POLITECNICO MILANO 1863 | DIPARTIMENTO DI MECCANICA

Design of a Cruise Control

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Contents

Contents

Part 1 – Mechanical system

01

1.a) Equation of motion non linearized

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{x}}\right)-\frac{\partial E_k}{\partial x}+\frac{\partial D}{\partial \dot{x}}+\frac{\partial V}{\partial x}=Q$$

$$\begin{cases}
E_k=\frac{1}{2}M\dot{x}^2+2J_w\left(\frac{\dot{x}}{R_w}\right)^2+\frac{1}{2}J_M\left(\frac{\tau_1\tau_2\dot{\beta}}{R_w}\right)^2=\frac{1}{2}\underbrace{\left(M+4\frac{J_w}{R_w^2}+J_M\frac{\tau_1^2\tau_2^2\beta}{R_w^2}\right)}_{m^*}\dot{x}^2 \\
V=0 \\
D=\frac{1}{2}\underbrace{\left(\frac{r_1\tau_2^2}{R_w^2}+\frac{r_2\tau_2^2}{R_w^2}\right)}_{r^*}\dot{x}^2 \\
Q=T_M\tau_1\tau_2\frac{1}{R_w}-\frac{1}{2}\rho AC_x(\dot{x}+w)^2-Mg\sin(\alpha)-C_rMg\left(1+K_{rr}\dot{x}\right)
\end{cases}$$

$$\boxed{m^*\ddot{x}+r^*\dot{x}=\left[T_M\tau_1\tau_2\frac{1}{R_w}-\frac{1}{2}\rho AC_x(\dot{x}+w)^2-Mg\sin(\alpha)-C_rMg\left(1+K_{rr}\dot{x}\right)\right]}$$

NB: in the linearized equation, we have applied the transformation: $\ddot{x}=\dot{v}; \dot{x}=v$

1.a) Equation of motion linearized

- The only non-linear term is the drag force: $F_D = \frac{1}{2}\rho C_x (v + w)^2$
- $F_D \approx F_{D0} + \left.\frac{\partial F_D}{\partial v}\right|_{v_0,w_0} (v - v_0) + \left.\frac{\partial F_D}{\partial w}\right|_{v_0,w_0} (w - w_0)$

$$\boxed{m^* \delta\dot{v} + r^* \delta v = T_c \frac{\tau_1 \tau_2}{R_w} - \rho A C_x (v_0 + w_0) \delta w}$$

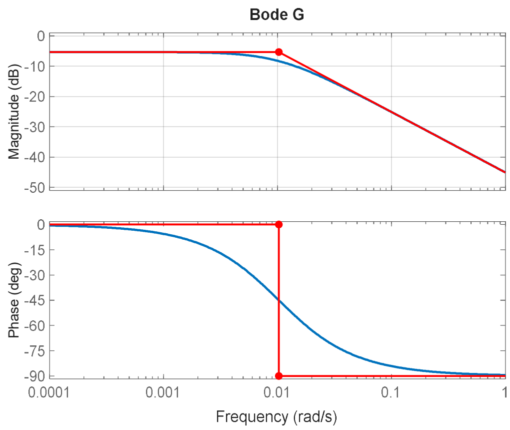
$$\mathrm{r}^* = \mathrm{r}^* + \rho A C_x (v_0 + w_0) + C_{rr} M g k_{rr}$$

m^* hasn't changed, r^*on the other hand is being overwritten by adding the terms deriving from the linearization

1.b) Steady state condition torque Tm0

$$\boxed{T_{M0} = [r^* v_0 + \frac{1}{2}\rho A C_x (v_0 + w_0)^2 + Mg \sin\alpha + C_{rr} Mg(1 + k_{rr} v_0)] \frac{R_w}{\tau_1 \tau_2} = 184{,}3\ N/m}$$

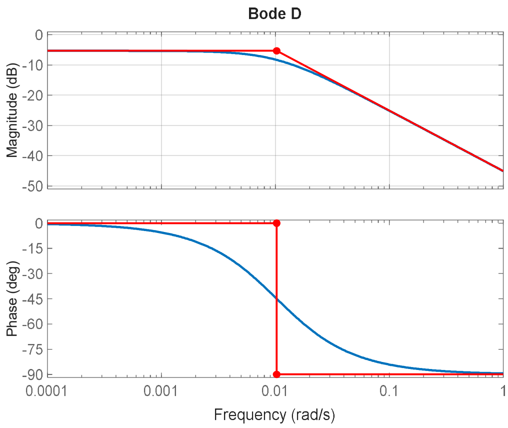
1.c) Transfer function G between v and Tm



- $G = \frac{v}{T_M} = \frac{\tau_1 \tau_2}{R_W(m^* * s + r^*)}$
- P1 = -0,0103 → stable passive system

