Building SUSY models II: using superfields

Seminar on Supersymmetry and its breaking

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Main points of the talk

- Apply the superspace formalism and show why it is useful to build SUSY theories
- Principles to construct SUSY lagrangians
- SUSY gauge theories: QED and QCD
- SUSY predictions: particles, interaction, masses, ...
- N=1 nonrenormalization theorem

How to build SUSY invariant lagrangians

- We want our theory to be SUSY invariant, that is

$$\left(\epsilon\hat{Q} + \epsilon^{\dagger}\hat{Q}^{\dagger}\right)S = \left(\epsilon\hat{Q} + \epsilon^{\dagger}\hat{Q}^{\dagger}\right)\int dx^{\mu}\mathcal{L} = 0$$

- We know that this condition is met if, under a given transformation

$$\mathcal{L} \to \mathcal{L} + \partial_{\mu} f$$

- Hence our goal is to build a lagrangian which transforms in this way, using **Chiral** ($\bar{D}_{\dot{\alpha}}\Phi = 0$ or $D_{\alpha}\Phi^{\dagger} = 0$) and **Vector** ($V = V^{\dagger}$) superfields

How to build SUSY invariant lagrangians

- Left-chiral field expansion (right-chiral is hermitian conjugate)

$$\Phi(x,\theta,\bar{\theta}) = \varphi(x) + i\bar{\theta}\bar{\sigma}^{\mu}\theta\partial_{\mu}\varphi(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial_{\mu}\partial^{\mu}\varphi(x) + \sqrt{2}\theta\psi(x) + \\ -\frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^{\mu}\partial_{\mu}\psi(x) + \theta\theta F(x)$$

- Vector field expansion

$$V\left(x,\theta,\bar{\theta}\right) = a + \theta\xi + \bar{\theta}\xi^{\dagger} + \theta\theta b + \bar{\theta}\bar{\theta}b^{\dagger} + \bar{\theta}\bar{\sigma}^{\mu}\theta A_{\mu} + \\ +\bar{\theta}\bar{\theta}\theta\left(\lambda - \frac{i}{2}\sigma^{\mu}\partial_{\mu}\xi^{\dagger}\right) + \theta\theta\bar{\theta}\left(\lambda^{\dagger} - \frac{i}{2}\sigma^{\mu}\partial_{\mu}\xi\right) + \theta\theta\bar{\theta}\bar{\theta}\left(\frac{1}{2}D + \frac{1}{4}\partial_{\mu}\partial^{\mu}a\right)$$

- They both carry no spinor, nor vector indices, the name is due to their particle content!

How to build SUSY invariant lagrangians

- For the components of a chiral field $\Phi(x, \theta, \bar{\theta})$ one has that

$$\delta_\epsilon \phi = \epsilon \psi$$

$$\delta_\epsilon \psi_\alpha = -i \left(\sigma^\mu \epsilon^\dagger\right)_\alpha \partial_\mu \phi + \epsilon_\alpha F,$$

$$\boxed{\delta_\epsilon F = -i \epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \psi}$$

- For the components of a vector field $V(x, \theta, \bar{\theta})$ one has

$$\sqrt{2}\delta_\epsilon a = \epsilon\xi + \epsilon^\dagger \xi^\dagger \qquad \sqrt{2}\delta_\epsilon \lambda_\alpha = \epsilon_a D + \frac{i}{2} \left(\sigma^\mu \sigma^\nu \epsilon\right)_\alpha \left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)$$

$$\sqrt{2}\delta_\epsilon b = \epsilon^\dagger \lambda^\dagger - i\epsilon^\dagger \sigma^\mu \partial_\mu \xi \qquad \sqrt{2}\delta_\epsilon \xi_\alpha = 2\epsilon_\alpha b - \left(\sigma^\mu \epsilon^\dagger\right)_\alpha \left(A_\mu + i\partial_\mu a\right)$$

$$\sqrt{2}\delta_\epsilon A^\mu = i\epsilon\partial^\mu \xi - i\epsilon^\dagger \partial^\mu \xi^\dagger + \epsilon\sigma^\mu \lambda^\dagger - \epsilon^\dagger \bar{\sigma}^\mu \lambda$$

$$\boxed{\sqrt{2}\delta_\epsilon D = -i\epsilon\sigma^\mu \partial_\mu \lambda^\dagger - i\epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \lambda}$$

