies most of the time—Enron not-withstanding.

Expected bankruptcy costs are probably small for healthy, large companies.

NERD NOTE: *Over the thumb, fewer than five Fortune-500 companies enter financial distress (either formal or informal) in a given year. Quadruple this number, to get an estimate of 4% probability of bankruptcy at the outset of the year. Further, presume that bankruptcy costs are 5% of the value of such Fortune-500 companies when they enter bankruptcy. Again, quadruple this number to presume a 20% distress cost. Finally, assume that a normal Fortune-500 company drops by about 80% in (market) value before entering financial distress. Again, quadruple this number, presuming that the firm only loses about 20% before entering financial distress, i.e., that the value being lost will be about 20% distress cost on the 80% remaining firm value. Then, at the outset of a year, i.e., before financial distress is indicated, the expected financial distress costs would be about* $4\% \cdot 20\% \cdot 80\%$ *or less than 1% of the firm's value. This can still be a large amount of money for lawyers to fend over, but it does not loom too large when a healthy firm considers taking on another loan in order to save on taxes.*

24·1.B. Underinvestment

The underinvestment problem is the concern that managers (acting on behalf of equity holders) slack off and ignore good projects when equity holders have no chance of receiving any proceeds when debt payments are due. Because bondholders would receive all the proceeds, the firm would not accept good projects *ex-post*, reducing the firm-value *ex-ante*. This reduces the value bondholders will receive, and thus the value bond purchasers are willing to pay for the firm today.

When there is debt, equity holders may not properly take care of the assets.

For example, consider managers with a firm worth \$100M, of which \$10M is in cash and \$90M in securetized assets. Assume further that the debt has a face value of \$150M due in one year, and that a new project comes along that costs \$10M but pays off \$30M after the debt comes due. Would equity holders prefer a \$10M dividend today, or the \$30M investment? The answer is the dividend, because equity holders would not receive any payoffs from the investment.

Grab as much while you can!

Clearly, this "underinvestment problem" is a cost of debt to the firm. If the firm chooses no debt *ex-ante*, such profitable future investments would not be ignored, which in turn would increase the value for which our hypothetical owner could sell the firm.

Fearing future lack of care again makes it more expensive for the firm to raise capital via debt.

IMPORTANT: *Ex-post reluctance to do the right thing (additional maintenance investment) in the presence of debt favors equity over debt as the cheaper financing vehicle.*

24·1.C. Reluctance to Liquidate

Managers may not want to liquidate the firm, even if they should. This is especially bad for debt holders.

A problem similar to the *ex-post* expropriation problem is the *no-liquidation problem*. Managers acting on behalf of equity holders may not wish to liquidate the firm when it has fallen onto hard times, even if this maximizes firm value. Equity holders always prefer more risky payoffs because equity is essentially like an option. If there is even a small chance of improvement, although deterioration is more likely, equity holders are better off to take their chance than to give up their option and to liquidate. For example, consider a firm that can be liquidated for $100M today, or can operate for another year and be worth either nothing or $150M (with equal probability). Together, the owners (bondholders, stockholders, etc.) would be better to liquidate. But if there is a bond coming due only next year with a face value between $100M and $150M, equity holders are better off waiting. Even simpler, if the firm is worth $60 today ("underwater"), and a "project-lover" were willing to overpay for the firm, offering say $90, equity holders would not want to accept this. After all, the $30 would accrue only to creditors (owed $100M), not to themselves. In effect, equity holders have an option on the firm at the cost of satisfying only the running interest and principal payments. This inefficient behavior, caused by the presence of debt in the capital structure, reduces the value of a firm with both debt and equity *today*.

IMPORTANT: *Ex-post reluctance to do the right thing (liquidation) in the presence of debt favors equity over debt as the cheaper financing vehicle.*

24·1.D. Strategic Considerations

Debt can change nature of the competition in the product market.

Finally, some financial academics have argued that debt can be used as a strategic device. This is perhaps easiest to understand by analogy. Consider two teenagers playing a game of chicken (two cars driving towards one another; the first to "chicken" out and get out of the way loses). How can a teenager ensure that he wins? All he needs to do is to remove the driving wheel and throw it visibly out of the window. It is easy to see why the other teenager will chicken out. The trick is of course to visibly commit yourself to not give way. (Some people have suggested that driving an old, large and apparently unstable Buick is the equivalent of throwing out the wheel; other cars will be in a hurry to get out of your way.)

An argument that leverage can make firms more aggressive and better off.

The same argument has been made for debt. Assume for a moment that a monopolist has levered up, that there are severe penalties for failing to meet interest payments, and that rearranging the capital structure is expensive. Consider the decision of a potential market entrant who knows that the monopolist is highly levered. The market entrant knows that it is in the interest of the monopolist's managers (acting on behalf of equity holders) to take on the most risky strategy—a price war. Consequently, he will not choose to enter, and the monopolist never has to start the price war—the risky strategy.

SIDE NOTE: Incidentally, if the market entrant is too stupid to understand the message, both players, the monopolist and the entrant, will be hurt badly. In my personal opinion, these arguments tend to be more clever than empirically relevant. Fee companies lever up *in order to* induce themselves to have management behave in a more risk-seeking fashion. This does not mean that companies, after levering up, may behave in a more risk-seeking fashion anyway.

However, there is evidence that debt can actually hurt corporate competitiveness. If debt is taken on because the owners are severely cash constrained, they may also not have the funds to compete properly. There is some evidence that supermarkets that underwent a **Leveraged Buyout** by their managers, also called a **Management Buyout** (that is, the managers borrowed a lot of money to buy their firm), did not have sufficient funds to optimally compete in pricing wars. To the extent that this reduced the NPV of the supermarket, this would count as a direct cost of a highly leveraged capital structure.

An argument that leverage makes firms less competitive and worse off.

IMPORTANT: *The competitive product market environment of the firm could favor either equity or debt.*

Solve Now!

Q 24.1 *Describe some non-tax based advantages of equity over debt.*

Q 24.2 *Give examples of bankruptcy costs, both direct and indirect.*

Q 24.3 *Give an example of an underinvestment problem.*

Q 24.4 *Give an example of a no-liquidation problem.*

Q 24.5 *Is leverage a strategic advantage? Describe the arguments on both sides.*

24·2. Operating Policy Distortions: Behavior in Good Times

24·2.A. Free Cash Flow and Agency Concerns

Chapter 25 will be exclusively devoted to agency and corporate governance concerns.

Managers like building empires and consumer perks; debt restrains them.

Unfortunately, optimal problems in managerial behavior do not only arise when the firm is in trouble. The so-called **Free Cash Flow** problem is that managers, no matter how well-intentioned at the outset, will prefer spending money internally on their pet projects once the opportunity arises instead of returning money to claimants. For example, in the 1980's, many large oil companies continued exploring for oil, even though it was well-known that oil companies could be bought at a stock exchange for significantly less than the expected cost of finding equivalent oil reserves. Debt which requires coupon payments forces managers to perform. Managers who fail to generate enough income to pay the coupon are subject to bankruptcy, and (as has been shown empirically) almost always lose their jobs. Therefore, firms with more debt may be worth more *today*.

Theft and Graft.

Another important problem is outright **theft**. If you are a passive partner, you are dependent on true and accurate reporting of what profits really are. The active partners or the managers, however, will just try to avoid reporting large profits: they would rather use corporate cash to build more of an empire, to compensate themselves better, or just to outright steal it! Debt has the advantage that the creditor does not need to know what the profits are: if the agreed-upon payments are not made, the creditors take over the company.

Hold Up Costs

Higher potential hold-up costs are another important drawback of equity. When a company, especially a public company, rolls in cash, anyone who has the power to hold up business will try to extort more of a share of these profits. For example, a supplier who delivers an important input, a wholesaler who is an important distributor, and any key employees who can bring production to a stop, will want to pressure the firm in order to renegotiate their deals. Yet, if the company is financed more via debt, these third parties will recognize that there is less cash to expropriate. After all, if the company does not pay the debt, it can go bankrupt. Thus, in a company with more debt, the equity earnings (over which renegotiation can occur ex-post) are smaller.

Important: *Free cash flow and agency concerns favor debt over equity as the cheaper financing vehicle.*

ANECDOTE: Airlines, Unions, and Shareholders
In September 2002, AMR operated over a thousand airplanes, and owned about half of them. It had assets valued about $30 billion, and debt valued at around $15 billion. Still, its equity market value was only $800 million—about the price of three of its forty top-of-the-line Boeing 777 airplanes. And it is not clear if American was worth even this $800 million: bankruptcy was imminent for American and all other major U.S. carriers (except Southwest).

If American were ever to make positive profits again (in 2002, it lost a significant amount of money operating), its unions will surely capture them. After all, it only takes one of its unions (e.g., pilots, flight attendants, mechanics) to ground a fleet worth $30 billion and to wreck customer loyalty. The unions will ultimately make sure that shareholders will receive just enough for them not to kill the golden goose. It remains a mystery to the author how airlines continue to exist as public corporations, instead of as employee-owned organizations.

(Debt may be the only chance that the major airlines have to restrain union demands.)

Solve Now!

Q 24.6 *Give some examples of perks that management might give up when debt load increases.*

24·3. DELIBERATE BONDHOLDER EXPROPRIATION

Although managers should structure the firm *at the outset* so as to make it in their interest to optimize overall firm value, when given the opportunity later, they might prefer to "steal from" bondholders in order to aid shareholders—after all, it is shareholders who vote them into office. So, the ex-post expropriation problem is that managers—acting on behalf of their employers, i.e. equity holders—will have the opportunity and the incentive to exploit debt holders once the debt is issued. Bondholders are cautious of these problems. They come in two flavors: first, bondholders can be expropriated by increasing the risk of the firm's projects (a change in operating policies); second, bondholders can be expropriated through further future bond issuing activity. This section describes these issues.

If there is debt, equity shareholders may want management to screw the debt holders. This has bad ex-ante value consequences.

SIDE NOTE: "Strip Financing" is the sale of securities in bundles. Each purchaser receives the same proportion of debt and equity. If the strips are not later unbundled and resold, there is no conflict between creditors and shareholders—the two parties are the same.

24·3.A. Project Risk Changes

For example, assume that a firm will pay off either $50M or $150M, with equal probability. Assume further that there is a senior bond with face value of $60M outstanding, plus equity. Finally assume that there are no taxes and everybody is risk-neutral. The payoffs to the project and its stakeholders are illustrated in Table 24.1.

An example: A plain benchmark firm.

Table 24.1. Bondholder Expropriation through Project Risk Changes

	Bad Luck	Good Luck	Expected Value
Prob	1/2	1/2	⇒ Today's Value
Project NPV	\$50M	\$150M	\$100M
Senior Bond (Promising F=\$60)	\$50M	\$60M	\$55M
Levered Equity Value	\$0	\$90M	\$45M

Adding a risky, but negative NPV project changes the state-contingent payoffs.

Would risk-neutral bondholders be willing to pay \$55M for the promise to receive \$60M? Ostensibly, yes. Now assume that half the time has passed, and the managers find a new, independent project that pays off either \$40 or loses \$50 with equal probability. Note that any Las Vegas casino provides better investment opportunities, so it would not be too hard for managers to find such projects. Table 24.2 shows the value of the firm if it were to add the new project, and the resulting value of equity and bond claims. There are now 4 states, each with probability one-quarter, and total firm value changes by the present value of the second project (-\$5).

Table 24.2. Bondholder Expropriation through Project Risk Changes

	Worst	Bad	Good	Best	Today's Exp. Val.
Prob	1/4	1/4	1/4	1/4	⇒ Today's Value
Old Project NPV	\$50	\$50	\$150	\$150	\$100
New Project NPV	−\$50	\$40	−\$50	\$40	−\$5
Senior Bond (Promising F=\$60)	0	\$60	\$60	\$60	\$45
Levered Equity Value	0	\$30	\$40	\$130	\$50

Managers can be like politicians—and do more harm than good.

Would the managers accept this project? Remember who votes them into and out of office! In essence, managers can expropriate \$10 from bondholders, waste \$5 of this money, and hand \$5 to equity holders. As explained on Page 497, although *ex-post* this is good for equity holders, *ex-ante* it is bad for them and the firm. If the managers expressed an intent not to accept the high-risk project, would bond purchasers pay \$55 for this bond, believing that management will not be able to create other higher-risk projects when the time comes?

SIDE NOTE: Luckily, managers may themselves be risk-averse, which may protect bondholders. In Chapter 11 bankruptcy, managers are fired about half the time. This may make them more reluctant to expropriate bondholders by taking on more risky projects.

Because "good faith" is difficult to convey, bondholders may not believe managers and demand a premium they would not demand if the firm could *commit* not to expropriate bondholders *ex-post*. If uncommitted even managers with the best intentions cannot shield themselves from the pressure of taking the negative NPV project. Obviously, this expropriation potential makes it difficult for the firm to raise funds through bonds at

fair prices, and tilts the optimal capital structure more towards equity. Who ultimately loses? Smart bond investors do not lose: *ex-ante* they get their fair compensation. However, the firm is robbed of the opportunity to issue debt (with its other advantages, such as tax savings), which tilts the optimal capital structure more towards equity.

IMPORTANT: *Inability to commit not to exploit bondholders (by taking on riskier projects in the future) favors equity over debt as the cheaper financing vehicle.*

Bond covenants reduce exploitational opportunities in the future—but at a cost in flexibility.

Would managers be willing to take *any* risky project? It depends on how much NPV stands to be lost and how much stands to be redistributed. If the above project gained only $20 in the good state, managers would not take it. In real life, managers try to protect bond purchasers from expropriation with **Bond Covenants**. For example, covenants can state that the firm cannot assume any new projects. However, these covenants impose costs, too. What would happen if managers suddenly found a project paying off one million dollars in a good state but had earlier committed not to take on any new projects? Clearly, a firm that sometimes cannot take profitable projects in the future is worth less today.

Convertible bonds allow bondholders to participate in the upside, and reduce exploitational incentives in the future.

Managers need to find a compromise that limits their incentives and/or their ability to *ex-post* expropriate bondholders, while still preserving the firm's option to accept new, possibly high-risk projects. One way to reduce this problem is to write bondholder covenants that allow new projects only if certain accounting ratios are met. Bondholders are aware that they may be expropriated (and demand up-front compensation for this possibility), but will only be expropriated if projects come around that allow management to meet the weaker bond covenants. Unfortunately, covenants are not perfect, and some good NPV projects may come around which do not satisfy the covenants. Partially to retain flexibility, many bonds with strong covenants today are "callable," allowing management to redeem the bonds before maturity at an agreed-upon price. Another solution is for the firm not to issue bonds with strong covenants, but instead to issue "convertible bonds" with weak covenants. Convertible bonds allow investors to participate in the one million dollar project, and thus reduces the risk expropriation problem. It is left to the reader as an exercise to show how a convertible bond can reduce the expropriation.

IMPORTANT: *Covenants, callability, and debt conversion features mitigate ex-post exploitation incentives. Their presence, ceteris paribus, helps render debt less disadvantageous.*

24·3.B. Issuance of Bonds of Similar Priority

Managers may also exploit bond holders by issuing more debt.

There is yet another problem of concern to bondholders. Assume again that our firm has a value today of \$100, and payoffs of \$50 or \$150 with equal probability. Assume that the firm has debt with face value of \$60 outstanding, and that covenants prevent the firm from taking any other project. Are bondholders now protected? The answer is no, if managers can issue another bond of similar (or higher) priority. For the sake of illustration, assume that they can issue a bond of equal priority to the old bond with a face value of \$60 and use the proceeds to pay a special dividend to equity holders. In bankruptcy (the bad state), the old senior bond would now no longer receive the entire \$50 (as per the example above), but would have to share proceeds with the new bond that has equal priority. In real life, the "spoils" are often allocated according to face-value within bonds of the same priority. So, because in our example both bonds have equal face value, each would receive \$25 in the bad state. Thus, the value of each bond is only $50\% \cdot \$25M + 50\% \cdot \$60M = \$42.5M$ today in expected value. The equity receives nothing in the bad state and \$30M in the good state—plus the one time dividend of \$42.5M. In total, by issuing new debt of equal priority, equity holders have increased their wealth from \$45M to \$57.5M, old senior bondholders have lost \$12.5M, and new senior bondholders just got a fair deal.

Again, fearing expropriation, the firm has to pay a higher interest upfront to potential bond holders.

Why is this expropriation a problem? Because the first bond's purchasers, will cautiously insist on assuming that expropriation will happen, and demand a higher interest rate. To recoup this higher interest rate, the firm indeed has to later issue more bonds to expropriate the first bond purchasers. In effect, the firm either issues no bonds, or it is dragged into a capital structure that contains *too much* leverage (perhaps unduly increasing bankruptcy risk). As before, it is the firm which loses *ex-ante* because of *ex-post* problems faced by its bond purchasers.

> **IMPORTANT**: *Inability to commit not to exploit bondholders (by issuing equally senior or shorter term debt in the future) favors equity over debt as the cheaper financing vehicle.*

And, again, covenants help reduce this problem.

In the real world, firms try to protect bondholders against such expropriation by including covenants that restrict dividend payout to equity holders and covenants that prevent the firm from later issuing debt with higher priority.

> **SIDE NOTE**: Debt with higher priority is automatically created when a firm is in Chapter 11 bankruptcy. Therefore, bondholders must take the possibility of *ex-post* expropriation if the firm goes bankrupt into account when negotiating an appropriate interest rate.

Are bondholders protected now? The answer is still no. In our example we used only two periods. Yet, in real life, when one bond becomes due before another, it effectively becomes more senior: it is paid first (except when the firm goes bankrupt within six months, in which case the firm's transactions can be unrolled by a bankruptcy court). Again, this problem can sometimes be remedied with appropriate covenants.

Without explicit covenants, promises can be broken. But sometimes, flexibility to refinance and take new risky projects is itself valuable, when it would allow firms to accept extraordinarily good projects that come its way. Therefore, successful firms often build a "reputation" that allows them to borrow at rates under which bond investors assume they will probably not be expropriated. Yet, when promises are broken, bondholders stand to lose substantially. For example, R.J.R. Nabisco in the 1980's was generally believed to be a safe investment for bondholders. However, when it was bought out in 1988 in the largest LBO ever, R.J.R. tripled its debt overnight, resulting in a 15% one-month loss for bondholders.

And, again, covenants reduce the flexibility of the firm to take advantage of other opportunities. Sometimes, reputation can substitute for covenants.

In the real world, firms have to undertake a delicate balancing act. When debt is issued, it can only be issued at favorable terms when the firm can promise not to exploit bondholders once bonds are issued. The loss of flexibility, however, can be so expensive that firms cannot issue debt—and thus have to forego some other beneficial effects of debt (such as tax advantages) which may put them at a competitive disadvantage.

Recap: It is in firm's interest to commit themselves not to take advantage of its creditors in the future. To the extent that they fail, debt is too expensive today.

Solve Now!

Q 24.7 *Why do bond covenants exist?*

Q 24.8 *What is the advantage of adding convertability features to a bond?*

Q 24.9 *Describe the two basic mechanisms of how unprotected bondholders can be expropriated by shareholders, preferably with a numerical example.*

24·4. Inside Information

Another important determinant of capital structure is inside information. This argument falls under the general rubric "Never bet with someone better informed than yourself." Typically, firm managers (acting on behalf of the old owners) have better information than new investors. New investors must be careful not to be exploited.

New potential partners/shareholders have less information than current managers and owners.

Consider: If you were a potential investor, and you knew that the current owner/manager (who has to raise new capital) already knows whether the project will be successful or not. What would you believe about the project if the present owner offered to make you a full partner sharing in all future profits, vs. if the present owner asked you for a loan to be paid back that she is willing to collateralize with her present assets? If you are offered to become a partner, you should be reluctant to believe that the project is profitable. If, however, the present owners want to keep the profits and simply borrow some working capital, they probably know that the project is profitable.

If owners want partners rather than lenders, the project may not be as good.

Side Note: This is sometimes referred to as the **Winner's Curse**. If you receive the offer to become partner, it does not help you very much. If you do not receive the offer to become partner, you would indeed be better off if you had received it.

A numerical example of the inside information problem.

This analogy is directly transferable to capital structure. Sharing in the firm's equity is the equivalent of becoming a partner. More specifically, consider a firm in which managers know whether a firm's projects will pay off \$100M or \$200M. Outsiders (you) know only that such projects pay off \$100M or \$200M roughly half of the time. Assume further that managers have to raise \$90M to take the project. They can either sell a percentage of the firm (equity), or promise to repay the \$90M (debt). Would an investor be willing to assume that when a firm offers to finance with equity, it would be a typical firm? Assume for a moment she would. At an expected value of \$150M, the firm would have to sell 60% of its equity to raise \$90M. The old investors would retain 40% of the firm, and would receive \$40M if the firm was a \$100M firm, \$80M if the firm was a \$200M firm. Yet, if the \$200M firm raised \$90M in debt (which it can always repay), it would keep \$110M. Because \$80M is less than \$110M, a \$200M firm will *never* use equity, but always use the more senior security, debt! (The reader can show that a \$100M firm will always prefer to use equity if investors were willing to assume that both types of firms use equity.) And therefore, no investor would be willing to assume that a firm that offered equity would be an "average" firm.

The "Pecking Order": More equity-like (partner-like) shares are bad news, and can only be sold at a discount.

Because new partners are not inclined to assume that the project is good, equity investors will assume that their new claims are on a project that will eventually develop problems. Thus, the announcement of a new equity offering releases information that the firm's projects are worse than generally believed, and the new equity can only be sold for a very low price. (In real life, we indeed observe that when firms announce that they plan to raise about \$1 by issuing new equity, their old public equity value declines by about 67 cents! Sharing in more junior (risky) bonds is the equivalent of the present owners making you a "little partner," when they are not willing to collateralize their loan. Consequently, the announcement of a risky junior security releases information that the firm's projects are not too great, but not too bad, either. In contrast, the new issue of a collateralized loan (or a risk-free senior bond) will indicate that the firm's projects are better than expected. The outcome is that the better the firm's projects are, the more senior the security the managers will offer for sale. This is the so-called **Pecking Order** view of capital structure: the best projects are financed by the most senior debt, worse projects by junior debt, and the worst projects by equity.

NERD NOTE: *An alternative explanation of the drop upon issuing new shares is that the demand for shares is not perfectly elastic (as implicitly assumed in Modigliani-Miller. In essence, assuming lack of elasticity is equivalent to assuming that investors care not just about their financial returns, with their risk-reward characteristics. This also means that investors could find very good bets in the neglected stocks.*

Firms may want to avoid issuing more shares of anything, and equity in particular.

