Computational Finance 2024/2025

Asset Allocation

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Abstract

The objective of this report is to build portfolios using different allocation strategies and discuss their performances, using as investment universe 11 sector indices and 5 factor indices of the S&P500. We will analyze three different categories: cyclical, defensive and sensitive.
We will be working with the prices of the 11 sectors and the 5 factors indices, focusing specifically on values for 2023.
In order to run the project, use the file *RunProject5.m*.

Information	0.0947	0.7591	0.0490	0.6854	0.0702	0.0688	0.4607	0.4102	0.0937	0.5781	0.0920	0.0494	0.2237	0.7546
Financials	0.0000	0.0000	0.0000	0.0000	0.0003	0.0002	0.0104	0.0044	0.0000	0.0000	0.0000	0.0579	0.0000	0.0000
HealthCare	0.2666	0.0000	0.5603	0.0000	0.2434	0.2291	0.0122	0.0080	0.2675	0.0000	0.0000	0.0816	0.0000	0.0000
Consumer	0.0000	0.0000	0.0000	0.0000	0.0003	0.0002	0.0910	0.0554	0.0000	0.0000	0.1494	0.0469	0.0000	0.0000
Communic	0.0081	0.1350	0.0000	0.1146	0.0184	0.0182	0.1772	0.1678	0.0080	0.0654	0.1200	0.0437	0.4025	0.1351
Industrials	0.0000	0.0000	0.0000	0.0000	0.0011	0.0009	0.0243	0.0133	0.0000	0.0000	0.0000	0.0639	0.0000	0.0000
ConsumerS	0.3791	0.0000	0.0000	0.0000	0.3791	0.3622	0.0137	0.0099	0.3796	0.0000	0.1343	0.0840	0.0796	0.0000
Energy	0.0657	0.0000	0.0510	0.0000	0.0565	0.0555	0.0355	0.0276	0.0656	0.0013	0.2588	0.0413	0.2556	0.0000
Utilities	0.0000	0.0000	0.0686	0.0000	0.0001	0.0000	0.0109	0.0081	0.0000	0.0000	0.1949	0.0529	0.0386	0.0000
RealEstate	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0142	0.0097	0.0000	0.0000	0.0506	0.0470	0.0000	0.0000
Materials	0.0000	0.0000	0.0000	0.0000	0.0002	0.0001	0.0074	0.0043	0.0000	0.0000	0.0000	0.0562	0.0000	0.0000
Momentum	0.0110	0.1059	0.0348	0.2000	0.0404	0.0429	0.1356	0.1924	0.0111	0.3552	0.0000	0.0724	0.0000	0.1100
Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0015	0.0138	0.0000	0.0000	0.0000	0.0705	0.0000	0.0000
Growth	0.0000	0.0000	0.0000	0.0000	0.0061	0.0123	0.0000	0.0428	0.0000	0.0000	0.0000	0.0703	0.0000	0.0000
Quality	0.0000	0.0000	0.2363	0.0000	0.0184	0.0190	0.0039	0.0237	0.0000	0.0000	0.0000	0.0756	0.0000	0.0000
LowVolatility	0.1747	0.0000	0.0000	0.0000	0.1655	0.1905	0.0017	0.0084	0.1745	0.0000	0.0000	0.0865	0.0000	0.0000

Table 1: Portfolio weights.

1 Efficient Frontier

We began by computing the efficient frontier under standard constraints. To do so we started by simulating $N = 100000$ random portfolios, each composed of the 16 assets available, to derive a range of possible values for the expected returns. We then computed the portfolio frontier as the set of all minimum variance portfolios among those with the same expected return. This is done by finding the weights that minimize the function:

$$\sigma = \sqrt{w^T \cdot V \cdot w}$$

where V is the covariance matrix obtained using the log-returns of the prices. We also imposed the standard constraints:

$$\sum_{i=1}^{N} w_i = 1 \qquad 0 \leq w_i \leq 1 \quad \forall i = 1, .., N$$

To compute the frontier, we used functions provided by the MATLAB Financial Toolbox. Throughout the analysis, we assumed the risk-free rate to be zero.
Having obtained the efficient frontier, we computed among this set of portfolios the one with minimum variance (Portfolio A) and the one that maximizes the Sharpe Ratio (Portfolio B). The weights of the two portfolios are shown in Table 1.

