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Lab 04: Tire Characterization

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Abstract

This lab analyses the characterization of a MF-Tire model using experimental data, focusing on relaxation length identification and MF coefficient fitting. Relaxation length is calculated with cross-correlation and FRF methods, while the forces and moment curves are fitted using MATLAB's toolbox and *fmincon* optimization. The study compares simplified and extended MF versions, highlighting the higher accuracy of the extended model.

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1. Introduction

The aim of this lab is to build an MF-Tire model starting from a set of data collected during an experimental activity performed in the LaST laboratory.

To characterize the tire model, the following quantities need to be determined:

- **Lateral relaxation length**: this is performed with two different approaches (Section 1)):
  - Using the cross correlation between time domain data
  - Using the FRF between the input (steering angle) and the output (lateral force)
- **MF-Tire 6.2 coefficients** to replicate the correct behavior of the F_x, F_y, M_z curves; this is also performed with two different approaches:
  - Using a built-in MATLAB toolbox (Section 3)
  - Implementing from scratch an algorithm to minimize the error between fitted and experimental data (Section 4)

1.1 Description of the experimental activity

The testing activity was performed on a drum machine, and it was divided into three phases:

1) Warmup (pure lateral slip condition)
   - A sinusoidal steering angle (= slip angle) was provided as input
   - Forces and moments were measured
   - The delay between the provided input and the measured output is used to compute the lateral relaxation length
2) Longitudinal characterization (pure longitudinal slip condition)
   - A braking torque was provided as input after having accelerated the tire up to a speed fixed by the standards
   - Forces and moments were measured and longitudinal slip was computed
   - The test was repeated for three different vertical loads acting on the tire and three different inflating pressures (different values of camber angle were not tested)
3) Lateral characterization (pure lateral slip condition)
   - A sine dwell steering angle (= slip angle) was provided as input
   - Forces and moments were measured
   - The test was repeated for three different vertical loads acting on the tire and three different camber angles (different values of pressure were not tested)

2. Relaxation length

The relaxation length is a parameter that can be used together with the MF to approximate the delay in the force response of the tire to the external input. When a tire is subjected to a slip angle the response has a certain delay due to the carcass compliance and the limited dimension of the contact patch.

The formula where the relaxation length is used is the following:

$$\frac{\lambda}{V_x}\,\dot{F_y} + F_y = \overline{F_y}$$

Equation 1

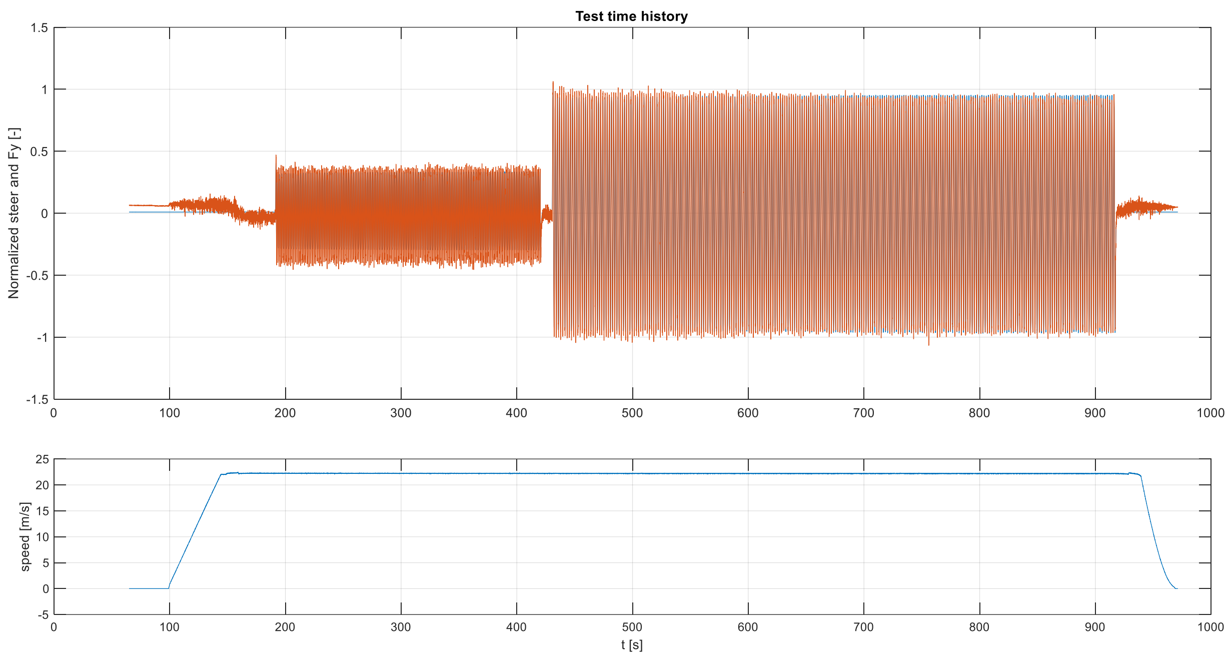

Figure 1

Given this time history, where input and output are available it is possible to evaluate the relaxation length both in time and frequency domain.

