FoP 3B Part II

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Room 151

Lecture 8: London equation and thermodynamics of the superconducting phase transition

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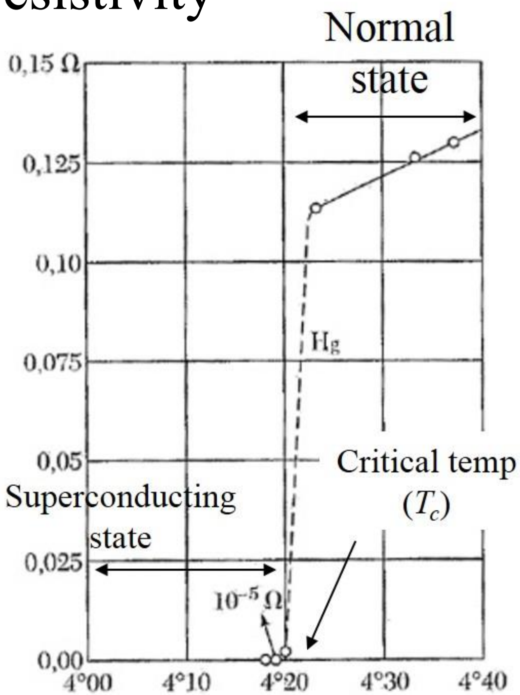
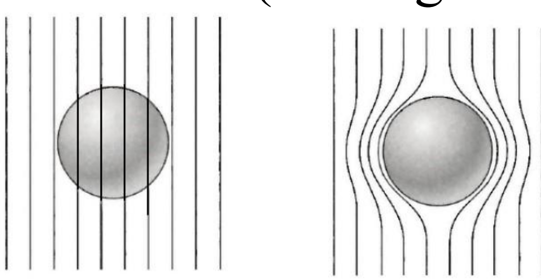
Summary of Lecture 7

Zero resistivity



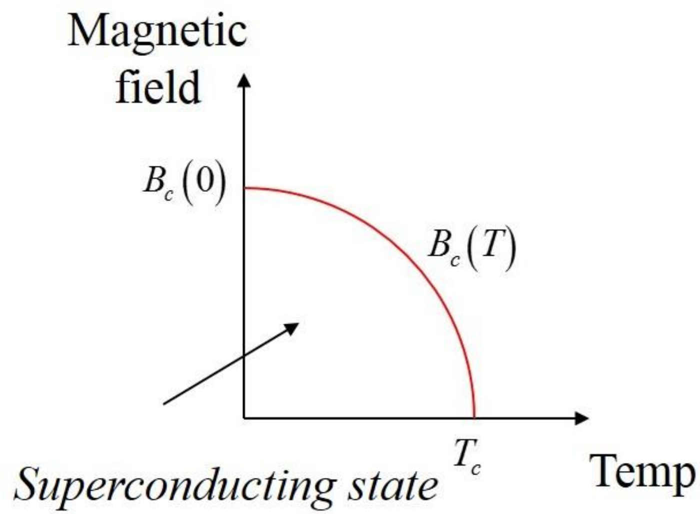
Meissner effect (diamagnetism)

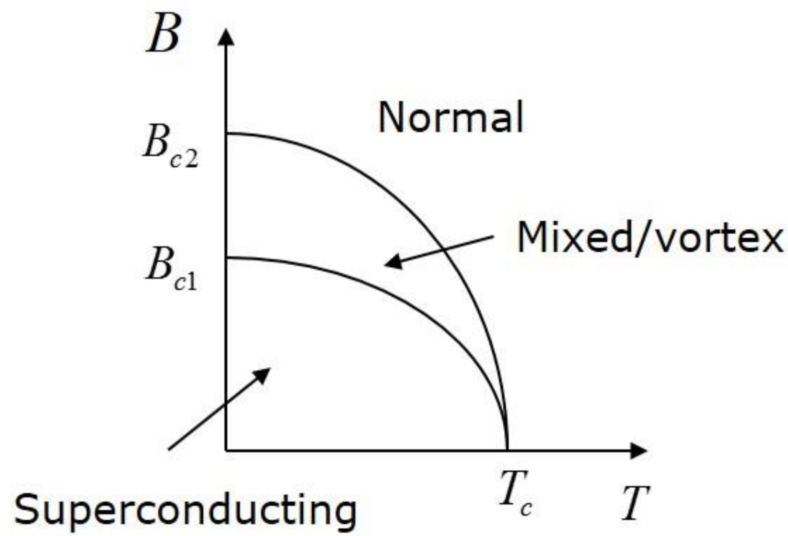


$T > T_c$
(**B** on)

$T < T_c$
(**B** on)

Type I vs Type II behaviour:





Aim of today's lecture

► Explain the origin of the Meissner effect and describe the thermodynamics of the superconducting phase transition

Key concepts:

-London equation: London penetration depth and supercurrents

-Thermodynamics of the superconducting phase transition: condensation energy and second order transition.

