

For $T_m = T_s$ we got a $\frac{0}{0}$ situation.

Solve this using L'Hospital by decomposition into 2 functions $f(T_s)$ and $g(T_s)$, such that $u(T_s) = \frac{f(T_s)}{g(T_s)}$:

$$f(T_s) = T_m \cdot T_s \cdot \underbrace{\sum_{\text{spikes } t_i} w_i \Theta(t-t_i) \left[\exp\left(-\frac{t-t_i}{T_m}\right) - \exp\left(-\frac{t-t_i}{T_s}\right) \right]}_{b(T_s)}$$

$$g(T_s) = C_m (T_m - T_s)$$

$$\text{we got } \lim_{T_s \rightarrow T_m} \frac{f(T_s)}{g(T_s)} = \frac{0}{0}$$

$$\text{L'Hospital: } \lim_{T_s \rightarrow T_m} \frac{f(T_s)}{g(T_s)} \approx \frac{f'(T_s)}{g'(T_s)}$$

$$f'(T_s) = T_m \cdot b + T_m T_s \cdot b'(T_s)$$

$$\text{with } b'(T_s) = \sum_{\text{spikes } t_i} w_i \Theta(t-t_i) \left[-\frac{t-t_i}{T_s^2} \cdot \exp\left(-\frac{t-t_i}{T_s}\right) \right]$$

$$g'(T_s) = -C_m$$

We obtain:

$$\lim_{T_s \rightarrow T_m} u(t) \approx \frac{-1}{C_m} \cdot \left[T_m \cdot b + T_m T_s \cdot b'(T_s) \right]$$