Credit Risk – Case study

Risk Controlling and Organisation of Credit Risk

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ARIMA 2023/24

1. Theoretical Part

1.1 Concept and requirements of IFRS 9 focusing on PD, LGD and CCF.

International Financial Reporting Standard 9 is a global accounting standard that was developed to replace IAS 39, the previous standard on financial instruments, and it became effective on January 1st, 2018 with exception of insurance companies (2023). IFRS 9 is significant for its impact on how entities account for and report their financial assets and liabilities. Its key components include the classification and measurement of financial instruments, impairment of financial assets, and hedge accounting. The focus on risk parameters like Probability of Default (PD), Loss Given Default (LGD), Exposure at Default (EAD), and Credit Conversion Factor (CCF) is primarily found in the impairment component of the standard.

In general, LGD represents the expected loss that a lender would suffer in the event of default. It is usually expressed as a percentage of the total exposure at the time of default. Credit Conversion Factor (CCF) is used primarily for off-balance sheet exposures. It estimates the portion of a contingent liability that might become an actual liability. In the context of IFRS 9, CCF is relevant for calculating the exposure at default (EAD) of credit commitments that are not yet drawn.

IFRS 9's impairment model is based on the concept of Expected Credit Losses, which is a significant shift from the incurred loss model used in IAS 39. This change was made to address criticisms that the previous standard recognized credit losses too late in the cycle. Lifetime Expected Loss (LEL) is a crucial concept in IFRS 9, particularly in the context of the Expected Credit Loss (ECL) model.

ECL is defined as the weighted average of credit losses with the respective risks of a default occurring as the weights. LEL represents the present value of all expected credit losses over the expected life of the financial instrument. LEL is calculated until maturity of the financial instrument and discounted to t=0. The calculation of LEL involves several components, including the Probability of Default (PD), Loss Given Default (LGD), and Exposure at Default (EAD). Formula is as follows;

$$\mathrm{LEL} = \sum_{t=0}^{T} \frac{(1 - MPD_{t-1}) \times PD_t \times \mathrm{LGD}_t \times EAD_t}{(1+r)^t}$$

Where, the $(1 - MPD_{t-1}) \times PD_t$ is the 1-year Point-in-Time PD conditional to non-default in $(t-1)$. The LGD covers all time-depending loss rates including loss of interest in alignment with the IFRS9 definition and the EAD covers the expected nominal profile based on expected cash flows including commitments and prepayment according to IFRS9.

Probability of Default (PD) is a key parameter in assessing credit risk. PD refers to the likelihood over a given time horizon that a borrower will be unable to meet its obligation and default on its debt. Under IFRS 9, entities are required to assess and measure the credit risk of financial assets not only at the time of initial recognition but also throughout their lifetime. This ongoing assessment must factor in changes in credit risk, and PD is a crucial metric in this analysis.

1.2 Through-the-Cycle and Point-in-Time differences

Differences between TTC and PIT risk parameters can be categorized into, for example, three categories, from the perspective of; Risk Management, Overall bank steering, and Modelling Aspects. From the perspective of Risk Management, Through-the-Cycle characterizes by allowing for short or medium term assessment of the current credit risk among other things limited. Also, usage of TTC reduces risk cyclicity. On the other hand, Point-in-time allows for more accurate estimation of the current risk but there is also a higher risk-volatility.

Through-the-Cycle from the perspective of overall bank steering differs by low precision of the internal control system and allows for (own) capital backing stability when, Point-in-Time is the opposite; capital backing fluctuates more pro-cyclical and there is a higher precision of the internal control system. PIT is more aligned with IFRS 9's requirement for timely recognition of changes in credit risk, while TTC is often used for strategic planning and capital adequacy assessments.

Furthermore, a few main differences can be distinguished from the perspective of modelling. Through-the-Cycle characterises by being a structural component of default risk, probability of default of a rating class is independent of the economic cycle. Also, its applicable for long-term average rating as well as long term of debtors in their rating classes. On the other side, Point-in-Time is structural and cyclic component of default risk. It allows for estimation of the true, annual default probability and possess strong fluctuations between rating classes.

1.3 What are non-linear effects and how to incorporate them in the model?

Incorporating non-linear effects into a model is crucial for improving its accuracy and predictive power, especially in cases where linear models fail to capture the underlying patterns in the data.

Credit losses and credit risk tends to have non-linear effects occurrence, when the change in the outcome variable is not proportional to the change in the predictor variables. Which means that exemplary additional losses in an optimistic scenario may overpass the reduced losses in a comparable pessimistic scenario. To measure the Expected Credit Loss it is viewed very positively to incorporate many forward-looking scenarios as it is far more effective. Especially when the connection between credit losses that corresponds to various scenarios is non-linear. To conduct appropriate calculations, the weighted average of credit losses determined for those various scenarios chosen, should be multiplied by the likelihood of occurrence of each scenario. This allows for comprehensive understanding of specific scenarios influence on Lifetime Expected Loss. Formula is as follows;

$$LEL_{total} = \omega_{Baseline} \times LEL_{Baseline} + \omega_{Optimistic} \times LEL_{Optimistic} + \omega_{Pessimistic} \times LEL_{Pessimistic}$$

When incorporating non-linear effects, it is important to avoid overfitting, where the model becomes too complex and starts capturing noise instead of the underlying pattern. Model validation techniques like cross-validation, along with regular monitoring and updating of the model, are essential to ensure its reliability and accuracy.

2. Practical Part

2.1 Description of provided data

The dataset 'CaseStudyData_2023.xlsx' encompasses multiple time series variables for upcoming analysis. This set comprises the 'Default Rate', designated as the primary variable, alongside various macroeconomic variables poised to serve as explanatory factors in the ensuing regression study. Commencing with the 'Default Rate', a table outlining summary statistics is presented below.

Table 1. Default Rate summary statistics

Variable analysed: Default Rate

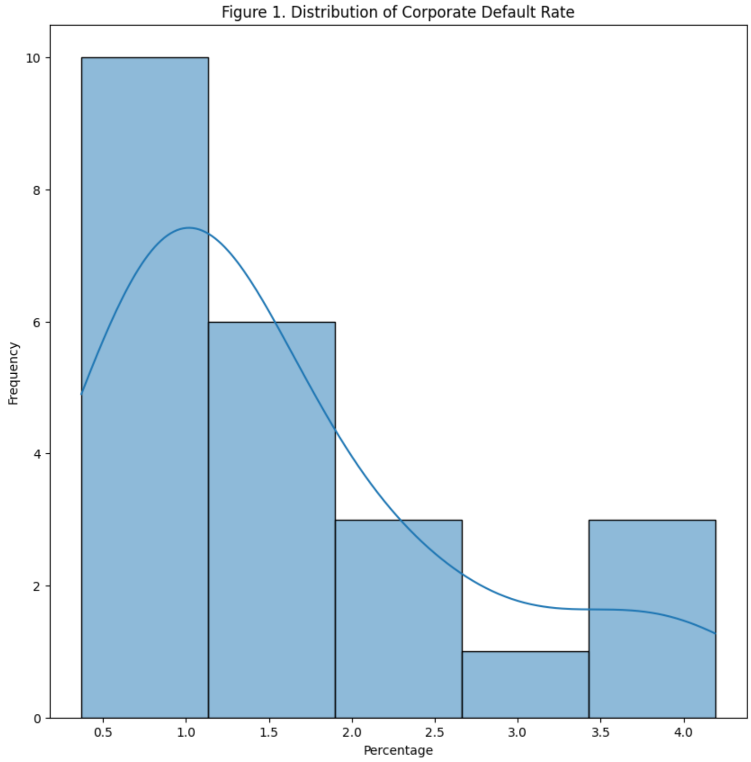
count	23.00	**max**	4.19
mean	1.5852	**Skewness**	1.2270
Std	1.0881	**Kurtosis**	0.5752
Min	0.37	**Variance**	1.1840
25%	0.82	**Standard Error**	0.2269
50%	1.21	**95% CI Lower**	1.1405
75%	2.01	**95% CI Upper**	2.0299

Source: author computation using Python libraries

The above table presents the main statistics like mean, standard deviation, minimum and maximum, quartiles, skewness, kurtosis, variance, standard error and both confidence intervals. It is important to notice that this dataset contains only 23 values. The central tendency of the data is around 1.59, minimum is 0.37 and maximum is 4.19. Standard Deviation (1. 0881) value suggests a moderate level of variability or dispersion around the mean. A standard deviation of about 1.09 indicates that, on average, the data points differ from the mean by this amount. A kurtosis value relatively far from 0 suggests that the data distribution has a shape somewhat unsimilar to a normal distribution in terms of its tails.

This positive skewness (1.2270) indicates that Default Rate dataset has a right-skewed distribution. In other words, there's a longer tail on the right side of the distribution, with more data points located on the lower value side. Which can be easily seen on the Figure 1 below.

Figure 1. Distribution of Corporate Default Rate



Source: author computation using Python libraries

Overall, these statistics suggest a dataset with moderate variability, a tendency to have values clustered around a mean of 1.59, and a right-skewed distribution. The confidence interval indicates a high probability range for the true mean of the underlying population.

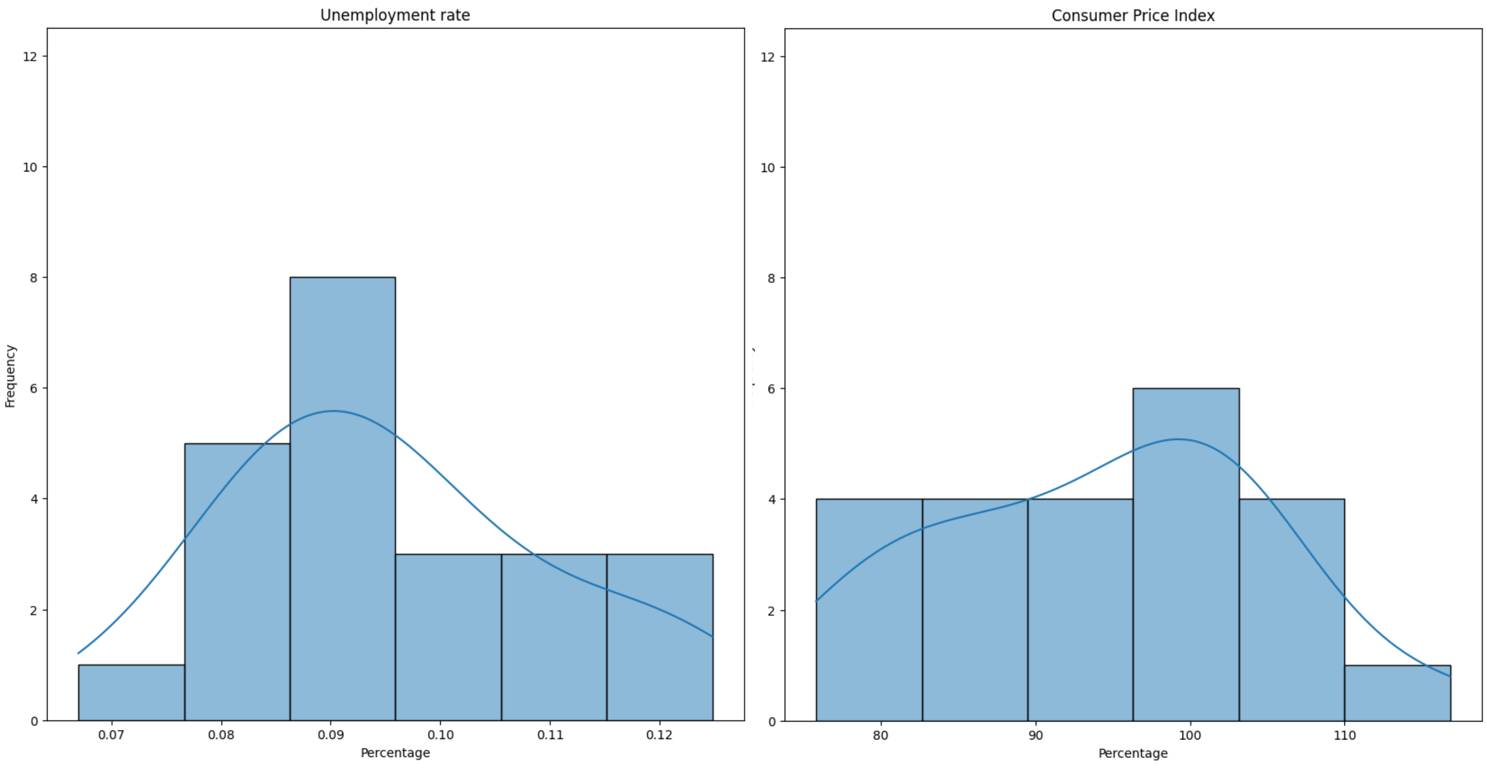
The Table 2 below shows the summary statistics of the data provided in the dataset and called historic data – macroeconomic variables which will be used further in model. The statistics presented below are mean, standard deviation, minimum, maximum and median value, as well as skewness, kurtosis, variance and standard error. It is important to notice that this dataset contains only 23 values. By looking at the skewness statistic it is possible to say that variables are relatively distributed close to the middle with the exception of GDP growth rate and CPI rate. This might be observed on the presented below plots of the distribution of data. All variables with the exception of EURIBOR 3-month, GDP growth rate and Yield structure are positive and their minimum value doesn't reach negative numbers.

