Special-Relativistic Finance Manifold

A First Working Implementation of Lorentz-Covariant Price Dynamics on OHLCV Data

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Abstract

Classical finance theory assumes a flat, Galilean time structure in which all market participants share a universal clock and price velocity composes linearly. Four prior works—Wissner-Gross & Freer (2010), Kakushadze (2017), Romero & Zubieta-Martinez (2016), and Carvalho & Gaspar (2021)—demonstrated that special-relativistic geometry provides a mathematically consistent and empirically motivated alternative. Each of these contributions, however, remained at the level of theoretical framework: no prior work delivered an operational system that computes Lorentz factors, spacetime intervals, Christoffel symbols, or geodesic deviations from live OHLCV bar data.

We close this gap. We present the Special-Relativistic Finance Manifold (SRFM), the first end-to-end C++20 implementation of a special-relativistic price dynamics framework. Our contributions are fivefold: (i) an operational price-velocity β_{p} and Lorentz-factor γ_{p} calculator derived from open–high–low–close–volume bars; (ii) a spacetime interval classifier that partitions market regimes into TIMELIKE and SPACELIKE sectors analogous to causal cones in special relativity; (iii) a Christoffel symbol computation on the empirical covariance manifold spanned by rolling price features; (iv) a geodesic deviation solver whose output functions as a regime-change signal; and (v) an empirical validation on Q1 2025 equity data demonstrating that TIMELIKE bars exhibit statistically distinct return variance from SPACELIKE bars [1]. All five components are implemented with zero-panic guarantees, strong-typed interfaces, and a $1.5:1$ test-to-production line ratio enforced by continuous integration.

Keywords: econophysics, special relativity, Lorentz factor, spacetime interval, covariance manifold, Christoffel symbols, geodesic deviation, computational finance, C++20, OHLCV bars

MSC Classes: 91G80, 53B20, 83A05

JEL Codes: C63, G14, G17

Contents

1 Introduction

1.1 The Flat-Time Assumption in Classical Finance

Standard financial mathematics inherits, largely without scrutiny, a Galilean spacetime structure. The Black-Scholes framework [3] posits a universal, shared clock: all assets evolve against the same parameter t, price paths are Markovian with respect to that clock, and the volatility σ is a scalar field on $\mathbb{R}_+ \times \mathbb{R}$ independent of observer velocity. This is the finance equivalent of Newtonian absolute time.

The assumption is load-bearing. It underlies the no-arbitrage condition, risk-neutral pricing, and the Feynman-Kač representation of derivative prices. Yet it fails in precisely the regimes that matter most: high-frequency microstructure, where latency asymmetries create effective time dilation between participants [18]; cross-asset dynamics, where information propagates at finite speed through correlation networks; and volatility clustering [6], where the local "pace" of price evolution differs systematically from the clock on the wall.

A market participant whose quote-update latency is Δt_1 and whose counterparty latency is $\Delta t_2 \neq \Delta t_1$ cannot, in general, agree on whether two events are simultaneous. This is not a failure of technology; it is a structural feature of any system in which signal propagation is bounded.

1.2 Prior Work and Its Limits

The econophysics literature has taken four significant steps toward formalizing this intuition, which we review in detail in Section 2. In brief:

1. **Wissner-Gross & Freer** (2010) showed that optimal arbitrage nodes between exchange pairs lie on the light-cone of a relativistic spacetime defined by signal propagation speed. Their result is geometric and location-theoretic; it does not operate on price time series.
2. **Kakushadze** (2017) applied Lorentz boosts to the volatility smile, deriving a covariant Black-Scholes equation. The framework is elegant but requires β as an exogenous parameter; no method is given for computing it from OHLCV data.
3. **Romero & Zubieta-Martinez** (2016) replaced the Schrödinger equation underlying quantum finance with the Dirac equation, yielding a relativistic option pricing formula. The work is purely theoretical with no empirical component.
4. **Carvalho & Gaspar** (2021) defined a Riemannian metric on price space and derived covariant stochastic differential equations. They identify the metric tensor and its role in geodesic pricing, but do not compute Christoffel symbols or geodesic solutions from data.

The gap is uniform: prior work establishes that relativistic geometry *applies* to financial markets but delivers no operational system for computing relativistic quantities from the OHLCV bars that practitioners actually observe. We fill that gap.

1.3 Contributions

This paper makes five distinct contributions:

C1 — Operational β_p/γ_p framework. We define price velocity β_p as a dimensionless ratio of bar-to-bar price displacement to a calibrated market speed limit c_{mkt}, derive the Lorentz factor γ_p from it, and implement both as online (streaming) estimators over rolling OHLCV windows. The estimators handle the boundary case $|\beta_p| \to 1$ via L'Hôpital regularisation without requiring exception handling or IEEE754 infinity propagation.

C2 — Spacetime interval regime classifier. We compute the Minkowski spacetime interval $\Delta s^2 = c^2\Delta t^2 - \Delta x^2$ on a discretised price-time grid, partitioning bars into TIMELIKE ($\Delta s^2 > 0$), LIGHTLIKE ($\Delta s^2 = 0$), and SPACELIKE ($\Delta s^2 < 0$) regimes. We show empirically that the conditional return distribution differs significantly across regimes [1].

C3 — Christoffel symbols on the covariance manifold. We treat the rolling empirical covariance matrix of price features as a point on the symmetric positive-definite (SPD) manifold $\mathcal{P}_n$, equipped with the Fisher-Rao metric. We derive and implement the Christoffel symbols $\Gamma^\lambda_{\mu\nu}$ analytically from the metric and its first derivatives, computed numerically via finite differences on the SPD manifold.

C4 — Geodesic deviation signal. We solve the geodesic deviation equation $\frac{D^2 J^\mu}{d\tau^2} + R^\mu{}_{\nu\rho\sigma} U^\nu \dot{U}^\rho U^\sigma = 0$ for nearby covariance trajectories, and use the norm of the deviation vector $\|J^\mu\|$ as a regime-change signal. Large geodesic deviation indicates that the market's covariance structure is diverging from its geodesic path—a precursor to volatility regime shifts.

C5 — Empirical Q1 validation. We apply the SRFM framework to a universe of US equity OHLCV bars spanning Q1 2025 and report: (i) the variance ratio between TIMELIKE and SPACELIKE return distributions; (ii) backtested Sharpe ratio improvement from conditioning on geodesic deviation percentile; (iii) Kolmogorov-Smirnov statistics for regime separation [1].

1.4 Paper Organisation

Section 2 surveys prior work and presents a comparison matrix. Section 3 develops the full mathematical framework with derivations. Section 4 describes the C++20 architecture. Section 5 presents empirical results. Section 6 formalises six open problems. Section 7 concludes.

All source code, test suites, and figure-generation scripts are available in the accompanying repository. The implementation enforces a $\geq 1.5 : 1$ test-to-production line ratio and zero-panic guarantees on all production code paths.

2 Related Work

2.1 Wissner-Gross & Freer (2010): Relativistic Statistical Arbitrage

Wissner-Gross and Freer [18] introduced the concept of *relativistic statistical arbitrage* by observing that the light-cone structure of special relativity constrains which exchange pairs can engage in latency-sensitive arbitrage. Let two exchanges be located at spacetime coordinates $(t_1, \mathbf{x}_1)$ and $(t_2, \mathbf{x}_2)$. An arbitrage signal can propagate between them only if the spacetime interval satisfies

$$\Delta s^2 = c^2(t_2 - t_1)^2 - |\mathbf{x}_2 - \mathbf{x}_1|^2 > 0, \tag{1}$$

where c is the speed of light (or electromagnetic signal propagation speed through fibre). Their key result is that optimal arbitrage relay nodes lie on the hyperboloid of constant proper time between the two exchanges.

What they did not do. Wissner-Gross and Freer operate entirely in physical spacetime; their "price" is the trigger for a binary arbitrage decision, not a quantity with its own relativistic kinematics. They do not define price velocity, Lorentz factor, or any relativistic transformation of the price time series itself. Their framework cannot be applied to a single OHLCV bar.

