

# Building SUSY models II: using superfields

Seminar on Supersymmetry and its breaking

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Matteo Zortea

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Coordinated by prof. Joerg Jaeckel

# Main points of the talk

- Apply the superspace formalism and show why it is useful to build SUSY theories
- Principles to construct SUSY lagrangians
- Feynman diagrams for supersymmetric theories
- SUSY gauge theories: QED and QCD
- SUSY predictions: particles, superparticles, interactions, ...

→ Search for a lagrangian that under a SUSY transformation

$$\mathcal{L} \rightarrow \mathcal{L} + \partial_\mu f$$

# How to build SUSY invariant lagrangians

- Left-chiral field expansion (right-chiral is hermitian conjugate)

$$\begin{aligned}\Phi(x, \theta, \bar{\theta}) = & \varphi(x) + i\bar{\theta}\bar{\sigma}^\mu\theta\partial_\mu\varphi(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial_\mu\partial^\mu\varphi(x) + \sqrt{2}\theta\psi(x) + \\ & -\frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi(x) + \theta\theta F(x)\end{aligned}$$

- Vector field expansion

$$\begin{aligned}V(x, \theta, \bar{\theta}) = & a + \theta\xi + \bar{\theta}\xi^\dagger + \theta\theta b + \bar{\theta}\bar{\theta}b^\dagger + \bar{\theta}\bar{\sigma}^\mu\theta A_\mu + \\ & + \bar{\theta}\bar{\theta}\theta\left(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\xi^\dagger\right) + \theta\theta\bar{\theta}\left(\lambda^\dagger - \frac{i}{2}\sigma^\mu\partial_\mu\xi\right) + \theta\theta\bar{\theta}\bar{\theta}\left(\frac{1}{2}D + \frac{1}{4}\partial_\mu\partial^\mu a\right)\end{aligned}$$

- They both carry no spinor, nor vector indices, the name is due to their particle content!

# How to build SUSY invariant lagrangians

- For the components of a chiral field  $\Phi(x, \theta, \bar{\theta})$  one has that

$$\begin{aligned}\delta_\epsilon \phi &= \epsilon \psi \\ \delta_\epsilon \psi_\alpha &= -i (\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi + \epsilon_\alpha F, \\ \delta_\epsilon F &= -i \epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \psi\end{aligned}$$

- For the components of a vector field  $V(x, \theta, \bar{\theta})$  one has

$$\begin{aligned}\sqrt{2} \delta_\epsilon a &= \epsilon \xi + \epsilon^\dagger \xi^\dagger & \sqrt{2} \delta_\epsilon \lambda_\alpha &= \epsilon_a D + \frac{i}{2} (\sigma^\mu \sigma^\nu \epsilon)_\alpha (\partial_\mu A_\nu - \partial_\nu A_\mu) \\ \sqrt{2} \delta_\epsilon b &= \epsilon^\dagger \lambda^\dagger - i \epsilon^\dagger \sigma^\mu \partial_\mu \xi & \sqrt{2} \delta_\epsilon \xi_\alpha &= 2 \epsilon_\alpha b - (\sigma^\mu \epsilon^\dagger)_\alpha (A_\mu + i \partial_\mu a) \\ \sqrt{2} \delta_\epsilon A^\mu &= i \epsilon \partial^\mu \xi - i \epsilon^\dagger \partial^\mu \xi^\dagger + \epsilon \sigma^\mu \lambda^\dagger - \epsilon^\dagger \bar{\sigma}^\mu \lambda \\ \sqrt{2} \delta_\epsilon D &= -i \epsilon \sigma^\mu \partial_\mu \lambda^\dagger - i \epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \lambda\end{aligned}$$

# How to build SUSY invariant lagrangians

- Idea  $\rightarrow$  "select" the components of the fields that transforms as total derivatives
- How can we "pick" only the terms we need?  $\rightarrow$  Grassman integration

$$\int d^2\theta \quad \theta^n f(x, \bar{\theta}) = f(x, \bar{\theta}) \delta_{n,2}$$
$$\int d^2\theta d^2\bar{\theta} \quad \bar{\theta}^m \theta^n f(x) = f(x) \delta_{m,2} \delta_{n,2}$$