What does this imply for the optimal capital structure? Consider a firm that cannot issue debt easily because it has little collateral or because additional debt would unduly increase expected bankruptcy costs. If it cannot issue equity because of these insider concerns, such a firm may have to pass up on some good (but perhaps not stellar) projects, simply because managers prefer that their owners should not to be perceived as holding low-quality projects. A publicly trading firm thus loses because inside information may force it to pass up on good projects in the future—and debt is the only viable financing alternative.

IMPORTANT: *Inside information concerns favor debt over equity as the cheaper financing vehicle.*

Solve Now!

Q 24.10 *A house up for auction can be worth either $500 or $1,000 with 50-50 probability. The other bidder knows the true value, you do not. If you bid for the house in an auction, what should you bid? If you bid $750, what is your expected rate of return?*

24·5. Transaction Costs and Behavioral Explanations

Transaction costs are everywhere. They definitely can prevent optimal adjustment.

Transaction costs have played an important role in all capital structure examples above: only by assuming a zero-transaction cost would one necessarily expect that the firm would have to do what is best. If transaction costs are high, managerial mistakes are difficult or impossible to correct for outsiders. It is not just enough for an outsider to purchase shares and then sell them, the appropriate corrective action requires accumulating enough shares and pressuring management to improve the situation. Without the discipline of external pressure, managers and investors can commit any mistakes they wish, and this can result in any sort of observed behavior.

Transaction costs "cause" behavioral finance concerns.

Section 19·2 has already described the link between high transaction costs and so-called **Behavioral Finance**. In the corporate finance context, the presence of high transaction costs—and, with it, the rarity of correcting mechanisms—means that Behavioral Finance plays an important role. It can explain a lot of managerial behavior, which is otherwise difficult to explain. Unfortunately, on this level, without a further description of what the mistakes are, it often offers little guidance as to how a smart manager should act differently (i.e., what the best debt/equity ratio is).

Specific behavioral errors can have specific consequences.

Behavioral finance is the most promising new area in corporate finance, but it is still too early to tell where and how it will help us understand the world. Some of its insights are already here. There are certain behavioral mistakes that appear more reasonable and common than others. For example, **Overconfidence** and **Overoptimism** are common traits among both managers and investors. To the extent that investors are overoptimistic, issuing equity may not be so disadvantageous as the inside information argument suggests. Investors may not believe the worst for some companies.

An Irrational Financial Market suggests other strategies.

Another view of behavioral finance is that it is not the managers that are committing the mistakes, but the financial markets. Although this is less likely than managers committing mistakes, there is good evidence that financial markets may be imperfect, too. Under such circumstances, or when managers have considerably inside information, managers may seek to time equity issuing activity.

Dividend policy is probably a behavioral managerial error.

An important behavioral effect may also be in the tendency of managers to want to pay dividends rather than to use share repurchases to send cash to their equity holders. This is not a wise choice from a tax perspective, but there is little one can do about it. The world is how it is.

And then there are also direct transaction costs...

A more traditional (but outmoded) view is that transaction costs can also play a direct role. For example, assume that investors' arthritic hands are hurt when they have to clip coupons off their corporate bonds. A more reasonable example of transaction costs are the reporting requirements and liabilities imposed by the *1933 Acts* for publicly traded securities. For small companies, these costs may be large enough to warrant a capital structure consisting exclusively of private securities and bank debt.

IMPORTANT: *Behavioral considerations could favor either debt or equity.*

Solve Now!

Q 24.11 *Why do firms pay dividends?*

24·6. Interactions: From Here to There

Executing the value-optimizing strategy may not be possible.

The real world is considerably more complex in that firms typically do not face each of the above problems in isolation, but all at the same time. The presence of one problem or attempts to reduce it may worsen another. That is, there can be significant costs to move from a suboptimal to an optimal capital structure.

IMPORTANT: *Interaction effects can make it difficult to optimally adjust capital structure in the future. This can favor a more flexible capital structure (more equity and slack) today.*

Conflict among different interest groups can prevent optimal solutions.

For example, we already know that if a firm is close to bankruptcy, issuing equity could avoid or reduce bankruptcy costs, which increases firm value. But the infusion of more equity may mostly benefit bondholders, so equity holders may not agree to put in more equity. So, although a reorganization (i.e., a new start) could install a capital structure to increase firm-value, there are problems to be resolved to get there, given the current capital structure.

NERD NOTE: *But, occasionally, infusing equity can preserve equity holder value. If a coupon comes due, the question for managers trying to maximize equity value is whether to raise more equity to pay the coupon, or to default. Raising more equity (or not paying out dividends to pay the coupon) is typically a wealth transfer from equity holders to bondholders—except that by paying the coupon, the equity (an option) is kept alive and thus perhaps worth more.*

24·7. Reputation

Sometimes, owners are best off building a corporate reputation, which can help alleviate investor worries.

Another important complication of the real world is that managers may be able to establish a reputation, which can lower financing costs, improve incentives, and maximize firm value. Remember the example that started the chapter: the presence of the *ex-post* ability to expropriate bondholders hurt the firm. If managers had a reputation not to take such bad projects, perhaps overly restrictive covenants could be avoided, in effect lowering financing costs *ex-ante*. More strongly, the example assumed that everyone knew exactly what expropriation opportunities (3-for-1) exist, and what their probabilities are. But, despite restrictive covenants, when exploitative opportunities can appear unforeseen, bondholders would always have the nagging suspicion that they may be expropriated after all. Only trust and reputation may overcome this suspicion with its associated increase in financing costs. But, do investors trust managers? Can investors trust managers? Should investors trust managers? And, when is it worthwhile for a manager/firm to build such a reputation? How can this effectively be accomplished? These are difficult questions to answer rigorously, but important in the real world.

24·8. Capital Structure Recommendations

Choosing the best Capital Structure is a combination of art and science.

Table 24.3 tries to summarize the arguments we made over the last three chapters. Ultimately, the trick in being a good manager is to judge and weigh the plethora of marginal costs and marginal benefits of projects, of debt, and of equity; and to have sound judgment in deciding on a good combination thereof. Although this article could describe many relevant forces, choosing a good capital structure remains as much an "art" as it is a "science." This is good news for today's business students: combined with an ever increasing arsenal of possible financing instruments, capital structure choices are unlikely to be taken over by a computer program anytime soon.

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24·9. Summary

The chapter covered the following major points:

- Deadweight bankruptcy costs—both direct costs (such as legal fees) and indirect costs (such as neglect and excessive risk-taking)—can make too much debt too expensive, and thus they can favor equity.
- Agency issues favor debt: managers will work harder if poor performance can lead to bankruptcy.
- Bondholders can fear expropriation.
- Inside information can favor debt: owners—or managers acting on behalf of owners—will prefer to sell partnership shares if the project is truly less valuable, and prefer to borrow if the project is truly more valuable.
- Managers can continue to use the WACC formula from Chapter 22, because the issues described in this chapter flow into the firm's cost of capital through the costs of capital quoted by the financial markets.

A summary of all capital structure effects is in Table 24.3.

Table 24.3. Summary of Capital Structure Effects

Effect	Favors
Corporate Income Taxes	Debt
Personal Income Taxes	Equity
Financial Distress Costs	Equity
Includes underinvestment problems, supplier and customer incentives, failure to liquidate or sell at appropriate price, predatory policies by competitors, etc.	
Too Much Cash Flow	Debt
Sometimes called **Moral Hazard**. Includes overinvestment, free cash flow, excessive managerial perks, verification, etc.	
Debt Expropriation	Equity
Includes costs arising from the interaction of borrower credibility and borrower flexibility. Includes complete contract specification costs.	
Inside Information	Debt
Sometimes called **Adverse Selection**. Sometimes called **Pecking Order**.	
Transaction Costs	Situation Dependent
Behavioral Finance	Situation Dependent

A. Q&A: Answers

1. Lower bankruptcy costs, both direct and indirect. Fewer incentive problems to put up extra "maintenance" money. Fewer incentive problems to avoid liquidation and drag on, instead.
2. For example, legal fees and management attention.
3. Neglected maintenance reduces the value of assets relative to the first-best.
4. Shareholders do not want to sell the firm if they are underwater, even if the offer is more than the firm is worth: all benefits would go to the shareholders.
5. It can get the firm to commit to undertake more risky projects. However, it can also make it more difficult to respond when the company is already levered up.
6. Corporate Planes. Large headquarters. Large staff.
7. They help reduce the incentives of equity shareholders to expropriate bondholders.
8. It reduces the need for some bond covenants and thus gives the firm more flexibility in case a great project were to suddenly appear. Bondholders would be happy, because they, too, would benefit.
9. First, issuance of other securities that have an earlier or equal dip on the firm's cash flows in distress. This could be other bonds of equal or higher priority, or a straight out dividend payment. Second, the adoption of risky projects. Numerical examples are in the text.
10. You should not bid anything (except perhaps $500). If you bid $750, then you will get the house only if it is worth $500, and you would therefore lose 33%.
11. One good explanation is that the managers are dumb. It is not the only good explanation, though.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 25

Corporate Governance

More Agency Conflict

last file change: Jun 15, 2004 (14:43h)

last major edit: n/a

Up to now, we have assumed that managers maximize firm value. In Chapter 20, we described the **cash flow rights** of securities: debt is paid first, equity receives the residual.

But we failed to ask the simplest of all questions: Why do managers return *any* money to investors? After all, the investors contribute little to the corporation *after* they have given their money. This question is the domain of **Corporate Governance**, which concerns itself with the conflict of interest between the corporation (its managers) and its capital providers.

This web chapter provides the answer: cash flow rights come with mechanisms, which alleviate investors' concerns and thereby allow firms to raise funds. The most prominent are so-called **control rights**, attached to securities and endorsed by the country's legal environment.

25·1. Less Fact, More Fiction: In Theory

Firms which start out as tightly controlled upstart firms become diffusely held, which brings many problems.

Most companies start out with few **conflict of interest** problems—if only because the entrepreneur owns the entire firm, provides all capital, and makes all decisions. As the company grows, the entrepreneur usually needs to raise more outside capital, either to expand the firm or just to start enjoying the new riches. To be able to get investors to part with their cash, the entrepreneur must create a corporate charter and install safeguards that satisfies potential investors, legal requirements, and common practice. Eventually, the founder's personal role begins to fade, more and more capital is raised from the outside, and management becomes "professional" in the sense that it no longer acts solely based on the whim of the entrepreneur. These professional managers also bring with them unique qualifications and specialization benefits. The legal theory is that these managers now become "agents" acting on behalf of the investors (primarily shareholders), the "principals." But over time, as the distinction between capital providers and decision makers grows, so do conflict of interest ("agency") problems. Potential problems abound. Although even two or three coowners can squabble, when there are thousands of shareholders, as in a publicly traded company, the coordination problems take on an entirely new dimension. Shareholders agree that they prefer more money to less money, but on little else. They are not capable of constantly voting and communicating their desires to their agent managers, much less checking over what their managers are doing day-to-day. (Managers know this, too.)

To get money back, investors need control rights. To get money in the first place, firms need to give control rights.

In some firms, raising capital means finding creditors; in other firms, this means finding shareholders. The promise of corporate value maximization and **cash flow rights**—first the promised payments to debt, then the residual cash flows to equity, as described in Chapter 20—by themselves mean little. Investors must somehow be able to coerce their managers to adhere by them. This is primarily (but not exclusively) done with **control rights**, which give investors future power over managers, especially when cash flow rights are not held up.

The equilibrium outcome is second-best, permitting unavoidable managerial self-enrichment.

In real life, it is impractical or impossible to protect capital providers *perfectly*. The expense would be prohibitive. So, corporations and capital providers must live with a **second-best** solution, in which there is a constant tension between investor protection and managerial self-enrichment. The better the control rights (protection) of creditors and shareholders, the better will be the terms under which the entrepreneur (or existing shareholders) can obtain capital in the first place.

Control rights differ for debt and equity.

The control rights are distinctly different for equity and debt investors. Shareholders are legally in charge of all decisions, but their prime power is their ability to vote and appoint representatives, usually once a year during the annual meetings. Their elected **Corporate Board** has the power to hire and fire managers. Creditors face similar but usually less pressing control problems. In theory, after the lender has ascertained whether a firm can meet the lending criteria, firm and lender sign a contract by which the firm obligates itself to repay a loan in the future and promises immediate legal remedy to the lender if it fails to do so. Usually, this means that the lender receives possession of the firm (collateral)—no ands, ifs, or buts.

When corporate governance usually works and when it does not work.

A firm that has no corporate board may not find shareholders willing to purchase equity shares; a firm that does not give the right to force bankruptcy upon default may not find creditors willing to lend money. But, control rights are not all black-and-white: if the firm does not offer perfect protection to its capital providers, it may still be able to obtain capital, albeit at worse terms (requiring the surrender of a higher percentage of the firm or the payment of a higher interest rate). So, the primary reason why entrepreneurs

and firms are interested in good corporate governance is that it enables them to raise capital at better terms. It also explains when corporate governance is likely to break down: when the entrepreneur (or shareholder-owners) are no longer solidly in charge, and managers have already taken control. The problem gets even worse when the firm generates a lot of internal cash and no longer needs to raise much new external capital.

25·2. Managerial Temptations

The human mind for scheming is infinite.

Although the legal fiction is that managers act solely on behalf of the firm and that shareholders own the firm after creditors are paid off, the fact is that all parties act primarily in their own interests. But, why is this a problem? How might a manager do harm? Unfortunately, there is a whole battery of things managers can do to enrich themselves at the expense of shareholders. This section outlines a slew of possible managerial self-enrichment schemes. In the next section, we will discuss mechansisms that seek to restrain such managerial behavior.

25·2.A. Illegal Temptations

Simple Theft is rare, but does occur.

Theft. The most obvious method is simple theft. What prevents the corporate managers from taking corporate diamonds out of the corporate safe? Though rare, simple theft does occur. For example, in March 2003, the Mercury News reported that 58-year-old C. Gregory Earls, head of an investment company called USV Partners, simply funneled investors money into a trust fund for his children. (Mr. Earls could win a prize for dumbest criminal. It is hard to leave a clearer paper trail than this.)

Complex theft through transfers is more common.

Transfers. More common than simple and obvious theft, the CEO of a public company can own private companies which do business with the public company on favorable terms. On occasion, the terms become *so* favorable that they warrant criminal indictment. For example, in May 1, 2003, the U.S. Department of Justice alleged that "In 1997, Fastow [CFO of Enron] conspired with others, including his wife, to create an [entity owned by the Fastows] in order to reap for themselves the profits generated by certain Enron wind farms, while simultaneously enabling Enron to fraudulently receive government financial benefits to which it was not entitled." Naturally, the smarter the manager, the more complex will be the arrangements, so that the true costs and true benefits to the public company are more difficult to assess. Criminal prosecution of such schemes is fairly rare, especially if the corporate executive has followed legal procedures to the letter (e.g., by asking for board approval).

Side Note: Transfers can also occur to friends of the management or to large shareholders. The role of large shareholders in corporate governance is described below.

Fraud is fairly common...

Fraud. An important and common problem is that managers can also commit fraud. Usually, this involves manipulation of financials, especially before the firm raises more capital. A stark example was *Hop-on Wireless*:

> Last year we had the story of how it looked like disposable cell phone company *Hop-On Wireless* was a scam. Since then, I've seen the company highly touted in many news stories, talking about how it was this great invention... but which no one seemed to be selling. Now, the CEO of Hop-On has been arrested for fraud, relating to work he did on an earlier company - but which brings up many parallels to Hop-On. The earlier company was an online gambling site, which he raised a lot of money for. However, they did so by showing

software that was really someone else's software "cosmetically altered" to look like their own. Hop-On's "disposable phones" were really Nokia phones with their own plastic casing put around them. Then, he promised not to take any salary until the company was making enough money, but did still use the investors' money to pay off his credit card debts and then gave another shell company he owned a $500,000 contract. Eventually, that gambling company was morphed into Hop-On. (Source: techdirt.com)

...but earnings management is legal.

Unlike in this extreme case, most accounting choices are not so black-and-white. The line between illegal accounting manipulation and legal earnings management is both wide and gray. There are many judgment calls that corporate executives have to make. There is evidence that, despite unusually strict SEC scrutiny, *legal* corporate earnings management is particularly aggressive when the corporation has to raise more capital—for obvious reasons. In any case, painting *too* bleak a picture is not in the interest of shareholders, either.

Insider trading is very common.

Insider Trading. A surprisingly common form of agency conflict is insider trading. CEOs have information before the public, and would love to use this information to trade. Naturally, this does not make shareholders better off. Strangely, the temptation to sell before bad news comes out is so strong that even smart executives can forget that the chance of being caught is relatively high. The most well publicized recent insider trading scandal involved Sam Waksal, CEO of ImClone (IMCL). Waksal received bad news about clinical tests of an ImClone cancer drug, and proceeded to tip his friends and family that they should immediately sell shares *before* this would become public news. His next seven years are all booked up now. But even with insider trading, there is a gray line. Not all insider trading is illegal. Executives are allowed to buy and sell shares, just not around major announcements.

ANECDOTE: Sexy Insider Trading
A juicier insider trading example comes from James McDermott Jr., former CEO at Keefe, Bruyette&Woods (a small investment bank). On January 21, ABC news reported that McDermott passed inside information to Kathryn Gannon, better known as porn star Marylin Star. The inside information allowed them to earn more than $170,000 in profits.

Third parties bribe managers all the time.

Bribes. Managers of publicly traded companies need not even create their own temptations: they practically come to them. For example, during the 1998-2000 technology bubble, receiving IPO share allocations was practically like getting free money. (Normal first-day rates of returns were around 50%. Ordinary brokerage clients would rarely get any allocations.) Citigroup was eager to do investment-banking business with WorldCom, a publicly traded company. It therefore allocated $17 million in 21 offerings into CEO Ebbers' *personal* account. In one IPO (*Rythms Net Connections*) alone, Ebbers was allegedly handed $16 million. De-facto, Ebbers was "nudged" to direct the business of the publicly traded company, WorldCom, to Citigroup.

SIDE NOTE: Preferential allocations to and treatment of executives' personal accounts were and continue to be common practice. Ebbers was an extreme, but not the only case. Lesser methods of bribing executives are so commonplace in business that they are considered outright ordinary. There is evidence that competitive bids for high-level professional services usually result in better contract terms than negotiated contracts, and yet most companies negotiate rather than bid contracts. Sometimes, negotiation is better for other reasons, but more commonly, the reason lies elsewhere. For example, CEOs naturally want to be on the list to become CEO candidate of bigger companies. It is therefore in the interest of a CEO to form a good relationship with an executive search firm. A CEO who uses competitive bids, which minimize executive search profits and alternate engaged search firms, is unlikely to build much loyalty and subsequent quid-pro-quo support.

25·2.B. Legal Temptations

If you now have the impression that fraud, theft, insider trading, and bribes are the most important agency conflicts between shareholders and managers, you would be wrong. The most important conflicts arise in the day-to-day execution of business, and are more judgment calls than outright unethical behavior.

Most managers are not criminals, though.

Empire Building. Most managers try to grow the firm. There are a whole slew of reasons. Most managers are loyal less to an abstract, ever-changing shareholder, as they are to their very tangible companies. Running bigger companies is also in the self-interest of managers: Executives of bigger companies are more prominent and receive more compensation. Some decades ago, this was even explicit: managerial compensation schemes were directly tied to sales, not earnings! These days, it is more implicit, and comes about through the choice of "comparable managers" when schemes are set. Unfortunately, corporate growth is not necessarily shareholder value maximizing. For example, in the 1980s, oil companies were flush with both cash and assets (oil reserves). The industry had overcapacity. Most oil companies did not return excess cash to shareholders, but chose to spend $20/barrel exploring for more oil reserves, even though proven oil could be purchased in the marketplace for $6/barrel. Managers naturally get paid for operating the company: oil exploration, growing the firm, acquiring other companies. It is not in the interests of managers to return cash to shareholders, especially when it entails drastic shrinking (asset sales) or when it means being taken over by another company. The reward for being a good manager, who maximizes shareholder wealth, would be the loss of one's job.

Most managers want to grow the firm at all cost.

Many academics believe that the highest agency costs in American companies today (in terms of expected costs to shareholders) are not illegal actions, but failure to direct corporate assets towards the activities that maximize shareholder wealth. The agency cost is particularly high for firms which have lots of cash and cash flow (e.g., from prior profitable activities) and few good new opportunities.

This growth is enormously costly to shareholders.

Corporate Perks. A closely related problem, although of lesser importance, is that managers disproportionately enjoy spending money on perks. A public company may buy a corporate airplane which costs $100 million and increases productivity by the equivalent of $10 million—just because it gives managers $1 million worth of pleasure. Plush corporate headquarters and fleets of corporate aircraft are usually sure signs of publicly traded companies, especially in slow growth industries.

Perks are goodies that managers have the firm buy for themselves.

Executive Pay. Naturally, executives also prefer higher pay. Executive compensation comes in many forms: salary, bonus, stock grants, option grants, retirement benefits, perks, and severance packages. According to Business Week, in 2002, A.W. Dunham of Conoco (COP) earned $30 million in salary and bonus alone; A. Lerner of MBNA (the credit card lender, KRB) earned $200 million in salary, bonus, and long-term compensation grants; J.C. Barbakov of Tenet Healthcare (THC) earned $300 million (Tenet is currently under indictment); and Larry Ellison of Oracle (ORCL) earned $800 million from 2000-2002, most from stock and options.

Executive Pay is often excessive.

SIDE NOTE: Economists' models also assume that executives prefer working less (called "shirking"). However, lack of work ethics among executives is rarely a problem in the real world—instead, it is self-enrichment that is the problem.

Questions about Executive Pay.

Why do U.S. shareholders pay so much money? How much worse a manager would the next executive choice have been, and would he have done the job for half? Why do second-in-commands usually earn only a very small fraction of the CEO's salary, even just one year before they succeed the CEO? And, why do German CEOs of even the most successful corporations earn only about 10% as much as their U.S. comparables?

Most likely, very large executive pay reflects the capture of the board and firm by management.