- Passive system has one pole with Re < 0 → stable passive system

1.d) Transfer function D between v and disturbance

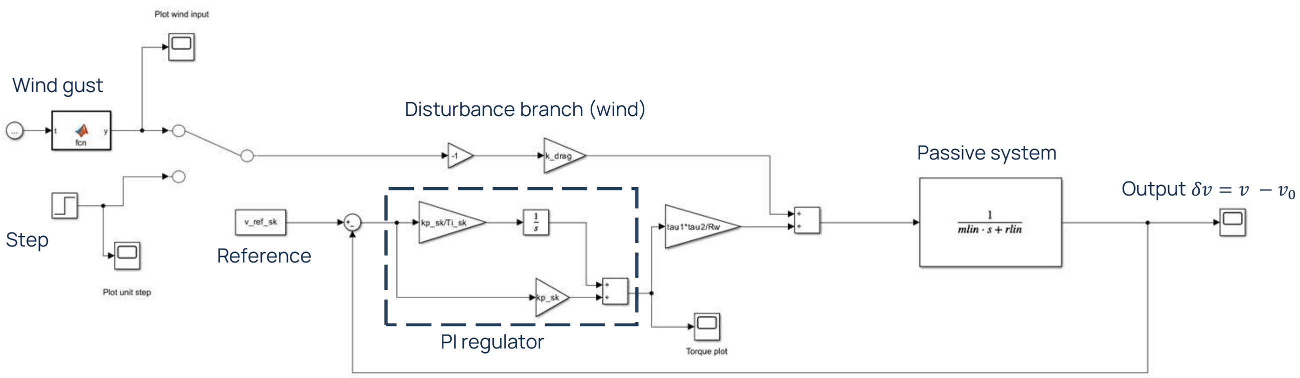


- $D = \frac{v}{w} = \frac{1}{(m^{*} * s + r^{*})}$

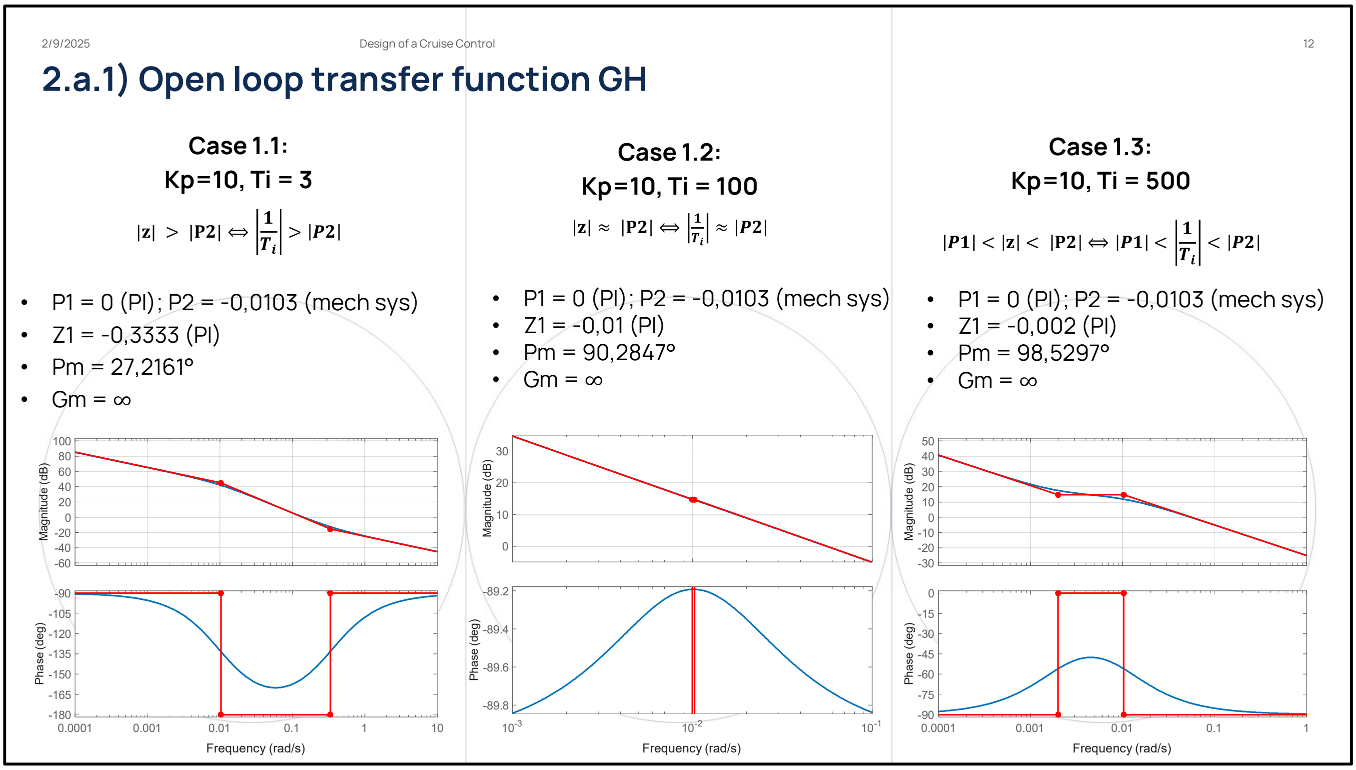
Part 2 – Design of the regulator

02

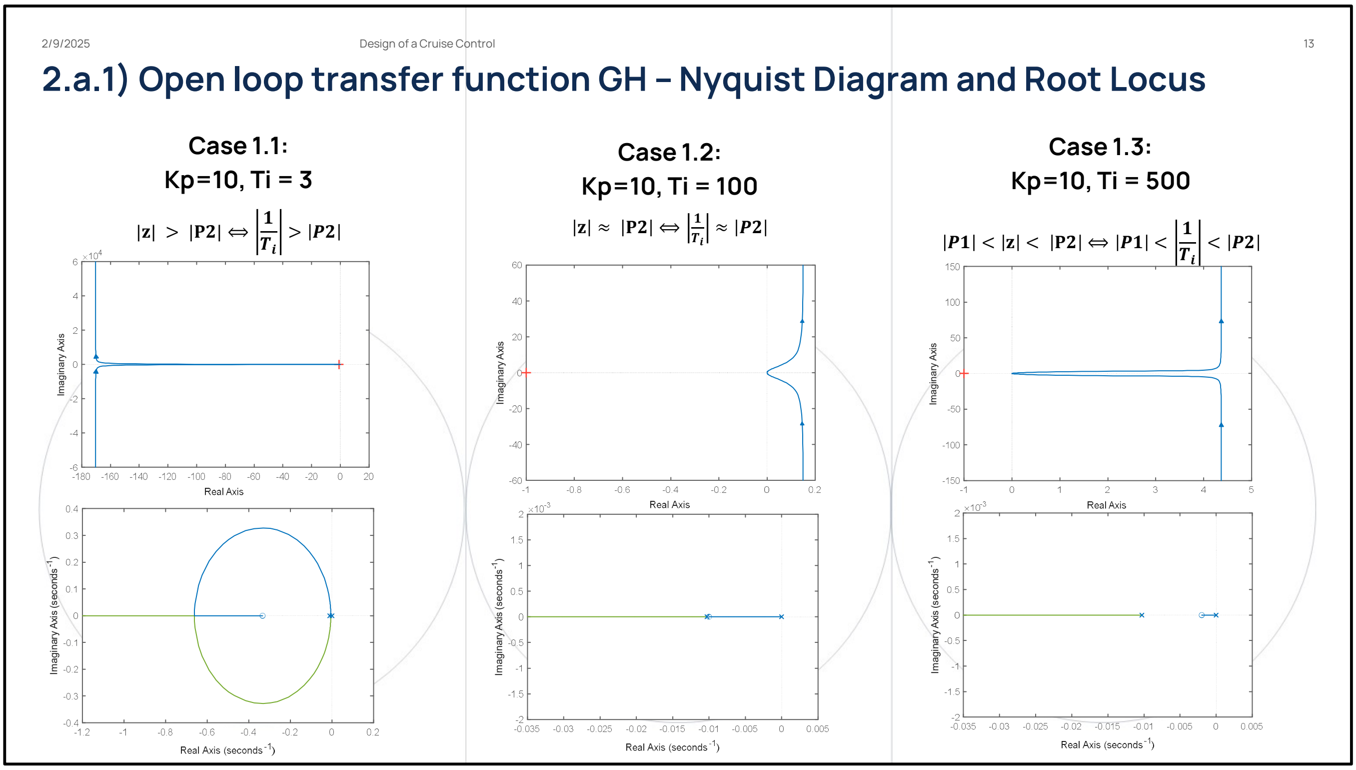
2) Block Diagram



- We needed to check the consistency of the results provided by simulink with the ones provided by matlab. We needed to switch between step and wind disturbance → this is the reason why we introduced a manual switch



- P2 comes from the mechanical system, P1 and Z1 are the pole and the zero introduced by the PI regulator
- 3 cases have been considered based on the position of the zero introduced by the PI regulator
- System is minumum phase → Bode criterion can be used to obtain information about stability → Pm and Gm > 0 → system is stable
  - PS: the reason why the system is never going to be unstable is that with only 2 stable poles the phase is never going to cross –pi (Gm always infinite)
- This same result is confirmed by the nyquist diagram and the root locus

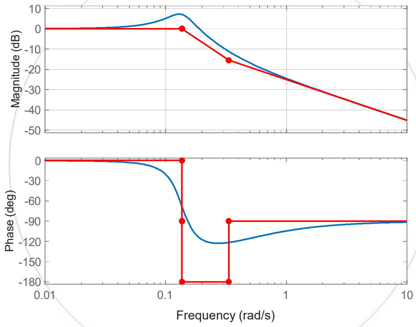


- P2 comes from the mechanical system, P1 and Z1 are the pole and the zero introduced by the PI regulator
- 3 cases have been considered based on the position of the zero introduced by the PI regulator
- System is minumum phase → Bode criterion can be used to obtain information about stability → Pm and Gm > 0 → system is stable
- This same result is confirmed by the nyquist diagram and the root locus (remember that with a pole in zero the circumference in Nyquist plot is ran in clockwise direction)
- NB: the reason why the system is never going to be unstable is that with only 2 stable poles the phase is never going to cross –pi (Gm always infinite)

2.b.1) Closed loop transfer function L

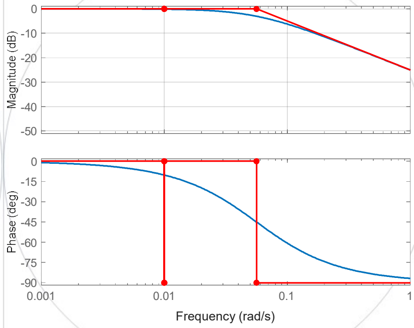
Case 1.1:
Kp=10, Ti = 3

- PC1,2 = -0.0330 ± 0.1323i;
- BW: 0,2155 rad/s



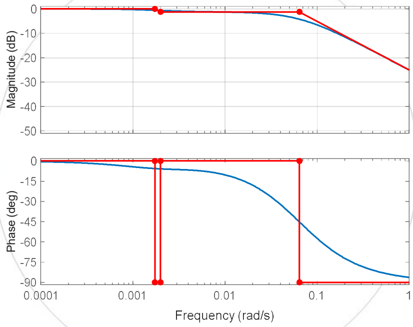
Case 1.2:
Kp=10, Ti = 100

- PC1 = = -0.0099 ; PC2 = -0.0561
- BW: 0,0553 rad/s

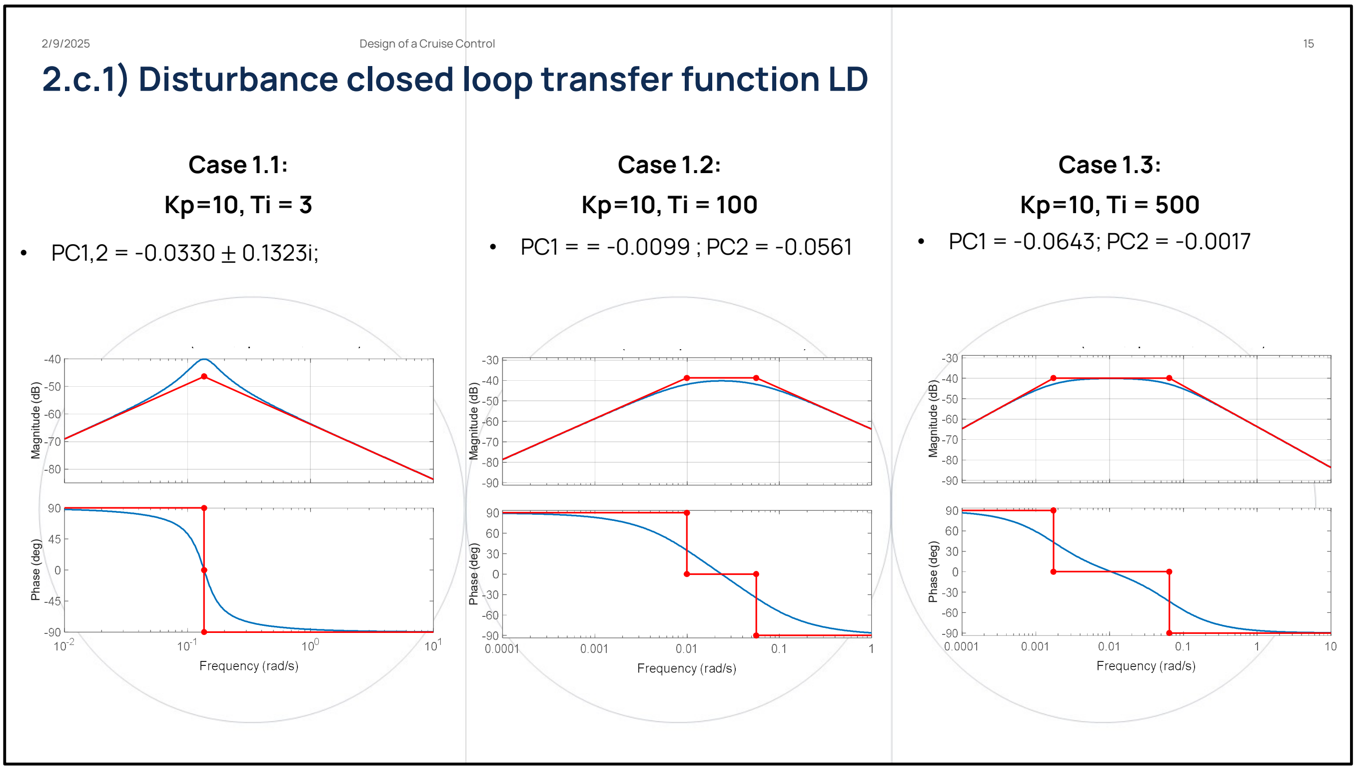


Case 1.3:
Kp=10, Ti = 500

- PC1 = -0.0643; PC2 = -0.0017
- BW: 0,0455 rad/s



- Transfer function L describes how well the reference is tracked → Ti = 3 allows having the largest bandwidth



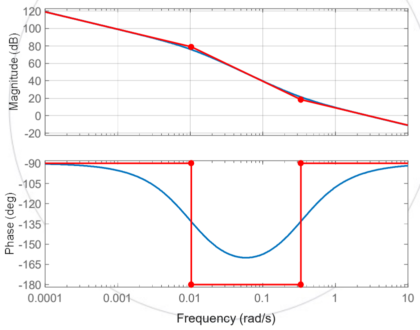
- Transfer function LD describes how well the effect of the disturbance is inhibited → note that in all the frequency range the transfer function is below 0dB axis → disturbance is filtered in all the frequency range
- Note that the poles are the same of L (expected result)

2.a.2) Open loop transfer function GH

Case 2.1:
Kp=500, Ti = 3

$|\mathbf{z}| > |\mathbf{P2}| \Leftrightarrow \left|\frac{1}{T_i}\right| > |P2|$

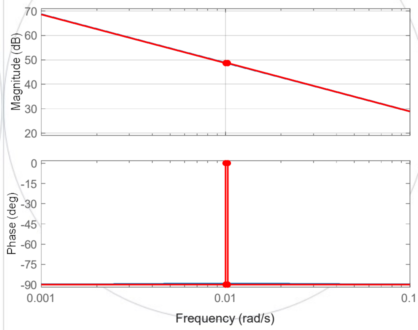
- P1 = 0 (PI); P2 = -0,0103 (mech sys)
- Z1 = -0,3333 (PI)
- Pm = 83,4436°
- Gm = ∞



Case 2.2:
Kp=500, Ti = 100

$|\mathbf{z}| \approx |\mathbf{P2}| \Leftrightarrow \left|\frac{1}{T_i}\right| \approx |P2|$

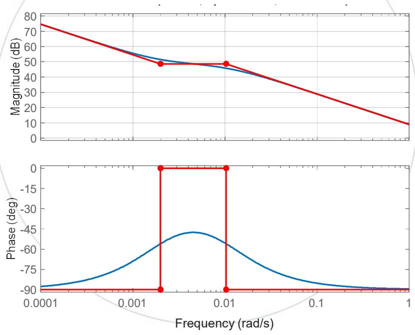
- P1 = 0 (PI); P2 = -0,0103 (mech sys)
- Z1 = -0,01 (PI)
- Pm = 90,0059°
- Gm = ∞



Case 2.3:
Kp=500, Ti = 500

$|P1| < |\mathbf{z}| < |\mathbf{P2}| \Leftrightarrow |P1| < \left|\frac{1}{T_i}\right| < |P2|$

- P1 = 0 (PI); P2 = -0,0103 (mech sys)
- Z1 = -0,002 (PI)
- Pm = 90,1702°
- Gm = ∞

