

Lab 02: Thermal models

20/10/2025

Abstract

This lab investigates the impact of temperature on tire performance and vehicle handling dynamics using a simplified thermal model. Through warmup cycles, the study explores how parameters such as frequency, amplitude, and vertical load influence thermal buildup and the resulting variation in peak grip and cornering stiffness. The addition of relaxation length further refines the model by introducing a delay in force development that influences heat generation.

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1. Introduction

The aim of this lab is to investigate the effect of temperature on the tire performance.

First, a warmup cycle is simulated and the effects of changing the settings on the steady state performance of a tire are considered.

Successively, the tire dynamics are included by means of the relaxation factor and, last, a steady state and transient maneuvers are simulated to investigate the effects on handling.

1.1 Simplified thermal model

The simplified thermal model used is based on the following assumptions:

- Pure lateral slip is considered
- All the thermal power (input/output) is exchanged simultaneously at the contact area between the tire and the ground
- Tire thermal properties are uniform across all different regions

Under these assumptions, the tire surface temperature is given by:

$$W \frac{dT}{dt} = q - \lambda A(T - T_0)$$

Equation 1

- $W \left[\frac{J}{K} \right] \rightarrow$ heat capacity
- $q[W] \rightarrow$ heat flux
- $\lambda \left[\frac{W}{m^2 K} \right] \rightarrow$ heat transfer coefficient
- $A[m^2] \rightarrow$ contact area
- $T_0[K] \rightarrow$ initial temperature

Equation 1 represents a **power balance**: the difference between the input power due to sliding ($q \approx |F_y \cdot \alpha \cdot V|$) and the output thermal power exchanged through the contact area ($-\lambda A(T - T_0)$) results in an increase of the tire surface temperature. W represents the thermal inertia of the tire (it is the energy needed to increase the temperature by 1 K).

It is also assumed that the dependence of grip on the temperature can be modeled by means of a symmetric curve reported in in Figure 1:

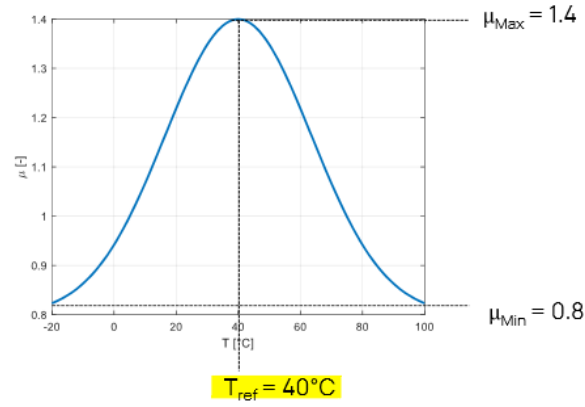


Figure 1

Last, the dependence of the lateral force on the temperature is included in the computation of the D_y, K_y terms of the MF-Tyre model used.

- D_y accounts for the variation of the peak lateral force with temperature:

$$D_y = D_{y0} \left\{ 1 + \frac{\partial \mu}{\partial T} (T - T_{ref}) \right\}$$

Equation 2

- The term $\frac{\partial \mu}{\partial T}$ is the slope of the curve in Figure 1 (it is not constant)
 - $\Rightarrow D_y$ has an **increasing-decreasing** trend with increasing temperature
- K_y accounts for the variation of the cornering stiffness with temperature:

$$K_y = K_{y0} \left\{ 1 + \frac{\partial C_p}{\partial T} (T - T_{ref}) \right\}$$

Equation 3

- The term $\frac{\partial C_p}{\partial T}$ is a fixed negative constant
 - $\Rightarrow K_y$ has a **monotonic decreasing** trend with increasing temperature

2. Parameters variation

This section focuses on investigating the effect of changing the parameters of a warmup cycle on the tire properties and behavior. The slip angle (SSA) is the input to the simulation, and it is an imposed triangular wave function of time. One parameter at a time is varied: at first the frequency (2.1) and the amplitude (2.2) of the input are varied. Then, also the speed (2.3) and the vertical load acting on the tire (2.4) are modified.

2.1 Varying input SSA frequency

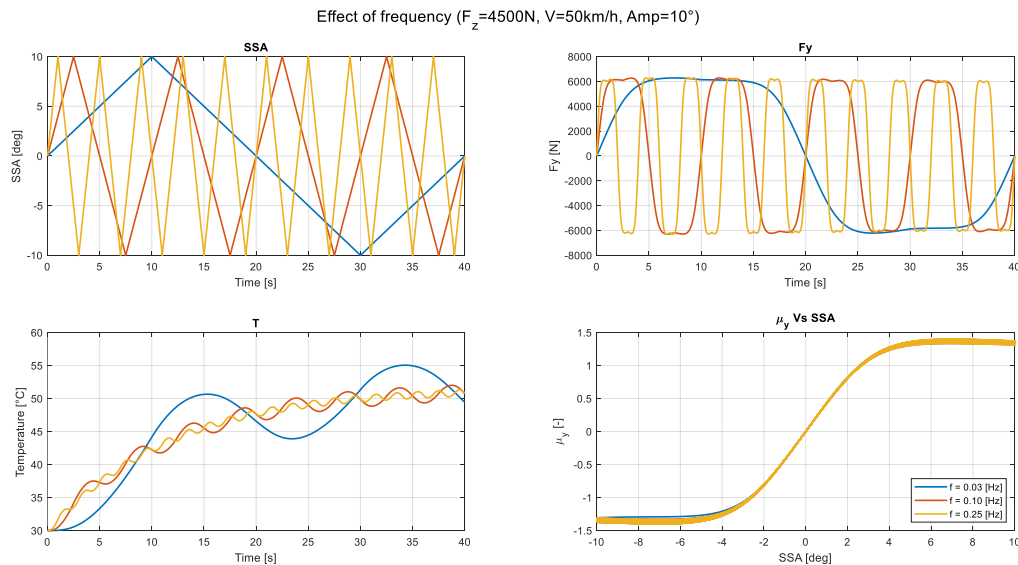


Figure 2

- Temperature-time diagram:
 - When the slip angle varies with very low frequency, the thermal dynamics of the tire are “faster” than the dynamics of the input signal → large oscillations about the increasing mean value
 - When the slip angle varies with higher frequency the thermal dynamics of the tire are “slower” than the dynamics of the input signal → small oscillations about the increasing mean value
 - In any case, the average temperature always overcomes the peak of lateral grip
- Lateral force-time diagram:
 - Coherently with the analysis of the temperature-time diagram, the maximum lateral force generated first increases and then decreases. This is not quite visible for the low frequency cycle whereas it can be easily highlighted by the **arrows** for the high frequency ones (Figure 3)

