FoP 3B Part II

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Room 151

Lecture 9: Ginzburg-Landau theory

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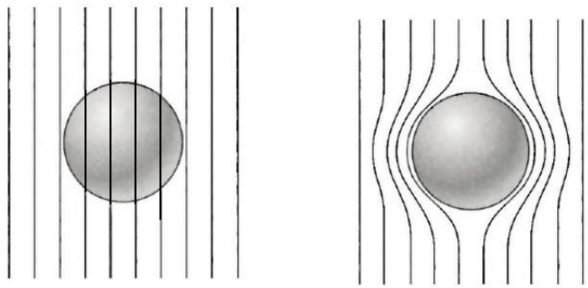
Summary of Lecture 8

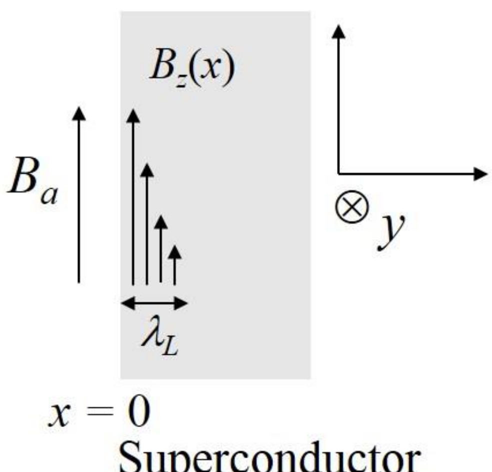
London equation



$T > T_c$
(**B** on)

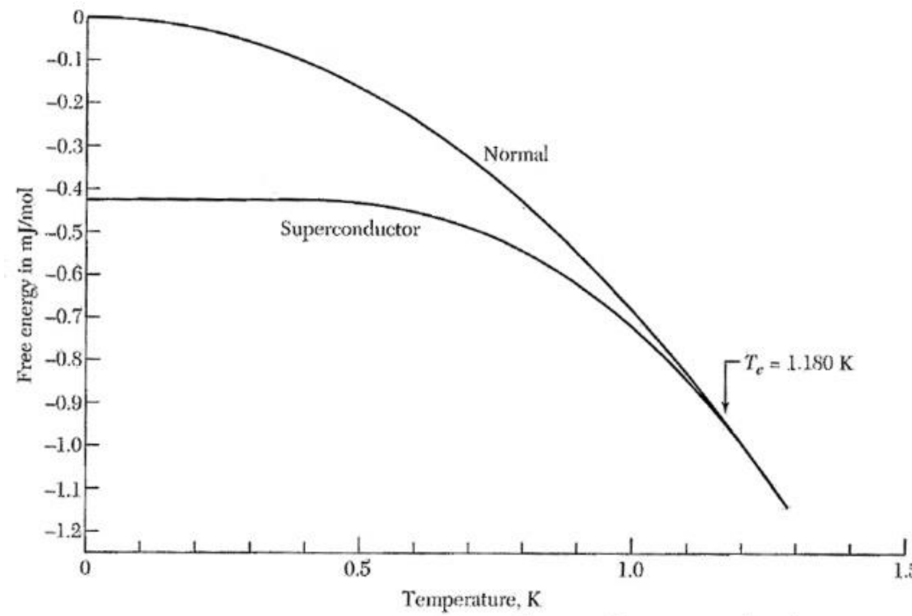
$T < T_c$
(**B** on)

$$\boldsymbol{\nabla}^2 \mathbf{B} = \frac{1}{\lambda_L^2} \mathbf{B}$$



Condensate energy

$$G_N[0] - G_s[0] = \frac{B_c(T)^2}{2\mu_o}$$



-No latent heat
-Second order phase transition

Aim of today's lecture

► What causes superconductivity and how do we model it?

