Optimization of parallel two-qubit gates in neutral atoms

QuEra challenge - ETH Quantum Hackathon

Embedded Memes
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Abstract

In this brief report we will discuss our achievements during the 2025 ETH Quantum Hackathon, which we took part in by joining the QuEra challenge. In short, we have found a general paradigm to increase the parallelization efficiency of ladders of CNOT gates, and we proved it by applying the general rule to four specific scenarios. he examples that were taken into account included the Quantum Fourier Transfrom and the Quantum Approximate Optimization Algorithm, both generalized to an arbitrary number of qubits, along with the encoding algorithms of the distance-5 color code and the distance-3 surface code[1].

0 A - Logarithmic-reduced depth of a CNOTs ladder

Given the relative frequency with which structures of CNOT gates resembling a ladder appeared, we focused on determining an efficient approach that could enable better parallelizability of the circuit. Drawing inspiration from the GHZ state preparation example, and noticing that in that case great simplification comes thanks to having identical target states prepared in $|0\rangle$, we realized that such improvement could only come at a price. We identified the cost of this optimization in the requirement of ancilla qubits, namely a number in the order of n ancillas for n qubits. CNOT gates amongs those "helpers", all initialized in $|0\rangle$, can be performed requiring logarithm depth in the number of qubits, as shown in the image below. After their initialization, each of them can control a

[1] We do acknowledge that the circuit schemes reported here can not be read easily. However, the code that produced them, as well as the .qasm files storing them can be found or recreated using the other material we are uploading. The circuits have been designed inside jupyter notebooks and they have then been saved in qasm files. From there, they were loaded into other notebooks to verify that they were indeed equivalent to the given circuit.

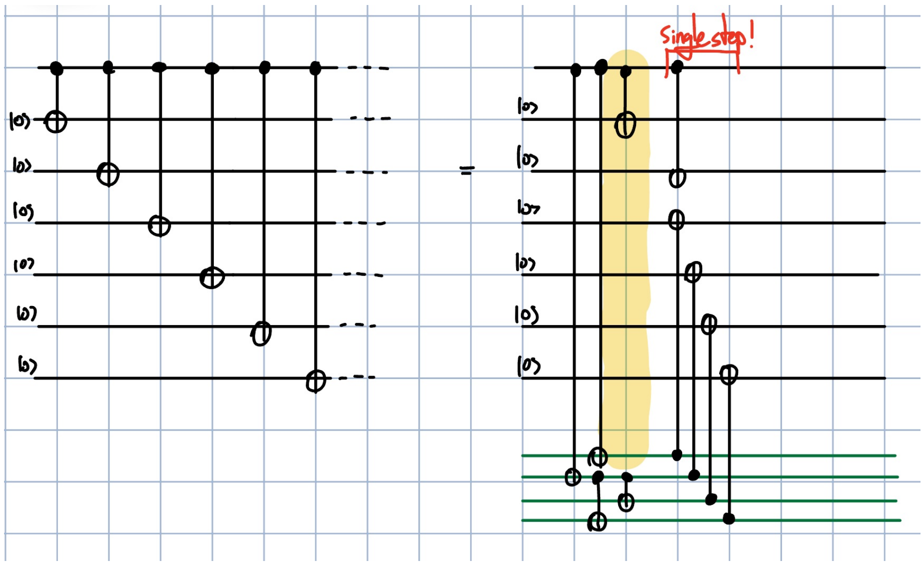


Figure 1: Logarithmic depth reduction of the CNOT ladder written on the right. The use of many ancilla qubits is necessary but provides an advantege at high qubit number.

CNOT targeting a qubit, allowing for a constant-depth final step of the protocol. Reasonably, this scheme becomes advantageous at large qubit numbers, reaching the same depth as the usual CNOTs ladder at $n = 5$ and improving on it from there on. It is worth noticing that, if the number of ancillas can't be expressed as $2^m - 1$, with $m \in \mathbf{N}$, the logarithmic advantage is not fully exploited; however, the blank spots highlighted in the picture can be filled with additional, parallel 2-qubit gates to further optimize the performances. Keep in mind that in some cases it may be needed to perform those CNOT cascades after performing some operations on the target qubits but *not* on the control qubit; in that case, the ancilla qubits are already initialized and the CNOT ladder can be performed again in a single step. You will find examples on that in the QFT and QAOA circuits.

