Statistical Physics: Weekly Problem 1 (SP1)

(a) Since $U = 3\epsilon$, the number of particles in states $k > 3$ is zero ($n_k = 0$ for $k > 3$). The number of microstates for each distribution are given by $4!/(n_0!n_1!n_2!n_3!)$. The possible distributions for the states (or levels) 0,1,2,3 are:

Distribution	(n_0, n_1, n_2, n_3)	Number of microstates
D_1	(3, 0, 0, 1)	4
D_2	(2, 1, 1, 0)	12
D_3	(1, 3, 0, 0)	4
Average [see (b)]	(2.0,1.2,0.6,0.2)	$\Omega = 20$

As the total number of microstates is $\Omega = 20$ then the total entropy is $S = k_B \ln(20) \simeq 3k_B$.

[4 mark]

(b) $\langle n_0 \rangle = (3 \times 4 + 2 \times 12 + 1 \times 4)/20 = 2$,
$\langle n_1 \rangle = (0 \times 4 + 1 \times 12 + 3 \times 4)/20 = 1.2$,
$\langle n_2 \rangle = (0 \times 4 + 1 \times 12 + 0 \times 4)/20 = 0.6$,
$\langle n_3 \rangle = (1 \times 4 + 0 \times 12 + 0 \times 4)/20 = 0.2$.

Hence (normalising back to 1):
$p_0 = 2/4 = 0.5$, $p_1 = 1.2/4 = 0.3$, $p_2 = 0.6/4 = 0.15$, $p_3 = 0.2/4 = 0.05$, and $p_k = 0$ for $k > 3$.

[3 mark]

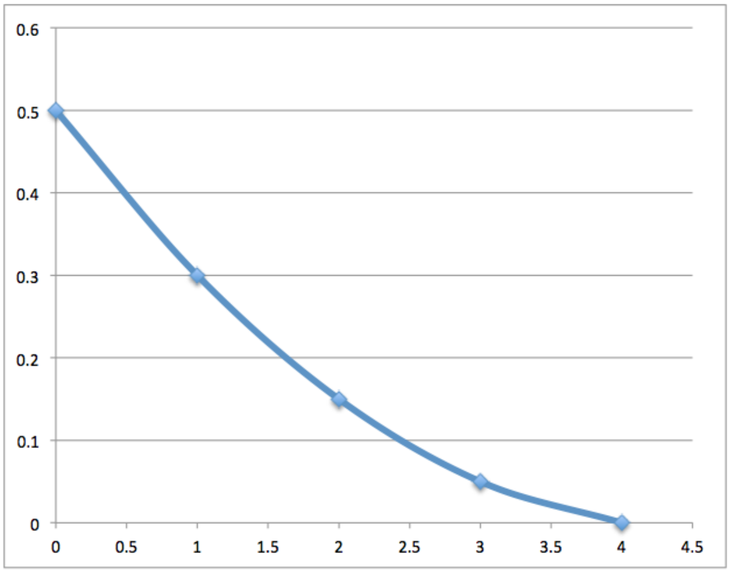

Figure 1: p_k vs k. The width, is where the curve drops to $1/e$ of the maximum (slightly below 0.2). So the width is about $\Delta = 1.5\epsilon$. Any answer between ϵ - 2ϵ is acceptable. The physical meaning of Δ is temperature.

(c) See Figure 1. [3 mark]