MSAS – Assignment #2: Simulation

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1 Exercise 1

The rocket engine in Figure 1 is fired in laboratory conditions. With reference to Figure 1, the nozzle is made up of an inner lining (k_1), an inner layer having specific heat c_2 and high conductivity k_2, an insulating layer having specific heat c_4 and low conductivity k_4, and an outer coating (k_5). The interface between the conductor and the insulator layers has thermal conductivity k_3.

1.1) Part 1: Parameters definition

Select the materials of which the nozzle is made of*, and therefore determine the values of k_i ($i = 1, \ldots, 5$), c_2, and c_4. Assign also the values of ℓ_i (i=1,. . . ,5), L, and A in Figure 1.

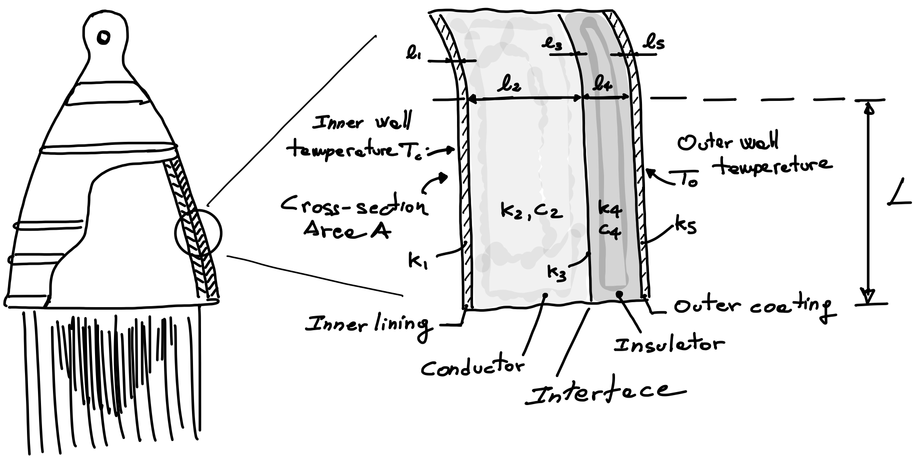

Figure 1: Real thermal system.

1.2) Part 2: Causal modeling

Derive a physical model and the associated mathematical model using one node per each of the five layers and considering that only the conductor and insulator layers have thermal capacitance. The inner wall temperature, T_i, as well as the outer wall temperature, T_o, are assigned. Using the mathematical model, carry out a dynamic simulation in MATLAB to show the temperature profiles across the different sections. At initial time, $T_i(t_0) = T_o(t) = 20$ C°. When the rocket is fired, $T_i(t) = 1000$ C°, $t \in [t_1, t_f]$, following a ramp profile in $[t_0, t_1]$. Integrate the system using $t_1 = 1$ s and $t_f = 60$ s.

1.3) Part 3: Acausal modeling

a) Reproduce in Simscape the physical model derived in Part 2. Run the simulation from $t_0 = 0$ s to $t_f = 60$ s and show the temperature profiles across the different sections. Compare the results with the ones obtained in point 1.2). b) Which solver would you choose? Justify

*The interface layer is not made of a physically existing material, though it produces a thermal resistance. For this layer, the value of the thermal resistance R_3 can be direcly assumed, so avoiding to choose k_3 and ℓ_3.

the selection based on the knowledge acquired from the first part of the course. c) Repeat the simulation in Simscape implementing two nodes for the conductor and insulator layers and show the temperature profiles across the different sections.

(15 points)

The nozzle materials are defined following approximatly the available data of the SpaceX Raptor engine and reported Table 1. The cross section area A and the length L of the nozzle are fixed and respectively equal to $4\,m^2$ and $1\,m$.

Table 1: Nozzle materials and properties

Layer	Material	l_i, m	$\rho, \frac{kg}{m^3}$	$c_{pi}, \frac{J}{kgK}$	$k_i, \frac{W}{mK}$
Inner lining	−−	−−	−−	−−	−−
Conductor	−−	−−	−−	−−	−−
Interface	−−	−−	−−	−−	−−
Insulator	−−	−−	−−	−−	−−
Outer coating	−−	−−	−−	−−	−−

Once defined all the nozzle dimensions and materials, the thermal resistance and capacitance are computed following Eq. (1) and Eq. (2).

$$R_i = \frac{l_i}{k_i\,A} \tag{1}$$

$$C_i = \rho_i A l_i c_{pi} \tag{2}$$

The different nozzle layers are modelled following the electric circuit analogy. A physical model of the system is derived considering one node for each of the five layers as rappresented in Fig. 2.

