Level 3 Condensed Matter Physics- Part II

Examples Class 2
Electrons and Holes

(1) *Conduction of electrons in a perfect crystal*

Consider the band structure of a one-dimensional perfect crystal which has the following dispersion (i.e. E vs k) relation:

$$E(k) = E_0 - 2I\cos(ka)$$

where E_0, I are constants and a is the inter-atomic spacing.

(i) derive an expression for the velocity (υ) of an electron as a function of k.
(ii) using $\mathbf{F} = \hbar(d\mathbf{k}/dt)$ express υ as a function of t when an electric field ε is applied in the positive k-direction (the electric field is applied at $t = 0$; you may assume $k = 0$ at $t = 0$).
(iii) show that the time averaged position of the electron is given by $\langle x \rangle = -(2I/e\varepsilon)$, where e is the charge of the electron (assume that $x = 0$ at $t = 0$)

The result in part (iii) reinforces the fact that electrons in a perfect crystal cannot carry a current (i.e. scattering is required for electrical conduction).

(2) *Properties of holes*

The valence band in silicon has the following dispersion relation:

$$E(\mathbf{k}) = \frac{\hbar^2}{2m}\left\{-4.29k^2 + \sqrt{\left[0.46k^4 + 23.72(k_x^2k_y^2 + k_y^2k_z^2 + k_z^2k_x^2)\right]}\right\}$$

where m is the rest mass of the electron.

Assume a photon promotes an electron at $\mathbf{k} = (0.02, 0, 0)$ Å^{-1} into the conduction band, leaving behind a hole.

For the hole calculate:

(i) the corresponding **k**-vector and energy
(ii) its velocity (use the vector form of the group velocity, $\mathbf{v} = (1/\hbar)\nabla_\mathbf{k}E$, where $\nabla_\mathbf{k}$ is the ∇ operator evaluated in reciprocal space)

(3) *Charge carriers in a magnetic field*

(i) Using $\mathbf{F} = \hbar(d\mathbf{k}/dt)$ write down an equation for the motion of particle of charge q and velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$.
(ii) By making use of the fact that $\mathbf{v} = (1/\hbar)\nabla_\mathbf{k}E$ show that the particle moves along a constant energy surface in k-space that is perpendicular to $\mathbf{B}$.