FoP 3B Part II

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Room 151

Lecture 10: Polarisation in Dielectrics

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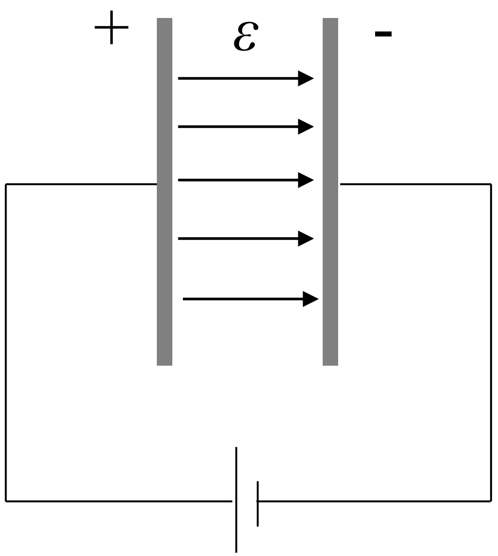
Dielectrics in capacitors



Capacitance is defined by:

capacitor plate area

$$C = \frac{Q}{V} = \epsilon_o \epsilon_r \frac{A}{d}$$

plate separation

dielectric constant

Material	Dielectric Constant	Material	Dielectric Constant
Vacuum	1	Benzene	2.28
Helium	1.000065	Diamond	5.7
Neon	1.00013	Salt	5.9
Hydrogen	1.00025	Silicon	11.8
Argon	1.00052	Methanol	33.0
Air (dry)	1.00054	Water	80.1
Nitrogen	1.00055	Ice (-30° C)	99
Water vapor (100° C)	1.00587	$KTaNbO_3$ (0° C)	34,000

$\left.\begin{array}{l} BaTiO_3 \\ PbTiO_3 \end{array}\right\}$ Ferroelectric 'perovskites'

-Experimentally it is found that inserting a dielectric in a capacitor causes a smaller potential drop across the plates (charge is however conserved).

-This is due to *polarisation* of the dielectric.

Aim of today's lecture

Definition: Dielectrics are insulator materials that can be polarised by an applied electric field.

► Develop the electrostatic framework used for describing dielectric media

Key concepts:

-Polarisation in dielectrics

-Microscopic vs macroscopic electric fields

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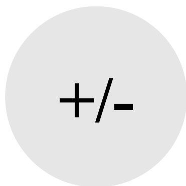
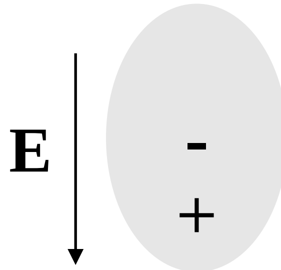
Definition of polarisation

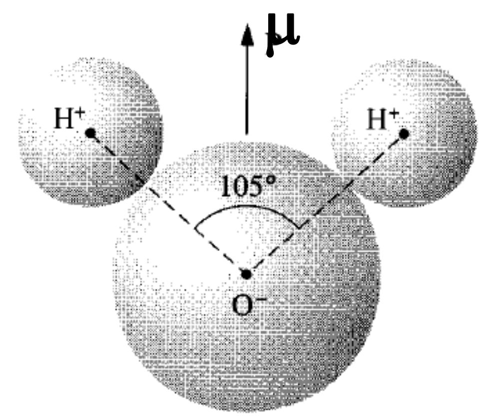


Unpolarised atom
Electric field OFF

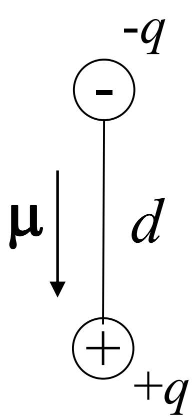


Polarised atom
Electric field ON

More 'complex' polarisations:



Water molecule: *permanent* electric dipole



Electric dipole moment defined by:

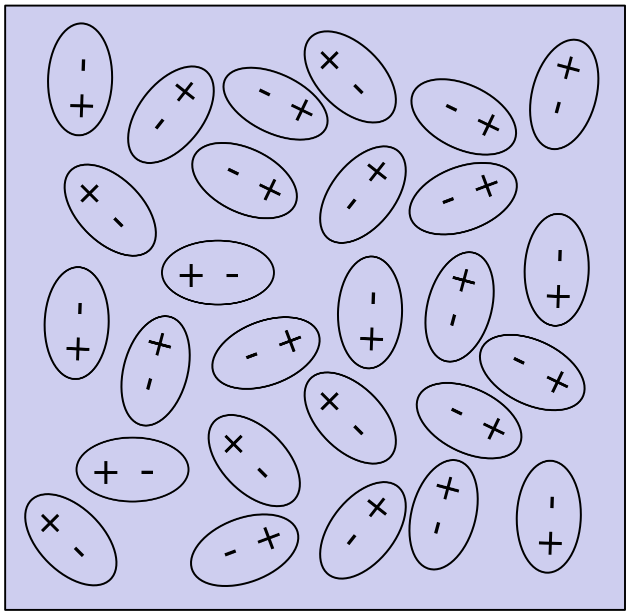
$$|\boldsymbol{\mu}| = qd$$

The direction of $\boldsymbol{\mu}$ is from negative to positive charge.

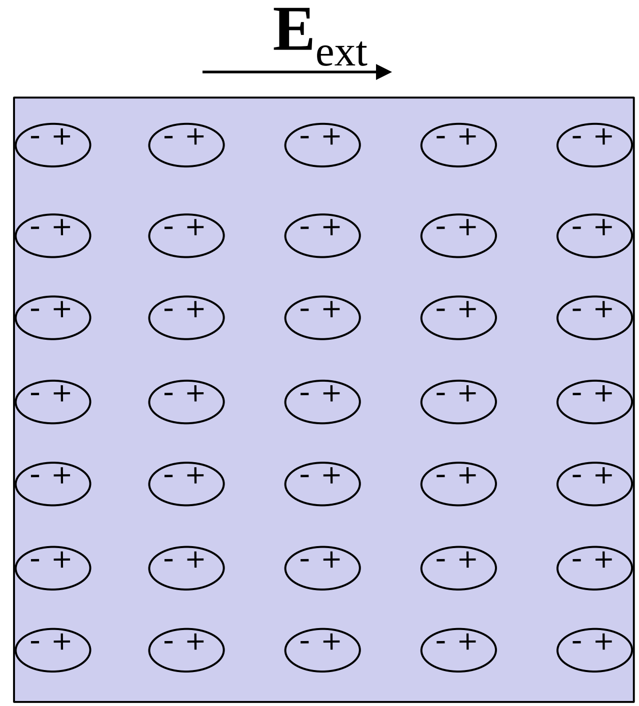
Torque due to electric field: $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{E}$

Potential energy due to electric field: $U = -\boldsymbol{\mu} \cdot \mathbf{E}$

Dielectric media in an electric field (e.g. capacitor)



