Building SUSY models II: using superfields

Seminar on Supersymmetry and its breaking

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Main points of the talk

- Apply the superspace formalism and show why it is useful to build SUSY theories
- Principles to construct SUSY lagrangians
- SUSY gauge theories: QED and QCD
- SUSY predictions: particles, interaction, masses, ...
- N=1 nonrenormalization theorem

How to build SUSY invariant lagrangians

We want our theory to be SUSY invariant, that is

$$\left(\epsilon\hat{Q}+\epsilon^\dagger\hat{Q}\right)S=\left(\epsilon\hat{Q}+\epsilon^\dagger\hat{Q}\right)\int dx^\mu\mathcal{L}=0$$

We know that this condition is met if, under a given transformation

$$\mathcal{L}\to\mathcal{L}+\partial_\mu f$$

Hence our goal is to build a lagrangin which transforms in this way, using **Chiral** ($\bar{D}_{\dot\alpha}\Phi=0$ or $D_\alpha\Phi^*=0$) and **Vector** ($V=V^*$) superfields

$$\Phi(x,\theta,\bar\theta)=\phi(x)+i\bar\theta\bar\sigma^\mu\theta\partial_\mu\phi(x)+\frac{1}{4}\theta\theta\bar\theta\bar\theta\partial_\mu\partial^\mu\phi(x)+\sqrt{2}\theta\psi(x)+$$

$$-\frac{i}{\sqrt{2}}\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu\psi(x)+\theta\theta\underbrace{F(x)}_{\text{F term}}$$

$$V\left(x,\theta,\bar\theta\right)=a+\theta\xi+\bar\theta\xi^*+\theta\theta b+\bar\theta\bar\theta b^*+\bar\theta\bar\sigma^\mu\theta A_\mu+$$

$$+\bar\theta\bar\theta\theta\left(\lambda-\frac{i}{2}\sigma^\mu\partial_\mu\xi^*\right)+\theta\theta\bar\theta\left(\lambda^*-\frac{i}{2}\sigma^\mu\partial_\mu\xi\right)+\theta\theta\bar\theta\bar\theta\underbrace{\left(\frac{1}{2}D+\frac{1}{4}\partial_\mu\partial^\mu a\right)}_{\text{D term}}$$

How to build SUSY invariant lagrangians

General superfield expansion

$S\left(x,\theta,\bar{\theta}\right)=a+\theta\xi+\bar{\theta}\chi^{\dagger}+\theta\theta b+\bar{\theta}\bar{\theta}c+\bar{\theta}\bar{\sigma}^{\mu}\theta v_{\mu}+\bar{\theta}\bar{\theta}\theta\eta+\theta\theta\bar{\theta}\zeta^{\dagger}+\theta\theta\bar{\theta}\bar{\theta}d$

For the chiral field $\Phi(x,\theta\bar{\theta})$ components one has the following transformation properties

$$\delta\phi=\sqrt{2}\zeta\psi$$

$$\delta\psi_{\alpha}=-\sqrt{2}F\zeta_{\alpha}-i\sqrt{2}\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\zeta}^{\dot{\alpha}}\partial_{\mu}\phi$$

$$\boxed{\delta_{\epsilon}F=-i\epsilon^{\dagger}\bar{\sigma}^{\mu}\partial_{\mu}\psi}$$

For the vector field $V(x,\theta,\bar{\theta})$ components one has

$$\sqrt{2}\delta_{\epsilon}a=\epsilon\xi+\epsilon^{\dagger}\xi^{\dagger}\qquad\sqrt{2}\delta_{\epsilon}\xi_{\alpha}=2\epsilon_{\alpha}b-\left(\sigma^{\mu}\epsilon^{\dagger}\right)_{\alpha}\left(A_{\mu}+i\partial_{\mu}a\right)$$

$$\sqrt{2}\delta_{\epsilon}b=\epsilon^{\dagger}\lambda^{\dagger}-i\epsilon^{\dagger}\sigma^{\mu}\partial_{\mu}\xi\qquad\sqrt{2}\delta_{\epsilon}A^{\mu}=i\epsilon\partial^{\mu}\xi-i\epsilon^{\dagger}\partial^{\mu}\xi^{\dagger}+\epsilon\sigma^{\mu}\lambda^{\dagger}-\epsilon^{\dagger}\bar{\sigma}^{\mu}\lambda$$

$$\sqrt{2}\delta_{\epsilon}\lambda_{\alpha}=\epsilon_{a}D+\frac{i}{2}\left(\sigma^{\mu}\sigma^{\nu}\epsilon\right)_{\alpha}\left(\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}\right)$$

$$\boxed{\sqrt{2}\delta_{\epsilon}D=-i\epsilon\sigma^{\mu}\partial_{\mu}\lambda^{\dagger}-i\epsilon^{\dagger}\bar{\sigma}^{\mu}\partial_{\mu}\lambda}$$

