MSAS – Assignment #2: Modeling

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Exercise 1

The rocket engine in Figure 1 is fired in laboratory conditions. With reference to Figure 1, the nozzle is made up of an inner lining (k_1), an inner layer having specific heat c_2 and high conductivity k_2, an insulating layer having specific heat c_4 and low conductivity k_4, and an outer coating (k_5). The interface between the conductor and the insulator layers has thermal conductivity k_3.

1.1) Part 1: Parameters definition

Select the materials of which the nozzle is made of*, and therefore determine the values of k_i ($i = 1, \dots, 5$), c_2, and c_4. Assign also the values of ℓ_i (i=1,...,5), L, and A in Figure 1.

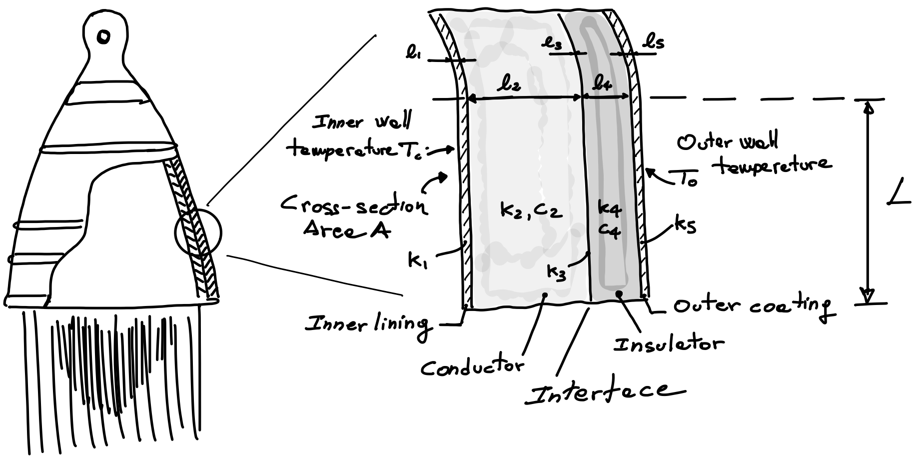

Figure 1: Real thermal system.

1.2) Part 2: Causal modeling

Derive a physical model and the associated mathematical model using one node per each of the five layers and considering that only the conductor and insulator layers have thermal capacitance. The inner wall temperature, T_i, as well as the outer wall temperature, T_o, are assigned. Using the mathematical model, carry out a dynamic simulation in MATLAB to show the temperature profiles across the different sections. At initial time, $T_i(t_0) = T_o(t) = 20$ C°. When the rocket is fired, $T_i(t) = 1000$ C°, $t \in [t_1, t_f]$, following a ramp profile in $[t_0, t_1]$. Integrate the system using $t_1 = 1$ s and $t_f = 60$ s.

1.3) Part 3: Acausal modeling

a) Reproduce in Simscape the physical model derived in Part 2. Run the simulation from $t_0 = 0$ s to $t_f = 60$ s and show the temperature profiles across the different sections. Compare the results with the ones obtained in point 1.2). b) Which solver would you choose? Justify

*The interface layer is not made of a physically existing material, though it produces a thermal resistance. For this layer, the value of the thermal resistance R_3 can be direcly assumed, so avoiding to choose k_3 and ℓ_3.

the selection based on the knowledge acquired from the first part of the course. c) Repeat the simulation in Simscape implementing two nodes for the conductor and insulator layers and show the temperature profiles across the different sections.

(15 points)

The surface and the length of the nozzle is fixed at $A = 1\ m^2$ and $L = 0.5\ m$ respectively. The chosen materials together with the respective properties chosen for every layer of the nozzle is listed in Tab.1 :

Layer	**Material**	$\mathbf{l_i}[m]$	$\mathbf{k_i}[\frac{W}{mK}]$	$\mathbf{c_{pi}}[\frac{J}{kgK}]$	$\rho_\mathbf{i}[\frac{kg}{m^3}]$	$\mathbf{R_i}[\frac{W}{K}]$	$\mathbf{C_i}[\frac{J}{K}]$
Inner lining	Silicon nitride	0.005	29	-	-	1.724e-4	-
Conductor	Tungsten	0.030	164	134	19300	1.829e-4	7.759e4
Interface	-	-	-	-	-	2.000e-3	-
Insulator	Graphene/CF/PAI	0.015	0.53	1113	1410	2.830e-2	2.354e4
Outer lining	C/C composite	0.005	5.2	-	-	9.615e-4	-

Table 1: Materials and properties of the nozzle layers

Note that the thermal resistance are calculated as $R_i = \frac{l_i}{k_i A}$, except for R_3 in the interface between the conductor and insulator layers which has been directly assumed as 2.000e-3 $\frac{W}{K}$; the thermal capacitances are computed as $C_i = \rho_i A l_i c_{pi}$.

