Politecnico di Milano

Final Project

Introduction to Space Mission Analysis

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1. Introduction

The mission behind this report was to design, describe and optimize an orbit transfer between two given positions of a generic satellite.
The initial condition has been assigned as two vector which describe position and velocity, while the final condition has been given as the six orbital parameters.
In order to accomplish the objective, there are an incredible amount of different strategies, our Team has considered a restricted group of them.

Each manoeuvre explained in the following pages can be execute in many ways. During the optimization phase our team finds the best values of the variables in order to obtain the best performance form each transfer technique.
The optimization has been executed considering three different parameters:

- Time duration: the time need to transfer our satellite form the initial to final position
- Cost: the total velocity variation that must be applied to the satellite during the manoeuvre
- The trade-off between time duration and cost

The trade-off value has been calculated with the product between time duration and cost:

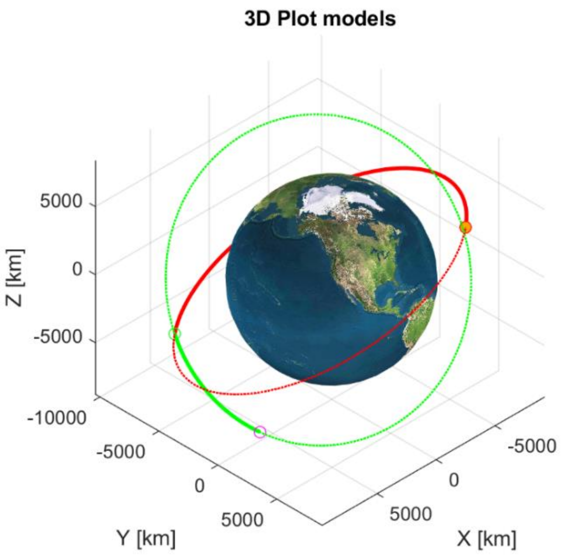
$$Tradeoff = \Delta v_{tot} \cdot \Delta T_{tot}$$

All the strategies presented in this report has been computed and plotted using MATLAB R2021a.
The entire work has been executed considering a couple of simplifying assumptions:

- All the analysis are based on the Keplerian model
- All the manoeuvres are assumed impulsive

The Team has used a couple of conventions to allow a better comprehension of the plots reported in the next pages:

- Each time the satellite change orbit the colour of its path changes
- The dashed parts of the orbits are those not covered by the satellite during its path
- The empty circular marker represents the starting point of the satellite in that orbit



2. Initial orbit characterisation

2.1. Initial condition

The initial conditions are defined by the following couple of vectors:

$$\vec{R} = \begin{Bmatrix} r_x \\ r_y \\ r_z \end{Bmatrix} = \begin{Bmatrix} -5183.4184 \\ 6189.4459 \\ 4334.1737 \end{Bmatrix} km \qquad \vec{V} = \begin{Bmatrix} v_x \\ v_y \\ v_z \end{Bmatrix} = \begin{Bmatrix} -5.3130 \\ -4.3350 \\ 0.2136 \end{Bmatrix} \frac{km}{s}$$

Using our MATLAB function “RV2ParObr.m” the corresponding orbital parameters that defines the initial position are:

Symbol	Parameter	Value
a	Semi-major Axis	9981.0950 [km]
e	Eccentricity	0.0859
i	Inclination	0.4931 [rad]
Ω	Right ascension of the ascendent node	0.7423 [rad]
ω	Argument of periapsis	1.1977 [rad]
θ	True anomaly	0.3332 [rad]

Table 2.1 – Initial orbit parameters

2.2. Other relevant orbit data

From the orbital parameters our Team has calculated other relevant parameters that has helped us to define some interesting characteristics of the orbit.

$\boldsymbol{p}$[km]	$\boldsymbol{r_p}$[km]	$\boldsymbol{r_a}$[km]	$\boldsymbol{E}\left[\mathrm{km}^2/_{s^2}\right]$	$\boldsymbol{T}$[sec]
9907.3557	9123.1908	10838.9993	-19.9678	9923.8062

Table 2.2 – Other relevant parameters

The initial orbit is a very low MEO, prograde and nearly circular due to a very low eccentricity. This last characteristic will be relevant in some choice that our Team has taken during the manoeuvre design.

