$$\begin{bmatrix} 2\alpha & & & 1 \\ 1 & \ddots & & \vdots \\ & \ddots & \ddots & 1 \\ & & 1 & 2\alpha \end{bmatrix}$$

$$A_{n-1} = \begin{bmatrix} 2\alpha & & \\ 1 & \ddots & \\ & 1 & 2\alpha \end{bmatrix} = \begin{bmatrix} 1 & & \\ \frac{1}{2\alpha} & \ddots & \\ & \frac{1}{2\alpha} & 1 \end{bmatrix} \begin{bmatrix} 2\alpha & & \\ & \ddots & \\ & & 2\alpha \end{bmatrix}$$

Se $\alpha \neq 0$ $\exists!$ LU

$|\alpha| \geq \frac{1}{2} \Rightarrow |2\alpha| \geq 1$

Se $\alpha = 0$ $\begin{bmatrix} 0 & & 1 \\ 1 & \ddots & 1 \\ & 1 & 0 \end{bmatrix}$ NO

$$A = \left[\begin{array}{ccc|c} 1 & & & \\ \frac{1}{2\alpha} & \ddots & & 0 \\ & \frac{1}{2\alpha} & 1 & \\ \hline 0 & \cdots 0 & \frac{1}{2\alpha} & 1 \end{array}\right] \left[\begin{array}{ccc|c} 2\alpha & & & \\ & \ddots & & z \\ & & 2\alpha & \\ \hline & 0 & & u \end{array}\right]$$

$$\begin{bmatrix} 1 & & \\ \frac{1}{\alpha} & \ddots & \\ & \frac{1}{\alpha} & 1 \end{bmatrix} z = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix},$$

$$\frac{1}{2\alpha} z_{n-1} + u = 2\alpha$$

$z_1 = 1$

$\frac{1}{\alpha} z_1 + z_2 = 1$

$\underline{z_2 = 1 - \frac{1}{\alpha}}$

$\frac{1}{\alpha} z_2 + z_3 = 1$

$= 1 - \frac{1}{\alpha}\left(1 - \frac{1}{\alpha}\right)$

$= 1 - \frac{1}{\alpha} + \frac{1}{\alpha^2}$

$$\begin{bmatrix} 2\alpha & & & 1 \\ 0 & 2\alpha & & 1-\frac{1}{\alpha} \\ & 1 & \ddots & \vdots \\ & & 1 & 2\alpha \end{bmatrix}$$

$|\alpha| \geq \frac{1}{2}$ $\quad |2\alpha| \geq 1$

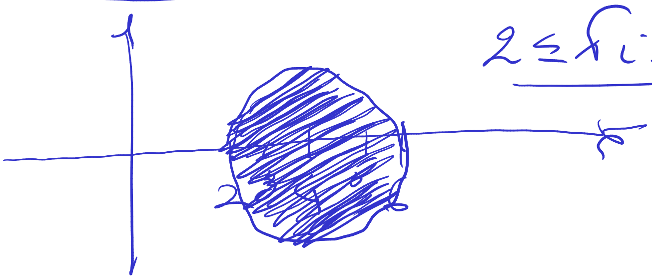
$$\begin{bmatrix} 1 & & \\ \frac{1}{2\alpha} & \ddots & \\ & \frac{1}{2\alpha} & 1 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 1 & & \\ -\frac{1}{2\alpha} & \ddots & \\ & & 1 \end{bmatrix}$$

$$A = \frac{1}{6}\begin{bmatrix} 4 & -1 & & \\ -1 & \ddots & \ddots & \\ & \ddots & \ddots & -1 \\ & & -1 & 4 \end{bmatrix} \qquad \|A\|_2 = \sqrt{\rho(A^T A)}$$

$$K_2(A) = \|A\|_2 \, \|A^{-1}\|_2 \le 3$$

$$\underline{K_2(A) = \frac{\max|\lambda_i|}{\min|\lambda_i|}} \qquad \lambda_i \text{ autovalori di } A$$



$$\underline{2 \le \lambda_i \le 6}$$

$$K_2(A) = \frac{\max|\lambda_i|}{\min|\lambda_i|} \le \frac{6}{2} \le 3$$

<u>A ben condizionata</u>

A p.d. $\Rightarrow$ A ammette unica LU

$$A^{(1)} = \frac{1}{6}\begin{bmatrix} 4 & -1 & & & \\ 0 & 4-\frac{1}{4} & -1 & & \\ & -1 & 4 & -1 & \\ & & \ddots & \ddots & \\ & & & -1 & 4 \end{bmatrix}$$