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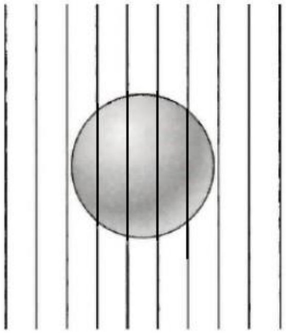
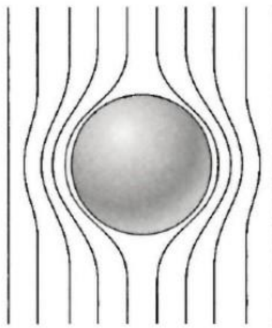
Meissner effect and supercurrent



$T > T_c$
(**B** on)



$T < T_c$
(**B** on)

- Magnetic field within a superconductor is always zero (perfect diamagnetism).
- Implies supercurrent **j** that depends on applied field (supercurrent cancels magnetic field within the superconductor).

Hence:

$$\boxed{\mathbf{j} = -\frac{1}{\mu_o \lambda_L^2}\mathbf{A}}$$

(λ_L = London penetration depth)

where **A** is the magnetic vector potential (i.e. $\mathbf{B} = \nabla \times \mathbf{A}$). Here the London gauge $\nabla \cdot \mathbf{A} = 0$ is used so that charge is conserved, i.e.:

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$$\frac{\partial n_s}{\partial t} = -\frac{1}{e}\vec{\nabla} \cdot \mathbf{j} = 0 \quad \text{(continuity equation)}$$

London equation

We have:

$$\vec{\boldsymbol{\nabla}} \times \mathbf{j} = -\frac{1}{\mu_o \lambda_L^2}\mathbf{B} \qquad \text{(London equation)}$$

$$\vec{\boldsymbol{\nabla}} \times \mathbf{B} = \mu_o \mathbf{j} \qquad \text{(Maxwell's equation; steady state)}$$

Therefore:

$$\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{\nabla}} \times \mathbf{B} = -\frac{1}{\lambda_L^2}\mathbf{B}$$

$$\Rightarrow \boxed{\boldsymbol{\nabla}^2 \mathbf{B} = \frac{1}{\lambda_L^2}\mathbf{B}} \; *$$

* NB: $\nabla \mathrm{x} \nabla \mathrm{x} \mathbf{B} = \cancel{\nabla(\nabla \cdot \mathbf{B})} - \nabla^2 \mathbf{B}$

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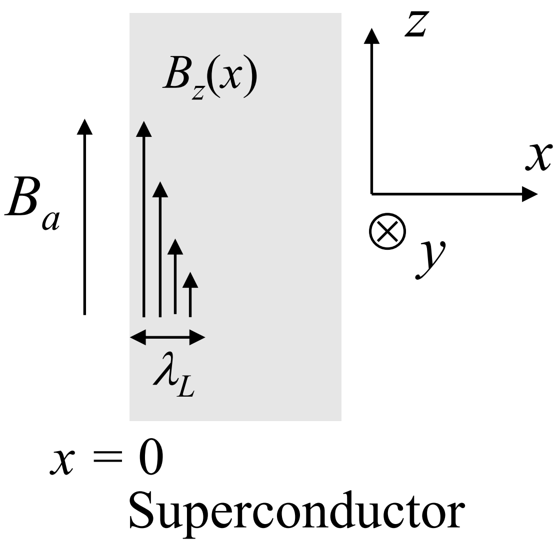
Application of the London equation (semi-infinite superconducting slab)



$$\frac{d^2 B_z}{dx^2} = \frac{B_z}{\lambda_L^2}$$

Solutions are of the form:

$$B_z(x) = \cancel{A\exp\left(\frac{x}{\lambda_L}\right)} + B\exp\left(-\frac{x}{\lambda_L}\right)$$