- Our terms of interest

$$[\Phi]_F = \int d^2\theta \quad \Phi(x, \theta, \bar{\theta}) = F + \text{total derivative}$$
$$[V]_D = \int d^2\theta d^2\bar{\theta} \quad V(x, \theta, \bar{\theta}) = \frac{1}{2} D + \text{total derivative}$$

# How to build SUSY invariant lagrangians

Let us focus for a moment on the D term.

$$\begin{aligned}
 \Phi(x, \theta, \bar{\theta}) &= \varphi(x) + i\bar{\theta}\bar{\sigma}^\mu\theta\partial_\mu\varphi(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial_\mu\partial^\mu\varphi(x) + \sqrt{2}\theta\psi(x) \\
 &\quad - \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi(x) + \theta\theta F(x) \\
 \Phi^{\dagger i}\Phi_j &= \varphi^{*i}\varphi_j + \sqrt{2}\theta\psi_j\varphi^{*i} + \sqrt{2}\bar{\theta}\psi^{\dagger i}\varphi_j + \theta\theta\varphi^{*i}F_j + \bar{\theta}\bar{\theta}\varphi_jF^{\dagger i} \\
 &\quad + \bar{\theta}\bar{\sigma}^\mu\theta [i\varphi^{*i}\partial_\mu\varphi_j - i\varphi_j\partial_\mu\varphi^{*i} - \psi^{\dagger i}\sigma_\mu\psi_j] \\
 &\quad + \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu (\psi_j\partial_\mu\varphi^{*i} - \partial_\mu\psi_j\varphi^{*i}) + \sqrt{2}\theta\theta\bar{\theta}\psi^{\dagger i}F_j \\
 &\quad + \frac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}\theta\sigma^\mu (\psi^{\dagger i}\partial_\mu\varphi_j - \partial_\mu\psi^{\dagger i}\varphi_j) + \sqrt{2}\bar{\theta}\bar{\theta}\theta\psi_jF^{*i} \\
 &\quad + \theta\theta\bar{\theta}\bar{\theta} \left[ F^{*i}F_j - \frac{1}{2}\partial^\mu\varphi^{*i}\partial_\mu\varphi_j + \frac{1}{4}\varphi^{*i}\partial^\mu\partial_\mu\varphi_j + \frac{1}{4}\varphi_j\partial^\mu\partial_\mu\varphi^{*i} \right. \\
 &\quad \left. + \frac{i}{2}\psi^{\dagger i}\bar{\sigma}^\mu\partial_\mu\psi_j + \frac{i}{2}\psi_j\sigma^\mu\partial_\mu\psi^{\dagger i} \right]
 \end{aligned}$$

One can note that for  $i = j$  one has that  $(\Phi^\dagger\Phi)^\dagger = \Phi^\dagger\Phi \Rightarrow$  vector field!

# How to build SUSY invariant lagrangians

- Now "select" the SUSY invariant component (D component)

$$\begin{aligned} [\Phi^\dagger \Phi]_D &= \int d^2\theta d^2\bar{\theta} \Phi^\dagger(x, \theta, \bar{\theta}) \Phi(x, \theta, \bar{\theta}) \\ &\left[ F^* F - \frac{1}{2} \partial^\mu \varphi^* \partial_\mu \varphi + \frac{1}{4} \varphi^* \partial^\mu \partial_\mu \varphi + \frac{1}{4} \varphi \partial^\mu \partial_\mu \varphi^* \right. \\ &\quad \left. + \frac{i}{2} \psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + \frac{i}{2} \psi \sigma^\mu \partial_\mu \psi^\dagger \right] = \\ &= -\partial^\mu \varphi^* \partial_\mu \varphi + i \psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + F^* F + \text{total derivative} \end{aligned}$$

