Master's Thesis Exam
Verification of the Blocking and Non-Blocking Michael-Scott Queue Algorithms

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Overview of the Project and Contributions

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    - Useful for sequential clients
  - Concurrent specification
    - Proves safety of concurrent queues
    - Useful for *some* concurrent clients
    - Doesn't track queue contents
  - HOCAP-style specification
    - Stronger specification, useful for more complex clients
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    - Demonstrated with a PoC queue client, QueueAdd

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- All proofs have been mechanised in the Coq proof assistant

Outline

Queue Specifications (HOCAP)

Specifications for Queues

Assumptions on Queues

- Queues consists of initialize, enqueue, and dequeue
- initialize creates an empty queue: []
- enqueue adds a value, v, to the beginning of the queue xs_v: $v :: xs_v$
- dequeue depends on whether queue is empty:
  - If non-empty, $xs_v ++ [v]$, remove value v at end of queue and return Some v
  - If empty, [], return None

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  - If empty, $[]$, return None

Nature of Specifications

- Specifications written in Iris, a higher order CSL
- Expressed in terms of *Hoare triples*: $\{P\}\ e\ \{v.\Phi\ v\}$
- Hoare triples prove partial correctness of programs, e
- In particular: safety

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- Construction ensures:
  - authoritative and fragmental views always agree on abstract state of queue
  - views can only be updated in unison
- Implemented using the resource algebra: $\textsc{Auth}((\textsc{Frac} \times \textsc{Ag}(\mathit{List\ Val}))^?)$
- The desirables are captured by the following lemmas

Lemmas on the Abstract State RA

$$\vdash \Rrightarrow \exists \gamma.\, \gamma \Mapsto_{\bullet} xs_v * \gamma \Mapsto_{\circ} xs_v \qquad \text{(Abstract State Alloc)}$$

$$\gamma \Mapsto_{\bullet} xs_v' * \gamma \Mapsto_{\circ} xs_v \vdash xs_v = xs_v' \qquad \text{(Abstract State Agree)}$$

$$\gamma \Mapsto_{\bullet} xs_v' * \gamma \Mapsto_{\circ} xs_v \Rrightarrow \gamma \Mapsto_{\bullet} xs_v'' * \gamma \Mapsto_{\circ} xs_v'' \qquad \text{(Abstract State Update)}$$

HOCAP-style Specification

- Post-condition of initialize specification gives fragmental view to clients
- Hoare triples for enqueue and dequeue are conditioned on view-shifts
- Clients must show that they can supply the fragmental view, so that the abstract (and concrete) state can be updated
- View-shifts and Hoare-triples parametrised by predicates P and Q
  - Client might have resources that need to be updated as a result of enqueue/dequeue
  - P is the clients resources before enqueue/dequeue and Q the resources after

Definition (HOCAP Specification)

$\exists\, \text{isQueue} : \mathit{Val} \to \mathit{Qgnames} \to \text{Prop}.$

$$\forall v_q, G.\ \text{isQueue}(v_q, G) \implies \square\, \text{isQueue}(v_q, G)$$

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$$\wedge\quad \forall v_q, v, G, P, Q.\ \ \left(\forall xs_v.\ G.\gamma_{\mathrm{Abst}} \Mapsto_{\bullet} xs_v * P \Rrightarrow_{\mathcal{E}\setminus\mathcal{N}.i\uparrow} \triangleright G.\gamma_{\mathrm{Abst}} \Mapsto_{\bullet} (v :: xs_v) * Q\right) \twoheadrightarrow$$
$$\{\mathsf{isQueue}(v_q, G) * P\}\ \mathsf{enqueue}\ \ v_q\ \ v\ \{w.Q\}$$

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& \{\mathrm{isQueue}(v_q, G) * P\}\ \mathrm{dequeue}\ \ v_q\ \{w.Q(w)\}
\end{aligned}
$$

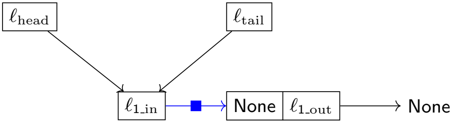
The Two-Lock Michael-Scott Queue

Implementation: initialize

- The queue data structure is a linked list

initialize $\triangleq$

  let *node* $=$ ref (None, ref (None)) in
  let *H_lock* $=$ newLock() in
  let *T_lock* $=$ newLock() in
  ref ((ref (*node*), ref (*node*)), (*H_lock*, *T_lock*))