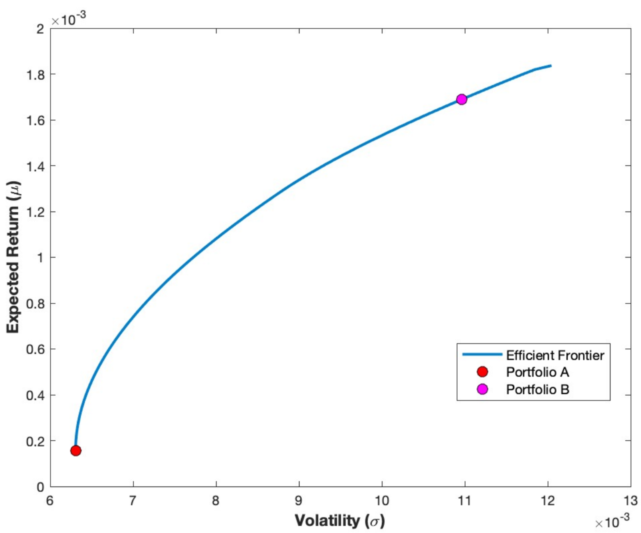


Figure 1: Efficient frontier, Portfolio A and Portfolio B

From the plot we can clearly see how Portfolio A is the one that minimizes the volatility, whereas Portfolio B is the one that indicates the optimal balance between risk and return.

2 Efficient Frontier under constraints

We then computed the efficient frontier under new constraints:

- standard constraints
- The total exposure to sensible sectors set to be greater than 10% and $\sum_{i \in SS} w_i > 0.10\%$
- the total exposure on cyclical sectors set to be lower than 30% $\sum_{i \in CS} w_i < 0.30\%$
- The weights of the sector Consumer Staples and the factor Low Volatility set to be set equal to 0. $w_{CS} = 0 \quad w_{LV} = 0$
- The maximum exposure on sectors set to be lower than 80%.

To add these constraints in the identification of the frontier, we first identified the indices of the sectors to which a constraint was applied. Secondly, we computed matrix A and vector b, which we used to solve the system $Ax \leq b$, by adding a row for each constraint.

By comparing this new frontier with the efficient frontier found previously, we can notice how, for low values of expected returns, the previous portfolios have lower volatility; for higher values of volatilities, the previous frontier has portfolios with higher expected retuns.

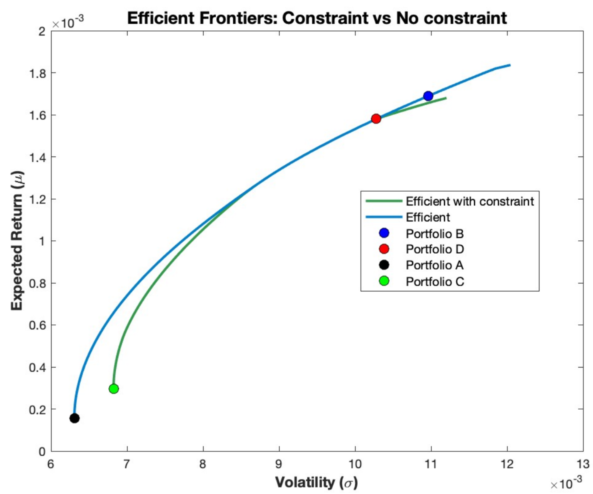


Figure 2: Efficient frontier and efficient frontier with constraints

Subsequently, we identified Portfolio C, which corresponds to the portfolio with minimum volatility, and Portfolio D, which is the portfolio that maximizes the Sharpe Ratio.

Both from the figure and the computations, we can see that the Minimum Variance portfolio on the standard efficient frontier shows lower volatility and lower expected return compared to its counterpart on the constrained frontier. Similarly, portfolio D has both a lower volatility and a lower expected return compared to portfolio B.

3 Robust Frontier

In this section, we compute the frontiers from sections 1 and 2 using a re-sampling method to obtain robust frontiers. By employing robust estimators, we can construct portfolios that are more resilient to extreme events, potentially leading to more stable and reliable investment strategies even in the presence of noisy or atypical data.
We conducted 500 simulations. We noticed that the results are highly sensitive on the number of simulations used. Therefore, we opted for a higher number of simulations, prioritizing precision over computation speed. The computational time is approximately **50 seconds**.
The weights presented in the initial table, particularly the largest ones, may vary with a maximum error of 0.01. In order to compare the results with the report we fixed the seed only in this particular case.
To obtain the robust frontier, for each simulation, we sampled the returns and the multivariate normal variables. Then we built the efficient frontier, as in section 1, and calculated the robust frontier as the mean of all the simulated frontiers.

