Operazioni_polinomiali_e_fattorizzazione

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1 Operations on Monomials and Polynomials

This document outlines the fundamental operations involving monomials and polynomials, including product, addition, and multiplication.

1.1 1. Monomials

A monomial is an algebraic expression consisting of a single term. A term can be a number, a variable, or the product of numbers and variables. Examples of monomials include:

- 5
- x
- $3xy$
- $-2a^2b$

1.1.1 1.1 Product of Monomials

To multiply monomials, multiply the coefficients and add the exponents of like variables.

Example 1:

$(3x^2y) \cdot (2xy^3) = (3 \cdot 2) \cdot (x^2 \cdot x) \cdot (y \cdot y^3) = 6x^3y^4$

Example 2:

$(-4ab^2) \cdot (5a^3c) = (-4 \cdot 5) \cdot (a \cdot a^3) \cdot b^2 \cdot c = -20a^4b^2c$

1.1.2 1.2 Addition and Subtraction of Monomials

Monomials can only be added or subtracted if they are *like terms*. Like terms have the same variables raised to the same powers.

Example 1 (Like terms):

$3x^2y + 5x^2y = (3 + 5)x^2y = 8x^2y$

Example 2 (Unlike terms - cannot be combined):

$3x^2y + 5xy^2$ (These cannot be simplified further)

Example 3 (Subtraction):

$7a^3b - 2a^3b = (7 - 2)a^3b = 5a^3b$

1.2 2. Polynomials

A polynomial is an algebraic expression consisting of one or more terms (monomials) connected by addition or subtraction. Examples of polynomials include:

- $2x + 1$
- $x^2 - 3x + 2$
- $4a^3b - 2ab^2 + 7$

1.2.1 2.1 Addition and Subtraction of Polynomials

To add or subtract polynomials, combine like terms.

Example 1:

$(2x^2 + 3x - 1) + (x^2 - x + 4) = (2x^2 + x^2) + (3x - x) + (-1 + 4) = 3x^2 + 2x + 3$

Example 2:

$(3a^3 - 2a + 5) - (a^3 + 4a - 2) = (3a^3 - a^3) + (-2a - 4a) + (5 + 2) = 2a^3 - 6a + 7$

1.2.2 2.2 Multiplication of Polynomials

The distributive property is used to multiply polynomials. Each term in the first polynomial is multiplied by each term in the second polynomial.

Example 1 (Monomial by Polynomial):

$2x(x^2 - 3x + 1) = 2x \cdot x^2 + 2x \cdot (-3x) + 2x \cdot 1 = 2x^3 - 6x^2 + 2x$

Example 2 (Binomial by Binomial - FOIL method):

$(x + 2)(x - 3) = x \cdot x + x \cdot (-3) + 2 \cdot x + 2 \cdot (-3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$

Example 3 (Binomial by Trinomial):

$(a + 2)(a^2 - a + 1) = a(a^2 - a + 1) + 2(a^2 - a + 1) = a^3 - a^2 + a + 2a^2 - 2a + 2 = a^3 + a^2 - a + 2$

General Rule: When multiplying polynomials, ensure that every term in the first polynomial is multiplied by every term in the second polynomial. Then, combine like terms to simplify the result.

1.3 Summary

- **Monomial Product:** Multiply coefficients, add exponents of like variables.
- **Monomial Addition/Subtraction:** Combine like terms only.
- **Polynomial Addition/Subtraction:** Combine like terms.
- **Polynomial Multiplication:** Use the distributive property, multiply each term in the first polynomial by each term in the second, and then combine like terms.

1.3.1 Checks calculations:

By modifying the inputs **poly1** and **poly2** you can check if your calculations are correct!

