G3A8 - Matteo Zortea, Alessandro Rizzi, Marvin Wolf
Sheet 9

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BIC #9

1) P Evolution Between spikes: from U_n to U_{n+1}

$$\frac{dU}{U} = -\frac{dt}{\tau_{rec}} \Rightarrow \ln\left(\frac{U_{n+1}}{U_n}\right) = -\frac{\Delta t}{\tau_{fac}} +$$

$$\Rightarrow U_{n+1} = U_n e^{-\Delta t/\tau_{fac}}$$

$$\frac{dR}{R-1} = -\frac{dt}{\tau_{rec}} \Rightarrow \ln\left(\frac{R_{n+1}-1}{R_n - 1}\right) = -\frac{\Delta t}{\tau_{rec}}$$

$$\Rightarrow R_{n+1} = (R_n - 1) e^{-\Delta t/\tau_{rec}} + 1$$

U_{n+1} after a spike:

$$U_{n+1} = U_n e^{-\Delta t/\tau_{fac}} + U_0 (1 - U_n e^{-\Delta t/\tau_{fac}})$$

$$= \boxed{U_n (1 - U_0) e^{-\Delta t/\tau_{fac}} + U_0}$$

$$\boxed{\Delta E_{n+1} = U_{n+1} \left[(R_n - 1) e^{-\Delta t/\tau_{rec}} + 1 \right]}$$

$$R_{n+1} = (R_n - 1) e^{-\Delta t/\tau_{rec}} + 1 - \Delta E_{n+1}$$

$$\boxed{R_{n+1} = (1 - U_{n+1}) \left[(R_n - 1) e^{-\Delta t/\tau} + 1 \right]}$$

b) $\Delta t = 1/r \qquad t_n = n/r$

$$\hat{U}(r) = \lim_{n \to \infty} U_n$$

$$\hat{U}_{n+1}(r) = \hat{U}_n(r)$$

$$\Rightarrow \hat{U}_n (1 - U_0) e^{-1/r\tau} + U_0 = \hat{U}_n$$

$$\boxed{\hat{U}_n = \frac{-U_0}{(1 - U_0) e^{-1/r\tau} - 1}}$$

$$\hat{R}_{n+1} = \hat{R}_n$$

$$\Rightarrow (1 - \overset{\overset{\hat{U}_n}{=}}{\hat{U}_{n+1}}) \left[(\hat{R}_n - 1) e^{-\Delta t/r\tau} + 1 \right] = \hat{R}_n$$

$$\hat{R}_n (1 - \hat{U}_n) e^{-1/r\tau} - \hat{R}_n = -(1 - \hat{U}_n) \left[1 - e^{-1/r\tau} \right]$$

$$\Rightarrow \boxed{\hat{R}_n = \frac{(\hat{U}_n - 1)(1 - e^{-1/r\tau})}{(1 - \hat{U}_n) e^{-1/r\tau} - 1}}$$

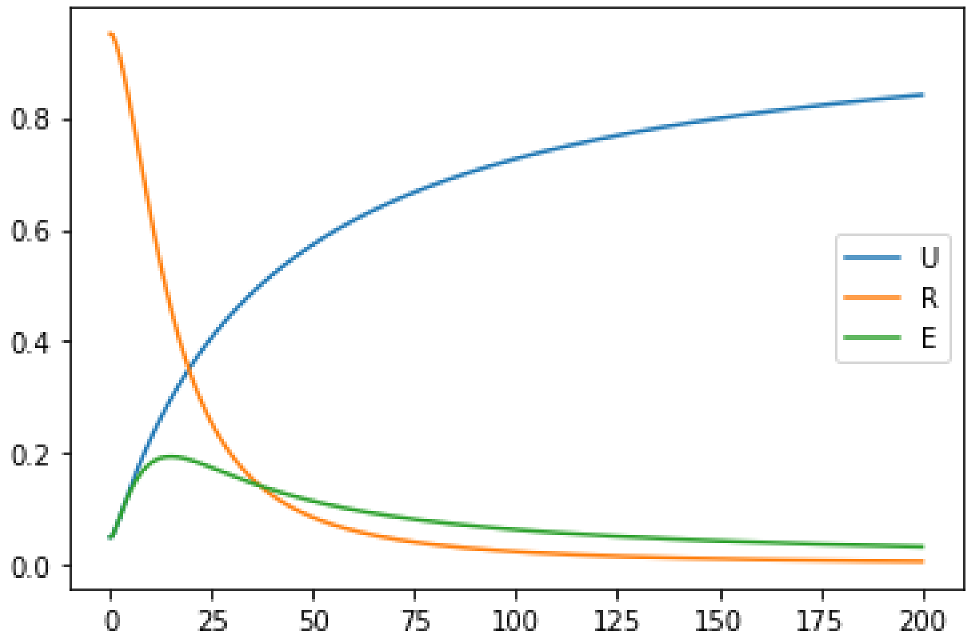
$$\boxed{\Delta \hat{E}_n = \hat{U}_n \left[(\hat{R}_n - 1) e^{-1/r\tau} + 1 \right]}$$