How to build SUSY invariant lagrangians

- Idea $\rightarrow$ "select" the components of the fields that transforms as total derivatives

$$\mathcal{L}[\Phi, V] = \alpha\, [V]_D + \beta\, [\Phi]_F + \gamma\, \left[\Phi^\dagger\right]_F$$

- How can we "pick" only the terms we need? $\rightarrow$ Grassman integration

$$\begin{aligned} \int d^2\theta \quad \theta^n\, f(x,\bar{\theta}) \;&=\; f(x,\bar{\theta})\, \delta_{n,2} \\ \int d^2\theta d^2\bar{\theta} \quad \bar{\theta}^m \theta^n\, f(x) \;&=\; f(x)\, \delta_{m,2}\, \delta_{n,2} \end{aligned}$$

- Our terms of interest

$$\begin{aligned} [\Phi]_F &= \int d^2\theta\ \Phi(x,\theta,\bar{\theta}) = F + \text{total derivative} \\ [V]_D &= \int d^2\theta d^2\bar{\theta}\ V(x,\theta,\bar{\theta}) = D + \text{total derivative} \end{aligned}$$

How to build SUSY invariant lagrangians

Let us focus for a moment on the D term.

A vector field can be obtained from 2 chiral superfields by taking the product $\Phi^\dagger\Phi$

$$\Phi(x,\theta,\bar\theta) = \varphi(x) + i\bar\theta\bar\sigma^\mu\theta\partial_\mu\varphi(x) + \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial_\mu\partial^\mu\varphi(x) + \sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu\psi(x) + \theta\theta F(x)$$

$$\begin{aligned}\Phi^{\dagger i}\Phi_j = {} & \varphi^{*i}\varphi_j + \sqrt{2}\theta\psi_j\varphi^{*i} + \sqrt{2}\theta^\dagger\psi^{\dagger i}\varphi_j + \theta\theta\varphi^{*i}F_j + \theta^\dagger\theta^\dagger\varphi_j F^{\dagger i} \\ & + \theta^\dagger\bar\sigma^\mu\theta\left[i\varphi^{\dagger i}\partial_\mu\varphi_j - i\varphi_j\partial_\mu\varphi^{*i} - \psi^{\dagger i}\sigma_\mu\psi_j\right] \\ & + \frac{i}{\sqrt{2}}\theta\theta\theta^\dagger\bar\sigma^\mu\left(\psi_j\partial_\mu\varphi^{*i} - \partial_\mu\psi_j\varphi^{*i}\right) + \sqrt{2}\theta\theta\theta^\dagger\psi^{\dagger i}F_j \\ & + \frac{i}{\sqrt{2}}\theta^\dagger\theta^\dagger\theta\sigma^\mu\left(\psi^{\dagger i}\partial_\mu\varphi_j - \partial_\mu\psi^{\dagger i}\varphi_j\right) + \sqrt{2}\theta^\dagger\theta^\dagger\theta\psi_j F^{*i} \\ & + \theta\theta\theta^\dagger\theta^\dagger\left[F^{*i}F_j - \frac{1}{2}\partial^\mu\varphi^{*i}\partial_\mu\varphi_j + \frac{1}{4}\varphi^{*i}\partial^\mu\partial_\mu\varphi_j + \frac{1}{4}\varphi_j\partial^\mu\partial_\mu\varphi^{*i}\right. \\ & \left. + \frac{i}{2}\psi^{\dagger i}\bar\sigma^\mu\partial_\mu\psi_j + \frac{i}{2}\psi_j\sigma^\mu\partial_\mu\psi^{\dagger i}\right]\end{aligned}$$

How to build SUSY invariant lagrangians

- Now "select" the SUSY invariant component (D component)

$$\left[\Phi^\dagger \Phi\right]_D = \int d^2\theta d^2\bar{\theta}\ \Phi^\dagger(x,\theta,\bar{\theta})\Phi(x,\theta,\bar{\theta})$$

$$\left[F^* F - \frac{1}{2}\partial^\mu \varphi^{*i}\partial_\mu \varphi_j + \frac{1}{4}\varphi^{*i}\partial^\mu\partial_\mu\varphi_j + \frac{1}{4}\varphi_j\partial^\mu\partial_\mu\varphi^{*i} \right.$$

$$\left. + \frac{i}{2}\psi^{\dagger i}\bar{\sigma}^\mu\partial_\mu\psi_j + \frac{i}{2}\psi_j\sigma^\mu\partial_\mu\psi^{\dagger i}\right] =$$

$$= -\partial^\mu\varphi^\dagger\partial_\mu\varphi + i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi + F^* F + \text{total derivative}$$

