Master's Thesis Exam
Verification of the Blocking and Non-Blocking Michael-Scott Queue Algorithms

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Overview of the Project and Contributions

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    - Useful for sequential clients
  - Concurrent specification
    - Proves safety of concurrent queues
    - Useful for *some* concurrent clients
    - Doesn't track queue contents
  - HOCAP-style specification
    - Stronger specification, useful for more complex clients
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- All proofs have been mechanised in the Coq proof assistant

Outline

1 Queue Specifications

2 The Two-Lock Michael-Scott Queue

3 Proving that the Two-Lock Michael-Scott Queue Satisfies the HOCAP-style Specification

4 The Lock-Free Michael-Scott Queue

5 Proving that the Lock-and-CC-Free Michael-Scott Queue Satisfies the HOCAP-style Specification

Queue Specifications

Specifications for Queues

Assumptions on Queues

- Queues consists of initialize, enqueue, and dequeue
- initialize creates an empty queue: []
- enqueue adds a value, v, to the beginning of the queue xs_v: $v :: xs_v$
- dequeue depends on whether queue is empty:
  - If non-empty, $xs_v ++ v$, remove value v at end of queue and return Some v
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Nature of Specifications

- Specifications written in Iris, a higher order CSL
- Expressed in terms of *Hoare triples*: $\{P\}\ e\ \{v.\Phi\ v\}$
- Hoare triples prove partial correctness of programs, e
- In particular: safety
- Idea: clients can use Hoare triples to prove results about their own code

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Owned by queue	Owned by client

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- Construction ensures:
  - authoritative and fragmental views always agree on abstract state of queue
  - views can only be updated in unison
- Implemented using the resource algebra: $\textsc{Auth}((\textsc{Frac} \times \textsc{Ag}(\mathit{List\ Val}))^?)$
- The desirables are captured by the following lemmas

Lemmas on the Abstract State RA

$$\vdash \Rrightarrow \exists \gamma.\, \gamma \Mapsto_{\bullet} xs_v * \gamma \Mapsto_{\circ} xs_v \qquad \text{(Abstract State Alloc)}$$

$$\gamma \Mapsto_{\bullet} xs_v' * \gamma \Mapsto_{\circ} xs_v \vdash xs_v = xs_v' \qquad \text{(Abstract State Agree)}$$

$$\gamma \Mapsto_{\bullet} xs_v' * \gamma \Mapsto_{\circ} xs_v \Rrightarrow \gamma \Mapsto_{\bullet} xs_v'' * \gamma \Mapsto_{\circ} xs_v'' \qquad \text{(Abstract State Update)}$$

HOCAP-style Specification

- Post-condition of initialize specification gives fragmental view to clients
- Hoare triples for enqueue and dequeue are conditioned on view-shifts
- Clients must show that they can supply the fragmental view, so that the abstract (and concrete) state can be updated
- View-shifts and Hoare-triples parametrised by predicates P and Q
  - Client might have resources that need to be updated as a result of enqueue/dequeue
  - P is the clients resources before enqueue/dequeue and Q the resources after

Definition (HOCAP Specification)

$\exists\, \text{isQueue} : \mathit{Val} \to \mathit{Qgnames} \to \text{Prop}.$

$$\forall v_q, G.\ \text{isQueue}(v_q, G) \implies \Box\, \text{isQueue}(v_q, G)$$

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& \forall v_q, G.\ \mathsf{isQueue}(v_q, G) \implies \Box\,\mathsf{isQueue}(v_q, G) \\
\wedge & \{\mathsf{True}\}\ \mathsf{initialize}\ ()\ \{v_q.\exists G.\ \mathsf{isQueue}(v_q, G) * G.\gamma_{\mathrm{Abst}} \Mapsto_\circ []\} \\
\wedge & \forall v_q, v, G, P, Q.\ \ \Big(\forall xs_v.\ G.\gamma_{\mathrm{Abst}} \Mapsto_\bullet xs_v * P \Rrightarrow_{\mathcal{E}\setminus\mathcal{N}.i\uparrow} \triangleright G.\gamma_{\mathrm{Abst}} \Mapsto_\bullet (v :: xs_v) * Q\Big) \mathrel{-\!\!*} \\
& \qquad \{\mathsf{isQueue}(v_q, G) * P\}\ \mathsf{enqueue}\ \ v_q\ \ v\ \{w.Q\}
\end{array}
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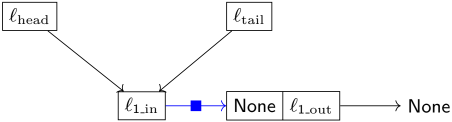
The Two-Lock Michael-Scott Queue

