<u>FoP3B Part I Lecture 4: Effective mass and Introduction to Magnetism</u>

Effective mass of electrons

An important concept is the **effective mass** of electrons in a solid. A simple way to think about effective mass is as follows. Consider first an electron in free space. We apply a force F and measure the acceleration a. The mass can then be determined from $F = ma$. Now consider an electron in a real crystal subject to the same external force. As the electron moves through the crystal it will undergo Coulomb scattering with other electrons as well as atomic nuclei. The presence of these *internal* forces means that $F \neq ma$ (recall that F is the *external* applied force). To satisfy Newton's second law we can therefore write $F = m^*a$, where m^* is the *effective* mass. It can be shown that for an isotropic crystal (see DUO supplementary material):

$$m^* = \frac{\hbar^2}{(d^2E/dk^2)}$$

where d^2E/dk^2 is the curvature of electronic band (note that for anisotropic crystals we have to define an *effective mass tensor* $m_{ij}^* = \hbar 2/\frac{d2E}{dkidkj}$).

<u>Introduction to Magnetism</u>

Magnetic dipole moment

According to the Biort-Savart law moving charge (i.e. an electric current) generates a magnetic field. A special case is an electric current flowing in a loop (Figure 1).

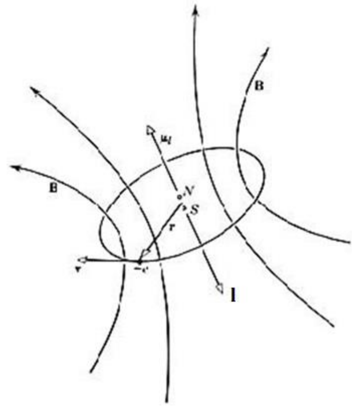

Figure 1

The magnetic field here is similar to that of a North-South pole bar magnet placed at the centre. We define a **magnetic dipole moment** to characterise the strength and orientation of the bar magnet. The magnetic moment $\boldsymbol{\mu} = I\mathbf{A}$, where I is the current (recall that current is defined as the flow of *positive* charge) and $\mathbf{A}$ is the *area vector* of the current loop, as defined by the right hand rule. The direction of $\boldsymbol{\mu}$ points from the South to North pole. Furthermore, for a negatively charged electron $\boldsymbol{\mu}$ is anti-parallel to the angular momentum $\mathbf{l} = \mathbf{r} \times \mathbf{p}$, where $\mathbf{r}$ is the radial vector and $\mathbf{p}$ is the linear momentum (Figure 1). Therefore, we may write $\boldsymbol{\mu} = -\gamma\mathbf{l}$, where γ is the **gyromagnetic ratio**.

It turns out that the magnetic moment for an electron has a fixed magnitude, called the **Bohr magneton** μ_B, irrespective of the size of the circular orbit. It is therefore a fundamental unit for quantifying magnetic moments in solids. Consider an electron in a circular orbit of radius r at speed v. The time taken for the electron to do a circular loop is $T = (2\pi r/v)$. The current is $I = e/T = ev/(2\pi r)$, so that:

$$\mu_B = IA = \frac{ev}{2\pi r} \cdot \pi r^2 = \frac{e}{2m}(mvr)$$

The term mvr is the angular momentum. It therefore follows that $\underline{\gamma = e/2m}$. The angular momentum can be calculated using some elementary quantum mechanics. The electron must form a standing wave around the circular orbit, so that the circumference must be equal to an integer number of wavelengths λ, i.e. $n\lambda = 2\pi r$. Furthermore, by de Broglie's equation $\lambda = \frac{h}{mv}$, so that for the ground state (n =1) we obtain $mvr = \hbar$, and consequently $\mu_B = \gamma\hbar = \frac{e\hbar}{2m}$. Note that this result is independent of the radius r. In fact $\mu_B = 9.3 \times 10^{-24}\mathrm{Am}^2$.