Cross correlation method (time domain)

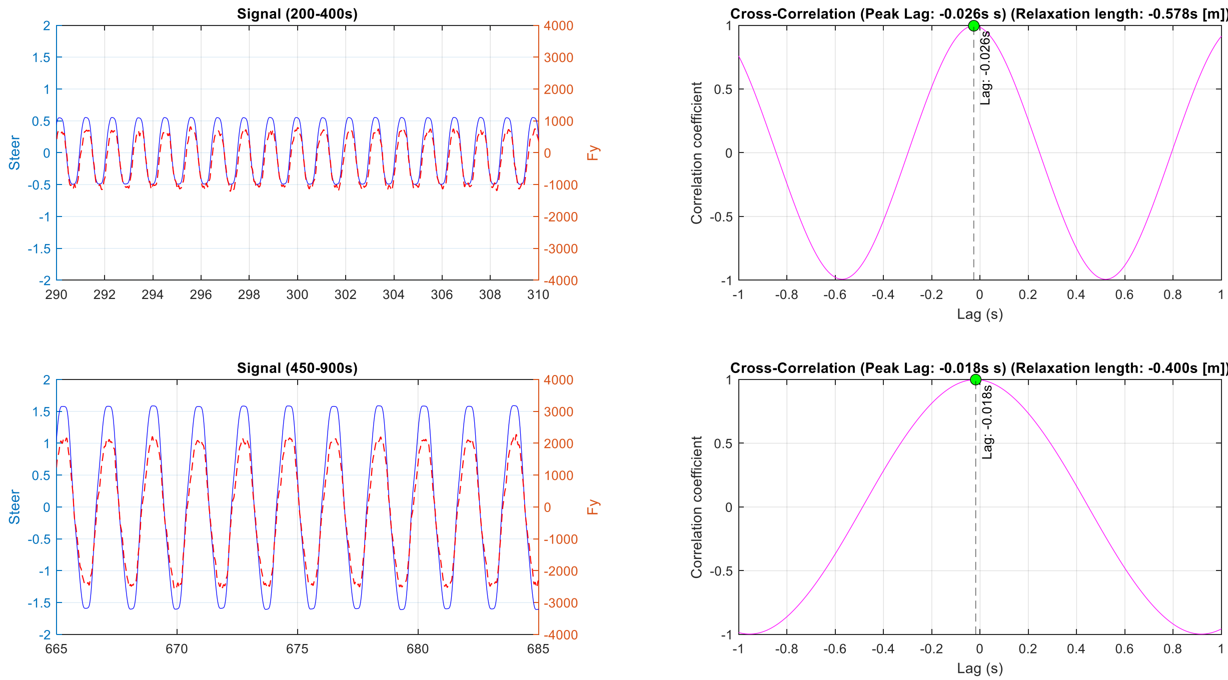

Figure 2

It is the most intuitive way, the time delay between the input and the output signal is measured through the **cross-correlation** function. In this case, the input is the imposed steer angle, and the output is the measured lateral force. The time delay obtained is than converted into distance by means of the wheel speed.

This operation is repeated for the two phases of the warmup phase.

Phase lag (frequency domain)

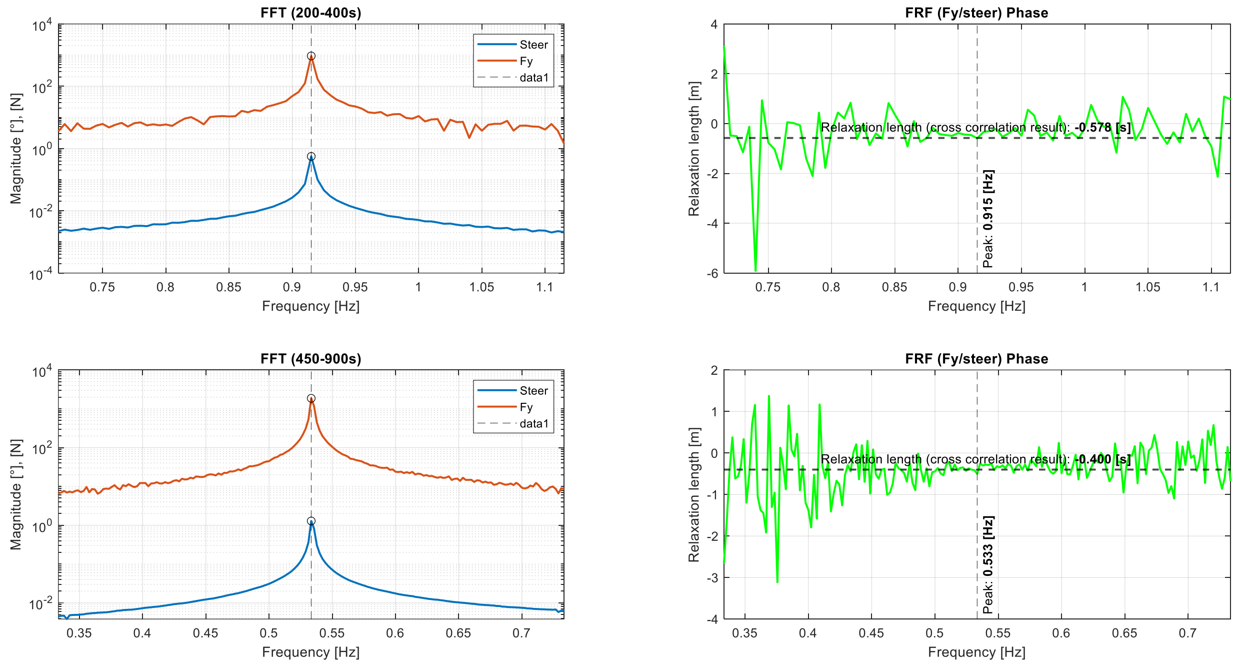

Figure 3

The relaxation length can also be evaluated for the frequency transformation of the two signals.

