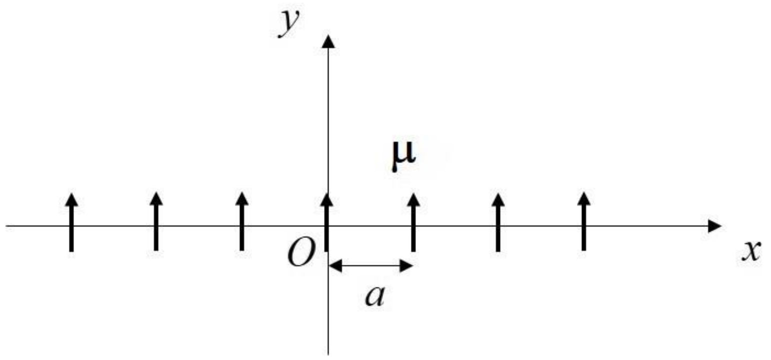
Level 3 Condensed Matter Physics- Part II
Supplementary Examples Class (Topic: Dielectrics)

(1) *Electric dipole moments*

Consider an infinite one dimensional row of electric dipoles as shown below. The moment for each dipole is $\boldsymbol{\mu}$ and is in the direction of the positive y-axis. The dipoles are arranged regularly along the x-axis with spacing 'a'. Calculate the electric field experienced by the dipole at the origin O due to all other dipoles.



[*Hint*: (i) the electric field due to a single dipole is $(\mathbf{r}) = \frac{3(\boldsymbol{\mu}\cdot\mathbf{r})\mathbf{r} - r^2\boldsymbol{\mu}}{4\pi\epsilon_o r^5}$, (ii) $\sum_{n=1}^{\infty}\frac{1}{n^3} \approx 1.2$

(2) *Electronic polarisation in a damped medium*

Assume polarisation of the electron cloud of an atom under a local electric field $\mathbf{E}_o\exp(i\omega t)$ can be modelled as simple harmonic motion (SHM) of a spring with spring constant $K = m\omega_o^2$. Assume also that the system is damped (this can happen through, say, electron-electron collisions). In SHM the damping force is proportional to the linear momentum of the oscillating particle. Hence we can define a positive constant $\gamma = F_d/mv$, where F_d and v are the magnitudes for the damping force and velocity respectively, while m is the particle mass.

i) Write down the equation of motion for the position $\mathbf{r}$ of the oscillating electron taking into account damping. By substituting $\mathbf{r} = \mathbf{r}_o\exp(i\omega t)$ solve this equation for $\mathbf{r}_o$.

ii) Show that the polarisability $\alpha(\omega)$ is given by:

$$\alpha(\omega) = \left(\frac{e^2}{m}\right)\left[\frac{(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2)^2 + (\gamma\omega)^2} - i\frac{\gamma\omega}{(\omega_o^2 - \omega^2)^2 + (\gamma\omega)^2}\right]$$

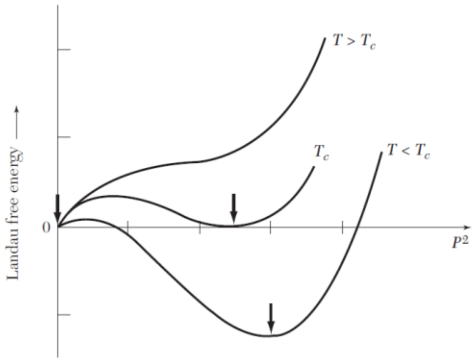
iii) The polarisability has an imaginary term compared to an undamped system. Comment on the physical significance of this extra term.

(3) *Ginzburg-Landau theory of first order ferroelectric transitions*

The Ginzburg-Landau free energy for a first order transition in the absence of an electric field is given by:

$$G_{FE}(T) = G_{PE}(T) + \frac{1}{2}g_2P^2 - \frac{1}{4}|g_4|P^4 + \frac{1}{6}g_6P^6$$

where $g_2 = \gamma(T\text{-}T_o)$ and γ, T_o, $g_6 > 0$. The free energy curves as the temperature passes through the Curie transition temperature are shown schematically below:



At the Curie temperature T_c the free energy of the ferroelectric phase is equal to the paraelectric phase.

i) By setting $G_{\mathrm{FE}}(T_c) = G_{\mathrm{PE}}(T_c)$ write down a polynomial equation for the ferroelectric polarisation P_c at the Curie temperature.

ii) By setting $d[G_{\mathrm{FE}}(T_c)]/dP = 0$ write down another polynomial equation for P_c.

iii) Using the two previous expressions show that:

$$P_c = \left(\frac{3|g_4|}{4g_6}\right)^{1/2}$$

iv) Show that the spontaneous polarisation P_s in the ferroelectric state at temperature T is given by:

$$P_s^2 = \frac{|g_4| + \sqrt{|g_4|^2 - 4g_6\gamma(T - T_o)}}{2g_6}$$

v) By comparing P_s at $T = T_o$ with the expression for P_c show that $T_o < T_c$ for a first order transition.