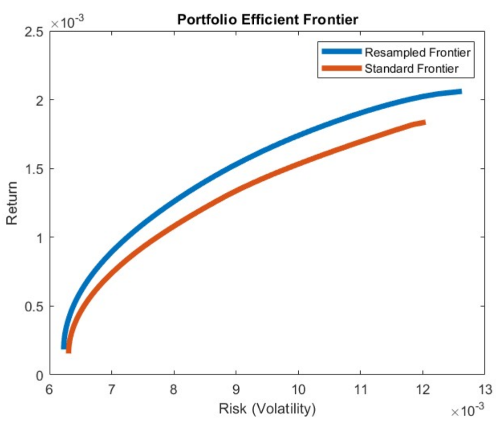


Figure 3: Efficient frontier and efficient robust frontier

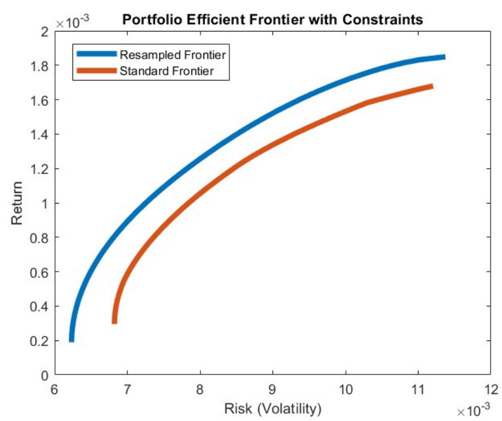


Figure 4: Efficient frontier with constraints and efficient robust frontier with constraints

As expected, the robust frontier allows for higher returns at the same level of volatility compared to the non-robust case.
We then proceeded by computing the Minimum Variance Portfolio (portfolio E and portfolio F) and the Maximum Sharpe Ratio (portfolio G and portfolio H) for both the robust frontier under standard constraints and adding additional constraints.

4 Black-Litterman Portfolio Frontier Analysis

The aim is to compute the portfolio frontier using the Black-Litterman model, incorporating two specific views on asset performance. From the frontier, we derive:

- **Portfolio I:** The Minimum Variance Portfolio (MVP).
- **Portfolio L:** The Maximum Sharpe Ratio Portfolio.

As initial data we have: **Market Capitalizations** which are data representing the overall market weight of assets, **Asset Return Covariance Matrix** which represents historical covariance matrix of returns (Cov_{ret}), **Risk Aversion Coefficient** set to $\lambda = 1.2$.

Views: Two views are incorporated:

- **View 1:** The Information Technology sector will outperform the Financials sector by 2% annually.
- **View 2:** The Momentum factor will outperform the Low Volatility factor by 1% annually.

We need some assumpions:

- **Confidence Level:** Represented by τ, a scalar reflecting uncertainty in the prior.
- **Independence of Views:** Variance-covariance matrix of the views (Ω) is diagonal.
- **Daily Returns:** Annualized views are converted into daily returns for consistency.

To calculate the Black-Litterman portfolio we proceed as follows. The market equilibrium return vector (μ_{mkt}) is computed using market capitalization weights (w_{caps}) and the covariance matrix of returns (Cov_{ret}):

$$\mu_{\text{mkt}} = \lambda \cdot \text{Cov}_{\text{ret}} \cdot w_{\text{caps}}$$

Using the P matrix (mapping views to assets) and the view expectations (q), the Black-Litterman posterior mean return (μ_{BL}) and covariance matrix (Cov_{BL}) are computed as:

$$\mu_{\text{BL}} = \left(C^{-1} + P^T \Omega^{-1} P\right)^{-1} \left(P^T \Omega^{-1} q + C^{-1} \mu_{\text{mkt}}\right)$$

$$\text{Cov}_{\text{BL}} = \left(P^T \Omega^{-1} P + C^{-1}\right)^{-1}$$

where $C = \tau \cdot \text{Cov}_{\text{ret}}$.

To build the portfolio we set some conditions:

- **Constraints:** Default constraints ensure long-only positions.
- **Optimal Weights:**
  - **Portfolio I (MVP):** Weights are derived to minimize portfolio variance.
  - **Portfolio L (Maximum Sharpe Ratio):** Weights are computed to maximize the Sharpe Ratio.

The Minimum Variance Portfolio minimizes risk regardless of returns. The allocation is derived using the posterior covariance matrix (Cov_{BL}) and posterior mean return vector (μ_{BL}). It includes assets with lower risk as determined by the posterior covariance matrix.

The Maximum Sharpe Ratio Portfolio optimizes the risk-adjusted return, incorporating both the posterior mean and covariance matrix. It allocates more to assets with higher expected returns and lower relative risk, reflecting confidence in the views.