Implementation: initialize

- The queue data structure is a linked list

initialize $\triangleq$
  let *node* = ref (None, ref (None)) in
  let *H_lock* = newLock() in
  let *T_lock* = newLock() in
  ref ((ref (*node*), ref (*node*)), (*H_lock*, *T_lock*))



Implementation: initialize

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- A node x in the linked list is a triple, $x = (\ell_{\text{in}}, w, \ell_{\text{out}})$, with $\ell_{\text{in}} \mapsto (w, \ell_{\text{out}})$
- We use the following notation for nodes

$$\mathsf{in}(x) = \ell_{\text{in}} \qquad \mathsf{val}(x) = w \qquad \mathsf{out}(x) = \ell_{\text{out}}$$

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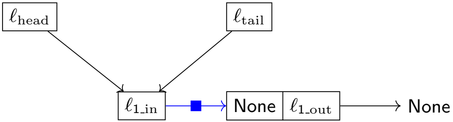

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- The initialize function first creates an initial head node, x_{head}
- Then, a lock protecting the head pointer, and a lock protecting the tail pointer
- Finally, it creates the head and tail pointers, ℓ_{head} and ℓ_{tail}, both pointing to x_{head}

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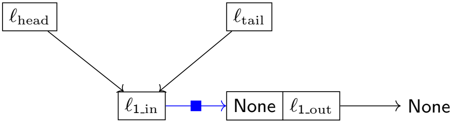

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  4. Swing tail pointer to x_{new}
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enqueue Q *value* $\triangleq$
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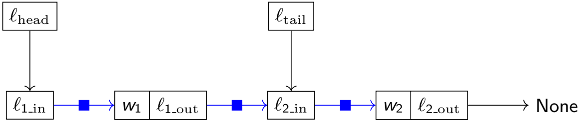

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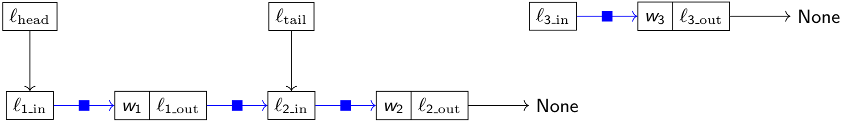

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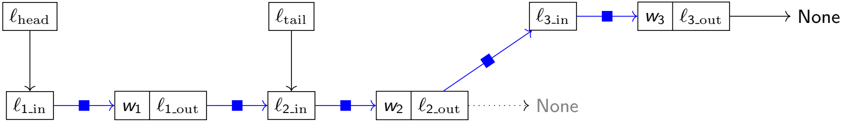

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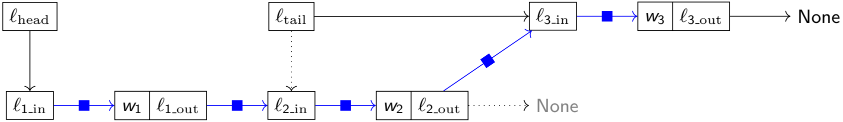

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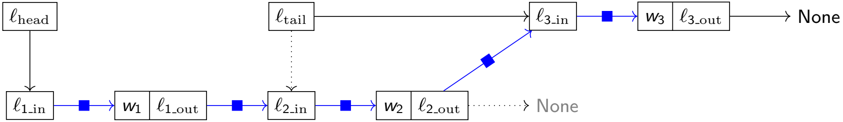