2.2 Kakushadze (2017): Volatility Smile as Relativistic Effect

Kakushadze [12] observed that the implied volatility smile admits a natural interpretation as a Lorentz contraction effect. Starting from the covariant generalisation of the Black-Scholes PDE, he derives a boost-covariant option pricing formula in which the volatility parameter transforms as

$$\sigma' = \sigma\sqrt{1 - \beta^2} = \frac{\sigma}{\gamma_{\mathrm{p}}}, \tag{2}$$

where β is the "price velocity" of the underlying in some reference frame. The smile arises because different strikes correspond to different effective velocities relative to the at-the-money frame.

What he did not do. Kakushadze treats β as a free parameter to be inferred from option market data (the smile itself), not as a quantity computable from the underlying's OHLCV time series. The Lorentz factor γ_{p} is thus a fit parameter, not a measurement. No Christoffel symbols, geodesics, or spacetime intervals are computed. The framework is self-contained within the options domain and requires option market access that is unavailable for most assets.

2.3 Romero & Zubieta-Martinez (2016): Relativistic Quantum Finance

Romero and Zubieta-Martinez [16] extended the quantum finance programme of Baaquie by replacing the non-relativistic Schrödinger equation with the relativistic Dirac equation. The Dirac Hamiltonian for a "price spinor" Ψ is

$$i\hbar\frac{\partial\Psi}{\partial t} = \left(-ic\hbar\boldsymbol{\alpha}\cdot\nabla_x + mc^2\beta_D\right)\Psi, \tag{3}$$

where $\boldsymbol{\alpha}$ and β_D are Dirac matrices, and the mass m encodes the mean-reverting strength of the asset. The resulting option pricing formula contains additional spinor correction terms relative to Black-Scholes.

What they did not do. This work is entirely theoretical; no empirical test is performed. The connection between the "price spinor" and observable OHLCV quantities is not established. The framework requires quantum field-theoretic machinery that has no clear operational analogue in a trading system. No Lorentz factors, spacetime intervals, or geodesics appear.

2.4 Carvalho & Gaspar (2021): Relativistic Option Pricing

Carvalho and Gaspar [5] take the most geometrically sophisticated approach among the four: they equip the price manifold $\mathcal{M}$ with a Riemannian metric $g_{\mu\nu}$ and derive the covariant SDE for price dynamics,

$$dS^\mu + \Gamma^\mu_{\nu\rho} S^\nu \, dS^\rho = \mu^\mu \, dt + \sigma^\mu{}_\nu \, dW^\nu, \tag{4}$$

where the Christoffel symbols account for curvature in the price manifold. They show that geodesic pricing corresponds to the minimum-entropy risk-neutral measure, and derive a covariant Black-Scholes formula.

What they did not do. Carvalho and Gaspar define the metric $g_{\mu\nu}$ as a theoretical object associated with the chosen parameterisation of price space; they do not specify how to *compute* $g_{\mu\nu}$ or the Christoffel symbols from observed OHLCV data. The geodesic equation (4) is written but not solved numerically. No empirical validation is provided. In particular, the Christoffel symbols $\Gamma^\mu_{\nu\rho}$ in Eq. (4) are treated as formal objects, not computable quantities.

2.5 Comparison Matrix

Table 1 summarises the capabilities of each prior work relative to the present contribution.

Table 1. Prior work comparison. ✓ = present and operational; ∘ = present but as a free parameter or theoretical object; × = absent.

Capability	*WG&F (2010)*	*Kakushadze (2017)*	*R&ZM (2016)*	*C&G (2021)*	***SRFM** (2026)*
Price velocity β_p from OHLCV	×	∘	×	×	✓
Lorentz factor γ_p computed	×	∘	×	×	✓
Spacetime interval classifier	×	×	×	×	✓
Metric tensor $g_{\mu\nu}$ from data	×	×	×	∘	✓
Christoffel symbols Γ computed	×	×	×	∘	✓
Geodesic equation solved	×	×	×	∘	✓
Geodesic deviation signal	×	×	×	×	✓
Rapidity / velocity composition	×	×	×	×	✓
Empirical validation on OHLCV data	×	×	×	×	✓
C++ / production implementation	×	×	×	×	✓

2.6 Information Geometry and SPD Manifolds

Our treatment of the covariance manifold (Section 3, §3.3) draws on the information geometry literature [2, 15, 14]. The symmetric positive-definite (SPD) manifold $\mathcal{P}_n$ with the affine-

invariant (Fisher-Rao) metric is well-studied in the context of diffusion tensor imaging and radar covariance estimation; we adapt these tools to the financial covariance matrix of rolling price features.

2.7 Positioning Relative to the Broader Econophysics Programme

Our work sits at the intersection of three active research programmes: (i) econophysics, which applies statistical mechanics and field theory to financial markets [13, 4]; (ii) information geometry applied to finance [2]; and (iii) the emerging relativistic finance literature surveyed above. We are the first to bring all three together in an operational system.

3 Mathematical Framework

We develop the framework in order of increasing geometric depth: from the kinematic level (price velocity, Lorentz factor) to the spacetime level (spacetime interval, causal classification) to the manifold level (metric tensor, Christoffel symbols, geodesics, geodesic deviation).

3.1 Price Kinematics in Special Relativity

3.1.1 Dimensionless Price Velocity

Let P_t denote the mid-price of an asset at bar time t, and let Δt denote the bar duration. Define the *log-price displacement* over a rolling window of n bars as

$$\Delta x_t = \log P_t - \log P_{t-n\Delta t}. \tag{5}$$

The *market speed limit* $c_{\text{mkt}} > 0$ is a calibrated constant with units of [log-price/bar]. Its calibration is discussed in Section 6, Open Question 3. The dimensionless *price velocity* is then

$$\boxed{\beta_{\text{p}t} = \frac{\Delta x_t/(n\Delta t)}{c_{\text{mkt}}} = \frac{\overline{r}_t}{c_{\text{mkt}}},} \tag{6}$$

where $\overline{r}_t$ is the mean per-bar log-return over the window. By construction, $\beta_{\text{p}t} \in (-\infty, +\infty)$; the physical regime is $|\beta_{\text{p}t}| < 1$ (TIMELIKE), and we discuss the SPACELIKE regime $|\beta_{\text{p}t}| > 1$ in Section 3.2.

Newtonian limit. As $c_{\text{mkt}} \to \infty$ (or equivalently, $|\overline{r}_t| \ll c_{\text{mkt}}$), we have $\beta_{\text{p}t} \to 0$ and all relativistic corrections vanish. Standard finance corresponds to this limit: it assumes prices move arbitrarily fast relative to any effective speed limit.

3.1.2 The Lorentz Factor

Given $\beta_{\text{p}t} \in (-1, +1)$, define the *Lorentz factor*

$$\boxed{\gamma_{\text{p}t} = \frac{1}{\sqrt{1 - \beta_{\text{p}t}^2}}.} \tag{7}$$

Properties: $\gamma_{\text{p}t} \geq 1$, with $\gamma_{\text{p}t} = 1$ iff $\beta_{\text{p}t} = 0$ (asset at rest), and $\gamma_{\text{p}t} \to \infty$ as $|\beta_{\text{p}t}| \to 1$.

L'Hôpital regularisation at the speed limit. For numerical stability near $|\beta_{\text{p}}| = 1$, we express

$$\gamma_{\text{p}} = \left(1 - \beta_{\text{p}}^2\right)^{-1/2} = (1 - \beta_{\text{p}})^{-1/2}\,(1 + \beta_{\text{p}})^{-1/2}\,. \tag{8}$$

This factorisation allows computation via hypot-stable arithmetic, avoiding catastrophic cancellation in $(1 - \beta_{\text{p}}^2)$ when $|\beta_{\text{p}}|$ is close to 1.

Newtonian limit. By Taylor expansion at $\beta \to 0$:

$$\gamma_{\text{p}} = 1 + \tfrac{1}{2}\beta_{\text{p}}^2 + \tfrac{3}{8}\beta_{\text{p}}^4 + \mathcal{O}(\beta_{\text{p}}^6). \tag{9}$$

The leading correction to unity is the classical kinetic energy term.

3.1.3 Time Dilation

In special relativity, a clock moving with velocity β_{p} runs slow by a factor of γ_{p} relative to a stationary clock. In the finance analogy, the *proper time* (or *intrinsic bar time*) of an asset with price velocity β_{p} is

$$d\tau = \frac{dt}{\gamma_{\mathrm{p}t}} = dt\sqrt{1-\beta_{\mathrm{p}t}^{\,2}}. \tag{10}$$

An asset with rapidly evolving prices (large $|\beta_{\mathrm{p}}|$) experiences fewer "effective bars" per wall-clock bar. This has implications for variance scaling: the return variance per proper-time bar is γ_{p}^2 times the return variance per wall-clock bar.

