1 Some shorter questions

a

This result can be explained by means of the nuclear shell model. According to this model each sub-level of the nucleus can be identified by 4 quantum numbers $\{n, j, l, m_l\}$. For fixed $\{n, j, l\}$ one has that the $2l+1$ sub-levels given by $m_l = -l, -l+1, \ldots, l-1, l$ are degenerate and each of this sub-levels can contain two nucleons with different spin z-projrection ($m_s = \pm 1$).
At first one can notice that if one sub-shell is filled with two nucleons (of the same type) then the spin contibution of that sub-shell is 0. Having an even number of protons and neutrons means that they combine in couples in the sub-shell in a way such that the contribution to the spin of each sub-shell is 0. For what concerns the parity, each particle in the nucleus gives a contribute of $(-1)^l$ (the intrinsic parity of the nucleons is +1) and the total parity is the product of single parities. The product of the parities of two protons (neutrons) in the same sub-shell is always +1 since they, in particular, share the same quantum number l. Hence it is legitimate to expect an even-even nucleus to be in the state 0^+.
Answer: spin 0, parity + .

2 Nuclear binding energy

a

The mass of the atom is given by

$$M(^{48}\text{CA}) = Zm_P + (A-Z)m_N - \frac{B(A,Z)}{c^2}$$

where B indicates the binding energy, which in turns can be calculated via the semi-empirical formula (atomic units)

$$B(A,Z) = a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym}\frac{(A-2Z)^2}{A} + \delta(A,Z) \tag{1}$$

One can simply pop the values of A and Z into the formula using the coefficients

$$a_v = \qquad a_s = \qquad a_c = \qquad a_{sym} = \qquad a_p =$$

and obtains $B(A,Z) = val$. Hence, inserting the result into 2 one obtains

$$M(^{48}\text{CA}) \simeq val$$

b

Since $B(A,Z) = \sum_i B_i(A_i, Z_i)$ one can calculate the contribution of the binding energy of the last neutron as

$$B_{neutron} = B(A,Z) - B(A-1,Z)$$

And using expression 1 to calculate the two quantities one ends up with an estimation for the energy required to extract the neutron

$$B_{neutron} = B(A,Z) - B(A-1,Z) \approx$$

3

4

5 Quantum numbers

a

Parity

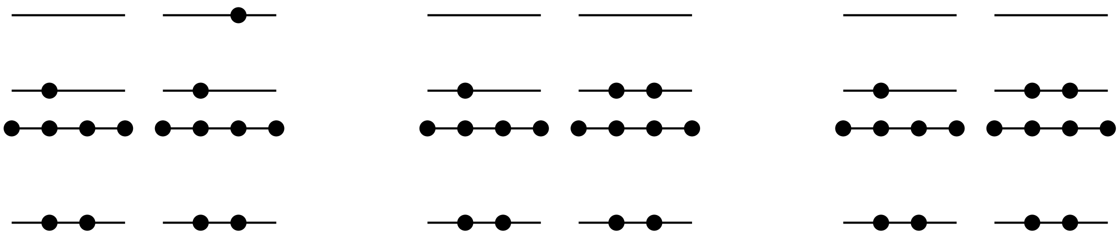

Figure 1: nothing

Let us consider a wavefunction $\psi(\mathbf{r})$. One can define the parity operator $\hat{P}$ as an operator such that

$$\hat{P}\,\psi(\mathbf{r}) = \psi(-\mathbf{r})$$

Since

$$\hat{P}^2\,\psi(\mathbf{r}) = \hat{P}\,\psi(-\mathbf{r}) = \psi(\mathbf{r}) \tag{2}$$

the eigenvalues of the parity operator are ± 1 and the corresponding eigenfunctions are the odd (eigenvalue -1) and even (eigenvalue $+1$) wavefunctions. If the wavefunction describes the state of particle, and the state is an eigenstate of the parity operator, the corresponding eigenvalue is also said to be the (intrinsic) parity of the particle.
Let us consider a system of two particles $A + B$ described by a wavefunction $\psi_{AB}(\mathbf{r}_A, \mathbf{r}_B)$. It can be proven that the parity of the system is given by

$$\hat{P}\psi_{AB} = \pi_A \pi_B (-1)^l \psi_{AB}$$

where l denotes the orbital angular momentum quantum number of the relative motion. Hence in a reaction of the type $A + B \to C + D$ described by a hamiltonian $\hat{H}$ that commutes with parity, parity is conserved or, in other words

$$\pi_A \pi_B (-1)^{l_{AB}} = \pi_C \pi_D (-1)^{l_{CD}}$$

This, for example, is not the case of the weak interaction where, in general, the hamiltonian operator does not commute with the parity operator.

Charge conjugation

C

^{15}N atom has 7 protons and 8 neutrons. Since the neutrons (in the ground state) are all coupled (each subshell admits two nucleons) they do not contribute to the spin of the nucleus and the parity contribution is $+1$. Instead the protons configuration of the ^{15}N atom in the ground state is $(1s)^2\,(1p_{3/2})^2\,(1p_{1/2})^1$: only the last proton contributes to the interested quantities. Since the orbital p represents the quantum number $l = 1$ the parity of the the nucleus is the product of the neutrons an protons contribution, that is $(+1) \cdot (-1)^1 = -1$. The spin is then 1/2.
There are three possible alternatives for the first excited state, all of which consists in moving a proton or a neutron to another level starting from the ground state configuration.

1. Move the $1p_{1/2}$ proton to the $1d_{5/2}$ level
   $\longrightarrow$ the new proton configuration is $(1s)^2\,(1p_{3/2})^2\,(1p_{1/2})^{-2}\,(1d_{5/2})^1$ and the spin-parity is $1/2^+$.
2. Move the $1p_{3/2}$ proton to the $1d_{5/2}$ level
   $\longrightarrow$ the new proton configuration is $(1s)^2\,(1p_{3/2})^{-1}\,(1p_{1/2})^2$ and the spin-parity is $3/2^-$.
3. Move the $1p_{1/2}$ neutron to the $1d_{5/2}$ level
   $\longrightarrow$ the new neutorn configuration is $(1s)^2\,(1p_{3/2})^2\,(1p_{1/2})^{-1}\,(1d_{5/2})^1$. The parity here is non-trivial because of the angular momemnta addition rules.

By looking at the suggested table, one can notice that the first excited state is described by configuration 3, the second by configuration 1 and the third excited state is described by configuration 2.