0 B - Sources of errors

In optimizing the given circuits, the following error sources, in decreasing order of importance, have been taken into account:

- Heating arising from moving the atoms around (two qubit gates are the major error source);
- Single qubit gates with individual ion addressability;

- Single qubit gates performed globally

Initially, we were considering also the inability to swap two ions at the same time (their trajectories can never cross). Thus, the algorithm could be in principle optimized to minimize the needed moves in a practical implementation. However, being this factor neglected in the simulation of errors provided, we did not stick to this plan till the end. Unfortunately, the software to simulate the errors produced during the execution of the circuit was not available, thus we ended up evaluating the efficiency of our results based on the depth of the circuit and the number of required gates (keeping in mind that two qubit gates are those that result in the largest errors).

1 QFT circuit

Consider the initial circuit in the figure 2. By replacing the controlled-rotation

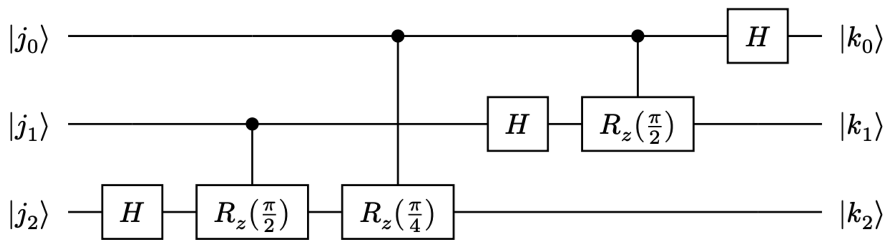


Figure 2: Circuit of the Quantum Fourier Transform on three qubits.

around the z axis with an equivalent circuit made of native gates (figure 3), exploiting the circuit identity of figure 4 (proof given in the appendix), and commuting the CNOT gates having the same control atom, we were able to obtain the CNOT ladder presented in the first section. That structure, can then be optimized as already discussed. After all these steps, the circuit was

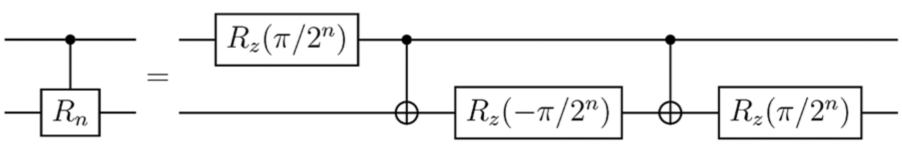


Figure 3: Circuitry equivalency used in the paper by B. Park and D. Ahn, Sci.Rep 13 (2023).

reduced to the one shown in figure 5. Bear in mind that the circuit becomes increasingly efficient for large numbers of qubits. Below, an approximate relation for the number of two-qubits layers needed for an optimal n-qubit QFT circuit.

$$\text{number of two-qubits layers} = \sum_{n=3} \big(2 + 2 * \lceil \log_2(n-1) \rceil\big) + 2 \tag{1}$$

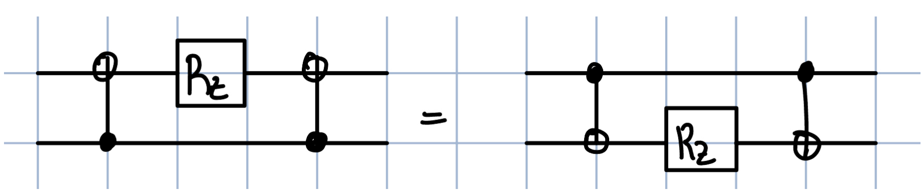


Figure 4: Circuit equivalency adopted in optimizing both the QFT and the QAOA algorithm. Proof given in the appendix

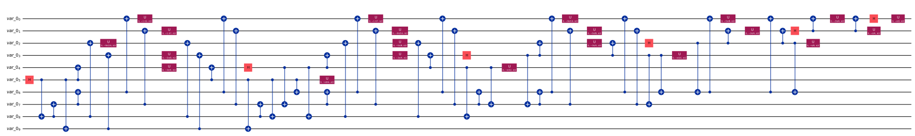


Figure 5: Circuit to perform the QFT on 6 qubits in a parallelized manner.

The term $\lceil \log_2(n-1) \rceil$ accounts for the layers needed to "copy" the information of the control qubit to the ancilla qubits (which is sped up exponentially); one step is needed for the parallel CNOTs towards the qubits; another one is identical after single qubit gates performed on the target qubits; lastly, the other term $\lceil \log_2(n-1) \rceil$ is the number of layers needed to disentangle all the ancillas from the qubits. The spare two is the number of layers for the 2 qubits case. Using the package qiskit, we have simulated the circuit starting from different states of the standard basis and we printed the density matrix of the state yielded as an output. By comparing this density matrix with that produced by the initial, unoptimized circuit, we have verified that our implementation is indeed equivalent to the desired QFT.

