UNDERSTANDING DEEP LEARNING REQUIRES RETHINKING GENERALIZATION

Aldo Lamarre [1] Matthew C. Scicluna [2]

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[1]Département d'Informatique et de Recherche Opérationnelle
Université de Montréal

[2]Montréal Institute of Learning Algorithms
Université de Montréal

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Introduction

Questions

Main Question

What distinguishes Neural Networks that generalize well from those that don't?

- Capacity ?
- Regularization ?
- How we train the model?

Questions

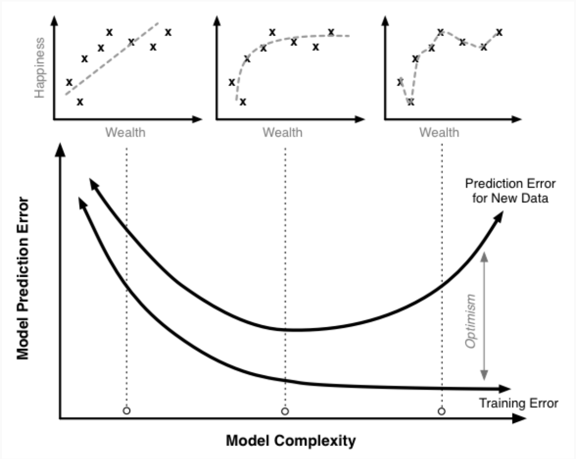

Figure 1: Traditional view of generalization. Image taken from [1]

Motivation

Why do we care about the problem?

- Make neural networks more interpretable
- May lead to more principled and reliable model architecture design

Background

Previous Approaches

Statistical Learning Theory gives bounds on the Generalization Error using:

- VC Dimension
- Rademacher Complexity
- Uniform Stability

Theory suggests that some regularization helps (including Early Stopping)

Related Work

In 2016 Hardt et al. gives an Upper bound on Generalization error on model using SGD using uniform stability [2]

BUT

Uniform stability is a property of a learning algorithm and is not affected by the labelling of the training data.

Limitations

Main Message
Statistical Learning Theory is insufficient in that it cannot distinguish between neural networks with dramatically different generalization performance.

This is demonstrated in the paper [3]. The central finding:

Deep neural networks easily fit random labels

Results

Experiment

Setup: trained several standard architectures on the data with various modifications:

1. True labels $\rightarrow$ No modifications
2. Random labels $\rightarrow$ randomly changed some labels
3. shuffled pixels $\rightarrow$ apply some fixed permutation of pixels to all images
4. Random pixels $\rightarrow$ apply some random permutation of pixels to all images
5. Gaussian $\rightarrow$ Generate pixels for all images from a Gaussian

Main Results

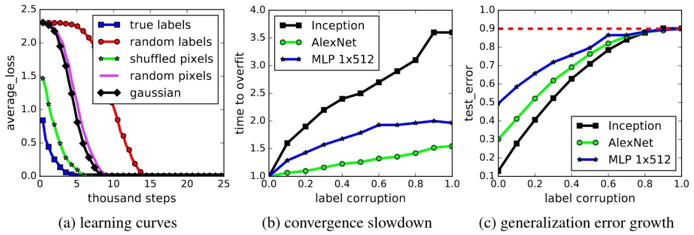

Figure 2: Fitting random labels and random pixels on CIFAR10.

Results

In most cases, the training error went to zero while test error was high

Notice:
the model capacity, hyperparameters, and the optimizer remained the same!

Results

Explicit regularization may improve generalization performance, but is neither necessary nor by itself sufficient for controlling generalization error

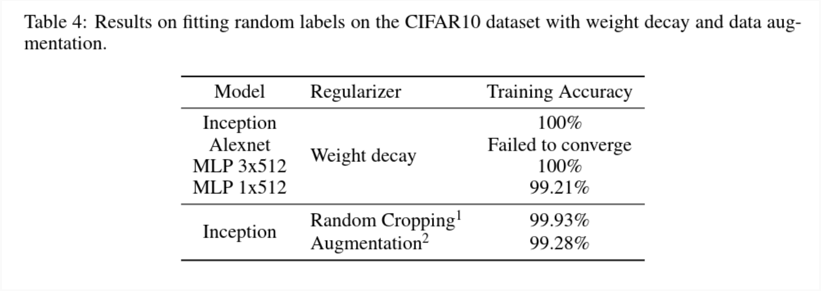
Table 4: Results on fitting random labels on the CIFAR10 dataset with weight decay and data augmentation.

