Dynamic Network Analysis on the Port Authority of Allegheny County Public Bus System

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Abstract

This research presents an empirical dynamic network analysis of the Allegheny County Bus System. Data, recorded and reported from the Port Authority of Allegheny County, of every bus trip in the month of April 2019 are analyzed to garner insights into aspects of the region's bus network. Key entity analysis with network centrality measures identify critical neighborhoods for transit within the county. Node removal impact analysis is used to quantify effects of proposed stop removals on the system's robustness. Louvain grouping method identifies major transit hubs in the county. A dynamic network comparison analysis quantifies drop in network robustness due to reduced service from weekday to weekend schedules. This paper applies dynamic network analysis methodology to the Allegheny County public bus network to propose and answer questions about the performance and connectivity of the bus network.

Keywords:
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1. Introduction

The public transportation network is a vital part of Allegheny County's citizens' economic and social life. Public transportation provides basic needs of accessibility and economic mobility for disadvantaged residents and communities. Thus, it is an important factor in fighting inequality in urban areas. When transit analysts typically evaluate transportation networks, they focus on specific aspects of the transportation system such as comparisons of the service levels in different areas, operational costs and inefficiencies therein, or ridership along different corridors. Additionally, system-wide analyses often only analyze distributions and descriptive statistics of these metrics. These analyses do not provide an all-encompassing understanding of a transportation system's effectiveness nor consequential impact to its riders. However, a dynamic network analysis fills these gaps.

The Port Authority of Allegheny County bus system is one of the few bus networks in the country that is still growing; additionally, it has a ridership rate that is ninety-two percent higher than the national average (Zanghi, 2017). Despite its significance and range of impact, the Allegheny County transportation network has not been subject to a comprehensive network analysis to date.

In this research, we perform a modern-day dynamic network analysis to understand the topology of Allegheny County's public bus network. Applying social network theory to a bus transportation system offers a different perspective in comparison to the traditional transit analysis methods. It also offers many advantages over the traditional transit analysis methods. For one, components of the network are evaluated in the context of the entire network rather than in isolation. We no longer consider individual connections but instead observe how the connections relate to one-another and compose an emergent system. This perspective allows us to evaluate connectivity at specific parts of the network, identify key entities, and holistically evaluate the network beyond just service level judgements.

The remainder of this paper is structured as follows. Section 2 covers background information on social network theory and how it applies to transportation networks. Section 3 defines five key research questions that our study investigates. Section 4 describes the data and pre-processing methods used in the upcoming analyses.

Sections 5-9 sequentially respond to the five defined research questions: (Section 5) How can we characterize the overall network topology? (Section 6) Which are the most highly connected regions in Allegheny County? (Section 7) Are disadvantaged regions being sufficiently served by the public transit system? (Section 8) How will the elimination of certain stop nodes impact the network and passenger travel? (Section 9) How does service compare from weekdays to weekends?

Section 10 discusses this study's limitations and suggested future work. Finally, Section 11 summarizes and concludes the methods and the findings of this study.

2. Background

A network is a description of a social structure at a given time. This description links connections between actors or agents that serve as nodes. The connections are based on resource flow between two agents. A network analysis is the process of analyzing a social network by identifying key actors, groups, measures of connectedness, redundancies, and other metrics. A dynamic network analysis involves evaluating the changes of a network across differing scenarios, such as over time (Altman et al., 2019). These concepts are most commonly applied to social networks.

Transportation networks are distinguishable from social networks because they account for the flows of people. Guillaume and Renato provide a foundation of the relevant measures in a transportation network analysis by comparing airline networks through connectivity measures in Connectivity in Air Transport Networks: An Assessment of Models and Applications (Burghouwt et al., 2013). While this analysis establishes the relevance of connectivity measures in transportation networks, it takes a granular analytical approach focusing on node-level (airport level) comparisons.

A network analysis on worldwide air transportation for the year 2000 into 2001 provides an independent transportation network analysis similar to the desired Allegheny County transportation network analysis. Through analyzing connectedness across regions by calculating degree and betweenness centralities, the paper unexpectedly demonstrates that the most connected regions were not the most centralized regions. Additionally, the study identifies the worldwide air transportation network to be a small-world network. This network's nodeset was airports based out cities around the world (Guimera, et al., 2005). Following this example, we will define the following terms that are relevant to the Allegheny County transportation network analysis.

Density: The ratio of the number of links and the number of total possible links within the network.

Diameter: The maximum most efficient path between two nodes.

Total-degree centrality: The total number of links directly to/from a node.

