

Level 3 Condensed Matter Physics- Part II

Examples Class 3 Answers

(1) (i) Conduction band density of states:

$$g_c(E) = \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2} \right)^{3/2} \sqrt{E - E_c}$$

m_e^* is the electron effective mass.

Valence band density of states:

$$g_v(E) = \frac{1}{2\pi^2} \left(\frac{2m_h^*}{\hbar^2} \right)^{3/2} \sqrt{E_v - E}$$

m_h^* is the hole effective mass.

(ii) The Fermi-Dirac distribution function $f(E)$ is defined as:

$$f(E) = \frac{1}{1 + \exp [(E - \mu)/kT]}$$

The electron concentration (n) between energy levels E and $E+dE$ is:

$$n(E) = g_c(E)f(E)dE = \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2} \right)^{3/2} \frac{\sqrt{E - E_c}}{1 + \exp [(E - \mu)/kT]} dE$$

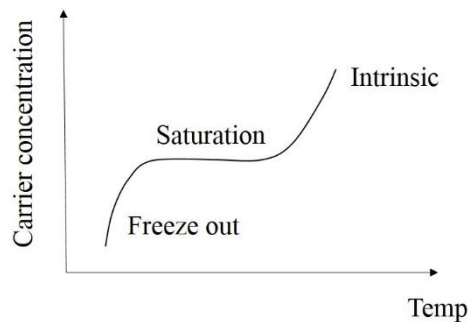
The hole concentration (p) between energy levels E and $E+dE$ is:

$$p(E) = g_v(E)[1 - f(E)]dE = \frac{1}{2\pi^2} \left(\frac{2m_h^*}{\hbar^2} \right)^{3/2} \frac{\sqrt{E_v - E}}{1 + \exp [(\mu - E)/kT]} dE$$

(iii) Shifting the Fermi level towards the valence band maximum would increase $(E - \mu)$ for energy levels within the conduction band. Therefore, from the above expression for $n(E)$ the electron concentration decreases.

On the other hand $(\mu - E)$ decreases for valence band energy levels and from the expression for $p(E)$ the hole concentration increases.

(iv)



(v) The Group III impurity has only 3 valence electrons, so substitution in the Group IV semiconductor crystal lattice will effectively generate a hole in the valence band. The majority carriers are therefore holes.

(vi) A diamond cubic crystal structure has 8 atoms per unit cell. The atomic volume density is therefore $8/[(0.357 \times 10^{-9})^3]$ or 5.1×10^{28} atoms/m³.

In the saturation regime every impurity atom is ionised and the majority carrier concentration is approximately equal to the impurity concentration.

Therefore the doping atomic fraction is $10^{20}/(5.1 \times 10^{28})$ or 2×10^{-9} (2 parts per billion).

(vii) At the intrinsic regime onset the intrinsic carrier concentration (n_i) equals the impurity concentration ($N_o = 10^{20}$ atoms/m³). Therefore:

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right) = N_o$$

or $T = E_g/[k \ln(N_c N_v / N_o^2)]$. Substituting values gives $T = \underline{530 \text{ K}}$

(viii) At the onset of freeze out the thermal energy kT is of the order of the impurity energy level (10 meV). Setting $kT = 10 \text{ meV}$ gives $T = \underline{116 \text{ K}}$

Note: a thermal energy of $(3/2)kT$ is also acceptable, in which case $T = \underline{77 \text{ K}}$.

(2) i) Using Gauss' law $d\varepsilon/dx = \rho(x)/\epsilon_r \epsilon_0$, where ε is the electric field, ρ the charge density, ϵ_r the relative permittivity (static dielectric constant), we have for the n -side:

$$\varepsilon(x) = \int \frac{\rho(x)}{\epsilon_r \epsilon_0} dx = \int \frac{e N_D x}{\epsilon_r \epsilon_0 w_n} dx = \frac{e N_D}{2 \epsilon_r \epsilon_0 w_n} (x^2 - w_n^2)$$

where the boundary condition that the electric field is zero at the space charge edge $x = w_n$ has been invoked.

Similarly for the p -side:

$$\varepsilon(x) = \int \frac{\rho(x)}{\epsilon_r \epsilon_0} dx = - \int \frac{-e N_A x}{\epsilon_r \epsilon_0 w_p} dx = \frac{e N_A}{2 \epsilon_r \epsilon_0 w_p} (x^2 - w_p^2)$$

The extra negative sign in front of the second integral takes into account the fact that ionised acceptors are negatively charged.

Furthermore the electric field within the quasi-neutral regions (i.e. $x > w_n$ and $x < -w_p$) is zero.

ii) From the continuity of the electric field at $x = 0$:

$$e N_D w_n = e N_A w_p$$

This implies charge conservation, since from the dopant profile the condition for equal amounts of positive and negative charge is:

$$\frac{1}{2}eN_Dw_n = \frac{1}{2}eN_Aw_p$$

iii) The potential ϕ is related to the electric field via $\varepsilon = -(d\phi/dx)$.

For the p -side:

$$\phi(x) = - \int \varepsilon(x)dx = \frac{eN_A}{2\epsilon_r\epsilon_0w_p} \int (w_p^2 - x^2)dx = \frac{eN_A}{2\epsilon_r\epsilon_0w_p} \left[w_p^2x - \frac{(x^3 - 2w_p^3)}{3} \right]$$

where we have arbitrarily set the potential to zero at $x = -w_p$.

For the n -side:

$$\phi(x) = - \int \varepsilon(x)dx = \frac{eN_D}{2\epsilon_r\epsilon_0w_n} \int (w_n^2 - x^2)dx = \phi_{bi} + \frac{eN_D}{2\epsilon_r\epsilon_0w_n} \left[w_n^2x - \frac{(x^3 + 2w_n^3)}{3} \right]$$

where ϕ_{bi} is the built-in potential at $x = w_n$. ϕ_{bi} is equal to the separation of n - and p -side Fermi levels within the quasi neutral regions before contact and therefore has the same form as that given in the lectures:

$$\phi_{bi} = \frac{kT}{e} \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

The potential is zero and ϕ_{bi} within the quasi-neutral regions on the p - and n -sides respectively.