It could be that it is necessary to attract and retain top CEOs; that it is necessary to have a large CEO salary as prize on the top in order to motivate lower-ranked executives; that the Germans have it wrong and are paying too little; or that American CEOs have somehow "engineered" the arrangements so as to maximize their own takes. Opinions differ on the causes. Personally, I find it hard to believe that executive talent is rarer in the U.S. than it is in Germany. I find it more plausible that German executives are less easily motivated by high top salaries or that German companies have chosen suboptimally low compensation levels. But I find it most plausible that other constraints (cultural, ethical, governance-structure) have prevented German executives from obtaining the same levels of **board capture** and high levels of executive pay as U.S. executives.

Entrenchment. Managers, like all employees, like to be indispensable. If they decide to take projects for which they are indispensable, their own personal valuations and therefore compensation will necessarily go up. If they decide to build redundancy, making themselves dispensible, their own personal values will necessarily go down—they might even be replaced by the board. The ability to **hold up** the company, once they have become indispensible (or at least, very difficult to replace), probably plays an important role in the awarding of high executive compensation contracts: In this case, no managerial board capture is necessary for high executive compensation to occur. The board will not have a choice and award high compensation "voluntarily."

ANECDOTE: Yassir
A reluctance to build redundancy is not limited to executives. For example, despite being born in 1929 (76 years of age in 2003), PLO leader Yassir Arafat has yet to build up a potential successor.

Nepotism abounds. Few boards have non-friends on them.

Friends. Most managers prefer to have loyal friends working around them, instead of gadflys and potential replacements. This naturally promotes nepotism (in the broad sense).

SIDE NOTE: Another common problem is the growth of bureaucracy, which discourages shareholder wealth maximization, but which helps managers to become indispensable (knowledgeable of the internal process), helps managers to undertake even the most bizarre projects, internally justified by "proper procedure." In contrast, fighting bureaucratization is a painful and prolonged process with few rewards for the involved executives.

Sometimes, managers even prefer low values.

Perversity. Though very rare, managers can even have the incentive to drive down firm value: they can then negotiate better incentive compensation contracts or even acquire the firm in a **levered buyout**, either of which is often followed by seemingly miraculous turnarounds. The most prominent example is Ross Johnson, CEO of RJR Nabisco, whose actions are chronicled in the bestselling book **Barbarians at the Gate**.

Ethics? A manager may feel special obligations towards the town in which the factories are located; towards the workers employed; towards charitable causes. Under these circumstances, managers may explicitly or implicitly donate the shareholders' money towards causes they deem to be more worthwhile.

The web chapter on ethics deals with this question in more detail.

25·2.C. Temptation Control Mechanisms

We now show that entrepreneurs (i.e., coordinated owners) benefit from good corporate governance.

Are all these problems real? Yes, they are. So, in order to get you, as a private investor, to give your money to a corporation, the current corporate owners must create mechanisms that will curtail its own misbehavior. If these mechanisms do not succeed, the firm will be unable to raise more capital at reasonable costs from investors. (Yet, as we shall see below, if manager-agents are already firmly in charge of shareholder-principals, they may not care about "reasonable" costs, either.)

- Start with a penniless entrepreneur who has an invention that requires $25 million investment but which will be worth $100 million in today's dollars. If the entrepreneur cannot raise the money, the project will be captured by the competition. Without capital, the entrepreneurs wealth would be $0. There are large gains to finding investors. This is why companies go public to begin with: gains from diversification and capital outweigh the costs of agency conflicts.

- If the manager had needed to raise no funds and could have avoided all professional management, his net worth would be $75 million. Similarly, if the manager can somehow commit to avoid all agency problems, investors would give him $25 million for 25% of the company, leaving him again with $75 million.

- Investors may rationally believe that agency problems will appear as soon as money will have been raised. The particular agency problem in our example is that the entrepreneur cannot prevent the grabbing of $30 million by management-in-charge after the capital has been raised. Further, to hide the managerial scheming, the theft requires another $10 million of waste.

  If the entrepreneur must now raise external capital, would investors be satisfied with 25% of the company for a $25 million investment? Probably not: any future manager—*including the entrepreneur himself*—will want to steal the money. Investors know this, and therefore value the company only at $60 million. To raise $25 million requires the entrepreneur to part with $25/$65 = 42% of the company. The entrepreneur's net worth, if he can remain in charge, will be the 58%·$60 = $35 million that he will own, plus the $30 million that he can steal. The $65 million is $10 million less than what he could have gotten if he could have committed to zero future managerial agency problems.

- The entrepreneur's situation becomes outright awful if the project duration exceeds his lifetime, and he must hand it over to professional managers. In this case, he will only earn $35 million—the $30 million in theft will go to then-management.

- In sum, our entrepreneur is best off if she can just prevent all future managerial theft (own theft and future manager's theft) by instituting good corporate governance. Our entrepreneur is worse off if she cannot prevent future theft by herself. Our entrepreneur is even worse off if she cannot prevent future theft by other managers. And our entrepreneur is worst off if she does not take the project.

  It is also not difficult to construct examples in which future theft completely prevents any ability to raise funds for otherwise productive projects.

Unfortunately, raising capital is no guarantee that managers will control agency concerns; quite the opposite: raising capital can be another method of managerial self-dealing.

The insight from this example is that owner-entrepreneurs have an incentive to control agency problems in order to be able to raise capital at good terms. Unfortunately, the need to raise capital is not enough to induce managers (rather than entrepreneurs) to institute good corporate governance.

- Assume that the above firm has been created, and professional managers are firmly in charge. They now happen to find another project that costs $50 million, which produces profits of $50 million in today's dollars, but which allows for an additional $20 million of managerial theft. (Such projects are easy to find.)

  The existing company is worth $60 million. If the new project is taken, shareholders will own $30 million in value from the new project and $60 million in value from the old project. To raise $50 million in capital from a $90 million company only requires issuing shares to account for 55% of the company: old share owners will now own only 55%·$90 = $40 million in the new firm, down from $60 million. The extra $10 million goes to management.

Thus, the need to raise capital alone is not a guarantee that management (the firm) has an incentive to control agency problems. On the contrary, raising capital becomes another mechanism which helps management extract shareholder wealth for themselves.

Thus, there is little hope for good internal corporate governance among large, old, cash-cow firms.

Managers who own little of the firm have few incentives to curtail agency issues, because they are the beneficiaries of agency problems. The need to raise more capital is a necessary, but not a sufficient condition for the desire for better corporate governance. If managers are in charge, their internal quest for power and control will quickly become stronger than their incentives to control agency issues. This is a situation that is especially applicable to large, old, cash-rich firms, in which the executives/boards have enjoyed long tenure. In this case, even though new capital can be difficult to raise, old capital (which cannot easily be recovered) can keep the company going for a long time. And this old capital allows new capital to be raised. Even if managerial looting has reduced the value of a $1 billion into a $1 million firm, managers may still be able to raise another $1 million in capital for their personal consumption by promising 51% of the new firm.

The control of agency problems is often neither easy nor worthwhile for the original entrepreneurs.

An important question is why the entrepreneur does not write contracts upfront (*ex-ante*) so as to control agency issues. After all, unlike hired executives, the original owner entrepreneur can capture the gains from agency controls. There are two reasons: First, it is impossible to write contracts for all future contingencies, especially insofar as managerial schemes are concerned. The human mind can be very creative. Second, the incentives to write the appropriate contracts may be modest. Most companies that go public will never face large agency problems—most upstart firms end up bankrupt. For example, assume that the problem is the future theft of 1% of the firm. An average entrepreneur with a $10 million company thus faces a $100,000 issue (in today's value). Only one in a thousand of these companies may grow to be a $10 billion company over the next decades. Managerial theft and looting of $100 million in this $10 billion company is large *ex-post*, but *ex-ante* the control incentives were not too large. Thomas A. Edison's pressing problems in 1880 was not to prevent theft by General Electric managers in the 21st century.

Sometimes, market-based mechanisms can step in to correct managerial misbehavior. The principal mechanism, especially in the 1980s, was the **leveraged buyout** (**LBO**), in which a small holding company (such as **Kohlberg-Kravis-Roberts** [**KKR**]) purchased potentially much larger publicly traded companies. The majority of financing was debt, which meant that KKR owned only a small slice of very high-powered equity. Even modest post-LBO underperformance could result in a total loss for them. This gave KKR enormous incentives to get everything right. KKR typically either fired existing management or completely restructured the existing management compensation contracts to dramatically change their incentives. In many companies, through better control of agency problems, KKR created great value. In the 1990s, however, public market valuations increased dramatically, and it became harder and harder for KKR to find companies which it could buy cheaply and then improve. In addition, companies became better in instituting takeover defenses. Thus, at least for the time being, this external market mechanisms have faded a little into background.

Some dynamic mechanisms.

Agency control in (old) public corporations is usually rather dismal, but rarely fully unsuccessful—perhaps because theft can only be *so* large. Capitalism will not collapse because of managerial theft. Agency control will be more successful in some companies and less successful in others. Like the agency problems themselves, the solutions to agency problems are complex. In broad strokes, the solutions involve the combination of corporate obligations (promises by the corporation), legal obligations, and informal and ethical obligations. If the executives have zero ethical scruples, even the best legal and corporate systems are unlikely to succeed. If there is no legal framework, any corporate promises would be worth no more than the paper that they are written on.

Various mechanisms try to constrain agency problems.

25·3. Equity Protection

25·3.A. Voting: Shareholder Proposals, Proxy Contests, and Hostile Takeovers

The primary control right of equity is its right to vote. By vote, shareholders can effect any changes they wish, including but not limited to replacing the **corporate board** (described below), which can replace the management or liquidate the firm, etc. Unfortunately, in a widely held company, gathering votes to control management and corporate change is not cheap. The right to vote is therefore of much use only to large block shareholders (also described below), and only during unusual situations.

The primary control right of equity is its vote.

Hostile Takeovers. The first unusual situation is a **hostile takeover** (formally called an **unsolicited bid**). Usually, an acquiror makes a **tender offer** to purchase shares in order to obtain either the whole firm or a voting majority. Obviously, engineering a hostile takeover is neither cheap nor easy.

Hostile takeovers are very effective.

Management can resist hostile takeover attempts through many actions. Value reducing actions range from greenmail (where the potential acquiror is bought off), to golden parachutes (where the management lets itself be bought off), to the acquisition of yet other companies (making its own acquisition more difficult), to antitrust and other litigation, to charter amendments and poison pills, to scorched earth activities, such as the issuance of additional shares (e.g., to management or employees). Value-increasing actions can also help: management may make a self-tender offer, or pay out free cash to shareholders.

Management can resist.

Staggered boards virtually eliminate all hostile takeovers.

But, the most effective takeover defense seems to be the introduction of a **staggered board**, where each year only a third of the directors are replaced. This means that even a 100% shareholder may have to wait for up to two years before he/she can take control of the firm. Needless to say, any executive bent on destruction could do quite some harm to the business over the course of a year. There have been no successful hostile takeovers of firms with effectively staggered boards.

SIDE NOTE: Not all takeovers are driven by poor target management: industry consolidation and acquisition of monopoly power, desire by acquirors to increase their own empires, and corporate taxes are other reasons.

ANECDOTE: **RJR, Ego, and Overpayment**
The bestseller **Barbarians at the Gate**, also made into a movie, describes the epic takeover battle for **RJR Nabisco** between **Kohlberg Kravis Roberts** (K.K.R.) and RJR management (supported by Shearson Lehman [now Lehman Brothers]). In October 1988, RJR's CEO, Ross Johnson and his predecessors, had mismanaged the company long enough to allow him to offer RJR shareholders the premium price of $17.6 billion in a leveraged management buyout. This required the resignation of Johnson from the board contemplating the offer, which in turn opened the door to a $20.6 billion counteroffer by KKR. Eventually, KKR purchased RJR for $25 billion, and Johnson got a $53 million golden parachute. This takeover was also probably KKR's biggest miscalculation, in that it overpaid for RJR. The reason were personal egos and animosities which fueled an irrational bidding war.

Shareholder Proposals and proxy contests just seek votes for a resolution.

Shareholder Proposals and Proxy Contests. A second unusual situation is a **shareholder proposal** or **proxy contest**. Typically, a large shareholder actively solicits other shareholders to vote against management on particular proposals during the annual meeting. The proxy resolution itself can be either modest (changes in the corporate charter) or drastic (total change of the corporate board and management). Only rarely do proxy contests seek to outright eliminate management. This is in contrast to hostile takeovers, which usually seek to eliminate current management. Proxy contests have been particularly successful in removing anti-takeover defenses, including in some cases the staggering of the board. So, they are useful in setting the stage for later actions against poorly performing management.

How to get a shareholder proposals up for vote.

Any shareholder can put forth a **shareholder proposal** for vote by all shareholders. The S.E.C. judges whether shareholder proxy suggestions are appropriate for shareholder vote. (The rules by which the S.E.C. accepts or rejects shareholder proposals are explained in www.sec.gov/interps/legal/cfslb14.htm.)

Proxy contests are costly and (only) modestly successful.

Proxy contests are a costly and only occasionally successful mechanism. ISSProxy reports that there were 17 proxy contests in the first 8 months of 2003, of which only 4 resulted in dissident victories. The average dissident's cost per proxy contest was about $1 million. (The highest cost was over $5 million.) Nevertheless, the small success ratio is misleading, because even the threat of a shareholder proxy contest can lead the executives to seek a compromise to rectify some of the problems. And, compared to hostile takeovers, proxy contests are outright cheap.

Gains must be very large to make voting action worthwhile.

Clearly, to wage either a proxy contest or a hostile takeover, the value gains from dislodging management must be large enough. "Modest" governance problems, such as an executive salary of 1% of a company worth $10 billion, are just not enough to make the expense worthwhile.

Takeover Activity in the U.S..

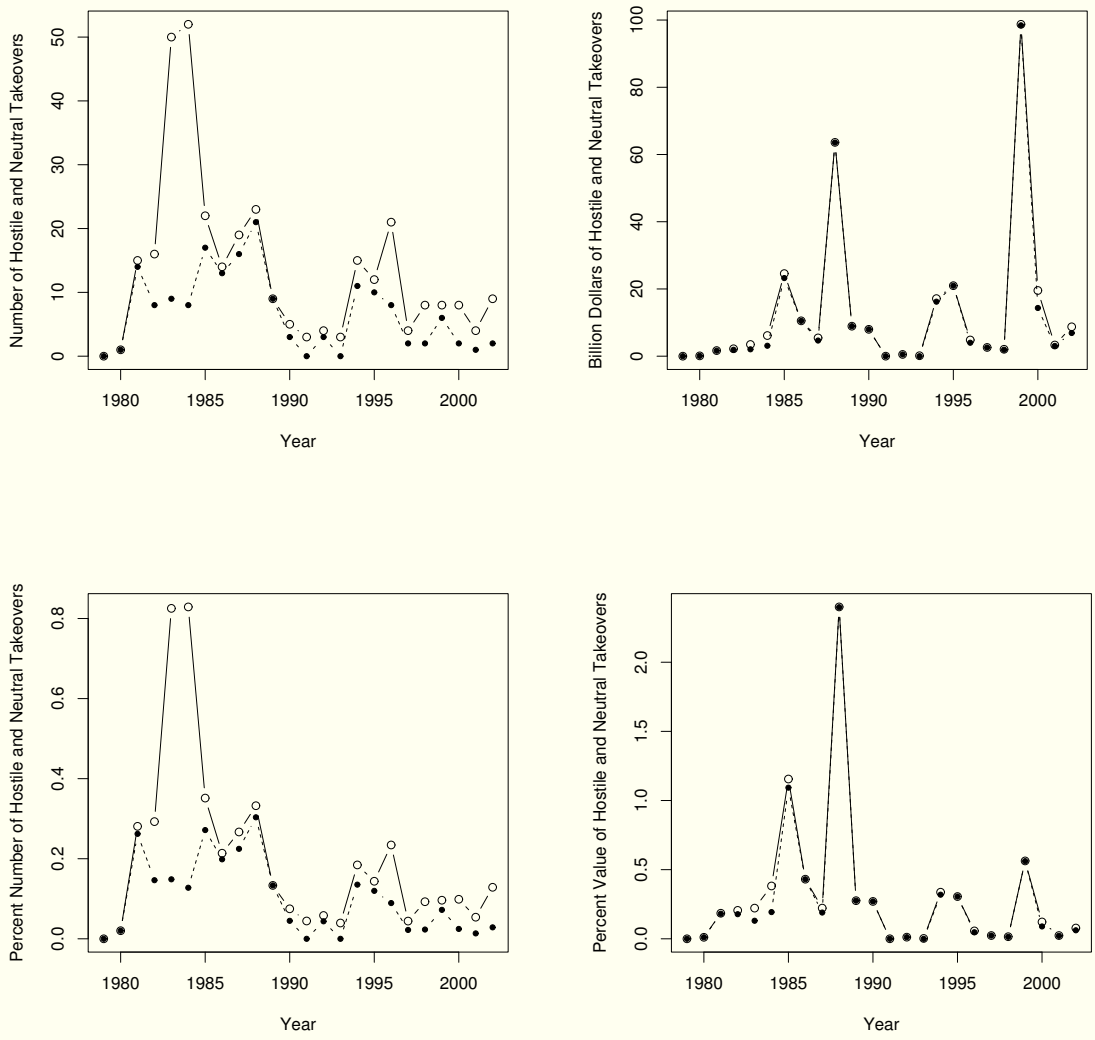
Securities Data Corp (SDC) reports that from 1979 to 2002, there were 40,983 domestic takeovers (incl. LBOs), with $7.6 trillion in total value Takeovers here include both public and private firms, as well as **Levered Buyouts** (in which a private acquiror borrows money to take a firm public). Most of this acquisition activity is solicited by, or occurs with the blessing of target management. Such takeovers are called "friendly." It is only the "hostile," or at least "neutral," takeovers that are likely to be a real threat to poorly performing management. In total, SDC classified 325 takeovers as neutral (159) or hostile (166), with $313 billion target value (neutral: $16 billion; hostile $297 billion).

Table 25.1. Hostile Takeovers with More Than $5 billion in Target Value, U.S. to 2003

Announced	Target	Acquiror	Value (billion-US$)
1999/11/04	Warner-Lambert	Pfizer	89.2
1988/10/24	RJR Nabisco	Kohlberg Kravis Roberts	30.6
1988/10/17	Kraft	Philip Morris	13.4
1995/10/18	First Interstate	Wells Fargo Capital	10.9
1994/08/02	American Cyanamid	American Home Products	9.6
2000/11/13	Willamette	Weyerhaeuser	7.9
1990/12/02	NCR	AT&T	7.8
2002/02/22	TRW	Northrop Grumman	6.7
2000/02/22	Mirage Resorts	MGM Grand	6.5
1999/08/11	Reynolds Metals	Alcoa	6.1
1985/10/16	Beatrice Co	Kohlberg Kravis Roberts	6.1
1985/09/24	General Foods	Philip Morris	5.7
1988/01/13	Farmer's Group	BAT PLC	5.2

Source: Securities Data Corp. Not inflation adjusted.

Figure 25.1. Takeover Activity in the United States



Source: Securities Data Corp. In the upper diagrams, the total number of both types are circles, the hostile type are fat dots. In the lower diagrams, these are reported as a percent of the total number and the total market value of all publicly traded companies on the NYSE, AMEX, and Nasdaq.

Takeover Activity in the US in the 1980s.

Figure 25.1 gives an idea of the relative importance of hostile and neutral takeover activity over time. In the 1980s, there were 8,360 takeovers with $1.2 trillion in total target value. Only about 1.4% of all takeovers were "hostile," another 1.2% were "neutral." But hostile acquirors managed to take over some pretty big fish relative to non-hostile acquirors: they accounted for 10% of all takeover value in the 1980s (neutral takeovers for less than 0.6%). How does this compare to the number and value of all publicly traded firms in the United States? In 1982, there were about 5,500 firms with just over $1 trillion in market value. The 50 or so hostile and neutral takeovers in 1982 and 1983 accounted for 0.8% of all publicly traded firms—in one year...and it was entirely a Kohlberg, Kravis, Roberts effect! This was also the peak of hostile takeover activity.

In the 1990s, the stock market boomed. General corporate takeover activity also heated up, with 25,493 takeovers and $4.4 trillion in total target value. But *hostile* takeover activity declined, accounting for only 0.2% of all takeovers (down from 1.4% in the 1980s), neutral takeovers for only 0.1% (down from 1.2%). In terms of target value, hostile takeovers accounted only for 3.5% (down from 10%), neutral takeovers only for 1.2%. Again, how does this compare to the number and value of all publicly traded firms in the United States? In 2000, for example, there were about 8,100 firms with about $16 trillion in market value. The 8 hostile takeovers with about $20 billion in market value now accounted for just about 0.1% of all publicly traded firms.

Takeover Activity in the US in the 1990s.

So, does the thread of a hostile takeover discipline managers? Probably at least somewhat, especially in the "old days" of the 1980s. One can argue that the sheer visibility and novelty of these takeovers were big enough to prevent the managers of many firms not to engage in the worst abuses. However, as Table 25.1 shows, KKR was pretty satiated after its takeover of RJR Nabisco. The hostile takeover threat rapidly receded after 1990. Any given year would typically see only about a handful of hostile takeovers. The number of hostile takeovers has become so small that, being acquired in a hostile transaction does not seem to be the sword of Damocles that is constantly hanging over and controlling management.

Does Takeover activity matter?

ANECDOTE: Bribing Shareholders in Proxy Fights
Karla Scherer led the *only* successful proxy contest of a major U.S. publicly held company in 1988. As a result, the company founded by her father in 1933 was sold in June 1989 at a price more than double the value of each shareholder's investment the year before, when the proxy contest began.

The most prominent recent proxy contest occurred in 2002, when Walter Hewlett, a Hewlett-Packard (HP) director and son of cofounder William Hewlett (holding 18% of HP), opposed HP's acquisition of Compaq. He lost the proxy vote after Deutsche Bank (DB) switched 17 million of the 25 million shares *of its clients* (held by the DB asset management division) in favor of the $22 billion merger—after DB had become the co-arranger of a new multibillion-dollar line of credit. In August 2003, the government fined DB $750,000 for failing to disclose another apparent conflict of interest to DB's asset management client. In a memo to her CEO, HP head Fiorina suggested HP do something "extraordinary" for DB and another firm. HP paid DB's investment banking arm $1 million for "market intelligence," with another $1 million contingent upon success. DB's investment banking arm then helped to convince DB's asset management group of DB's interest. And rightly so, during a conference call with DB money managers, Fiorina then reminded DB that their votes would be "of great importance to our ongoing relationship."