Eigenvector centrality: A normalized numerical metric for how well-connected each of its neighboring nodes are. Here, neighbors refer to nodes that are distanced from the considered node by one connection.

Betweenness centrality: The fraction of shortest paths in the network that pass through a given node.

Hub centrality: A normalized numerical metric for how many of a node's out-links have many in-links.

Characteristic path length: The average shortest path between two nodes, if the two nodes are somehow connected.

An akin network analysis on the Allegheny County bus network has not been done before. However, there have been a handful of similar analyses done on the worldwide air transportation network as previously discussed.

3. Research Questions

We seek to evaluate Allegheny County's public bus transportation system from a dynamic network perspective. Specifically, we aim to answer the following questions.

1. How can we characterize the overall network topology?
2. Which are the most highly connected regions in Allegheny County?

3. Are disadvantaged regions being sufficiently served by the public transit system?
4. How will the elimination of certain stop nodes impact the network and passenger travel?
5. How does service compare from weekdays to weekends?

4. Data

The data used in this study is the Port Authority of Allegheny County's automatic passenger counting (APC) dataset for the month of April 2019. These data are collected at the stop, route, and trip level every time a rider enters or exits the bus so it is extremely precise. We supplemented this data source with geographic data from the West Pennsylvania Regional Data Center to link stops to Allegheny County neighborhoods and county municipalities. Finally, we used Python to aggregate the networks at the region level and to convert the dataset into network data format. We used ORA to perform the network analysis, a social network analysis software tool developed at Carnegie Mellon University.

The data is formatted to include two nodesets, or ontology classes: stops and neighborhoods. The union set of stops has 7,032 nodes and the union set of neighborhoods has 183 nodes.

There are two link types: buses and loads. Buses give how many total buses travel on routes from one node to another per day. Loads represent how many people travel on routes from one node to another per day. While buses represent system capacity, loads represent actual utilization. The two can be compared to gain insights on network performance.

We created three meta-networks: one general bus network, one bus network that only includes information extracted from weekdays, and one bus network that only includes information extracted from weekends. We created the latter two networks because we realize that bus service changes from weekdays to weekends; the two individual networks enable us to compare the bus service from weekdays to weekends. Below, Table 1 summarizes the meta-network's nodeset statistics.

Table 1

Meta-network Nodeset Statistics.

Meta-Network	Number Stops	Number Neighborhoods
Bus Network	7032	183
Weekday	7032	183
Weekend	5101	183

For each meta-network, we created three networks: Hood x Hood, Stop x Stop, and Stop x Hood. Their densities are presented in Table 2 below. To better understand these networks, Table 3 and Table 4 provide additional network centrality statistics.

Table 2

Network densities.

Network	Density
Hood x Hood	0.030
Stop x Stop	2.720×10^{-4}
Stop x Hood	0.005

Table 3

Hood x Hood network centrality statistics

Centrality Measure	Min	Max	Mean	Median	STDV
Degree	0.002	0.003	0.00266667	0.003	0.00057735
Betweenness	0.028	0.029	0.03316667	0.028	0.00057735
Eigenvector	0.03	0.043	0.038	0.041	0.007

Table 4

Stop x Stop network centrality statistics.

Centrality Measure	Min	Max	Mean	Median	STDV
Degree	8.13E-06	1.73E-05	1.19E-05	1.04E-05	4.79393E-06
Betweenness	0.00E+00	1.00E-02	5.00E-03	5.00E-03	0.005
Eigenvector	0.00E+00	8.38E-04	4.19E-04	4.19E-04	0.000592697

Because the Stop x Stop network is so incredibly sparse, all centrality measures are very low. In both cases, weighted measures may be better to use instead of unweighted. For the Hood x Hood network, Eigenvector Centrality has the biggest spread and therefore might be one of the better measures to distinguish between our measures.

5. Topology

We will characterize this network's topologies. We note that this transportation system is a closed network, as opposed to an open network. This implies that the network can be designed for optimal usage. Since this network is a transportation system, it is useful to know that the results of a network analysis can influence the system's design for specific desired system performance. Additionally, this network is characterized by routinization and standardized processes since buses are scheduled to run along predefined routes.

Finally, we note that the system is a decentralized hybrid network since it includes star, bus, mesh, etc. sub-networks throughout the full system. Thus, we realize that each part of the system has unequal transportation options and services. This becomes transparent as we visualize the Stops x Stops and the Hoods x Hoods networks.

