CONVOLUTIONAL CODES FOR SHORT-PACKET COMMUNICATION OVER NONCOHERENT CHANNELS

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OUTLINE

INTRODUCTION

SCENARIO
Sporadic transmission of short data units

CONSTRAINT
Information size (e.g. 64 information bits)

PROBLEM
Synchronization requires a packet overhead → **RATE LOSS**

CASE OF STUDY
Noncoherent communication and noncoherent receiver

CONVOLUTIONAL CODES

CONVOLUTIONAL ENCODER

$$\Rightarrow \quad \boldsymbol{x}_t = u_t\boldsymbol{G}_0 + u_{t-1}\boldsymbol{G}_1 + \cdots + u_{t-\nu}\boldsymbol{G}_\nu = \sum_{i=0}^{\nu} u_{t-i}\boldsymbol{G}_i$$

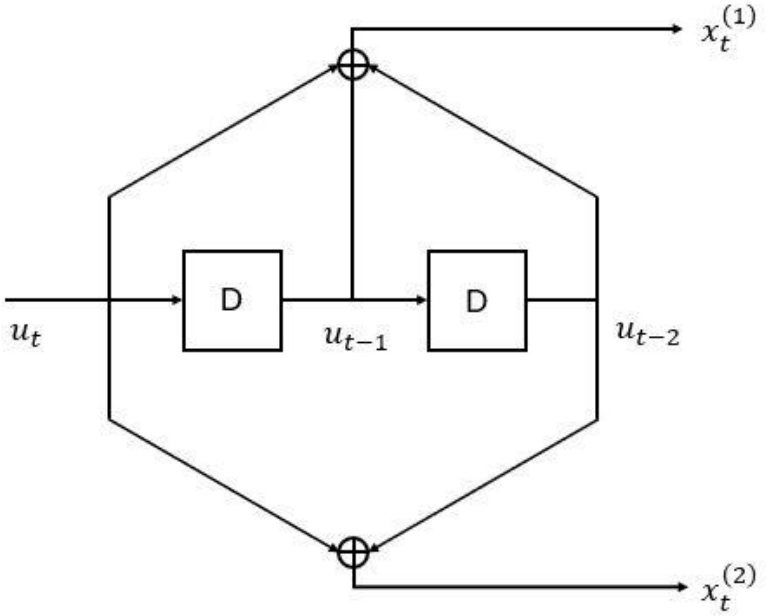
$\boldsymbol{x}_t$ → <u>output bits</u> (n)

u_t → <u>input bit</u> (k)

<u>Rate</u>: $R = \frac{k}{n}$

$\boldsymbol{G}_i, i = 0, \ldots, \nu$ → $k \times n$ generator matrices

ν = <u>code memory</u>



ENCODER OF [7, 5] CONVOLUTIONAL CODE

CONVOLUTIONAL CODES - TERMINATION

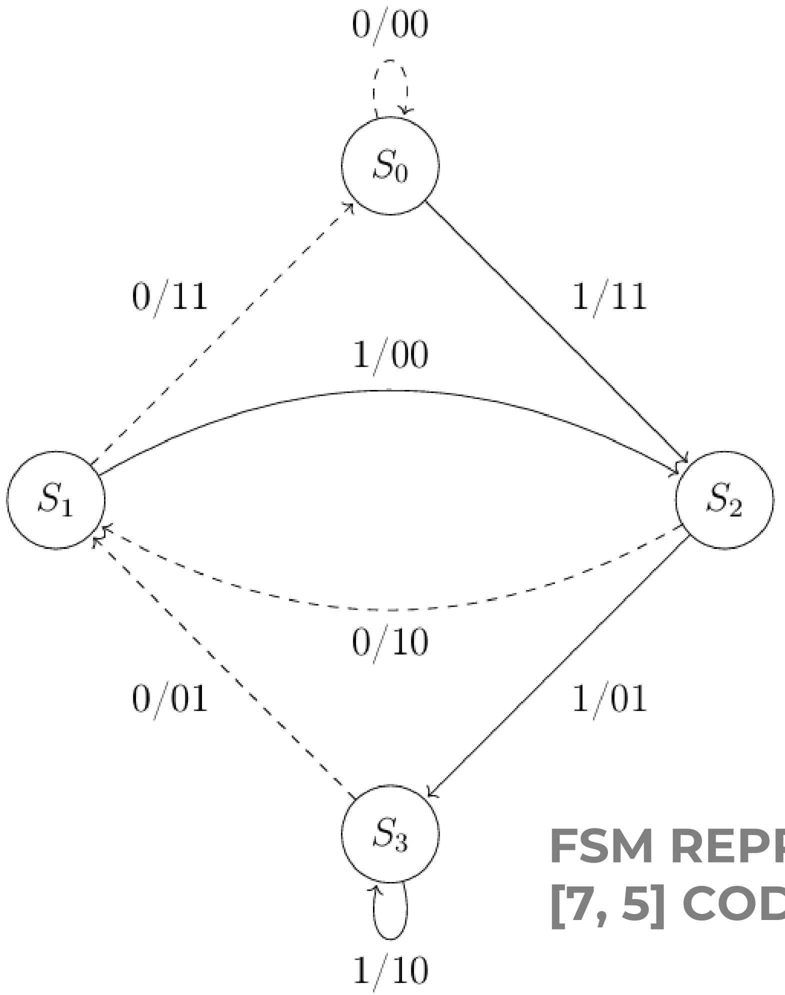
ZERO-TAIL

- Code is initialized in the zero state
- ν clock times needed to drive the encoder back to the zero state
- Overall rate: $R = \frac{K}{n(K+\nu)} \longrightarrow$ Rate Loss

