Building SUSY models II: using superfields

Seminar on Supersymmetry and its breaking

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Main points of the talk

- Apply the superspace formalism and show why it is useful to build SUSY theories
- Principles to construct SUSY lagrangians
- SUSY gauge theories: QED and QCD
- SUSY predictions: particles, interaction, masses, ...

How to build SUSY invariant lagrangians

- We want our theory to be SUSY invariant, that is if $S = \int dx^\mu \mathcal{L}$ is such that $\delta S = 0$, then

$$\delta S' = \delta \left[\left(\epsilon \hat{Q} + \epsilon^\dagger \hat{Q}^\dagger \right) S \right] = 0$$

- We know that this condition is met if, under a given transformation

$$\mathcal{L} \to \mathcal{L} + \partial_\mu f$$

- Hence our goal is to build a lagrangian which transforms in this way, using **Chiral** ($\bar{D}_{\dot{\alpha}} \Phi = 0$ or $D_\alpha \Phi^\dagger = 0$) and **Vector** ($V = V^\dagger$) superfields

How to build SUSY invariant lagrangians

- Left-chiral field expansion (right-chiral is hermitian conjugate)

$$\Phi(x,\theta,\bar\theta) = \varphi(x) + i\bar\theta\bar\sigma^\mu\theta\partial_\mu\varphi(x) + \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial_\mu\partial^\mu\varphi(x) + \sqrt{2}\theta\psi(x) + \\ -\frac{i}{\sqrt{2}}\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu\psi(x) + \theta\theta F(x)$$

- Vector field expansion

$$V\left(x,\theta,\bar\theta\right) = a + \theta\xi + \bar\theta\xi^\dagger + \theta\theta b + \bar\theta\bar\theta b^\dagger + \bar\theta\bar\sigma^\mu\theta A_\mu + \\ +\bar\theta\bar\theta\theta\left(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\xi^\dagger\right) + \theta\theta\bar\theta\left(\lambda^\dagger - \frac{i}{2}\sigma^\mu\partial_\mu\xi\right) + \theta\theta\bar\theta\bar\theta\left(\frac{1}{2}D + \frac{1}{4}\partial_\mu\partial^\mu a\right)$$

- They both carry no spinor, nor vector indices, the name is due to their particle content!

How to build SUSY invariant lagrangians

- For the components of a chiral field $\Phi(x,\theta,\bar{\theta})$ one has that

$$\delta_\epsilon \phi = \epsilon\psi$$
$$\delta_\epsilon \psi_\alpha = -i\left(\sigma^\mu \epsilon^\dagger\right)_\alpha \partial_\mu \phi + \epsilon_\alpha F,$$
$$\boxed{\delta_\epsilon F = -i\epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \psi}$$

- For the components of a vector field $V(x,\theta,\bar{\theta})$ one has

$$\sqrt{2}\delta_\epsilon a = \epsilon\xi + \epsilon^\dagger \xi^\dagger \qquad \sqrt{2}\delta_\epsilon \lambda_\alpha = \epsilon_a D + \frac{i}{2}\left(\sigma^\mu \sigma^\nu \epsilon\right)_\alpha \left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)$$
$$\sqrt{2}\delta_\epsilon b = \epsilon^\dagger \lambda^\dagger - i\epsilon^\dagger \sigma^\mu \partial_\mu \xi \qquad \sqrt{2}\delta_\epsilon \xi_\alpha = 2\epsilon_\alpha b - \left(\sigma^\mu \epsilon^\dagger\right)_\alpha \left(A_\mu + i\partial_\mu a\right)$$
$$\sqrt{2}\delta_\epsilon A^\mu = i\epsilon\partial^\mu \xi - i\epsilon^\dagger \partial^\mu \xi^\dagger + \epsilon\sigma^\mu \lambda^\dagger - \epsilon^\dagger \bar{\sigma}^\mu \lambda$$
$$\boxed{\sqrt{2}\delta_\epsilon D = -i\epsilon\sigma^\mu \partial_\mu \lambda^\dagger - i\epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \lambda}$$