Electric field OFF

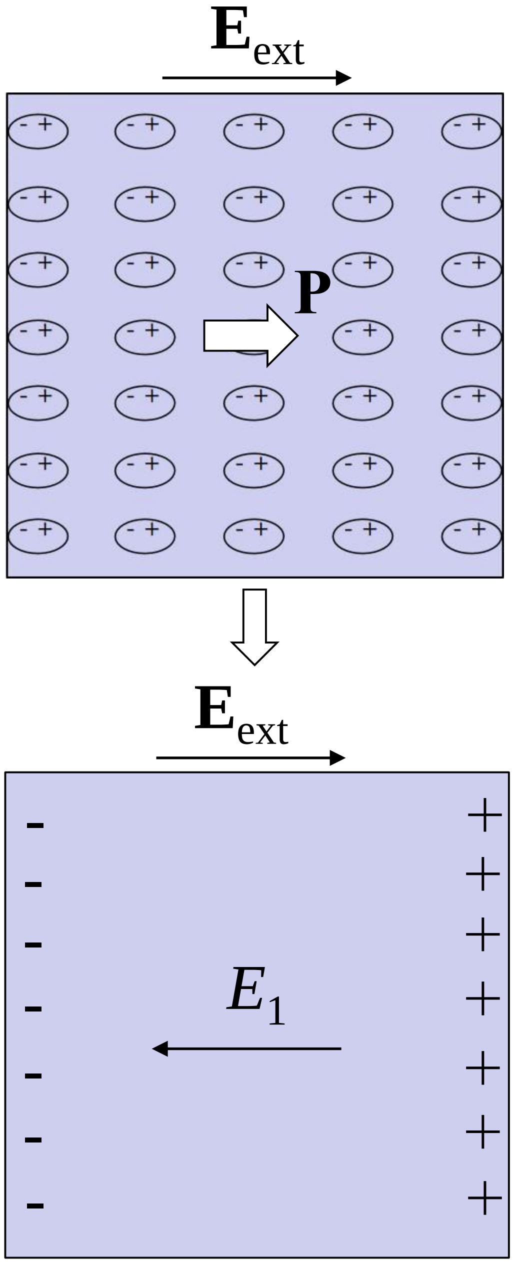


Electric field ON

-Torque on permanent dipoles rotate them to the minimum energy configuration.

-Define polarisation **P** as dipole moment per unit volume, i.e. $\mathbf{P} = N\boldsymbol{\mu}$, where N is the number density of dipoles.

Macroscopic electric field



Electric field due to a single dipole is given by*:

$$\mathbf{E}(\mathbf{r}) = \frac{3(\boldsymbol{\mu}\cdot\mathbf{r})\mathbf{r} - r^2\boldsymbol{\mu}}{4\pi\epsilon_o r^5}$$

For *uniform* polarisation **P** the collective effect of all dipoles can be modelled by a surface charge density (σ) *:

$$\sigma = \mathbf{P}\cdot\hat{\mathbf{n}}$$

Unit surface normal vector

A depolarisation field is therefore present:

$$E_1 = -\frac{\sigma}{\epsilon_o} \qquad \text{(Gauss' law)}$$

Internal field is (E_{ext}-$|E_1|$).

* See for example Griffiths, *Introduction to Electrodynamics,* Chapters 3, 4 (non-examinable)

Macroscopic vs microscopic fields

E_1 is a *macroscopic* electric field 'smoothed' over many dipoles. The local *microscopic* field at an individual dipole can however defer significantly from E_1.

The polarisability (α) of a single dipole is defined as:

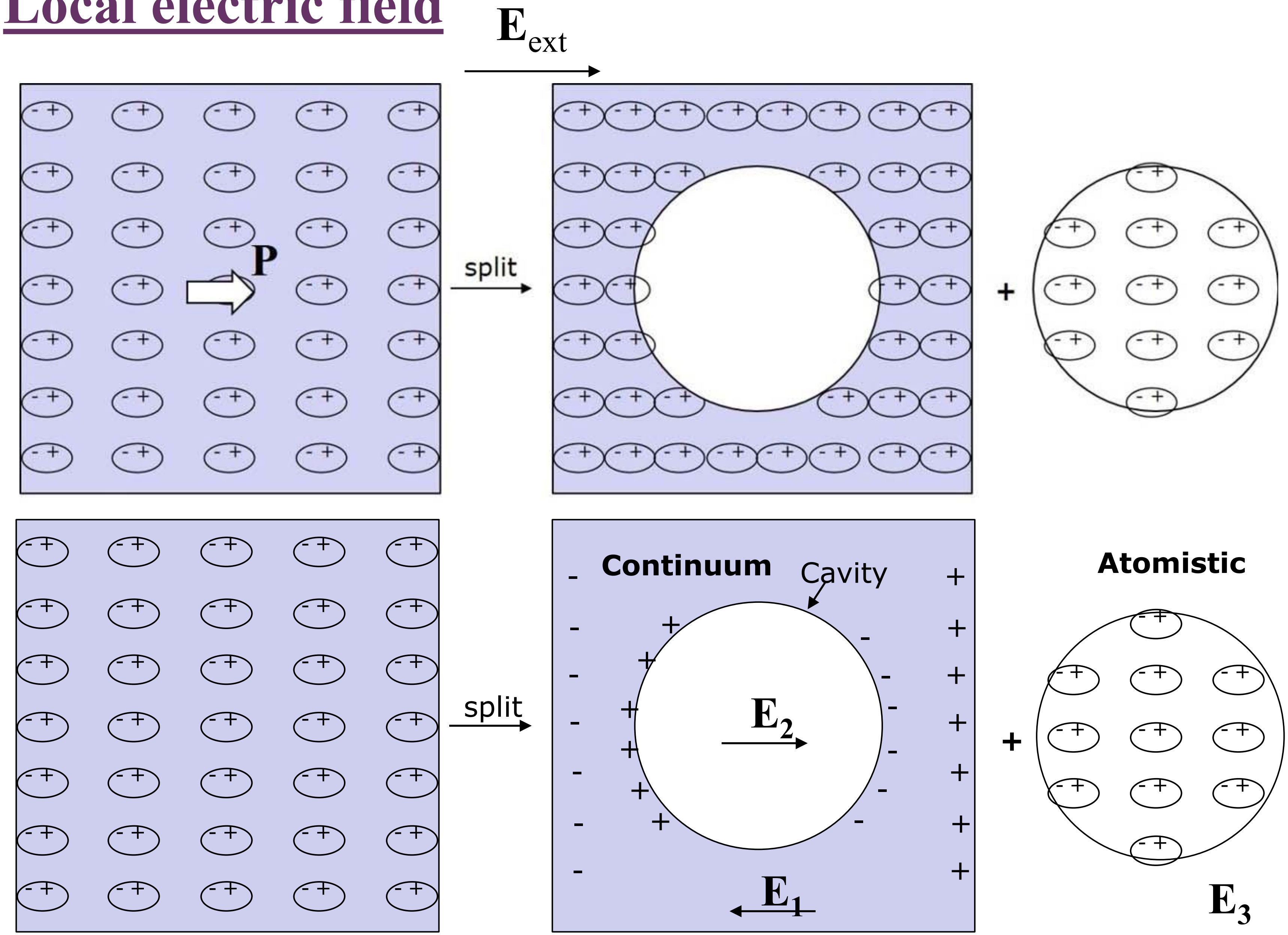
$$\boldsymbol{\mu} = \alpha \mathbf{E}_{\text{local}}$$

