#1 Redo $W^{(\#)}$ calculation:

$M \hat{=}$ Mathematica aided

$M \hat{=}$ equal to former calc

08/27/14

- $\varkappa^2 = \frac{1}{r^3}\partial_r\left[(r^2\Omega)^2\right] \Rightarrow \boxed{\varkappa = \sqrt{2}\cdot\sqrt{\Omega(2\Omega + r\Omega')}}$

$$\Rightarrow \boxed{\varkappa' = \partial_r \varkappa = \frac{r(\Omega')^2 + \Omega(5\Omega' + r\Omega'')}{\sqrt{2}\cdot\sqrt{\Omega(2\Omega+r\Omega')}} = \frac{r(\Omega')^2+\Omega(5\Omega'+r\Omega'')}{\varkappa}}$$

- using: $\log x \equiv \log_e x = a\cdot\log_{10} x$ with $a = \frac{\log_e x}{\log_{10} x} = \log_e 10 \approx 2{,}3026$

  ↳ so if to base 10, just use factor!

  ↳ better: $\boxed{\log x \equiv \log_a x \Rightarrow \partial_x \log x = \frac{1}{x}\cdot\frac{1}{\ln(a)}}$ & a chosen later!

- $$\boxed{v(1-v^2) = \frac{1}{\varkappa^3}\cdot\Big[\underbrace{(-m\varkappa^2\Omega + m^3\Omega^3)}_{\equiv B_0} + \underbrace{(\varkappa^2 - 3m^2\Omega^2)}_{\equiv B_1}\omega + \underbrace{(3m\Omega)}_{\equiv B_2}\omega^2 + \underbrace{(-1)}_{\equiv B_3}\omega^3\Big] = \frac{1}{\varkappa^3}\sum_{i=0}^{3} B_i\omega^i}$$

- $$v(1-v^2)^2 = \frac{1}{\varkappa^5}\cdot\sum_{i=0}^{5} C_i\omega^i \quad \text{with} \quad C_5 = 1;\ C_4 = -5m\Omega;\ C_3 = -2\varkappa^2 + 10m^2\Omega^2$$

$$C_2 = 6m\varkappa^2\Omega - 10m^3\Omega^3;\ C_1 = \varkappa^4 - 6m^2\Omega^2\varkappa^2 + 5m^4\Omega^4;$$

$$C_0 = -m\varkappa^4\Omega + 2m^3\Omega^3\varkappa^2 - m^5\Omega^5$$

- $$W_{1k} = (\omega - m\Omega)\mathcal{L}_{1k} = (\omega - m\Omega)\cdot\Big[\Big\{D^{(1)}_{ij} + \Big[(1-p) - \frac{2m\Omega}{\omega - m\Omega}\Big]\delta_{ij}\Big\}\mathcal{J}_{jk} + d_{im}\left(\frac{\omega - 3\Omega}{\omega - m\Omega}\right)\frac{\omega^2}{2}\frac{r^3}{G(M_* + M_D)}\mathcal{J}_{ik}$$

$$+ \frac{1}{\Sigma r}\Big\{D^{(1)}_{ik} - \Big[q + \frac{2m\Omega}{\omega - m\Omega}\Big]\delta_{ik}\Big\}\Big]$$

$$= \left(\frac{m\Omega\cdot(-D^{(1)}_{ik} - r\Sigma\cdot(D^{(1)}_{ij} - (-3+p)\delta_{ij})\mathcal{J}_{jk} + (-2+q)\delta_{ik})}{r\Sigma}\right)$$

$$+ \left(\frac{D^{(1)}_{ik} + r\Sigma(D^{(1)}_{ij} - (p-1)\delta_{ij})\mathcal{J}_{jk} - q\,\delta_{ik}}{r\Sigma}\right)\omega$$

$$+ \left(\frac{-3\Omega r^3 d_{im}\mathcal{J}_{ik}}{2G(M_D + M_*)}\right)\omega^2 + \left(\frac{r^3 d_{im}\mathcal{J}_{ik}}{2G(M_D + M_*)}\right)\omega^3 + 0\cdot\omega^4 + 0\cdot\omega^5 = \sum_{i=0}^{5} W^{(i)}_{1k}\cdot\omega^i$$