2.3. Graphic representation

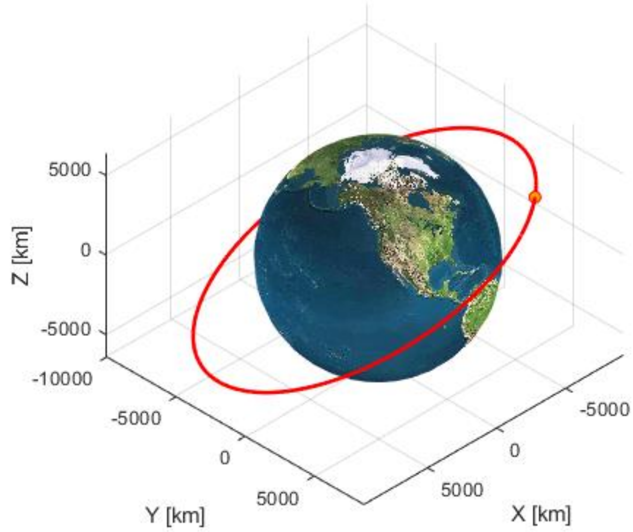


Figure 2.1 - Initial orbit plot

3. Final orbit characterisation

3.1. Final condition

The final conditions are defined by the following orbit parameters:

Symbol	Parameter	Value
a	Semi-major Axis	12610.0 [km]
e	Eccentricity	0.2656
i	Inclination	1.0430 [rad]
Ω	Right ascension of the ascendent node	1.8790 [rad]
ω	Argument of periapsis	2.4080 [rad]
θ	True anomaly	2.3880 [rad]

Table 3.1 – Final orbit parameters

Using our MATLAB function "ParObr2RV.m" the corresponding vectors those defines the final position are:

$$\vec{R} = \begin{Bmatrix} r_x \\ r_y \\ r_z \end{Bmatrix} = \begin{Bmatrix} 6583.1197 \\ 3369.6911 \\ -12513.8989 \end{Bmatrix} km \qquad \vec{V} = \begin{Bmatrix} v_x \\ v_y \\ v_z \end{Bmatrix} = \begin{Bmatrix} -1.1298 \\ 4.6506 \\ -0.5731 \end{Bmatrix} \frac{km}{s}$$

3.2. Other relevant orbit data

From the orbital parameters our Team has calculated other relevant parameters that has helped us to define some interesting characteristics of the orbit.

$\boldsymbol{p}$[km]	$\boldsymbol{r_p}$[km]	$\boldsymbol{r_a}$[km]	$\boldsymbol{E}\left[{}^{\mathbf{km^2}}/_{\boldsymbol{s^2}}\right]$	$\boldsymbol{T}$[sec]
11720.4482	9260.7840	15959.2160	-15.8049	14092.3563

Table 3.2 – Other relevant parameters

The final orbit is MEO and prograde.

3.3. Graphic representation

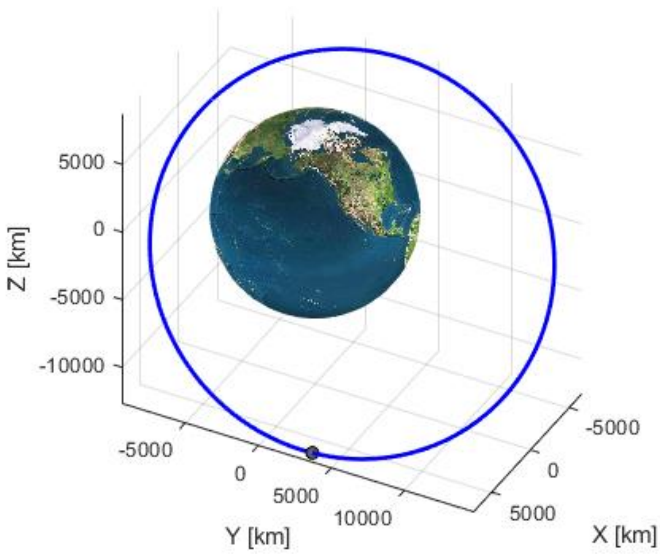


Figure 3.3.1 – Final orbit plot

4. Transfer trajectory definition and analysis

4.1. PAS Manoeuvre

The final position and velocity are achieved thanks to a three steps manoeuvre:

1. Change of plane that brings the satellite on an orbit with final inclination (i) and RAAN (Ω)
2. Change of periapsis anomaly (ω) through a single impulse
3. Change of shape by a bitangent transfer that takes satellite on the final orbit (a, e)

Those three steps can be executed in many ways depending on the points in which the impulses are applied to the satellite or the phase displacement of periapsis anomaly.
All the possible cases are defined through four different Boolean variables which are:

- Change of plane point
- Change of periapsis anomaly point
- Displacement of periapsis anomaly
- Type[1] of bitangent

Our team has optimized this manoeuvre by finding the best combination of these Boolean variables for each optimization parameter considered in this report (time duration, cost and trade-off).
The optimization has been executed in MATLAB by computing all the 16 linear combinations of the Boolean variables (fig 4.1)

