FoP 3B Part II

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Room 151

Lecture 3: Statistical Physics of semiconductors

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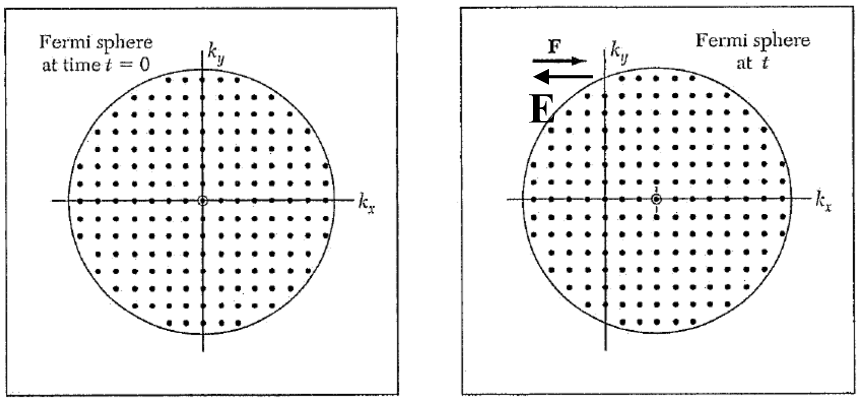
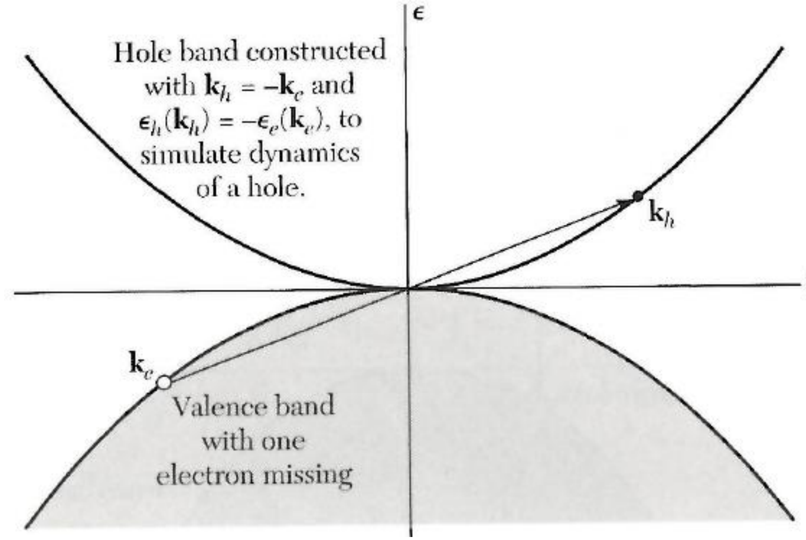
Summary of Lecture 2

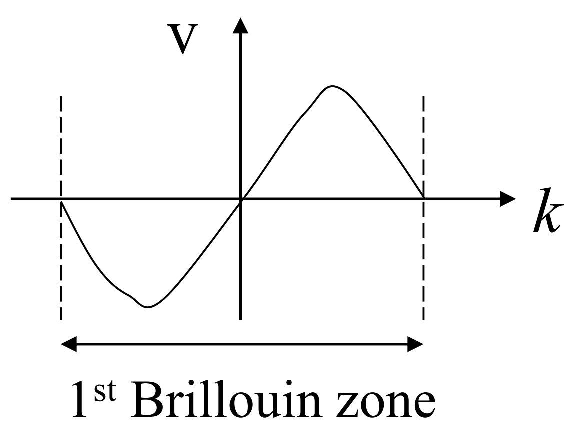


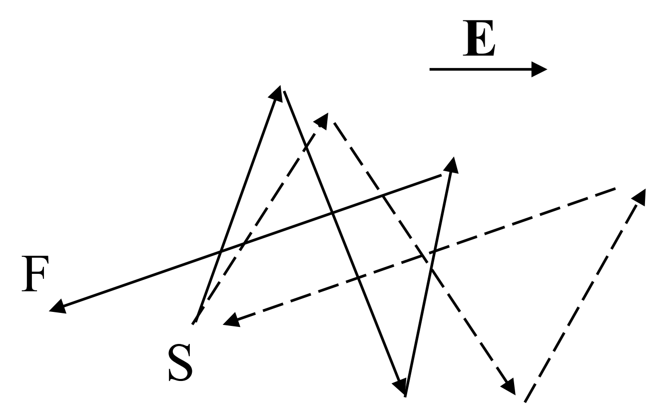
Conduction: nearly empty band (electrons)



Conduction: nearly full band (holes)

Role of scattering in electron/hole conduction:





Aim of today's lecture

Q: How to calculate electron and hole populations in a semiconductor at a given temperature?