- $$W_{Nk} = \sum_{i=0}^{5} W^{(i)}_{Nk}\cdot\omega^i \quad \text{with:} \quad W^{(3)}_{Nk} = \frac{-r\,\delta_{N,k}}{p\pi\sigma_0 2G} + \frac{r^3 d_{im}\mathcal{J}_{Nk}}{2G(M_D + M_*)};\ W^{(4)}_{Nk} = W^{(5)}_{Nk} = 0$$

$$W^{(2)}_{Nk} = \frac{r\,m\,\delta_{Nk}3\Omega}{2\pi G\sigma_0 p} - \frac{3r^3\Omega d_{im}\mathcal{J}_{Nk}}{2G(M_D + M_*)}$$

$$W^{(1)}_{Nk} = \frac{(\varkappa^2 - 3m^2\Omega^2)r^2\Sigma\delta_{Nk} + 2Gp\pi\sigma_0[D^{(1)}_{Nk} - q\delta_{Nk} + r\Sigma(D^{(1)}_{Nj} + \delta_{Nj} - p\,\delta_{Nj})\mathcal{J}_{jk})}{2Gp\pi r\Sigma\sigma_0}$$

$$W^{(0)}_{Nk} = \frac{m\Omega[(m^2\Omega^2 - \varkappa^2)r^2\Sigma\delta_{Nk} + 2Gp\pi\sigma_0(-D^{(1)}_{Nk} + (-2+q)\delta_{Nk} - r\Sigma(D^{(1)}_{Nj} - (-3+p)\delta_{Nj})\mathcal{J}_{jk})]}{2Gp\pi r\Sigma\sigma_0}$$

$$-W_{ik} = \nu(1-\nu^2)\,\mathcal{L}_{ik} = \nu(1-\nu^2)\cdot\Big[D^{(2)}_{ij}\partial_{jk} + \{2(1-p)-1\} D^{(1)}_{ij}\partial_{jk} + p(p-1)\delta_{ij}\partial_{jk} + \frac{1}{\varepsilon r}\Big(D^{(2)}_{ik} + (-2q-1)D^{(1)}_{ik} + q(q+1)\,\delta_{ik}\Big)\Big]$$

$$+\nu(1-\nu^2)^2\cdot\left(\frac{-\kappa^2 r\,\delta_{ik}}{2\pi G\sigma_0}\right)$$

$$+\nu(1-\nu^2)A\cdot\left(r D^{(1)}_{ij}\partial_{jk} + r(1-p)\delta_{ij}\partial_{jk} + \frac{1}{\varepsilon r}\left(r D^{(1)}_{ik} - rq\,\delta_{ik}\right) + \frac{\delta_{im}\, r\,\omega^2 r^3 \partial_{ik}}{2\cdot G(M_* + M_D)}\right)$$

$$+\nu(1-\nu^2)B\cdot\left(r^2\delta_{ij}\partial_{jk} + \frac{1}{\varepsilon r}\cdot(r^2\delta_{ik}) + \frac{\delta_{im}\, r^2\cdot\omega^2 r^3 \partial_{ik}}{2G(M_D+M_*)}\right)$$

$$\equiv \nu(1-\nu^2)\cdot F^1 + \nu(1-\nu^2)^2\cdot F^2 + \nu(1-\nu^2)A\cdot(F^{A_0} + F^{A_2}\omega^2) + \nu(1-\nu^2)B\,(F^{B_0}+F^{B_2}\omega^2)$$

− $\nu(1-\nu^2)\cdot A$: − because of B3 $\Rightarrow \mathcal{R} = e$ from here on!

$$A = \partial_r \log\left(\frac{\sigma_0 r}{\kappa^2(1-\nu^2)}\right) = \frac{\cancel{\kappa^2(1-\nu^2)}}{\sigma_0 r}\cdot\frac{(\sigma_0 + r\sigma_0')\,\kappa^2(1-\nu^2) - \sigma_0 r\cdot[(1-\nu^2)\cdot 2\kappa\kappa' + \kappa^2\cdot(-2\nu\nu')]}{\kappa^{\cancel{4}2}(1-\nu^2)^{\cancel{2}}}$$

$$= \frac{(\sigma_0 + r\sigma_0')\kappa^2(1-\nu^2) - \sigma_0 r\cdot[(1-\nu^2)\cdot 2\kappa\kappa' - \kappa^2\cdot 2\nu\nu']}{\sigma_0 r\,\kappa^2\,(1-\nu^2)} \quad \& \quad \nu' = \partial_r\left(\frac{\omega - m\Omega}{\kappa}\right) = \frac{-m\Omega'\kappa - \kappa'(\omega - m\Omega)}{\kappa^2}$$

