

**Level 3 Condensed Matter Physics- Part I**  
**Weekly problem 2**

(1) Define the term *group velocity* for nearly-free electrons in a metal. Explain how a nearly-free electron can have a velocity which changes with increasing wavevector  $k$ . Draw a sketch of a typical energy – wavevector  $E(k)$  relationship for a nearly-free conduction electron in a metal across the first Brillouin zone. Use your diagram to show where the group velocity of electrons is zero and where it is a maximum. **[4 marks]**

(2) A metal has an energy – wavevector  $E(k)$  relationship given by:  $E(k) = C(k^2 - Dk^4)$  where  $C$  and  $D$  are constants. From this obtain an expression for the group velocity of an electron,  $v_g$ . From what you know about the behavior of  $v_g$  across the Brillouin zone, determine the value of  $D$ . **[3 marks]**

(3) For the above metal the effective mass at the first Brillouin zone boundary ( $k = \pi/a$ ) is -0.5 times the effective mass at the centre of the Brillouin zone ( $k = 0$ ), where  $a$  is the lattice spacing. Find the values for the effective masses at  $k = 0$  and at  $k = \pi/a$  in terms of the constant  $C$ . **[3 marks]**