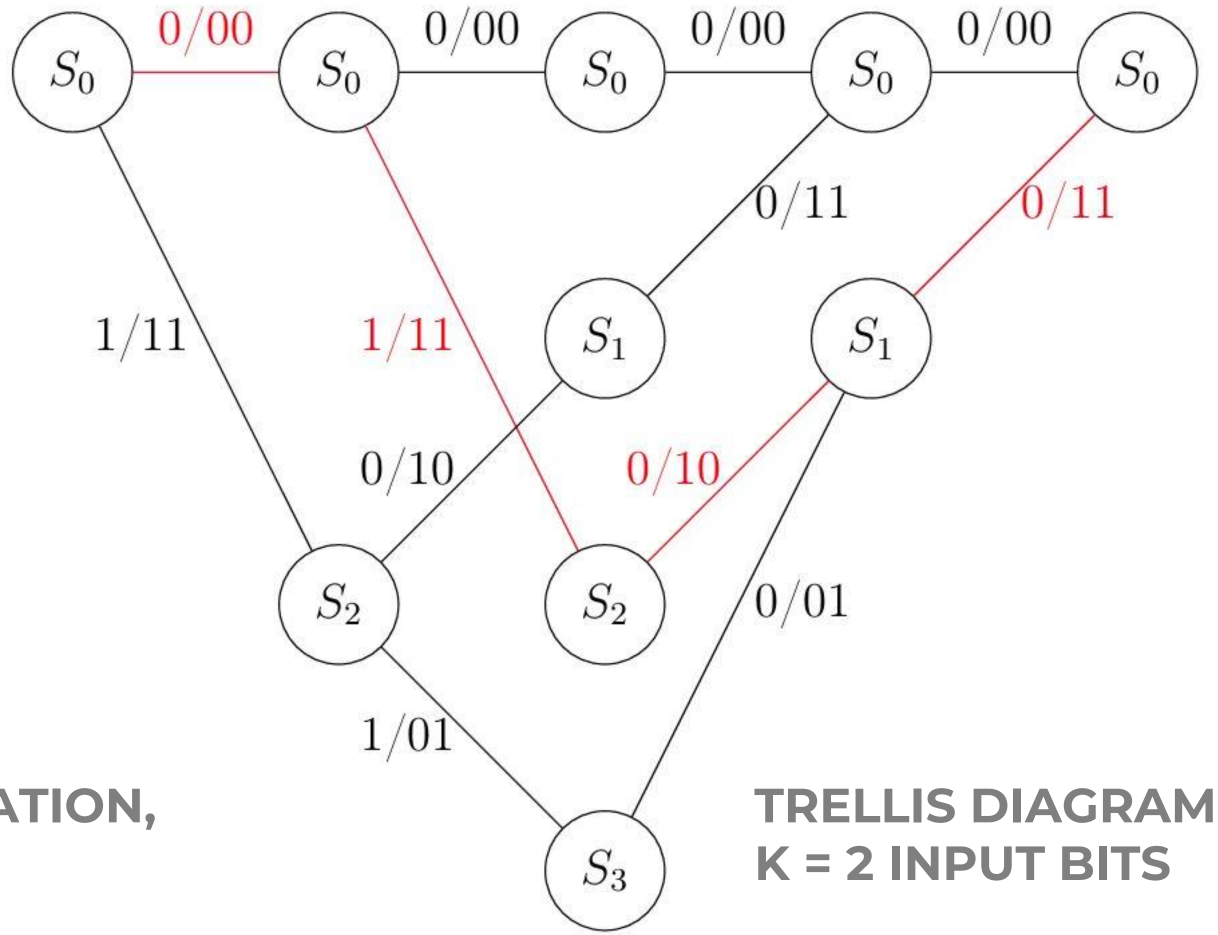
TAILBITING

- Code can be initialized in every possible state
- The ending state is the same as the starting one
- Overall rate: $R = \frac{K}{nK} = \frac{1}{n} \longrightarrow$ No Rate Loss

TRELLIS DIAGRAM & FSM REPRESENTATION



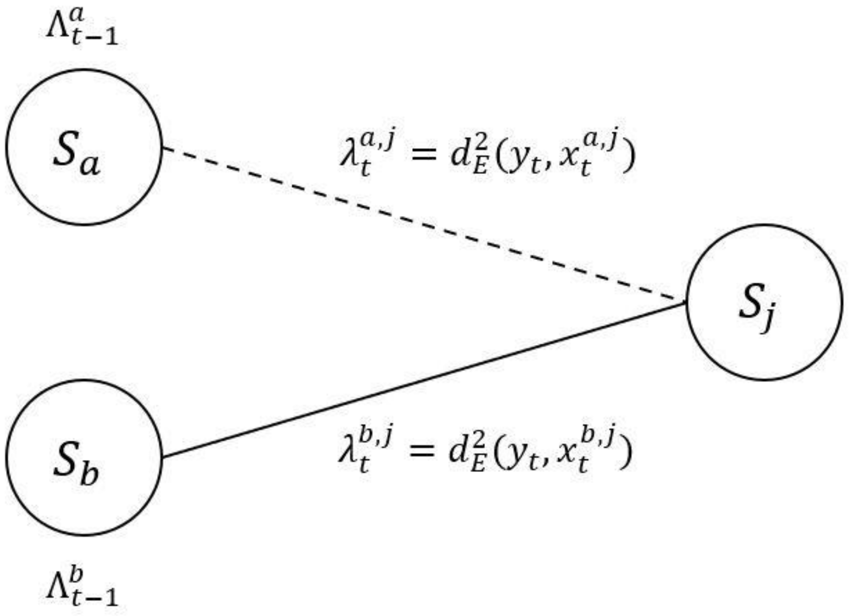
FSM REPRESENTATION,
[7, 5] CODE



TRELLIS DIAGRAM,
K = 2 INPUT BITS

VITERBI ALGORITHM

1. Scan the trellis diagram
2. Two paths converge in the same state $\longrightarrow$ select the one with the best metric



$$\Lambda_t^j = \min(\Lambda_{t-1}^a + \lambda_t^{a\,j}, \Lambda_{t-1}^b + \lambda_t^{b,j})$$

$\Lambda \longrightarrow$ **CUMULATIVE METRIC**

$\lambda \longrightarrow$ **TRANSITION METRIC**

PHASE ROTATION CHANNEL

NONCOHERENT CHANNEL

$$\boxed{y_i = hx_i + n_i}$$

- $n_i \sim CN\{0, 2\sigma^2\}$
- $h = e^{j\phi},\ \phi \sim U\,[-\pi, \pi)$
- $x_i \in \{-1, +1\}$ → BPSK modulation