# How to build SUSY invariant lagrangians

- Now "select" the SUSY invariant component (D component)

$$\begin{aligned} [\Phi^\dagger \Phi]_D &= \int d^2\theta d^2\bar{\theta} \Phi^\dagger(x, \theta, \bar{\theta}) \Phi(x, \theta, \bar{\theta}) \\ &= \left[ F^* F - \frac{1}{2} \partial^\mu \varphi^* \partial_\mu \varphi + \frac{1}{4} \varphi^* \partial^\mu \partial_\mu \varphi + \frac{1}{4} \varphi \partial^\mu \partial_\mu \varphi^* \right. \\ &\quad \left. + \frac{i}{2} \psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + \frac{i}{2} \psi \sigma^\mu \partial_\mu \psi^\dagger \right] = \\ &= -\partial^\mu \varphi^* \partial_\mu \varphi + i \psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + F^* F + \text{total derivative} \end{aligned}$$

- Hence

$$S = \int dx^\mu \mathcal{L} = \int dx^\mu \left( -\partial^\mu \varphi^* \partial_\mu \varphi + i \psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + F^* F \right)$$

→ **Free Wess-Zumino model!**



# How to build SUSY invariant lagrangians

- In order to add SUSY invariant interactions, let us recall the definitions of chiral superfields

$$\bar{D}_{\dot{\alpha}}\Phi = 0 \quad D_{\alpha}\Phi^{\dagger} = 0$$

- Note that any analytic function of chiral superfields is in turn a chiral superfield (power series expansion and product rule).
- $\Rightarrow$  Write our chiral term of  $N$  fields as

$$W(\{\Phi_k\}) = \sum_i^N a_i \Phi_i + \sum_{i,j}^N \frac{1}{2!} m_{ij} \Phi_i \Phi_j + \sum_{i,j,k}^N \frac{1}{3!} y_{ijk} \Phi_i \Phi_j \Phi_k$$

- Higher order terms are non-renormalisable
- Reality of the action requires us to take  $W + W^*$
- $W$  is called *superpotential*

# How to build SUSY invariant lagrangians

The interacting lagrangian becomes

$$\begin{aligned}\mathcal{L}_{WZ}(\{\Phi_i\}, \{\Phi_i^\dagger\}) &= \mathcal{L}_{WZ,D} + \mathcal{L}_{WZ,F} = \\ &= [\Phi^{\dagger i} \Phi^i]_D + [W(\{\Phi_i\})]_F + [W^\dagger(\{\Phi_i^\dagger\})]_F = \\ &= \int d^2\theta \left( -\frac{1}{4} \overline{D} \overline{D} \Phi^{\dagger i} \Phi_i + W(\{\Phi_i\}) \right) + \int d^2\bar{\theta} W^\dagger(\{\Phi_i^\dagger\}) = \\ &= \int d^2\bar{\theta} \left( -\frac{1}{4} D D \Phi^{\dagger i} \Phi_i + W(\{\Phi_i\}) \right) + \int d^2\bar{\theta} W^\dagger(\{\Phi_i^\dagger\})\end{aligned}$$

where I used that

$$\int d^2\theta d^2\bar{\theta} \Phi^\dagger \Phi = - \int d^2\theta \frac{1}{4} \overline{D} \overline{D} \Phi^\dagger \Phi = - \int d^2\bar{\theta} \frac{1}{4} D D (\Phi^\dagger \Phi)$$

Equations of motion varying w.r.t.  $\Phi_i$  and  $\Phi_i^\dagger$

$$0 = -\frac{1}{4} \overline{D} \overline{D} \Phi^{\dagger i} + \frac{\delta W}{\delta \Phi_i} \quad 0 = -\frac{1}{4} D D \Phi_i + \frac{\delta W^\dagger}{\delta \Phi_i^\dagger}$$

# Wess-Zumino model

- We are now interested in studying better the components of  $\Phi$ , hence let us focus on the case  $i = j$  with  $a_i = a$ ,  $m_{ij} = m$ ,  $y_{ijk} = y$  and the superpotential

$$W(\Phi) = \frac{1}{2} m \Phi \Phi + \frac{1}{3!} y \Phi \Phi \Phi$$

where we dropped the linear term.

