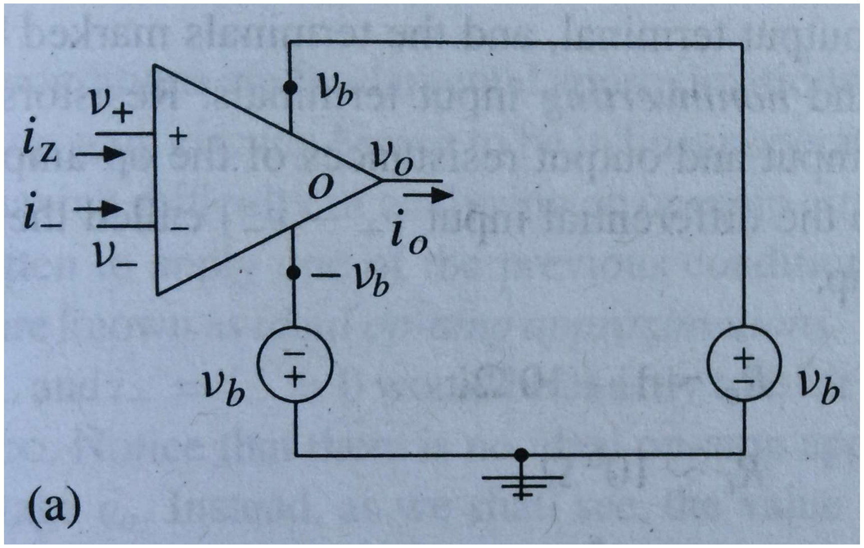
Chapter 3

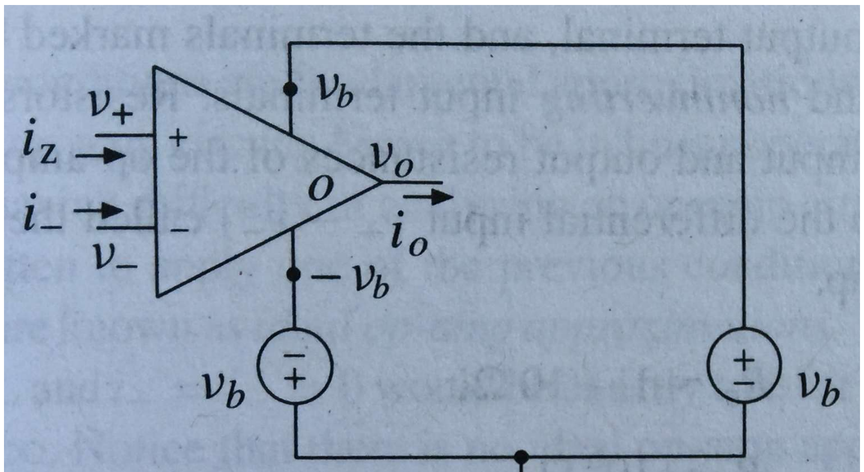
Circuit for Signal Processing

3.1 op-amp & signal arithmatic

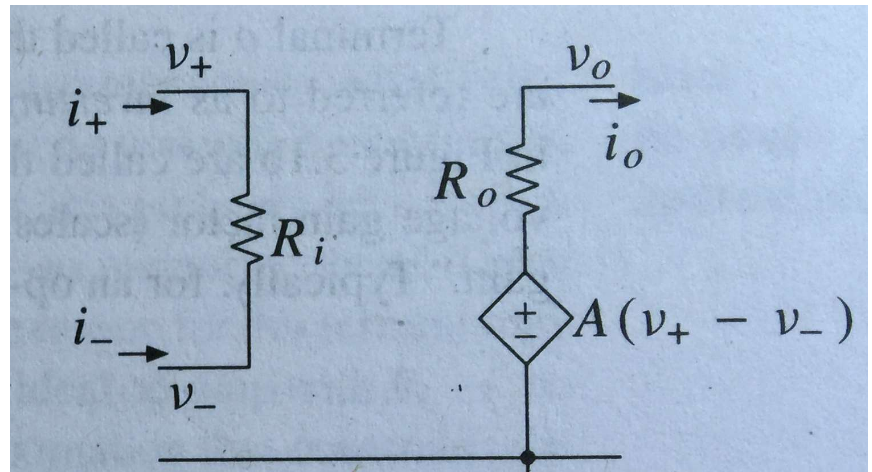
- Four inputs: terminals with node voltage V+ and V-; terminals with biasing voltage V_b and $-V_b$ (V_b is positive)
- One output: terminal with node voltage V_o



- $V_o = A(V_+ - V_-) - R_o i_o$

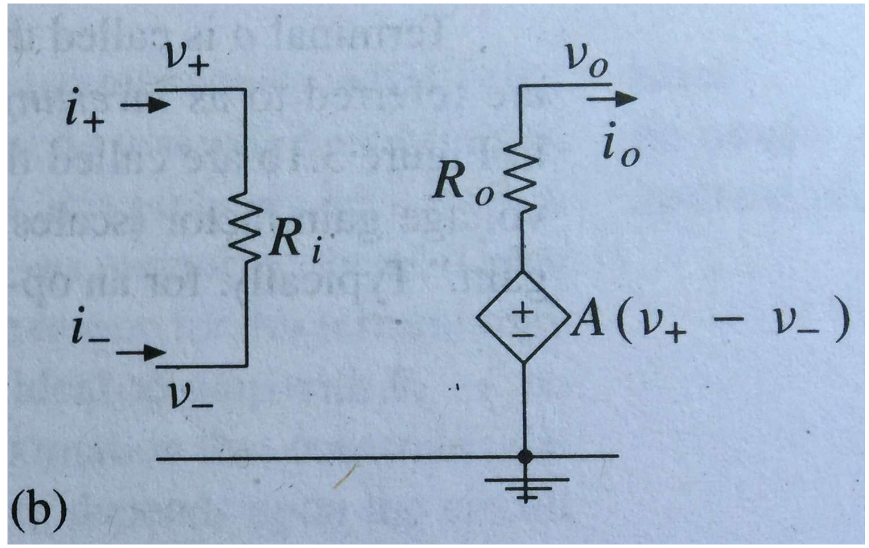


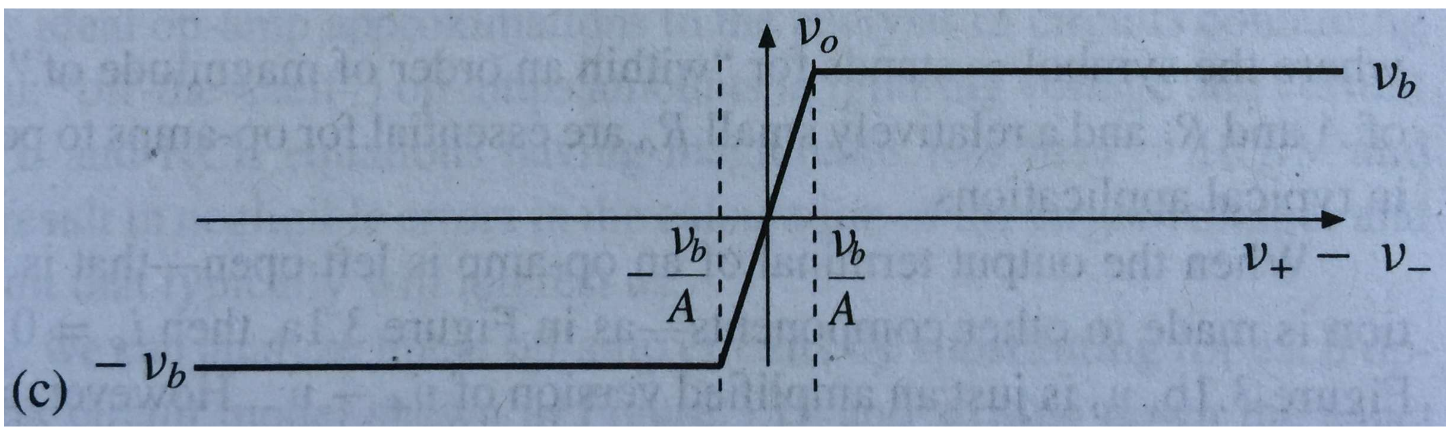
(a)

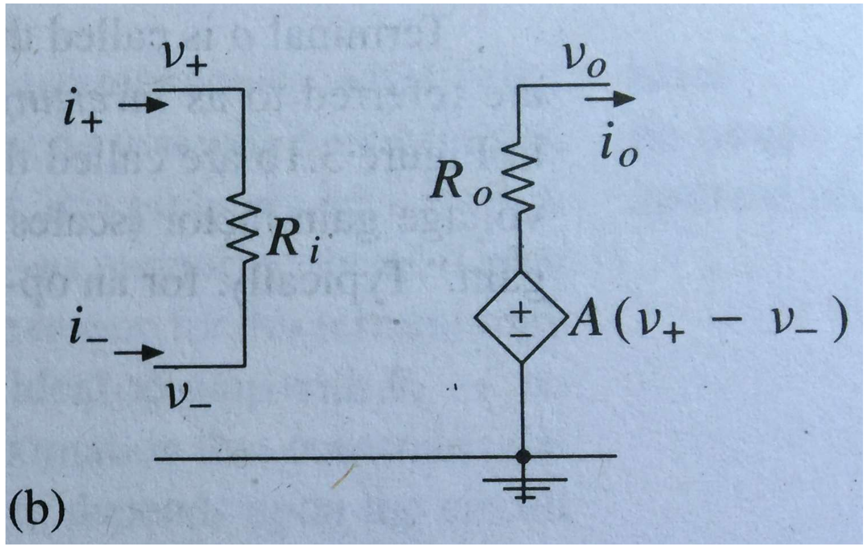


(b)

