University of Durham

EXAMINATION PAPER

Examination session:	Year:	Examination code:
May/June	2022	PHYS3631-WE01

Title:
Foundations of Physics 3B

Instructions to candidates:

- Attempt **all** questions. The short-answer questions at the start of each section carry 50% of the total marks for the paper. The remaining 50% of the marks are carried by the longer questions, which are equally weighted.
- The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.
- Write your answers on A4 paper. Begin your answer to each question on a new page.
- You do not need to write your name or your anonymous exam code or any other identifier on your answer.
- Make a scan/photo of your work and convert this to pdf.
- Submit your answers via Gradescope. For details see the Gradescope instructions document, which contains scanning tips and detailed submission instructions.

Information

Section A: Statistical Physics
Section B: Condensed Matter Physics part 1
Section C: Condensed Matter Physics part 2

A list of physical constants is provided on the next page.

Revision:

Information

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Speed of light:	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Boltzmann constant:	$k_{\mathrm{B}} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Bohr magneton:	$\mu_{\mathrm{B}} = 9.27 \times 10^{-24}\ \mathrm{J\,T^{-1}}$
Electron mass:	$m_{\mathrm{e}} = 9.11 \times 10^{-31}$ kg
Gravitational constant:	$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^{2}\,kg^{-2}}$
Proton mass:	$m_{\mathrm{p}} = 1.67 \times 10^{-27}$ kg
Planck constant:	$h = 6.63 \times 10^{-34}\ \mathrm{J\,s}$
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Molar gas constant:	$R = 8.31\ \mathrm{J\,K^{-1}\,mol^{-1}}$
Avogadro's constant:	$N_{\mathrm{A}} = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Gravitational acceleration at Earth's surface:	$g = 9.81\ \mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$
Astronomical Unit:	$\mathrm{AU} = 1.50 \times 10^{11}$ m
Parsec:	$\mathrm{pc} = 3.09 \times 10^{16}$ m
Solar Mass:	$M_{\odot} = 1.99 \times 10^{30}$ kg
Solar Luminosity:	$L_{\odot} = 3.84 \times 10^{26}$ W

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SECTION A: STATISTICAL PHYSICS

1. (a) (i) How many unique permutations of the letters in the word STATISTICAL are there?

   (ii) A system has three energy levels containing n_1, n_2 and n_3 distinguishable particles. What is the entropy of the system?

   [4 marks]

   (b) A system has states of energies 0, ϵ, 2ϵ, ... containing 4 particles and has a total energy $U = 4\epsilon$. Identify the possible distributions of particles in these states and calculate the entropy of the system if the particles are (i) classical, (ii) fermions and (iii) bosons. [4 marks]

   (c) A solid of volume V contains spin-1/2 non-interacting fermions of mass m with an energy density of states

$$g(\epsilon)d\epsilon \;=\; \frac{4\pi V}{h^3}\,(2m)^{3/2}\,\sqrt{\epsilon}d\epsilon.$$

   If there are N fermions, obtain an expression for the Fermi energy at 0 K in terms of N and V. [4 marks]

   (d) A system has two energy levels with energies 0 and ϵ and contains one particle. If we have N such systems in thermal equilibrium at temperature T what fraction is expected to be found in an excited state? [4 marks]

   (e) A system has 3 energy levels, ϵ, 2ϵ and 3ϵ. The system contains 2 particles. What is the partition function for the system if the particles are (i) classical distinguishable, (ii) classical indistinguishable, (iii) bosons and (iv) fermions? [4 marks]

   (f) The energy levels of a one dimensional harmonic oscillator are

$$\epsilon_n \;=\; \left(n+\frac{1}{2}\right)\hbar\omega.$$

   Calculate its partition function and then, using your result for the one-dimensional case, state the partition function of a three-dimensional harmonic oscillator, explaining your reasoning. [4 marks]

   (g) Liquid helium-4 at a temperature of 2 K has a density of 0.1248 g/cm^3. Do we need to treat this system quantum mechanically or can we use Maxwell-Boltzmann statistics? Justify your answer. [4 marks]

2. (a) The quantum states of a solid can be modeled using the states of the 3-dimensional infinite square well of a cube with sides of length a. Given the solutions to this system are when wave-vector coefficients $k_\alpha = n_\alpha \pi/a$ ($\alpha = x$, y or z), with quantum numbers $n_\alpha = 1, 2, 3, ...$, calculate the density of states $g(k)dk$ for this system. [4 marks]

   (b) A gas of classical particles at relativistic speeds have the energy(ϵ)-momentum(p) relation $\epsilon = cp$, where c is the speed of light. Using your result from part (a), calculate the energy density of states $g(\epsilon)d\epsilon$. [4 marks]

   (c) Calculate the partition function of N particles in this system. [4 marks]

   (d) What is the internal energy, free energy, entropy and specific heat of this N-particle system as a function of temperature? [4 marks]

   (e) Calculate the pressure of this system. [4 marks]

SECTION B: CONDENSED MATTER PHYSICS part 1

3. (a) Qualitatively explain why the group velocity of an electron with wave vector at the Brillouin zone boundary is zero. [4 marks]