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- A node x in the linked list is a triple, $x = (\ell_{\text{in}}, w, \ell_{\text{out}})$, with $\ell_{\text{in}} \mapsto (w, \ell_{\text{out}})$
- We use the following notation for nodes

$$\mathsf{in}(x) = \ell_{\text{in}} \qquad \mathsf{val}(x) = w \qquad \mathsf{out}(x) = \ell_{\text{out}}$$

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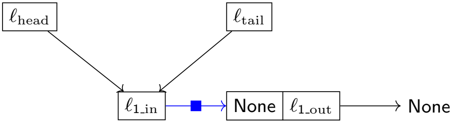

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- The initialize function first creates an initial head node, x_{head}
- Then, a lock protecting the head pointer, and a lock protecting the tail pointer
- Finally, it creates the head and tail pointers, ℓ_{head} and ℓ_{tail}, both pointing to x_{head}

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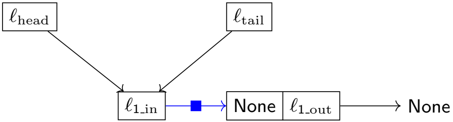

Implementation: enqueue

- The enqueue function consists of the following steps
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enqueue Q *value* $\triangleq$
  let *node* $=$ ref (Some *value*, ref (None)) in
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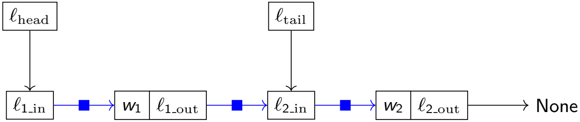

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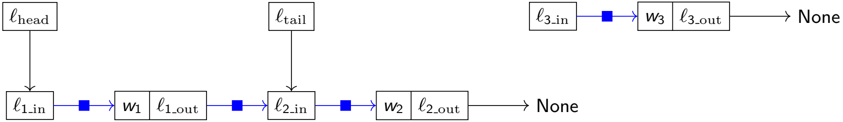

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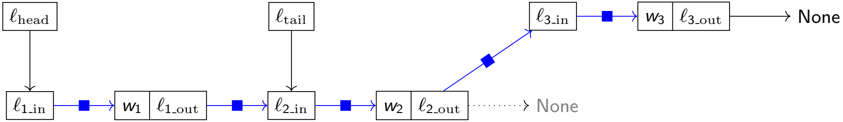

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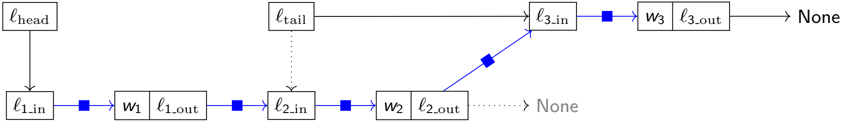

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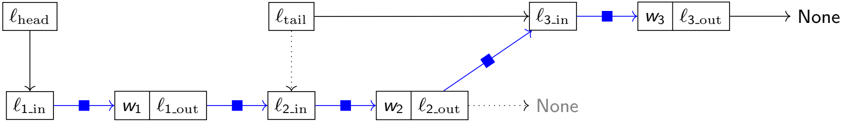

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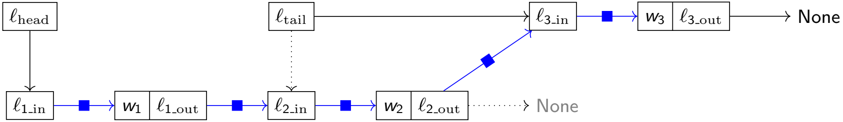

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- dequeue ignores tail pointer $\rightarrow$ Tail node can lag behind head node

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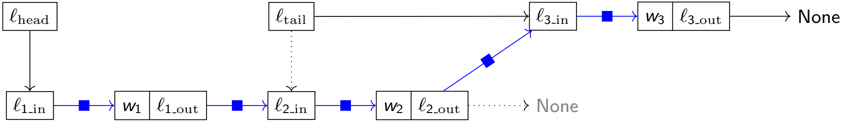