- $V_o = A(V_+ - V_-) - R_o i_o$
- |Vo| cannot be larger than Vb







R_o ~ 1 – 10 Ω

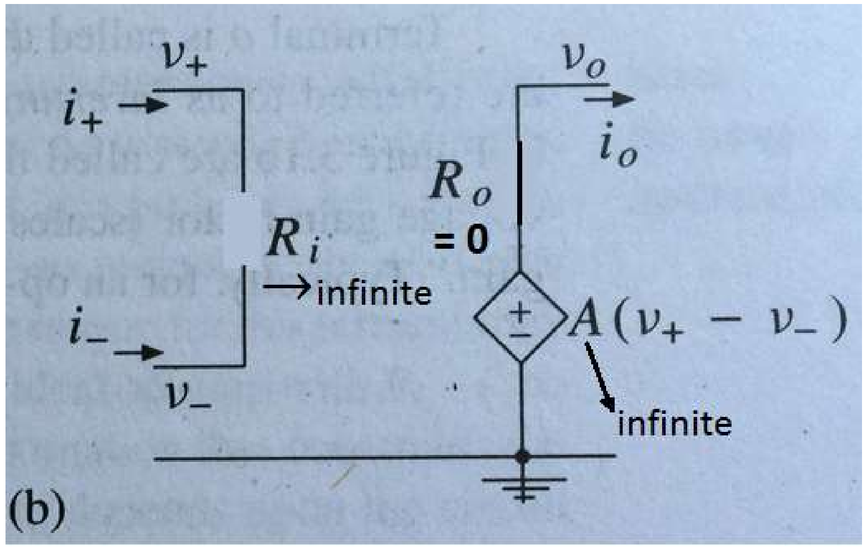
R_i ~ 10^6 Ω

A ~ 10^6

R_o = 0 Ω

R_i → ∞ Ω

A → ∞



$R_o \sim 1 - 10\ \Omega$

$R_i \sim 10^6\ \Omega$

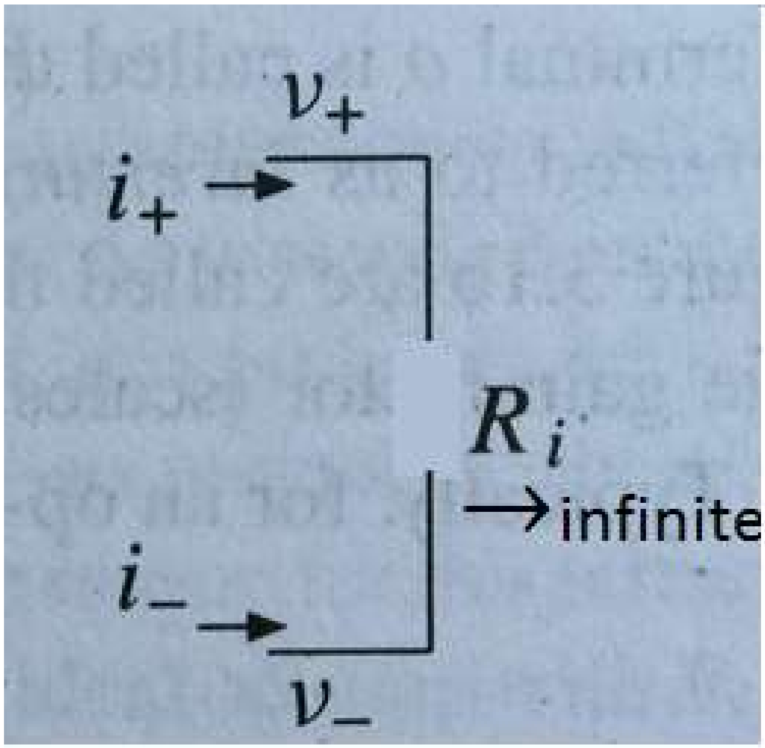
$A \sim 10^6$

$\Rightarrow$

$R_o = 0\ \Omega$

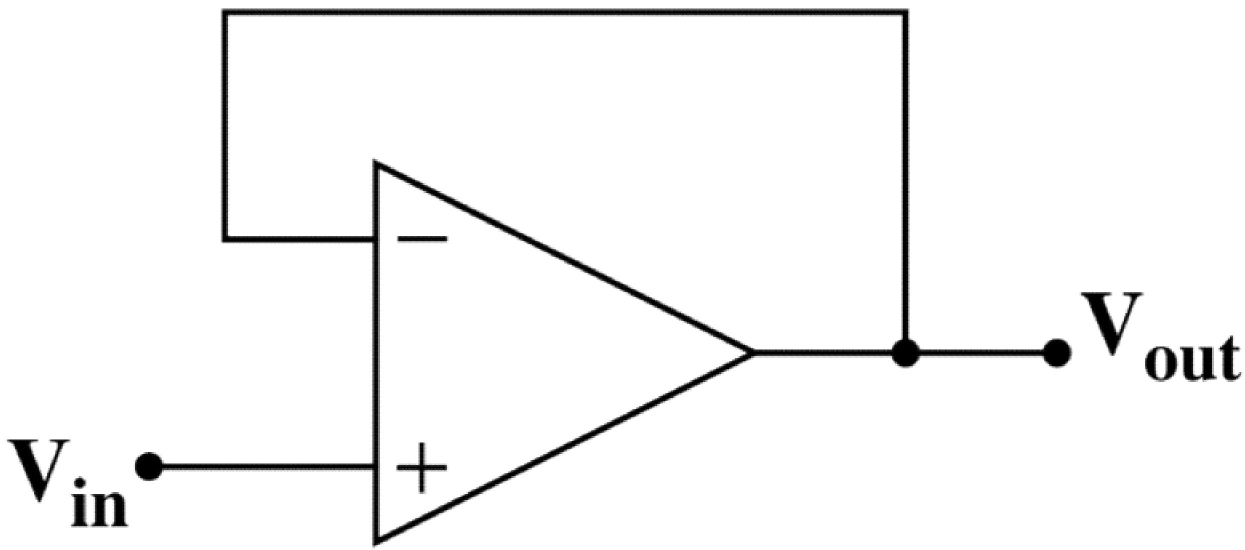
$R_i \rightarrow \infty\ \Omega$

$A \rightarrow \infty$



- $i_+ = i_- = 0$
- $V_+ = V_-$

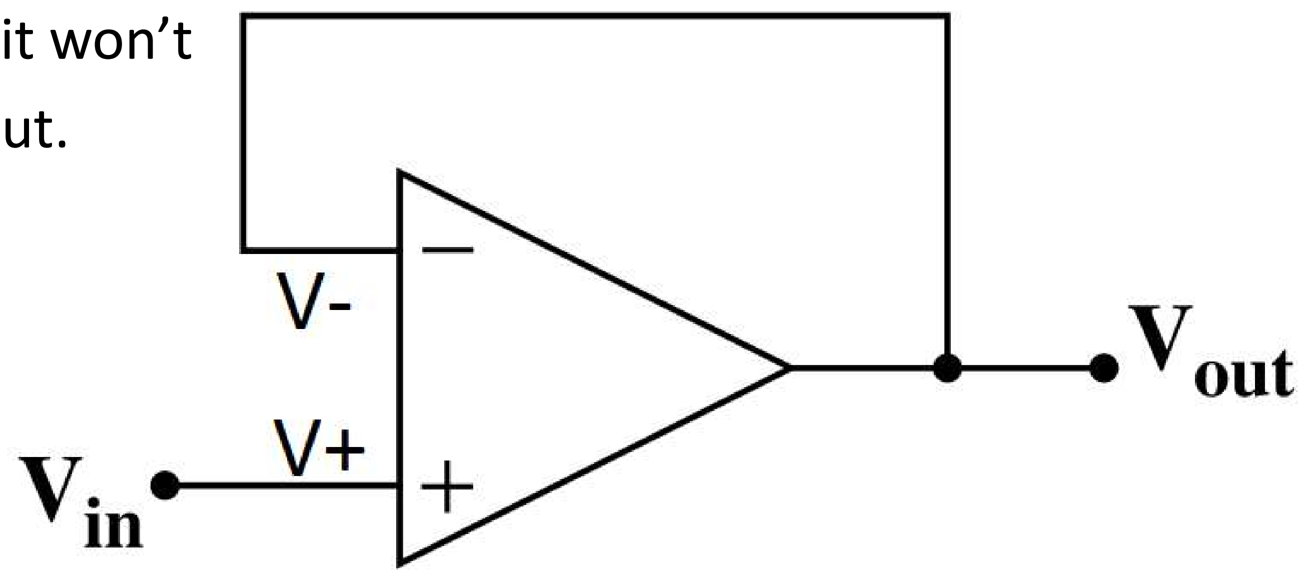
- Voltage follower



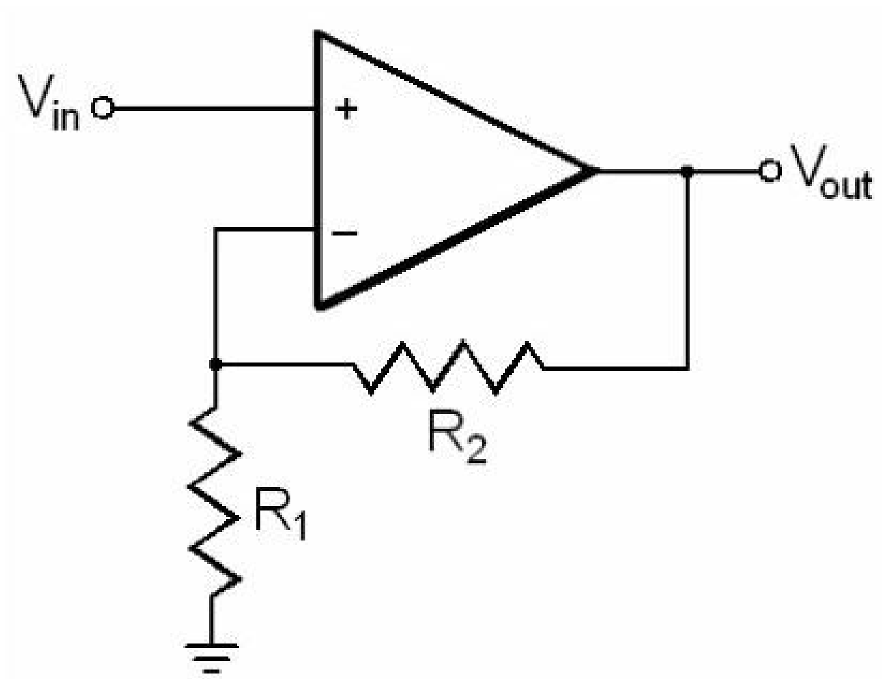
- Voltage follower

- V+ = V-
- $V_{in} = V_{out}$

- The output side circuit won't affect the voltage output.



- Noninverting amplifier

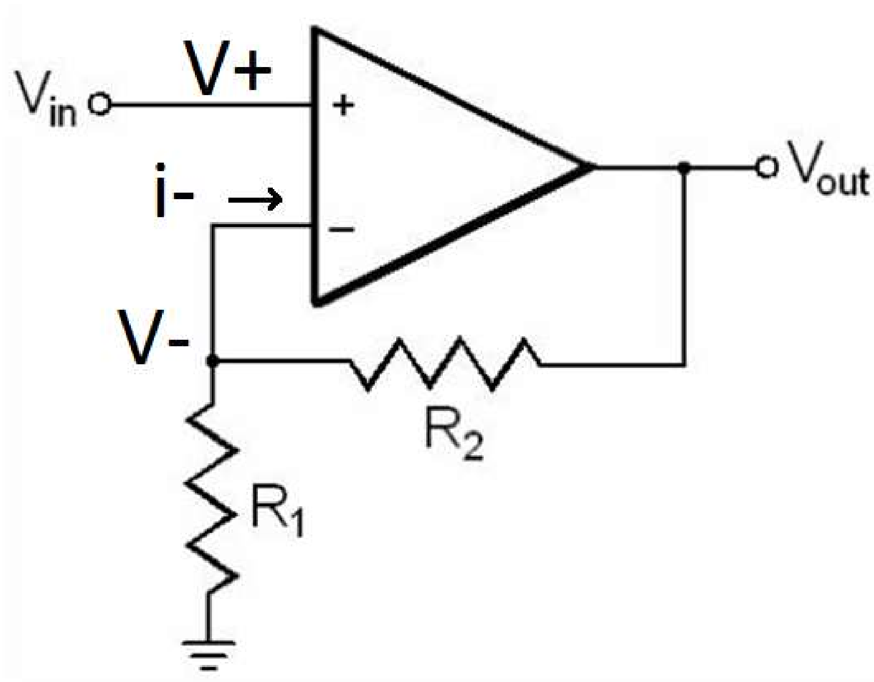


- Noninverting amplifier

- V+ = V- = Vin
- i- = 0
- According to KCL,

$$\frac{V_-}{R_1} = \frac{V_{out} - V_-}{R_2}$$

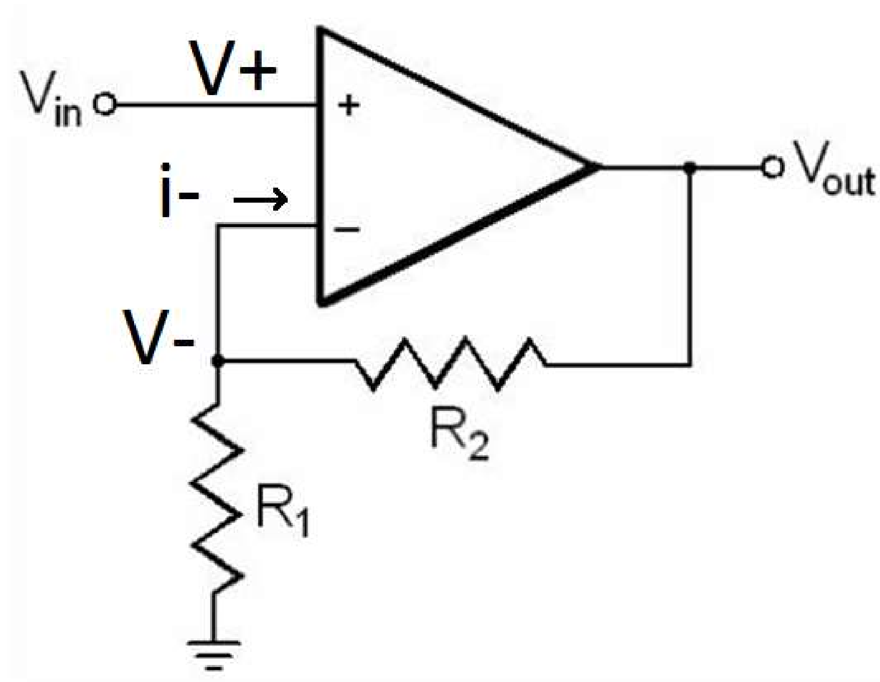
- $V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in}$