Some definitions and terminology

Definitions and terminology in magnetism can sometimes be rather confusing. Here we will use SI units, rather than CGS units still used in some books! The equation for magnetic fields in vacuum is $\mathbf{B} = \mu_0\mathbf{H}$, where **H** is the magnetic field (units Am^{-1}), $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Hm}^{-1}$ (Henry per metre) is the permeability of free space and **B** is the magnetic *induction* field (units Tesla, T)[1]. In vacuum **B** and **H** are simply proportional to one another, so that there is no significant difference between the two. However, in the interior of a magnetic solid we have $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$, where **M** is the *magnetisation* or net magnetic moment per unit volume (recall that the magnetic moment $\boldsymbol{\mu}$ is a vector, so both the direction and magnitude must be taken into account when determining **M**). **M** has the same units as **H** (i.e. Am^{-1}) and means **B** is no longer proportional to **H**.

The **magnetic susceptibility** is given by $\chi = \mathbf{M}/\mathbf{H}$. χ is a dimensionless quantity and is defined for small values of **H** where the *magnetisation curve* (i.e. M vs. H curve) shows linear behaviour[2]. From $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$ it is clear that if $\chi < 0$ the **B**-field will be smaller within the solid compared to vacuum, i.e. the material shows **diamagnetism** and repels magnetic fields. The opposite situation arises for $\chi > 0$. Typically, these would be materials showing weak magnetism (**paramagnetism**) or strong magnetism (**ferromagnetism**), although other forms of magnetic ordering are also possible as we shall see later in the course. The supplementary reading in DUO describes how we can measure the magnetic properties of materials using techniques such as vibrating sample magnetometry (VSM).

Magnetism as a quantum mechanical phenomenon

The potential energy (E) of a magnetic moment $\boldsymbol{\mu}$ in a **B**-field is $E = -\boldsymbol{\mu}\cdot\mathbf{B}$. The energy is therefore a minimum when $\boldsymbol{\mu}$ is parallel to **B**. Therefore, the magnetisation $M = -\frac{dE_{\mathrm{vol}}}{dB}$, where E_{vol} is the energy per unit volume. Now because the Lorentz force $\mathbf{F} = -e(\mathbf{v} \times \mathbf{B})$ is normal to the velocity **v** a magnetic field cannot do any work on a moving electron. Therefore, according to classical physics the magnetisation must be zero! The response of a magnetic moment $\boldsymbol{\mu}$ to a **B**-field is illustrated in Figure 2.

[1] To further complicate matters **H** is sometimes referred to as the magnetic field strength and **B** the magnetic flux density.

[2] In certain strong magnetic materials, called ferromagnets (e.g. Fe, Co and Ni), the magnetisation graph is non-linear. The magnetic susceptibility is then defined as the gradient of the graph, i.e. $\chi = d\mathbf{M}/d\mathbf{H}$ for small **H**.

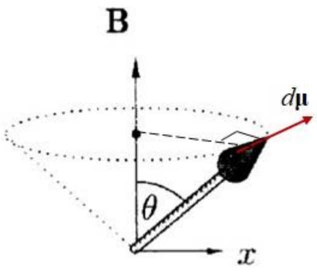

Figure 2

The torque $\boldsymbol{\tau}$ on the moment is $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$. Since $\boldsymbol{\tau} = d\mathbf{l}/dt$ (rate of change of angular momentum) and $\boldsymbol{\mu} = -\gamma\mathbf{l}$:

$$\frac{d\boldsymbol{\mu}}{dt} = -\gamma(\boldsymbol{\mu} \times \mathbf{B})$$

The change in moment $d\boldsymbol{\mu}$ is therefore perpendicular to $\boldsymbol{\mu}$, so that the magnetic moment *precesses* around the **B**-field (Figure 2). Because the angle θ between $\boldsymbol{\mu}$ and $\mathbf{B}$ does not change the potential energy is constant. Note that this behaviour is very different to electric dipoles, which do not have angular momentum and therefore *can* rotate under an applied electric field to minimise its energy.

The **Bohr-van Leeuwen theorem** states the above more rigorously: the magnetisation at *thermal equilibrium* for a solid obeying classical statistical mechanics (i.e. Maxwell-Boltzmann distribution) is zero. This is essentially due to the fact that the partition function for the electrons is not dependent on the magnetic field. The implication is that magnetism is Quantum Mechanical in origin.