Level 3 Condensed Matter Physics

Example Workshop 4

1. Hund's rules and magnetic levels in paramagnetic Cu^{2+} ions

(*a*) Calculate the principal quantum numbers *S*, *L*, and *J* of a Cu^{2+} ion ($1s^2 2s^2 2p^6 3s^2 3p^6 3d^9$) using Hund's rules. Make a sketch of the (ground state energy) levels associated with the magnetic moment of a Cu^{2+} ion in a magnetic field of flux density of 1 Tesla. Label each of the levels with the quantum number representing the projection of the total angular momentum on the field direction. Calculate the energy separation between the levels.

(*b*) How would your answer differ in Cu^{2+} was orbitally quenched (i.e. L = 0 and hence $J = S$)? Again calculate Make a sketch of the (ground state energy) levels associated with the magnetic moment of a Cu^{2+} ion in a magnetic field of flux density of 1 Tesla. Label the levels and calculate the energy separation between the levels.

2. Paramagnetism, Hund's rules and spin-orbit coupling in Sm^{3+} ions.

(a) Calculate *S*, *L* and *J* of the ground state of an Sm^{3+} ion ($4f^5$) stating any assumptions that you make.

(b) Calculate the magnitudes of the atomic spin, orbital and total angular momenta of the Sm^{3+} ion in its ground state (in units of $\hbar$). What are the magnitudes of the three corresponding magnetic moments (in units of μ_B).

(c) State the number of energy levels that correspond to the magnetic moment of an Sm^{3+} ion in the presence of a magnetic field. Calculate the smallest energy separation between two of these levels if the applied magnetic field strength is $H = 1.6 \times 10^6$ A m^{-1}. What frequency of electromagnetic radiation could be used to excite a transition between two of these levels?

(d) Assuming the ions are magnetically isolated from one another in the solid, comment on the applicability of Curie's law to the paramagnetic susceptibility at 1 K and 300 K (**Hint**: Use the condition adopted to obtain Curie's law from the Brillouin function form of the magnetisation).

(e) Calculate the maximum measureable magnetisation of a solid consisting of 1 mole of Sm^{3+} ions.

(f) The spin-orbit interaction that couples $\underline{S}$ and $\underline{L}$ gives rise to the ground state (of part (a)) and to several excited states. The spin-orbit energy $E_{SO} = \lambda(\underline{L} \cdot \underline{S})$. If the spin-orbit coupling constant for Sm^{3+} is λ = 28 meV calculate the energy difference between the ground state and the first excited state. (**Hint**: Take the dot product of $\underline{J} = \underline{L} + \underline{S}$ with itself and re-arrange to obtain an expression for $\underline{L} \cdot \underline{S}$).

(g) The effective number of Bohr magnetons of Sm^{3+} at room temperature is 1.5. Compare this value to your result from part (*b*) and explain any difference there may be.

(**Hint**: The effective number of Bohr magnetons, μ_{eff}, is defined via the equation $|\mu_J| = \{J(J+1)\}^{\frac{1}{2}} g_J \mu_{\text{B}} = \mu_{\text{eff}} \mu_{\text{B}}$).

$e = 1.60 \times 10^{-19}$ C
$\mu_{\text{B}} = 9.27 \times 10^{-24}$ J T^{-1}
$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
$k_{\text{B}} = 1.38 \times 10^{-23}$ J K^{-1}
$h = 6.63 \times 10^{-34}$ J s^{-1}
$N_{\text{A}} = 6.022 \times 10^{23}$
$m_e = 9.11 \times 10^{-31}$ kg