Level 3 Condensed Matter Physics

Example Workshop 5

1. Ferromagnetism

(a) Sketch the general form of the temperature dependence of the spontaneous magnetisation of a ferromagnet. Indicate both the Curie temperature, T_c, and the saturation magnetisation, M_{sat}, on you sketch.

(b) A ferromagnetic solid contains Gd^{3+} ions arranged in a primitive cubic arrangement with a unit cell of 0.75 nm. Each gadolinium ion has 7 electrons in its 4f shell. At very low temperatures close to absolute zero the magnetocrystalline anisotropy is described by the constants K_1 = 5.4 x 10^5 J m^{-3} and K_2 = 5.1 x 10^3 J m^{-3}. Use this information to:

i. Calculate the saturation magnetisation of the ferromagnetic solid at absolute zero.

ii. Demonstrate that <100> directions in the crystal are '*easy*' axes for magnetisation, whereas <111> are '*hard*' directions for magnetisation.

(**Hint**: For a cubic crystal the anisotropic energy density is:

$$U_{anis} = K_1(\alpha_1^2\alpha_2^2 + \alpha_1^2\alpha_3^2 + \alpha_2^2\alpha_3^2) + K_2(\alpha_1^2\alpha_2^2\alpha_3^2)$$

where $\alpha_1 = \cos\theta_1, \alpha_2 = \cos\theta_2$ and $\alpha_3 = \cos\theta_3$, are the direction cosines of the magnetisation with respect to the [100], [010] and [001] directions).

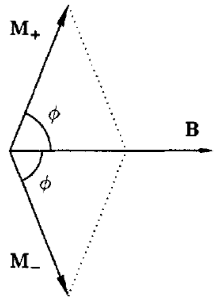
(c) The Hamiltonian for a cubic ferromagnetic material is given by:

$$\hat{H}_{\text{mag}} = \sum_{i,j} -J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j - \kappa \sum_i \left[(S_i^x)^4 + \left(S_i^y\right)^4 + (S_i^z)^4\right]$$

where the first term represents the exchange interaction and the second term is the magnetocrystalline anisotropy, with κ being a positive constant. S_i^x, S_i^y, S_i^z are the *x,y,z* components of the spin angular momentum for atomic site '*i*'. If the spin angular momentum magnitude S is constant for all atomic sites, show that the easy axis for magnetisation is along the cubic <100> directions.

2. Antiferromagnetism and Ferrimagnetism

(a)



The figure opposite shows the stable configuration for an antiferromagnet with external **B**-field applied at right angles to the easy magnetisation axis. The magnetisations M^+ and M^- of the spin 'up' and spin 'down' sub-lattices are at an angle ϕ to the **B**-field.

i. Write down an expression for the total energy, taking into account contributions due to the exchange interaction, magnetocrystalline anisotropy and (Zeeman) potential energy. You may assume that $J = S$ and that the anisotropy energy density has the form $K\sin^2\theta$, where K is a positive constant and θ is the angle between the magnetisation vector and easy axis of the crystal.

ii. Using your expression for the total energy determine the equilibrium value for the angle ϕ.

(b) A ferrimagnet displays antiferromagnetic ordering, but with the magnetisation of the spin 'up' and spin 'down' sub-lattice being unequal. Ferrimagnets therefore have a net magnetisation (unlike antiferromagnets). For some ferrimagnets it is found that at the so-called 'compensation temperature' the net magnetisation drops to zero.

i. Using the Weiss model explain qualitatively the origin of the compensation temperature.

ii. At high temperature the ferrimagnet undergoes a transformation to the paramagnetic state. Show that the paramagnetic susceptibility is given by:

$$\chi = \frac{\mu_0}{T^2 - \theta^2}\left[(C_+ + C_-)T - 2\lambda C_+ C_-\right]$$

where θ is a constant, T is temperature and the Curie constant $C = \frac{g_J \mu_B (J+1) M_S}{3k_B}$, with the subscripts '+' and '-' denoting spin 'up' and spin 'down' sub-lattices with corresponding saturation magnetisation M_S. The molecular field due to a sub-lattice of magnetisation M is represented by $-\lambda M$, where λ is a constant.

(**Hint**: For small y the Brillouin function $B_J(y) \approx \frac{(J+1)}{3J} y$).

$e = 1.60 \times 10^{-19}$ C
$\mu_B = 9.27 \times 10^{-24}$ J T^{-1}
$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
$k_B = 1.38 \times 10^{-23}$ J K^{-1}
$h = 6.63 \times 10^{-34}$ J s^{-1}
$N_A = 6.022 \times 10^{23}$
$m_e = 9.11 \times 10^{-31}$ kg