- The lagrangian becomes

$$\begin{aligned} \mathcal{L}(\Phi, \Phi^\dagger) &= \mathcal{L}_{WZ,D} + \mathcal{L}_{WZ,F} = \\ &= F^* F + (\partial_\mu \varphi) (\partial^\mu \varphi)^* + \frac{i}{2} \psi \sigma^\mu (\partial_\mu \psi^\dagger) - \frac{i}{2} (\partial_\mu \psi) \sigma^\mu \psi^\dagger + \\ &\quad - m \varphi F - \frac{m}{2} (\psi \psi) - \frac{y}{2} \varphi \varphi F - \frac{y}{2} \varphi (\psi \psi) + \text{h.c.} \end{aligned}$$

- E.o.m. for  $F$  (analogous for  $F^*$ ) is

$$0 = \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu F)} - \frac{\partial \mathcal{L}}{\partial F} = -\frac{\partial \mathcal{L}}{\partial F} = -F^* + m\varphi + \frac{y}{2} \varphi \varphi$$

- $\Rightarrow$  algebraic equation  $\Rightarrow F, F^*$  are unphysical.

$$F^* = m\varphi + \frac{y}{2}\varphi\varphi \quad F = m\varphi^* + \frac{y}{2}\varphi\varphi$$

- One can rewrite the terms containing  $F, F^*$  as

$$F^*F - \left( m\varphi F + \frac{y}{2}\varphi\varphi F + hc \right) = - \left| m\varphi + \frac{y}{2}\varphi\varphi \right|^2 = - \left| \frac{\partial W(\varphi)}{\partial \varphi} \right|^2$$

that is, the superpotential evaluated at the scalar field value  $F = \varphi$ !

- The lagrangian then becomes

$$\begin{aligned} \mathcal{L}_{\text{Wz}} = & (\partial_\mu \varphi) (\partial^\mu \varphi)^\dagger + \frac{i}{2} \psi \sigma^\mu (\partial_\mu \psi^\dagger) - \frac{i}{2} (\partial_\mu \psi) \sigma^\mu \psi^\dagger \\ & - |M|^2 \varphi \varphi^* - \frac{|y|^2}{4} \varphi \varphi \varphi^* \varphi^* \\ & - \left( \frac{M}{2} \psi \psi + \frac{M \cdot y}{2} \varphi \varphi \varphi^* + \frac{y}{2} \varphi \psi \psi + \text{h.c.} \right) \end{aligned}$$

The lagrangian can also be written as

$$\begin{aligned}
 \mathcal{L}_{\text{WZ}} &= (\partial_\mu \varphi) (\partial^\mu \varphi)^\dagger + \frac{i}{2} \psi \sigma^\mu (\partial_\mu \psi^\dagger) - \frac{i}{2} (\partial_\mu \psi) \sigma^\mu \psi^\dagger \\
 &\quad - |M|^2 \varphi \varphi^* - \frac{|y|^2}{4} \varphi \varphi \varphi^* \varphi^* - \left( \frac{M}{2} \psi \psi + \frac{M \cdot y}{2} \varphi \varphi \varphi^* + \frac{y}{2} \varphi \psi \psi + \text{h.c.} \right) \\
 &= (\partial_\mu \varphi) (\partial^\mu \varphi)^* + \frac{i}{2} \psi \sigma^\mu (\partial_\mu \psi^\dagger) - \frac{i}{2} (\partial_\mu \psi) \sigma^\mu \psi^\dagger \\
 &\quad - \left| \frac{\partial W(\varphi)}{\partial \varphi} \right|^2 - \frac{1}{2} \left( \frac{\partial^2 W(\varphi)}{\partial \varphi \partial \varphi} \right) \psi \psi - \frac{1}{2} \left( \frac{\partial^2 W^*(\varphi)}{\partial \varphi^* \partial \varphi^*} \right) \psi^\dagger \psi^\dagger
 \end{aligned}$$

