Level 3 Condensed Matter Physics- Part II
Weekly problem 1

(1) Find the composition of $Ga_xIn_{1-x}As$ such that its lattice constant matches that of InP. What is the band gap at that composition? [4 marks]

You may use the following. GaAs: a = 5.65 Å, E_g = 1.42 eV. InAs: a = 6.06 Å, E_g = 0.36 eV. InP: a = 5.87 Å, E_g = 1.27 eV. The bowing parameter for the alloy $Ga_xIn_{1-x}As$ is b = 0.475 eV.

(2) The energy of an electron in the conduction band of a two-dimensional semiconductor is given by:

$$E(\mathbf{k}) = Ak^2 + Bk_x^2$$

where A and B are positive constants and $\mathbf{k} = (k_x, k_y)$ is the wavevector. Derive expressions of the effective mass in k_x and k_y directions. [4 marks]

(3) Silicon is made up of two interpenetrating face centred cubic (fcc) lattices displaced by a (¼,¼,¼) fractional lattice vector. Derive the (fractional) position coordinates of all 8 atoms in the unit cell. [2 marks]

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Weekly problem 2

(1) Silicon has a relative permittivity $\varepsilon_r = 11.7$ and electron effective mass $m_e = 0.2m$. Calculate:

i) the donor ionisation energy [1 mark]
ii) the radius of the ground state orbit [1 mark]
iii) the donor concentration at which the orbits begin to overlap [1 mark]

(2) A sample of silicon is purified until it contains only 10^{18} donors/m^3. Below what temperature does it cease to show intrinsic behaviour? The band gap $E_g = 1.1$ eV and the intrinsic carrier concentration at 300 K is 5 x10^{15} m^{-3}. For simplicity assume that the effective density of states N_c and N_v are independent of temperature. [4 marks]

(3) 10^{20} atoms/m^3 of phosphorus is added as dopant to pure silicon. What is the conductivity of the material at room temperature, assuming the sample is in the saturation regime? State any assumptions you make. The electron mobility in silicon is 0.16 m^{2}V^{-1}s^{-1}. [3 marks]

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Weekly problem 3

(1) *n*- and *p*-type silicon with doping concentrations of 5 x 10^{21} m^{-3} and 2 x 10^{22} m^{-3} are brought together to form a pn-junction at room temperature (300 K). Calculate:

i) the built-in potential [2 marks]
ii) the space charge width on the *n*- and *p*-side [2 marks]
iii) the magnitude of the maximum electric field [1 mark]

For silicon, band gap E_g = 1.1 eV, permittivity $\varepsilon = 11.7\varepsilon_0$, effective density of states of conduction band N_c = 2.8 x 10^{25} m^{-3} and valence band N_v = 1.0 x 10^{25} m^{-3} (ε_0 = permittivity of free space).

(2) Using the values for parts (i) and (ii) from the previous question estimate the capacitance per unit area of the pn-junction. [2 marks]

(3) When a forward bias of 0.2 V is applied to a pn-junction at 300 K a current of 25 mA flows. Calculate the reverse saturation current. [3 marks]

Level 3 Condensed Matter Physics- Part II
Weekly problem 4

(1) In a superconductor persistent current measurement no noticeable decrease in the current is observed after 1 year. The precision of the measurement is estimated at 0.1% of the current measured. Calculate a lower limit for the average scattering time τ in the superconductor.
[3 marks]

(2) Lead is a Type I superconductor with critical temperature 7.2 K and critical field 0.080 T at 0 K. Calculate the maximum current that can be carried in a 1 mm diameter Pb wire at liquid-He temperature (4.2 K). [4 marks]

(3) The magnetic susceptibility of a Type II superconductor is measured to be -0.7 in the mixed state. If the susceptibility of the normal phase is 2×10^{-5} calculate the volume fraction of superconducting and normal phases within the material. For simplicity assume the magnetic susceptibility follows a rule of mixtures type law for composite materials, and that the London penetration depth is small compared to the spacing between normal material filaments.
[3 marks]