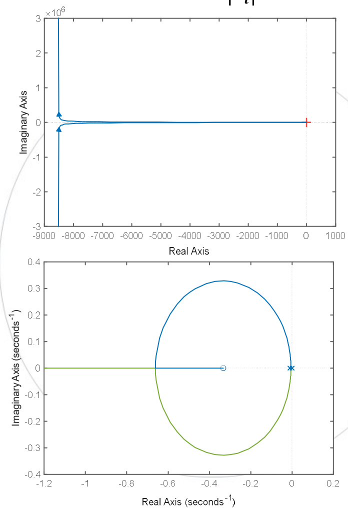


- Increasing kp shifts the GH diagram upwards → in case 1 the phase margin increases because the 0dB axis is crossed with slope -20 instead of -40 (stable zero is placed before the diagram crosses 0dB)

2.a.2) Open loop transfer function GH – Nyquist Diagram and Root Locus

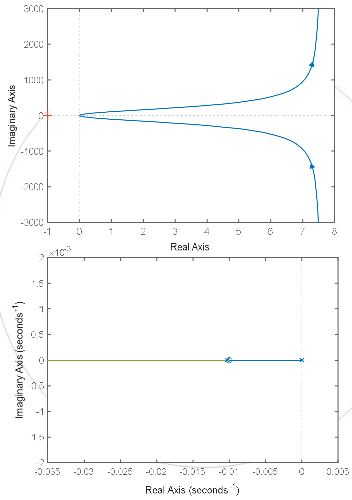
Case 2.1:
Kp=500, Ti = 3

$|z| > |P2| \Leftrightarrow \left|\frac{1}{T_i}\right| > |P2|$



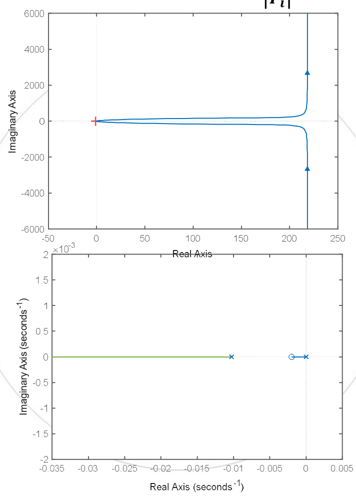
Case 2.2:
Kp=500, Ti = 100

$|z| \approx |P2| \Leftrightarrow \left|\frac{1}{T_i}\right| \approx |P2|$



Case 2.3:
Kp=500, Ti = 500

$|P1| < |z| < |P2| \Leftrightarrow |P1| < \left|\frac{1}{T_i}\right| < |P2|$



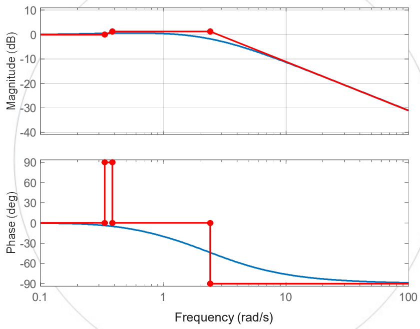
- The system is stable: this result is confirmed by the nyquist diagram and the root locus

2.b.2) Closed loop transfer function L

Case 2.1:

Kp=500, Ti = 3

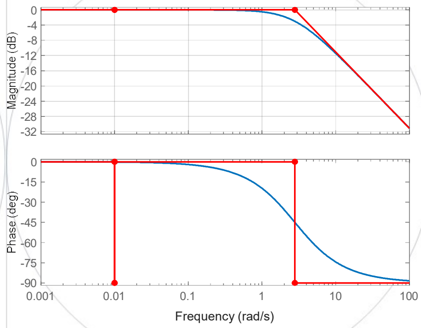
- PC1 = -2,4148; PC2 = -0,3851
- BW: 3,1043 rad/s



Case 2.2:

Kp=500, Ti = 100

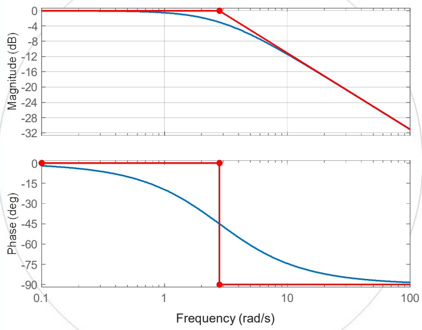
- PC1 = = -2,7899 ; PC2 = -0.01
- BW: 2,7827 rad/s



Case 2.3:

Kp=500, Ti = 500

- PC1 = -2,7979; PC2 = -0.002
- BW: 2,7746 rad/s



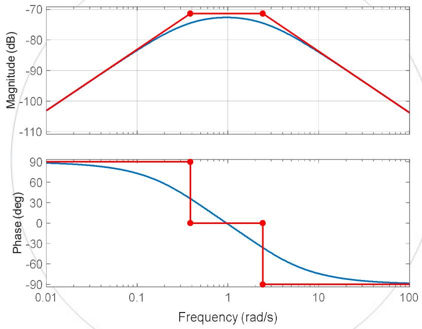
- Coherently to what has been seen at lecture, increasing the Kp increases the bandwidth (see asymptotic behaviour of L with respect to GH)

2.c.2) Closed loop transfer function LD

Case 2.1:

Kp=500, Ti = 3

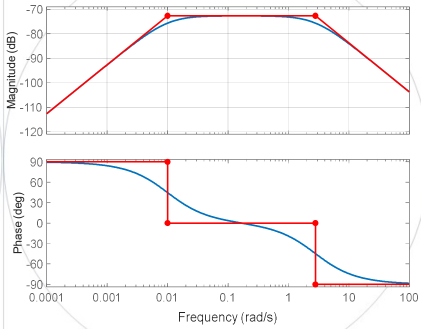
- PC1 = -2,4148; PC2 = -0,3851



Case 2.2:

Kp=500, Ti = 100

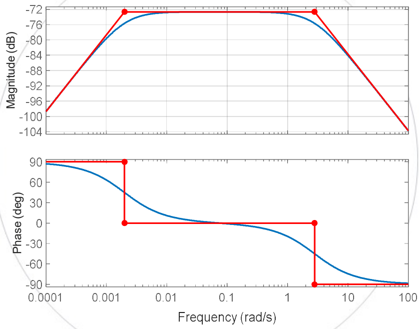
- PC1 = = -2,7899 ; PC2 = -0.01



Case 2.3:

Kp=500, Ti = 500

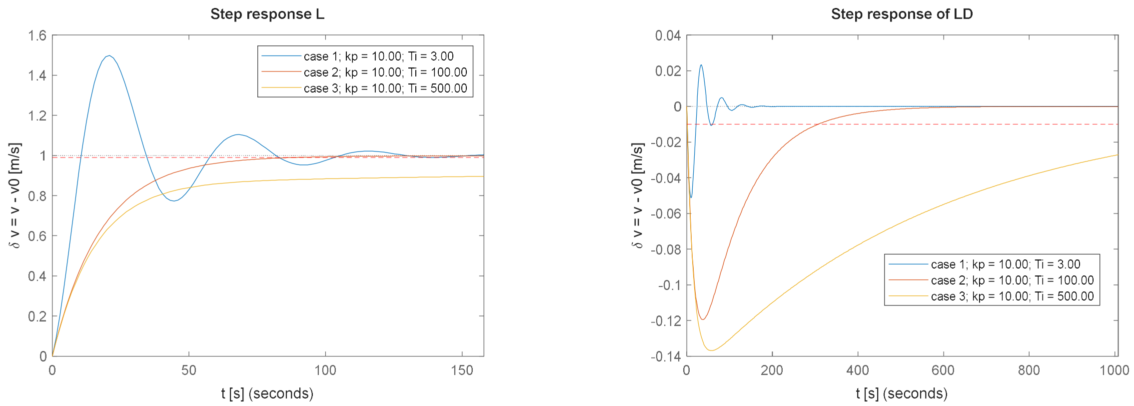
- PC1 = -2,7979; PC2 = -0.002



- Transfer function LD describes how well the effect of the disturbance is inhibited → note that in all the frequency range the transfer function is below 0dB axis → disturbance is filtered in all the frequency range
- Increasing kp has an effect on the poles (note that for Ti = 3) they are no longer complex conjugate and it shifts the diagram downwards → more filtering action of the disturbance
- Note that the poles are the same of L (expected result)
- Results obtained up to this point:
    - Increasing kp seems to have only positive effects in terms of stability
    - Increasing Ti doesn't seem to have significant effects on the stability (except for what concerns the bandwidth of L) → it is worth looking at how Ti affect the performance of the system

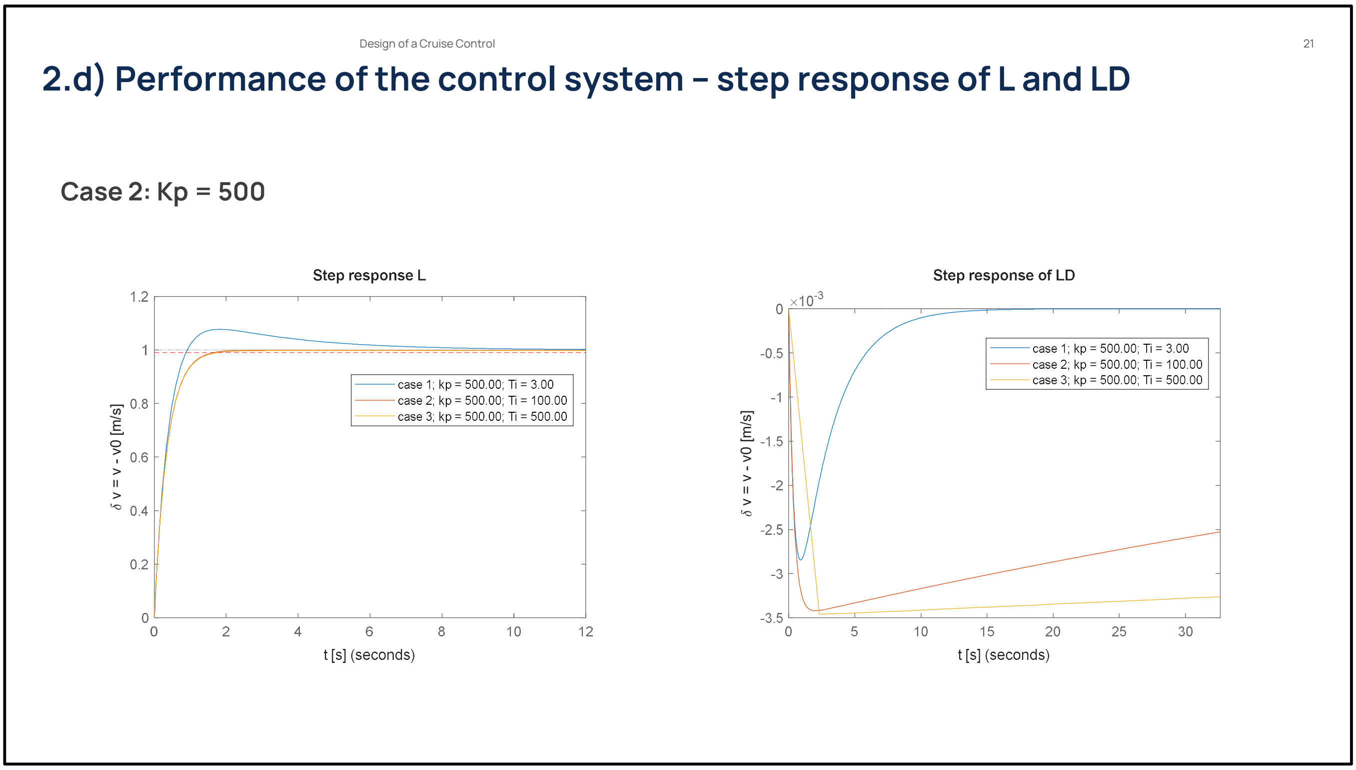
2.d) Performance of the control system – step response of L and LD