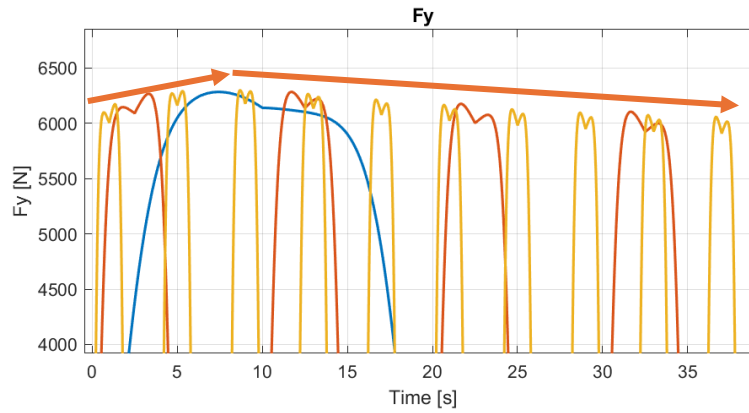


Figure 3

- Grip-slip angle diagram:
 - Two separate details of the diagram show how larger hysteresis cycles are visible for the low frequency warmup compared to the high frequency one (Figure 4)

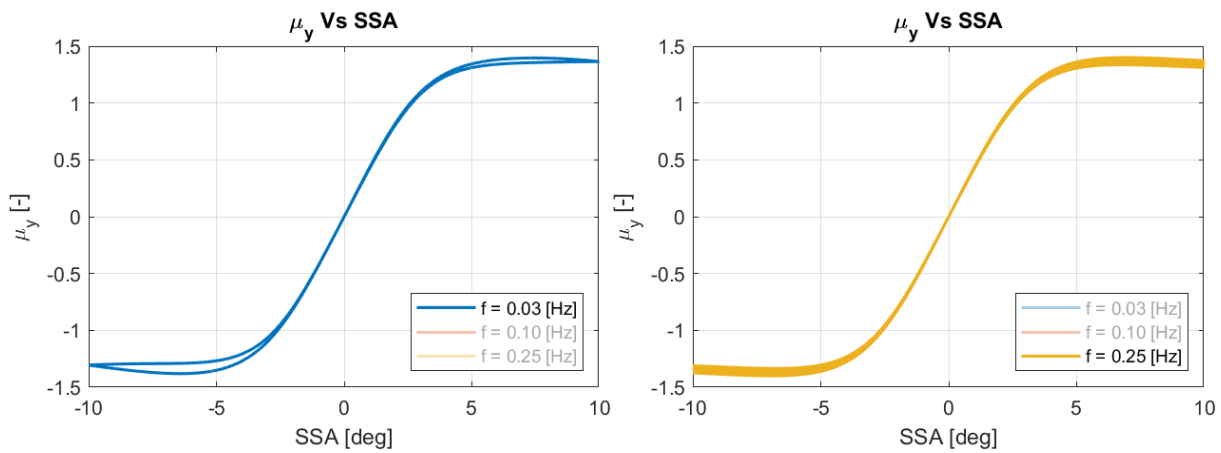


Figure 4

2.2 Varying input SSA amplitude

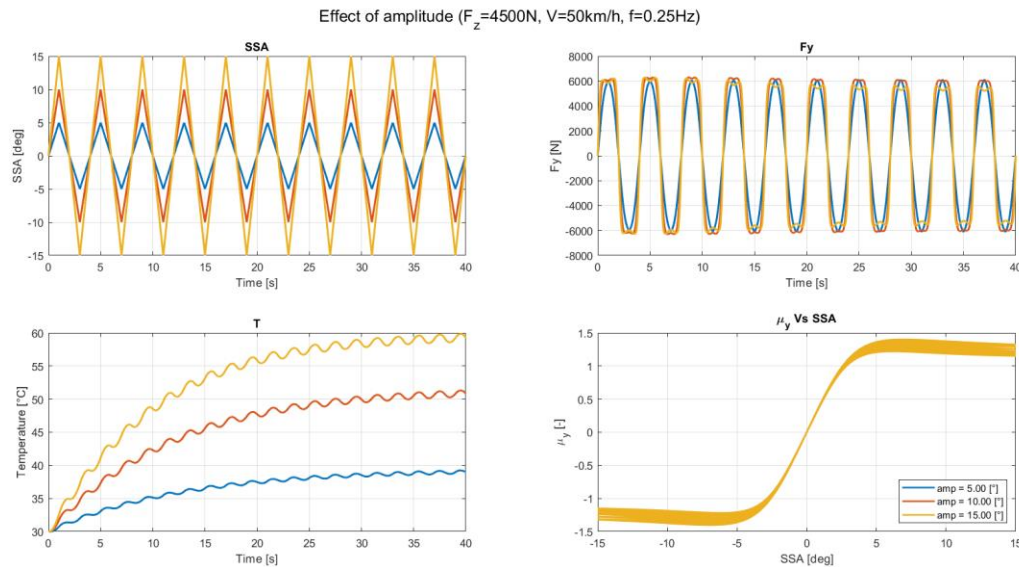


Figure 5

- Temperature-time diagram:
 - o The average temperature of the lowest amplitude cycle doesn't reach the peak of lateral grip while the high amplitude cycles overcome it
- Lateral force-time diagram:
 - o Coherently with the analysis of the temperature-time diagram, the maximum lateral force generated **monotonically increases** for the low amplitude cycle while it **reaches a maximum and then decreases** for the high amplitude ones (Figure 6)

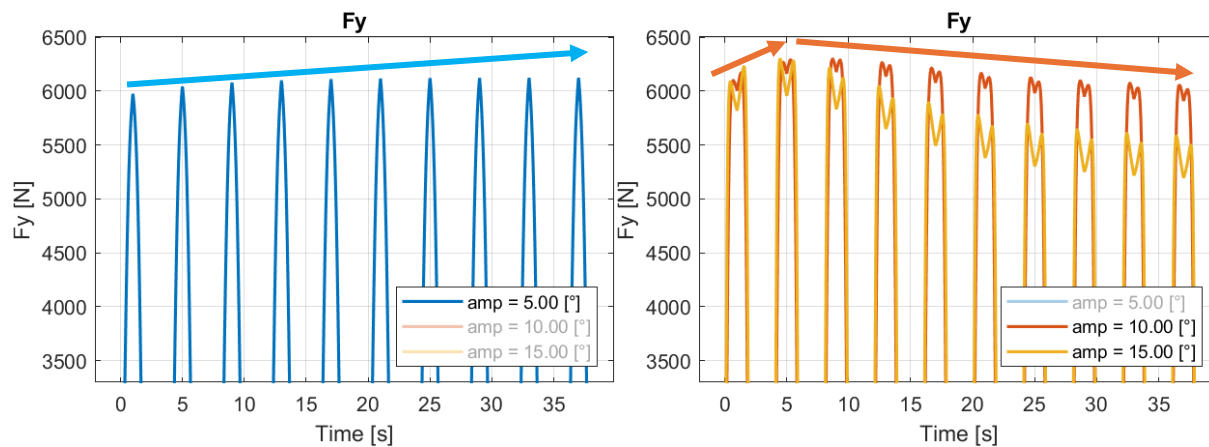


Figure 6

- Grip-slip angle diagram:
 - o Coherently with the analysis of the temperature-time diagram, the grip generated by the tire with a low amplitude warmup cycle is lower compared to the one generated by the high grip cycles

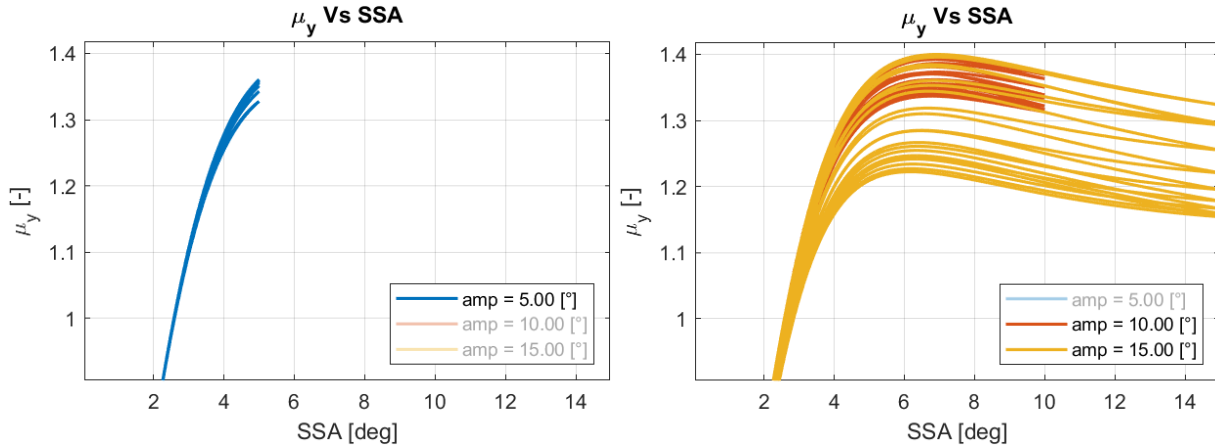


Figure 7

2.3 Varying speed

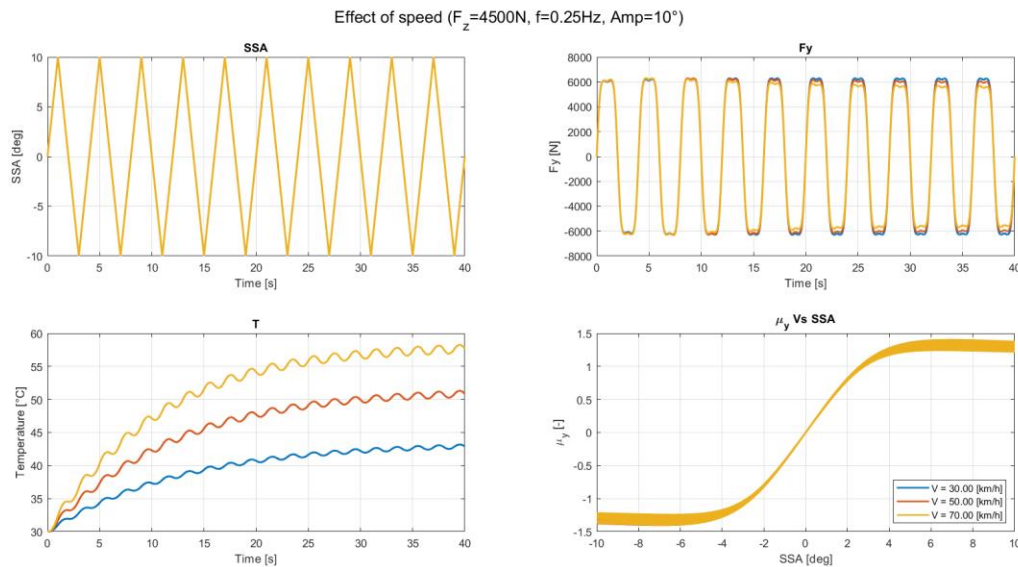


Figure 8

- Temperature-time diagram:
 - o Higher velocity \rightarrow higher heat introduced into the tire (recall $q \propto V$) \rightarrow higher temperature is reached
 - o In any case, the average temperature always overcomes the peak of lateral grip

- Lateral force-time diagram:
 - Coherently with the analysis of the temperature-time diagram, the maximum lateral force generated first increases and then decreases (Figure 9). This is only less visible for the 30 km/h curve because the friction limit is only overcome slightly

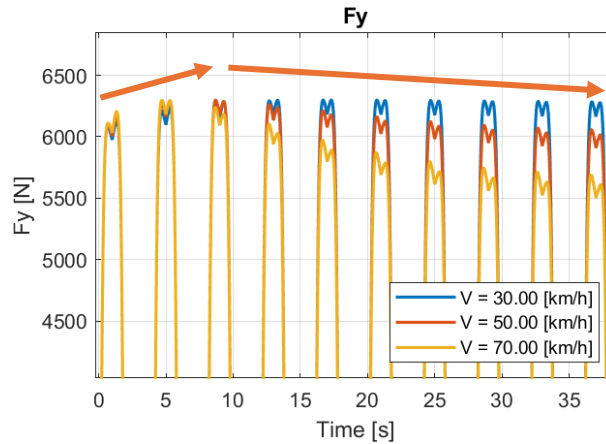


Figure 9

- Grip-slip angle diagram:
 - Coherently with the analysis of the temperature-time diagram, the decrease in grip for the high-speed cycle is much more significant
 - Larger hysteresis cycles can be observed in the high-speed warmup compared to the low speed one

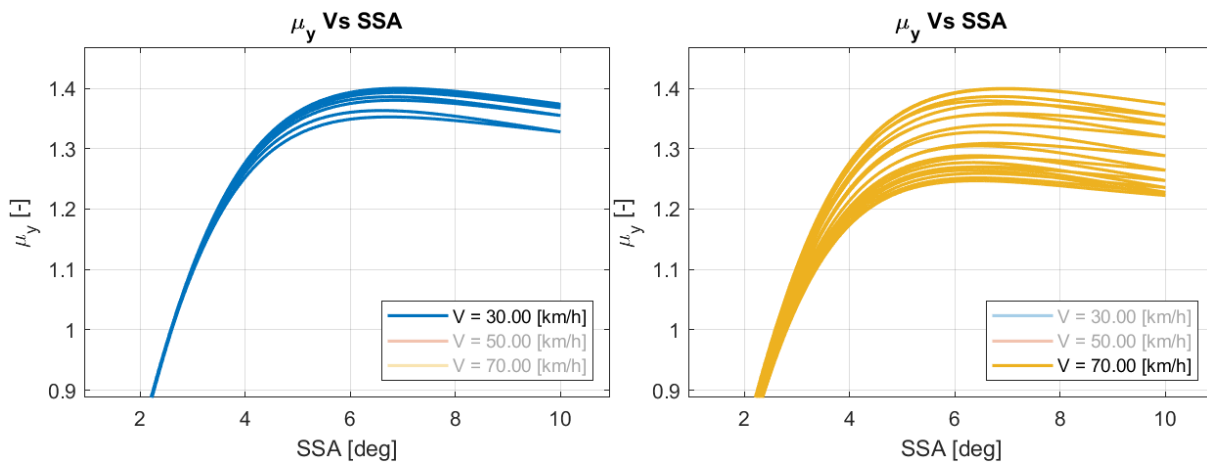


Figure 10

2.4 Varying radial load

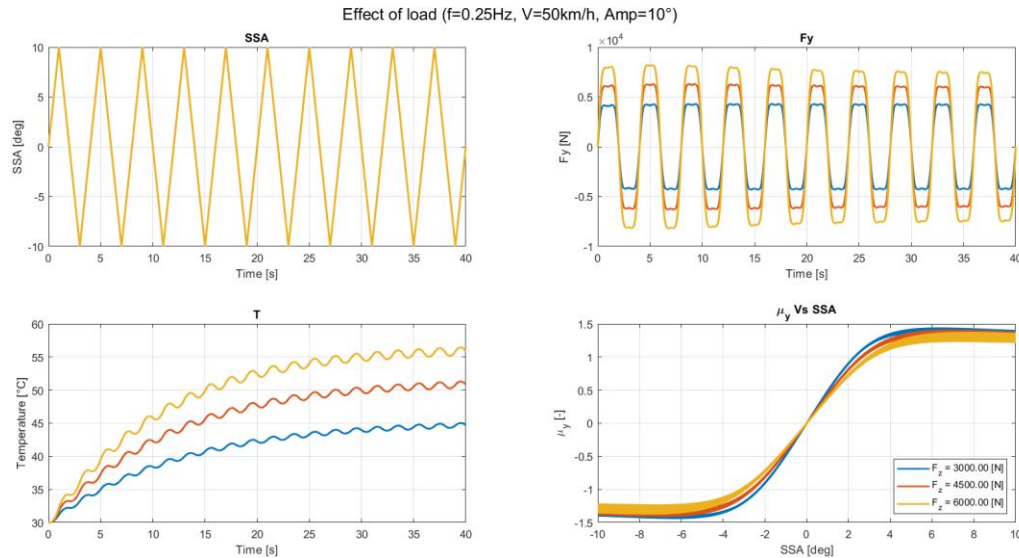


Figure 11

- Temperature-time diagram:
 - The average temperature always overcomes the peak of lateral grip
 - Higher radial load \rightarrow the tire can generate a higher lateral force (for fixed slip angle) \rightarrow higher heat introduced into the tire (recall $q \propto F_y$) \rightarrow higher temperature is reached
- Lateral force-time diagram:
 - Once again, coherently with the analysis of the temperature-time diagram, the lateral force shows an increasing-decreasing trend
- Grip-slip angle diagram:
 - The peak grip decreases with increasing radial load acting on the tire
 - The radial load has a very significant effect on the slope, that decreases for increasing radial load

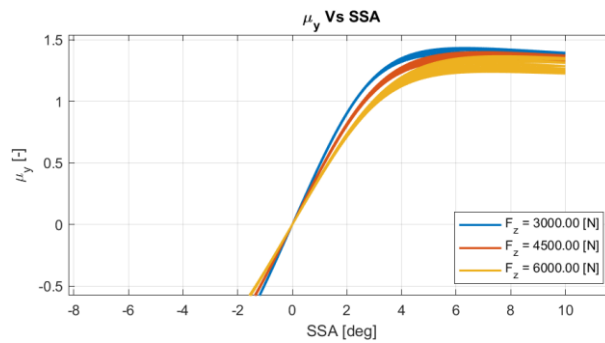


Figure 12

3. Tire relaxation length

Comparison with and without relaxation length

Tire relaxation length represents the space delay between the application of a steering angle to the tire and the associated development of lateral force.

In the model previously described, it was implemented as a first order ODE:

$$L_y \frac{\delta F_y}{\delta t} + V F_y = V F_{y,SS}(\alpha, T)$$

Equation 4

In detail, in the Simulink model, this formula is used to evaluate the derivative of the lateral force that is afterwards integrated and used in the thermal model.

3.1 Varying input SSA frequency

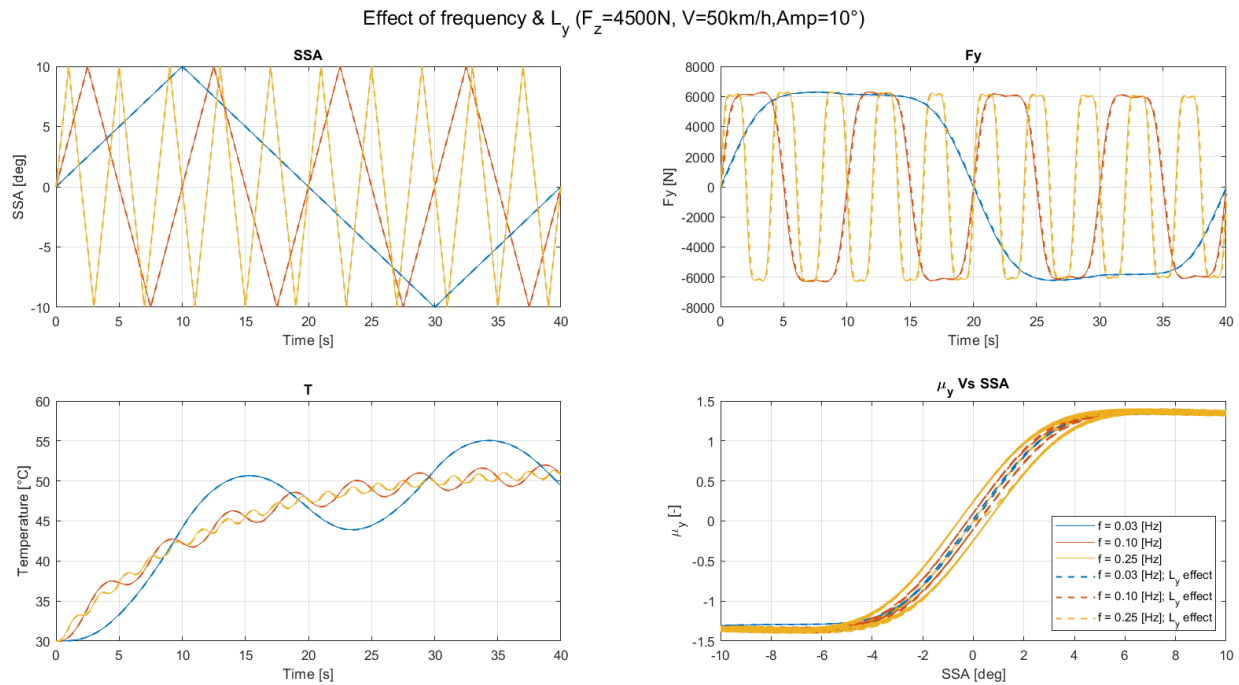


Figure 13

Changing the frequency of application of the steering wheel angle, the considerations previously done in section 2.1 remain valid: observing the temperature vs time plot we can still notice that for low frequency the thermal dynamics is faster and larger oscillations are produced with respect to the one associated with a higher frequency of the SSA. Moreover,

by looking at the Lateral force vs time diagram it is still noticeable the ascending and descending trend of lateral force.

The main effect of **adding the relaxation length** is that the lateral force will not be developed instantaneously as the SSA is applied. Another difference of this model compared with the one without relaxation length is that as the frequency of the triangular input increases, the lateral force peaks become more rounded and the maximum amplitude decreases. The tire does not have enough time to develop the steady-state theoretical force before the input reverses direction. Since the lateral force does not reach its steady-state peak at high frequencies, this will result in less rapid heating during very fast transients.

3.2 Varying input SSA amplitude

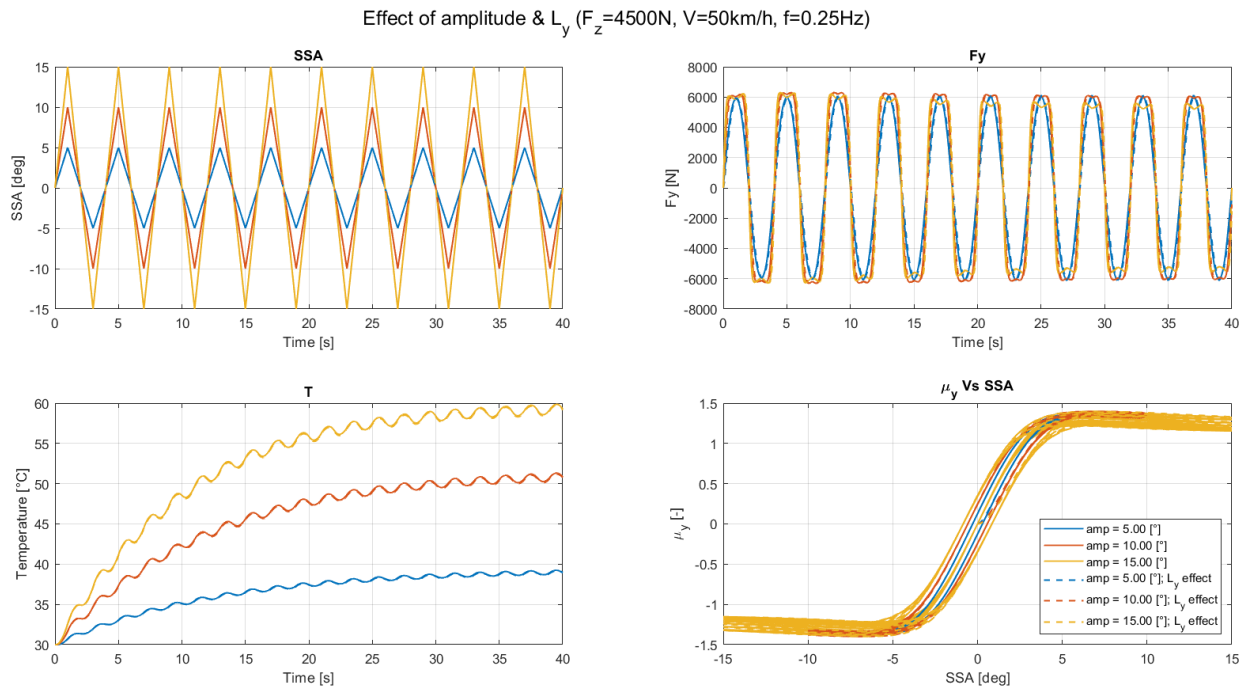


Figure 14

The effect of adding the relaxation length is always a delay between the application of the SSA and the generation of the lateral force. As the amplitude increases, the temperature increases accordingly and the force does not have anymore a triangular shape due to saturation.