Key concepts:

-Statistical physics of electrons and holes

-Fermi level/chemical potential in an intrinsic semiconductor

-Law of mass action

-Effect of temperature on conductivity

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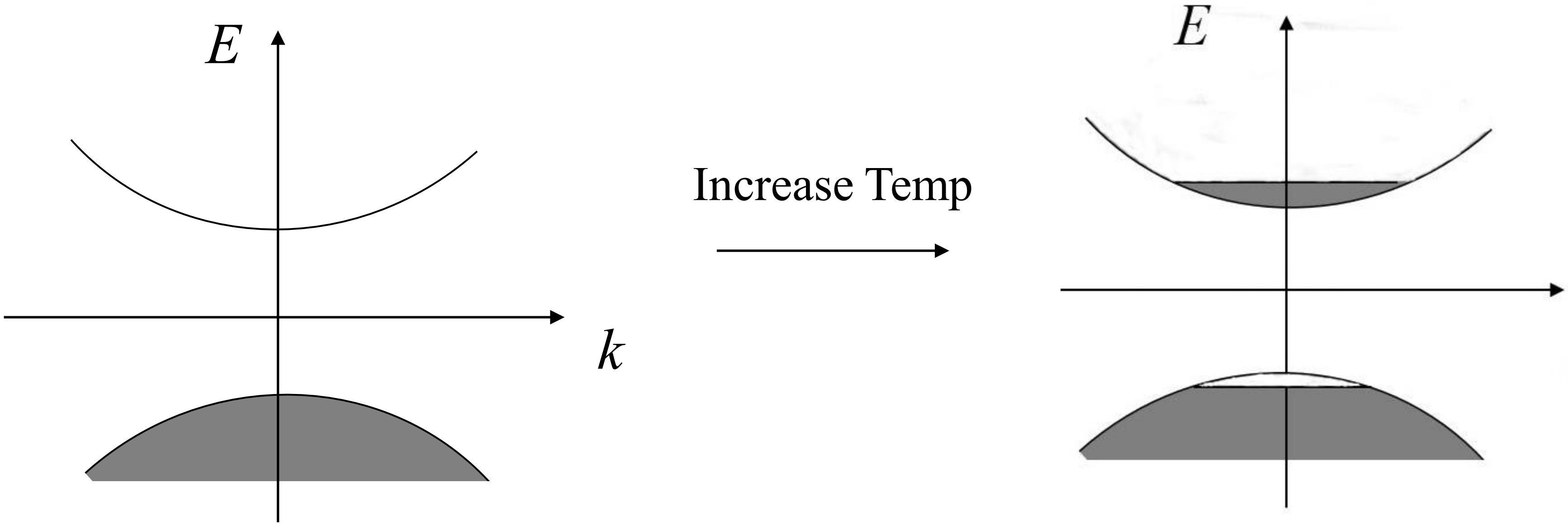
Semiconductor in equilibrium: effect of temperature



Absolute zero (T = 0 K) Finite temp

- Electrons and holes thermally generated.
- For thermal generation number of electrons in the conduction band (n) = number of holes in the valence band (p).
- n and p at room temp are small since kT << band gap.

Calculating *equilibrium* electron, hole concentrations at a given temperature

(1) Since the semiconductor is in equilibrium Fermi-Dirac statistics apply.

(2) The number of electrons between energy levels E and $E+dE$ is:

$$f(E)g(E)dE$$

(3) Number of electrons *missing* between energy levels E and $E+dE$:

$$[1-f(E)]g(E)dE$$

This is equal to the number of holes.