From the graphs on the left of Figure 3 the peak of the **FFT** is computed, that **peak** corresponds to the frequency associated with the steering input, which is the one that will present the higher amplitude for both steer and force.

Looking at the **phase lag** of the **transfer function** between the steer angle and the lateral force, it is possible to obtain the relaxation length. In the graphs presented above on the right, the phase is already converted in meters, so to be easily compared to the result of the cross correlation. In those two graphs it is possible to see that, in correspondence of the peaks frequency, the phase lag in meters corresponds to the relaxation length.

3. MATLAB fitting tool

The first request asked to implement the fitting of the parameters through a toolbox already implemented in MATLAB. This method requires, given a TYDEX file, to use the function `fit` to fit the parameters to the provided data. The required toolbox implemented the Pacejka MF 6.2 version and performs the minimization using the *fmincon* function.

3.1 Longitudinal force

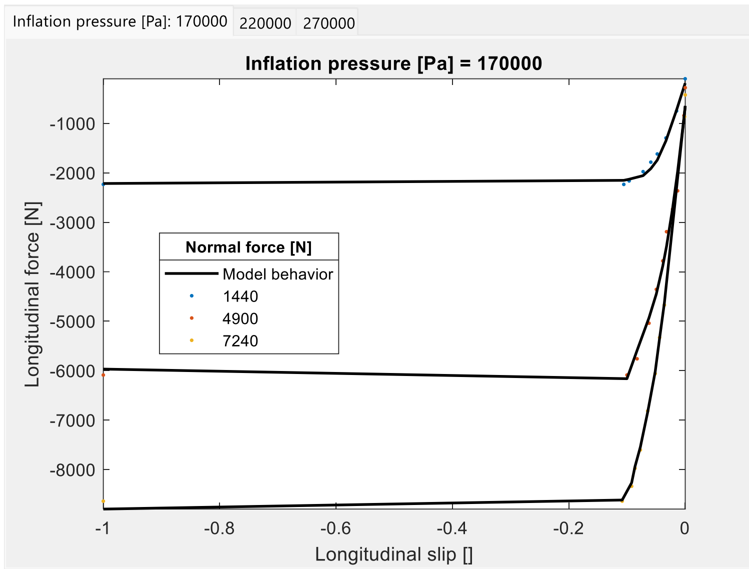

Figure 4

The results for the fitting of the parameters of the Pacejka model for the longitudinal test are reported in Figure 4. As it can be seen, the modelled behavior reflects well the provided data.

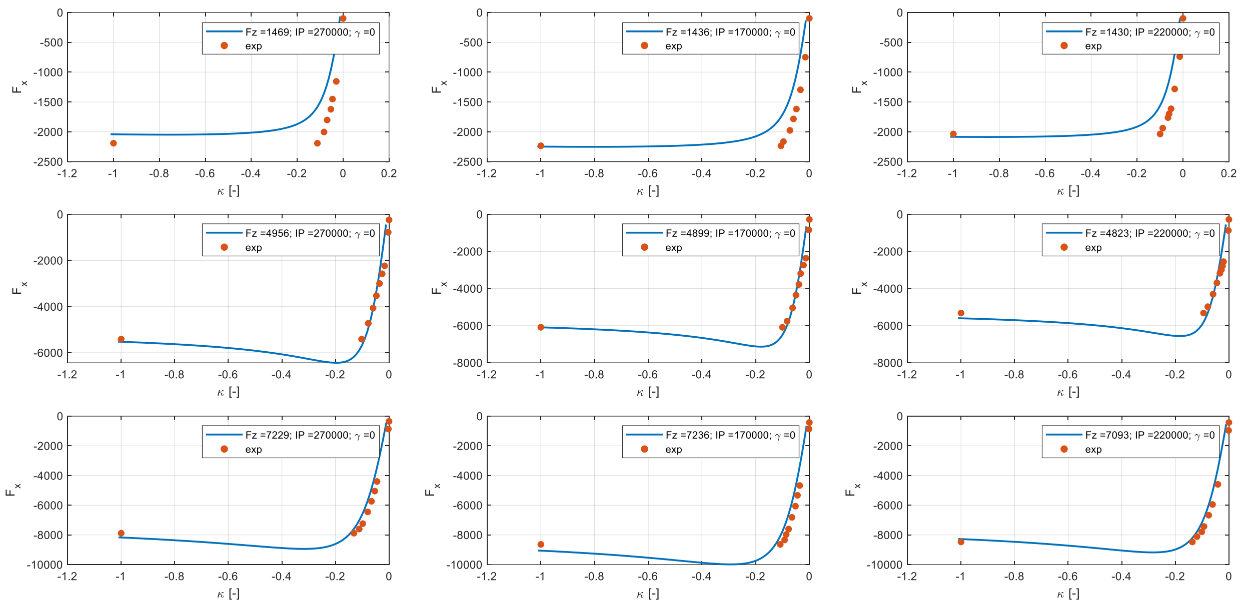

Figure 5

In figure 5, using the parameter extracted from the fitting, the longitudinal force was evaluated implementing the MF formula 6.2 and is plotted along with the original data.