- Hence

$$\boxed{S = \int dx^\mu \mathcal{L} = \int dx^\mu - \partial^\mu\varphi^*\partial_\mu\varphi + i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi + F^* F}$$

$\rightarrow$ **Free Wess-Zumino model!**

How to build SUSY invariant lagrangians

- In order to add SUSY invariant interactions, let us recall the definitions of chiral superfields

$$\bar{D}_{\dot{\alpha}}\Phi = 0 \qquad D_{\alpha}\Phi^{\dagger} = 0$$

- Note that any analytic function of chiral superfields is in turn a chiral superfield (power series expansion and product rule).
- $\Rightarrow$ Write our chiral term of N fields as

$$W(\{\Phi_k\}) = \sum_{i}^{N} a_i \Phi_i + \sum_{i,j}^{N} \frac{1}{2!} m_{ij} \Phi_i \Phi_j + \sum_{i,j,k}^{N} \frac{1}{3!} y_{ijk} \Phi_i \Phi_j \Phi_k$$

- Higher order terms are non-renormalisable
- Reality of the action requires us to take $W + W^*$

How to buld SUSY invariant lagrangians

The interacting lagrangian becomes

$$\mathcal{L}_{WZ}\left(\{\Phi_i\},\{\Phi_i^\dagger\}\right)=\mathcal{L}_{WZ,D}+\mathcal{L}_{WZ,F}=$$
$$=\left[\Phi^{\dagger i}\Phi^i\right]_D+\left[W(\{\Phi_i\})\right]_F+\left[W^\dagger(\{\Phi_i^\dagger\})\right]_F=$$
$$=\int d^2\theta\ \left(-\frac{1}{4}\overline{DD}\Phi^{\dagger i}\Phi_i+W(\{\Phi_i\})\right)+\int d^2\bar{\theta}\ W^\dagger(\{\Phi_i^\dagger\})$$

Equations of motion varying w.r.t. Φ_i and $\Phi_i^\dagger$

$$0=-\frac{1}{4}\overline{DD}\Phi^{\dagger i}+\frac{\delta W}{\delta\Phi_i}$$
$$0=-\frac{1}{4}DD\Phi_i+\frac{\delta W^\dagger}{\delta\Phi^{\dagger i}}$$

Wess-Zumino model

- We are now interested in studying better the components of Φ, hence let us focus on the case $i = j$ with $a_i = a, m_{ij} = m, y_{ijk} = y$ and the superpotential

$$W(\Phi) = \frac{1}{2}\, m\, \Phi\Phi + \frac{1}{3!}\, y\, \Phi\Phi\Phi$$

where we dropped the linear term.

- The lagrangian becomes

$$\mathcal{L}(\Phi, \Phi^\dagger) = \mathcal{L}_{WZ,D} + \mathcal{L}_{WZ,F} =$$
$$= F^* F + (\partial_\mu \varphi)(\partial^\mu \varphi)^* + \frac{i}{2}\psi\sigma^\mu \left(\partial_\mu \bar{\psi}\right) - \frac{i}{2}(\partial_\mu \psi)\,\sigma^\mu \bar{\psi} +$$
$$-m\varphi F - \frac{m}{2}(\psi\psi) - \frac{y}{2}\varphi\varphi F - \frac{y}{2}\varphi(\psi\psi) + \text{ h.c.}$$

- E.o.m. for F (analogous for F^*) is

$$0 = \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu F)} - \frac{\partial \mathcal{L}}{\partial F} = -\frac{\partial \mathcal{L}}{\partial F} = -F^* + m\varphi + \frac{y}{2}\varphi\varphi$$

- $\Rightarrow$ algebraic equation $\Rightarrow F, F^*$ are unphysical.