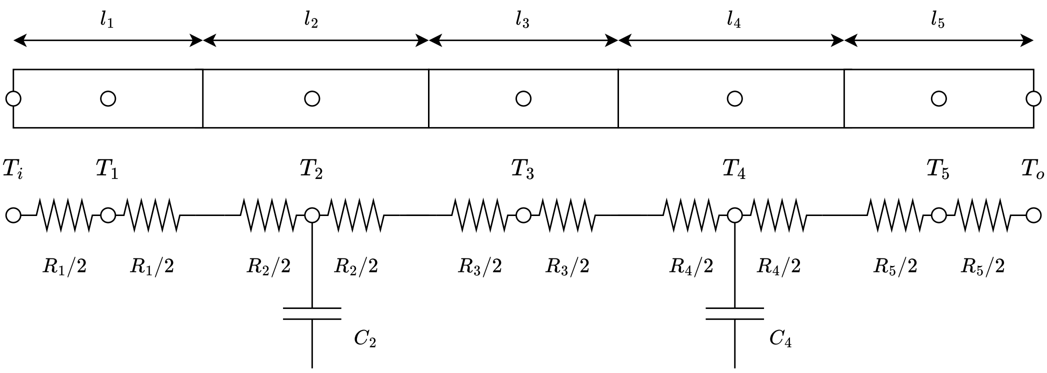

Figure 2: TBD

The model can be simplified by computing the equivalent resistance between the different nodes; the math is reported in Eq. (3).

$$R_{i2} = R_1 + \frac{R_2}{2} \tag{3a}$$

$$R_{24} = \frac{R_2}{2} + R_3 + \frac{R_4}{2} \tag{3b}$$

$$R_{4o} = \frac{R_4}{2} + R_5 \tag{3c}$$

Note that only conductor and insulator layers have thermal capacitance as suggested from the exercise instruction. This assumption simplifies quite a lot the mathematical formulation since, in absence of capacitance, the heat flow can be assumed constant. As consequence the Eq. (4) can be written.

$$\begin{cases} Q_{i1} = Q_{i2} = Q_{12} \\ Q_{23} = Q_{24} = Q_{34} \\ Q_{45} = Q_{4o} = Q_{5o} \end{cases} \tag{4}$$

All the heatflows can be easly put in relationship with the nodes temperature value by means of Eq. (5).

$$Q_{i2} = \frac{T_i - T_2}{R_{i2}} \qquad Q_{24} = \frac{T_2 - T_4}{R_{24}} \qquad Q_{4o} = \frac{T_4 - T_o}{R_{4o}} \tag{5}$$

The temperature evolution in time of nodes T_2 and T_4 is obtained solving numerically the ODEs Eq. (6) and Eq. (7).

$$\dot{T}_2 = \frac{Q_{i2} - Q_{24}}{C_2} \tag{6}$$

$$\dot{T}_4 = \frac{Q_{24} - Q_{4o}}{C_4} \tag{7}$$

Finally even T_1, T_3 and T_5 nodes temperature can be evalutaed over time by exploting the heatflows relationship previously written (Eq. (4) and Eq. (5)) as done in Eq. (8), Eq. (9) and Eq. (10).

$$T_1 = T_i - Q_{i2}\frac{R_1}{2} \tag{8}$$

$$T_3 = T_4 + Q_{24}\frac{R_3 + R_4}{2} \tag{9}$$

$$T_5 = T_o + Q_{4o}\frac{R_5}{2} \tag{10}$$

The IVP obtained combining Eq. (6) and Eq. (7) can be integrated in MATLAB using different algorithms. To understand which is the best method to use an eigenvalues analysis is performed to find out if the system analyzed has some particular characteristics.
Firstly the Eq. (6) and Eq. (7) are rewritten in matrix form considering $\mathbf{T} = [T_2; T_4]$ as presented in Eq. (11).

$$\frac{d\mathbf{T}}{dt} = \mathbf{AT} = \begin{bmatrix} -\left(\frac{1}{C_2 R_{i2}} + \frac{1}{C_2 R_{24}}\right) & \frac{1}{C_2 R_{24}} \\ \frac{1}{C_4 R_{24}} & -\left(\frac{1}{C_4 R_{24}} + \frac{1}{C_4 R_{4o}}\right) \end{bmatrix} \mathbf{T} \tag{11}$$

The eigenvalues of matrix $\mathbf{A}$ are shown in Fig. 3 and results equal to $\lambda_1 = xxxx$ and $\lambda_2 = xxxx$.