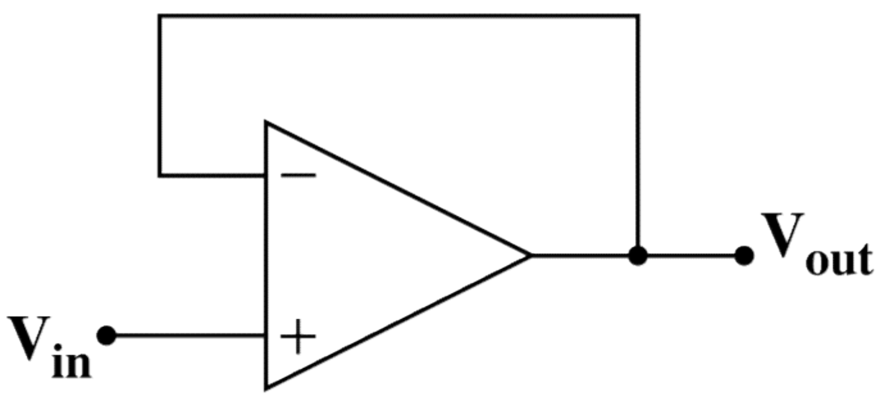
- Noninverting amplifier

- Negative feedback

Connecting the output of an op-amp to its inverting (-) input is called *negative feedback*.

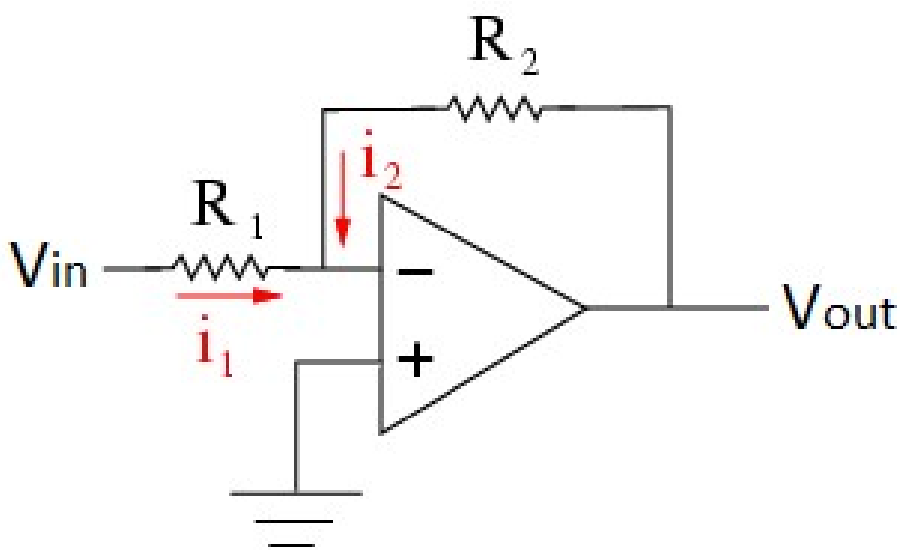


- Why negative feedback?
- When the output of an op-amp is *directly* connected to its inverting (-) input, a *voltage follower* will be created. An op-amp with negative feedback will try to drive its output voltage to whatever level necessary so that the differential voltage between the two inputs is practically zero.



$$V_o = A(V_+ - V_-) - R_o i_o$$

- Inverting amplifier

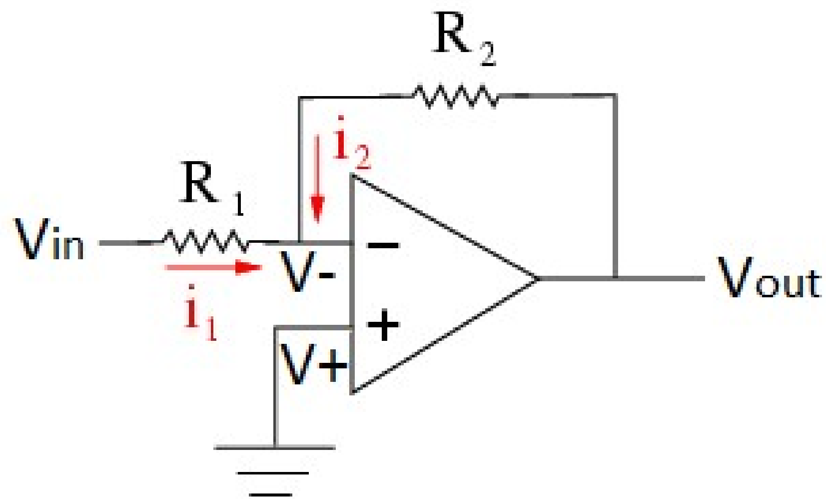


- Inverting amplifier

$i_1 + i_2 = 0$ $\quad$ $V+ = V- = 0$

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} = 0$$

$$V_{out} = -\frac{R_2}{R_1} V_{in}$$

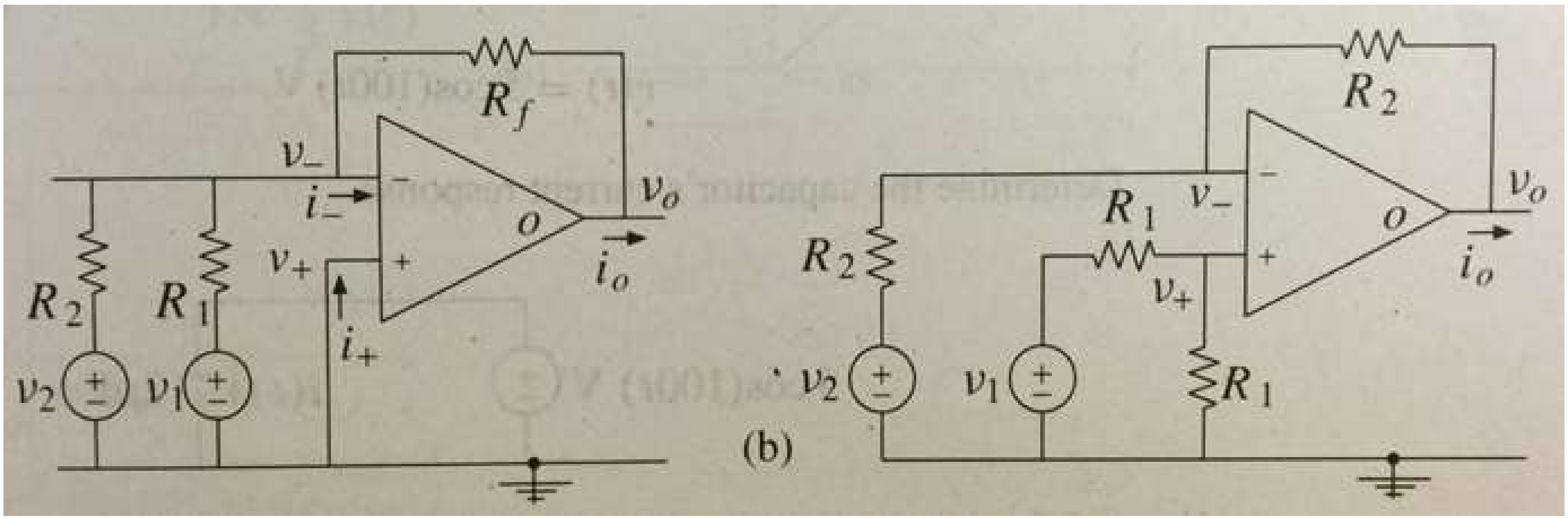


- Voltage gain

$$G = \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

- Sum and difference calculators

sum difference

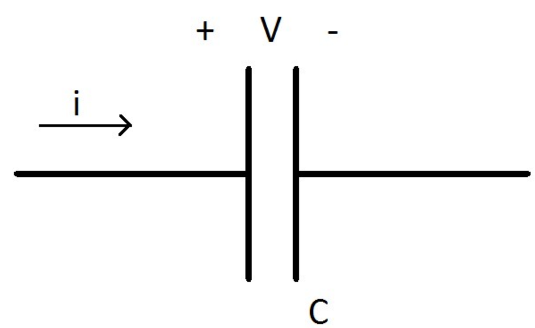


3.2 Differentiators and Integrators

- Capacitor and inductor

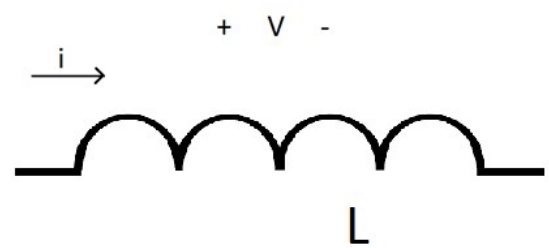
Capacitor

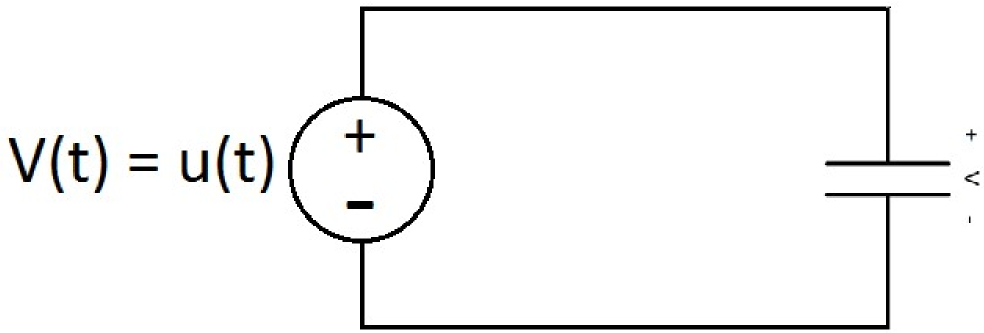
- $i(t) = C\frac{dv(t)}{dt}$
- $V(t) = (\int_{t_0}^{t}\frac{i(t)}{C}) + V(t_0)$



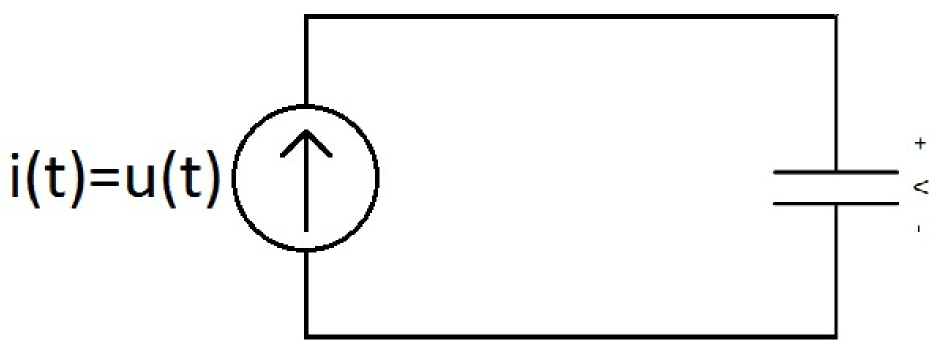
Inductor

- V(t)= $L\frac{di(t)}{dt}$
- $i(t) = (\int_{t_0}^{t}\frac{V(t)}{L}) + i(t_0)$





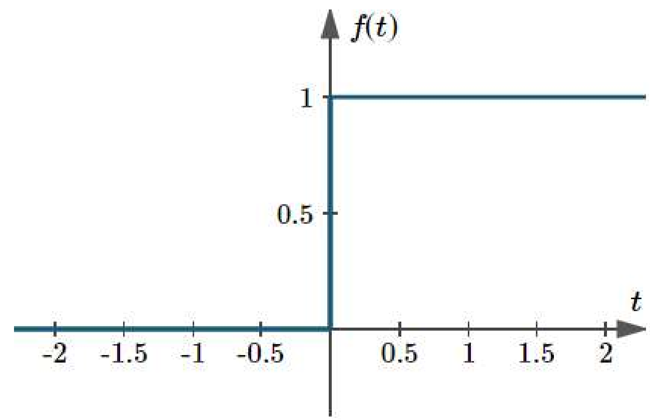
"unhealthy" differentiator circuit.