How to build SUSY invariant lagrangians

Idea $\rightarrow$ "select" the components of the fields that transforms as total derivatives

$$\mathcal{L}[\Phi, V] = \alpha\ [V]_D + \beta\ [\Phi]_F + \gamma\ [\Phi^*]_F$$

How can we "pick" only the terms we need? $\rightarrow$ Grassman integration

$$\int d^2\theta\ \theta^\alpha\, f(x, \bar{\theta}) = f(x, \bar{\theta})\, \delta_{\alpha,2}$$

$$\int d^2\theta d^2\bar{\theta}\ \bar{\theta}^\alpha \theta^\beta f(x) = f(x)\, \delta_{\alpha,2}\, \delta_{\beta,2}$$

Hence we are interested in

$$[\Phi]_F = \int d^2\theta \Phi + \text{total derivative}$$

$$[V]_D = \int d^2\theta d^2\bar{\theta} V(x, \theta, \bar{\theta}) + \text{total derivative}$$

How to build SUSY invariant lagrangians

Let us focus for a moment on the D term.

A vector field can be obtained from 2 chiral superfields by taking the product $\Phi^*\Phi$

$$\Phi(x,\theta,\bar\theta) = \phi(x) + i\bar\theta\bar\sigma^\mu\theta\partial_\mu\phi(x) + \frac{1}{4}\theta\theta\bar\theta\bar\theta\partial_\mu\partial^\mu\phi(x) + \sqrt{2}\theta\psi(x)$$
$$-\frac{i}{\sqrt{2}}\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu\psi(x) + \theta\theta F(x)$$

$$\Phi^{*i}\Phi_j = \phi^{*i}\phi_j + \sqrt{2}\theta\psi_j\phi^{*i} + \sqrt{2}\theta^\dagger\psi^{\dagger i}\phi_j + \theta\theta\phi^{*i}F_j + \theta^\dagger\theta^\dagger\phi_j F^{*i}$$
$$+\theta^\dagger\bar\sigma^\mu\theta\left[i\phi^{*i}\partial_\mu\phi_j - i\phi_j\partial_\mu\phi^{*i} - \psi^{\dagger i}\sigma_\mu\psi_j\right]$$
$$+\frac{i}{\sqrt{2}}\theta\theta\theta^\dagger\bar\sigma^\mu\left(\psi_j\partial_\mu\phi^{*i} - \partial_\mu\psi_j\phi^{*i}\right) + \sqrt{2}\theta\theta\theta^\dagger\psi^{\dagger i}F_j$$
$$+\frac{i}{\sqrt{2}}\theta^\dagger\theta^\dagger\theta\sigma^\mu\left(\psi^{\dagger i}\partial_\mu\phi_j - \partial_\mu\psi^{\dagger i}\phi_j\right) + \sqrt{2}\theta^\dagger\theta^\dagger\theta\psi_j F^{*i}$$
$$+\theta\theta\theta^\dagger\theta^\dagger\left[F^{*i}F_j - \frac{1}{2}\partial^\mu\phi^{*i}\partial_\mu\phi_j + \frac{1}{4}\phi^{*i}\partial^\mu\partial_\mu\phi_j + \frac{1}{4}\phi_j\partial^\mu\partial_\mu\phi^{*i}\right.$$
$$\left.+\frac{i}{2}\psi^{\dagger i}\bar\sigma^\mu\partial_\mu\psi_j + \frac{i}{2}\psi_j\sigma^\mu\partial_\mu\psi^{\dagger i}\right]$$