Two approaches:*

(i) Bardeen, Cooper and Schrieffer (BCS theory)
-microscopic theory of superconductivity (1957)
-*Key concepts:* Cooper pairs, band gap

(ii) Ginzburg-Landau (GL) theory
-phenomenological theory of superconductivity (1950)
-*Key concepts:* coherence length, Type I vs Type II behaviour

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(Note that Ginzburg-Landau theory can be derived from BCS, so the two are formally equivalent)

GL theory and modelling of second order transitions

-Assume phase transition characterised by an order parameter (e.g. magnetisation for a ferromagnetic-paramagnetic transition)

-For superconductivity GL postulated order parameter is $|\psi|^2$, where ψ is a complex number. In the normal state $|\psi|^2 = 0$, while $|\psi|^2 \neq 0$ for the superconducting state.

-Assume zero magnetic field and spatially uniform material*. Around the transition temperature T_c:

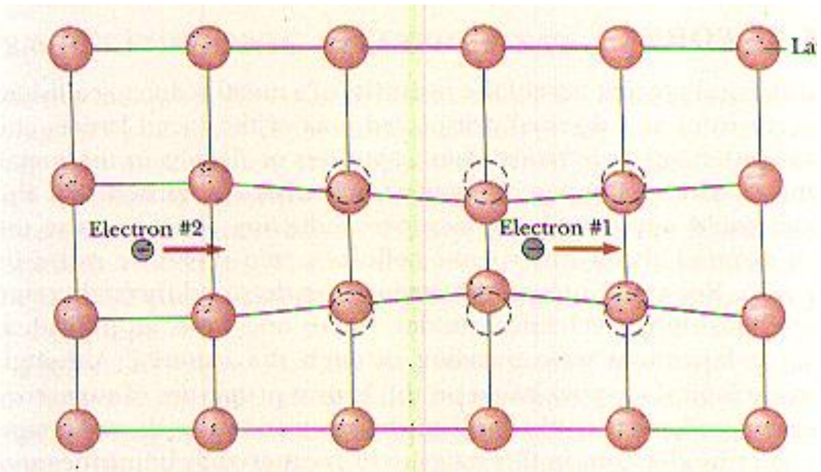
$$G_s(T) = G_N(T) + a(T)|\psi|^2 + \frac{b(T)}{2}|\psi|^4$$

$a(T)$ and $b(T)$ are constants that vary with temperature. $G_{s,N}$ is the free energy of superconducting and normal phases respectively.

*GL theory can be extended to include magnetic fields and spatially non-uniform materials