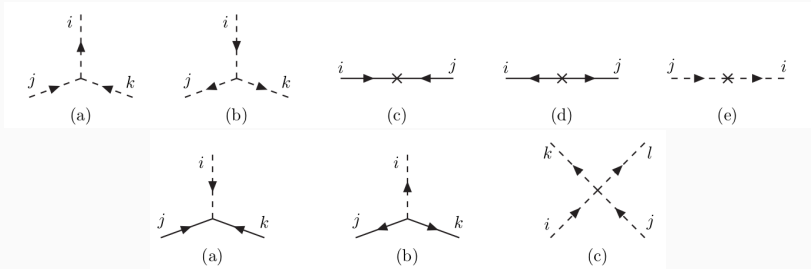
where, I remember,

$$W(\Phi) = \frac{1}{2} M \Phi \Phi + \frac{1}{3!} y \Phi \Phi \Phi$$

# Wess-Zumino model

$$\mathcal{L}_{\text{WZ}} = (\partial_\mu \varphi) (\partial^\mu \varphi)^* + \frac{i}{2} \psi \sigma^\mu (\partial_\mu \psi^\dagger) - \frac{i}{2} (\partial_\mu \psi) \sigma^\mu \psi^\dagger \\ - |M|^2 \varphi \varphi^* - \frac{|y|^2}{4} \varphi \varphi \varphi^* \varphi^* - \left( \frac{M}{2} \psi \psi + \frac{M \cdot y}{2} \varphi \varphi \varphi^* + \frac{y}{2} \varphi \psi \psi + \text{h.c.} \right)$$

Let us give a look at the interactions (diagrams taken from [3])



# Nonrenormalization theorem

One can check straightforwardly that the quadratic divergence in the boson's mass is cancelled!

One can prove that in general the following theorem holds

*The superpotential is not renormalised at any order in perturbation theory. Thus it might get affected by nonperturbative effects such as instantons*

This is the so called **N=1 nonrenormalization theorem**

# Abelian gauge theories

Let us now introduce (abelian) gauge interactions

- Let us start with U(1) global symmetry

$$\Phi_i \rightarrow e^{iq_i \Lambda_i} \Phi_i$$

- The kinetic part of the lagrangian is always invariant

$$\mathcal{L}_K = \mathcal{L}_{WZ,D} = \int d^2\theta d^2\bar{\theta} \Phi^\dagger \Phi = \int d^2\theta -\frac{1}{4} \overline{D D} \Phi^\dagger \Phi$$

- The interaction part

$$\mathcal{L}_{int} = \mathcal{L}_{WZ,F} = \int d^2\theta \frac{1}{2} \sum_{ij} m_{ij} \Phi_i \Phi_j + \frac{1}{3!} \sum_{ijk} y_{ijk} \Phi_i \Phi_j \Phi_k + \text{h.c.}$$

requires

$$m_{ij} = 0 \quad \text{or} \quad y_{ijk} = 0$$

whenever

$$q_i + q_j \neq 0 \quad \text{or} \quad q_i + q_j + q_k \neq 0$$



# Abelian gauge theories

Promote to a local gauge symmetry

$$\Lambda \rightarrow \Lambda(x, \theta, \bar{\theta})$$

- The gauge parameter is now a supergauge field  $\Lambda = \Lambda(x, \theta, \bar{\theta})$
- We need  $\Lambda(x, \theta, \bar{\theta})$  to be a left-chiral superfield if we want  $\Phi'$  to be a left-chiral superfield.
- Thus this causes a problem in the kinetic term because  $\Lambda^\dagger$  is a right-chiral superfield hence obviously  $\Phi'^\dagger \Phi' \neq \Phi^\dagger \Phi$
- The problem is analogous to the kinetic term in "normal" QFT when  $\partial_\mu \varphi^* \partial^\mu \varphi$  was not gauge invariant
- Solution: add a term that compensate the gauge for the non invariant terms

Recall from the previous talk that a gauge transformation on a vector field  $V$  reads

$$V \rightarrow V' = V + i(\Lambda^\dagger - \Lambda)$$

one can modify the kinetic term to

$$\Phi^\dagger e^V \Phi$$

so that it is invariant under the above gauge transformations

$$\Phi^\dagger e^V \Phi \rightarrow \Phi'^\dagger e^{V'} \Phi' = \Phi^\dagger e^{-i\Lambda^\dagger} e^{i\Lambda^*} e^V e^{-i\Lambda} e^{i\Lambda} \Phi = \Phi^\dagger e^V \Phi$$

# Abelian gauge theories

Let us now define the two chiral fields

$$\mathcal{W}_\alpha = -\frac{1}{4}\overline{D}\overline{D}D_\alpha V, \quad \overline{\mathcal{W}}_{\dot{\alpha}} = -\frac{1}{4}DD\bar{D}_{\dot{\alpha}} V$$

where  $V$  is a vector field.