It is possible to derive the physical model from the real system exploiting the electrical circuit analogy, using one node per each of the five layers: the single node assumption consists into concentrate all the mass of the layer in a node placed in the middle of it. The model is shown in Fig. 2:

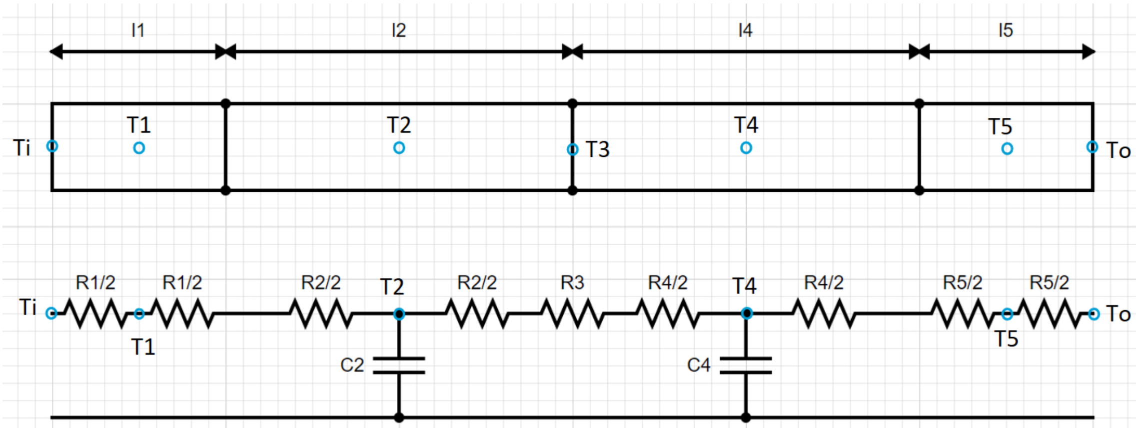

Figure 2: Physical model with one node per layer

It is possible to retrieve the related mathematical model. In absence of capacitance, the heat flow can be assumed as constant, so that in this model three different heat flows can be defined as shown in Eq. 1:

$$\begin{cases} Q_{i2} = Q_{i1} = Q_{12} \\ Q_{24} = Q_{23} = Q_{34} \\ Q_{4o} = Q_{45} = Q_{5o} \end{cases} \tag{1}$$

The equivalent thermal resistances for the traits i-2,2-4 and 4-o are figured out in Eq.2 :

$$\begin{cases} R_{i2} = R_1 + \frac{R_2}{2} \\ R_{24} = \frac{R_2}{2} + R_3 + \frac{R_4}{2} \\ R_{4o} = R_5 + \frac{R_4}{2} \end{cases} \tag{2}$$

The presence of two capacitances makes possible to express the variation in time of T_2 and T_4 as two ODEs. Furthermore, the expressions for the heat fluxes defined in 1 are retrieved. The model is expressed in Eq.3 :

$$\begin{cases} Q_{i2} = \frac{T_i - T_2}{R_{i2}} \\ Q_{24} = \frac{T_2 - T_4}{R_{24}} \\ Q_{4o} = \frac{T_4 - T_o}{R_{4o}} \\ \dot{T}_2 = \frac{Q_{i2} - Q_{24}}{C_2} \\ \dot{T}_4 = \frac{Q_{24} - Q_{4o}}{C_4} \end{cases} \tag{3}$$

Once the evolution in time of T_2 and T_4 is obtained through the built-in function `ode23t`, T_1, T_3 and T_5 are found as shown in Eq. 4:

$$\begin{cases} T_1 = T_i - Q_{i2}\frac{R_1}{2} \\ T_3 = T_4 + Q_{24}\frac{R_3 + R_4}{2} \\ T_5 = T_o + Q_{4o}\frac{R_5}{2} \end{cases} \tag{4}$$

The evolution in time of temperature at the nodes is displayed in Fig.3 :

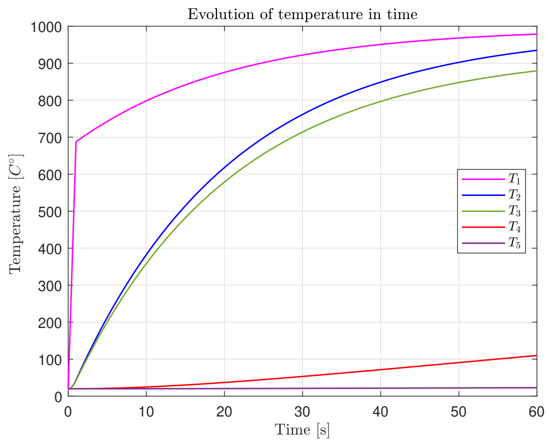

Figure 3: Evolution of the temperatures at each node

The same physical model is replicated in Simscape, as shown in Fig.4 :