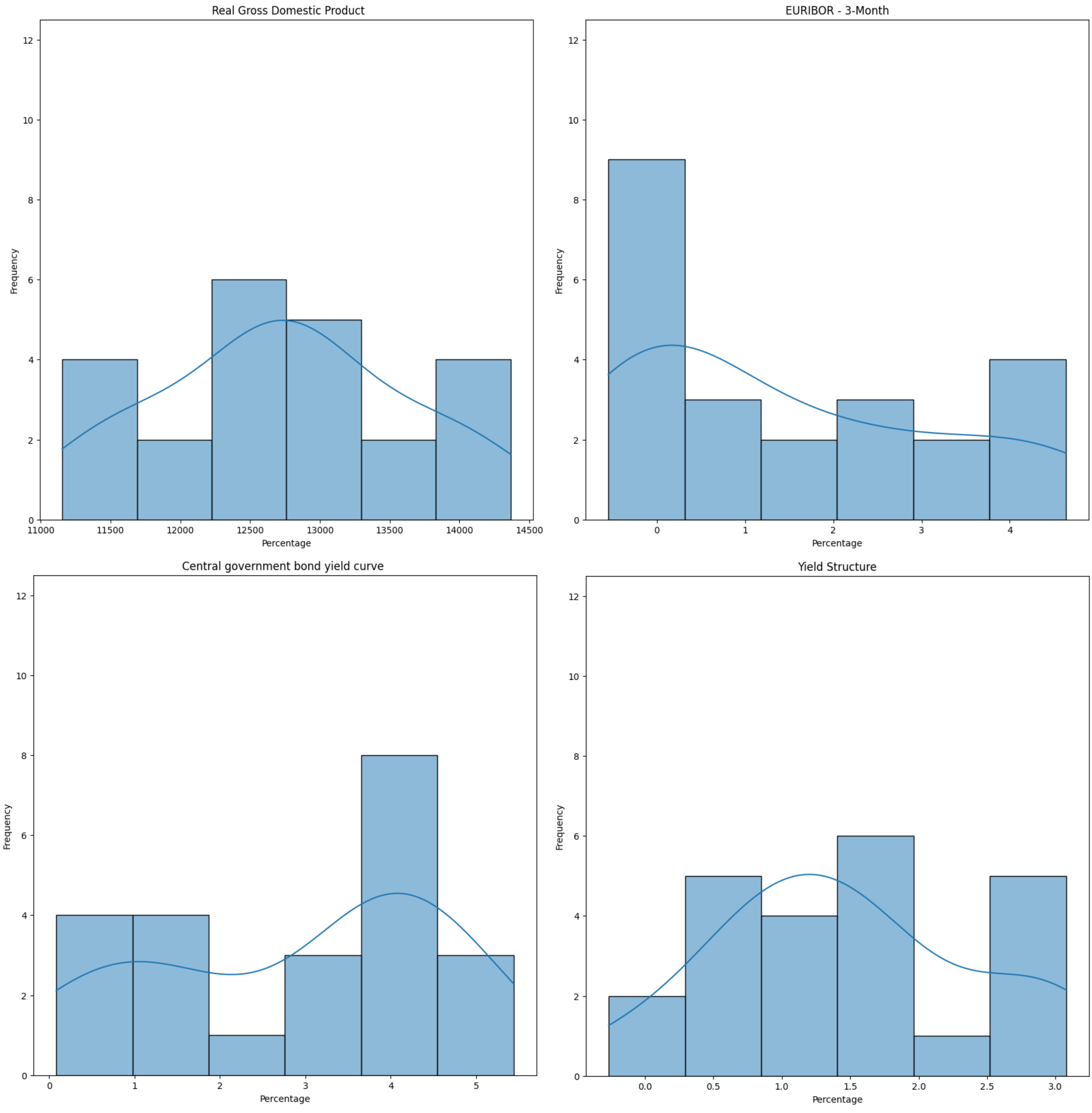
Table 2. Macroeconomic variables summary statistics

	Unemployment rate	Consumer Price Index	Real Gross Domestic Product	EURIBOR - 3-Month	Central gov. bond yield curve	Yield Structure
count	23	23	23	23	23	23
mean	0.0950	94.1514	12733.707	1.4394	2.9174	1.4780
std	0.0149	10.7648	892.6501	1.7870	1.7210	0.9455
min	0.0670	75.8075	11155.393	-0.5488	0.0836	-0.2645
median	0.0936	95.6675	12757.556	0.8110	3.6505	1.4400
max	0.1248	116.8359	14365.524	4.6342	5.4389	3.0774
Skewness	0.3822	-0.0112	0.0426	0.6204	-0.3898	0.2134
Kurtosis	-0.3295	-0.6305	-0.6658	-1.0762	-1.3268	-0.6578
Variance	0.0002	115.8800	796824.17	3.1933	2.9618	0.8940
Standard Error	0.0031	2.2446	186.1304	0.3726	0.3588	0.1971

Source: author computation using Python libraries

Figure 2. Macroeconomic variables distribution plots





Source: author computation using Python libraries

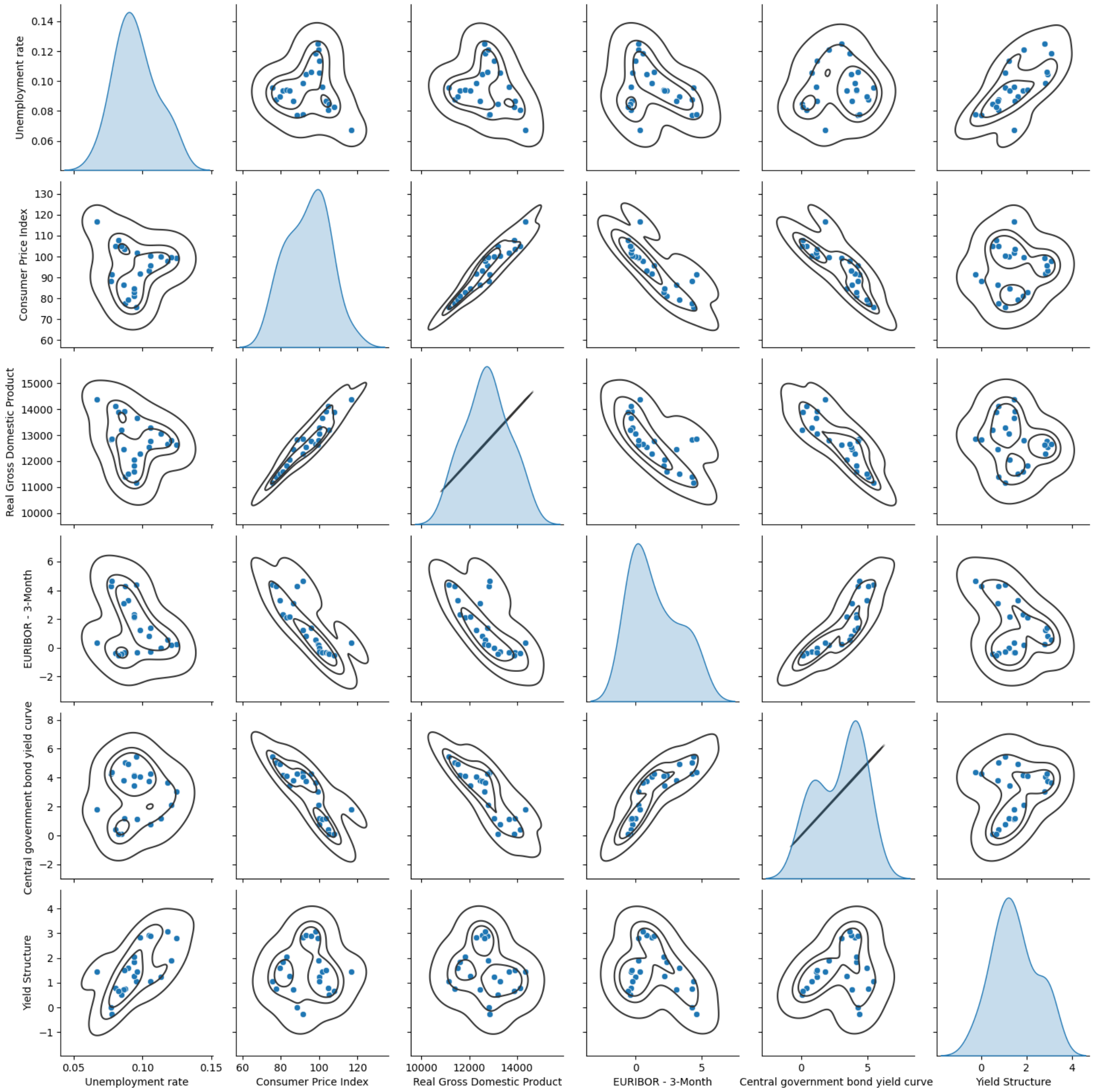
The table presented below shows the matrix of correlations between variables. It can be observed that there is a strong positive correlation between variables Real Gross Domestic Product and Consumer Price Index, as well as for Central Government bond yield curve and EURIBOR – 3 month. Rather strong negative correlation can be observed for the EURIBOR – 3 month and Consumer Price Index, also for the Central Government bond yield curve with Consumer Price Index and with Real Gross Domestic Product. Other variables tends to be no significantly correlated or both low positively and negatively correlated. This results can be graphically observed on the presented below Figure 3. The more narrower the cycles the stronger the positive correlation, the farer the figures are from smooth oval – the stronger the negative correlation.

Table 3. Macroeconomic variables correlation

	Unemployment rate	**Consumer Price Index**	**Real Gross Domestic Product**	**EURIBOR - 3-Month**	**Central government bond yield curve**	**Yield Structure**
Unemployment rate	100%	-3.62%	-24.06%	-30.79%	5.33%	67.90%
Consumer Price Index	-3.62%	100.00%	94.10%	-82.31%	-83.91%	2.84%
Real Gross Domestic Product	-24.06%	94.10%	100.00%	-71.01%	-83.45%	-17.69%
EURIBOR - 3-Month	-30.79%	-82.31%	-71.01%	100.00%	85.54%	-33.31%
Central gov. bond yield curve	5.33%	-83.91%	-83.45%	85.54%	100.00%	20.35%
Yield Structure	67.90%	2.84%	-17.69%	-33.31%	20.35%	100.00%

Source: author computation using Python libraries

Figure 4. Matrix plot of correlation



Source: author computation using Python libraries

2.2 Estimation of TTC Probability of Default model

The dataset provided for this study delineates the Global Corporate Average One-year Transition Rates Matrix covering the period from 1981 to 2022. This matrix segregates non-defaulting debtors into seven credit ratings: AAA, AA, A, BBB, BB, B, CCC, and adds a distinct classification for defaults, marked as D. Close examination of this matrix suggests its derivation using the cohort method, characterizing it as a discrete migration matrix. A notable characteristic of a discrete migration matrix is the occurrence of zero values in some migration probabilities, implying that the matrix tracks the year-end positions of borrowers rather than their transitions throughout the year. Furthermore, an analysis of the matrix's last column discloses the default probabilities for each credit rating, indicative of the 1-year Probability of Default (PDs). The formula to calculate such matrix is as follows[1];

$$P_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}$$

Resulting from the above, the probability of migrating *i* to *j* is the proportion of the number n_{ij} of all credits that migrated from *i* to j, to all credits which rating was *i* at *t-1*. The Cohort method used in calculation is the industry standard for count data but it also has drawbacks. Any rating change activity which occurs withing the period Δt is ignored[2]. In result, this type of discrete migration matrix become unreliable when credits change their rating class frequently.