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- The dequeue function checks if queue is empty
  - If empty, return None
  - Else, swing head pointer to new head node, and return its value

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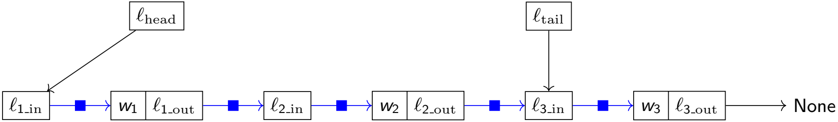

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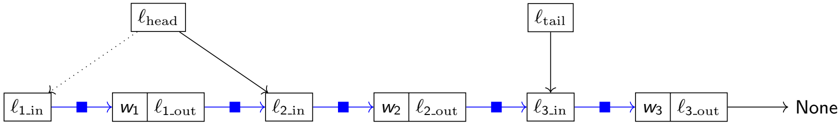

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- Dequeued node not freed → Linked list only grows

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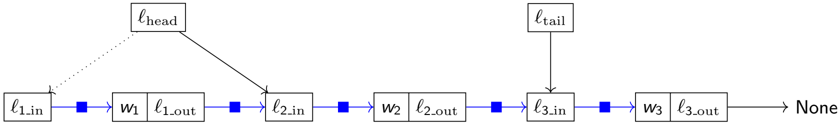

Proving that the Two-Lock Michael-Scott Queue Satisfies the HOCAP-style Specification

The isLL Predicate

- Idea: express the structure of the linked list in terms of points-to predicates
- Also captures persistent and non-persistent parts of the linked list

Definition (Linked List Predicate)

$$
\begin{aligned}
\text{isLL\_chain}([]) &\triangleq \text{True} \\
\text{isLL\_chain}([x]) &\triangleq \text{in}(x) \mapsto^{\square} (\text{val}(x), \text{out}(x)) \\
\text{isLL\_chain}(x :: x' :: xs) &\triangleq \text{in}(x) \mapsto^{\square} (\text{val}(x), \text{out}(x)) * \text{out}(x') \mapsto^{\square} \text{in}(x) * \text{isLL\_chain}(x' :: xs)
\end{aligned}
$$

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\begin{aligned}
\text{isLL}([]) &\triangleq \text{True} \\
\text{isLL}(x :: xs) &\triangleq \text{out}(x) \mapsto \text{None} * \text{isLL\_chain}(x :: xs)
\end{aligned}
$$

Example

Consider the list: $xs = [(\ell_{3\_\mathrm{in}}, w_3, \ell_{3\_\mathrm{out}}); (\ell_{2\_\mathrm{in}}, w_2, \ell_{2\_\mathrm{out}}); (\ell_{1\_\mathrm{in}}, w_1, \ell_{1\_\mathrm{out}})]$.

$$
\begin{aligned}
\text{isLL}(xs) = \ \ell_{3\_\mathrm{out}} \mapsto \text{None} * {} & \ell_{3\_\mathrm{in}} \mapsto^{\square} (w_3, \ell_{3\_\mathrm{out}}) * \ell_{2\_\mathrm{out}} \mapsto^{\square} \ell_{3\_\mathrm{in}} * \\
& \ell_{2\_\mathrm{in}} \mapsto^{\square} (w_2, \ell_{2\_\mathrm{out}}) * \ell_{1\_\mathrm{out}} \mapsto^{\square} \ell_{2\_\mathrm{in}} * \\
& \ell_{1\_\mathrm{in}} \mapsto^{\square} (w_1, \ell_{1\_\mathrm{out}})
\end{aligned}
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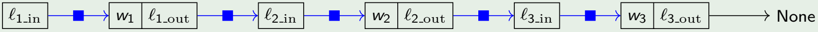

Invariant

- Queue predicate must be persistent (according to specification)
- Problem: the queue relies on non-persistent resources (e.g. $\ell_{\mathrm{head}} \mapsto \mathsf{in}(x_{\mathrm{head}})$)
- Solution: identify an *invariant* (persistent), describing the resources

