Statistical Physics: Weekly Problem 5 (SP5)

(1) (a) Consider a system of spin one particles localised on a lattice (i.e. distinguishable) in a magnetic field. The energy levels associated with the three spins states $(-1, 0, 1)$ have energies $(-\epsilon, 0, \epsilon)$. Calculate the single-particle partition function Z. [2 mark]

(b) The same system is at a temperature T. Calculate the internal energy U, heat capacity C_V, free energy F and entropy S. [2 marks]

(2) (a) In a hypothetical system of identical particles, restricted to move in two dimensions, each single particle state may hold up to η particles, where η is a fixed positive integer. The degeneracy of each single-particle energy ϵ_i is g_i. (This hypothetical system is a model for 2D particles known as anyons, or the fractional statistics gas.)

Justify that the number of microstates $\Omega(n_1, n_2, \ldots)$ for the distribution $(n_1, n_2, \ldots)$ of N such identical particles in the single-particle energies ϵ_i of the system is

$$\Omega(\{n_i\}) = \prod_j \frac{(\eta \times g_j)!}{n_j!\,(\eta \times g_j - n_j)!}$$

[2 marks]

(b) For the same hypothetical system of identical particles show that the distribution function, or fractional occupancy, $f_i = n_i/g_i$, of each single particle energy is

$$\frac{n_i}{g_i} = \frac{1}{A\, e^{\beta\epsilon_i} + (1/\eta)}$$

where A and β are Lagrange multipliers. [2 marks]

(c) What are the limits of the distribution function for small and large η? [2 marks]