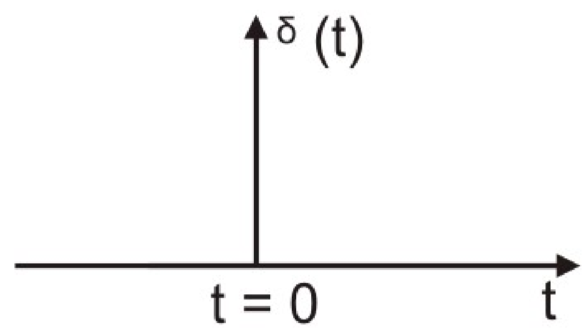


integrator

Capacitor

- $i(t) = C\,\frac{dv(t)}{dt}$
- $V(t) = \left(\int_{t_0}^{t} \frac{i(t)}{C}\right) + V(t_0)$





i(t)=u(t)

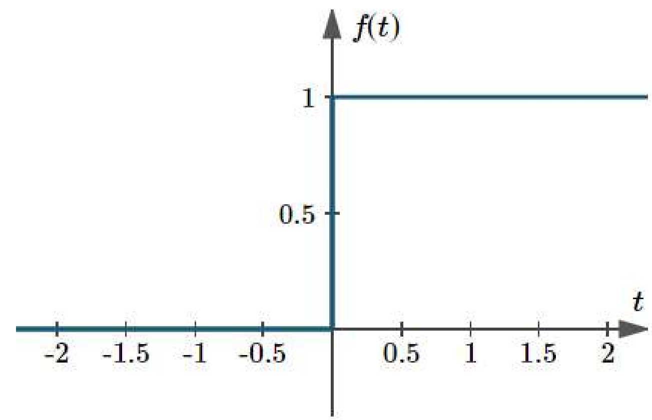
“unhealthy” differentiator circuit.

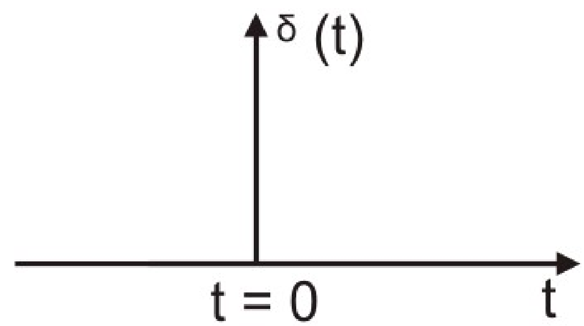
V(t) = u(t)

integrator

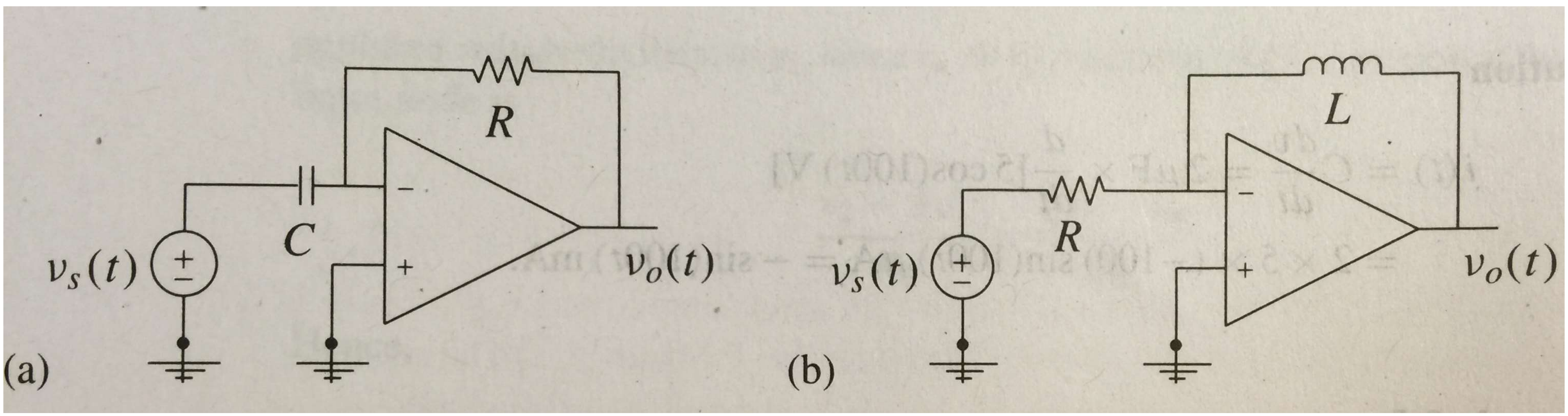
V(t)= $L\frac{di(t)}{dt}$

$$i(t) = (\int_{t_0}^{t}\frac{V(t)}{L}) + i(t_0)$$

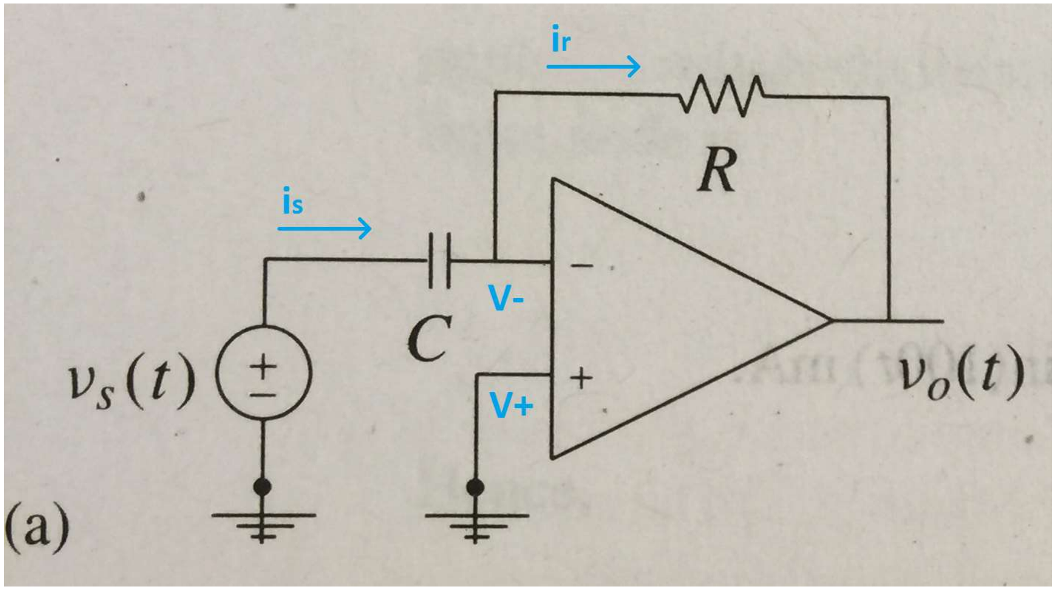




• Op-amp differentiator



- V- = V+ = 0
- $i_s = C\frac{d(V_s - V_-)}{dt} = C\frac{dV_s}{dt}$
- $i_s = i_r$
- $V_o = V_- - i_r R = -RC\frac{dV_s}{dt}$

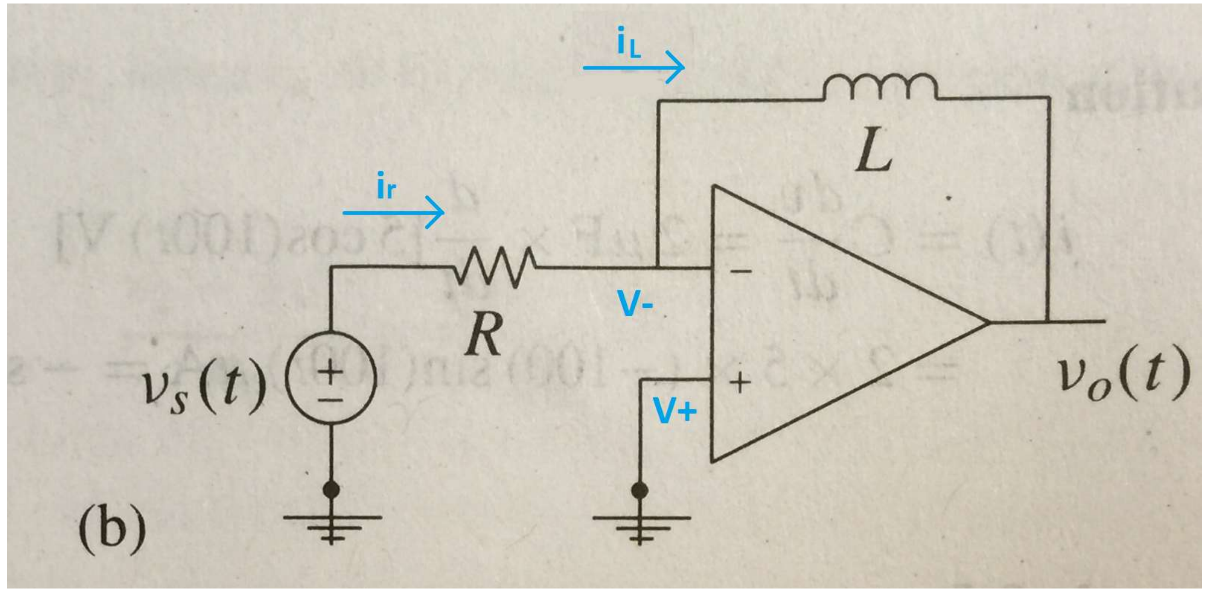


Capacitor

$$i(t) = C\frac{dv(t)}{dt}$$

$$V(t) = (\int_{t_0}^{t}\frac{i(t)}{C}) + V(t_0)$$

- V- = V+ = 0
- $i_r = \frac{V_s}{R}$
- $i_L = i_r$
- $V_- - V_o = L\frac{di_L}{dt}$
- $V_o = -\frac{L}{R}\frac{dV_s}{dt}$

