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# **Project 2 Documentation**

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**Jun 01, 2021**



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**class** `do.GHF` (*l=10, grid\_length=10, num\_grid\_points=201, alpha=1.0, a=0.25, Omega=0.25, omega=2, epsilon0=1, nparticles=2, antisymmetrize=True*)

Initializes a system in the general spin representation. The default values in the creator are referred to the main analyzed case. A more detailed description is provided in the paper folder at <https://github.com/Matteo294/FYS4411>. The quantum-system documentation appears in <https://schoyen.github.io/quantum-systems/>

#### Args:

`l` (int) : number of harmonic oscillator eigenstates used for the expansion of the radial part of the single-particle wavefunctions  
`grid_length` (int) : extension of the mesh goes from `-grid_length` to `grid_length`  
`num_grid_points` (int) : number of points in the mesh  
`alpha` (float) : strength parameter in the shielded Coulomb potential  
`a` (float) : shielding parameter in the shielded Coulomb potential  
`Omega` (float) : frequency of the harmonic oscillator potential in which the electrons are trapped  
`omega` (float) : frequency of the laser source  
`epsilon0` (float) : amplitude of the sinusoidal potential associated to the laser  
`nparticles` (int) : number of electrons in the system  
`antisymmetrize` (bool) : if True, antisymmetrizes the system.u matrix

#### Attributes:

`potential` (func) : `ODQD.HOPotential(Omega)`  
`system` : `GeneralOrbitalSystem` object (see <https://schoyen.github.io/quantum-systems/>)  
`Omega` (float) : frequency of the harmonic oscillator potential in which the electrons are trapped  
`omega` (float) : frequency of the laser source  
`epsilon0` (float) : amplitude of the sinusoidal potential associated to the laser  
`nparticles` (int) : number of electrons in the system

**anima** (*i, text, line1, line2, integrator, dt, t\_max, save\_every\_n*)  
 Iterative function needed for `self.gif_generator`

#### **eval\_dipole** (*C*)

Returns the expected value of the position operator for the system, given by

$$\bar{x}(t) = \sum_i^{occ} \sum_{\alpha\beta} C_{\alpha,i}^*(t) C_{\beta,i}(t) \langle \chi_\alpha(x) | \hat{x} | \chi_\beta(x) \rangle$$

**Args:** `C` (np.ndarray) : coefficient matrix

**Returns:** dipole (complex) : expected value for position operator

#### **eval\_one\_body\_density** (*C*)

Returns the one-body density of the system, according to

$$\rho(x) = \sum_i^{occ} \sum_{\alpha\beta} C_{\alpha,i}^*(x) C_{\beta,i}(x) \chi_\alpha(x) \chi_\beta(x)$$

**Args:** `C` (np.ndarray) : coefficient matrix

**Returns:** one\_body\_density (np.array)

#### **eval\_total\_energy** (*C*)

Returns the total energy of the system, given by

$$E[H] = \sum_{i=1}^2 \sum_{\alpha,\beta} C_{\alpha,i}^* C_{\beta,i} h_{\alpha\beta}^{ho} + \frac{1}{2} \sum_{i,j=1}^2 \sum_{\alpha\beta\gamma\delta} C_{\alpha,i}^* C_{\gamma,j}^* C_{\beta,i} C_{\delta,j} u_{\beta\delta,AS}^{\alpha\gamma}$$

**Args:** C (np.ndarray) : coefficient matrix

**Returns:** energy (complex)

**fill\_density\_matrix**(C)

Returns the density matrix evaluated using the coefficient matrix C, according to

$$M_{\alpha\beta} = \sum_i^{occ} C_{\alpha,i}^* C_{\beta,i}$$

**Args:** C (np.ndarray) : coefficient matrix

**Returns:** density\_matrix (np.ndarray)

**fill\_fock\_matrix**(C, t)

Returns the Fock matrix evaluated using the coefficient matrix C evaluated at time t, according to

$$f_{\mu\nu}(t) = h_{\mu\nu}^{ho} + x_{\mu\nu}\varepsilon_0 \sin(\omega t) + \sum_j^{occ} \sum_{\gamma\delta} C_{\gamma,j}^* C_{\delta,j} u_{\nu\delta,AS}^{\mu\gamma}$$

**Args:** C (np.ndarray) : coefficient matrix

t (float) : time instant

**Returns:** fock\_matrix (np.ndarray)

**fourier\_analysis**(tolerance, max\_iter, t\_laser\_ON, t\_max, dt, eval\_energy=False)

Solves the time-independent Ruthaan-Hall equations and then performs a time-evolution of the system switching off the laser source at a certain time. Performs the Fourier analysis on the curves for overlap and dipole moment obtained for t>t\_laser\_ON.