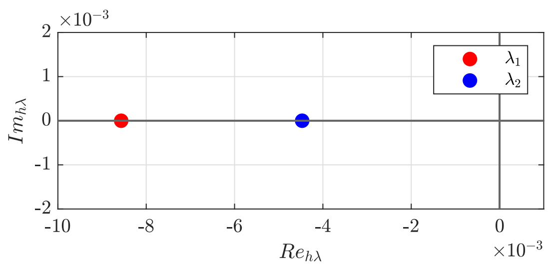

Figure 3: TBD

The problem turns out to be stable and only marginally stiff. The second eigenvalue results about half of the first one, with both presenting the same order of magnitude. From this analysis can be concluded that a nonstiff method such as *ode*45 might be suitable for integration. Another good candidate might also be *ode*23*t*, which is ideal for solving only moderatly stiff problems.
The choice of *ode*45 is also confirmed by the automatic ODE selector built-in in MATLAB.
The thermal problem is also solved in Simscape by building the physical model shown in Fig. 4.

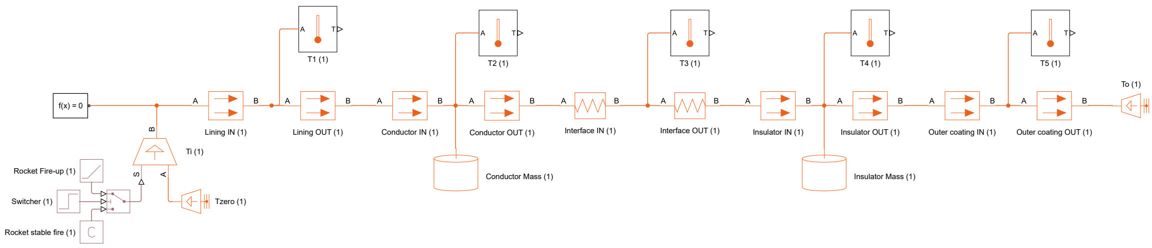

Figure 4: TBD

To solve the problem Simscape builds a DAEs system to compute simultaneously all the nodes temperature. By doing so methods that guarantees efficiency in solving Differential-algebraic system of equations, as *daessc* or *ode*23*t*, are suggested.
In Fig. 5 and Fig. 6 the temperature profiles of the different nodes obtained on MATLAB with *ode*45 and *ode*23*t* are reported and compared with the one obtained on Simscape using *ode*23*t*.