The Stops x Stops network, shown below in Fig. 1, shows a birds-eye view of this network overlaid on Allegheny County's geography. This figure helps visualize the network at this high-level. It also visually emphasizes that the network is geographically unequal as there are clearly regions with a greater density or number of stops. Additionally, the nodes are colored by betweenness centrality. The nodes of higher betweenness are colored red, those with lower betweenness scores are colored blue. Fig. 1 shows an uneven distribution of these colors throughout Allegheny County - while there are many red nodes in Downtown Pittsburgh, there are few anywhere else in the region.

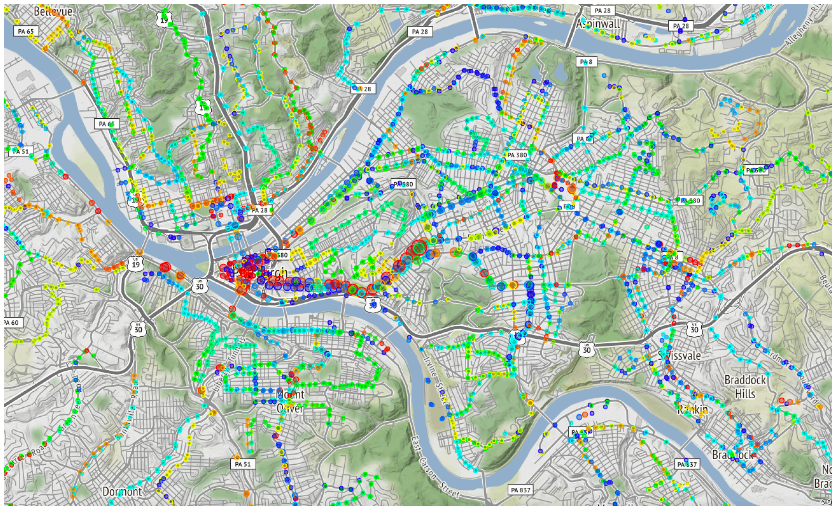


Fig. 1. Stops x Stops Network (colored by betweenness centrality, sized by degree centrality)

The Hoods x Hoods network, shown below in Fig. 2, shows a birds-eye view of this network overlaid on Allegheny County's geography. The nodes in this visual are sized by eigenvector centrality. This visual depicts the scarcity of defined neighborhoods external to the city of Pittsburgh. The nodes in these sparse regions are smaller than those in the city of Pittsburgh showing that these nodes in the outer regions are less connected than those larger nodes which reside in the city of Pittsburgh. This further displays the geographically unequal nature of the system.

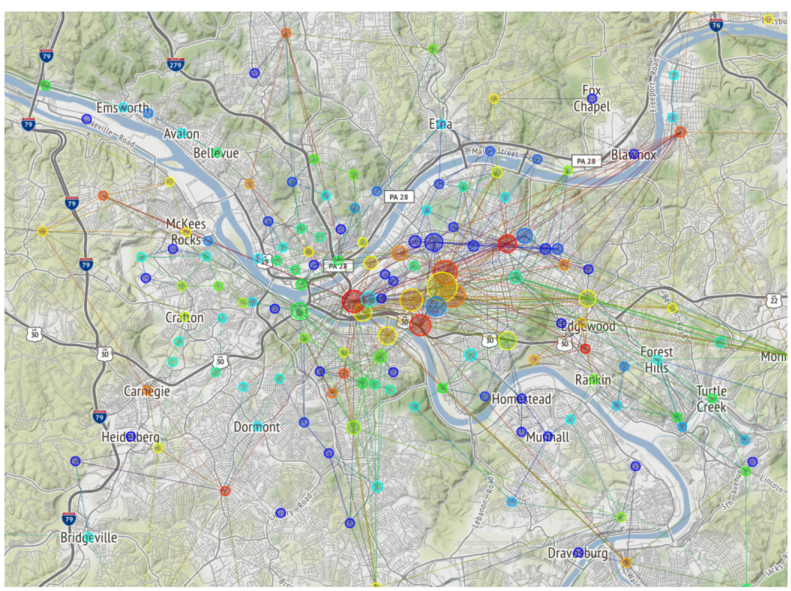


Fig. 2. Hoods x Hoods Network (colored by betweenness centrality, sized by eigenvector centrality)

There are nodes in each network, the bus network and the loads network, that contribute significantly to the network's success and functionality. We will identify and rank the ten most important neighborhoods in the Allegheny County public transit system for the bus network as seen through both the stop and passenger links. Namely, we will calculate the following centrality measures for the Hood x Bus network and the Hood x Load network: Degree Centrality, Betweenness Centrality, Eigenvector Centrality, and Hub Centrality. Degree centrality is important since it conveys the total number of connections between the two considered nodes. This relays the magnitude of importance of a hood as seen through the considered nodeset. Betweenness Centrality shows the importance of a neighborhood as a general intermediary along a bus route or passenger travel path. Eigenvector Centrality relays the importance of each neighborhood as a transfer point between two other well-connected hoods. Hub Centrality manifests the importance of each neighborhood in terms of how well it connects to other well-connected hoods. These measures are shown in Table 5 and Table 6, and creates an exhausted set of important neighborhoods in the transportation network as seen through bus routes and passenger travel, respectively.

