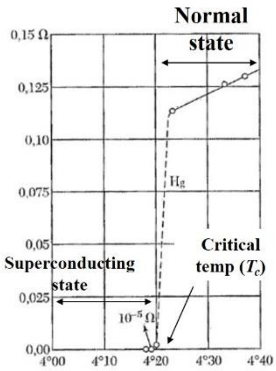
FoP3B Part II Lecture 7: Introduction to Superconductors

Superconductivity is the phenomenon of *zero electrical resistivity* below a **critical temperature** T_c. It was first discovered by Kammerling Onnes in 1911 when carrying out cryogenic experiments on the resistivity of mercury. In metals the resistivity (ρ)-temperature (T) dependence has the form:

$$\rho(T) = \rho_o + aT^2 + bT^5$$

where a and b are constants. The individual terms are due to (i) scattering of conduction electrons by impurities (ρ_o), (ii) electron-electron scattering (aT^2) and (iii) electron-phonon scattering (bT^5). Provided the material is pure the resistivity should therefore approach zero <u>*gradually*</u> as the temperature is lowered.



Onnes' result for mercury is shown in Figure 1. Zero resistivity is obtained <u>*suddenly*</u> and <u>*above*</u> absolute zero temperature. This implies that a new **phase** of matter is obtained below the transition (or critical) temperature T_c; this is the **superconducting state**. Above T_c the material is said to be in the **normal state**.

Figure 1: Resistivity as a function of temperature (kelvin) for mercury. The critical temperature (T_c), superconducting and normal states are indicated.

Besides zero resistivity there are several other characteristics common to superconductors. The first is the generation of **persistent currents** and the second is the **Meissner effect**. These will be described below. Finally, *magnetic fields also have an important effect on the stability of the superconducting state and this leads to a classification of superconductors according to* **Type I** *and* **Type II**, which is also discussed.

Persistent Currents

According to Ohm's law $\mathbf{j} = \sigma\varepsilon$, where $\sigma = 1/\rho$ is the conductivity, *the electric field ε within a zero resistivity material must be zero* in order to maintain a constant current density. Hence from **Faraday's law**, i.e. $\vec{\nabla} \times \boldsymbol{\varepsilon} = -\partial \mathbf{B}/\partial t = 0$, i.e. the *magnetic induction field* **B** *is time invariant*.

Consider the following experiment: a magnetic induction field **B** is passed through a ring which is held above T_c (Figure 2a). Next cool the material below T_c in the presence of the **B**-field so that the material transitions from normal to superconducting state. The external **B**-field is then switched off. However, because the **B**-field is time invariant the superconductor must generate an electric current such that it produces an identical magnetic field (Figure 2b)[1]. The sign of the current is determined by **Lenz's law**. Furthermore, *because the resistivity is zero there will be no damping of the current with time*. It is therefore called a **persistent current**.

[1] As will be shown later, the Meissner effect indicates that the **B**-field cannot penetrate the superconductor. Therefore, strictly speaking a current is generated in order to preserve the magnetic flux through the loop (see Supplementary notes).

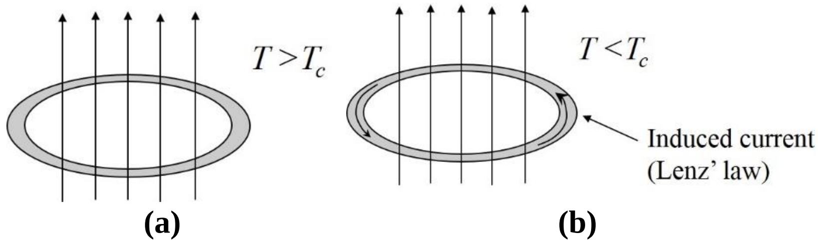


Figure 2: (a) **B**-field passing through a ring in the normal state ($T > T_c$). In (b) the material is cooled below T_c and the external **B**-field is switched off. The superconducting ring generates a persistent current to maintain a constant **B**-field.

It could be questioned whether the resistivity of the superconducting state is really zero or alternatively a small, but non-zero value. Persistent currents provide a more accurate method for measuring resistivity compared to the standard four point probe technique. An electric current decays exponentially with the scattering time τ, which is a measure of the resistivity (in fact $\rho \propto 1/\tau$). With superconductors persistent currents have been monitored over periods of years with no noticeable decay. This places an *upper limit* of 10^{-25} Ωm for the resistivity of a superconductor; by comparison the resistivity of a metal such as copper is significantly higher at 10^{-8} Ωm. Hence the best experimental measurements suggest that the resistivity of a superconductor is in indeed zero for all practical purposes.

Meissner Effect

A further property of superconductors is the expulsion of a magnetic field from within the material. This is known as **diamagnetism** and is the origin of the **Meissner effect**, where a magnet can be levitated above a superconductor. In fact, *superconductors are perfect diamagnets*, i.e. the **B**-field is completely excluded from the material. This means that **ferromagnetic** materials (e.g. Fe, Co, Ni) are typically not superconductors at low temperatures, due to the large internal fields within magnetic domains. Since $\mathbf{B} = \mu_o(\mathbf{M} + \mathbf{H}) = 0$, the **magnetic susceptibility** of a superconductor $\chi = M/H = -1$ (here **M** is the magnetisation, **H** is the magnetic field and μ_o the permeability of free space).

