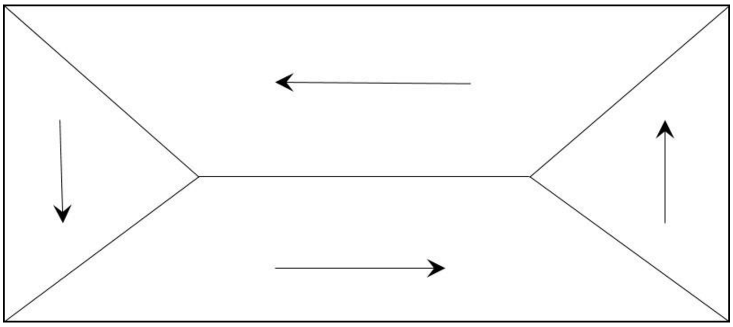
Level 3 Condensed Matter Physics- Part II
Examples Class 4 Answers

(1) Applying $B = \mu_o M_r$, with $M_r = 1.72 \times 10^6$ Am^{-1} gives $B = 2.16$ T. This is considerably larger than the critical magnetic field and therefore a superconducting transition is not expected.

Heating above the Curie temperature and cooling in a zero field will remove the *overall* magnetisation, although within individual magnetic domains (see below) the B-field will be large. Therefore the superconducting transition still cannot take place.

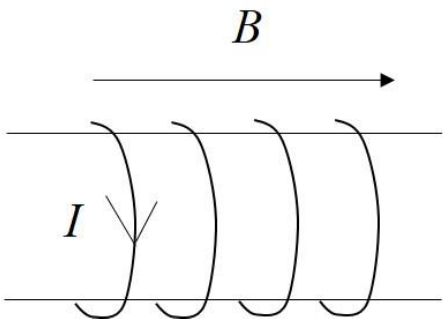


(2) i) For a Type I superconductor:

$$B_c(T) = B_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

Substituting $B_c(T) = ½\ B_c(0)$ gives $T = T_c/\sqrt{2}$, the superconducting transition temperature.

ii) The solenoid magnetic field is given by $B = (\mu_o NI)/L$, and is directed along the solenoid axis (see below).



iii) Reversing the direction of the solenoid current will reverse its magnetic field. Therefore we need to select the current direction such that the magnetic field from the solenoid is anti-parallel to the applied field $½B_c(0)$. At 0 K the maximum B-field that can be generated by the solenoid is therefore $3B_c(0)/2$. The maximum current is given by:

$$I_{max} = \frac{3LB_c(0)}{2\mu_o N}$$

(3) i) Re-arranging for the superconducting electron density gives:

$$n_s = \frac{m}{\mu_o e^2 \lambda_L^2}$$

Substituting values gives $n_s = 1.4 \times 10^{28}$ electrons/m^3.

ii) The number of Cooper pairs is ($n_s/2$). If the Cooper pair separation is x then $(n_s/2) = 1/x^3$ (imagine a cube of side x with a Cooper pair at each cube corner. Each Cooper pair contributes 1/8th to the cube, so there is effectively only one Cooper pair within the cube volume). Using the value for n_s we obtain $x = 0.53$ nm as the Cooper pair separation.

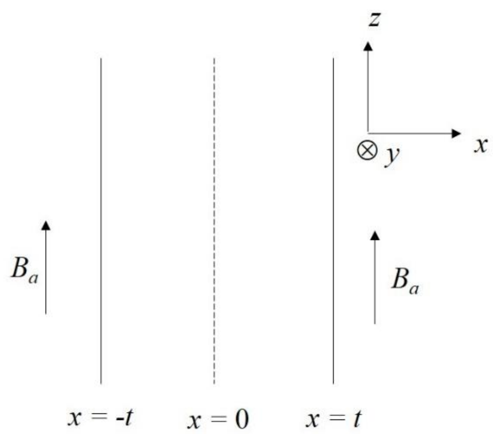
The coherence length for Al is 1550 nm which is much larger than the average Cooper pair separation. It is therefore clear than many Cooper pairs occupy the same region of material.

iii) There are 4 Al atoms within a face centred cubic crystal and therefore 12 valence electrons per unit cell. If n_v is the density of valence electrons then:

$$n_v = 12/(4.05 \times 10^{-10})^3 = 1.8 \times 10^{29} \text{ electrons/m}^3$$

iv) n_v is an order of magnitude larger than n_s. The valence electrons are responsible for atomic bonding, so that only a small fraction can form Cooper pairs while still preserving the structural integrity of the solid.

(4) The slab geometry is indicated below:



Since $\mathbf{B} = (0,0,B_a)$ the London equation becomes:

$$\frac{d^2 B_z}{dx^2} = \frac{B_z}{\lambda_L^2}$$

Solutions to this equation are of the form $B_z(x) = A\exp(x/\lambda_L) + B\exp(-x/\lambda_L)$, where A and B are constants to be determined. From the boundary conditions $B_z(-t) = B_a$ and $B_z(t) = B_a$ we find:

$$A = B = \frac{B_a}{\left[\exp\left(\frac{t}{\lambda_L}\right) + \exp\left(-\frac{t}{\lambda_L}\right)\right]}$$

Using the relation $\cosh\theta = (e^{\theta} + e^{-\theta})/2$, we finally obtain:

$$B_z(x) = B_a \frac{\cosh\left(\frac{x}{\lambda_L}\right)}{\cosh\left(\frac{t}{\lambda_L}\right)}$$

A schematic of the graph is shown below: