FoP3B Part I Lecture 11: Domain Walls

Bloch's $T^{3/2}$ law

Thermal disorder of magnetic moments in a ferromagnet is due to spin waves or magnons. Let us therefore calculate the magnetisation as a function of temperature using a magnon-based model. Excitation of a single magnon will reduce the net magnetic moment by 1 unit. The equilibrium magnon concentration, n_{magnon}, therefore determines the magnetisation as a function of temperature. Since magnons are bosons:

$$n_{magnon} = \int \left[\frac{1}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1}\right] g_m(\hbar\omega)d(\hbar\omega)$$

where the term within the square brackets is the Bose-Einstein distribution and $g_m(\hbar\omega)d(\hbar\omega)$ is the density of states, i.e. number of magnon states per unit volume between energy $E = \hbar\omega$ and $[\hbar\omega + d(\hbar\omega)]$. At low temperatures only low energy magnons can be excited. Since the magnon energy $\hbar\omega = 4J_{\text{ex}}S(1 - \cos qa)$ it follows that q must be small at low temperature. Using the small angle approximation $\cos(qa) = 1 - \frac{(qa)^2}{2}$ we have $\hbar\omega \propto q^2$. This is a similar energy dependence to electrons in a free electron solid (i.e. $E \propto k^2$, where k is the electron wavenumber). Furthermore, since magnons are also plane waves they satisfy the same boundary conditions as a free electron solid (i.e. $\mathbf{k} = 2\pi/L$, where L is the dimension of the solid). We can use the density of states result from free electron theory to obtain $g_m(\hbar\omega)d(\hbar\omega) \propto \sqrt{\omega}d(\hbar\omega)$ (see Lecture 1), so that:

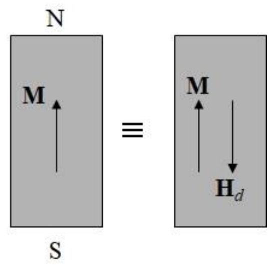
$$n_{magnon} = \int \left[\frac{1}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1}\right] g_m(\hbar\omega)d(\hbar\omega) \propto \int_0^\infty \frac{\sqrt{\omega}d\omega}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1}$$

$$\text{or } n_{magnon} \propto \left(\frac{kT}{\hbar}\right)^{3/2} \int_0^\infty \frac{\sqrt{x}dx}{e^x - 1} \propto T^{3/2}$$

with $x = \hbar\omega/kT$. The magnetisation is then given by $\frac{M_s - M}{M_s} \propto n_{\text{magnon}} \propto T^{3/2}$. This is known as **Bloch's $T^{3/2}$ law**. Experimentally it is found that the magnetisation of a ferromagnet at low temperature obeys Bloch's $T^{3/2}$ law rather than the Weiss model prediction (see previous lecture). This confirms that spin disorder is due to magnons rather than discrete 180° spin flips.

Domain Walls

We will now discuss **domain walls** in ferromagnetic materials.



Consider a ferromagnetic solid with uniform magnetisation **M** (Figure 1). Since $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$ and $\nabla \cdot \mathbf{B} = 0$, we have $\nabla \cdot \mathbf{M} = -\nabla \cdot \mathbf{H}$. The divergence in **M** is non-zero at the free surfaces, so that a **demagnetising field** $\mathbf{H}_d$ that is anti-parallel to **M** must be present in order to satisfy Maxwell's equations (Figure 1). Recall however that the energy of a magnetic moment $\boldsymbol{\mu}$ in a **B**-field is $-\boldsymbol{\mu} \cdot \mathbf{B}$. Since **M** and $\mathbf{H}_d$ are anti-parallel the energy is <u>increased</u>.

