Building supersymmetric models using superfields

Seminar on Supersimmetry

University of Heidelberg, Summer Semester 2022

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Part on $\mathcal{L} \to \mathcal{L} + \partial_\mu f$

The two main ingredients of a supersymmetric theory in the superspace formalism, are the chiral and vector superfields. Let us briefly recall some of their properties, useful for the subsequent reasonings.

A left-chiral superfield Φ (right-chiral superfield χ) is obtained by imposing the constrain $\bar{D}_{\dot{\alpha}}\Phi(x,\theta,\bar{\theta}) = 0$ ($D_\alpha \chi(x,\theta,\bar{\theta}) = 0$), and a general expansion in powers of $\theta, \bar{\theta}$ reads

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) = \varphi(x) + i\bar{\theta}\bar{\sigma}^\mu\theta\partial_\mu\varphi(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial_\mu\partial^\mu\varphi(x) + \sqrt{2}\theta\psi(x) + \\ -\frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi(x) + \theta\theta F(x)\end{aligned} \tag{1}$$

The hermitian conjugate of a left-chiral superfield $\Phi^\dagger$ is a right-chiral superfield and vice-versa. The field ϕ entering equation **??** is the bosonic field of the theory, ψ is its fermionic supersymmetric partner and F will simply turn out to be unphysical. This will become clearer when looking at the interactions between the fields and at the equations of motion.

Under a SUSY transformation the components transform as

$$\delta_\epsilon\phi = \epsilon\psi \qquad \delta_\epsilon\psi_\alpha = -i\left(\sigma^\mu\epsilon^\dagger\right)_\alpha\partial_\mu\phi + \epsilon_\alpha F, \qquad \boxed{\delta_\epsilon F = -i\epsilon^\dagger\bar{\sigma}^\mu\partial_\mu\psi}$$

The transformation law of F is of particular interest, since it is precisely a total derivative, that is what one looks for to build invariant lagrangians.

A vector superfield V is obtained by imposing the reality condition $V = V^*$, and an expansion in powers of $\theta, \bar{\theta}$ reads

$$\begin{aligned}V\left(x,\theta,\bar{\theta}\right) = a + \theta\xi + \bar{\theta}\xi^\dagger + \theta\theta b + \bar{\theta}\bar{\theta}b^\dagger + \bar{\theta}\bar{\sigma}^\mu\theta A_\mu + \\ +\bar{\theta}\bar{\theta}\theta\left(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\xi^\dagger\right) + \theta\theta\bar{\theta}\left(\lambda^\dagger - \frac{i}{2}\sigma^\mu\partial_\mu\xi\right) + \theta\theta\bar{\theta}\bar{\theta}\left(\frac{1}{2}D + \frac{1}{4}\partial_\mu\partial^\mu a\right)\end{aligned}$$

Here A_μ will be the spin-1 gauge field, with λ beeing its fermionic supersymmetric partner. The field D will dropout when looking at the equations of mottion, while all the others degrees of freedom can be supergagued away by going in the Wess-Zumino gauge.

Under a SUSY transformation the components transform as

$$\begin{gathered}\sqrt{2}\delta_\epsilon a = \epsilon\xi + \epsilon^\dagger\xi^\dagger \qquad \sqrt{2}\delta_\epsilon\lambda_\alpha = \epsilon_a D + \frac{i}{2}\left(\sigma^\mu\sigma^\nu\epsilon\right)_\alpha\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right) \\ \sqrt{2}\delta_\epsilon b = \epsilon^\dagger\lambda^\dagger - i\epsilon^\dagger\sigma^\mu\partial_\mu\xi \qquad \sqrt{2}\delta_\epsilon\xi_\alpha = 2\epsilon_\alpha b - \left(\sigma^\mu\epsilon^\dagger\right)_\alpha\left(A_\mu + i\partial_\mu a\right) \\ \sqrt{2}\delta_\epsilon A^\mu = i\epsilon\partial^\mu\xi - i\epsilon^\dagger\partial^\mu\xi^\dagger + \epsilon\sigma^\mu\lambda^\dagger - \epsilon^\dagger\bar{\sigma}^\mu\lambda \\ \boxed{\sqrt{2}\delta_\epsilon D = -i\epsilon\sigma^\mu\partial_\mu\lambda^\dagger - i\epsilon^\dagger\bar{\sigma}^\mu\partial_\mu\lambda}\end{gathered}$$

Here the term of our interest is the field D because it transforms precisely as a total derivative, as desired.

Hence a lagrangian of the type

$$\mathcal{L} = \int d^4x \left[\Phi\right]_F + \left[\Phi^\dagger\right]_F + \left[V\right]_D$$

where $[\Phi]_F$, $\left[\Phi^\dagger\right]_F$ and $[V]_D$ denote respectively the F component of the chiral fields and the D component of a vector field, would certainly be SUSY invariant due to the transformation properties of the selected fields.

