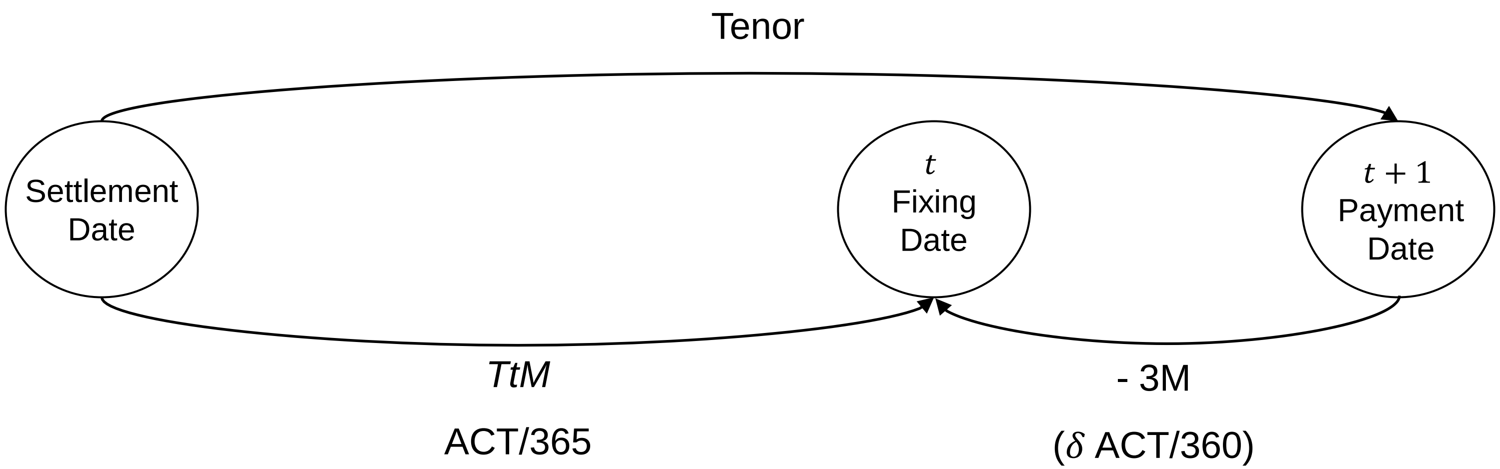
Assignment 6: Take Home Messages

Caplet volatility surface bootstrap:

- ✓ Mid-term volatility interpolation on fixing dates and not on payment dates
- ✓ Bachelier pricer: (*t*, *t+1*)-caplet fixes in advance, therefore time to maturity refers to *t*



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Managing caplet volatility surface:

✓ Interpolation rules: linear on fixing dates, cubic spline on strikes

Example: consider $\frac{\partial \sigma}{\partial K}$ that is the derivative of caplet spot volatility with respect to the strike. Would you prefer a linear or spline interpolation?

✓ Extrapolation rules: flat on fixing dates and strikes

Example: consider a floating rate loan Libor 3m + spol (i.e. 3%) with a global floor:

$$max\{L^{3m} + 3\%; 0\%\} = L^{3m} + 3\% + max\{-3\% - L^{3m}; 0\%\}$$

So that in the payoff there is an embedded floor option with -3% strike. In order to price this option, would you prefer a linear or a flat extrapolation rule considering the market data of the current assignment?

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Algorithm to boostrap spot volatility surface:

Given $\{L(T_0, T_i, T_{i+1}), \delta(T_i, T_{i+1}), ttm(T_i, T_{i+1})\}$ for each $i+1$ quarterly payment date, repeat the following procedure for each quoted strike K:

For each m in maturities:

  $cap_m = \sum_i caplet_i\left(L(T_0, T_i, T_{i+1}), K, ttm(T_i, T_{i+1}), \sigma_m^{flat}\right)$

  if m == “1y”:

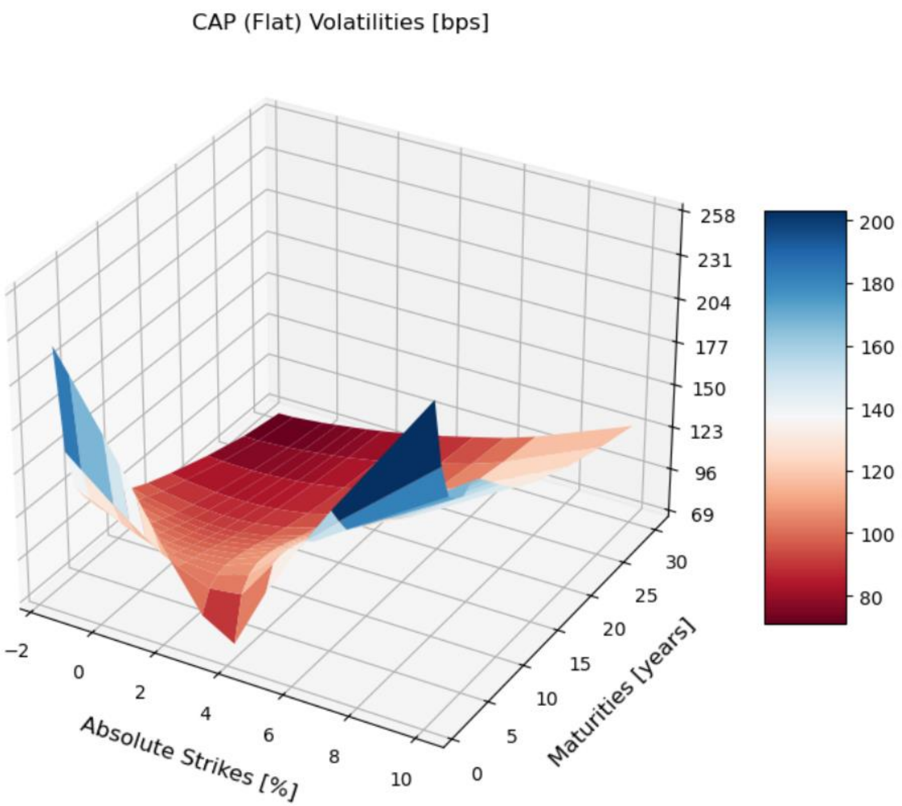
    $\sigma_i^{spot} = \sigma_{1y}^{flat}$ for i $\leq$ 1y

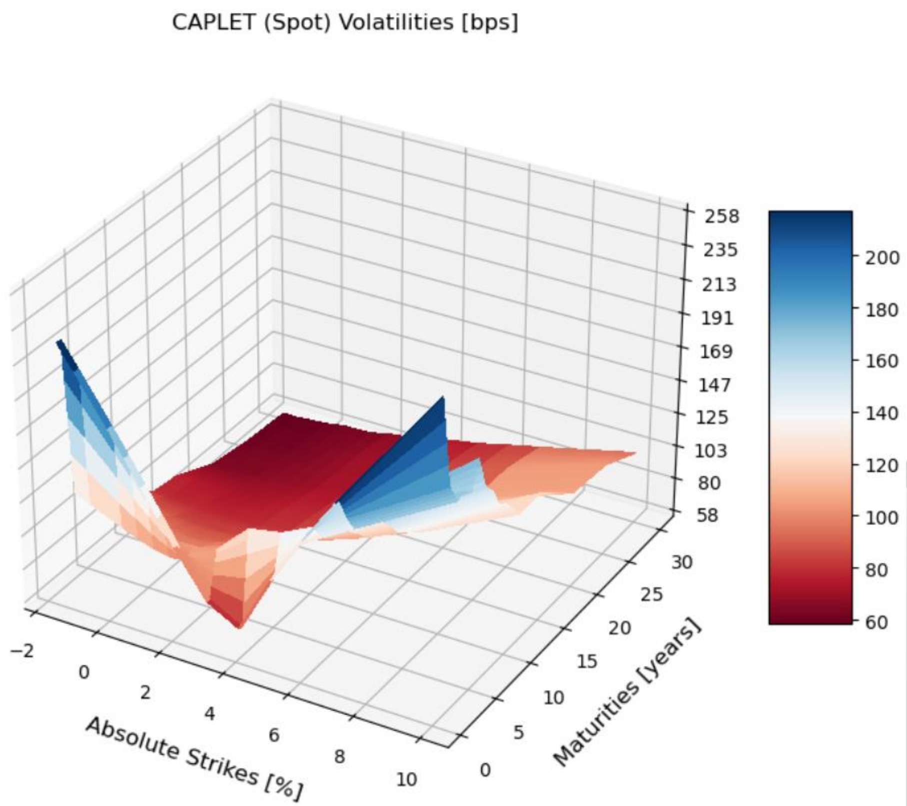
  else:

    define σ_i^{spot} as a linear interpolation in terms of $ttm(T_i, T_{i+1})$ between previous terminal σ_{m-1}^{spot} and unknown terminal σ_m^{spot}

    define $\Delta cap = \sum_{i>m-1}^{m} caplet_i\left(L(T_0, T_i, T_{i+1}), K, ttm(T_i, T_{i+1}), \sigma_i^{spot}\right)$

    solve via Newton imposing $\Delta cap = cap_m - cap_{m-1}$





How do the two interpolations methods (i.e. linear or cubic spline) take into account volatility skew for each fixed maturity?