SHORT PACKET TRANSMISSION ⟶ **CONSTANT CHANNEL** OVER A TX TIME

PILOT-AIDED DECODER

PERFECT CSI $\longrightarrow$ $\hat{\boldsymbol{x}}_{ML} = p(\boldsymbol{y}|\boldsymbol{x}, h) = \cdots = \arg\min_{\boldsymbol{x}\in C} d_E(\boldsymbol{y}, h\boldsymbol{x})^2$

PILOT-AIDED DECODER $\longrightarrow$ $\boldsymbol{x}_p = (x_p^1, x_p^2, \ldots, x_p^L)$ → PILOT SEQUENCE, $(x_p^i = 1\ \forall i)$

$$\hat{h} = \frac{\langle \boldsymbol{x}_p, \boldsymbol{y}_p \rangle}{\left|\left|\boldsymbol{x}_p\right|\right|^2} = \frac{\sum_{i=1}^{L} y_p^i}{L}$$

VITERBI UPDATE RULE $\longrightarrow$ $\lambda_t^{i,j} = d_E\left(\hat{h}^* \boldsymbol{y}_t, \boldsymbol{x}_t^{i,j}\right)^2$

ML DECODER – NONCOHERENT CHANNEL

$$\hat{\boldsymbol{x}}_{ML} = \arg\max_{\boldsymbol{x}\in C} p(\boldsymbol{y}|\boldsymbol{x})$$

$$p(\boldsymbol{y}|\boldsymbol{x}) = \int_{-\pi}^{\pi} p(\boldsymbol{y}|\boldsymbol{x},\phi)\cdot p(\phi)d\phi\,, \qquad p(\phi) = \frac{1}{2\pi}$$

$$p(\boldsymbol{y}|\boldsymbol{x},\phi) = \prod_{i=1}^{N} p(y_i|x_i,\phi) = \prod_{i=1}^{N} \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2\sigma^2}\left|y_i - e^{j\phi}x_i\right|^2}$$

$$\hat{\boldsymbol{x}} = \arg\max_{\boldsymbol{x}\in C} \int_{-\pi}^{\pi} e^{\sum_i x_i Re\{y_i e^{-j\phi}\}/\sigma^2}\cdot\frac{1}{2\pi}d\phi = \arg\max_{\boldsymbol{x}\in C} I_0\left(\frac{1}{\sigma^4}\cdot\left|\sum_{i=1}^{N} x_i y_i\right|^2\right)$$

$$\hat{\boldsymbol{x}} = \boxed{\arg\max_{\boldsymbol{x}\in C} \left|\sum_{i=1}^{N} y_i x_i\right|} \longrightarrow$$

NONCOHERENT CORRELATION METRIC

BLIND VITERBI

Viterbi update rules with **noncoherent correlation** as metric

1. Compute the **branch metric**

$$\lambda_t^{i,j} = \langle \boldsymbol{x}_t^{i,j}, \boldsymbol{y}_t \rangle$$

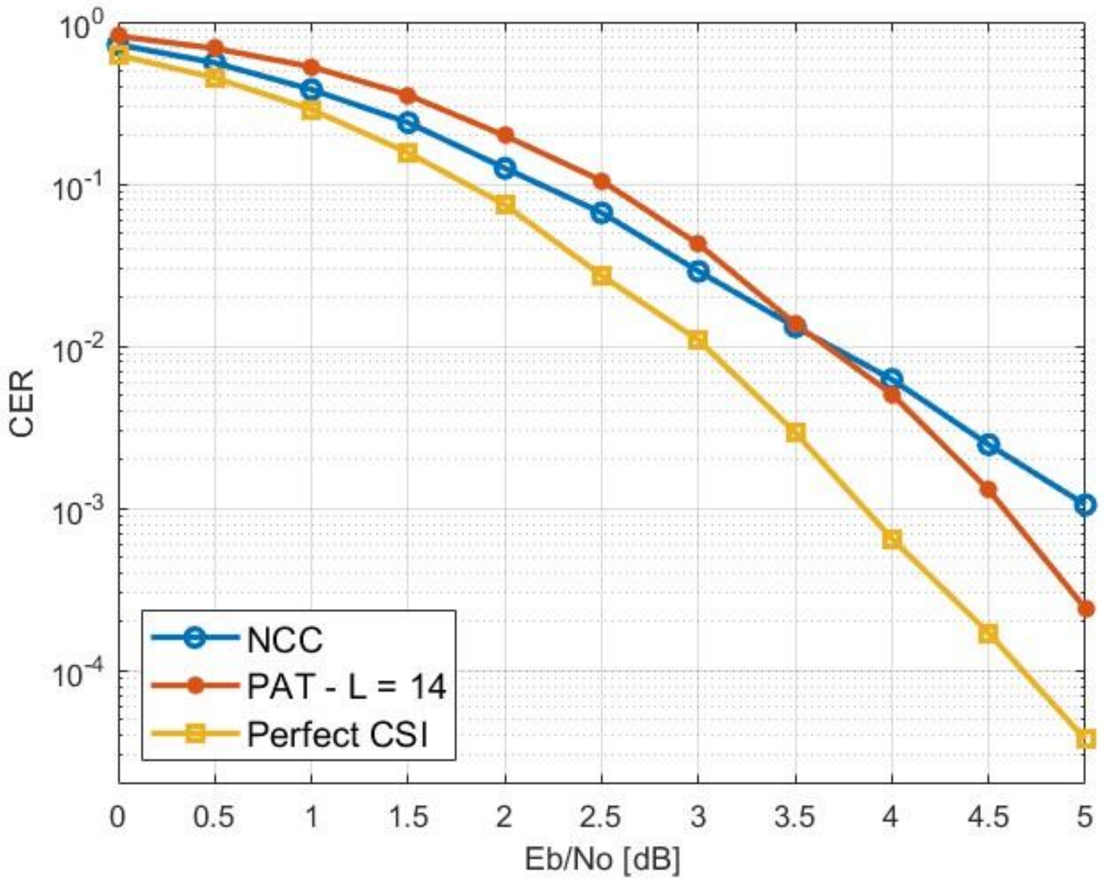
2. Select the **best path**

$$\hat{\imath} = \arg\max_i \left(\left| \Lambda_{t-1}^i + \lambda_t^{i,j} \right| \right)^2$$

