Statistical Physics: Weekly Problem 3 (SP3)

(1) (a) In the *microcanonical ensemble*, or (N, U, V) macrostate, are the various microstates that are consistent with the (N, U, V) macrostate equally probable, or do they have different in general probabilities? [1 mark]

(b) Similarly, in the *canonical ensemble*, or (N, T, V) macrostate, describing a system in thermodynamic equilibrium, are the various accessible microstates equally probable, or do they have, in general, different probabilities? [1 mark]

(2) In a system of N weakly interacting particles in thermal equilibrium at temperature T, the probability that a particle will be in (single-particle) state i with energy ϵ_i is proportional to

$$p_i \propto \exp\left[-\frac{\epsilon_i}{k_\mathrm{B}T}\right],$$

i.e. the Boltzmann probability.

The two lowest-lying energy levels of a hydrogen atom have energies $\epsilon_0 = -13.6$ eV and $\epsilon_1 = -3.4$ eV. Ignoring degeneracies, at what temperature would we find one hundredth as many hydrogen atoms in the first excited state as in the ground state?
($k_\mathrm{B} = 8.617 \times 10^{-5}$ eV K^{-1}) [2 marks]

(3) A paramagnetic solid consists of N ions with spin 1/2 and magnetic moment μ_B. The system lies in a magnetic field B and each magnetic moment is oriented either parallel to the field (up), with energy $\epsilon_\uparrow = -\mu_\mathrm{B}B$, or antiparallel (down) with energy $\epsilon_\downarrow = +\mu_\mathrm{B}B$.

(a) What is the probability that an ion will have its magnetic moment oriented parallel to B? [2 marks]

(b) What is the internal energy U, entropy S and the temperature T of the system in the limit where all the magnetic moments are parallel to B? [3 marks]
Hint: do not try to think of complicated equations. Use your physical intuition.

(c) The system of ions is brought into a state where the internal energy U is positive. Show that the temperature of the system is negative. Is a negative temperature "hotter" (i.e. of higher energy) or "colder" than infinite temperature? [1 mark]