HKN ECE 310 Quiz 5 Review Session

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Linear Phase Filters

- Type 1
  - Odd Length and Even Symmetric
- Type 2
  - Even Length and Even Symmetric
- Type 3
  - Odd Length and Odd Symmetric
- Type 4
  - Even Length and Odd Symmetric

Linear Phase Filter Design

- Why linear phase?
  - Satisfy causality and filter should have finite number of terms
- Generalized Linear Phase vs. Linear Phase
- Begin with ideal magnitude response, D(ω), for a length N filter
- Introduce linear phase
  - $G(\omega) = D(\omega)e^{-j\omega M}, where\ M = \frac{N-1}{2}$
- Perform inverse DTFT of $G(\omega)$ to obtain $g[n]$
  - $g[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} G(\omega)\, e^{j\omega n} d\omega$
- Window with window function $w[n]$
  - $h[n] = g[n]w[n]$

Exercise 1

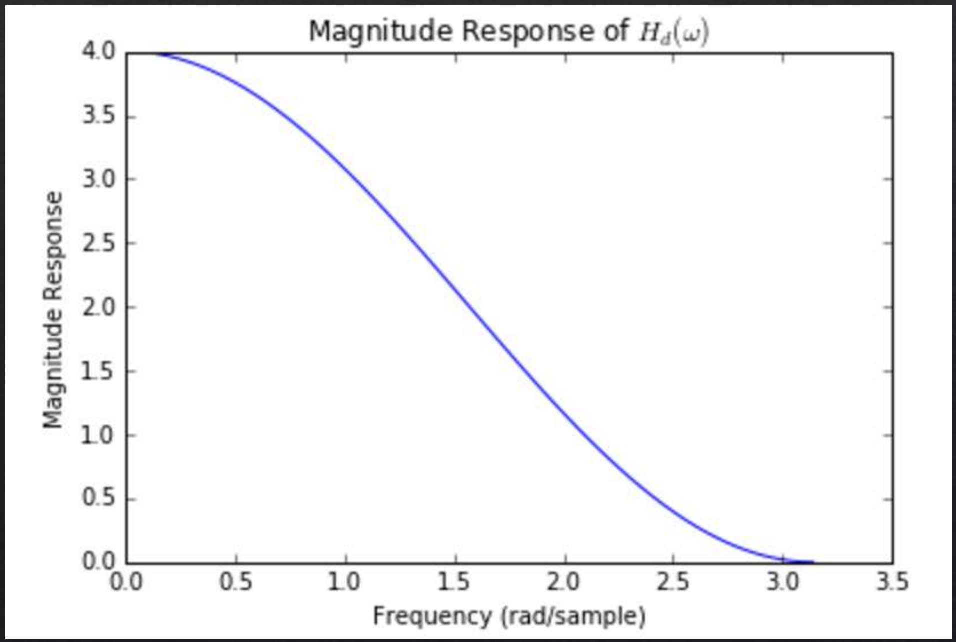
- Determine the frequency response of a filter represented by:
  - $h[n] = \{a_0, a_1, a_0\}$
  - How can we make this a low-pass filter?
  - Does this filter have linear phase or generalized linear phase?

Exercise 1 Solution

- $H_d(\omega) = e^{-j\omega}(2a_0\cos(\omega) + a_1)$

- $a_1 = 2a_0$
  - This forces $H_d(\omega) = 0$ at $\omega = \pm\pi$

- Depends on the range of your phase plot!
  - If we limit from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$ $\rightarrow$ GLP
  - If we limit from $-\pi$ to π $\rightarrow$ Linear

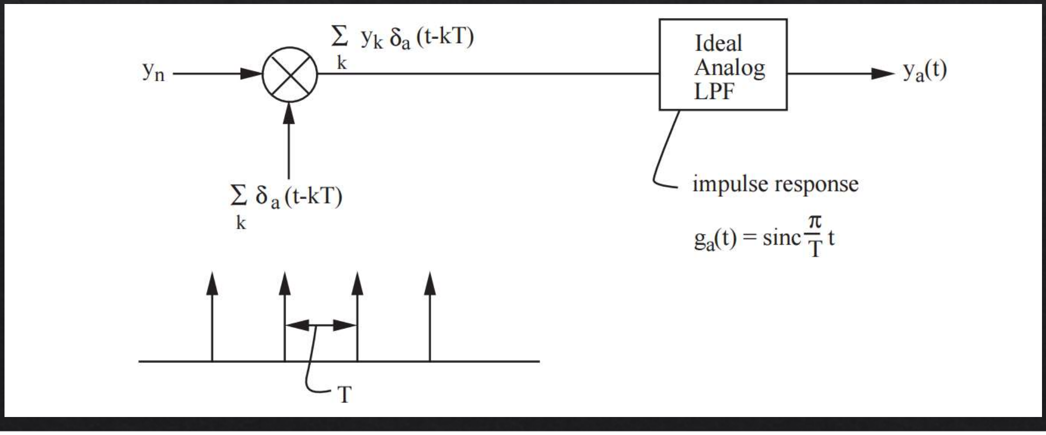


Tips for Linear Phase Filter Design

- Perform the IDTFT integral for high-pass filters from $0\ to\ 2\pi$.
  - This will allow you to do one integral instead of two
- Only design low-pass filters!
  - Why?
    - Easier
  - How?
    - Use modulation property (multiplication by cosine) to obtain other filters from low-pass filters

