Level 3 Condensed Matter Physics- Part II
Examples Class 2 Answers

(1) i) Using $\upsilon = (dE/dk)/\hbar$ we get:

$$\upsilon = \frac{2Ia}{\hbar}\sin(ka)$$

(ii) $F = \hbar\frac{dk}{dt} = -e\varepsilon$ gives:

$$k(t) = -\frac{e\varepsilon t}{\hbar}$$

where we have made use of the fact that $k(0) = 0$. Substituting in the expression for υ:

$$\upsilon(t) = -\frac{2Ia}{\hbar}\sin\left(\frac{ea\varepsilon t}{\hbar}\right)$$

(iii) From $\upsilon = dx/dt$:

$$x(t) = \frac{2I}{e\varepsilon}\left[\cos\left(\frac{ea\varepsilon t}{\hbar}\right) - 1\right]$$

where we have made use of the boundary condition $x(0) = 0$. During time averaging the cosine term averages to zero, so that:

$$< x > = -\frac{2I}{e\varepsilon}$$

(2) i) Using the fact that $\mathbf{k}_h = -\mathbf{k}_e$ the **k**-vector for the hole is (-0.02, 0, 0) Å^{-1}. The energy can be determined by substituting $\mathbf{k}$ = (0.02, 0, 0) Å^{-1} into the energy expression and noting that the hole energy is the negative of the electron energy. The value is 5.5 meV.

ii) Since $\nabla_{\mathbf{k}} = (\partial/\partial k_x)\mathbf{i} + (\partial/\partial k_y)\mathbf{j} + (\partial/\partial k_z)\mathbf{k}$ we have to first find the directional derivatives. For a direction p, where $p = x$, y or z:

$$\frac{\partial E}{\partial k_p} = \frac{\hbar^2}{2m}\left\{-8.58k_p + \frac{0.92k^2k_p + 23.72k_p(k_q^2 + k_r^2)}{\sqrt{[0.46k^4 + 23.72(k_p^2k_q^2 + k_q^2k_r^2 + k_r^2k_p^2)]}}\right\}$$

where k_q and k_r are the **k**-vector components perpendicular to k_p. For $\mathbf{k}$ = (0.02, 0, 0) Å^{-1} it is clear that the directional derivative is non-zero only along the x-direction, so that:

$$\mathbf{v} = -83{,}543\ \mathbf{i}\ \text{(m/s)}$$

Note that we have used the fact that the hole velocity is equal to the electron velocity.

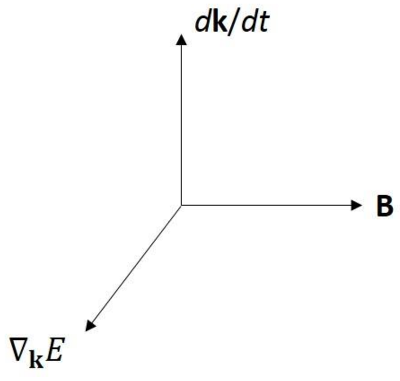
(4) i) introducing the Lorentz force due to the magnetic field:

$$\hbar\frac{d\mathbf{k}}{dt} = q(\mathbf{v} \times \mathbf{B})$$

ii) substituting the expression for the velocity gives:

$$\hbar^2 \frac{d\mathbf{k}}{dt} = q(\nabla_{\mathbf{k}} E \times \mathbf{B})$$

The above equation can be represented diagrammatically as a right-handed system:



Since (d**k**/dt) is perpendicular to **B** this means that the component of **k** parallel to **B** in unchanged. Furthermore the change in energy (δE) in time δt is given by:

$$\delta E = \frac{dE}{dt}\delta t = \left(\nabla_{\mathbf{k}} E \cdot \frac{d\mathbf{k}}{dt}\right)\delta t = 0$$

Therefor the particle moves along a constant energy surface in k-space that is perpendicular to **B**.