Beginning with the one-year migration matrix, we can expand it to longer periods like 2 or even 10 years. However, this process is contingent on an important caveat: the discrete migration matrix is not inherently time-homogeneous. Therefore, assuming time-homogeneity becomes a prerequisite before we can utilize the formula needed to derive the discrete migration matrix for these extended periods.

$$\begin{pmatrix} p_{11} & \cdots & p1_n \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{pmatrix}^{i+j} = \begin{pmatrix} p_{11} & \cdots & p1_n \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{pmatrix}^{i\,(years)} \times \begin{pmatrix} p_{11} & \cdots & p1_n \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{pmatrix}^{j\,(years)}$$

The following 1-year migration matrix was scaled to 10 years using the above formula. After having a complete migration matrix for each year 1 to 10, it is possible to calculate the multi-year probability of default for each rating class which is basically presented in each migration matrix under column D representing default. A multi-year probability of Default for the rating class D is always 100% because it is assumed that its not possible to migrate to a higher, non-default class back again, once graded D.

Table 3. 1-year migration matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	87.09%	9.05%	0.52%	0.05%	0.10%	0.03%	0.05%	3.11%
AA	0.47%	87.42%	7.64%	0.46%	0.05%	0.06%	0.02%	3.88%
A	0.02%	1.54%	88.96%	4.86%	0.25%	0.10%	0.01%	4.26%
BBB	0.00%	0.08%	3.13%	86.95%	3.38%	0.40%	0.09%	5.97%
BB	0.01%	0.02%	0.06%	4.50%	78.30%	6.50%	0.53%	10.08%
B	0.00%	0.02%	0.06%	0.15%	4.50%	74.82%	4.81%	15.64%
CCC	0.00%	0.00%	0.08%	0.15%	0.46%	13.72%	44.74%	40.85%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Source: provided dataset

[1] U. Grzybowskaa, M. Karwański, A. Orłowski, „Examples of Migration Matrices Models and their Performance in Credit Risk Analysis", Proceedings of the 5th Symposium on Physics in Economics and Social Sciences, Warszawa, Poland, November 25–27, [2010], LINK: Accessed, 14.01.2023

[2] U. Grzybowskaa, M. Karwański, A. Orłowski, „Examples of Migration Matrices Models and their Performance in Credit Risk Analysis", Proceedings of the 5th Symposium on Physics in Economics and Social Sciences, Warszawa, Poland, November 25–27, [2010], LINK: Accessed, 14.01.2023

Table 4. 2-year migration matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	75.89%	15.80%	1.61%	0.16%	0.17%	0.07%	0.07%	6.23%
AA	0.82%	76.58%	13.49%	1.18%	0.12%	0.11%	0.03%	7.66%
A	0.04%	2.72%	79.41%	8.57%	0.59%	0.20%	0.02%	8.45%
BBB	0.00%	0.19%	5.51%	75.91%	5.61%	0.88%	0.16%	11.74%
BB	0.02%	0.04%	0.25%	7.45%	61.76%	10.04%	0.97%	19.48%
B	0.00%	0.03%	0.11%	0.46%	6.92%	56.93%	5.77%	29.77%
CCC	0.00%	0.00%	0.12%	0.24%	1.19%	16.43%	20.68%	61.33%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Source: author computation using Python libraries

Table 5. 3-year migration matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	66.17%	20.71%	3.04%	0.33%	0.23%	0.11%	0.08%	9.34%
AA	1.08%	67.23%	17.89%	2.04%	0.21%	0.16%	0.04%	11.35%
A	0.07%	3.61%	71.12%	11.35%	0.96%	0.31%	0.04%	12.55%
BBB	0.00%	0.31%	7.30%	66.53%	7.01%	1.36%	0.21%	17.28%
BB	0.02%	0.06%	0.50%	9.29%	49.06%	11.69%	1.25%	28.13%
B	0.00%	0.04%	0.16%	0.81%	8.02%	43.84%	5.36%	41.77%
CCC	0.00%	0.01%	0.14%	0.33%	1.77%	15.21%	10.05%	72.49%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Source: author computation using Python libraries

Table 6. 4-year migration matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	57.72%	24.14%	4.64%	0.58%	0.28%	0.14%	0.08%	12.42%
AA	1.26%	59.15%	21.13%	2.96%	0.32%	0.21%	0.04%	14.93%
A	0.09%	4.27%	63.90%	13.38%	1.33%	0.42%	0.06%	16.56%
BBB	0.01%	0.44%	8.61%	58.52%	7.82%	1.77%	0.26%	22.58%
BB	0.02%	0.08%	0.78%	10.33%	39.26%	12.15%	1.39%	35.99%
B	0.00%	0.05%	0.21%	1.14%	8.31%	34.06%	4.55%	51.68%
CCC	0.00%	0.01%	0.16%	0.41%	2.13%	12.88%	5.24%	79.18%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Source: author computation using Python libraries

Table 7. 5-year migration matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	50.38%	26.40%	6.29%	0.88%	0.33%	0.17%	0.08%	15.47%
AA	1.38%	52.15%	23.41%	3.89%	0.45%	0.25%	0.05%	18.43%
A	0.11%	4.74%	57.59%	14.82%	1.67%	0.53%	0.07%	20.47%
BBB	0.01%	0.57%	9.53%	51.66%	8.20%	2.11%	0.30%	27.63%
BB	0.02%	0.10%	1.05%	10.80%	31.65%	11.87%	1.42%	43.07%
B	0.00%	0.06%	0.25%	1.44%	8.10%	26.65%	3.72%	59.78%
CCC	0.00%	0.02%	0.17%	0.49%	2.29%	10.49%	2.97%	83.58%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Source: author computation using Python libraries

Table 8. 6-year migration matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	44.01%	27.73%	7.90%	1.23%	0.38%	0.20%	0.08%	18.47%
AA	1.45%	46.08%	24.94%	4.78%	0.58%	0.29%	0.05%	21.83%
A	0.13%	5.05%	52.06%	15.79%	1.98%	0.63%	0.09%	24.27%
BBB	0.01%	0.69%	10.14%	45.75%	8.29%	2.37%	0.32%	32.42%
BB	0.03%	0.13%	1.31%	10.89%	25.69%	11.18%	1.39%	49.39%
B	0.00%	0.06%	0.30%	1.67%	7.61%	20.99%	2.99%	66.38%
CCC	0.00%	0.02%	0.17%	0.55%	2.29%	8.41%	1.85%	86.70%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Source: author computation using Python libraries

Table 9. 7-year migration matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	38.46%	28.35%	9.42%	1.62%	0.42%	0.23%	0.08%	21.43%
AA	1.48%	40.80%	25.87%	5.61%	0.71%	0.34%	0.06%	25.13%
A	0.15%	5.24%	47.20%	16.37%	2.25%	0.73%	0.10%	27.97%
BBB	0.02%	0.80%	10.51%	40.66%	8.17%	2.55%	0.35%	36.95%
BB	0.03%	0.15%	1.54%	10.71%	21.00%	10.27%	1.30%	55.01%
B	0.00%	0.07%	0.34%	1.85%	6.97%	16.61%	2.39%	71.77%
CCC	0.00%	0.02%	0.18%	0.61%	2.20%	6.70%	1.24%	89.04%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Source: author computation using Python libraries

Table 10. 8-year migration matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	33.63%	28.41%	10.79%	2.04%	0.47%	0.25%	0.07%	24.33%
AA	1.49%	36.21%	26.31%	6.35%	0.85%	0.38%	0.06%	28.35%
A	0.16%	5.33%	42.90%	16.65%	2.47%	0.82%	0.11%	31.55%
BBB	0.02%	0.89%	10.69%	36.24%	7.92%	2.66%	0.36%	41.22%
BB	0.03%	0.17%	1.74%	10.35%	17.27%	9.27%	1.20%	59.98%
B	0.00%	0.07%	0.38%	1.96%	6.28%	13.22%	1.91%	76.17%
CCC	0.00%	0.03%	0.19%	0.65%	2.05%	5.33%	0.89%	90.87%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Source: author computation using Python libraries

Table 11. 9-year migration matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	29.42%	28.05%	12.01%	2.47%	0.53%	0.28%	0.07%	27.18%
AA	1.47%	32.20%	26.38%	7.01%	0.98%	0.42%	0.07%	31.47%
A	0.18%	5.35%	39.10%	16.70%	2.64%	0.90%	0.12%	35.00%
BBB	0.03%	0.98%	10.72%	32.39%	7.57%	2.71%	0.36%	45.23%
BB	0.03%	0.19%	1.90%	9.88%	14.30%	8.27%	1.08%	64.36%
B	0.01%	0.07%	0.42%	2.03%	5.59%	10.57%	1.52%	79.79%
CCC	0.00%	0.03%	0.20%	0.68%	1.87%	4.24%	0.67%	92.32%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Source: author computation using Python libraries

Table 12. 10-year migration matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	25.76%	27.37%	13.06%	2.90%	0.58%	0.30%	0.07%	29.97%
AA	1.44%	28.69%	26.16%	7.57%	1.11%	0.46%	0.07%	34.50%
A	0.19%	5.31%	35.72%	16.57%	2.78%	0.97%	0.13%	38.34%
BBB	0.03%	1.05%	10.63%	29.04%	7.17%	2.71%	0.36%	49.00%
BB	0.02%	0.21%	2.03%	9.34%	11.91%	7.31%	0.97%	68.21%
B	0.01%	0.08%	0.45%	2.06%	4.93%	8.49%	1.22%	82.77%
CCC	0.00%	0.03%	0.20%	0.69%	1.68%	3.39%	0.51%	93.49%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Source: author computation using Python libraries

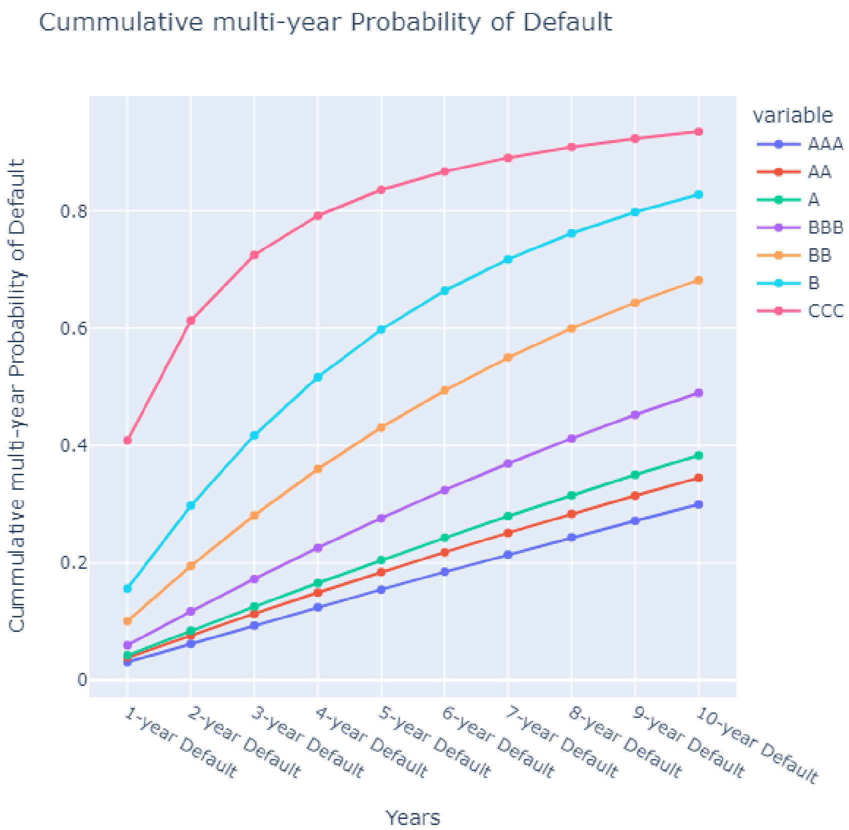
As mentioned before, using data form columns D from each year migration matrix, the below cumulative multi-year Probability of Default table is created. It can be seen, especially on the below Figure 4 plot of the probabilities that they possess a tendency of monotonic increase and that they are non-overlapping. Left side Y axis represents the percentage (PD) as proportion of 1.

