1 The concept of temperature

The purpose of this section is to introduce the concept of temperature in an intuitive manner and give a intuition on what higher/lower temperature means. In addition I will present the basic thermodynamics relations used in the following sections

Introduction to temperature: temperature as a quantity that describes equilibrium between two systems. Concepts of hot and cold.
Thermodynamic temperature: definition ($\frac{1}{T} = \frac{\partial S}{\partial E}$) and physical meaning. According to this definition what does $T_1 > T_2$ mean?
Thermodynamic basic concepts and relations between quantities (entropy, temperature, internal energy, ...)
Connection between thermodynamic definition of temperature and intuitive idea of temperature.
Comparison between different scales of temperature.

2 Two-level system

This is the simplest system that can exhibit negative temperatures, but still it contains many of the fundamental concepts. The purpose of the analysis is to introduce these concepts emphasising the physics thanks to the simple maths in the treatment.

Statistical analysis: show that this model admits negative temperatures
Physical meaning of negative temperature: why does this system admit negative temperature (upper bound on the energy)? What is the behaviour of the two-level system in the region of negative temperature? How do I bring it in this state and how does it evolve?
Ramsey's criteria: generalized criteria for a system to admit negative temperatures [4]
Negative temperatures are hotter than positive ones

3 Purcell-Pound experiment

Presentation of the historically fundamental experiment carried by Purcell and Pound in 1951 [3]

Spin temperature: definition of spin temperature according to [1]
Model: analytical approximated model (spin relaxation time $\ll$ spin-lattice interaction typical time) for the experiment
Experimental procedure

4 Boltzmann vs Gibbs

Boltzmann definition of entropy: definition and motivations $\longrightarrow$ admits negative temperatures
Gibbs definition of entropy: definition and motivations $\longrightarrow$ does not admit negative temperatures
Consistent thermostatistics forbids negative absolute temperatures [5]
Comment on "Consistent thermostatistics forbids negative absolute temperatures": show that the Boltzmann entropy is instead thermostatistically consistent referring e.g. to article [2]

5 Carnot cycles at negative temperatures

efficiency > 1: show that the efficiency formula of the Carnot cycle allows an efficiency > 1
confutation: show that the Carnot efficiency formula is not valid for negative temperatures by referring e.g. to [2]
Possible cycles to keep negative temperature: refer again to [2]

6 System analysis and simulation

Reproduce experimental data or simply simulate a system that admits negative temperatures?

References

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