Table 5

Most important neighborhoods – buses.

Rank	Degree	Betweenness	Eigenvector	Hub
1	Central Business District	Central Business District	North Oakland	Central Business District
2	Shadyside Neighborhood	Penn Hills Municipality	West Oakland	Shadyside
3	North Oakland	Churchill Borough	Central Business District	North Oakland
4	Wilkinsburg Borough	East Liberty	Shadyside Neighborhood	West Oakland
5	Squirrel Hill North	South Oakland	South Oakland	Bloomfield
6	East Liberty	Kennedy Township	Squirrel Hill North	South Oakland
7	South Oakland	Polish Hill Neighborhood	Central Oakland	East Liberty
8	West Oakland	Plum Borough	East Liberty	Squirrel Hill North

Table 6

Most important neighborhoods – loads.

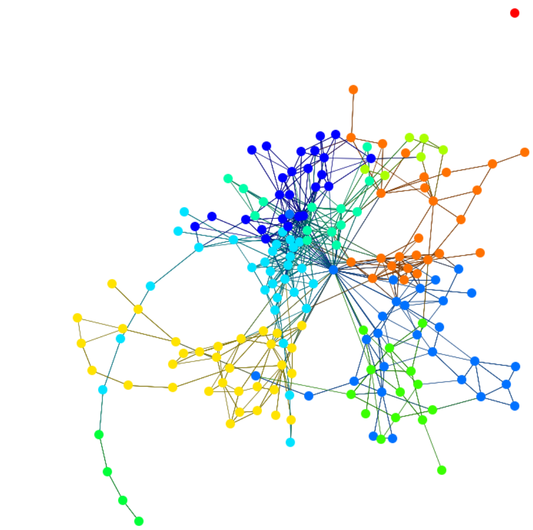
Rank	Degree	Betweenness	Eigenvector	Hub
1	Central Business District	Central Business District	North Oakland	North Oakland
2	Shadyside	Swissvale Borough	Shadyside	Central Business District
3	North Oakland	Shadyside	Squirrel Hill North	Shadyside
4	Squirrel Hill North	Knoxville	Central Business District	West Oakland
5	Wilkinsburg Borough	South Oakland	West Oakland	Bloomfield
6	South Shore	East Liberty	South Oakland	Squirrel Hill South
7	Bloomfield	Mount Lebanon Township	Central Oakland	South Oakland
8	South Oakland	Kennedy Township	Southside Flats	Wilkinsburg Borough

The tables above show the differences in the top ranked hoods based on the various centrality measures observed. These rankings differ between the loads and buses networks. This is because the buses represent the existing routes or the capacity of travel/connections. The load, on the other hand, represents the actual passenger usage of the buses to commute between the hoods.

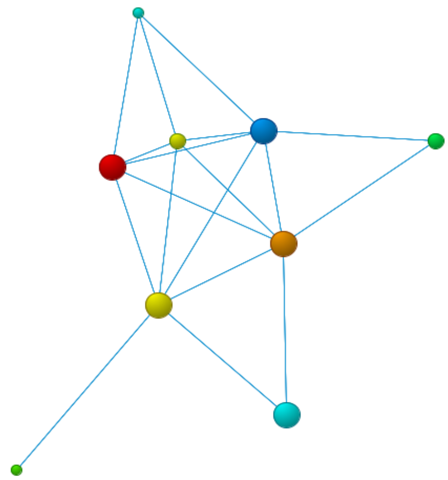
6. Transit Clusters

In this section, we identify "transit clusters" within Allegheny County. In context, clusters are groupings of neighborhoods that have significant levels of human and bus connection between them. This analysis provides a better understanding of the travel and social habits of residents within the county. Oftentimes, residents of a given region spend the majority of their time in only a subset of neighborhoods that the region has to offer. This transit cluster analysis defines a subset of pockets within the Allegheny County that public transit riders travel within.