Grassman integration provides a natural way to select such components. Indeed

$$\int d^2\theta\, \Phi(x,\theta,\bar{\theta}) = \frac{1}{4}\bar{\theta}\bar{\theta}\partial_\mu\partial^\mu\phi(x) - \frac{i}{\sqrt{2}}\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi(x) + F(x) = F(x) + \text{total derivative} \equiv [\Phi]_F$$

$$\int d^2\theta d^2\bar{\theta}\, V(x,\theta,\bar{\theta}) = \frac{1}{2}D + \frac{1}{4}\partial_\mu\partial^\mu a = \frac{1}{2}D + \text{total derivative} \equiv [V]_D$$

The first meaningful supersymmetric lagrangian can be obtained by noting that the product $\Phi^\dagger\Phi$ is real, hence a vector field. In particular, this implies that one can select the D component $\left[\Phi^\dagger\Phi\right]_D$ via Grassman integration to obtain a SUSY invariant lagrangian.
Let us write the product $\Phi^\dagger\Phi$ in components and let us in particulat look at the D term (i.e. the coefficient of $\theta\theta\bar{\theta}\bar{\theta}$, compare with expansion of V)

$$\Phi^\dagger\Phi = greatmess$$

$$\boxed{\mathcal{L} = \int d^4x \left[\Phi^\dagger\Phi\right]_D = \int d^4x \left(-\partial^\mu\phi^*\partial_\mu\phi + i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\phi + F^*F\right)}$$

$\longrightarrow$ **Free Wess-Zumino model**

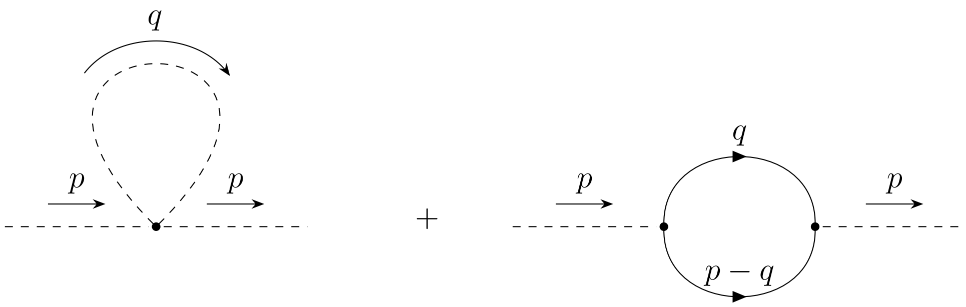

Figure 1: 1-loop corrections to the scalar propagator with the lagrangian give by equation ??????????????????

The contribution from the first diagram is

$$I_1 = 4\,\frac{i|y|^2}{4}\int \frac{d^4q}{(2\pi)^4}\,\frac{i}{q^2-m^2} = -|y|^2\int \frac{d^4q}{(2\pi)^4}\,\frac{1}{q^2-m^2}$$

By performing a Wick rotation $q \to iq$ and putting a cutoff Λ, the integral can be evaluated exactly in polar coordinates

$$I_1 = \frac{i|y|^2}{(2\pi)^3}\int_0^\Lambda dq\int_0^\pi d\varphi_1\int_0^\pi d\varphi_2\,\frac{1}{q^2+m^2}\,q^3 sin^2\varphi_1 sin\varphi_2 =$$

$$= \frac{i|y|^2}{8\pi^2}\int_m^{(\Lambda^2+m^2)^{1/2}} dt\quad \frac{t^2-m^2}{t} = \frac{i|y|^2}{16\pi^2}\left(\Lambda^2 - m^2\ln\left(\frac{\Lambda^2+m^2}{m^2}\right)\right)$$

The contribution from the second diagram is

$$I_2 = -2\left(\frac{iy}{2}\right)\left(\frac{iy^*}{2}\right)\int \frac{d^4q}{(2\pi)^4}\,\mathrm{Tr}\left[\frac{i(\sigma\cdot q+m)}{q^2-m^2}\,\frac{i(\bar\sigma\cdot(p-q)+m)}{(p-q)^2-m^2}\right] =$$
$$-\frac{1}{2}|y|^2\int\frac{d^4q}{(2\pi)^4}\,(-2)\,\frac{q\cdot p - q^2 - 2m^2}{(q^2-m^2)((q-p)^2-m^2)}$$

Where the property $\mathrm{Tr}[\sigma_\mu\bar\sigma_\nu] = -2\eta_{\mu\nu}$ has been used. This integral is not easilly evaluable, but for the current purpose it is sufficient to determine its leading behaviour for $|q| \to +\infty$.
After performing the Wick rotation and writing the integral in polar coordinates, the leading term is

$$I_2 = \frac{|y|^2}{(2\pi)^3}\int_0^\Lambda dq\int_0^\pi d\varphi_1\int_0^\pi d\varphi_2\,\frac{1}{q^2}\,q^3\sin^2\varphi_1\sin\varphi_2 =$$
$$= \frac{i|y|^2}{(2\pi)^3}\,\pi\int_0^\Lambda dq\,q = \frac{i|y|^2}{16\pi^2}\,\Lambda^2$$

This term cancels precisely the quadratic term in ?????? as desired