FoP3B – Statistical Physics

Questions received by email

Q: Currently the only notes we have for the Statistical Physics part of FoP 3B are the handwritten ones from the lecture. I was wondering if there were any typed up notes available anywhere?

A: The recommended course book "Statistical Physics" by Tony Guenault is very short and on Blackboard there is a document "Course outline and reading material" which says which sections/pages correspond to each lecture.

Q: The solution for Q1 of homework 3 says that all microstates are assumed to be equally probable for the microcanonical ensemble and that the probabilities are different for the canonical ensemble. However, in the solution for Q1 of homework 4, where a system of N particles with fixed volume and internal energy is considered, it says that the microstates all have different probabilities. I assumed that this is a microcanonical ensemble and so the microstate probabilities should be equal.

A: Double check your reading of homework 4 Q1. It asks if the various **distributions** are equally probably, not if the **microstates** are equally probable, the answer states that the distributions are not equally likely. I think you've mixed up distributions and microstates. Remember that a distribution is a collection of different microstates. My favourite example, 2 distinguishable dice. The microstates are all of the possible outcomes of rolling the two dice, hence 36 equally likely microstates. A distribution is a collection of microstates that exhibits a particular property. Let's say the property is the sum of the two numbers. Therefore the probability of a distribution having a sum being 2 is 1/36 (1+1 roll only), the probability of distribution 3 is 2/36 (1+2, 2+1),…, probability of distribution 7 is 6/36 (1+6, 2+5, 3+4, 4+3, 5+2, 6+1), etc. The microstates are equally likely, the distributions are not.

Q: Question 1 (a) 2021 calls for the sketching of the Maxwell-Boltzmann distribution function at different temperatures. I have sketched it in a way I believe to be correct however the mark scheme appears to have the curves at different temperatures intersect the y axis at different points. The equation $F_{MB}(E) = \exp(-BE)$ suggests at this point of E=0 the functions should be equal.

A: Either is fine - your way is plotting $\exp[-E/kT]$ which is fine, the other way is to renormalise that so that there are equal areas (constant particle number), which is in the solutions. Both are equivalent and correct ways of showing the plots.

Q: May I ask you a question about the derivation for the Boltzmann distribution in lecture 4? You derived the expression for exp(A) as exp(-1-alpha), but in the latter derivation for the Boltzmann distribution involving degeneracy g_j , and also on the recommended statistical physics textbook, there is not any additional 1 involved in the expression for expA. And I found if we do not eliminate the N in (NlogN-N) with the summation term sum(n_j) initially, the additional 1 can be canceled in the derivative.

A: There are many ways to derive results in Physics. Often I use a different way from the book to show other ways. Both are equivalent. If alpha is a constant then -1-alpha is a constant. Let this constant be A.

Q: Question 4 from workshop 3. I am not entirely sure how the solution works. We take the ratios of the populations from each pair of energy states and rearrange these to yield some temperature values, which we then average to obtain the system's mean temperature. However, I am not sure what the temperatures obtained from rearranging the ratios of the populations (i.e. 99.0 K, 100.2 K, and 99.6 K) physically correspond to.

A: Given any excitation in the 3-level system, we can work out the temperature given the energies and populations. The numbers in the question are given to around 3 significant figures. No matter which if the three possible excitations we look at (first to second, first to third, second to third) we get the same temperature to this number of significant figures. This is the temperature at which the whole system is at.

Q: I was wondering why the Boltzmann distribution doesn't have a chemical potential included in it

A: You can put one in, $e^{-\beta(\varepsilon-\mu)}$, but it then factors out, $e^{\beta\mu}e^{-\beta\varepsilon}$, and is really just incorporated into the number constraint (see proof, it's just the α Lagrange multiplier constraint).

Q: Workshop 7 Q2(d) asks to calculate the fermi energy of electrons in aluminium which has valency 3 and then gives you a density of 2.7 gcm^-3 I know how to calculate the fermi energy of elements, but I'm not sure how the valency of aluminium comes into it? I've tried it by having only the mass of aluminium in the number density equation, then the mass of of aluminium plus the mass of the three electrons, but these both gave me answers which were multiple orders of 10 away from the answer.

A: 2(a) gives the Fermi energy in terms of number of particles and volume. 2(d)(ii) says Al is 2.7g/cm^3, valance 3. So we need to turn this into how many electrons (N) in a volume (V). Mass of aluminium is 27 amu and 1 amu is 1.66x10^{-27}kg, so one aluminium atom has a mass of 4.482x10^{-26}kg. Therefore 0.0027kg (2.7g) of Al has 6.02x10^{22} atoms. Each atom has 3 valence electrons hence there are 1.81x10^{23} electrons (N) in 1 cm^3, i.e. 10^{-6} m^3 (V) of aluminium. Plug into 2(a) formula.
2(d)(i) and (iii) are similar, enough information is given to work out how many particles (N) are in some volume (V).

Q: I have a question about Q1g in the 2020 past paper, regarding the BE distribution to find the number of photons in a cavity. In this type of question, do you assume the chemical potential is zero when using the BE distribution?

A: It's more than an assumption, the chemical potential for photons is zero. The number of photons is not conserved hence costs zero to add/remove them from a system.

