Statistical Physics: Weekly Problem 7 (SP7)

(1) (a) Derive the single-particle partition function, Z_1, for a free particle of mass M in three dimensions, constrained to a box of volume V, in thermal equilibrium at temperature T. You will need common integrals of the form $\int_0^\infty x^n e^{-bx^2} dx$ which you can look up and state without proof. [1 mark]

(b) Give a criterion for the dilute gas limit in terms of the thermal de Broglie wavelength $\lambda_{\rm D} = \sqrt{h^2/2\pi M k_{\rm B} T}$. [1 mark]

(c) The free energy for a gas of N weakly interacting particles in a volume V at temperature T is given by $F = -k_{\rm B} T \ln Z_N$, where Z_N is the N-particle partition function. Find the free energy of

(i) A gas of distinguishable particles. [1 mark]

(ii) A gas of indistinguishable particles. [1 mark]

(iii) Explain why there is a difference between the answers to parts (i) and (ii). [1 mark]

(d) Show that the entropy for a classical gas of monoatomic distinguishable particles is

$$S = N k_{\rm B} \ln V T^{3/2} + \frac{3}{2} N k_{\rm B} \left[\ln \frac{2\pi M k_{\rm B}}{h^2} + 1 \right].$$

[2 marks]

(e) A classical gas of distinguishable monoatomic particles is in thermal equilibrium at temperature $T = 300$ K in a container of volume V. The gas is allowed to expand adiabatically (without change in its entropy) until it occupies twice the initial volume. What is the temperature of the gas after the expansion? [2 marks]

(f) Mention two examples of cooling under constant entropy. One example example was given in lectures, you'll have to research for another. [1 mark]