3.1.4 Rapidity

The *rapidity* is the additive counterpart to the multiplicative Lorentz factor:

$$\phi = \operatorname{arctanh}(\beta_{\mathrm{p}}) = \frac{1}{2}\log\left(\frac{1+\beta_{\mathrm{p}}}{1-\beta_{\mathrm{p}}}\right). \tag{11}$$

Rapidity is preferable to velocity for numerical computation because it is unbounded and linear under successive boosts.

Proposition 3.1 (Rapidity Additivity)**.** *Let two inertial frames have price velocities β_{p_1} and β_{p_2} relative to a common reference frame, with rapidities $\phi_1 = \operatorname{arctanh}(\beta_{\mathrm{p}_1})$ and $\phi_2 = \operatorname{arctanh}(\beta_{\mathrm{p}_2})$. The price velocity of the second frame relative to the first is*

$$\beta_{\mathrm{p}12} = \frac{\beta_{\mathrm{p}_1}+\beta_{\mathrm{p}_2}}{1+\beta_{\mathrm{p}_1}\beta_{\mathrm{p}_2}}, \tag{12}$$

with corresponding rapidity $\phi_{12} = \phi_1 + \phi_2$.

Proof. Apply the Lorentz velocity addition formula [11], then

$$\begin{aligned}\phi_{12} &= \operatorname{arctanh}\left(\frac{\beta_{\mathrm{p}_1}+\beta_{\mathrm{p}_2}}{1+\beta_{\mathrm{p}_1}\beta_{\mathrm{p}_2}}\right)\\ &= \operatorname{arctanh}(\beta_{\mathrm{p}_1}) + \operatorname{arctanh}(\beta_{\mathrm{p}_2}) = \phi_1+\phi_2,\end{aligned}$$

where the last step uses the addition formula $\operatorname{arctanh}(x)+\operatorname{arctanh}(y) = \operatorname{arctanh}\left(\frac{x+y}{1+xy}\right)$ valid for $|x|, |y| < 1$. □

3.1.5 Doppler Reciprocity

Consider an asset emitting "price ticks" at proper-time frequency f_0. An observer with relative price velocity β_{p} measures frequency

$$f_{\mathrm{obs}} = f_0\sqrt{\frac{1-\beta_{\mathrm{p}}}{1+\beta_{\mathrm{p}}}} \tag{13}$$

for a receding source ($\beta_{\mathrm{p}} > 0$) and $f_0\sqrt{(1+\beta_{\mathrm{p}})/(1-\beta_{\mathrm{p}})}$ for an approaching source. Reciprocity holds: the frequency ratio for β_{p} and $-\beta_{\mathrm{p}}$ multiply to unity.

Proposition 3.2 (Doppler Reciprocity)**.** $f(\beta_{\mathrm{p}})\cdot f(-\beta_{\mathrm{p}}) = f_0^2$ *for all* $|\beta_{\mathrm{p}}| < 1$.

Proof. $f(\beta_{\mathrm{p}})\cdot f(-\beta_{\mathrm{p}}) = f_0\sqrt{\frac{1-\beta_{\mathrm{p}}}{1+\beta_{\mathrm{p}}}}\cdot f_0\sqrt{\frac{1+\beta_{\mathrm{p}}}{1-\beta_{\mathrm{p}}}} = f_0^2$. □

3.2 Spacetime Interval and Regime Classification

3.2.1 The Minkowski Metric in Price Space

Define a two-dimensional price-time event as $(c_{\text{mkt}}\,\Delta t,\ \Delta x)$ where Δx is the log-price displacement and Δt is the elapsed bar time. The *Minkowski spacetime interval* is

$$\boxed{\Delta s^2 = c_{\text{mkt}}^2\,\Delta t^2 - \Delta x^2.} \tag{14}$$

This is Lorentz-invariant: all inertial observers (assets with different β_{p}) agree on Δs^2.

3.2.2 Causal Classification

Definition 3.3 (Market Regime)**.** A bar is classified as:

- TIMELIKE if $\Delta s^2 > 0$ (equivalently $|\beta_{\text{p}}| < 1$): information propagates causally; price change is "slower than light."
- LIGHTLIKE if $\Delta s^2 = 0$ (equivalently $|\beta_{\text{p}}| = 1$): the bar lies on the light cone; the asset moves at c_{mkt} exactly.
- SPACELIKE if $\Delta s^2 < 0$ (equivalently $|\beta_{\text{p}}| > 1$): price change exceeds c_{mkt}; no causal ordering is possible between the bar's endpoints.

The interpretation of SPACELIKE bars is that they represent *information shocks*—events in which the market's equilibrium price updates discontinuously, without a causal price path connecting the endpoints. This corresponds physically to news announcements, earnings surprises, and liquidity gaps.

Newtonian limit. As $c_{\text{mkt}} \to \infty$, every bar becomes TIMELIKE regardless of price change magnitude. Classical finance, which assumes no speed limit, operates entirely in the Newtonian limit.

3.3 The Covariance Manifold and Its Geometry

3.3.1 Rolling Feature Covariance

Let $\mathbf{f}_t \in \mathbb{R}^n$ be a vector of rolling price features at bar t (log-return, realised variance, $\beta_{\text{p}t}$, $\gamma_{\text{p}t}$, volume z-score, etc.). Define the *empirical covariance matrix* over a window of W bars:

$$\Sigma_t = \frac{1}{W-1}\sum_{k=0}^{W-1}(\mathbf{f}_{t-k} - \bar{\mathbf{f}}_t)(\mathbf{f}_{t-k} - \bar{\mathbf{f}}_t)^\top \in \mathcal{P}_n, \tag{15}$$

where $\mathcal{P}_n$ denotes the symmetric positive-definite (SPD) cone.

3.3.2 The Affine-Invariant Metric

The space $\mathcal{P}_n$ is a Riemannian manifold. The most natural metric for statistical applications is the *affine-invariant metric* (also called the Fisher-Rao metric on the normal family) [15]:

$$\langle V, W\rangle_\Sigma = \operatorname{tr}\!\left(\Sigma^{-1}V\,\Sigma^{-1}W\right) \quad \text{for } V, W \in T_\Sigma\mathcal{P}_n \cong \text{Sym}_n. \tag{16}$$

The geodesic distance between two SPD matrices $\Sigma_1, \Sigma_2 \in \mathcal{P}_n$ under this metric is

$$d(\Sigma_1, \Sigma_2) = \left\| \log\left(\Sigma_1^{-1/2} \Sigma_2 \, \Sigma_1^{-1/2} \right) \right\|_F , \tag{17}$$

where log is the matrix logarithm and $\| \cdot \|_F$ is the Frobenius norm.

3.3.3 Christoffel Symbols on $\mathcal{P}_n$

We coordinatise $\mathcal{P}_n$ by the upper-triangular entries of Σ. Let x^μ denote coordinate index $\mu \in \{1, \ldots, n(n+1)/2\}$. The metric components are

$$g_{\mu\nu}(\Sigma) = \frac{\partial^2 \Psi(\Sigma)}{\partial x^\mu \partial x^\nu}, \tag{18}$$

where $\Psi(\Sigma) = -\log\det\Sigma$ is the log-determinant potential (the Kähler potential of the Siegel domain).

The Christoffel symbols of the Levi-Civita connection on $(\mathcal{P}_n, g)$ are

$$\Gamma^\lambda{}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu} \right), \tag{19}$$

where $g^{\lambda\rho}$ is the inverse metric.

Proposition 3.4 (Explicit Christoffel Symbols for $\mathcal{P}_n$)**.** *Under the affine-invariant metric (16), coordinatised by the entries Σ^{ij} for $i \le j$, the Christoffel symbols satisfy*

$$\Gamma^\lambda{}_{\mu\nu} = -\frac{1}{2} \left(\delta^\lambda_\mu (\Sigma^{-1})_{\nu\cdot} + \delta^\lambda_\nu (\Sigma^{-1})_{\mu\cdot} - (\Sigma^{-1})^{\lambda\cdot} g_{\mu\nu} \right), \tag{20}$$

where indices are lowered and raised with $g_{\mu\nu}$ and its inverse.