How to build SUSY invariant lagrangians

- Idea $\rightarrow$ "select" the components of the fields that transforms as total derivatives
- How can we "pick" only the terms we need? $\rightarrow$ Grassman integration

$$\int d^2\theta \quad \theta^n f(x,\bar{\theta}) \;=\; f(x,\bar{\theta})\,\delta_{n,2}$$

$$\int d^2\theta d^2\bar{\theta} \quad \bar{\theta}^m \theta^n f(x) \;=\; f(x)\,\delta_{m,2}\,\delta_{n,2}$$

- Our terms of interest

$$[\Phi]_F = \int d^2\theta\ \Phi(x,\theta,\bar{\theta}) = F + \text{total derivative}$$

$$[V]_D = \int d^2\theta d^2\bar{\theta}\ V(x,\theta,\bar{\theta}) = \frac{1}{2}\,D + \text{total derivative}$$

How to build SUSY invariant lagrangians

Let us focus for a moment on the D term.

$$\Phi(x,\theta,\bar\theta) = \varphi(x) + i\bar\theta\bar\sigma^\mu\theta\partial_\mu\varphi(x) + \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial_\mu\partial^\mu\varphi(x) + \sqrt{2}\theta\psi(x)$$

$$-\frac{i}{\sqrt{2}}\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu\psi(x) + \theta\theta F(x)$$

$$\Phi^{\dagger i}\Phi_j = \varphi^{*i}\varphi_j + \sqrt{2}\theta\psi_j\varphi^{*i} + \sqrt{2}\bar\theta\psi^{\dagger i}\varphi_j + \theta\theta\varphi^{*i}F_j + \bar\theta\bar\theta\varphi_j F^{\dagger i}$$

$$+\bar\theta\bar\sigma^\mu\theta\left[i\varphi^{*i}\partial_\mu\varphi_j - i\varphi_j\partial_\mu\varphi^{*i} - \psi^{\dagger i}\sigma_\mu\psi_j\right]$$

$$+\frac{i}{\sqrt{2}}\theta\theta\bar\theta\bar\sigma^\mu\left(\psi_j\partial_\mu\varphi^{*i} - \partial_\mu\psi_j\varphi^{*i}\right) + \sqrt{2}\theta\theta\bar\theta\psi^{\dagger i}F_j$$

$$+\frac{i}{\sqrt{2}}\bar\theta\bar\theta\theta\sigma^\mu\left(\psi^{\dagger i}\partial_\mu\varphi_j - \partial_\mu\psi^{\dagger i}\varphi_j\right) + \sqrt{2}\bar\theta\bar\theta\theta\psi_j F^{*i}$$

$$+\theta\theta\bar\theta\bar\theta\left[F^{*i}F_j - \frac{1}{2}\partial^\mu\varphi^{*i}\partial_\mu\varphi_j + \frac{1}{4}\varphi^{*i}\partial^\mu\partial_\mu\varphi_j + \frac{1}{4}\varphi_j\partial^\mu\partial_\mu\varphi^{*i}\right.$$

$$\left.+\frac{i}{2}\psi^{\dagger i}\bar\sigma^\mu\partial_\mu\psi_j + \frac{i}{2}\psi_j\sigma^\mu\partial_\mu\psi^{\dagger i}\right]$$

One can note that for $i=j$ one has that $(\Phi^\dagger\Phi)^\dagger = \Phi^\dagger\Phi \Rightarrow$ vector field!

How to build SUSY invariant lagrangians

- Now "select" the SUSY invariant component (D component)

$$\left[\Phi^\dagger\Phi\right]_D = \int d^2\theta d^2\bar\theta\ \Phi^\dagger(x,\theta,\bar\theta)\Phi(x,\theta,\bar\theta)$$

$$\Big[F^*F - \frac{1}{2}\partial^\mu\varphi^*\partial_\mu\varphi + \frac{1}{4}\varphi^*\partial^\mu\partial_\mu\varphi + \frac{1}{4}\varphi\partial^\mu\partial_\mu\varphi^*$$

$$+\frac{i}{2}\psi^\dagger\bar\sigma^\mu\partial_\mu\psi + \frac{i}{2}\psi\sigma^\mu\partial_\mu\psi^\dagger\Big] =$$

$$= -\partial^\mu\varphi^*\partial_\mu\varphi + i\psi^\dagger\bar\sigma^\mu\partial_\mu\psi + F^*F + \text{total derivative}$$

