Level 3 Condensed Matter Physics

Example Workshop 1

These are some questions to test your recall and understanding of Foundations of Physics 2B material. In the first couple of lectures I will be covering free electron theory and the nearly free electron model, using your newly-acquired quantum mechanical abilities. If you have any problems with the questions below, then maybe some reading of Simon (Solid State Basics) would be a good idea.

1. Fermi energy

Give a definition of the *Fermi energy*. Describe, with the aid of a sketch, the Fermi surface in the free electron approximation for a three-dimensional solid. Explain why it has this shape.

2. From FoP2B Exam 2015

(a) What does the *energy density of states* function $g(E)$ describe? How does the distribution of particles dN/dE vary with energy in three dimensions at low temperatures? Illustrate your answer with a diagram.

(b) Explain the significance of the *Fermi energy* in a metal. Draw a diagram to illustrate the behavior of the Fermi-Dirac distribution function for a typical metal at both low temperatures close to absolute zero, and also at room temperature?

(c) Silver has a free electron density of 6×10^{28} m^{-3} at 300 K. Determine the Fermi energy, Fermi velocity and Fermi temperature of free electrons in silver. Comment on your result in comparison to the thermal energy of electrons at room temperature.

3. Square lattice, free electron energies.

Show that for a primitive square lattice (in two dimensions) that the kinetic energy of a free electron at a corner of the first Brillouin zone is higher than that of an electron at the midpoint of a side face of the zone by a factor of 2.
What is the corresponding factor for a primitive cubic lattice in three dimensions?

4. Kinetic energy of a free electron gas.

Show that the kinetic energy of a three-dimensional gas of N free electrons is

$$U_0 = \frac{3}{5} N E_{\mathrm{F}} \ .$$