Level 3 Condensed Matter Physics- Part II
Supplementary Example Class 5 Answers

(1) Writing **i**, **j** and **k** for the unit vectors along the *x, y* and *z*-axes, we have $\boldsymbol{\mu} = \mu\mathbf{j}$. Furthermore, $\mathbf{r} = (na)\mathbf{i}$, where **r** is the position vector between a given dipole at *x*-position *na* and the origin *O* (here *n* is any non-zero integer). Since $\boldsymbol{\mu}\cdot\mathbf{r} = 0$, the electric field due to a single dipole is:

$$\mathbf{E}(\mathbf{r}) = \frac{-\boldsymbol{\mu}}{4\pi\epsilon_o r^3}$$

Any electric field at the origin should therefore be parallel to the dipole moment $\boldsymbol{\mu}$, i.e. along the *y*-axis. Its magnitude is given by:

$$E = -\frac{2\mu}{4\pi\epsilon_o a^3}\sum_{n=1}^{\infty}\frac{1}{n^3}$$

The additional factor of 2 takes into account dipoles on the left and right hand sides. Substituting the value for $\Sigma(1/n^3)$ gives $E = -0.6\mu/(\pi\epsilon_o a^3)$. The electric field direction is along the negative *y*-axis.

(2) i) The force ($\mathbf{F}_d$) due to damping is $\mathbf{F}_d = -\gamma m\mathbf{v} = -\gamma m(d\mathbf{r}/dt)$. Note that since $\gamma>0$ a minus sign is used to indicate that the damping acts against the electron velocity **v**. The equation of motion for the electron is therefore:

$$m\frac{d^2\mathbf{r}}{dt^2} = -K\mathbf{r} - \gamma m\frac{d\mathbf{r}}{dt} - e\mathbf{E}_o\exp(i\omega t)$$

Substituting $\mathbf{r} = \mathbf{r}_o\exp(i\omega t)$ and solving for $\mathbf{r}_o$:

$$\mathbf{r}_o = -\left(\frac{e\mathbf{E}_o}{m}\right)\frac{(\omega_o^2 - \omega^2) - i\gamma\omega}{(\omega_o^2 - \omega^2)^2 + (\gamma\omega)^2}$$

where the substitution $K = m\omega_o^2$ has been made.

ii) Using the fact that $\boldsymbol{\mu} = \alpha(\omega)\mathbf{E}_{local} = -e\mathbf{r}$ gives:

$$-e\mathbf{r}_o = \alpha(\omega)\mathbf{E}_o$$

Substituting the above expression for $\mathbf{r}_o$ results in:

$$\alpha(\omega) = \left(\frac{e^2}{m}\right)\left[\frac{(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2)^2 + (\gamma\omega)^2} - i\frac{\gamma\omega}{(\omega_o^2 - \omega^2)^2 + (\gamma\omega)^2}\right]$$

iii) In an undamped medium, where there is no imaginary term in the polarisability, the polarisation ($\boldsymbol{\mu} = \alpha(\omega)\mathbf{E}_{local}$) of an individual dipole is in phase with the local electric field. In a damped medium the presence of an imaginary term means that the polarisation is out of phase with the electric field, i.e. the electrons cannot oscillate with the same frequency as the electric field. Effectively this gives rise to an energy loss. Examples of this phenomenon are microwave heating of food and absorption of light by matter.

(3) i) From $G_{FE}(T_c) = G_{PE}(T_c)$ we have:

$$\frac{1}{2}g_2P_c^2 - \frac{1}{4}|g_4|P_c^4 + \frac{1}{6}g_6P_c^6 = \frac{1}{2}P_c^2\left[g_2 - \frac{1}{2}|g_4|P_c^2 + \frac{1}{3}g_6P_c^4\right] = 0$$

Since $P_c \neq 0$ (ferroelectric phase) the terms within the square brackets must be zero, i.e.

$$g_2 - \frac{1}{2}|g_4|P_c^2 + \frac{1}{3}g_6P_c^4 = 0 \qquad \ldots (1)$$

ii) From $d[G_{FE}(T_c)]/dP = 0$ we have:

$$g_2P_c - |g_4|P_c^3 + g_6P_c^6 = P_c[g_2 - |g_4|P_c^2 + g_6P_c^4] = 0$$

Again since $P_c \neq 0$ the terms within the square brackets must be zero, i.e.

$$g_2 - |g_4|P_c^2 + g_6P_c^4 = 0 \qquad \ldots (2)$$

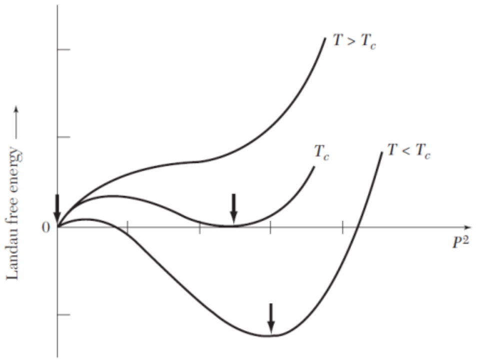
iii) Subtracting (1) from (2) and re-arranging for P_c gives:

$$P_c = \left(\frac{3|g_4|}{4g_6}\right)^{1/2}$$

iv) The spontaneous polarisation (P_s) has the same form as equation (2) with P_s substituted for P_c. The roots of the quadratic equation are given by:

$$P_s^2 = \frac{|g_4| \pm \sqrt{|g_4|^2 - 4g_6g_2}}{2g_6}$$

As can be seen from the free energy diagram below the larger value of P_s^2 corresponds to a minimum (the smaller value is a maximum). The spontaneous polarisation is therefore:



$$P_s^2 = \frac{|g_4| + \sqrt{|g_4|^2 - 4g_6 g_2}}{2g_6}$$

Or after substituting $g_2 = \gamma(T\text{-}T_o)$:

$$P_s^2 = \frac{|g_4| + \sqrt{|g_4|^2 - 4g_6\gamma(T - T_o)}}{2g_6}$$

v) At $T = T_o$ the above equation predicts:

$$P_s(T = T_o) = \left(\frac{|g_4|}{g_6}\right)^{1/2}$$

This is larger than the value of P_c. Since the spontaneous polarisation of the ferroelectric phase decreases monotonically with temperature (see figure above) this means that $T_o < T_c$.