3. Update the **cumulative metric**

$$\Lambda_t^j = \Lambda_{t-1}^{\hat{\imath}} + \lambda_t^{\hat{\imath},j}$$

NCC METRIC PERFORMANCE



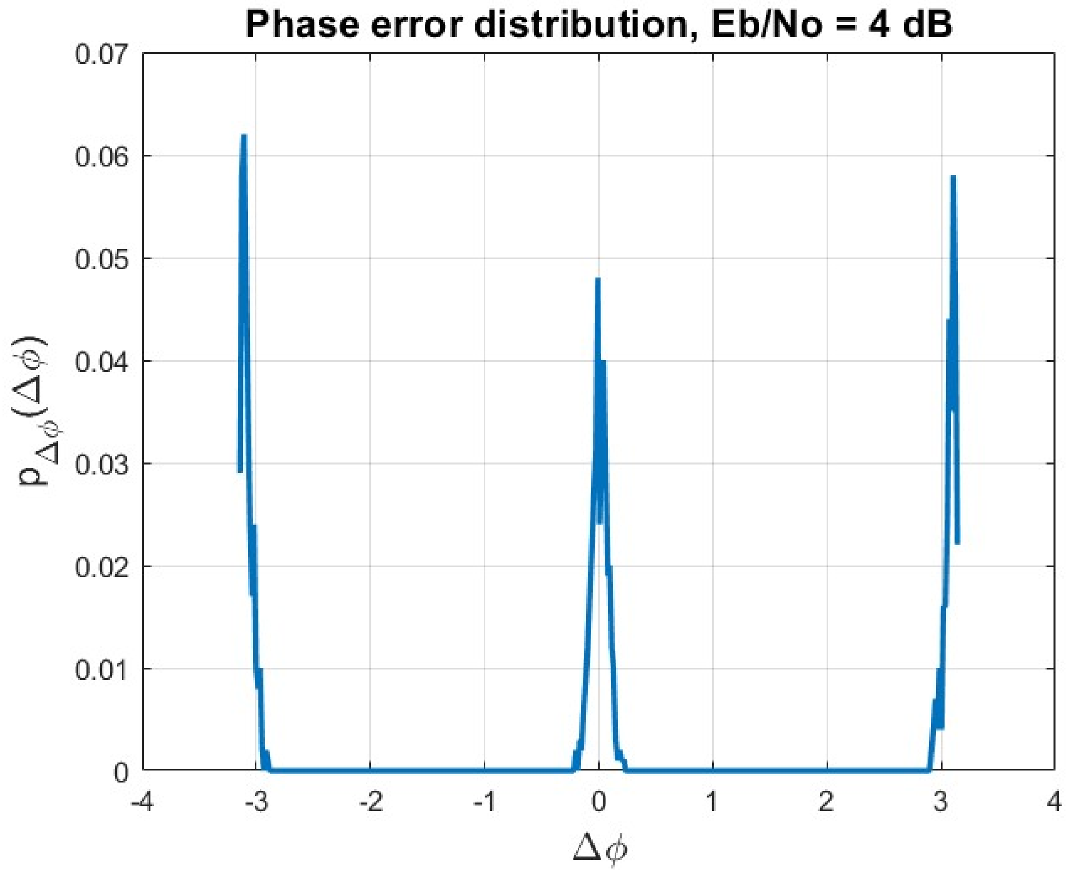
ZTCC, [133, 171] - ($\nu = 6$)

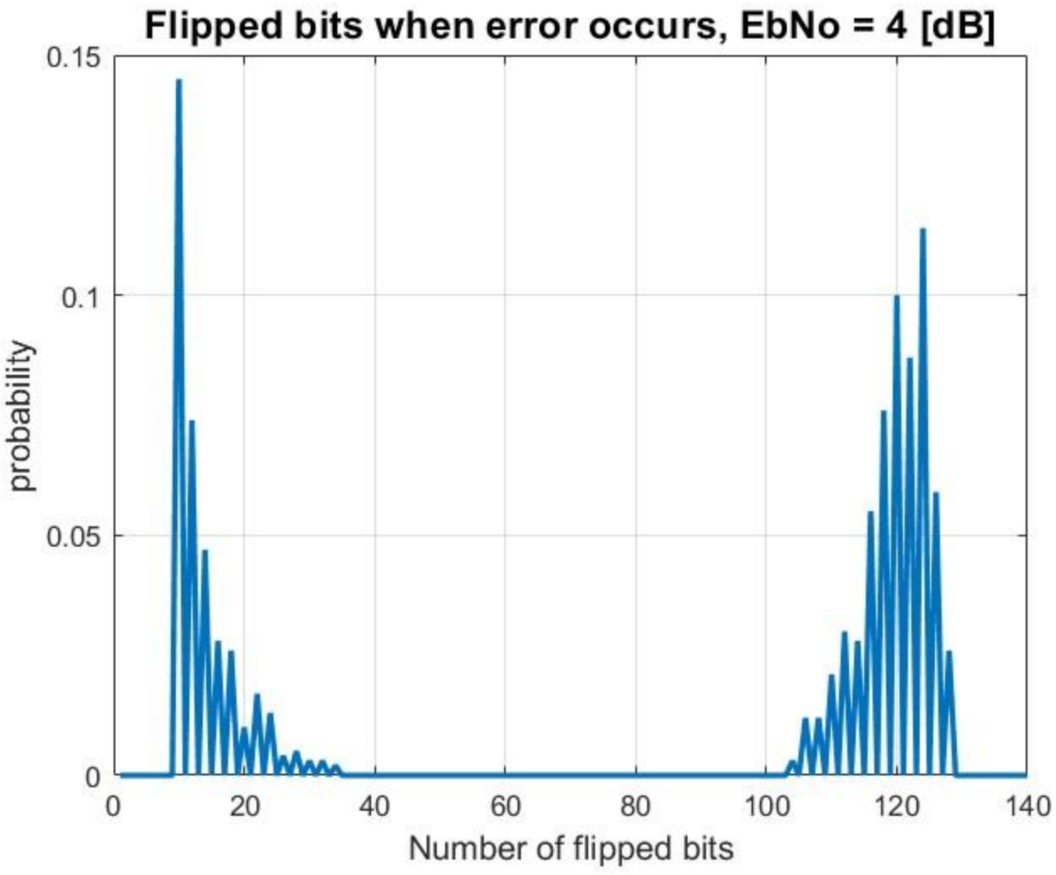
1. **NCC**
   Decoder with noncoherent correlation as metric