In Fig. 3 below, we see the outcomes of clustering the neighborhoods by the Louvain clustering technique. This method created nine groups, four of significant size and five of marginal size. The large clusters are all internally mutually connected, are based within Pittsburgh, and roughly correspond to the Pittsburgh Northside, Pittsburgh Southside, East Pittsburgh and Northeast Pittsburgh.



(a)



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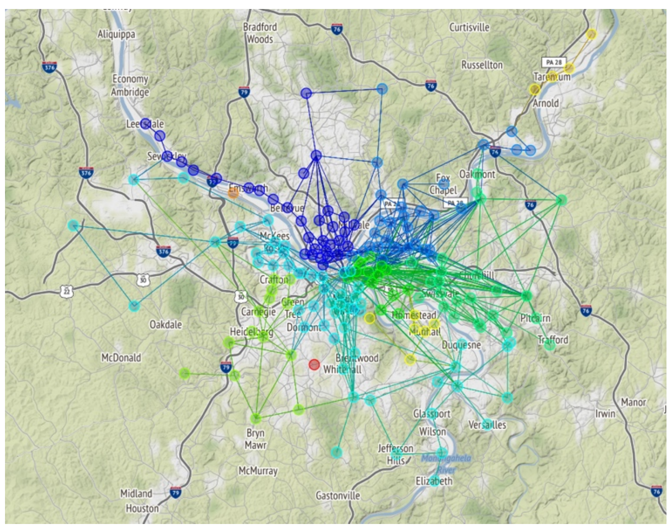


(c)

Fig. 3. Louvain clustering of neighborhood network colored (a), grouped (b) and mapped (c).

7. Equity

To define “in need” areas and therefore see if these areas are serviced fairly, we use Allegheny County of Human Services’ Need Index. Some of the variables that compose this index include percent single parent households, percent of households under 200% of the poverty line, and percent of population age 25 or above with bachelor’s degrees. One important caveat of this index is that it only applies to neighborhoods within Pittsburgh whereas "neighborhood" nodes within our dataset include not only urban neighborhoods but also municipalities and townships outside of the city limits.

Port Authority has identified the following seven neighborhoods as most in need neighborhoods: California-KirkBride, Crawford-Roberts, Garfield, Homewood North, Homewood West, Larimer, and Sheraden. Table 7 shows their corresponding rankings along six different centrality measures: degree, betweenness, and eigenvector for both bus and load links. In this table, *x* is a stand-in where the neighborhood's calculated metric was too low to be ranked by ORA.

Table 7

In need neighborhoods rankings.

Neighborhood	Bus			Load		
	Degree	Betweenness	Eigenvector	Degree	Betweenness	Eigenvector
California-KirkBride	97	85	x	80	x	x
Crawford-Roberts	31	x	12	23	x	16
Garfield	80	x	46	83	x	50
Homewood North	33	x	31	30	x	35
Homewood West	59	x	38	48	x	31
Larimer	16	x	14	13	x	15
Sheraden	69	96	68	44	96	45
Avg Rank	**55**	**126**	**50**	**46**	**134**	**47**

For both degree and eigenvector centrality, the average load rankings are higher than average bus rankings. This may be because these neighborhoods are being systematically underserved - they are more important in terms of actually connecting riders than they are in terms of the total buses allocated to these areas. Betweenness centrality is extremely low for both buses and loads since it is not even ranked. This implies that the underserved neighborhoods are out of the way of most riders' typical routes.

These results validate that the following identified neighborhoods are in fact inequitably underserved: Crawford-Roberts, Garfield, Homewood North, Homewood West, and Larimer. This list consists of the union between the neighborhoods that are considered underserved by

