FYS 3500 - Midterm Exam Spring 2021 (home exam)

Deadline to hand it in is 26 of March 2021, 2 PM

The delivery will be in Inspera

Useful resources:

- Books presented in the lecture/problem sessions
- Chart of nuclides on http://www.nndc.bnl.gov/
  Eg masses, lifetimes, energy levels, decay schemes, …
- Also: Livechart – Table of Nuclides:

  https://www.nndc.bnl.gov/nudat2/

- Particle physics: Lifetimes, decay widths (...)
  http://pdg.lbl.gov

Please cite external references where used.

Please site colleague(s) if you have worked intensively together on a problem.

1. **Some shorter questions (14p)**
Answer the following questions (you're welcome to be brief where possible).

a) What is the ground-state spin and parity of even-even nuclei? Why? (2)

b) A hadron is observed to decay by the strong interaction to $\pi^+\pi^-\pi^0$. Work out and list the possible isospin states of this hadron, i.e. the possible values of I and I_3. (2)

c) Sketch the nuclear potential for both neutrons and protons. Comment on the differences. (2)

d) Why are there only two $J = 1/2$baryons (n and p) with $S = 0$ (strangeness zero)? (2)

e) Is it harder to steal a neutron from ^{118}Sn, ^{16}O, or ^{119}Sn? Explain your thinking, not just look up the S_n. (2)

f) In a dataset corresponding to 5/fb (5 inverse femtobarns) of integrated luminosity, the number of observed events of $pp \rightarrow X$ is 1234. Given a detection efficiency of 25% and a predicted background of 250 events, what is the cross-section for $pp \rightarrow X$? (2)

g) What are the approximate ratios of the cross sections
$\sigma(\gamma e^- \rightarrow \gamma e^-) : \sigma(\gamma e^- \rightarrow \gamma\gamma e^-) : \sigma(\gamma e^- \rightarrow \gamma\gamma\gamma e^-)$ at high energy? (2)

2. **Nuclear binding energy (15p)**

   a) Use the semi-empirical mass formula to determine the mass of ^{48}Ca. How well does it agree with measured value(s)? If deviation, what are possible explanations?

   b) Calculate the energy needed to remove one neutron from the ^{44}Ca nucleus.

   c) For A=136 find the most stable nucleus using the semi-empirical mass formula. Sketch the mass as a function of Z for the A=136 isobars ("mass parabola") and explain how the beta decay flows between nuclei with A=136. Make also a sketch for a case where A is an odd number, for example A=135, explain the difference from the even A case.

3. Higgs boson decay - (20p)

Higgs bosons are produced in high-energy pp collisions at the Large Hadron Collider (LHC). In one of the pp collisions, two pairs of oppositely charged electrons (i.e. twoe^+e^- pairs, total of four leptons) are observed (see the table below). The candidate is hypothesized to be a $H \rightarrow ZZ^*$decay followed by $Z \rightarrow e^+e^-$and $Z^* \rightarrow e^+e^-$ (the "*" indicates that this is a "highly virtual" or "off-shell" Z boson).

Particle	Type	p_x(GeV)	p_y(GeV)	p_z(GeV)
1	e^+	0.814	-13.810	-8.409
2	e^+	-27.649	-1.511	-20.665
3	e^-	38.632	13.361	44.775
4	e^-	-13.443	2.514	-5.853

a) What is the invariant mass of this Higgs boson candidate? (6)
b) There are multiple ways the e^+ and e^- can be paired. How many? Which one is the most compatible with the hypothesis? To derive the answer you should determine the invariant masses of the various Z boson candidates. (6)
c) Given the invariant masses of the Higgs boson M_H and the two Z bosons m_1 and m_2, what are the energies E_1 and E_2 of the two Z bosons in the rest frame of the Higgs boson? (6)
d) Don't do the work, but describe how you would go about determining the 4 lepton energies in the rest frame of the Higgs boson (a formula and how to apply it would be helpful). (2)

4. Lepton universality - (10p)

Using dimensional arguments, derive an approximate expression for the ratio of the decay rates $\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)/\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)$. (7) What observable quantities do we need to measure in experiments to test this ratio for lepton universality? (3)