Some other institutional shareholders held shares in the target, Compaq, and therefore also voted in favor. (CalPERS, a prominent pension fund and advocate of better corporate governance, voted with Hewlett.) Net, 838 million shares voted in favor, 793 million shares against the deal. Hewlett alleged that HP spent roughly $150 million of shareholders' money on the proxy fight against him (18% of which he had to effectively pay for).

25·3.B. Large Shareholders

The Good

It is not worth the time of small diverse shareholders to attempt to vote and/or to influence management. The costs of meaningful action and coordination are too high, the benefits to each individual shareholder are too low. (Economists call this the **free-rider problem**.) Consequently, any active role is most likely to originate from large shareholders both with enough votes to scare management and with enough value-at-stake to take an active interest. But to become a large shareholder is itself costly, foregoing significant diversification benefits. Typically, the larger the firm, the smaller

Only large shareholders have an incentive to control agency problems.

the stakes of the largest outside shareholders and therefore the smaller the largest shareholder's influence. The shareholders tend to be most dispersed among the firms with the most severe agency problems.

Large shareholder influence is limited.

The power of large shareholders is limited. First, even if large shareholders have some incentives to control management, it is not enough. A shareholder who owns 5% of a firm suffers 100% of the cost of any effort to influence the management, yet reaps only 5% of the benefit. Second, votes are not secret: managers know exactly how their shareholders vote and can seek retribution lateron. Third, if the large shareholder is a mutual fund, it cannot actively seek to influence corporate behavior. If it does, it would run into insider trading laws when it wanted to divest itself of its stake. Therefore, many large institutional shareholders abstain from actively seeking corporate influence. However, many passive institutional shareholders still can and often do tend to vote their shares *against* management if a third party were to seek an active influence, e.g., in a proxy contest. The presence of large blocks of shares, even passive shares, which could potentially overwhelm the voting power held by management and their allies, is therefore a low-level, but constant restraint on management.

ANECDOTE: CalPERS Top-10 List
The most visible corporate governance activist in the United States is the California Public Employee Retirement System. CalPERS publishes an annual focus list of worst corporate governance companies (in its portfolio). Among its 2003 winners were Gemstar (GMSTE), JDS Uniphase (JDSU), Manugistics (MANU), Midway Games (MWY), Parametric Technology (PMTC), and Xerox (XRX). The detailed corporate governance shortcomings make interesting reading.

But even CalPERS rarely takes on Fortune-100 companies (which are most prone to suffer from agency conflicts). The reason may not only be political, but the fact that CalPERS' ownership share in Fortune-100 companies is too low to make much difference.

The Bad

Large shareholders may not seek better governance, but better treatment for themselves!

It is not in the interest of the executives to pick a fight with their largest shareholders. It could be publicly embarrassing, especially in light of management's fiduciary responsibility towards their shareholders (see below). Instead, most corporate executives seek a cordial arrangement with their large shareholders. Special treatment of large shareholders—from special access to information and the sharing of corporate perks (especially easy if the large shareholder has other relationships with the company, e.g., for an employee shareholder group, or for an investment bank, or for friends of the original entrepreneur), all the way to so-called "targeted share repurchases" (greenmail), in which the company management uses shareholder money to repurchase pesky institutions' shares at a higher price—usually are more effective than confrontation. Company founders often have a special relation with the company. They typically always consider the company to be their own, and usually hold enough stock to control it. There is strong evidence which suggests that founders are often detrimental to shareholders *on average*: when the founder of a company suddenly dies, the stock price of the company usually goes up, not down!

SIDE NOTE: Similarly, shareholdings by management have an ambiguous effects on corporate governance. On the one hand, they can incentivice managers to be more eager maximizers of share value: they can benefit more. On the other hand, more shares means more votes, which means that they are more likely to be able to win any votes. In perspective, the best control of agency problems may be managerial retirement and death—the new manager is unlikely to obtain the same high levels of capture immediately after succession.

ANECDOTE: Graft in Action: Panavision
In August 2002, Businessweek reported the end of a two-year drama. In 2001, Ronald Perelman had a 53% stake in M&F Worldwide Group (MFW), a publicly traded tobacco ingredient company. Perelman initiated an M&F purchase of Perelman's Panavision shares at Perelman's cost of $17/share. At the time, Panavision (PVIS.OB), a movie camera maker, traded for $4/share. After more than a year in court with a minority shareholder (a hedge fund which had to pay for its court costs), Perelman graciously agreed to reverse the transaction.

ANECDOTE: Board Courage at Citigroup
Although biased, the PBS series Frontline episode *The Wall Street Fix* (www.pbs.org/wgbh/pages/frontline/shows/wallstreet/) illuminates many of the conflicts of interest between ordinary shareholders and larger stakeholders. It details how Jack Grubman, star analyst for the investment bank of Salomon Smith Barney, hyped Worldcom in 2000 to its brokerage's small retail investors. At the same time, the CEO of Worldcom, Bernie Ebbers, held a personal $1 billion mortgage from Travelers. Both SSB and Travelers are owned by Citigroup (C). Ebbers' wealth, and therefore his $1 billion mortgage, was closely tied to the Worldcom stock value.

(In a display of ordinary courage, after the indictment of Citigroup for a variety of questionable activities, the Citigroup board voted its full support and confidence in its CEO, Sandy Weill. Business Week was not so generous: in January 2003, it ranked Sandy Weil as the worst manager in America.)

The Evidence

Some large shareholders help, other large shareholders hurt.

The degree of power of large shareholders to restrain management and the degree to which the presence of a large shareholder aids small shareholders remains a matter of opinion. In some firms, large shareholders serve a useful role in constraining management, and thereby aid small shareholders. In other firms, large shareholders help themselves to corporate assets, and thereby hurt small shareholders. There is some evidence that firms with large public pension fund investors tend to engage in fewer value-reducing takeovers; that firms with external 5% owners tend to perform better than firms without such; and that managers in poorly performing companies are more often replaced when there are large shareholders. In my opinion, large shareholders are such a diverse group that it is not possible to generalize much here.

ANECDOTE: Credibility of Shareholders in East Asia
In East Asia, non-profit shareholder organizations seek to protect shareholders right. They purchase shares in many corporations and bring suit to defend shareholder rights when violated. The non-profit nature means that they are not perceived as frivolous self-serving litigants.

25·3.C. The Corporate Board

The board is supposed to represent shareholders and control managers.

During the annual meetings, shareholders get to vote on the **corporate board**. During the year, it is the board's legal responsibility to oversee management and to ensure that managers are acting in the interests of shareholders. The **chairman of the board**'s and managements' most important powers are the ability to choose the information fed to the board and the determination of the agenda. After all, with only a couple of days per year on the job, and with their own other jobs to attend to, board members cannot possibly know the business as well as management. Thousands of pages of readings as preparation for the board meeting—possibly with, possibly without key information—are often just as useful to board members as zero pages. And board members fully well know that if they do not stick to the specific agenda, the risk is that the discussion degenerates into endless unfocused conversations. Not surprisingly, large boards are usually less effective.

The board is often controlled by the CEO.

In most corporations, this does not matter, because the CEO is also the chairman of the board. For example, in 2003, out of the thirty Dow-Jones Index companies, only four (GM, Intel, Microsoft, and Walmart) had both a CEO and a chairman. It should be obvious that this identity crisis severely limits the ability of an ordinary board to oversee and potentially discipline and control management.

The Chairman/CEO can stack the board. There usually is no meaningful shareholder vote.

New members of the board are usually either proposed by the chairman of the board or nominated by an independent committee sanctioned by the chairman. The board is then elected *as a slate* with an up-or-down vote at the annual shareholder's meeting. Shareholders cannot vote for or against particular candidates; and, in the extreme, a shareholder with 49% of the shares may find herself with zero board representation. Most corporate board elections are about as democratic as (and the outcome is as full of tension) as elections in North Korea. The common outcome is that between one-third and two-thirds of the board are also employees of the company, and thus under the direct day-to-day control of the CEO. Finally, to avoid an external board coup, a good number of companies have adopted **staggered boards**, already explained above.

Many firms have a majority of corporate employees on the board. (Such) boards tend not to fire poorly performing managers.

A Weisbach study of 495 corporate boards from 1974 to 1983 classified directors as insiders if they were full employees of the company—which puts them under the direct control of the CEO. Only about one-half of these 495 NYSE corporate boards even had a majority of outside directors! Only 128 had boards with clear majorities of outside directors, many of whom had their own dealings with the company and were thus also conflicted. Fortunately, the presence of inside rather than outside directors does not seem to matter much, either. In the ten years from 1974 to 1983, the probability that a manager would depart increased only from only 5% to 6% when a company lost 33% (!) of its stock market value (market adjusted)—and this retirement increase may have been because the manager was already close to retirement (and maximized his own take). Firms with more than 60% outside directors only had an additional 1% resignation frequency; and for firms which lost "only" 10% to 25% of their values, a majority of outside directors did not even increase the resignation frequency at all.

ANECDOTE: Executive Succession in Action

Through board control, CEOs usually can even determine their successors (and more than one successor has found huge skeletons in the closet). Also, recall that in many boards, the Chairman/CEO has considerable influence over which board members should retire and who the next board's members should be. Of course, these board members in turn nominate the executive compensation committee, who in turn decide on the Chairman/CEO's compensation.

A study of compensation committee membership found that when a director its on multiple executive compensation committees, the firms tend to then have similar executive severance pay packages. For example, the chair of IBM's compensation committee is Charles Knight, whose exit package from Emersen Electric had similar provision to Jack Welch's package from General Electric. Not surprisingly, IBM CEO Lou Gerstner's separation package was similar to Jack Welch's. (Both Knight's package and Gerstner's package were among the most generous around.) Ivan Seidenberg of Verizon, singled out for enjoying among the most egregious severance packages by the report, sits on Honeywell's compensation committee—*also singled out*. Source: corporatelibrary.com.

As corporate control mechanisms, except in extreme situations, corporate boards are a joke.

In sum, almost any CEO and chairman, who was originally successful and who is eager to gain full control, can stack the board with dependents and friends within fairly short order. Consequently, in many but not all corporations, the boards are effective control mechanisms only in two cases: first, when the CEO/Chairman is fairly new and has not yet taken full control of the board (or has not shown the appropriate interest to do so, a rare but occasional condition). Second, when the CEO's misbehavior is so egregious that board members begin to fear negative publicity and personal legal liability. (The ability of the media to embarrass managers should not to be underestimated.)

Now, corporate boards can and do serve other useful functions. For example, they can advise the executives, they can help build relationships with suppliers and customers, and they can help to find a new CEO if the current CEO suddenly "evaporates." But when it comes to proactive control of managerial misbehavior, most corporate boards in the United States today are more theatrical stages than effective corporate control mechanisms.

Corporate boards can serve other mechanisms.

SIDE NOTE: Better corporate boards and better corporate governance would not come without cost. If legal reform were to reduce the cozy relationship between board and management, management could indeed be better controlled. But it might allow large shareholders to extort more value for themselves at the expense of small shareholders, it might reduce other beneficial functions of the board (better relation with suppliers, etc.), and it could even destroy the company, if the relationship between management and board degenerates were to degenerate into a war. I personally would assess these costs to be rather modest, and considerably outweighed by the potential benefits of legal reform.

25·3.D. The Legal Environment

The United States' best aspect of corporate governance is probably its legal environment. Investors are protected by a set of laws, regulations, and court rulings, plus appropriate legal enforcement. Much law has come about through court rulings and judicial precedence. This process has an intrinsic flexibility, which continues to fill gaps created by new problems, which is more difficult to accomplish by statutory law. In civil law countries, like Belgium, where regulations have to be legislated from the top, investor protections tend to be worse.

U.S. law is ever-evolving.

To become a successful publicly traded company, a U.S. company usually has to satisfy state shareholder laws, federal shareholder laws, the Securities Exchange Commission (S.E.C.), the rules of the National Association of Securities Dealers (N.A.S.D.), the Financial Accounting Standards Board (F.A.S.B.). It also has to try to avoid class action lawsuits, which have bankrupted more than one company. These needs together set minimum standards on corporate behavior—especially on appropriate information disclosure and self-dealing—that are not easy to skirt.

There are many regulations that try to ensure minimum decent corporate governance.

Shareholders' single most important and broadest legal protection is management's legal fiduciary responsibility to act on shareholders' behalves. Black's Law Dictionary defines a fiduciary relationship as one "in which one person is under a duty to act for the benefit of the others." The seminal opinion was written by the New York Court of Appeals in 1984:

Shareholder's best protection is fiduciary responsibility.

> Because the power to manage the affairs of a corporation is vested in the directors and majority shareholders, they are cast in the fiduciary role of "guardians of the corporate welfare." In this position of trust, they have an obligation to all shareholders to adhere to fiduciary standards of conduct and to exercise their responsibilities in good faith when undertaking any corporate action. Actions that may accord with statutory requirements are still subject to the limitation that such conduct may not be for the aggrandizement or undue advantage of the fiduciary to the exclusion or detriment of the stockholders.
>
> The fiduciary must treat all shareholders, majority and minority, fairly. Moreover, all corporate responsibilities must be discharged in good faith and with "conscientious fairness, morality and honesty in purpose." Also imposed are

the obligations of candor and of good and prudent management of the corporation. When a breach of fiduciary duty occurs, that action will be considered unlawful and the aggrieved shareholder may be entitled to equitable relief.

Management's fiduciary responsibility thus limits primarily excessive self-dealing, especially transactions between the management with the public company and the public company itself.

SIDE NOTE: The **Business Judgment Rule** protects managers against lawsuits if they make poor choices in the execution of most other company affairs. (Otherwise, the litigious climate in the U.S. would paralyze them!)

Actual enforcement is important, too.

The importance of *enforcement* of laws (rather than just what is on the books) is not to be overlooked. The U.S. has strong civil (financial) and criminal penalties and enforcement for the range of actions detailed in Section 25·2.A. (Although the wheels of American justice grind slowly, usually taking years to resolve even clearcut cases [itself a big problem, both for guilty and innocent parties], they do grind.)

ANECDOTE: Disclosure Rights Outside the U.S.A.
If you believe the U.S. corporate governance situation is bad, wait until you learn the situation in other countries.

In Germany, until recently, insider trading was legal. Disclosure standards are modest. Minority shareholders have few rights against self-dealing by majority shareholders, which are themselves often other corporations. Executives have legal obligations not only to shareholders, but also to employees. But the most amazing fact is that many German firms are owned by complex webs of other firms, which in turn are owned by yet other sets of firms. Ultimately, most large publicly traded firms are owned by the banks. The banks in turn are owned by...themselves! Deutsche Bank holds voting rights for 47.2% of its shares; Dresdner for 59.25%, Commerzbank for 30.29%. (Source: Charkham (1994).) This makes effective control by the ultimate owners very difficult. Many German banks even own themselves!

And Germany looks like investor heaven relative to Russia. In Russia, shares can be declared void by the board at any time; majority share owners cannot force an issue onto the corporate agenda; and even physical threats against pesky shareholders are not unheard of. (And do not look to courts and police for protection: judicial and political corruption in Russia is rampant.)

Firms can choose some then-legally-binding mechanisms.

Some corporate (legal) choices are optional. For example, firms can choose a particular auditor, stock exchange (with exchange rules), a particular investment banker, a particular set of warrantees, a particular collateral. Large multinational firms can even choose which country to incorporate in (or reincorporate in). Of course, a firm that reincorporates itself in Russia, with Arthur Anderson as its auditor, lists on the Moscow Stock Exchange, self-underwrites securities, and gives no promises or collateral is likely not to be able to raise much equity capital.

25·3.E. Ethics, Publicity, and Reputation

Managers are self-interested, but most are not criminals or unethical snails.

Many CEOs *want* to do the right thing. Ethics is an important factor that constrains many managers (and often underestimated by economists). Most CEOs want to do well for themselves, but do so "within the bounds of the normal, ethical range." Staying within the bounds of the ordinary also reduces the concern for negative publicity and legal liability for violation of their fiduciary duties.

Ethical standards are relative and changing.

Yet, ethical standards are themselves defined by the behavior of a CEO relative to those of her peers—and the race has been to the bottom. For example, one-hundred years ago, the financier J.P. Morgan argued that no CEO should make more than 20 times what the average company employee earns. The average today is almost 200 times. Consequently, being paid 200 times an average worker's pay does not violate the standard of ethics today. Similar arguments apply to almost every other issue in corporate governance: if a practice is commonplace, it is unlikely to violate an executive's sense of appropriateness.

SIDE NOTE: It is my personal opinion that the need to avoid negative publicity is today's primary constraint on executive compensation. Negative publicity is also the reason why managerial compensation has come to consist of many components. They make it difficult to ascertain the true size of the executive pay package, and they make it difficult to determine whether executive pay is primarily due to corporate performance rather than due to managerial board capture. (Some evidence of how boards change compensation after poor performance, e.g., where executive options would expire worthless, suggests that it is more of the latter than the former.)

Reputation sometimes constrains managers.

Corporations can also reduce their financing credibility problem through building **reputations**. A manager who has once harmed investors is much less likely to be able to raise capital in the future. Conversely, a company that has a long history of treating investors well (e.g., paying dividends and repurchasing shares) often has an easier time raising capital than a company that has just started up. Reputation may also play a role when a manager is CEO of only a small company, and has his sights set on being selected manager of a larger company in the future. To receive a higher call (with more ability to steal!), the manager must constrain his self-interest for a while. One problem with reputation as an agency control mechanism is that managers close to retirement no longer care as much about their reputation as they care about their severance packages (which are therefore often large). Most CEOs retire, rather than graduate to bigger companies.

ANECDOTE: **The fox guarding the henhouse: The N.Y.S.E.**
The New York Stock Exchange (NYSE) is not a publicly traded company, but owned by its members, primarily the likes of Goldman-Sachs. The members appoint the NYSE board.

The NYSE is an odd creature in another respect. It is both a stock exchange and a regulatory agency. The SEC heavily relies on the NYSE to ensure good corporate governance among its members and its traded firms, which represent almost all large U.S. corporations (with the exception of the technology sector).

The NYSE board decides on its chairman's compensation package. The chairman regulates its members. The NYSE members appoint the board. The board appoints the chairman and sets the chairman's pay package. The chairman regulates the members who appoint the board. The board pays the chairman. And so on.

In August 2003, the media found out that Richard Grasso, the NYSE Chairman, held a retirement package worth $140 million—about four times the annual profits of the NYSE. The media later found an additional $48 million in pay, which Grasso then promptly and graciously declined (but never in writing, so he will likely accept it, after all). After more digging, it was revealed that Grasso also helped pick the executive compensation committee. Then, many large institutional shareholders joined the chorus, publicly demanding Grasso's resignation. On September 17, 2003, Grasso finally believed the board's discontent, but did not resign outright. Meeting with his lawyers, he learned that by forcing the board to terminate him (rather than by resigning), he would receive an additional $57.7 million, in addition to the $140 million deferred compensation—which he did.

25·4. Debt Protection

Equity needs constant expensive supervision.

Equity payoffs depend very sensitively on good management control and actions and accurate accounting (verification). Even if they are firmly in charge, equityholders have the unenviable task of determining whether poor performance was the fault of management, the fault of the market, or both.

Debt has a much easier task: collect promised amounts, or seize assets.

Unlike equity, creditors do not need to play a large role in the day-to-day operations of the company in order to receive most of their due. Ascertaining the value of collateral is cheaper than ascertaining the value of equity (with its future growth options). And if cash is not paid when promised—regardless of whether it is because the market environment was bad, or because management performed poorly, or because management just hides assets—the company falls into automatic default (usually bankruptcy and/or corporate liquidation), and creditors can take control of the company and/or the collateral. Therefore, creditors need not spend much time and money investigating managers. (Some researchers believe that equity both should be and is mostly sold only after all collateralizable assets have been used up to obtain credit.)

Some typical covenants.

Creditors usually demand and receive covenants, by which the firm must live. Covenants may include collateral, priority, the naming of an auditor, the specification of minimal financial ratios (e.g., dividend payout ratio), and many more. Default occurs when covenants are not met. Importantly, creditor action coordination upon delinquence is not required, because coordination mechanisms are designed at inception. If the creditor is a single large bank, this is not necessary, but if the creditor are public bonds, then the bond covenants designate a trustee to oversee performance of covenants. The trustee has the obligation to declare a bond in default when the covenants are not met. (The process is mechanical.) Therefore, in contrast to equity holders, bond holders do not commonly suffer from free-rider problems.

Bankruptcy is bad news for management, which is good for creditors.

Equity owners, on whose behalves managers supposedly act, lose access to the firm's future projects, one reason why management dislikes default. But the important reason is that in the real world, corporate management is replaced in virtually all bankrupt companies. This gives management and shareholder-owners an enormous incentive to avoid default/bankruptcy.

Screwing bondholders is possible, but not easy.

Although there are some escape mechanisms that permit management to manipulate the covenants, these are rare and slow. The first such mechanism is a "forced exchange offer," in which managers set up a prisoner's dilemma that makes it in the interest of every individual bondholder to exchange for a less worthy bond but of higher seniority, even though it is not in their collective interest. The second mechanism is a covenant amendmend, which must be approved by the bond trustee and voted on by bondholders. The third mechanism requires major corporate surgery: When Marriott Corporation announced that it would split into two companies (hotel operator *Marriott International*, MRT, and a real-estate investment trust *Host Mariott*, HMT) in 1992, its share price rose by 10%. Marriott's bondholders sued, because the old Marriott debt now would be owed only by one descendent, *Host Marriott*. Moody's Special Report covering 1970–1992 wrote on page 4 that:

> Perhaps the most notorious fallen angel of the year was Marriott Corp., which alone accounted for $2.6 billion of downgraded debt. In October, Marriott announced a controversial spin-off that would relieve the profitable hotel operations business of the heavily indebted real estate and concessions business. Such a move would have the effect of creating one very healthy and essen-

tially debt-free company, Marriott International Inc., and another substantially weaker debt-laded firm, Host Marriott Corp. While issuer-bondholder talks are ongoing in the Marriott case, investors worry that such lopsided spin-offs may become more popular in the future.

Nevertheless, these are the exceptions rather than the rule. It is generally much harder for management to escape bondholder discipline than it is for them to escape stockholder discipline.