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$$d_i = \frac{L(T_0, T_i, T_{i+1}) - K}{\sigma_i \sqrt{T_{i+1} - T_i}}$$

$$caplet_i(\sigma_i) = B(T_0, T_{i+1}) \cdot \delta(T_i, T_{i+1}) \cdot \left[(L(T_0, T_i, T_{i+1}) - K) \cdot N(d_i) + \sigma_i \sqrt{T_{i+1} - T_i} \cdot \phi(d_i) \right]$$

$$vega_i(\sigma_i) = B(T_0, T_{i+1}) \cdot \delta(T_i, T_{i+1}) \cdot \left[\sqrt{T_{i+1} - T_i} \cdot \phi(d_i) \right] \cdot 1bp$$

In order to compute the total vega sensitivity, what would you prefer?

- ❑ Use the closed formula that relies on (interpolated) spot volatilities, which results in 56.7 k€ sensitivity
- ❑ Use the numerical finite difference method via 1bp-parallel shift of flat volatility surface and subsequent boostrap, which results in 55.8 k€ sensitivity

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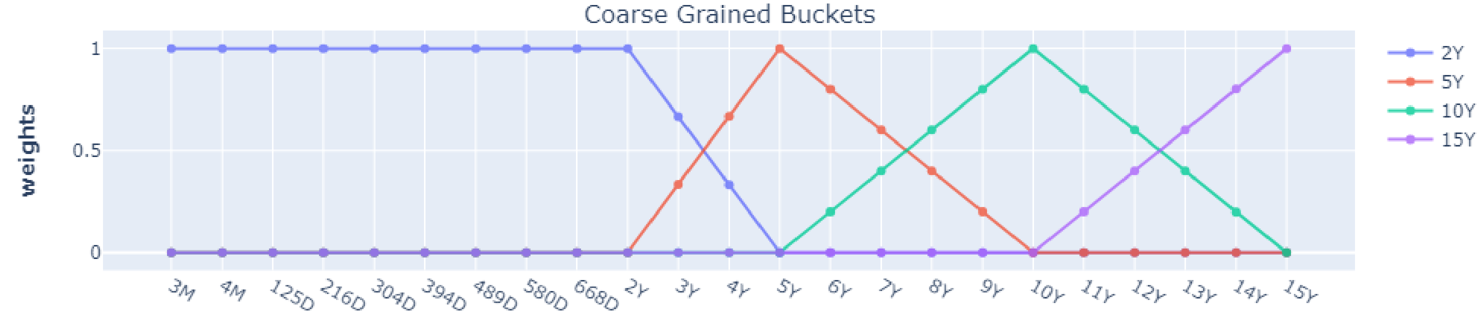
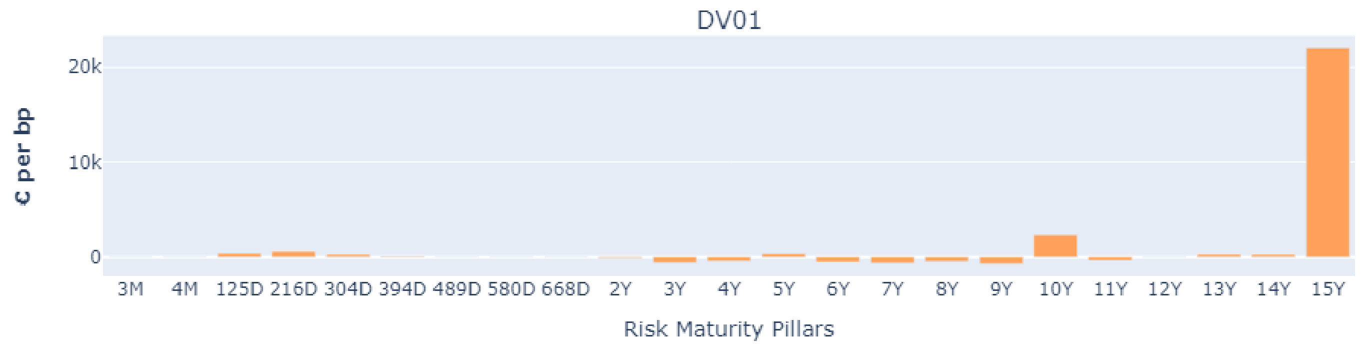
In order to hedge the bucketed delta (vega) risk via coarse grained technique, you should take into account the following quantities:

- The coarse grained bucketed weights $\{\beta_i, i = 2y, 5y, 10y, 15y\}$
- Fixed the i year, compute the delta (vega) risk r_j^i of each hedging instrument whose maturity is $j \geq i, j \in \{2y, 5y, 10y, 15y\}$. Therefore, bumping $10y$ par instrument does not have any effect on $2y$ and $5y$ instruments
- Invert the upper triangolar matrix to detect the notional $\{\omega_i, i = 2y, 5y, 10y, 15y\}$

$$
\begin{cases}
\omega_{2y} \cdot r_{2y}^{2y} + \omega_{5y} \cdot r_{5y}^{2y} + \omega_{10y} \cdot r_{10y}^{2y} + \omega_{15y} \cdot r_{15y}^{2y} = \beta_{2y} \\
\omega_{5y} \cdot r_{5y}^{5y} + \omega_{10y} \cdot r_{10y}^{5y} + \omega_{15y} \cdot r_{15y}^{5y} = \beta_{5y} \\
\omega_{10y} \cdot r_{10y}^{10y} + \omega_{15y} \cdot r_{15y}^{10y} = \beta_{10y} \\
\omega_{15y} \cdot r_{15y}^{15y} = \beta_{15y}
\end{cases}
$$

DV01 sensitivity

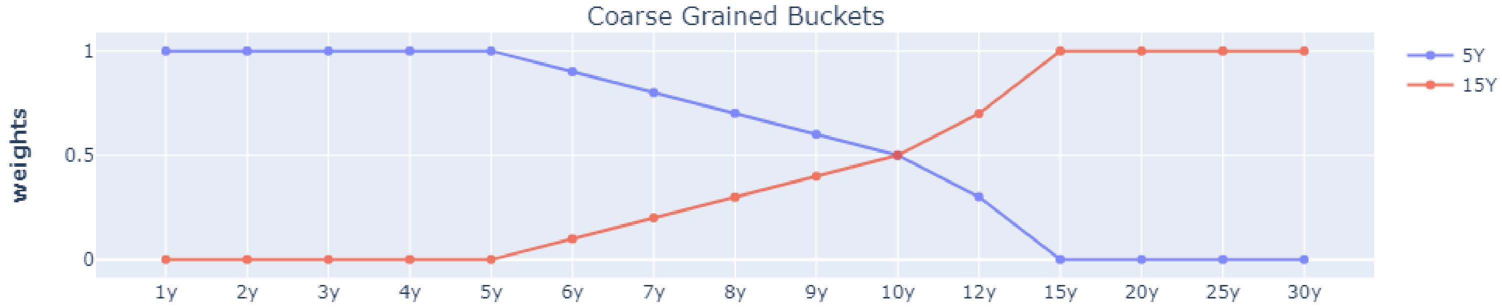


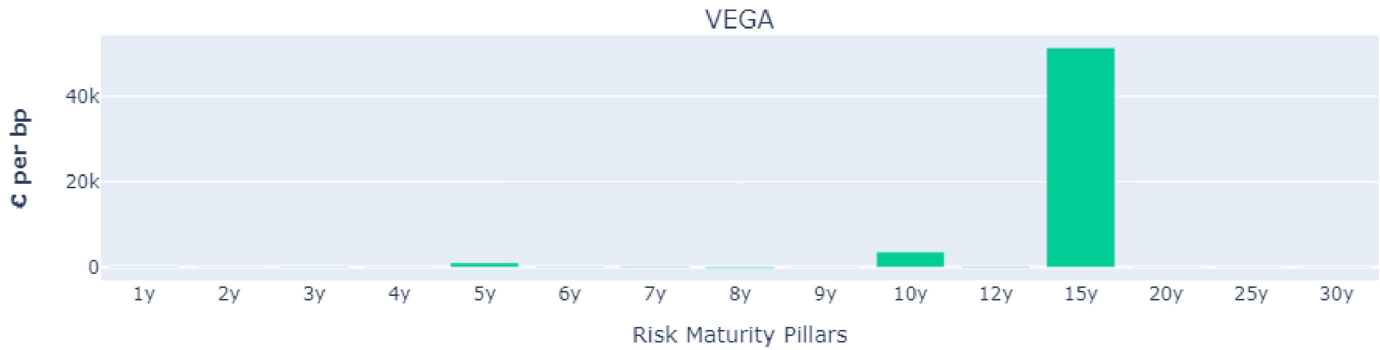


	2y	5y	10y	15y
Coarse Grained Sensitivity	0.6 k€	-1.3 k€	0.9 k€	22.3 k€
Hedging Swap Notional	3.1 Mio	2.9 Mio	1 Mio	18.4 Mio
Hedging Swap Type	Receiver	Payer	Receiver	Receiver

Total Delta = 22.6 k€ per 1bp

Vega sensitivity





	5y	15y
Coarse Grained Sensitivity	2.7 k€	53.1 k€
Hedging Atm Cap Notional	10.9 Mio	45.1 Mio
Hedging Atm Cap Side	Sell	Sell

Total Vega = 55.9 k€ per 1bp