Cooper pairs

What does $|\psi|^2$ represent? *Answer:* Cooper pairs

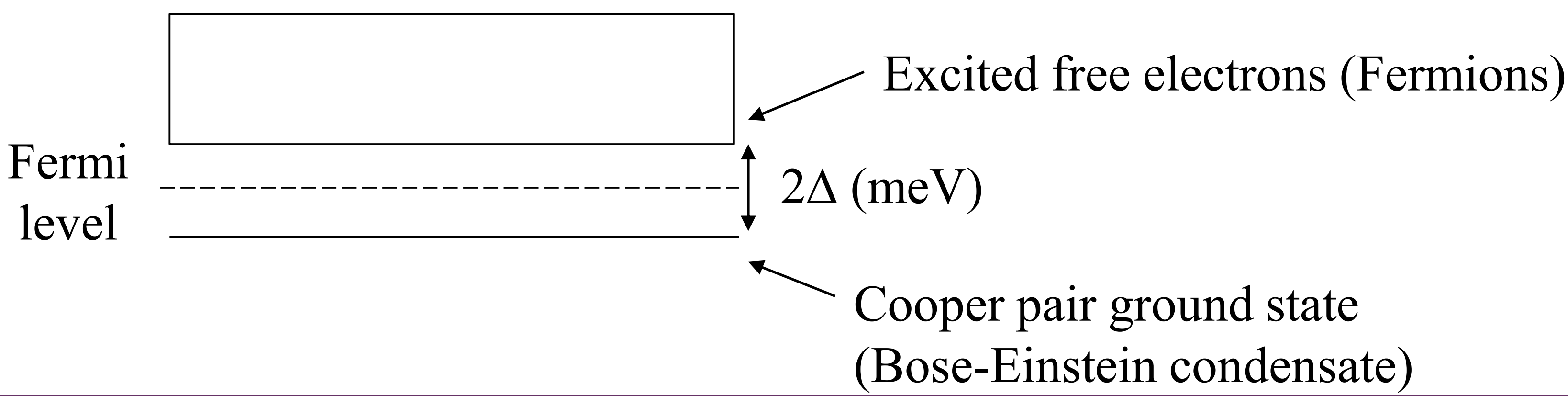


Electrons can *attract* via electron-phonon interactions.

$$|\psi|^2 = n_s/2$$

(n_s = density of superconducting electrons)

Superconducting band gap (2Δ) predicted:

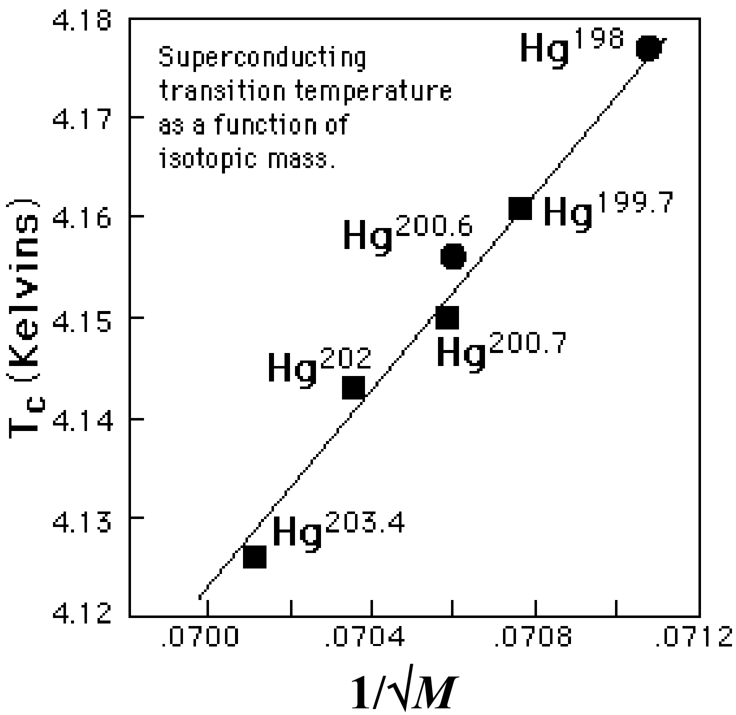


Evidence for Cooper pairs and band gaps

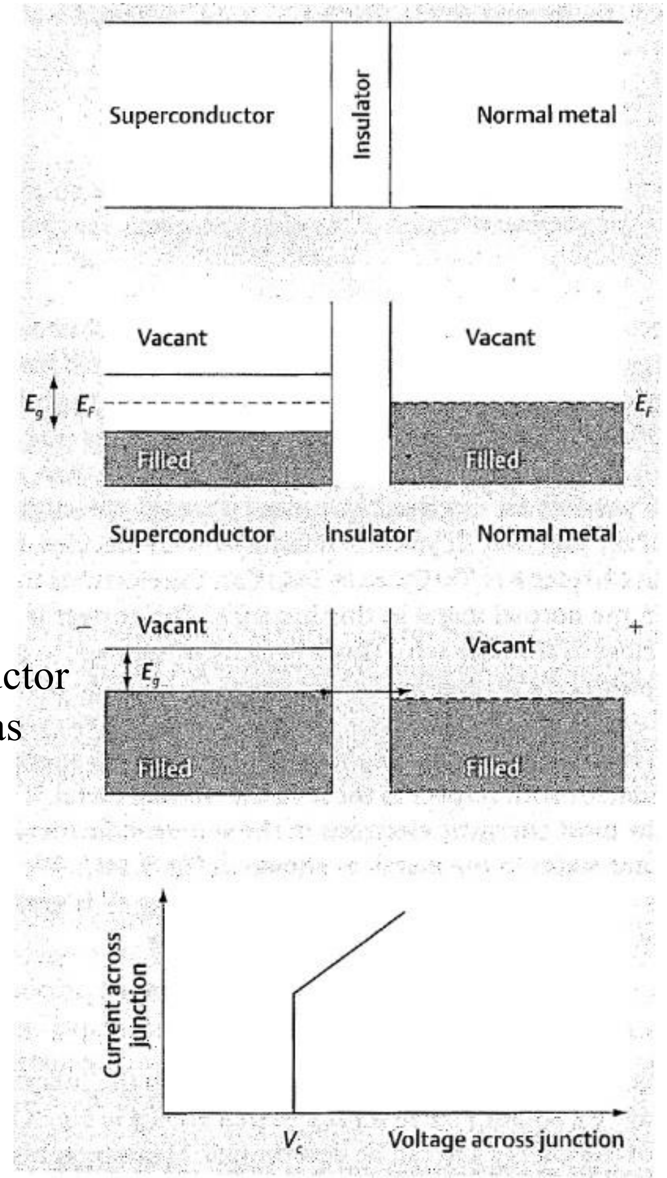
(1) Isotope effect

$$T_c \propto \frac{1}{\sqrt{M}}$$

(M = atomic mass)



(2) Tunneling current



Superconductor negative bias

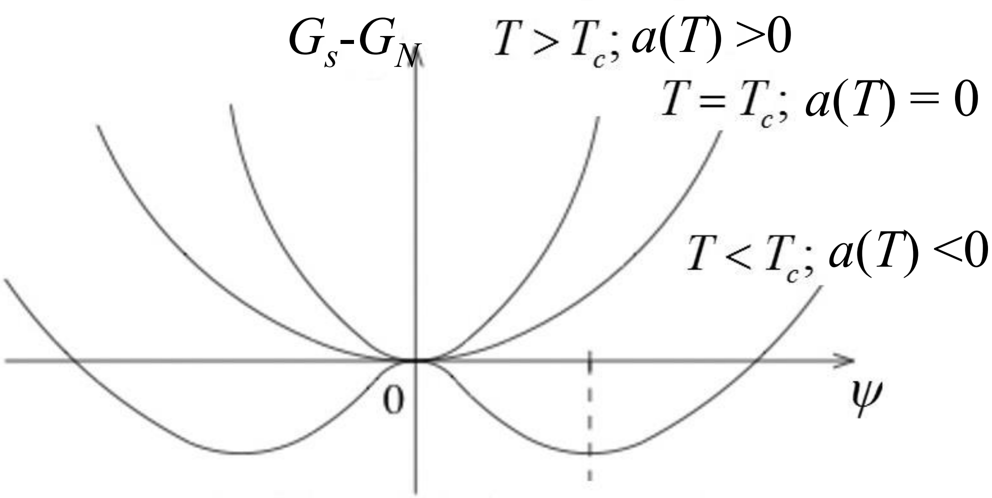
Metal positive bias

GL energy

$$G_s(T) = G_N(T) + a(T)|\psi|^2 + \frac{b(T)}{2}|\psi|^4$$

(i) For energy minimum require $b(T) > 0$.