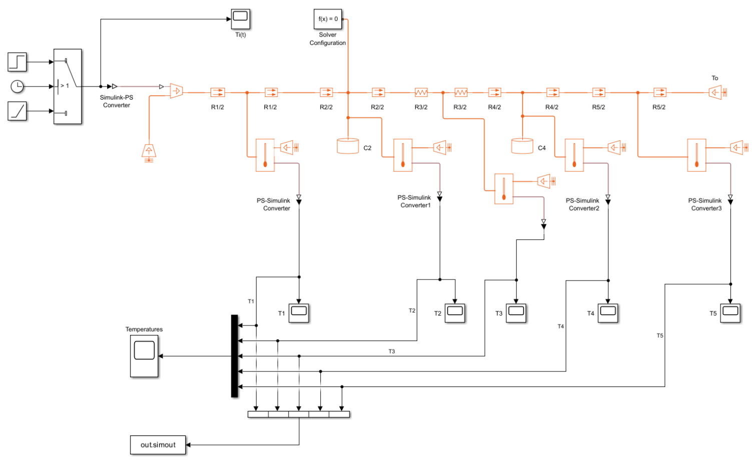

Figure 4: Simscape model with one node per layer

The temperature profiles at each node are portrayed in Fig.5:

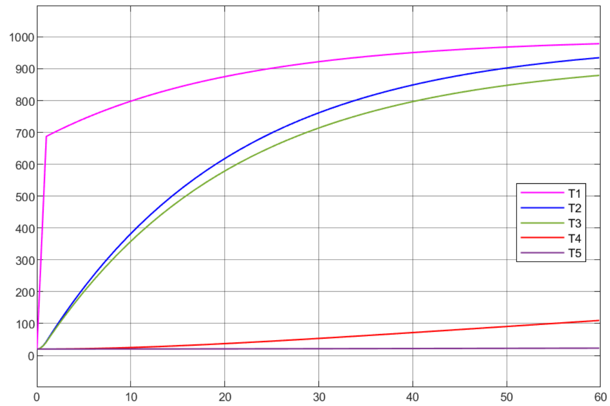

Figure 5: Evolution of the temperatures at each node

Both the temperature profiles shown in Fig.3 and Fig.5 are retrieved exploiting `ode23t`, which is a multi-step method that implements the trapezoidal rule, and it is appropriate for moderately stiff differential equations. Prior to this choice, the two ODEs in Eq. 3 expressing the evolution in time of T_2 and T_4 have been reformulated as:

$$\frac{dT}{dt} = AT \tag{5}$$

where A is the following matrix:

$$\begin{bmatrix} -(\frac{1}{C_2R_{i2}} + \frac{1}{C_2R_{24}}) & \frac{1}{C_2R_{24}} \\ \frac{1}{C_4R_{24}} & -(\frac{1}{C_4R_{24}} + \frac{1}{C_4R_{4o}}) \end{bmatrix} \tag{6}$$

The eigenvalues of A are $\lambda_1 = -0.0497$ and $\lambda_2 = -0.0054$; these values do not differ by much. However, the choice fell on `ode23t` because of the different behaviours among the evolution trends of T_1,T_2,T_3,T_4 and T_5. The max stepsize imposed to the solver scheme is of h = 0.01. The temperature curves obtained through Matlab and Simscape are successively compared to each other; for this purpose, at each time instant in a time span from 0 to 60 s, the temperature values are linearly interpolated. Afterward, it is possible to plot the distance between the two processes in terms of errors, for each temperature profile as displayed in Fig.6:

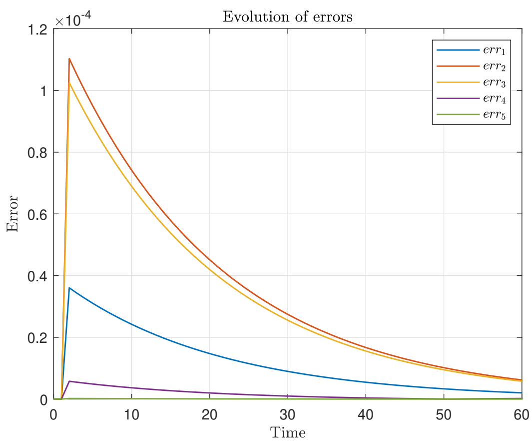

Figure 6: Evolution of the distance between the two results

A more refined analysis can be performed by increasing the number of nodes to two for the conductor and insulator layers. In this thermal model, the mass of the two layers is collapsed in two points that are located respectively at $\frac{l_i}{4}$ and $\frac{3}{4}l_i$. The physical model is presented in Fig.7 :

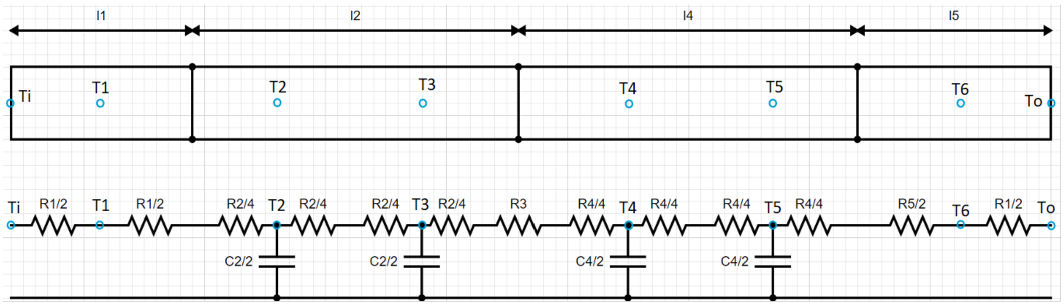

Figure 7: Physical model with two nodes in insulator and conductor layers

The model is represented in Simscape as shown in Fig. 8:

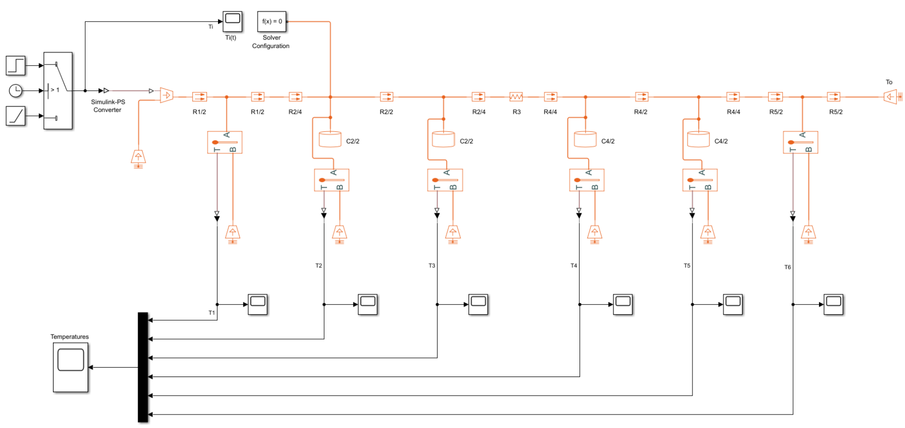

Figure 8: Simscape model with two nodes in insulator and conductor layers

Finally, the temperature profiles at each nodeof the new model are plotted in Fig.9 :

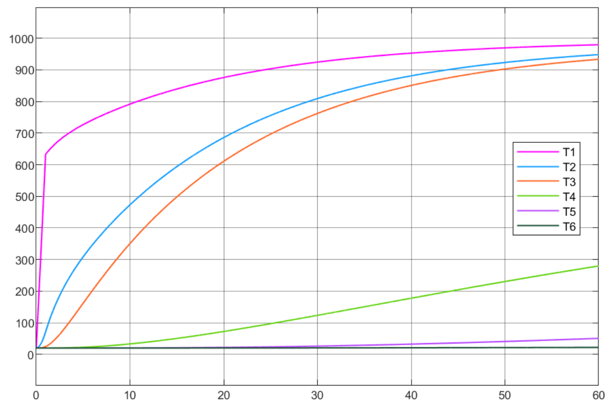

Figure 9: Evolution of the temperature at each node

Exercise 2

The real system of an electric propeller engine is depicted in Figure 10. It is composed by a DC permanent magnet motor which drives a propeller shaft. Between the motor and propeller shaft there is a single stage gear box to regulate the angular speed ratio. Moreover, to avoid overheating of the gear unit, the system is augmented by a cooling system where a fluid exchanges heat with the gear box itself. In Figure 11 a functional breakdown structure of the system is shown.

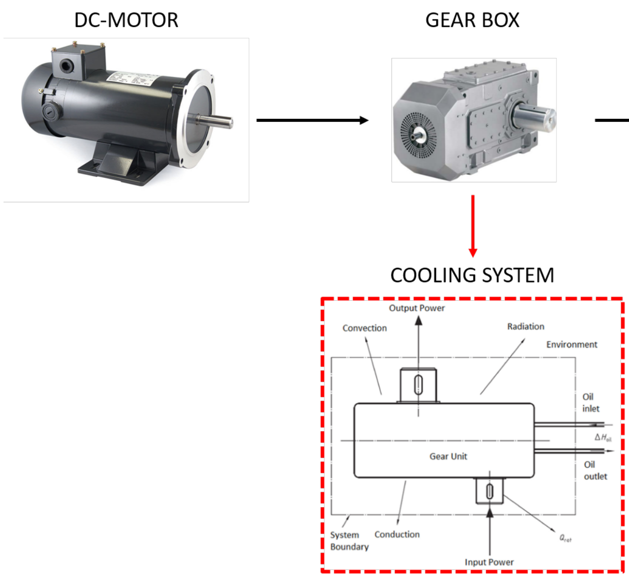

Figure 10: Real system.

