Level 3 Condensed Matter Physics

Example Workshop 3

1. Larmor precession

A uniform magnetic field **B** is applied along the z-axis. A magnetic moment $\boldsymbol{\mu}$ due to an electron lies along the xz-plane at angle θ to the **B**-field.

(a) Derive the x,y and z-components of the magnetic moment $\boldsymbol{\mu}$ as a function of time in the presence of the **B**-field.
(b) Show that the magnetic moment precesses about the **B**-field at frequency:

$$\omega = \frac{eB}{2m}$$

where e and m are the charge and mass of the precessing electron.
(c) Comment on the work done by the **B**-field on the precessing magnetic moment.

2. Diamagnetism in Germanium.

Germanium is a diamagnetic semiconductor. The diamagnetic susceptibility is independent of temperature. When arsenic dopant atoms are added to the solid they behave as hydrogenic donors and it is assumed that the donors act paramagnetically at all temperatures. The paramagnetic susceptibility of the As donors follows Curie's Law, $\chi = {}^{C}/_{T}$, where Curie's constant, $C = N \times 2.356 \times 10^{-22}$ K, and N is the number of donor atoms per unit volume.

(a) Calculate the diamagnetic susceptibility of undoped germanium given that the root mean square radius of the atom is 0.12 nm. The density of germanium is 5.323×10^3 kg m^{-3}.

[*Hint:* the diamagnetic susceptibility is given by:

$$\chi = -\frac{ne^2\mu_0 Z\langle r^2\rangle}{6m}$$

where n is the number of atoms per unit volume, Z the atomic number, $\langle r^2\rangle$ the mean square radius of an electron orbit and m the electron mass].

(b) A sample of germanium is doped with As atoms. A measurement of the magnetisation for H = 100 A m^{-1} indicates that at 2 K, M = 0 A m^{-1}. Use this information to calculate the number of donor atoms per unit volume stating any assumptions that you have made.

(c) The relative permeability, $\mu_r = 1 + \chi$, of iron is about 10^5. At what temperature would the arsenic doped germanium sample achieve such a value of μ_r? If a sample with this relative permeability was placed at the centre of a very long solenoid having 1000 turns of wire per metre, what solenoid current would

be required to produce a flux density of 1 Tesla?

Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
Bohr magneton:	$\mu_B = 9.27 \times 10^{-24}$ A m^2
Boltzmann constant:	$k = 1.38 \times 10^{-23}$ J K^{-1}

Relative atomic mass of germanium is 72.63×10^{-3} kg
Atomic number of germanium is $Z_{Ge} = 32$

Avogadro's number	$N_A = 6.022 \times 10^{23}$ mol^{-1}
Mass of electron	$me = 9.11 \times 10^{-31}$ kg

3. Paramagnetism in a solid with angular momentum *J* (to be completed after Lecture 7).

The paramagnetic susceptibility in the classical limit (Langévin theory) and a J = ½ solid have been derived in the lectures. This question will examine paramagnetic susceptibility for intermediate values of J using quantum mechanics. The derivation is an extension of the methods used for a J = ½ solid.

(a) For a given value of J write down the values the z-component, J_z, can take. Show that the partition function Z for the solid in a magnetic field B is:

$$Z = \sum_{m_J=-J}^{J} \exp(m_J x) = \frac{\sinh[(2J+1)x/2]}{\sinh[x/2]}$$

where $x = g_J \mu_B B/kT$, with g_J being the Landé g-factor and m_J the quantum number for J_z.

(b) Show that the average magnetic moment $\langle \mu_z \rangle$ is given by:

$$\langle \mu_z \rangle = \frac{-\mu_B g_J}{Z} \frac{\partial Z}{\partial x}$$

(c) Hence if $y = xJ$ show that:

$$\frac{M}{M_s} = B_J(y)$$

where M_s is the saturation magnetisation and the Brillouin function is given by:

$$B_J(y) = \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J} y\right) - \frac{1}{2J} \coth\left(\frac{y}{2J}\right)$$

(d) Show that for $J = \infty$ the result in (c) leads to the Langévin equation:

$$\frac{M}{M_s} = \coth y - \frac{1}{y}$$