(ii) When $a(T) > 0$ only minimum is at $|\psi| = 0$; (normal state)
When $a(T) < 0$ minima at $|\psi|^2 = -a(T)/b(T)$; (superconducting state)



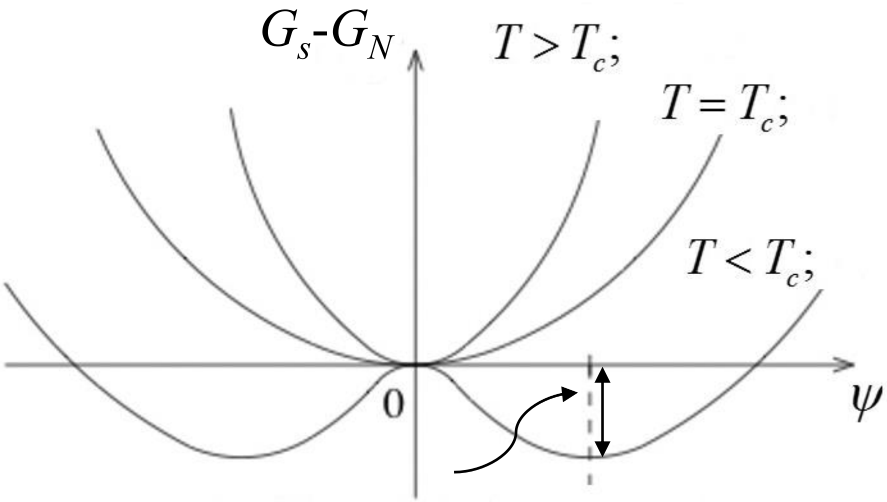
Therefore:

$$a(T) \approx \dot{a}(T - T_c) + \cdots$$
$$(\dot{a} > 0)$$
$$b(T) \approx b + \cdots$$
$$(b > 0)$$

And:

$$|\psi|^2 = \begin{cases} \left[\dfrac{\dot{a}(T_c - T)}{b}\right] & T < T_c \\ 0 & T > T_c \end{cases}$$

Predicting superconducting properties using GL theory



Condensation energy

Equating condensation energies:

$$\frac{[\dot{a}(T-T_c)]^2}{2b} \quad \text{(GL)}$$

$$\frac{B_c(T)^2}{2\mu_o} \quad \text{(Thermodynamics)}$$

$$\frac{[\dot{a}(T-T_c)]^2}{2b} = \frac{B_c(T)^2}{2\mu_o}$$

⇨ Gives value for $\dot{a}^2/b$

From $S = -(dG/dT)$ the entropy change:

$$S_s(T) - S_N(T) = -\frac{\dot{a}^2}{b}(T_c - T)$$

⇨ No latent heat/entropy change at T_c (second order transition)

GL theory in inhomogenous systems*

$$G_s(T,\mathbf{r}) = G_N(T,\mathbf{r}) + \frac{\hbar^2}{2m}|\nabla\psi(\mathbf{r})|^2 + a(T)|\psi(\mathbf{r})|^2 + \frac{b(T)}{2}|\psi(\mathbf{r})|^4$$

Minimise:

$$G_s(T) = G_N(T) + \int\left(\frac{\hbar^2}{2m}|\nabla\psi(\mathbf{r})|^2 + a(T)|\psi(\mathbf{r})|^2 + \frac{b(T)}{2}|\psi(\mathbf{r})|^4\right)d\mathbf{r}$$

$$\Rightarrow \quad \frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + \left[a(T) + \frac{b(T)}{2}|\psi(\mathbf{r})|^2\right]\psi(\mathbf{r}) = 0$$