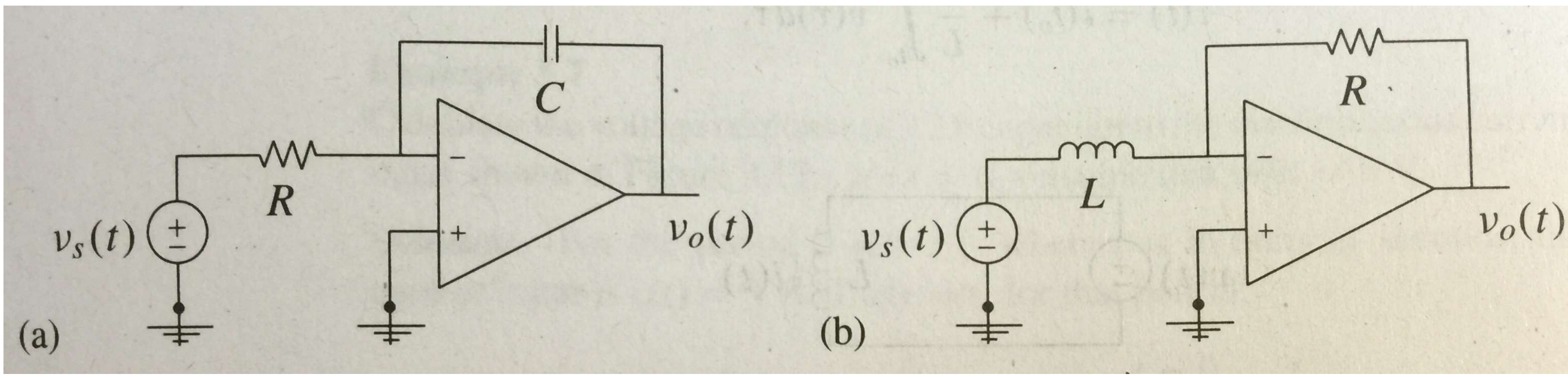


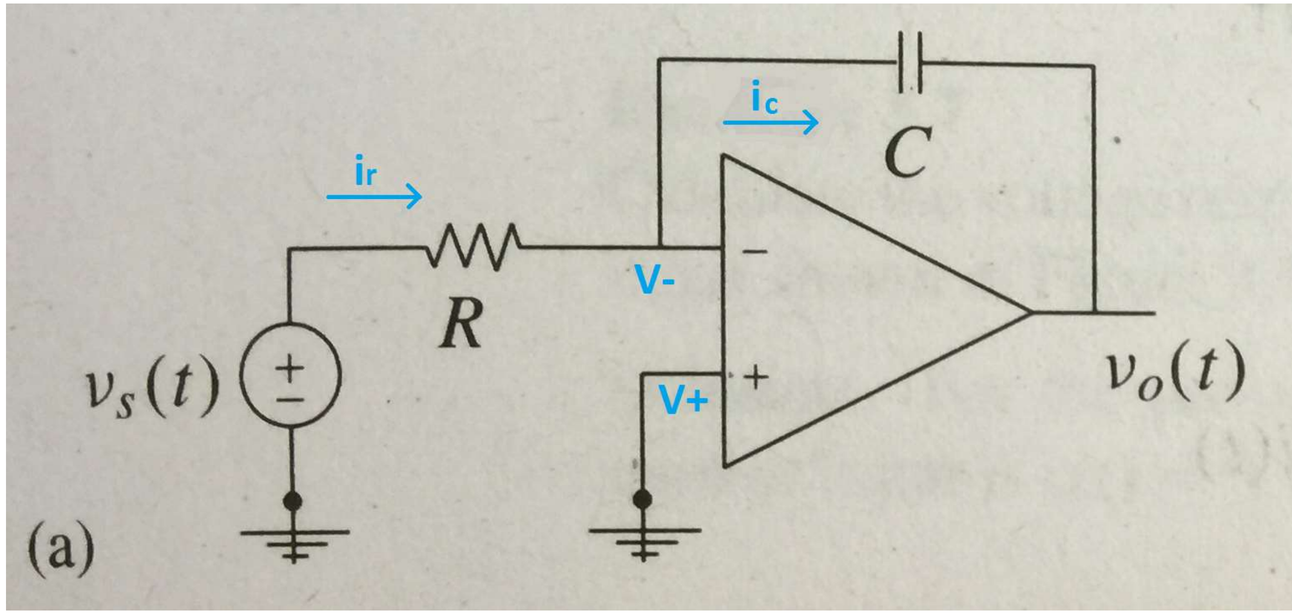
Inductor

V(t)= $L\frac{di(t)}{dt}$

$i(t) = (\int_{t_0}^{t} \frac{V(t)}{L}) + i(t_0)$

• Op-amp integrator

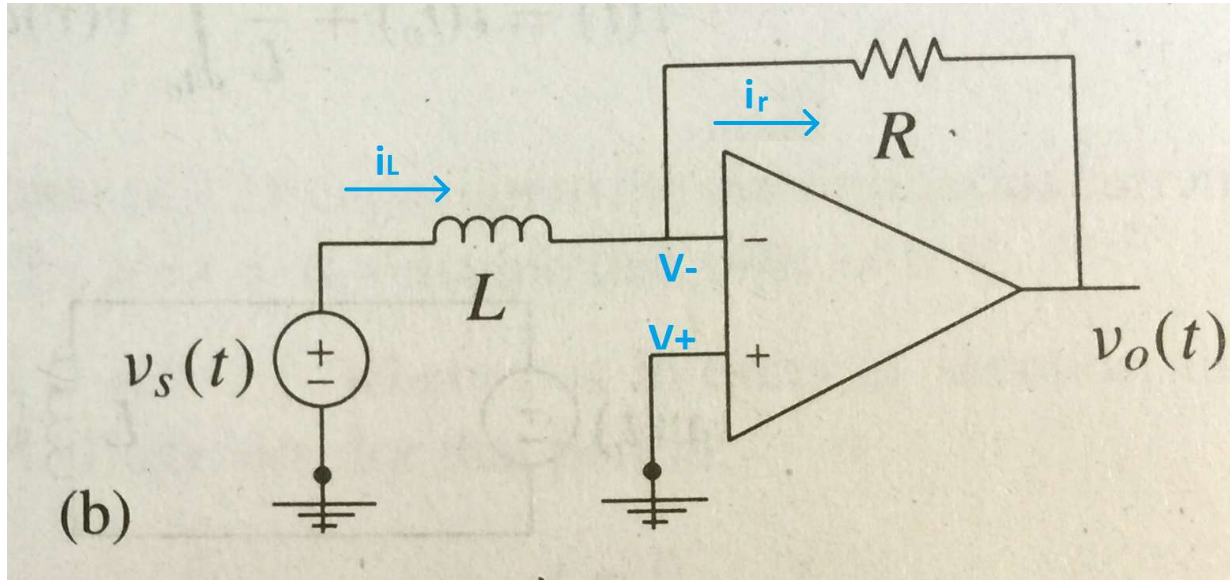




- V- = V+ = 0
- $i_r = \frac{V_s}{R}$
- $i_c = i_r$
- $V_{-} - V_o = \left(\int_{t_0}^{t} \frac{i_c\,(t)}{C}\right) + V_c\,(t_0)$
- $V_o = -\left(\int_{t_0}^{t} \frac{V_s\,(t)}{RC}\right) - V_c\,(t_0)$

$$i(t) = C\,\frac{dv(t)}{dt}$$

$$V(t) = \left(\int_{t_0}^{t} \frac{i(t)}{C}\right) + V(t_0)$$



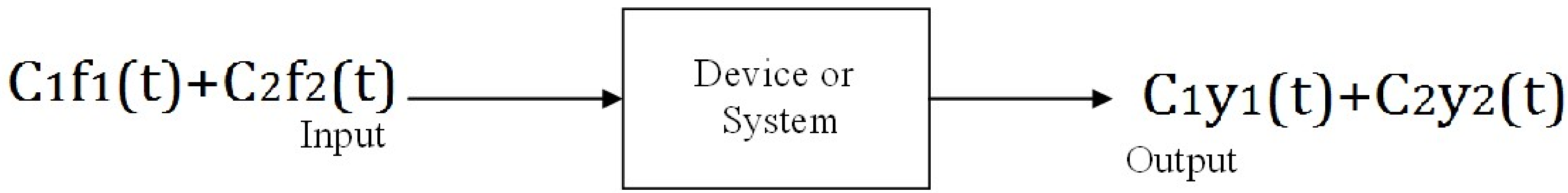
- V- = V+ = 0
- $i_L = (\int_{t_0}^{t} \frac{V_S(t)}{L}) + i(t_0)$
- $i_L = i_r$
- $V_{-} - V_o = i_r R = \left((\int_{t_0}^{t} \frac{V_S(t)}{L}) + i(t_0) \right) R$
- $V_o = -\left((\int_{t_0}^{t} \frac{V_S(t)}{L}) + i(t_0) \right) R$

V(t)= $L\frac{di(t)}{dt}$

$i(t) = (\int_{t_0}^{t} \frac{V(t)}{L}) + i(t_0)$

3.3 Linearity, Time Invariance, & LTI System

- Linearity
- $y(t) = \quad \underbrace{y(t_0)}_{\text{Zero-input response}} \quad + \quad \underbrace{\int_{t_0}^{t} f(\tau) d\tau}_{\text{Zero-state response}}$
- A system is said to be linear if its output is a sum of distinct zero-input and zero-state responses that vary linearly with the initial state of the system and linearly with the system input, respectively.



- How to decide whether linear or not?
- Assume
  - $f_1(t) \rightarrow y_1(t)$
  - $f_2(t) \rightarrow y_2(t)$
  - $f_3(t) = C_1 f_1(t) + C_2 f_2(t)$
- If $y_3(t) = C_1 y_1(t) + C_2 y_2(t)$ holds true, then we say the system is linear

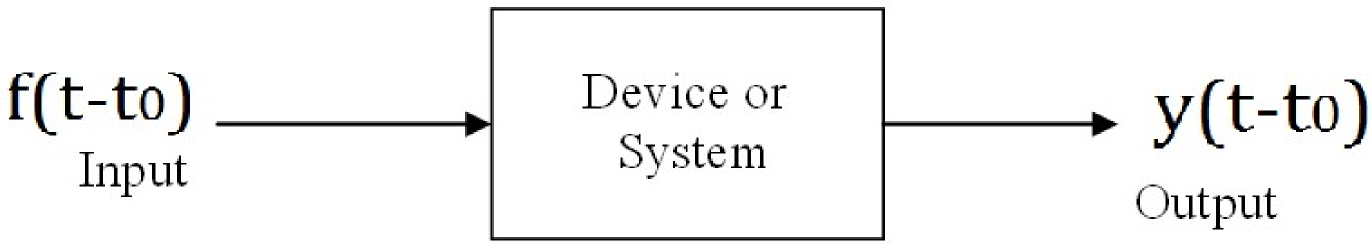
- E.g. Verify $y(t) = \int_{t_0}^{t} f(t)dt$ is linear.
- Assume
  - $f_1(t) \to y_1(t)$, $y_1(t) = \int_{t_0}^{t} f_1(t)dt$
  - $f_2(t) \to y_2(t)$, $y_2(t) = \int_{t_0}^{t} f_2(t)dt$
  - $f_3(t) = C_1 f_1(t) + C_2 f_2(t)$
- $y_3(t) = \int_{t_0}^{t} f_3(t)dt = \int_{t_0}^{t} (C_1 f_1(t) + C_2 f_2(t))dt$

$$= C_1 \int_{t_0}^{t} f_1(t)dt + C_2 \int_{t_0}^{t} f_2(t)dt$$

$$= C_1 y_1(t) + C_2 y_2(t)$$

- That this system is linear is hereby proved.

- E.g. Verify $y(t) = f^2(t)$ is not linear.
- Assume
  - $f_1(t) \rightarrow y_1(t)$, $y_1(t) = f_1^2(t)$
  - $f_2(t) \rightarrow y_2(t)$, $y_2(t) = f_2^2(t)$
  - $f_3(t) = C_1 f_1(t) + C_2 f_2(t)$
- $y_3(t) = f_3^2(t) = (C_1 f_1(t) + C_2 f_2(t))^2$
  $\neq C_1 y_1(t) + C_2 y_2(t)$
- That this system is not linear is hereby proved.