Case 1: Kp = 10



- Step response L:
    - Settling time is high for every Ti
    - In case 1 the solution is oscillating (from root locus we can tell that by increasing kp it is no longer going to happen)

- Step response LD:
    - Settling time is high for every Ti (note that time needed to compensate disturbance is even higher than time needed to track the reference)
    - In case 1 the solution is oscillating but percentage overshoot is very low
- With low kp, the best solution in terms of stability (Ti = 3), doesn't seem to be very promising → kp = 500

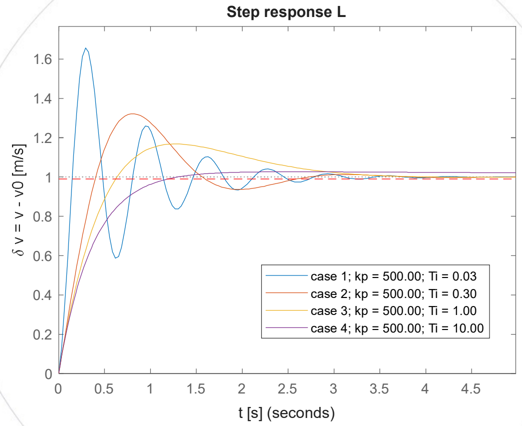


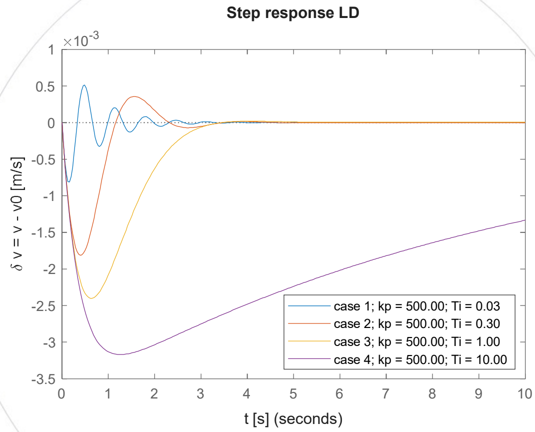
- Step response L:
    - The output is no longer oscilating (as predicted from the root locus) even though it still has a small overshoot
    - Higher Ti allows to track the reference better
- Step response LD:
    - The amplitude of the overshoot is extremely small (already within 99% of steady state value)
    - Low Ti allows to converge to reference value faster, in this case, high Ti implies high settling time (the zero is at low frequency)

- → high Ti allows tracking the reference better but it decreases the capability of mitigating the effect of the disturbance, on the other hand, low Ti implies a small oscillation when tracking the reference but the response to the disturbance is very good
- Trade off is needed → step response plot for multiple Ti

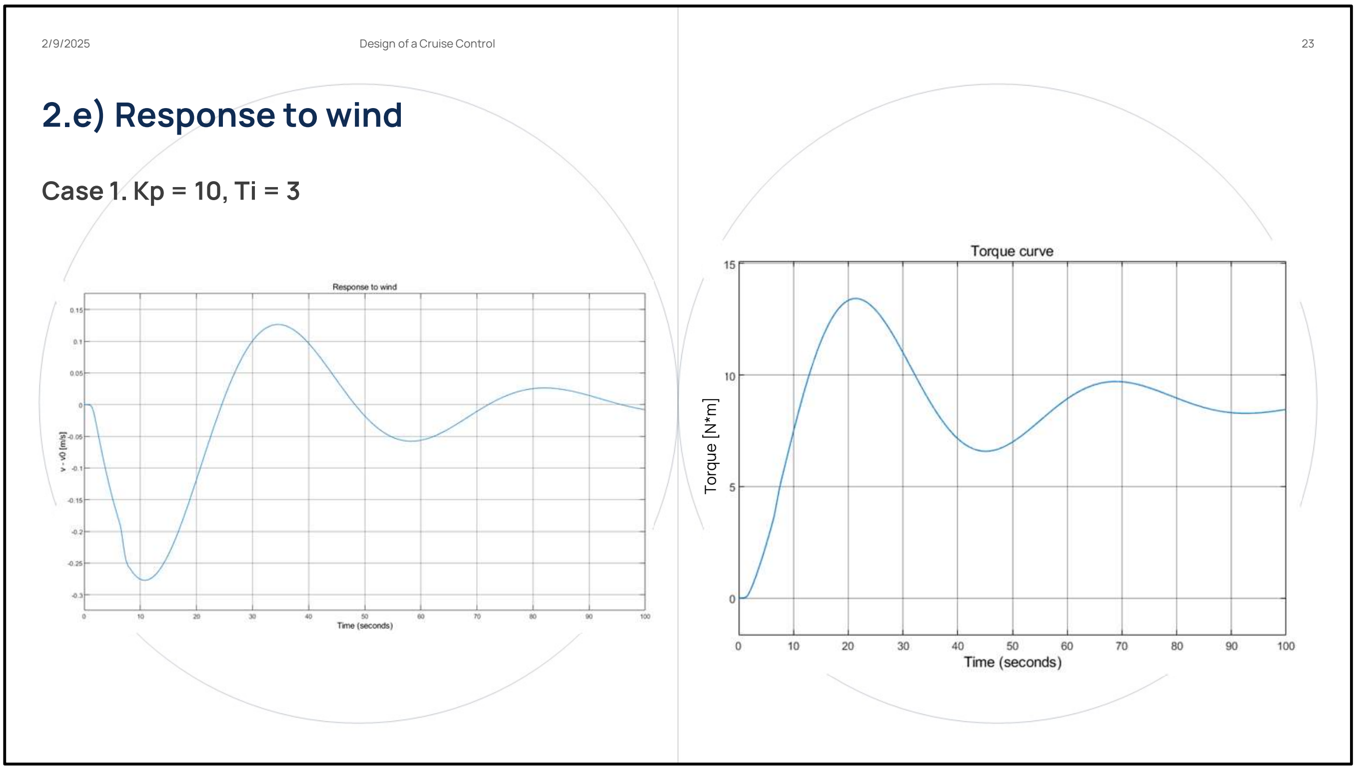
2/9/2025 Design of a Cruise Control 22

2.d) Performance of the control system – step response of L and LD

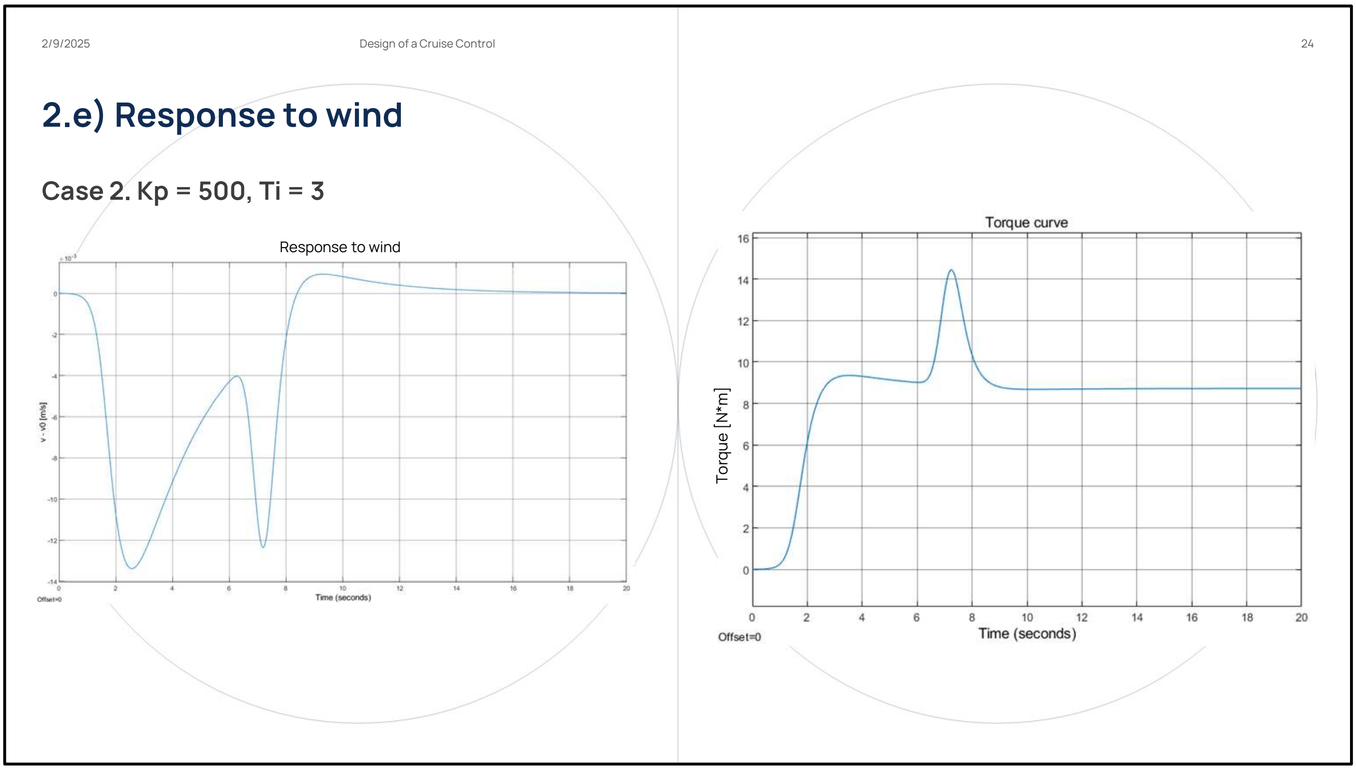




- From these plots, it is even more clear how a trade off is needed in the choice of Ti
- From this plots, having a higher Ti seems to be acceptable (even if settling time in step(LD) is higher, the amplitude of the oscillations is extremely low
- However, considering the stability analysis, Ti is going to be chosen low (= 3); we expect to decrease the overshoot in of step(L) with the PID regulator in part 3
- For the last part of the stability analysis, we are going to investigate how does the system respond to the disturbance



- As suggested in slide 20 by the step response, for low kp the output associated to the disturbance is oscillating
- Settling time is high with small overshoot, note that the amount of torque that the motor needs to provide is reasonable in terms of maximum value and time needed