From the boundary condition $B_z(0) = B_a$, we have:

$$B_z(x) = B_a \exp\left(-\frac{x}{\lambda_L}\right)$$

The supercurrent is derived from $\nabla \times \mathbf{B} = \mu_o \mathbf{j}$:

$$j_y(x) = \frac{B_a}{\mu_o \lambda_L} \exp\left(-\frac{x}{\lambda_L}\right)$$

Magnetic field and supercurrents decay over λ_L distance from surface

The London penetration depth

It can be shown that*:

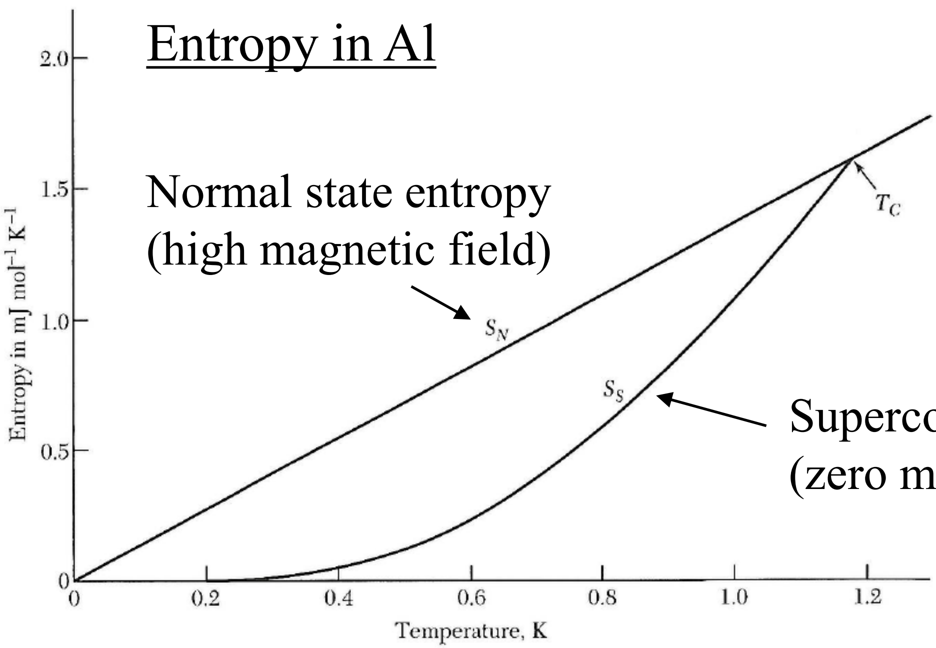
$$\lambda_L = \sqrt{\frac{m}{\mu_o n_s e^2}}$$

electron mass

electron charge

n_s = number density of *superconducting* electrons (increases with cooling below T_c; see below)

Entropy in Al



Normal state entropy (high magnetic field)

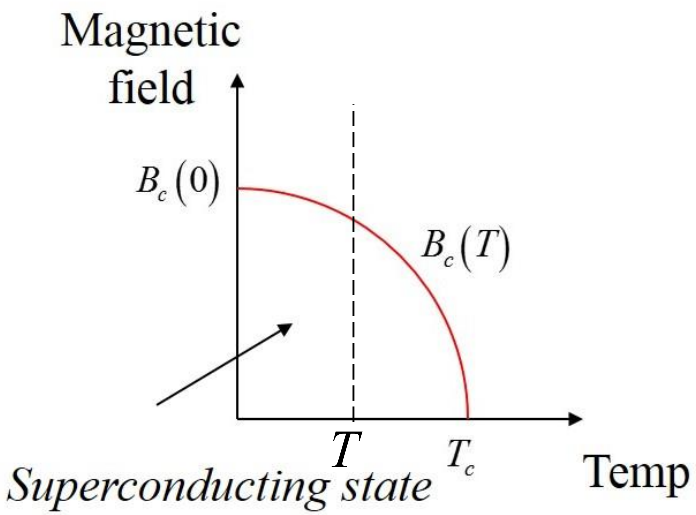
Superconducting state entropy (zero magnetic field)

- Superconducting electrons more ordered than in normal state

* See 'derivations' of London equation on DUO; non-examinable

Thermodynamics of superconducting phase transition

Consider a Type I superconductor:



What is the free energy difference between superconducting and normal state?