- Hence

$$\boxed{S = \int dx^\mu \mathcal{L} = \int dx^\mu \left(-\partial^\mu\varphi^*\partial_\mu\varphi + i\psi^\dagger\bar\sigma^\mu\partial_\mu\psi + F^*F\right)}$$

$\rightarrow$ **Free Wess-Zumino model!**

How to build SUSY invariant lagrangians

- In order to add SUSY invariant interactions, let us recall the definitions of chiral superfields

$$\bar{D}_{\dot{\alpha}}\Phi = 0 \qquad D_{\alpha}\Phi^{\dagger} = 0$$

- Note that any analytic function of chiral superfields is in turn a chiral superfield (power series expansion and product rule).
- $\Rightarrow$ Write our chiral term of N fields as

$$W(\{\Phi_k\}) = \sum_{i}^{N} a_i \Phi_i + \sum_{i,j}^{N} \frac{1}{2!} m_{ij} \Phi_i \Phi_j + \sum_{i,j,k}^{N} \frac{1}{3!} y_{ijk} \Phi_i \Phi_j \Phi_k$$

- Higher order terms are non-renormalisable
- Reality of the action requires us to take $W + W^*$

How to buld SUSY invariant lagrangians

The interacting lagrangian becomes

$$\mathcal{L}_{WZ}\left(\{\Phi_i\},\{\Phi_i^\dagger\}\right) = \mathcal{L}_{WZ,D} + \mathcal{L}_{WZ,F} =$$

$$= \left[\Phi^{\dagger i}\Phi^i\right]_D + \left[W(\{\Phi_i\})\right]_F + \left[W^\dagger(\{\Phi_i^\dagger\})\right]_F =$$

$$= \int d^2\theta \left(-\frac{1}{4}\overline{DD}\Phi^{\dagger i}\Phi_i + W(\{\Phi_i\})\right) + \int d^2\bar{\theta}\; W^\dagger(\{\Phi_i^\dagger\}) =$$

$$= \int d^2\bar{\theta} \left(-\frac{1}{4}DD\Phi^{\dagger i}\Phi_i + W(\{\Phi_i\})\right) + \int d^2\bar{\theta}\; W^\dagger(\{\Phi_i^\dagger\})$$

where I used that

$$\int d^2\theta d^2\bar{\theta}\Phi^\dagger\Phi = -\int d^2\theta\,\frac{1}{4}\,\overline{DD}\Phi^\dagger\Phi = -\int d^2\bar{\theta}\,\frac{1}{4}\,DD(\Phi^\dagger\Phi)$$

Equations of motion varying w.r.t. Φ_i and $\Phi_i^\dagger$

$$0 = -\frac{1}{4}\overline{DD}\Phi^{\dagger i} + \frac{\delta W}{\delta\Phi_i} \qquad 0 = -\frac{1}{4}DD\Phi_i + \frac{\delta W^\dagger}{\delta\Phi^{\dagger i}}$$

Wess-Zumino model

- We are now interested in studying better the components of Φ, hence let us focus on the case $i = j$ with $a_i = a, m_{ij} = m, y_{ijk} = y$ and the superpotential
$$W(\Phi) = \frac{1}{2}\, m\, \Phi\Phi + \frac{1}{3!}\, y\, \Phi\Phi\Phi$$
where we dropped the linear term.
- The lagrangian becomes
$$\mathcal{L}(\Phi, \Phi^\dagger) = \mathcal{L}_{WZ,D} + \mathcal{L}_{WZ,F} =$$
$$= F^* F + (\partial_\mu \varphi)(\partial^\mu \varphi)^* + \frac{i}{2}\psi\sigma^\mu \left(\partial_\mu \bar{\psi}\right) - \frac{i}{2}(\partial_\mu \psi)\,\sigma^\mu \psi^\dagger +$$
$$-m\varphi F - \frac{m}{2}(\psi\psi) - \frac{y}{2}\varphi\varphi F - \frac{y}{2}\varphi(\psi\psi) + \text{ h.c.}$$
- E.o.m. for F (analogous for F^*) is
$$0 = \partial_\mu \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu F\right)} - \frac{\partial \mathcal{L}}{\partial F} = -\frac{\partial \mathcal{L}}{\partial F} = -F^* + m\varphi + \frac{y}{2}\varphi\varphi$$
- $\Rightarrow$ algebraic equation $\Rightarrow F, F^*$ are unphysical.

Wess-Zumino model

$$F^* = m\varphi + \frac{y}{2}\varphi\varphi \qquad F = m\varphi^* + \frac{y}{2}\varphi\varphi$$

- One can rewrite the terms containing F, F^* as

$$F^*F - \left(m\varphi F + \frac{y}{2}\varphi\varphi F + hc\right) = -\left|m\varphi + \frac{y}{2}\varphi\varphi\right|^2 = -\left|\frac{\partial W(\varphi)}{\partial\varphi}\right|^2$$

  that is, the superpotential evaluated at the scalar field value $F = \varphi$!
- The lagrangian then becomes

$$\begin{aligned}\mathcal{L}_{\mathrm{Wz}} = & \left(\partial_\mu\varphi\right)\left(\partial^\mu\varphi\right)^\dagger + \frac{i}{2}\psi\sigma^\mu\left(\partial_\mu\bar{\psi}\right) - \frac{i}{2}\left(\partial_\mu\psi\right)\sigma^\mu\bar{\psi} \\ & - |M|^2\varphi\varphi^* - \frac{|y|^2}{4}\varphi\varphi\varphi^*\varphi^* \\ & - \left(\frac{M}{2}\psi\psi + \frac{M\cdot y}{2}\varphi\varphi\varphi^* + \frac{y}{2}\varphi\psi\psi + \text{ h.c. }\right)\end{aligned}$$

Wess-Zumino model

The lagrangian can also be written as

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Wz}} &= (\partial_\mu \varphi)(\partial^\mu \varphi)^\dagger + \frac{i}{2}\psi\sigma^\mu\left(\partial_\mu\bar{\psi}\right) - \frac{i}{2}(\partial_\mu\psi)\,\sigma^\mu\bar{\psi} \\
&\quad - |M|^2\varphi\varphi^* - \frac{|y|^2}{4}\varphi\varphi\varphi^*\varphi^* - \left(\frac{M}{2}\psi\psi + \frac{M\cdot y}{2}\varphi\varphi\varphi^* + \frac{y}{2}\varphi\psi\psi + \text{ h.c. }\right) \\
&= (\partial_\mu \varphi)(\partial^\mu \varphi)^* + \frac{i}{2}\psi\sigma^\mu\left(\partial_\mu\bar{\psi}\right) - \frac{i}{2}(\partial_\mu\psi)\,\sigma^\mu\bar{\psi} \\
&\quad - \left|\frac{\partial W(\varphi)}{\partial\varphi}\right|^2 - \frac{1}{2}\left(\frac{\partial^2 W(\varphi)}{\partial\varphi\partial\varphi}\right)\psi\psi - \frac{1}{2}\left(\frac{\partial^2 W^*(\varphi)}{\partial\varphi^*\partial\varphi^*}\right)\bar{\psi}\bar{\psi}
\end{aligned}
$$

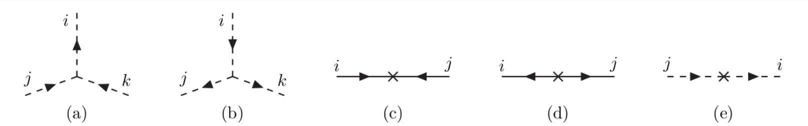
where, I remember,

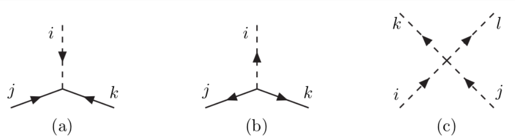
$$
W(\Phi) = \frac{1}{2}\,M\,\Phi\Phi + \frac{1}{3!}\,y\,\Phi\Phi\Phi
$$

Wess-Zumino model

$$
\begin{aligned}
\mathcal{L}_{\mathrm{WZ}} = {} & (\partial_\mu \varphi)(\partial^\mu \varphi)^* + \frac{i}{2}\psi\sigma^\mu \left(\partial_\mu \bar{\psi}\right) - \frac{i}{2}(\partial_\mu \psi)\,\sigma^\mu \bar{\psi} \\
& - |M|^2 \varphi\varphi^* - \frac{|y|^2}{4}\varphi\varphi\varphi^*\varphi^* - \left(\frac{M}{2}\psi\psi + \frac{M \cdot y}{2}\varphi\varphi\varphi^* + \frac{y}{2}\varphi\psi\psi + \text{ h.c. }\right)
\end{aligned}
$$

Let us give a look at the interactions (diagrams taken from [2])





Nonrenormalization theorem

One can check straighforwardly that the quadratic divergence in the boson's mass is cancelled!

One can prove that in general the following theorem holds

The superpotential is not renormalised at any order in perturbation theory. Thus it might get affected by nonperturbative effects such as instantons

This is the so called **N=1 nonrenormalization theorem**

Abelian gauge theories

Let us now introduce (abelian) gauge interactions

- Let us start with U(1) global symmetry

$$\Phi_i \to e^{iq_i\Lambda_i}\Phi_i$$

- The kinetic part of the lagrangian is always invariant

$$\mathcal{L}_K = \mathcal{L}_{WZ,D} = \int d^2\theta d^2\bar{\theta}\ \Phi^\dagger\Phi = \int d^2\theta - \frac{1}{4}\overline{DD}\Phi^\dagger\Phi$$

- The interaction part

$$\mathcal{L}_{int} = \mathcal{L}_{WZ,F} = \int d^2\theta\ \frac{1}{2}\sum_{ij}\Phi_i\Phi_j + \frac{1}{3!}\sum_{ijk}\Phi_i\Phi_j\Phi_k + \text{complex. conj.}$$

requires

$$m_{ij} = 0 \qquad \text{or} \qquad y_{ijk} = 0$$

whenever

$$q_i + q_j \neq 0 \qquad \text{or} \qquad q_i + q_j + q_k \neq 0$$

Abelian gauge theories

Promote to a local gauge symmetry

$$\Lambda \to \Lambda(x, \theta, \bar{\theta})$$

- The gauge parameter is now a supergauge field $\Lambda = \Lambda(x, \theta, \bar{\theta})$
- We need $\Lambda(x, \theta, \bar{\theta})$ to be a left-chiral superfield if we want Φ' to be a left-chiral superfield.
- Thus this causes a problem in the kinetic term because $\Lambda^\dagger$ is a right-chiral superfield hence obviously $\Phi'^\dagger \Phi' \neq \Phi^\dagger \Phi$
- The problem is analogous to the kinetc term in "normal" QFT when $\partial_u \varphi^* \partial^\mu \varphi$ was not gauge invariant
- Solution: add a term that compensate the gauge for the non invariant terms

Abelian gauge theories

Recall from the previous talk that a gauge transformation on a vector field V reads

$$V \to V' = V + i(\Lambda^\dagger - \Lambda)$$

one can modify the kinetic term to

$$\Phi^\dagger e^V \Phi$$

so that it is invariant under the above gauge transformations

$$\Phi^\dagger e^V \Phi \to \Phi'^\dagger e^{V'} \Phi' = \Phi^\dagger e^{-i\Lambda^\dagger} e^{i\Lambda *} e^V e^{-i\Lambda} e^{i\Lambda} \Phi = \Phi^\dagger e^V \Phi$$

A particularly common gauge choice is the Wess-Zumino in which one ends with

$$V_{\mathrm{WZ}}(x, \theta, \bar{\theta}) = \theta \sigma^\mu \bar{\theta} v_\mu(x) + i(\theta\theta)\bar{\theta}\bar{\lambda}(x) - i(\bar{\theta}\bar{\theta})\theta\lambda(x) + \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})D(x)$$

Abelian gauge theories

Let us now define the two chiral fields

$$\mathcal{W}_\alpha = -\frac{1}{4}\overline{DD}D_\alpha V, \quad \overline{\mathcal{W}}_{\dot\alpha} = -\frac{1}{4}DD\bar{D}_{\dot\alpha}V$$

where V is a vector field.

The gauge-invariant dynamical term (equivalent to $F_{\mu\nu}F^{\mu\nu}$) is

$$[\mathcal{W}\mathcal{W}]_F + [\overline{\mathcal{W}\mathcal{W}}]_F = \int d^2\theta\ \mathcal{W}_\alpha\mathcal{W}^\alpha + \int d^2\bar\theta\ \overline{\mathcal{W}}_{\dot\alpha}\overline{\mathcal{W}}^{\dot\alpha}$$

The explicit derivation is quite long but we can make two checks to get more convinced

- Check that it is indeed gauge invariant
- Check that it contains the "normal" gauge strength field $F_{\mu\nu}F^{\mu\nu}$ after integrating out θ and $\bar\theta$

Abelian gauge theories

To see that it is gauge invariant remember that under a $U(1)$ transformation $V \to V + i\left(\Omega^\dagger - \Omega\right)$ and that $D_\alpha \Omega = 0, \bar{D}^{\dot{\alpha}}\Omega = 0$

$$\begin{aligned}
\mathcal{W}_\alpha \to -\frac{1}{4}\overline{DD}D_\alpha\left[V + i\left(\Omega^\dagger - \Omega\right)\right] &= \mathcal{W}_\alpha + \frac{i}{4}\overline{DD}D_\alpha\Omega \\
&= \mathcal{W}_\alpha - \frac{i}{4}\bar{D}^{\dot{\beta}}\left\{\bar{D}_{\dot{\beta}}, D_\alpha\right\}\Omega \\
&= \mathcal{W}_\alpha + \frac{1}{2}\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu\bar{D}^{\dot{\beta}}\Omega \\
&= \mathcal{W}_\alpha
\end{aligned}$$

where I also used that

$$\left\{\bar{D}_{\bar{\beta}}, D_\alpha\right\} = -2i\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu,$$

Abelian gauge theories

Remember that in the Wess-Zumino gauge the field expansion takes the form

$$V(y,\theta,\bar{\theta}) = \bar{\theta}\bar{\sigma}^{\mu}\theta A_{\mu}(y) + \bar{\theta}\bar{\theta}\theta\lambda(y) + \theta\theta\bar{\theta}\lambda^{\dagger}(y) +$$
$$\frac{1}{2}\theta\theta\theta\bar{\theta}\bar{\theta}\left[D(y) + i\theta_{\mu}A^{\mu}(y)\right]$$

Hence

$$\mathcal{W}_{\alpha}\left(y,\theta,\bar{\theta}\right) = \lambda_{a} + \theta_{\alpha}D + \frac{i}{2}\left(\sigma^{\mu}\bar{\sigma}^{\nu}\theta\right)_{\alpha}F_{\mu\nu} + i\theta\theta\left(\sigma^{\mu}\partial_{\mu}\lambda^{\dagger}\right)_{\alpha}$$

Finally

$$\frac{1}{4}\left[\mathcal{W}\mathcal{W}\right]_{F} + \frac{1}{4}\left[\overline{\mathcal{W}\mathcal{W}}\right]_{F} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\lambda^{\dagger}\bar{\sigma}^{\mu}\partial_{\mu}\lambda + \frac{1}{2}D^{2}$$

We recovered the desired term $F_{\mu\nu}F^{\mu\nu}$, but what are the other two terms?

Abelian gauge theories

The term

$$i\lambda^\dagger\bar{\sigma}^\mu\partial_\mu\lambda$$

is just the superpartner of the photon, the *photino*!

For the other term, we can show that it plays no physical role (as the F term in the chiral fields). To do this we need to spot all the D dependence in our lagrangian.

Remembering that up to now our lagrangian is

$$\mathcal{L} = \frac{1}{4}\left[\mathcal{W}\mathcal{W}\right]_F + \frac{1}{4}\left[\overline{\mathcal{W}\mathcal{W}}\right]_F + \left[\Phi^\dagger e^V\Phi\right]_D + [W(\Phi)]_F + [\bar{W}(\Phi^\dagger)]_F$$

once can note that the only other dependence on D is in $\left[\Phi^\dagger e^V\Phi\right]_D = \Phi^\dagger\Phi D$. Hence the equation of motions for D are

$$0 = \frac{\partial\mathcal{L}}{\partial D} = D + \Phi^\dagger\Phi$$

Abelian gauge theories

Putting all together we get the SUSY QED lagrangian

$$\mathcal{L} = \mathcal{L} = \left[\Phi^{*i} e^{2gq_i V_{\Phi_i}}\right]_D + \left([W(\Phi_i)]_F + \text{ c.c. }\right) + \frac{1}{4}\left([\mathcal{W}^\alpha \mathcal{W}_\alpha]_F + \text{ c.c. }\right)$$

To this we add another SUSY and supergauge invariant term $2[kV]_D = 2kD$ (e.o.m. is still algebraic). This term is called *Fayet-Iliopoulos* and it will play an important role in the spontaneous SUSY breaking (next talks)

$$\boxed{\begin{gathered}\mathcal{L}_{SQED} = \left[\Phi^{*i} e^{2gq_i V_{\Phi_i}}\right]_D + \left([W(\Phi_i)]_F + \text{ c.c. }\right) + \\ + \frac{1}{4}\left([\mathcal{W}^\alpha \mathcal{W}_\alpha]_F + \text{ c.c. }\right) - 2\kappa[V]_D\end{gathered}}$$

Non abelian gauge theories

Now extend to generic gauge theories, in particular SU(n). In spacetime:

- $n^2 - 1$ generators $T_1, \dots T_{n^2-1}$ (gauge fields)
- for $n = 3$ we have 8 fields (gluons)
- $[T_a, T_b] = i\, f_{abc}\, T_c$ where f_{abc} are called structure constants
- $U \in SU(n) \Rightarrow U = \exp(igA^a T^a)$
- Covariant derivatives in spacetime are
  $D_\mu = \partial_\mu - igA_\mu = \partial_\mu - igA^a_u T^a$

In spacetime, guided by the fact that for $U(1)$ we had

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \frac{i}{e}\left[D_\mu, D_\nu\right]$$

we *define* our field strength tensor for a general gauge symmetry via

$$F_{\mu\nu} \equiv \frac{i}{g}\left[D_\mu, D_\nu\right] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right]$$

In superspace proceed analogously by "generalising" the definition

Non abelian gauge theories

We note that

$$e^V \to e^{V+i(\Lambda^\dagger - \Lambda)}$$

Is a particular case of

$$e^V \to e^{i\Lambda^\dagger} e^V e^{-i\Lambda}$$

when the commutators vanish $[\Lambda^\dagger, V] = 0 = [\Lambda, V]$.
In fact

$$\begin{aligned} V \to V + i\left(\Omega^\dagger - \Omega\right) - \frac{i}{2}\left[V, \Omega + \Omega^\dagger\right] + \\ + i \sum_{k=1}^{\infty} \frac{B_{2k}}{(2k)!}\left[V, \left[V, \ldots \left[V, \Omega^\dagger - \Omega\right] \ldots\right]\right] = \\ = + i\left(\Omega^{a*} - \Omega^a\right) + g_a f^{abc} V^b \left(\Omega^{c*} + \Omega^c\right) \\ - \frac{i}{3} g_a^2 f^{abc} f^{cd^b} V^b V^d \left(\Omega^c - \Omega^r\right) + \ldots \end{aligned}$$

where B_n defined by $\frac{x}{e^x - 1} = \sum_{n=0}^{\infty} \frac{B_n}{n!} x^n$ are the Bernoulli numbers

Non abelian gauge theories

We have to generalise also the kinetic term

$$\mathcal{W}_\alpha = -\frac{1}{4}\overline{DD}\left(e^{-V}D_\alpha e^{V}\right)$$

so that it is invariant under

$$\mathcal{W}_\alpha \to e^{i\Omega}\mathcal{W}_\alpha e^{-i\Omega}$$

The lagrangian then becomes

$$\mathcal{L} = \frac{1}{4}\left[\mathcal{W}^{ax} W^a_a\right]_F + c.c. + \left[\Phi^{-i}\left(e^{2g\cdot T^\alpha V^a}\right)_i \omega_{\Phi_j}\right]_D + \left(\left[W\left(\Psi_i\right)\right]F + c.c.\right).$$

Let us write it in components in the case of SU(3) (QCD) after eliminating D via e.o.m.

NSQCD interactions

In case of QCD

$$
\begin{aligned}
\mathcal{L} = & \left(D_\mu \varphi_i\right)^\dagger \left(D^\mu \varphi\right)_i + \frac{i}{2}\psi_i \sigma^\mu \left(D_\mu \bar{\psi}\right)_i - \frac{i}{2}\left(D_\mu \psi\right)_i \sigma^\mu \bar{\psi}_i \\
& - \frac{1}{4} F^a_{\mu\nu} \left(F^a\right)^{\mu\nu} + \frac{i}{2}\lambda^a \sigma^\mu \left(D_\mu \bar{\lambda}\right)^a - \frac{i}{2}\left(D_\mu \lambda\right)^a \sigma^\mu \bar{\lambda}^a \\
& - \sqrt{2} i g \bar{\psi}_i \bar{\lambda}^a T^a_{ij} \varphi_j + \sqrt{2 i g} \varphi_i^\dagger T^a_{ij} \psi_j \lambda^a \\
& - \frac{1}{2} \frac{\partial^2 W}{\partial \varphi_i \partial \varphi_j} \psi_i \psi_j - \frac{1}{2} \frac{\partial^2 W^\dagger}{\partial \varphi_i^\dagger \partial \varphi_j^\dagger} \bar{\psi}_i \bar{\psi}_j - V\left(\varphi_i, \varphi_j^\dagger\right)
\end{aligned}
$$

where

$$
V\left(\varphi_i, \varphi_j^\dagger\right) = F_i^\dagger F_i + \frac{1}{2}\left(D^a\right)^2 = \sum_i \left|\frac{\partial W}{\partial \varphi_i}\right|^2 + \frac{1}{2}\sum_a \left(g \varphi_i^\dagger T^a_{ij} \varphi_j + k^a\right)^2
$$

$$
W\left(\varphi_i\right) = a_i \varphi_i + \frac{1}{2} m_{ij} \varphi_i \varphi_j + \frac{1}{3!} y_{ijk} \varphi_i \varphi_j \varphi_k
$$

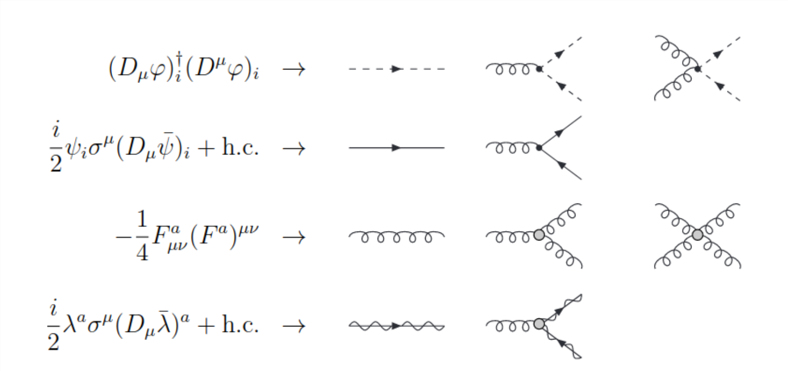
SQCD interactions

Where the gauge covariant derivatives are

$$\begin{aligned}(D_\mu\varphi)_i &= \partial_\mu\varphi_i + igv_\mu^a T_{ij}^a\varphi_j\\ (D_\mu\psi)_i &= \partial_\mu\psi_i + igv_\mu^a T_{ij}^a\psi_j\\ (D_\mu\lambda)^a &= \partial_\mu\lambda^a - gf^{abc}v_\mu^b\lambda^c\end{aligned}$$

SQCD interactions

This gives very cool interactions (diagrams from [3])



SQCD interactions

$$-\sqrt{2}ig\,\bar{\psi}_i\bar{\lambda}^a T^a_{ij}\varphi_j + \text{h.c.} \quad \rightarrow$$

$$\frac{1}{2}(g\,\varphi_i^\dagger\,T^a_{ij}\,\varphi_j)^2 \quad \rightarrow$$

$$-\frac{1}{2}\frac{\partial^2 W}{\partial\varphi_i\partial\varphi_j}\psi_i\psi_j + \text{h.c.} \quad \rightarrow$$

$$\left|\frac{\partial W}{\partial\varphi_i}\right|^2 \quad \rightarrow$$

Thank you for the attention

References

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