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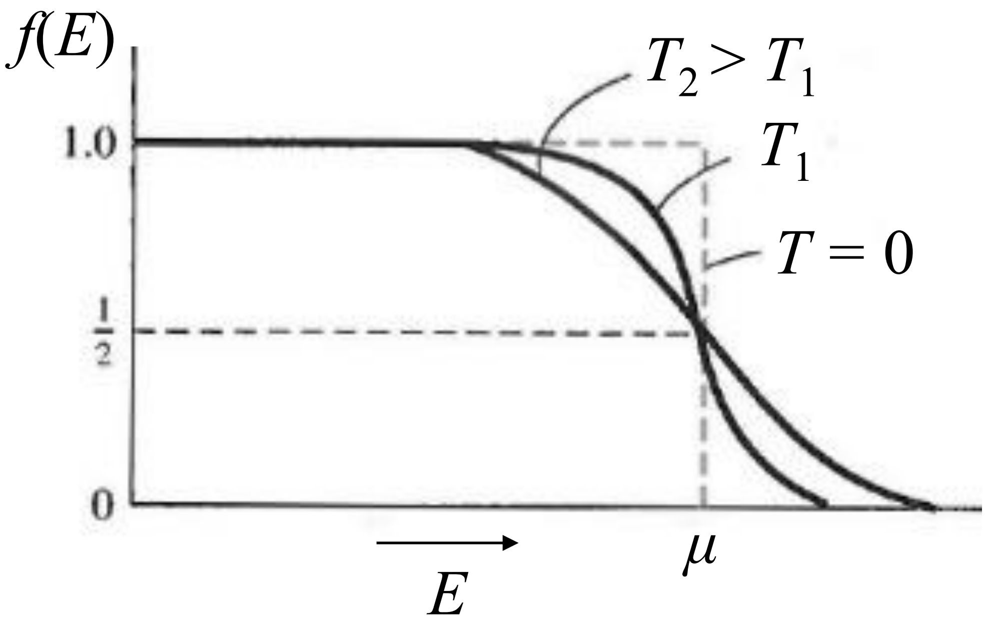
$f(E)$: Fermi-Dirac distribution function
$g(E)$: density of states of energy band

Pre-requisites: Fermi-Dirac distribution function



$$f(E) = \frac{1}{1 + \exp\left(\frac{E-\mu}{kT}\right)}$$

- μ: chemical potential
- $0 \le f(E) \le 1$ (Pauli exclusion principle)

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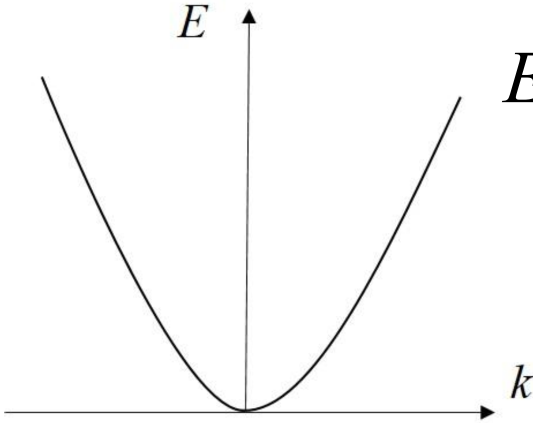
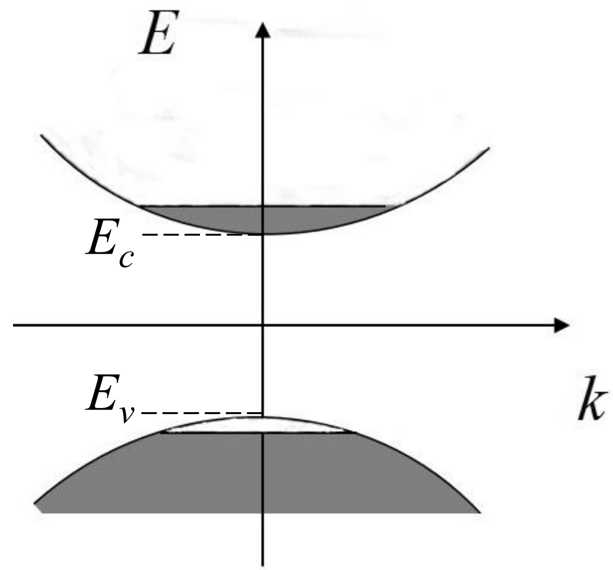
Pre-requisites: Density of states

Assume a free electron solid.
Then:



$$E = \frac{(\hbar k)^2}{2m}$$

$$g_e(E) = \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}\sqrt{E - E_c}$$



$$g_h(E) = \frac{1}{2\pi^2}\left(\frac{2m_h^*}{\hbar^2}\right)^{3/2}\sqrt{E_v - E}$$