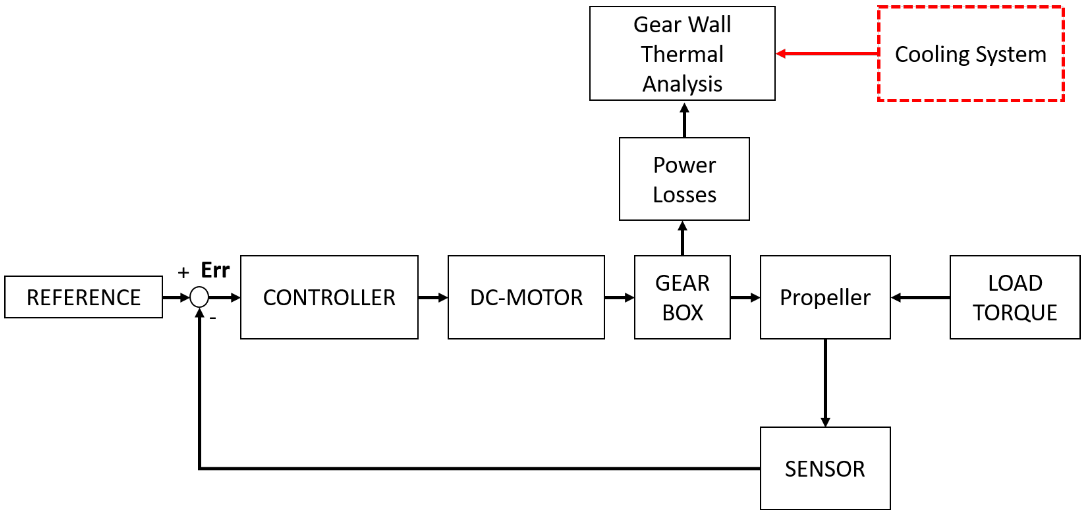

Figure 11: Functional block scheme of the system.

2.1) Part 1: Propeller Electric Engine

Considering the real system in 10 **without** the cooling part, you are asked to:

1. Extract a physical model highlighting assumptions and simplifications.
2. Reproduce the model in acausal manner in Dymola.

3. According to the block scheme in 11, tune a controller (e.g., a PID controller) such that the motor input voltage remains less than 200 V and the error signal **Err** is less than 0.1 rad/s after 10 s.
4. Study the Gear box temperature and heat flux for a simulation time of $\mathbf{t_f} = 120$ s (considering only conduction as heat transfer).
5. Discuss the simulation results and the integration scheme used

For the simulation part, you shall consider: the DC motor data listed in Table 2; the gear box data listed in Table 3, with loss parameters in Table 4; a propeller made of **aluminium** with nominal angular speed $\hat{\omega}$ and a nominal quadratic speed load torque $\hat{T}_{load}$ acting on it (Table 5). The reference angular speed signal to be tracked by the propeller is given in Figure 12.

Table 2: DC motor data

Parameter	**Value**	**Unit**
Coil Resistance	0.1	Ω
Inductance	0.01	H
Motor Inertia	0.001	kg m^2
Motor Constant	0.3	Nm/A

Table 3: Gear Box data

Parameter	**Value**	**Unit**
Mass	3	kg
Gear ratio	2	[-]
Specific heat	1000	J/(kg K)
Thermal Conductivity	100	Wm/K

Table 4: Gear Box Loss Table

Driver angular speed [rad/s]	**Mesh efficiency[-]**	**Bearing friction torque [Nm]**
0	0.99	0
50	0.98	0.5
100	0.97	1
210	0.96	1.5

Table 5: Propeller data

Parameter	**Value**	**Unit**
Diameter	0.8	m
Thickness	0.01	m
$\hat{\omega}$	210	rad/s
$\hat{T}_{load}$	100	Nm

2.2) Part 2: Cooling System

After the previous gear unit thermal analysis, now consider the steady-state condition reached by the propeller engine at the end of the simulation to model and simulate a single **fixed** volume flow rate cooling system (as shown in Figure 10) for the gear unit and considering only **convection** as heat transfer. In particular, you are asked to:

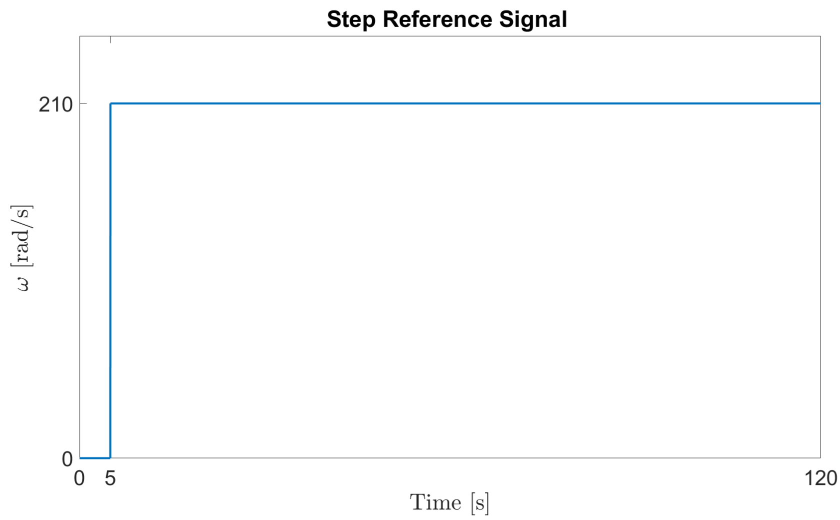

Figure 12: Angular speed reference for the propeller.

1. Derive a physical model highlighting assumptions and simplifications.
2. Reproduce the acausal model in Dymola.
3. Tune the cooling system in terms of volume flow rate, control logics, and initial fluid storage temperature such that:
   (a) the gear unit is kept between 40°C and 60°C.
   (b) the source tank does not get empty before the end simulation time
   (c) the storage tanks have a maximum height of 0.8 m and cross section area of 0.01 m^2
   (d) the system shall have a recirculating capability in order to exploit the outlet fluid for a next cooling process (when the source tank get empty)
   (e) the sink heated fluid is kept between 5°C and 10°C.
   (f) the power consumption of the thermal system shall be no more than 6 kW
4. Discuss the simulation results and the integration scheme used

For the simulation part consider properties of water at 10°C as cooling incompressible fluid (convective thermal conductance λ_{conv} = 300 W/K) and the cylindrical pipe line data listed in Table 6. The simulation shall last at least $\mathbf{t_{sim}}$ = 300 s starting with no water along the pipe.