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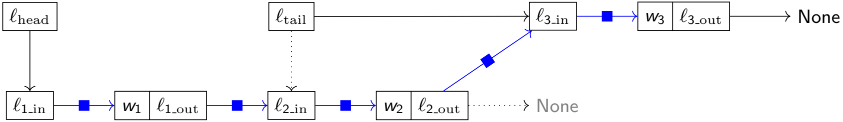

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- dequeue ignores tail pointer $\rightarrow$ Tail node can lag behind head node

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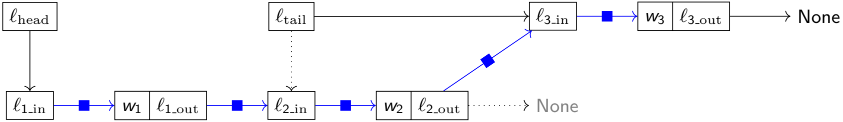

Implementation: dequeue

- The dequeue function checks if queue is empty
  - If empty, return None
  - Else, swing head pointer to new head node, and return its value

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  acquire(fst(snd(! Q)));
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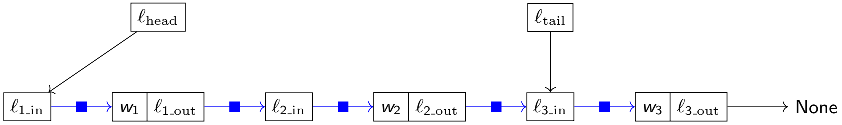

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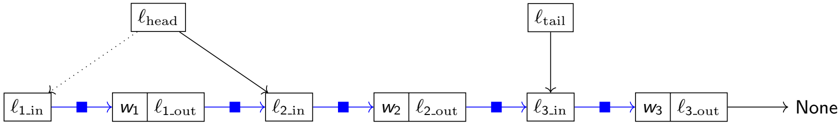

Implementation: dequeue

- The dequeue function checks if queue is empty
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- Dequeued node not freed → Linked list only grows

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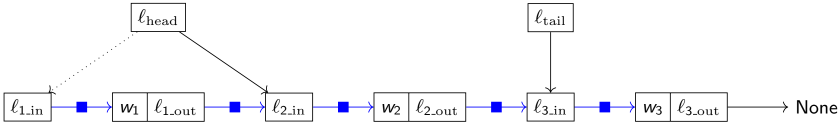


Proving that the Two-Lock Michael-Scott Queue Satisfies the HOCAP-style Specification

The isLL Predicate

- Idea: express the structure of the linked list in terms of points-to predicates
- Also captures persistent and non-persistent parts of the linked list

Definition (Linked List Predicate)

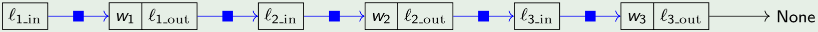
$$\begin{aligned}
\text{isLL_chain}([]) &\triangleq \text{True} \\
\text{isLL_chain}([x]) &\triangleq \text{in}(x) \mapsto^{\square} (\text{val}(x), \text{out}(x)) \\
\text{isLL_chain}(x :: x' :: xs) &\triangleq \text{in}(x) \mapsto^{\square} (\text{val}(x), \text{out}(x)) * \text{out}(x') \mapsto^{\square} \text{in}(x) * \text{isLL_chain}(x' :: xs)
\end{aligned}$$

$$\begin{aligned}
\text{isLL}([]) &\triangleq \text{True} \\
\text{isLL}(x :: xs) &\triangleq \text{out}(x) \mapsto \text{None} * \text{isLL_chain}(x :: xs)
\end{aligned}$$

Example

Consider the list: $xs = [(\ell_{3\_\text{in}}, w_3, \ell_{3\_\text{out}}); (\ell_{2\_\text{in}}, w_2, \ell_{2\_\text{out}}); (\ell_{1\_\text{in}}, w_1, \ell_{1\_\text{out}})]$.

