

Building supersymmetric models using superfields

Seminar on Supersimmetry

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Part on $\mathcal{L} \rightarrow \mathcal{L} + \partial_\mu f$

The two main ingredients of a supersymmetric theory in the superspace formalism, are the chiral and vector superfields. Let us briefly recall some of their properties, useful for the subsequent reasonings.

A left-chiral superfield Φ (right-chiral superfield χ) is obtained by imposing the constrain $\bar{D}_{\dot{\alpha}}\Phi(x, \theta, \bar{\theta}) = 0$ ($D_\alpha\chi(x, \theta, \bar{\theta}) = 0$), and a general expansion in powers of $\theta, \bar{\theta}$ reads

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) = & \varphi(x) + i\bar{\theta}\bar{\sigma}^\mu\theta\partial_\mu\varphi(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial_\mu\partial^\mu\varphi(x) + \sqrt{2}\theta\psi(x) + \\ & -\frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi(x) + \theta\theta F(x) \end{aligned} \quad (1)$$

The hermitian conjugate of a left-chiral superfield Φ^\dagger is a right-chiral superfield and vice-versa. The field ϕ entering equation ?? is the bosonic field of the theory, ψ is its fermionic supersymmetric partner and F will simply turn out to be unphysical. This will become clearer when looking at the interactions between the fields and at the equations of motion.

Under a SUSY transformation the components transform as

$$\delta_\epsilon\phi = \epsilon\psi \quad \delta_\epsilon\psi_\alpha = -i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi + \epsilon_\alpha F, \quad \boxed{\delta_\epsilon F = -i\epsilon^\dagger\bar{\sigma}^\mu\partial_\mu\psi}$$

The transformation law of F is of particular interest, since it is precisely a total derivative, that is what one looks for to build invariant lagrangians.

A vector superfield V is obtained by imposing the reality condition $V = V^*$, and an expansion in powers of $\theta, \bar{\theta}$ reads

$$\begin{aligned} V(x, \theta, \bar{\theta}) = & a + \theta\xi + \bar{\theta}\xi^\dagger + \theta\theta b + \bar{\theta}\bar{\theta}b^\dagger + \bar{\theta}\bar{\sigma}^\mu\theta A_\mu + \\ & + \bar{\theta}\bar{\theta}\theta\left(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\xi^\dagger\right) + \theta\theta\bar{\theta}\left(\lambda^\dagger - \frac{i}{2}\sigma^\mu\partial_\mu\xi\right) + \theta\theta\bar{\theta}\bar{\theta}\left(\frac{1}{2}D + \frac{1}{4}\partial_\mu\partial^\mu a\right) \end{aligned}$$

Here A_μ will be the spin-1 gauge field, with λ beeing its fermionic supersymmetric partner. The field D will dropout when looking at the equations of mottion, while all the others degrees of freedom can be supergagued away by going in the Wess-Zumino gauge.

Under a SUSY transformation the components transform as

$$\begin{aligned} \sqrt{2}\delta_\epsilon a = \epsilon\xi + \epsilon^\dagger\xi^\dagger \quad \sqrt{2}\delta_\epsilon\lambda_\alpha = \epsilon_\alpha D + \frac{i}{2}(\sigma^\mu\sigma^\nu\epsilon)_\alpha(\partial_\mu A_\nu - \partial_\nu A_\mu) \\ \sqrt{2}\delta_\epsilon b = \epsilon^\dagger\lambda^\dagger - i\epsilon^\dagger\sigma^\mu\partial_\mu\xi \quad \sqrt{2}\delta_\epsilon\xi_\alpha = 2\epsilon_\alpha b - (\sigma^\mu\epsilon^\dagger)_\alpha(A_\mu + i\partial_\mu a) \\ \sqrt{2}\delta_\epsilon A^\mu = i\epsilon\partial^\mu\xi - i\epsilon^\dagger\partial^\mu\xi^\dagger + \epsilon\sigma^\mu\lambda^\dagger - \epsilon^\dagger\bar{\sigma}^\mu\lambda \\ \boxed{\sqrt{2}\delta_\epsilon D = -i\epsilon\sigma^\mu\partial_\mu\lambda^\dagger - i\epsilon^\dagger\bar{\sigma}^\mu\partial_\mu\lambda} \end{aligned}$$

Here the term of our interest is the field D because it transforms precisely as a total derivative, as desired.