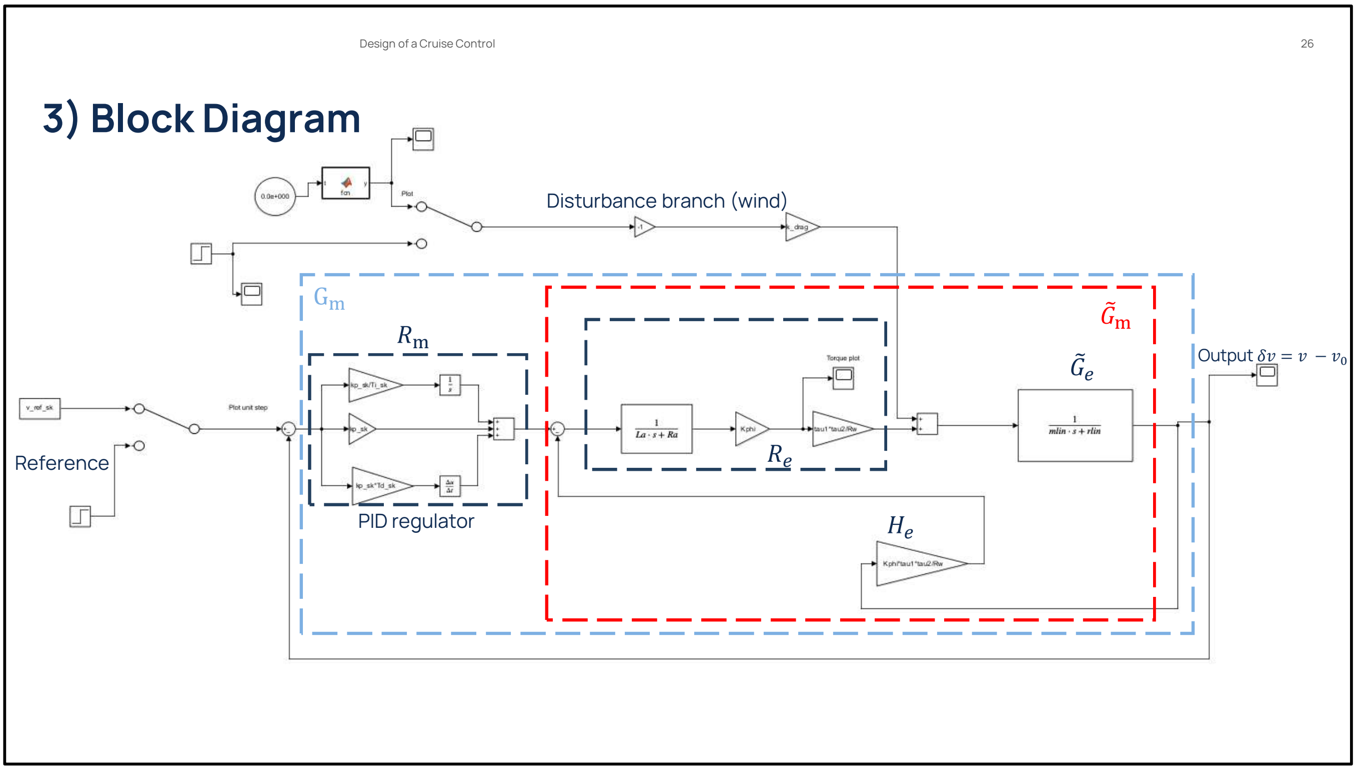


The response exhibits variations that align with the wind function we introduced as the disturbance, demonstrating coherence with the temporal correlation of the wind changes.

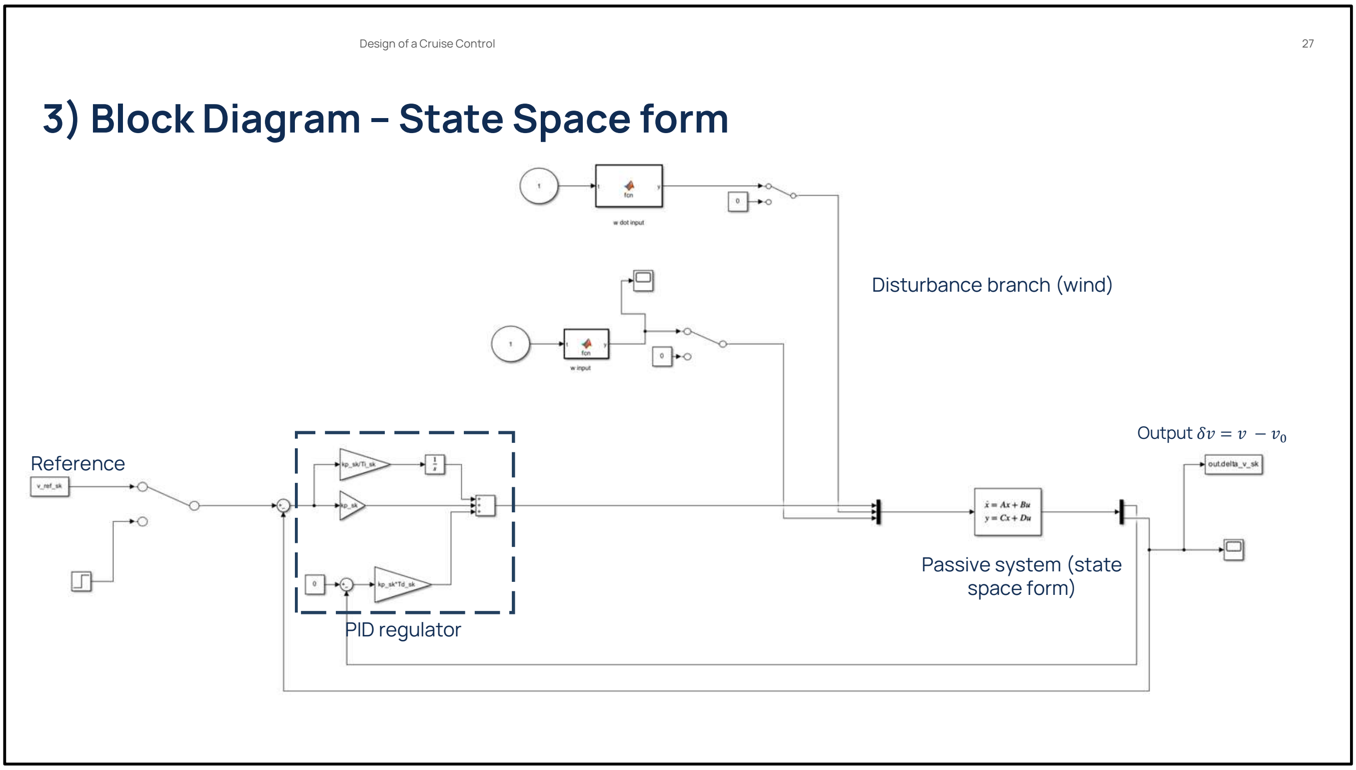
The maximum torque value is approximately 14 N/m, which we consider to be reasonable.

Part 3 – Permanent magnet DC motor

03



- We needed to check the consistency of the results provided by simulink with the ones provided by matlab. We needed to switch between step and wind disturbance → this is the reason why we introduced a manual switch



- We needed to check the consistency of the results provided by simulink with the ones provided by matlab. We needed to switch between step and wind disturbance → this is the reason why we introduced a manual switch
- Note that we have verified in the Matlab code that the eigenvalues of state matrix A are equal to the poles of G_tilde_m

3.a) Linearized equation of the electro-mechanical system

$$
\begin{cases}
m^*\delta\dot{v} + r^*(\delta v + v0) = T_M \dfrac{\tau_1\tau_2}{R_w} - F_{D0} - \rho A C_x(v_0 + w_0)\delta v \ - \rho A\, C_x(v_0 + w_0)\delta w - Mgsin\alpha - C_{rr}Mg - C_{rr}MgK_{rr}(\delta v + v_0) \\
T_M = k\_\phi(\delta i_a + i_{a0}) \\
\delta V_a + V_{a0} = L_a \dfrac{d(\delta i_a + i_{a0})}{dt} + R_a(\delta i_a + i_{a0})
\end{cases}
$$

- Rewriting the equation of motion previously computed
- DC motor equation is also introduced; relationship between theta dot and v of the car is known

3.b) Steady state condition voltage Va0

$$V_{a_0} = \left\{\left[r^* v_0 + \frac{1}{2}\rho A C_x (v_0 + w_0)^2 + Mg sin\alpha + C_{rr} Mg cos\alpha(1 + \ k_{rr} v_0)\right]\frac{R_w}{k_{\phi \tau_1 \tau_2}} R_a\right\} + k_\phi \frac{\tau_1 \tau_2}{R_w} v_0 \approx 114\, V$$

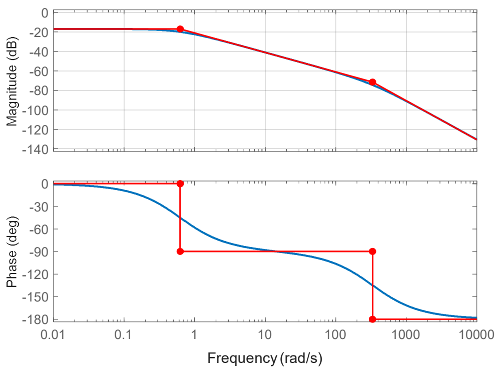
- Steady state is imposed in the equation of motion to get the steady state value of the current
- Steady state current is substituted into the equation of the DC motor to compute the steady state value of the tension Va0, which is Va0

3.c) Transfer function $\widetilde{G}_m$ between voltage and speed

$$\frac{\Delta V}{V_a} = \frac{k_\phi \tau_1 \tau_2 R_w}{(m^* s + r)(L_a s + R_a) R_w^2 + K_\phi^2 (\tau_1 \tau_2)^2}$$

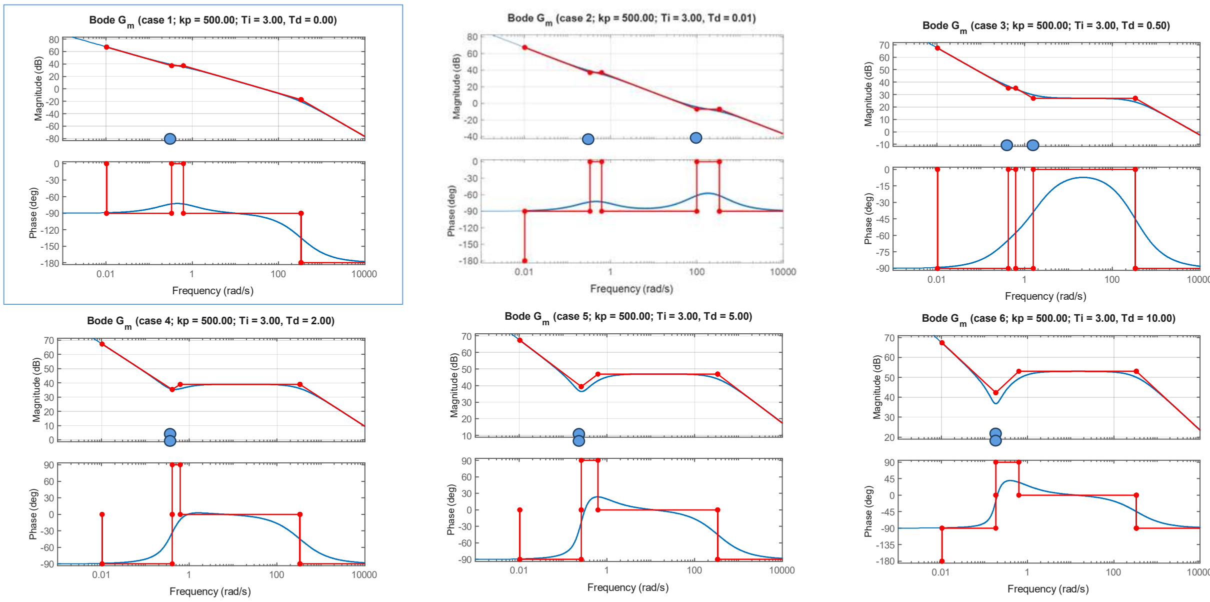
3.c) Transfer function $\tilde{G}_m$ between voltage and speed