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- Contains abstract state of queue – existentially quantified as it can change

Definition (Two-Lock M&S Queue HOCAP Invariant)

$$\mathrm{I}_{\mathrm{TLH}}(\ell_{\mathrm{head}}, \ell_{\mathrm{tail}}, G) \triangleq \exists xs_v.\ G.\gamma_{\mathrm{Abst}} \Mapsto_{\bullet} xs_v \ast \qquad \text{(abstract state)}$$

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- Defines structure of the concrete linked list, xs

Definition (Two-Lock M&S Queue HOCAP Invariant)

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\begin{aligned}
\mathsf{I}_{\mathsf{TLH}}(\ell_{\mathrm{head}}, \ell_{\mathrm{tail}}, G) \triangleq\ & \exists xs_v.\ G.\gamma_{\mathrm{Abst}} \Mapsto_\bullet xs_v \ * && \text{(abstract state)} \\
& \exists xs, xs_{\mathrm{queue}}, xs_{\mathrm{old}}, x_{\mathrm{head}}, x_{\mathrm{tail}}. && \text{(concrete state)} \\
& xs = xs_{\mathrm{queue}} +\!\!+ [x_{\mathrm{head}}] +\!\!+ xs_{\mathrm{old}} \ * \\
& \mathsf{isLL}(xs) \ *
\end{aligned}
$$

Invariant

- Queue predicate must be persistent (according to specification)
- Problem: the queue relies on non-persistent resources (e.g. $\ell_{\mathrm{head}} \mapsto \mathsf{in}(x_{\mathrm{head}})$)
- Solution: identify an *invariant* (persistent), describing the resources

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- Asserts relation between abstract state and concrete state

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- Contains abstract state of queue – existentially quantified as it can change
- Defines structure of the concrete linked list, xs
- Asserts relation between abstract state and concrete state
- Identifies possible queue states: **Static**, **Enqueue**, **Dequeue**, and **Both**
  - Two locks → Four queue states
  - Invariant describes the queue resources in each state
  - See next slide

Definition (Two-Lock M&S Queue HOCAP Invariant)

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& xs = xs_{\text{queue}} ++ [x_{\text{head}}] ++ xs_{\text{old}} * \\
& \text{isLL}(xs) * \\
& \text{projVal}(xs_{\text{queue}}) = \text{wrapSome}(xs_v) * \\
& \ldots
\end{aligned}
$$

Invariant (Queue States)

- Idea: the enqueueing thread keeps half of tail pointer between invariant openings
- Guarantees that the pointer is not updated (full pointer needed for update)
- Similarly for the dequeueing thread
- **Enqueue** and **Both** also captures "gap" between adding x_{new} and swinging ℓ_{tail}
- Tokens used to reason about which state queue is in

Definition (Two-Lock M&S Queue HOCAP Invariant – continued)

. . .

$$\begin{aligned}
&\ell_{\mathrm{head}} \mapsto \mathrm{in}(x_{\mathrm{head}}) * \ell_{\mathrm{tail}} \mapsto \mathrm{in}(x_{\mathrm{tail}}) * \mathrm{isLast}(x_{\mathrm{tail}}, xs) * && \textbf{(Static)}\\
&\quad \mathrm{TokNE}\ G * \mathrm{TokND}\ G * \mathrm{TokUpdated}\ G \\
\vee\ &\ell_{\mathrm{head}} \mapsto \mathrm{in}(x_{\mathrm{head}}) * \ell_{\mathrm{tail}} \mapsto^{\frac{1}{2}} \mathrm{in}(x_{\mathrm{tail}}) * && \textbf{(Enqueue)}\\
&\quad (\mathrm{isLast}(x_{\mathrm{tail}}, xs) * \mathrm{TokBefore}\ G \vee \mathrm{isSndLast}(x_{\mathrm{tail}}, xs) * \mathrm{TokAfter}\ G) * \\
&\quad \mathrm{TokE}\ G * \mathrm{TokND}\ G \\
\vee\ &\ell_{\mathrm{head}} \mapsto^{\frac{1}{2}} \mathrm{in}(x_{\mathrm{head}}) * \ell_{\mathrm{tail}} \mapsto \mathrm{in}(x_{\mathrm{tail}}) * \mathrm{isLast}(x_{\mathrm{tail}}, xs) * && \textbf{(Dequeue)}\\
&\quad \mathrm{TokNE}\ G * \mathrm{TokD}\ G * \mathrm{TokUpdated}\ G \\
\vee\ &\ell_{\mathrm{head}} \mapsto^{\frac{1}{2}} \mathrm{in}(x_{\mathrm{head}}) * \ell_{\mathrm{tail}} \mapsto^{\frac{1}{2}} \mathrm{in}(x_{\mathrm{tail}}) * && \textbf{(Both)}\\
&\quad (\mathrm{isLast}(x_{\mathrm{tail}}, xs) * \mathrm{TokBefore}\ G \vee \mathrm{isSndLast}(x_{\mathrm{tail}}, xs) * \mathrm{TokAfter}\ G) * \\
&\quad \mathrm{TokE}\ G * \mathrm{TokD}\ G
\end{aligned}$$