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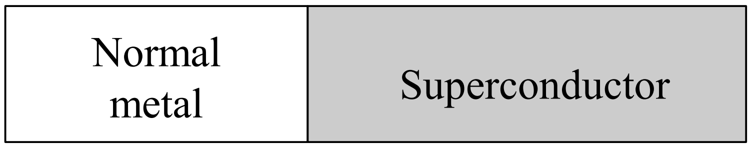
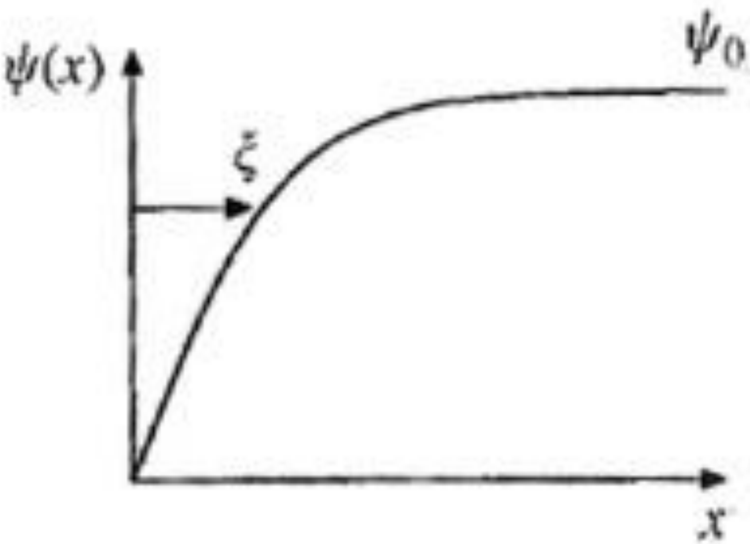
* Equations non-examinable

Application of GL theory to normal metal-superconductor interface*





$$\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + a(T)\psi(x) + \frac{b(T)}{2}\psi(x)^2 = 0$$

$$\psi(x) = \psi_0 \tanh\left(\frac{x}{\sqrt{2}\xi(T)}\right)$$

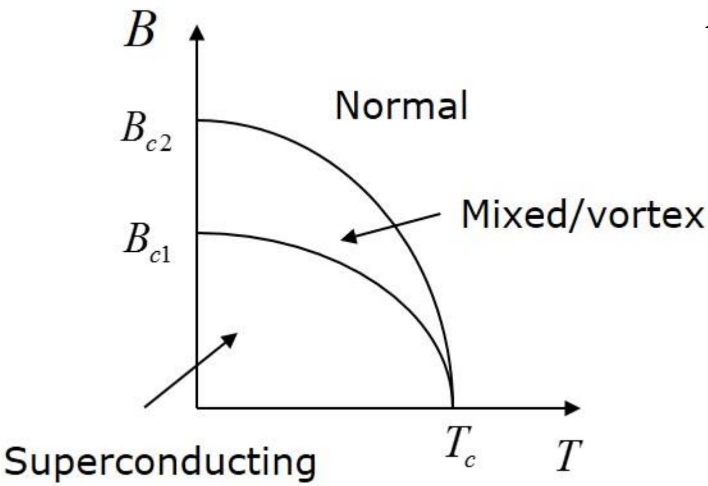
$$\xi(T) = \left(\frac{\hbar^2}{2m|a(T)|}\right)^{1/2}$$

-$\xi(T)$ is the coherence length, i.e. distance of separation of Cooper pair electrons

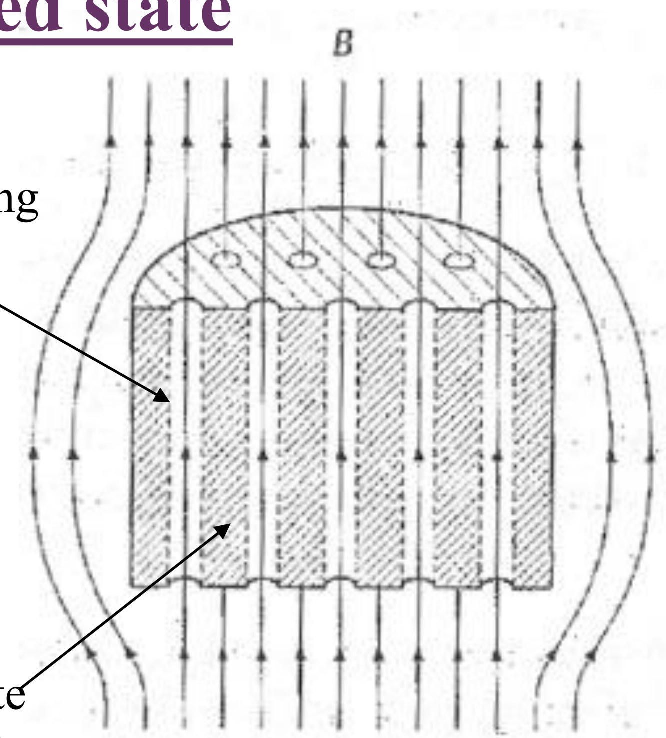
- $\xi(T)$ decreases with T due to $|a(T)|$ term (at T_c coherence length is divergent)

