Level 3 Condensed Matter Physics- Part II

Examples Class 4
Topic: Superconductors in magnetic fields

(1) *Superconductivity and ferromagnetism*

Iron has a saturation magnetisation (M_s) of 1.72 x 10^6 Am^{-1}. In Type I elemental superconductors the critical magnetic field is typically found to be smaller than 0.1 T.

Using Fe as an example discuss the possibility of ferromagnetic materials becoming superconducting at low temperatures (you may assume that the remnance magnetisation has a similar value to M_s).

Assume now that Fe is heated above the Curie temperature and cooled in a zero magnetic field, so that it has zero magnetisation. Would the material become superconducting at low temperature?

(2) *Critical currents and fields*

A Type I superconductor is cooled from room temperature under a magnetic field $\frac{1}{2}B_c(0)$, where $B_c(0)$ is the critical magnetic field at 0 K.

i) At which temperature will the normal to superconducting transition be observed? You may express the result in terms of the zero field transition temperature T_c.
ii) Write down an expression for the magnetic field generated by a solenoid with (N/L) number of turns per unit length and carrying a current I. You may refer to old lecture notes or the web.
iii) If the solenoid is made of the superconducting material in part (i) calculate the maximum current that can be carried at 0 K without destroying the superconducting state, assuming the external magnetic field $\frac{1}{2}B_c(0)$ is applied along the solenoid axis.
[*Hint*: pay attention to the current direction- does this have an effect on the critical current?]

(3) *London Penetration depth*

The formula for the London penetration depth λ_L is given by:

$$\lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

where μ_0 is the permeability of free space, m,e the mass and charge of an electron and n_s is the number density of superconducting electrons.

Aluminium is a Type I superconductor with T_c value of 1.2 K. λ_L for Al at 0 K temperature is measured to be 45 nm.

i) Calculate the density of superconducting electrons at 0K.
ii) Calculate the average spacing between Cooper pairs.
iii) Al has a face centred cubic crystal structure with lattice parameter 4.05 Å. Calculate the density of valence electrons (There are 3 valence electrons per Al atom).
iv) Compare the result of (iii) with (i). Why is it important that the density of valence electrons is larger than superconducting electrons?

(4) *London equation- slab of finite thickness*

The London equation describes the penetration of a magnetic field **B** into a superconductor and is given by:

$$\nabla^2 \mathbf{B} = \frac{\mathbf{B}}{\lambda_L^2}$$

where λ_L is the London penetration depth (a constant). Consider a superconducting slab of thickness $2t$. The plane of the slab is in the yz-plane and an external field B_a is applied in the z-direction. Show that the magnetic field within the slab is given by:

$$B_z(x) = B_a \frac{\cosh\left(\frac{x}{\lambda_L}\right)}{\cosh\left(\frac{t}{\lambda_L}\right)}$$

where $x = 0$ passes through the centre of the slab. Sketch the shape of this function.