3.2 Lateral force

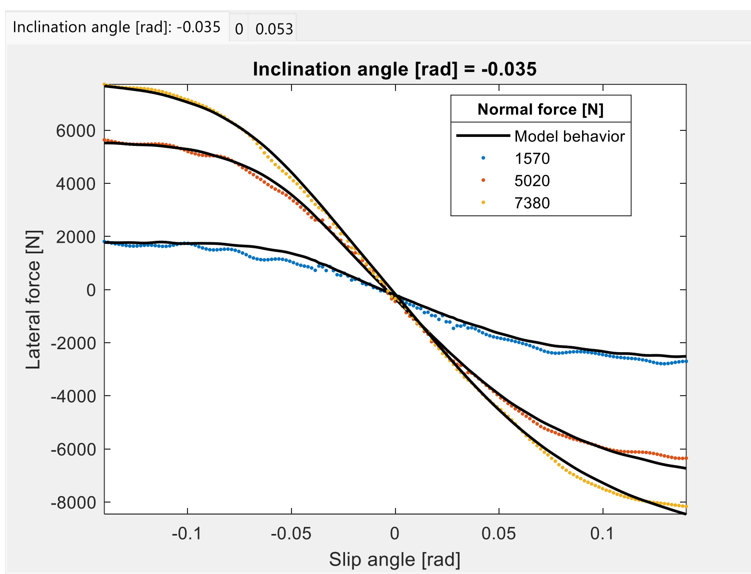

Figure 6

The same procedure of fitting was repeated also on the data provided for the lateral force as shown in Figure 6.

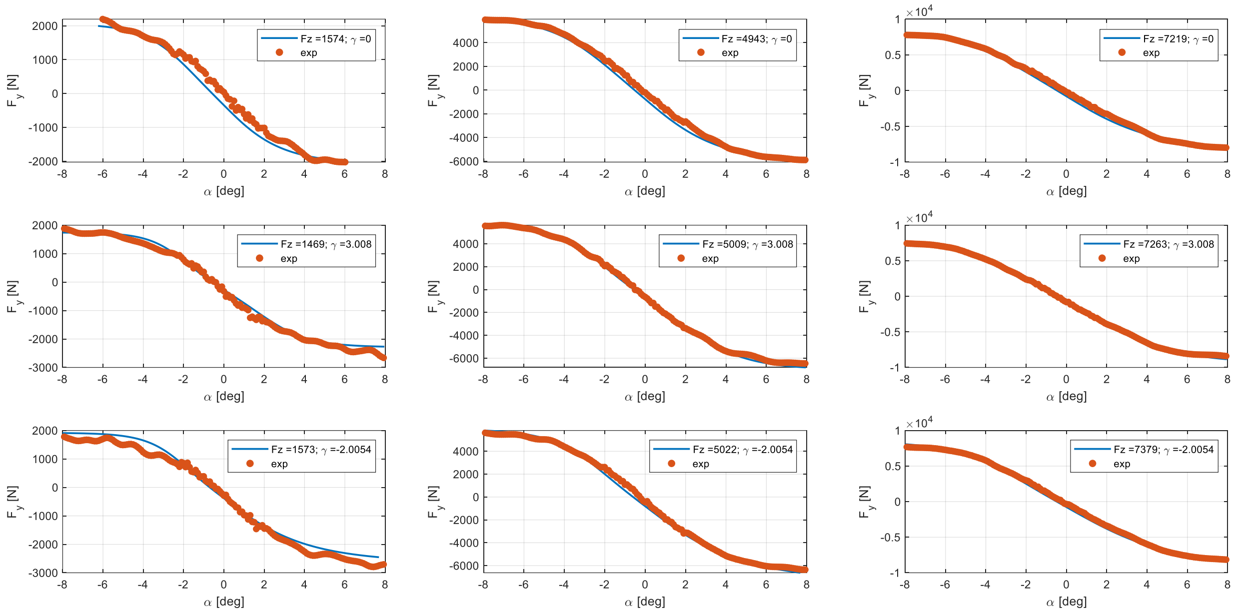

Figure 7

Once again the MF 6.2 was implemented, and the provided data was compared with the one evaluated with the MF combined with the parameters exiting from the fitting: Here the fitting results more accurate since the number of provided points is higher.

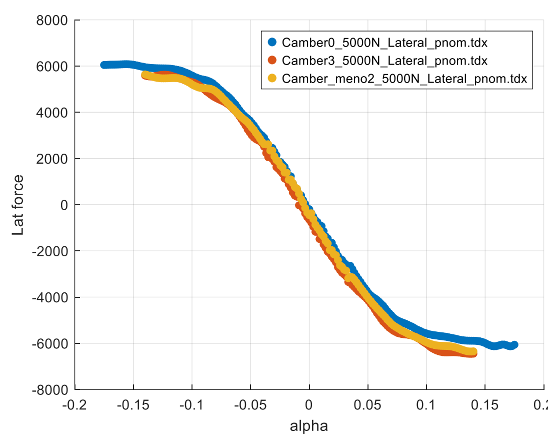

Figure 8

3.3 Self-aligning moment

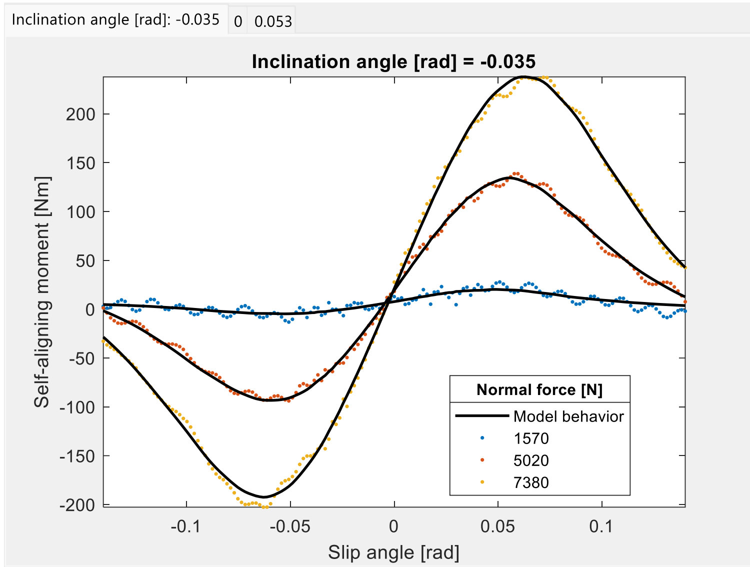

Figure 9

The fitting using the MATLAB toolbox was implemented also for the self-aligning moment (Figure 9).