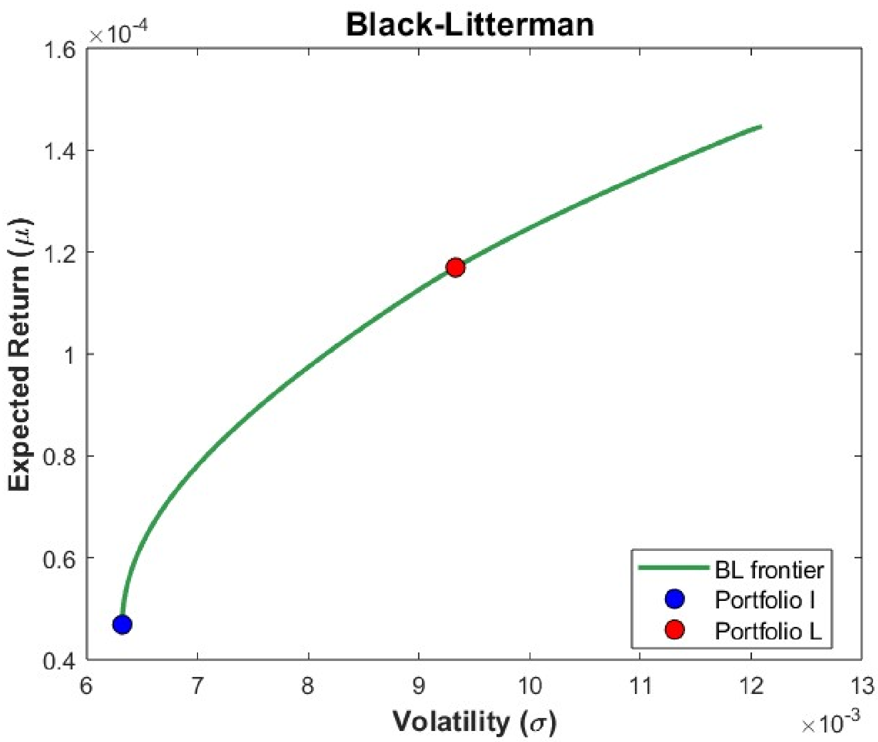


Figure 5: Black-Litterman Frontier, Maximum Sharpe Ratio Portfolio (L) and Minimum Variance Portfolio (I)

In conclusion, the Black-Litterman model integrates market equilibrium data with investor views to create robust portfolio allocations. This demonstrates the utility of the Black-Litterman approach in modern portfolio theory.

5 Maximum Diversified Portfolio and Maximum Entropy Portfolio

In this section, we compute the Maximum Diversified Portfolio and the Maximum Entropy Portfolio under the following constraints:

- Standard constraints
- The total exposure on cyclicals set to be greater than 20%
- The sum of the difference, in absolute value, of the weights of the benchmark portfolio, which is capitalization weighted portfolio, and the optimal weights set to be greater than 20%.

Both strategies prioritize diversification but differ in their objectives: the Maximum Diversified Portfolio focuses on optimizing risk distribution, while the Maximum Entropy Portfolio maximizes unpredictability in portfolio composition.
Both approaches are effective alternatives to traditional mean-variance optimization, especially when return estimates are unreliable. The Maximum Diversified Portfolio aims to optimize the risk distribution by maximizing the diversification ratio (DR). This approach ensures a spread of risks across portfolio assets. The diversification ratio is defined as:

$$DR = \frac{\sum_{i=1}^{N} w_i \sigma_i}{\sigma_p}$$

The Maximum Entropy Portfolio is inspired by principles from information theory, where entropy measures the level of unpredictability or uncertainty. In this context, portfolio entropy maximization emphasizes diversification through an even weight distribution or risk contributions. The entropy is defined as:

$$H_w = -\sum_{i=1}^{N} w_i ln(w_i)$$

To compute the results, we used the *fmincon* function. We imposed all the standard constraints as in previous sections and we incorporated the non linear ones as $nonlincon = @(w)deal(0.20 - total\_difference(Benchmark', w))$ where $total\_difference = @(benchmark, w)sum(abs(benchmark - w))$.