   (b) Calculate the paramagnetic volume susceptibility of hydrogen atoms at a temperature of 100 K. Assume that the hydrogen atoms do not form any molecules and that the atomic number density is 1.0×10^{20} m^{-3}. [4 marks]

   (c) Xenon is an inert gas with atomic number 54. The magnetic susceptibility of 1.0 m^3 volume of xenon at 1.0×10^5 Pa pressure and 300 K temperature is -2.0×10^{-8}. Assuming xenon shows ideal gas behaviour, estimate the average size of an electron orbit in a xenon atom. [4 marks]

   (d) A ferromagnet with cubic crystal structure has an easy magnetisation axis along the [100] crystallographic direction. The magnetocrystalline anisotropic energy density is $K \sin^2 \theta$, where the anisotropy constant $K = 5.0 \times 10^4$ Jm^{-3} and θ is the angle between the applied magnetic field and easy magnetisation axis. A magnetic field is applied along the [111] crystallographic direction. Calculate the magnetocrystalline anisotropic energy gain for the ferromagnet, which has the shape of a cube with side lengths of 5.0×10^{-2} m. [4 marks]

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4. The Ginzburg-Landau free energy for a ferromagnet under zero applied magnetic field is given by

$$G_{\mathrm{FM}}(T) = G_{\mathrm{PM}}(T) + a\,(T - T_{\mathrm{c}})\,M^2 + bM^4,$$

where $G_{\mathrm{FM}}(T)$ and $G_{\mathrm{PM}}(T)$ are the free energies for the ferromagnetic and paramagnetic phases at temperature T, T_{c} is the Curie temperature, M is the magnetisation and a, b are constants.

(a) Explain why the free energy depends only on even powered terms in M. Determine the sign of the constants a and b. [4 marks]

(b) Determine if the ferromagnetic to paramagnetic phase transition is first or second order. [2 marks]

(c) An external magnetic field $\mu_0 H$ is applied parallel to the magnetisation vector. Write down an expression for the Ginzburg-Landau free energy. Show that the magnetisation has the form

$$M^2 = u + v\frac{H}{M},$$

where u and v are variables that depend on a and b. [6 marks]

(d) Use Ginzburg-Landau theory to derive the Curie-Weiss law for ferromagnets. [4 marks]

(e) By comparing the magnetic susceptibility with the Weiss model for a $J = 1/2$ ferromagnet, deduce an expression for the saturation magnetisation in terms of the Ginzburg-Landau parameters a and b. [4 marks]

SECTION C: CONDENSED MATTER PHYSICS part 2

5. (a) Silicon has a lattice parameter of 5.4×10^{-10} m. In a sample of p-doped silicon, there are 6 acceptor ions in every million atoms. Calculate the acceptor concentration in units of m^{-3}. Calculate the average distance between neighbouring acceptor ions. [4 marks]

   (b) GaAs is a semiconductor with zinc blende crystal structure. A group IV silicon atom is substituted on a gallium site. Is the silicon atom a donor or acceptor? How does this compare with a silicon atom in an arsenic site? State your reasoning. [4 marks]

   (c) A superconducting material has a 100 nm coherence length and a 250 nm London penetration depth. An external magnetic field is applied to the superconductor. Describe any phase transformations of the superconducting state as the strength of the magnetic field is progressively increased. [4 marks]

   (d) A uniform, external magnetic field $\underline{B}$ is applied parallel to a semi-infinite superconducting slab of thickness $2t$. Derive an expression for the supercurrent generated within the slab. [4 marks]

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6. A molecule in zero electric field consists of a $-2q$ charged ion of mass M and two positively charged ions each with mass m and charge $+q$. The m-M-m bond angle is $\pi/3$ radians. The m-M bond has length a, and can be modelled as a simple harmonic oscillator with stiffness constant K. Assume that $M \gg m$ and any coupling between the positively charged ions is negligible. A uniform, external static electric field, E, is applied to the molecule.

(a) Sketch the equilibrium orientation of the molecule with respect to the electric field direction and annotate the forces acting on each ion. Explain, briefly, your reasoning for the sketch. [4 marks]

(b) Determine the equilibrium separation between the two positively charged ions of mass m, clearly explaining your reasoning in determining this value. [2 marks]

(c) Show that the equilibrium m-M-m bond angle θ satisfies the relationship

$$\left[a + \frac{Eq\cos(\theta/2)}{K}\right]\sin(\theta/2) = \frac{a}{2}.$$

[6 marks]

(d) Let $\delta\theta$ be the change in the equilibrium m-M-m bond angle θ when the electric field is applied. Using the small angle approximation show that $\delta\theta$, for small electric fields, is given by

$$\delta\theta \approx -\frac{\sqrt{3}Eq}{\sqrt{3}aK + Eq}.$$

[4 marks]

(e) The static electric field is switched off and a dynamic electric field oscillating at frequency ω is immediately switched on. Qualitatively describe the changes in molecular configuration as a function of the frequency ω of the dynamic electric field. [4 marks]