Table 13. Cumulative multi-year Probability of Default

	AAA	AA	A	BBB	BB	B	CCC
1-year Default	3.11%	3.88%	4.26%	5.97%	10.08%	15.64%	40.85%
2-year Default	6.23%	7.66%	8.45%	11.74%	19.48%	29.77%	61.33%
3-year Default	9.34%	11.35%	12.55%	17.28%	28.13%	41.77%	72.49%
4-year Default	12.42%	14.93%	16.56%	22.58%	35.99%	51.68%	79.18%
5-year Default	15.47%	18.43%	20.47%	27.63%	43.07%	59.78%	83.58%
6-year Default	18.47%	21.83%	24.27%	32.42%	49.39%	66.38%	86.70%
7-year Default	21.43%	25.13%	27.97%	36.95%	55.01%	71.77%	89.04%
8-year Default	24.33%	28.35%	31.55%	41.22%	59.98%	76.17%	90.87%
9-year Default	27.18%	31.47%	35.00%	45.23%	64.36%	79.79%	92.32%
10-year Default	29.97%	34.50%	38.34%	49.00%	68.21%	82.77%	93.49%

Source: author computation using Python libraries

Figure 4. Cumulative multi-year Probability of Default plot



Source: author computation using Python libraries

A further step that can be taken is the calculation the Marginal Probabilities of Default using the Cumulative multi-year PDs (MPD) obtained before. This will allow, and also simplify, the calculation of Conditional Probabilities of Default. The formulas for the mentioned calculations are as follows:

$$PD_t^{Marginal} = MPD_t - MPD_{t-1}$$

$$PD_t^{Conditional} = \frac{PD_t^{Marginal}}{1 - MPD_{t-1}}$$

Table 14. Through-the-Cycle Marginal PD

	AAA	AA	A	BBB	BB	B	CCC
1-year Default	3.11%	3.88%	4.26%	5.97%	10.08%	15.64%	40.85%
2-year Default	3.12%	3.78%	4.19%	5.77%	9.40%	14.13%	20.48%
3-year Default	3.11%	3.68%	4.10%	5.54%	8.65%	11.99%	11.16%
4-year Default	3.08%	3.59%	4.01%	5.30%	7.86%	9.91%	6.69%
5-year Default	3.05%	3.49%	3.91%	5.05%	7.08%	8.10%	4.40%
6-year Default	3.00%	3.40%	3.81%	4.79%	6.32%	6.60%	3.12%
7-year Default	2.96%	3.31%	3.69%	4.53%	5.62%	5.39%	2.34%
8-year Default	2.90%	3.21%	3.58%	4.27%	4.97%	4.40%	1.82%
9-year Default	2.85%	3.12%	3.46%	4.01%	4.38%	3.62%	1.45%
10-year Default	2.79%	3.03%	3.33%	3.77%	3.86%	2.98%	1.17%

Source: author computation using Python libraries

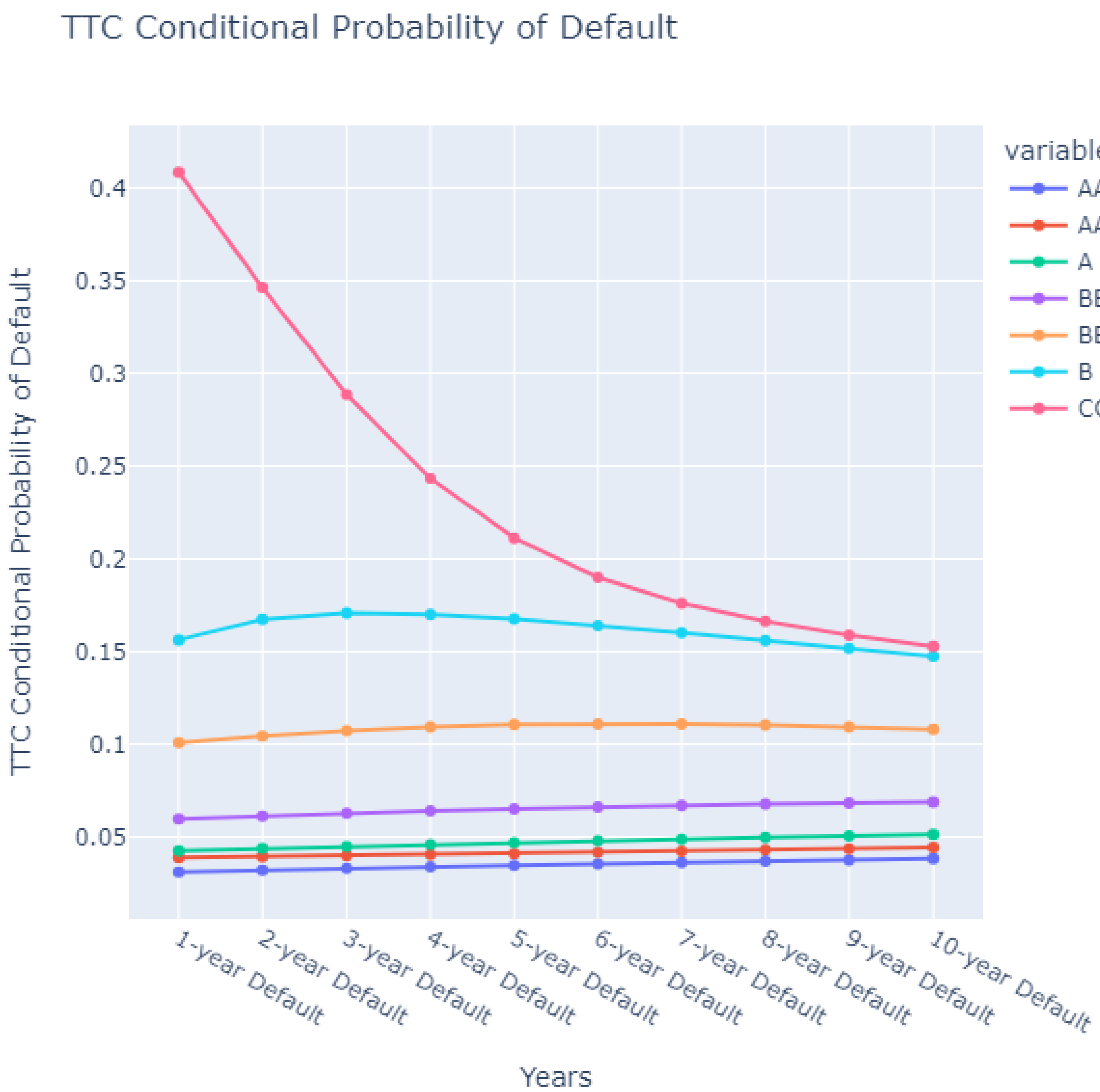
Table 15. Through-the-Cycle Conditional PD

	AAA	AA	A	BBB	BB	B	CCC
1-year Default	3.11%	3.88%	4.26%	5.97%	10.08%	15.64%	40.85%
2-year Default	3.22%	3.93%	4.37%	6.13%	10.45%	16.75%	34.63%
3-year Default	3.31%	3.99%	4.48%	6.28%	10.74%	17.08%	28.85%
4-year Default	3.40%	4.05%	4.59%	6.41%	10.94%	17.02%	24.31%
5-year Default	3.48%	4.11%	4.69%	6.52%	11.06%	16.77%	21.13%
6-year Default	3.55%	4.17%	4.79%	6.62%	11.11%	16.42%	19.02%
7-year Default	3.63%	4.23%	4.88%	6.70%	11.10%	16.02%	17.61%
8-year Default	3.70%	4.29%	4.97%	6.77%	11.04%	15.60%	16.63%
9-year Default	3.76%	4.36%	5.05%	6.83%	10.94%	15.17%	15.89%
10-year Default	3.83%	4.42%	5.13%	6.88%	10.82%	14.75%	15.29%

Source: author computation using Python libraries

The below graph represents the TTC Conditional Probability of Default. It can be observed that the CCC rating has constantly decreasing it's probably of default with the relatively high starting point. It is due to being the last non-default rating class before the actual default and with the time, it can be assumed that less and less CCC rated credits are left in the pool. All other rating classes possess a very slight upward trend. All PDs are non-overlapping.

Figure 4. Through-the-Cycle Conditional Probability of Default plot



Source: author computation using Python libraries

2.3 Estimation of PIT Probability of Default model

The further presented attempt to create a macro economic Point-in-Time PD model will be based on the assumption of having Default Rate as a target variable and the usage of Historical Macroeconomic Data as explanatory variables. Both Default Rate and Macroeconomic variables will be discussed.

First step in preparing the Default Rate data for modelling is to ensure that predicted DR are always between the 0 and 1 to be applicable for use in a regression. In order to do that, Log Odds formula is used and results are presented in the below Table 16. For purpose of calculation, DR are presented in decimal numbers.

$$\widehat{DR_{t+h}} = \text{logit}^{-1} = \frac{\exp\left(\ln\left(\frac{DR_t}{1 - DR_t}\right)\right)}{1 + \exp\left(\ln\left(\frac{DR_t}{1 - DR_t}\right)\right)}$$

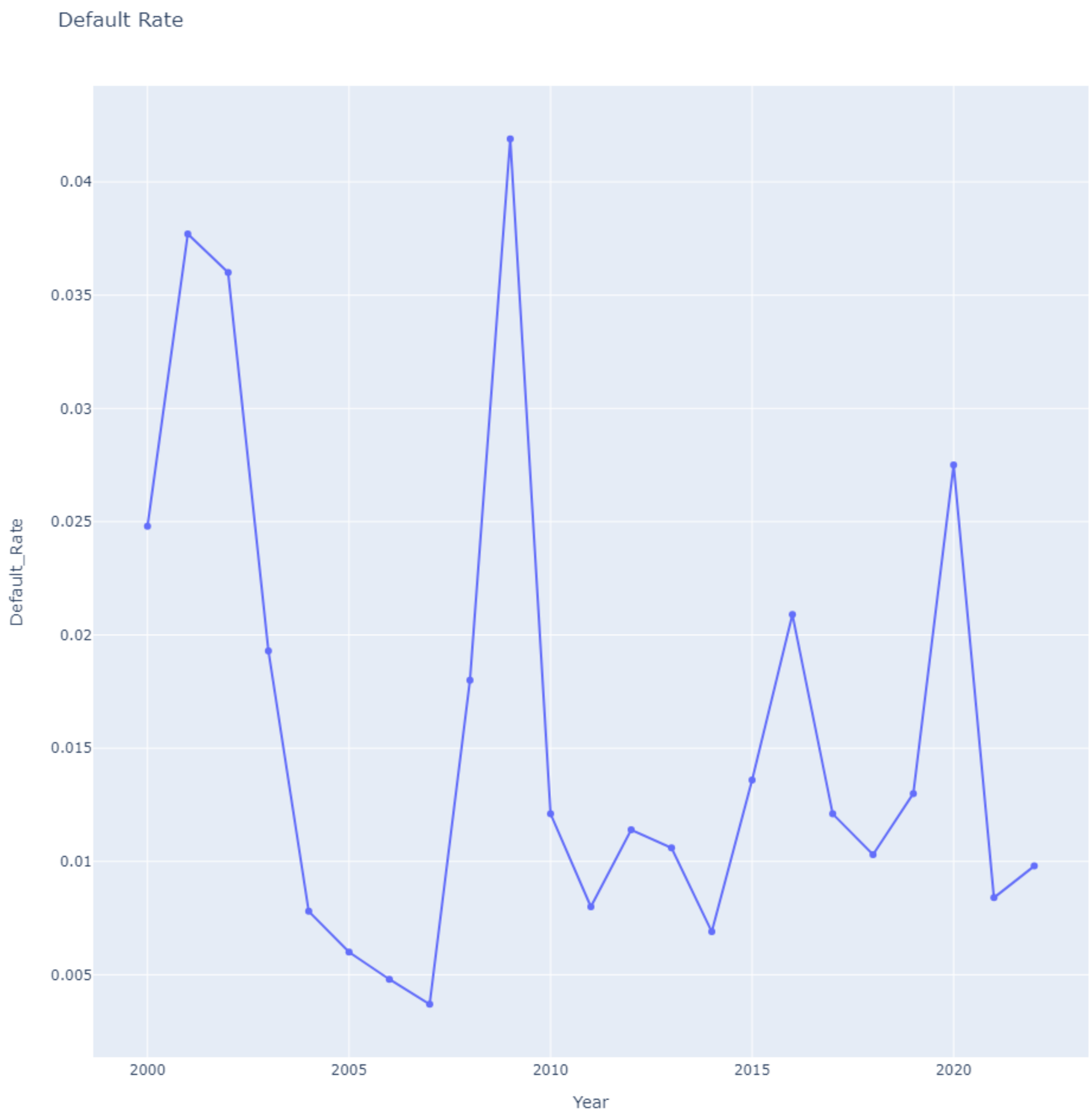
Table 16. Default Rate and Default Rate Log Odds

Year	Default Rate	Logistic Odds
2000	0.0248	-3.671799
2001	0.0377	-3.239666
2002	0.036	-3.287572
2003	0.0193	-3.928162
2004	0.0078	-4.845801
2005	0.006	-5.109978
2006	0.0048	-5.334328
2007	0.0037	-5.595716
2008	0.018	-3.99922
2009	0.0419	-3.129666
2010	0.0121	-4.402376
2011	0.008	-4.820282
2012	0.0114	-4.462676
2013	0.0106	-4.536245
2014	0.0069	-4.96931
2015	0.0136	-4.283992
2016	0.0209	-3.846885
2017	0.0121	-4.402376
2018	0.0103	-4.565258
2019	0.013	-4.329721
2020	0.0275	-3.565684
2021	0.0084	-4.771088
2022	0.0098	-4.615525

Source: author computation using Python libraries

Prior to the estimation of an econometric model, a critical preparatory step is the examination of stationarity in Default Rate and macroeconomic variables. Stationarity, a fundamental premise in regression analysis, necessitates the constancy of mean, variance, and autocorrelation over time. Non-adherence to this assumption can lead to erroneous interpretations and misleading conclusions regarding the interrelationships among variables. Ensuring stationarity is pivotal for the integrity and reliability of regression outcomes, as any deviations may result in spurious correlations and diminished predictive precision of the model. A preliminary assessment of stationarity can be conducted visually by analysing the time series graph, where a stationary series is indicated by a consistent mean and variance throughout, absent of any significant long-term trends, and exhibiting stable fluctuations around the mean.