Comparison between mechanical and electro-mechanical system



- $\begin{cases} P1 = -332.7201 \\ P2 = -0.6235 \end{cases} \rightarrow$ stable passive system

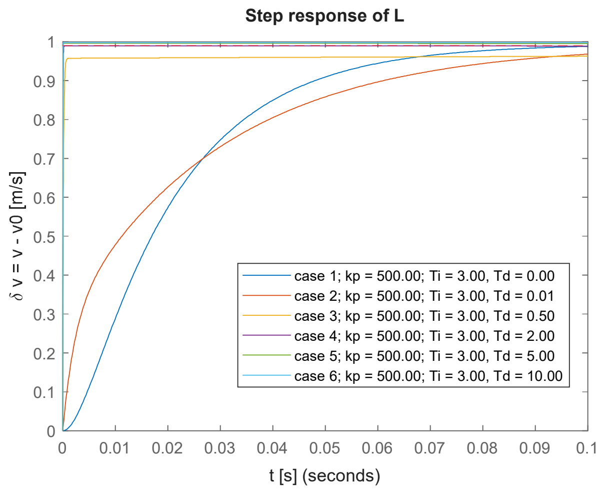
3.d) Preliminary comparison between PI and PID

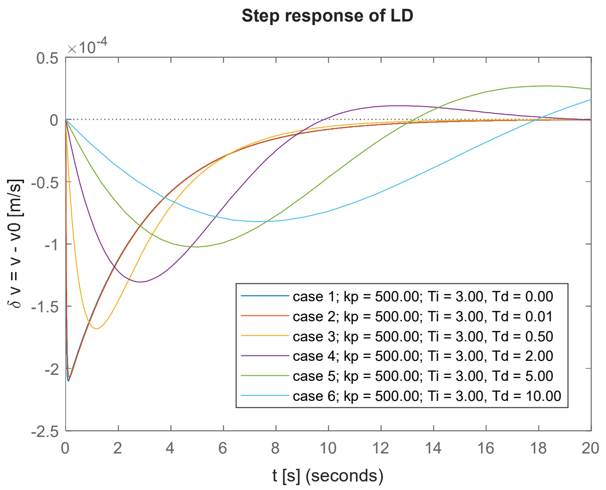


- The starting point for part 3 is the same parameter combination we identified at the end of part 2: kp = 500; Ti = 3. Td needs to be identified
- From the expression of $G_m = R_m * \tilde{G}_m$. Note that $R_m = \frac{T_i T_d * s^2 + T_i * s + 1}{T_i * s}$, therefore we conclude that the PID regulator introduces 2 zeros (that can be real or complex conjugate) and one pole in s = 0 (this pole is always introduced for every value of Td)
- In case 1 the PID provides only one zero in -1/Ti
- In case 2 the PID introduces two real zeros
- As Td increases both the poles move at lower frequency until they become complex conjugated (case 4)
- In the next slides, the effect of Td is also evaluated in terms of performance
- Among all these cases, we analyzed the stability for cases 1, 3 , 6