The optimized performance of the PAS manoeuvre has been summarised in the table 4.1.1

Optimized for	dT $[sec]$	dv $[km/s]$
Time duration	17746.5452	6.6372
Cost	31729.9277	6.6350
Trade-off	17746.5452	6.6372

Table 4.1 - PAS optimization data

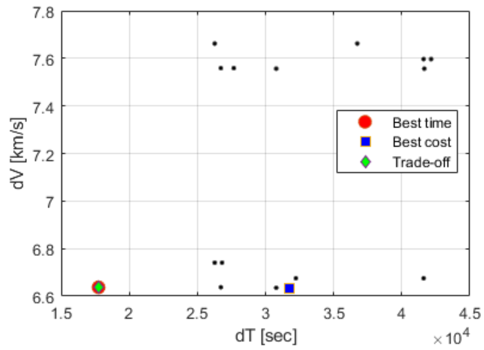


Figure 4.1.1 - PAS optimization plot

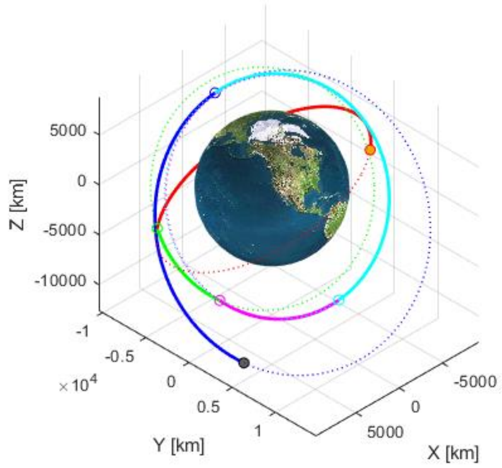


Figure 4.1.4.2 - PAS optimized on time duration

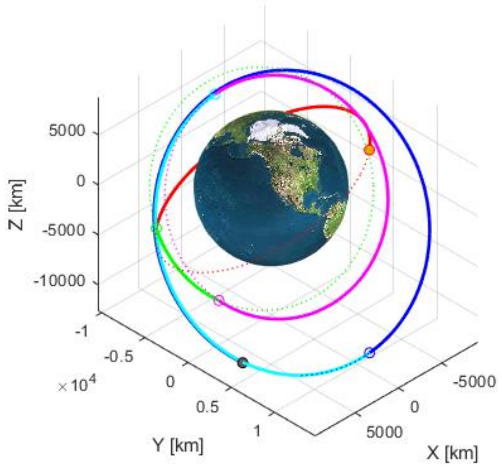


Figure 4.1.4.3 - PAS optimized on cost

[1] The bitangent transfer in the apsidal point has four different paths: (peri → peri, apo → apo, peri → apo, apo → peri)

4.2. PCS Manoeuvre

The final position and velocity are achieved thanks to a three steps manoeuvre:

1. Change of plane that brings the satellite on an orbit with final inclination (i) and RAAN (Ω)
2. Change of periapsis anomaly (ω) through a circular transfer orbit
3. Change of shape by a bitangent transfer that takes satellite on the final orbit (a, e)

Those three steps, as it was in PAS manoeuvre, can be executed in different ways.
All the possible cases are defined through four different Boolean variables which are:

- Change of plane point
- Circularization point
- Displacement of periapsis anomaly
- Type of bitangent

Our goal was to find an alternative technique which has better performance than the 4.1 one. The Team has thought that the very low eccentricity of the initial could be used to obtain a circular transfer orbit with low effort in term of cost (Δv).

The optimization has been executed with the same procedure used to optimize the PAS manoeuvre. The optimized performance of the PCS manoeuvre has been summarised in the table 4.2.1:

Optimized for	$dT\ [sec]$	$dv\ [km/s]$
Time duration	24282.6823	6.7549
Cost	40012.6287	6.2219
Trade-off	25536.5020	6.2476