The gauge-invariant dynamical term (equivalent to  $F_{\mu\nu}F^{\mu\nu}$ ) is

$$[\mathcal{W}\mathcal{W}]_F + [\overline{\mathcal{W}}\overline{\mathcal{W}}]_F = \int d^2\theta \mathcal{W}_\alpha \mathcal{W}^\alpha + \int d^2\bar{\theta} \overline{\mathcal{W}}_{\dot{\alpha}} \overline{\mathcal{W}}^{\dot{\alpha}}$$

The explicit derivation is quite long but we can make two checks to get more convinced

- Check that it is indeed gauge invariant
- Check that it contains the "normal" gauge strength field  $F_{\mu\nu}F^{\mu\nu}$  after integrating out  $\theta$  and  $\bar{\theta}$

# Abelian gauge theories

To see that it is gauge invariant remember that under a  $U(1)$  transformation  $V \rightarrow V + i(\Omega^\dagger - \Omega)$  and that  $D_\alpha \Omega = 0, \bar{D}^{\dot{\alpha}} \Omega = 0$

$$\begin{aligned}\mathcal{W}_\alpha &\rightarrow -\frac{1}{4}\overline{D}D D_\alpha [V + i(\Omega^\dagger - \Omega)] = \mathcal{W}_\alpha + \frac{i}{4}\overline{D}D D_\alpha \Omega \\ &= \mathcal{W}_\alpha - \frac{i}{4}\bar{D}^{\dot{\beta}} \left\{ \bar{D}_{\dot{\beta}}, D_\alpha \right\} \Omega \\ &= \mathcal{W}_\alpha + \frac{1}{2}\sigma_{\alpha\dot{\beta}}^\mu \partial_\mu \bar{D}^{\dot{\beta}} \Omega \\ &= \mathcal{W}_\alpha\end{aligned}$$

where I also used that

$$\{\bar{D}_{\dot{\beta}}, D_\alpha\} = -2i\sigma_{\alpha\dot{\beta}}^\mu \partial_\mu,$$

# Abelian gauge theories

Remember that in the Wess-Zumino gauge the field expansion takes the form

$$V(y, \theta, \bar{\theta}) = \bar{\theta} \bar{\sigma}^\mu \theta A_\mu(y) + \bar{\theta} \bar{\theta} \theta \lambda(y) + \theta \theta \bar{\theta} \lambda^\dagger(y) + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} [D(y) + i \theta_\mu A^\mu(y)]$$

Hence

$$\mathcal{W}_\alpha(y, \theta, \bar{\theta}) = \lambda_a + \theta_\alpha D + \frac{i}{2} (\sigma^\mu \bar{\sigma}^\nu \theta)_\alpha F_{\mu\nu} + i \theta \theta (\sigma^\mu \partial_\mu \lambda^\dagger)_\alpha$$

Finally

$$\frac{1}{4} [\mathcal{W}\mathcal{W}]_F + \frac{1}{4} [\overline{\mathcal{W}\mathcal{W}}]_F = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \lambda^\dagger \bar{\sigma}^\mu \partial_\mu \lambda + \frac{1}{2} D^2$$

We recovered the desired term  $F_{\mu\nu} F^{\mu\nu}$ , but what are the other two terms?

# Abelian gauge theories

The term

$$i\lambda^\dagger \bar{\sigma}^\mu \partial_\mu \lambda$$

is just the superpartner of the photon, the *photino*!

For the other term, we can show that it plays no physical role (as the  $F$  term in the chiral fields). To do this we need to spot all the  $D$  dependence in our lagrangian.

