HKN ECE 310 Quiz 4 Review Session

Corey Snyder

Z-Transform

- We will focus on the one-sided, or unilateral, z-transform
- $X(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$
- Typically perform inverse z-transform by inspection or by Partial Fraction Decomposition
- Important properties:
  - Multiplication by n: $nx[n] \leftrightarrow -z(\frac{dX(z)}{dz})$
  - Delay Property #1: $y[n-k]u[n-k] \leftrightarrow z^{-k}Y(z)$
- Make sure to note the Region of Convergence (ROC) for your transforms!
- DTFT is only defined if the ROC contains the unit circle

Difference Equations and Transfer Functions

- Given a LCCDE, perform the z-transform on the system.

- Remember to apply Delay Property #1 to delayed inputs and outputs
  - Ex: $ay[n-k] \leftrightarrow aY(z)z^{-k}$

- $H(z) = \frac{Y(z)}{X(z)}$

BIBO Stability

- Three ways to check for BIBO Stability:
  - Pole-Zero Plot
  - Absolute integrability
  - Given $|x[n]| < \alpha,\ if\ |y[n]| < \beta < \infty$, then the system is BIBO stable
- Pole-Zero Plot
  - For an LSI system: if the ROC contains the unit circle, this system is BIBO stable
  - What if the ROC is $|z| > 1$?
    - This is *marginally stable*, but unstable for ECE 310 purposes
- Absolute Integrability
  - $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

Frequency Response

- For any stable LSI system: $H_d(\omega) = H(z)|_{z=e^{j\omega}}$
- What is the physical interpretation of this?
  - The DTFT is simply the z-transform evaluated along the unit circle!
  - It makes sense that the system must be stable and LSI since the ROC will contain the unit circle, thus ensuring that the DTFT is well defined
- Why is the frequency response nice to use in addition to the z-transform?
  - $e^{j\omega}$ is an *eigenfunction* of LSI systems
  - By extension: $x[n] = \cos(\omega_0 n + \theta) \rightarrow y[n] = |H_d(\omega_0)| \cos(\omega_0 n + \theta + \angle H_d(\omega_0))$

Magnitude and Phase Response

- Very similar to ECE 210
- Frequency response, and all DTFTs for that matter, are 2π periodic
- Magnitude response is fairly straightforward
  - Take the magnitude of the frequency response, remembering that $|e^{j\omega}| = 1$
- For phase response:
  - Phase is “contained” in $e^{j\omega}$ terms
  - Remember that cosine and sine introduce sign changes in the phase
  - Limit your domain from $\frac{-\pi}{2}\ to\ \frac{\pi}{2}$
- For real-valued systems:
  - Magnitude response is even-symmetric
  - Phase response is odd-symmetric