— calcolo i moltiplicatori

— calcolo elemento pivotale

$\underline{O(n)}$

$$\begin{bmatrix} 1 & \cdots & \cdots & 1 \\ \alpha & 0 & & \\ & \ddots & \ddots & \\ & & \alpha & 0 \end{bmatrix}$$

Per quali α è p.d.?

nessun val di α

$$\downarrow$$

$$\begin{bmatrix} \alpha & & \\ & \ddots & \\ & & \alpha \\ 1 & \cdots & 1 \end{bmatrix}$$

$$\det = \alpha^{n-1} = 0 \iff \alpha = 0$$

$$\begin{bmatrix} 1 & \cdots & \cdots & 1 \\ \alpha & 0 & & \\ & \ddots & \ddots & \\ & & \alpha & 0 \end{bmatrix}$$

$\alpha \neq 0$ le sottomatrici principali di testa sono invertibili $\Rightarrow \exists!\ LU$

$\alpha = 0$ $\exists$ infinite LU

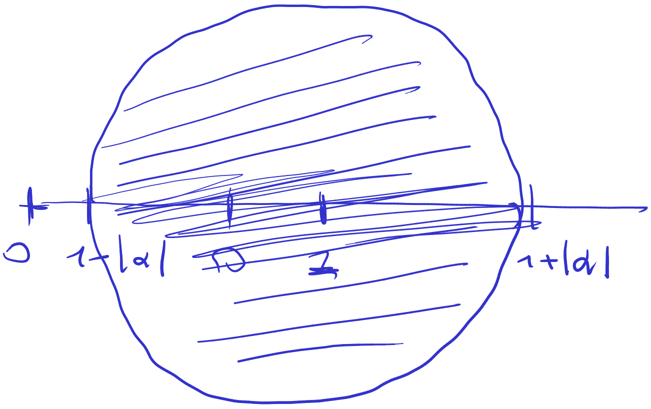
$$\begin{bmatrix} 1 & \cdots & \cdots & 1 \\ \alpha & 0 & & \\ & \ddots & \ddots & \\ & & \alpha & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \cdots & \cdots & 1 \\ 0 & -\alpha & \cdots & -\alpha \\ 0 & \alpha & & \\ & & \ddots & \\ & & & \alpha \end{bmatrix} \qquad E_1 = \begin{bmatrix} 1 & & \\ -\alpha & \ddots & \\ & & 1 \end{bmatrix}$$

$$\begin{bmatrix} -\alpha & \cdots & -\alpha \\ \alpha & & \\ & \ddots & \\ & & \alpha \end{bmatrix} \rightarrow \begin{bmatrix} -\alpha & \cdots & \cdots & -\alpha \\ 0 & -\alpha & \cdots & -\alpha \\ & \alpha & & \\ & & \ddots & \\ & & & \alpha \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & & \\ \alpha & -1 & \\ & \ddots & \ddots \\ & & -1 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & \cdots & \cdots & 1 \\ 0 & -\alpha & \cdots & \alpha \\ & & \ddots & \\ & & & -\alpha \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & & \alpha \\ & \ddots & \\ \alpha & & 1 \end{bmatrix}$$

A



$$K_2(A) = \frac{\max|\lambda_i|}{\min|\lambda_i|}$$

$|\alpha| < 1$ $\qquad K_2(A) \le \dfrac{1+|\alpha|}{1-|\alpha|}$

$$\det(\lambda I - A) = \det \begin{bmatrix} \lambda-1 & & -\alpha \\ & \ddots & \\ -\alpha & & \lambda-1 \end{bmatrix}$$

$$= (\lambda-1) \det \begin{bmatrix} \lambda-1 & & -\alpha \\ & \ddots & \\ -\alpha & & \lambda-1 \end{bmatrix}$$

$$= (\lambda-1)^2 \det \begin{bmatrix} \lambda-1 & & -\alpha \\ & \ddots & \\ \alpha & & \lambda-1 \end{bmatrix}$$

$$= (\lambda-1)^{n-2} \det \begin{bmatrix} \lambda-1 & -\alpha \\ -\alpha & \lambda-1 \end{bmatrix}$$