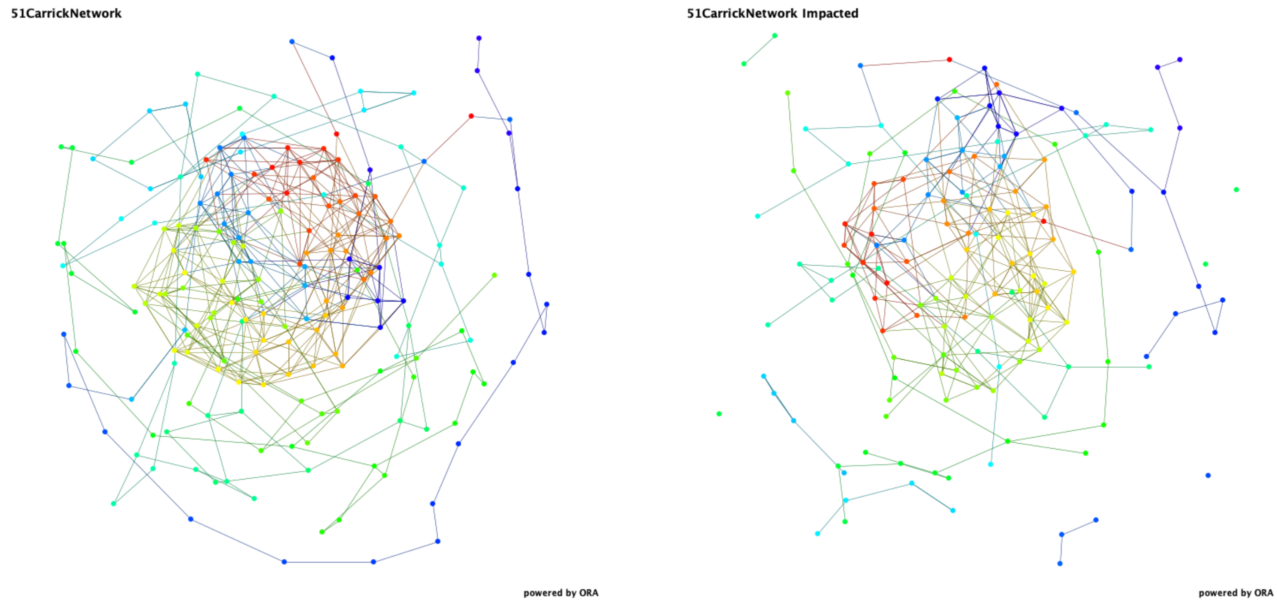
Port Authority, and the neighborhoods that are ranked with extremely low betweenness centrality in the bus network.

8. Stop Eliminations

As all systems evolve over time, Port Authority is constantly making changes to Allegheny County's public transit system. The organization recently eliminated dozens of stops from four major lines: 48-Arlington, 88-Penn, 16-Brighton, and 51-Carrick. Port Authority's goal is to sustain a connected public transit system in the region. Its limited budget serves as a challenge in accomplishing this goal. As Port Authority continues to exclude bus stops from its network, it needs to strive towards an equitable network for residents from all regions and classes.

To ensure this goal is met, this analysis examines the impact of these stop eliminations on the transportation network. The eliminated stops were manually acquired from Port Authority's website (Bus Stop Consolidation, 2020). For each of the four lines, we can perform an immediate impact change analysis by removing all bus stops that were removed in March 2020. These immediate impact analyses are, in theory, also dynamic network analyses as they compare networks changed over time. The timeframes in these dynamic network analyses are: (1) February 2020 which was prior to bus stop eliminations, and (2) March 2020 which was after bus stop eliminations.

To detail this process and the effect of this analysis, we will perform the immediate impact change analysis only on the 51-Carrick line as an example. We perform two iterations of the immediate impact report. The first is accomplished by removing all bus stops along the 51-Carrick line that were eliminated in March 2020 during the immediate impact change analysis. The resulting report manifests the effect on the network as a whole, and each impacted neighborhood. The second method is performed by first filtering the whole network to only include the bus stops that are along the 51-Carrick route. During the immediate impact change analysis, we then remove all nodes that were eliminated in March 2020. The completed report details the effect of these stop eliminations to the relevant bus line. We see the immediate impact of the change analysis on, specifically, the 51-Carrick line network by comparing Fig. 4 (a) to Fig. 4 (b) below.



(a) (b)

Fig. 4. 51-Carrick bus line network of stops (a) before and (b) after stop eliminations.

It is important to note that in Fig. 4 (a), it is seen that some nodes have multiple links and routes between them. This is because these stops are mutually connected along other bus lines as well as through the 51-Carrick line.

From the 184 nodes in the original 51-Carrick network, 34 nodes were removed in the immediate impact change analysis. After the stop removals, there are drastically less links in the network as seen in Fig. 4 (b). There are also drastically less nodes in the center mass of the network visualization in Fig. 4 (b). As a result from the stop eliminations, there are many new broken routes and isolated bus stops. This means that these nodes are still connected in the larger Allegheny County bus network through other bus routes and lines. However, 51-Carrick simply no longer has access to these nodes.

Table 8 below presents numerical network characteristics before and after the stop eliminations for the 51-Carrick line network. The two characteristics considered are characteristic path length and average communication speed. These two characteristics provide insight into how the stop eliminations impact the time the 51-Carrick buses take to traverse the network. This delta in time is directly relational to the extra time a passenger on the 51-Carrick line would save or consume after the stop eliminations. Therefore, in Table 8 we specifically note these characteristics for the Stop x Load 51-Carrick network.

