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# Lab 01: Brake distribution

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## Abstract

The braking distribution of a simplified car model was optimized by varying brake piston diameters. The objective was to minimize the error between ideal and real braking curves using the Simplex Method. The problem was found to be not well-posed because an infinite number of solutions exist, all defined by a constant ratio of piston areas, which dictates the optimal performance.

# Contents

# 1. Introduction

The aim of this first lab was to optimize the braking distribution of a simplified model of a car by varying the diameters of the pistons of the braking system.

## 1.1 Description of the problem

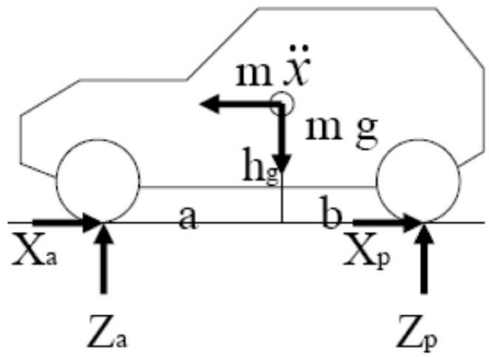


*Figure 1*

The model of the car considered is the one reported in *figure 1*.

### *Ideal distribution*

The longitudinal forces, vertical forces and momentum equilibria are written and the expression of the vertical forces acting on the rear and front tyre is derived. Through the friction coefficient μ (equal for front and rear axle), the maximum longitudinal forces can be estimated. The ideal braking force distribution is finally determined by considering the longitudinal force equilibrium and it is expressed as a function of the vehicle's characteristics:

$$X_{A,\max} = m\ddot{x}\left(\frac{b}{a+b} + \frac{\ddot{x}}{g}\frac{h_G}{a+b}\right)$$

$$X_{P,\max} = m\ddot{x}\left(\frac{a}{a+b} - \frac{\ddot{x}}{g}\frac{h_G}{a+b}\right)$$

- $a, b$ define the position of the center of mass with respect to the front and rear axle
- $m$ is the mass of the vehicle
- $\ddot{x}$ is the vehicle longitudinal acceleration
- $h_G$ is the height of the center of mass

These expressions represent a quadratic function in the $X_A, X_P$ plane, where the braking force at the front $X_A$ has been assumed as independent variable.

### *Real distribution*

The real distribution cannot trace perfectly the ideal one that can only be approximated. The linear approximation of the curve is obtained by expressing the braking force as a function of the design variables, that are the front and rear diameters of the pistons pushing the pad against the disk:

$$X = \frac{r}{R}T = \frac{r}{R}\mu_d N = \frac{r}{R}\mu_d * pA * 2n$$

- $r$ is the effective radius
- $R$ is the wheel radius
- $T$ is the braking torque
- $\mu_d$ is the friction coefficient between pad and disk
- $p$ is the pressure in the hydraulic system
- $A$ is the area of a single piston
- $n$ is the number of pistons on one side of the caliper

Assuming the same effective radius for front and rear disks, the same number of pistons and the same friction coefficient between disk and pads, the relationship between front/rear braking force is obtained:

$$X_P = X_A * A_P / A_A$$

- $A_P$ is the area of the pistons at the rear
- $A_A$ is the area of the pistons at the front

The braking force at the rear axle must not exceed the ideal distribution curve, since this would lead to a potentially dangerous situation in which the rear tires might lock up before the front ones, thus making the vehicle unstable. At the intersection point between the two curves, the brake splitter operates by closing part of the hydraulic system and maintaining the rear force constant.

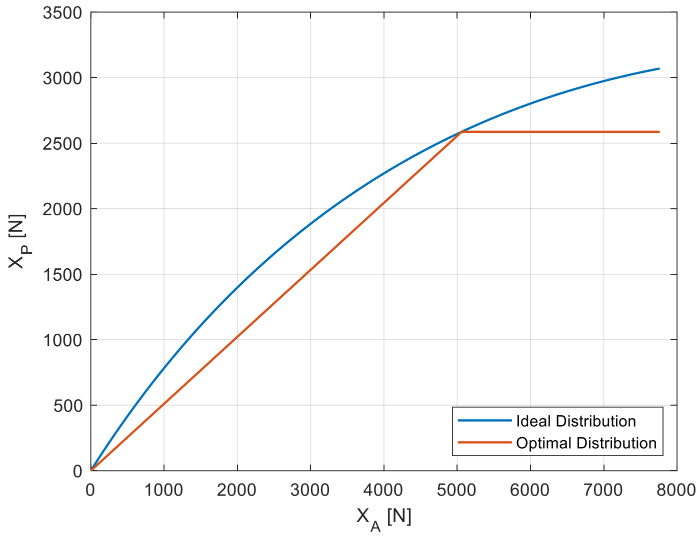
3500
3000
2500
2000
1500
1000
500
0
$X_P$ [N]
0
1000
2000
3000
4000
5000
6000
7000
8000
$X_A$ [N]
Ideal Distribution
Optimal Distribution

*Figure 2*

# 2. Optimization

## 2.1 Objective function and optimization method

The problem that needs to be solved is nonlinear and single objective; the optimization task was programmed in MATLAB using the optimization function *fminsearch*. This function implements the **Simplex Method**. This method is not efficient but frequently used since it does not require the evaluation of derivatives.

