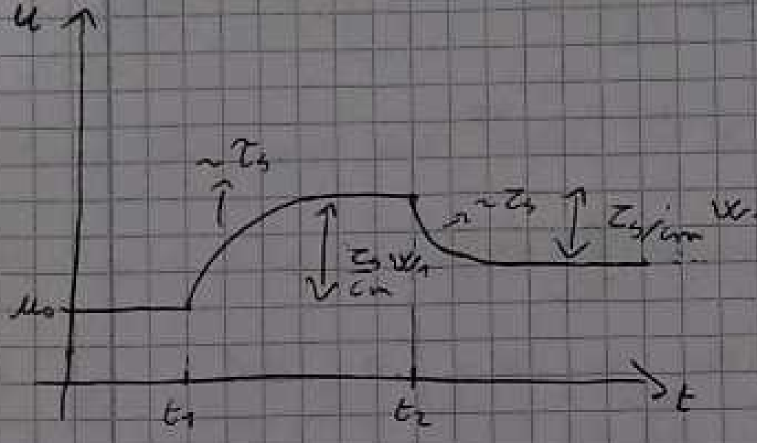
BK #6

$\tau_m = \infty \qquad u(t)_{\tau_m=\infty} = \frac{1}{C_m}\tau_s \sum_{t_s} w_i \,\theta(t-t_i)\left[-\exp\left(-\frac{t-t_i}{\tau_s}\right)+1\right]$

a)



b) $u(t)_{\tau_m=\infty} = \frac{\tau_s}{C_m}\sum_{t_i} w_i\,\theta(t-t_i)\left[1-\exp\left(-\frac{t-t_i}{\tau_s}\right)\right]$

c) $z = e^{t/\tau_s}$ Note that $\theta(t-t_i) = \theta(e^{t/\tau_s} - e^{t_i/\tau_s})$ for the monotony of the exponential function

$\Rightarrow u(z_{out})_{\tau_m=\infty} = \frac{\tau_s}{C_m}\sum_{z_i} w_i\,\theta(z_{out}-z_i)\left[1-\frac{z_i}{z_{out}}\right]$

Now, introduce the set G that contains all only the t_i before the first spike, i.e. with $z_i < z_{out}$ $\rightarrow$ get rid of θ function

Then $u(z_{out}) = \frac{\tau_s}{C_m}\sum_{z_i\in G} w_i\left[1-\frac{z_i}{z_{out}}\right] = \Theta \hookrightarrow$ threshold

$\Theta\frac{C_m}{\tau_s} - \sum w_i = -\frac{1}{z_{out}}\sum w_i z_i \Rightarrow$

$\Rightarrow z_{out} = \frac{\sum z_i w_i}{\sum w_i - \Theta\frac{C_m}{\tau_s}}$ Where the sums are carried out on the set G

d) $\frac{\partial z_{out}}{\partial w_s} = \frac{z_s\left(\sum w_i - \Theta\frac{C_m}{\tau_s}\right) - \sum z_i w_i}{\left[\sum w_i - \Theta\frac{C_m}{\tau_s}\right]^2}$ using that $\frac{\partial}{\partial w_s}\sum_i w_i = 1$ and $\frac{\partial}{\partial w_s}\sum_i w_i z_i = z_s$

because $\frac{\partial w_i}{\partial w_s} = \delta_i^s$

$\Rightarrow \frac{\partial z_{out}}{\partial w_s} = \frac{z_s}{[\quad]} - \frac{\sum z_i w_i}{[\quad]^2}$

$$\frac{\partial z_{out}}{\partial z_j} = \frac{w_j}{\sum_i w_i - \theta \ln / z_j} \quad \text{because } \frac{\partial z_i}{\partial z_j} = \delta_{ij}$$