Table 8

Metrics for 51-Carrick bus line Stop x Load network, before and after stop eliminations.

Characteristic	Before	After
Characteristic Path Length	8293.083	967.152
Average Communication Speed	1.206e-04	0.001

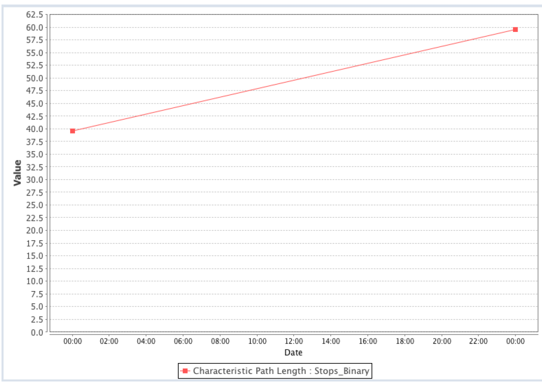
Table 8 shows that after stop removals, the 51-Carrick line Stop x Load network decreases in characteristic path length by about 88%. This smaller number for characteristic path length implies that the average shortest path between stop nodes has decreased. Table 8 also shows that after stop eliminations, the 51-Carrick line Stop x Load network increases in average communication speed by about 757%. This larger number for average communication speed means that the passenger travel speed between two stop nodes has increased. These two characteristics were the only measures that were significantly impacted in this change analysis.

Overall, the results of this immediate impact change analysis of the stop removals along the 51-Carrick line show that the stop eliminations decrease the connectivity of the 51-Carrick line. However, the stop eliminations do not completely deter the binary ability of reaching one stop from another. With decreased connectivity and less stops along its route, the 51-Carrick line network benefits from stop eliminations because the impact decreases distances between nodes, and thus decreases time and distance of travel along this bus route.

9. Weekday Weekend Network Comparisons

Typically, transportation planners create different schedules for weekdays and weekends. Weekdays have higher levels of service overall and have service concentrated around times that workers typically commute to and from work. Thus, we compare the weekday and weekend networks of buses to understand how they differ for the Allegheny County bus transportation system. To do this, we compare various network and centrality measures between the two states.

Fig. 5 below depicts the changes in characteristic path length and diameter for each of the two networks, independently: stops and hoods. Each graph plots the weekday network measure on the left, and the corresponding weekend network measure on the right. Thus, the change in the network's performance measure from weekday to weekend is discernable.



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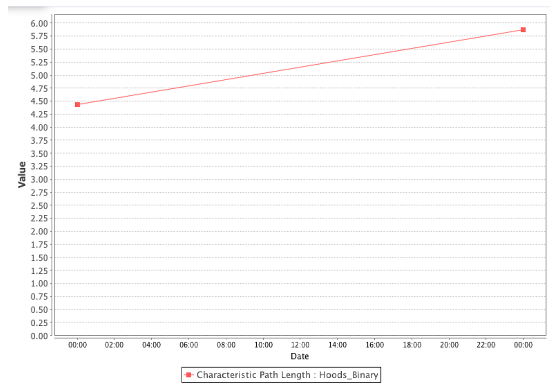

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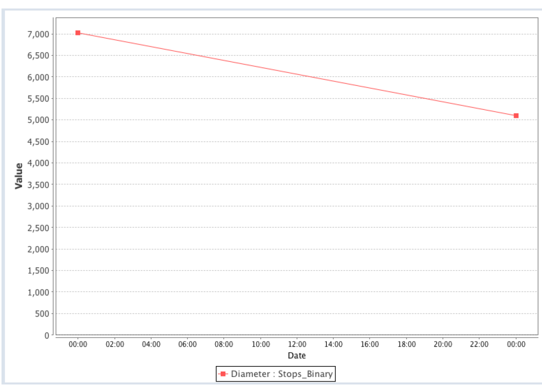

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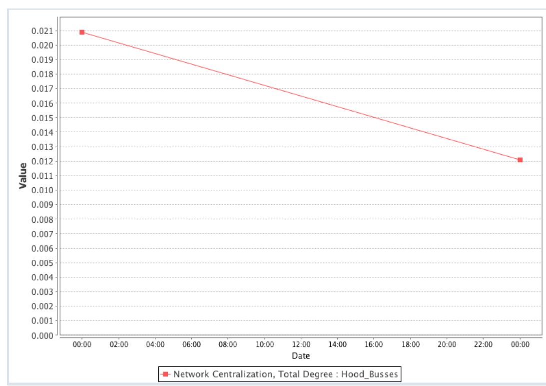

(d)

Fig. 5. Weekday vs Weekend centrality metrics comparisons; (a) stops characteristic path length, (b) hoods characteristic path length, (c) stops diameter, (d) hoods total degree centrality.