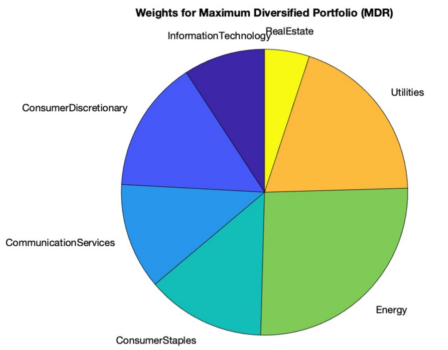


Figure 6: Maximum Diversified Portfolio Weights

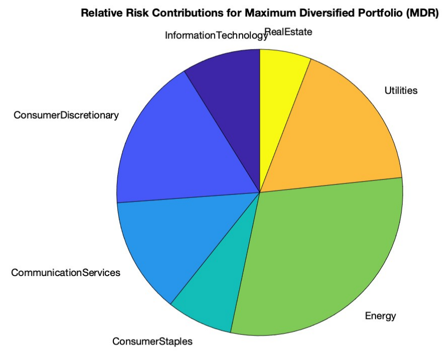


Figure 7: Maximum Diversified Portfolio risk contribution

As observed in the pie plots, not all weights are greater than zero, which deviates from our initial expectation of achieving a fully diversified portfolio. This outcome can be attributed to the applied constraints.

Furthermore, the weights that are greater than zero are relatively similar, aligning with our expectation for a well-diversified portfolio.

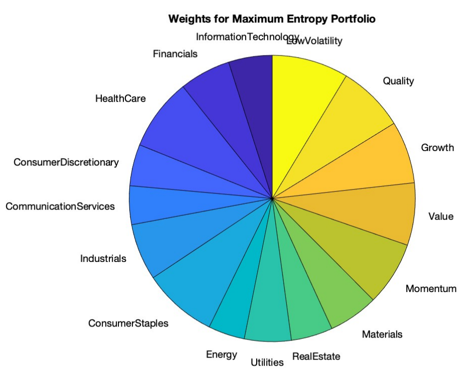


Figure 8: Maximum Entropy Portfolio Weights

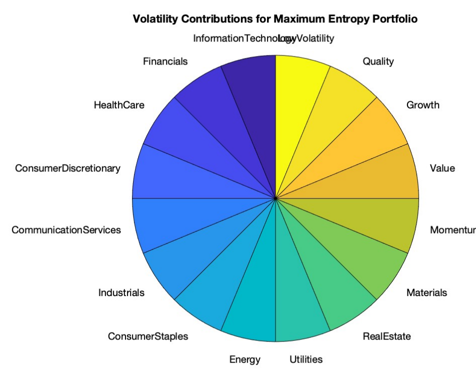


Figure 9: Maximum Entropy Portfolio risk contribution

Conversely, in the second portfolio under consideration, we observe a well-diversified allocation where all weights are greater than zero. Additionally, the risk contributions exhibit a consistent pattern, further supporting this behavior.

6 Principal Component Analysys

Principal Component Analysis (PCA) is a statistical method for dimensionality reduction that transforms a set of features into a smaller one while retaining a given amount of information. It achieves this by linearly transforming data into a new coordinate system defined by orthogonal axes called principal components. The first principal component captures the direction of the maximum variance in the data, the second the remaining variance orthogonal to the first, and so on. This method removes multicollinearity, improves data interpretability and reduces complexity in large datasets.

Our objective was to compute the portfolio, using the Principal Component Analysis, that maximizes its expected return under the following constraints (to be considered all at once):

- Standard constraints
- The volatility of the portfolio set to be equal to or less than a target volatility:

  $$\sigma_P \leq \sigma_{\text{tgt}} = 0.7$$

  where σ_P is the portfolio volatility.

Moreover, we aimed to use the minimum number of principal components that cumulatively explain at least the 90% of the variance.

To achieve this, we implemented Principal Component Analysis (PCA) in MATLAB. The key steps in our approach are outlined below.

Before applying PCA, the logarithmic returns were standardized to ensure that the variables were on the same scale. Standardization is achieved by subtracting the mean and dividing by the standard deviation of each variable:

$$R_{\text{std}} = \frac{R - \mu}{\sigma}$$

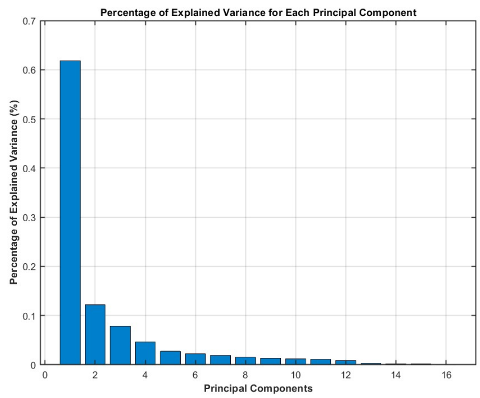
where R represents the logarithmic returns, μ the expected return, and σ the standard deviation.