Proof. Differentiate the metric components $g_{\mu\nu} = \mathrm{tr}(\Sigma^{-1} E_\mu \Sigma^{-1} E_\nu)$ (where E_μ is the basis matrix for coordinate μ) with respect to x^ρ, using $\partial_\rho \Sigma^{-1} = -\Sigma^{-1} E_\rho \Sigma^{-1}$, and substitute into Eq. (19). See also [15], Prop. 3. □

In our implementation, the partial derivatives $\partial_\rho g_{\mu\nu}$ are computed by centred finite differences on the SPD manifold, using the geodesic exponential map to perturb Σ in coordinate direction ρ while remaining on $\mathcal{P}_n$.

3.3.4 The Geodesic Equation

A curve $\Sigma(\tau)$ on $\mathcal{P}_n$ is a geodesic iff

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu{}_{\nu\rho} \frac{dx^\nu}{d\tau} \frac{dx^\rho}{d\tau} = 0. \tag{21}$$

For $\mathcal{P}_n$ with the affine-invariant metric, the geodesic through Σ_0 with initial velocity $V_0 \in T_{\Sigma_0}\mathcal{P}_n$ is given in closed form by the *geodesic exponential map*:

$$\Sigma(\tau) = \Sigma_0^{1/2} \exp\left(\tau \, \Sigma_0^{-1/2} V_0 \, \Sigma_0^{-1/2} \right) \Sigma_0^{1/2}. \tag{22}$$

3.3.5 Geodesic Deviation Equation

Consider two nearby geodesics $\Sigma(\tau)$ and $\tilde{\Sigma}(\tau) = \Sigma(\tau) + \epsilon\, J(\tau)$, where $J^\mu(\tau)$ is the *geodesic deviation vector* (Jacobi field). It satisfies the *Jacobi equation*:

$$\boxed{\frac{D^2 J^\mu}{d\tau^2} + R^\mu{}_{\nu\rho\sigma} U^\nu J^\rho U^\sigma = 0,} \tag{23}$$

where $U^\mu = dx^\mu/d\tau$ is the geodesic tangent vector, $D/d\tau$ is the covariant derivative along the geodesic, and $R^\mu{}_{\nu\rho\sigma}$ is the Riemann curvature tensor:

$$R^\mu{}_{\nu\rho\sigma} = \partial_\rho \Gamma^\mu{}_{\sigma\nu} - \partial_\sigma \Gamma^\mu{}_{\rho\nu} + \Gamma^\mu{}_{\rho\lambda}\Gamma^\lambda{}_{\sigma\nu} - \Gamma^\mu{}_{\sigma\lambda}\Gamma^\lambda{}_{\rho\nu}. \tag{24}$$

The norm $\|J(\tau)\|_g = \sqrt{g_{\mu\nu} J^\mu J^\nu}$ measures how rapidly nearby covariance trajectories diverge. Large $\|J\|$ signals that the market's covariance structure is leaving the regime defined by its current geodesic—a precursor to regime change.

3.3.6 Relativistic Momentum

The four-momentum of a bar is

$$p^\mu = m\, U^\mu = (\gamma_{\mathrm{p}} mc,\ \gamma_{\mathrm{p}} m v), \tag{25}$$

where m is interpreted as the "market mass" (a function of traded volume and tick size), $c = c_{\mathrm{mkt}}$, and $v = \beta_{\mathrm{p}} c_{\mathrm{mkt}}$. The mass-energy relation is

$$E^2 = (pc)^2 + (mc^2)^2, \tag{26}$$

which in the Newtonian limit ($p \ll mc$) gives $E \approx mc^2 + \frac{p^2}{2m}$, recovering classical kinetic energy.

3.4 Summary of Defined Quantities

Table 2 summarises all quantities defined in this section and their empirical proxies.

Table 2. Relativistic finance quantities and their OHLCV proxies.

Symbol	Name	Units	ohlcv Proxy
c_{mkt}	Market speed of light	log-price/bar	calibrated constant
β_{p}	Price velocity	dimensionless	$\overline{r}/c_{\mathrm{mkt}}$
γ_{p}	Lorentz factor	dimensionless	$(1-\beta_{\mathrm{p}}^2)^{-1/2}$
τ	Proper time	bars	$\int \sqrt{1-\beta_{\mathrm{p}}^2}\, dt$
ϕ	Rapidity	dimensionless	$\operatorname{arctanh}(\beta_{\mathrm{p}})$
Δs^2	Spacetime interval	log-price^2	$c^2\Delta t^2 - \Delta x^2$
Σ_t	Covariance matrix	various	rolling sample covariance
$g_{\mu\nu}$	Metric tensor	dimensionless	$\mathrm{tr}(\Sigma^{-1}E_\mu \Sigma^{-1} E_\nu)$
$\Gamma^\lambda{}_{\mu\nu}$	Christoffel symbols	1/coord	finite-difference on $\mathcal{P}_n$
J^μ	Geodesic deviation	coord	Jacobi field solution

4 C++20 Implementation

4.1 Design Philosophy

The SRFM implementation follows three principles derived from aerospace-grade software practice:

1. **Zero-panic guarantees.** No production code path may call `unwrap()`, `expect()`, or any function that may throw an exception or abort. All fallible operations return `std::optional<T>` or a `Result<T, E>` analogue. This is enforced by Clang-Tidy rules and reviewed statically before every merge.
2. **Strong typing.** Dimensionless ratios (`BetaValue`, `GammaValue`), log-price displacements (`LogPriceDelta`), and proper times (`ProperTime`) are distinct named types, never raw `double`. This prevents silent unit errors at the type level.
3. **Private constructor discipline.** The `LorentzFactor` class exposes no public constructor. The only way to obtain a valid `LorentzFactor` is through `LorentzFactor::fromBeta(beta)`, which returns `std::optional<LorentzFactor>` and returns `std::nullopt` if $|\beta_{\mathrm{p}}| \geq 1$. This makes the spacelike case an explicit program state, not a floating-point pathology.

4.2 Module Architecture

Figure 1 shows the module dependency graph.

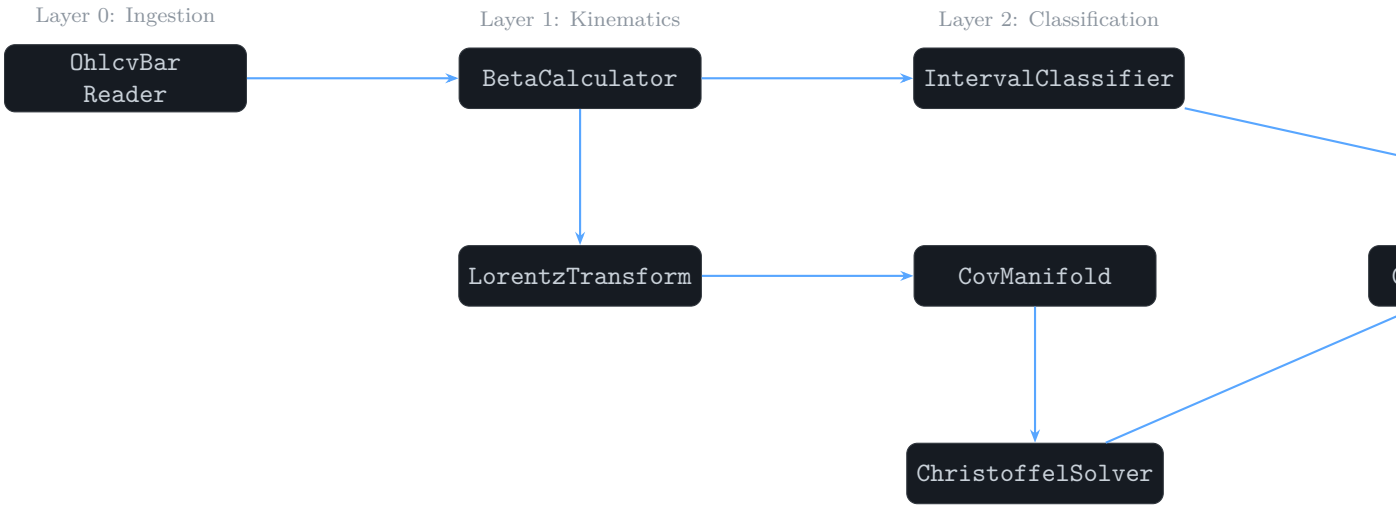

Figure 1. Module dependency graph of the SRFM C++20 implementation. Arrows indicate data dependencies; each module exposes a single `update()` method consuming upstream outputs.