* Equations non-examinable

Type II superconductors in the mixed state

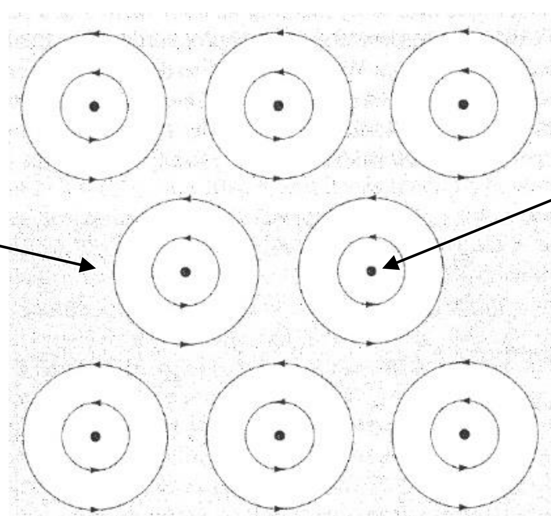


Magnetic field penetrating through Normal state 'filaments'



Superconducting state (no field penetration)

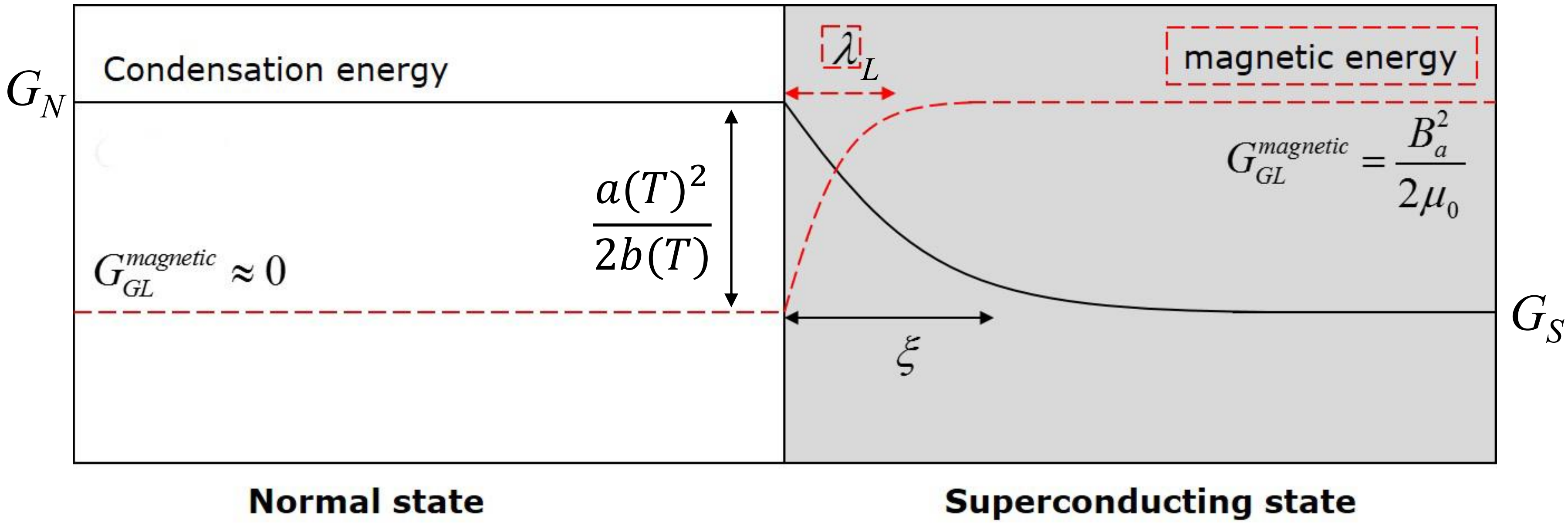
Supercurrents (see London equation)