3.3 Varying speed

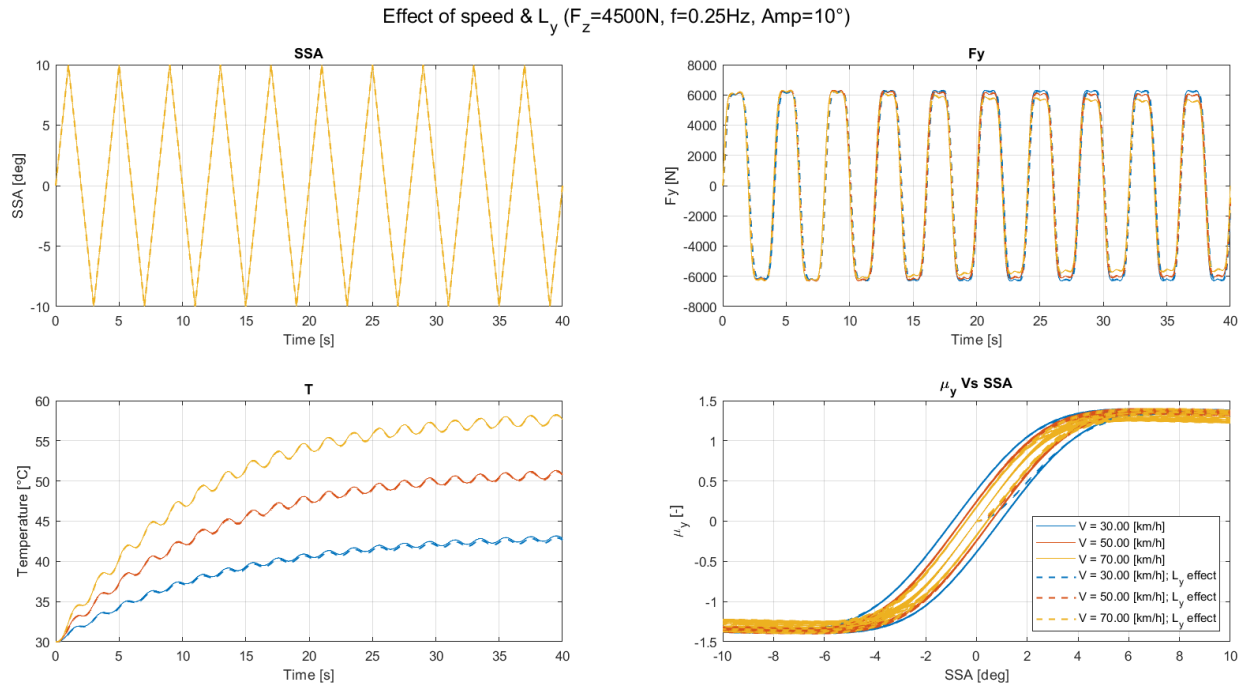


Figure 15

By looking at the formula implemented in the model, increasing the speed causes the relaxation length to be covered in a shorter time and so the lateral force follows the reference one more accurately meaning the drop at the peak is less severe (Figure 16).

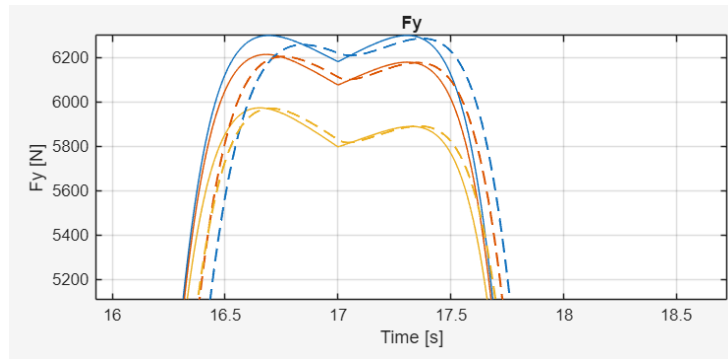


Figure 16

3.4 Varying radial load

Effect of load & L_y ($f=0.25\text{Hz}$, $V=50\text{km/h}$, $\text{Amp}=10^\circ$)

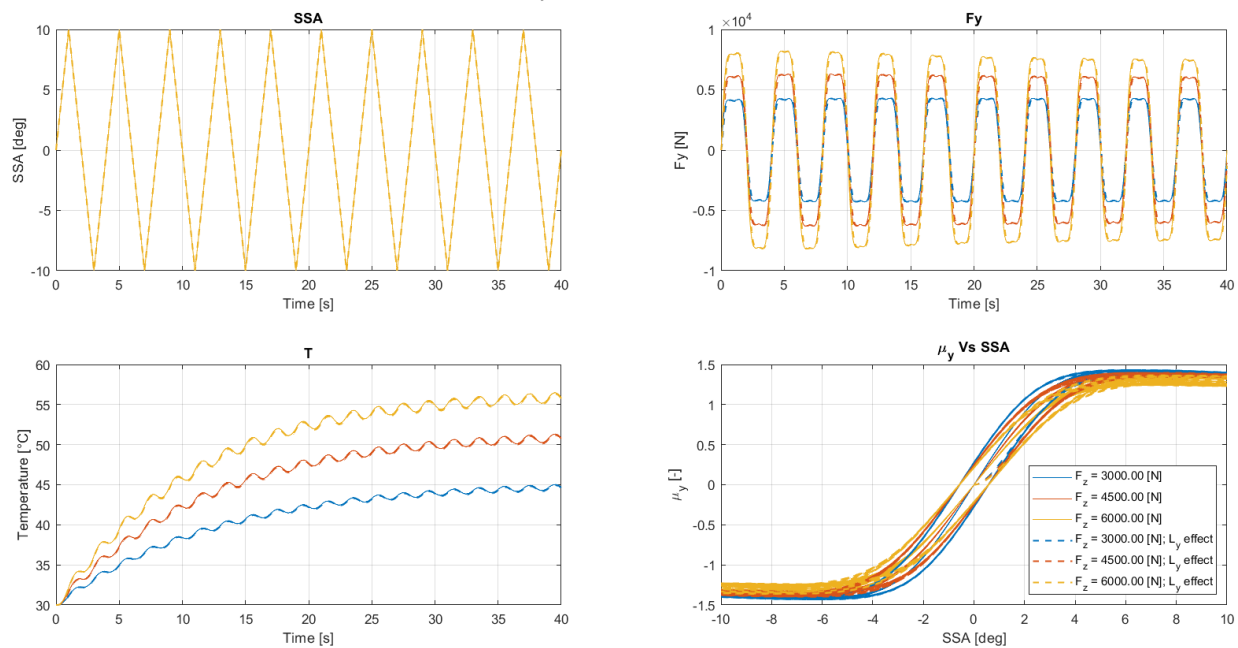


Figure 17

In a model that includes relaxation length, the radial load acts as a multiplier for both the lateral force and the response delay. Moreover, as the load increases, the difference between the temperature between the two models becomes more marked.

