Level 3 Condensed Matter Physics

Example Workshop 2

1. Properties of electrons in Bloch energy bands.

This problem will look at the properties of electrons, and how they can be determined from knowledge of the E-k relationship (the energy band structure).

(a) Write down the formula for the effective mass of an electron within an energy band $E(k)$ of a simple one-dimensional crystal.

(b) Obtain an expression for the effective mass of electrons in an energy band described by the energy wavevector relationship $E(k) = A\cos(ka)$ where $A > 0$ at the points *i)* $k = 0$, and *ii)* $k = \pi/a$.

(c) What is the group velocity of electrons at $k = 0$ and $k = \pi/a$?

(d) Consider the situation where the energy band is almost full except for a small number of states n_h per unit length near the top of the energy band ($k = 0$). Show that the total current carried by the band is equal to $+en_h v$ where v is the velocity of the missing electrons and $-e$ is the charge on an electron.

(e) How do you interpret the result in part *(d)* in terms of positively charged pseudo-particles (called 'holes')?

(f) Draw a simple sketch of the energy band structure for the first Brillouin zone indicating the position of electrons and holes as described in part *(d)*.

2. The important ingredients of the quantum nearly free electron theory

(*a*) Consider a one-dimensional chain of atoms of lattice constant a. Starting from the energy - wavevector $E(k)$ relationship for free electrons explain why the introduction of a periodic potential causes band gaps to open up at the wavevectors $k = \pm\pi/a$.

(*b*) Show that the wavefunctions at $k = \pm\pi/a$ are not travelling waves of the form $\exp(\pm i\pi x/a)$ but are instead standing waves.

(*c*) By considering the distribution of probability densities of these standing waves show that the energy gap, U, is equal to the Fourier component of the crystal potential.

(*d*) Show that the group velocity of an electron at the bottom of an energy band in the nearly-free electron model is

$$v_{\text{group}} = \frac{\hbar k}{m^*}$$

where $\hbar k$ is the crystal momentum of the Bloch electrons and m^* is the effective mass of the electron. Use this to show that a completely filled band makes no contribution to the current carried by a crystal.