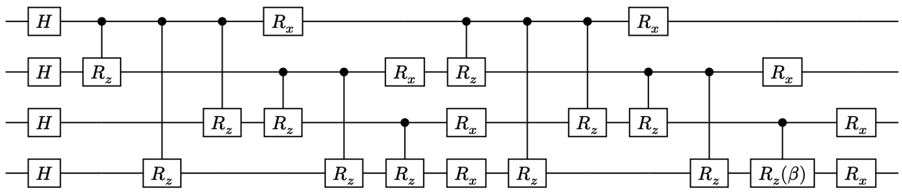


Figure 6: Starting point for the QAOA circuit.

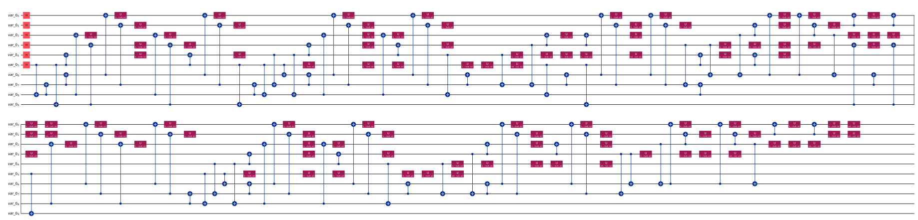


Figure 7: Optimized QAOA circuit, reducing the depth from 28 to 10.

2 QAOA circuit

Very much alike the previous section, we took as a starting point the circuit in figure 6 and translated the control-Z rotations into native gates as previously shown. From there, we were able to gather the CNOTs sharing the same control qubit together, recreating the ladder-like structure. It is worth noticing that the Z-rotation on the control qubit that is needed according to figure 3 can be rearranged so that it aligns with the last Z-rotation on the target qubit of the same figure. With a bit of care, one can ensure that those single qubit gates are performed as global single qubit gates, with an additional advantage on the error committed by the circuit. The resulting circuit is drawn in figure 7. Again, the efficiency of the circuit grows at higher qubit numbers, according to the same relation for the number of two-qubit gates layers of the QFT circuit, written in eqution 1. Similarly to what we have discussed before, we simulated our circuit starting from different initial states of the standard basis and printed the density matrix of the final state. By comparison with the one yielded by the initial, unoptimized circuit, we confirmed that our implementation is equivalent to the one we were given.

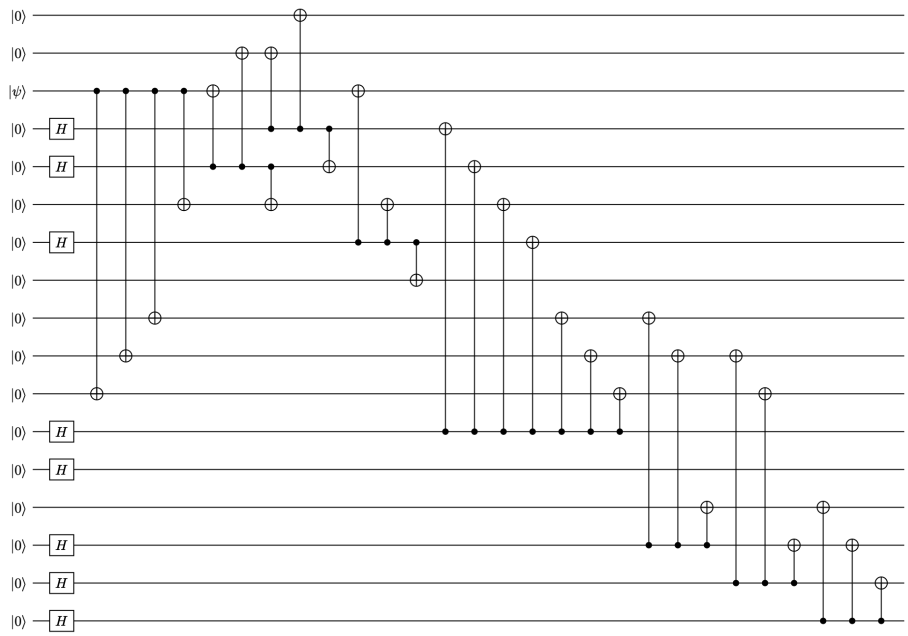


Figure 8: Starting point of task 3

3 Distance-5 color code

The third task did require a different approach than the previous two. Its main focus was on reorganizing the two-qubit gates according to allowed rules so that we could produce an equivalent circuit that could implement more two-qubit gates simultaneously. The circuit we have been facing is reproduced in figure 8. Keeping in mind that CNOTs with the same control qubit do commute, while those who are concatenated (control-target shared) or share the target qubit do not, we arrived to the circuit displayed in figure 9.

