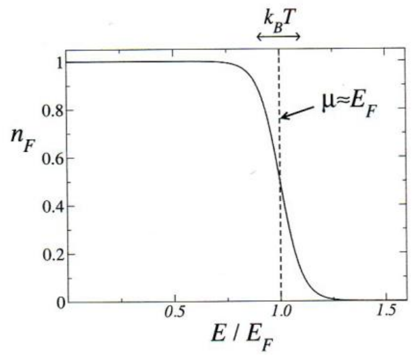
Level 3 Condensed Matter Physics- Part I
Weekly problem 1 solutions

(1) Given a system of free electrons with chemical potential μ the probability of an eigenstate of energy E being occupied is given by the Fermi-Dirac distribution

$$f(E) = \frac{1}{e^{(E-\mu/k_{\mathrm{B}}T)} + 1} \qquad (1.1)$$

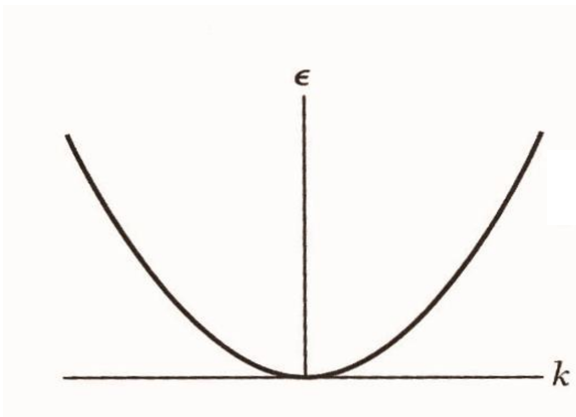


The Fermi-Dirac distribution for $k_BT \ll E_F$. The dashed line marks the chemical potential μ, which is approximately E_F. At $T = 0$ the distribution is a step function but at finite temperature it gets smeared out over a range of energies of width a few k_BT.

At low temperature the Fermi function becomes a step function (electron energy states below the chemical potential are filled, those above are empty), whereas at finite temperatures, the step function becomes progressively more smeared out. **[2 marks]**

(2) We will consider electron waves in a box of size $V = L^3$ with the box having periodic boundary conditions. The plane wavefunctions of the electron waves are of the form $e^{i\mathbf{k}\cdot\mathbf{r}}$ with the wavevectors of the electrons taking the values $(2\pi/L)(n_1, n_2, n_3)$. These plane waves have corresponding energies

$$E(k) = \frac{\hbar^2 |k|^2}{2m}$$



where m is the electron mass. We can define the Fermi energy, E_F as equal to the chemical potential at $T = 0$ and related to the Fermi wavevector k_F via

$$E_{\mathrm{F}} = \frac{\hbar^2 k_{\mathrm{F}}^2}{2m}$$

A graph of energy against wavevector gives a parabola centred at the origin. **[3 marks]**

(3) There are 4 monovalent copper atoms in a cubic unit cell of volume a^3 giving a free electron density of $n = 8.50 \times 10^{28}$ and a corresponding Fermi wavevector of $k_F = 1.35 \times 10^{10}$ m^{-1}, a Fermi energy E_F of 7.00 eV and a corresponding Fermi temperature ($T_{\mathrm{F}} = E_{\mathrm{F}}/k_{\mathrm{B}}$) of 81,200 K. This shows that at any temperature below T_F copper will behave as a quantum degenerate system i.e. a quantum system even at room temperature and up to its melting point. **[3 marks]**

(4) The energy eigenvalues are

$$E = \frac{\hbar^2 k^2}{2m}$$

The mean value over the volume of the Fermi sphere in k space is

$$\langle E \rangle = \frac{\int_0^{k_F} \frac{\hbar^2 k^2}{2m} (4\pi k^2) dk}{\left(\frac{4\pi}{3} k_F^3\right)} = \frac{3}{5} \frac{\hbar^2}{2m} {k_{\mathrm{F}}}^2 = \frac{3}{5} E_{\mathrm{F}} \,.$$