ANECDOTE: Creditor Protection Outside the U.S.A.
In the United States, management can file for Chapter 11 protection, which can delay the turning over off assets to creditors. This option does not exist in many other countries. For example, in Germany, creditors can force practically immediate liquidation of the firm upon non-payment. As a result of poor shareholder protection and strong creditor protection, many German companies are heavily creditor financed: it is far more difficult for them to find shareholders than it is to find creditors. Many of the largest German companies remain founding-family financed.

The worst creditor protection usually occurs in the case of sovereign debt (debt issued by countries). There is very little other than a country's desire for a good name and its foreign assets that prevents it from simply repudiating its debt. For example, Argentina owed about $220 billion in 2001, with required repayments of $22 billion a year—during the worst economic crisis the country had ever experienced. Interestingly, in July 2000, an Argentinian Judge named Jorge Ballestero even sent down an interesting ruling on the foreign debt: the ruling attributed responsibility for the debt to the civil servants during the previous dictatorship that contracted it *and co-responsibility* to the international organisations like the IMF, who approved these loans, now declared illegal and fraudulent. Would you lend money to a country?

Source: odiousdebts.org.

SIDE NOTE: Although we discuss primarily the case where creditors cannot trust corporations, the opposite can also be the case. (And it can just as much prevent viable debt financing.) A creditor may be able to pull its line of credit and thereby expropriate the firm's equity (receiving control of the firm). Banks attempt to build a reputation for not doing so in order to reduce such borrower concerns.

SIDE NOTE: Even though liquidation will hurt shareholders, the threat of future liquidation upon poor managerial performance can *help* dispersed public shareholders up-front.

Also, some researchers believe that regular dividends can serve a similar function as coupon payments. When a firm omits paying dividends, the board is more likely to fire the managers.

25·5. The Effectiveness of Corporate Governance

25·5.A. An Opinion: What Works, What Doesn't Work.

Give up all hope—or don't give up hope.

In the real world, it is impossible to design corporate contracts and arrangements which result in perfect ("first-best") managerial behavior. If we want to maximize wealth, we have to live with imperfection ("second-best"). In equilibrium, we must trade off the advantages of being public (such as access to more capital and better diversification) against the disadvantages (managerial misbehavior). This is not to condone the latter: just because some shoplifting may be unavoidable does not mean it is right.

There are many mechanisms which work in concert. Now comes my opinion.

Our ultimate tradeoff is achieved not through one, but through a variety of mechanisms. Corporate governance consists of many component, of which the corporate and legal structure are perhaps the most important. Although the many mechanisms all need one another, we can ask which ones work better and which ones work worse. This is a matter of opinion, and here is my own.

Corporate self-governance in the U.S. is almost totally broken.

Even if corporate governance in the United States works better than it does elsewhere, it is largely broken. A manager who starts out with a couple of good years and who is bent on taking over control of the company will encounter only mild internal resistance. Once entrenched, it is not corporate self-governance, but only legal and public relations concerns which are likely to constrain the manager. Fortunately, our corporate governance problems are not big enough to destroy most of the wealth created by our multi-billion dollar publicly traded companies, and they won't bring down capitalism, either. But in terms of the wealth siphoned off from the corporate sector into individual pockets, the problem is big

Ethical constraints are decaying or decayed.

The ethical aspect of corporate governance has also begun to erode, perhaps because the corporate governance aspect has largely broken down. For example, even as late as 1980, the typical manager earned only 40 times of what the average employee earned. Most managers would have felt uncomfortable earning more than 100 times. Nowadays, the average executive earns over 400 times what the average employee earns, and few executives would deem pay packages of $100 million or more to be obscene. The standards of appropriate managerial behavior today are not the same as they were in the past—and the past itself is nowhere near as rosy as it is often painted.

Legal protection is the only half-way intact mechanism.

It is the legal structure in the U.S. that is the saving grace. It is the standard of disclosure, the requirement of fiduciary responsibility, the effectively enforced prohibition of theft, fraud, and insider trading, the personalized legal liability, and the strong enforcement of its laws. Oddly, this structure is enough to rank the United States at the top of locales for equity investors.

Legal protection as a corporate governance mechanism carries a real danger: in the future, it could screw up more than helps.

This situation is perplexing. There is a widespread philosophical view, which I share and which is backed up by much good empirical evidence, that almost everything that the government touches comes out for the worse. Private companies usually tend to do better than government endeavors. And yet, it is precisely the legal structure in the United States that has become the most effective corporate governance mechanism. So, should we ask the government to take a more active role in corporate affairs? There is a very real and very grave danger that the government could potentially screw up, and misregulate or overregulate. A government cure could easily end up worse than the disease. The appropriate remedy for managerial abuse is therefore a difficult question for someone with my philosophical bent.

Other researchers do not share my view. In particular, some researchers believe that large shareholders—though conflicted in many situations and therefore themselves a problem—can serve as an effective constraint in some situations. Other researchers are so loathe to any government intervention that they even believe that if the market had been left alone, it would have come up with better solutions than what we got with the mixed-solution situation—some government control, some internal control—that we are in today.

My perspective may be unusually stark.

25·5.B. Quo Vadis?

It is a real danger that if no action is taken and corporate governance becomes worse in the United States than in other countries, investors may wander off to other locales. The recent corporate scandals in the United States have helped to highlight the need for corporate governance reform. Ironically, these scandals were the results of already illegal actions, and many perpetrators may end up spending many years in prison. Current reform attempts will not eliminate such scandals in the future: Just as bank robberies exist despite laws against bank robbery, so will illegal managerial looting continue despite laws against it. Fortunately, some good may yet come out of the current attempts at corporate reform.

The corporate scandals of 2001-2003 ironically are not the result of inadequate corporate governance laws, and reform efforts are unlikely to prevent them from repeating.

The main corporate governance legal regulatory functions are served by the S.E.C. and the stock exchanges. The **Sarbanes-Oxley Act of 2002** further reinforces this system. In line with this act, the stock exchanges are trying to tighten their rules for listed companies. The NYSE usually has the tightest corporate governance rules, NASDAQ's are less tight. The remainder of this subsection therefore describes the joint set of proposed changes.

Reforms are proposed by Sarbanes-Oxley, the NYSE, and NASDAQ.

Side Note: In addition to these legal regulators, there have also been a whole range of institutions which have proposed "best practice" guidelines for corporate governance. The most prominent are the *GM Board Guidelines* (since 1994), the *American Law Institute Principles* (since 1992), the *Business Roundtable Principles* (since 2002), the *National Association of Corporate Directors Report* (since 1996), the *Conference Board Recommendations* (since 2002), the *CalPERS Principles/Guidelines* (at least since 1998), the *Council of Institutional Investors Principles and Positions* (since 1998), the *TIAA-CREF Policy Statement* (since 1997), the *AFL-CIO Voting Guidelines* (since 1997), and the *OECD Principles/Millstein Report* (since 1998).

Most of the post-Enron changes seek to strengthen the independence and function of the corporate board, especially insofar as the audit, executive compensation, and nomination committees are concerned. Here is a selection of the most important reforms of 2003:

Listing the current changes to corporate governance.

- There will be a clear definition of what an independent director is: individuals who have no current or recent material relationship with the company.

  (There is a modest requirement against board reciprocity by executives [on NASDAQ, related to executive compensation committees], and only minimal requirements against material relationships between the executive and the board member.)

- Independent directors must meet in regularly scheduled executive sessions without management.

- A large part of the Sarbanes-Oxley Act pertains to the audit committee, as the Act itself was sparked by accounting scandals.

- The audit committee, checking over the company's financial reports, must consist entirely of independent directors. There are additional special rules for the audit committee pertaining to large shareholders.
- The audit committee must have choice of, oversight of, and compensation responsibility for the company's auditors. It can engage additional advisors, and it must institute procedures to handle complaints and whistleblowers.
- External auditors are also to be limited in the amount of consulting work they can do for companies, which has historically been a great source of conflict for public auditors. In addition, the audit committee must approve any remaining non-audit consulting work by the auditor.
- The audit committee must identify which of its member is a financial expert, and at least one required.
- The audit committee also has "code of ethics" responsibility now.
- Auditors must be rotated on regular basis, reducing the tendencies of relationships between firms and auditors to become too cozy. (Of course, this has costs, too: new auditors have to first learn more about the firm, and may be less apt in detecting unusual behavior.)

· The C.E.O. and C.F.O. must certify to the audit committee the accuracy of the company's financial reports/condition.

SIDE NOTE: This is a new feature of Sarbanes-Oxley—or is it? Executives were responsible for the reported financials of their companies even before its enactment. It made for good television, though.

· Attorneys must alert the SEC if they learn of credible evidence of U.S. securities law or material breach of fiduciary duty.

· Companies can select the members of their executive compensation committee and board-nominating committee, but these committees must be majority independent (NASDAQ) or fully independent (NYSE).

Here is what I think is missing.

My personal view is that the current corporate governance reform wave is a good thing: it is a step in the right direction—but only a small step. Many of the Sarbanes-Oxley prescriptions are more image then substance, focus too much on process over outcome, and require yet more bureaucracy. We need more reform—and more intelligent reform. What would I love to see? I believe the following four simple reforms would go a long way:

1. Proposed by Ira Millstein, the position of Chairman of the Board should be separate from that of Chief Executive Officer. It should be obvious that if the Chairman is also the C.E.O., the board at best can only struggle to assert influence over management, rather than act as the primary employer, directing management to act in the interest of shareholders. Today, in executive circles, a company that has a separate chairman is seen as not trusting its C.E.O. It must become an accepted corporate norm for these two positions to be separate.

2. The voting system should be changed to a proportional system: a shareholder with 10% of the shares should be able to obtain 10% of the seats, if this shareholder so desires. This would allow large institutional shareholders to create mechanisms of "professional trustees" that are not beholden to management.

3. Any insider trading should be disclosed *prior to* trade, not post trade.

4. Large publicly traded companies should be forced to disclose their tax financials.

I still have a real fear of too much government intervention, so I would prefer the following compromise: The government should tighten the legal liability of corporations and individuals that do not follow these recommendations, and offer a "safe harbor" to corporations and individuals that do follow their recommendations. This would put the appropriate pressure on firms to follow them, without absolutely requiring it.

Better than forcible regulation is carrot-and-stick.

ANECDOTE: The Corporate Governance Consulting Industry
A recent phenomenon has been the emergence of Corporate Governance Consultants. Georgeson publishes on www.georgeson.com/pdf/02wrapup.pdf an interesting year end wrap up of shareholder proposals and proxy contests. Some of these consultants also publish ratings of how well publicly traded companies are governed. Unfortunately, they often suffer tremendous conflicts of interest themselves: they sell "advice services" to companies, and when a company retains them, they are likely to be ranked better in the future.

SIDE NOTE: Weil, Gotshal & Manges is the premier law firm dealing with matters of corporate governance. The final section describing Sarbanes-Oxley was largely based on their documents, authored by Holly Gregory, and available at the class websites with their generous permission. Newer versions can be obtained from www.weil.com/weil/corpgov_frames.html.

25·6. SUMMARY

The chapter covered the following major points:

- Control rights are important components of any security.
- Managers can—and often do—"steal" resources from corporate owners.
- Mechanisms have evolved to reduce or reign in managerial theft—such as corporate takeovers, large shareholders, corporate boards, legal environments, ethics, debt.
- There are a number of possible mechanisms to improve corporate governance in the United States.

Special thanks to Florencio Lopez-De-Silanes, Paul Macavoy, Ira Millstein and Holly Gregory (both of Weil, Gotshal&Manges) for the ideas in, and their help with, this chapter.

Part V

Appendices

APPENDIX A

EPILOGUE

Afterthoughts and Opinions. **Preliminary**

last file change: Jun 10, 2004 (17:25h)

last major edit: n/a

Much in this chapter is highly personal. That is, it is more opinion than fact. It covers

1. Thoughts on Business and Finance Education.
2. Finance as a Discipline.
3. Finance Research.

References to and suggestions of other sources of information (such as books, magazines, journals, and websites), are now on my website, http://welch.som.yale.edu/book/.

ANECDOTE: Yogi Berra's Theory and Practice

In theory, there is no difference between theory and practice. In practice, there is.

— Yogi Berra

A·1. Thoughts on Business and Finance Education

By nature, most disciplines in business schools, but especially finance, are closely related to practice. It is not an overstatement to claim that the majority of ideas in finance were either invented or developed in Academia, before they crossed over into practice. Unfortunately, over the years, fundamental misunderstandings have developed, which have become the source of much frustration among both faculty and students. Let me try to correct some of them.

A·1.A. Common Student Misconceptions

A good number of students seem to believe that business schools exist primarily to increase salaries and to enhance job opportunities. As a result, such students expect a "vocational education." It is no wonder that they are especially fond of some adjunct professors and lecturers, who can share plenty of war stories, "vouch" for the importance of their own teaching in their business environment, and may even help some students to get a job at their own or their friends' businesses.

This is a sad and limited view of what business schools have to offer, and it will necessarily cause your finance education to be a rather unrewarding experience. Vocational training is not what top business schools are good at. The top business schools are without exception *not* vocational training centers, but research centers. The University of New Haven teaches job-specific skills; Yale University in New Haven does not!

Business schools are, or at least should be, a profoundly intellectual experience, allowing students to take a fresh look at the world, to explore other business areas for the first time, to learn how to think in economic and business terms, to consider the intellectual foundations of business, and to learn about the most novel ideas—those that have not yet permeated practice. Chances are that the practice in any given company is based on knowledge that the previous generation of managers learned in business schools *10-20 years ago*.

So, the value of an M.B.A. graduate—even to the first employer—is not his/her immediate business knowledge. It makes no sense for our M.B.A. students to learn how the fixed-income department at Goldman-Sachs works *this year*, which is well known by anyone working there (including the secretaries), and which will surely be best explained by the Goldman-Sachs traders to any new hire upon arrival. Instead, the value of our M.B.A. students to an employer is the intellectual ability; knowledge of the fundamentals, basic theories and their application; cutting edge ideas; human skills; team skills; sales skills, etc. Some of these skills are native, but most can be taught or at least improved upon by studying. In the end, it is an individual's versatility and curiosity, an ability to generalize, and an ability to bring an aerial perspective to a specific problem that will allow the newly minted M.B.A. to be of value for many years to come.

Naturally, many students feel a great deal of anxiety about job prospects, and therefore they tend to prefer skills that they believe will facilitate immediate placement upon graduation. Trust me: pretending to have been taught business practice in business schools is not what employers want. Employers first and foremost want to hire smart, curious, and enthusiastic students, who are solid on the basic concepts and who can apply them to new situations. They can teach their own practices better than business schools can.

ANECDOTE: The Time Warp
Do you really want to just learn what the CFO knows *today*? In October 2003, City&Guilds (U.K.) released their study of 405 random financial directors. One in seven needs help even switching his or her computer on and off. One in five struggle to save a document. More than one in five need assistance printing. And a quarter cannot understand spreadsheets. (Source: The Register.)

A·1.B. Common Faculty Misconceptions

A good number of faculty are as mistaken as many students. They seem to believe that ideas in Academia are too difficult to communicate to M.B.A. students in an exciting and interesting fashion. They deemphasize current academic research in their classes. They rarely talk about what it is that drew themselves to business schools rather than to practice: the excitement of new knowledge and research, and the opportunity to convey ideas to students and the world at large. If academic research is not universally incorporated into the curriculum and identified as such, then it is not surprising that students find little value in it. In fact, if the research ideas are so obscure that they cannot be explained to and appeal to M.B.A. students, they probably are of little interest to begin with.

So, here is my personal appeal to faculty in core courses: in addition to integrating current research throughout the curriculum, please reserve your final teaching session of class to talk about academic research in finance in general terms—and the academic research in *your* own department, specifically. My own experience tells me that students will find this to be the single most popular session of the entire course.

A·1.C. Business School vs. Practice

Table A.1. Advantages and Disadvantages of Business Schools over Business Practice

Some Examples of	
What Business School Teaches Better Than Practice	What Practice Teaches Better Than Business School
General, universal knowledge	Job specific knowledge
Concepts of business	The specific business
General tools (statistics, data, economics, etc.)	Specific tools (e.g., a particular accounting system)
Marketing methods	Our product or service marketing
Method of thinking	Method of company's practice
Concepts of ideas for the next 20 years	Implementation of ideas from the last 10 years
Knowledge for a lifetime	Knowledge for this year
Leadership principles and theories	Learning how to lead a particular Set of people
Source of conflict	Conflict resolution with a specific person
Learning by study	Learning by doing
Reflection	Action
Selling principles	Selling our product or service
Negotiation principles	Negotiating with specific customers
Forests	trees

This is *not* to say that practice is any less interesting than Academia. It *is* to say that practice is best taught by practice (the employer) than by business schools. As an M.B.A. student, be patient: the fixed income department at Goldman-Sachs will explain in its own training program the specialized fixed income and institutional knowledge that it will require. The fixed income department does not seek individuals who already know what Goldman will teach in its first week. Instead, the fixed income department seeks smart, flexible, and open-minded individuals, with a solid understanding of fundamentals—of forests, not of trees.

A good curriculum relies on topics that business schools *both can* teach well, and can teach *better* than practice. One or the other is not enough—or it would be a waste of students' time and money. Take the first for example: There is ample research that has shown that taller people are more successful. But height is not something that business schools can contribute much to, so we should not teach it. Take the second: I wish I knew how to teach you how to "sell" anything—products, services, ideas. In my opinion, the ability to sell to other people—to get them excited—may be the single most important skill and key for success in life. Now, some people are naturally adept at selling, others can learn it, and still others will never be good at it. Unfortunately, although selling ability is undoubtedly enormously important, this does not mean that business schools can and should teach it. It may be better learned by following the company's best salesperson. (I will let you know when I figure this one out!)

ANECDOTE: Success in Business: Grow up!
Timothy Judge, a University of Florida management professor, finds that controlling for gender, weight, and age, each inch in height seems to add about $789 a year in salary. In his study, greater height boosted subjective ratings of work performance, including supervisors' evaluations of how effective someone was on the job. It also raised objective measures of performance, such as sales volume. The relationship between height and earnings was particularly strong in sales and management, but was also present in less social occupations such as engineering, accounting and computer programming.
(Source: Yahoo.

Fortunately, there are many important subjects that business schools can teach well: finance, accounting, marketing, economics, statistical thinking, ways of thinking and analyzing problems. These are important factors for success. But, do not expect to learn *everything* you need for success either only in practice or only in school! If you do, you will be disappointed.

A·1.D. The Rankings

In 1988, *Business Week* (BW) began to publish a bi-annual ranking of business schools. This business school rankings issue has become one of BW's top sellers. Unfortunately, despite the visibility of this issue, the quality of the rankings is only mediocre. Worse, the influence of the rankings on business education has been both enormous and negative.

The BW rankings are based primarily on "customer satisfaction" surveys of students and recruiters. Consequently, the BW ratings end up mostly as a popularity contest, and are not based on criteria that measure the quality of education. For example, take another prominent survey: students at California State University at Chico were #1 in Playboy's Party School Rankings. They would probably rate their satisfaction very highly—but this does not make CalState Chico a good school. The same issue applies to recruiters sampled by BW. Most recruiters are themselves alums of *one* of the schools they are asked to rank. Most business school alums have never studied at any school beyond their own, which naturally makes them relatively ill-equipped to make comparisons. (They also see themselves reflected in the students from their alma mater.) Because larger schools have more alums that are sampled, the size of the pool of alums ends up being the primary predictor of "recruiter opinion" in the BW survey. The result is inevitable: the average recruiter ranks his or her own alma mater highest (or at least very highly). Finally, all schools, students, and alums are now catering to and manipulating the BW rankings. Students and alums know that if they do not rank their own school highly, the values of their degrees will go down. And in almost every school, some faculty member will explain this to those students who have not yet understood this basic fact. In sum, popularity ratings are not a great measure of educational quality.

But the most important error of the BW survey is that it treats education as if it were a consumption good, for which it rates vendors. Instead, education is something that is coproduced by the school *and the student*. Almost anyone with an above-average IQ can get a degree in a business school today, but its usefulness is largely determined by the depth of engagement of the student. A student who coasts will gain little, no matter how good the school is.

This is not to say that there are no quality differences between schools. There are big differences, but the BW rankings do not fairly reflect them. My advice to any student is to consider many rankings, and to realize that rankings are only approximations. For example, Harvard Business School (HBS) should probably be ranked as the #1 business

school for a general M.B.A. education today, although a ranking somewhere between #1 and #5 would be more appropriate. But HBS is not #1 in every field. Its finance education, though very good, is not the world's best. In contrast, HBS' strategy education—where its world-renowned case method works well—is undoubtedly #1. Yale, my own school, may not boast a top 3 M.B.A. program, but it offers the #1 ranked education for not-for-profit management today. And so on. Finally, quality differences among similarly ranked schools are often modest: most schools teach similar curricula. The material in our book should appeal to students of any school. My personal guess is that the educational quality difference between the #1 school and the #10 school is very small (as it would be between #10 and #30, or between #30 and #100). The variation in what an individuals student gets out of a program within one individual school probably swamps the average quality variations across schools.

Fortunately, although deciding on the right school is a tough problem, there are many good choices to pick from. It is especially encouraging to me that many schools that never show up in any of the rankings are offering excellent business educations. Again, by selection of classes and instructors, a student can easily get a worse business education at, say, Harvard Business School, than at, say, Notre Dame, even though Harvard clearly outranks Notre Dame.

SIDE NOTE: In my opinion, there are no good distance-learning universities in existence today. (This may change in the future.) The most prominent, the so-called University of Phoenix, is a great business for its owners, but not for its students. Its degrees are not recognized by others and it is not accredited by the AACSB. (This is not an absolute necessity for an established top-10 school, but it is necessary for an upstart school.)

A·2. FINANCE: AS A DISCIPLINE

A·2.A. Art or Science?

I have stated several times throughout the book that finance is as much an art as it is a science. All three parts of finance—valuation, investments, and financing—have simple conceptual underpinnings, but their applications in real life are difficult. And for all three of them, there is no alternative: finding the proper value, the proper portfolio, the proper capital structure may be tough, but this is what it is all about. The difficulty of these questions is good news for practitioners and academics alike: it means that computers will not replace them for a long time to come.

What to do for now? Given that all methods have their errors, the best advice is to use common sense, to employ a number of different techniques to come up with a whole range of possible answers, and then to make a judgment at the end of the day as to what appears most reasonable in light of different models and estimates.