Table 6: Pipe line properties

Parameter	Value	Unit
Diameter	4	cm
Length	40	cm
Geodetic height	0	m
Friction losses	0	[-]

(15 points)

The physical model is retrieved from the real model through a process of abstraction, which is done exploiting the following simplifications:

- The DC motor is composed by a circuit composed by a voltage source which receives a signal as input which is translated in voltage, a resistor, an inductor and a rotor which transforms electrical energy into rotational mechanical energy;
- The inductor and the resistor are purely inductive and resistive respectely; the small inductance of the resistor is neglected, as well as the small resistance of the inductor;
- The total resistance of the circuit is represented by the resistor, and the total inductance of the circuit is represented by the inductor;
- The shafts are rigid and have no inertia;
- The propeller is represented as rigid disc;
- The bearing friction losses in the gear box are the only losses among the systems;
- The systems do not alter the environment around them;
- Every form of uncertainty and noise is neglected;
- The gear box data, the DC motor data and the propeller data do not change in time;
- There is no delay between the input and the state;
- The system presents a lumping approach.

Once the physical model is defined, the acasual model is represented in Dymola as shown in Fig.13 :

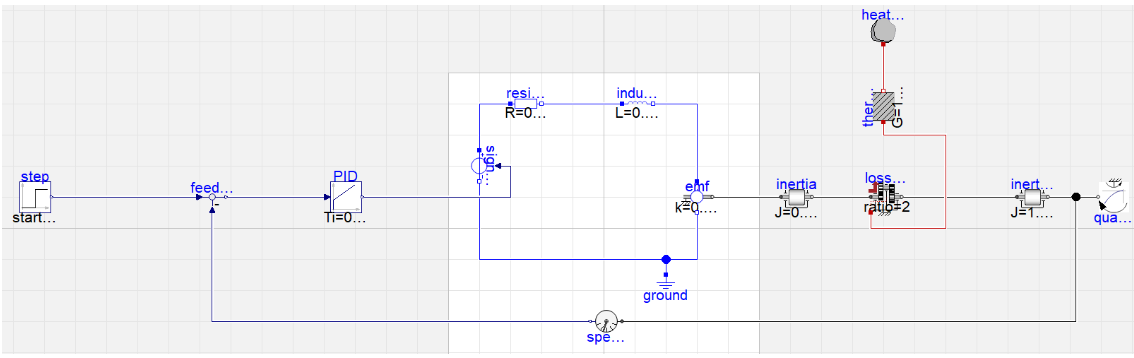

Figure 13: Acasual model of the system

The motor voltage source receives as an input a real signal which is the output of a PID controller, which is tuned as follows:

- Gain: $k = 0.08$;
- Time constant of integrator: $T_i = 0.17\ s$;
- Time constant of derivative block: $T_d = 0.50\ s$.

The PID controller makes sure that the motor input voltage and the signal **Err**, which is the distance between the commanded angular velocity and the actual one, stay within specified limits: as it is displayed in Fig.14 and Fig.15 , the voltage never rises over 200 V and **Err** is less than 0.1 $\frac{rad}{s}$.