Figure 5. Default Rate plot



Source: author computation using Python libraries

It is possible to see on the above presented graph that Default Rates seems to be without an obvious trend which would suggest that it is a stationary time series. To make sure, one of the most popular among unit root tests is performed. Statistical test called Augmented Dickey-Fuller (ADF) test is used and the same is performed for the other variables i.e Macroeconomic Variables. This test assesses the H_0 hypothesis – time series is non-stationary, where the alternative hypothesis H_1 means that time series is stationary. To indicate that, the p-value of the test lower than 5% allow for rejection of null hypothesis.

Table 17. Results of ADF test for stationarity

	ADF Statistic:	p-value:	Used lags:	nobs:	Conclusion:
Unemployment rate	-1.20577	0.909313	1	21	Non-stationary
Consumer Price Index	-2.12079	0.534435	1	21	Non-stationary
Real Gross Domestic Product	-3.00499	0.130693	1	21	Non-stationary
EURIBOR - 3-Month	-3.00545	0.130565	1	21	Non-stationary
Central government bond yield curve	-1.96725	0.619155	1	21	Non-stationary
Yield Structure	-2.75172	0.215243	1	21	Non-stationary
Default_Rate	-3.73232	0.020315	1	21	Stationary

Source: author computation using Python libraries

As expected, the test results presented in the above table allow for rejection of null hypothesis for the Default Rate, however null hypothesis is confirmed for the rest of variables tested. In order to make macroeconomic variables stationary which is needed for further model, a simple differencing method is used. By looking at the data and even on the Figure 2 it can be seen that unfortunately a negative values occur for example for EURIBOR – 3-month which wont allow to perform log differencing. To maintain consistency, simple differencing with the below presented formula is performed.

$$y_t = y_t - y_{t-1}$$

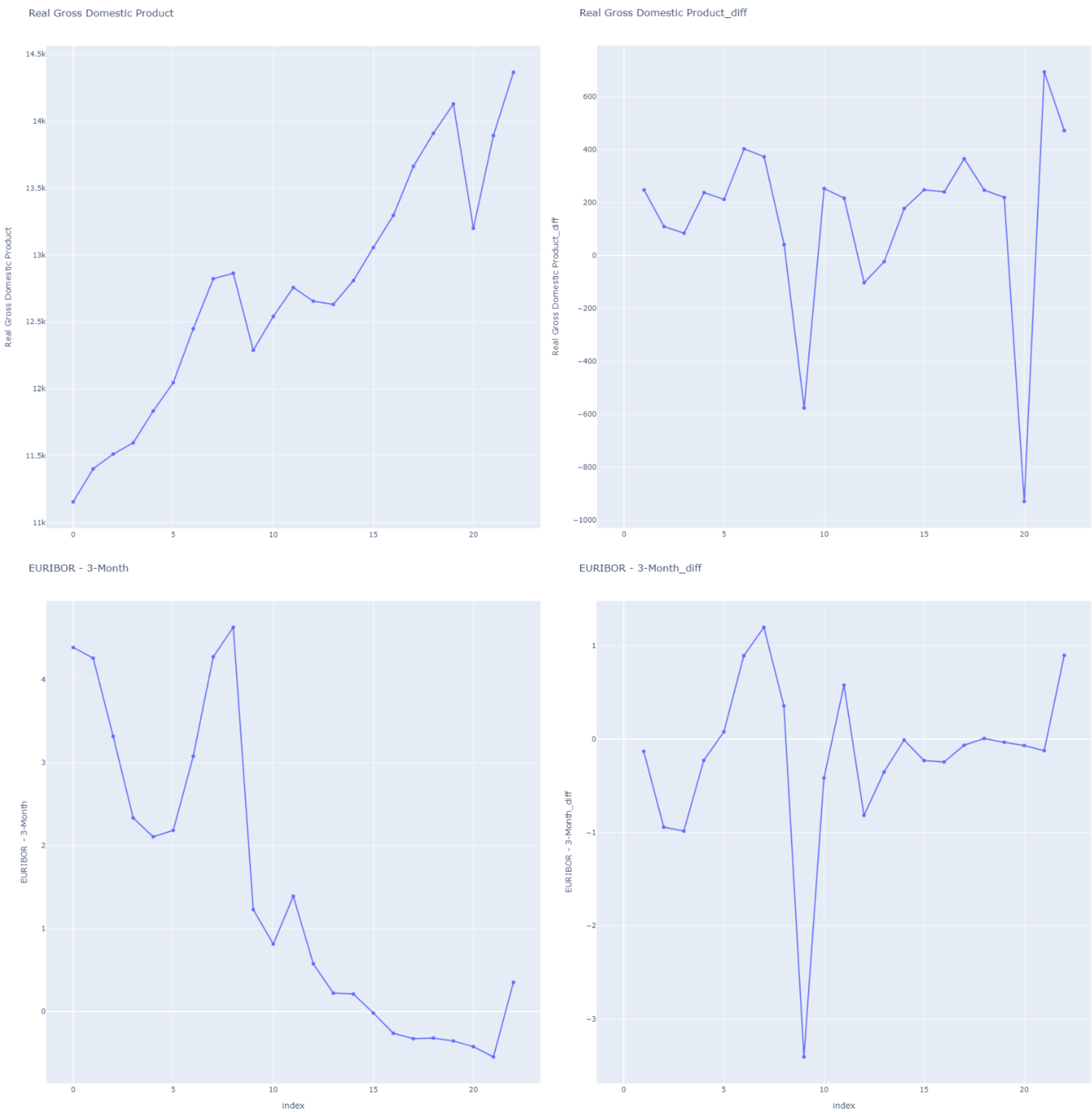
Application of this differencing formula should make variables stationary. To confirm that the ADF test was performed again and the results are presented on the below Table 18.

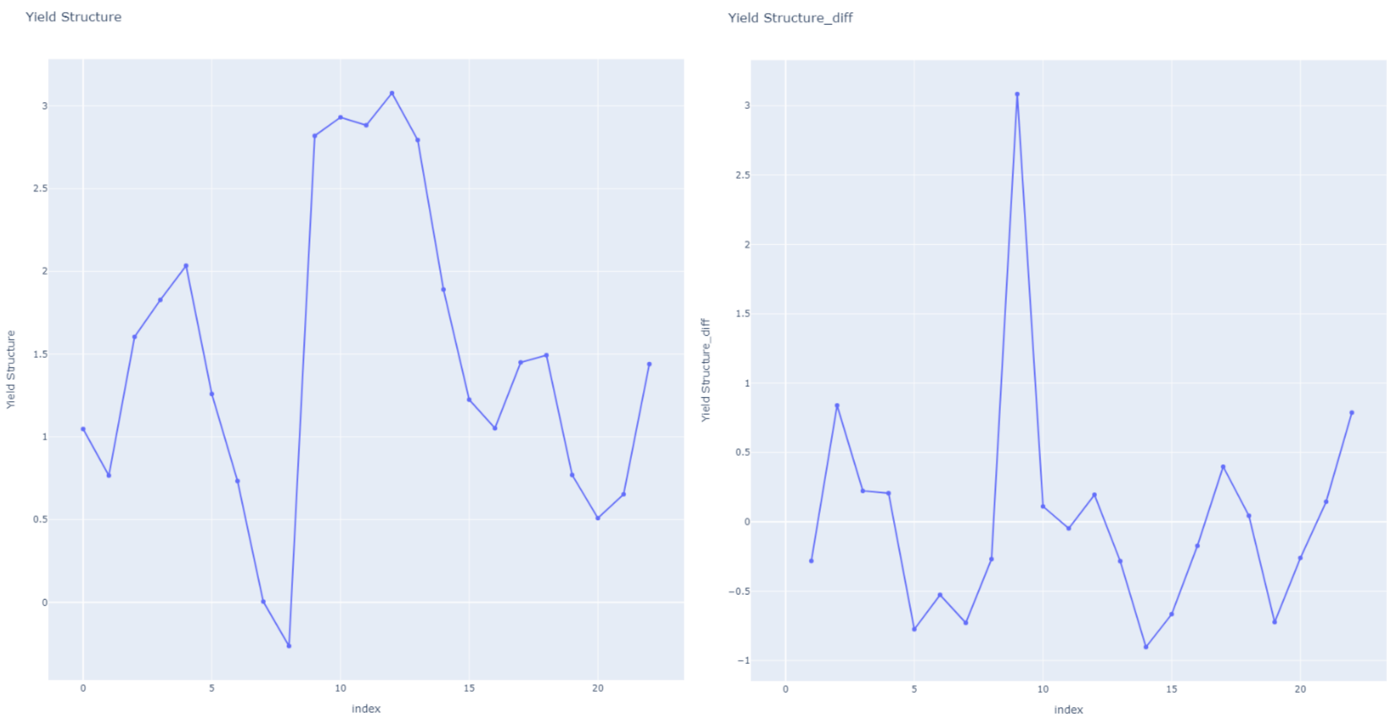
Table 18. Results of ADF test for stationarity after differencing

	ADF Statistic:	p-value:	Used lags:	nobs:	Conclusion:
Unemployment rate_diff	-3.02354	0.12562	1	20	Non-stationary
Consumer Price Index_diff	-1.07218	0.93359	1	20	Non-stationary
Real Gross Domestic Product_diff	-4.04744	0.00752	1	20	Stationary
EURIBOR - 3-Month_diff	-4.09378	0.00644	1	20	Stationary
Central government bond yield curve _diff	-1.80341	0.70311	1	20	Non-stationary
Yield Structure_diff	-3.15732	0.09325	1	20	Stationary

Source: author computation using Python libraries

Figure 6. Macroeconomic variables plot to compare after differencing





Source: author computation using Python libraries

Estimation of the Default rate was performed using three differenced macroeconomic variables i.e. Real Gross Domestic Product_diff, EURIBOR – 3-month_diff and Yield Structure_diff as the explanatory variables. The below tables present the regression results.