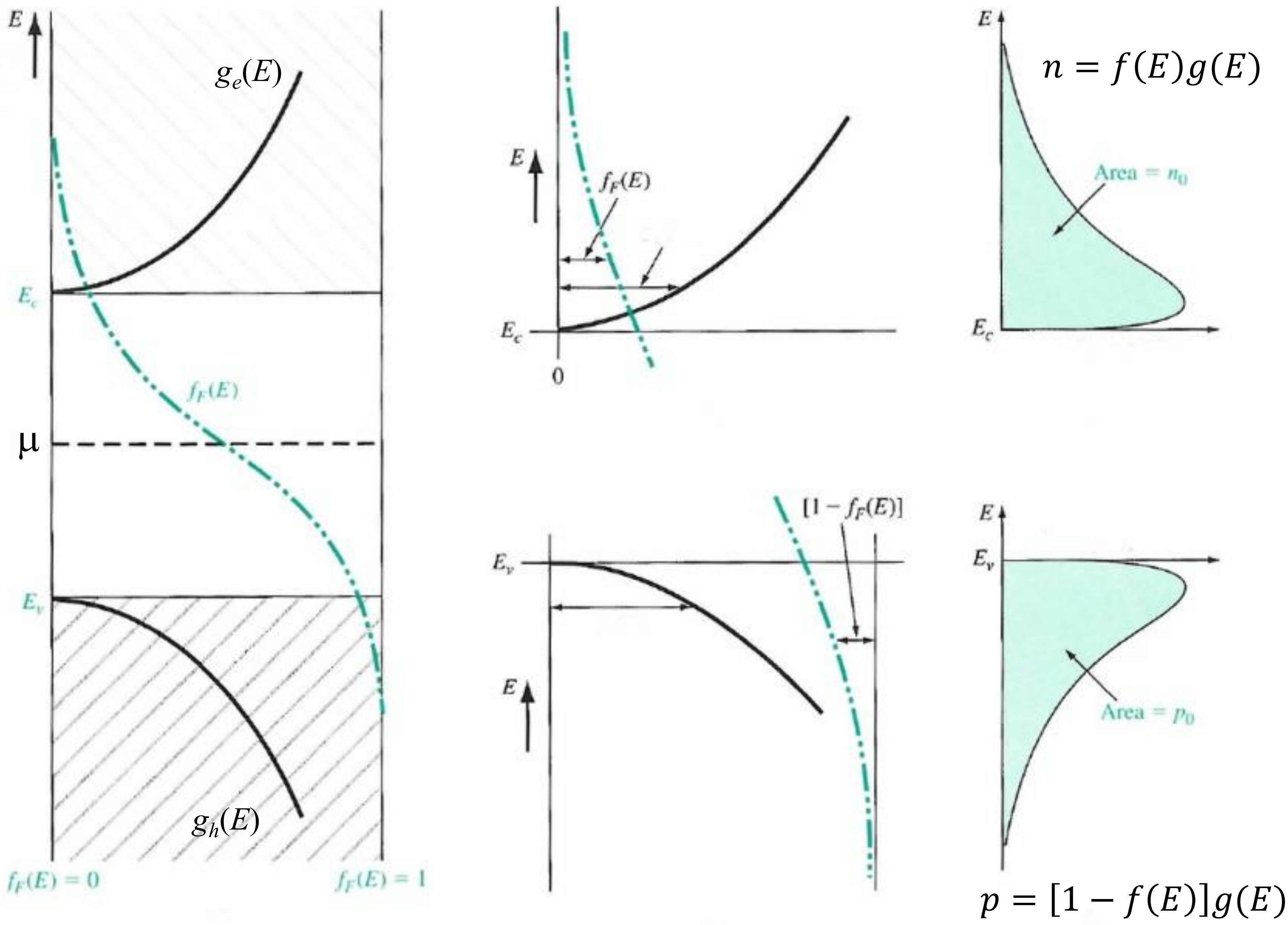
Density of states:

$$\boxed{g(E) = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E}}$$

See Kittel, Chapter 6

$g_e(E)$: density of states for conduction band
$g_h(E)$: density of states for valence band

Equilibrium electron, hole concentrations (I)



Equilibrium electron, hole concentrations (II)

Substituting expressions for $f(E)$, $g(E)$ gives*:

$$\boxed{n = N_c \exp\left[-\frac{(E_c-\mu)}{kT}\right]} \quad \text{where} \quad N_c = 2\left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$\boxed{p = N_v \exp\left[-\frac{(\mu-E_v)}{kT}\right]} \quad \text{where} \quad N_v = 2\left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{3/2}$$

N_c , N_v are the effective density of states for the conduction and valence band respectively.