$$\begin{aligned}
\text{isLL}(xs) = \; & \ell_{3_\text{out}} \mapsto \text{None} * \ell_{3_\text{in}} \mapsto^{\square} (w_3, \ell_{3_\text{out}}) * \ell_{2_\text{out}} \mapsto^{\square} \ell_{3_\text{in}} * \\
& \ell_{2_\text{in}} \mapsto^{\square} (w_2, \ell_{2_\text{out}}) * \ell_{1_\text{out}} \mapsto^{\square} \ell_{2_\text{in}} * \\
& \ell_{1_\text{in}} \mapsto^{\square} (w_1, \ell_{1_\text{out}})
\end{aligned}$$



Invariant

- Queue predicate must be persistent (according to specification)
- Problem: the queue relies on non-persistent resources (e.g. $\ell_{\mathrm{head}} \mapsto \mathsf{in}(x_{\mathrm{head}})$)
- Solution: identify an *invariant* (persistent), describing the resources

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- Contains abstract state of queue – existentially quantified as it can change

Definition (Two-Lock M&S Queue HOCAP Invariant)

$$\mathsf{I}_{\mathsf{TLH}}(\ell_{\mathrm{head}}, \ell_{\mathrm{tail}}, G) \triangleq \exists xs_v.\ G.\gamma_{\mathrm{Abst}} \Mapsto_\bullet xs_v \ast \qquad \text{(abstract state)}$$

Invariant

- Queue predicate must be persistent (according to specification)
- Problem: the queue relies on non-persistent resources (e.g. $\ell_{\mathrm{head}} \mapsto \mathsf{in}(x_{\mathrm{head}})$)
- Solution: identify an *invariant* (persistent), describing the resources

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$$\begin{aligned} I_{\mathrm{TLH}}(\ell_{\mathrm{head}}, \ell_{\mathrm{tail}}, G) \triangleq\ & \exists xs_v.\ G.\gamma_{\mathrm{Abst}} \Mapsto_\bullet xs_v * && \text{(abstract state)} \\ & \exists xs, xs_{\mathrm{queue}}, xs_{\mathrm{old}}, x_{\mathrm{head}}, x_{\mathrm{tail}}. && \text{(concrete state)} \\ & xs = xs_{\mathrm{queue}} +\!\!+ [x_{\mathrm{head}}] +\!\!+ xs_{\mathrm{old}} * \\ & \mathsf{isLL}(xs) * \end{aligned}$$

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- Contains abstract state of queue – existentially quantified as it can change
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- Asserts relation between abstract state and concrete state

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\mathsf{I}_{\mathsf{TLH}}(\ell_{\text{head}}, \ell_{\text{tail}}, G) \triangleq\ & \exists xs_v.\ G.\gamma_{\text{Abst}} \Mapsto_\bullet xs_v \ast & \text{(abstract state)}\\
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& xs = xs_{\text{queue}} +\!\!+ [x_{\text{head}}] +\!\!+ xs_{\text{old}} \ast &\\
& \text{isLL}(xs) \ast &\\
& \text{projVal}(xs_{\text{queue}}) = \text{wrapSome}(xs_v) \ast &
\end{aligned}
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- Queue predicate must be persistent (according to specification)
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- Contains abstract state of queue – existentially quantified as it can change
- Defines structure of the concrete linked list, *xsc*
- Asserts relation between abstract state and concrete state
- Identifies possible queue states: **Static**, **Enqueue**, **Dequeue**, and **Both**
  - Two locks → Four queue states
  - Invariant describes the queue resources in each state
  - See next slide

Definition (Two-Lock M&S Queue HOCAP Invariant)

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& \mathsf{isLL}(xs) * \\
& \mathsf{projVal}(xs_{\text{queue}}) = \mathsf{wrapSome}(xs_v) * \\
& \ldots
\end{aligned}
$$

Invariant (Queue States)

- Idea: the enqueueing thread keeps half of tail pointer between invariant openings
- Guarantees that the pointer is not updated (full pointer needed for update)
- Similarly for the dequeueing thread
- **Enqueue** and **Both** also captures "gap" between adding x_{new} and swinging ℓ_{tail}
- Tokens used to reason about which state queue is in

Definition (Two-Lock M&S Queue HOCAP Invariant – continued)