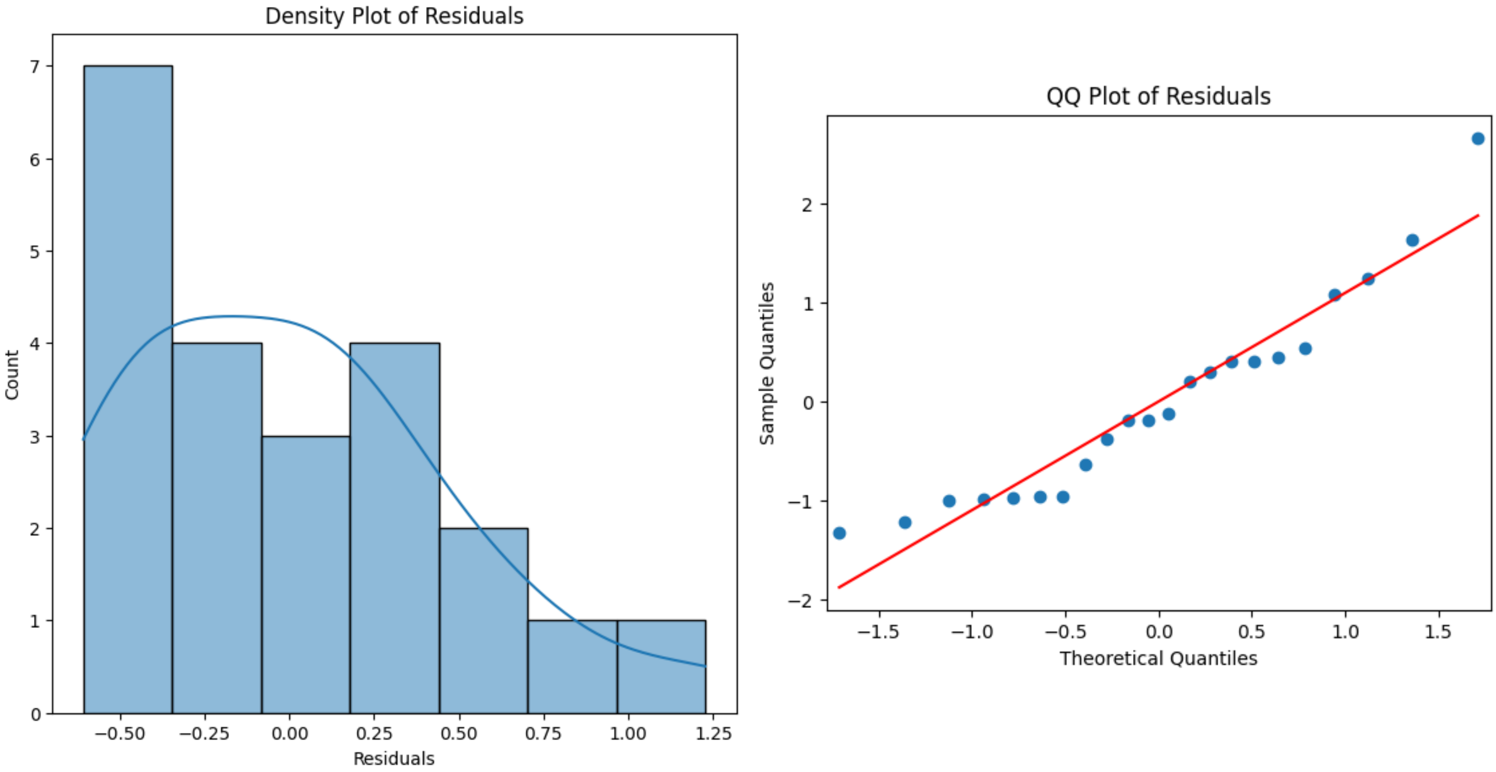
Table 19. Results of ADF test for stationarity after differencing

R-squared:	0.500	**F-statistic:**	5.998
Adj. R-squared:	0.417	**Prob (F-statistic):**	0.00511
Df Model:	3	**Log-Likelohood:**	-14.141
No. Observations:	22	**AIC:**	36.28
Df Residuals:	18	**BIC:**	40.65

	coef	**Std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	-4.3417	0.135	-32.164	0.000	-4.625	-4.058
R. GDP _diff	-0.0006	0.000	-1.455	0.163	-0.001	0.000
EUR_diff	-0.3112	0.211	-1.476	0.157	-0.754	0.132
Yield_diff	0.0863	0.206	0.418	0.681	-0.348	0.520

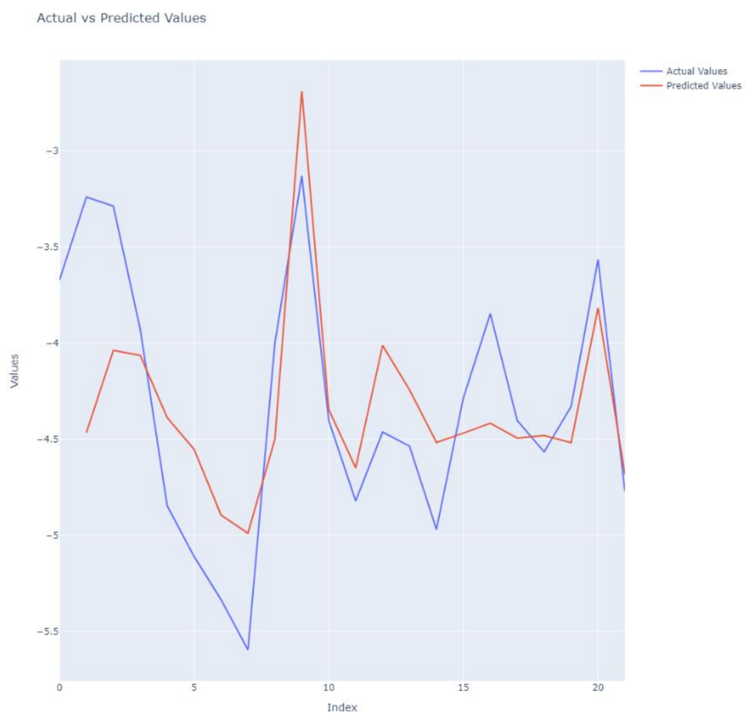
Source: author computation using Python libraries

Figure 7. Residuals plotted



Source: author computation using Python libraries

Figure 8. Actual and Predicted Values



Source: author computation using Python libraries

The QQ plot above presents, that the dataset follows a theoretical normal distribution as points on the graph seems to be not far from the line so this may suggest that residuals are normally distributed. The Figure 8 on the left shows the estimated values together with the historical provided data of Default Rates which were taken as dependent variable. By looking at the graph, estimated model fits good the data with minor exceptions. This model is simplified as possible which allows this well fitting of data and easiness of interpretation.

Following the application of the econometric model, the subsequent task is to estimate Default Rates for the ensuing three-year prediction period (2023, 2024, 2024). These estimations are outlined in table

20, reflecting values under three projected scenarios: optimistic, baseline, and pessimistic. The differing Default Rate predictions for each scenario arise from the unique behaviour of the macroeconomic variables in the model. An integral step in calculating Point-in-Time PDs is the derivation of Shift Factors, which is done using the formula provided below:

$$Shift\ Factor_{t+h} = \frac{\widehat{DR_{t+h}}}{\overline{DR}}$$

Table 20. Estimated DR end Shift Factors for three Scenarios

Scenario	Year	Estimated DR	Average DR (2000-22)	Shift Factor
Base	2023	0.48%	1.59%	0.30
Base	2024	1.20%	1.59%	0.75
Base	2025	1.23%	1.59%	0.77
Optimistic	2023	0.64%	1.59%	0.40
Optimistic	2024	0.64%	1.59%	0.40
Optimistic	2025	1.11%	1.59%	0.70
Pessimistic	2023	0.46%	1.59%	0.29
Pessimistic	2024	2.03%	1.59%	1.28
Pessimistic	2025	1.32%	1.59%	0.83

Source: author computation using Python libraries

It can be seen that Base scenario presents higher estimations of Default Rate than the Optimistic Scenario. Also the Pessimistic Scenario estimates higher PD than in Optimistic Scenario only for the year 2024. Value of Shift Factor for that year is also relatively much higher (1.28) than for other years and scenarios. It tends to be a common problem when estimating PiT PD model that results are not intuitive as explained above i.e. Base Scenario higher estimates than optimistic.

Applying the above-calculated shift factors, the formula for adjusting the Conditional Through-the-Cycle PDs (Table 15.) is as follows:

$$PiT\ PD_t^{Conditional} = TTC\ PD_t^{Conditional} \times Shift\ Factor_t$$

In Table 21 below, the Conditional Point-in-Time-PDs are enumerated. To enable a comprehensive comparison with the Conditional TTC-PDs, both are estimated for a period of ten years. Nevertheless, given that shift factors were determined exclusively for the initial trio of years, the PDs for the remaining seven-year period (2026-2032) are identical to those in the TTC model for each assessed scenario.

Table 21. Conditional Point-in-Time PDs for each assessed scenario

Base Scenario:

years	AAA	AA	A	BBB	BB	B	CCC
1-year Default	0.94%	1.18%	1.29%	1.81%	3.06%	4.75%	12.41%
2-year Default	2.42%	2.96%	3.29%	4.61%	7.86%	12.60%	26.05%
3-year Default	2.56%	3.08%	3.45%	4.84%	8.28%	13.17%	22.24%
4-year Default	3.40%	4.05%	4.59%	6.41%	10.94%	17.02%	24.31%
5-year Default	3.48%	4.11%	4.69%	6.52%	11.06%	16.77%	21.13%
6-year Default	3.55%	4.17%	4.79%	6.62%	11.11%	16.42%	19.02%
7-year Default	3.63%	4.23%	4.88%	6.70%	11.10%	16.02%	17.61%
8-year Default	3.70%	4.29%	4.97%	6.77%	11.04%	15.60%	16.63%
9-year Default	3.76%	4.36%	5.05%	6.83%	10.94%	15.17%	15.89%
10-year Default	3.83%	4.42%	5.13%	6.88%	10.82%	14.75%	15.29%

Optimistic Scenario:

years	AAA	AA	A	BBB	BB	B	CCC
1-year Default	1.26%	1.57%	1.72%	2.41%	4.07%	6.32%	16.51%
2-year Default	1.29%	1.57%	1.75%	2.45%	4.18%	6.69%	13.84%
3-year Default	2.31%	2.78%	3.13%	4.38%	7.50%	11.92%	20.13%
4-year Default	3.40%	4.05%	4.59%	6.41%	10.94%	17.02%	24.31%
5-year Default	3.48%	4.11%	4.69%	6.52%	11.06%	16.77%	21.13%
6-year Default	3.55%	4.17%	4.79%	6.62%	11.11%	16.42%	19.02%
7-year Default	3.63%	4.23%	4.88%	6.70%	11.10%	16.02%	17.61%
8-year Default	3.70%	4.29%	4.97%	6.77%	11.04%	15.60%	16.63%
9-year Default	3.76%	4.36%	5.05%	6.83%	10.94%	15.17%	15.89%
10-year Default	3.83%	4.42%	5.13%	6.88%	10.82%	14.75%	15.29%

Pessimistic Scenario:

	AAA	AA	A	BBB	BB	B	CCC
years							
1-year Default	0.90%	1.13%	1.24%	1.73%	2.93%	4.54%	11.86%
2-year Default	4.12%	5.03%	5.59%	7.85%	13.37%	21.43%	44.29%
3-year Default	2.74%	3.30%	3.71%	5.20%	8.89%	14.14%	23.88%
4-year Default	3.40%	4.05%	4.59%	6.41%	10.94%	17.02%	24.31%
5-year Default	3.48%	4.11%	4.69%	6.52%	11.06%	16.77%	21.13%
6-year Default	3.55%	4.17%	4.79%	6.62%	11.11%	16.42%	19.02%
7-year Default	3.63%	4.23%	4.88%	6.70%	11.10%	16.02%	17.61%
8-year Default	3.70%	4.29%	4.97%	6.77%	11.04%	15.60%	16.63%
9-year Default	3.76%	4.36%	5.05%	6.83%	10.94%	15.17%	15.89%
10-year Default	3.83%	4.42%	5.13%	6.88%	10.82%	14.75%	15.29%

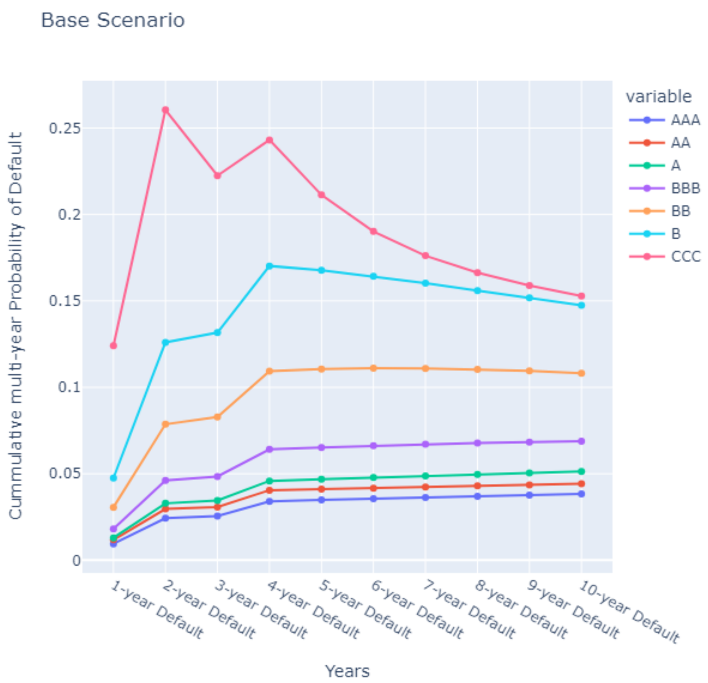
Source: author computation using Python libraries

By looking at the values presented in above tables in can be observed that PiT PDs are lower in first 3 years than in TTC PDs, all else for each scenario is the same. This can be seen on the below presented figures.