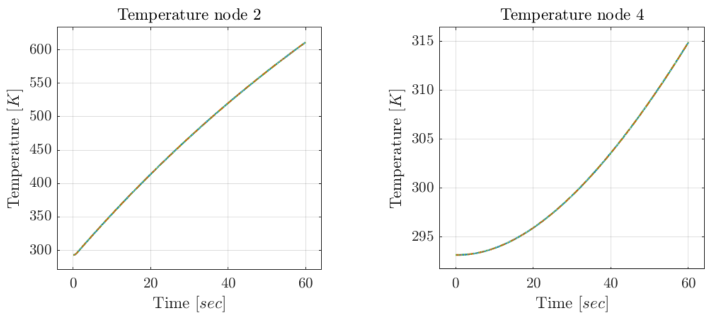

Figure 5: TBD

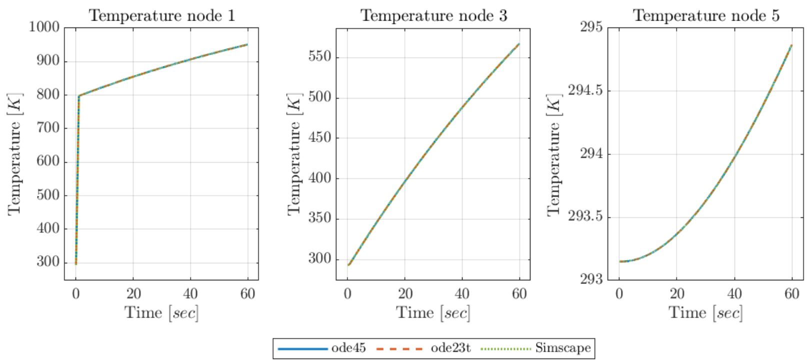

Figure 6: TBD

Both in MATLAB and Simscape the maximum step size has been manually set to $h = 0.01$. Note that the temperature curves obtained with the different methods are perfectly overlapped. The same conclusion can be made observing Fig. 7 where all the nodes temperature are plotted on the same graph. On the left the temperature obtained integrating the problem on MATLAB with *ode*45 is shown, while on the right the results obtained on Simscape with *ode*23*t* are reported.

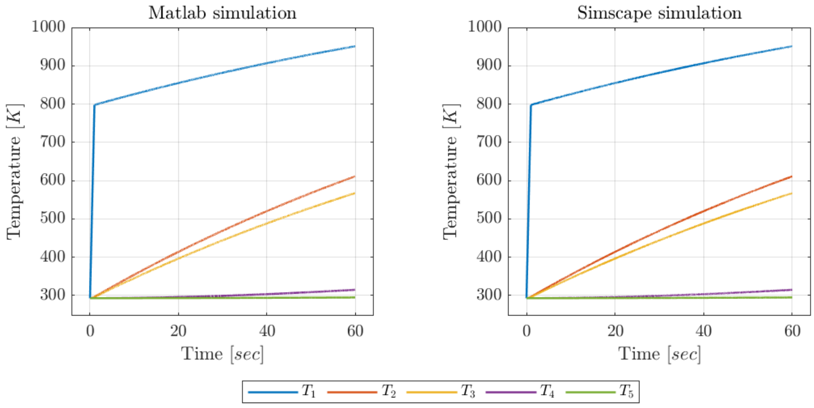

Figure 7: TBD

Note that *ode*23*t* is picked, instead of *daessc*, for the Simscape simulation because of the important differences of the temperature evolution trends over time.
An analysis on the time cost of the different methods has also been performed. For each method a cycle of 50 runs is exectued, the time needed to complete these tasks is measured and post processed by means of an arithmetic mean. In this phase a bigger pool of integration method is considered, this allows to verify if the considerations previously done, based on the eigenanalysis results, are correct. The integration methods tested on MATLAB are *ode*45, *ode*89, *ode*113, *ode*23*t* and *ode*15*s*, while the Simscape simulaton is, once again, done with *ode*23*t*. The results of the analysis are reported in the histogram shown in Fig. 8.

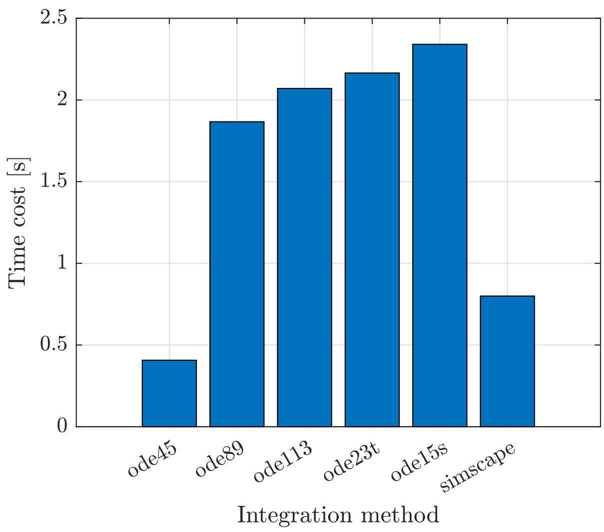

Figure 8: TBD

As expected *ode*45 turns out to be the best method to integrate the thermal problem on MATLAB. *ode*23*t* applied on MATLAB model performs a little bit worse than expected, probably due to the fact the problem isn't sufficiently stiff, while it works well on Simscape to solve the DAE system.

The temperature curves obtained through MATLAB *ode*45 and *ode*89 are then compared with the Simscape results. This analysis is done to check if the bigger computational cost is justified by a higher precision. To do so a time grid of 30 elements is built and the temperature value in each point is retrived by means of a linear interpolation. The results are shown in Fig. 9.

