**Level 3 Condensed Matter Physics- Part II**
**Weekly problem 1 solutions**

(1) The lattice constant for InP is 5.87 Å. We want to find $x$ such that $a(Ga_xIn_{1-x}As)$ = 5.87 Å.

$$a(Ga_xIn_{1-x}As) = xa(GaAs) + (1-x)a(InAs)$$
$$5.87\ \text{Å} = x5.65\text{Å} + (1-x)6.06\text{Å}$$
$$x = 0.46$$

[2 marks]

The energy gap at this composition is:

$$E_g(Ga_xIn_{1-x}As) = xE_g\,(GaAs) + (1-x)E_g(InAs) - bx(1-x)$$
$$= 0.73\ \text{eV}$$

[2 marks]

(2) We first express the energy of the conduction electron in the following manner:

$$E(k_x,k_y) = (A+B)k_x^2 + Ak_y^2$$

From the definition of the effective mass tensor $(m^*)_{ij} = \hbar^2/(d^2E/dk_idk_j)$ it follows that:

$$(m^*)_{xx} = \hbar^2/2(A+B)$$

[2 marks]

$$(m^*)_{yy} = \hbar^2/2A$$

[2 marks]

(3) The fractional atom coordinates for a fcc lattice is [0,0,0], [½,½,0], [½,0,½] and [0,½,½]. Adding the [¼,¼,¼] vector gives the other four atom positions, i.e. [¼,¼,¼], [¾,¾,¼], [¾,¼,¾] and [¼,¾,¾].

[2 marks]

**Level 3 Condensed Matter Physics- Part II**
**Weekly problem 2 solutions**

(1) i) The donor ionisation energy is given by [1 mark]

$$E_D = \frac{e^4 m_e^*}{2(4\pi\varepsilon\varepsilon_0\hbar)^2} = \frac{m_e^*}{m\varepsilon^2}\left[\frac{e^4 m}{2(4\pi\varepsilon_0\hbar)^2}\right]$$
$$= \frac{0.2}{(11.7)^2} \times 13.6\ eV = 19.9\ meV$$

ii) The radius of the orbit [1 mark]

$$r = \frac{4\pi\varepsilon\varepsilon_0\hbar^2}{m_e^* e^2} = \frac{\varepsilon m}{m_e^*}\left(\frac{4\pi\varepsilon_0\hbar^2}{me^2}\right)$$
$$= \frac{11.7}{0.2} \times 0.53\ \text{Å} = 31.0\ \text{Å}$$

iii) The overlap occurs when [1 mark]

$$N_D \sim \frac{1}{\frac{4}{3}\pi r^3} = 8.0 \times 10^{24}\ \mathrm{m}^{-3}$$

(2) For an intrinsic semiconductor:

$$n = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right)$$

so that

$$\frac{n(T)}{n(300K)} = \exp\left[-\frac{E_g}{2k}\left(\frac{1}{T} - \frac{1}{300}\right)\right]$$

[1 mark]

Extrinsic behaviour occurs when $n(T) \leq 10^{18}\ \mathrm{m}^{-3}$ [1 mark]. Substituting into the above equation:

$$\frac{10^{18}}{5 \times 10^{15}} = \exp\left[-\frac{E_g}{2k}\left(\frac{1}{T} - \frac{1}{300}\right)\right]$$

we have $T \leq 400$ K. [2 marks]

(3) Since the semiconductor is in the saturation regime all phosphorus donor atoms are ionised and we assume the conductivity is dominated by the majority carrier electrons. [1 mark]

From $\sigma = en\mu_e$ with $n \sim 10^{20}\ \mathrm{m}^{-3}$ and $\mu_e = 0.16\ \mathrm{m}^2\mathrm{V}^{-1}\mathrm{s}^{-1}$ we get $\sigma = 2.56$ S/m. [2 marks]

**Level 3 Condensed Matter Physics- Part II**
**Weekly problem 3 solutions**

(1) i) The intrinsic carrier concentration ($n_i$) is given by [1 mark]:

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right) = 9.7\ \times 10^{15}\ \mathrm{m}^{-3}$$

The built-in voltage ($V_{\mathrm{bi}}$) is therefore [1 mark]:

$$\phi_{bi} = \frac{kT}{e} \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.71\ \mathrm{V}$$

ii) The space charge width on the *n*-side [1 mark]:

$$w_n = \left[\frac{2\epsilon_r \epsilon_0 \phi_{bi}}{e}\left(\frac{N_A}{N_D}\right)\left(\frac{1}{N_A + N_D}\right)\right]^{1/2} = 0.38\ \mu\mathrm{m}$$

and on the *p*-side [1 mark]:

$$w_p = \left[\frac{2\epsilon_r \epsilon_0 \phi_{bi}}{e}\left(\frac{N_D}{N_A}\right)\left(\frac{1}{N_A + N_D}\right)\right]^{1/2} = 0.12\ \mu\mathrm{m}$$

iii) The magnitude of the maximum electric field is [1 mark]:

$$\mathcal{E}_{\max} = \frac{eN_D w_n}{\epsilon_r \epsilon_0} = \frac{eN_A w_p}{\epsilon_r \epsilon_0} = 2.9\ \times 10^6\ \mathrm{V/m}$$

(2) The charge $Q$ stored per unit junction area is:

$$Q = eN_A w_p = eN_D w_n = 3.0 \times 10^{-4} \mathrm{C/m^2}$$

The capacitance per unit area is therefore $C = Q/\phi_{\mathrm{bi}} = 4.3 \times 10^{-4}\ \mathrm{F/m^2}$. [2 marks]

An alternative, but equivalent, definition of capacitance per unit area is:

$$C = \frac{\epsilon_r \epsilon_0}{d}$$

where $d$ is the capacitor plate spacing. Substituting $d = (w_n + w_p)$ gives $C = 2.1 \times 10^{-4}\ \mathrm{F/m^2}$. This is different by a factor of 2 to the previous answer due to the approximate nature of the calculation. Both approaches are nevertheless acceptable. [2 marks]

(3) The current is given by [1 mark]:

$$I(V) = I_0 \left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

Substituting $I = 25$ mA for 0.2 V forward bias gives $I_0 = 11$ μA [2 marks].

**Level 3 Condensed Matter Physics- Part II**
**Weekly problem 4 solutions**

(1) The change in the current ($\Delta I$) cannot be larger than the precision of the measurement and therefore $\Delta I = I(t) - I_o = -0.001 I_o$, where $I_o$ is the current at $t = 0$ [1 mark].

Using $I(t) = I_o \exp(-t/\tau)$:

$$\Delta I = I_o \left[ \exp\left(-\frac{t}{\tau}\right) - 1 \right] = -0.001 I_o$$

Substituting $t = 1$ year gives $\tau = 999$ years. This is a lower limit for the average scattering time. [2 marks]

(2) For a Type I superconductor:

$$B_c(T) = B_c(0) \left[ 1 - \left(\frac{T}{T_c}\right)^2 \right]$$

[1 mark]

Substituting $T = 4.2$ K, $T_c = 7.2$ K and $B_c(0) = 0.080$ T gives $B_c(T = 4.2\ \text{K}) = 0.053$ T. [1 mark]

The maximum current is:

$$I_{max} = \frac{2\pi R B_c(T)}{\mu_o} = \frac{2\pi \times (0.5 \times 10^{-3}\ \text{m}) \times 0.053\text{T}}{4\pi \times 10^{-7}\ \text{H/m}} = 132.5\ \text{A}$$

[2 marks]

(3) The superconducting phase (beyond the London penetration depth) is a perfect diamagnet and has susceptibility -1. [1 mark]

If the volume fraction of superconducting and normal phase is $V_{sc}$ and $V_n$ respectively, with $V_{sc} + V_n = 1$, then:

$$\chi = -V_{sc} + (2 \times 10^{-5}) V_n = -0.7$$

Solving for $V_n$ gives $V_n = 0.3$ (30%) and hence $V_{sc} = 0.7$ (70%). [2 marks]