Normal core region

-Magnetic field from supercurrent cancels external field passing through normal metal core

Type I superconductors: ξ > λ_L



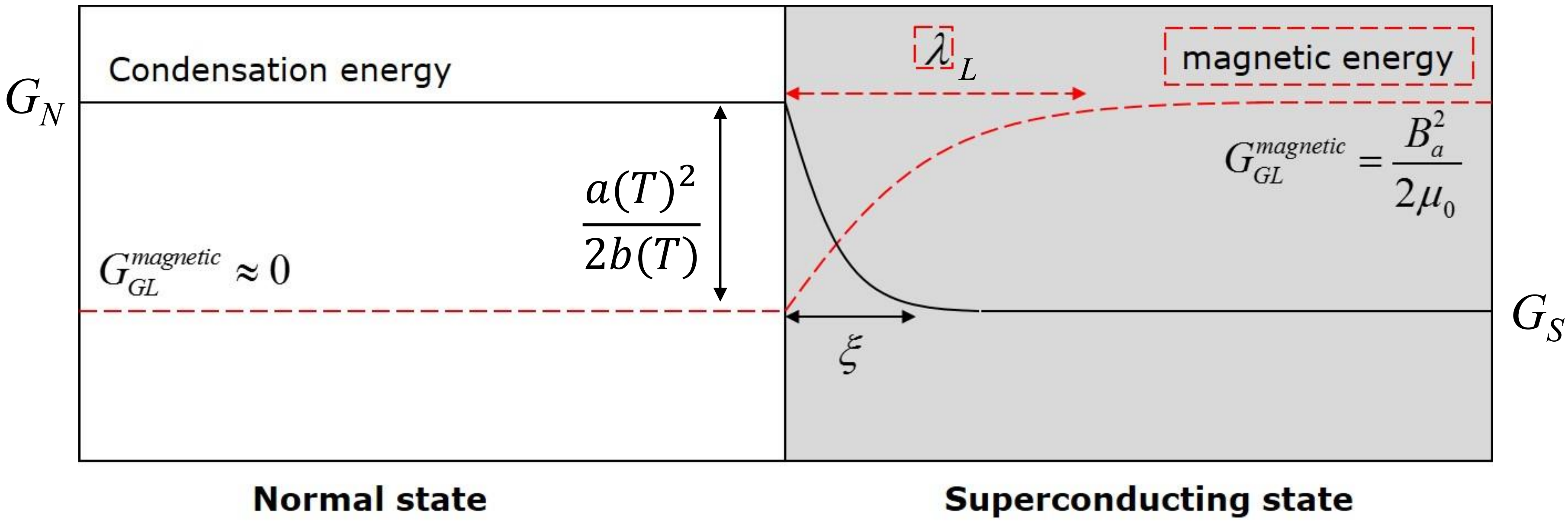
-Magnetic energy due to field penetration increases at a *faster* rate than energy decrease due to Cooper pair condensation

-Leads to *high* interfacial energy

i.e. co-existence of normal and superconducting regions *not allowed* in Type I

-Coherence length ξ typically larger for elements: reason for Type I behaviour in elements and Type II behaviour in compounds

Type II superconductors: ξ < λ_L



-Magnetic energy due to field penetration increases at a *slower* rate than energy decrease due to Cooper pair condensation

-Leads to *low* interfacial energy

i.e. co-existence of normal and superconducting regions *allowed* in Type II