

Level 3 Condensed Matter Physics- Part II

Weekly problem 1 solutions

(1) The lattice constant for InP is 5.87 Å. We want to find x such that $a(\text{Ga}_x\text{In}_{1-x}\text{As}) = 5.87 \text{ Å}$.

$$\begin{aligned}a(\text{Ga}_x\text{In}_{1-x}\text{As}) &= xa(\text{GaAs}) + (1-x)a(\text{InAs}) \\5.87 \text{ Å} &= x5.65\text{Å} + (1-x)6.06\text{Å} \\x &= 0.46\end{aligned}$$

[2 marks]

The energy gap at this composition is:

$$\begin{aligned}E_g(\text{Ga}_x\text{In}_{1-x}\text{As}) &= xE_g(\text{GaAs}) + (1-x)E_g(\text{InAs}) - bx(1-x) \\&= 0.73 \text{ eV}\end{aligned}$$

[2 marks]

(2) We first express the energy of the conduction electron in the following manner:

$$E(k_x, k_y) = (A+B)k_x^2 + Ak_y^2$$

From the definition of the effective mass tensor $(m^*)_{ij} = \hbar^2/(d^2E/dk_i dk_j)$ it follows that:

$$(m^*)_{xx} = \hbar^2/2(A+B)$$

[2 marks]

$$(m^*)_{yy} = \hbar^2/2A$$

[2 marks]

(3) The fractional atom coordinates for a fcc lattice is $[0,0,0]$, $[\frac{1}{2},\frac{1}{2},0]$, $[\frac{1}{2},0,\frac{1}{2}]$ and $[0,\frac{1}{2},\frac{1}{2}]$. Adding the $[\frac{1}{4},\frac{1}{4},\frac{1}{4}]$ vector gives the other four atom positions, i.e. $[\frac{1}{4},\frac{1}{4},\frac{1}{4}]$, $[\frac{3}{4},\frac{3}{4},\frac{1}{4}]$, $[\frac{3}{4},\frac{1}{4},\frac{3}{4}]$ and $[\frac{1}{4},\frac{3}{4},\frac{3}{4}]$.

[2 marks]