Queue Predicate

- We now define the queue predicate in terms of our invariant

Definition (Two-Lock M&S Queue - isQueue Predicate)

$$
\begin{aligned}
\mathsf{isQueue}(v_q, G) \triangleq\ & \exists \ell_{\text{queue}}, \ell_{\text{head}}, \ell_{\text{tail}} \in Loc.\ \exists h_{\text{lock}}, t_{\text{lock}} \in Val. \\
& v_q = \ell_{\text{queue}} * \ell_{\text{queue}} \mapsto^{\square} ((\ell_{\text{head}}, \ell_{\text{tail}}), (h_{\text{lock}}, t_{\text{lock}})) * \\
& \boxed{\mathsf{I}_{\mathsf{TLH}}(\ell_{\text{head}}, \ell_{\text{tail}}, G)}^{\mathcal{N}.queue} * \\
& \mathsf{isLock}(G.\gamma_{\text{Hlock}}, h_{\text{lock}}, \mathsf{TokD}\ \ G) * \\
& \mathsf{isLock}(G.\gamma_{\text{Tlock}}, t_{\text{lock}}, \mathsf{TokE}\ \ G)
\end{aligned}
$$

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Definition (Two-Lock M&S Queue - isQueue Predicate)

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- The queue predicate is persistent, as all its constituents are

Queue Predicate

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& \boxed{\mathsf{I}_{\mathsf{TLH}}(\ell_{\text{head}}, \ell_{\text{tail}}, G)}^{\mathcal{N}.\mathit{queue}} * \\
& \mathsf{isLock}(G.\gamma_{\text{Hlock}}, h_{\text{lock}}, \mathsf{TokD}\ \ G) * \\
& \mathsf{isLock}(G.\gamma_{\text{Tlock}}, t_{\text{lock}}, \mathsf{TokE}\ \ G)
\end{aligned}
$$

- The queue predicate is persistent, as all its constituents are
- Proving that Two-Lock M&S Queue satisfies the HOCAP-style specification then consists of proving the Hoare triples for initialize, enqueue, and dequeue
- We here focus on enqueue

Proof Sketch of the Hoare Triple for enqueue

$$\forall v_q, v, G, P, Q. \quad \left(\forall xs_v.\ G.\gamma_{\mathrm{Abst}} \Mapsto_{\bullet} xs_v * P \Rrightarrow_{\mathcal{E}\setminus\mathcal{N}.i\uparrow} \triangleright G.\gamma_{\mathrm{Abst}} \Mapsto_{\bullet} (v :: xs_v) * Q\right) \twoheadrightarrow$$
$$\{\mathrm{isQueue}(v_q, G) * P\}\ \mathsf{enqueue}\ \ v_q\ \ v\ \{w.Q\}$$

Proof Sketch of the Hoare Triple for enqueue

$\forall v_q, v, G, P, Q.\quad (\forall xs_v.\ G.\gamma_{\mathrm{Abst}} \Rrightarrow_{\bullet} xs_v * P \Rrightarrow_{\mathcal{E}\setminus\mathcal{N}.i\uparrow} \triangleright G.\gamma_{\mathrm{Abst}} \Rrightarrow_{\bullet} (v :: xs_v) * Q) \twoheadrightarrow$
$\{\mathsf{isQueue}(v_q, G) * P\}$ enqueue $v_q\ v\ \{w.Q\}$