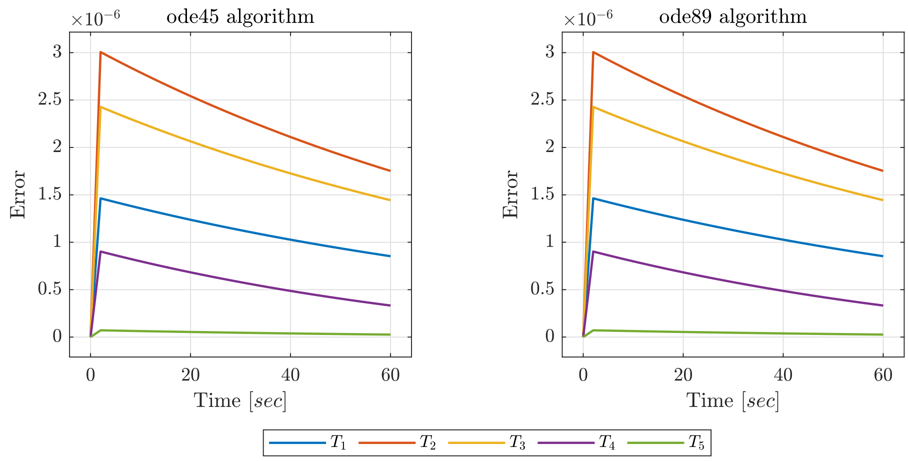

Figure 9: TBD

Both methods, as expected, present a very small error over all the time intervall but, unfortunatly, *ode*89 doesn't perform clearly better than *ode*45.
Finally a more precise analysis is performed modifying the simscape model. The number of nodes for conductor and insulator layers are increased to two and the mass of these two layers is considered located in two points, respectivly at 1/4 and 3/4 of l_i (Fig. 10 and Fig. 11).

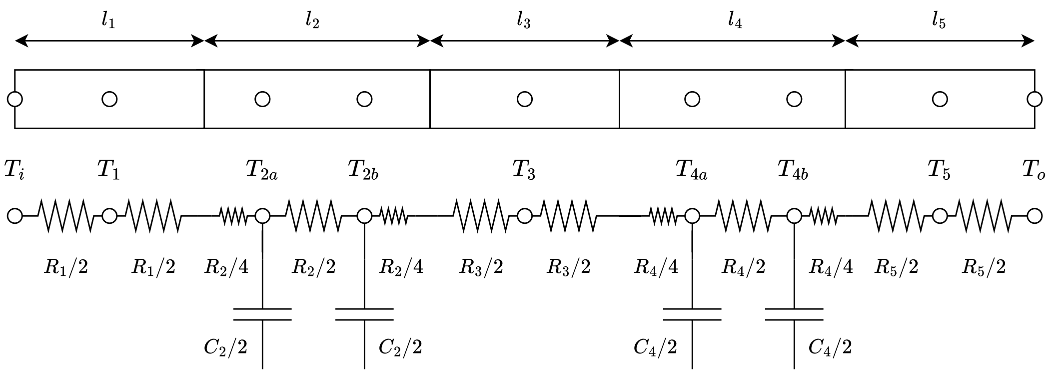

Figure 10: TBD

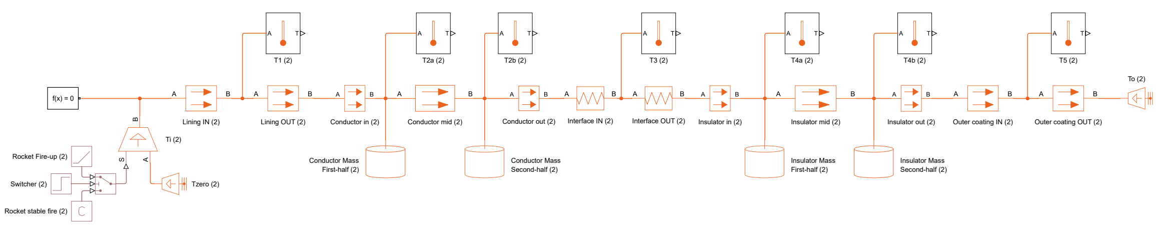

Figure 11: TBD

Using, once again, *ode*23*t* as integration method and $h = 0.01$ as maximum step size the new model nodes temperature are obtained and reported in Fig. 12.