ANECDOTE: The History of Triangulation
The process of honing in on a best estimate, given many noisy estimates, is sometimes called "triangulation"—a reference to pinpointing one's geographic location in relation to multiple landmarks. Each pinpointing is not perfectly accurate, but hopefully, the intersection of many triangulated estimates provides the best overall estimate.

Strictly speaking, triangulation is not even the correct word. Triangulation is merely the use of triangles for surveying, not the use of multiple methods. Triangulation works by measuring the angle to a distant landmark from two different local points. This permits estimating the distances between the three corners.

A remarkable accomplishment of triangulation occurred in the 2nd Century B.C.E.. Eratosthenes managed to estimate the earth's size by comparing the angles of the midsummer sun at two locations! He observed that in Syene, Egypt, the sun shone vertically into a well. In Alexandria, 800km north, the sun shone at an inclination of 7.2 degrees. He computed that if the earth were a perfect sphere, it would have a circumference of 250,000 stadia (40,000km). The correct number is 40,077km!

Eratosthenes' estimate is all the more remarkable because his inputs were not perfect: the earth is not a perfect sphere, the distance between Alexandria and Syene is 729km, not 800km; the two cities are not on the same meridian, Syene is not exactly on the Tropic of Cancer, and the angular difference is not 7° 12', but $7^o 5'$. Some errors increased the estimate, some errors decreased the estimate—but all in all, the ultimate estimate ended up not so bad.

Averaging away errors is also why the idea of using multiple valuation estimates (and methods) to come up with a reasonable average estimate makes sense. The average estimate tends to have less error than any individual estimates.

A·2.B. Will We Ever Fully Understand Finance?

No! It is the nature of the beast. Most of finance is a social science. When there are no arbitrage conditions to constrain permissible behavior and prices, behavior and prices can and will change. On occasion, this leads some to conclude that economics is less worthy of study or even a lesser science than, say, mathematics or physics. This is a mistake. The questions are different. Economics is not interested in the big bang, and physics is not interested in the behavior of C.F.O.'s. The study of one is not more or less worthy than the study of the other.

Finance and physics even share many similar philosophical issues: Some questions permit more precise answers than others. Some systems (like the weather and stock prices) are chaotic and difficult to predict, while others (like Newtonian mechanics and option prices) are more exact. It may even surprise the reader that I am comfortable stating that economics and finance ask many questions to which the answers are more difficult and complex than those often pondered in mathematics and physics. For example, economic agents can react to economic forecasts: imagine how much more difficult it would be for atmospheric physicists to predict the weather if the weather read the weather forecast, and changed its behavior after reading the weather forecast!

Unfortunately, we are now encountering a new hindrance to progress in finance. Many of the most interesting questions cannot be answered, because the data are difficult to come by. Financial institutions have come to consider their data to be their proprietary competitive advantage. Fear of legal liability is further limiting the data that becomes available for public study—and given the litigiousness of U.S. society, justly so. Some questions in finance may have become as unanswerable as the questions of whether multiple universes exist.

This is all good news and bad news. The bad news is that we will never fully understand financial markets and individuals. The good news is that our knowledge will continue

to improve, and that there is plenty of space for new and exciting research in finance. For me, this means finance is still intellectually challenging enough to remain "fun."

A·3. Finance Research

Finance research is not just for aspiring academics: consulting firms are basically research firms. Academics and consultants may have different audiences, production speeds, team systems, and evaluation processes, but they both research issues of interest to business and use similar methods. There is also much cross-fertilization: many professors work regularly with major consulting firms—and some have even quit Academia altogether and departed for higher paying jobs in consulting.

A·3.A. Accomplishments of Finance

Rather than taking up space here, let me just refer you to a paper I wrote called The Top Achievements, Challenges, and Failures of Finance, available for free download at the book's website or the Social Science Research Network (www.ssrn.com).

A·3.B. Interesting Current Academic Research

Fortunately, finance is by nature a very applied discipline. If you have read this book, you already understand the main questions and problems in finance and finance research today. You do not need a higher finance degree. Unfortunately, academic finance journals (and many academics) love obscure jargon and algebra. It may or may not require some extra training in "language" for you to follow the writeups of academic papers in academic journals. But, in the end, with just a little bit of extra jargon, you should be able to pick up the important journals and understand the most cutting-edge and interesting research ideas in finance today.

A·3.C. Getting Involved in Academic Research

My own recommendation to an aspiring students of finance is first to learn what the top professors (and especially the younger professors) in your own school are working on. Then, I would browse SSRN for current working papers. Finally, I would suggest you work for a professor in your finance or economics department, even if it is unpaid—though I would pick a professor who does not have too many assistants already. You will learn more in this one-on-one contact than you will learn from taking many classes.

A·3.D. Finance Degrees

The most common finance degree in many of the top schools is the M.B.A. with a specialization in finance. But increasingly many universities, such as UC/Berkeley, Princeton, and Wharton, are offering undergraduate degrees in finance. The Harvard economics department may well feature the best finance department in the world, and it really only teaches undergraduates and Ph.D. students. Similarly, Yale is now beginning to expand its curriculum. Finance definitely qualifies as a subject with no less intellectual rigor than economics, and no more of a specialization/vocational education component than, say, pre-med or biochemistry.

There are also some other programs that offer masters programs in finance, e.g., the N.Y.U. program in mathematical finance, offered by the Courant Institute. Typically, these programs have a bent towards financial engineering. Their graduates tend to come from specific backgrounds (usually some other engineering discipline), and their graduates tend to work in specific types of jobs (typically in derivatives and fixed income modeling). Finally, there is the Ph.D. track, discussed next.

A·3.E. Academic Careers in Finance and Economics: A Ph.D.?

Finance is a subfield of economics. About one-third of its professors have an economics Ph.D. instead of a finance Ph.D. Either degree is sufficient—although it is imperative for the future academic to have solid grounding in both disciplines.

The typical Ph.D. program in finance takes between 4 and 8 years. Unlike most degree programs, success is not guaranteed. About one-third of accepted students drop out, typically after 2 to 4 years—not a cheap outcome. Although qualifying exams, usually taken in the first two years of the program, are very challenging, the biggest hurdle for almost every Ph.D. student to overcome is the transition from classroom work to academic research. This is a Gordian knot, and success is difficult to predict. Although intelligence and smarts are necessary, it is not mathematical sophistication that determines success. Very little of finance uses more than plain algebra—although it does use lots of it. Instead, the successful Ph.D. student must develop a problem-relevant intuition and creativity. If I only knew how to translate this skill into a recipé!

Although the first 4 years in Ph.D. programs are usually paid for by full stipends by the university, the opportunity costs and the uncertainty of ultimate success mean that only the most intellectually interested students will find a Ph.D. program to be a rewarding endeavor. For the successful graduate, job opportunities tend to be plenty and lucrative. Even academic careers are not exactly a vow of poverty. In 2003, the typical first year Assistant Professor in a top business school earned somewhere between $130,000 and $180,000 per year. Industry jobs in financial or consulting institutions sometimes pay more even in the first year, but their big advantage is that salaries tend to escalate far more rapidly than those in Academia in subsequent years. Finally, many economics and finance Ph.D.s pursue governmental careers, e.g., at the **International Monetary Fund** or the **World Bank**. (I sometimes wonder whether this is why these institutions have developed such bad reputations over time.)

I find it very encouraging that many universities and institutions today conduct terrific academic research in finance and economics. Thirty years ago, only a handful of schools were able to produce great papers, but this time of exclusivity has passed. Nevertheless, there are differences in average academic quality. I will volunteer here my personal impression of the rank order of academic finance departments today, which is based on the tendency of departments to successfully attract faculty from other departments. In my opinion, the top academic department today is the University of Chicago. It is followed closely by "Cambridge," which is really the combination of Harvard (economics and finance) and M.I.T. (economics and finance). A large number of schools vie for the ranking spots right after. Among them, but not exclusively, are (in alphabetical order) Columbia, Duke, N.Y.U., Northwestern, Stanford, U.C./Berkeley, U.C.L.A., Wharton, and Yale. These schools each have their unique advantages and disadvantages, and regularly succeed in stealing faculty from one another.[1] There are also a large number of excellent schools, many of which have individual faculty who are every bit as good as some faculty

[1] I almost surely have omitted some schools by mistake.

at, say, Chicago, but which typically do not have the same overall average academic quality or resources.

The average quality of a finance or economics department can make an important difference for Ph.D. students, however. They benefit greatly from the variety of interaction. Therefore, a Ph.D. from any top academic institution would make an excellent springboard into a top-notch academic economics or finance department, or into a very high-quality investment or consulting career.

A·3.F. Top Finance Journals

The top academic journals in finance today are *The Journal of Finance*, the *Journal of Financial Economics*, the *Journal of Financial and Quantitative Analysis*, the *Review of Financial Studies*, and the *Journal of Business*. However, there are also many other good outlets for academic research. For example, economics journals have published some of the most influential work in finance. Other journals are written with more of a practitioner audience in mind, such as *Financial Analysts Journal*.

Although numbers do not tell the whole story (it is impact that counts!), the tenure standards for professors range from about 8–10 papers in the top journals for a Chicago professor, to 5–7 papers for a school ranking at around #10, to 3–5 for a school ranking at around #30. The top journals have rejection rates of about 90%. A successful academic will write about 2–3 papers per year, but publish only one of them in a top journal.

A·4. Bon Voyage

Our book has covered the principles of finance in some depth and breadth. You can trust me when I say that if you have read and understood these chapters, you are very well prepared for the next steps in your finance/business education. (You can choose your next courses á la carte: investments, derivatives, corporate finance, fixed income, financial institutions, international finance, or something else. If you are still curious to learn more, visit the book's web site at http://welch.som.yale.edu/book/.)

I have enjoyed writing this book in the same way that I enjoy writing my academic research papers, and pretty much for the same reason: it has been like solving an intriguing puzzle that no one else has figured out quite the same—a particular way to see and explain finance. Of course, writing it has taken me far longer than I had anticipated—four years and still counting just for the first edition.

But it will all have been worth it if you have learned from my presentation. If you have studied the book, you should now know about 90% of what I know about finance. You might also find it interesting that only after I had figured out how to explain a number of topics to you did I really understand them clearly myself. And this brings me to a key point that I want to leave you with—never be afraid to ask questions, even about first principles. To do so is not a sign of stupidity—on the contrary, it is often a sign of deepening awareness and understanding.

I have no illusions that you will remember all the fine details as time passes—nor will I. But more than the details, I hope that I will have left you with an appreciation for the big ideas, an arsenal of tools, a method to approach novel problems, and a new perspective—you can now think like a financier.

Ivo Welch

APPENDIX B

More Resources

NPV Checklist, Some Used Data Links, Algebra, Statistics!

last file change: Jun 14, 2004 (14:04h)

last major edit: n/a

2·1. An NPV Checklist

Here is an abbreviated list of issues to worry about when using NPV.

The NPV formula is easy. For most projects, its application is hard. It is usually very difficult to estimate future cash flows (and even their appropriate interest rates), especially for far-in-the-future returns. It is usually more important and more difficult to avoid errors for the expected cash flow (the NPV numerator) than it is for the cost of capital (the NPV denominator). The NPV formula is less robust to cash flow errors than it is to cost of capital (r) errors, and it is "easier" to commit dramatic errors in the cash flow estimation than in the cost of capital estimation.

Here is an abbreviated checklist of items to consider when working out NPV estimates.

- ✓ **Real After-Tax Dollars** (Page 69, Page 142, Page 142):
  - ✓ Have all relevant inputs and outputs been quoted in what-is-relevant-to-you after-tax dollars? This applies to both expected cash flows and to appropriate discount rates.
  - ✓ Has inflation been properly included? Preferably, have all computations used nominal expected future cash flows and nominal costs of capital, with inflation used only to gross up nominal cash flows appropriately?
- ✓ **Interactions** (Page 146, Page 419):
  - ✓ Have all projects been properly credited with their contributions, positive or negative, to the values of other projects (externalities)?
  - ✓ Have all projects been judged "on the margin," i.e., without charging them for unalterable or previously made choices, such as sunk costs, overhead, etc.?
  - ✓ Has the cost of capital applicable to each project component, respectively, been used, and not the (incorrect) overall average cost of capital? (Note: some errors and simplifications here are unavoidable in the real world, because it is impossible to put a different cost of capital on each paper clip.)
- ✓ **Conditionals (Strategic Options)** (Page 153, and in the Web Chapter on Options and Derivatives):
  - ✓ Have all possible future options been considered (using scenario analyses) in order to find the correct *expected* cash flows, e.g.:
    - ✓ The ability to leverage a product into future markets?
    - ✓ The ability to find product spinoffs?
    - ✓ The ability to learn about (how to do) future products?
    - ✓ The ability to stop the project if conditions are bad.
    - ✓ The ability to delay the project if conditions are bad.
    - ✓ The ability to mothball the project if conditions are bad and restart the project if conditions improve.
    - ✓ The ability to accelerate the project if conditions are good.
    - ✓ The ability to expand the project if conditions are good.

✓ **Accuracy** (Page 136, Page 154, Page 156, Page 180, Page 413):

- ✓ How accurate are the estimated project cash flows?
- ✓ If project success and project cash flows were estimated by someone else, what are the motives of the estimator? Does the estimator want the project taken or rejected?
- ✓ Can the cash flow estimates be improved by doing more research?
- ✓ Is it possible to get another independent evaluation/audit of the project estimates?
- ✓ Given unavoidable simplifications, assumptions, and errors, how sensitive/robust is the NPV computation to changes?

✓ **Correct Inputs** (Page 152):

- ✓ Are the cash flows *expected*, rather than just *promised*? Are the interest rates *expected*, rather than *promised*? (Recall: expected interest rates are below promised interest rates due to default premia, not just due to risk premia.)
- ✓ Are the expected cash flows the "average outcome" (correct!), and not the "most likely outcome"?
- ✓ Do the expected cash flow estimates include the correct weighted probabilities of low-probability events, especially for negative outcomes?
- ✓ If money needs to be borrowed to execute the project, is the used cost of capital r the borrowing rate? If capital is already available, is the used cost of capital r the lending (investments) rate?

✓ **Corporate Income Taxes** (Page 520):

- ✓ For use of WACC and APV, is the numerator in the NPV calculation the expected cash flows "as if all equity financed"? (This means that the company bears the full brunt of its corporate income tax load.)
- – In the weighted cost of capital, is the debt cost of capital the *expected* (not the promised!) interest rate on debt? Is the numerator the *expected* cash flow, not the *promised* cash flow?

A final warning: although many of these issues seem obvious, in complex real-world situations, they are much harder to spot and take care of than they were in our highlighted expositions. Watch out! The most common error is worth its own box:

IMPORTANT: *The most common NPV method is to estimate cash flows for the numerator, and to use an expected rate of return (cost of capital) from the CAPM formula (see Chapter 17).*

- ✓ *The default risk is handled only in the numerator, i.e., in the computation of expected cash flows.*
- ✓ *The time-premium and risk-premium are handled only in the denominator. The CAPM formula provides an expected rate of return, which contains only these two components.*
- ✓ *Do not try to adjust the numerator for the time premium or the risk premium. Do not try to add a default-premium to the rate of return in the denominator. (This would yield a promised, not an expected rate of return on capital.) Do not believe that by using the CAPM expected rate of return, you have taken care of the default risk.*

Q B.1 *Recall as many items from the NPV checklist as you can remember. Which are you most likely to forget?*

2·2. Prominently Used Data Websites

The following data and information websites have been prominently used in this book. (If you are reading this on the Acrobat reader , you can click on the links!) Please note that the list is not complete, and that the links may have changed by the time you read this.

	Overall Market Information	
Marketgauge.com	Various market gauges (incl. S&P500 dividend and earnings yields).	http://tal.marketgauge.com/dvmgpro/gauges/dvplast.htm
Yahoo!*Finance*	Stock and Index Quotes, Current and Historical	http://quote.yahoo.com
Yahoo!*Finance*	Current Interest Rates	http://bonds.yahoo.com/rates.html
Federal Reserve Data	Historical Interest Rates	http://www.federalreserve.gov/releases/h15/data.htm
Fred (Federal Reserve)	U.S. economic time series, macroeconomic and financial.	http://research.stlouisfed.org/fred
SmartMoney	Animated Yield Curve	http://www.smartmoney.com/onebond/index.cfm?story=yieldcurve
Treasury Direct	Inflation Protected Interest Rates	http://www.publicdebt.treas.gov/gsr/gsrlist.htm
Bloomberg	Index rates (incl. muni bonds)	http://www.bloomberg.com/markets/rates/index.html
R. Shiller's Website	Very long-run indexes	http://aida.econ.yale.edu/~shiller/
Yahoo!*Finance*	Foreign financial market websites	e.g., Germany: http://de.finance.yahoo.com/
	Individual Stock and Fund Information	
Yahoo!*Biz*	Firm-specific corporate profiles (earnings, sales, etc.). Here, IBM.	http://biz.yahoo.com/p/i/ibm.html
Edgar	All public corporate S.E.C. filings	http://www.edgar.sec.gov
PWC	Price-Waterhouse-Coopers' Edgarscan	http://edgarscan.pwcglobal.com/EdgarScan/
Vanguard	Funds Information	http://www.vanguard.com
PepsiCo	Investor Information	http://www.pepsico.com/investors/
PepsiCo	Annual Reports	http://www.pepsico.com/investors/annual-reports/
PepsiCo	2000 10-K Filing	http://www.pepsico.com/filings/200010k.shtml
	Other Information	
Ivo Welch	General Website	http://welch.som.yale.edu/
SSRN	Finance Working Papers	http://www.ssrn.com
AFA	The American Finance Association	http://www.afajof.org
Moody's	Monthly bond default reports.	http://riskcalc.moodysrms.com/us/research/mdr.asp
Moody's	Extended report on default rates, 1992.	http://riskcalc.moodysrms.com/us/research/defrate/0085.pdf
	Governance Web Chapter	
CalPers	Corporate Governance Focus	http://www.calpers-governance.org/alert/focus/
	International Web Chapter	
Barchart	Currencies	http://www2.barchart.com/mktcom.asp?section=currencies
Bloomberg	Market Indices	http://www.bloomberg.com/markets/rates/index.html
The Economist	The "Big-Mac" Price Index	http://www.economist.com/markets/Bigmac/Index.cfm

2·3. Necessary Algebraic Background

- Finding a base:

$$\begin{aligned} 3^2 &= 9 \quad \Leftrightarrow \quad 3 = 9^{1/2} \\ x^a &= b \quad \Leftrightarrow \quad x = b^{1/a} \ . \end{aligned} \tag{B.1}$$

- Finding an exponent:

$$\begin{aligned} 3^2 &= 9 \quad \Leftrightarrow \quad 2 = \frac{\ln(9)}{\ln(3)} \\ a^x &= b \quad \Leftrightarrow \quad x = \frac{\ln(b)}{\ln(a)} \ . \end{aligned} \tag{B.2}$$

- Summation Notation:

$$\sum_{i=1}^{N} f(i) = f(1) + f(2) + \cdots + f(N) \ . \tag{B.3}$$

This should be read as the "sum over all i from 1 to N." There are N terms in this sum. i is not a real variable: it is simply a dummy counter to abbreviate the notation. When 1 and N are omitted, it usually means "over all possible i."

- Summation Rules:

$$\begin{aligned} \sum_{i=1}^{N} [a \cdot f(i) + b] &= [a \cdot f(1) + b] + [a \cdot f(2) + b] + \cdots + [a \cdot f(N) + b] \\ &= a \cdot \left[\sum_{i=1}^{N} f(i) \right] + N \cdot b \ . \end{aligned} \tag{B.4}$$

Illustration:

$$\sum_{i=1}^{3} \left[5 \cdot i^i + 2 \right] = [5 \cdot 1^1 + 2] + [5 \cdot 2^2 + 2] + [5 \cdot 3^3 + 2] = 7 + 22 + 137 = 166 \ . \tag{B.5}$$

Nerd Note: *A function $l(\cdot)$ is called a linear function, if and only if $l(a+b\cdot x) = a+\cdot l(b\cdot x) = a+b\cdot l(x)$, where a and b are constants. Summation is indeed a linear function (or "operator"). Further, the function $f(y) = c+d\cdot y$ is linear. So are expected values. This is what has permitted us to interchange expectations and linear functions:*

$$\mathcal{E}(a + b \cdot \tilde{X}) = a + b \cdot \mathcal{E}(\tilde{X}) \ . \tag{B.6}$$

Solve Now!

Q B.2 *If* $(1 + x)^{10} = (1 + 50\%)$, *what is* x*?*

Q B.3 *If* $(1 + 10\%)^x = (1 + 50\%)$, *what is* x*?*

Q B.4 *Write out and compute* $\sum_{x=1}^{3}(3+5\cdot x)$*. Is* x *a variable or just a placeholder to write the expression more conveniently?*

Q B.5 *Write out and compute* $\left(\sum_{y=1}^{3} 3\right)+5\cdot\left(\sum_{x=1}^{3} y\right)$*. Compare the result to the previous expression.*

Q B.6 *Is* $\sum_{i=1}^{3}(i\cdot i)$ *the same as* $\left(\sum_{i=1}^{3}\right)\cdot\left(\sum_{i=1}^{3} i\right)$*?*

2·4. Laws of Probability, Portfolios, and Expectations

2·4.A. Single Random Variables

The **Laws of Expectations** for single random variables (illustration will follow):

- Definition of Expectation

$$\mathcal{E}(\tilde{X}) \;=\; \sum_{i=1}^{N} \mathcal{P}rob(i)\cdot[\tilde{X}=X(i)] \tag{B.7}$$

- The expected value of a linear transformation (a and b are known constants):

$$\mathcal{E}(a\cdot\tilde{X}+b) \;=\; a\cdot\mathcal{E}(\tilde{X})+b \;\;. \tag{B.8}$$

Side Note: This only works of the special linear form. For example,

$$\mathcal{E}(\tilde{X}^2) \;\neq\; \cdot\mathcal{E}(\tilde{X})^2 \;\;. \tag{B.9}$$

To see this, consider a fair coin that can be either 0 or 1. $\mathcal{E}(\tilde{X}^2)=0.5\cdot 0^2+0.5\cdot 1^2=0.5$, but $\mathcal{E}(\tilde{X})^2=(0.5\cdot 0+0.5\cdot 1)^2=0.25$.

