Level 3 Condensed Matter Physics- Part I
Weekly problem 5 solutions

(1) **L-S coupling**; here the coupling of the **l** vectors of the individual electrons combine to form a total angular momentum **L**, while the spin momentum vectors combine independently to form a total spin momentum vector **S**. This means that the calculated **S** is not affected by the value of **L** i.e. the spin-spin and orbit-orbit couplings are much stronger than the spin-orbit coupling for each electron. This leads to a total angular momentum **J** of an atom is the vector sum of the two non-interacting momenta **L** and **S**, $\mathbf{J} = \mathbf{L} + \mathbf{S}$. [1 mark]

(2) We are given that,

$$M = M_S\left[\frac{(2J+1)}{2J}\coth\left(\frac{(2J+1)}{2J}y\right) - \frac{1}{2J}\coth\left(\frac{y}{2J}\right)\right]$$

where $y = \frac{g_J \mu_B J B}{(k_B T)}$ and $M_S = ng\mu_B J$.At small magnetic fields and/or high temperatures y is very small. This means we can use the result,

$$\coth x \approx \frac{1}{x} + \frac{x}{3} \text{ for } x \to 0$$

To obtain,

$$M \approx n g_J \mu_B J\left[\frac{2J+1}{2J}\left(\frac{2J}{(2J+1)y} + \frac{(2J+1)}{2J} \times \frac{y}{3}\right) - \frac{1}{2J}\left(\frac{2J}{y} + \frac{1}{2J} \times \frac{y}{3}\right)\right]$$
$$= n g_J \mu_B J\left(\frac{(J+1)}{J} \times \frac{y}{3}\right).$$

Substituting in for y gives,

$$M \approx \frac{n {g_J}^2 {\mu_B}^2 J(J+1)B}{3k_B T}.$$

[1 mark]

Substituting $B = \mu_0 H$ into this expression and re-arranging gives the paramagnetic susceptibility,

$$\chi_P = \frac{M}{H} = \frac{n {g_J}^2 \mu_0 {\mu_B}^2 J(J+1)}{3k_B T},$$

Which has the form of Curie's law, $\chi = {}^{C}\!/_{T}$ [1 mark]

(3) Ti^{2+} $3d^2$ (note that the azimuthal quantum number $l = 2$ for the *d*-shell):

m_l	-2	-1	0	1	2
s				+1/2	+1/2

Ground state:

$$S = \sum s = 2 \times \frac{1}{2} = 1\,, \qquad L = \sum m_l = 2 + 1 = 3\,,$$
$$J = |L - S| = |3 - 1| = 2\,.$$

Excited states: $J = |L - S| + 1 = 3$, $J = L + S = 4$ [2 marks]

Total magnetic moment of an atom, $|m_J| = g_J \mu_B \sqrt{J(J+1)}$ and

$$g_J = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}.$$

$$J = 2\,, g_J = 1 + \frac{2(2+1) - 3(3+1) + 1(1+1)}{4(2+1)} = 0.67\,, |m_2| = 0.67 \times \sqrt{2(2+1)}\,\mu_B$$
$$= 1.64\,\mu_B.$$

$$J = 3\,, g_J = 1 + \frac{3(3+1) - 3(3+1) + 1(1+1)}{6(3+1)} = 1.08\,, |m_2| = 1.08 \times \sqrt{3(3+1)}\,\mu_B$$
$$= 3.74\,\mu_B.$$

$$J = 4\,, g_J = 1 + \frac{4(4+1) - 3(3+1) + 1(1+1)}{8(4+1)} = 1.25\,, |m_2| = 1.25 \times \sqrt{4(4+1)}\,\mu_B$$
$$= 5.59\,\mu_B.$$

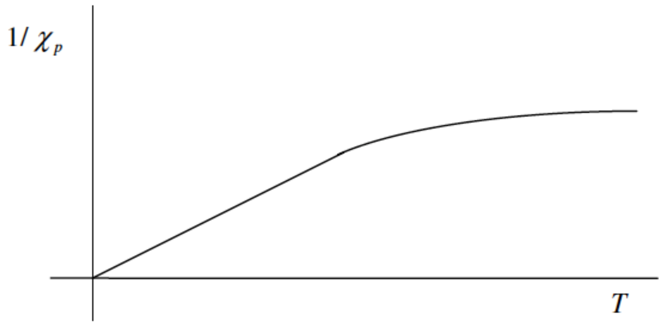
[1 mark]

(4) Energy states: $E_{SO}(J) = \frac{\lambda}{2}[J(J+1) - L(L+1) - S(S+1)]$

$$J = 2, E_{SO}(2) = \frac{4.5}{2}[2(2+1) - 3(3+1) - 1(1+1)] = -18 \text{ meV}$$
$$J = 3, E_{SO}(3) = \frac{4.5}{2}[3(3+1) - 3(3+1) - 1(1+1)] = -4.5 \text{ meV}$$
$$J = 4, E_{SO}(4) = \frac{4.5}{2}[4(4+1) - 3(3+1) - 1(1+1)] = 13.5 \text{ meV}$$

[2 marks]

(5) Given the form of the probability, $P(J) \propto (2J+1)\exp(-E_{SO}(J)/k_B T)$, it would be expected that, at very low temperatures, only the ground J = 2 state, would be substantially populated and the ground state magnetic moment would dominate the paramagnetic susceptibility with $\frac{1}{\chi_P} = \frac{3k_B T}{\mu_0 N m_2^2}$. At higher temperatures we would expect the contribution from the J = 3 state, and given the greater atomic moment, this would cause the measured paramagnetic susceptibility to be larger than that expected for the J = 2 state. Hence the inverse susceptibility would have the form:



[2 marks]