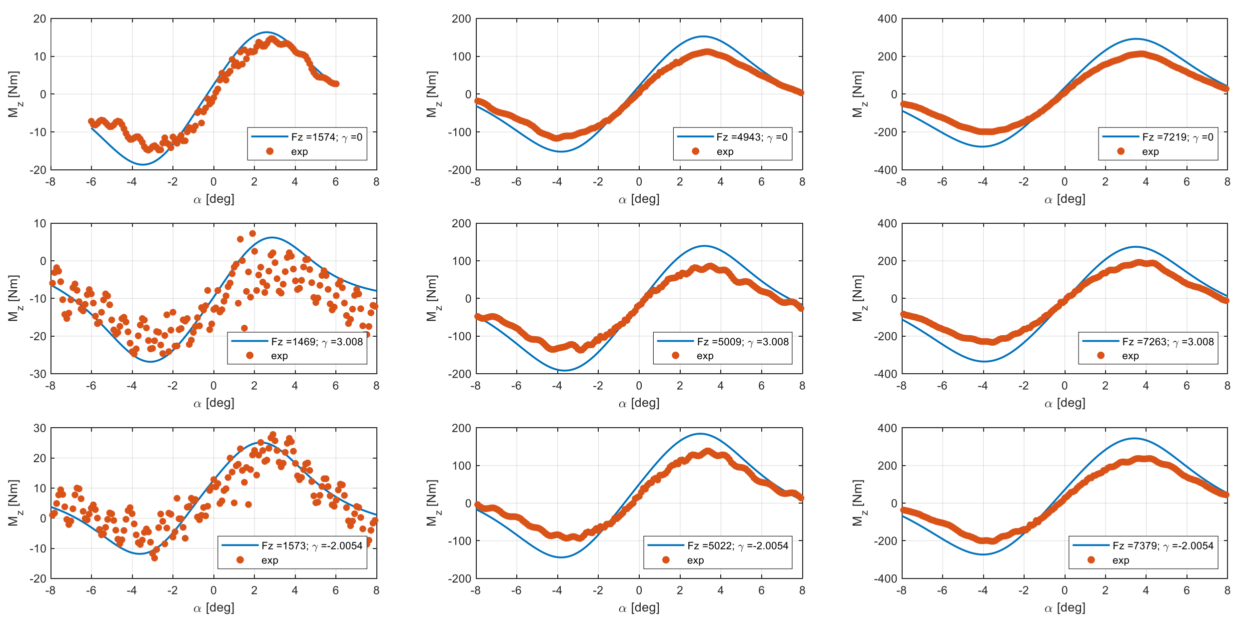

Figure 10

In this case, due to the limited access to the MF 6.2 formulas it is not possible to reconstruct the trend of the self-aligning moment even if the MATLAB toolbox fitting is correct as Figure 9 suggests.

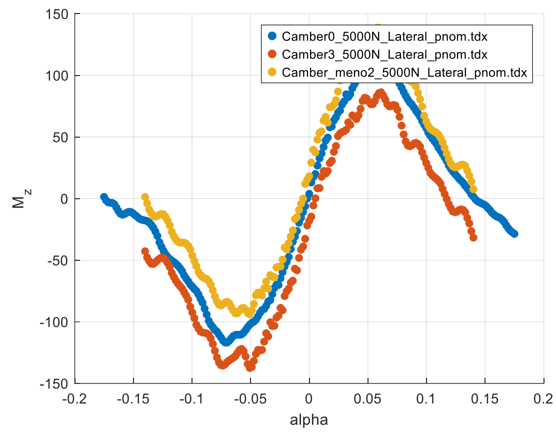

Figure 11

4. *fmincon* fitting

This section focuses on the implementation from scratch of an optimization algorithm to fit the experimental data with the Magic Formula; as in the previous section, this has been done for the longitudinal force, lateral force and for the self-aligning moment.

- The objective function to be minimized is conceptually the same for all the cases considered:

$$err = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{\left[\left(F_{exp} - F_{MF}\right)\right]}{F_z}\right]^2$$

Equation 2

  - N → number of samples
  - F_z → tire vertical load
  - F_{exp} → experimental data
  - F_{MF} → MF-Tire
- Two different sets of design variables are considered to highlight the importance of introducing enough coefficients to track the effect of the vertical load, pressure and camber on the forces and moments.
- *fmincon* was used with the following settings
  - *interior-point algorithm*
  - *StepTolerance = 1e-16*
  - *MaxFunctionEvaluations = 100000*

4.1 Longitudinal force

4.1.1 Simplified MF

- The MF used to fit the experimental data is the following:

$$F_{x,MF} = D_x \sin\{C_x \, atan[B_x\kappa - E_x(B_x\kappa - atan(B_x\kappa))]\} + S_{vy}$$