Q: Density of states derivation in 2D, covered in the 2019/2020 F3B paper. In this question the constant multiplication factor was ¼ rather than 1/8 we used in the 3D derivation in lectures. Is this because in 2D there's a different number of positive quantum numbers to in 3D?

A: Yes. The quantum numbers of the infinite square well are all positive. Draw a circle (2 dimensions) centred at (0,0), and then more that only 1/4 of it has coordinates that are all positive. Now draw a sphere (3 dimensions) centred at (0,0,0). Only 1/8 of the sphere has coordinates that are all positive.

Q: Q2c of workshop 4. My understanding is that an extensive property is one that changes when the size of the substance changes. My question is what specifically is meant by size- is it synonymous with particle number? If this is the case, then I'm confused as to why the question implies that we should check how the quantity changes when the volume and the particle number change. If particle number is held constant and volume increases, would the value of an extensive property increase?

A: An extensive property depends on the amount of matter, for example volume and mass. An intensive property doesn't depend on the amount of matter, for example density and temperature. The question suggests doubling both the volume and number of particles (not holding either fixed), i.e. having twice the amount of matter in a system of twice the size. A two pint cup of coffee versus a one pint cup of coffee, having twice the amount of matter in the system what happens to the internal energy, free energy and entropy?

Q: I just had a quick question about the 2020 1e question if you don't mind. When saying what the energy is to sub into the FD distribution we say it's E = chemical potential + epsilon. Why the chemical potential? I just wrote E = E_f + epsilon. E_f being the fermi energy. What am I missing?

A: Chemical potential and Fermi energy are the same thing.

Q: In the Statistical Physics textbook, they define β as $-1/k_BT$ whereas in your lectures you've defined it without a minus. I was just wondering which one I should use for the exam?

A: I don't mind which one you use, just state what you're doing. Personally I prefer using 1/kT without the minus so the minus is not "hidden" in expressions such as $e^{-\beta \epsilon}$.

Q: I was just doing the 2020 past paper and on 1g they use the bose-einstein distribution but they don't have the usual e^α factor before the e^βε Is it ok to disregard this factor?

A: This factor is related to particle number, we did a change of variable to obtain μ, the chemical potential (Fermi level). For photons the chemical potential is zero.

Q: I was confused about a couple of questions going through the maths of the mark scheme of the 2016/17 exam. These were Q2b and 2d. For Q2b, I was confused as to why taking the high temperature limit resulted in the denominator exponentials being taken as 1 then the

numerator exponentials are uB/kbT? For Q2d, I am not sure where the factor of T^-2 has come from in the differentiation wrt T.

A: The limit, remove all of the extra details to clarify what's happening. In the high temperature limit exp(-1/T) is a small number. Tidy up even more and just call this x. The thing you\re wanting to approximate for small x is $\frac{x}{1+x} \sim x$ (put in some small numbers for x and convince yourself). For the $1/T^2$, it's just the chain rule, take the derivative of $e^{-1/T}$ ...

Q: In the solution for part f of question 2 in the 2021 3B exam, I was wondering how we are able to treat ln(n1/n2) as a conserved quantity during the adiabatic cooling

A: Be carerful on definitions. We have n_1 referring to T_1 and n_2 referring to T_2. So that term is the (log of) the ratio of change in population – it is the change.

Q: I was having a look at the exercises in workshop 8 and had a question about the 2nd to last step of the answer to question 1.a). Here, the following formula is used: Cv ~ (k_B)^2 * T * g(E_F). I was wondering where exactly this formula came from? I had a look if there was a good way to approximate the integral for the internal energy, U though was not too sure. I thought it instead might have come from the idea that U can be approximated as follows U(T) ~ U(T=0) + ½ (k_B)^2 * T^2 * g(E_F), which would lead to the Cv the answer gave. I could justify this by saying that the region of ½ k_B * T below E_F gets increased with k_B * T and that at this energy the system has g(E_F) states ((k_B * T) * g(E) dE = (k_B * T) * g(E_F) * (½ k_B * T)). Is this statement correct to begin with? And is this how the equation for Cv was found?

A: Thank you and well done for answering your own question, you are correct. Essentially U(T) is needed so you can get Cv via dU/dT, so we need the integral of e.g(e).f(e) which a pain - there are many ways to approach/approximate this, just as you have done. Note workshop 8 (we only had 7 timetabled) is intentionally harder and optional. I won't be this difficult in an exam!

Q: I am a bit confused by Q1(d) in the 2021 past paper. In the revision lecture and the solution to the paper it says that: <h> = integral{h*exp(-mgh/KbT)*dh}/int{exp(-mgh/KbT)*dh}. However in my notes I have written that the average of a quantity Q: <Q> = integral{g(E)*f(E)*Q(E)*dE}/integral{g(E)*f(E)*dE}. Surely then for the g(E)dE part we should get it proportional to E^(1/2)dE and so in the question there should be an extra factor of h^(1/2). Could you explain where I am going wrong?

A: Yes, this is the average of a quantity, but you need to use the correct f and g. You seem to be using the density of electronic states in a quantum 3D solid (proportional to sqrt(E)) for the density of states of a particle in a classical gravitational field. We're just after the expectation value of height, so height times Boltzmann factor (normalised by partition function).