

The Best Queue Specifications You Will Ever See today probably

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November 2025



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 - Sequential specification
 - Concurrent specification
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 - HOCAP-style specification
 - Tracks queue contents with added complexity

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 - **Concurrent** specification
 - **Doesn't track** queue contents
 - **HOCAP-style** specification
 - **Tracks** queue contents with added **complexity**
- Uses **HeapLang**, but should be mostly language-agnostic
- Project was advised by Amin

Specifications for Queues

Informal Queue Specification

- Queues consists of **initialize**, **enqueue**, and **dequeue**
- **initialize** creates an **empty queue**: `[]`
- **enqueue** adds a value, v , to the **beginning of the queue** $xs_v: v :: xs_v$
- **dequeue** depends on whether queue is empty:
 - If **non-empty**, $xs_v ++ [v]$, remove value v at **end of queue** and return **Some v**
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Nature of Specifications

- Specifications written in **Iris**, a **higher order CSL**
- Expressed in terms of **Hoare triples**: $\{P\} e \{v. \Phi v\}$
- Hoare triples prove **partial correctness** of programs, e
- In particular: **safety**

Sequential Specification

Definition (Sequential Specification)

$\exists \text{isQueue}_S : \text{Val} \rightarrow \text{List Val} \rightarrow \text{SeqQnames} \rightarrow \text{Prop.}$

- The proposition $\text{isQueue}_S(v_q, xs_v, G)$, states that value v_q represents the queue, which contains elements xs_v
- $G \in \text{SeqQnames}$ is a collection of ghost names (depends on specific queue)
- Specification consists of three Hoare triples – one for each queue function
- **Important:** isQueue_S not required to be persistent!

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$\left\{ w. \begin{array}{l} (xs_v = [] * w = \text{None} * \text{isQueue}_S(v_q, xs_v, G)) \vee \\ (\exists v, xs'_v. xs_v = xs'_v ++ [v] * w = \text{Some } v * \text{isQueue}_S(v_q, xs'_v, G)) \end{array} \right\}$

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Concurrent Specification

- To support **concurrent clients**, we shall require the **queue predicate** be **persistent**
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- Idea: have **two** “**views**” of the **abstract state** of the queue

Authoritative view

$$\gamma \Vdash_{\bullet} x s_v$$

Owned by queue

Fragmental view

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- Construction **ensures**:
 - **authoritative** and **fragmental** views always **agree** on abstract state of queue
 - views can only be **updated** in **unison**
- **Implemented** using the **resource algebra**: $\text{AUTH}((\text{FRAC} \times \text{AG}(\text{List Val}))^?)$
- The **desirables** are captured by the following **lemmas**

Lemmas on the Abstract State RA

$$\vdash \Vdash \exists \gamma. \gamma \Vdash_{\bullet} xs_v * \gamma \Vdash_{\circ} xs_v \quad (\text{Abstract State Alloc})$$

$$\gamma \Vdash_{\bullet} xs'_v * \gamma \Vdash_{\circ} xs_v \vdash xs_v = xs'_v \quad (\text{Abstract State Agree})$$

$$\gamma \Vdash_{\bullet} xs'_v * \gamma \Vdash_{\circ} xs_v \Rightarrow \gamma \Vdash_{\bullet} xs''_v * \gamma \Vdash_{\circ} xs''_v \quad (\text{Abstract State Update})$$

HOCAP-style Specification

- Post-condition of **initialize** specification gives **fragmental view** to **clients**
- Hoare triples for **enqueue** and **dequeue** are conditioned on **view-shifts**
- Clients must show that they can **supply** the **fragmental view**, so that the **abstract** (and concrete) **state** can be **updated**
- View-shifts and Hoare-triples **parametrised** by predicates **P** and **Q**
 - Client might have **resources** that need to be **updated** as a result of **enqueue/dequeue**
 - **P** is the clients resources **before enqueue/dequeue** and **Q** the resources **after**