2. **PAT**
   Pilot-aided (optimized pilot length)

3. **PERFECT CSI**
   Perfect channel knowledge

PHASE ESTIMATE ERROR





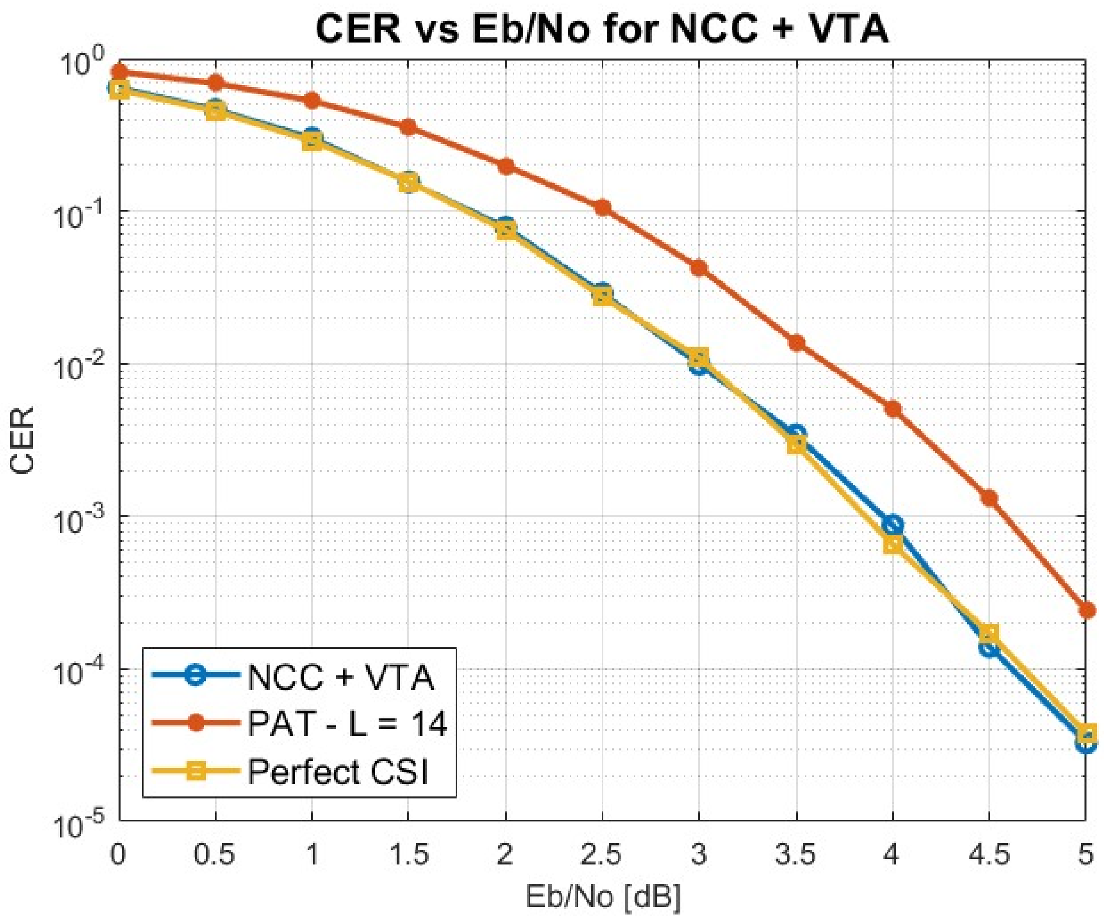
VITERBI TRACKING ALGORITHM

DECODING ERROR $\longrightarrow$ estimate channel through $\hat{\boldsymbol{x}}$

$$\hat{h} = \frac{\langle \boldsymbol{y}, \hat{\boldsymbol{x}} \rangle}{N}$$

1. Run **two instances** of the Viterbi algorithm with CSI
   - $\hat{\boldsymbol{x}}_1 \longrightarrow$ channel = $\hat{h}$
   - $\hat{\boldsymbol{x}}_2 \longrightarrow$ channel = $-\hat{h}$
2. Select the one with the best metric $\Lambda = |\langle \boldsymbol{y}, \hat{\boldsymbol{x}}_i \rangle|$

VITERBI TRACKING ALGORITHM



NCC + VTA

Errors are supposed to be known by the decoder; gap to perfect CSI closed

How can we detect errors in real systems?

- **CRC codes**
- **Threshold on Λ**

CRC CODES

Add parity bits at the end of the encoded word

Polynomial generator

$$g(x) = g_0 + g_1 \cdot x + g_2 \cdot x^2 + \ldots + g_m \cdot x^m = 1 + \sum_{i=1}^{m-1} g_i \cdot x^i + x^m$$

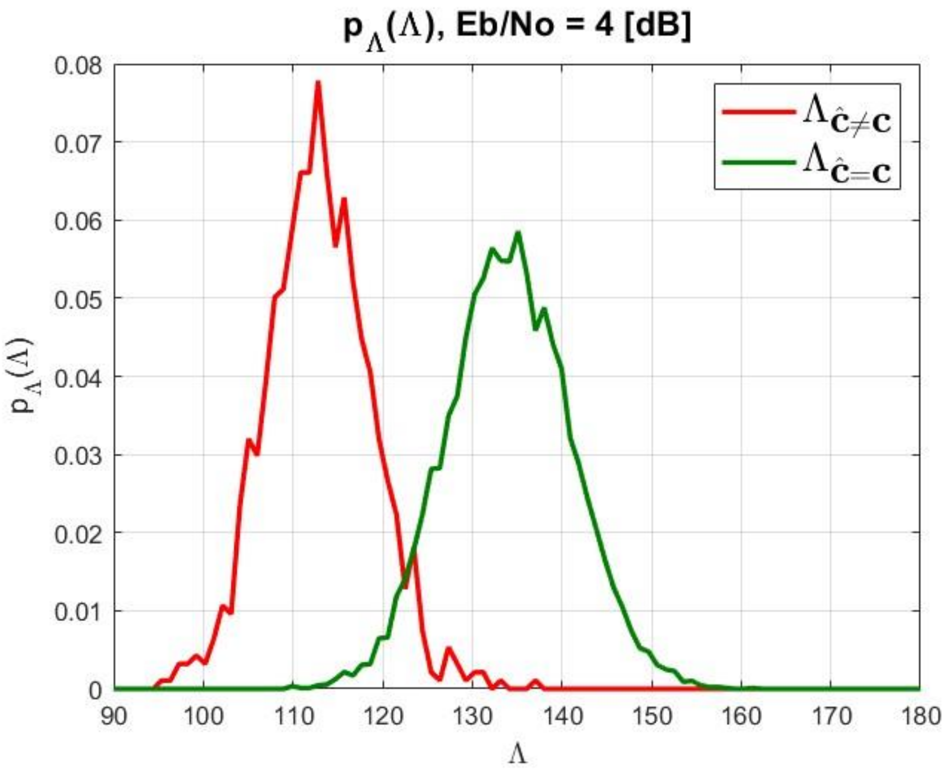
Parity sequence $\longrightarrow \frac{u(x) \cdot x^m}{g(x)}$ $u(x)$ input sequence, m CRC code degree

