TITLE HERE

Mathias Pedersen, 201808137

Master's Thesis, Computer Science
February 2024
Advisor: Amin Timany

AARHUS
UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE

Abstract

►in English…◄

Resumé

►in Danish. . . ◄

Acknowledgments

►...◄

Mathias Pedersen
Aarhus, February 2024.

Contents

Chapter 1

Introduction

►motivate and explain the problem to be addressed◄
►example of a citation: [1]◄ ►get your bibtex entries from `https://dblp.org/`◄

Chapter 2

The Great Ideas

The Two-Lock Michael Scott Queue

I present here he an implementation of the Two-lock MS-Queue in HeapLang. This implementation differs slightly from the original, presented in [1], but most changes simply reflect the differences in the two languages.

The queue consists of 3 functions: initialize, enqueue, and dequeue, as well as an auxiliary function get_some. get_some simply extracts the contents of an option value, crashing if it is none. I now present each of the three main functions in turn.

initialize

The underlying data structure making up the queue is a singly-linked list. The linked-list will always contain at least one element, called the *sentinel* node, marking the beginning of the queue. Note that the sentinel node is itself not part of the queue, but all nodes following it are. The queue keeps a head pointer (ℓ_{head}) which always points to the sentinel, and a tail pointer (ℓ_{tail}) which points to some node in the linked list.

initialize will first create a single node – the sentinel – marking the start of the linked list. It then creates two locks, H_lock and T_lock, protecting the head and tail pointers, respectively. Finally, it creates the head and tail pointers, both pointing to the sentinel. The queue is then a pointer to a structure containing the head, the tail, and the two locks.

Figure 2.1 illustrates the structure of the queue after initialisation.

Note that in the original specification, a queue is a pointer to a 4-tuple $(\ell_{head}, \ell_{tail}, H\_lock, T\_lock)$. Since HeapLang doesn't support 4-tuples, we instead represent the queue as a pointer to a pair of pairs: $((\ell_{head}, \ell_{tail}), (H\_lock, T\_lock))$.

enqueue

►explain enqueue◄

The reader may wonder why there is an extra, intermediary pointer, between nodes of the linked list, and why the head and tail pointers point to these instead of the pairs making up the linked list. In the original implementation, nodes are allocated on the heap. To simulate this in HeapLang, when creating a new node, we create a pointer to a pair making up the node. Now, in the C-like language used in the original specification, an assignment operator is available, which is not present in HeapLang. So in order to mimic this behaviour, we model variables as pointers. In this way, we can model a variable x as a location ℓ_x, and the value stored at ℓ_x is the current value of x. This

means that the variable head in the original becomes a location ℓ_{head}, and the value stored at the location is what head is currently assigned to. Since head is a pointer, then the value saved at that location will also be a pointer.

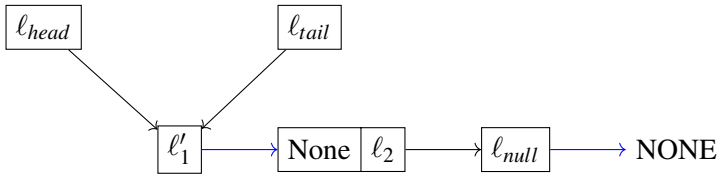

Figure 2.1: Queue after initialisation

$$
\begin{array}{l}
\texttt{let}\,\text{initialize} := \\
\qquad \texttt{let}\, node = \texttt{ref}\,(\text{Some}(\text{None}, \texttt{ref}\,(\texttt{ref}\,(\text{None})))) \,\texttt{in} \\
\qquad \texttt{let}\, H\_lock = newlock() \,\texttt{in} \\
\qquad \texttt{let}\, T\_lock = newlock() \,\texttt{in} \\
\qquad \texttt{ref}\,((\texttt{ref}\,(node), \texttt{ref}\,(node)), (H\_lock, T\_lock))
\end{array}
$$

$$
\begin{array}{l}
\texttt{let}\,\text{enqueue}\ Q\ value := \\
\qquad \texttt{let}\, node = \texttt{ref}\,(\text{Some}(\text{Some}\, value, \texttt{ref}\,(\texttt{ref}\,(\text{None})))) \,\texttt{in} \\
\qquad \text{acquire}(\texttt{snd}(\texttt{snd}(!Q))); \\
\qquad \texttt{snd}(\text{get\_some}\,!(!(\texttt{snd}(\texttt{fst}(!Q))))) \leftarrow node; \\
\qquad \texttt{snd}(\texttt{fst}(!Q)) \leftarrow node; \\
\qquad \text{release}(\texttt{snd}(\texttt{snd}(!Q)))
\end{array}
$$