PCA was applied to the standardized data to extract the latent variables (principal components) and their explained variances. The cumulative variance explained by the first n components was calculated to determine the minimum number of components required.

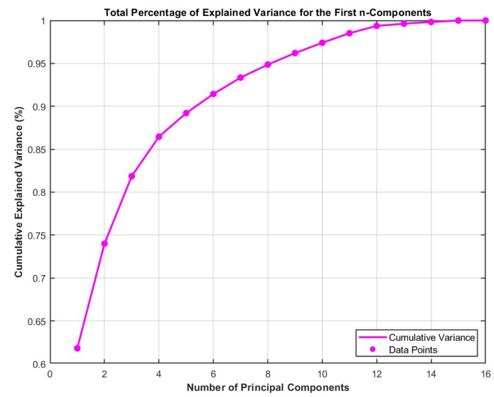
The results (Figure 10b) indicate that using the first 6 principal components meets the variance retention requirement.

PCA was used to compute the factor loadings and the reduced covariance matrix. The optimization problem was formulated to maximize the portfolio's expected return under the following constraints:

- The total portfolio weight set to sum to one.



(a) Figure 9: Percentage of explained variances for each principal component.



(b) Figure 10: Cumulative variance explained as a function of the number of principal components.

- Portfolio volatility set not to exceed the target volatility ($\sigma_{\text{tgt}} = 0.7$).
- Factor weights set to be consistent with asset weights via the factor loadings.

This is mathematically represented as:

$$\text{Max} \sum_{i=1}^{p} \mu_i w_i \quad \text{subject to:}$$

$$\sum_{i=1}^{p} w_i = 1, \quad w^T \Sigma w \leq \sigma_{\text{tgt}}^2, \quad L^T w = f$$

where w represents asset weights, L the factor loadings, f the factor weights, and Σ the covariance matrix.

The problem was solved using MATLAB's optimization toolbox. Initial weights were set equally across assets, and constraints were enforced using the Sequential Quadratic Programming (SQP) algorithm. The final weights for each asset were obtained and can be used to construct the optimal portfolio.

The optimal portfolio derived from PCA demonstrates improved efficiency by leveraging dimensionality reduction while meeting all specified constraints.

7 VaR-modified Sharpe Ratio

In this section, we aim to evaluate the portfolio that maximizes the VaR-modified Sharpe Ratio under standard constraints.
The VaR-modified Sharpe Ratio focuses on downside risk, specifically the risk of extreme losses.

This is particularly valuable for stress testing scenarios.Notable, the VaR (Value at Risk) captures tail risk and provides a realistic reflection of the risk in portfolios with non-normal return distributions.

To compute the modified Sharpe Ratio, we began by evaluating the VaR at the 95th percentile. We then calculated the objective function as follows:

$$obj(w) = -\frac{\mu \cdot w}{\mu w + z_\alpha \sqrt{w' \cdot C \cdot w}}$$

where C is the covariance matrix of the log-returns and z_α is the quantile of the standard normal distribution. To find the weights for portfolio Q - the one that maximizes the modified Sharpe Ratio - we minimized the objective function using MATLAB's *fmincon* function.

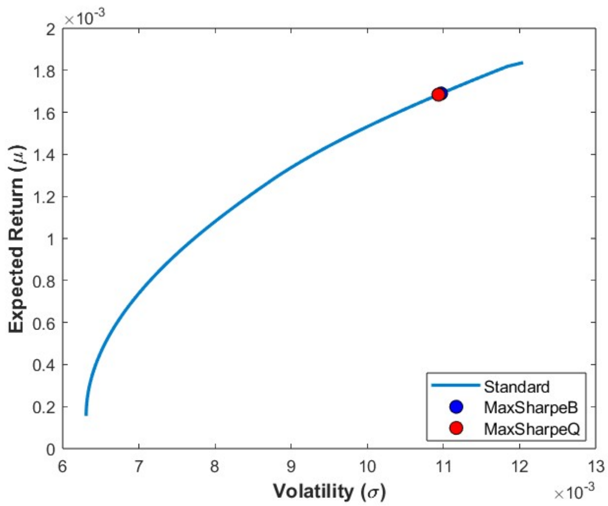


Figure 11: Efficient frontier, Portfolio B and Q

This figure illustrates a comparison between Portfolio B and Portfolio Q, with the aim of highlighting the differences between the traditional Sharpe Ratio and its modified version. From the plot we can see that the two portfolios appear to be quite similar. Notably, the modified Sharpe Ratio Portfolio has a slightly lower volatility which results in smaller expected return.

