Lecture 1 - 20/01/2021

Parity

The action of the parity operator $\mathcal{P}$ is defined on the coordinate vector as

$$\mathcal{P}\,|x\rangle = |-x\rangle$$

. For a generical state $|\psi\rangle$ it automatically follows that

$$\mathcal{P}\,|\psi\rangle = \mathcal{P}\int dx\;|x\rangle < x|\psi >= \int dx\;|-x\rangle < x|\psi >= \int dx'\;|x'\rangle < -x'|\psi >$$

so that

$$< x|\,\mathcal{P}\,|\psi >= \psi(-x)$$

In spherical coordinates the action of the parity operator causes $r \to r$, $\theta \to \pi - \theta$ and $\phi \to \pi + \phi$.
Suppose now that $|\psi\rangle$ is an eigenstate of the parity operator so that

$$\mathcal{P}\,|\psi\rangle = c\,|\psi\rangle$$

By applying $\mathcal{P}$ to both member of the last expression, one obtains that

$$\mathcal{P}^2\,|\psi\rangle = c^2\,|\psi\rangle \tag{1}$$

but also

$$\mathcal{P}^2\,|\psi\rangle = \mathcal{P}\int dx\;|-x\rangle < x|\psi >= \int dx\;|-x\rangle < x|\psi >= |\psi\rangle \tag{2}$$

and combining 1 and 2 we conclude that $c = \pm 1$. If the eigenvalue is $+1$ (-1) we say that the corresponding eigenstate has even (odd) parity.