Same operation at the receiver side:

- Remainder $= 0$ → decoded word is considered correct
- Remainder $\neq 0$ → decoded word is considered wrong $\longrightarrow$ enter loop

THRESHOLD MECHANISM

Select a value Λ_{thr} such that, if $\Lambda < \Lambda_{thr}$, the word is supposed to be wrong

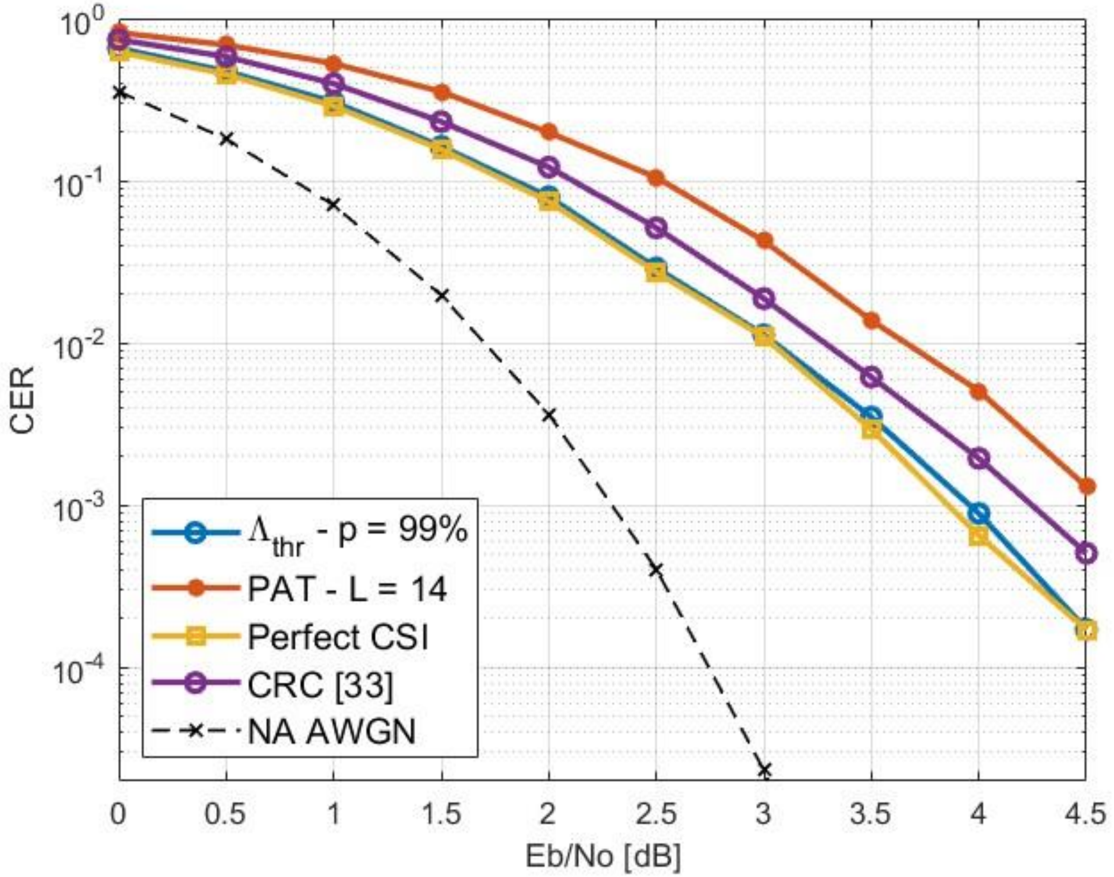


1. Fix a percentage of errors
2. Convert it into a Λ_{thr} value

UPSIDE
No overhead introduced

DOWNSIDE
The decoder is reiterating over some correct words

PERFORMANCE



$\Lambda_{thr} - p = 99\%$

The decoder enters the Viterbi tracking loop over 99% of decoding errors

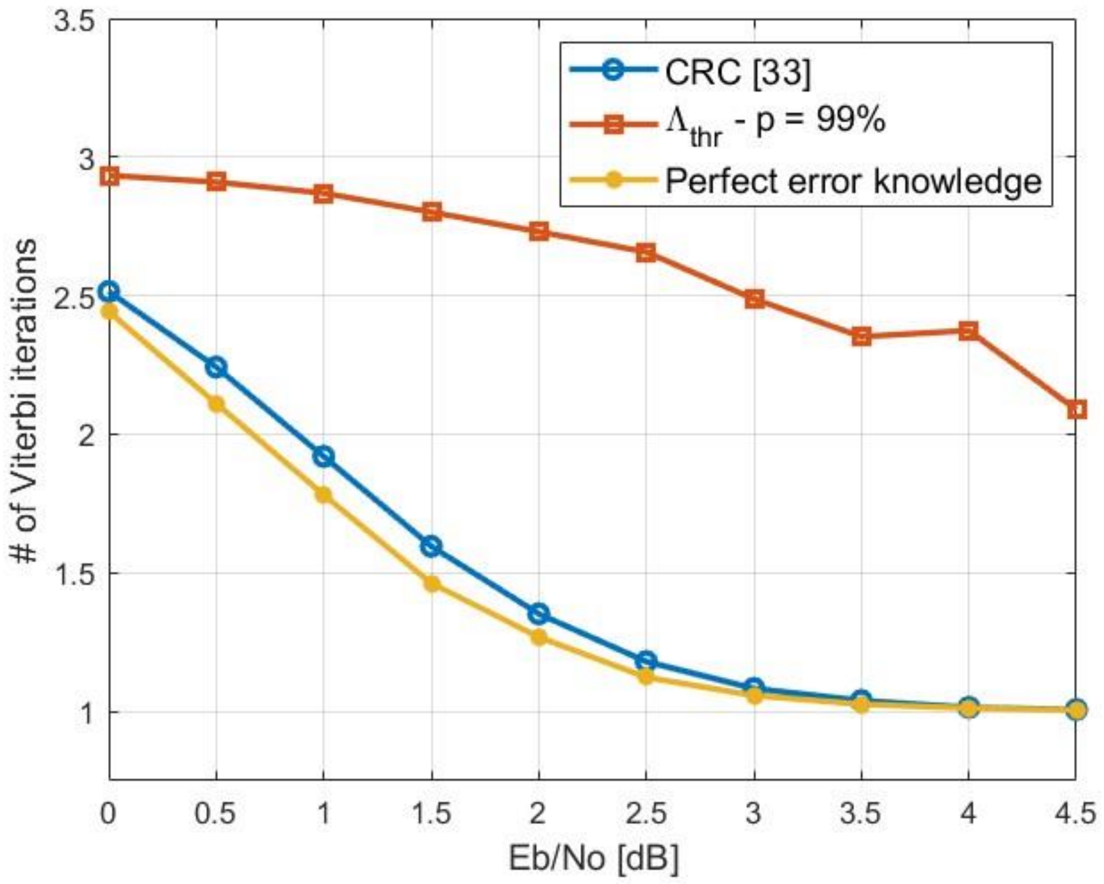
CRC [33]