Electric dipole moment

Local microscopic electric field

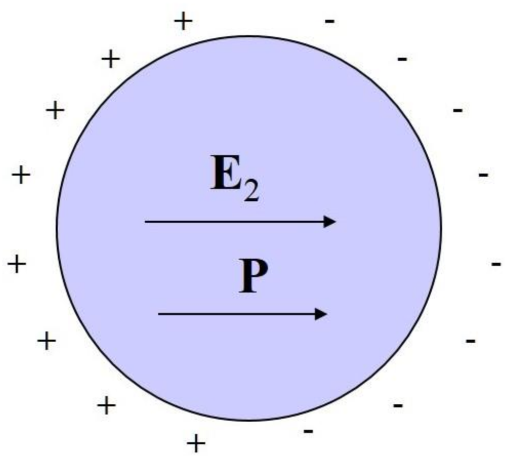
Q: What is the local electric field at a dipole?
Q: How do you calculate polarisability?
(Clausius-Mossotti relation- next lecture)

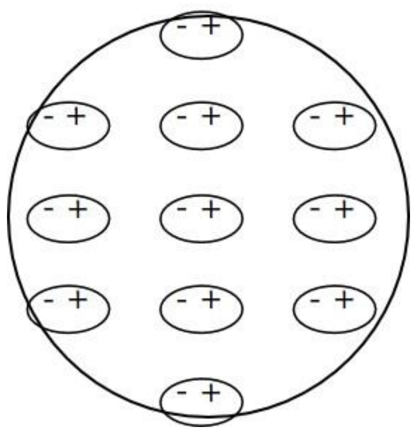
Local electric field



Local electric field $\mathbf{E}_{local} = (\mathbf{E}_{ext} + \mathbf{E}_1) + \mathbf{E}_2 + \mathbf{E}_3$

Lorentz (E_2) and atomistic (E_3) fields





Lorentz field is given by*:

$$\mathbf{E}_2 = \frac{\mathbf{P}}{3\epsilon_o}$$

Atomistic field is given by:

$$\mathbf{E}_3 = \sum_{\text{dipoles } i} \frac{3(\boldsymbol{\mu}_i \cdot \mathbf{r}_i)\mathbf{r}_i - r_i^2 \boldsymbol{\mu}_i}{4\pi\epsilon_o r_i^5}$$

$$= \mu \sum_i \frac{3x_i^2 - r_i^2}{4\pi\epsilon_o r_i^5} = 0$$

For polarisation along *x* and *cubic* crystal.