5. Quantum numbers - (12p)

a) Describe the parity (P) and charge conjugation (C) quantum numbers. (2)

b) What does the quark model predict for the C-parity of the $J/\Psi(3097)$ meson, given that it is a $^3S_1$ state? (2)

c) Find and explain the spin and parity of ^{15}N for the ground state and 1st excited state. Compare to experimental results found here:
https://www.nndc.bnl.gov/nudat2/getdatasetClassic.jsp?nucleus=15N&unc=nds.
Using the experimentally determined second and third excited states, are you able to identify their nucleon configurations? (4)

d) What are the possible gamma transitions from the third excited state? Explain. For each transition, circle the most probable one. Why does the second excited state prefer to decay directly to the ground state, and not through the first excited state? (4)

6. Allowed, suppressed and forbidden processes (20p)

1) $\Psi(3686) \rightarrow \chi_{c0}(3415)\gamma$ 2) $pp \rightarrow \Sigma^{+}p\overline{K}^{0}\pi^{-}$ 3) $b \rightarrow s\gamma$

4) $\Lambda^{0} \rightarrow pe^{-}\overline{\nu}_{e}$ 5) $\Delta^{+} \rightarrow p\pi^{0}$ 6) $e^{+}e^{-} \rightarrow q\overline{q}g$

7) ${}^{82}_{34}Se \rightarrow {}^{82}_{36}Kr + 2e^{-} + 2\overline{\nu}_{e}$ 8) ${}^{82}_{34}Se \rightarrow {}^{82}_{36}Kr + 2e^{-}$

a) Which of the above processes are allowed and which are forbidden?
b) If allowed, draw the (dominant) Feynman diagram(s) and state which interaction(s) is (are) at work.
c) For allowed *decays* check that the interaction type and lifetime are compatible.
d) If suppressed or forbidden, give a reason. You may consider new physics scenarios if and when appropriate.
e) Assuming processes 7 and 8 are allowed, how would you separate them experimentally? What would be the consequences of such an observation?

7. Radioactive decay chain (20p)

NB: Remember to include your code and the figures in your hand-in!
Consider a chain of radioactive decays of nucleus 1, 2, and 3:

$1 \rightarrow 2 \rightarrow 3$,

where nucleus 3 is stable and the decays $1 \rightarrow 2$ and $2 \rightarrow 3$ are characterised by the decay constants λ_1 and λ_2 respectively. The amount of nuclei 1, 2, and 3 that we have at time t is described by $N_1(t)$, $N_2(t)$, and $N_3(t)$. At $t = 0$, $N_1(t = 0) = N_0$ and $N_2(t = 0) = N_3(t = 0) = 0$.

a) Use the radioactive decay law to show that $N_1(t)$ and $N_2(t)$ can be written as:

$$N_1(t) = N_0 e^{-\lambda_1 t}$$

$$N_2(t) = N_0 \frac{\lambda_1 (e^{-\lambda_1 t} - e^{-\lambda_2 t})}{\lambda_2 - \lambda_1}$$

It is also possible to show (but you don't need to show it!) that

$$N_3(t) = N_0 \frac{\lambda_1 (1 - e^{-\lambda_2 t}) - \lambda_2 (1 - e^{-\lambda_1 t})}{\lambda_1 - \lambda_2}$$

b) What are the relationships between average lifetime τ, half-life $t_{1/2}$, decay rate λ, and natural width Γ of unstable nuclei and particles? What are their units?

We now look at the decay chain ^{139}Cs → ^{139}Ba → ^{139}La, observed from an initially pure sample of 1mCi ^{139}Cs.

c) Calculate the amount of ^{139}Cs present at $t = 0$.
d) Write a script that plots the number of ^{139}Cs, ^{139}Ba, and ^{139}La as a function of time. Also plot the activities, and let the time axis run from 0 to 12 hours in both cases. Explain what you see.
e) What is the maximum activity of ^{139}Ba and when does it occur?
f) At what time are the activities of ^{139}Cs and ^{139}Ba equally large?

Good Luck!