Building SUSY models II: using superfields

Seminar on Supersymmetry and its breaking

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Main points of the talk

- Apply the superspace formalism and show why it is useful to build SUSY theories

- Principles to construct SUSY lagrangians

- SUSY gauge theories: QED and QCD

- SUSY predictions: particles, interaction, masses, ...

D and F terms of the superfields

Chiral superfields $\bar{D}_{\dot{\alpha}}\Phi = 0$ or $D_\alpha \Phi^* = 0$

$$\Phi(x,\theta,\bar{\theta}) = \phi(x) + i\bar{\theta}\bar{\sigma}^\mu\theta\partial_\mu\phi(x) + \frac{1}{4}\theta\theta\theta\dagger\bar{\theta}\partial_\mu\partial^\mu\phi(x) + \sqrt{2}\theta\psi(x)$$
$$-\frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi(x) + \theta\theta F(x)$$
$$\Phi^* = blablabla$$

A priori Φ and Φ^* (hence the components) are independent.

Vector superfields

$$V(x,\theta,\bar{\theta}) = longstuff$$

Remember that

$$\delta_\epsilon F, \delta_\epsilon F', \delta_\epsilon D$$

are total derivatives!

$\Rightarrow$ Use F terms of chiral superfields and D terms of vector superfields to build SUSY invariant lagrangians!

$$\mathcal{L}[\Phi, V] = [V]_D + [\Phi]_F + [\Phi^*]_F$$

How can we "pick" only the terms we need? $\rightarrow$ Grassman integration

$$[V]_D = \int d^2\theta \; d^2\bar{\theta} V(\theta, \bar{\theta})$$

$$[\Phi]_D = \int d^2\theta \; \Phi(\theta, \bar{\theta})_{|_{\bar{\theta}=0}}$$

Thus, our lagrangian will be of the form

$$\mathcal{L}[\Phi, V] = \int d^2\theta \; d^2\bar{\theta} V(\theta, \bar{\theta}) + \int d^2\theta \; \Phi(\theta, \bar{\theta})_{|_{\bar{\theta}=0}} + \int d^2\bar{\theta} \; \Phi^*(\theta, \bar{\theta})_{|_{\theta=0}}$$

Let us focus for a moment on the first term

Our building blocks are the chiral superfields and a vector field can be obtained by taking the product $\Phi^*\Phi$

$$\Phi(x,\theta,\bar{\theta}) = \phi(x) + i\bar{\theta}\bar{\sigma}^\mu\theta\partial_\mu\phi(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial_\mu\partial^\mu\phi(x) + \sqrt{2}\theta\psi(x)$$
$$-\frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi(x) + \theta\theta F(x)$$
$$\Phi^*\Phi = \dots$$

Now "select" the SUSY invariant component

$$[\Phi^*\Phi]_D = \int d^2\theta d^2\bar{\theta}\ \Phi^*(x,\theta,\bar{\theta})\Phi(x,\theta,\bar{\theta})$$
$$= \boxed{-\partial^\mu\phi^*\partial_\mu\phi + i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi + F^*F} + \text{total derivative}$$

$\rightarrow$ **Free Wess-Zumino model!**

Let us introduce again the F-terms and let us try to generalize a bit. Remembering the definitions of chiral superfields

$$\bar{D}_{\dot{\alpha}}\Phi = 0 \qquad D_\alpha \Phi^* = 0$$

we note that any analytic function of chiral superfields is in turn a chiral superfield (power series expansion and product rule).
$\Rightarrow$ Write our chiral term of N fields as

$$W(\{\Phi_k\}) = \sum_i^N M_i \Phi_i + \sum_{i,j}^N \frac{1}{2!} M_{ij} \Phi_i \Phi_j + \sum_{i,j,k}^N \frac{1}{3!} M_{ijk} \Phi_i \Phi_j \Phi_k$$