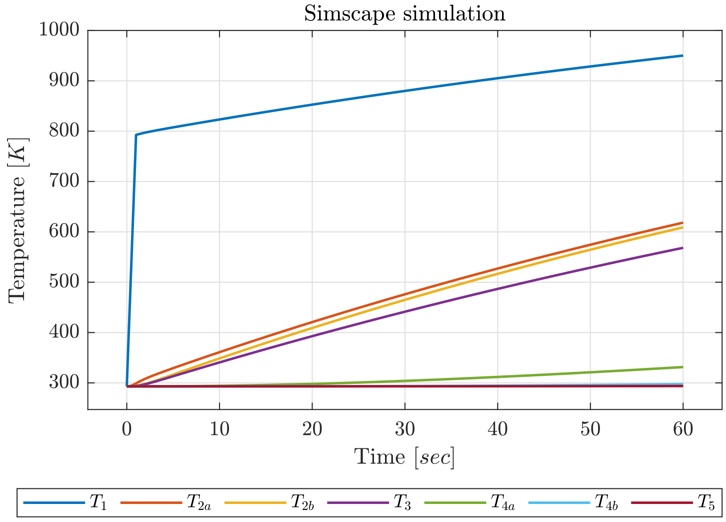

Figure 12: TBD

2 Exercise 2

The real system of an electric propeller engine is depicted in Figure 13. It is composed by a DC permanent magnet motor which drives a propeller shaft. Between the motor and propeller shaft there is a single stage gear box to regulate the angular speed ratio. Moreover, to avoid overheating of the gear unit, the system is augmented by a cooling system where a fluid exchanges heat with the gear box itself. In Figure 14 a functional breakdown structure of the system is shown.

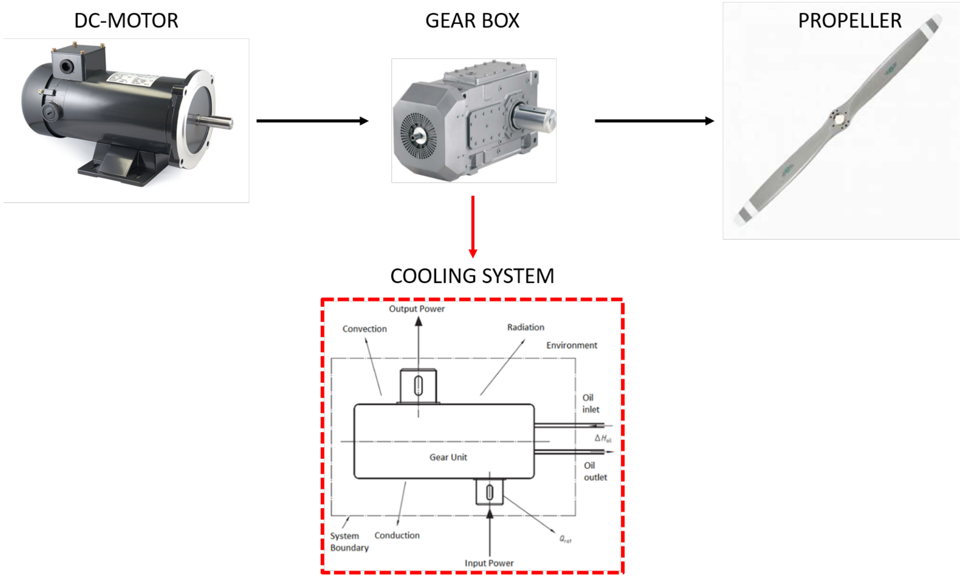

Figure 13: Real system.