Table 4.2 - PCS optimization data

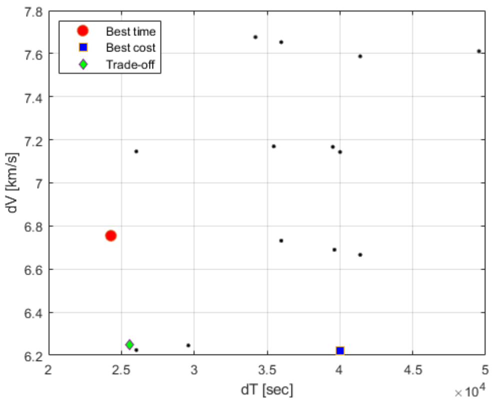


Figure 4.2.1 - PCS optimization plot

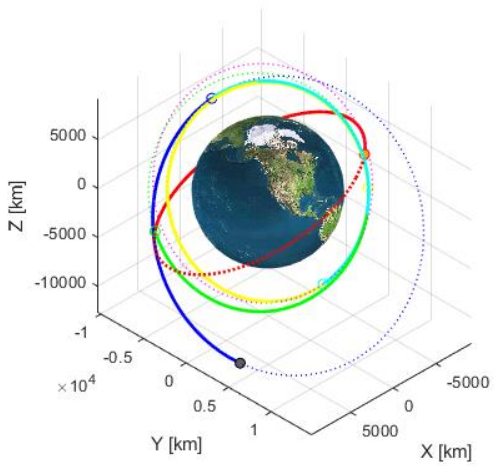


Figure 4.4.2 - PCS optimized on time duration

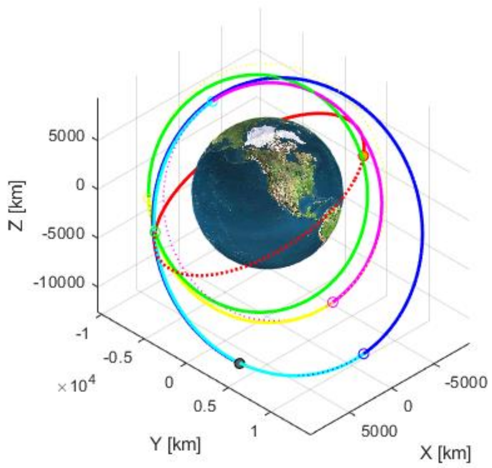


Figure 4.2.3 - PCS optimized on cost

4.3. SPA and SPC Manoeuvre

To find other manoeuvres with better performance our Team thought that it would be interesting to analyse what happens when the order of the steps that characterize PAS and PCS transfer changes.
With this idea we have designed SPA and SPC manoeuvres. The difference between PAS and PCS is that, instead of starting with the change of plane, the first step is to transform the orbit shape into the final one with a bielliptic transfer path.

The three steps of these two alternative techniques are:

SPA manoeuvre:
1. Change of shape (a, e)
2. Change of plane (i)
3. Change of "ω" (single impulse)

SPC manoeuvre:
1. Change of shape (a, e)
2. Change of plane (i)
3. Change of "ω" (circularization)

For each manoeuvre all the possible cases are defined through three different Boolean variables:

SPA manoeuvre:
- Change of plane point
- Change of "ω" point
- Type of bitangent

SPC manoeuvre:
- Change of plane point
- Circularization point
- Type of bitangent

The optimized performance of these two manoeuvres has been summarised in the tables 4.3.1 and 4.3.2:

Optimized for	dT $[sec]$	dv $[km/s]$
Time duration	16791.4343	9.7006
Cost	20815.8865	7.0802
Trade-off	20815.8865	7.0802

Table 4.3.1 - SPA optimization data

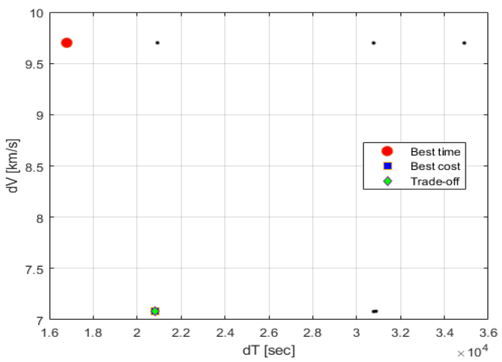


Figure 4.3.1 - SPA optimization plot

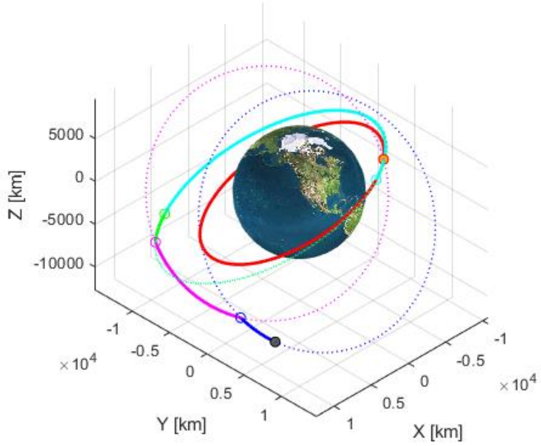


Figure 4.3.3 - SPA optimized on Trade-off

Optimized for	dT $[sec]$	dv $[km/s]$
Time duration	29634.0096	6.0325
Cost	45882.7810	5.8221
Trade-off	29634.0096	6.0325