(Proof)

Assume the view-shift, and the persistent information in isQueue(v_q, *Qgnames*):

- $v_q = \ell_{\mathrm{queue}} * \ell_{\mathrm{queue}} \mapsto^{\Box} ((\ell_{\mathrm{head}}, \ell_{\mathrm{tail}}), (h_{\mathrm{lock}}, t_{\mathrm{lock}}))$
- $\boxed{\mathsf{I}_{\mathsf{TLH}}(\ell_{\mathrm{head}}, \ell_{\mathrm{tail}}, G)}^{\mathcal{N}.queue}$
- $\mathsf{isLock}(G.\gamma_{\mathrm{Tlock}}, t_{\mathrm{lock}}, \mathsf{TokE}\ \ G)$

Proof Sketch of the Hoare Triple for enqueue

$$\forall v_q, v, G, P, Q. \quad \left(\forall xs_v.\ G.\gamma_{\text{Abst}} \Mapsto_\bullet xs_v * P \Rrightarrow_{\mathcal{E}\setminus\mathcal{N}.i\uparrow} \triangleright G.\gamma_{\text{Abst}} \Mapsto_\bullet (v :: xs_v) * Q\right) \twoheadrightarrow$$
$$\{\text{isQueue}(v_q, G) * P\} \text{ enqueue } v_q\ v\ \{w.Q\}$$

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Assume the view-shift, and the persistent information in $\text{isQueue}(v_q, Qgnames)$:

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$\{P\}$

```
let node = ref (Some v, ref (None)) in

acquire(snd(snd(! v_q)));

e_t = !(snd(fst(! v_q)))

snd(!(e_t)) ← node;

snd(fst(! v_q)) ← node;

release(snd(snd(! v_q)))
```

$\{Q\}$

Proof Sketch of the Hoare Triple for enqueue

$$\forall v_q, v, G, P, Q.\quad (\forall xs_v.\ G.\gamma_{\mathrm{Abst}} \Rrightarrow_{\bullet} xs_v * P \Rrightarrow_{\mathcal{E}\setminus\mathcal{N}.i\uparrow} \triangleright G.\gamma_{\mathrm{Abst}} \Rrightarrow_{\bullet} (v :: xs_v) * Q) \twoheadrightarrow$$
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- $\mathrm{isLock}(G.\gamma_{\mathrm{Tlock}}, t_{\mathrm{lock}}, \mathrm{TokE}\ \ G)$

```
{P}
  let node = ref (Some v, ref (None)) in     (create node x_new)
{P * out(x_new) ↦ None}
  acquire(snd(snd(! v_q)));

  e_t = !(snd(fst(! v_q)))

  snd(!(e_t)) ← node;

  snd(fst(! v_q)) ← node;

  release(snd(snd(! v_q)))
{Q}
```

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$\forall v_q, v, G, P, Q.\quad (\forall xs_v.\ G.\gamma_{\mathrm{Abst}} \Mapsto_\bullet xs_v * P \Rrightarrow_{\mathcal{E}\setminus\mathcal{N}.i\uparrow} \triangleright G.\gamma_{\mathrm{Abst}} \Mapsto_\bullet (v :: xs_v) * Q) \mathrel{-\!\!*}$
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  let $node$ = ref (Some v, ref (None)) in   (create node x_{new})
$\{P * \mathrm{out}(x_{\mathrm{new}}) \mapsto \mathrm{None}\}$
  acquire(snd(snd(! v_q)));   (acquire tail lock)
$\{P * \mathrm{out}(x_{\mathrm{new}}) \mapsto \mathrm{None} * \mathrm{TokE}\ \ G\}$
  e_t = !(snd(fst(! v_q)))

  snd(!(e_t)) $\leftarrow$ $node$;

  snd(fst(! v_q)) $\leftarrow$ $node$;

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  $e_t = !(\text{snd}(\text{fst}(!\ v_q)))$ (find current tail, x_{tail}. $\mathrm{I}_{\mathrm{TLH}}$: **Static**/**Dequeue** $\to$ **Enqueue**/**Both** (before))

