Statistical Physics: Workshop Problems 1

- (1) In a population of fifty million people, on average, two hundred people have the same DNA profile.
 - (a) What is the probability of two people with the same profile?
 - (b) What is the probability that the police, only knowing that a suspect from the population has a DNA profile that matches that found at the scene of crime, has found the guilty party?
- (2) What is the probability that out of five people, none have the same birthday?
- (3) Consider a system in which the allowed (non-degenerate) one-particle states have energies $0, \epsilon, 2\epsilon, 3\epsilon, \ldots$ The assembly has four distinguishable (localised) particles (N=4) and a total energy of $U=6\epsilon$. Identify the possible distributions, evaluate the number of microstates Ω , and statistical entropy S, and work out the average distribution (i.e., the average occupation $\langle n_i \rangle$ of each energy level) and the probability p_i that each one-particle energy level ϵ_i is occupied.
- (4) A manufacturer knows that their resistors have values which are distributed as a Gaussian probability distribution with a mean resistance of 100Ω and standard deviation of 5Ω . What percentage of resistors have resistances between 95 and 105 Ω ? What is the probability of selecting a resistor with resistance less than 80 Ω ?

Reminder: For a continuous probability distribution, the probability $p(a \le x \le b) = \int_a^b dx f(x)$. For the Gaussian probability distribution,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

You may use:

$$\frac{1}{\sqrt{2\pi}} \int_{-1}^{1} dx \, \exp\left(-\frac{x^2}{2}\right) = 0.68269 \,, \quad \frac{1}{\sqrt{2\pi}} \int_{-2}^{2} dx \, \exp\left(-\frac{x^2}{2}\right) = 0.95450$$

$$\frac{1}{\sqrt{2\pi}} \int_{-3}^{3} dx \, \exp\left(-\frac{x^{2}}{2}\right) = 0.99730 \,, \quad \frac{1}{\sqrt{2\pi}} \int_{-4}^{4} dx \, \exp\left(-\frac{x^{2}}{2}\right) = 0.99994$$

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