Table 4.3.2 - SPC optimization data

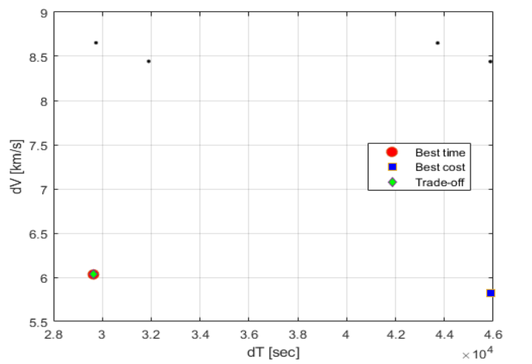


Figure 4.3.2 - SPC optimization plot

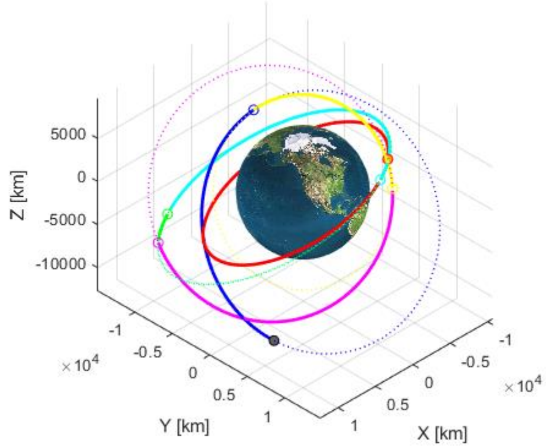


Figure 4.3.4 - SPC optimized on Trade-off

4.4. CB Manoeuvre

The following strategy has been ideated to reduce the total cost by executing the change of plane in the apoapsis of an orbit arbitrarily[2] created (the ray of this apoapsis is a variable).
Unfortunately, it tuns out that even with the best optimization possible the cost of this manoeuvre is higher than the SPC one.

The final position and velocity are achieved thanks to a five steps manoeuvre:

1. Circularization of the initial orbit
2. Single tangent burn to apoapsis of the orbit arbitrarily created
3. Change of plane that brings the satellite on an orbit with final inclination (i) and RAAN (Ω)
4. Change of shape by a bitangent transfer that takes satellite on the final orbit (a, e)
5. Change of periapsis anomaly (ω) through a circular transfer orbit

All the possible cases are defined through the apoapsis ray of the step 2 and three different Boolean variables which are:

- Apocenter direction
- Change of periapsis anomaly point
- Type of bitangent (arrive in apoapsis or periapsis of the final orbit)

The optimization has been executed in MATLAB by computing all the 8 linear combinations of the Boolean variables with a vector of different apoapsis ray[3] (step 2). This vector has been created with an iterative procedure in ordered to report in the plot all the best performance cases.
The optimized performance of the CB manoeuvre has been summarised in the table 4.4.1

Optimized for	$dT\ [sec]$	$dv\ [km/s]$
Time duration	28635.8135	8.8887
Cost	52689.9045	6.9252
Trade-off	35695.1268	6.9667

$R_{apoapsis}$ **optimized for**	$R_a\ [km]$
Time duration	20000
Cost	34000
Trade-off	20000

