Without interaction between the particles the general expression for the potential of the *elliptical trap* is

$$V(\mathbf{r}) = \frac{1}{2}m\left[\omega_{ho}^2(x^2+y^2)+w_z z^2\right]$$

and the case in which $\omega_z = \omega_{ho}$ is called *spherical trap*. The trial function for this potential is

$$\Psi_T(\mathbf{r}) = \prod_{i=1}^{N} g(\alpha,\beta,\mathbf{r}_i)$$

where $g(\alpha,\beta,\mathbf{r}_i) = \prod_{i=1}^{N} exp\left[-\alpha\left(x_i^2+y_i^2+\beta z_i^2\right)\right]$, so that the local energy becomes

$$\begin{aligned}E_L(\mathbf{r}) &= \frac{1}{\Psi_T(\mathbf{r})}H\Psi_T(\mathbf{r})\\ &= \left(\prod_{i=1}^{N} g(\alpha,\beta,\mathbf{r}_i)\right)^{-1}\left(-\frac{\hbar^2}{2m}\nabla_{\mathbf{r}}^2+V(\mathbf{r})\right)\left(\prod_{i=1}^{N} g(\alpha,\beta,\mathbf{r}_i)\right)\\ &= \left(\prod_{i=1}^{N} g(\alpha,\beta,\mathbf{r}_i)\right)^{-1}\left\{V(\mathbf{r})\prod_{i=1}^{N} g(\alpha,\beta,\mathbf{r}_i) - \frac{\hbar^2}{2m}\nabla_{\mathbf{r}}^2\prod_{i=1}^{N} g(\alpha,\beta,\mathbf{r}_i)\right\}\end{aligned} \tag{1}$$

with $\nabla_{\mathbf{r}} = \sum_{i=1}^{N}\nabla_{\mathbf{r}_i}$. Let now be $\beta = 1$.

$$\frac{\partial^2}{\partial x^2}\left(e^{-\alpha\left(x_i^2+y_i^2+z_i^2\right)}\right) = -2\alpha e^{-\alpha\left(x_i^2+y_i^2+z_i^2\right)} + 4\alpha^2x^2e^{-\alpha\left(x_i^2+y_i^2+z_i^2\right)} = 2\alpha\left(2\alpha x^2-1\right)g(\alpha,\mathbf{r}_i)$$

Combining with the analogous expressions for y and z:

$$\nabla_i^2\Psi_i(\mathbf{r}_i) = \left[4\alpha^2(x_i^2+y_i^2+z_i^2)-6\alpha\right]g(\alpha,\mathbf{r}_i)$$

Using this result one can write expression 1 as follows

$$\begin{aligned}E_L(\mathbf{r}) = V(\mathbf{r}) - \frac{\hbar^2}{2m}\left(\prod_{i=1}^{N} g(\alpha,\beta,\mathbf{r}_i)\right)^{-1}\sum_{i=1}^{N}\left\{g(\alpha,\mathbf{r}_i)\left[4\alpha^2(x_i^2+y_i^2+z_i^2)-6\alpha\right]\prod_{j\neq i}^{N} g(\alpha,\mathbf{r}_j)\right\} =\\ = V(\mathbf{r}) - \frac{\hbar^2}{2m}\sum_{i=1}^{N}\left[4\alpha^2(x_i^2+y_i^2+z_i^2)-6\alpha\right]\\ = V(\mathbf{r}) + \frac{\alpha\hbar^2}{m}\left(3-2\alpha\sum_{i=1}^{N}\left(x_i^2+y_i^2+z_i^2\right)\right)\end{aligned}$$