Higher order terms are non-renormalisable

Full Wess-Zumino model

$$\mathcal{L}_{WZ}\left(\{\Phi_i\},\{\Phi_i^*\}\right) = \mathcal{L}_{WZ,D} + \mathcal{L}_{WZ,F} =$$
$$= \left[\Phi^{*i}\Phi^i\right]_D + \left[W(\{\Phi_i\})\right]_F + \left[W^*(\{\Phi_i^*\})\right]_F =$$
$$= \boxed{\int d^2\theta \; -\frac{1}{4}\overline{DD}\Phi^{*i}\Phi_i + \left[W(\{\Phi_i\})\right]_F + \int d^2\bar{\theta} \; \left[W^*(\{\Phi_i^*\})\right]_F}$$

Equations of motion varying w.r.t. Φ_i and Φ_i^*

$$0 = -\frac{1}{4}\overline{DD}\Phi^{*i} + \frac{\delta W}{\delta \Phi_i}$$
$$0 = -\frac{1}{4}DD\Phi_i + \frac{\delta W^*}{\delta \Phi^{*i}}$$

Abelian gauge theories

Let us introduce (abelian) gauge interactions.

Let us start with U(1) global symmetry

$$\Phi_i \to e^{iq_i\Lambda_i}\Phi_i$$

The kinetic part of the lagrangian is always invariant

$$\mathcal{L}_K = \mathcal{L}_{WZ,D} = \int d^2\theta d^2\bar{\theta}\Phi^*\Phi = \int d^2\theta - \frac{1}{4}\overline{DD}\Phi^*\Phi$$

The interaction part

$$\mathcal{L}_{int} = \mathcal{L}_{WZ,F} = \int d^2\theta \frac{1}{2}\sum_{ij}\phi_i\phi_j + \frac{1}{3!}\sum_{ijk}\phi_i\phi_j\phi_k + \text{complex. conj.}$$

requires

$$m_{ij} = 0 \qquad \text{or} \qquad y_{ijk} = 0$$

whenever

$$q_i + q_j \neq 0 \qquad \text{or} \qquad q_i + q_j + q_k \neq 0$$

Abelian gauge theories

Promote to a local gauge symmetry

$$\Lambda \to \Lambda(x, \theta, \bar{\theta})$$

- Gauge parameter is now a supergauge field $\Lambda = \Lambda(x, \theta, \bar{\theta})$
- Promote derivatives to covariant derivatives $\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$
- We need $\Lambda(x, \theta, \bar{\theta})$ to be a left-chiral superfield if we want Φ' to be a left-chiral superfield (chain rule).
- Thus this causes a problem in the kinetic term because Λ^* is a right-chiral superfield hence obviously $\Phi'^*\Phi' \neq \Phi^*\Phi$
- The problem is analogous to the kinetc term in "normal" when $\partial_u \phi^* \partial^\mu \phi$ was not gauge invariant
- Solution: add a term that compensate the gauge for the non invariant terms

$$\Phi^{+}\Phi \rightarrow \Phi^{*} e^{-i\Lambda^{*}(x)} e^{i\Lambda(x)}\Phi$$

$\Rightarrow$ need an object A such that $A' = e^{i\Lambda *} A e^{-i\Lambda}$ and the quantity $\Phi^{*} A \Phi *$ is then gauge invariant.

Remember that a vector superfield V transforms according to

$$V \rightarrow V' = V + i\Lambda * - \Lambda$$

$$\Rightarrow \qquad A = e^{V} \text{ is what we need.}$$

Now we are just left with finding the gauge-invariant strength field term (euivalent of $F_{\mu\nu}F^{\mu\nu}$)

Abelian gauge theories

Let us start by defining the two chiral fields

$$\mathcal{W}_\alpha = -\frac{1}{4}\overline{DD}D_\alpha V, \quad \overline{\mathcal{W}}_{\dot{\alpha}} = -\frac{1}{4}DD\bar{D}_{\dot{\alpha}}V$$

where V is a vector field.
The gauge-invariant dynamical term (equivalent to $F_{\mu\nu}F^{\mu\nu}$) is

$$[W]_F + [\bar{W}]_F = \int d^2\theta W_\alpha W^\alpha + \int d^2\bar{\theta}\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}$$

The explicit derivation is quite long (it will be put in the report appendix) but we can make two checks to get more convinced

- Check that it is indeed gauge invariant
- Check that it contains the "normal" gauge strength field $F_{\mu\nu}F^{\mu\nu}$ after integrating out θ and $\bar{\theta}$