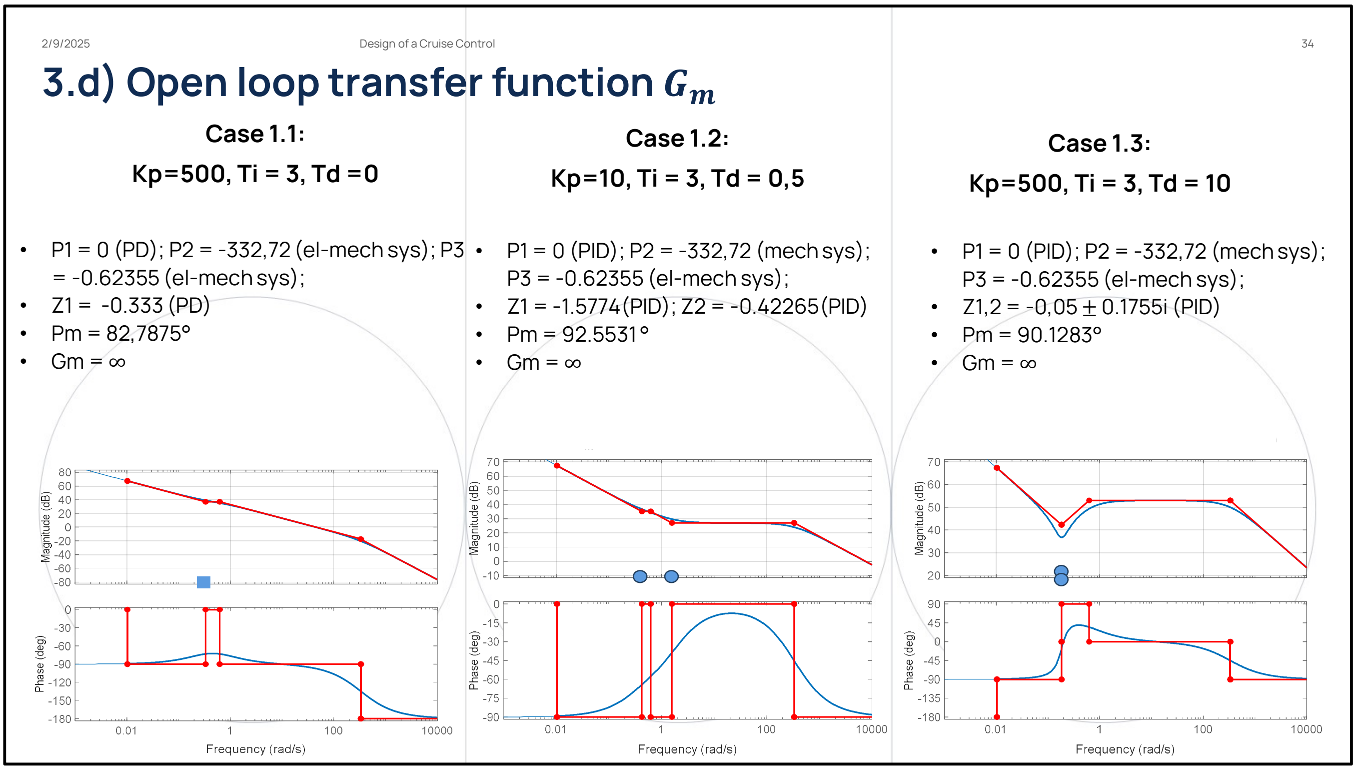
Design of a Cruise Control 33

3.d) Preliminary comparison between PI and PID

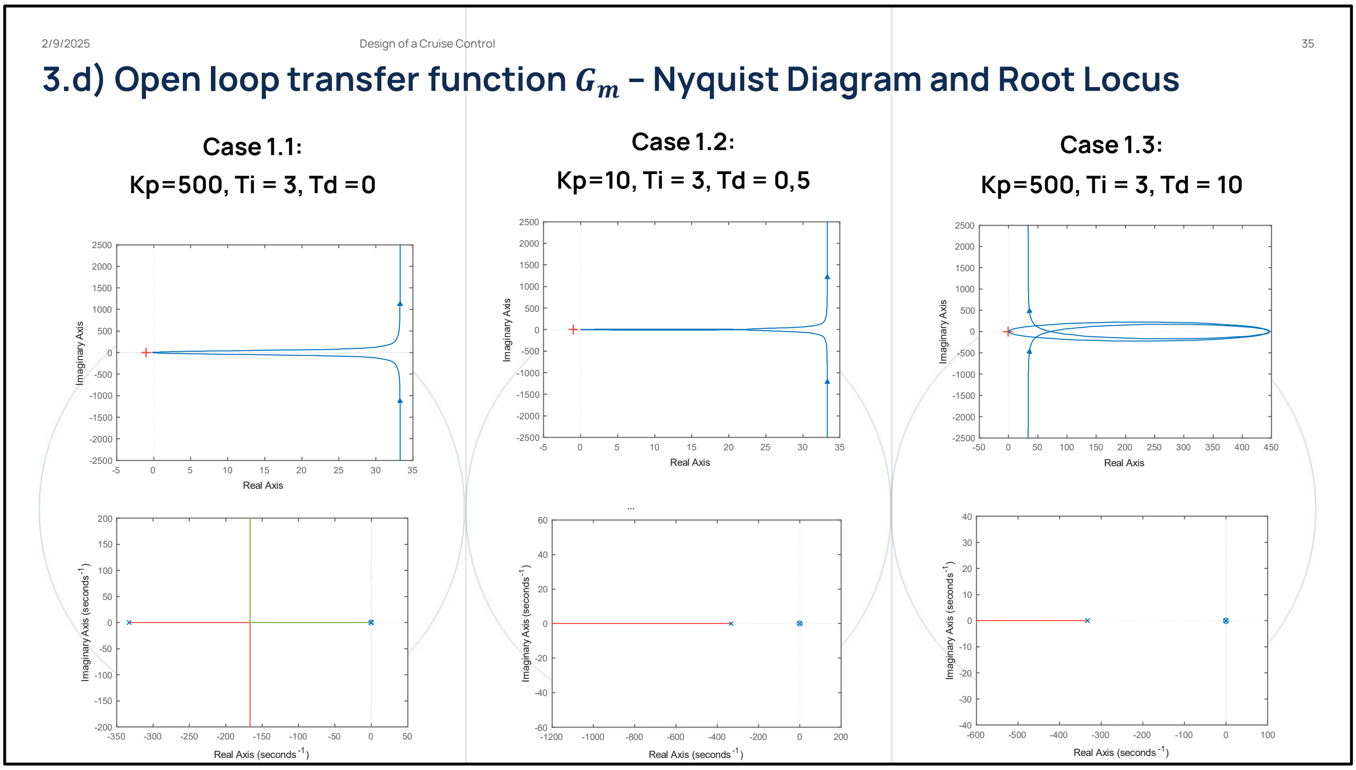




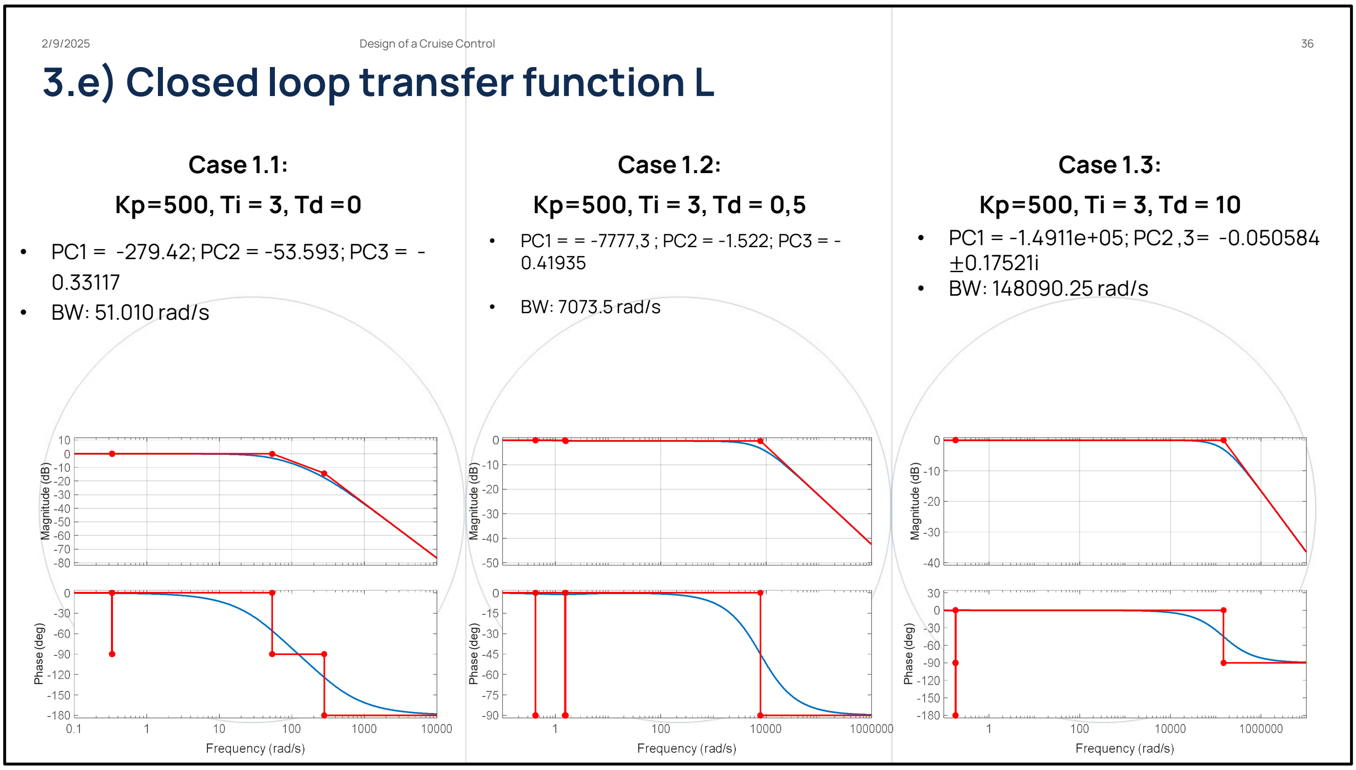
- We also wanted to see how do the performance change for different values of Td
- LD: from Td=2 the response starts to have some oscillations
- the oscillation is due to the complex coniugated poles in the closed loop TF, if there are complex conjugated zeros in GH there could be complex conjugated poles in L (den(L) = 1 + num(GH))



- G_m is the open loop TF of the controlled elettromechanic system.
- We change Td to change the position of the zeros introduced by the PID
- The system is minimum phase, looking at Gm and Pm the system is stable for each case of Td



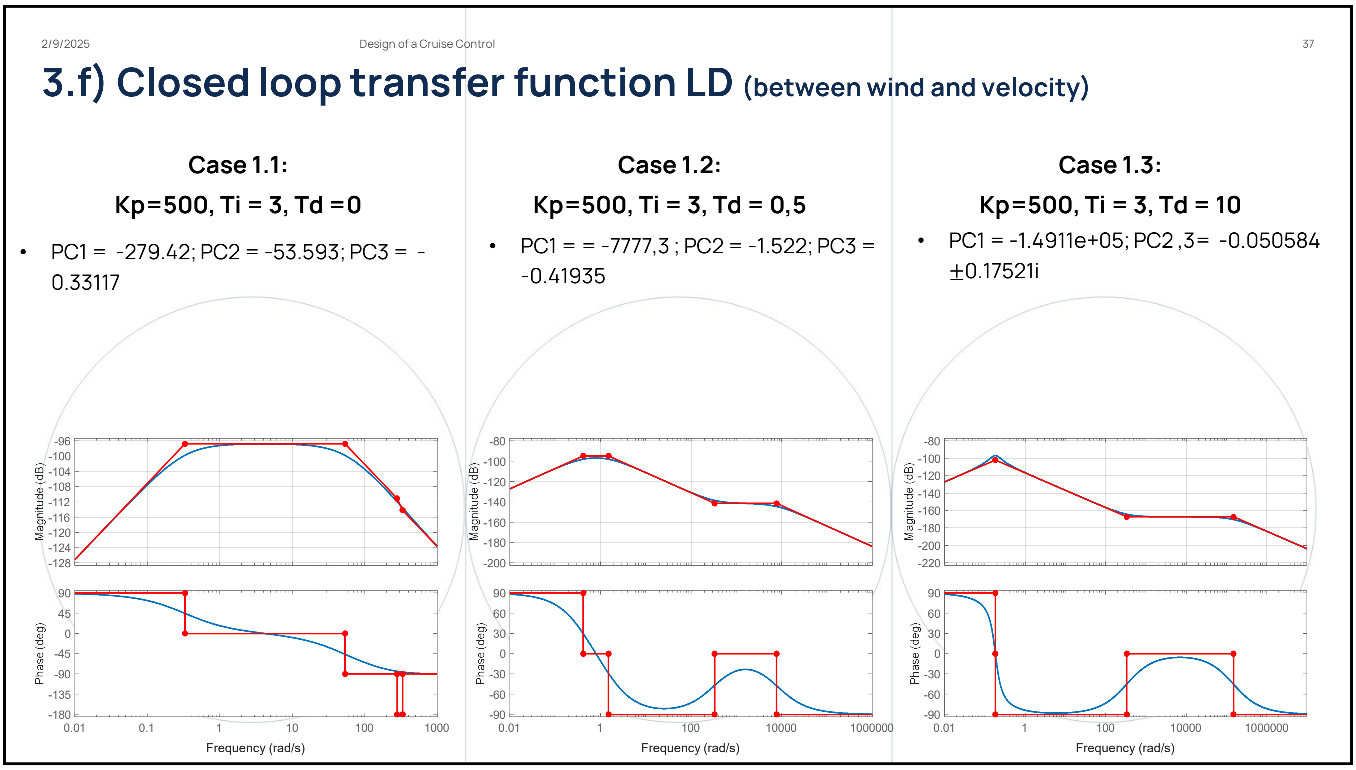
- The system is stable for the Td selected : this result is confirmed by the nyquist diagram and the root locus



Poles and zeros are very close

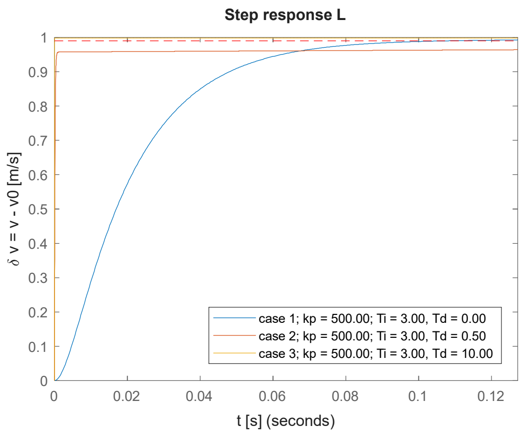
In the last case is confirmed that there are complex conjugated poles

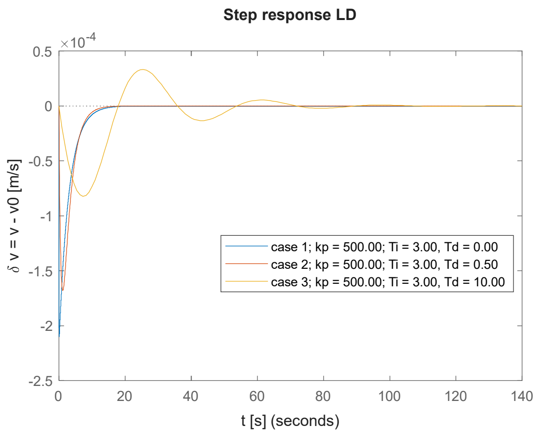
BW increases



The frequency range where the maximum value is reached, as Td increases, decreases in magnitude and also the frequency range decreases

3.g) Performance of the control system – step response of L and LD

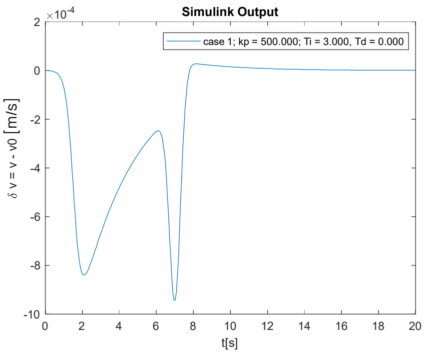




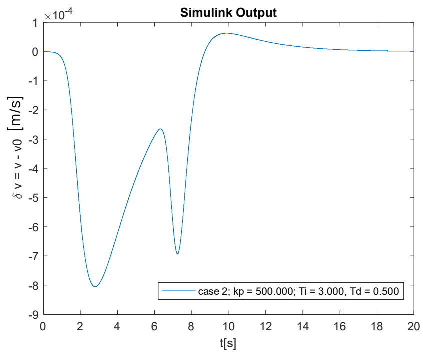
- Here it is clear that as TD increases the system will experience oscillations and settling time also increases
- Step(L) doesn't provide any useful information

3.h) Response to wind: speed

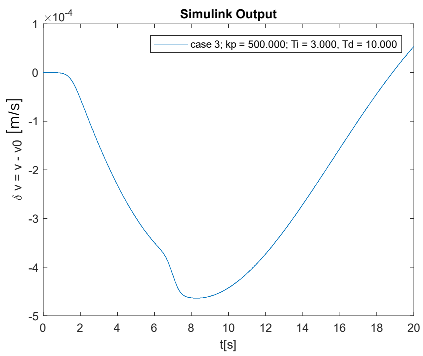
Case 1
Kp = 500, Ti = 3,00, Td = 0



Case 2
Kp = 500, Ti = 3,00, Td = 0,50

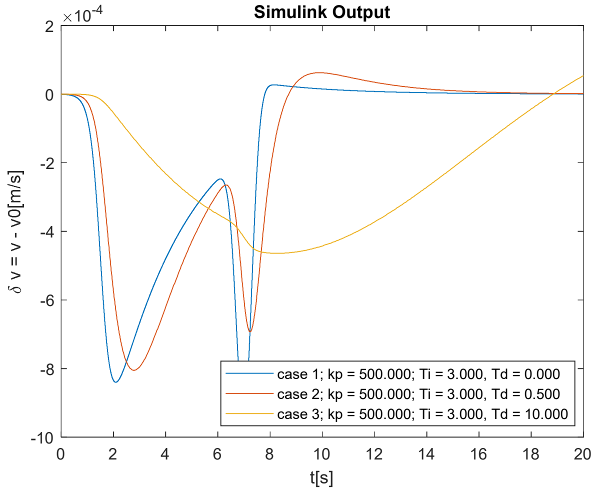


Case 3
Kp = 500, Ti = 3,00, Td = 10,0



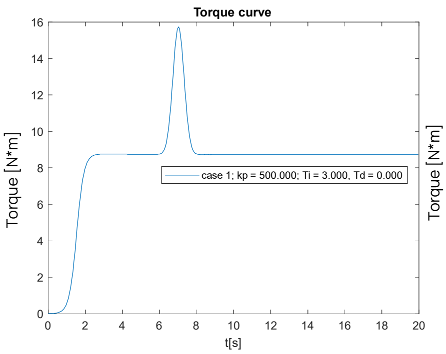
Up to a certain Td the response exhibits variations that align with the wind function we introduced as the disturbance, demonstrating coherence with the wind changes. However, we are in range of 10e-4 magnitude so we will not experience differences if we were in the car