By investigating the weights of the two portoflios, they demonstrate significant similarity, both in the sectors that have values different from 0 and in the values of the weights themselves.

8 Metrics Analysis

This section of the report evaluates the performance, risk, and diversification of the previous portfolios. The portfolios were optimized with various allocation strategies, which included constraints that reflect economic behavior. The metrics used to analyze portfolios include annual returns, volatility, Sharpe Ratios, Maximum Drawdowns and Calmar Ratios. The equally weighted portfolio serves as a benchmark. Similarly to Section 3, the results related to portfolios E, F, G, and H can differ by up to 0.001 compared to those in the code.

The results, summarized in Table 2, illustrate the distinct characteristics of each portfolio. Portfolio B (Maximum Sharpe Ratio Portfolio under standard constraints) achieved the highest Sharpe ratio (3.03), reflecting exceptional risk-adjusted performance. Similarly, Portfolio Q, optimized based on

VaR-modified Sharpe Ratio, closely matched Portfolio B in both Sharpe ratio and returns (52.8%), confirming the robustness of this optimization method.

In contrast, Portfolio A (Minimum Variance Portfolio) had the lowest risk (10.03% volatility) but correspondingly modest returns (4.05%). This aligns with the expected trade-off between risk and return in efficient frontier theory.

Portfolios E and F, derived using resampling, showed slightly enhanced Sharpe ratios compared to Portfolio A, demonstrating the utility of robust optimization techniques in reducing estimation errors. The Maximum Diversified Portfolio (M) and Maximum Entropy Portfolio (N) exhibited balanced performance with returns of around 13.8% and moderate volatility.

Table 2: Portfolio Performance Metrics

Portfolio	Annual Returns	Volatility	Sharpe Ratio	Max Drawdown	Calmar Ratio
EW	0.1536	0.1216	1.2634	-0.0982	1.5647
A	0.0405	0.1003	0.4041	-0.0965	0.4199
B	0.5300	0.1749	3.0296	-0.0936	5.6624
C	0.0768	0.1085	0.7082	-0.0875	0.8781
D	0.4903	0.1640	2.9901	-0.0850	5.7698
E	0.0442	0.1004	0.4408	-0.0942	0.4695
F	0.0453	0.1004	0.4516	-0.0939	0.4830
G	0.4174	0.1536	2.7181	-0.0887	4.7037
H	0.3961	0.1475	2.6861	-0.0824	4.8094
I	0.0400	0.1003	0.3991	-0.0965	0.4146
L	0.4262	0.1482	2.8754	-0.0771	5.5300
M	0.1358	0.1271	1.0681	-0.0929	1.4612
N	0.1381	0.1173	1.1770	-0.0974	1.4185
P	0.3109	0.1467	2.1191	-0.0864	3.5987
Q	0.5280	0.1744	3.0279	-0.0931	5.6705

The diversification metrics highlight the benefits of strategies such as Maximum Entropy (N) and Maximum Diversification (M), which ensure balanced allocation across indices. Portfolio P, constructed via Principal Component Analysis, achieved substantial returns (31.1%) while maintaining a relatively low volatility (14.67%), underscoring the effectiveness of factor-based optimization.

Compared to the benchmark (Equally Weighted Portfolio), Portfolio Q and B substantially outperformed in terms of both risk-adjusted returns and drawdowns. This demonstrates the advantage of tailored optimization strategies over naive allocation. The equally weighted portfolio (EW), while not optimized, showed a reasonable Sharpe ratio (1.26) and a Calmar ratio (1.56), outperforming several portfolios in terms of diversification and stability.

The analysis reveals that portfolios optimized for Sharpe Ratio and diversification outperform in terms of risk-adjusted returns and resilience to market fluctuations. Resampling techniques (E, F) and advanced models like Black-Litterman (I, L) or PCA (P) provide robust and adaptive frameworks for portfolio construction.