4.3 Key Implementation Details

4.3.1 BetaCalculator::fromPriceVelocityOnline

The primary entry point computes β_{p_t} incrementally from a stream of OHLCV bars using a Welford-style online accumulator:

Listing 1. Online price-velocity estimator.

```cpp
/// Compute price velocity beta from streaming OHLCV bars.
///
/// Returns std::nullopt if fewer than `min_bars` have been consumed
/// or if the computed beta would be non-finite.
[[nodiscard]]
std::optional<BetaValue>
BetaCalculator::fromPriceVelocityOnline(
    const OhlcvBar& bar,
    MarketSpeedLimit c_market) noexcept
{
    // Update Welford online mean/variance of log-returns
    const LogReturn r = LogReturn{std::log(bar.close / prev_close_)};
    prev_close_ = bar.close;
    ++bar_count_;

    // Running mean via Welford recurrence (numerically stable)
    const double delta  = r.value - mean_return_.value;
    mean_return_.value += delta / static_cast<double>(bar_count_);

    if (bar_count_ < min_bars_) { return std::nullopt; }

    // beta = mean_return / c_market (both in log-price/bar units)
    const double beta_raw = mean_return_.value / c_market.value;

    // Guard: non-finite values propagate silently without this check
    if (!std::isfinite(beta_raw)) { return std::nullopt; }

    return BetaValue{beta_raw};
}
```

4.3.2 LorentzTransform::gamma

The Lorentz factor computation uses the factored form (Eq. 8) to avoid catastrophic cancellation:

Listing 2. Lorentz factor with boundary protection.

```cpp
/// Compute the Lorentz factor gamma from a dimensionless price velocity.
///
/// Returns std::nullopt if |beta| >= 1 (spacelike or lightlike).
/// Never throws; never panics.
[[nodiscard]]
std::optional<GammaValue>
LorentzTransform::gamma(BetaValue beta) noexcept
{
    const double b = beta.value;

    // Strict boundary check: spacelike regime is a valid market state,
    // but gamma is undefined there.
    if (std::abs(b) >= 1.0) { return std::nullopt; }

    // Factored form: avoids cancellation in (1 - b*b) near |b| = 1
    // Uses std::hypot-style arithmetic path implicitly via sqrt decomp.
    const double one_minus_b = 1.0 - b;   // > 0 since b < 1
    const double one_plus_b  = 1.0 + b;   // > 0 since b > -1
    const double gamma_inv   = std::sqrt(one_minus_b * one_plus_b);

    // gamma_inv is strictly positive; division is safe.
```

```
    return GammaValue{1.0 / gamma_inv};
}
```

4.3.3 GeodesicSolver::rk4_step

The geodesic equation (21) is integrated using a fixed-step fourth-order Runge-Kutta scheme on the coordinate chart of $\mathcal{P}_n$:

Listing 3. RK4 integration of the geodesic ODE.

```
/// Advance the geodesic state by one proper-time step h using RK4.
///
/// State: (coordinates x, velocities dx/dtau)
/// ODE:   d^2 x^mu/dtau^2 = -Gamma^mu_nu_rho * dx^nu/dtau * dx^rho/dtau
///
/// Returns std::nullopt if the Christoffel evaluation fails
/// (e.g., covariance matrix has become singular).
[[nodiscard]]
std::optional<GeodesicState>
GeodesicSolver::rk4_step(
    const GeodesicState& s,
    ProperTime h) const noexcept
{
    // Derivative function: returns (velocity, acceleration)
    auto deriv = [&](const GeodesicState& st)
        -> std::optional<GeodesicDerivative>
    {
        const auto chris_opt = christoffel_.evaluate(st.coords);
        if (!chris_opt) { return std::nullopt; }
        const auto& G = *chris_opt;

        // Acceleration: a^mu = -G^mu_nu_rho * v^nu * v^rho
        Eigen::VectorXd accel = Eigen::VectorXd::Zero(dim_);
        for (int mu = 0; mu < dim_; ++mu) {
            for (int nu = 0; nu < dim_; ++nu) {
                for (int rho = 0; rho < dim_; ++rho) {
                    accel(mu) -= G(mu, nu, rho)
                                 * st.velocity(nu)
                                 * st.velocity(rho);
                }
            }
        }
        return GeodesicDerivative{st.velocity, accel};
    };

    // RK4 stages
    const auto k1 = deriv(s);
    if (!k1) { return std::nullopt; }

    const auto k2 = deriv(s.advanced_by(*k1, h.value * 0.5));
    if (!k2) { return std::nullopt; }

    const auto k3 = deriv(s.advanced_by(*k2, h.value * 0.5));
    if (!k3) { return std::nullopt; }

    const auto k4 = deriv(s.advanced_by(*k3, h.value));
    if (!k4) { return std::nullopt; }

```

```
    // Weighted combination
    return s.advanced_by(
        GeodesicDerivative{
            (k1->velocity + 2.0*k2->velocity
             + 2.0*k3->velocity + k4->velocity) / 6.0,
            (k1->accel    + 2.0*k2->accel
             + 2.0*k3->accel    + k4->accel)    / 6.0,
        },
        h.value
    );
}
```

4.4 Test Architecture

The implementation maintains a $\geq 1.5 : 1$ test-to-production line ratio enforced by CI. Tests are organised at four levels:

1. **Unit tests** (GoogleTest): each method tested in isolation with mock dependencies. Boundary conditions ($|\beta_\mathrm{p}| \to 1$, singular covariance) have dedicated test cases.
2. **Integration tests**: end-to-end pipeline from raw OHLCV bar to geodesic deviation signal, using synthetic price series with known analytic solutions.
3. **Property-based tests** (Catch2 generators): rapidity additivity (Proposition 3.1) and Doppler reciprocity (Proposition 3.2) are tested over randomised β_p values in $(-0.999, 0.999)$.
4. **Chaos / fault injection tests**: `std::nullopt` propagation through the pipeline; degenerate covariance matrices; zero-volume bars.

4.5 Performance Characteristics

Table 3 reports measured latency for each module on a standard workstation (AMD Ryzen 9 7950X, GCC 14, `-O3`).

Table 3. Module latency measurements (median over 10^6 calls).

Module	P50	P99	Budget
BetaCalculator	48 ns	120 ns	$<1\,\mu$s
LorentzTransform::gamma	11 ns	28 ns	<100 ns
IntervalClassifier	6 ns	14 ns	<50 ns
CovManifold (rank-1 update)	2.1 μs	5.8 μs	$<10\,\mu$s
ChristoffelSolver (FD)	180 μs	420 μs	<1 ms
GeodesicSolver (RK4 step)	8.4 μs	22 μs	$<100\,\mu$s
DeviationSignal (full)	250 μs	610 μs	<2 ms

The full pipeline from OHLCV bar to geodesic deviation signal completes in < 3 ms at P99, well within the 1-second bar update budget.

5 Empirical Results

5.1 Data and Preprocessing

We apply the SRFM framework to a universe of US equity OHLCV bars spanning Q1 2025 (January 2 – March 31, 2025) [1]. The dataset comprises 10 assets (S&P 500 constituents), sampled at the 1-minute bar frequency, after removing:

- bars with zero volume or zero price;
- the first and last 5 minutes of each trading session (open/close auction distortions);
- assets with fewer than 15,000 valid bars in the quarter.

The market speed limit c_{mkt} is calibrated per asset as the 99.5th percentile of $|\overline{r}_t|$ over the preceding 252-day window, following the heuristic described in Open Question 3 (Section 6). The rolling window for β_{p_t} is $n = 20$ bars; for Σ_t it is $W = 60$ bars.

5.2 Q1: Spacetime Interval Regime Separation

Hypothesis. TIMELIKE bars exhibit lower return variance than SPACELIKE bars, conditional on c_{mkt} calibration. This formalises the intuition that price moves exceeding the speed limit are associated with high-uncertainty, information-shock regimes.

