Level 3 Condensed Matter Physics- Part II

Examples Class 5
Topic: Thermodynamics of Superconductors

(1) *Ginzburg-Landau model*

In the Ginzburg-Landau (GL) model the Gibbs free energy for a homogeneous superconductor (G_s) in a magnetic field free environment is expressed as:

$$G_s(T) = G_N(T) + a(T)|\psi|^2 + \frac{b}{2}|\psi|^4$$

where G_N is the free energy of the normal phase and $a(T) = \dot{a}(T - T_c)$, with $\dot{a} > 0$. Due to the limited number of terms in the Taylor expansion the GL model is only valid close to T_c.

i) The critical magnetic field for a Type I superconductor is given by:

$$B_c(T) = B_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

Show that close to T_c the critical magnetic field is approximately:

$$B_c(T) \approx \frac{2B_c(0)}{T_c}(T_c - T)$$

ii) Starting from the expression for condensation energy derive an expression for $B_c(T)$ using GL-theory. Show that this is consistent with the approximation in (i).

iii) Express the superconductor coherence length (ξ) and London penetration depth (λ_L) in terms of the constants $\dot{a}$ and b in the GL model (please refer to notes for definitions of ξ and λ_L). From this show that GL theory predicts the ratio $\kappa = \lambda_L/\xi$ to be temperature independent.

iv) Using your result for κ express the conditions for Type I and Type II behaviour in terms of the GL model parameters $\dot{a}$ and b.

(2) *Magnetic energy in superconductors*

A superconducting cylinder with radius $R << \lambda_L$ is placed in an external magnetic field $\mathbf{B}_a$. The magnetic field within the cylinder can be approximated by:

$$\mathbf{B}(r) = \mathbf{B}_a\left(1 + \alpha\frac{r^2}{\lambda_L^2} - \alpha\frac{R^2}{\lambda_L^2}\right)$$

where α is a constant.

i) Using the expression for magnetic work done per unit volume $dW = -\mathbf{M}\cdot d\mathbf{B}_a$ and the relation $\mathbf{B}(r) = \mathbf{B}_a + \mu_o\mathbf{M}$, show that the Gibbs free energy density of the superconducting phase in the cylinder is given by:

$$G_s(r, B_a) = G_s(0) + \frac{\alpha B_a^2}{2\mu_o\lambda_L^2}(R^2 - r^2)$$

where $G_s(0)$ is the free energy density at zero applied field.

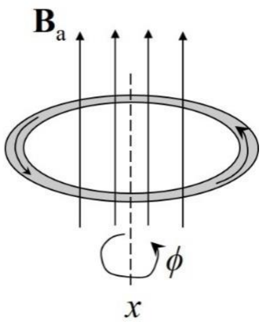
ii) At the critical field B_c the average free energy across the cylinder $<G_s>$ is equal to the free energy of the normal state G_N. Show that this leads to the condition:

$$G_N - G_s(0) = \frac{\alpha B_c^2 R^2}{4\mu_o\lambda_L^2}$$

iii) If $\alpha<1$ is the critical magnetic field for the cylinder smaller or larger than the bulk superconductor?

(3) *Flux through a superconductor ring*

A superconducting ring in the *xy*-plane has a persistent current (magnitude *j*) which generates a magnetic field $\mathbf{B}_a$ along the z-direction. A feature of superconductivity is that the magnetic flux passing through the ring must be constant (magnetic flux is the **B**-field integrated over the area enclosed by the ring). The ring can be rotated about the *x*-axis through an arbitrary angle ϕ. After rotation an external magnetic field is applied along the z-direction with the same magnitude as $\mathbf{B}_a$.



(i) Using the fact that the magnetic flux is constant determine the supercurrent for $\phi = 0^o$ (starting position), 60^o and 90^o. Express your answer in terms of the supercurrent *j* before the external field was applied.

(ii) What would the supercurrents for the different ϕ-values be if the external field was still along the z-direction but had a larger magnitude of $(3B_a/2)$?