ROTOTRASLATION

using EULER ANGLE

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ROTATIONS

Different quantities expressed in different references $\longrightarrow$ must be INTERFACED

$\Rightarrow$ 3D ROTATIONS

↳ In aerospace expressed using:
- DIRECTION cos MATRIX
- QUATERNIONS
- EULER ANGLE $\longrightarrow$ Tait-Bryan sequence

One operation different descriptions

Euler angle : link $\mathcal{B}(O, \underline{i}_1, \underline{i}_2, \underline{i}_3)$ to $K(O, \underline{k}_1, \underline{k}_2, \underline{k}_3)$ using 3 PLANAR ROTATIONS

↳ Defined by: AXIS + INTENSITY

↳ Particular case: **TAIT-BRYAN**

1) Rotation $\underline{i}_3$, σ_1 $\longrightarrow$ $\mathcal{B}'(\underline{i}_1', \underline{i}_2', \underline{i}_3')$, $\sigma_1 \in [-\pi, \pi]$
2) Rotation $\underline{i}_2$, σ_2 $\longrightarrow$ $\mathcal{B}''(\underline{i}_1'', \underline{i}_2'', \underline{i}_3'')$, $\sigma_2 \in [-\pi/2, \pi/2]$
3) Rotation $\underline{i}_1$, σ_3 $\longrightarrow$ $K(\underline{k}_1, \underline{k}_2, \underline{k}_3)$, $\sigma_3 \in [-\pi, \pi]$

Propriety of ROTATIONS

ORTHOGONALITY: $\underline{\underline{R}}^T_{\mathcal{B}\to K} = \underline{\underline{R}}^{-1}_{\mathcal{B}\to K}$, PROOF $\longrightarrow$ $\underset{K\text{ axis}}{\underline{w}} = \underline{\underline{R}}_{\mathcal{B}\to K}\, \underset{\mathcal{B}\text{ axis}}{\underline{v}}$ and $\underline{w}^T = \underline{v}^T \underline{\underline{R}}^T_{\mathcal{B}\to K}$

but... $\underline{w}^T\underline{w} = \underline{v}^T\underline{v}$ $\longrightarrow$ In rotations $|\cdot| = $ cost.

$\Rightarrow$ $\underline{\underline{R}}^T_{\mathcal{B}\to K} \cdot \underline{\underline{R}}_{\mathcal{B}\to K} = \underline{\underline{I}}$

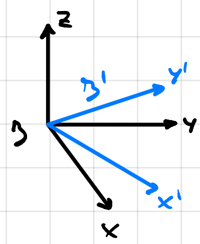
CHANGE REF. of a TENSOR: TENSOR = operator that change a vector in another vector: $\underline{w} = \underline{\underline{T}}\,\underline{v}$

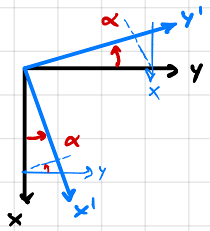
↳ $\begin{cases} \underline{w}^{\mathcal{B}} = \underline{\underline{T}}^{\mathcal{B}}\,\underline{v}^{\mathcal{B}} \\ \underline{w}^{K} = \underline{\underline{T}}^{K}\,\underline{v}^{K} \end{cases}$ but... $\begin{matrix} \underline{w}^{\mathcal{B}} = \underline{\underline{R}}^{\mathcal{B}|K}_{\mathcal{B}\to K}\,\underline{w}^K \\ \underline{v}^{\mathcal{B}} = \underline{\underline{R}}^{\mathcal{B}|K}_{\mathcal{B}\to K}\,\underline{v}^K \end{matrix}$ $\Rightarrow$ $\underline{\underline{T}}^K = \underline{\underline{R}}^{\mathcal{B}|K\,T}_{\mathcal{B}\to K}\,\underline{\underline{T}}^{\mathcal{B}}\,\underline{\underline{R}}^{\mathcal{B}|K}_{\mathcal{B}\to K}$

Tait-Bryan sequence MATRIX

$$\underline{\underline{R}}_{\mathcal{B}\to\mathcal{B}'}(\sigma_I) = \begin{bmatrix} \cos & -\sin & \varnothing \\ \sin & \cos & \varnothing \\ \varnothing & \varnothing & 1 \end{bmatrix} \qquad \underline{\underline{R}}_{\mathcal{B}'\to\mathcal{B}''}(\sigma_{II}) = \begin{bmatrix} \cos & \varnothing & \sin \\ \varnothing & 1 & \varnothing \\ -\sin & \varnothing & \cos \end{bmatrix} \qquad \underline{\underline{R}}_{\mathcal{B}''\to K}(\sigma_{III}) = \begin{bmatrix} 1 & \varnothing & \varnothing \\ \varnothing & \cos & -\sin \\ \varnothing & \sin & \cos \end{bmatrix}$$

$$\Rightarrow \underline{\underline{R}}_{\mathcal{B}\to K} = \underline{\underline{R}}_{\mathcal{B}\to\mathcal{B}'}(\sigma_I) \cdot \underline{\underline{R}}_{\mathcal{B}'\to\mathcal{B}''}(\sigma_{II}) \cdot \underline{\underline{R}}_{\mathcal{B}''\to K}(\sigma_{III})$$