[2]:
```
import sympy
```

```python
def multiply_multivariable_polynomials(poly_str1, poly_str2, variables_str):
    """Multiplies two multivariable polynomials.

    Args:
        poly_str1: String representation of the first polynomial.
        poly_str2: String representation of the second polynomial.
        variables_str: A string containing the variables, separated by spaces␣
↪(e.g., "x y z").

    Returns:
        String representation of the resulting polynomial, or an error message.
    """
    try:
        variables = [sympy.Symbol(var) for var in variables_str.split()]  #␣
↪Create symbolic variables
        poly1 = sympy.sympify(poly_str1)
        poly2 = sympy.sympify(poly_str2)
        result = sympy.expand(poly1 * poly2)
        return str(result)
    except (sympy.SympifyError, TypeError):
        return "Invalid polynomial or variable input. Please use valid␣
↪mathematical expressions."


def factor_multivariable_polynomial(poly_str, variables_str):
    """Factors a multivariable polynomial.

    Args:
        poly_str: String representation of the polynomial.
        variables_str: A string containing the variables, separated by spaces␣
↪(e.g., "x y z").

    Returns:
        String representation of the factored polynomial, or an error message.
    """
    try:
        variables = [sympy.Symbol(var) for var in variables_str.split()]
        poly = sympy.sympify(poly_str)
        result = sympy.factor(poly)
        return str(result)
    except (sympy.SympifyError, TypeError):
        return "Invalid polynomial or variable input. Please use valid␣
↪mathematical expressions."
    except sympy.PolynomialError:
        return "Polynomial is not factorable by the current method or is too␣
↪complex."
```

```python
# Example usage (easily changeable inputs):
variables = "x y z"  # Define your variables here!

# Multiplication Examples
poly1 = "x + y"
poly2 = "x - y"
result = multiply_multivariable_polynomials(poly1, poly2, variables)
print(f"({poly1}) * ({poly2}) = {result}")  # Output: x**2 - y**2

poly1 = "x + 2*y"
poly2 = "x**2 - x*y + y**2"
result = multiply_multivariable_polynomials(poly1, poly2, variables)
print(f"({poly1}) * ({poly2}) = {result}")  # Output: x**3 + y**3

# Factoring Examples
poly = "x**2 - y**2"
result = factor_multivariable_polynomial(poly, variables)
print(f"Factoring {poly} = {result}")  # Output: (x - y)*(x + y)

poly = "x**3 + y**3"
result = factor_multivariable_polynomial(poly, variables)
print(f"Factoring {poly} = {result}")  # Output: (x + y)*(x**2 - x*y + y**2)

poly = "x**2 + 2*x*y + y**2"
result = factor_multivariable_polynomial(poly, variables)
print(f"Factoring {poly} = {result}")  # Output: (x + y)**2

poly = "x**4 - y**4"
result = factor_multivariable_polynomial(poly, variables)
print(f"Factoring {poly} = {result}")  # Output: (x - y)*(x + y)*(x**2 + y**2)
```

```
(x + y) * (x - y) = x**2 - y**2
(x + 2*y) * (x**2 - x*y + y**2) = x**3 + x**2*y - x*y**2 + 2*y**3
Factoring x**2 - y**2 = (x - y)*(x + y)
Factoring x**3 + y**3 = (x + y)*(x**2 - x*y + y**2)
Factoring x**2 + 2*x*y + y**2 = (x + y)**2
Factoring x**4 - y**4 = (x - y)*(x + y)*(x**2 + y**2)
```

1.3.2 Common Factoring Techniques in High School

This document outlines common factoring techniques taught in high school algebra, along with examples.

1.4 1. Greatest Common Factor (GCF) Factoring

- **Concept:** Find the largest factor that divides all terms in the polynomial and factor it out.

- **Example 1:** $6x^3 + 9x^2 = 3x^2(2x + 3)$
- **Example 2:** $4a^2b - 6ab^2 + 2ab = 2ab(2a - 3b + 1)$

1.5 2. Factoring Trinomials (Quadratic Expressions)

- **a) Simple Trinomials (leading coefficient is 1):**
  - **Concept:** Find two numbers that add up to the coefficient of the middle term and multiply to the constant term.
  - **Example 1:** $x^2 + 5x + 6 = (x + 2)(x + 3)$ (2 + 3 = 5, 2 * 3 = 6)
  - **Example 2:** $x^2 - 7x + 10 = (x - 2)(x - 5)$ (-2 + -5 = -7, -2 * -5 = 10)
  - **Example 3:** $x^2 + 2x - 8 = (x + 4)(x - 2)$ (4 + -2 = 2, 4 * -2 = -8)
- **b) Trinomials with a Leading Coefficient (a 1):**
  - **Concept:** Use various methods (e.g., "ac method," grouping) to factor.
  - **Example 1:** $2x^2 + 5x + 2 = (2x + 1)(x + 2)$
  - **Example 2:** $3x^2 - 10x + 3 = (3x - 1)(x - 3)$

1.6 3. Difference of Squares

- **Concept:** $a^2 - b^2 = (a - b)(a + b)$
- **Example 1:** $x^2 - 9 = (x - 3)(x + 3)$
- **Example 2:** $4x^2 - 25y^2 = (2x - 5y)(2x + 5y)$

1.7 4. Sum and Difference of Cubes

- **Concept:**
  - $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
  - $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- **Example 1:** $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$
- **Example 2:** $27x^3 - 1 = (3x - 1)(9x^2 + 3x + 1)$

1.8 5. Factoring by Grouping

- **Concept:** Group terms and factor out common factors.
- **Example 1:** $xy + 2x + 3y + 6 = x(y + 2) + 3(y + 2) = (x + 3)(y + 2)$
- **Example 2:** $x^3 - 2x^2 + 5x - 10 = x^2(x - 2) + 5(x - 2) = (x^2 + 5)(x - 2)$

1.9 6. Perfect Square Trinomials

- **Concept:**
  - $a^2 + 2ab + b^2 = (a + b)^2$
  - $a^2 - 2ab + b^2 = (a - b)^2$
- **Example 1:** $x^2 + 6x + 9 = (x + 3)^2$
- **Example 2:** $4x^2 - 12x + 9 = (2x - 3)^2$

1.10 7. Factoring by Substitution (u-substitution)

- **Concept:** Replace a complex expression with a single variable (often "u") to simplify the factoring process.
- **Example:** $(x^2+1)^2 - 4(x^2+1) + 3$. Let $u = x^2 + 1$. Then we have $u^2 - 4u + 3 = (u-1)(u-3)$. Now substitute back: $(x^2 + 1 - 1)(x^2 + 1 - 3) = x^2(x^2 - 2)$

1.11 Important Notes

- **Always look for a GCF first!** This often simplifies the remaining factoring.
- **Factoring completely:** Make sure the polynomial is factored until it can't be factored any further.
- **Practice is key!** The more you practice, the better you'll become at recognizing which factoring technique to use.

This list covers most of the common factoring methods taught in high school algebra. The complexity and specific techniques used might vary slightly depending on the curriculum.