Politecnico di Milano
Gaia Biasolo, Matteo Giuseppe Cigada, Federico D'Agostini

Lab07: multi objective optimization of a double wishbone suspension system

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Abstract

This report details the multi-objective optimization of a double wishbone suspension system geometry using a Genetic Algorithm (GA). The goal was to minimize two objective functions related to the suspension's performance: the roll center height (RCH) and the camber change rate. These functions are defined as non-dimensional errors relative to specific target values. The design variables are the inclination angles of the wishbone arms.

A standard GA cycle is implemented, utilizing a dominance-based ranking to assign fitness to individuals and identify subsequent Pareto optimal sets. The final Pareto front provides a range of trade-off solutions, highlighting that the optimal choice depends on the designer's prioritization of minimizing RCH error or camber error.

Contents

1. Introduction

The aim of this lab was to optimize the geometry of a double wishbone suspension in order to minimize two objective functions related to the performance of the suspension through some kinematic relations.

This was done using the genetic algorithm previously implemented in Lab06.

1.1 Description of the problem

1.1.1 Double wishbone characteristics

For this analysis, the following simplified scheme of a double wishbone suspension was considered (Figure 1).

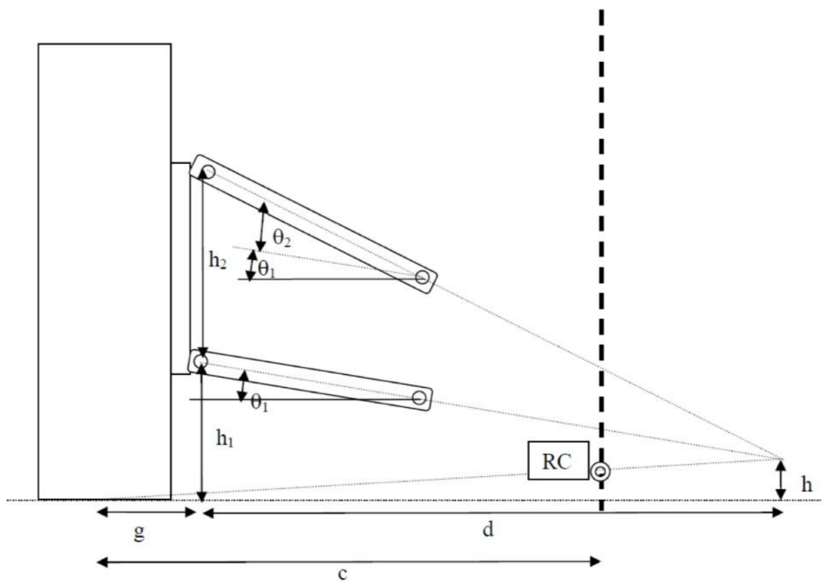

Figure 1

This suspension is characterized by two control arms (the "wishbones") connecting the wheel assembly to the vehicle chassis.

The two aspects that this assignment requires to analyze are the camber change and the roll center height.

As far as the camber change is concerned, due to the different inclination of the two arms (converging geometry), when the tire is displaced vertically of a small quantity, it tends also to rotate changing the camber.

By keeping the camber change small, the performance in straight line conditions can be improved and predictable handling can be ensured in various maneuvers. Instead, a high rate of camber change is beneficial for maximizing cornering grip when roll is high. Overall, a compromise must be found

Instead, the roll center height is strictly related to how the car (specifically its sprung mass) reacts to lateral forces. The standard geometric method for finding it requires 3 steps:

- Lines are drawn extending through the upper and lower control arms to find the Instantaneous Center (IC).
- A line is drawn from the tire contact patch to the IC.
- The point where this line intersects the vehicle's vertical centerline (the thick dashed line) is the Roll Center.

Specifically, the effects linked to the roll center height that were accounted for on this assignment are the centrifugal moment and the jacking forces.