Equation 3

- The parameters highlighted in red are the design variables to be tuned

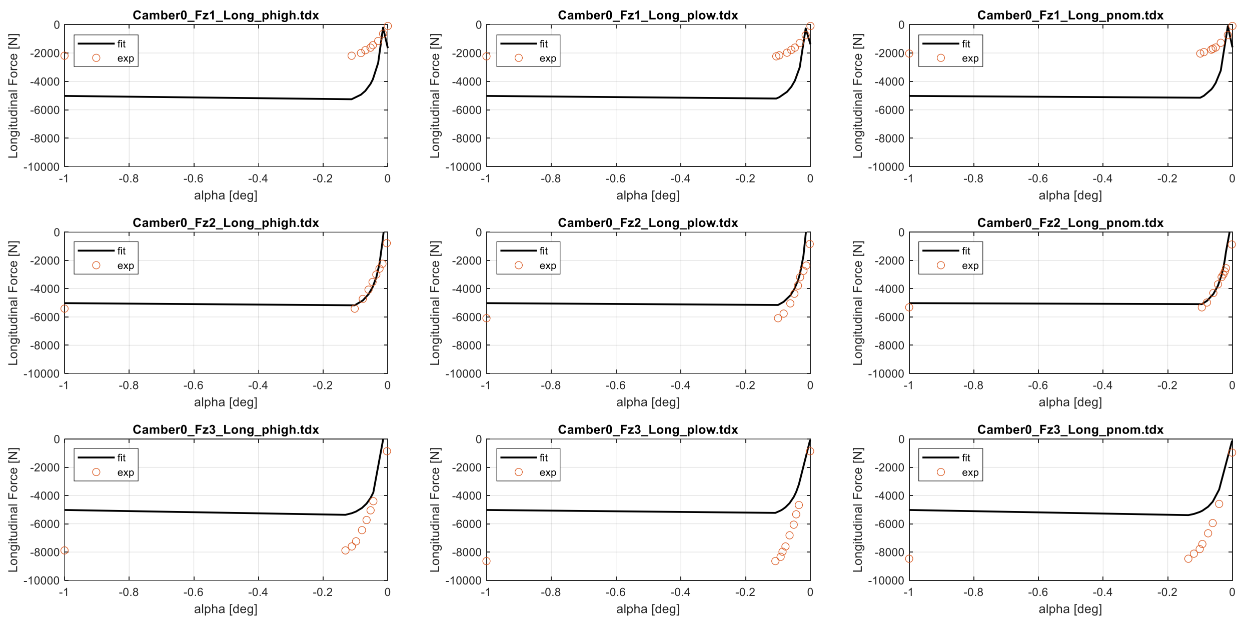

Figure 12

- The results of the optimization are very poor: the fitted curve is not able to track the experimental data
    - In addition to the limitations introduced by the usage of the simplified MF formulation it is important to specify that the number of measurements available for the longitudinal force was very limited (the very last point wasn't even acquired experimentally but was introduced a posteriori with $\kappa = -1$ and F_x value equal to last measurement; no experimental information about the curvature after the peak was available). These two aspects both have a strong negative impact on the result
- **The average RMS error is $\approx 1600\ N$**

4.1.2 Extended MF

- In the extended version of the MF, the dependence of all the coefficients on the vertical load and on the pressure are included

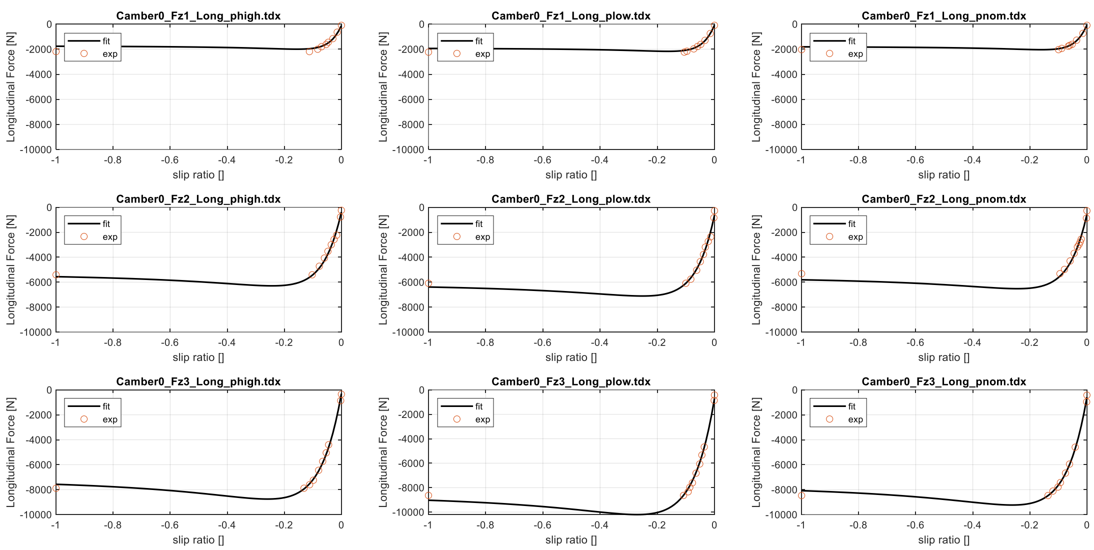

Figure 13

- The results of the optimization are much more promising in this case; the model is able to track the trend of the experimental data and the effect of inflation pressure and vertical load (recall that during the test was not repeated for different values of camber)
- **The average RMS error is** $\approx \mathbf{500}\ \boldsymbol{N}$
    - The error is indeed large but it can be seen that it is surely due to the lack of data after the peak → the error is visibly large in correspondence of the last point while the fitting is very good in the initial part of the curve