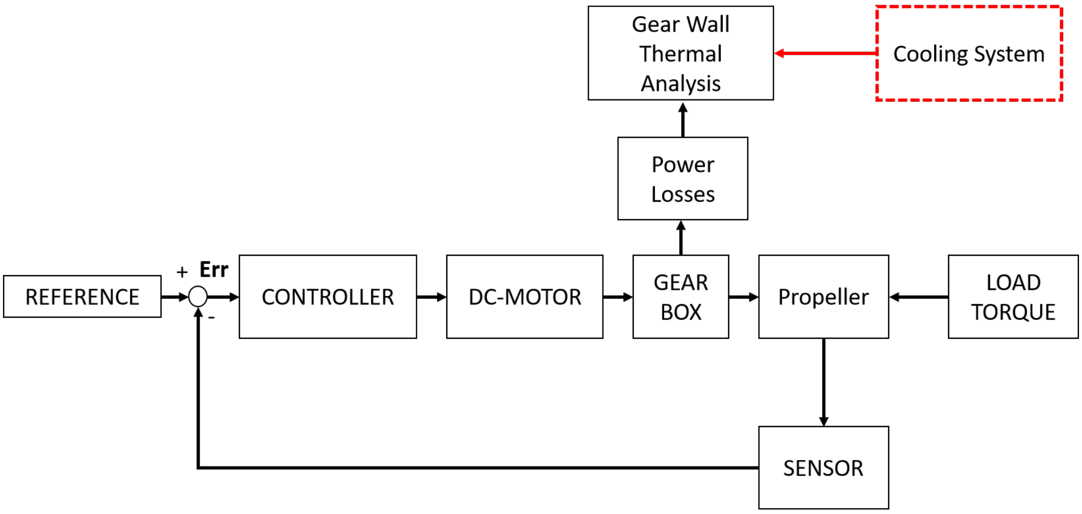

Figure 14: Functional block scheme of the system.

2.1) Part 1: Propeller Electric Engine

Considering the real system in 13 **without** the cooling part, you are asked to:

1. Extract a physical model highlighting assumptions and simplifications.

2. Reproduce the model in acausal manner in Dymola.

3. According to the block scheme in 14, tune a controller (e.g., a PID controller) such that the motor input voltage remains less than 200 V and the error signal **Err** is less than 0.1 rad/s after 10 s.

4. Study the Gear box temperature and heat flux for a simulation time of $\mathbf{t_f} = 120$ s (considering only conduction as heat transfer).

5. Discuss the simulation results and the integration scheme used

For the simulation part, you shall consider: the DC motor data listed in Table 2; the gear box data listed in Table 3, with loss parameters in Table 4; a propeller made of **aluminium** with nominal angular speed $\hat{\omega}$ and a nominal quadratic speed load torque $\hat{T}_{load}$ acting on it (Table 5). The reference angular speed signal to be tracked by the propeller is given in Figure 15.

Table 2: DC motor data

Parameter	Value	Unit
Coil Resistance	0.1	Ω
Inductance	0.01	H
Motor Inertia	0.001	kg m^2
Motor Constant	0.3	Nm/A

Table 3: Gear Box data

Parameter	Value	Unit
Mass	3	kg
Gear ratio	2	[-]
Specific heat	1000	J/(kg K)
Thermal Conductivity	100	Wm/K

Table 4: Gear Box Loss Table

Driver angular speed [rad/s]	Mesh efficiency[-]	Bearing friction torque [Nm]
0	0.99	0
50	0.98	0.5
100	0.97	1
210	0.96	1.5

Table 5: Propeller data

Parameter	Value	Unit
Diameter	0.8	m
Thickness	0.01	m
$\hat{\omega}$	210	rad/s
$\hat{T}_{load}$	100	Nm

2.2) Part 2: Cooling System

After the previous gear unit thermal analysis, now consider the steady-state condition reached by the propeller engine at the end of the simulation to model and simulate a single **fixed** volume flow rate cooling system (as shown in Figure 13) for the gear unit and considering only **convection** as heat transfer. In particular, you are asked to:

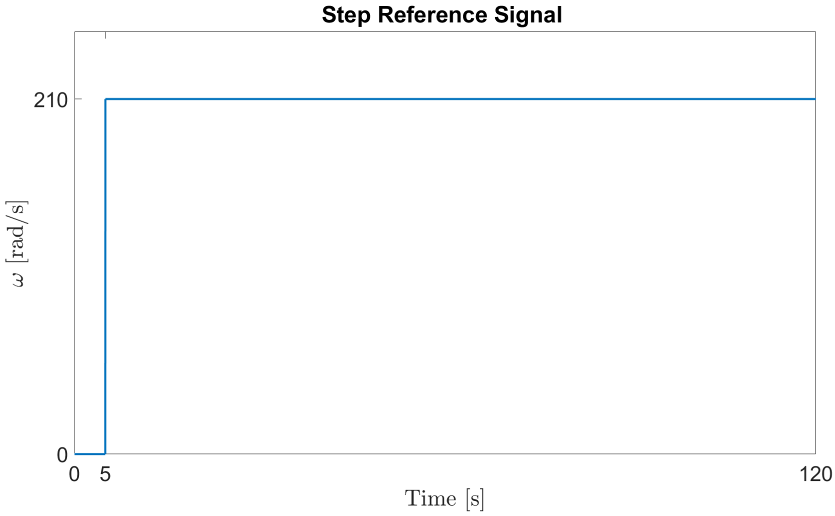

Figure 15: Angular speed reference for the propeller.