...

$$\ell_{\mathrm{head}} \mapsto \mathrm{in}(x_{\mathrm{head}}) * \ell_{\mathrm{tail}} \mapsto \mathrm{in}(x_{\mathrm{tail}}) * \mathrm{isLast}(x_{\mathrm{tail}}, xs) * \mathrm{TokNE}\ G * \mathrm{TokND}\ G * \mathrm{TokUpdated}\ G \qquad \text{(Static)}$$

$$\vee\ \ell_{\mathrm{head}} \mapsto \mathrm{in}(x_{\mathrm{head}}) * \ell_{\mathrm{tail}} \mapsto^{\frac{1}{2}} \mathrm{in}(x_{\mathrm{tail}}) * (\mathrm{isLast}(x_{\mathrm{tail}}, xs) * \mathrm{TokBefore}\ G \vee \mathrm{isSndLast}(x_{\mathrm{tail}}, xs) * \mathrm{TokAfter}\ G) * \mathrm{TokE}\ G * \mathrm{TokND}\ G \qquad \text{(Enqueue)}$$

$$\vee\ \ell_{\mathrm{head}} \mapsto^{\frac{1}{2}} \mathrm{in}(x_{\mathrm{head}}) * \ell_{\mathrm{tail}} \mapsto \mathrm{in}(x_{\mathrm{tail}}) * \mathrm{isLast}(x_{\mathrm{tail}}, xs) * \mathrm{TokNE}\ G * \mathrm{TokD}\ G * \mathrm{TokUpdated}\ G \qquad \text{(Dequeue)}$$

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Queue Predicate

- HOCAP-style specification requires the existence of a persistent queue predicate
- We define it in terms of our invariant

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Definition (Two-Lock M&S Queue - isQueue Predicate)

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\begin{aligned}
\mathsf{isQueue}(v_q, G) \triangleq\ & \exists \ell_{\mathrm{queue}}, \ell_{\mathrm{head}}, \ell_{\mathrm{tail}} \in \mathit{Loc}.\ \exists h_{\mathrm{lock}}, t_{\mathrm{lock}} \in \mathit{Val}. \\
& v_q = \ell_{\mathrm{queue}} * \ell_{\mathrm{queue}} \mapsto^{\square} ((\ell_{\mathrm{head}}, \ell_{\mathrm{tail}}), (h_{\mathrm{lock}}, t_{\mathrm{lock}})) * \\
& \boxed{\mathsf{I}_{\mathsf{TLH}}(\ell_{\mathrm{head}}, \ell_{\mathrm{tail}}, G)}^{\mathcal{N}.\mathit{queue}} * \\
& \mathsf{isLock}(G.\gamma_{\mathrm{Hlock}}, h_{\mathrm{lock}}, \mathsf{TokD}\ \ G) * \\
& \mathsf{isLock}(G.\gamma_{\mathrm{Tlock}}, t_{\mathrm{lock}}, \mathsf{TokE}\ \ G)
\end{aligned}
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Definition (Two-Lock M&S Queue - isQueue Predicate)

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& v_q = \ell_{\text{queue}} * \ell_{\text{queue}} \mapsto^{\Box} ((\ell_{\text{head}}, \ell_{\text{tail}}), (h_{\text{lock}}, t_{\text{lock}})) * \\
& \boxed{\mathsf{I}_{\mathsf{TLH}}(\ell_{\text{head}}, \ell_{\text{tail}}, G)}^{\mathcal{N}.\mathit{queue}} * \\
& \text{isLock}(G.\gamma_{\text{Hlock}}, h_{\text{lock}}, \text{TokD}\ \ G) * \\
& \text{isLock}(G.\gamma_{\text{Tlock}}, t_{\text{lock}}, \text{TokE}\ \ G)
\end{aligned}
$$

- The queue predicate is persistent, as all its constituents are
- Proving that Two-Lock M&S Queue satisfies the HOCAP-style specification then consists of proving the Hoare triples for initialize, enqueue, and dequeue
- We here focus on enqueue