**Args:** tolerance (float) : stopping condition for the time-independent solver.

max\_iter (int) : maximum number of iterations before the time-independent solver stops.

t\_laser\_ON (float) : instant at which the laser is switched off

t\_max (float) : final instant for time evolution

dt (float) : time step

eval\_energy (bool) : if True, evaluates energy at every time step (for faster computations, set to False)

**Returns:** C2 (np.ndarray) : coefficient matrix for t=t\_max

time (np.array) : time instants from tstart to t\_max spaced by dt

dipole (np.array) : dipole evaluated at each time instant

overlap (np.array) : overlap evaluated at each time instant

dipoleFFT (np.array) : fft of dipole values for t>t\_laser\_ON

dipolefreqFFT (np.array) : array of frequencies corresponding to dipoleFFT

overlapFFT (np.array) : fft of overlap values for t>t\_laser\_ON

overlapfreqFFT (np.array) : array of frequencies corresponding to overlapFFT

energy (np.array) : total energy at each time instant

**gif\_generator**(dt, t\_max, C0, save\_every\_n=500)

Generates an animated image (.gif) with the time evolution of the one-body density and the time evolution of the one body potential.

**Args:** dt (float) : time step

t\_max (float) : total duration of the time evolution

C0 (np.ndarray) : coefficient matrix at time t=0

`save_every_n` (int) : number of time steps between two successive frames in the final .gif file

**laser\_potential** (*t*)

Returns the value of the laser potential at time *t*.

**Args:** *t* (float) : time instant

**Returns:**  $\epsilon_0 \sin(\omega \cdot t)$  (float)

**plot\_AO** ()

Plots the square modulus of the Atomic Orbitals (harmonic oscillator eigenstates). The zero-level for each single particle state is its corresponding eigenvalue.

**plot\_MO** (*epsilon*, *C*)

Plots the square modulus of the Molecular Orbitals (after the basis change). The zero-level for each single particle state is its corresponding eigenvalue.

**plot\_fourier\_analysis** (*time*, *dipole*, *overlap*, *xFFT*, *xfreqFFT*, *overlapFFT*, *overlapfreqFFT*, *energy=None*)

Plots the results of the Fourier analysis.

**Args:** *time* (np.array) : array of time instants

*dipole* (np.array) : dipole for each time instant

*overlap* (np.array) : overlap for each time instant

*xFFT* (np.array) : fast Fourier transform of the dipole signal

*xfreqFFT* (np.array) : frequency spectrum of the dipole signal (useful for plotting)

*overlapFFT* (np.array) : fast Fourier transform of the overlap signal

*overlapfreqFFT* (np.array) : frequency spectrum of the overlap signal (useful for plotting)

*energy* (np.array) : energy for each time instant, may not be included

**plot\_one\_body\_density** (*one\_body\_density*)

Plots the comparison between the one-body density and the result by Zanghellini et al..

**Args:** *one\_body\_density* (np.array) : result of `self.eval_one_body_density()`

**plot\_overlap** (*time*, *overlap*)

Plots the comparison between the overlaps evaluated in the time-dependent solver and by Zanghellini et al..

**Args:** *time* (np.array) : array of time instants

*overlap* (np.array) : overlap for each time instant

**rhsf** (*t*, *C*)

Returns the right-hand side of the Ruthaan-Hall equations with the laser source on, corresponding to

$$\dot{C}(t) = -if(t)C(t)$$

The output is reshaped into an array in order to be used into `self.solve_TDHF()`

**Args:** *t* (float) : time

*C* (np.array) : reshaped coefficient matrix into array

**Returns:** *rhs* (np.array)

**rhsf\_OFF** (*t*, *C*)

Returns the right-hand side of the Ruthaan-Hall equations with the laser source off, given by

$$\dot{C}(t) = -if(t)C(t)$$

The output is reshaped into an array in order to be used into `self.solve_TDHF()`

**Args:** `t` (float) : time

`C` (np.array) : coefficient matrix reshaped into array

**Returns:** `rhs` (np.array)

**`solve_TDHF`** (*tstart*, *dt*, *t\_max*, *C0*, *eval\_overlap=False*, *eval\_dipole=False*, *eval\_energy=False*,  
*laser\_ON=True*)

Solves iteratively the time-dependent Ruthaan-Hall equations for the system.

**Args:** `t_start` (float) : time at which the evolution begins

`dt` (float) : time step

`t_max` (float) : time at which the evolution ends

`C0` (np.ndarray) : coefficient matrix at time `t_start`

`eval_overlap` (bool) : if True, the overlap with the wavefunction at `t=t_start` is evaluated at every instant

`eval_dipole` (bool) : if True, the expected value for the dipole operator is evaluated at every instant

`eval_energy` (bool) : if True, the total energy is evaluated at every instant

`laser_ON` (bool) : if True, the laser source is on

**Return:** `C` (np.ndarray) : coefficient matrix at `t=t_max`

`time` (np.array) : time instants from `tstart` to `t_max` spaced by `dt`

`overlap` (np.array) : None if `eval_overlap==False`

`dipole` (np.array) : None if `eval_dipole==False`

`energy` (np.array) : None if `eval_energy==False`

**`solve_TIHF`** (*tolerance*, *max\_iter*, *print\_ON=False*, *eval\_energy\_per\_step=False*,  
*eval\_delta\_per\_step=False*)

Solves the Ruthaan-Hall equations for the system.

