Level 3 Condensed Matter Physics- Part I
Weekly problem 4 solutions

(1) How much work has been done?
A magnetic field acting upon a magnetic dipole will produce a torque, $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$. When the magnetic dipole rotates through an angle $d\theta$ the torque does work,

$dW = \tau d\theta = \mu B \sin\theta d\theta.$ [1 mark]

Setting this work equal to the change in potential energy we obtain,
$dE = dW = \mu B \sin\theta d\theta$, and by integrating we obtain,
$E = -\mu B[\cos\theta]_{\theta_0}^{\theta} + E_0$ [1 mark]

Choosing the potential energy to be zero when the magnetic dipole is perpendicular to the field, i.e. $E = 0$ when $\theta_0 = 90°$, we obtain the potential energy of the magnetic dipole as required,
$E = -\mu B\cos\theta = -\boldsymbol{\mu} \cdot \mathbf{B}$ [1 mark]

(2) Energy differences due to magnetism

The potential energy is at its minimum when $\theta = 0°$ and at its maximum when $\theta = 180°$.

$E_{\text{max}} = -\mu B\cos\theta = -2 \times \mu_{\text{B}} \times 1 \times \cos(180°) = -2 \times 9.27 \times 10^{-24} \times 1 \times -1 = 18.5 \times 10^{-24}$ J

Similarly, the minimum potential energy when θ = 0° yields $E_{\text{min}} = -18.5 \times 10^{-24}$J. [2 marks]

The potential energy difference between these two states of the atom is therefore

$\Delta E = 37.1 \times 10^{-24}$J , which at a room temperature of 300 K compares to a thermal energy of

$k_B T \approx 4.14 \times 10^{-21}$J . Thus, the potential energy difference is only approx. 1% of the thermal energy available at room temperature. This means both states will exist with almost equal populations at room temperature. [1 mark]

(3) **Diamagnetism in Bismuth**

(*a*) We use the Langévin's expression for diamagnetic susceptibility $\chi_{\text{d}} = -\frac{\mu_0 N Z e^2 \langle r^2 \rangle}{6 m_e}$

We determine *N* from the equation where *m* is the molar mass of bismuth.

$N = \frac{N_{\text{A}}\rho}{m} = \frac{6.023\times10^{23}\times9.78\times10^{3}}{208.98\times10^{-3}} = 2.82 \times 10^{28}\ \text{m}^{-3}$ [1 mark]

Therefore

$$\chi_d = -\frac{\mu_0 N Z e^2 \langle r^2 \rangle}{6 m_e} = -\frac{4\pi\times10^{-7}\times2.82\times10^{28}\times83\times\left(1.6\times10^{-19}\right)^2\times\left(0.16\times10^{-9}\right)^2}{6\times9.11\times10^{-31}} = -3.5 \times 10^{-4}$$

[1 mark]

(b) The magnetic moment (*m*) of the sample can be calculated from the magnetisation (total magnetic moment per unit volume), *M*, but we need to convert from sample mass (m_s) to sample volume using the density ρ involving an equation which contains too many m's

$m = \frac{Mm_s}{\rho}$ and using the definition for susceptibility

$$m = \frac{\chi_{\mathrm{d}} H m_s}{\rho} = \frac{-3.5\times10^{-4}\times3\times10^{3}\times1\times10^{-3}}{9.78\times10^{3}} = -1.07 \times 10^{-7}\ \mathrm{A\ m^2}$$

[2 marks]