Hence a lagrangian of the type

$$\mathcal{L} = \int d^4x [\Phi]_F + [\Phi^\dagger]_F + [V]_D$$

where $[\Phi]_F$, $[\Phi^\dagger]_F$ and $[V]_D$ denote respectively the F component of the chiral fields and the D component of a vector field, would certainly be SUSY invariant due to the transformation properties of the selected fields.

Grassman integration provides a natural way to select such components. Indeed

$$\int d^2\theta \Phi(x, \theta, \bar{\theta}) = \frac{1}{4} \bar{\theta}\bar{\theta} \partial_\mu \partial^\mu \phi(x) - \frac{i}{\sqrt{2}} \bar{\theta} \bar{\sigma}^\mu \partial_\mu \psi(x) + F(x) = F(x) + \text{total derivative} \equiv [\Phi]_F$$

$$\int d^2\theta d^2\bar{\theta} V(x, \theta, \bar{\theta}) = \frac{1}{2} D + \frac{1}{4} \partial_\mu \partial^\mu a = \frac{1}{2} D + \text{total derivative} \equiv [V]_D$$

The first meaningful supersymmetric lagrangian can be obtained by noting that the product $\Phi^\dagger \Phi$ is real, hence a vector field. In particular, this implies that one can select the D component $[\Phi^\dagger \Phi]_D$ via Grassman integration to obtain a SUSY invariant lagrangian.

Let us write the product $\Phi^\dagger \Phi$ in components and let us in particular look at the D term (i.e. the coefficient of $\theta\theta\bar{\theta}\bar{\theta}$, compare with expansion of V)

$$\Phi^\dagger \Phi = \text{greatmess}$$

$$\mathcal{L} = \int d^4x [\Phi^\dagger \Phi]_D = \int d^4x \left(-\partial^\mu \phi^* \partial_\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + F^* F \right)$$

→ **Free Wess-Zumino model**

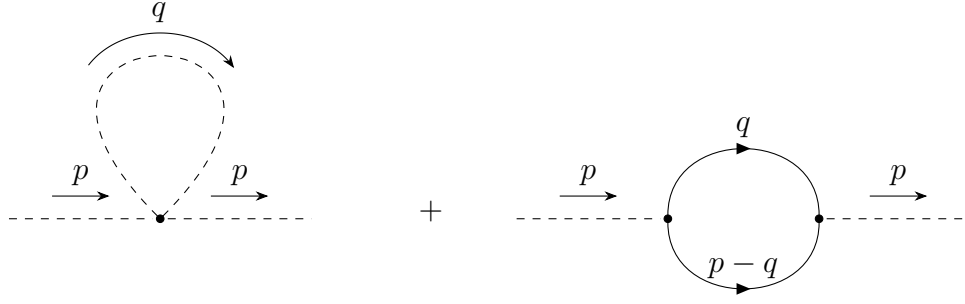


Figure 1: 1-loop corrections to the scalar propagator with the lagrangian give by equation
 ????????????????????

The contribution from the first diagram is

$$-4 \frac{i|y|^2}{4} \int \frac{d^4 q}{(2\pi)^4} \frac{i}{q^2 - m^2}$$

The contribution from the second diagram is

$$-4 \left(\frac{iy}{2} \right) \left(\frac{iy^*}{2} \right) \int \frac{d^4 q}{(2\pi)^4} \text{Tr} \left[\frac{i\sigma \cdot q}{q^2 - m^2} \frac{i\bar{\sigma} \cdot (p - q)}{(p - q)^2 - m^2} \right]$$