4.2 Lateral force

4.2.1 Simplified MF

- The MF used to fit the experimental data is the following:

$$F_{y,MF} = D_y \sin\{C_y \, atan[B_y\alpha - E_y(B_y\alpha - atan(B_y\alpha))]\} + S_{vy}$$

Equation 4

- The parameters highlighted in red are the design variables to be tuned

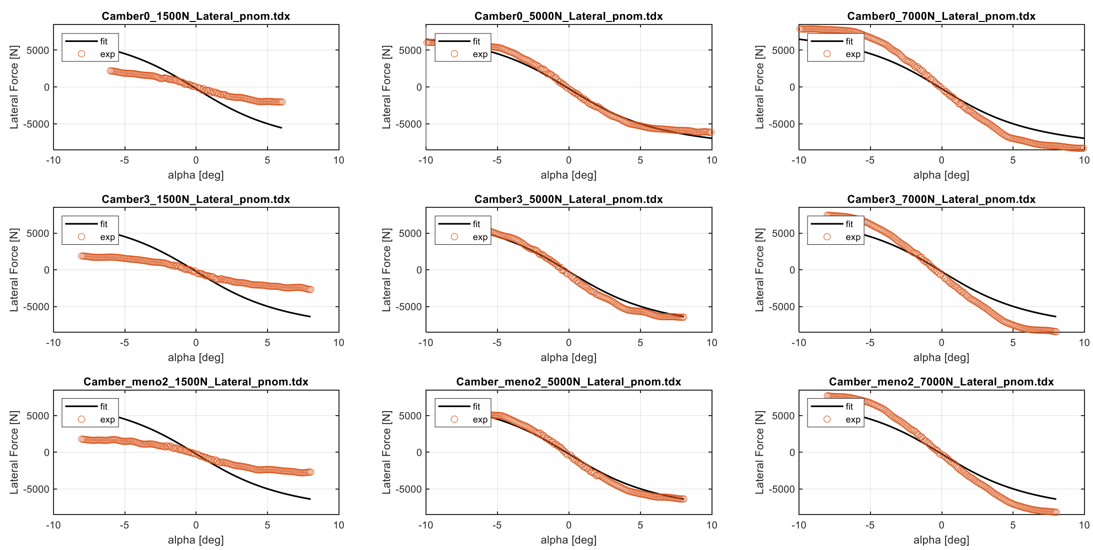

Figure 14

- The quality of the fitting is not good, however, due to the larger number of measurements available, it is better compared to the result obtained previously with the same formulation of the MF in Section 4.1.1
- **The average RMS error is** $\approx \mathbf{1300\ N}$

4.2.1 Extended MF

- In the extended version of the MF, the dependence of all the coefficients on the vertical load and on the camber are included

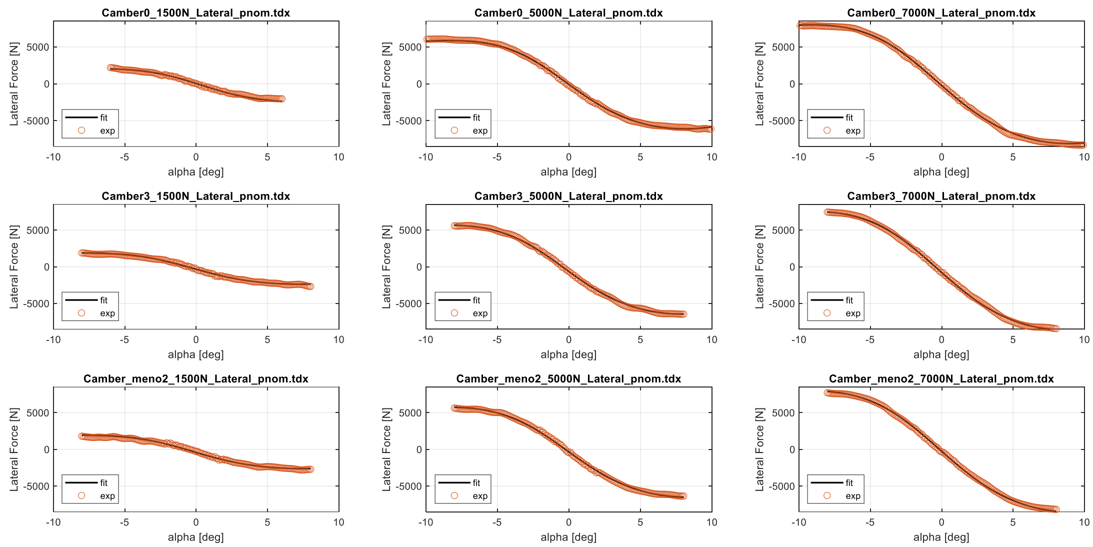

Figure 15

- The results of the optimization are much more promising in this case; the model is able to track the trend of experimental data and the effect of camber angle and vertical load (recall that during the test was not repeated for different values of inflation pressure)
- **The average RMS error is $\approx 80\ N$**

4.3 Self-aligning moment

4.3.1 Simplified MF

- The MF used to fit the experimental data is the following:

$$\begin{cases} M_{z,MF} = -tF_y + M_{zr} \\ t(\alpha) = D_t \cos\{C_t \, atan[B_t\alpha_t - E_t[B_t\alpha_t - atan(B_t\alpha_t)]]\} \\ M_{zr}(\alpha) = D_r \cos\{atan[B_r\alpha_r]\} \\ \alpha_t = \alpha + S_{Ht} \\ \alpha_r = \alpha + S_{Hf} \end{cases}$$

