HKN ECE 310 Quiz 2 Review Session

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Discrete Time Fourier Transform

- $X_d(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$
- $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_d(\omega)e^{j\omega n}d\omega$
- Important Properties:
  - Periodicity!
  - Linearity
  - Symmetries (Magnitude, angle, real part, imaginary part)
  - Time shift and modulation
  - Product of signals and convolution
  - Parseval's Relation
- Know your geometric series sums!

Discrete Fourier Transform

- $X[k] = \sum_{n=0}^{N-1} x[n]e^{\frac{-j2\pi kn}{N}}$
- $x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{\frac{j2\pi kn}{N}}$

- What is the relationship between the DTFT and the DFT?

- $\omega_k = \frac{2\pi k}{N}$

Discrete Fourier Transform Properties

- *Circular* shift
- *Circular* modulation
- *Circular* convolution
- Parseval's Relation

Windowing and Spectral Analysis

- Signals cannot go to infinity
  - Therefore, we need to window
- There are many different windows
  - Rectangular (boxcar)
  - Hamming
  - Hanning
  - Triangular
  - Kaiser
- More on advantages/disadvantages later

Windowing and Spectral Analysis

- What happens when we dictate that $x[n] = \cos(\omega_0 n)$ is of finite duration N?
  - Derivation on page 54 of textbook

- Spectral Analysis: Resolving different sinusoidal frequency components in a signal
- The DTFT of a finite sinusoidal signal has main lobe width of $\frac{4\pi}{N}$ where N is the # of samples in the signal
- Resolution can be defined in different ways
  - Full lobe resolution vs. Half lobe resolution

Full-Lobe vs. Half-Lobe Resolution

- Suppose we represent a cosinusoid as $Acos(\Omega T)$
- The lobe centers of two cosinusoids will be located at $\Omega_0 T \ and \ \Omega_1 T$
  - Remember that the half-width of each lobe is $\frac{2\pi}{N}$
- Full-Lobe
  - To prevent crossover: $\Omega_0 T + \frac{2\pi}{N} < \Omega_1 T - \frac{2\pi}{N} \rightarrow \Omega_1 - \Omega_0 > \frac{4\pi}{NT}$
- Half-Lobe
  - $\Omega_0 T < \Omega_1 T - \frac{2\pi}{N} \rightarrow \Omega_1 - \Omega_0 > \frac{2\pi}{NT}$

Zero-Padding

- We can improve the resolution of the DFT simply by adding zeros to the end of the signal
- This doesn’t change the frequency content of the DTFT!
- Instead, it increases the number of samples the DFT takes of the DTFT
- This can be used to improve spectral analysis

Window Comparisons

- Rectangular (boxcar)
  - Maintains width of the main lobe , thus better resolution
  - Poor side lobe attenuation, can lead to resolution errors
- Hamming
  - Doubles the width of the main lobe, thus poorer resolution
  - Greatly reduces side lobes, prevents mistaking side lobes as main lobes of other frequencies
- Kaiser
  - Optimal