Model	Regularizer	Training Accuracy
Inception	Weight decay	100%
Alexnet		Failed to converge
MLP 3x512		100%
MLP 1x512		99.21%
Inception	Random Cropping[1]	99.93%
	Augmentation[2]	99.28%

Technical dive

Technical dive

Some definitions:

- **Representational Capacity**: A models ability to fit a wide variety of functions:
- **Effective Capacity**: The functions that the Learning Algorithm is capable of learning e.g. imperfection of optimization algorithm.

Finite-sample expressivity

Theorem
There exists a two-layer neural network with ReLU activations and $2n + d$ weights that can represent any function on a sample of size n in d dimensions.

Proof

Lemma 1
For any two interleaving sequences of n real numbers
$b_1 < x_1 < b_2 < x_2 \cdots < b_n < x_n$, the $n \times n$ matrix
$A = [max\{x_i - b_j, 0\}]_i j$ has full rank. Its smallest eigenvalue is
$min_i\{x_i - b_i\}$

Proof

For weight vectors $w, b \in R^n$ and $a \in R^d$, consider the function $c : R^n \to R$,

$$c(x) = \sum_{j=1} w_j \, max\{a^T x - bj, 0\}$$

Proof

For weight vectors $w, b \in R^n$ and $a \in R^d$, consider the function $c : R^n \rightarrow R$,

$$c(x) = \sum_{j=1} w_j \, max\{a^T x - bj, 0\}$$

- This can be done trivially with a depth 2 neural network with relu.

Proof

For weight vectors $w, b \in R^n$ and $a \in R^d$, consider the function $c : R^n \to R$,

$$c(x) = \sum_{j=1} w_j \, max\{a^T x - bj, 0\}$$

- Now, fixing a sample $S = z_1, \ldots, z_n$ of size n and a target vector $y \in R_n$. We need to find weights a, b, w so that $y_i = c(z_i)$ for all $i \in \{1, \ldots, n\}$

Proof

For weight vectors $w, b \in R^n$ and $a \in R^d$, consider the function $c : R^n \to R$,

$$c(x) = \sum_{j=1} w_j \, max\{a^T x - bj, 0\}$$

- First, choose a and b such that with $x_i = a_i^T z_i$ we have the interleaving property $b_1 < x_1 < b_2 < \cdots < b_n < x_n$ Next, consider the set of n equations in the n unknowns w,

  $$y_i = c(z_i), i \in \{1, \ldots, n\}$$

  We have $c(z_i) = Aw$, where $A = [max\{x_i b_i, 0\}]_{ij}$ is the matrix of Lemma 1.

Proof

For weight vectors $w, b \in R^n$ and $a \in R^d$, consider the function $c : R^n \to R$,

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- Now, fixing a sample $S = z_1, \ldots, z_n$ of size n and a target vector $y \in R_n$. We need to find weights a, b, w so that $y_i = c(z_i)$ for all $i \in \{1, \ldots, n\}$
- We chose a and b so that the lemma applies and hence A has full rank. We can now solve the linear system $y = Aw$ to find suitable weights w.

Discussion

Some Thoughts...

1. Our favourite papers are the ones that shed light on truths that are taken for granted.
2. Its obvious that randomizing the labels would eliminate generalizability, but explaining why is not!
3. The paper doesn't really make many conclusions of its own.
4. The most important result is that a depth 2 neural network with relu activation can learn well overlearn any function.
5. This result is the most promising to improve with says a test set and other technique to prevent simply learning every point.

References

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