Equation 5

- The parameters highlighted in red are the design variables to be tuned
- Note that the coefficients that define the lateral force are not optimized again, the F_y model computed in the previous section is used

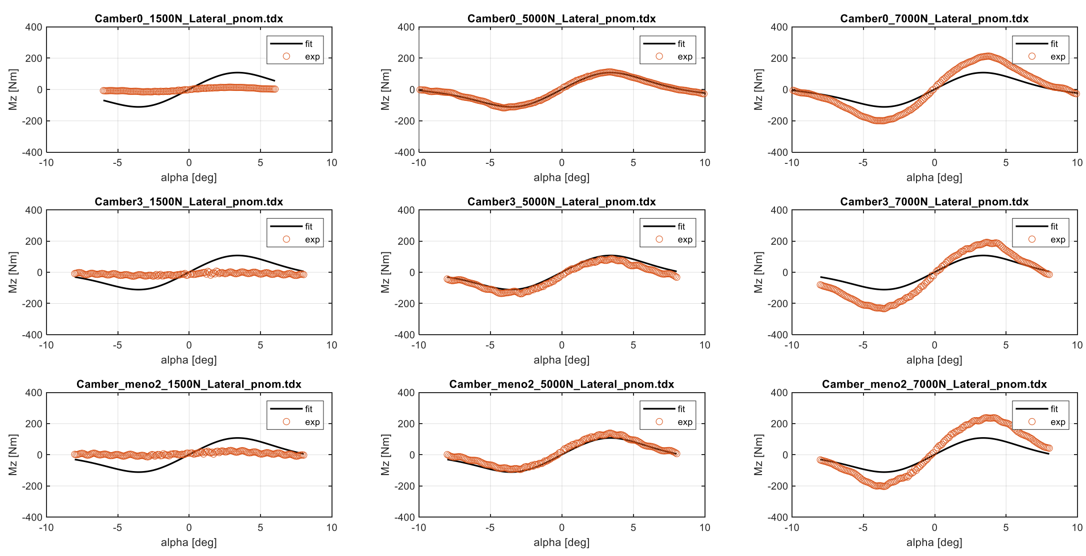

Figure 16

- The quality of the optimization for M_z is clearly related to the quality of the optimization for F_y (4.2.1) and the same trend is observed: the optimization of the data collected at nominal load (central column) are reasonably good while the other ones are bad
- **The average RMS error is** $\approx 75\ Nm$

4.3.2 Extended MF

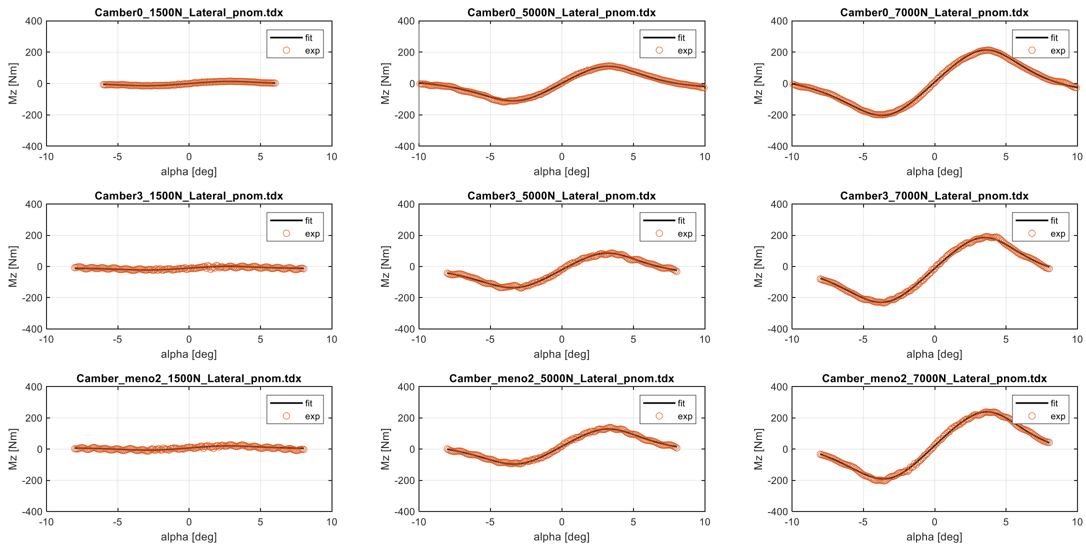

Figure 17

- The results of the optimization are much more promising in this case; the model is able to track the trend of experimental data and the effect of camber angle and vertical load (recall that during the test was not repeated for different values of inflation pressure)
- **The average RMS error is** $\approx 2\ Nm$

5. Conclusion

The laboratory successfully a fitted the data in a MF-Tire model by combining experimental data processed with numerical optimization techniques. The identification of the lateral relaxation length through both time-domain cross-correlation and frequency-domain FRF provided a baseline for describing the transient behaviour of the tire.

The implementation of the fmincon algorithm allowed for more precise tuning of scaling factors and coefficients, with respect to MATLAB toolbox, resulting in a model capable of following accurately experimental trends. These findings confirm that for advanced vehicle dynamics applications, tire models must incorporate multi-variable sensitivities to provide reliable predictions across the entire operating range.