

1 Some shorter questions

a

This result can be explained by means of the nuclear shell model. According to this model each sub-level of the nucleus can be identified by 4 quantum numbers $\{n, j, l, m_l\}$. For fixed $\{n, j, l\}$ one has that the $2l+1$ sub-levels given by $m_l = -l, -l+1, \dots, l-1, l$ are degenerate and each of this sub-levels can contain two nucleons with different spin z-projection ($m_s = \pm 1$).

At first one can notice that if one sub-shell is filled with two nucleons (of the same type) then the spin contribution of that sub-shell is 0. Having an even number of protons and neutrons means that they combine in couples in the sub-shell in a way such that the contribution to the spin of each sub-shell is 0. For what concerns the parity, each particle in the nucleus gives a contribute of $(-1)^l$ (the intrinsic parity of the nucleons is +1) and the total parity is the product of single parities. The product of the parities of two protons (neutrons) in the same sub-shell is always +1 since they, in particular, share the same quantum number l . Hence it is legitimate to expect an even-even nucleus to be in the state 0^+ .

Answer: spin 0, parity +

2 Nuclear binding energy

a

The mass of the atom is given by

$$M(^{48}\text{Ca}) = Zm_P + (A - Z)m_N - \frac{B(A, Z)}{c^2}$$

where B indicates the binding energy, which in turns can be calculated via the semi-empirical formula (atomic units)

$$B(A, Z) = a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{A} + \delta(A, Z) \quad (1)$$

One can simply pop the values of A and Z into the formula using the coefficients

$$a_v = \quad a_s = \quad a_c = \quad a_{sym} = \quad a_p =$$

and obtains $B(A, Z) = val$. Hence, inserting the result into 2 one obtains

$$M(^{48}\text{Ca}) \simeq val$$

b

Since $B(A, Z) = \sum_i B_i(A_i, Z_i)$ one can calculate the contribution of the binding energy of the last neutron as

$$B_{neutron} = B(A, Z) - B(A-1, Z)$$

And using expression 1 to calculate the two quantities one ends up with an estimation for the energy required to extract the neutron

$$B_{neutron} = B(A, Z) - B(A-1, Z) \approx$$