This values can be used to compute the Cumulative multi-year Point-in-Time PD (Table 22 below the graphs) by the use of the following formula:

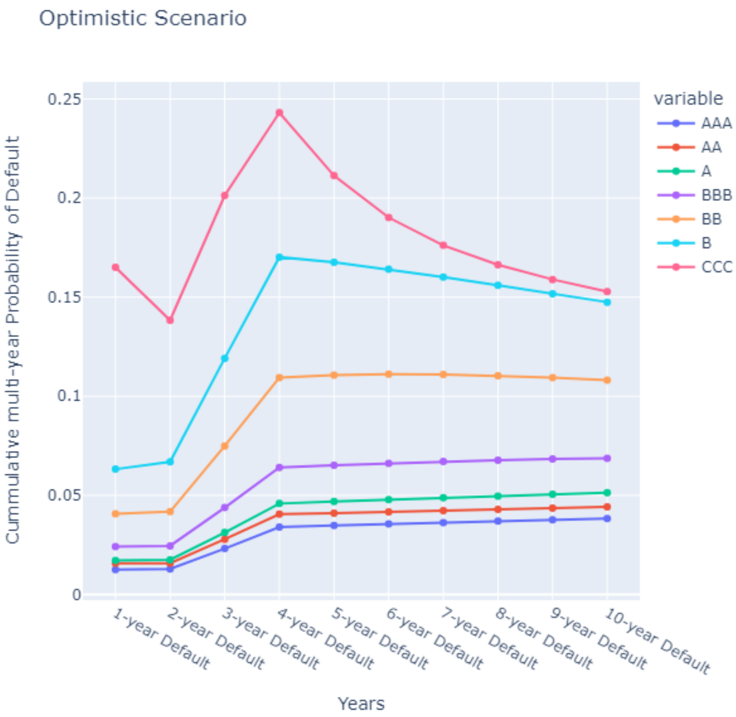
$$PiT\ PD_t^{Cumulative} = PiT\ PD_t^{Conditional} \times \left(1 - PiT\ PD_{t-1}^{Cumulative}\right) + PiT\ PD_{t-1}^{Cumulative}$$

Figure 9. Base Scenario Conditional PiT PD



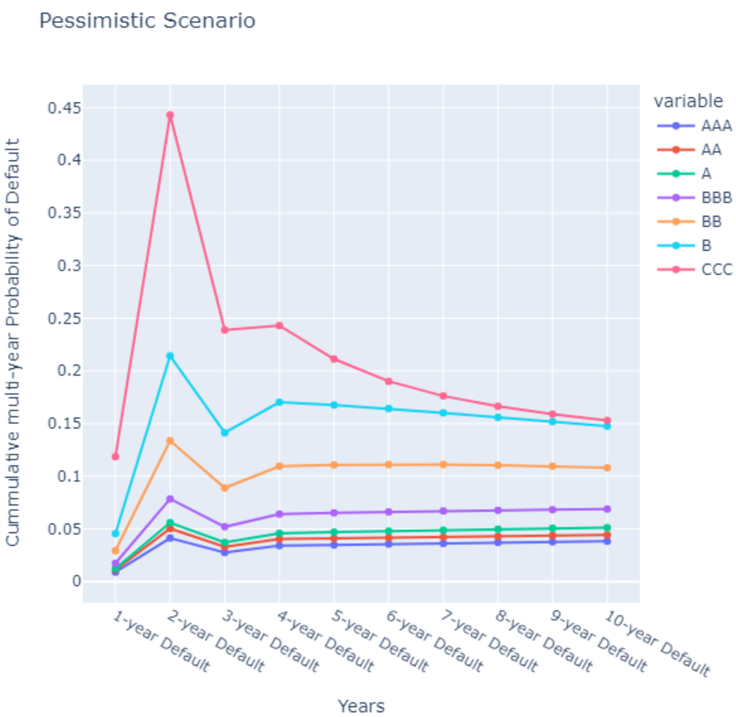
Source: author computation using Python libraries

Figure 10. Optimistic Scenario Conditional PiT PD



Source: author computation using Python libraries

Figure 11. Pessimistic Scenario Conditional PiT PD



Source: author computation using Python libraries

Table 22. Cumulative Point-in-Time PDs for each assessed scenario

Base Scenario:

	AAA	AA	A	BBB	BB	B	CCC
0	0.94%	1.18%	1.29%	1.81%	3.06%	4.75%	12.41%
1	1.88%	2.34%	2.57%	3.59%	6.03%	9.27%	23.28%
2	4.26%	5.23%	5.78%	8.04%	13.42%	20.71%	43.26%
3	6.70%	8.15%	9.03%	12.49%	20.59%	31.15%	55.88%
4	9.88%	11.87%	13.20%	18.10%	29.28%	42.87%	66.61%
5	13.01%	15.49%	17.27%	23.44%	37.10%	52.45%	73.66%
6	16.10%	19.01%	21.23%	28.51%	44.08%	60.25%	78.67%
7	19.14%	22.43%	25.07%	33.30%	50.29%	66.62%	82.43%
8	22.13%	25.76%	28.79%	37.82%	55.78%	71.83%	85.35%
9	25.06%	29.00%	32.39%	42.06%	60.62%	76.10%	87.68%

Optimistic Scenario:

	AAA	AA	A	BBB	BB	B	CCC
0	1.26%	1.57%	1.72%	2.41%	4.07%	6.32%	16.51%
1	2.50%	3.11%	3.41%	4.77%	7.98%	12.24%	30.29%
2	3.75%	4.63%	5.10%	7.10%	11.82%	18.11%	39.93%
3	5.98%	7.29%	8.07%	11.17%	18.43%	27.87%	52.03%
4	9.17%	11.04%	12.28%	16.86%	27.36%	40.15%	63.69%
5	12.33%	14.70%	16.39%	22.28%	35.39%	50.18%	71.36%
6	15.45%	18.25%	20.40%	27.43%	42.57%	58.36%	76.81%
7	18.51%	21.71%	24.28%	32.29%	48.94%	65.03%	80.89%
8	21.53%	25.07%	28.04%	36.88%	54.58%	70.49%	84.07%
9	24.48%	28.33%	31.67%	41.19%	59.55%	74.97%	86.60%

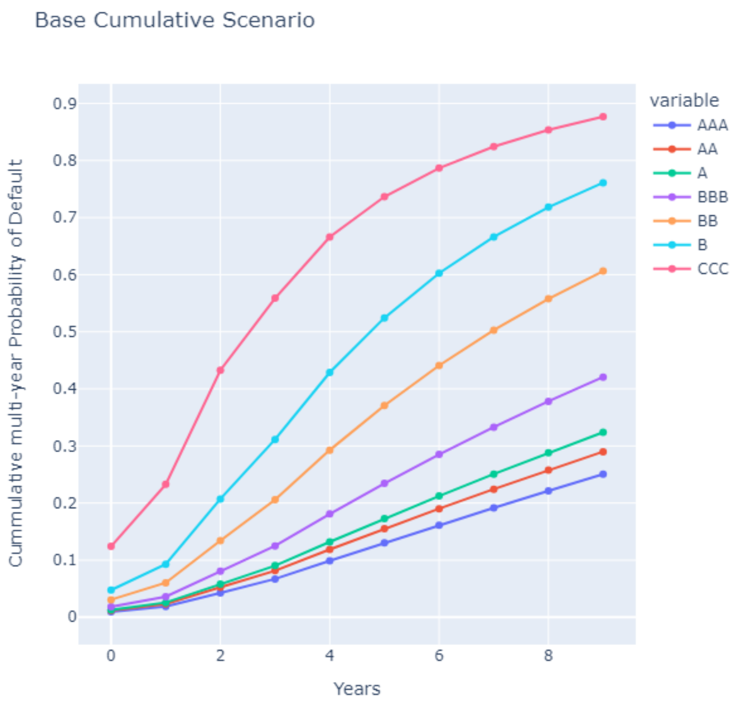
Pessimistic Scenario:

	AAA	AA	A	BBB	BB	B	CCC
0	0.90%	1.13%	1.24%	1.73%	2.93%	4.54%	11.86%
1	1.80%	2.24%	2.46%	3.44%	5.77%	8.87%	22.31%
2	5.84%	7.16%	7.91%	11.01%	18.36%	28.40%	56.72%
3	8.43%	10.23%	11.33%	15.64%	25.62%	38.52%	67.05%
4	11.54%	13.86%	15.39%	21.04%	33.76%	48.98%	75.06%
5	14.62%	17.40%	19.36%	26.19%	41.08%	57.54%	80.33%
6	17.65%	20.84%	23.22%	31.07%	47.63%	64.51%	84.07%
7	20.64%	24.19%	26.96%	35.69%	53.44%	70.19%	86.88%
8	23.57%	27.44%	30.59%	40.05%	58.58%	74.84%	89.06%
9	26.44%	30.60%	34.10%	44.14%	63.11%	78.66%	90.80%

Source: author computation using Python libraries

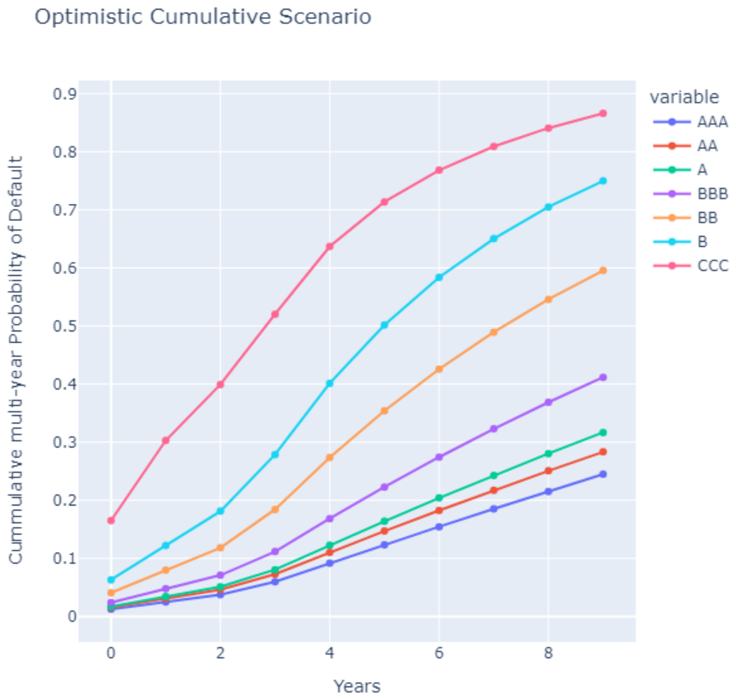
Figures 12, 13 and 14 present an a plot of the cumulative multi-year Point-in-Time Probability of Defaults (PIT PDs) across the three delineated scenarios. A detailed examination reveals that the curves corresponding to each rating class are distinctly non-overlapping, indicating clear demarcation in risk assessments. Moreover, in the initial three-year span, these curves exhibit different shapes, a variation attributable to the application of scenario-specific shift factors.