* Derivation given in lecture and DUO notes.

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Fermi level position in an intrinsic semiconductor

-For an *intrinsic* semiconductor (i.e. no impurities) the number of electrons and holes are equal, since both are produced by thermal excitation.

- Equating $n = p$ gives:

$$\mu = E_{mid-gap} + \frac{3}{4} kT \ln\left(\frac{m_h^*}{m_e^*}\right)$$

Fermi level approximately in the middle of the band gap for $m_h \sim m_e$. Note that $E_{mid\text{-}gap} = (E_c + E_v)/2$.

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Law of mass action

-The product np is given by:

$$np = N_c N_v \exp\left(-\frac{E_g}{kT}\right) \propto T^3 \exp\left(-\frac{E_g}{kT}\right)$$

-In an intrinsic semiconductor ($n = p = n_i$) both electron and hole concentrations increase with temperature.

-Carrier concentrations are larger for small band gap semiconductors.

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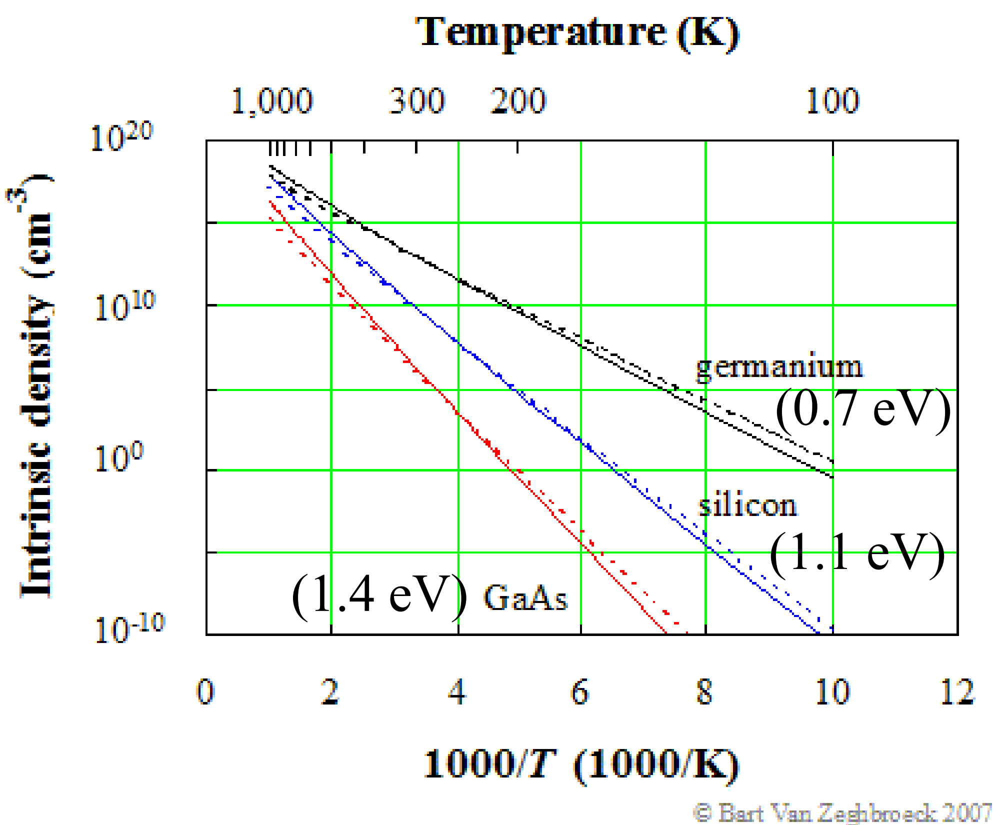
Effect on conductivity of semiconductor



From $\mathbf{J} = qn\mathbf{v}$:

$$\mathbf{J} = -en\mathbf{v}_e + ep\mathbf{v}_h$$

Electron current　　　Hole current

For electrons $\mathbf{v}_e = -\mu_e\mathbf{E}$

holes $\mathbf{v}_h = \mu_h\mathbf{E}$

$$\therefore\ \mathbf{J} = (en\mu_e + ep\mu_h)\mathbf{E}$$
$$= \sigma\mathbf{E}$$

For an intrinsic semiconductor *n* and *p* increase exponentially with temp. This means conductivity (σ) increases with temp; cf. metals*.

* See supplementary reading for more info.