- Time-invariance
- Delayed inputs cause equally delayed outputs

- How to decide whether time-invariant or not?
- Assume
  - $f_1(t) \rightarrow y_1(t)$
  - $f_2(t) = f_1(t - t_0)$
  - $f_2(t) \rightarrow y_2(t)$
- If $y_2(t) = y_1(t - t_0)$ holds true, then we say the system is time-invariant

- E.g. Verify $y(t) = \int_{t_0}^{t} f(t)dt$ is time-invariant.
- Assume
  - $f_1(t) \to y_1(t),\, y_1(t) = \int_{t_0}^{t} f_1(t)$
  - $f_2(t) = f_1(t - t_0)$
  - $f_2(t) \to y_2(t)$
- $y_2(t) = \int_{t_0}^{t} f_2(t)dt = \int_{t_0}^{t} f_1(t - t_0)dt$

  $y_1(t - t_0) = \int_{t_0}^{t} f_1(t - t_0)$

  $y_2(t) = y_1(t - t_0)$

  That this system is time-invariant is hereby proved.

- E.g. Verify $y(t) = \int_{t_0}^{t} f(3-t)dt$ is not time-invariant.
- Assume
  - $f_1(t) \rightarrow y_1(t)$, $y_1(t) = \int_{t_0}^{t} f_1(3-t)$
  - $f_2(t) = f_1(t-t_0)$
  - $f_2(t) \rightarrow y_2(t)$
- $y_2(t) = \int_{t_0}^{t} f_2(t)dt = \int_{t_0}^{t} f_1((3-t)-t_0)dt$

  $y_1(t-t_0) = \int_{t_0}^{t} f_1(3-(t-t_0))$

  $y_2(t) \neq y_1(t-t_0)$

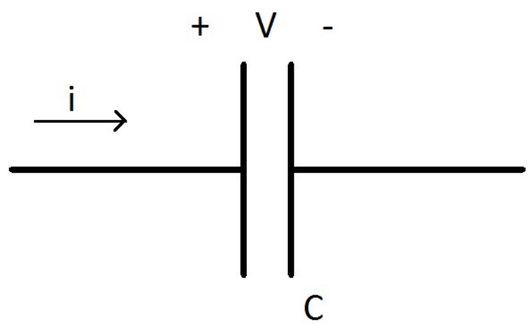
  That this system is not time-invariant is hereby proved.

3.4 First-order RC & RL Circuits

- Capacitor and inductor

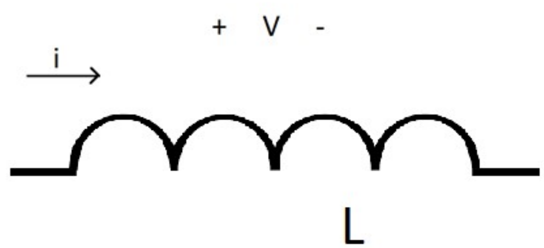
Capacitor

- $i(t) = C\frac{dv(t)}{dt}$
- $V(t) = (\int_{t_0}^{t}\frac{i(t)}{C}) + V(t_0)$

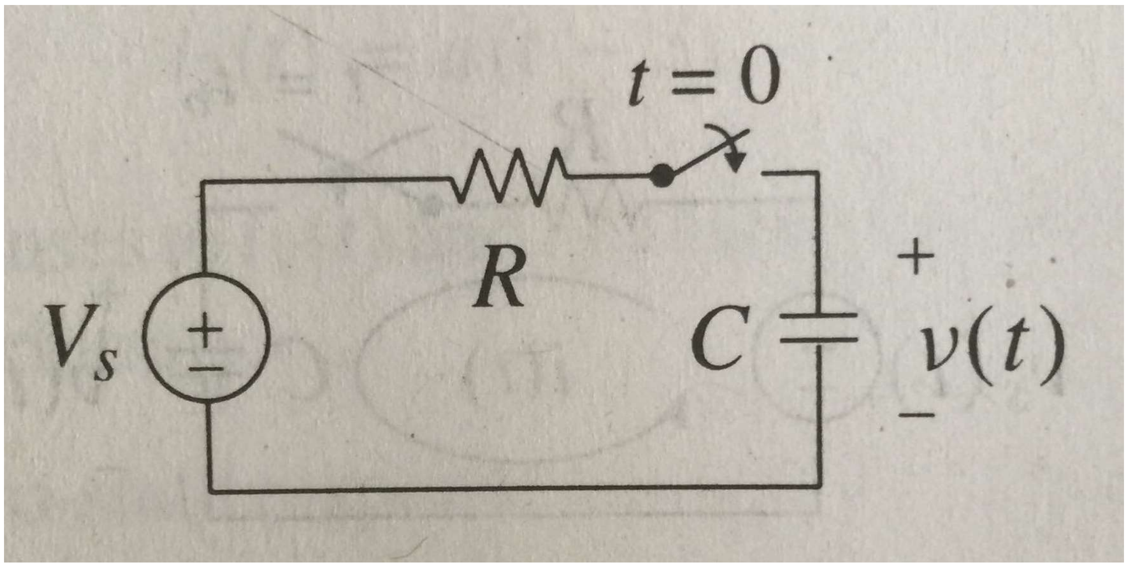


Inductor

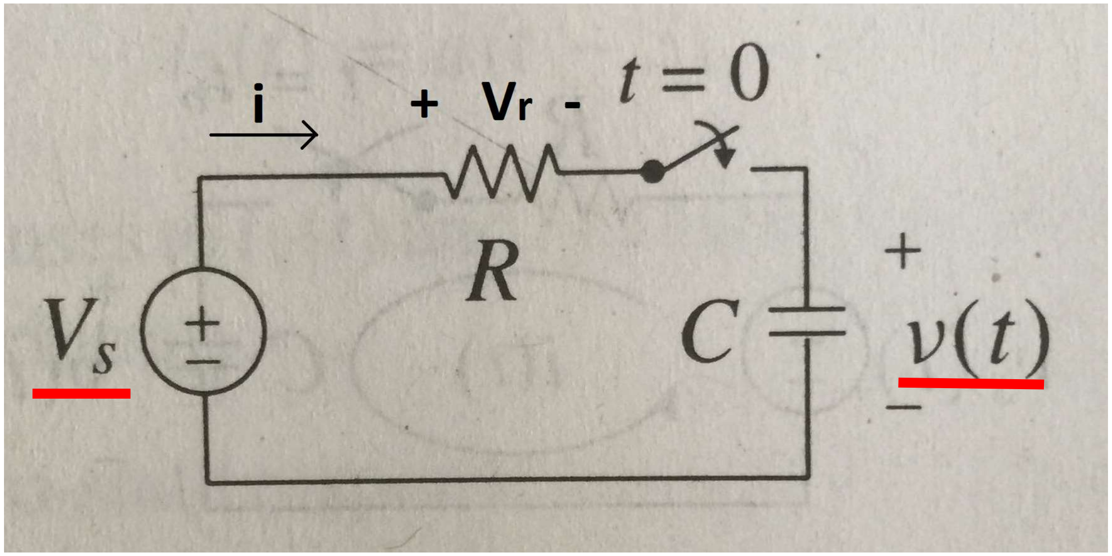
- V(t)= $L\frac{di(t)}{dt}$
- $i(t) = (\int_{t_0}^{t}\frac{V(t)}{L}) + i(t_0)$



- RC-circuit response to constant sources

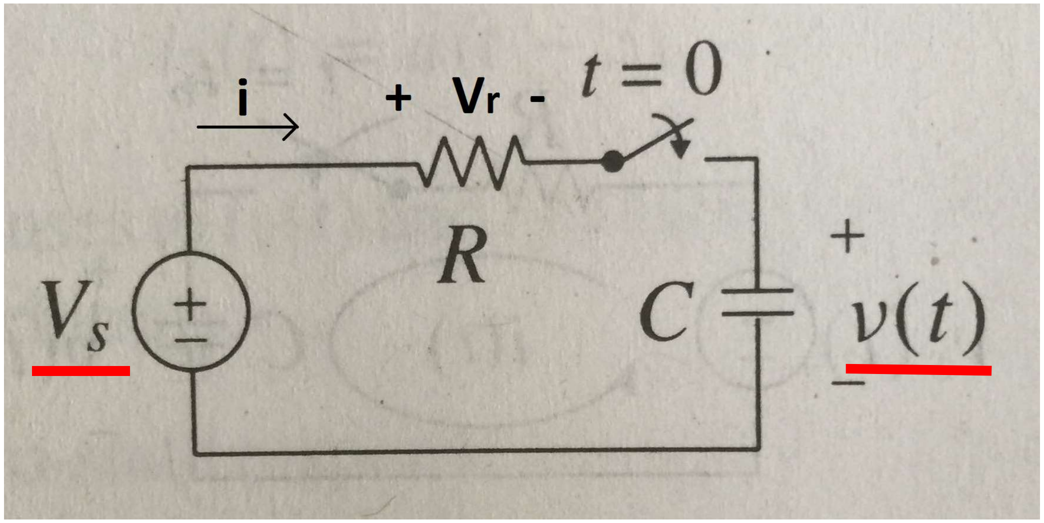


- KVL
  - Vs = Vr + V
- Vs = iR + V
- Vs = $(C\frac{dv(t)}{dt})$R + V
- $\frac{dv}{dt} + \frac{1}{RC}v = \frac{1}{RC}V_s$



$$i(t) = C\frac{dv(t)}{dt}$$

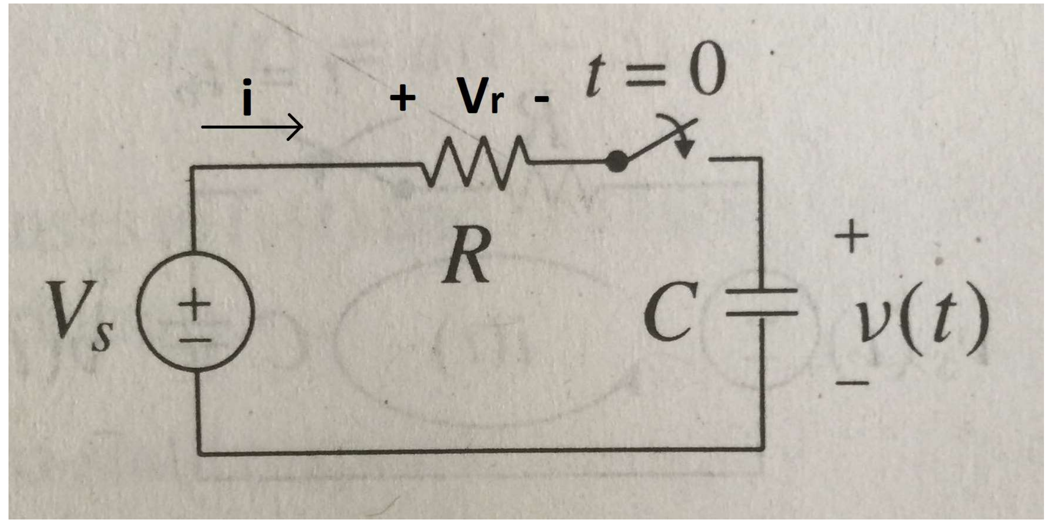
$$V(t) = (\int_{t_0}^{t}\frac{i(t)}{C}) + V(t_0)$$



- $\frac{dv}{dt} + \frac{1}{RC} v = \frac{1}{RC} V_s$
- $v = Ae^{-\frac{t}{RC}} + V_s$
- When t = 0, $v(0) = v(0-) = A + V_s$
- A = v(0-) - Vs

3.3 Linearity, Time Invariance, & LTI System

- Linearity
- $y(t) = \underbrace{y(t_0)}_{\text{Zero-input response}} + \underbrace{\int_{t_0}^{t} f(\tau)d\,\tau}_{\text{Zero-state response}}$
- A system is said to be linear if its output is a sum of distinct zero-input and zero-state responses that vary linearly with the initial state of the system and linearly with the system input, respectively.



