Statistical Physics: Workshop Problems 8

(1) Take a system with N free electrons in a metal with volume V. Calculate to within a reasonable approximation the following.

(a) The specific heat C_V.

(b) The magnetic susceptibility which can defined as

$$\chi = \mu_B^2 g(E_F)$$

where μ_B is the Bohr magneton and E_F is the Fermi energy.

(c) The average kinetic energy of the (non-interacting) electrons.

(d) The electron pressure (remembering how pressure is related to kinetic energy will be useful).

(2) In a Fermi gas model of atomic nuclei, except for the Pauli exclusion principle, the nucleons can be assumed to be a completely degenerate Fermi gas in a sphere of volume V. Let N be the number of neutrons and Z be the number of protons (and for simplicity let $N = Z$). If $A = N + Z$ then compute the kinetic energy per nucleon in this model if the volume of the atomic nucleus is $V = 4\pi R_0^3 A/3$ where $R_0 = 1.4 \times 10^{-13}$ cm.

(3) Non-relativistically, at what particle density does a gas of free electrons at $T = 0$ have enough kinetic energy (i.e. Fermi energy) to allow the reaction

$$\text{electron} + \text{proton} + 0.8\text{MeV} \quad \rightarrow \quad \text{neutron}.$$

Use this to estimate the minimum density of a neutron star. Look up the minimum density of a neutron star and decide whether this non-relativistic result is valid.

(4) Consider a $T = 0$ gas of N non-interacting electrons in a volume V.

(a) Find an equations that relates the volume, energy and pressure of the gas in the extreme relativistic case ($\epsilon = cp$).

(b) Estimate when the result in (a) is approximately valid.

(5) In the very early universe $k_B T$ is large and so we can assume an extreme relativistic limit where particle masses and chemical potentials are approximately zero. Calculate the average number density and energy density of a gas of Fermions in these conditions.