∴ only the Lorentz field alters the microscopic field from the macroscopic field.

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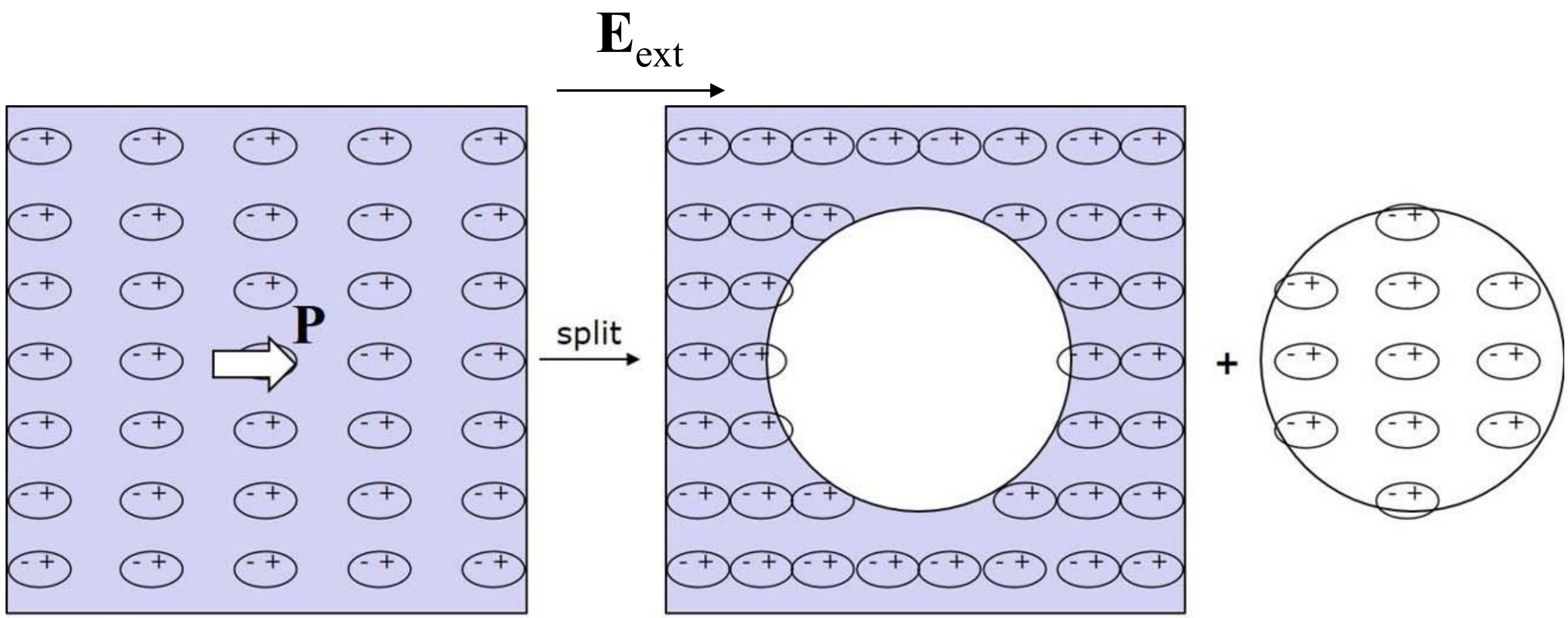
* See for example Griffiths, *Introduction to Electrodynamics,* Chapter 4 (non-examinable)

Local electric field



Local electric field $\mathbf{E}_{local} = (\mathbf{E}_{ext} + \mathbf{E}_1) + \mathbf{E}_2 + \mathbf{E}_3$

$E_1 = -\sigma/\epsilon_0$, $E_2 = P/3\epsilon_0$, $E_3 = 0$ gives:

$$\mathbf{E}_{local} = \mathbf{E}_{ext} - \frac{\sigma}{\epsilon_o} + \frac{P}{3\epsilon_o}$$

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