1 BIC EX 9.1

[92]:
```
import numpy as np
from matplotlib import pyplot as plt
%matplotlib inline
```

[93]:
```
U_0 = 0.05
tau_fac = 0.5
tau_rec = 0.15

v_min = 0.1
v_max = 200

def U(v):
    return U_0 / ( 1 - (1-U_0) * np.exp(-1/(v*tau_fac)) )

def R(v):
    return (U(v)-1) * (1-np.exp(-1/(v*tau_rec)) ) / ( (1-U(v))*np.exp(-1/
 ↪(v*tau_rec)) -1)

def E(v):
    return U(v)* ( (R(v)-1)*np.exp(-1/(v*tau_rec)) +1 )
```

[94]:
```
freq = np.linspace(v_min ,v_max , 1000 )
U1 , R1 , E1 = U(freq), R(freq), E(freq)

plt.plot(freq,U1, label='U')
plt.plot(freq,R1, label='R')
plt.plot(freq,E1, label='E')
plt.legend()
plt.show()
```

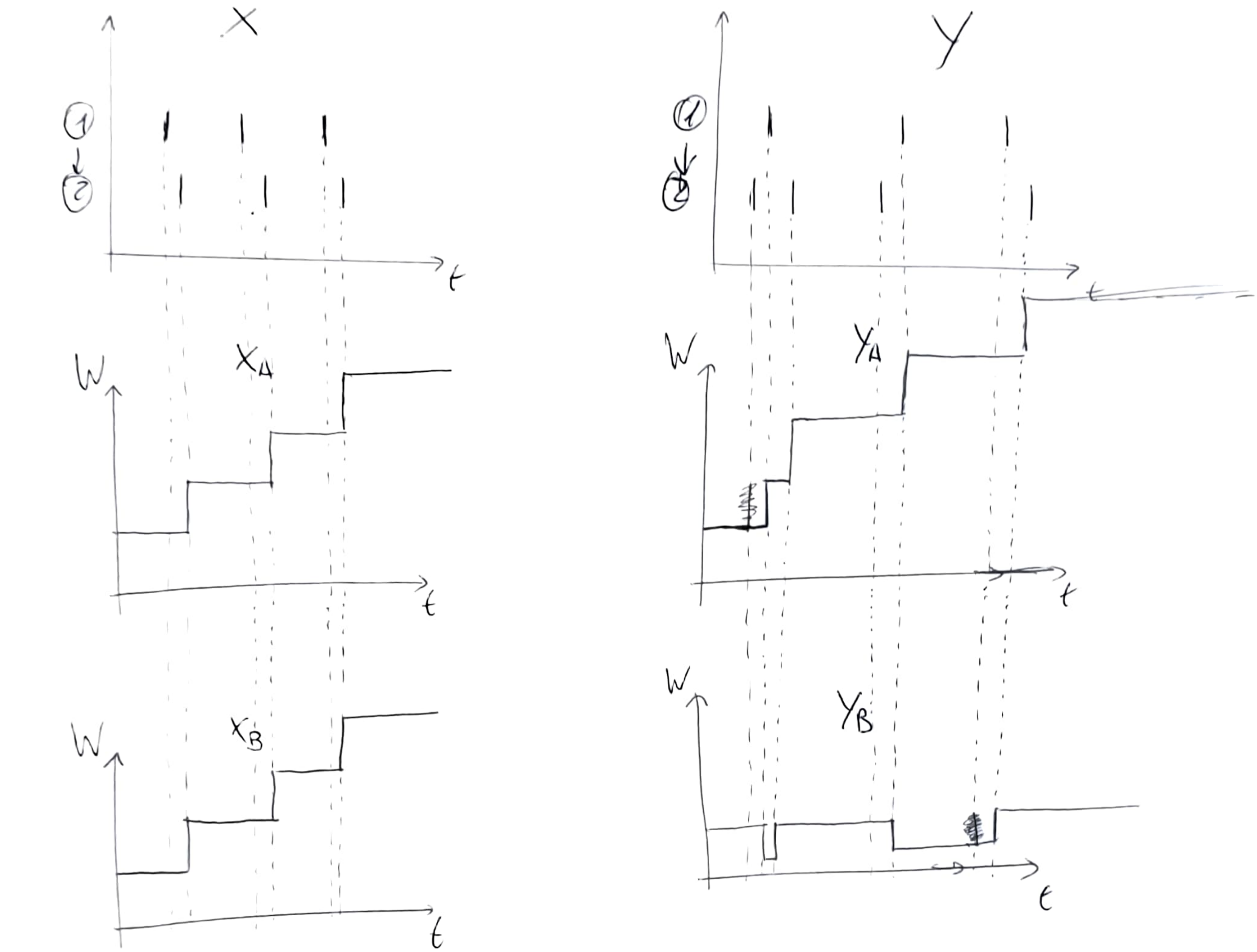


0.8
0.6
0.4
0.2
0.0
0
25
50
75
100
125
150
175
200
U
R
E

[ ]:

[ ]:

Exercise 2



I neglected all the changes between two distant events, for example like afther the time between the spike of the second neuron and the successive spike of the first neuron (this should be a long negative Δt).