$\{P * \text{out}(x_{\text{new}}) \mapsto \text{None} * \ell_{\text{tail}} \mapsto^{\frac{1}{2}} \text{in}(x_{\text{tail}}) * \text{TokNE}\ \ G * \text{TokAfter}\ \ G\}$

  snd(!(e_t)) $\leftarrow$ *node*;

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snd(!(e_t)) $\leftarrow$ $node$; (make x_{tail} point to x_{new}. VS. $\mathrm{I_{TLH}}$: **Enqueue**/**Both** (before) $\to$ **Enqueue**/**Both** (after))

$\{Q * \ell_{\mathrm{tail}} \mapsto^{\frac{1}{2}} \mathrm{in}(x_{\mathrm{tail}}) * \mathrm{TokNE}\ \ G * \mathrm{TokBefore}\ \ G\}$

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Proof Sketch of the Hoare Triple for enqueue

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  e_t = !(snd(fst(! v_q)))   (find current tail, x_{tail}. $\mathrm{I_{TLH}}$: **Static**/**Dequeue** $\to$ **Enqueue**/**Both** (before))
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  snd(!(e_t)) $\leftarrow$ $node$;   (make x_{tail} point to x_{new}. VS. $\mathrm{I_{TLH}}$: **Enqueue**/**Both** (before) $\to$ **Enqueue**/**Both** (after))
$\{Q * \ell_{\mathrm{tail}} \mapsto^{\frac{1}{2}} \mathrm{in}(x_{\mathrm{tail}}) * \mathrm{TokNE}\ \ G * \mathrm{TokBefore}\ \ G\}$
  snd(fst(! v_q)) $\leftarrow$ $node$;   (swing tail pointer to x_{new}. $\mathrm{I_{TLH}}$: **Enqueue**/**Both** (after) $\to$ **Static**/**Dequeue**)
$\{Q * \mathrm{TokE}\ \ G\}$
  release(snd(snd(! v_q)))
$\{Q\}$

Proof Sketch of the Hoare Triple for enqueue

$$\forall v_q, v, G, P, Q. \quad (\forall xs_v.\ G.\gamma_{\mathrm{Abst}} \Mapsto_\bullet xs_v * P \Rrightarrow_{\mathcal{E}\setminus\mathcal{N}.i\uparrow} \triangleright G.\gamma_{\mathrm{Abst}} \Mapsto_\bullet (v :: xs_v) * Q) \twoheadrightarrow$$
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Assume the view-shift, and the persistent information in $\mathrm{isQueue}(v_q, Qgnames)$:

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  let *node* = ref (Some v, ref (None)) in (create node x_{new})

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  $e_t = !(\mathrm{snd}(\mathrm{fst}(!\ v_q)))$ (find current tail, x_{tail}. $\mathrm{I_{TLH}}$: **Static**/**Dequeue** $\rightarrow$ **Enqueue**/**Both** (before))

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  $\mathrm{snd}(!(e_t)) \leftarrow node;$ (make x_{tail} point to x_{new}. VS. $\mathrm{I_{TLH}}$: **Enqueue**/**Both** (before) $\rightarrow$ **Enqueue**/**Both** (after))

$\{Q * \ell_{\mathrm{tail}} \mapsto^{\frac{1}{2}} \mathrm{in}(x_{\mathrm{tail}}) * \mathrm{TokNE}\ \ G * \mathrm{TokBefore}\ \ G\}$

  $\mathrm{snd}(\mathrm{fst}(!\ v_q)) \leftarrow node;$ (swing tail pointer to x_{new}. $\mathrm{I_{TLH}}$: **Enqueue**/**Both** (after) $\rightarrow$ **Static**/**Dequeue**)

$\{Q * \mathrm{TokE}\ \ G\}$

  release(snd(snd(! v_q))) (release tail lock)

$\{Q\}$

The Lock-Free Michael-Scott Queue

Implementation (Consistency-Check Free)

- Queue data structure is still a linked list
- The lock-free versions of initialize, enqueue, and dequeue perform the same manipulations of the linked list as two-lock versions
- Difference is how the manipulations take place