How to build SUSY invariant lagrangians

Now "select" the SUSY invariant component (D component)

$$[\Phi^*\Phi]_D = \int d^2\theta d^2\bar{\theta}\ \Phi^*(x,\theta,\bar{\theta})\Phi(x,\theta,\bar{\theta})$$

$$\Big[F^*F - \frac{1}{2}\partial^\mu\phi^{*i}\partial_\mu\phi_j + \frac{1}{4}\phi^{*i}\partial^\mu\partial_\mu\phi_j + \frac{1}{4}\phi_j\partial^\mu\partial_\mu\phi^{*i}$$

$$+\frac{i}{2}\psi^{\dagger i}\bar{\sigma}^\mu\partial_\mu\psi_j + \frac{i}{2}\psi_j\sigma^\mu\partial_\mu\psi^{\dagger i}\Big] =$$

$$= -\partial^\mu\phi^*\partial_\mu\phi + i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi + F^*F + \text{total derivative}$$

Hence

$$\boxed{S = \int dx^\mu \mathcal{L} = \int dx^\mu - \partial^\mu\phi^*\partial_\mu\phi + i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi + F^*F}$$

$\rightarrow$ **Free Wess-Zumino model!**

How to build SUSY invariant lagrangians

In order to add SUSY invariant interactions, let us recall the definitions of chiral superfields

$$\bar{D}_{\dot{\alpha}}\Phi = 0 \qquad D_\alpha \Phi^* = 0$$

Noote that any analytic function of chiral superfields is in turn a chiral superfield (power series expansion and product rule).
$\Rightarrow$ Write our chiral term of N fields as

$$W(\{\Phi_k\}) = \sum_i^N a_i \Phi_i + \sum_{i,j}^N \frac{1}{2!} m_{ij} \Phi_i \Phi_j + \sum_{i,j,k}^N \frac{1}{3!} y_{ijk} \Phi_i \Phi_j \Phi_k$$

Higher order terms are non-renormalisable.
Reality of the action requires us to take $W + W^*$

How to buld SUSY invariant lagrangians

$$\mathcal{L}_{WZ}\left(\{\Phi_i\},\{\Phi_i^*\}\right) = \mathcal{L}_{WZ,D} + \mathcal{L}_{WZ,F} =$$
$$= \left[\Phi^{*i}\Phi^i\right]_D + [W(\{\Phi_i\})]_F + [W^*(\{\Phi_i^*\})]_F =$$
$$= \boxed{\int d^2\theta\ \left(-\frac{1}{4}\overline{DD}\Phi^{*i}\Phi_i + W(\{\Phi_i\})\right) + \int d^2\bar{\theta}\ W^*(\{\Phi_i^*\})}$$

Equations of motion varying w.r.t. Φ_i and Φ_i^*

$$0 = -\frac{1}{4}\overline{DD}\Phi^{*i} + \frac{\delta W}{\delta \Phi_i}$$
$$0 = -\frac{1}{4}DD\Phi_i + \frac{\delta W^*}{\delta \Phi^{*i}}$$

Wess-Zumino model

We are now interested in studying better the components of Φ, hence let us focus on the case $i = j$ with $a_i = a, m_{ij} = m, y_{ijk} = y$ and the superpotential

$$W(\Phi) = \frac{1}{2} m\Phi\Phi + \frac{1}{3!}\Phi\Phi\Phi$$

where we dropped the linear term

$$\begin{gathered}\mathcal{L}(\Phi, \Phi^*) = \mathcal{L}_{WZ,D} + \mathcal{L}_{WZ,F} = \\ = F^\dagger F + (\partial_\mu \phi)(\partial^\mu \phi)^\dagger + \frac{i}{2}\psi\sigma^\mu \left(\partial_\mu \bar{\psi}\right) - \frac{i}{2}(\partial_\mu \psi)\sigma^\mu \tilde{\psi} + \\ -m\phi F - \frac{m}{2}(\psi\psi) - \frac{y}{2}\phi\phi F - \frac{y}{2}\phi(\psi\psi) + \text{ h.c.}\end{gathered}$$

E.o.m. for F (analogous for F^*) is

$$0 = \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu F)} - \frac{\partial \mathcal{L}}{\partial F} = -\frac{\partial \mathcal{L}}{\partial F} = -F^\dagger + m\phi + \frac{y}{2}\phi\phi$$

$\Rightarrow$ algebraic equation $\Rightarrow$ F, F^* are unphysical.