- Definition of Variance:

$$\mathcal{V}ar(\tilde{X}) \;=\; \mathcal{E}\left([\tilde{X}-\mathcal{E}(\tilde{X})]^2\right) \;\;. \tag{B.10}$$

- Definition of a Standard Deviation:

$$\text{Standard Deviation}(\tilde{X}) = \sqrt{\mathcal{V}ar(\tilde{X})} \;\;. \tag{B.11}$$

- The variance of a linear transformation (a and b are known constants):

$$\mathcal{V}ar(a \cdot \tilde{X} + b) \;=\; a^2 \cdot \mathcal{V}ar(\tilde{X}) \;. \tag{B.12}$$

Illustration: A coin, outcome called $\tilde{X}$, with 4 and 8 written on the two sides. The outcomes can be written as $4 \cdot i$ where i is either 1 or 2. So, the expected value of $\tilde{X}$ is

$$\begin{aligned} \mathcal{E}(\tilde{X}) &= \sum_{i=1}^{2} \mathcal{P}rob[\tilde{X} = (4 \cdot i)] \cdot (4 \cdot i) \\ &= \mathcal{P}rob(\tilde{X} = 4) \cdot (4) \;+\; \mathcal{P}rob(\tilde{X} = 8) \cdot (8) \\ &= 50\% \cdot 4 \;+\; 50\% \cdot 8 \qquad = \; 6 \;. \end{aligned} \tag{B.13}$$

$$\begin{aligned} \mathcal{V}ar(\tilde{X}) &= \sum_{i=1}^{2} \mathcal{P}rob[\tilde{X} = (4 \cdot i)] \cdot [(4 \cdot i) - 6]^2 \\ &= \mathcal{P}rob(\tilde{X} = 4) \cdot (4-6)^2 \;+\; \mathcal{P}rob(\tilde{X} = 8) \cdot (8-6)^2 \\ &= 50\% \cdot 4 \;+\; 50\% \cdot 4 \qquad = \; 4 \;. \end{aligned} \tag{B.14}$$

The standard deviation is the square root of the variance, here 2.

Side Note:

$$\begin{aligned} \mathcal{E}(\tilde{X}^2) &= \sum_{i=1}^{2} \mathcal{P}rob[\tilde{X} = (2 \cdot i)] \cdot (2 \cdot i)^2 \\ &= \mathcal{P}rob(\tilde{X} = 2) \cdot (2^2) \;+\; \mathcal{P}rob(\tilde{X} = 4) \cdot (4^2) \\ &= 50\% \cdot 4 \;+\; 50\% \cdot 16 = 10 \;. \end{aligned} \tag{B.15}$$

This is not the same as $\mathcal{E}(\tilde{X})^2$, which is $3^2 = 9$.

Now define a linear transformation of the X, say $\tilde{Z} = \$2.5 \cdot \tilde{X} + \$10$. Thus,

$\mathcal{P}rob$	Coin	$\tilde{X}$	$\tilde{Z}$
$1/2$	Heads	4	\$20
$1/2$	Tail	8	\$30

We want to show that the expected value of $\tilde{Z}$, defined as is $\$2.5 \cdot \tilde{X} + \$10$, is $\$2.5 \cdot \mathcal{E}(\tilde{X}) + \$10 = \$25$. So, we hand-compute the expected value the long way from $\tilde{Z}$,

$$\begin{aligned} \mathcal{E}(\tilde{Z}) &= \sum_{i=1}^{2} \mathcal{P}rob[\tilde{X} = (4 \cdot i) \text{ same as } \tilde{Z} = \$2.5 \cdot X + \$10] \cdot (Z_i) \\ &= \mathcal{P}rob(\tilde{X} = 4 \text{ same as } \tilde{Z} = \$20) \cdot (\$20) \\ &\quad + \mathcal{P}rob(\tilde{X} = 8 \text{ same as } \tilde{Z} = \$30) \cdot (\$30) \\ &= 50\% \cdot \$20 \;+\; 50\% \cdot \$30 \;=\; \$25 \;. \end{aligned} \tag{B.16}$$

Unlike the mean (the expected value), the variance is *not* a linear function, so the variance of $\tilde{Z} = \$2.5 \cdot \tilde{X} + \$10$ is *not* $\$2.5 \cdot \mathcal{V}ar(\tilde{X}) + \$10 = \$2.5 \cdot 4 + \$10 = \$20$. Instead,

$\mathcal{V}ar(\tilde{Z}) = \mathcal{V}ar(a \cdot \tilde{X} + c) = a^2 \cdot \mathcal{V}ar(\tilde{X}) = (\$2.5)^2 \cdot \mathcal{V}ar(\tilde{X}) = \$^2 \cdot 2.5^2 \cdot 4 = \$^2 25$. We can confirm this working with $\tilde{Z}$ directly:

$$\begin{aligned} \mathcal{V}ar(\tilde{Z}) &= \sum_{i=1}^{2} \mathcal{P}rob[\tilde{X} = (4 \cdot i)] \cdot [(\tilde{Z}_i) - \mathcal{E}(\tilde{Z})]^2 \\ &= \mathcal{P}rob(\tilde{X} = 4 \text{ same as } \tilde{Z} = \$20) \cdot (\$20 - \$25)^2 \\ &\quad + \mathcal{P}rob(\tilde{X} = 8 \text{ same as } \tilde{Z} = \$30) \cdot (\$30 - \$25)^2 \\ &= 50\% \cdot (\$5)^2 + 50\% \cdot (\$5)^2 = \$^2 25 \quad . \end{aligned} \tag{B.17}$$

The standard deviation of $\tilde{Z}$ is therefore $\sqrt{\$^2 \cdot 25} = \$5$.

You should confirm Formula B.8: the expected value of $\tilde{Z}$ should be ($5) times the expected value of $\tilde{X}$ plus $10. Confirm Formula B.12: the variance of $\tilde{Z}$ should be the variance of $\tilde{X}$ multiplied by ($5) squared.

Solve Now!

Q B.7 *What is the expected value and standard deviation of a bet B that pays off the number of points on a fair die, squared? For example, if the die comes down 3, you receive $9.*

Q B.8 *Assume that you have to pay $30, but you receive twice the outcome of the previous bet $\tilde{B}$. This is a new bet, called $\tilde{C}$. That is, your payoff is $\tilde{C} = -\$30 + 2 \cdot \tilde{B}$. What is the expected payoff and risk of your position? Make your life easy!*

2·4.B. Portfolios

Portfolios are defined as follows (illustration will follow):

$$\tilde{r}_p \equiv \sum_i w_i \cdot \tilde{r}_i \quad , \tag{B.18}$$

where w_i is the known investment weights in security i and $\tilde{r}_i$ is the security return on security i. Unlike the above, simpler definitions, portfolios are the weighted sum of multiple random variables.

- Portfolio Expectations

$$\mathcal{E}\left(\sum_i w_i \cdot \tilde{r}_i\right) = \sum_i w_i \cdot \mathcal{E}(\tilde{r}_i) \quad . \tag{B.19}$$

  Although the weights are fixed and known constants, they cannot be pulled out of the summation, because they are indexed by i (each could be different from the others).

-

$$\begin{aligned} \mathcal{V}ar\left(\sum_i w_i \cdot \tilde{r}_i\right) &= \sum_{i=1}^{N} \left\{ \sum_{j=1}^{N} \left[w_i \cdot w_j \cdot Cov(\tilde{r}_i, \tilde{r}_j) \right] \right\} \\ &= \sum_{i=1}^{N} \sum_{j=1}^{N} \left[w_i \cdot w_j \cdot Cov(\tilde{r}_i, \tilde{r}_j) \right] \end{aligned} \tag{B.20}$$

Illustration: A coin toss outcome is a random variable, $\tilde{T}_i$, and it will return either \$2 (head) or \$4 (tail). You have to pay \$2 to receive this outcome. This looks like a great bet: The mean rate of return on each coin toss is 50%, the variance on *each* coin toss is

$$\mathcal{V}ar(\tilde{r}_1) \;=\; {}^1/_2 \cdot (0\% - 50\%)^2 + {}^1/_2 \cdot (100\% - 50\%)^2 \;=\; 0.50 \;. \tag{B.21}$$

Therefore, the standard deviation of each coin toss is 0.707.

Now, bet on two independent such coin toss outcomes. You have \$10 invested on the first bet and \$20 on the second bet. In other words, your overall actual and unknown rates of return R are

$$\begin{aligned} r &= \sum_{i=1}^{2} w_i \cdot r_i \;, \\ \tilde{r} &= \sum_{i=1}^{2} w_i \cdot \tilde{r}_i \;. \end{aligned} \tag{B.22}$$

(The second equation is in random variable terms.) Now, your investment portfolio consists of the following investments

$$\begin{aligned} w_1 &= \frac{\$10}{\$30} = 0.33 \;, \\ w_2 &= \frac{\$20}{\$30} = 0.67 \;. \end{aligned} \tag{B.23}$$

We can now use the formulas to compute your expected rate of return ($\mathcal{E}(\tilde{r})$) and risk ($Sdv(\tilde{r})$). To compute your expected rate of return, use

$$\begin{aligned} \mathcal{E}(\tilde{r}) = \sum_{i=1}^{2} w_i \cdot \mathcal{E}(\tilde{r}_i) &= w_1 \cdot \mathcal{E}(\tilde{r}_1) + w_2 \cdot \mathcal{E}(\tilde{r}_2) \\ &= {}^1/_3 \cdot (50\%) + {}^2/_3 \cdot (50\%) \;=\; 50\% \;. \end{aligned} \tag{B.24}$$

To compute your variance, use

$$\begin{aligned} \mathcal{V}ar(\tilde{r}) &= \sum_{i=1}^{2}\sum_{j=1}^{2} w_i \cdot w_j \cdot Cov(\tilde{r}_i, \tilde{r}_j) \\ &= w_1 \cdot w_1 \cdot Cov(\tilde{r}_1, \tilde{r}_1) + w_1 \cdot w_2 \cdot Cov(\tilde{r}_1, \tilde{r}_2) \\ &= w_2 \cdot w_1 \cdot Cov(\tilde{r}_2, \tilde{r}_1) + w_2 \cdot w_2 \cdot Cov(\tilde{r}_2, \tilde{r}_2) \\ &= w_1^2 \cdot Cov(\tilde{r}_1, \tilde{r}_1) + 2 \cdot w_1 \cdot w_2 \cdot Cov(\tilde{r}_1, \tilde{r}_2) \\ &\quad + w_2^2 \cdot Cov(\tilde{r}_2, \tilde{r}_2) \\ &= w_1^2 \cdot \mathcal{V}ar(\tilde{r}_1) + 2 \cdot w_1 \cdot w_2 \cdot Cov(\tilde{r}_1, \tilde{r}_2) \\ &\quad + w_2^2 \cdot \mathcal{V}ar(\tilde{r}_2) \\ &= ({}^1/_3)^2 \cdot \mathcal{V}ar(\tilde{r}_1) + 2 \cdot w_1 \cdot w_2 \cdot 0 + ({}^2/_3)^2 \cdot \mathcal{V}ar(\tilde{r}_2) \\ &= ({}^1/_9) \cdot \mathcal{V}ar(\tilde{r}_1) + ({}^4/_9) \cdot \mathcal{V}ar(\tilde{r}_2) \\ &= ({}^1/_9) \cdot 0.5 + ({}^4/_9) \cdot 0.5 \;=\; 0.278 \;. \end{aligned} \tag{B.25}$$

The standard deviation is therefore $\sqrt{0.278} = 52.7\%$. This is lower than the 70.7% that a single coin toss would provide you with.

Solve Now!

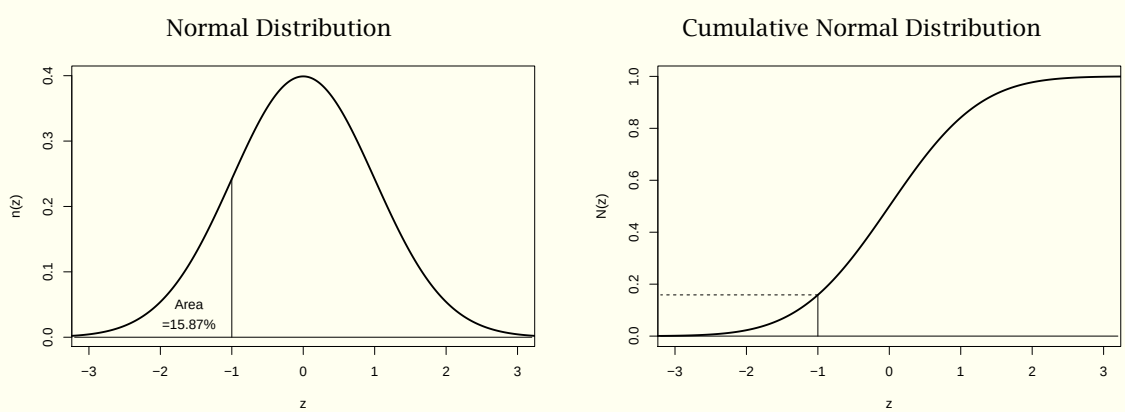
Q B.9 *Repeat the example, but assume that you invest $15 into each coin toss, rather than $10 and $20 respectively. Would you expect the risk to be higher or lower? (Hint: What happens if you choose a portfolio that invests more and more into just one of the two bets.)*

2·5. Cumulative Normal Distribution Table

Table B.1. Cumulative Normal Distribution Table

z	$\mathcal{N}(z)$	z	$\mathcal{N}(z)$	z	$\mathcal{N}(z)$	z	$\mathcal{N}(z)$	z	$\mathcal{N}(z)$	z	$\mathcal{N}(z)$
-4.0	0.00003										
-3.5	0.00023										
-3.0	0.0013	-2.0	0.0228	-1.0	0.1587	0.0	0.5000	1.0	0.8413	2.0	0.9772
-2.9	0.0019	-1.9	0.0287	-0.9	0.1841	0.1	0.5398	1.1	0.8643	2.1	0.9821
-2.8	0.0026	-1.8	0.0359	-0.8	0.2119	0.2	0.5793	1.2	0.8849	2.2	0.9861
-2.7	0.0035	-1.7	0.0446	-0.7	0.2420	0.3	0.6179	1.3	0.9032	2.3	0.9893
-2.6	0.0047	-1.6	0.0548	-0.6	0.2743	0.4	0.6554	1.4	0.9192	2.4	0.9918
-2.5	0.0062	-1.5	0.0668	-0.5	0.3085	0.5	0.6915	1.5	0.9332	2.5	0.9938
-2.4	0.0082	-1.4	0.0808	-0.4	0.3446	0.6	0.7257	1.6	0.9452	2.6	0.9953
-2.3	0.0107	-1.3	0.0968	-0.3	0.3821	0.7	0.7580	1.7	0.9554	2.7	0.9965
-2.2	0.0139	-1.2	0.1151	-0.2	0.4207	0.8	0.7881	1.8	0.9641	2.8	0.9974
-2.1	0.0179	-1.1	0.1357	-0.1	0.4602	0.9	0.8159	1.9	0.9713	2.9	0.9981
										3.5	0.99977
										4.0	0.99997

Normal Score (z) vs. standardized Normal Cumulative Distribution Probability $\mathcal{N}(z)$ Table: This table allows determining the probability that an outcome X will be less than a prespecified value x, when standardized into the score z. For example, if the mean is 15 and the standard deviation is 5, an outcome of $X = 10$ is one standard deviation below the mean. This standardized score can be obtained by computing $z(x) = [x - \mathcal{E}(x)]/Sdv(x) = (x - 15)/5 = (10 - 15)/5 = (-1)$. This table then indicates that the probability that the outcome of $\tilde{X}$ (drawn from this distribution with mean 15 and standard deviation 5) will be less than 10 (or less than its score of $z = -1$) is 15.87%.



These two figures show what the table represents. The left-side is the classical bell curve. Recall that at $z = -1$, the table gave $N(z = -1) = 15.87\%$. This 15.87% is the area under the left curve up to an including $z = -1$. The right figure just plots the values in the table itself, i.e., the area under the graph to the left of each value from the left-side figure.

We used the cumulative normal distribution in Subsection d. It is also used in the famous Black-Scholes formula, explained in the Web Chapter on Options and Derivatives.

A. Q&A: Answers

1. See text for list. Your personal propensity to forget is probably unique to yourself.

2. $x \approx 4.138\%$. Check: $(1 + 4.138\%)^{10} \approx 1.5$.
3. $x \approx 4.254$. Check: $1.1^{4.254} \approx 1.5$.
4. The expression is

$$\sum_{x=1}^{3} (3 + 5 \cdot x) \;=\; (3 + 5 \cdot 1) + (3 + 5 \cdot 2) + (3 + 5 \cdot 3) \;=\; 8 + 13 + 18 \;=\; 39 \;. \tag{B.26}$$

x is not an unknown. It is simply a counter dummy used for writing convenience. It is not a part of the expression itself.

5. The expression is

$$\left(\sum_{y=1}^{3} 3\right) + 5 \cdot \left(\sum_{y=1}^{3} y\right) \;=\; (3 + 3 + 3) + 5 \cdot (1 + 2 + 3) \;=\; 39 \;. \tag{B.27}$$

The result is the same. This is an example why $\sum_i a + b \cdot x = \left(\sum_i a\right) + b \cdot \sum_i x$.

6.

$$\begin{aligned} \sum_{i=1}^{3} (i \cdot i) &= 1 + 4 + 9 = 14 \;. \\ \left(\sum_{i=1}^{3} i\right) \cdot \left(\sum_{i=1}^{3} i\right) &= (1 + 2 + 3) \cdot (1 + 2 + 3) = 36 \;. \end{aligned} \tag{B.28}$$

The two are not the same. So, be careful not to try to pull out multiplying i's! You can only pull out constants, not counters. Incidentally, is also why $\mathcal{E}(\tilde{X}^2) \neq \mathcal{E}(\tilde{X})^2$, as stated in the next section.

7. The expected value is

$$\begin{aligned} \mathcal{E}(\tilde{B}) &= (^1/_6) \cdot \$1 + (^1/_6) \cdot \$4 + (^1/_6) \cdot \$9 + (^1/_6) \cdot \$16 + (^1/_6) \cdot \$25 + (^1/_6) \cdot \$36 \\ &= \$15.^{17} \end{aligned} \tag{B.29}$$

The variance is

$$\begin{aligned} \mathcal{V}ar(\tilde{B}) &= (^1/_6) \cdot (\$1 - \$15.^{17})^2 + (^1/_6)(\$4 - \$15.^{17})^2 + (^1/_6) \cdot (\$9 - \$15.^{17})^2 \\ &+ (^1/_6) \cdot (\$16 - \$15.^{17})^2 + (^1/_6) \cdot (\$25 - \$15.^{17})^2 + (^1/_6) \cdot (\$36 - \$15.^{17})^2 \\ &= \$^2 149.14 \end{aligned} \tag{B.30}$$

The standard deviation is therefore

$$\mathcal{S}dv(\tilde{B}) \;=\; \sqrt{\mathcal{V}ar(\tilde{B})} \;=\; \sqrt{149.14} \;=\; 12.21 \tag{B.31}$$

8. You expect to receive

$$\begin{aligned}
\mathcal{E}(\tilde{C}) &= -\$30 + 2 \cdot \mathcal{E}(\tilde{B}) = -\$30 + 2 \cdot \$15.^{17} &= \$0.^{34} \quad , \\
\mathcal{V}ar(\tilde{C}) &= 2^2 \cdot \mathcal{E}(\tilde{B}) = 4 \cdot \$149.^{14} &= \$595.^{56} \quad , \\
\mathcal{S}dv(\tilde{C}) &= \sqrt{\mathcal{V}ar(\tilde{B})} &= \$24.^{42} \quad .
\end{aligned} \tag{B.32}$$

9. Your investment weights are now $w_1 = w_2 = 0.5$.) The mean rate of return remains the same 50%. The variance of the rate of return is computed similarly to the example in the text,

$$\mathcal{V}ar(\tilde{r}) = ({}^1/_4) \cdot 0.5 + ({}^1/_4) \cdot 0.5 = 0.25 \quad . \tag{B.33}$$

Therefore, the risk (standard deviation) is 50%. This is lower than where you put more weight on one of the coin tosses. This makes sense: as you put more and more into one of the two coin tosses, you lose the benefit of diversification.!

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

APPENDIX C

(Sample Exams)

Applied Torture!

last file change: Jun 14, 2004 (14:04h)

last major edit: n/a

Addison-Wesley-Pearson will eventually release extensive instructor and student test and study materials.