Implementation (Consistency-Check Free)

- Queue data structure is still a linked list
- The lock-free versions of initialize, enqueue, and dequeue perform the same manipulations of the linked list as two-lock versions
- Difference is how the manipulations take place – now with CAS instructions
- Ensures no overwrites of enqueued nodes, and no node dequeued twice

```
initialize ≜
  let node = ref (None, ref (None)) in
  ref (ref (node), ref (node))

enqueue  Q  value ≜
  let node = ref (Some value, ref (None)) in
  (rec  loop_ =
    let tail = !(snd(! Q)) in
    let next = !(snd(! tail)) in
    if next = None then
      if CAS (snd(! tail)) next node then
        CAS (snd(! Q)) tail node
      else loop ()
    else CAS (snd(! Q)) tail next; loop ()
  ) ()
```

```
dequeue  Q ≜
  (rec  loop_ =
    let head = !(fst(! Q)) in
    let tail = !(snd(! Q)) in
    let next = !(snd(! head)) in
    if head = tail then
      if next = None then
        None
      else
        CAS(snd(! Q)) tail next; loop ()
    else
      let value = fst(! next) in
      if CAS (fst(! Q)) head next then
        value
      else loop ()
    )()
```

Consistency Checks and Prophecies

- Original implementation has consistency checks to deal with ABA problem
- Creates complications when proving adherence to HOCAP-style specification
- In dequeue, the point at which to apply the view-shift (to update P to Q) depends on whether queue is empty and result of future consistency check
- Prophecies allow us to reason about future computations (e.g. consistency check)

```
initialize ≜
  let node = ref (None, ref (None)) in
  ref (ref (node), ref (node))

enqueue  Q value ≜
  let node = ref (Some value, ref (None)) in
  (rec  loop_ =
    let tail = !(snd(! Q)) in
    let next = !(snd(! tail)) in
    if tail = !(snd(! Q)) then
      if next = None then
        if CAS (snd(! tail)) next node then
          CAS (snd(! Q)) tail node
         else loop ()
       else CAS (snd(! Q)) tail next; loop ()
     else loop ()
  ) ()
```

```
dequeue  Q ≜
  (rec  loop_ =
    let head = !(fst(! Q)) in
    let tail = !(snd(! Q)) in
    let p = newproph in
    let next = !(snd(! head)) in
    if head = Resolve(!(fst(! Q)), p, ()) then
      if head = tail then
        if next = None then
          None
         else
          CAS(snd(! Q)) tail next; loop ()
       else
        let value = fst(! next) in
        if CAS (fst(! Q)) head next then
          value
         else loop ()
     else loop ()
    )()
```

Proving that the Lock-Free Michael-Scott Queue Satisfies the HOCAP-style Specification

Reachability

- The queue relies on some important properties to function correctly:
  - The head and tail are only moved forward in the linked list
  - The tail cannot lag behind the head (unlike in the two-lock version)

Reachability

- The queue relies on some important properties to function correctly:
  - The head and tail are only moved forward in the linked list
  - The tail cannot lag behind the head (unlike in the two-lock version)
- We capture these properties with a notion of *reachability*
- Consists of a concrete and abstract version of reachability

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- Concrete reachability is reflexive and transitive

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Lemmas for Reachability (simplified)

$$x \rightsquigarrow x \Rrightarrow \exists \gamma.\, \gamma \rightarrowtail x \qquad \text{(Abs Reach Alloc)}$$

$$x_n \dashrightarrow \gamma_m * \gamma_m \rightarrowtail x_m \twoheadrightarrow x_n \rightsquigarrow x_m \qquad \text{(Abs Reach Concr)}$$

$$x_n \rightsquigarrow x_m * \gamma_m \rightarrowtail x_m \Rrightarrow x_n \dashrightarrow \gamma_m \qquad \text{(Abs Reach Abs)}$$

$$\gamma_m \rightarrowtail x_m * x_m \rightsquigarrow x_o \Rrightarrow \gamma_m \rightarrowtail x_o \qquad \text{(Abs Reach Advance)}$$

Queue Predicate, Invariant, and Proof of Hoare triples

In Coq!