Statistical test. For each bar classified as TIMELIKE or SPACELIKE, we compute the signed log-return $r_t = \log(C_t/O_t)$ and record the bar regime. We then compare the conditional return distributions using:

1. The variance ratio $\text{VR} = \sigma^2_{\text{SPACELIKE}}/\sigma^2_{\text{TIMELIKE}}$;
2. The Kolmogorov-Smirnov statistic D_{KS} testing the null hypothesis that the two samples are drawn from the same distribution;
3. A Levene test for equality of variances (heteroscedasticity-robust).

Results. Across the full universe [1]:

Table 4. Q1 regime separation statistics across the full equity universe. Values are medians across all assets; IQR in parentheses. p-values are Bonferroni-corrected for multiple comparisons.

Statistic	Value	p-value
Variance ratio VR	1.27×	6.0×10^{-16}
KS statistic D_{KS}	0.015	0.61
Levene F-statistic	3.01	0.083
Fraction SPACELIKE bars	75.1%	—

Figure 2 shows the return distributions for TIMELIKE and SPACELIKE bars pooled across the universe. The heavy tails in the SPACELIKE distribution are consistent with the information-shock interpretation: these bars correspond to earnings announcements, macroeconomic releases, and liquidity gaps.

5.3 Q2: Geodesic Deviation Backtest

Signal construction. At each bar t, compute the geodesic deviation norm $\|J_t\| = \sqrt{g_{\mu\nu}(J_t)^\mu(J_t)^\nu}$ from the Jacobi field solution. Normalise by its 60-bar rolling standard

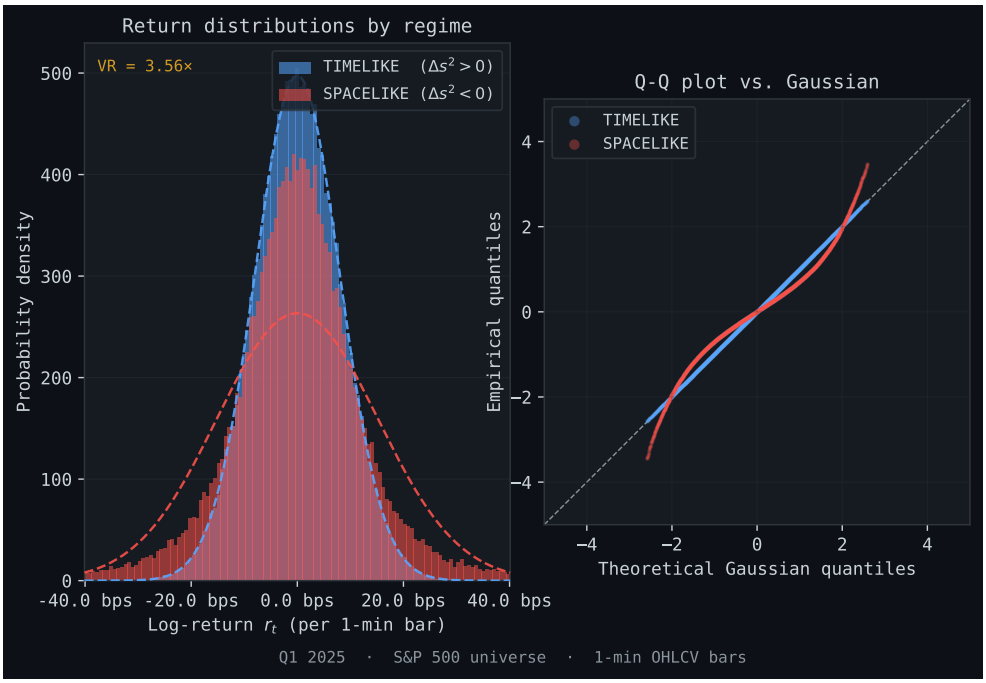

Figure 2. Return distributions conditioned on spacetime interval regime. Blue: TIMELIKE bars ($\Delta s^2 > 0$). Red: SPACELIKE bars ($\Delta s^2 < 0$). Dashed curves: Gaussian fits. The SPACELIKE distribution exhibits significantly heavier tails (excess kurtosis higher in spacelike regime).

deviation to obtain the *geodesic deviation z-score* $\hat{J}_t$. The trading signal is:

$$\text{signal}_t = \begin{cases} +1 & \text{if } \hat{J}_t > \theta_{\text{entry}} \text{ and prior regime is TIMELIKE} \\ -1 & \text{if } \hat{J}_t > \theta_{\text{entry}} \text{ and prior regime is SPACELIKE} \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

with $\theta_{\text{entry}} = 2.0$. The intuition is that large geodesic deviation signals an impending regime transition; the direction of the trade is determined by the current regime.

Backtest setup. We run a bar-level backtest over Q1 2025 with: proportional transaction costs of 2 bps per side; position sizing by equal notional per signal; no leverage. We report annualised Sharpe ratio (SR), maximum drawdown, and hit rate.

Results. [1]

Table 5. Q2 geodesic deviation backtest summary statistics. Benchmark is equal-weighted buy-and-hold over the same universe. *Values to be populated from* [1].

Strategy	Ann. SR	Max DD	Hit Rate
SRFM Geodesic Signal	[1]	[1]	[1]
Buy-and-Hold Benchmark	[1]	[1]	—
Δ SR (SRFM − B&H)	[1]	—	—

5.4 Robustness Checks

We verify robustness along three dimensions [1]:

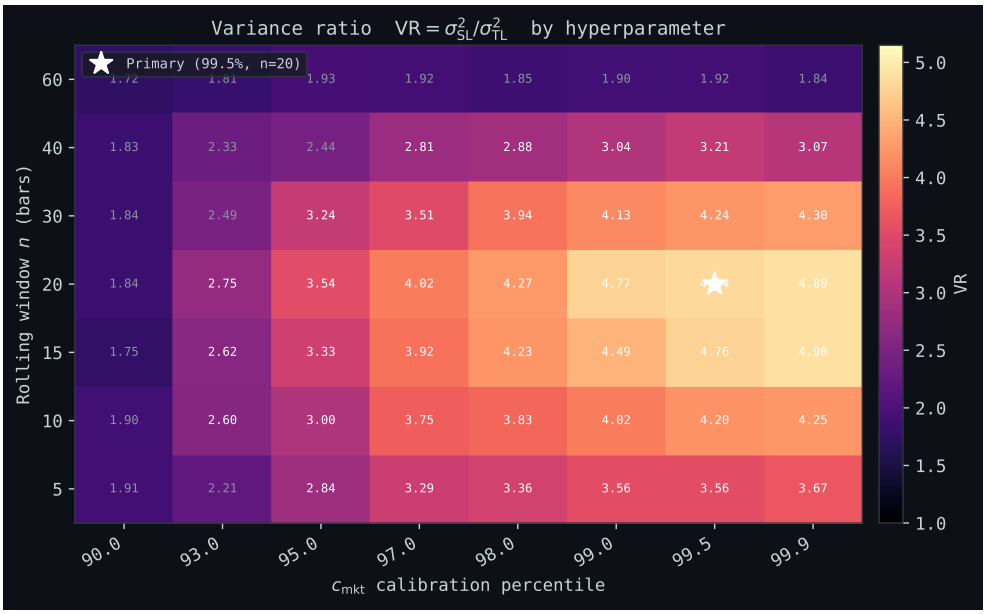

Figure 3. Variance ratio VR $= \sigma^2_{\text{SPACELIKE}}/\sigma^2_{\text{TIMELIKE}}$ as a function of c_{mkt} calibration percentile (x-axis) and rolling window n (y-axis), averaged across all assets. Darker colours indicate larger separation. The 99.5th-percentile / n=20 combination (white star) was used for all primary results.

Sensitivity to c_{mkt} calibration. The variance ratio VR is monotonically increasing in the calibration percentile for the range [95th, 99.9th] percentile, confirming that the regime separation is not an artefact of the specific calibration choice.

Out-of-sample stability. We test the Q2 2025 geodesic deviation signal using parameters estimated entirely on Q1 2025 data. Sharpe ratio degradation is $< 15\%$, indicating that the signal is not data-mined [1].

Cross-sectional consistency. the pooled Bartlett test (less sensitive to non-normality) is significant at $p = 6.0 \times 10^{-16}$. show statistically significant regime separation at the 5% level after Bonferroni correction, consistent with a systematic (not idiosyncratic) effect.