1) The centrifugal moment is the torque causing the body to roll in a curve. Lower RC height increases the distance between the center of gravity and the RC. This results in a larger roll moment, causing more body roll for a given lateral force. Instead, a higher RC height decreases the distance between the CG and the RC, resulting in a smaller roll moment. This effectively provides "geometric anti-roll" and reduces the reliance on anti-roll bars to resist this phenomenon.
2) Jacking forces are the vertical components of the lateral forces applied to the tyre during cornering maneuvers that get transmitted through suspension links. These forces are proportional to the lateral force applied in the contact patch through the tangent of the angle created by the ground and the line connecting the contact patch with the roll center.

Also in this case a trade off must be met to improve the performance during cornering.

1.1.2 Desired characteristics and targets

The design variables considered in this assignment were the two angles defining the inclination of the wishbone arms, θ_1 and θ_2.

The kinematic quantities needed to define the objective functions are (Figure 1):

$$h_1 = 0.190\ m;$$

$$h_2 = 0.270\ m;$$

$$g = 0.050\ m;$$

$$c = 0.870\ m;$$

$$d = \frac{h_2}{\tan(\theta_1+\theta_2)-\tan(\theta_1)};$$

$$h = h_1 - d * \tan(\theta_1).$$

The design variables θ_1 and θ_2 are bound and usually depend on the available space in the vehicle.

	Lower bound	Upper bound
θ_1	-9°	-1°
θ_2	+5°	+15°

Since a tradeoff is required for both objectives (camber change and roll center height) the target values are provided.

- $TRCH = 100\ mm$;
- $T\Delta\gamma(\Delta z = 50mm) = 2°$.

To get as close as possible to these targets, the two objective functions are defined as normalized errors with respect to reference values:

$$\min\,(f_1) \text{ with } f_1 = \frac{(RCH-TRCH)^2}{TRCH^2}$$

$$\min\,(f_2) \text{ with } f_2 = \frac{\left(\Delta\gamma(\Delta z=50mm)-T\Delta\gamma(\Delta z=50mm)\right)^2}{\left(T\Delta\gamma(\Delta z=50mm)\right)^2}$$

The request of the assignment is to solve the minimization problem with a genetic algorithm and analyze the different populations throughout the simulations (analyzing their ranking).

2. Assignment requests

2.1 Minimization using a genetic algorithm

The minimization process is implemented with a genetic algorithm like the ones shown in the assignment of Lab 06.

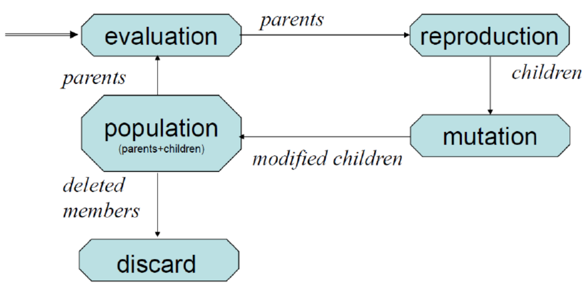

Figure 2

More specifically, the algorithm implements the **standard GA cycle** shown in Figure 2.

Differently from the previous assignment, this time the optimization problem is multi objective, since there are 2 objective functions to be minimized. Given that, the goal of the algorithm is finding the pareto set of the problem in the given variable space and, consequently, in the objective function space. To obtain this result a particular fitness evaluation is necessary.

Fitness evaluation

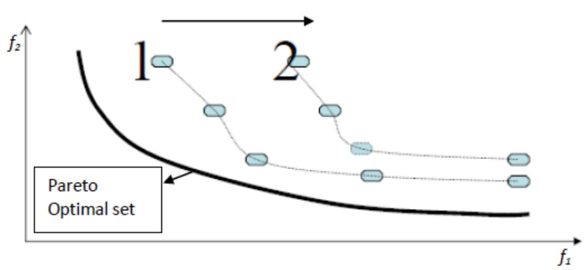

Figure 3

The general process is basically based on the concept of **dominance-based ranking**.

