

Building SUSY models II: using superfields

Seminar on Supersymmetry and its breaking

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Main points of the talk

- Apply the superspace formalism and show why it is useful to build SUSY theories
- Principles to construct SUSY lagrangians
- SUSY gauge theories: QED and QCD
- SUSY predictions: particles, interaction, masses, ...

D and F terms of the superfields

Chiral superfields $\bar{D}_{\dot{\alpha}}\Phi = 0$ or $D_{\alpha}\Phi^* = 0$

$$\begin{aligned}\Phi(x, \theta, \bar{\theta}) = & \phi(x) + i\bar{\theta}\bar{\sigma}^{\mu}\theta\partial_{\mu}\phi(x) + \frac{1}{4}\theta\theta\theta^{\dagger}\bar{\theta}\partial_{\mu}\partial^{\mu}\phi(x) + \sqrt{2}\theta\psi(x) \\ & - \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^{\mu}\partial_{\mu}\psi(x) + \theta\theta F(x) \\ & \Phi^* = \text{blablabla}\end{aligned}$$

A priori Φ and Φ^* (hence the components) are independent.

Vector superfields

$$V(x, \theta, \bar{\theta}) = \text{longstuff}$$

Remember that

$$\delta_{\epsilon}F, \delta_{\epsilon}F', \delta_{\epsilon}D$$

are total derivatives!

\Rightarrow Use F terms of chiral superfields and D terms of vector superfields to build SUSY invariant lagrangians!

$$\mathcal{L}[\Phi, V] = [V]_D + [\Phi]_F + [\Phi^*]_F$$

How can we "pick" only the terms we need? \rightarrow Grassman integration

$$[V]_D = \int d^2\theta \, d^2\bar{\theta} V(\theta, \bar{\theta})$$

$$[\Phi]_D = \int d^2\theta \, \Phi(\theta, \bar{\theta})|_{\bar{\theta}=0}$$

Thus, our lagrangian will be of the form

$$\mathcal{L}[\Phi, V] = \int d^2\theta \, d^2\bar{\theta} V(\theta, \bar{\theta}) + \int d^2\theta \, \Phi(\theta, \bar{\theta})|_{\bar{\theta}=0} + \int d^2\bar{\theta} \, \Phi^*(\theta, \bar{\theta})|_{\theta=0}$$

Let us focus for a moment on the first term

Our building blocks are the chiral superfields and a vector field can be obtained by taking the product $\Phi^* \Phi$

$$\begin{aligned}\Phi(x, \theta, \bar{\theta}) &= \phi(x) + i\bar{\theta}\bar{\sigma}^\mu\theta\partial_\mu\phi(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial_\mu\partial^\mu\phi(x) + \sqrt{2}\theta\psi(x) \\ &\quad - \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi(x) + \theta\theta F(x) \\ \Phi^*\Phi &= \dots\end{aligned}$$

Now "select" the SUSY invariant component

$$\begin{aligned}[\Phi^*\Phi]_D &= \int d^2\theta d^2\bar{\theta} \Phi^*(x, \theta, \bar{\theta})\Phi(x, \theta, \bar{\theta}) \\ &= \boxed{-\partial^\mu\phi^*\partial_\mu\phi + i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi + F^*F} + \text{total derivative}\end{aligned}$$

→ **Free Wess-Zumino model!**

Let us introduce again the F-terms and let us try to generalize a bit.
Remembering the definitions of chiral superfields

$$\bar{D}_{\dot{\alpha}}\Phi = 0 \quad D_{\alpha}\Phi^* = 0$$

we note that any analytic function of chiral superfields is in turn a chiral superfield (power series expansion and product rule).

