Level 3 Condensed Matter Physics- Part I
Weekly problem 5

(1) What is meant by L-S (or Russell Saunders) coupling and under what conditions is it applicable? **[1 mark]**

(2) The magnetisation of a paramagnet consisting of isolated atoms in their ground state with total angular momentum J is given by the Brillouin function,

$$M = M_S\left[\frac{(2J+1)}{2J}\coth\left(\frac{(2J+1)}{2J}y\right) - \frac{1}{2J}\coth\left(\frac{y}{2J}\right)\right]$$

where M_S is the saturated magnetisation value $ng\mu_B J$, $y = g_J\mu_B JB/(k_B T)$, B is the magnetic induction field and g_J is the Landé g-factor which is given by the expression,

$$g_J = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}.$$

Use the expression for M to derive Curie's law, stating any assumptions that you make.

[Hint: $\coth x \simeq \frac{1}{x} + \frac{x}{3}$ for $x \to 0$]

[2 marks]

(3) Use Hund's rules to determine the ground state and excited total angular momentum states of an isolated Ti^{2+} ion which has a $3d^2$ electronic structure. Calculate the total magnetic moments of each of these states in units of μ_B. **[3 marks]**

(4) The spin-orbit energy is given by $E_{SO}(J) = \frac{\lambda}{2}[J(J+1) - L(L+1) - S(S+1)]$. For a Ti^{2+} ion the spin-orbit coupling constant λ = 4.5 meV. Calculate the energies $E_{SO}(J)$ of the total angular momentum states determined in part (3). **[2 marks]**

(5) Sketch the form of the temperature dependence of the inverse paramagnetic susceptibility of a solid of non-interacting Ti^{2+} ions, measured at low magnetic fields. Note that the probability of thermal occupation of the energy levels, determined in part (4), is proportional to $(2J+1)\exp(-E_{SO}(J)/k_B T)$. **[2 marks]**