Wess-Zumino model

$$F^* = m\phi + \frac{y}{2}\phi\phi \qquad F = m\phi^* + \frac{y}{2}\phi\phi$$

One can rewrite the terms containing F, F^* as

$$F^\dagger F - \left(m\phi F + \frac{y}{2}\phi\phi F + hc\right) = -\left|m\phi + \frac{y}{2}\phi\phi\right|^2 = -\left|\frac{\partial W(\phi)}{\partial\phi}\right|^2$$

that is, the superpotential evaluated at the scalar field ϕ! The lagrangian then becomes

$$\mathcal{L}_{\mathrm{Wz}} = (\partial_\mu\varphi)(\partial^\mu\varphi)^\dagger + \frac{i}{2}\psi\sigma^\mu\left(\partial_\mu\bar{\psi}\right) - \frac{i}{2}(\partial_\mu\psi)\sigma^\mu\bar{\psi}$$
$$- |M|^2\varphi\varphi^\dagger - \frac{|y|^2}{4}\varphi\varphi\varphi^\dagger\varphi^\dagger - \left(\frac{M}{2}\psi\psi + \frac{M\cdot y}{2}\varphi\varphi\varphi^\dagger + \frac{y}{2}\varphi\psi\psi + \text{ h.c. }\right)$$

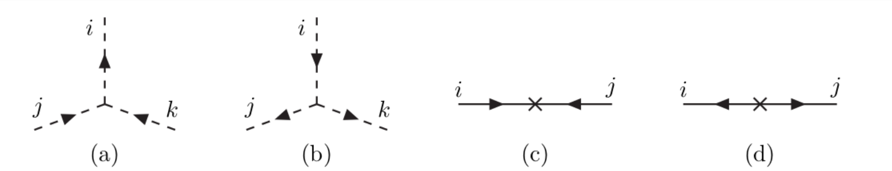
and one can prove that in the case of N fields it can be written as

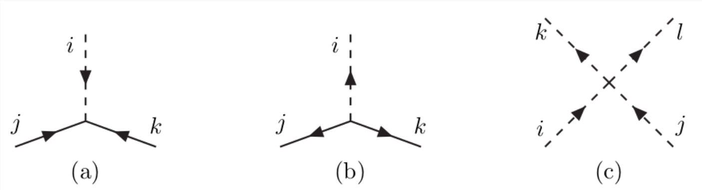
$$\mathcal{L}_{WZ} = (\partial_\mu\varphi_i)(\partial^\mu\varphi_i)^\dagger + \frac{i}{2}\psi_i\sigma^\mu\left(\partial_\mu\bar{\psi}_i\right) - \frac{i}{2}(\partial_\mu\psi_i)\sigma^\mu\bar{\psi}_i$$
$$-\sum_i\left|\frac{\partial W(\varphi_i)}{\partial\varphi_i}\right|^2 - \frac{1}{2}\left(\frac{\partial^2 W(\varphi_i)}{\partial\varphi_i\partial\varphi_j}\right)\psi_i\psi_j - \frac{1}{2}\left(\frac{\partial^2 W^\dagger(\varphi_i)}{\partial\varphi_i^*\partial\varphi_j^*}\right)\bar{\psi}_i\bar{\psi}_j$$

Wess-Zumino model

$$\mathcal{L}_{\mathrm{Wz}} = \left(\partial_\mu \varphi\right)\left(\partial^\mu \varphi\right)^\dagger + \frac{i}{2}\psi\sigma^\mu\left(\partial_\mu\bar{\psi}\right) - \frac{i}{2}\left(\partial_\mu\psi\right)\sigma^\mu\bar{\psi}$$
$$- |M|^2\varphi\varphi^\dagger - \frac{|y|^2}{4}\varphi\varphi\varphi^\dagger\varphi^\dagger - \left(\frac{M}{2}\psi\psi + \frac{M\cdot y}{2}\varphi\varphi\varphi^\dagger + \frac{y}{2}\varphi\psi\psi + \text{ h.c. }\right)$$

Let us give a look at the interactions





Abelian gauge theories

Let us now introduce (abelian) gauge interactions.