⇒ Write our chiral term of N fields as

$$W(\{\Phi_k\}) = \sum_i^N M_i \Phi_i + \sum_{i,j}^N \frac{1}{2!} M_{ij} \Phi_i \Phi_j + \sum_{i,j,k}^N \frac{1}{3!} M_{ijk} \Phi_i \Phi_j \Phi_k$$

Higher order terms are non-renormalisable

Full Wess-Zumino model

$$\begin{aligned}\mathcal{L}_{WZ}(\{\Phi_i\}, \{\Phi_i^*\}) &= \mathcal{L}_{WZ,D} + \mathcal{L}_{WZ,F} = \\ &= [\Phi^{*i}\Phi^i]_D + [W(\{\Phi_i\})]_F + [W^*(\{\Phi_i^*\})]_F = \\ &= \boxed{\int d^2\theta \left[-\frac{1}{4}\overline{D}\overline{D}\Phi^{*i}\Phi_i + [W(\{\Phi_i\})]_F \right] + \int d^2\bar{\theta} \left[W^*(\{\Phi_i^*\}) \right]_F}\end{aligned}$$

Equations of motion varying w.r.t. Φ_i and Φ_i^*

$$\begin{aligned}0 &= -\frac{1}{4}\overline{D}\overline{D}\Phi^{*i} + \frac{\delta W}{\delta\Phi_i} \\ 0 &= -\frac{1}{4}D\overline{D}\Phi_i + \frac{\delta W^*}{\delta\Phi^{*i}}\end{aligned}$$

Gauge theories

Let us introduce (abelian) gauge interactions.

Example: $U(1)$ global symmetry

$$\Phi_i \rightarrow e^{iq_i \Lambda_i} \Phi_i$$

The kinetic part of the lagrangian is always invariant

$$\mathcal{L}_K = \mathcal{L}_{WZ,D} = \int d^2\theta d^2\bar{\theta} \Phi^* \Phi = \int d^2\theta - \frac{1}{4} \overline{DD} \Phi^* \Phi$$

The interaction part

$$\mathcal{L}_{int} = \mathcal{L}_{WZ,F} = \int d^2\theta \frac{1}{2} \sum_{ij} \phi_i \phi_j + \frac{1}{3!} \sum_{ijk} \phi_i \phi_j \phi_k + \text{complex. conj.}$$

requires

$$m_{ij} = 0 \quad \text{or} \quad y_{ijk} = 0$$

whenever

$$q_i + q_j \neq 0 \quad \text{or} \quad q_i + q_j + q_k \neq 0$$

Abelian gauge theories

Promote to a local gauge symmetry

$$\phi \rightarrow \phi' = \phi e^{i\Lambda(x)}$$

- is now a supergauge field $\Lambda = \Lambda(x, \theta, \bar{\theta})$
- We need $\Lambda(x)$ to be a left-chiral superfield if we want Φ' to be a left-chiral superfield (chain rule).
- Thus this causes a problem in the kinetic term because Λ^* is a right-chiral superfield hence obviously $\Phi'^* \Phi' \neq \Phi^* \Phi$
- The problem is analogous to the kinetic term in "normal" when $\partial_\mu \phi^* \partial^\mu \phi$ was not gauge invariant
- Solution: add a term that compensate the gauge for the non invariant terms (in normal QFT was the covariant derivative)

Abelian gauge theories

$$\Phi^\dagger \Phi \rightarrow \Phi^* e^{-i\Lambda^*(x)} e^{i\Lambda(x)} \Phi$$

\Rightarrow need an object A such that $A' = e^{i\Lambda^*} A e^{-i\Lambda}$ and the quantity $\Phi^* A \Phi$ is then gauge invariant.

Remember that a vector superfield V transforms according to

$$V \rightarrow V' = V + i\Lambda * -\Lambda$$

$$\Rightarrow \quad A = e^V \text{ is what we need.}$$

Now we are just left with finding the gauge-invariant strength field term (equivalent of $F_{\mu\nu} F^{\mu\nu}$)

Abelian gauge theories

Let us start by defining the two chiral fields

$$\mathcal{W}_\alpha = -\frac{1}{4}\overline{D}\overline{D}D_\alpha V, \quad \overline{\mathcal{W}}_{\dot{\alpha}} = -\frac{1}{4}DD\bar{D}_{\dot{\alpha}} V$$

where V is a vector field.