$$= (\lambda-1)^{n-2} \left[(\lambda-1)^2 - \alpha^2\right]$$

$\alpha \simeq 1$ $\qquad K_2(A) \simeq \dfrac{2}{|1-\alpha|} \to +\infty$

$\alpha \simeq -1$ $\qquad K_2(A) \simeq \dfrac{2}{|1+\alpha|} \to +\infty$

$|\alpha| \to +\infty$ $\qquad K_2(A) \simeq \dfrac{|1+\alpha|}{1} \to +\infty$

$\lambda = 1 \qquad \lambda = 1-\alpha \qquad \lambda = 1+\alpha$

$$K_2(A) = \frac{\max\{1, |1-\alpha|, |1+\alpha|\}}{\min\{1, |1-\alpha|, |1+\alpha|\}}$$

$$\begin{bmatrix} 1 & & & -\alpha \\ 1 & \ddots & & \\ & \ddots & \ddots & \\ & & 1 & 1 \end{bmatrix}$$

α: è p.d.

p. simme: α

$$\begin{bmatrix} 1 & & \\ 1 & \ddots & \\ & \ddots & 1 \end{bmatrix} z = \begin{bmatrix} -\alpha \\ \vdots \\ 0 \end{bmatrix}$$

$\forall \alpha$: $\exists!$ LU

$z_1 = -\alpha$

$z_1 + z_2 = 0 \quad z_2 = \alpha$

$z_2 + z_3 = 0 \quad z_3 = -\alpha$

$$\left[\begin{array}{ccc|c} 1 & & & \\ 1 & \ddots & & 0 \\ & \ddots & 1 & \\ \hline 0 & \cdots & 1 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & & & -\alpha \\ & \ddots & & \alpha \\ & & 1 & -\alpha \\ & & & \vdots \\ & & & \pm\alpha \\ \hline & 0 & & 1 \mp \alpha \end{array}\right]$$

$\pm\alpha + u = 1$

$\underline{u = 1 \mp \alpha}$

$$M = \begin{bmatrix} 1 & & \\ 1 & \ddots & \\ & \ddots & 1 \end{bmatrix} \quad N = \begin{bmatrix} & & \alpha \\ & 0 & 0 \\ & & \vdots \\ & & 0 \end{bmatrix} \quad G\text{-}S$$

$$G = M^{-1} N = \left[\begin{array}{ccc|c} 0 & \cdots & 0 & \underbrace{M^{-1}\begin{bmatrix} \alpha \\ 0 \\ \vdots \\ 0 \end{bmatrix}} \end{array}\right]$$

$$= \underbrace{\left[\begin{array}{ccc|c} & & & \alpha \\ & & & -\alpha \\ 0 & \cdots & 0 & \vdots \\ & & & \pm\alpha \end{array}\right]} \qquad \rho(G) = |\alpha|$$

G-S. converge $\Leftrightarrow$ $\underline{|\alpha| < 1}$

$$M = \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \quad N = \begin{bmatrix} 0 & & \alpha \\ -1 & \ddots & \\ & \ddots & \\ & & -1 \end{bmatrix} \quad \text{Jacobi}$$

$$M^{-1} N = \begin{bmatrix} 0 & & \alpha \\ -1 & & \\ & \ddots & \\ & & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & & & \alpha \\ -1 & \ddots & & \\ & \ddots & \ddots & \\ & & -1 & 0 \end{bmatrix}$$

$$\det(\lambda I - A) = \det \begin{bmatrix} \lambda & & & \alpha \\ 1 & \ddots & & \\ & \ddots & \ddots & \\ & & 1 & \lambda \end{bmatrix}$$

$$= \left[\begin{array}{cccc|c} 1 & & & & \\ \frac{1}{\lambda} & \ddots & & & \\ & \ddots & \ddots & & \\ & & \frac{1}{\lambda} & 1 & \\ \hline & & & \frac{1}{\lambda} & 1 \end{array}\right] \left[\begin{array}{cccc|c} \lambda & 1 & & & \pm\alpha \\ & \ddots & & & -\frac{\alpha}{\lambda} \\ & & \ddots & & \frac{\alpha}{\lambda^2} \\ & & & \lambda & \vdots \\ \hline & & & & w \end{array}\right] \quad \frac{\pm\alpha}{\lambda^{n-2}}$$

$$\begin{bmatrix} 1 & & \\ \frac{1}{\lambda} & \ddots & \\ & \frac{1}{\lambda} & 1 \end{bmatrix} Z = \begin{bmatrix} \alpha \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad \begin{aligned} Z_1 &= \alpha \\ Z_2 &= -\frac{1}{\lambda}\alpha \\ Z_3 &= \frac{1}{\lambda^2}\alpha \end{aligned}$$