Wess-Zumino model

$$F^* = m\varphi + \frac{y}{2}\varphi\varphi \qquad F = m\varphi^* + \frac{y}{2}\varphi\varphi$$

- One can rewrite the terms containing F, F^* as

$$F^*F - \left(m\varphi F + \frac{y}{2}\varphi\varphi F + hc\right) = -\left|m\varphi + \frac{y}{2}\varphi\varphi\right|^2 = -\left|\frac{\partial W(\varphi)}{\partial\varphi}\right|^2$$

  that is, the superpotential evaluated at the scalar field value $F = \varphi$!
- The lagrangian then becomes

$$\begin{aligned}\mathcal{L}_{\mathrm{Wz}} &= (\partial_\mu\varphi)(\partial^\mu\varphi)^\dagger + \frac{i}{2}\psi\sigma^\mu\left(\partial_\mu\bar{\psi}\right) - \frac{i}{2}(\partial_\mu\psi)\sigma^\mu\bar{\psi} \\ &- |M|^2\varphi\varphi^\dagger - \frac{|y|^2}{4}\varphi\varphi\varphi^\dagger\varphi^\dagger \\ &- \left(\frac{M}{2}\psi\psi + \frac{M\cdot y}{2}\varphi\varphi\varphi^\dagger + \frac{y}{2}\varphi\psi\psi + \text{ h.c. }\right)\end{aligned}$$

Wess-Zumino model

The lagrangian can also be written as

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Wz}} &= \left(\partial_\mu \varphi\right)\left(\partial^\mu \varphi\right)^\dagger + \frac{i}{2}\psi\sigma^\mu\left(\partial_\mu \bar{\psi}\right) - \frac{i}{2}\left(\partial_\mu \psi\right)\sigma^\mu \bar{\psi} \\
&- |M|^2 \varphi\varphi^\dagger - \frac{|y|^2}{4}\varphi\varphi\varphi^\dagger\varphi^\dagger - \left(\frac{M}{2}\psi\psi + \frac{M \cdot y}{2}\varphi\varphi\varphi^\dagger + \frac{y}{2}\varphi\psi\psi + \text{ h.c. }\right) \\
&= \left(\partial_\mu \varphi\right)\left(\partial^\mu \varphi\right)^* + \frac{i}{2}\psi\sigma^\mu\left(\partial_\mu \bar{\psi}\right) - \frac{i}{2}\left(\partial_\mu \psi\right)\sigma^\mu \bar{\psi} \\
&- \left|\frac{\partial W(\varphi)}{\partial \varphi}\right|^2 - \frac{1}{2}\left(\frac{\partial^2 W(\varphi)}{\partial \varphi \partial \varphi}\right)\psi\psi - \frac{1}{2}\left(\frac{\partial^2 W^*(\varphi)}{\partial \varphi^* \partial \varphi^*}\right)\bar{\psi}\bar{\psi}
\end{aligned}
$$

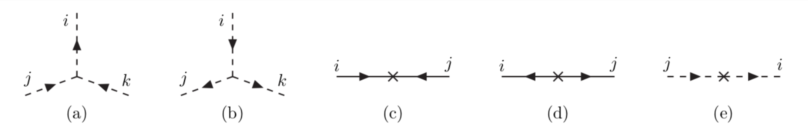
where, I remember,

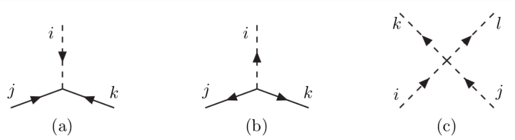
$$
W(\Phi) = \frac{1}{2}\,\Phi\Phi + \frac{1}{3!}\,\Phi\Phi\Phi
$$

Wess-Zumino model

$$\mathcal{L}_{\mathrm{Wz}} = (\partial_\mu \varphi)(\partial^\mu \varphi)^* + \frac{i}{2}\psi\sigma^\mu\left(\partial_\mu \bar{\psi}\right) - \frac{i}{2}(\partial_\mu \psi)\,\sigma^\mu \bar{\psi}$$
$$- |M|^2 \varphi\varphi^* - \frac{|y|^2}{4}\varphi\varphi\varphi^*\varphi^* - \left(\frac{M}{2}\psi\psi + \frac{M \cdot y}{2}\varphi\varphi\varphi^* + \frac{y}{2}\varphi\psi\psi + \text{ h.c. }\right)$$

Let us give a look at the interactions (diagrams taken from [2])





Wess Zumino model

Superpotential never renormalised in perturbation theory [1]
Condition for SUSY not spontaneously broken