The gauge-invariant dynamical term (equivalent to $F_{\mu\nu}F^{\mu\nu}$) is

$$[W]_F + [\bar{W}]_F = \int d^2\theta W_\alpha W^\alpha + \int d^2\bar{\theta} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}$$

The explicit derivation is quite long (it will be put in the report appendix) but we can make two checks to get more convinced

- Check that it is indeed gauge invariant
- Check that it contains the "normal" gauge strength field $F_{\mu\nu}F^{\mu\nu}$ after integrating out θ and $\bar{\theta}$

To see that it is gauge invariant:

$$\begin{aligned}\mathcal{W}_\alpha &\rightarrow -\frac{1}{4}\overline{D}D D_\alpha [V + i(\Omega^* - \Omega)] = \mathcal{W}_\alpha + \frac{i}{4}\overline{D}D D_\alpha \Omega \\ &= \mathcal{W}_\alpha - \frac{i}{4}\bar{D}^{\dot{\beta}} \left\{ \bar{D}_{\dot{\beta}}, D_\alpha \right\} \Omega \\ &= \mathcal{W}_\alpha + \frac{1}{2}\sigma_{\alpha\dot{\beta}}^\mu \partial_\mu \bar{D}^{\dot{\beta}} \Omega \\ &= \mathcal{W}_\alpha\end{aligned}$$

Abelian gauge theories

Remember that in the Wess-Zumino gauge the field expansion takes the form

$$V(y, \theta, \bar{\theta}) = \theta^\dagger \bar{\sigma}^\mu \theta A_\mu(y) + \theta^\dagger \theta^\dagger \theta \lambda(y) + \theta \theta \theta^\dagger \lambda^\dagger(y) + \frac{1}{2} \theta \theta \theta^\dagger \theta^\dagger [D(y) + i \theta_\mu A^\mu(y)]$$

Hence

$$W = \dots$$

Finally

$$\frac{1}{4} [\mathcal{W}\mathcal{W}]_F + \frac{1}{4} [\overline{\mathcal{W}}\overline{\mathcal{W}}]_F = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \lambda^\dagger \bar{\sigma}^\mu \partial_\mu \lambda + \frac{1}{2} D^2$$

We recovered the desired term $F_{\mu\nu} F^{\mu\nu}$, but what are the other two terms?

Abelian gauge theories

The term

$$i\lambda^* \bar{\sigma}^\mu \partial_\mu \lambda$$

is just the superpartner of the photon, the *photino*!

For the other term, we can show that it plays no physical role (as the F term in the chiral fields). To do this we need to spot all the D dependence in our lagrangian.

Remembering that up to now our lagrangian is

$$\mathcal{L} = \frac{1}{4} [\mathcal{W}\mathcal{W}]_F + \frac{1}{4} [\overline{\mathcal{W}\mathcal{W}}]_F + [\Phi^* e^V \Phi]_D + [W(\Phi)]_F + [\bar{W}(\Phi^*)]_F$$

once can note that the only other dependence on D is in $[\Phi^* e^V \Phi]_D = \Phi^* \Phi D$. Hence the equation of motions for D are

$$0 = \frac{\partial \mathcal{L}}{\partial D} = D + \Phi * \Phi$$

Putting all together we get the SUSY QED lagrangian

$$\mathcal{L} = \dots$$

To this we add another SUSY and supergauge invariant term $2[kV]_D = 2kD$ (e.o.m. is still algebraic) This term is called *Fayet-Iliopoulos* and it will play an important role in the spontaneous SUSY breaking (next talks)

$$\mathcal{L}_{SQED} = \dots$$

Non abelian gauge theories