-At $T<T_c$ start from zero magnetic field and gradually increase field up to $B_c(T)$

-Potential energy of a magnetic dipole moment μ in a **B**-field is -μ·**B**. Therefore the work done per unit volume on a given material by changing the magnetic field is :

$$dW = -\mathbf{M}\cdot d\mathbf{B}$$

$$\Rightarrow G_s[B_c(T)] - G_s[0] = -\int_0^{B_c(T)} \mathbf{M}\cdot d\mathbf{B}$$

$$= \int_0^{B_c(T)} \frac{B}{\mu_o} dB = \frac{B_c(T)^2}{2\mu_o}$$

(Assuming perfect diamagnetism, i.e. $M = -H$, for superconducting state)

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Condensation energy

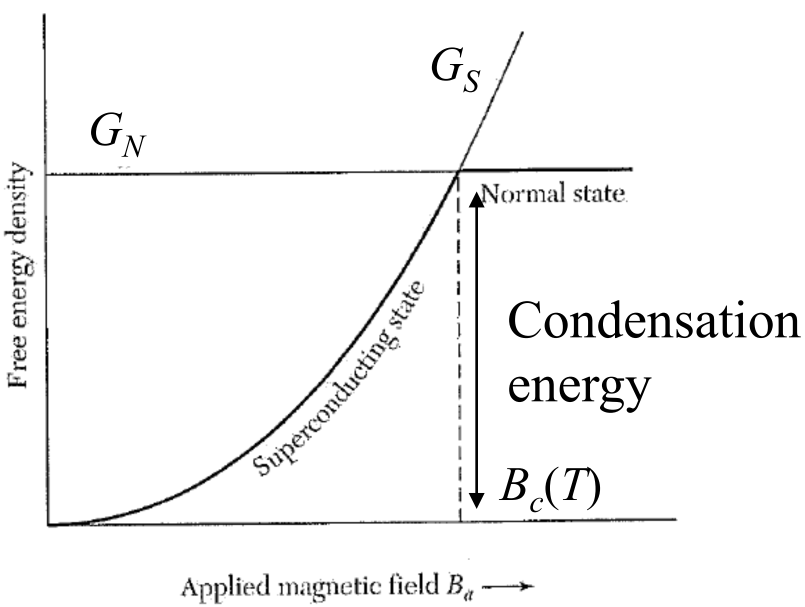
For the normal state assuming negligible magnetisation $\mathbf{M}$:

$$G_N[B_c(T)] - G_N[0] = -\int_0^{B_c(T)} \mathbf{M} \cdot d\mathbf{B} = 0$$

At critical field $G_s[B_c(T)] = G_N[B_c(T)] = G_N[0]$, so that

$$\boxed{G_N[0] - G_s[0] = \frac{B_c(T)^2}{2\mu_o}}$$

Condensation energy

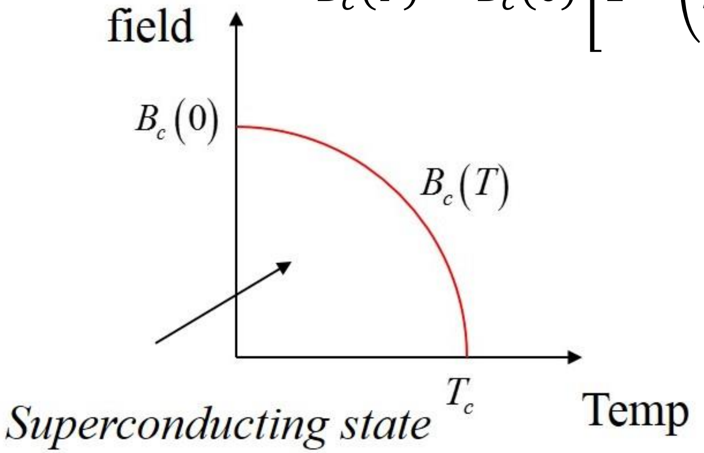


Note:

(i) G_S quadratic dependence on B-field
(ii) G_N independent of B-field
(iii) Superconductor has lower free energy below $B_c(T)$
(iv) Condensation energy extremely small (μeV/atom)

Order of phase transformation

$$B_c(T) = B_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$



$$\boxed{G_N[0] - G_s[0] = \frac{B_c(T)^2}{2\mu_o}}$$

-Free energy of superconducting and normal states equal at T_c, i.e. no latent heat/entropy change.

-Hence second order phase transition. Link to temp dependence of superconducting electrons n_s