Let us start with U(1) global symmetry

$$\Phi_i \to e^{iq_i\Lambda_i}\Phi_i$$

The kinetic part of the lagrangian is always invariant

$$\mathcal{L}_K = \mathcal{L}_{WZ,D} = \int d^2\theta d^2\bar{\theta}\Phi^*\Phi = \int d^2\theta - \frac{1}{4}\overline{DD}\Phi^*\Phi$$

The interaction part

$$\mathcal{L}_{int} = \mathcal{L}_{WZ,F} = \int d^2\theta\frac{1}{2}\sum_{ij}\phi_i\phi_j + \frac{1}{3!}\sum_{ijk}\phi_i\phi_j\phi_k + \text{complex. conj.}$$

requires

$$m_{ij} = 0 \qquad \text{or} \qquad y_{ijk} = 0$$

whenever

$$q_i + q_j \neq 0 \qquad \text{or} \qquad q_i + q_j + q_k \neq 0$$

Abelian gauge theories

Promote to a local gauge symmetry

$$\Lambda \to \Lambda(x, \theta, \bar{\theta})$$

- Gauge parameter is now a supergauge field $\Lambda = \Lambda(x, \theta, \bar{\theta})$
- Promote derivatives to covariant derivatives $\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$
- We need $\Lambda(x, \theta, \bar{\theta})$ to be a left-chiral superfield if we want Φ' to be a left-chiral superfield (chain rule).
- Thus this causes a problem in the kinetic term because Λ^* is a right-chiral superfield hence obviously $\Phi'^*\Phi' \neq \Phi^*\Phi$
- The problem is analogous to the kinetc term in "normal" QFT when $\partial_u \phi^* \partial^\mu \phi$ was not gauge invariant
- Solution: add a term that compensate the gauge for the non invariant terms

Abelian gauge theories

Recall from the previous talk that a gauge transformation on a vector field V reads

$$V \to V' = V + i(\Lambda^* - \Lambda)$$

one can modify the kinetic term to

$$\Phi^* e^V \Phi$$

so that it is invariant under the above gauge transformations

$$\Phi^* e^V \Phi \to \Phi'^* e^{V'} \Phi' = \Phi^* e^{-i\Lambda^*} e^{i\Lambda *} e^V e^{-i\Lambda} e^{i\Lambda} \Phi = \Phi^* e^V \Phi$$

A particularly common gauge choice is the Wess-Zumino in which one ends with

$$V_{\mathrm{WZ}}(x, \theta, \bar{\theta}) = \theta \sigma^\mu \bar{\theta} v_\mu(x) + i(\theta\theta)\bar{\theta}\bar{\lambda}(x) - i(\bar{\theta}\bar{\theta})\theta\lambda(x) + \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})D(x)$$

Abelian gauge theories

Let us now define the two chiral fields

$$\mathcal{W}_\alpha = -\frac{1}{4}\overline{DD}D_\alpha V, \quad \overline{\mathcal{W}}_{\dot{\alpha}} = -\frac{1}{4}DD\bar{D}_{\dot{\alpha}}V$$

where V is a vector field.

The gauge-invariant dynamical term (equivalent to $F_{\mu\nu}F^{\mu\nu}$) is

$$[\mathcal{W}]_F + [\overline{\mathcal{W}}]_F = \int d^2\theta\ \mathcal{W}_\alpha \mathcal{W}^\alpha + \int d^2\bar{\theta}\ \overline{\mathcal{W}}_{\dot{\alpha}}\overline{\mathcal{W}}^{\dot{\alpha}}$$

The explicit derivation is quite long but we can make two checks to get more convinced

- Check that it is indeed gauge invariant
- Check that it contains the "normal" gauge strength field $F_{\mu\nu}F^{\mu\nu}$ after integrating out θ and $\bar{\theta}$

To see that it is gauge invariant remember that under a $U(1)$ transformation $V \to V + i\left(\Omega^{*} - \Omega\right)$ and that $D_\alpha \Omega = 0, \bar{D}^{\dot{\alpha}}\Omega = 0$

$$\begin{aligned}\mathcal{W}_\alpha \to -\frac{1}{4}\overline{DD}D_\alpha\left[V + i\left(\Omega^{*} - \Omega\right)\right] &= \mathcal{W}_\alpha + \frac{i}{4}\overline{DD}D_\alpha\Omega \\ &= \mathcal{W}_\alpha - \frac{i}{4}\bar{D}^{\dot{\beta}}\left\{\bar{D}_{\dot{\beta}}, D_\alpha\right\}\Omega \\ &= \mathcal{W}_\alpha + \frac{1}{2}\sigma^{\mu}_{\alpha\dot{\beta}}\partial_\mu\bar{D}^{\dot{\beta}}\Omega \\ &= \mathcal{W}_\alpha\end{aligned}$$

where I also used that

$$\left\{\bar{D}_{\bar{\beta}}, D_\alpha\right\} = -2i\sigma^{\mu}_{\alpha\dot{\beta}}\partial_\mu,$$