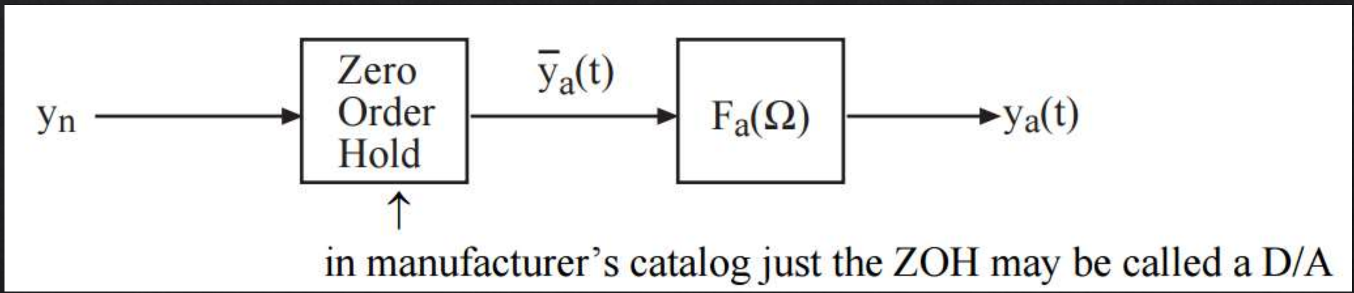
Ideal D/A

- Want $Y_a(\Omega) = Y_d(\Omega T)$, but we want to only take one copy of the DTFT
  - Thus, we should low-pass filter from $\Omega = \pm\frac{\pi}{T}$ (domain of the central copy of the DTFT)
- Inverse CTFT of $T rect\left(\frac{\Omega}{\frac{2\pi}{T}}\right) = sinc(\frac{\pi}{T}t)$
- Remember that multiplication in the frequency domain is convolution in the time domain
  - Thus, $y_a(t) = \sum_{n=-\infty}^{\infty} y_n sinc[\frac{\pi}{T}(t - nT)]$, where y_n is obtained by multiplying y[n] by an impulse train



Realizable D/A: Zero-Order Hold

- Ideal D/A is not practical because generating delta impulses is not achievable
- Zero-Order Hold (ZOH) gives us a suitable approximation to the Ideal D/A
- The ZOH multiplies each sample by a rectangular pulse of width T (our sampling rate)
  - Thus, $\overline{y_a}(t) = \sum_n y_n p_a(t - nT)$ where p_a is the rectangular pulse provided by the ZOH
- $F_a(\Omega)$ is an analog filter that corrects the distortion presented by the ZOH



Exercise 2

- Derive the expression for $\overline{Y_a}(\Omega)$ given $y_n \xleftrightarrow{DTFT} Y_d(\omega) = Y_d(\Omega T)$

Exercise 2 Solution

- Derive the expression for $\overline{Y_a}(\Omega)$ given $y_n \overset{DTFT}{\longleftrightarrow} Y_d(\omega) = Y_d(\Omega T)$

$$G_a(\Omega) = \int_0^T 1 \bullet e^{-j\Omega t}\, dt$$

$$= \left. \frac{e^{-j\Omega t}}{-j\Omega} \right|_0^T = \frac{e^{-j\Omega T} - 1}{-j\Omega}$$

$$= \frac{e^{-j\Omega \frac{T}{2}} \left(e^{-j\Omega \frac{T}{2}} - e^{j\Omega \frac{T}{2}} \right)}{-j\Omega} = e^{-j\Omega \frac{T}{2}} \frac{2 \sin \frac{\Omega T}{2}}{\Omega}$$

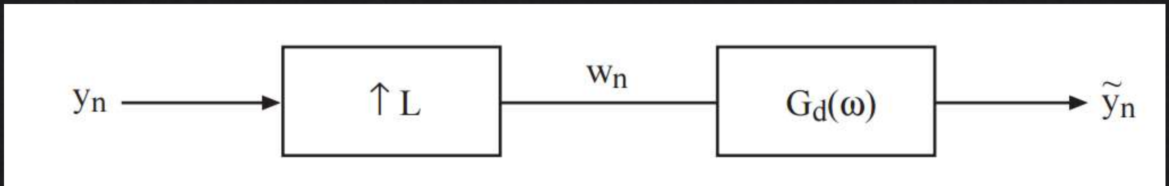
$$= T\, e^{-j\Omega \frac{T}{2}} \operatorname{sinc} \frac{\Omega T}{2}$$

Thus,

$$\overline{Y}_a(\Omega) = T\, e^{-j\Omega \frac{T}{2}} \operatorname{sinc} \left(\frac{\Omega T}{2} \right) Y_d(\Omega T)$$

Upsampling

- If we upsample by L, we will interpolate L – 1 zeros between each sample
  - $y[n] = x[\frac{n}{L}]$ if n mod L = 0
  - 0 else
- What happens in the frequency domain?
  - Think about what happens when we oversample and signal, i.e. above Nyquist?
- What does the frequency response look like after upsampling?
  - Shrink x-axis by factor of L
- What should $G_D(\omega)$ be in order to obtain a desirable frequency response?
  - Remove extra copies and correct amplitude for conservation of energy



Upsampled D/A

- Upsampling prior to D/A conversion can make recovery simpler
  - i.e. Compensator $F_a(\Omega)$ can be simpler to implement
- Upsampling effectively increases our sampling frequency, thus our ZOH pulse can be narrower and give us a better staircase approximation
- This 'smoother staircase' will be easier to rectify with the compensator
  - i.e. the transition bandwidth will be larger
- In the frequency domain, we see the frequency axis compress by L; however, the analog frequencies upon recovery do not change!

Downsampling

- If we downsample by D, we keep every D^{th} sample (decimate the rest)
  - $y[n] = x[Dn]$
- What happens in the frequency domain?
  - Frequency response stretches by a factor of D
  - Amplitude reduces by a factor of D (think conservation of energy)
- Anti-aliasing filter prevents downsampling from aliasing our signal
  - $A(\omega) = LPF \ with \ \omega_c = \frac{\pi}{D}$