Statistical Physics: Weekly Problem 6 (SP6)

(1) In this question you will need to evaluate integrals of the form

$$I_n(\alpha) \quad = \quad \int_0^\infty x^n e^{-\alpha x^2} dx.$$

Simply look up the expressions for these standard integrals and use them (they are also given in the lectures).

The probability distribution of speeds of particles in a gas is given by the Maxwell-Boltzmann (MB) distribution

$$p(v)dv = \sqrt{\frac{2}{\pi}} \left(\frac{m}{k_B T}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2k_B T}\right) dv.$$

(a) Sketch $p(v)$ as a function of v for (i) a low temperature and (ii) a high temperature. Indicate the v dependence for small v, and the trend at large v. [2 marks]

(b) The expressions for the most probable speed v_{max}, the mean speed $\bar{v}$, and the r.m.s. speed v_{rms} were evaluated in lectures. State them here and mark them on the graph. [2 mark]

(c) Calculate v_{max}, $\bar{v}$, and v_{rms} for a system composed of gaseous neon atoms at room temperature. The mass of a neon atom is 3.37×10^{-26} kg. [2 marks]

(d) In two dimensions the probability distribution of speeds of particles in a gas is

$$p(v)\, dv = C\, v \, \exp\left(-\frac{mv^2}{2k_B T}\right) dv$$

where C normalises the probability.

(i) Calculate C. [2 marks]

(ii) Find the expressions for the most probable speed v_{max}, the mean speed $\bar{v}$, and the r.m.s. speed v_{rms} of particles in this two dimensional gas. [2 marks]