Abelian gauge theories

Let us now introduce (abelian) gauge interactions

- Let us start with U(1) global symmetry

$$\Phi_i \to e^{iq_i\Lambda_i}\Phi_i$$

- The kinetic part of the lagrangian is always invariant

$$\mathcal{L}_K = \mathcal{L}_{WZ,D} = \int d^2\theta d^2\bar{\theta}\ \Phi^\dagger\Phi = \int d^2\theta - \frac{1}{4}\overline{DD}\Phi^\dagger\Phi$$

- The interaction part

$$\mathcal{L}_{int} = \mathcal{L}_{WZ,F} = \int d^2\theta\ \frac{1}{2}\sum_{ij}\Phi_i\Phi_j + \frac{1}{3!}\sum_{ijk}\Phi_i\Phi_j\Phi_k + \text{complex. conj.}$$

requires

$$m_{ij} = 0 \qquad \text{or} \qquad y_{ijk} = 0$$

whenever

$$q_i + q_j \neq 0 \qquad \text{or} \qquad q_i + q_j + q_k \neq 0$$

Abelian gauge theories

Promote to a local gauge symmetry

$$\Lambda \to \Lambda(x, \theta, \bar{\theta})$$

- Gauge parameter is now a supergauge field $\Lambda = \Lambda(x, \theta, \bar{\theta})$
- Promote derivatives to covariant derivatives $\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$
- We need $\Lambda(x, \theta, \bar{\theta})$ to be a left-chiral superfield if we want Φ' to be a left-chiral superfield (chain rule).
- Thus this causes a problem in the kinetic term because $\Lambda^\dagger$ is a right-chiral superfield hence obviously $\Phi'^\dagger \Phi' \neq \Phi^\dagger \Phi$
- The problem is analogous to the kinetc term in "normal" QFT when $\partial_u \varphi^\dagger \partial^\mu \varphi$ was not gauge invariant
- Solution: add a term that compensate the gauge for the non invariant terms

Abelian gauge theories

Recall from the previous talk that a gauge transformation on a vector field V reads

$$V \to V' = V + i(\Lambda^\dagger - \Lambda)$$

one can modify the kinetic term to

$$\Phi^\dagger e^V \Phi$$

so that it is invariant under the above gauge transformations

$$\Phi^\dagger e^V \Phi \to \Phi'^\dagger e^{V'} \Phi' = \Phi^\dagger e^{-i\Lambda^\dagger} e^{i\Lambda *} e^V e^{-i\Lambda} e^{i\Lambda} \Phi = \Phi^\dagger e^V \Phi$$

A particularly common gauge choice is the Wess-Zumino in which one ends with

$$V_{\mathrm{WZ}}(x, \theta, \bar{\theta}) = \theta \sigma^\mu \bar{\theta} v_\mu(x) + i(\theta\theta)\bar{\theta}\bar{\lambda}(x) - i(\bar{\theta}\bar{\theta})\theta\lambda(x) + \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})D(x)$$

Abelian gauge theories

Let us now define the two chiral fields

$$\mathcal{W}_\alpha = -\frac{1}{4}\overline{DD}D_\alpha V, \quad \overline{\mathcal{W}}_{\dot{\alpha}} = -\frac{1}{4}DD\bar{D}_{\dot{\alpha}}V$$

where V is a vector field.

The gauge-invariant dynamical term (equivalent to $F_{\mu\nu}F^{\mu\nu}$) is

$$[\mathcal{W}]_F + [\overline{\mathcal{W}}]_F = \int d^2\theta\ \mathcal{W}_\alpha \mathcal{W}^\alpha + \int d^2\bar{\theta}\ \overline{\mathcal{W}}_{\dot{\alpha}}\overline{\mathcal{W}}^{\dot{\alpha}}$$

The explicit derivation is quite long but we can make two checks to get more convinced

- Check that it is indeed gauge invariant
- Check that it contains the "normal" gauge strength field $F_{\mu\nu}F^{\mu\nu}$ after integrating out θ and $\bar{\theta}$

Abelian gauge theories

To see that it is gauge invariant remember that under a $U(1)$ transformation $V \to V + i\left(\Omega^\dagger - \Omega\right)$ and that $D_\alpha \Omega = 0, \bar{D}^{\dot{\alpha}}\Omega = 0$

$$\begin{aligned} \mathcal{W}_\alpha \to -\frac{1}{4}\overline{DD}D_\alpha\left[V + i\left(\Omega^\dagger - \Omega\right)\right] &= \mathcal{W}_\alpha + \frac{i}{4}\overline{DD}D_\alpha\Omega \\ &= \mathcal{W}_\alpha - \frac{i}{4}\bar{D}^{\dot{\beta}}\left\{\bar{D}_{\dot{\beta}}, D_\alpha\right\}\Omega \\ &= \mathcal{W}_\alpha + \frac{1}{2}\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu\bar{D}^{\dot{\beta}}\Omega \\ &= \mathcal{W}_\alpha \end{aligned}$$

where I also used that

$$\left\{\bar{D}_{\bar{\beta}}, D_\alpha\right\} = -2i\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu,$$