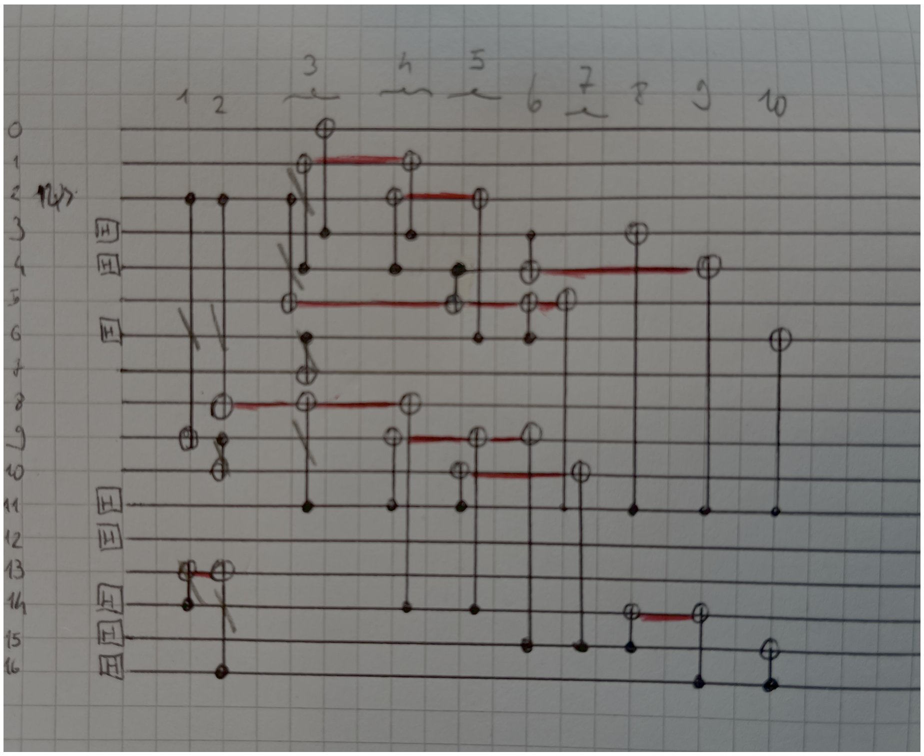


Figure 9: Optimized circuit for task 3. As you can see, many two-qubit gates have been gathered because they address different qubits, and thus they can be performed simultaneously. Red lines highlight further optimization opportunities that are specific for the neutral atom platform: taking into account that CNOT gates are translated into CZ gates with Hadamard gates on the target qubit, some Hadamard can be simplified. Notice the marks on top of the circuit: they highlight those two-qubit gates that can be performed simultaneously, providing an advantage over the previous architecture.

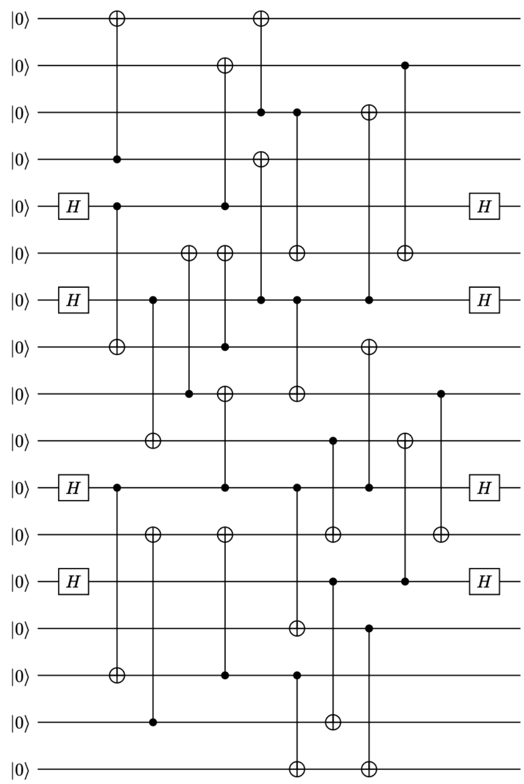


Figure 10: Initial, unoptimized circuit for the distance-3 color code.

4 Distance-3 surface code

Similarly to the third task, this also required to find an optimal configuration of the two qubit gates, to perform the highest possible number of CNOTs at the same time. The starting point is reported in figure 10. Unfortunately, the optimization we performed did not provided an improvement as large as those we experienced in the previous task. The circuit we obtained is shown in figure 11.