Proof Sketch of the Hoare Triple for enqueue

$$\forall v_q, v, G, P, Q.\quad \left(\forall xs_v.\ G.\gamma_{\mathrm{Abst}} \Mapsto_{\bullet} xs_v * P \Rrightarrow_{\mathcal{E}\setminus\mathcal{N}.i\uparrow} \triangleright G.\gamma_{\mathrm{Abst}} \Mapsto_{\bullet} (v :: xs_v) * Q\right) \twoheadrightarrow$$
$$\{\mathrm{isQueue}(v_q, G) * P\}\ \mathrm{enqueue}\ \ v_q\ \ v\ \{w.Q\}$$

Proof Sketch of the Hoare Triple for enqueue

$\forall v_q, v, G, P, Q.\quad (\forall xs_v.\ G.\gamma_{\mathrm{Abst}} \Mapsto_\bullet xs_v * P \Rrightarrow_{\mathcal{E}\setminus\mathcal{N}.i\uparrow} \triangleright G.\gamma_{\mathrm{Abst}} \Mapsto_\bullet (v :: xs_v) * Q) \twoheadrightarrow$
$\{\mathrm{isQueue}(v_q, G) * P\}$ enqueue $v_q\ v\ \{w.Q\}$

(Proof)

Assume the view-shift, and the persistent information in $\mathrm{isQueue}(v_q, Qgnames)$:

- $v_q = \ell_{\mathrm{queue}} * \ell_{\mathrm{queue}} \mapsto^{\square} ((\ell_{\mathrm{head}}, \ell_{\mathrm{tail}}), (h_{\mathrm{lock}}, t_{\mathrm{lock}}))$
- $\boxed{\mathrm{I}_{\mathrm{TLH}}(\ell_{\mathrm{head}}, \ell_{\mathrm{tail}}, G)}^{\mathcal{N}.queue}$
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$\forall v_q, v, G, P, Q.\quad (\forall xs_v.\ G.\gamma_{\text{Abst}} \Mapsto_\bullet xs_v * P \Rrightarrow_{\mathcal{E}\setminus\mathcal{N}.i\uparrow} \triangleright G.\gamma_{\text{Abst}} \Mapsto_\bullet (v :: xs_v) * Q) \twoheadrightarrow$
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$\{P\}$
```
let node = ref (Some v, ref (None)) in

acquire(snd(snd(! v_q)));

e_t = !(snd(fst(! v_q)))

snd(!(e_t)) ← node;

snd(fst(! v_q)) ← node;

release(snd(snd(! v_q)))
```
$\{Q\}$

Proof Sketch of the Hoare Triple for enqueue

$$\forall v_q, v, G, P, Q.\quad \dfrac{\begin{array}{l}(\forall xs_v.\ G.\gamma_{\mathrm{Abst}} \Mapsto_\bullet xs_v * P \Rrightarrow_{\mathcal{E}\setminus\mathcal{N}.i\uparrow} \triangleright G.\gamma_{\mathrm{Abst}} \Mapsto_\bullet (v :: xs_v) * Q) \twoheadrightarrow \\ \{\mathrm{isQueue}(v_q, G) * P\}\ \mathrm{enqueue}\ \ v_q\ \ v\ \{w.Q\}\end{array}}{}$$

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$\{P\}$

  let $node$ = ref (Some v, ref (None)) in   (create node x_{new})

$\{P * \mathrm{out}(x_{\mathrm{new}}) \mapsto \mathrm{None}\}$

  acquire(snd(snd(! v_q)));

  e_t = !(snd(fst(! v_q)))

  snd(!(e_t)) ← $node$;

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Assume the view-shift, and the persistent information in $\mathsf{isQueue}(v_q, Qgnames)$:

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  let $node$ = ref (Some v, ref (None)) in   (create node x_{new})
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  acquire(snd(snd(! v_q)));   (acquire tail lock)
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  e_t = !(snd(fst(! v_q)))   (find current tail, x_{tail}. I_{TLH}: **Static**/**Dequeue** → **Enqueue**/**Both** (before))
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  snd(!(e_t)) $\leftarrow$ $node$;   (make x_{tail} point to x_{new}. $\mathsf{I_{TLH}}$: **Enqueue**/**Both** (before) $\to$ **Enqueue**/**Both** (after))

