Lecture 1 - 20/01/2021

Parity

The action of the parity operator $\mathcal{P}$ is defined on the coordinate vector as

$$\mathcal{P}\,|x\rangle = |-x\rangle$$

. For a generical state $|\psi\rangle$ it automatically follows that

$$\mathcal{P}\,|\psi\rangle = \mathcal{P}\int dx\ |x\rangle < x|\psi >= \int dx\ |-x\rangle < x|\psi >= \int dx'\ |x'\rangle < -x'|\psi >$$

so that

$$< x|\,\mathcal{P}\,|\psi >= \psi(-x)$$

In spherical coordinates the action of the parity operator causes $r \to r$, $\theta \to \pi - \theta$ and $\phi \to \pi + \phi$.
Suppose now that $|\psi\rangle$ is an eigenstate of the parity operator so that

$$\mathcal{P}\,|\psi\rangle = c\,|\psi\rangle$$

By applying $\mathcal{P}$ to both member of the last expression, one obtains that

$$\mathcal{P}^2\,|\psi\rangle = c^2\,|\psi\rangle \tag{1}$$

but also, from what previously said

$$\mathcal{P}^2\,|\psi\rangle = \mathcal{P}\int dx\ |-x\rangle < x|\psi >= \int dx\ |-x\rangle < x|\psi >= |\psi\rangle \tag{2}$$

and combining 1 and 2 we conclude that $c = \pm 1$. If the eigenvalue is $+1$ (-1) we say that the corresponding eigenstate has even (odd) parity.

Cross section

Let us define the flux J as a quantity that quantifies the rate of production of new particles in the experiment (in the sense of things coming out of the target)

$$J = n_b\, v_i$$

where n_b is the number density of particles in the beam (number of particles per unit volume) and v_i is the speed of each particle in the rest frame of the target. J identifies the number of particles per second per unit area in the beam

$$[J] = m^{-2}s^{-1}$$

Call N the number of particles in the target "illuminated" by the beam: we can then guess that the number of scattered particle per second per unit area is of the form

$$W_r = JN\sigma_r$$

where σ_r is a proportionality constant called *cross section*. One has that

$$[\sigma_r] = m^2$$

The cross section can be thought as the equivalent area that allows the process to occur. For example if both the target and the beam consist in a single particle, the equivalent area at which the collison may occur is a circle of radius $2r$, hence $\sigma_r = \pi\,(2r)^2 = 4\pi r$. To visualise it keep one sphere fixed, put the other in contact and move it around keeping one contact point: this is the area that allows the collison.

Lifetimes

Decay is a stochastic process: hence we can only know the probability of an atom to decay. By observing a population of nuclei one describe the behaviour of the systems in therms of a differential equation

$$\text{Activity} \equiv -\frac{dN}{dt} = \lambda N$$

from which

$$N(t) = N_0 e^{-\lambda t}$$

the time at which the population is reduced by a factor 2 is

$$t_{N/2} = \frac{1}{\lambda} \ln 2 \equiv \tau \ln 2$$

and we call τ (or equivalently $1/\lambda$) the decay characteristic time.

Decay width, branching ratio

The probability distribution over time of the decay to occur is normally a gaussian-like distribution. Let us then define a parameter Γ called the *decay width* that is the time difference of the two points in which we have $N(t) = N/2$ (otherwise called FWHM).
If a particle may decay into multiple particles, we define a probability distribution for each event, and we can calculate the factor Γ for each of them (call it Γ_k). We then define the *branching ratio* as

$$B_k = \frac{\Gamma_k}{\Gamma}$$

to compare the different cases.

Q-value

The Q-value express the mass energy difference between the states before and after the decay. For example suppose that a particle x may decay into x and z. Then the Q-value for this event is

$$Q = (m_x - m_y - m_z)\, c^2$$

. If Q is positive energy is release during the process, otherwise we should add energy in order to make the decay happen.