Level 3 Condensed Matter Physics- Part I
Weekly problem 1

A Resume of the Properties of Free Electrons in Metals from FoP2B

(1) Given a system of free electrons in a metal with chemical potential μ write an expression which gives the probability of an eigenstate of energy *E* being occupied. Draw this distribution as a graph of the probability of occupation against the free electron energy, labelling the position of the chemical potential and the Fermi energy. **[2 marks]**

(2) By considering the free electron plane wavefunctions write an expression which relates the eigenstate energy to the wavevector. Use this expression to derive an expression for the Fermi energy and Fermi wavevector giving a definition of both. Draw a graph showing the $E(k)$ dispersion curve. **[3 marks]**

(3) Calculate the Fermi wavevector, Fermi energy and Fermi temperature of free electrons in copper metal given that copper crystallises in a fcc structure with a = 0.361 nm and is monovalent. What can you deduce about the properties of copper from these values? **[3 marks]**

(4) Show that the kinetic energy of a three-dimensional gas of *N* free electrons is

$$U_0 = \frac{3}{5} N E_F \ .$$

[2 marks]