4. Temperature model for vehicle dynamics

In this section the thermal model presented in the report is implemented in the Simulink for the previous report, allowing to simulate different maneuvers by considering the tyre temperature evolution. In this section the temperature effect on vehicle dynamics is also analyzed.

4.1 Steering pad

Steering pad maneuver at constant speed is presented.

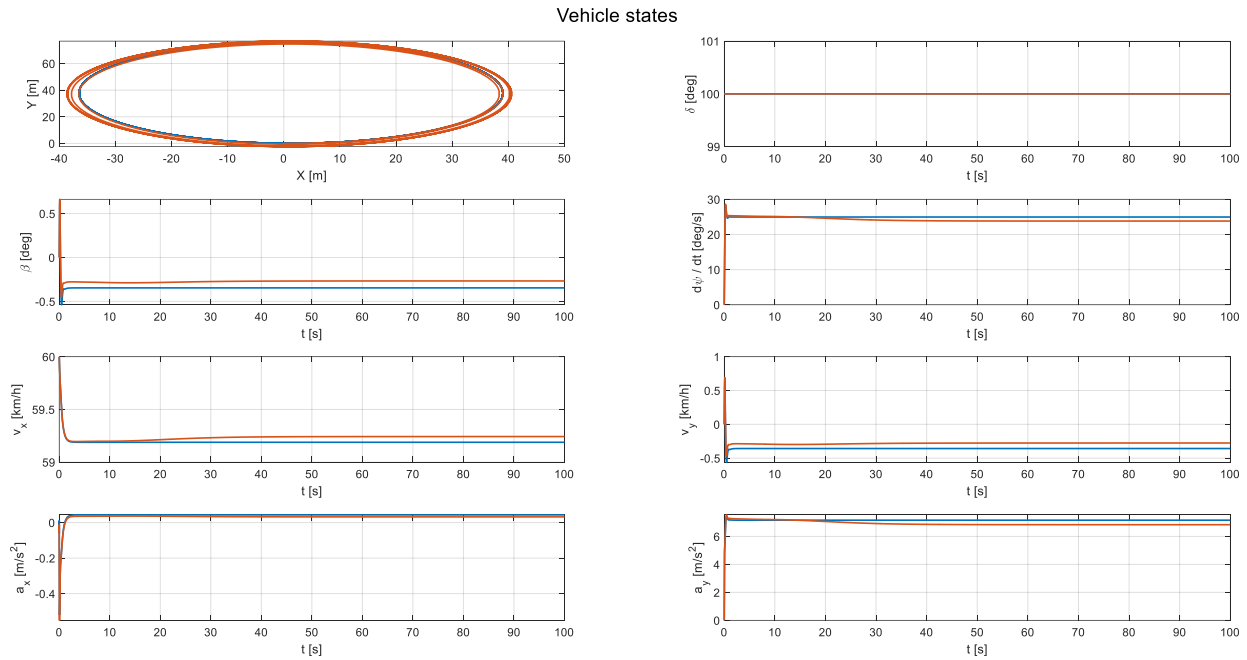


Figure 18

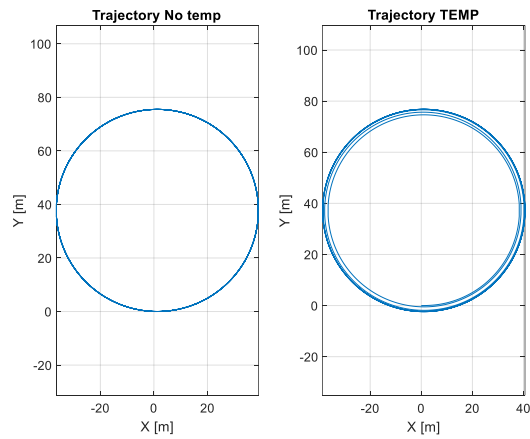


Figure 19

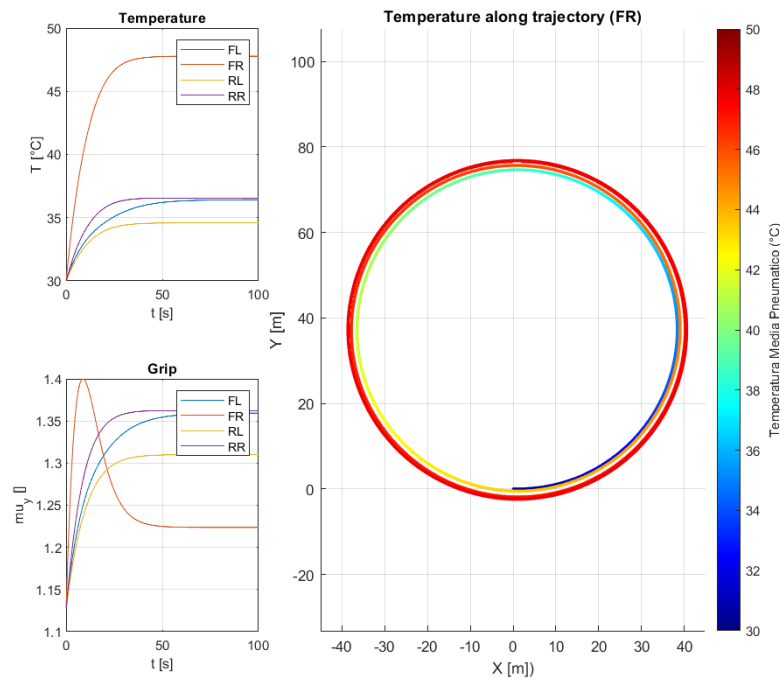


Figure 20

- **Temperature evolution**
 - From the last figure it is possible to see that the most stressed tire is the front right, which is obvious given the maneuver. Its temperature rises above 47°C.
 - All the other tires stay below 37°C during the whole maneuver
- **Grip evolution**
 - The grip available in the tire starts reducing for higher temperature values (higher than 40°C, from the tire data). This behavior depends on the specific compound and tire characteristics.
 - Front right: it slows the mentioned decreasing trend
 - All other: the grip keeps growing until the temperature stabilizes
- **Vehicle dynamics**
 - Because of the grip evolution, the behavior of the vehicle along the steering pad gets more understeering.