Type I vs Type II behaviour

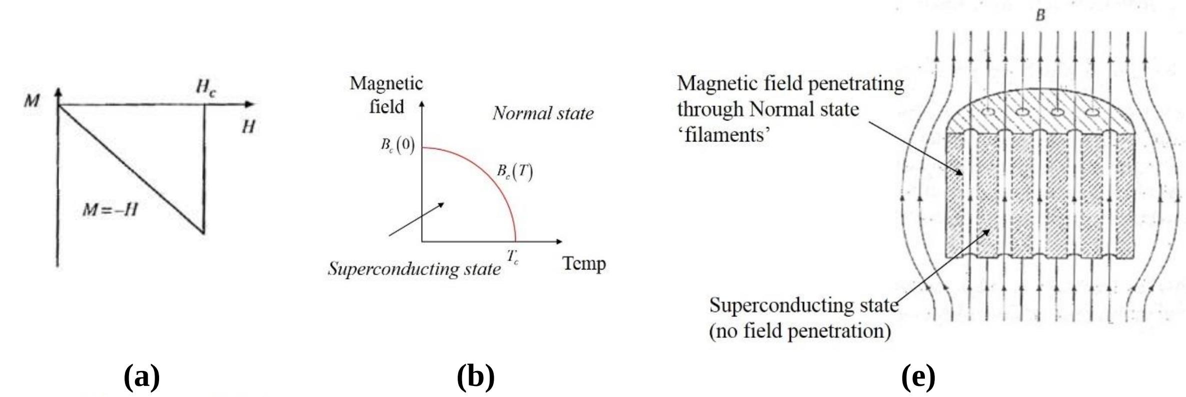
An increase in temperature above a critical value destroys the superconducting state (Figure 1). A similar trend is also observed with magnetic fields, i.e. *the superconducting state breaks down in strong magnetic fields*. The transition from superconducting to normal state under increasing magnetic field has two forms. *In* **Type I** *behaviour the transition happens suddenly at a critical field*. This is illustrated in the magnetisation curve of Figure 3a. At low magnetic fields the material shows diamagnetic behaviour characteristic of a superconductor. However, at a critical field H_c the magnetic susceptibility, given by the gradient of the graph, abruptly changes to the small, but positive value of the **paramagnetic** normal state. *A* **phase diagram** *can be constructed showing the stability regions of the superconducting and normal states w.r.t temperature and magnetic field* (Figure 3b). For a Type I superconductor the critical **B**-field at temperature T, $B_c(T)$, has the following empirical relationship:

$$B_c(T) = B_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right] \quad \ldots (1)$$

where $B_c(0)$ is the critical field at absolute zero. *The critical magnetic field also has implications for the maximum current that can pass through a superconducting wire*. This follows from **Ampere's law**, which states that the (radial) magnetic field in the vicinity of a current carrying wire of radius R is given by:

$$B = \frac{\mu_o I}{2\pi R} \qquad \dots (2)$$

The maximum current I that can be passed through a superconducting wire is that which generates a magnetic field equal to $B_c(T)$.



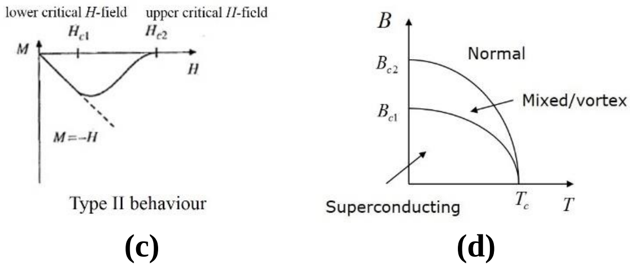


Figure 3: (a) Magnetisation curve and (b) phase diagram for a Type I superconductor. The equivalent diagrams for a Type II superconductor are shown in (c) and (d). (e) is a schematic of a Type II superconductor in the mixed state, containing filaments of normal state material.

A second type of superconducting behaviour, called **Type II** behaviour, is shown in Figure 3c. *Here the transition from superconducting to normal state is gradual, starting at a lower critical field H_{c1} and ending at an upper critical field H_{c2}*. The corresponding phase diagram is shown in Figure 3d. *For magnetic fields between H_{c1} and H_{c2} the material is in a* **mixed** *or* **vortex state**, *where both superconducting and normal regions of material exist side by side.* The normal state exists as 'filaments' extending through the material in the direction of the magnetic field (Figure 3e). <u>*The magnetic field penetrates the normal state filaments, but not the surrounding diamagnetic superconducting regions*</u>. *As the magnetic field is increased from H_{c1} to H_{c2} the density of normal state filaments increases, until finally above H_{c2} the material has completely transformed into the normal state*.