Figure 1

A ferromagnet minimises this excess energy by forming **domains**. The concept can be illustrated using Figure 2. A solid with uniform magnetisation has magnetic field lines that look like a bar magnet with North-South poles (compare the field lines in Figure 2 (left) with the North-South poles for uniform magnetisation; Figure 1). These magnetic field lines carry excess energy. If we split the solid into two domains with anti-parallel **M** then the field lines, and therefore the excess energy, is reduced somewhat (Figure 2 middle). The lowest energy configuration is shown in Figure 2 (right).

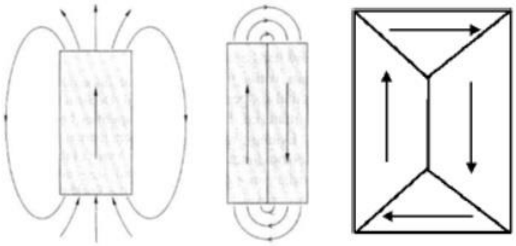


Figure 2

Here the domain configuration is such that **M** is parallel to the free surfaces, so that the demagnetising field due to $\nabla \cdot \mathbf{M} \neq 0$ is avoided. The magnetic field lines outside the solid are absent. The net magnetisation for the entire solid is also zero, but this is a consequence of the domain structure. Within a single domain the magnetisation has the value expected of a ferromagnetic material.

The **domain wall** is the interface separating two neighbouring magnetic domains. On crossing the domain wall the magnetic moments or spins must change their orientation. This process happens gradually over a certain distance. Consider the case where the domain wall is between two neighbouring domains that have anti-parallel spins. As shown in Figure 3 the re-orienting of spins can either happen in a plane parallel to the domain wall (so-called **Bloch walls**) or in a plane perpendicular to the domain wall (**Néel walls**).

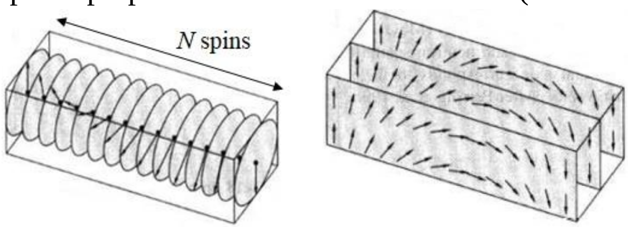
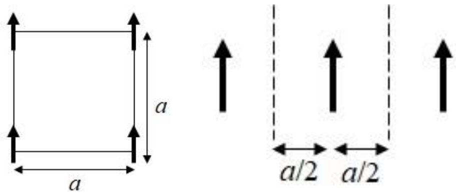


Figure 3: Bloch (left) and Néel (right) domain walls

The spin re-orienting happens over N atoms or N spins. The value of N is determined by two opposing factors: exchange energy and anisotropic energy density. Let us examine how these two factors affect the domain wall energy and width.

First consider the exchange energy contribution $-2J_{\mathrm{ex}}\boldsymbol{S}_i \cdot \boldsymbol{S}_j$. We will assume the atoms have periodic spacing a in a direction normal to the domain wall, and that the exchange interaction is only present between neighbouring atoms. For N perfectly aligned spins the energy $E_0 = -2J_{\mathrm{ex}}NS^2$. On the other hand, if the spin misalignment between neighbouring atoms is $\Delta\theta$, the energy $E_1 = -2J_{\mathrm{ex}}NS^2\cos\Delta\theta$. The energy gain is therefore $\Delta E = E_1 - E_0 = -2J_{\mathrm{ex}}NS^2(\cos\Delta\theta - 1)$ or $J_{\mathrm{ex}}NS^2\Delta\theta^2$ for small $\Delta\theta$ (recall the small angle approximation: $\cos\Delta\theta \approx 1-\Delta\theta^2/2$). Furthermore, $N\Delta\theta = \pi$, so that $\Delta E \approx J_{\mathrm{ex}}\frac{\pi^2S^2}{N}$. The domain wall energy is expressed as an energy per unit area. We can assume that in the plane of the domain wall the spins are arranged in a square lattice (Figure 4a) and that there will be $(1/a^2)$ spins per unit area. The domain wall energy due to exchange is then $\sigma_1 = \frac{\Delta E}{a^2} = J_{\mathrm{ex}}\frac{\pi^2S^2}{Na^2}$. It follows that the exchange interaction favours a wide domain wall, i.e. large N.