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Queue Client - A PoC Client

- Add two numbers after having two threads enqueue and subsequently dequeue them

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unwrap w  $\triangleq$  match w with None  $\Rightarrow$  () () | Some v  $\Rightarrow$  v end
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enqdeq vq c  $\triangleq$  enqueue vq c; unwrap(dequeue vq)
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queueAdd a b  $\triangleq$ 
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- HOCAP-style specification supports consistency and tracks queue contents, allowing us to exclude cases where dequeue returns None

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Queue Client - A PoC Client (continued)

Lemma (QueueAdd Specification)

$$\forall a, b \in \mathbb{Z}. \{True\} \text{queueAdd } a \ b \{v.v = a + b\}$$

Queue Client - A PoC Client (continued)

Lemma (QueueAdd Specification)

$$\forall a, b \in \mathbb{Z}. \{True\} \text{ queueAdd } a \ b \{v.v = a + b\}$$

- **Proof idea:** create **invariant** capturing possible **states of queue contents**
- **Tokens** are used to reason about which **state** we are in

Definition (Invariant for QueueAdd)

$$\begin{aligned} I_{QA}(G, Ga, a, b) &\triangleq G.\gamma_{\text{Abst}} \Rightarrow_{\circ} [] * \text{TokD1 } Ga * \text{TokD2 } Ga \vee \\ &\quad G.\gamma_{\text{Abst}} \Rightarrow_{\circ} [a] * \text{TokA } Ga * (\text{TokD1 } Ga \vee \text{TokD2 } Ga) \vee \\ &\quad G.\gamma_{\text{Abst}} \Rightarrow_{\circ} [b] * \text{TokB } Ga * (\text{TokD1 } Ga \vee \text{TokD2 } Ga) \vee \\ &\quad G.\gamma_{\text{Abst}} \Rightarrow_{\circ} [a; b] * \text{TokA } Ga * \text{TokB } Ga \vee \\ &\quad G.\gamma_{\text{Abst}} \Rightarrow_{\circ} [b; a] * \text{TokB } Ga * \text{TokA } Ga \end{aligned}$$

- When using the HOCAP-style Queue specification to prove the above, we will make P and Q talk about the tokens.
- E.g for enqueue:
 - $P = \text{TokA } Ga \vee \text{TokB } Ga$
 - $Q = \text{TokD1 } Ga \vee \text{TokD2 } Ga$

Queue Specifications Overview

Spec\Feature	Supports Tracking	Supports Concurrency	
Sequential	✓	✗	
Concurrent	✗	✓	
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- HOCAP generalises Sequential and Concurrent Specs
- In fact, they are provably derivable from HOCAP

HOCAP derives Sequential

Definition (Sequential Specification (reminder))

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$\wedge \{ \text{True} \} \text{initialize } () \{ v_q, \exists G. \text{isQueue}(v_q, G) * G.\gamma_{\text{Abst}} \mapsto_{\circ} [] \}$

$\wedge \forall v_q, v, G, P, Q. \left(\forall x_{s_v}. G.\gamma_{\text{Abst}} \mapsto_{\bullet} x_{s_v} * P \Rightarrow_{\mathcal{E} \setminus \mathcal{N}.i \uparrow} \triangleright G.\gamma_{\text{Abst}} \mapsto_{\bullet} (v :: x_{s_v}) * Q \right) \multimap$
 $\{ \text{isQueue}(v_q, G) * P \} \text{enqueue } v_q \ v \{ w.Q \}$

