Sequential Specification

Definition (Sequential Specification)

$\exists\, \text{isQueue}_S : \mathit{Val} \rightarrow \mathit{List}\ \mathit{Val} \rightarrow \mathit{SeqQgnames} \rightarrow \text{Prop}.$

- The proposition $\text{isQueue}_S(v_q, xs_v, G)$, states that value v_q represents the queue, which contains elements xs_v
- $G \in \mathit{SeqQgnames}$ is a collection of ghost names (depends on specific queue)
- Specification consists of three Hoare triples – one for each queue function
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$$\mathrm{dequeue}\ \ v_q$$

$$\left\{ w.\ \begin{array}{l} (xs_v = [] * w = \mathrm{None} * \mathrm{isQueue}_S(v_q, xs_v, G)) \vee \\ (\exists v, xs'_v.\ xs_v = xs'_v +\!\!+ [v] * w = \mathrm{Some}\ v * \mathrm{isQueue}_S(v_q, xs'_v, G)) \end{array} \right\}$$

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Queue Client - A PoC Client

- Add two numbers after having two threads enqueue and subsequently dequeue them

$$\text{unwrap}\ \ w \triangleq \textbf{match}\ w\ \textbf{with}\ \text{None} \Rightarrow ()\ () \mid \text{Some}\ v \Rightarrow v\ \textbf{end}$$

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$$\begin{aligned}&\text{queueAdd}\ \ a\ b \triangleq\\ &\quad \textbf{let}\ v_q = \text{initialize}\ ()\ \textbf{in}\\ &\quad \textbf{let}\ p = (\text{enqdeq}\ \ v_q\ \ a) \mid\mid (\text{enqdeq}\ \ v_q\ \ b)\ \textbf{in}\\ &\quad \textbf{fst}\ p + \textbf{snd}\ p\end{aligned}$$

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- HOCAP-style specification supports consistency and tracks queue contents, allowing us to exclude cases where dequeue returns None

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Queue Client - A PoC Client (continued)

Lemma (QueueAdd Specification)

$$\forall a, b \in \mathbb{Z}.\ \{True\}\ \text{queueAdd}\ \ a\ \ b\,\{v.v = a + b\}$$

Queue Client - A PoC Client (continued)

Lemma (QueueAdd Specification)

$$\forall a, b \in \mathbb{Z}. \{True\} \text{ queueAdd } a\ b \{v.v = a + b\}$$

- Proof idea: create invariant capturing possible states of queue contents
- Tokens are used to reason about which state we are in

Definition (Invariant for QueueAdd)

$$\begin{aligned} I_{QA}(G, Ga, a, b) \triangleq\ & G.\gamma_{\mathrm{Abst}} \mapsto_{\circ} [] \ast \text{TokD1 } Ga \ast \text{TokD2 } Ga \vee \\ & G.\gamma_{\mathrm{Abst}} \mapsto_{\circ} [a] \ast \text{TokA } Ga \ast (\text{TokD1 } Ga \vee \text{TokD2 } Ga) \vee \\ & G.\gamma_{\mathrm{Abst}} \mapsto_{\circ} [b] \ast \text{TokB } Ga \ast (\text{TokD1 } Ga \vee \text{TokD2 } Ga) \vee \\ & G.\gamma_{\mathrm{Abst}} \mapsto_{\circ} [a; b] \ast \text{TokA } Ga \ast \text{TokB } Ga \vee \\ & G.\gamma_{\mathrm{Abst}} \mapsto_{\circ} [b; a] \ast \text{TokB } Ga \ast \text{TokA } Ga \end{aligned}$$

Implementation: initialize

- Queue data structure is still a linked list
- The lock-free versions of initialize, enqueue, and dequeue perform the same manipulations of the linked list as two-lock versions
- Difference is how the manipulations take place – now with CAS instructions

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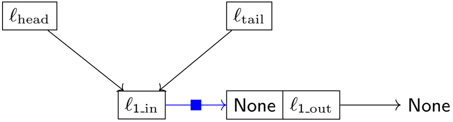
- The lock-free versions of initialize, enqueue, and dequeue perform the same manipulations of the linked list as two-lock versions
- Difference is how the manipulations take place – now with CAS instructions
- No longer need locks

---

initialize $\triangleq$

$\quad$ let *node* $=$ ref (None, ref (None)) in

$\quad$ ref (ref (*node*), ref (*node*))



Implementation: enqueue

- Appending x_{new} to linked list is now done with CAS
- Ensures that no other thread has performed an enqueue during own enqueue
  - Otherwise, we might "overwrite" another threads enqueued node
- Swinging tail to x_{new} might fail – another thread has helped us

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enqueue  Q value ≜
  let node = ref (Some value, ref (None)) in
  (rec  loop_ =
    let tail = !(snd(! Q)) in
    let next = !(snd(! tail)) in
    if tail = !(snd(! Q)) then
      if next = None then
        if CAS (snd(! tail)) next node then
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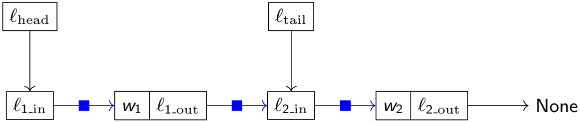

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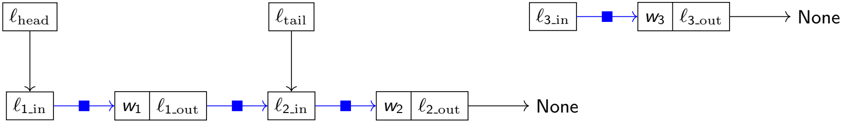

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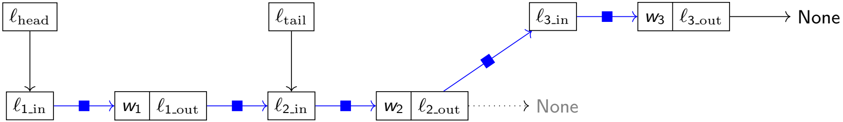

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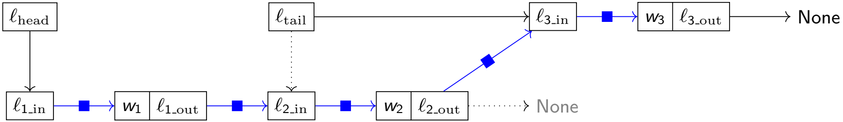

Implementation: dequeue

- Head now swung with CAS instruction
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    let p = newproph in
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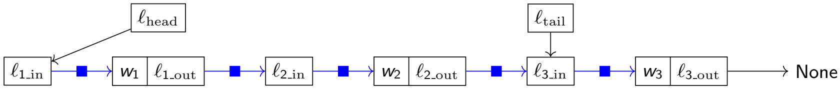

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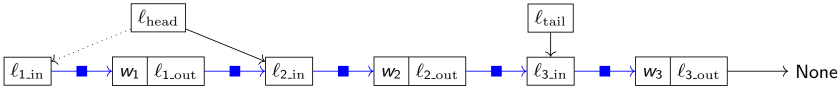

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- Proving adherence to HOCAP-style specification requires applying the view-shift at some point (must update P to Q)

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  - Consistency check on next line succeeds
- Prophecies: reason about future computations (e.g. the consistency check)
  - $!(\mathsf{fst}(!\,Q))$ will evaluate to some v_p (later proof obligation)
  - Before reading *next*, reason about whether $head = v_p$

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