1. Derive a physical model highlighting assumptions and simplifications.
2. Reproduce the acausal model in Dymola.
3. Tune the cooling system in terms of volume flow rate, control logics, and initial fluid storage temperature such that:
   (a) the gear unit is kept between 40°C and 60°C.
   (b) the source tank does not get empty before the end simulation time
   (c) the storage tanks have a maximum height of 0.8 m and cross section area of 0.01 m^2
   (d) the system shall have a recirculating capability in order to exploit the outlet fluid for a next cooling process (when the source tank get empty)
   (e) the sink heated fluid is kept between 5°C and 10°C.
   (f) the power consumption of the thermal system shall be no more than 6 kW
4. Discuss the simulation results and the integration scheme used

For the simulation part consider properties of water at 10°C as cooling incompressible fluid (convective thermal conductance λ_{conv} = 300 W/K) and the cylindrical pipe line data listed in Table 6. The simulation shall last at least $\mathbf{t_{sim}}$ = 300 s starting with no water along the pipe.

Table 6: Pipe line properties

Parameter	Value	Unit
Diameter	4	cm
Length	40	cm
Geodetic height	0	m
Friction losses	0	[-]

(15 points)

Starting from the real system a physical model is retrived by means of some assumption and semplification on its components.

The DC motor is modelled as a circuit composed by a voltage source, a fixed resistor, a fixed inductor and rotor. Both the inductor resistance and resistor inductance are considerd negligiable.

The voltage source recives as input a signal which is then translated in voltage while the rotor converts the voltage in rotational mechanical energy. ANALIZZARE BENE IL DISCORSO INERZIE DA MODELICA

The gearbox is modelled as a "lossy gear", which is a built-in block in modelica, connected to a thermal conductivity and capacitance block. All the data, including the loss table, is given as user input to these blocks to correctly model the gearbox behavior.

The propeller is considered rigid and its inertia is modelled VEDERE DA MODELICA COME.

The physical system has no delay between input and the state, uncertainty and noise are negletted and all input data are considerd constant in time. Finally the just presented model, built following a lumping approach, is shown in **??**.

CONSIDERAZIONI SU SOLUTORE

mettere magari una tabella che confronta le performance dei diversi solutori che modelica consente di usare. scegliere quindi il migliore (prendi spunto da quel che dice il bo)

Note that the input signal to the voltage source is regulated by a PID which has been manually tuned to match the system requirements (given in the exercise instruction). The obtained tuning of the PID is reported in **??**

The motor input voltage and the difference between the reference angular velocity and the actual one (defined as $error$) are shown in cref. Their envelop over time confirm that the voltage is always less than $200V$ and the $error$, a part from the first transitory, is always less than $0.1rad/s$.

The gear box is the only component of the physical system that heats up, for this reason its thermal propriety are analyzed more in depth. The heat flux, after a small transitory, reaches a constant value of $Q = xxxxW$ (cref), while the temperature, as expected, linearly increases over time. After $120s$, as shown in cref, the temperature reaches $T = xxxxK$.

From this analysis is clear that a cooling system is needed, otherwise the temperature would increase indefinitely leading to component failure.

INIZIA PARTE 2 DELL'ASSIGNMENT

- Develop one Matlab script for Exercise 1; name the file `lastname123456_Assign2.m`. If needed, organize the script in sections and use local functions.
- Develop one Simulink model for Exercise 1; name the file `lastname123456_Assign2.slx`.
- Develop two Dymola models for Exercise 2; name the files `lastname123456_Assign2_Part_1.mo` and `lastname123456_Assign2_Part_2.mo`.
- Create a single .zip file containing the report in PDF, the MATLAB file, the Simulink model, and the two Dymola models. The name shall be `lastname123456_Assign2.zip`.
- Red text indicates where answers are needed; be sure there is no red stuff in your report.
- In your answers, be concise: to the point.
- **Deadline for the submission: Dec 18 2023, 23:59**.
- **Load the compressed file to the Homework folder on Webeep**.