(a)

$$= \frac{1}{r} - \frac{2\kappa'}{\kappa} - \frac{2\nu\nu'}{\nu^2-1} + \frac{\sigma_0'}{\sigma_0}$$

$$\Rightarrow \underbrace{\nu(1-\nu^2)A}_{\hat{A}\,\equiv} = \sum_{i=0}^{3}\hat{X}_i\,\omega^i \quad \& \quad \hat{X}_0 = \frac{m\Omega\left[\sigma_0' r(m^2\Omega^2-\kappa^2) + \sigma_0\left(-\kappa^2 + 2\kappa\kappa' r + m^2\Omega(\Omega - 2\Omega' r)\right)\right]}{\kappa^3 r\sigma_0};$$

(A)

$$\hat{X}_1 = \frac{\sigma_0'(\kappa^2 - 3m^2\Omega^2)r + \left(\kappa^2 - 2\kappa\kappa' r + m^2\Omega(-3\Omega + 4\Omega' r)\right)\sigma_0}{\kappa^3 r\sigma_0};$$

$$\hat{X}_2 = \frac{m(3\sigma_0'\Omega r + 3\Omega\sigma_0 - 2\Omega' r\sigma_0)}{\kappa^3 r\sigma_0}; \quad \hat{X}_3 = -\frac{\sigma_0' r + \sigma_0}{\kappa^3 r\sigma_0}.$$

− $\nu(1-\nu^2)B$:

$$-B = -\frac{m^2}{r^2} - \frac{4m}{r^2}\frac{\Omega}{\kappa}\frac{r}{(1-\nu^2)}\underbrace{\partial_r\nu}_{\frac{-m\Omega'\kappa - \kappa'(\omega - m\Omega)}{\kappa^2}} + \frac{2m}{r\nu}\frac{\Omega}{\kappa}\underbrace{\partial_r\log\left(\frac{\kappa^2}{\Omega\sigma_0}\right)}_{\frac{\Omega\sigma_0}{\kappa^2}\frac{2\kappa\kappa'\Omega\sigma_0 - \kappa^2(\Omega\sigma_0' + \Omega'\sigma_0)}{\Omega^2\sigma_0^2} = \frac{2\kappa\kappa'\Omega\sigma_0 - \kappa^2(\Omega\sigma_0' + \Omega'\sigma_0)}{\kappa^2\Omega\sigma_0}}$$

$$= -\frac{m^2}{r^2} - \frac{4m}{r^2}\frac{\Omega}{\kappa}\frac{r}{1-\nu^2}\nu' + \frac{2m}{r\nu}\frac{\Omega}{\kappa}\frac{2\kappa\kappa'\Omega\sigma_0 - \kappa^2(\Omega\sigma_0' + \Omega'\sigma_0)}{\kappa^2\Omega\sigma_0}$$

$$\Rightarrow \hat{B} = \nu(1-\nu^2)B = \sum_{i=0}^{3}\hat{Y}_i\,\omega^i \quad \& \quad \hat{Y}_3 = \frac{m^2}{\kappa^3 r^2}; \quad \hat{Y}_2 = \frac{m(2\sigma_0'\Omega r - 3m^2\Omega\sigma_0 + 2\Omega' r\sigma_0)}{\kappa^3 r^2\sigma_0}$$

$$\hat{Y}_1 = \frac{m^2\left(-\kappa^2\sigma_0 + \Omega^2(-4\sigma_0' r + 3m^2\sigma_0)\right)}{\kappa^3 r^2\sigma_0}; \quad \hat{Y}_0 = \frac{m\left[2\sigma_0'\Omega(m^2\Omega^2 - \kappa^2)r + \sigma_0\left(4\kappa'\kappa\Omega r + \kappa^2(m^2\Omega - 2\Omega' r) - m^2\Omega^2(m^2\Omega + 2r\Omega')\right)\right]}{\kappa^3 r^2\sigma_0}$$