$\wedge \forall v_q, G, P, Q.$
 $\left(\forall x_{s_v}. G.\gamma_{\text{Abst}} \mapsto_{\bullet} x_{s_v} * P \Rightarrow_{\mathcal{E} \setminus \mathcal{N}.i \uparrow} \triangleright \left(\begin{array}{l} (x_{s_v} = [] * G.\gamma_{\text{Abst}} \mapsto_{\bullet} x_{s_v} * Q(\text{None})) \\ \vee \left(\begin{array}{l} \exists v, x'_{s_v}. x_{s_v} = x'_{s_v} ++ [v] * \\ G.\gamma_{\text{Abst}} \mapsto_{\bullet} x'_{s_v} * Q(\text{Some } v) \end{array} \right) \end{array} \right) \right) \multimap$
 $\{ \text{isQueue}(v_q, G) * P \} \text{dequeue } v_q \{ w.Q(w) \}$

- Chose $\text{isQueues}(v_q, x_{s_v}, G) = \text{isQueue}(v_q, G) * G.\gamma_{\text{Abst}} \mapsto_{\circ} x_{s_v}$
- For enqueue, pick
 - $P = G.\gamma_{\text{Abst}} \mapsto_{\circ} x_{s_v}$
 - $Q = G.\gamma_{\text{Abst}} \mapsto_{\circ} v :: x_{s_v}$

HOCAP derives Sequential

Definition (HOCAP Specification)

$\exists \text{isQueue} : \text{Val} \rightarrow \text{Qgnames} \rightarrow \text{Prop.}$

$$\forall v_q, G. \text{isQueue}(v_q, G) \implies \Box \text{isQueue}(v_q, G)$$

$$\wedge \{ \text{True} \} \text{initialize } () \{ v_q, \exists G. \text{isQueue}(v_q, G) * G.\gamma_{\text{Abst}} \mapsto_{\circ} [] \}$$

$$\wedge \forall v_q, v, G, P, Q. \left(\forall x_{s_v}. G.\gamma_{\text{Abst}} \mapsto_{\bullet} x_{s_v} * P \Rightarrow_{\mathcal{E} \setminus \mathcal{N}.i \uparrow} \triangleright G.\gamma_{\text{Abst}} \mapsto_{\bullet} (v :: x_{s_v}) * Q \right) \multimap \{ \text{isQueue}(v_q, G) * P \} \text{enqueue } v_q \ v \{ w.Q \}$$

$$\wedge \forall v_q, G, P, Q. \left(\forall x_{s_v}. G.\gamma_{\text{Abst}} \mapsto_{\bullet} x_{s_v} * P \Rightarrow_{\mathcal{E} \setminus \mathcal{N}.i \uparrow} \triangleright \left(\begin{array}{c} (x_{s_v} = [] * G.\gamma_{\text{Abst}} \mapsto_{\bullet} x_{s_v} * Q(\text{None})) \\ \vee \left(\begin{array}{c} \exists v, x_{s'_v}. x_{s_v} = x_{s'_v} ++ [v] * \\ G.\gamma_{\text{Abst}} \mapsto_{\bullet} x_{s'_v} * Q(\text{Some } v) \end{array} \right) \end{array} \right) \right) \multimap \{ \text{isQueue}(v_q, G) * P \} \text{dequeue } v_q \{ w.Q(w) \}$$

■ Chose $\text{isQueues}(v_q, x_{s_v}, G) = \text{isQueue}(v_q, G) * G.\gamma_{\text{Abst}} \mapsto_{\circ} x_{s_v}$

■ For enqueue, pick

■ $P = G.\gamma_{\text{Abst}} \mapsto_{\circ} x_{s_v}$

■ $Q = G.\gamma_{\text{Abst}} \mapsto_{\circ} v :: x_{s_v}$

■ For dequeue, pick

■ $P = G.\gamma_{\text{Abst}} \mapsto_{\circ} x_{s_v}$

■ $Q = G.\gamma_{\text{Abst}} \mapsto_{\circ} [] \vee (\exists v, x_{s'_v}. x_{s_v} = x_{s'_v} ++ [v] * G.\gamma_{\text{Abst}} \mapsto_{\circ} x_{s'_v})$

HOCAP derives Concurrent

Left as an exercise :)

Thanks for your time

Questions?