T: ***Be aware that these questions are not solid, and the answers are even less solid. This section is currently under construction.***

3·1. An Exam for Part I (Basics)

Q C.1 *Roughly, what is the highest marginal federal income tax rate in the U.S. in 2003?*

Q C.2 *Is the highest marginal income tax rate higher or lower than the highest average income tax rate?*

Q C.3 *What makes a "perfect market"?*

Q C.4 *A die has numbers 4, 10, -2, 3, 1, and 1 written on it. Compute the expected value if you throw the die once. Is it the most likely outcome? Is this number a random variable?*

Q C.5 *At an interest rate of 10% per annum, how long does it take you to double your investment?*

Q C.6 *The annualized 5-year interest rate is 6%. What is the 5-year holding rate of return?*

Q C.7 *A project that costs $100 will provide a safe $400 in 30 years. What is its annualized rate of return?*

Q C.8 *(This question requires coverage of a nerd-box in Section 8·4.A.) What is the duration of a 0% zero coupon bond: year 1=$0, year 2=$0, year 3=$100.*

Q C.9 *(This question requires coverage of a nerd-box in Section 8·4.A.) What is the duration of a 15% coupon bond with payments: year 1=$15, year 2=$15, year 3=$115.*

Q C.10 *Can a yield curve slope down? Yes or no.*

Q C.11 *One minute after you purchase a 30-year bond at an interest rate of 6.4%, the bond yield has gone up by 10 basis points. What is your holding rate of return?*

Q C.12 *The bank posts an interest rate of 10% per year. Is this an interest rate or an interest quote? If it is the former, what is the interest quote? If it is the latter, what is the interest rate?*

Q C.13 *(This question requires coverage of a nerd-topic, forward rates, from Section 3·3.) The yield curve (per annum rates) is 5% over 1 year, 10% over 2 years, 20% over 3 years.*

1. *Compute the three holding rates of return.*
2. *Compute the two forward rates.*
3. *Solve this last (difficult): can you lock in borrowing of $100 at the year 1 to year 2 forward rate?*
4. *You have a project with cash flows −$25 now, +$33 next year, and +$50 the following year. What is the NPV?*

Q C.14 *At a zero-tax rate of return of 1% per month, what is the payoff on a $50 investment in 20 years?*

Q C.15 *(Continued) At an inflation rate of 30 basis points per month, what is this payoff in 20 years worth in today's purchasing power?*

Q C.16 *(Continued) At a tax rate of 35%, what is your after-tax payoff in today's purchasing power?*

3·2. An Exam for Part II (Valuation and Capital Budgeting)

Q C.17 *The annualized interest rates are 5% for 1 year, 6% for 2 years, and 7% for 3 years. What is the NPV of a project that costs $120 now, and returns $70 each in years 2 and 3?*

Q C.18 *What can happen if borrowing rates and lending rates are different, which is precluded by the assumption of perfect markets?*

Q C.19 *A stable project has stopped growing in real terms, but not in nominal terms. It has yielded $100 today, and the owner now wants to sell it. The inflation rate is 2% per annum, the discount rate is 8% per annum. What is (an upper bound for) the value of this project?*

Q C.20 *What is the monthly payment on a fixed 30-year 12% home mortgage for $100,000?*

Q C.21 *A firm has bank debt worth $50 billion and equity worth $20 billion. The overall firm can be worth either $120 billion or $40 billion next year, with equal probability. The world is risk-neutral. What interest rate does the firm have to promise the bank?*

Q C.22 *How do long-term and short-term projects differ in terms of robustness to errors?*

Q C.23 *The project's expected cash flow next year is $100. The real inflation-adjusted interest rate is 4%. What is the present value of the project?*

Q C.24 *Describe 3 biases in capital budgeting.*

Q C.25 *Who usually provides cash flow estimates to headquarters? Can you trust them? Why?*

Q C.26 *Is a higher NPV project always better?*

Q C.27 *Give a numerical example of a project that has no IRR, and a numerical example of a project that has multiple IRRs.*

Q C.28 *What is required of a project to be guaranteed a single and unique IRR?*

Q C.29 *Can a positive NPV project have a negative IRR?*

Q C.30 *Is a higher IRR project always better?*

Q C.31 *Consider the following project:*

Project	
Real Life	6 Years
Cost	$60
Raw Output	$80 per year for 6 years
– Input Costs (cash)	$5/year
– Selling Costs (cash)	$5/year
Overall Cost of Capital	12%/year
Corporate Tax Rate (τ)	40%/year

Taken Financing	
Debt Capacity	$100
Debt Interest Rate	10%/year

Accounting Treatment	
Accounting Life	6 Years

The project starts in year 1. We are standing in year 0. The first depreciation can be taken in the year of the capital expenditure (year 1). The first interest rate appears one year after the loan is taken (i.e., loan in year 1, interest in year 2). In the liquidation stage (year 7), you need no expenses, such as cogs or sg&a, or loan. Assume customers pay (the gross sales) one year after delivery. Assume taxes are paid immediately when they are incurred (i.e., the same year as the income that generates it).

1. *Construct the income statement, and the relevant items of the cash flow statement.*
2. *Assume that debt obligations must be discounted at 10%. The project and tax cost of capital is 12%. Treat the above as projections and value the firm, as of period 0.*

Q C.32 *A project has the following:*

	Year 1	Year 2	Year 3	Beyond
Sales = Income	$200	$300	$500	$0
A/R	$100	$200	$50	$0

What are the cash flows?

Q C.33 *From memory, write down the basic financial statements components (starting with earnings before interest and taxes) that go into the cash flow formula.*

Q C.34 *Why is the cash flow that we use for the project NPV not equal to the* CASH FLOW FROM OPERATONS *plus* CASH FLOW FROM INVESTING*? What is missing and why?*

Q C.35 *This question works with two firms, X and Y. The interest rate is 10%.*

1. *What is the P/E ratio of firm X that grows at a rate of 5%? (Assume perpetual life.) What is the PVGO relative to the firm's value?*
2. *What is the P/E ratio of firm Y that shrinks at a rate of 5%? (Assume perpetual life.) What is the PVGO relative to the firm's value?*
3. *Assume that the market values of two firms, X and Y, are the same today. The two merge. What is the P/E ratio of the merged firm?*
4. *Would the new firm XY's earnings fit the assumptions of our growing perpetuity formula? What will be XY's very long-run growth rate?*

Q C.36 *Compare two equal underlying firms (projects). One, however, is levered. Which one has the higher P/E ratio. Explain.*

3·3. An Exam for Part III

T: ***no solutions here.***

Funds A and B had the following rates of return:

Year	A	B
2000	-4%	+6%
2001	+4%	+19%
2002	0%	-2%
2003	+13%	+3%

Q C.37 *What would be your estimates of the expected rates of return for A and B in 2004?*

Q C.38 *Using the "divide by $N-1$" version of the variance, what is your expected standard deviation?*

Q C.39 *What is the historical covariance between A and B?*

Q C.40 *What would you expect the risk of a portfolio of 25% in A and 75% in B to be? Can you compute it "both ways"? (How often do you rebalance?)*

Q C.41 *What was the historical mean and standard deviation of the S&P500? You can choose the period.*

Q C.42 *How accurate do you believe your expected returns would be? Are historical statistics representative of the future? Which ones are more, which ones are less trustworthy?*

Q C.43 *If you believed the historical numbers to be representative, estimate the one-week risk (standard deviation) of a portfolio of 50% in A and 50% in B.*

Q C.44 *What is the lowest risk combination of funds A and B?*

Q C.45 *Draw the efficient frontier.*

Q C.46 *Introduce a risk-free bond that offers 1% per year. Now draw the efficient frontier.*

Q C.47 *What portfolio would you choose that has an expected rate of return of 20%?*

Q C.48 *Assume that Fund A is worth $100 billion, Fund B is worth $200 billion. Is this a* CAPM *world? Use both the* MVE *frontier and the security markets line to prove your answer.*

Q C.49 *Write down the* CAPM *equation. What are reasonable inputs? Where can they be obtained?*

Q C.50 *A firm has $200 million in debt, and a stock market capitalization of $500. Yahoo quotes its stock market-beta as 0.9. Our own firm is private, but otherwise very similar to this publicly traded firm. What would you expect our firm beta to be? Would this beta be a good measure of our risk exposure?*

Q C.51 *Give an example of how a firm can destroy its own shareholder value if it uses its own cost of capital to value another firm in a takeover.*

Q C.52 *You are receiving a promise to get $1,000 if the market goes up by 20% or more, but nothing if the market goes up by less. Assume that you can model the market as returning either 25% or 5% with equal probability. The risk-free rate is 3%, the equity premium is 5%. Value your gift.*

Q C.53 *Is the empirical evidence in favor of the* CAPM*? Discuss.*

Q C.54 *What is the difference between arbitrage and a great bet?*

Q C.55 *Concisely, what is an efficient stock market? Can a stock market be efficient without following the* CAPM*?*

Q C.56 *What is an eventstudy?*

3·4. An Exam for Part IV (Financing)

T: ***no solutions here.***

Q C.57 *Draw the payoff diagram for a stock and a bond promising to pay off $100 next year.*

Q C.58 *What is the difference between a convertible bond and a callable bond?*

Q C.59 *What is the Modigliani-Miller insight?*

Q C.60 *In a Modigliani-Miller world, does an increase in dividends produce shareholder value?*

Q C.61 *A long WACC, APV, Financials question is needed.*

Q C.62 *How does an increase in (deadweight) bankruptcy costs due to excessive leverage show up in the* WACC *formula?*

Q C.63 *How does a change in the dividend tax rate show up in the* WACC *formula?*

Q C.64 *On tax grounds, how should clientele and sorting effects work?*

Q C.65 *Name three underlying factors pulling firms towards higher debt ratios. (Note: do not name secondary consequences, such as changes in earnings-per-share or price-earnings ratios.)*

Q C.66 *Name three factors pulling firms towards lower debt ratios.*

Q C.67 *How can managers expropriate creditors?*

T: ***To be written***

A. Q&A: Answers

1. Page 15: About 35%. (30% to 40% are correct answers.)
2. Page 15: Higher.
3. Page 23: Perfect Competition. Zero Frictions (taxes, x-costs). No information differences.
4. Page 26, Section 1·8: 2.83. Obviously this is not the most likely, but an impossible outcome. 2.83 is not a random variable; it is a known number. (The outcome of the die throw is a random variable.)
5. Page 40: $(1+10\%)^x = (1+100\%) \to x = \log(2)/\log(1.1) = 7.27$ years. Check: $(1+10\%)^{7.27} \approx 2$. (Logarithms are reviewed in Appendix 2·3. You could also solve this question through trial and error, but it would be more painful.)
6. Page 42, Section 2·3: $1 + r_{0,5} = (1 + r_{\overline{5}})^5 = (1+6\%)^5 = 1 + 33.82\%$.
7. Page 42, Section 2·3: $r_{0,30} = \$400/\$100 - 1 = 300\%$. Thus, $(1+3) = (1 + r_{0,30}) = (1 + r_{\overline{30}})^{30}$, which solves to $r_{\overline{30}} = \sqrt[30]{1+3} = 4.73\%$.
8. Page 166: 3
9. Page 166, Section 8·4.A: $\dfrac{\$15 + \$15 \cdot 2 + \$115 \cdot 3}{\$145} = 2.69$. This is the simplest duration formula. The Macauley duration requires knowledge of the prevailing interest rates, which we do not have.
10. Page 52: Yes.
11. Page 60, Section 3·4: $P_0 = \$100/(1.064^{30}) = \$15.^{55}$. $P_1 = \$100/(1.067^{30}) = \$14.^{29}$. $r_{0,1} = -8.1\%$.
12. Page 44, Section 2·4: A quote. The interest rate is $10\%/365 = 0.0274\%$ per day. The interest rate is $(1 + .0274\%)^{365} \approx 10.52\%$.
13. Page 53:
    (a) 5%. 21%. 72.8%.
    (b) 1 to 2: 1.21/1.05 -1 = 15.2%.
    2 to 3: 1.728/1.21 -1 = 42.81%.
    (c) $-\$25 + \$33/1.05 + \$50/1.10^2 = \$47.^{75}$
14. Page 40: $\$50 \cdot 1.01^{12\cdot 20} = \$50 \cdot 10.89 = \$544.^{63}$.
15. Page 69: $\$544.63/1.003^{240} = \$265.^{38}$.
16. Page 71: Restart: $\$50 \cdot 1.0065^{12\cdot 20} = \$236.^{74}$. This is $\$115.^{36}$.
17. Page 98, Section 5·2:
$$-\$120 + \frac{\$70}{1+5\%} + \frac{\$70}{(1+6\%)^2} = +\$8.^{97} \quad .$$
18. Page 109: You decide on projects based on your wealth, rather than based on the project alone.
19. Page 114, Section 5·4.C: $\$102/0.06 = \$1,700$.
20. Page 116, Section 5·4.E: Solve the annuity: $\$100,000 = \dfrac{C}{12\%} \cdot \left(1 - \dfrac{1}{(1+12\%)^{30}}\right)$. The solution is $C = \$12,414.^{40}$.
21. Page 124, Section 6·1: Firm worth today: \$70 billion.
    Expected Firm worth next year: \$80 billion.
    Expected rate of return: 14.28%. In a risk-neutral world, all investments offer the same expected rate of return, so
$$50\% \cdot P + 50\% \cdot \$40 = \$50 \cdot (1 + 14.28\%) \qquad \Rightarrow \qquad P = \$74.^{28}.$$
    Thus, the promised interest rate r is $\$74.^{28}/\$50 = 48.56\%$.

22. Page 136, Section 6·3: Long-term projects are sensitive to both expected cash flows and interest rates, while short-term projects are sensitive primarily to expected cash flow problems. So, the different is that for long-term projects, interest rates matter, too.
23. Page 142, Section 7·1.A: This question cannot be answered. You cannot use a real interest rate on a nominal cash flow.
24. Page 154 (Section 7·3.C) and Page 156 (Section 7·3.D): For example, agency bias, overconfidence bias, mental budgets, sunk cost fallacies, etc. See Sections subsect:humanbias and 7·3.D.
25. Page 156, Section 7·3.D: From subsidiary managers. They are very untrustworthy.
26. Page 161: Yes.
27. Page 163, Section 8·2: All positive or all negative cash flows. Multiple IRRs: many positive and many negative cash flows.
28. Page 163: It is sufficient if you have one positive cash flow upfront, and all other negative in the future, or vice-versa. (The sign changes only once.)
29. Page 163, Section 8·2: Yes. For example, cash flows of −$100, +$250, −$100 have either an IRR of -50% or +100%. So, if the interest rate is, say, +50%, the IRR (or at least one IRR) could be negative, while the NPV would be positive. Full credit for an answer that states that this is not possible if there is a unique IRR.
30. Page 164, Subsection 8·2.B: No. A $1 project with a 100% IRR is worse than a $100 project with a 10% IRR.
31. Page 176, Section 9·2.A:

(a)

	Year	1	2	3	4	5	6	7
	gross sales (revenues)	$80	$80	$80	$80	$80	$80	
–	cost of goods sold (cogs)	$5	$5	$5	$5	$5	$0	
–	sg&a	$5	$5	$5	$5	$5	$0	
=	ebitda (net sales)	$70	$70	$70	$70	$70	$70	
–	depreciation	$10	$10	$10	$10	$10	$10	
=	ebit (operating income)	$60	$60	$60	$60	$60	$60	
–	interest expense	$0	$10	$10	$10	$10	$10	
=	eaibt (or EBT)	$60	$50	$50	$50	$50	$50	
–	corporate income tax (at 40%)	$24	$20	$20	$20	$20	$20	
=	net income	$36	$30	$30	$30	$30	$30	

Excerpts From the Cash Flow Statement							
Year	1	2	3	4	5	6	
capital expenditures	–$60	-	-	-	-	-	
NET DEBT ISSUE	+$100	-	-	-	-	–$100	
Changes in Work Cap	–$80	$0	$0	$0	$0	$0	+$80

(b)

Cash Flow	1	2	3	4	5	6	7	DR	NPV
Machine if payment were immediate	+$10	+$70	+$70	+$70	+$70	+$70			
WKC Adj	–$80	$0	$0	$0	$0	$0	+$80	-	
= Machine w/ WKC Adj.	–$70	+$70	+$70	+$70	+$70	+$70	+$80	12%	$198.97
Uncle Sam	$24	$20	$20	$20	$20	$20	–	12%	$85.80
Project	–$94	+$50	+$50	+$50	+$50	+$50	+$80		$113.17
Loan	+$100	–$10	–$10	–$10	–$10	–$110		10%	$0
Levered Ownership	+$6	+$40	+$40	+$40	+$40	–$60	+$80		$113.17

(Give credit regardless of whether the loan is repaid in year 6 or year 7.)

32. Page 184, Section 9·3.A: Cash Flows: \$100. \$300 − \$100 = \$200.\$500 − (−\$150) = \$650.\$0 − (−\$50) = \$50.
33. See Page 188.
34. Page 195: Interest expense is missing. It needs to be added back, because it is a distribution to the owners (in the broad sense: debt plus equity), even though accountants treat it as a cost of operations.
35. Page 228, Section 11·1:
    (a) If E is \$1, then P is \$20. The P/E ratio is therefore 20. The PVGO is \$20-\$1/.1=\$10, or one-half.
    (b) If E is \$1, then P is \$6.67. The P/E ratio is therefore 6.67. The PVGO is \$6.67−\$1/.1 = −\$3.33, or minus one half.
    (c) We need to make them equal sized. So, if Y has earnings of \$3 today, it would also be worth \$20. In total, the two firms would have earnings of \$4. The price would be \$40. The earnings would be \$4. So, the P/E ratio would be 10.
    (d) No! \$4. Then \$3 · 0.95 + \$1 · 1.05 = \$3.90. Then 3.81. Then \$3.72975. Then \$3.65903. The first growth rate is –2.5%. The fourth growth rate is –1.9%. In many periods, after Y will have disappeared, the growth rate will asymptote to 5%.
36. Page 242, Section 11·2.E: The unlevered firm has the higher P/E ratio.
37.
38.
39.
40.
41.
42. Bad. No. Means=awful. Covariances=ok.
43.
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67.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

WEB CHAPTER A

INDEX

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last file change: Jun 11, 2004 (10:03h)

last major edit: n/a

1·1. MAIN INDEX

- Please note that page numbers here can be off by a couple of pages (most hopefully no more than 1 page). This has to do both with infrequent updating of the index by myself, and with LaTeX's way of processing lines and pages. Underline means frequent mention on the same page.
- **Boldface** of a page number (or range) means an important occurrance (or specific definition) of the phrase on the particular page.
- <u>underline</u> of a page number (or range) means multiple occurrances.

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1·2. Internal: Planned Improvements and To-Dos

1·2.A. To Do List

The following list is for myself, but it may help you realize where the book is still flawed.

1. More questions at the end of each section.
2. I need a larger number of student questions, both in the book and in a student self-test bank. This is of first importance.

   I am working on questions. These questions will appear on the website (not in the book), in order to permit adding more questions, corrections, etc. As an instructor, if you want to assign specific questions, you will need to copy them from the website into your own assignments. Apologies: the questions will not be stable for at least another year.
3. I need good instructor notes/slides as templates. Unlike class notes from book publishers, these notes will be freely changeable and without copyright notice for any instructors who wish to adopt the book.
4. Eventually, I want to put together reference lists and further reading lists. It is also almost embarrassing that many of the academic inventors of the concepts in the book are not even mentioned. This may have decluttered the book, but, as an academic, I am not fond of not giving credit.

   Therefore, I also need to build the book website, which will describe the main sources (academic articles), and where to find further information on the web.
5. I need to make a high-level decision whether side notes and nerd notes should appear like footnotes. Right now they break up the text, perhaps too much so. (Would it not be nice if one could print books to notes expand when touched?)
6. Write a perl program to check that guiders have a percent at the end, that every paragraph has a guider, that there are no duplicate words, that checks spelling, that checks sentence lengths, that checks for non-use of plurals in textterms for the index, etc.
7. Specific check list for this book:

   * I need tons and tons of rereading, rechecking, rewriting of paragraphs and sentences, etc.
   * fix second-level enumii counters in queries' answers.
   * All approximate calculations should use ≈ rather than =. This is inconsistent at the moment.
   * Each paragraph with more than 2 sentences should have a guider. Guiders should be consistent in terms of their descriptiveness.
   * Work over each and every guider.
   * I should have guiders on the formulas when I proceed forward to solving something.
   * The number of pages per section varies, perhaps too much. I may need to split some chapters. (The longer chapters now are financials [existing vs. pro-forma] , statistics [univariate vs. bivariate], and optimalpfio [efficient frontier vs. security markets line]).
   * The index at the end of the book is pretty decent, but it can be further improved.
   * I need a more consistent frequency of anecdote boxes and important boxes. Note that chapters lack a final summary, so Importants must substitute for now. Eventually, I may create a final summary for each chapter.
   * Work over the chapter introductions. Maybe add keywords in front of each chapter introduction. Make explanations, headers, etc., consistent.
   * Reread each and every sidenote, nerdnote, and anecdote for relevance.
   * The use of Important boxes is inconsistent. Sometimes it is a question, sometimes a formula, sometimes a list-to-remember. I may also need to close each chapter with a list of important things to take away (though this is less important in my book than in other books, because the material is drastically shorter to begin with; hopefully students can now remember at the end what they read at the beginning).
   * All graphics should be produced by R. Right now, I mix R and gnuplot.
   * Describe the institutional detail on how corporate bonds are quoted in file 'returns.tex' via a nerdbox.
   * Ask David H. and Richard T. on whether there are other good places to mention specific Behavioral Finance findings. Maybe define Behav Fin.

* Explain more intuition on immediate stock market reactions to major corporate events before the eventstudy.
* Add one figure with historical equity premia, and another figure with equity premium by horizon.
* Use percent notation everywhere. Right now, I sometimes use fractions, I sometimes use percent.
* Smooth transition from "holding diversified pfio like the mkt" to "hold the mkt."
* Put growth annuities into a nerd box. (pensions and SS payments linked to COLA may be a good real-world link here.)
* Mention more prominently that the bond credit spread can estimate the default premium. give a historical figure.
* Maybe cover a little more payback and profitability methods.
* Maybe add unequal-lived annuities.
* In npvcertain/npvuncertain, drop in a forward reference to cash flows extraction from financials.tex.
* Comb for return (vs. net return, vs. rate of return).
* (For [mortgage] loans,) lay out APR, effective, stated, annual return, compounded monthly return, true annual return.
* The stats chapter only misses more regression details, and it would almost be a complete stats intro, rather than just the necessary background. Should I add it?
* I need more anecdotes in the Investments part.
* The diversification example may read a bit academic. (As $N \to \infty$, risk goes to zero.) Maybe I should use real securities. The problem is how to illustrate effects of correlations of $-1, 0, +1$ with real data rather than made up data.
* Look at ruback, FM 2002, Capital Cash Flow method.
* Mkt Efficiency: give some more evidence or at least anecdotes on how many investors do not believe RW, and why past performance is no predictor for future performance. look at mark simonson's comments on improving the market efficiency chapter. (also, some good suggestions on comparables restructuring.)
* Improve the last section in comparables.tex (on other financial ratios).
* in the CAPM section: describe more explicitly what an investor should do if one stock offers too much exp return for the CAPM tradeoff. (not arbitrage, but just earn excess returns and choose a better portfolio.) we already have this somewhat.
* add more stuff on dividends, especially empirical evidence. the problem is, we know so little, that there is more mystery than knowledge here.
* Jay R. suggested an anecdote: Buffet, Warren (1999) "Mr. Buffet on the Stock Market," Fortune November 22, 1999, in which Buffet warned about the overvaluations of tech stocks and internet stock, was the most reprinted article in the history of Fortune magazine.

8. Create a rich website. http://welch.som.yale.edu/book/ is the current website for this book. *After* the more standard ancillary materials are produced, the website will contain additional resources, such as links to academic papers, a bibliography, pertinent related websites, cross-references to other textbook chapters covering the same topic, etc. This website will be organized with direct reference to the book's sections and pages. Thus, a reader who wants to find out more about the material covered in Chapter 20 can go to the website's Chapter 20 to locate additional material. The book itself explicitly tries to avoid individual author and web references, if only because the printed version does not have the possibility of updating and continuously adding to this background information.

Web coverage will link directly.

1·2.B. Text Analysis

[perl bookstatistics.pl –quiet]