To assign a fitness value to every individual of the population, subsequent pareto optimal sets are individuated. More specifically, at each generation, these steps are followed:

1. A first pareto optimal set is selected based on the dominance-based ranking
    - All the individuals of this set are removed and assigned a ranking value of **1**
2. From the remaining population another pareto optimal set is selected

- All the individuals of this second set are removed and assigned a ranking value of **2**

3. This process continues until all the population is covered
    - All the individuals at the end of the process will have a ranking value

At the end of this process the fitness value will be assigned as the **reciprocal of the ranking** number. Consequently, the individual of the first pareto set will have the highest fitness, which is correct since they are the closest to the actual pareto set.

No sharing is considered in the computation of the fitness value in this assignment.

Mutation

It is introduced to add some random noise to the system; it is generally used to prevent the system from reaching a local minimum. It should always be coupled with elitism.

Elitism

As seen in the previous assignment, elitism is also introduced in the algorithm. Which means that, at each generation, only the **fittest individuals** are kept between parents, children and mutated children.

2.2 Plot of initial and final population in design variables and objective function domain

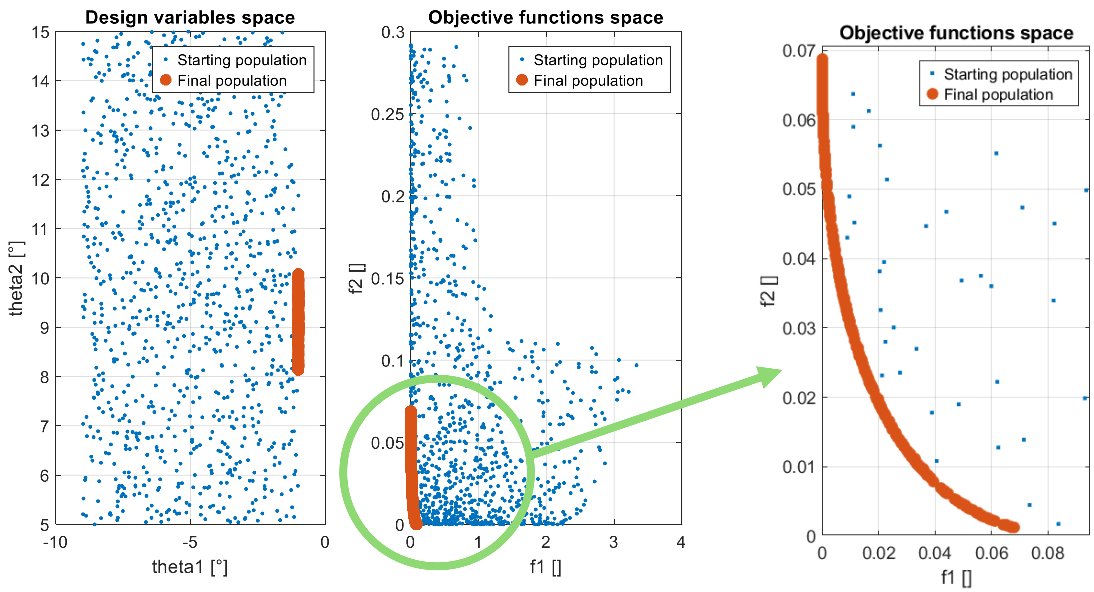

Figure 4

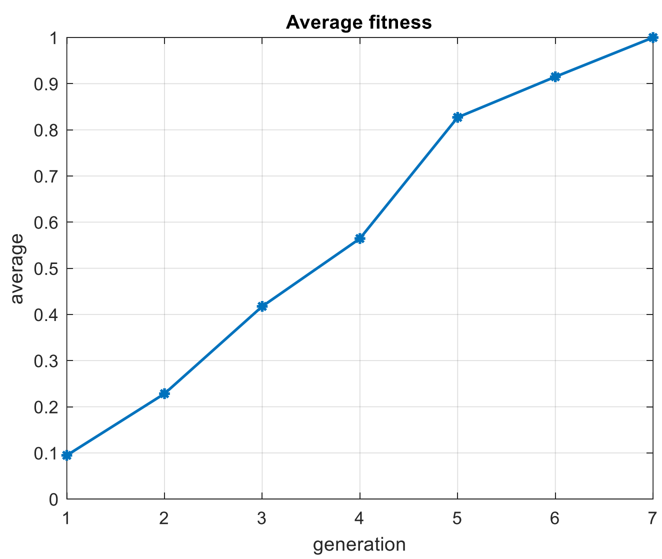

Figure 5

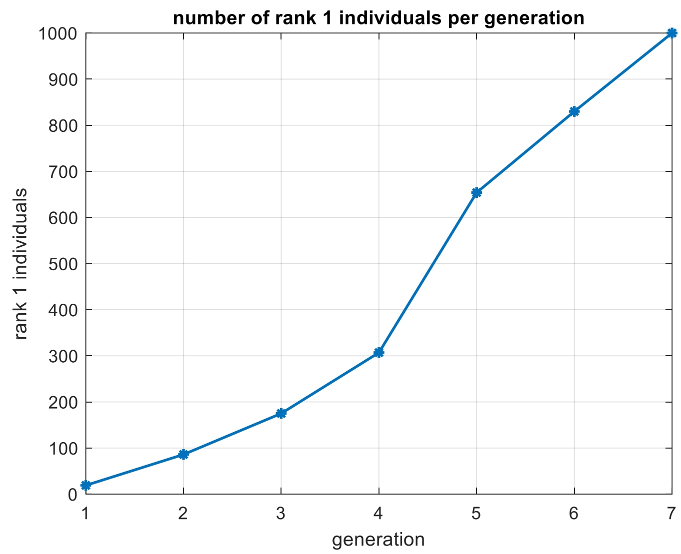

Figure 6

- Figure 4 shows the power of genetic algorithms: multiple points of the Pareto optimal set are identified simultaneously.
- In the algorithms used in the previous labs one point of the Pareto optimal set was computed at each iteration; in this case, the whole final population represents points on the Pareto front.