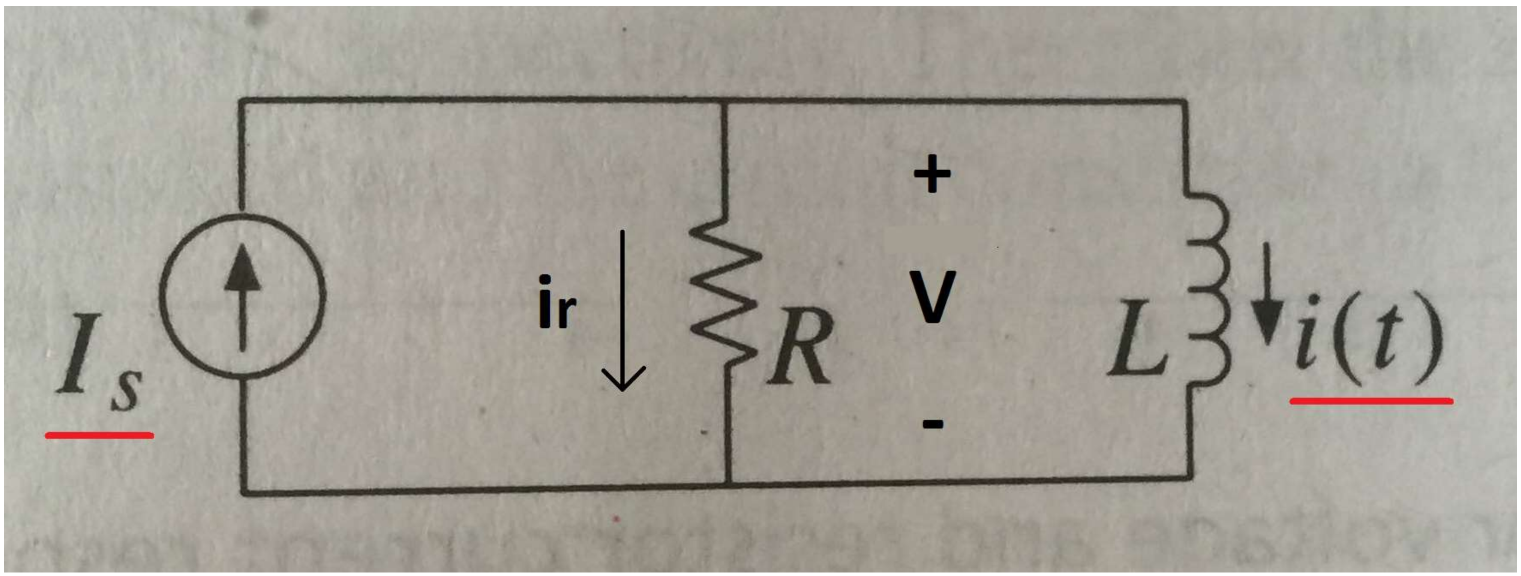
- $\frac{dv}{dt} + \frac{1}{RC} v = \frac{1}{RC} V_s$
- $v = Ae^{-\frac{t}{RC}} + V_s$
- A = v(0-) – Vs

$$v = (v(0-) - V_s)e^{-\frac{t}{RC}} + V_s$$

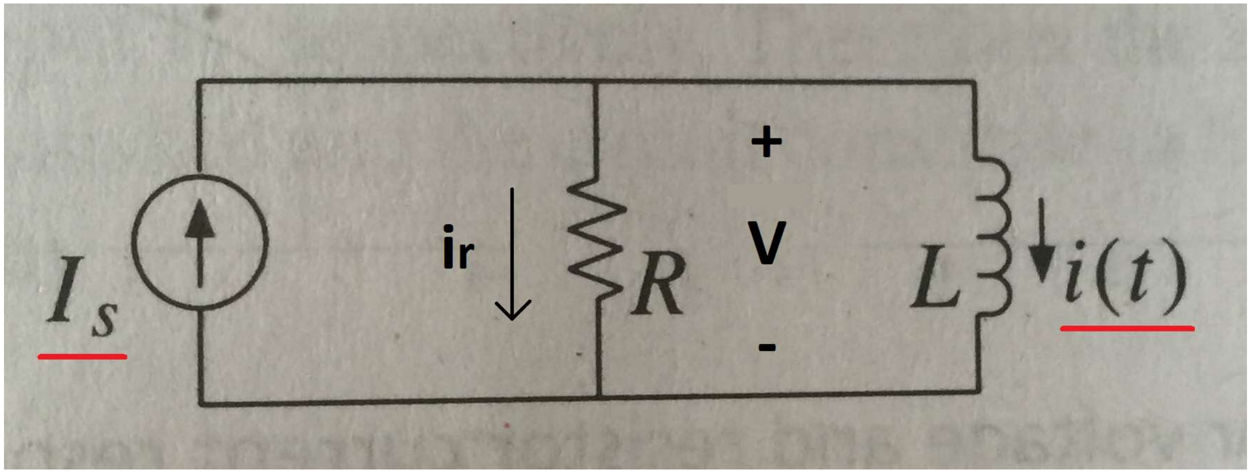
$$v = e^{-\frac{t}{RC}} v(0-) + (1 - e^{-\frac{t}{RC}})V_s$$

$$v = v_{zero-inpu} + v_{zero-state}$$

- RL-circuit response to constant sources



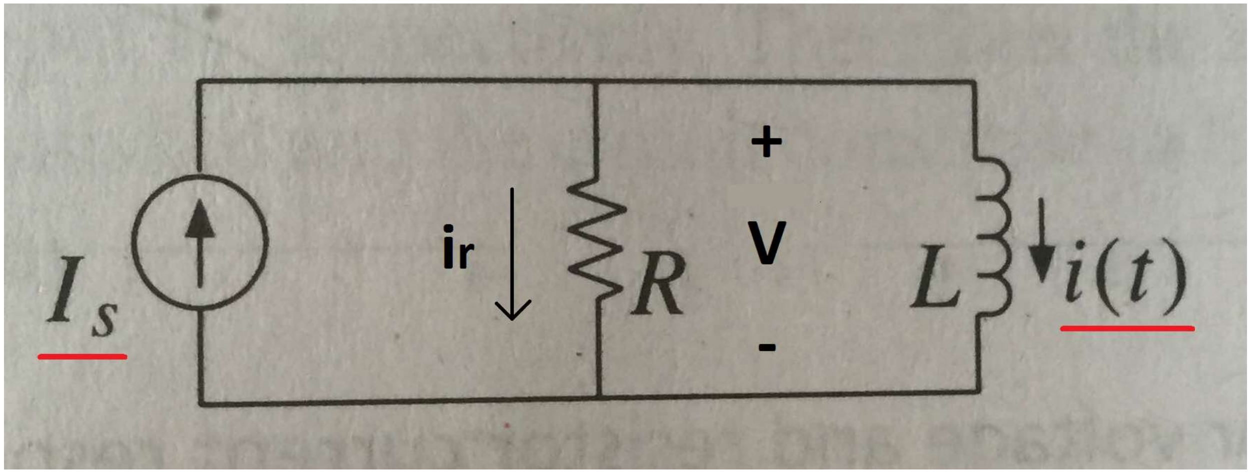
- KCL
  - $I_s = i_r + i$
- $I_s = \frac{V}{R} + i$
- $I_s = \frac{(L\frac{di(t)}{dt})}{R} + i$
- $\frac{di}{dt} + \frac{R}{L} i(t) = \frac{R}{L} I_s$



$V(t) = L\frac{di(t)}{dt}$

$i(t) = (\int_{t_0}^{t} \frac{V(t)}{L}) + i(t_0)$

- $\frac{di}{dt} + \frac{R}{L} i(t) = \frac{R}{L} I_s$

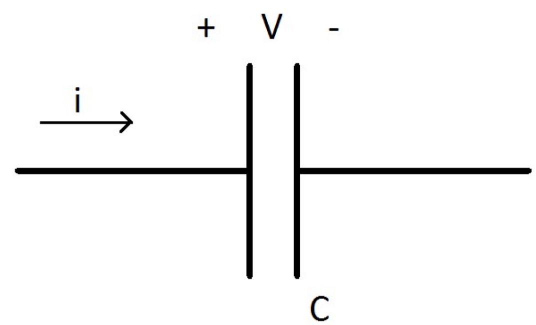


- $i = (i(0-) - I_s)e^{-\frac{t}{L/R}} + I_s$
- $i = e^{-\frac{t}{L/R}} i(0-) + (1 - e^{-\frac{t}{L/R}}) I_s$

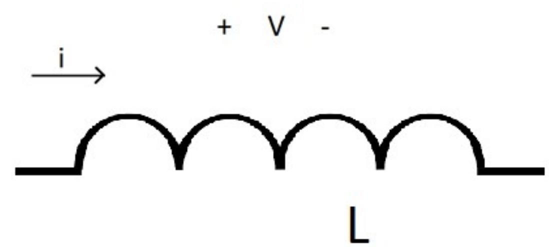
$$i = i_{zero-input} + i_{zero-state}$$

- Initial energy

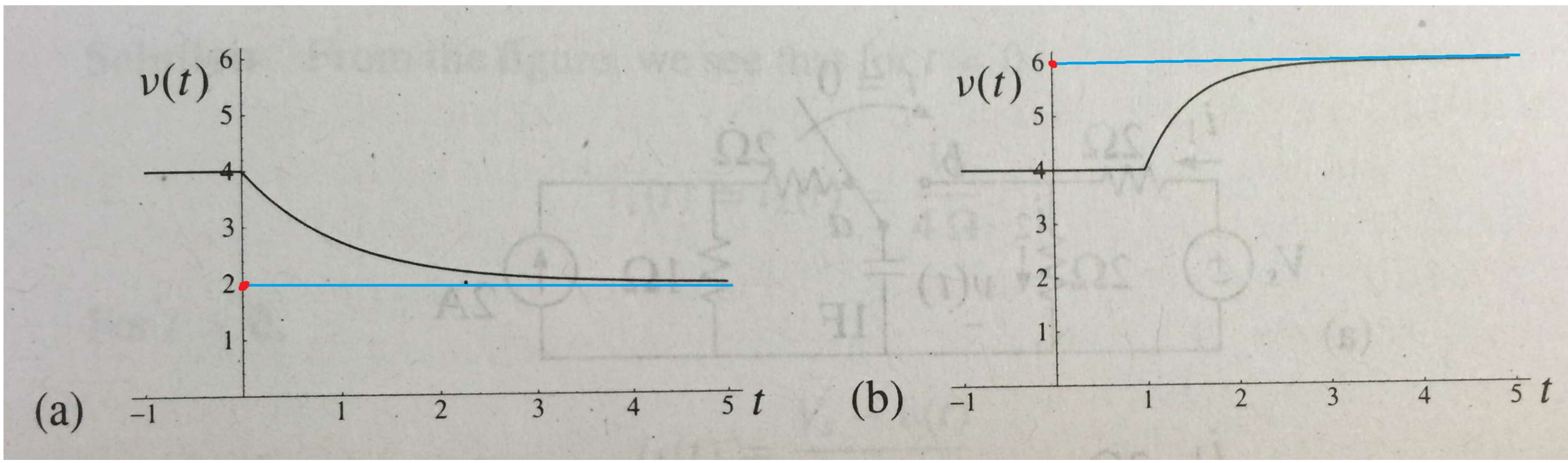
- $E_c = \frac{1}{2}CV^2$
- $E_c = \frac{1}{2}CV(0-)^2$