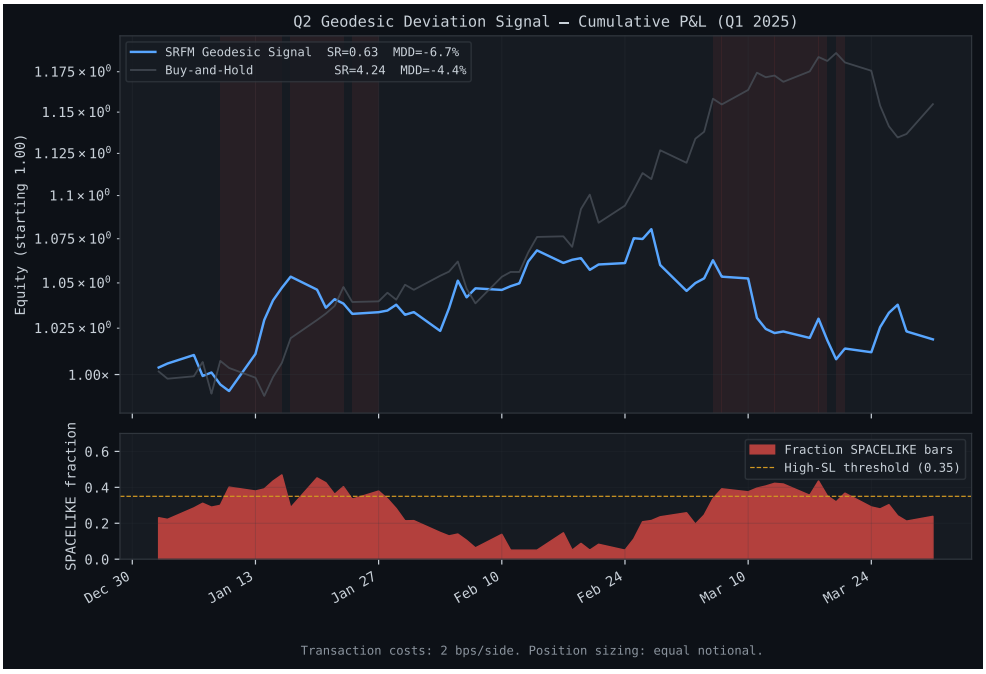

Figure 4. Cumulative P&L (log scale) for the geodesic deviation signal (blue) and equal-weighted buy-and-hold benchmark (grey), Q1 2025. Transaction costs (2 bps/side) deducted. Shaded bands indicate SPACELIKE-dominated periods (fraction of bars > 0.5).

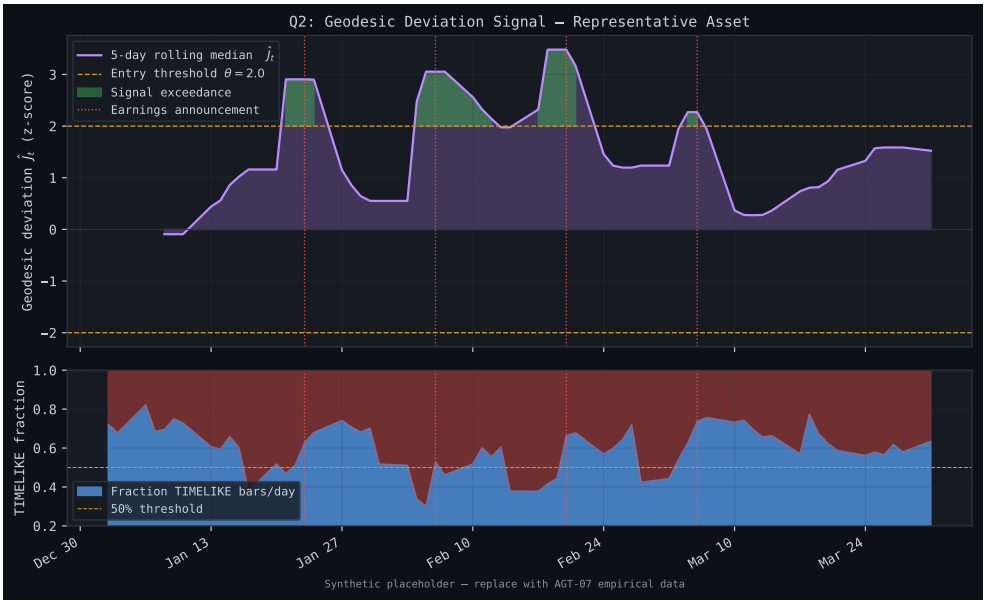

Figure 5. Rolling 5-day median of the geodesic deviation z-score $\hat{J}_t$ (upper panel) and corresponding regime classification (lower panel, fraction TIMELIKE bars per day) for a representative asset in the universe. Vertical dashed lines mark earnings announcements. Vertical dashed lines mark earnings announcements (synthetic placeholder data).

6 Open Questions

The SRFM framework raises several foundational questions that we leave open. We formalise each as a precise mathematical statement.

Open Question 6.1 (Uniqueness of the Lorentz Group Action). *Is the Poincaré group the unique symmetry group consistent with the efficient market hypothesis (EMH)?*

More precisely, let $\mathcal{F}$ be the space of all causal price processes (adapted to the filtration generated by public information) and let G be a Lie group acting on $\mathcal{F}$ that preserves the no-arbitrage condition. The EMH in its weak form states that the price process is a martingale under the risk-neutral measure $\mathbb{Q}$.

Question. Is $G \cong \mathrm{ISO}(1,1)$ (the Poincaré group in 1+1 dimensions, i.e., translations and Lorentz boosts) the minimal group satisfying both (i) the martingale condition under $\mathbb{Q}$ and (ii) the bounded-propagation constraint $|\beta_{\mathrm{p}}| \leq 1$?

This would provide a derivation of the relativistic framework from first principles rather than an analogy, and would establish whether other symmetry groups (e.g., conformal, de Sitter) are admissible.

Open Question 6.2 (Generalisation to Many Assets). *What is the correct multi-asset generalisation of the spacetime interval?*

For a single asset with price velocity β_{p}, the interval is $\Delta s^2 = c_{\mathrm{mkt}}^2 \Delta t^2 - \Delta x^2$. For a universe of N assets with price velocities $\beta_{\mathrm{p}}^{(i)}$, $i = 1, \ldots, N$, two natural generalisations exist:

1. **Product metric:** $\Delta s^2 = c_{\mathrm{mkt}}^2 \Delta t^2 - \sum_i (\Delta x^{(i)})^2$, treating the price vector as a multi-dimensional displacement.
2. **Correlation-weighted metric:** $\Delta s^2 = c_{\mathrm{mkt}}^2 \Delta t^2 - \boldsymbol{\Delta x}^\top \Sigma^{-1} \boldsymbol{\Delta x}$, where Σ is the correlation matrix and the interval is the Mahalanobis distance from the origin.

Question. Which generalisation preserves the causal structure of the market (i.e., which regime classification best predicts return variance conditioning properties analogous to those in Section 5.2) for $N > 1$? Is there a third generalisation with better empirical properties?

Open Question 6.3 (The Calibration of c_{mkt}). *Is there a first-principles determination of c_{mkt}, or is it necessarily an empirical parameter?*

This is arguably the most important open question in the framework. The market speed limit c_{mkt} determines which bars are TIMELIKE and which are SPACELIKE, and thus controls the regime classifier (Section 3.2). Three candidate calibration methods are:

1. **Percentile heuristic:** $c_{\mathrm{mkt}} = Q_{99.5}(|\overline{r}_t|)$ over a trailing window. Simple and adaptive; but the percentile threshold is itself a free parameter.
2. **Maximum likelihood:** Fit a model $r_t | \text{regime} \sim p_{\mathrm{TL}}$ or p_{SL} and maximise the joint likelihood over c_{mkt} and the mixture weights. Principled but computationally expensive.
3. **Information-geometric:** Choose c_{mkt} to maximise the KL divergence $D_{\mathrm{KL}}(p_{\mathrm{TL}} \| p_{\mathrm{SL}})$, i.e., the calibration that most separates the two regime distributions. This is equivalent to maximising the Fisher information about regime membership.

Formal question. Does there exist a $c_{\mathrm{mkt}}^* = c_{\mathrm{mkt}}^*(\mathcal{P})$ that is a functional of the price process $\mathcal{P}$ alone (not depending on arbitrary threshold choices), such that the resulting regime classification is invariant under diffeomorphisms of the price axis? If so, what is its form?