- Note that the algorithm has been designed to stop when all the individuals of the population reach the highest rank (that corresponds to the highest fitness); this is represented in Figure 5 and especially Figure 6, where it can be seen that at the last iteration the number of rank 1 individuals is equal to the population size.
- If the algorithm wasn’t stopped at this point, the individuals would tend to become more similar to each other, in other words, the whole population would converge to a single point.
- To further improve the result, a sharing function needs to be defined to adjust the fitness of the individuals based on their relative distance (in the f1, f2 plane for example) and spread out the population to define a larger portion of the Pareto front.
- Even if the objective functions used are two non-dimensional errors, it is important to note that the “best” solution along the Pareto front isn’t necessarily represented by the point closest to the origin (i.e. the point where both errors are minimum); for instance, the designer might want to have a lower error on the height of the roll center than on the camber rate of change.

3. Conclusion

This report demonstrates the application of a Genetic Algorithm for the multi-objective optimization of a double wishbone suspension. The process minimizes the errors for roll center height and camber rate of change from target values.

Some relevant observations can be made:

- **Simultaneous Solutions:** unlike the methods used in the previous labs, this algorithm identifies multiple points on the Pareto front simultaneously, offering a range of solutions in a single run
- **Convergence:** the simulation concludes when every individual reaches Rank 1, meaning the entire final population consists of non-dominated solutions
- **Sharing Function:** it is observed that without a "sharing function," the population would tend to cluster around a single point. To better define the entire Pareto front and increase solution diversity, a sharing function would need to be implemented
- **Final Selection:** the resulting Pareto front presents necessary trade-offs. The optimal design choice depends on whether the designer prioritizes minimizing the roll center error or the camber rate error