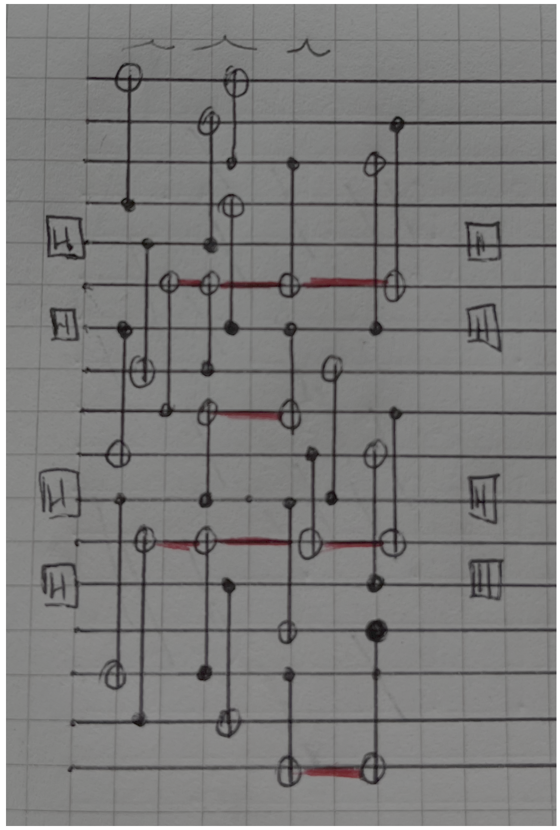


Figure 11: Optimized version of the task 4 circuit. Notice both the red marks we have explained at the previous point and the signs on top marking those gates that can be performed simultaneously.

A Propagating Z rotations through CNOT's

If there is a z-rotation, it can be propagated without collecting any commutators across an even number of CNOT's because Z commutes with x^2. To use this trick you have to verify that the CNOTs are not interleaved with something that does not commute with Z.

B Swapping CNOTs with Z rotations only inside

We have proved that the pair of CNOTs interleaved with a Z rotation can be swapped (meaning that the control qubit can now become the target qubit and vice versa, given that also the Z rotation is switched). Note that if the Z rotation

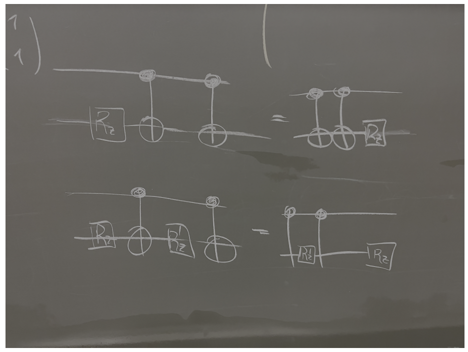


Figure 12: Example of propagation of a Z rotation towards even number of CNOTs.

is on the control qubit, it can be trivially taken outside of the CNOTs and the two of them cancel out.

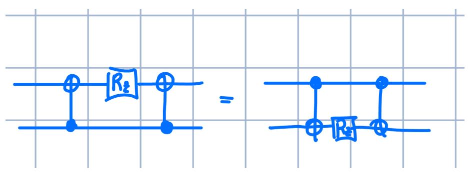
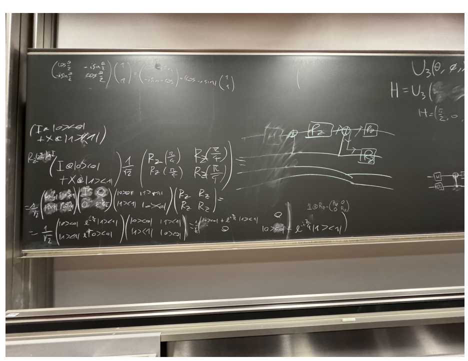


Figure 13: Example of swapping the control and the target qubit when there is only a Z rotation between them.

C Proof of the equality of the two block forms in the QFT



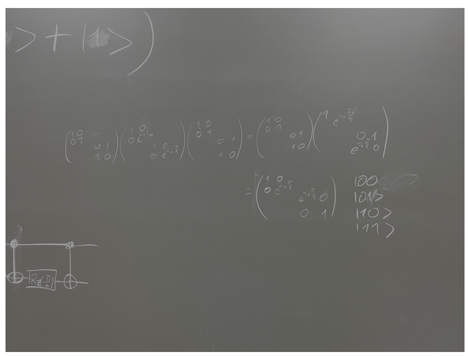


Figure 14: Here are the two images, showing by brute force that the two circuits are equivalent.