Table 4.4.1 – CB Optimization data

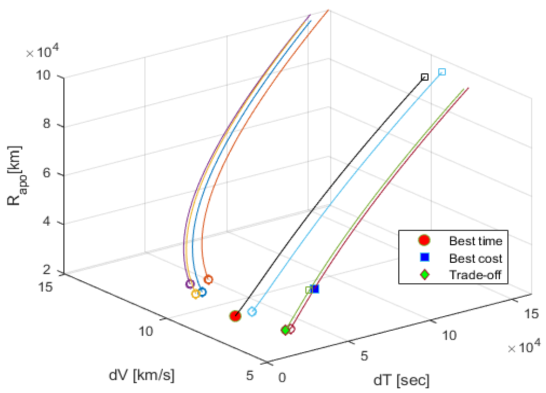


Figure 4.4.1 - CB optimization plot

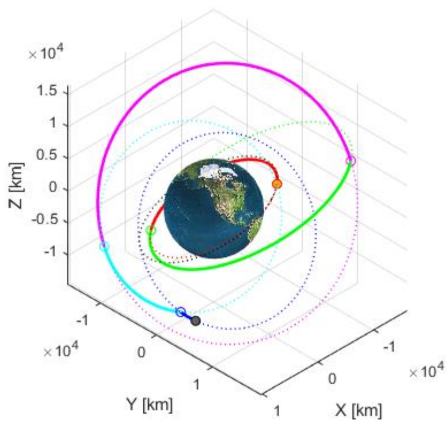


Figure 4.4.2 – CB optimized on time duration

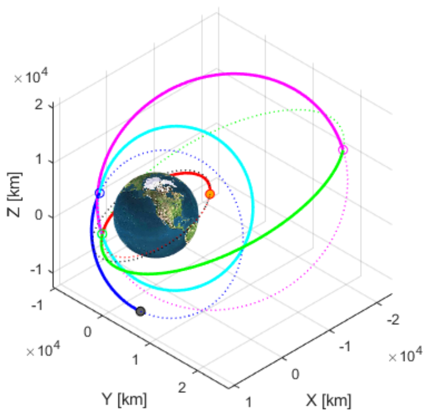


Figure 4.4.3 – CB optimized on cost

[2] The only constraint is that the apoapsis must be chosen on the intersection line between the planes of initial and final orbit
[3] The value indicated as “apoapsis ray” is the distance between the center of the Earth and the point where the change of plane manoeuvre will take place

4.5. Direct Manoeuvre

This strategy has been ideated to find the best transfer orbit in terms of time duration. Since in any case this manoeuvre would be the best in terms of time duration, our team has decided to optimize it in terms of Δv. The satellite's path consists in a direct transfer orbit between the initial and the final position, so it is just composed by two impulses: insertion in the transfer orbit and insertion in the final orbit.

The transfer orbit is characterized by its own unique orbital parameters which has been found with an optimization based on an iterative algorithm as it follows:

1. Identifying the unique inclination and RAAN of the direct transfer orbit
2. Analysing all the possible transfer orbit between the initial and final point by permuting the argument of periapsis with the following restrictions
    - The orbit needs to be closed ($e < 1$)
    - The orbit must not intersect Earth (plus its atmosphere which is considered up to 100km above the sea level)

Two additional notes regarding the transfer orbit

I. After identifying the orbital plane (1.) the path of the transfer orbit can be travelled along two different verses. Both cases are defined through a Boolean variable (the sign of the orbital plane's normal versor), so at the end, we obtain two different optimizations for the two different travel directions.

II. In those cases where the eccentricity is negative (blue region), the eccentricity vector points towards the apoapsis. Thus, this orbit is equal to an orbit with the same eccentricity, but positive, with ω and θ rotated of 180°. Anyway, we have already analysed this class of cases through the Boolean variable explained in I.

Optimized for	$dT\ [sec]$	$dv\ [km/s]$
Time duration	1803.9996	21.4947
Cost	9668.6036	7.1734
Trade-off	1803.9996	21.4947