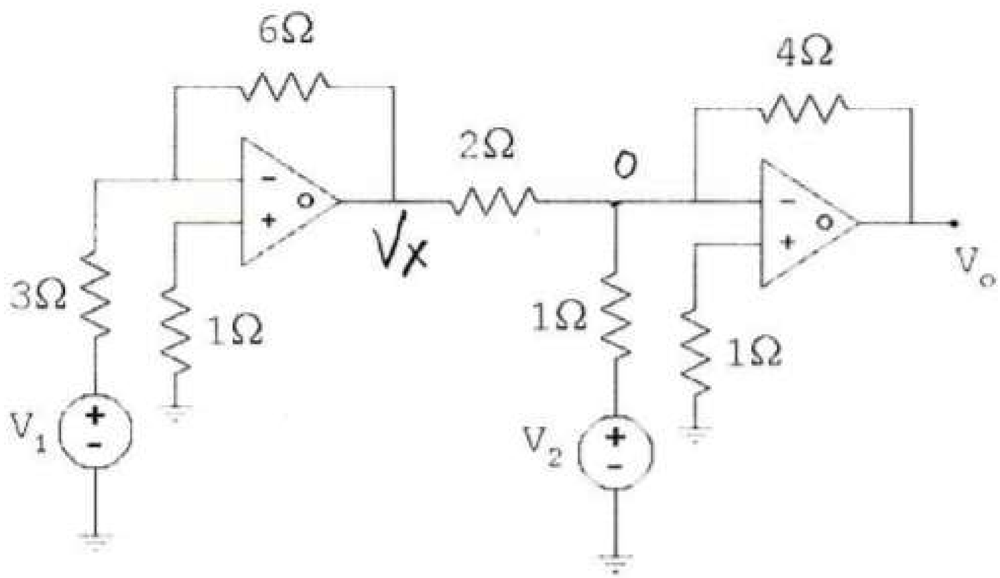
- $E_L = \frac{1}{2}LI^2$
- $E_L = \frac{1}{2}LI(0-)^2$



• Steady state

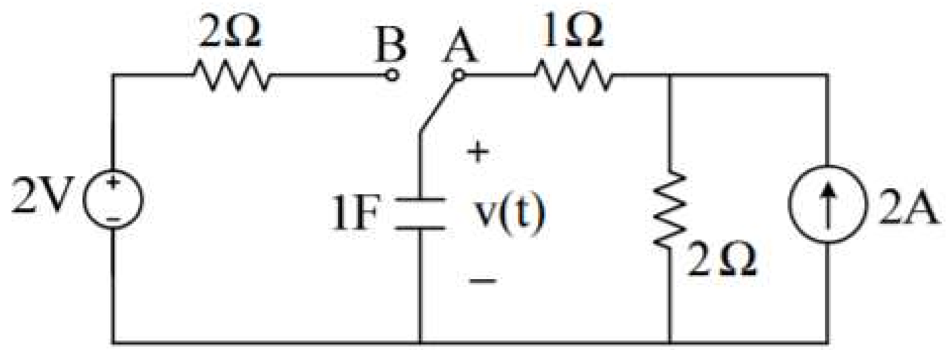


4. (10 pts) In the following circuit, assuming linear operation and ideal op-amp approximation, express the output voltage V_o, in terms of V_1 and V_2.



$V_o =$ $4V_1 - 4V_2$

Problem 4 (25 points)



Assume the switch has been in position A for a long time. It moves to position B at $t = 0$.

a) (5 points) Write the 1st order ODE of v(t) for $t > 0$.

b) (3 points) Find the initial value of v(t) at $t = 0^-$.

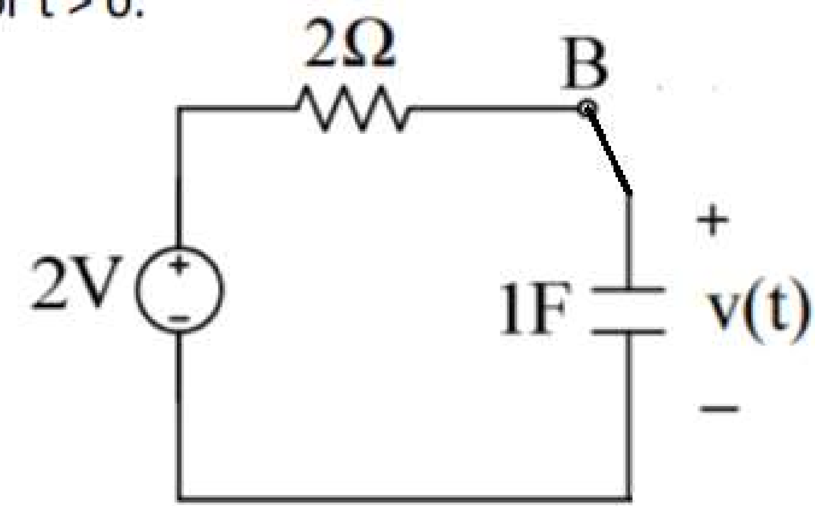
c) (8 points) Solve v(t) for $t > 0$.

d) (3 points) What is the zero input component of v(t)?

e) (3 points) What is the zero state component of v(t)?

f) (3 points) What is the steady state value of v(t) for $t > 0$?

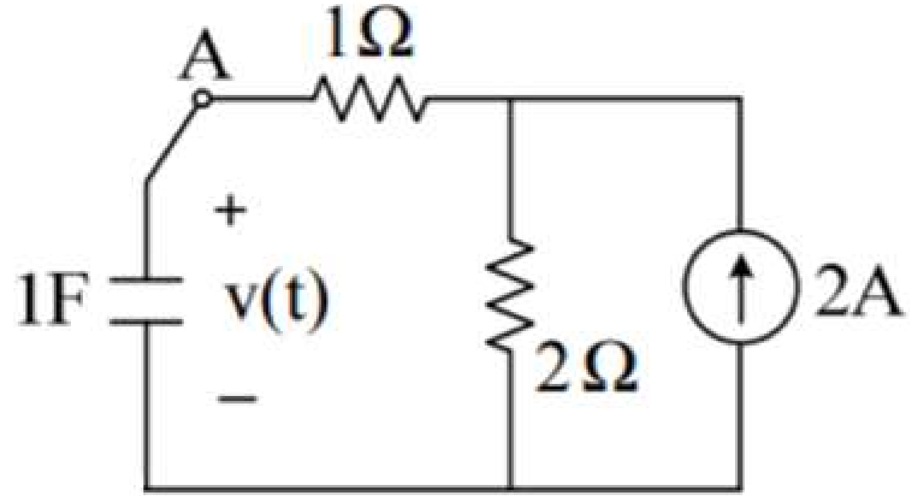
a) (5 points) Write the 1st order ODE of v(t) for t > 0.



$$i(t) = C\frac{dv(t)}{dt}$$

$$V(t) = \left(\int_{t_0}^{t} \frac{i(t)}{C}\right) + V(t_0)$$

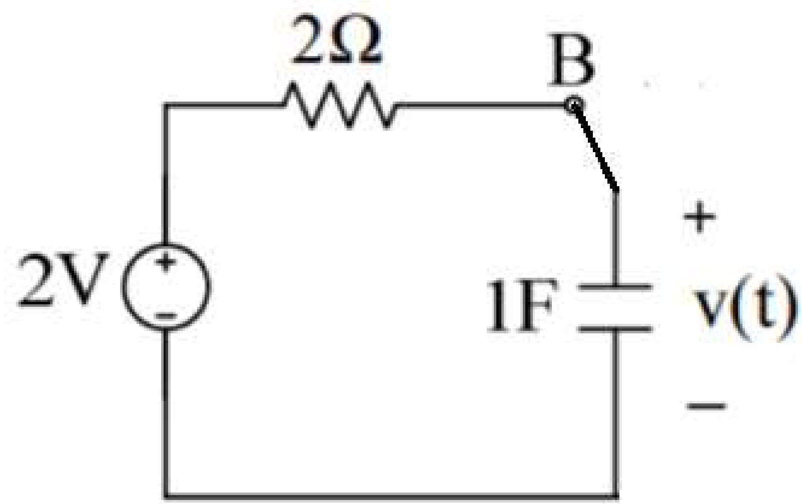
b) (3 points) Find the initial value of v(t) at $t = 0^-$.



$$i(t) = C\frac{dv(t)}{dt}$$

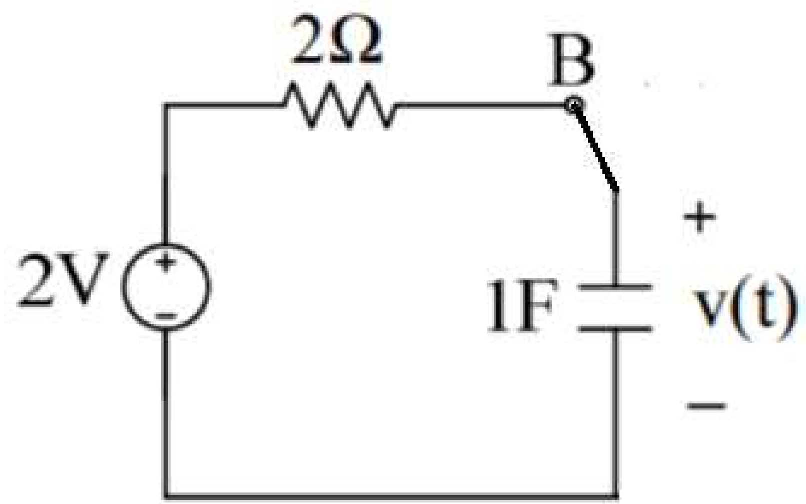
$$V(t) = \left(\int_{t_0}^{t}\frac{i(t)}{C}\right) + V(t_0)$$

c) (8 points) Solve v(t) for t > 0.



- $\frac{dv}{dt} + \frac{1}{2}v = 1$
- $v_{0-} = 4$
- $v = Ae^{-\frac{t}{RC}} + V_s$
- A = v(0-) - Vs

c) (8 points) Solve v(t) for t > 0.



- $\frac{dv}{dt} + \frac{1}{2}v = 1$
- $v_{0-} = 4$
- $v = Ae^{-\frac{t}{RC}} + V_s$
- A = v(0-) - Vs

d) (3 points) What is the zero input component of v(t)?

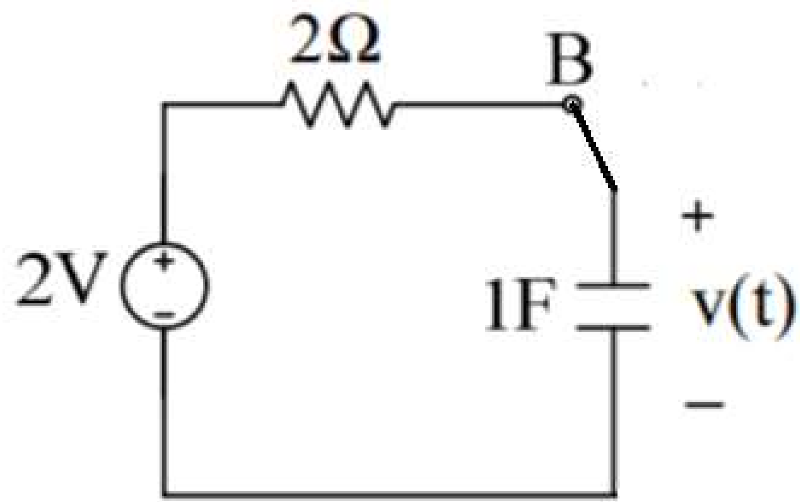
e) (3 points) What is the zero state component of v(t)?

$$v = (v(0-) - V_s)e^{-\frac{t}{RC}} + V_s$$

$$v = e^{-\frac{t}{RC}}v(0-) + (1 - e^{-\frac{t}{RC}})V_s$$

$$v = v_{zero-input} + v_{zero-state}$$

f) (3 points) What is the steady state value of v(t) for t > 0?



Questions?


THANK YOU