$$
\begin{array}{l}
\texttt{let}\,\text{dequeue}\ Q := \\
\qquad \text{acquire}(\texttt{fst}(\texttt{snd}(!Q))); \\
\qquad \texttt{let}\, node = !(\texttt{fst}(\texttt{fst}(!Q))) \,\texttt{in} \\
\qquad \texttt{let}\, new\_head = !(\texttt{snd}(\text{get\_some}(!node))) \,\texttt{in} \\
\qquad \texttt{if}\, !new\_head = \text{None}\ \texttt{then} \\
\qquad\qquad \text{release}(\texttt{fst}(\texttt{snd}(!Q))); \\
\qquad\qquad \text{None} \\
\qquad \texttt{else} \\
\qquad\qquad \texttt{let}\, value = \texttt{fst}(\text{get\_some}(!new\_head)) \,\texttt{in} \\
\qquad\qquad \texttt{fst}(\texttt{fst}(!Q)) \leftarrow new\_head; \\
\qquad\qquad \text{release}(\texttt{fst}(\texttt{snd}(!Q))); \\
\qquad\qquad value
\end{array}
$$

Key insights

1. Head always points to the first node in the queue.

2. Tail always point to either the last or second last node in the queue.

3. All but the last pointer in the queue (the pointer to null) never change

Insight 2 is true, as it holds initially, and every time a new node is enqueued, the tail pointer is updated to point to the last node, and no other nodes can be enqueue before the update takes place. However, just after the new node has been linked to the queue, the tail queue will be pointing to the second last node in the queue, until it is updated in the next line.

Insight 3 means that we can mark all pointers in the queue (except the pointer to the null node) as persistent.

Key insights (concurrent)

1. All the same as before

2. The tail can lag one node behind Head

3. At any given time, the queue is in one of four states:

   (a) No threads are interacting with the queue (**Static**)

   (b) A thread is enqueueing (**Enqueue**)

   (c) A thread is dequeuing (**Dequeue**)

   (d) A thread is enqueueing and a thread is dequeuing (**Both**)

Insight 2 might seem a little surprising, and indeed it stands in contrast to property 5 in [1], which states that the tail always points to a node in the linked list. I also didn't realise this possibility until a proof attempt using a model that "forgot" old nodes lead to an unprovable case (see section 2). The situation can occur when the queue is empty, and a thread performs an incomplete enqueue; it attaches the new node to the end, but before it can swing the tail node to this new node, another thread performs a dequeue, which dequeues this new node, swings the head to it. Now the tail is lagging a node behind the head.

It is not possible for the tail to point more than one node behind the head, as in order for this to happen, more nodes must be enqueued, but this can't happen before the current enqueue finishes, which will update the tail and bring it up to speed with the head.

Fortunately, this isn't an issue for safety, but a consequence of this possibility is that when modelling the queue, we must remember at least one "old" node (i.e. a dequeued node), as the tail might be pointing to this node. For the sake of simplicity in the model, the choice is made to remember an arbitrary amount of old nodes, which is represented by the list xs_{old}.

Discussing the need for xs_{old}

As mentioned in the insights, it is possible for the tail to lag one node behind the head. This insight lead to including the old nodes of the queue in the queue invariant. This addition manifests in the end of the proof of dequeue. When we open the invariant to

swing the head to the new node, we get that the entire queue is xs. After performing the write, we can then close the invariant with the same xs that we opened the queue to (just written differently to signify that x_{head} is now "old"). Because of this, we can supply the same predicate concerning the $tail$ (the or) that we opened the queue with, since these only mention xs, which remains the same.

Had we not used an xs_{old} and essentially just "forgotten" old nodes in the queue, we couldn't have done this. Say that we defined xs as $xs = x_{head} :: xs_{rest}$ instead. Then, once we have to close the invariant, we cannot supply the xs, which we got when we opened the invariant. Our only choice (due to the fact that $head$ must point to x_{n_head}) is to close the invariant with $xs' = xs_{rest} = x_{n_head} :: xs''_{n_rest}$. However, clearly $xs' \neq xs$, so we cannot supply the same predicate concerning the $tail$ (the or) that we got when opening the invariant, since this predicate talks about xs, not xs'. Now, if we opened the invariant in the Dequeue case, then we could assert that $lastxs = lastxs'$, and hence still be able close the invariant. However, if we opened the invariant in the Both case, then we would need to assert that $2lastxs = 2lastxs'$. This is however not provable, since it might be the case that xs''_{n_rest} is empty, and hence $2lastxs'$ is $None$, whereas $2lastxs = x_{n_head}$.

Chapter 3

Conclusion

►conclude on the problem statement from the introduction◄

Bibliography

[1] Maged M. Michael and Michael L. Scott. Simple, fast, and practical non-blocking and blocking concurrent queue algorithms. In James E. Burns and Yoram Moses, editors, *Proceedings of the Fifteenth Annual ACM Symposium on Principles of Distributed Computing, Philadelphia, Pennsylvania, USA, May 23-26, 1996*, pages 267–275. ACM, 1996.

Appendix A

The Technical Details

►…◄