**Args:** `tolerance` (float) : stopping condition for the algorithm.

`max_iter` (int) : maximum number of iterations before the algorithm stops

`print_ON` (bool) : if True prints if the convergence or the max number of iterations has been reached.

`eval_energy_per_step` (bool) : if True, the energy of the system is evaluated at every step

`eval_delta_per_step` (bool) : if True, the  $\Delta$  parameter of the system is evaluated at every step

**Returns:** `epsilon` (np.array) : final eigenvalues

`C` (np.ndarray) : final coefficient matrix

`energy_per_step` (np.array) : None if `energy_per_step==False`

`delta_per_step` (np.array) : None if `delta_per_step==False`

**`class do.RHF`** (*l=10*, *grid\_length=10*, *num\_grid\_points=201*, *alpha=1.0*, *a=0.25*, *Omega=0.25*, *omega=2*,  
*epsilon0=1*, *nparticles=2*, *potential=None*, *antisymmetrize=False*)

Initializes a system in the restricted spin representation. The default values in the creator are referred to the analyzed case. A more detailed description is provided in the paper folder at <https://github.com/Matteo294/FYS4411>. The quantum-system documentation appears in <https://schoyen.github.io/quantum-systems/>



**Args:** l (int) : number of harmonic oscillator eigenstates used for the expansion of the radial part of the single-particle wavefunctions

grid\_length (int) : extension of the mesh goes from -grid\_length to grid\_length

num\_grid\_points (int) : number of points in the mesh

alpha (float) : strength parameter in the shielded Coulomb potential

a (float) : shielding parameter in the shielded Coulomb potential

Omega (float) : frequency of the harmonic oscillator potential in which the electrons are trapped

omega (float) : frequency of the laser source

epsilon0 (float) : amplitude of the sinusoidal potential associated to the laser

nparticles (int) : number of electrons in the system

antisymmetrize (bool) : MUST remain False

**Attributes:** potential (func) : ODQD.HOPotential(Omega)

system : ODQD object (see <https://schoyen.github.io/quantum-systems/>)

Omega (float) : frequency of the harmonic oscillator potential in which the electrons are trapped

omega (float) : frequency of the laser source

epsilon0 (float) : amplitude of the sinusoidal potential associated to the laser

nparticles (int) : number of electrons in the system

**eval\_one\_body\_density** (C)

Returns the one-body density of the system, according to

$$\rho(x) = 2 \sum_i^{n/2} \sum_{\alpha\beta} C_{\alpha,i}^* C_{\beta,i} \chi_{\alpha}^*(x) \chi_{\beta}(x)$$

**Args:** C (np.ndarray) : coefficient matrix

**Returns:** one\_body\_density (np.array)

**eval\_total\_energy** (C)

Returns the total energy of the system, given by

$$E[H] = 2 \sum_{i=1}^{n/2} \sum_{\alpha,\beta} C_{\alpha,i}^* C_{\beta,i} h_{\alpha\beta}^{ho} + 2 \sum_{i,j=1}^{n/2} \sum_{\alpha\beta\gamma\delta} C_{\alpha,i}^* C_{\gamma,j} C_{\beta,i} C_{\delta,j} u_{\beta\delta}^{\alpha\gamma} - \sum_{i,j=1}^{n/2} \sum_{\alpha\beta\gamma\delta} C_{\alpha,i}^* C_{\gamma,j} C_{\beta,i} C_{\delta,j} u_{\delta\beta}^{\alpha\gamma}$$

**Args:** C (np.ndarray) : coefficient matrix

**Returns:** energy (complex)

**fill\_density\_matrix** (C)

Returns the density matrix evaluated using the coefficient matrix C, according to

$$M_{\alpha\beta} = \sum_i^{nparticles/2} C_{\alpha,i}^* C_{\beta,i}$$

**Args:** C (np.ndarray) : coefficient matrix

**Returns:** density\_matrix (np.ndarray)

**fill\_fock\_matrix** (C, t)

Returns the Fock matrix evaluated using the coefficient matrix C evaluated at time t, according to

$$f_{\mu\nu} = h_{\mu\nu}^{ho} + 2 \sum_j^{n/2} \sum_{\gamma\delta} C_{\gamma,j}^* C_{\delta,j} u_{\nu\delta}^{\mu\gamma} - \sum_j^{n/2} \sum_{\gamma\delta} C_{\gamma,j}^* C_{\delta,j} u_{\delta\nu}^{\mu\gamma}$$

**Args:** C (np.ndarray) : coefficient matrix

t (float) : time instant

**Returns:** fock\_matrix (np.ndarray)

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