$$w = \lambda \pm \frac{\alpha}{\lambda^{n-1}} \qquad \varphi_i(J) = \sqrt[n]{|\alpha|}$$

$$\det(\lambda I - A) = \underline{\lambda^{n-1}}\left(\lambda \pm \frac{\alpha}{\lambda^{n-1}}\right) = \qquad \sqrt[n]{|\alpha|} < 1 \iff |\alpha| < 1$$

$$\underbrace{\lambda^n \pm \alpha}_{} = 0$$

$$|\lambda^n| = |\mp \alpha| \iff |\lambda|^n = |\alpha|$$

$$|\lambda| = \sqrt[n]{|\alpha|}$$

$$A = \begin{bmatrix} n & -1 & \cdots & -1 \\ -1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 \\ -1 & \cdots & -1 & n \end{bmatrix} \qquad F = \begin{bmatrix} a & b & \cdots & b \\ b & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & b \\ b & \cdots & b & a \end{bmatrix}$$

$$A \cdot F = I$$

$$\begin{cases} na - (n-1)b = 1 \\ nb - a - (n-2)b = 0 \end{cases}$$

$$\begin{cases} na - (n-1)b = 1 \\ -a + 2b = 0 \end{cases} \qquad b = \frac{a}{2}$$

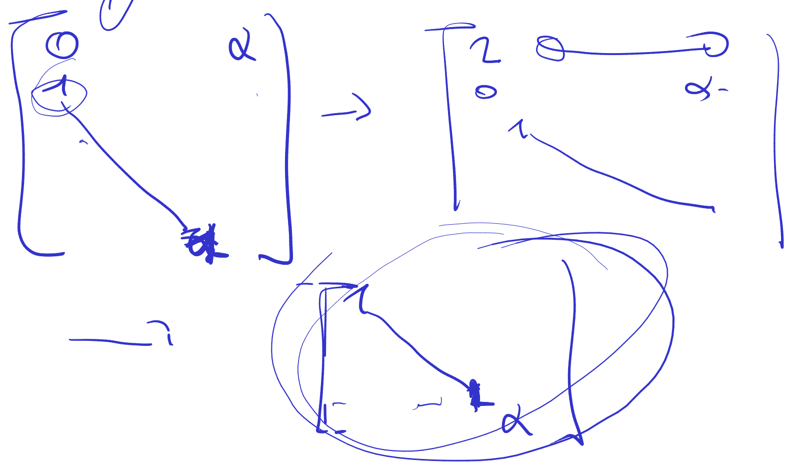
$$na - (n-1)\frac{a}{2} = 1$$

$$2na - na + a = 2$$

$$(n+1)a = 2$$

$$a = \frac{2}{n+1} \qquad b = \frac{1}{n+1}$$

$$K_\infty(A) = (2n-1)\left[\frac{2}{n+1} + \frac{n-1}{n+1}\right]$$



$$f(x) = \sqrt{\log(e^x)} = \sqrt{x}$$

$$\log(e^x) = x \log e = x$$

$$\varepsilon_u = \frac{f'(x)}{f(x)} x \cdot \varepsilon_x$$

$$\varepsilon_u = \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}} \cdot x \; \varepsilon_x$$

$$= \frac{1}{2\cancel{x}} \cancel{x} \; \varepsilon_x$$

$$|\varepsilon_u| \leq \frac{1}{2} u$$

$$x \longrightarrow \underset{G_1}{e^x} \xrightarrow{\frac{1}{\log(e^x)} = \frac{1}{x}} \underset{G_2}{\log(e^x)} \xrightarrow{\frac{1}{2}} \underset{G_3}{\sqrt{\log(e^x)}}$$

$$t \to \log t$$

$$\frac{\frac{1}{t}}{\log t} \, t = \frac{1}{\log t}$$

$$G_{og} = G_3 + \frac{1}{2}\left[G_2 + \frac{1}{x} G_1\right]$$

$$|G_{og}| \leq u + \frac{1}{2} u + \frac{1}{2|x|} u$$

$$= \left(\frac{3}{2} + \frac{1}{2|x|}\right) u \qquad \underline{x \to 0^+}$$

$$x \qquad f'(x) = 1$$

$$\log(e^x) = \frac{1}{e^x} e^x = 1$$