**Level 3 Condensed Matter Physics- Part I**
**Weekly problem 3**

This weekly problem concerns the derivation of the Hamiltonian for an atomic electron in a uniform magnetic field **B** (Lecture 5). It explores some facts implicitly assumed in the derivation, as well as some implications of the theory.

(1) For a uniform magnetic field **B** show that the magnetic vector potential is given by the so-called symmetric gauge:

$$\mathbf{A}(\mathbf{r}) = \frac{\mathbf{B} \times \mathbf{r}}{2}$$

where **r** is the position vector.

[2 marks]

(2) Show that the symmetric gauge is rotationally invariant.

[2 marks]

(3) Show that the momentum operator $\mathbf{p} = -i\hbar\vec{\nabla}$ commutes with **A**, i.e.

$$[\mathbf{p}, \mathbf{A}] = \mathbf{p} \cdot \mathbf{A} - \mathbf{A} \cdot \mathbf{p} = 0$$

[2 marks]

(4) In the presence of a magnetic field the velocity operator for an electron is given by:

$$\frac{(\mathbf{p} + e\mathbf{A})}{m}$$

where *e* and *m* are the charge magnitude and mass of an electron. Show that when an external magnetic field is applied to an atom a current density $\mathbf{J}(\mathbf{r})$ is generated, which is given by:

$$\mathbf{J}(\mathbf{r}) = \frac{e^2}{m}\rho(\mathbf{r})[\mathbf{r} \times \mathbf{B}]$$

where $\rho(\mathbf{r})$ is the electron density within the atom. Is there a link between $\mathbf{J}(\mathbf{r})$ and diamagnetism?

[4 marks]