9 Part B

In this last section, we want to use the portfolio allocations computed from steps 1 to 7 (from portfolio A to Q) to evaluate the out-of-sample performance of the portfolios in the period 01/01/2024 to the end of the available data, which corresponds to 25/10/2024.
The importance of this additional analysis is related to the possibility of verifying whether the assumptions and simplifications we made during our computations are reasonable or not.
To do so, we computed the metrics, as done in point 8, for the new values we are considering. We obtained the following results:

Table 3: Portfolio Performance Metrics

Portfolio	Annual Return	Annual Volatility	Sharpe Ratio	Max Drawdown	Calmar Ratio
EW	0.2612	0.1028	2.5409	-0.0569	4.5936
A	0.1776	0.0812	2.1865	-0.0445	3.9858
B	0.4823	0.2106	2.2902	-0.1520	3.1728
C	0.1875	0.0945	1.9829	-0.0529	3.5433
D	0.4912	0.2077	2.3649	-0.1497	3.2819
E	0.1896	0.0817	2.3195	-0.0436	4.3530
F	0.1927	0.0818	2.3566	-0.0434	4.4381
G	0.4098	0.1722	2.3791	-0.1247	3.2875
H	0.4246	0.1731	2.4528	-0.1247	3.4049
I	0.1772	0.0812	2.1826	-0.0446	3.9764
L	0.5069	0.2044	2.4798	-0.1469	3.4503
M	0.2315	0.1033	2.2416	-0.0520	4.4564
N	0.2589	0.0999	2.5928	-0.0536	4.8343
P	0.3007	0.1328	2.2637	-0.0884	3.4038
Q	0.4825	0.2103	2.2941	-0.1518	3.1781

When comparing the portfolio performance metrics for 2023 and 2024, several key differences emerge that reflect the impact of changing market conditions.

Firstly, annual returns show a marked difference between the two years. In 2023, the portfolios generally exhibited lower returns, particularly in Portfolio A (0.04052) and C (0.07683), compared to the stronger performance in 2024, where Portfolio A returned 0.17756 and Portfolio C 0.18746. Notably, Portfolio B also showed a significant decline in returns from 0.52996 in 2023 to 0.48226 in 2024. This may indicate that the markets in 2023 were more volatile, but offered opportunities for higher returns, while in 2024, the performance was more moderate and stable.

In terms of volatility, Table 2 shows generally lower values compared to Table 1 (2023). This suggests that the market in 2024 was likely less volatile, reflecting perhaps greater market stability or more favorable economic conditions.

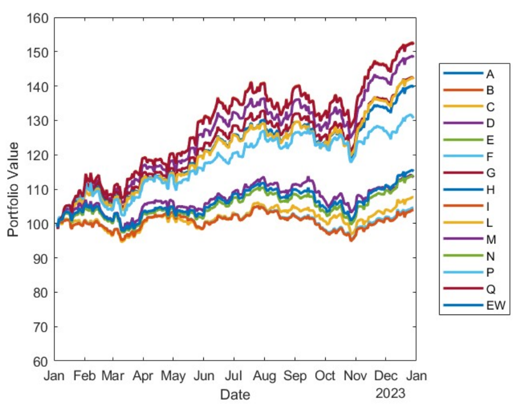
The Sharpe ratio, a measure of risk-adjusted return, is significantly higher in 2024 for most portfolios. This indicates that, despite lower returns in 2024, these portfolios were more efficient in delivering higher returns for each unit of risk compared to 2023. The higher Sharpe ratios in 2024 may reflect a more favorable risk-return environment, or better diversification strategies implemented in the portfolios.

Looking at Max Drawdown, the figures for 2024 show a less severe downside risk compared to 2023. This is consistent with the lower overall volatility, indicating that while the market was less volatile in 2024, there were still periods of significant market downturns.

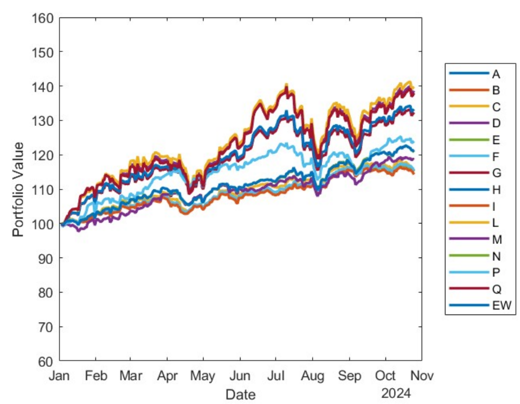
Lastly, the Calmar ratio was generally higher in 2024, reflecting more efficient risk management.

Overall, the differences between 2023 and 2024 highlight the impact of varying economic and market conditions, with 2023 seeing higher returns and volatility, while 2024 offered a more stable, risk-adjusted performance. These differences may be attributable to factors such as macroeconomic trends, market sentiment and the effectiveness of portfolio strategies in adapting to these conditions.

The last plots represent the standardized paths of the during the analyzed periods (considering 100 as the initial value for all the portfolios). These plots confirm our observations since the second year, in general, is more stable but less profitable compared to the first one.



(a) Figure 9: Standardized portfolios values 2023.



(b) Figure 10: Standardized portfolios values 2024.