Table 4.5.1 – DIRECT Optimization data

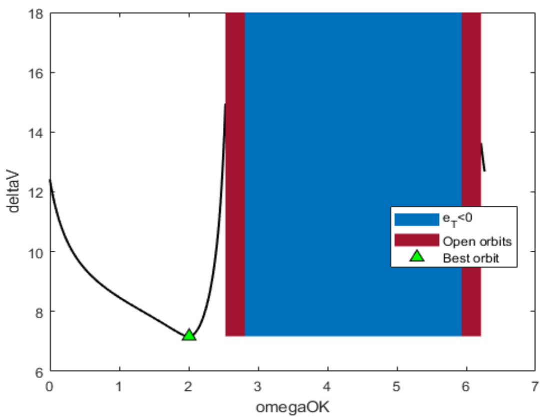


Figure 4.5.1 - DIRECT optimization plot

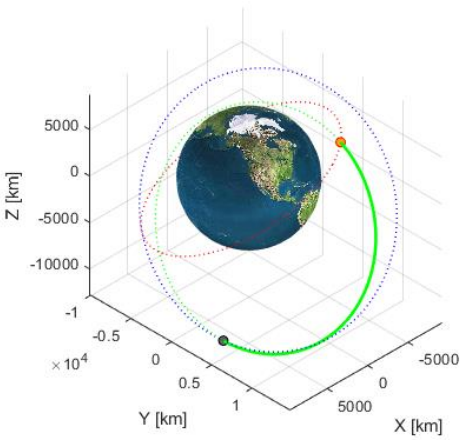


Figure 4.5.3 – DIRECT optimized on time duration

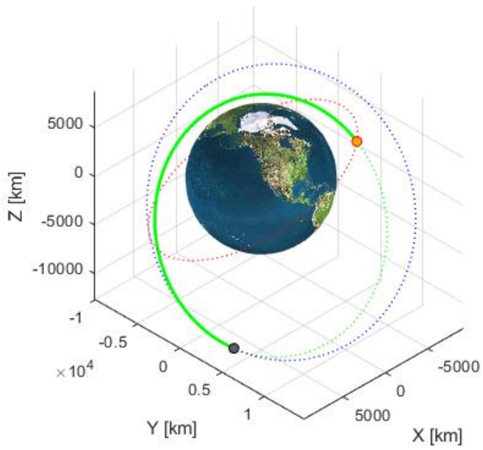


Figure 4.5.4 – DIRECT optimized on cost

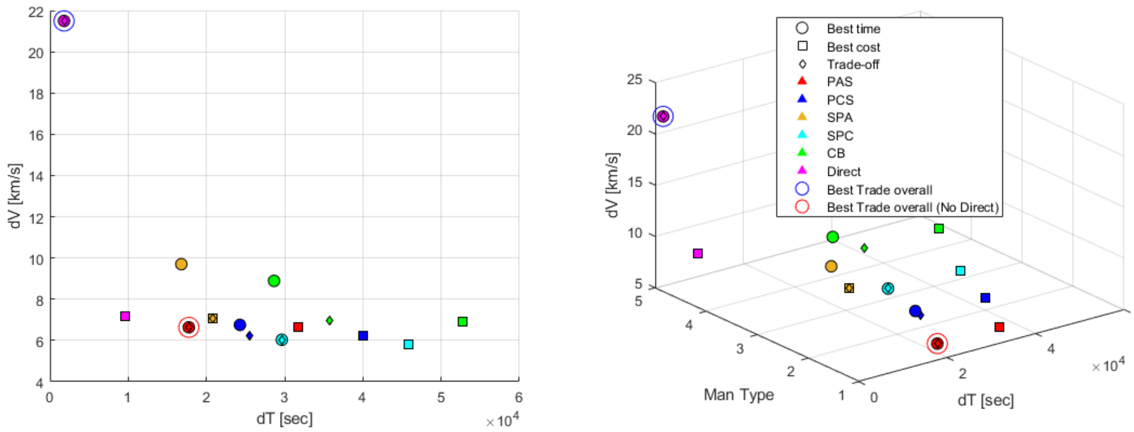
5. Conclusions

Now, considering all the data obtained from the different strategies, the team has optimized it through an iterative process and by doing so, the following conclusions has been made:

A simple bielliptic change of shape maneuverer was not worth to be considered since the ratio of semi major axes of the initial and final orbits is less than 11.94 (which means that an Hohmann transfer is better).

We noticed that the most expensive manoeuvre in terms of Δv was the change of plane because of the big difference between the initial and final inclination of the orbit. It could be seen in PAS and PCS, that the change of plane is expensive if it is done "near" the earth. In the CB manoeuvre we tried to do the change of plane as far away as we could always considering the fact that the manoeuvre which brings the satellite away from earth has also a cost.

It can be observed that the circularization of the initial orbit has a low cost because it has a low eccentricity. For example, by changing the anomaly of periapsis through a circular transfer orbit is cheaper than using a single impulse.



The cheapest manoeuvre in terms of Δv is SPC (6.0325 $\frac{km}{s}$), the most efficient strategy in terms of ΔT (1803.9996 s) and $Tradeoff$ is clearly the direct manoeuvre. Nevertheless, it requires a huge total Δv (21.4947 $\frac{km}{s}$) in just two impulses, furthermore we would go against the hypothesis of instantaneous manoeuvre, and we would increase the possibility of making errors.
By excluding the direct manoeuvre, the best in terms of ΔT becomes the SPA (16791.4343 s) and PAS for the best $Tradeoff$

6. Appendix

Table 6.1: PAS Manoeuvre

t (s)	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv (km/s)
0	9981.0950	0.0859	28.2525	42.5306	68.6231	19.0909	-
5021.353	9981.0950	0.0859	28.2525	42.5306	68.6231	195.4244	5.1003
	9981.0950	0.0859	59.7595	107.6588	17.6013	195.4244	
6421.3999	9981.0950	0.0859	59.7595	107.6588	17.6013	240.1839	0.946
	9981.0950	0.0859	59.7595	107.6588	137.9682	119.8169	
8323.5359	9981.0950	0.0859	59.7595	107.6588	137.9682	180	0.0235
	10049.8917	0.0785	59.7595	107.6588	137.9682	180	
13336.8286	10049.8917	0.0785	59.7595	107.6588	137.9682	0	0.5673
	12610.0	0.2656	59.7595	107.6588	137.9682	0	
17746.5452	12610.0	0.2656	59.7595	107.6588	137.9682	136.8223	TOT: 6.6372