Remembering that up to now our lagrangian is

$$\mathcal{L} = \frac{1}{4} [\mathcal{W}\mathcal{W}]_F + \frac{1}{4} [\overline{\mathcal{W}\mathcal{W}}]_F + [\Phi^\dagger e^V \Phi]_D + [W(\Phi)]_F + [\bar{W}(\Phi^\dagger)]_F$$

once can note that the only other dependence on  $D$  is in

$[\Phi^\dagger e^V \Phi]_D = \Phi^\dagger \Phi D$ . Hence the equation of motions for  $D$  are

$$0 = \frac{\partial \mathcal{L}}{\partial D} = D + \Phi^\dagger \Phi$$

Putting all together we get the SUSY QED lagrangian

$$\mathcal{L} = [\Phi^{*i} e^{q_i V} \Phi_i]_D + ([W(\Phi_i)]_F + \text{c.c.}) + \frac{1}{4} ([\mathcal{W}^\alpha \mathcal{W}_\alpha]_F + \text{c.c.})$$

To this we add another SUSY and supergauge invariant term  $2[kV]_D = 2kD$  (e.o.m. is still algebraic). This term is called *Fayet-Iliopoulos* and it will play an important role in the spontaneous SUSY breaking (next talks)

$$\mathcal{L}_{SQED} = [\Phi^{*i} e^{q_i V} \Phi_i]_D + ([W(\Phi_i)]_F + \text{c.c.}) + \frac{1}{4} ([\mathcal{W}^\alpha \mathcal{W}_\alpha]_F + \text{c.c.}) - 2\kappa[V]_D$$

# Non abelian gauge theories

Now extend to generic gauge theories, in particular  $SU(n)$ . In spacetime:

- $n^2 - 1$  generators  $T_1, \dots, T_{n^2-1}$  (gauge fields)
- for  $n = 3$  we have 8 fields (gluons)
- $[T_a, T_b] = i f_{abc} T_c$  where  $f_{abc}$  are called structure constants
- $U \in SU(n) \Rightarrow U = \exp(igA^a T^a)$
- Covariant derivatives in spacetime are
$$D_\mu = \partial_\mu - igA_\mu = \partial_\mu - igA_\mu^a T^a$$

In spacetime, guided by the fact that for  $U(1)$  we had

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \frac{i}{e} [D_\mu, D_\nu]$$

we *define* our field strength tensor for a general gauge symmetry via

$$F_{\mu\nu} \equiv \frac{i}{g} [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig [A_\mu, A_\nu]$$

In superspace proceed analogously by "generalising" the definition



# Non abelian gauge theories

We note that

$$e^V \rightarrow e^{V+i(\Lambda^\dagger-\Lambda)}$$

Is a particular case of

$$e^V \rightarrow e^{i\Lambda^\dagger} e^V e^{-i\Lambda}$$

when the commutators vanish  $[\Lambda^\dagger, V] = 0 = [\Lambda, V]$ .

In fact

$$\begin{aligned} V &\rightarrow V + i(\Omega^\dagger - \Omega) - \frac{i}{2} [V, \Omega + \Omega^\dagger] + \\ &+ i \sum_{k=1}^{\infty} \frac{B_{2k}}{(2k)!} [V, [V, \dots [V, \Omega^\dagger - \Omega] \dots]] = \\ &= + i(\Omega^{a*} - \Omega^a) + g_a f^{abc} V^b (\Omega^{c*} + \Omega^c) \\ &- \frac{i}{3} g_a^2 f^{abc} f^{cd^b} V^b V^d (\Omega^c - \Omega^r) + \dots \end{aligned}$$

where  $B_n$  defined by  $\frac{x}{e^x-1} = \sum_{n=0}^{\infty} \frac{B_n}{n!} x^n$  are the Bernoulli numbers

# Non abelian gauge theories

We have to generalise also the kinetic term

$$\mathcal{W}_\alpha = -\frac{1}{4} \overline{D D} (e^{-V} D_\alpha e^V)$$

so that it is invariant under

$$\mathcal{W}_\alpha \rightarrow e^{i\Omega} \mathcal{W}_\alpha e^{-i\Omega}$$

The lagrangian then becomes

$$\mathcal{L} = \frac{1}{4} [\mathcal{W}^{a\alpha} \mathcal{W}_\alpha^a]_F + c.c. + \left[ \Phi^{*i} \left( e^{2g_a T^a V^a} \right)_i^j \Phi_j \right]_D + ([W(\Phi_i)]_F + c.c.).$$

Let us write it in components in the case of SU(3) (QCD) after eliminating  $F, D$  via e.o.m.