Abelian gauge theories

Remember that in the Wess-Zumino gauge the field expansion takes the form

$$V(y,\theta,\bar{\theta}) = \bar{\theta}\bar{\sigma}^{\mu}\theta A_{\mu}(y) + \bar{\theta}\bar{\theta}\theta\lambda(y) + \theta\theta\bar{\theta}\lambda^{\dagger}(y) + \\ \frac{1}{2}\theta\theta\theta\bar{\theta}\bar{\theta}\left[D(y) + i\theta_{\mu}A^{\mu}(y)\right]$$

Hence

$$\mathcal{W}_{\alpha}\left(y,\theta,\theta^{\dagger}\right) = \lambda_{a} + \theta_{\alpha}D + \frac{i}{2}\left(\sigma^{\mu}\bar{\sigma}^{\nu}\theta\right)_{\alpha}F_{\mu\nu} + i\theta\theta\left(\sigma^{\mu}\partial_{\mu}\lambda^{\dagger}\right)_{\alpha}$$

Finally

$$\frac{1}{4}\left[\mathcal{W}\mathcal{W}\right]_{F} + \frac{1}{4}\left[\overline{\mathcal{W}\mathcal{W}}\right]_{F} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\lambda^{\dagger}\bar{\sigma}^{\mu}\partial_{\mu}\lambda + \frac{1}{2}D^{2}$$

We recovered the desired term $F_{\mu\nu}F^{\mu\nu}$, but what are the other two terms?

Abelian gauge theories

The term

$$i\lambda^*\bar{\sigma}^\mu\partial_\mu\lambda$$

is just the superpartner of the photon, the *photino*!

For the other term, we can show that it plays no physical role (as the F term in the chiral fields). To do this we need to spot all the D dependence in our lagrangian.

Remembering that up to now our lagrangian is

$$\mathcal{L} = \frac{1}{4}\left[\mathcal{W}\mathcal{W}\right]_F + \frac{1}{4}\left[\overline{\mathcal{W}\mathcal{W}}\right]_F + \left[\Phi^* e^V \Phi\right]_D + \left[W(\Phi)\right]_F + \left[\bar{W}(\Phi^*)\right]_F$$

once can note that the only other dependence on D is in $\left[\Phi^* e^V \Phi\right]_D = \Phi^*\Phi D$. Hence the equation of motions for D are

$$0 = \frac{\partial \mathcal{L}}{\partial D} = D + \Phi^*\Phi$$

Abelian gauge theories

Putting all together we get the SUSY QED lagrangian

$$\mathcal{L} = \dots$$

To this we add another SUSY and supergauge invariant term $2[kV]_D = 2kD$ (e.o.m. is still algebraic). This term is called *Fayet-Iliopoulos* and it will play an important role in the spontaneous SUSY breaking (next talks)

$$\boxed{\mathcal{L}_{SQED} = \dots}$$

Non abelian gauge theories

Now extend to generic gauge theories, in particular SU(n). In spacetime:

- $n^2 - 1$ generators $T_1, \dots T_{n^2-1}$ (gauge fields)
- for $n = 3$ we have 8 fields (gluons)
- $[T_a, T_b] = i\, f_{abc}\, T_c$ where f_{abc} are called structure constants
- $U \in SU(n) \Rightarrow U = \exp(igA^a T^a)$
- Covariant derivatives in spacetime are
  $D_\mu = \partial_\mu - igA_\mu = \partial_\mu - igA^a_u T^a$

In spacetime, guided by the fact that for $U(1)$ we had

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \frac{i}{e}\,[D_\mu, D_\nu]$$

we *define* our field strength tensor for a general gauge symmetry via

$$F_{\mu\nu} \equiv \frac{i}{g}\,[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\,[A_\mu, A_\nu]$$

$\Rightarrow$ how to extend to superspace?

Non abelian gauge theories