Figure 12. Base Scenario Cumulative PiT PD



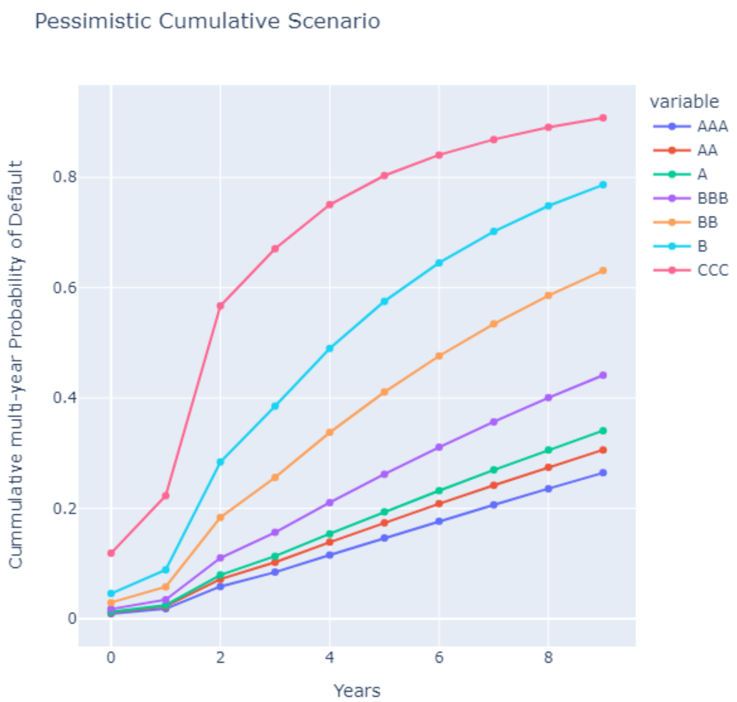
Source: author computation using Python libraries

Figure 13. Optimistic Scenario Cumulative PiT PD



Source: author computation using Python libraries

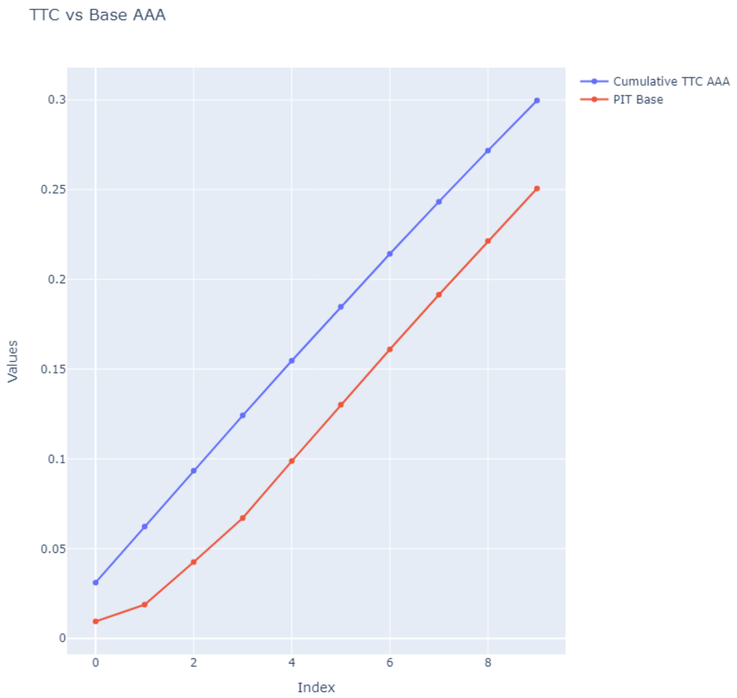
Figure 14. Pessimistic Scenario Cumulative PiT PD



Source: author computation using Python libraries

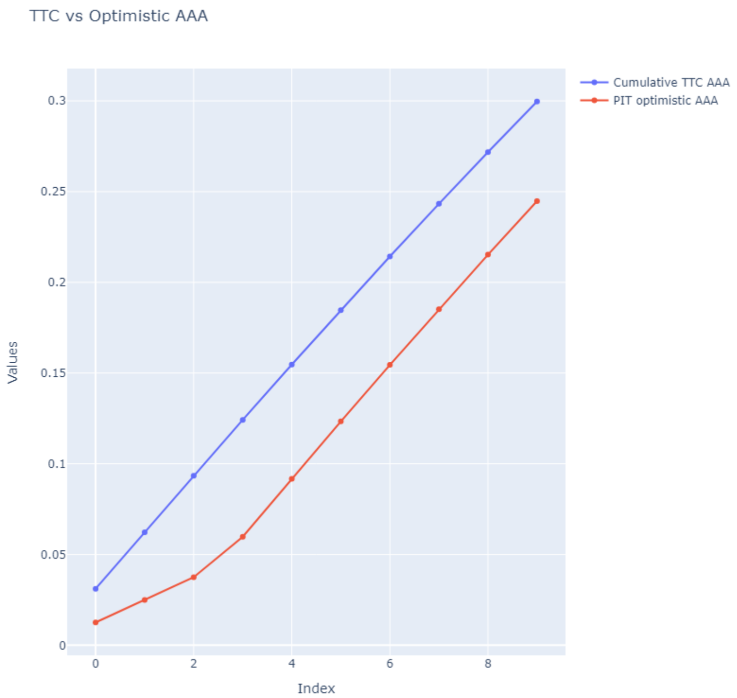
Conversely, Figures 15, 16 and 17 provide an intricate comparison of the cumulative PIT versus Through-the-Cycle Probability of Defaults (TTC PDs) for the highest credit rating category, AAA, under all three scenarios: optimistic, baseline, and pessimistic. A meticulous analysis of these figures shows that, irrespective of the scenario, the PIT PDs consistently register at lower values compared to their TTC counterparts. This disparity underscores the intrinsic differences in the probabilistic modelling approaches between PIT and TTC PDs under varying economic conditions. For further reference, below are also figures comparing the Conditional PiT PD with TTC PD.

Figure 15. Base Scenario Cumulative PiT PD vs. TTC PD



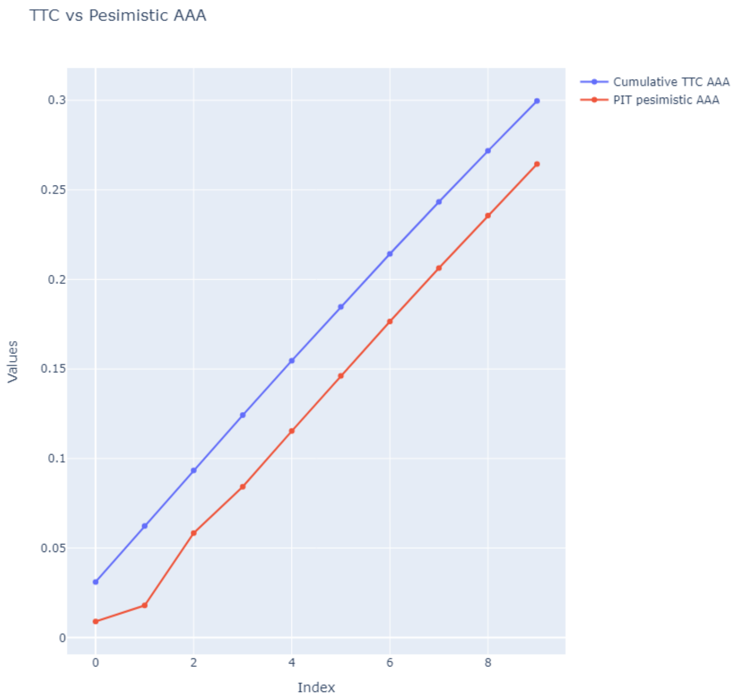
Source: author computation using Python libraries

Figure 16. Optimistic Scenario Cumulative PiT PD vs. TTC PD



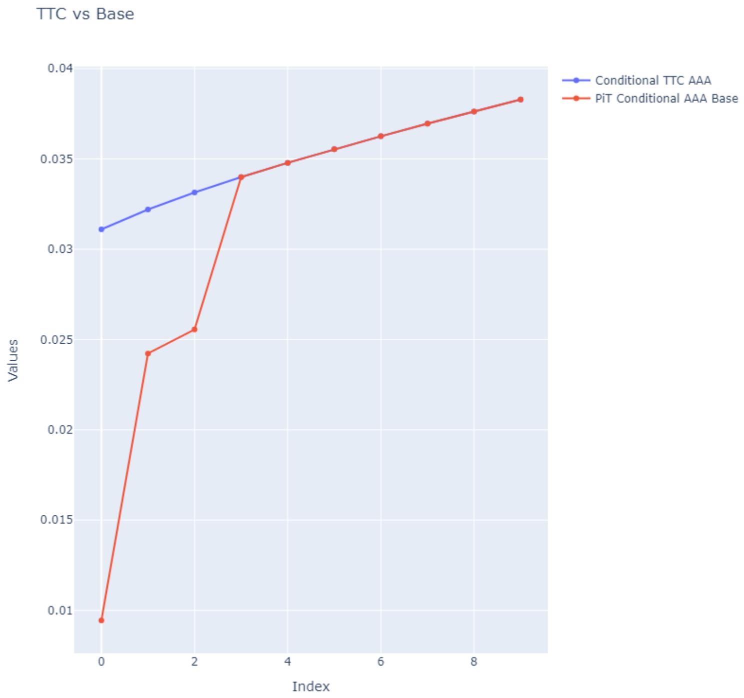
Source: author computation using Python libraries

Figure 17. Pessimistic Scenario Cumulative PiT PD vs. TTC PD



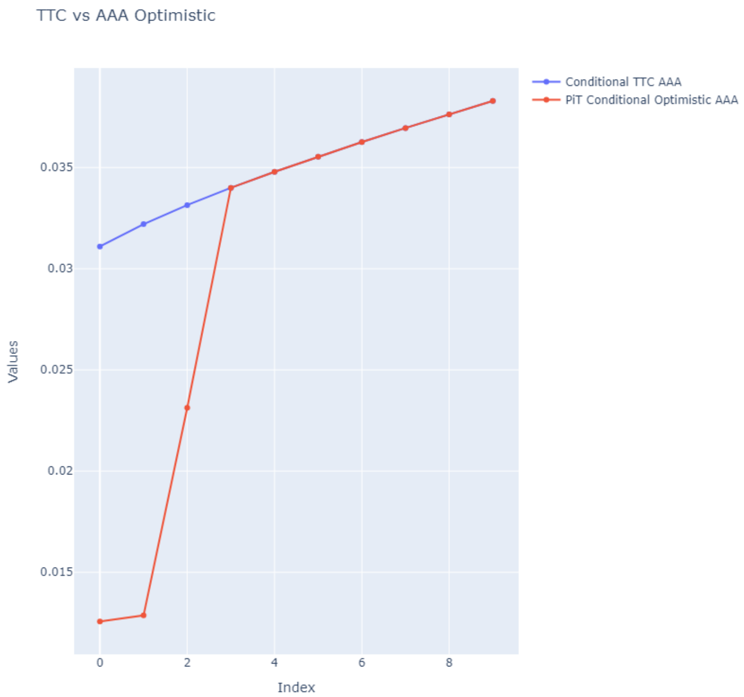
Source: author computation using Python libraries

Figure 18. Base Scenario Conditional PiT PD vs. TTC PD



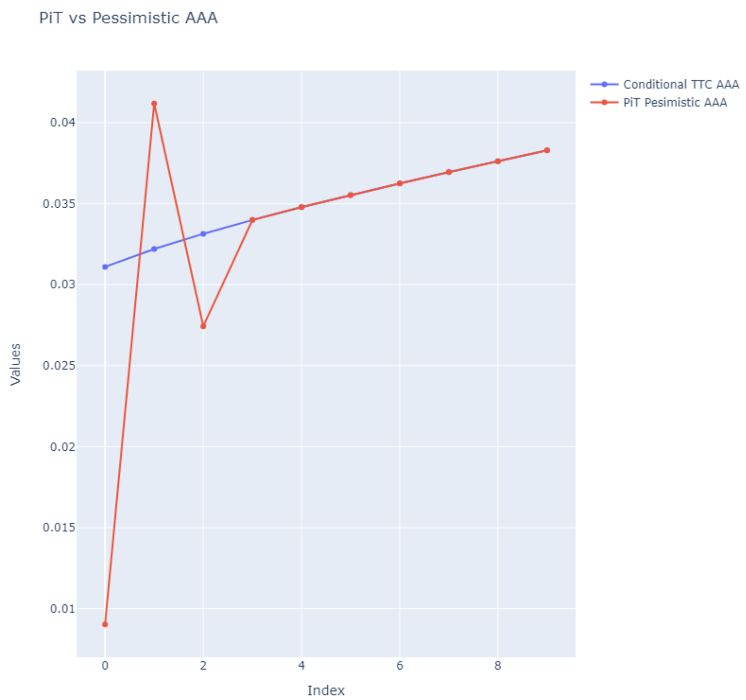
Source: author computation using Python libraries

Figure 19. Optimistic Scenario Conditional PiT PD vs. TTC PD



Source: author computation using Python libraries

Figure 20. Pessimistic Scenario Conditional PiT PD vs. TTC PD



Source: author computation using Python libraries