4.2 Slalom

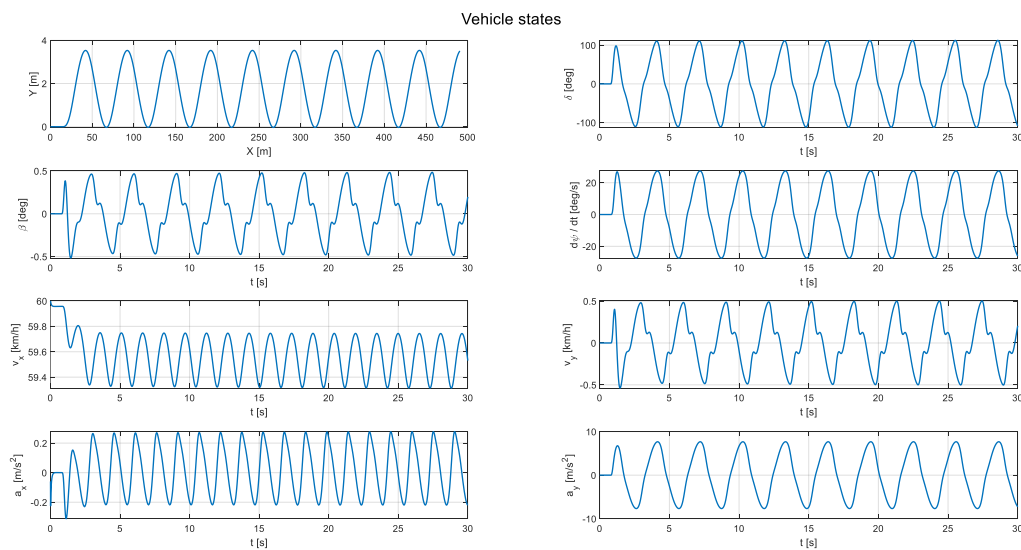


Figure 21

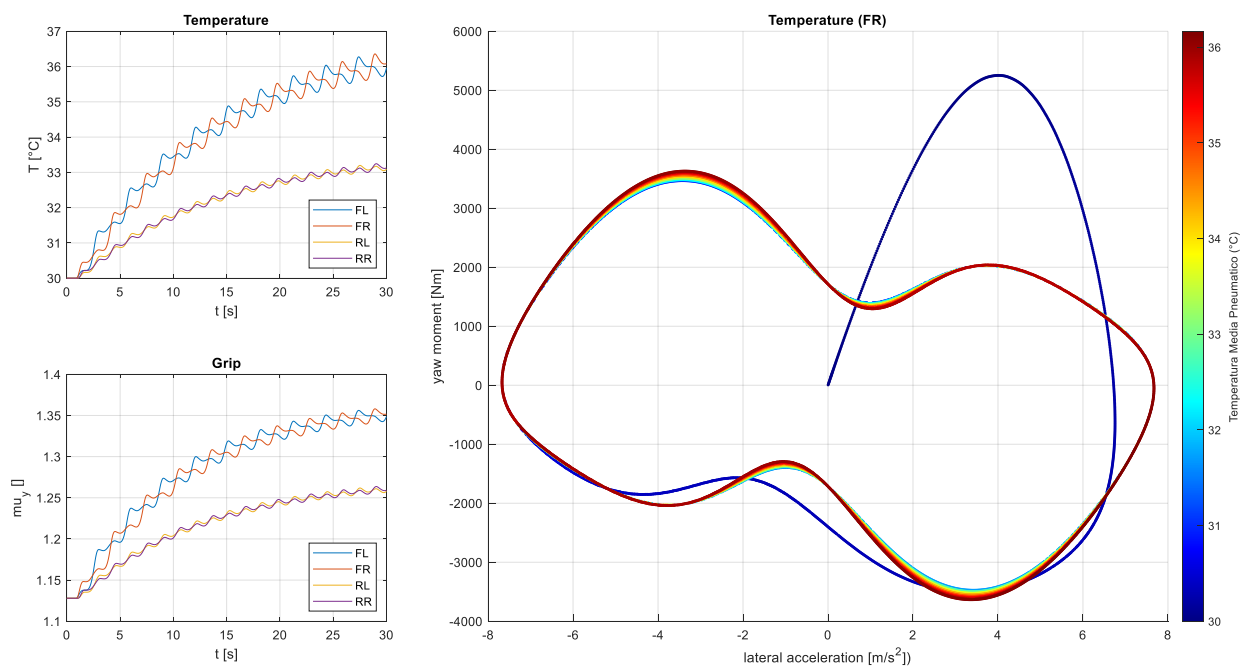


Figure 22

- **Temperature evolution**

- Front axis (FL and FR) temperatures rise more and faster than the rear tires, probably more energy is introduced in the front tires by this maneuver (given that the tires are equal or similar)
- Temperatures in the front axis oscillate more than the rear ones. There are probably two main reasons:
 - There is more **load transfer** in the front axis than the rear, this can explain the understeering behavior (this is not sufficient condition, but it can be an indication). More load transfer implies more oscillations in the lateral forces during the maneuver
 - **Input dynamics**
 - **Front:** during a slalom, the front tires are subjected to direct, rapid steering inputs. This creates a high-frequency excitation where the input dynamics are relatively fast compared to the thermal dynamics but the high magnitude of the heat flux is enough to drive significant oscillations
 - **Rear:** the rear tires experience slip angles that are largely a result of the vehicle's yaw motion and lateral acceleration, which are "smoother" and delayed compared to the front. Because the "input" to the rear tires is filtered by the vehicle, the dynamics are slower and less aggressive, leading to the smaller temperature oscillations observed
- Temperature oscillations
 - Front: counterphase
 - Rear: almost in phase
- **Grip evolution**
 - Given the simple relationship implemented, grip behaves accordingly to temperature, especially since all tires are working below the optimal working temperature of 40°C
- **Vehicle dynamics**
 - As tire temperature and grip increase, the maximum lateral acceleration remains constant, since it is probably not needed more for the maneuver.
 - The maximum yaw rate increases with increasing temperature, which is also a significant indication of increased grip.

5. Conclusion

The laboratory confirmed that tire temperature is a critical factor in determining vehicle handling limits, with grip exhibiting a non-monotonic trend centered at an optimal reference temperature. Further analysis revealed that higher speeds and radial loads lead to more rapid temperature increases and performance degradation. The inclusion of tire relaxation length provided a more realistic representation of transient behavior, showing that the delay in lateral force development at high frequencies reduces the rate of heat generation compared to instantaneous models.