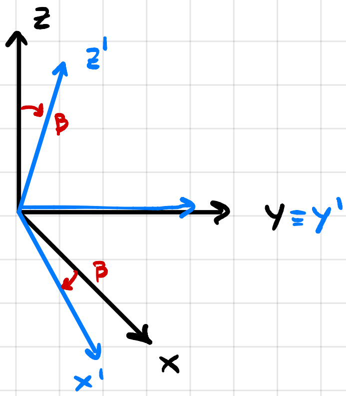
$$\underline{x}'^{B} = \underline{\underline{R}}_{B\to B'}\ \underline{x}^{B} \longrightarrow \text{ROTAZIONE}$$

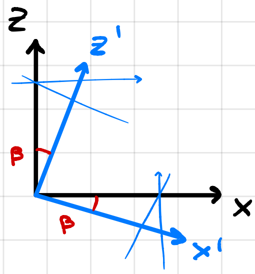
asse $x = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}^B$ | asse $x' = \begin{Bmatrix} \cos\alpha \\ \sin\alpha \\ 0 \end{Bmatrix} = \begin{bmatrix} \cos\alpha & \cdot & \cdot \\ \sin\alpha & \cdot & \cdot \\ 0 & \cdot & \cdot \end{bmatrix} \cdot \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}^B$

asse $y = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}^B$ | asse $y' = \begin{Bmatrix} -\sin\alpha \\ \cos\alpha \\ 0 \end{Bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & \cdot \\ \sin\alpha & \cos\alpha & \cdot \\ \varnothing & \varnothing & \cdot \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}^B$

asse $z = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}^B$ | asse $z' = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & \varnothing \\ \sin\alpha & \cos\alpha & \varnothing \\ \varnothing & \varnothing & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}^B$

↳ From here can write change of reference : $\underline{x}^{B'} = \underbrace{\underline{\underline{R}}^{T}_{B\to B'}}_{\text{ORTHOGONAL}}\ \underline{x}^{B}$





$$\underline{x}'^{B} = \underline{\underline{R}}_{B\to B'} \cdot \underline{x}^{B} \longrightarrow \text{ROTAZIONE}$$

asse $x = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}^B$ | asse $x' = \begin{Bmatrix} \cos\beta \\ 0 \\ -\sin\beta \end{Bmatrix}^B = \begin{bmatrix} \cos\beta & \cdot & \cdot \\ 0 & \cdot & \cdot \\ -\sin\beta & \cdot & \cdot \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}^B$

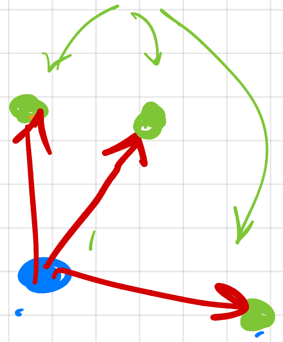
asse $y = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$ | asse $y' = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}^B = \begin{bmatrix} \cos\beta & 0 & \cdot \\ 0 & 1 & \cdot \\ -\sin\beta & 0 & \cdot \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}^B$

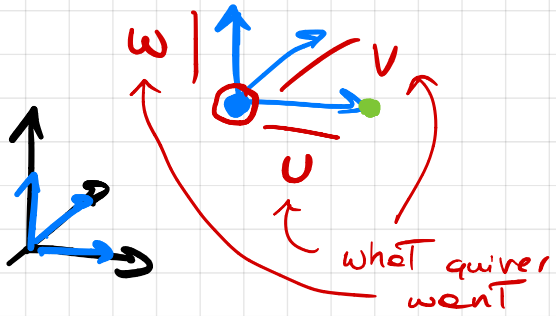
asse $z = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$ | asse $z' = \begin{Bmatrix} \sin\beta \\ 0 \\ \cos\beta \end{Bmatrix}^B = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}^B$

exploiting ORTHOGONALITY

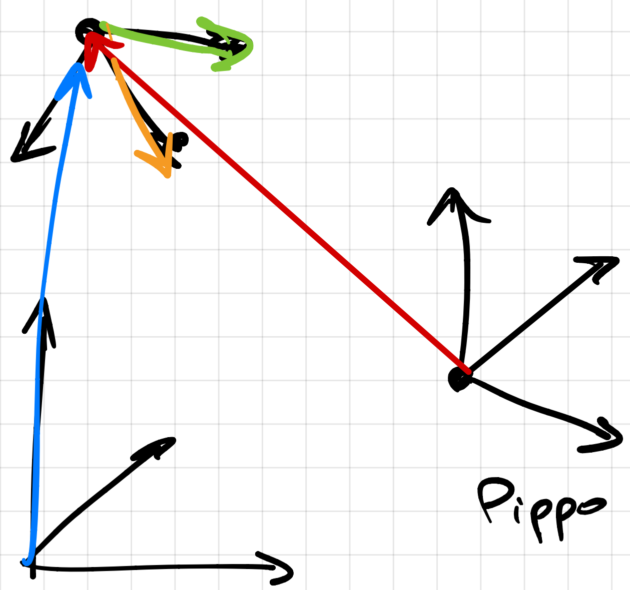
↳ From here can write change of reference : $\underline{x}^{B} = \underline{\underline{R}}_{B\to B'} \cdot \underline{x}^{B'}$

MATLAB





world
~~DACOM~~ → CUSTOM → Anodo
→ shutter
→ detettore



sys custom. Ref Custom : $\begin{pmatrix}0\\0\\0\end{pmatrix}$ $\begin{bmatrix}1&&\\&1&\\&&1\end{bmatrix}$

. Ref Pippo : $\begin{pmatrix}2\\1\\0\end{pmatrix}$ $\begin{bmatrix}\vdots&\vdots&\vdots\end{bmatrix}$

. Ref Pepero : $\begin{pmatrix}0\\0\\0\end{pmatrix}$