Brain Inspired Computing (WS 21): Exercise sheet 2

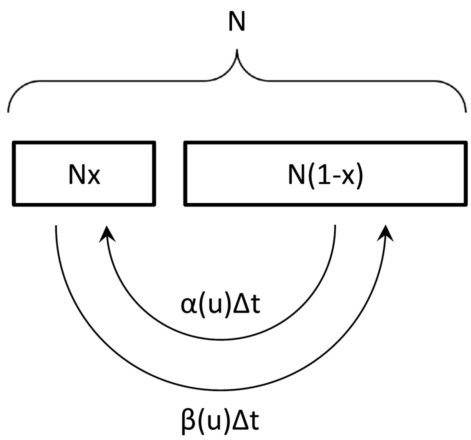
Hand in on 09.11.2021, 14:00

Name(s): Group:

Question:	1	2	3	4	Total
Points:	40	40	10	10	100
Score:					

Exercise 1: Channel activation functions (40 points)

Since the variables m, n and h represent probabilities of particular molecular gates being open, some authors prefer describing them with a different set of equations that more intuitively depict their stochastic switching between "open" and "closed" states.



Assume that a protein is "open" with the probability x. Out of N such proteins that are embedded in a membrane, Nx will be open and $N(1-x)$ closed, on average, at any point in time. If, during a very short time interval Δt, the protein may switch from "open" to "closed" with the probability $\beta_x(u)\Delta t$ and from "closed" to "open" with the probability $\alpha_x(u)\Delta t$, then the population of open proteins will lose $Nx \cdot \beta_x(u)\Delta t$ and gain $N(1-x) \cdot \alpha_x(u)\Delta t$ members during this Δt.

(a) (10 points) Write this down as a differential equation for x (we can replace Δs with "d"s in the limit of $\Delta t \to 0$).

Hint: the total number of channels does not change, so $\Delta(Nx) = N\Delta x$.

(b) (15 points) Transform the ODE in a) to the form

$$\dot{x} = \frac{1}{\tau_x(u)}[x_0(u) - x]$$

(which we used in the lecture to describe the dynamics of the activation variables m, n and h).

What are the required transformations from α_x and β_x to τ_x and x_0?

(c) (15 points) Show that with the switching rates

$$\alpha(u) = \frac{1}{1+e^{-\frac{u+a}{b}}} \quad \text{and} \quad \beta(u) = \frac{1}{1+e^{\frac{u+a}{b}}}$$

the stationary value of the activation variables x can be written as

$$x_0(u) = \frac{1}{2}\left[1 + \tanh\left[\beta(u - \Theta_{\text{act}})\right]\right].$$

Determine the activation threshold Θ_{act} and the activation slope β.

Exercise 2: Euler moving forward (1.3 continued) (40 points)

(a) (30 points) **4D case (Hodgkin-Huxley neuron)**
Simulate a Hodgkin-Huxley neuron with forward Euler for a cell with membrane capacitance $C_\text{m} = 1\,\text{nF}$. As an external stimulus $I^\text{ext}(t)$, use a step current of amplitude 7.5 nA and width 50 ms. This stimulus is strong enough to elicit spikes. Use integration time step $\Delta t = 0.01\,\text{ms}$, and initial conditions $u(0) = -65\,\text{mV}$, $n(0) = 0.3$, $m(0) = 0.1$, $h(0) = 0.6$. Plot the time course of the input, the membrane potential and the gating variables.

The dynamics of the gating variables $x \in \{\text{n,m,h}\}$ are provided in the table below. They are given in the original formulation of A. Hodgkin and A. Huxley: Gates open and close stochastically with switching probabilities $\alpha_x(u)$ and $\beta_x(u)$ respectively. In this formulation, the differential equations of the gating variables read $\dot{x} = \alpha_x(u)(1 - x) - \beta_x(u)x$. The formulation used in the lecture (and that you should use in your implementation) is $\dot{x} = \tau_x^{-1}(u)(x_0(u) - x)$. Both formulations are equivalent, and $x_0(u)$ and $\tau_x^{-1}(u)$ can be expressed in terms of $\alpha_x(u)$ and $\beta_x(u)$ as follows:

$$x_0(u) = \alpha_x(u)/(\alpha_x(u) + \beta_x(u)) \qquad \text{and} \qquad \tau_x^{-1}(u) = \alpha_x(u) + \beta_x(u) \quad . \tag{15}$$

The resulting values for τ_x with the equations in the table are in ms. Hint: First define functions for all $\alpha_x(u)$ and $\beta_x(u)$ by copying the equations below. Then define functions for all six variables $x_0(u)$ and $\tau_x^{-1}(u)$ using the translation eq. (15).

x	E_x	g_x
Na	50 mV	120 µS
K	−77 mV	36 µS
l	−54.4 mV	0.3 µS

Reversal potentials and conductances.

x	α_x(u[mV])	β_x(u[mV])
n	$\frac{-0.55-0.01u}{\exp(-5.5-0.1u)-1.}$	$0.125\exp(-(u+65)/80.0)$
m	$\frac{-4.0-0.1u}{\exp(-4.0-0.1u)-1.0}$	$4.0\exp(-(u+65)/18.0)$
h	$0.07\exp(-(u+65)/20.0)$	$1.0/(\exp(-3.5-0.1u)+1.0)$

Gating variable ODEs, given in the original formulation of the HH model as chemical kinetic equations.

(b) (10 points) **Hodgkin-Huxley neuron: Post-inhibitory rebound spike**
Again, use your forward Euler Hodgkin-Huxley integrator to reproduce another effect mentioned in the lecture called "Post-inhibitory rebound". To this end, apply a step current of amplitude −3.0 nA for 30 ms. Plot the stimulus, the membrane potential and the gating variables.

Exercise 3: LIF Firing Rate (10 points)

Derive the following equation for the firing rate ν of a LIF neuron stimulated by the constant current I^{ext}:

$$\nu(I^{\text{ext}}) = \left(\tau_{\text{ref}} + \tau_{\text{m}} \ln\left(\frac{E_{\text{reset}} - E_{\text{leak}} - \frac{I^{\text{ext}}}{g_{\text{l}}}}{\theta - E_{\text{leak}} - \frac{I^{\text{ext}}}{g_{\text{l}}}}\right)\right)^{-1}, \tag{16}$$

with the refractory period τ_{ref}, the membrane time constant τ_{m}, the reset potential E_{reset}, the resting potential E_{leak}, the firing threshold θ and the leak conductance g_{l}.

Exercise 4: Asymptotic behavior of LIF neurons (10 points)

Use the activation function $\nu(I^{\text{ext}})$ of LIF neurons in response to a constant input current I^{ext} from the previous exercise. For vanishing refractory period, $\tau_{\text{ref}} = 0$, the activation $\nu(I^{\text{ext}})$ diverges in the limit of large input currents $I^{\text{ext}} \to \infty$. However, this divergence happens linearly (the activation function has an oblique asymptote).

Find the asymptotic behavior of $\nu(I^{\text{ext}})$ as $I^{\text{ext}} \to \infty$ in the limit of $\tau_{\text{ref}} \to 0$. Give the full function in the form $\nu_{\text{asymp}}(I^{\text{ext}}) = K \cdot I^{\text{ext}} + C$, with the slope K and the y-intercept C.

Hint: Consider the taylor series for $\ln(1-y)$ and $\frac{1}{1-y}$:

$$\ln(1-y) = -y - \frac{y^2}{2} - \frac{y^3}{3} + O(y^4), \tag{32}$$

$$\frac{1}{1-z} = 1 + z + z^2 + z^3 + O(z^4). \tag{33}$$