The simplex method is:

- Zero order method
- Direct method

The steps of the simplex method algorithm can be schematized as follows:

1. Simplexes are generated in the design variables space;
2. The function to be minimized is evaluated in the points that represent the vertices of the simplexes;
3. The point where its value is largest is discarded. This point is then reflected with respect to the opposite side and the process repeated.

The most important step in the design of an optimization process is the definition of the objective functions. In this case, we only have one, which is the square of the difference between the ideal and real distribution curve:

$$e = \frac{1}{N} * \sum\left(X_{p,ideal} - X_p\right)^2$$

The fact that the real force distribution is a broken line has been considered in the definition of the objective function.

Two constraints need to be respected in the range of values that the front and rear piston diameters can assume: they need to be in the range $15mm \div 50mm$. To select the initial guesses, two values in the provided range have been selected making sure that the front diameter is larger than the rear diameter, otherwise the line would always be above the ideal distribution.

Since *fminsearch* is not designed to operate with constraints, a check on the simulation output is performed to verify if the constraints have been respected.

## 2.2 Different initial conditions choice

After the first optimization process, some other trials were conducted. It was observed that different initial conditions brought different solutions. This occurs with a constat value of the ratio of the two pistons areas, which relates to the fact that, in the real brake force distribution, the areas ratio is the only parameter which influences the initial slope.

The computed value for this ratio is: $\frac{A_p}{A_A} = 0{,}5110$

## 2.3 Well-posedness of the problem

### *Definition of well-posedness of a problem*

A problem is well-posed if it satisfies three essential criteria:

1. **Existence**: A solution to the problem exists.
2. **Uniqueness**: The solution is unique.
3. **Stability**: The solution depends continuously on the input of the data (initial conditions, boundary conditions, or parameters). This means that a small change in the input data only leads to a small change in the solution.

Given the previous observations, the problem is not well posed.

By plotting in Figure 3 the surface representing the objective function as function of the two variables it is possible to comment:

- The solution is not unique; there are multiple and equal minimum values (in red);
- The solutions all lay on a constant slope line, which represents the areas ratio which minimizes the chosen objective function;
- Some values where excluded
    - $A_P > A_A$ : it is not possible to find an intersection with the ideal curve, moreover it would not make sense physically as it opposes to the load transfer that occurs in braking
    - $\frac{A_A}{A_P} > 2.5$ : the solution would be clearly not optimal, as an intersection with the optimal curve is not feasible in that range of longitudinal accelerations

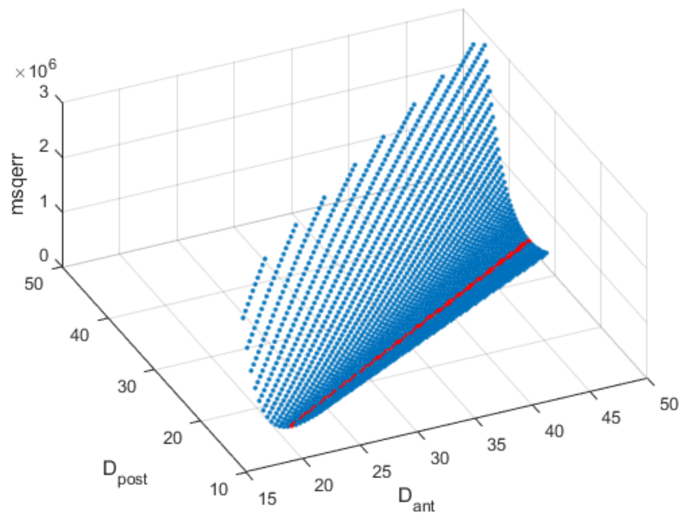


*Figure 3*

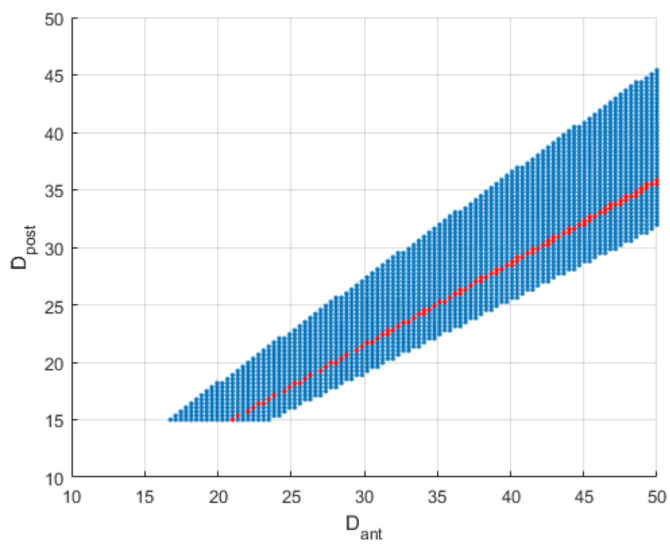


*Figure 4*

# 3. Conclusion

The optimization successfully minimized the distribution error but proved to be not well-posed (lacking uniqueness). Performance is solely determined by the piston area ratio, which controls the initial braking slope. Future work requires a secondary objective (e.g., minimum weight) to select a unique, physically optimal design, along with a constrained optimization solver.