Abelian gauge theories

Remember that in the Wess-Zumino gauge the field expansion takes the form

$$V(y,\theta,\bar{\theta}) = \bar{\theta}\bar{\sigma}^{\mu}\theta A_{\mu}(y) + \bar{\theta}\bar{\theta}\theta\lambda(y) + \theta\theta\bar{\theta}\lambda^{\dagger}(y) +$$
$$\frac{1}{2}\theta\theta\theta\bar{\theta}\bar{\theta}\left[D(y) + i\theta_{\mu}A^{\mu}(y)\right]$$

Hence

$$\mathcal{W}_{\alpha}\left(y,\theta,\theta^{\dagger}\right) = \lambda_{a} + \theta_{\alpha}D + \frac{i}{2}\left(\sigma^{\mu}\bar{\sigma}^{\nu}\theta\right)_{\alpha}F_{\mu\nu} + i\theta\theta\left(\sigma^{\mu}\partial_{\mu}\lambda^{\dagger}\right)_{\alpha}$$

Finally

$$\frac{1}{4}\left[\mathcal{W}\mathcal{W}\right]_{F} + \frac{1}{4}\left[\overline{\mathcal{W}\mathcal{W}}\right]_{F} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\lambda^{\dagger}\bar{\sigma}^{\mu}\partial_{\mu}\lambda + \frac{1}{2}D^{2}$$

We recovered the desired term $F_{\mu\nu}F^{\mu\nu}$, but what are the other two terms?

Abelian gauge theories

The term

$$i\lambda^\dagger \bar{\sigma}^\mu \partial_\mu \lambda$$

is just the superpartner of the photon, the *photino*!

For the other term, we can show that it plays no physical role (as the F term in the chiral fields). To do this we need to spot all the D dependence in our lagrangian.

Remembering that up to now our lagrangian is

$$\mathcal{L} = \frac{1}{4}\left[\mathcal{W}\mathcal{W}\right]_F + \frac{1}{4}\left[\overline{\mathcal{W}\mathcal{W}}\right]_F + \left[\Phi^\dagger e^V \Phi\right]_D + [W(\Phi)]_F + [\bar{W}(\Phi^\dagger)]_F$$

once can note that the only other dependence on D is in $\left[\Phi^\dagger e^V \Phi\right]_D = \Phi^\dagger \Phi D$. Hence the equation of motions for D are

$$0 = \frac{\partial \mathcal{L}}{\partial D} = D + \Phi^\dagger \Phi$$

Abelian gauge theories

Putting all together we get the SUSY QED lagrangian

$$\mathcal{L} = \dots$$

To this we add another SUSY and supergauge invariant term $2[kV]_D = 2kD$ (e.o.m. is still algebraic). This term is called *Fayet-Iliopoulos* and it will play an important role in the spontaneous SUSY breaking (next talks)

$$\boxed{\mathcal{L}_{SQED} = \dots}$$

Non abelian gauge theories

Now extend to generic gauge theories, in particular SU(n). In spacetime:

- $n^2 - 1$ generators $T_1, \dots T_{n^2-1}$ (gauge fields)
- for $n = 3$ we have 8 fields (gluons)
- $[T_a, T_b] = i\, f_{abc}\, T_c$ where f_{abc} are called structure constants
- $U \in SU(n) \Rightarrow U = \exp(igA^a T^a)$
- Covariant derivatives in spacetime are
  $D_\mu = \partial_\mu - igA_\mu = \partial_\mu - igA_u^a T^a$

In spacetime, guided by the fact that for $U(1)$ we had

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \frac{i}{e}\,[D_\mu, D_\nu]$$

we *define* our field strength tensor for a general gauge symmetry via

$$F_{\mu\nu} \equiv \frac{i}{g}\,[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\,[A_\mu, A_\nu]$$

$\Rightarrow$ how to extend to superspace?

Non abelian gauge theories

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References

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