Abelian gauge theories

To see that it is gauge invariant:

$$
\begin{aligned}
\mathcal{W}_\alpha \to -\frac{1}{4}\overline{DD}D_\alpha\left[V + i\left(\Omega^* - \Omega\right)\right] &= \mathcal{W}_\alpha + \frac{i}{4}\overline{DD}D_\alpha\Omega \\
&= \mathcal{W}_\alpha - \frac{i}{4}\bar{D}^{\dot\beta}\left\{\bar{D}_{\dot\beta}, D_\alpha\right\}\Omega \\
&= \mathcal{W}_\alpha + \frac{1}{2}\sigma^\mu_{\alpha\dot\beta}\partial_\mu\bar{D}^{\dot\beta}\Omega \\
&= \mathcal{W}_\alpha
\end{aligned}
$$

Abelian gauge theories

Remember that in the Wess-Zumino gauge the field expansion takes the form

$$V(y,\theta,\bar{\theta}) = \theta^\dagger\bar{\sigma}^\mu\theta A_\mu(y) + \theta^\dagger\theta^\dagger\theta\lambda(y) + \theta\theta\theta^\dagger\lambda^\dagger(y) + \\ \frac{1}{2}\theta\theta\theta\theta^\dagger\theta^\dagger\left[D(y) + i\theta_\mu A^\mu(y)\right]$$

Hence

$$W = \dots$$

Finally

$$\frac{1}{4}\left[\mathcal{W}\mathcal{W}\right]_F + \frac{1}{4}\left[\overline{\mathcal{W}\mathcal{W}}\right]_F = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\lambda^\dagger\bar{\sigma}^\mu\partial_\mu\lambda + \frac{1}{2}D^2$$

We recovered the desired term $F_{\mu\nu}F^{\mu\nu}$, but what are the other two terms?

Abelian gauge theories

The term

$$i\lambda^* \bar{\sigma}^\mu \partial_\mu \lambda$$

is just the superpartner of the photon, the *photino*!
For the other term, we can show that it palys no physical role (as the F term in the chiral fields). To do this we need to spot all the D dependence in our lagrangian.
Remembering that up to now our lagrangian is

$$\mathcal{L} = \frac{1}{4}\left[\mathcal{W}\mathcal{W}\right]_F + \frac{1}{4}\left[\overline{\mathcal{W}\mathcal{W}}\right]_F + \left[\Phi^* e^V \Phi\right]_D + \left[W(\Phi)\right]_F + \left[\bar{W}(\Phi^*)\right]_F$$

once can note that the only other dependence on D is in $\left[\Phi^* e^V \Phi\right]_D = \Phi^* \Phi D$. Hence the equation of motions for D are

$$0 = \frac{\partial \mathcal{L}}{\partial D} = D + \Phi * \Phi$$

Putting all together we get the SUSY QED lagrangian

$$\mathcal{L} = \dots$$

To this we add another SUSY and supergauge invariant term $2[kV]_D = 2kD$ (e.o.m. is still algebraic) This term is called *Fayet-Iliopoulos* and it will play an important role in the spontaneous SUSY breaking (next talks)

$$\boxed{\mathcal{L}_{SQED} = \dots}$$

Non abelian gauge theories

Now extend to generic gauge theories, in particular SU(n). In spacetime:

- $n^2 - 1$ generators $T_1, \ldots T_{n^2-1}$ (gauge fields)
- for $n = 3$ we have 8 fields (gluons)
- $[T_a, T_b] = i\, f_{abc}\, T_c$ where f_{abc} are called structure constants
- $U \in SU(n) \Rightarrow U = \exp(igA^a T^a)$
- Covariant derivatives in spacetime are
  $D_\mu = \partial_\mu - igA_\mu = \partial_\mu - igA_u^a T^a$

In spacetime, guided by the fact that for $U(1)$ we had

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \frac{i}{e}\,[D_\mu, D_\nu]$$

we *define* our field strength tensor for a general gauge symmetry via

$$F_{\mu\nu} \equiv \frac{i}{g}\,[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\,[A_\mu, A_\nu]$$

$\Rightarrow$ how to extend to superspace?

Non abelian gauge theories

Our starting point is again a term like

$$\Phi^* e^A \Phi$$

$\rightarrow$ we need to figure out how to set A