$\{Q * \ell_{\mathrm{tail}} \mapsto^{\frac{1}{2}} \mathsf{in}(x_{\mathrm{tail}}) * \mathsf{TokNE}\ \ G * \mathsf{TokBefore}\ \ G\}$

  snd(fst(! v_q)) $\leftarrow$ $node$;

  release(snd(snd(! v_q)))

$\{Q\}$

Proof Sketch of the Hoare Triple for enqueue

$$\forall v_q, v, G, P, Q.\quad (\forall xs_v.\ G.\gamma_{\mathrm{Abst}} \Mapsto_\bullet xs_v * P \Rrightarrow_{\mathcal{E}\setminus\mathcal{N}.i\uparrow} \triangleright G.\gamma_{\mathrm{Abst}} \Mapsto_\bullet (v :: xs_v) * Q) \twoheadrightarrow \{\mathrm{isQueue}(v_q, G) * P\}\ \mathrm{enqueue}\ \ v_q\ v\ \{w.Q\}$$

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Assume the view-shift, and the persistent information in $\mathrm{isQueue}(v_q, Qgnames)$:

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  e_t = !(snd(fst(! v_q))) (find current tail, x_{tail}. $\mathrm{I_{TLH}}$: **Static/Dequeue** → **Enqueue/Both** (before))

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  snd(!(e_t)) ← *node*; (make x_{tail} point to x_{new}. $\mathrm{I_{TLH}}$: **Enqueue/Both** (before) → **Enqueue/Both** (after))

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  snd(fst(! v_q)) ← *node*; (swing tail pointer to x_{new}. $\mathrm{I_{TLH}}$: **Enqueue/Both** (after) → **Static/Dequeue**)

$\{Q * \mathrm{TokE}\ \ G\}$

  release(snd(snd(! v_q)))

$\{Q\}$

Proof Sketch of the Hoare Triple for enqueue

$\forall v_q, v, G, P, Q.\quad (\forall xs_v.\ G.\gamma_{\text{Abst}} \Mapsto_{\bullet} xs_v * P \Rrightarrow_{\mathcal{E}\setminus\mathcal{N}.i\uparrow} \triangleright G.\gamma_{\text{Abst}} \Mapsto_{\bullet} (v :: xs_v) * Q) \twoheadrightarrow$
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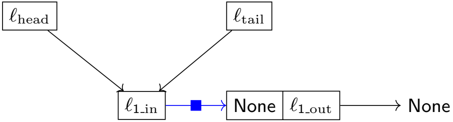
  snd(fst(! v_q)) ← $node$;   (swing tail pointer to x_{new}. I_{TLH}: **Enqueue/Both** (after) → **Static/Dequeue**)
$\{Q * \ \text{TokE}\ \ G\}$
  release(snd(snd(! v_q)))   (release tail lock)
$\{Q\}$

The Lock-Free Michael-Scott Queue

Implementation: initialize

- Queue data structure is still a linked list
- The lock-free versions of initialize, enqueue, and dequeue perform the same manipulations of the linked list as two-lock versions
- Difference is how the manipulations take place – now with CAS instructions
- No longer need locks

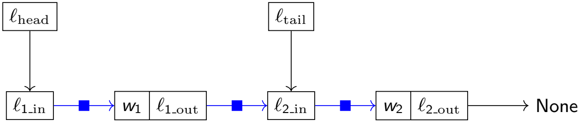
initialize $\triangleq$
  let *node* = ref (None, ref (None)) in
  ref (ref (*node*), ref (*node*))