As seen in Fig. 5 (a), the characteristic path length to get from one stop to another increases from 40 to almost 60 going from the weekday network to the weekend network. In Fig. 5 (b) we see that the characteristic path length to get from one neighborhood to another increases from 4.5 to almost 6 going from the weekday network to the weekend network. These results signify longer commute times and paths during weekend travel in comparison to weekday travel.

Fig. 5 (c) shows that the network diameter decreases from 7,000 to just above 5,000 going from weekday to weekend. This manifests that the weekend network has a significantly less range of accessibility. This is seen further in Fig. 5 (d) which shows that the average total degree centrality is almost halved going from weekday to weekend. Fig. 5 (d) also illustrates the sheer disparity in the system's capacity from weekday to weekend, where the weekend service offers less capacity.

As a whole, this analysis demonstrates that the weekend Allegheny County bus transportation network is less connected, provides less accessibility, and thus has lower performance than that of the weekday network. For network design purposes, Port Authority would benefit from using these results and a similar analysis to understand the true magnitude and impact of their scaled down weekend transportation system.

10. Limitations and Future Work

A key limitation in the original dataset used in this study is that it is live, on-bus tracking data; the dataset does not include individualized origin-destination pairs. That is to say that if passenger A took a bus ride from Squirrel Hill to Lawrenceville, passenger A would be counted as a bus rider along all the intermittent neighborhoods as well. Future work would benefit from implementing a method to collect this key data value for more detailed passenger network analysis as an extension to our work.

The only notable bias within our data is that we only have data from the month of April 2019. Bus ridership in Allegheny County varies significantly based on factors like season and whether school is in session. Other than that caveat, the data should be exhaustive and therefore unbiased. Future work should entail performing similar analyses on data that takes seasonal traffic variations into account. This could include independent and comparative analyses for a winter month such as January or February, a summer month when Pittsburgh's large student population vacates the region such as June, a summer month when Pittsburgh's large student population is within the region such as August, and a fall month such as November.

A restriction in our analyses was that it only included data and transportation routes of Port Authority buses. However, there are other transit methods in the region such as Port Authority's

Light Rail, and university shuttle services. Including these exclusions would provide a more cohesive understanding of the Allegheny County full transportation network. This extended study would be more comparable to the Department of Transportation's national transportation network analysis through multiple travel domains such as highways, rails, and air. A corresponding research question for this extended analysis is: What is the synergy between Allegheny County's Port Authority bus route and other local transportation systems such as Carnegie Mellon or University of Pittsburgh Shuttle Services?

Further research could include an abstracted and extended version of the stop eliminations immediate impact change analysis. An extended analysis could focus on and provide the most travelled stops and neighborhoods from a given bus line, the farthest stops and neighborhoods by public transit accessibility, the stops that are essential to each neighborhood, and the stops that are not frequently visited along a bus route that do not provide residents with greater reachability. These possibilities provide an all-encompassing understanding of the impact of stop removals in any of the bus sub-networks, and would provide numerical measures that could help advise Port Authority on its bus stop prioritization list.

To further understand the Allegheny County bus transportation system's network, the following research questions can be considered: What are the most under-connected regions in Allegheny County? What are the most valued stops/regions in the Allegheny County public bus transportation system? How safe are stops at a given time range based on passenger traffic patterns? Is this a successful transportation network? How can this network topology help define the characteristics of a successful transportation network model? How can Port Authority optimize the network design while simultaneously optimizing financial investment phasing strategies (Dantzig et al., 1976)?

11. Conclusion

This paper applied several dynamic network analysis techniques to the Port Authority of Allegheny County public bus system. It identified the system's network characteristics to be a hybrid, closed network with routinization; it found which neighborhoods were most important based on numerous centrality measures for both the network capacity, and the actual utilization;

it found evidence that high-need areas were systematically underserved by the network; it proposed a method to, and provided an example to, quantify the positive and negative impacts of removing stops that were slated for elimination to certain routes; and it compared the weekday and weekend networks to learn that the weekend network provided less accessibility and performance than the weekday network. The contributions of this study provide an understanding of the Allegheny County public bus transportation system design performance, and pave way to novel methods of analyzing transportation networks.

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