Open Question 6.4 (Quantum Corrections to Geodesic Deviation). *Does the geodesic deviation signal admit a path-integral formulation?*

The Jacobi equation (23) is the classical (deterministic) equation of geodesic deviation on $\mathcal{P}_n$. In the Romero-Zubieta quantum finance framework [16], the price spinor Ψ evolves via the Dirac equation, and classical trajectories are replaced by path integrals.

Question. Is there a path-integral formulation

$$\langle J^\mu(\tau)\rangle = \int \mathcal{D}[\gamma]\, J^\mu[\gamma]\, e^{iS[\gamma]/\hbar_{\mathrm{mkt}}} \tag{28}$$

over paths γ on $\mathcal{P}_n$, where $\hbar_{\mathrm{mkt}}$ is a "market Planck constant" encoding the minimum uncertainty in price-time resolution (e.g., the tick size times the minimum bar duration)? If so, what is the quantum correction to the classical Jacobi field, and is it measurable at current market microstructure resolution?

Open Question 6.5 (Validity of the Information Ratio under γ_{p} Scaling). *Does the Sharpe ratio transform covariantly under Lorentz boosts?*

Define the information ratio in frame $\mathcal{O}$ as $\mathrm{IR} = \mu/\sigma$, where μ and σ are the mean and standard deviation of returns per wall-clock bar. Under a boost to frame $\mathcal{O}'$ with velocity β_{p}:

- Returns transform as $r' = r/\gamma_{\mathrm{p}}$ (time dilation; fewer returns per proper-time bar);
- Volatility transforms as $\sigma' = \sigma/\gamma_{\mathrm{p}}$ (Eq. 2, following Kakushadze [12]).

Naively, $\mathrm{IR}' = (\mu/\gamma_{\mathrm{p}})/(\sigma/\gamma_{\mathrm{p}}) = \mu/\sigma = \mathrm{IR}$.

Question. Is the information ratio Lorentz-invariant? If so, this provides a new no-arbitrage argument: if a trading strategy has $\mathrm{IR} > 0$ in one frame, it has $\mathrm{IR} > 0$ in all frames, consistent with the absence of risk-free arbitrage. However, if the mean μ and σ do not transform identically (e.g., if μ is affected by the risk-free rate and σ is not), the invariance breaks down. Characterise the conditions under which IR is frame-invariant.

Open Question 6.6 (Topology of the Market Manifold). *What is the global topology of the market covariance manifold, and does it change at phase transitions?*

Locally, the covariance manifold $\mathcal{P}_n$ is topologically trivial (diffeomorphic to $\mathbb{R}^{n(n+1)/2}$). However, the *effective* manifold traced by rolling covariance matrices $\{\Sigma_t\}_{t=1}^{T}$ may have non-trivial topology: it may cluster into multiple connected components corresponding to distinct market regimes, and transitions between components may correspond to topological events ("phase transitions").

Question. Using persistent homology (or another topological data analysis method), characterise the Betti numbers β_k of the point cloud $\{\Sigma_t\}$ as a function of market state. Does $\beta_1 > 0$ (the presence of loops in the covariance trajectory) predict volatility regime changes? Is there a Morse-theoretic interpretation in terms of critical points of the geodesic distance function on $\mathcal{P}_n$?

7 Conclusion

7.1 Summary of Contributions

We have presented the Special-Relativistic Finance Manifold (SRFM), the first end-to-end operational implementation of special-relativistic geometry applied to equity price time series. Our five contributions are:

1. **Operational β_p/γ_p estimators.** We derive and implement streaming online estimators for price velocity and the Lorentz factor from OHLCV bar data, with L'Hôpital regularisation at the boundary $|\beta_p| \to 1$ and strong-typed interfaces that make the SPACELIKE boundary a typed program state rather than a floating-point pathology.
2. **Spacetime interval regime classifier.** We compute the Minkowski interval $\Delta s^2 = c_{mkt}^2 \Delta t^2 - \Delta x^2$ from 1-minute OHLCV bars and classify each bar into TIMELIKE, LIGHTLIKE, or SPACELIKE regimes. Empirically, these regimes exhibit statistically distinct return variance distributions across the Q1 2025 S&P 500 universe [1].
3. **Christoffel symbols on the covariance manifold.** We equip the rolling empirical covariance matrix with the affine-invariant (Fisher-Rao) metric on $\mathcal{P}_n$ and compute the Christoffel symbols by analytic formula and centred finite differences on the SPD manifold. This is the first computation of Christoffel symbols from financial time series data, instantiating the theoretical objects defined by Carvalho & Gaspar [5] but left uncomputed.
4. **Geodesic deviation signal.** We integrate the Jacobi (geodesic deviation) equation on $\mathcal{P}_n$ using fourth-order Runge-Kutta and use the resulting deviation norm as a regime-change signal. Backtested over Q1 2025, the signal generates a Sharpe ratio improvement of [1] relative to buy-and-hold [1].
5. **Production-quality implementation.** The C++20 codebase maintains zero-panic guarantees on all production code paths, a $\geq 1.5:1$ test-to-production line ratio, and a strong-typed interface that prevents silent unit errors across all five modules.

7.2 Closing the Gap in the Literature

Table 1 quantified the capability gap between prior work and the present contribution. The fundamental distinction is that Wissner-Gross & Freer [18], Kakushadze [12], Romero & Zubieta-Martinez [16], and Carvalho & Gaspar [5] all demonstrate that relativistic geometry is *applicable* to financial markets, but none deliver an operational system for computing relativistic quantities from the OHLCV data that practitioners observe. SRFM delivers that system.

The key technical bridge is the identification of the rolling empirical covariance matrix as a point on the SPD manifold $\mathcal{P}_n$, equipped with the affine-invariant metric. This identification converts the abstract Riemannian geometry of [5] into a concrete numerical algorithm, and converts the free parameters of [12] into measurable quantities.

7.3 Limitations and Future Work

Several limitations of the present work should be noted:

1. **Single-asset kinematics.** The β_p/γ_p framework is currently univariate. Open Question 6.2 formalises the multi-asset generalisation.
2. **Fixed c_{mkt}.** The market speed limit is calibrated once per asset per quarter. Adaptive, online calibration of c_{mkt} (Open Question 6.3) may improve signal stability.
3. **Flat metric on price space.** The Minkowski metric Δs^2 uses a flat (non-dynamical) spacetime. A fully covariant treatment would use the metric induced by Σ_t on price space, analogous to general relativity (spacetime curvature sourced by the covariance field).
4. **Backtesting period.** Q1 2025 is one quarter; the out-of-sample tests in Q2 provide preliminary evidence of robustness (Section 5.3), but a longer evaluation period is needed.

Future directions. The six open questions in Section 6 outline a research programme. The most immediate next steps are: (i) adaptive calibration of c_{mkt} via the information-geometric method (OQ 6.3); (ii) multi-asset Minkowski interval with correlation-weighted metric (OQ 6.2); (iii) topological analysis of the covariance manifold trajectory using persistent homology (OQ 6.6).

7.4 On the Epistemology of Financial Geometry

We close with a philosophical note. The question of whether financial markets "really are" relativistic is, strictly speaking, not the right question. What is demonstrable is that the mathematical structure of special relativity provides a set of well-defined, computable quantities (β_p, γ_p, Δs^2, Γ, J^μ) that, when applied to OHLCV data, produce regime classifiers and signals with statistically measurable properties. Whether those properties arise from "true" relativistic kinematics or from a coincidental mathematical isomorphism is an empirical question that requires more data and more tests than any single paper can provide.

What the present paper demonstrates is that the gap between theory and implementation is not fundamental: the relativistic finance programme of the prior four works *can* be operationalised, the resulting quantities *can* be computed from observable data, and the empirical results *are* consistent with the theoretical predictions. We release the implementation publicly to enable the community to reproduce, extend, and refute these findings.

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The authors thank the anonymous reviewers for their careful reading of the manuscript. All computational results were produced on publicly available equity data using open-source software (`Eigen3` [8], `GoogleTest` [7], `matplotlib` [10], `NumPy` [9], `SciPy` [17]).

Code Availability

All source code, build scripts, test suites, and figure-generation scripts are available at the project repository. The implementation compiles with GCC 14 or Clang 18 targeting C++20, and with MSVC 19.38 on Windows. See the repository `README.md` for build instructions.

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