Table 6.2: PCS Manoeuvre

t (s)	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv (km/s)
0	9981.0950	0.0859	28.2525	42.5306	68.6231	19.0909	
5021.3530dt	9981.0950	0.0859	28.2525	42.5306	68.6231	195.4244	5.1003
	9981.0950	0.0859	59.7595	107.6588	17.6013	195.4244	
9481.1670	9981.0950	0.0859	59.7595	107.6588	17.6013	0	0.2782
	9123.1908	0	59.7595	107.6588	0	17.6013	
11151.5896	9123.1908	0	59.7595	107.6588	0	137.9682	0.2782
	9981.0950	0.0859	59.7595	107.6588	137.9682	0	
16113.4927	9981.0950	0.0859	59.7595	107.6588	137.9682	180	0.0235
	10049.8917	0.0785	59.7595	107.6588	137.9682	180	
21126.7853	10049.8917	0.0785	59.7595	107.6588	137.9682	0	0.5673
	12610.0	0.2656	59.7595	107.6588	137.9682	0	
25536.5020	12610.0	0.2656	59.7595	107.6588	137.9682	136.8223	TOT: 6.2476

Table 6.3.1: SPA Manoeuvre

t (s)	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv (km/s)
0	9981.0950	0.0859	28.2525	42.5306	68.6231	19.0909	
9481.1670	9981.0950	0.0859	28.2525	42.5306	68.6231	0	0.5683
	12541.2034	0.2725	28.2525	42.5306	68.6231	0	
16469.7609	12541.2034	0.2725	28.2525	42.5306	68.6231	180	0.0203
	12610.0	0.2656	28.2525	42.5306	68.6231	180	
17464.1611	12610.0	0.2656	28.2525	42.5306	68.6231	195.4244	3.8038
	12610.0	0.2656	59.7595	107.6588	17.6013	195.4244	
19961.0545	12610.0	0.2656	59.7595	107.6588	17.6013	240.1839	2.6877
	12610.0	0.2656	59.7595	107.6588	137.9682	119.8169	
20815.8865	12610.0	0.2656	59.7595	107.6588	137.9682	136.8223	TOT: 7.0802

Table 6.3.2: SPC Manoeuvre

t (s)	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv (km/s)
0	9981.0950	0.0859	28.2525	42.5306	68.6231	19.0909	
9481.1670	9981.0950	0.0859	28.2525	42.5306	68.6231	0	0.5683
	12541.2034	0.2725	28.2525	42.5306	68.6231	0	
16469.7609	12541.2034	0.2725	28.2525	42.5306	68.6231	180	0.0203
	12610.0	0.2656	28.2525	42.5306	68.6231	180	
17464.1611	12610.0	0.2656	28.2525	42.5306	68.6231	195.4244	3.8038
	12610.0	0.2656	59.7595	107.6588	17.6013	195.4244	
23515.9391	12610.0	0.2656	59.7595	107.6588	17.6013	0	0.82
	9260.7840	0	59.7595	107.6588	0	17.6013	
25224.2930	9260.7840	0	59.7595	107.6588	0	137.9682	0.82
	12610.0	0.2656	59.7595	107.6588	137.9682	0	
29634.0096	12610.0	0.2656	59.7595	107.6588	137.9682	136.8223	TOT: 6.0325

Table 6.4: CB Manoeuvre

t (s)	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv (km/s)
0	9981.0950	0.0859	28.2525	42.5306	68.6231	19.0909	
0	9981.0950	0.0859	28.2525	42.5306	68.6231	19.0909	0.5452
	9163.0850	0	28.2525	42.5306	0	87.7198	
4275.6057	9163.0850	0	28.2525	42.5306	0	264.0476	1.1288
	14581.5425	0.3716	28.2525	42.5306	264.0476	0	
13037.2617	14581.5425	0.3716	28.2525	42.5306	264.0476	180	3.1156
	14630.3920	0.3670	59.7595	107.6588	213.0257	180	
21842.9829	14630.3920	0.3670	59.7595	107.6588	213.0257	180	0.2900
	12610.0	0.2656	59.7595	107.6588	213.0257	0	
26564.1965	12610.0	0.2656	59.7595	107.6588	213.0257	142.4716	1.8870
	12610.0	0.2656	59.7595	107.6588	137.9682	217.5296	
35695.1268	12610.0	0.2656	59.7595	107.6588	137.9682	136.8223	TOT: 6.9667

Table 6.5: Direct Manoeuvre

t (s)	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv (km/s)
0	9981.0950	0.0859	28.2525	42.5306	68.6231	19.0909	
0	9981.0950	0.0859	28.2525	42.5306	68.6231	19.0909	12.3024
	11190.4165	0.4166	120.4644	291.5381	58.0005	88.7168	
1803.9996	11190.4165	0.4166	120.4644	291.5381	58.0005	209.1697	9.1923
	12610.0	0.2656	59.7595	107.6588	137.9682	136.8223	
1803.9996	12610.0	0.2656	59.7595	107.6588	137.9682	136.8223	TOT: 21.4947