(a) **(b)**

Figure 4: Spin arrangement (a) in the plane of the domain wall and (b) along a perpendicular direction to the domain wall.

Next consider the contribution due to the anisotropic energy density, which is assumed to take the form $K\sin^2\alpha$ (K = anisotropy constant, α = angle w.r.t. easy axis). The anisotropy energy is zero far away from the domain wall (i.e. $\alpha = 0, \pi$). For the intermediate N spins:

$$\Delta E = \sum_{i=1}^{N} K\sin^2(i\Delta\theta)$$

Passing onto the continuum limit for small $\Delta\theta$ and keeping in mind that $N\Delta\theta = \pi$ gives:

$$\Delta E = \frac{1}{\Delta\theta}\int_0^{\pi} K\sin^2\theta d\theta = \frac{N}{\pi}\int_0^{\pi} K\sin^2\theta d\theta = \frac{NK}{2}$$

ΔE is an energy density (i.e. energy per unit volume). The anisotropy energy for a given spin is defined over a distance $\pm a/2$ in the direction normal to the domain wall (Figure 4b). The anisotropy energy per unit domain wall area is therefore $\sigma_2 = a\Delta E = \frac{NKa}{2}$. Note that the anisotropy energy favours a narrow domain wall (i.e. small N), unlike the exchange interaction.

The total domain wall energy is $\sigma = \sigma_1 + \sigma_2 = J_{\text{ex}}\frac{\pi^2 S^2}{Na^2} + \frac{NKa}{2}$. The equilibrium width is given by the condition $\frac{d\sigma}{dN} = 0$ which has the solution $N = \pi S\sqrt{\frac{2J_{\text{ex}}}{Ka^3}}$. Substituting into the expression for σ we obtain the equilibrium domain wall energy $\sigma = \pi S\sqrt{\frac{2J_{\text{ex}}K}{a}}$.

Magnetic Hysteresis Loops

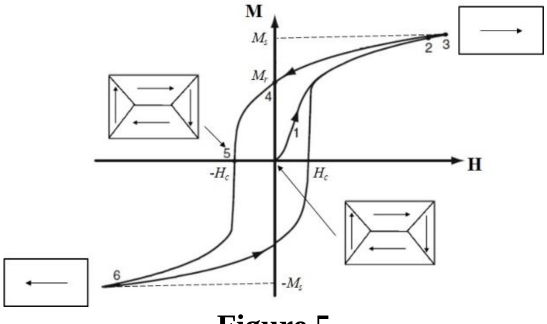


Figure 5

The presence of domains gives rise to **hysteresis** behaviour in **M** vs. **H** magnetisation curves (Figure 5). The shape of the hysteresis loop is explained by the energy of individual domains in the presence of an applied magnetic field, i.e. domains where the magnetisation **M** is parallel to **H** have the lowest energy. On reversing the direction of the magnetic field the magnetisation shows hysteresis due to **domain wall pinning** by impurities in the material.

From $E = -\mathbf{M}\cdot\mathbf{B}$ the area enclosed by a hysteresis loop equals the energy lost during a magnetisation cycle. **Soft magnetic materials** (e.g. Ni-Fe permalloy) have a small hysteresis loop and are therefore used in low power loss applications, such as transformer coils, generators and motors. Soft magnetic behaviour is favoured by wide domain walls (large J_{ex}, small K), due to domain pinning being harder. **Hard magnetic materials** (e.g. NdFeB, SmCo alloys) have a large hysteresis loop and are used as permanent magnets for magnetic storage, fridge magnets, loudspeakers etc. They have narrow domain walls (small J_{ex}, large K), that make domain pinning is easier.

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