Statistical Physics: Workshop Problems 1

(1) In a population of fifty million people, on average, two hundred people have the same DNA profile.

   (a) What is the probability of two people with the same profile?

   (b) What is the probability that the police, only knowing that a suspect from the population has a DNA profile that matches that found at the scene of crime, has found the guilty party?

(2) What is the probability that out of five people, none have the same birthday?

(3) Consider a system in which the allowed (non-degenerate) one-particle states have energies $0, \epsilon, 2\epsilon, 3\epsilon, \ldots$. The assembly has four distinguishable (localised) particles ($N = 4$) and a total energy of $U = 6\epsilon$. Identify the possible distributions, evaluate the number of microstates Ω, and statistical entropy S, and work out the average distribution (i.e., the average occupation $\langle n_i \rangle$ of each energy level) and the probability p_i that each one-particle energy level ϵ_i is occupied.

(4) A manufacturer knows that their resistors have values which are distributed as a Gaussian probability distribution with a mean resistance of 100 Ω and standard deviation of 5 Ω. What percentage of resistors have resistances between 95 and 105 Ω? What is the probability of selecting a resistor with resistance less than 80 Ω?

   Reminder: For a continuous probability distribution, the probability $p(a \leq x \leq b) = \int_a^b dx\, f(x)$. For the Gaussian probability distribution,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

   You may use:

$$\frac{1}{\sqrt{2\pi}} \int_{-1}^{1} dx\, \exp\left(-\frac{x^2}{2}\right) = 0.68269\,, \quad \frac{1}{\sqrt{2\pi}} \int_{-2}^{2} dx\, \exp\left(-\frac{x^2}{2}\right) = 0.95450$$

$$\frac{1}{\sqrt{2\pi}} \int_{-3}^{3} dx\, \exp\left(-\frac{x^2}{2}\right) = 0.99730\,, \quad \frac{1}{\sqrt{2\pi}} \int_{-4}^{4} dx\, \exp\left(-\frac{x^2}{2}\right) = 0.99994$$