CRC code of degree 4 $(g(x) = x^4 + x^3 + x + 1)$

NA AWGN

Normal approximation bound for the AWGN channel

COMPLEXITY



THRESHOLD

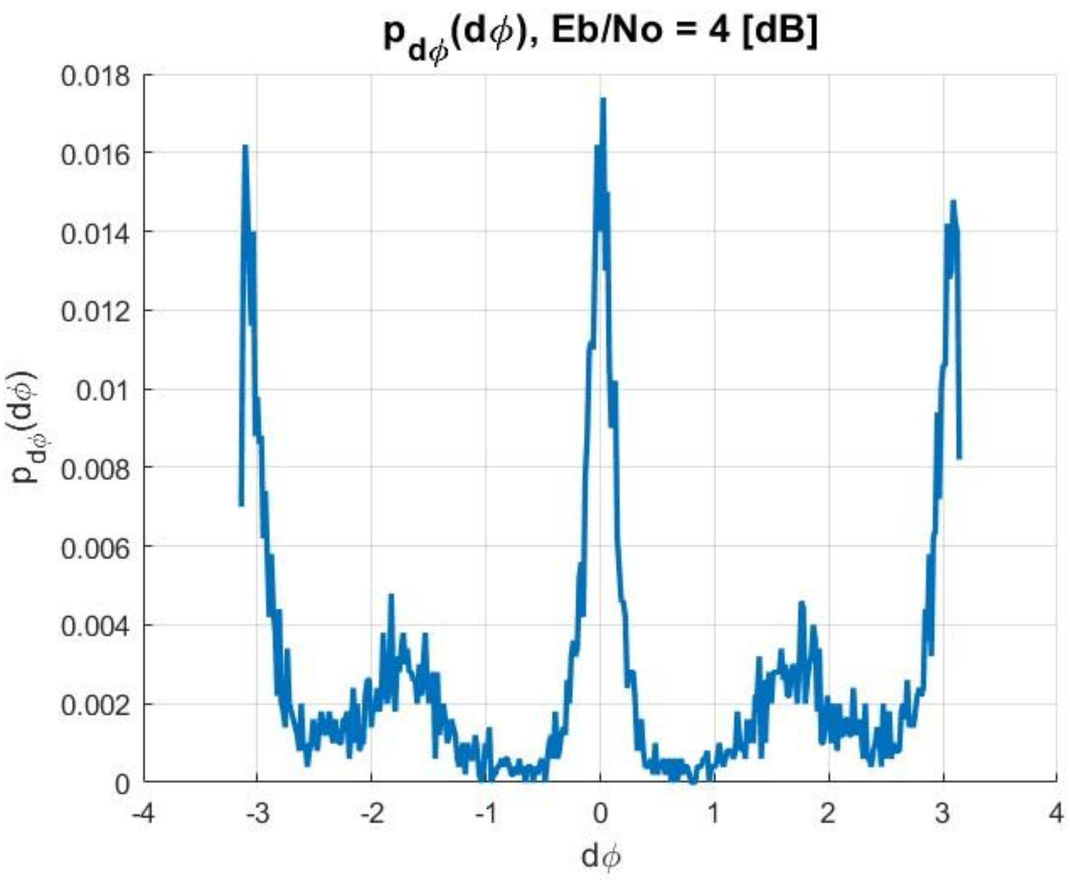
+ No overhead
– Redundant Viterbi runs

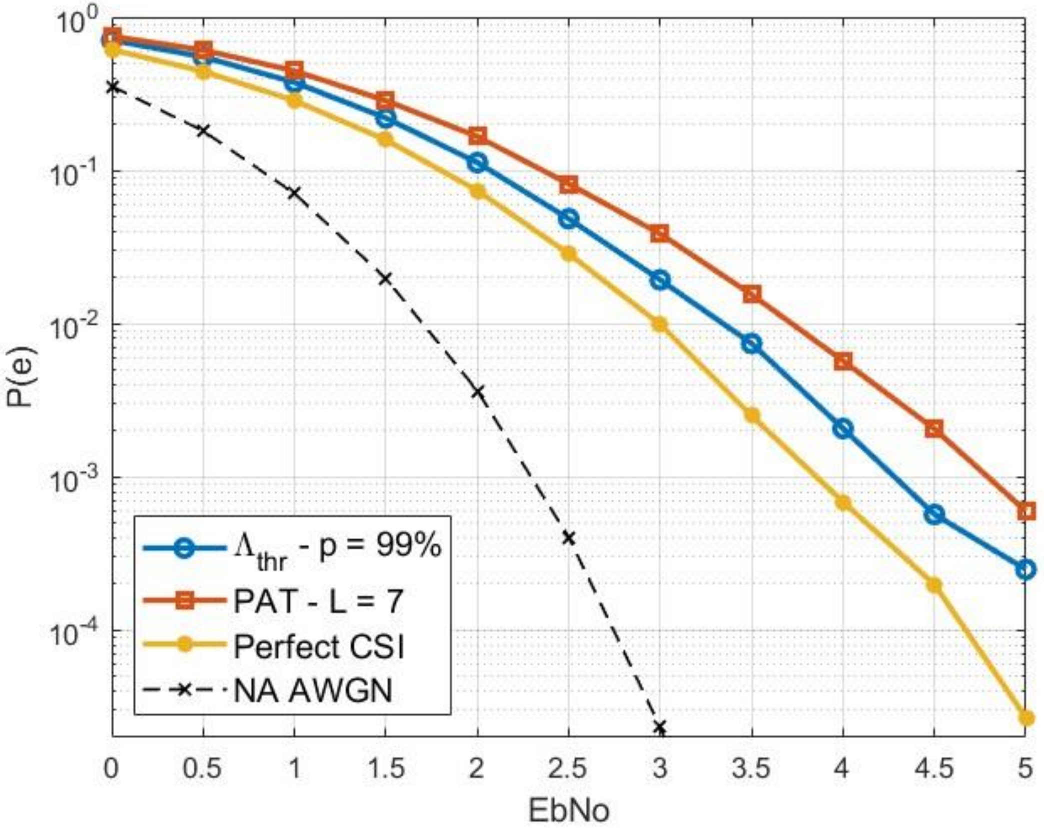
CRC CODE

+ Excellent error identification
– Introduces overhead

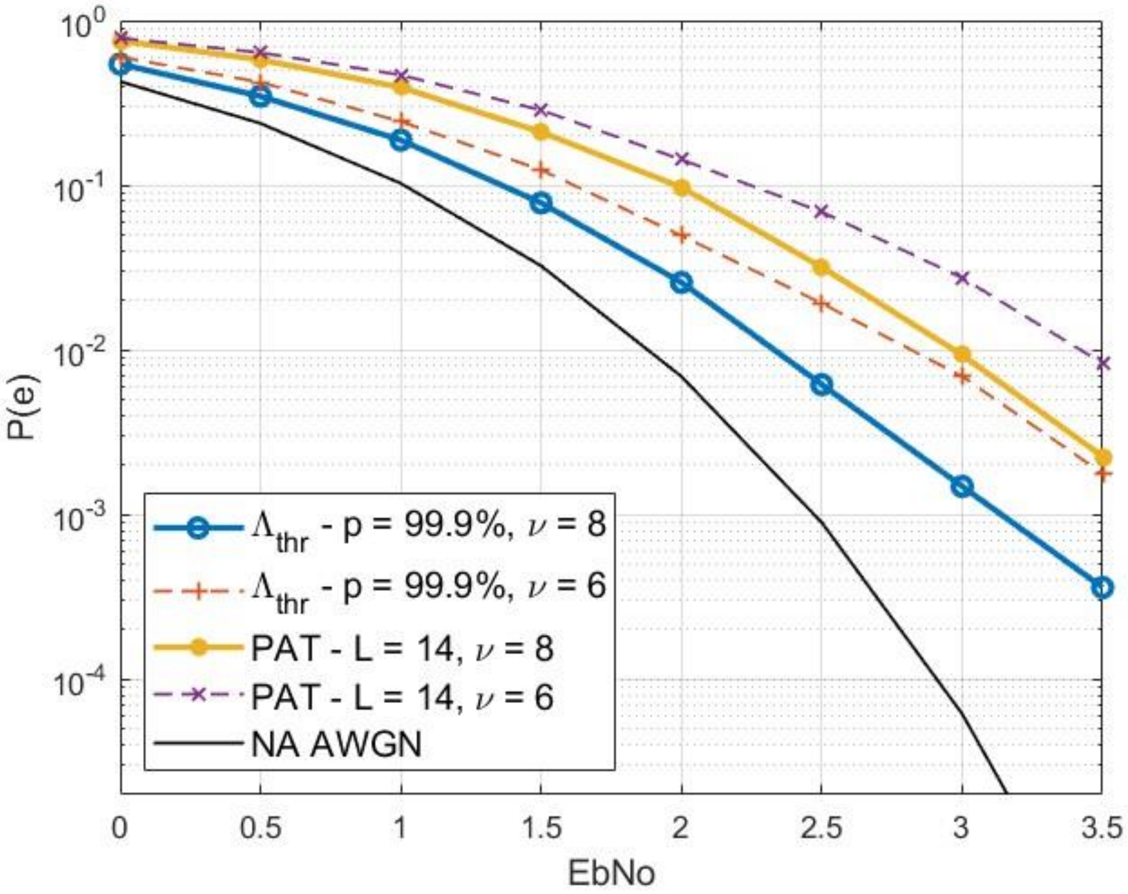
TRADE-OFF ACCURACY/COMPLEXITY

QPSK MODULATION





MEMORY 8 – TBCC



TBCC, [663, 711] - $\nu = 8$

Suboptimal code due to symmetry concerns

MEMORY GAIN

Memory 6 NCC performs better than memory 8 PAT

EXCELLENT ERROR RATE

Gap to the NA AWGN bound drops to 0.5 dB at $p(e) = 10^{-3}$

CONCLUSIONS

1. CONSISTENT **CODING GAINS** OF **0.3 TO 0.5 dB**
2. PROVEN FOR DIFFERENT MEMORY VALUES AND CODE TERMINATIONS
3. VALUABLE FOR BOTH **BPSK AND QPSK** MODULATION
4. IMPROVED ACCURACY WITH **MANAGEABLE COMPLEXITY**
5. MEMORY 8 TBCC CLOSE TO NA AWGN BOUND

THANK YOU FOR YOUR ATTENTION!