# NSQCD interactions

In case of QCD

$$\begin{aligned}\mathcal{L} = & (D_\mu \varphi_i)^* (D^\mu \varphi)_i + \frac{i}{2} \psi_i \sigma^\mu (D_\mu \psi^\dagger)_i - \frac{i}{2} (D_\mu \psi)_i \sigma^\mu \psi_i^\dagger \\ & - \frac{1}{4} F_{\mu\nu}^a (F^a)^{\mu\nu} + \frac{i}{2} \lambda^a \sigma^\mu (D_\mu \bar{\lambda})^a - \frac{i}{2} (D_\mu \lambda)^a \sigma^\mu \bar{\lambda}^a \\ & - \sqrt{2} i g \psi_i^\dagger \bar{\lambda}^a T_{ij}^a \varphi_j + \sqrt{2} i g \varphi_i^\dagger T_{ij}^a \psi_j \lambda^a \\ & - \frac{1}{2} \frac{\partial^2 W}{\partial \varphi_i \partial \varphi_j} \psi_i \psi_j - \frac{1}{2} \frac{\partial^2 W^\dagger}{\partial \varphi_i^* \partial \varphi_j^*} \psi_i^\dagger \psi_j^\dagger - V(\varphi_i, \varphi_j^*)\end{aligned}$$

where

$$V(\varphi_i, \varphi_j^*) = F_i^* F_i + \frac{1}{2} (D^a)^2 = \sum_i \left| \frac{\partial W}{\partial \varphi_i} \right|^2 + \frac{1}{2} \sum_a (g \varphi_i^* T_{ij}^a \varphi_j + k^a)^2$$

$$W(\varphi_i) = a_i \varphi_i + \frac{1}{2} m_{ij} \varphi_i \varphi_j + \frac{1}{3!} y_{ijk} \varphi_i \varphi_j \varphi_k$$

Where the gauge covariant derivatives are

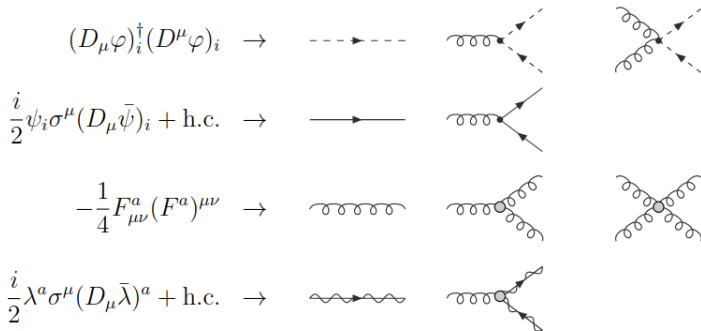
$$(D_\mu \varphi)_i = \partial_\mu \varphi_i + ig v_\mu^a T_{ij}^a \varphi_j$$

$$(D_\mu \psi)_i = \partial_\mu \psi_i + ig v_\mu^a T_{ij}^a \psi_j$$

$$(D_\mu \lambda)^a = \partial_\mu \lambda^a - gf^{abc} v_\mu^b \lambda^c$$

# SQCD interactions

This gives very cool interactions (diagrams from [5])



# SQCD interactions

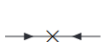
$$- \sqrt{2}ig \bar{\psi}_i \bar{\lambda}^a T_{ij}^a \varphi_j + \text{h.c.} \rightarrow$$



$$\frac{1}{2}(g \varphi_i^\dagger T_{ij}^a \varphi_j)^2 \rightarrow$$



$$-\frac{1}{2} \frac{\partial^2 W}{\partial \varphi_i \partial \varphi_j} \psi_i \psi_j + \text{h.c.} \rightarrow$$



$$\left| \frac{\partial W}{\partial \varphi_i} \right|^2 \rightarrow$$



Thank you for your attention

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