Implementation: enqueue

- Appending x_{new} to linked list is now done with CAS
- Ensures that no other thread has performed an enqueue during own enqueue
  - Otherwise, we might "overwrite" another threads enqueued node
- Swinging tail to x_{new} might fail – another thread has helped us

```
enqueue  Q value ≜
  let node = ref (Some value, ref (None)) in
  (rec  loop_ =
    let tail = !(snd(! Q)) in
    let next = !(snd(! tail)) in
    if tail = !(snd(! Q)) then
      if next = None then
        if CAS (snd(! tail)) next node then
          CAS (snd(! Q)) tail node
        else loop ()
      else CAS (snd(! Q)) tail next; loop ()
    else loop ()
  ) ()
```



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         else loop ()
       else CAS (snd(! Q)) tail next; loop ()
     else loop ()
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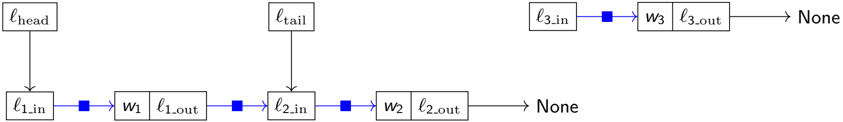

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    let next = !(snd(! tail)) in
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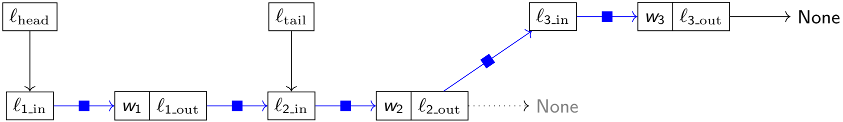

Implementation: enqueue

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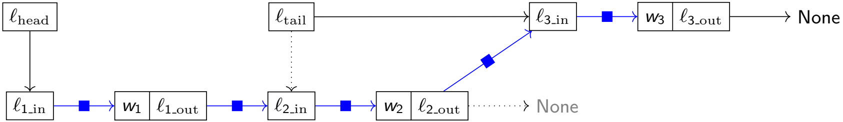

Implementation: dequeue

- Head now swung with CAS instruction
- Ensures that no other thread has dequeued the element we are trying to dequeue

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    let p = newproph in
    let next = !(snd(! head)) in
    if head = Resolve(!(fst(! Q)), p, ()) then
      if head = tail then
        if next = None then
          None
        else
          CAS(snd(! Q)) tail next; loop ()
      else
        let value = fst(! next) in
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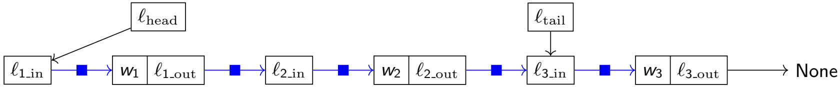

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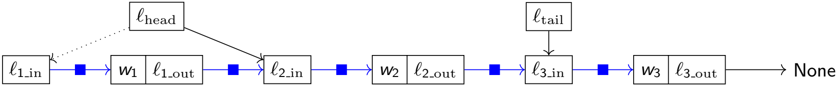

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- Prophecies: reason about future computations (e.g. the consistency check)
  - $!(\mathsf{fst}(!\,Q))$ will evaluate to some v_p (later proof obligation)
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Proving that the Lock-and-CC-Free Michael-Scott Queue Satisfies the HOCAP-style Specification

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- The queue relies on some important properties to function correctly:
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- Concrete reachability is reflexive and transitive

Reachability (continued)

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Lemmas for Reachability (simplified)

$$x \rightsquigarrow x \Rrightarrow \exists \gamma.\, \gamma \rightarrowtail x \qquad \text{(Abs Reach Alloc)}$$

$$x_n \dashrightarrow \gamma_m * \gamma_m \rightarrowtail x_m \mathrel{-\!\!*} x_n \rightsquigarrow x_m \qquad \text{(Abs Reach Concr)}$$

$$x_n \rightsquigarrow x_m * \gamma_m \rightarrowtail x_m \Rrightarrow x_n \dashrightarrow \gamma_m \qquad \text{(Abs Reach Abs)}$$

$$\gamma_m \rightarrowtail x_m * x_m \rightsquigarrow x_o \Rrightarrow \gamma_m \rightarrowtail x_o \qquad \text{(Abs Reach Advance)}$$

In Coq!