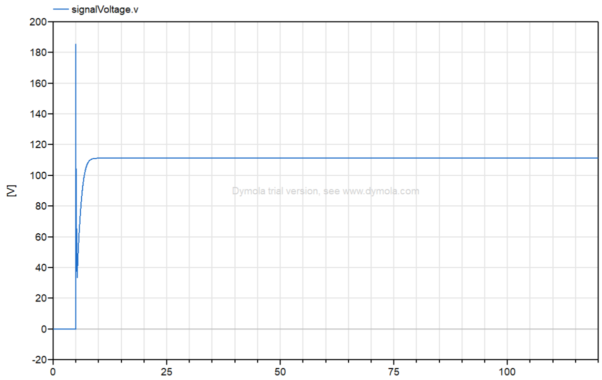

Figure 14: Motor input voltage

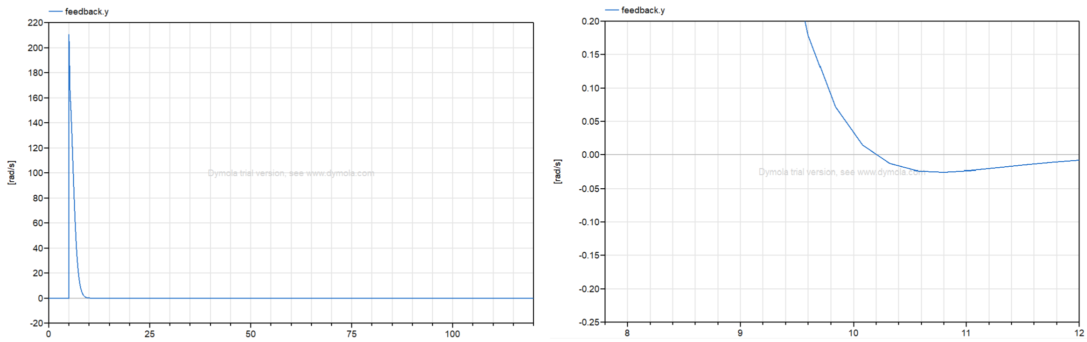

Figure 15: Error signal **Err**

The gear box is the component of the engine that heat up and that shows an increment in the temperature levels. Considering only conduction as heat transfer, in Fig.16 and Fig.17 are displayed the temperature profile and the heat flux of the gear box: it is clear from the graphs that the heat flux settles at a value of $Q = 2210.9\ W$, while the temperature steadily increases in absence of a cooling system, reaching a value of $T = 103.328\ C^{\circ}$ after 120 s.

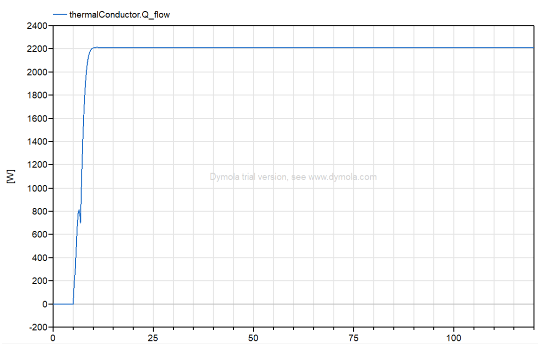

Figure 16: Heat flux through the gear box walls

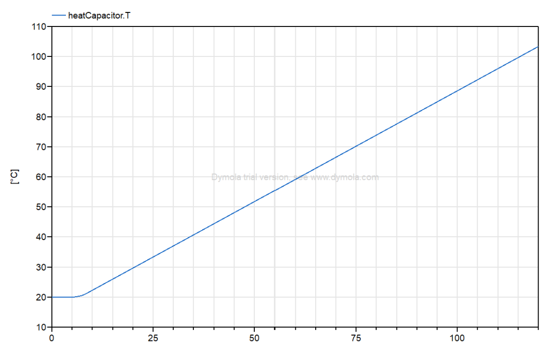

Figure 17: Temperature profile of the gear box

Once the PID controller is tuned properly, the system quickly adapts to the given step reference signal as the angular velocity of the propeller reaches the steady value of $\omega = 210\frac{rad}{s}$ after 10 s. However the implementation of the cooling system becomes mandatory in order to maintain the gearbox within an acceptable temperature range.
For the simulation, the integration scheme `dassl` has been used, as it is widely used for a large number of applications characterized by highly non-linear systems of equations, due to its capability to deal with discontinuities in the system's behaviour.

As it is clear from the previous analysis, in the steady-state condition an adequate cooling system is required. The real system consists into a fluid exchange between the gear box and a fluid, with convection as heat transfer process. The physical model is obtained from the real one after having stated the ensuing assumptions:

- The friction in the pipe line is neglected;
- The parameters of the tanks and the pipe line do not change in time;
- The gear box is the only heat source that is exchanging heat with the cooling system, and this process does not depend on temperature; there is no heat exchange with the environment;
- The system does not alter the environment around it; there is no mass exchange with the environment;
- Every form of uncertainty and noise is neglected;
- There volume flow does not present a transient phase;
- The convective element has a purely convective behaviour;
- The system presents a lumping approach.

The physical model is successively reproduced in Dymola in Fig. 18:

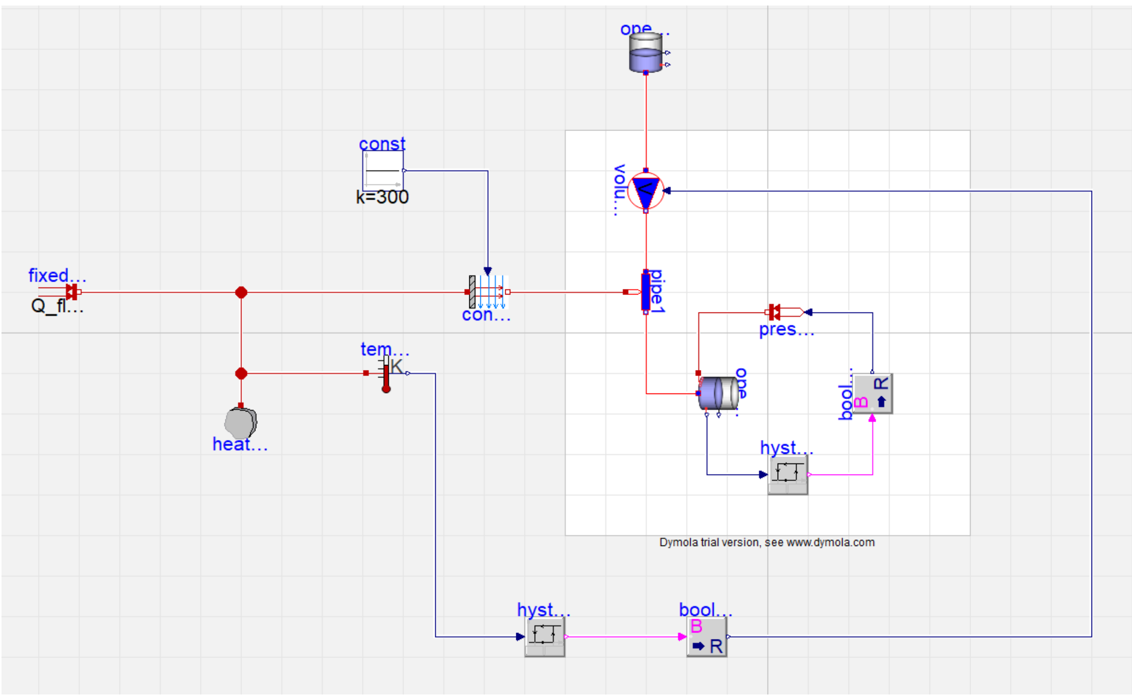

Figure 18: Model of the system in Dymola

The cooling system shall be calibrated in order to meet the requirements on the gear box temperature range and on the cooling system itself. The control logic of the volume flow rate across the pipe is strictly linked to the gear box temperature and based on a hysteresis logic, as the temperature has to stay between 40 $C°$ and 60 $C°$: if the temperature rises above 60 $C°$, the volume flow rate passes from 0 $\frac{m^3}{s}$ to a fixed value, carefully chosen in order to not get the storage tank completely empty. Moreover, the sink heated fluid shall be kept between 5 $C°$ and 10 $C°$: in order to do so, a thermal system is implemented such that no more than 6000 W of heat are removed from the storage tank; once again, an hysteresis based logic has been implemented. The results are shown below in Fig.19 and Fig.20: the graphs show how effectively the temperature of the gear box falls within the required range before 20 s have passed, while the heated fluid shortly before 40 s.

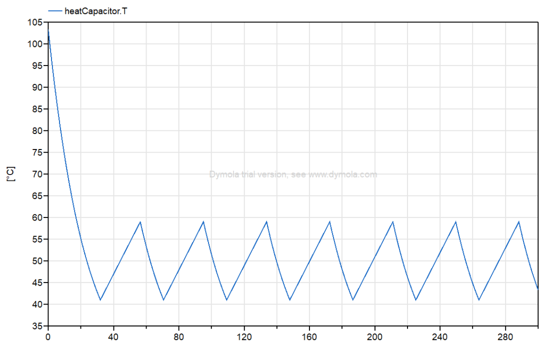

Figure 19: Temperature profile of the gear box

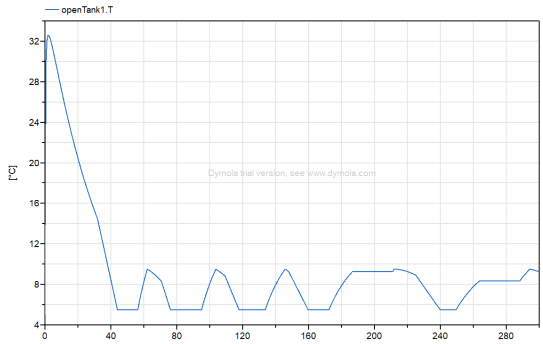

Figure 20: Temperature profile of the sink heated fluid

The volume flow of the fluid across the pipe is displayed below in Fig. 21 and Fig. 22. If the fluid is flowing, the volume flow rate assumes a fixed constant; otherwise no fluid is moving across the line. Regarding the source tank, it can be seen that the level slowly decreases during the simulation until it stops at $h = 0.027\ m$ once 300 s have passed.

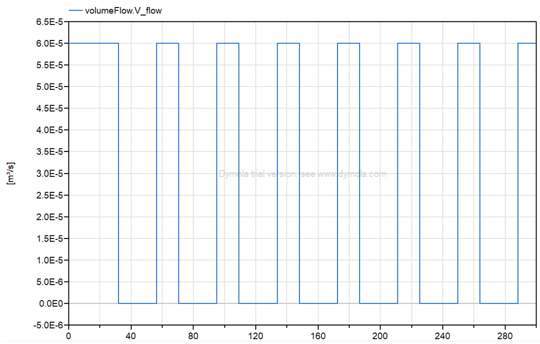

Figure 21: Volume flow across the pipe line

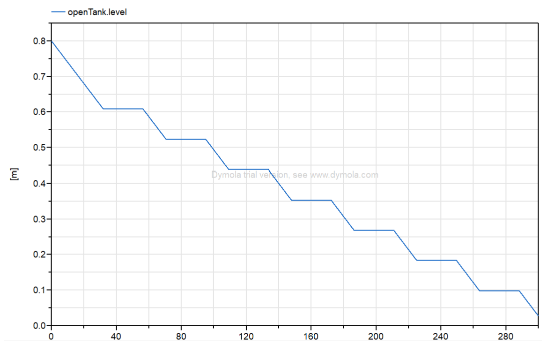

Figure 22: Level of medium in the source tank

Once more, `dassl` has been chosen again as integration scheme for the simulation, as it has been for the analysis of the engine.