3.h) Response to wind: speed

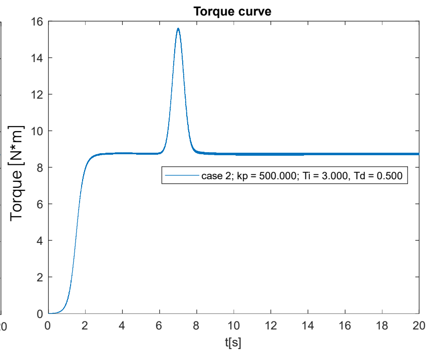


3.h) Response to wind: torque required

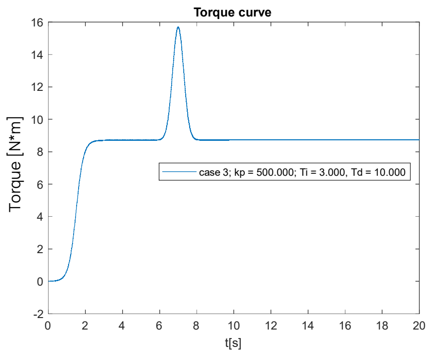
Case 1
Kp = 500, Ti = 3,00, Td = 0



Case 2
Kp = 500, Ti = 3,00, Td = 0,50

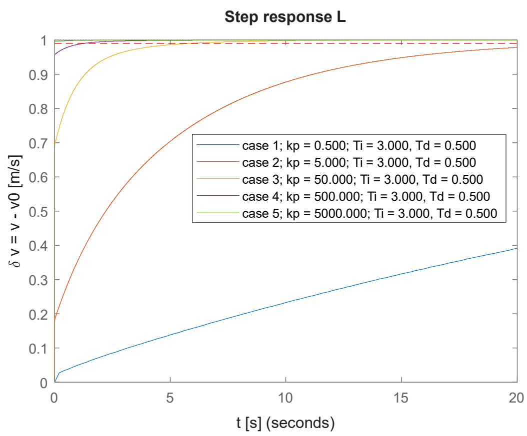


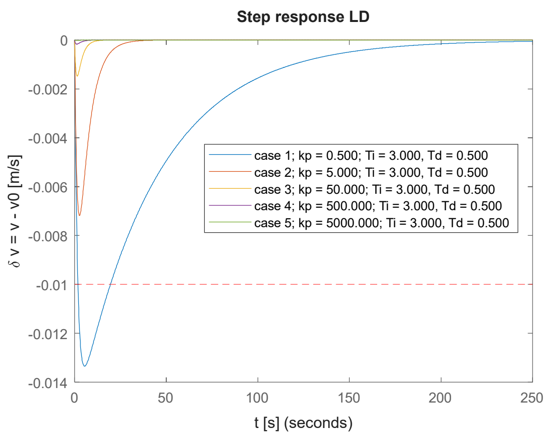
Case 3
Kp = 500, Ti = 3,00, Td = 10,0



The maximum torque value is approximately 15 N/m, which we consider to be reasonable
It is not really affected by the Td coefficient (to see effect of kp, see part 2)

3.i) Performance of the control system – effect of kp on step response

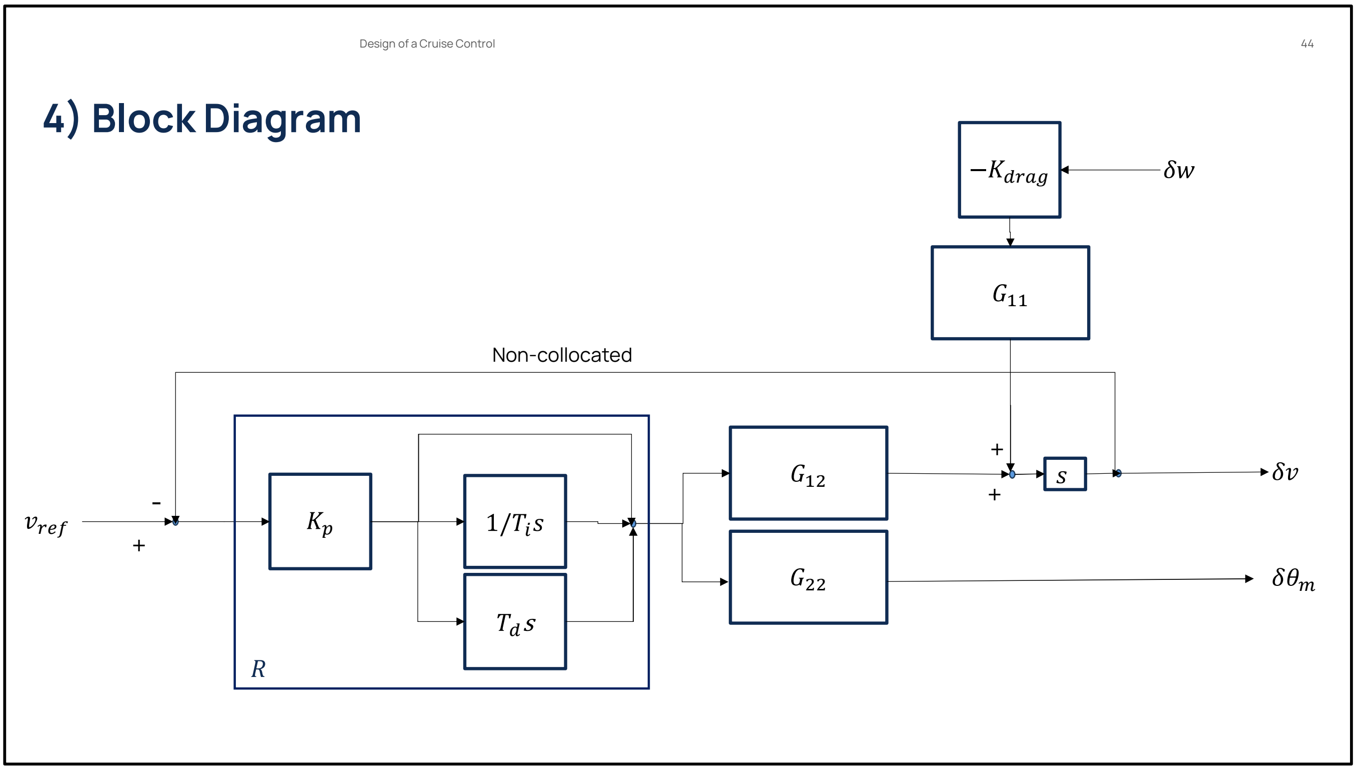




- For increasing kp, the rise time is lower (converges faster to reference)

Part 4 - Deformable drive shaft

04



- We needed to check the consistency of the results provided by simulink with the ones provided by matlab. We needed to switch between step and wind disturbance → this is the reason why we introduced a manual switch
- Note that we have verified in the Matlab code that the eigenvalues of state matrix A are equal to the poles of G_tilde_m

4.a) Linearized equations of motion

$$\begin{bmatrix} M + 4\left(\frac{J_w}{R_w^2}\right) & 0 \\ 0 & J_M \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta}_M \end{bmatrix} + \begin{bmatrix} \left(\frac{\tau_2}{R_w}\right)^2 (r_2 + c_t) + \rho A_f C_x (v_0 + w_0) + C_{rr} k_{rr} M g \cos(\alpha) & -c_t \frac{\tau_2}{\tau_1 R_w} \\ -c_t \frac{\tau_2}{\tau_1 R_w} & \left(\frac{1}{\tau_1}\right)^2 (r_1 + c_t) \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\theta}_M \end{bmatrix}$$

$$+ \begin{bmatrix} k_t \left(\frac{\tau_2}{R_w}\right)^2 & -k_t \frac{\tau_2}{\tau_1 R_w} \\ -k_t \frac{\tau_2}{\tau_1 R_w} & \frac{k_t}{\tau_1^2} \end{bmatrix} \begin{bmatrix} x \\ \theta_M \end{bmatrix} = \begin{bmatrix} -\rho A_f C_x (v_0 + w_0) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta w \\ T_M \end{bmatrix}$$

4.b) Transfer function G_{22} between torque and motor angular speed

$$D = \begin{bmatrix} \left[M + 4\left(\frac{J_w}{R_w^2}\right)\right]s^2 + \left[\left(\frac{\tau_2}{R_w}\right)^2 (r_2 + c_t) + \rho A_f C_x (v_0 + w_0) + C_{rr} k_{rr} M g \cos(\alpha)\right]s + k_t\left(\frac{\tau_2}{R_w}\right)^2 & \left(-c_t \frac{\tau_2}{\tau_1 R_w}\right)s - k_t \frac{\tau_2}{\tau_1 R_w} \\ \left(-c_t \frac{\tau_2}{\tau_1 R_w}\right)s - k_t \frac{\tau_2}{\tau_1 R_w} & J_M s^2 + \left[\left(\frac{1}{\tau_1}\right)^2 (r_1 + c_t)\right]s + \frac{k_t}{\tau_1^2} \end{bmatrix}$$

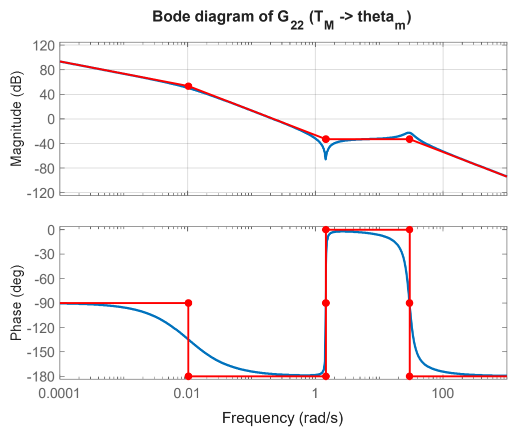
$$\det(D) = \left[\left[M + 4\left(\frac{J_w}{R_w^2}\right)\right]s^2 + \left[\left(\frac{\tau_2}{R_w}\right)^2 (r_2 + c_t) + \rho A_f C_x (v_0 + w_0) + C_{rr} k_{rr} M g \cos(\alpha)\right]s + k_t\left(\frac{\tau_2}{R_w}\right)^2\right] * \left[J_M s^2 + \left[\left(\frac{1}{\tau_1}\right)^2 (r_1 + c_t)\right]s + \frac{k_t}{\tau_1^2}\right] - \left[\left(-c_t \frac{\tau_2}{\tau_1 R_w}\right)s - k_t \frac{\tau_2}{\tau_1 R_w}\right] * \left[\left(-c_t \frac{\tau_2}{\tau_1 R_w}\right)s - k_t \frac{\tau_2}{\tau_1 R_w}\right]$$

4.b) Transfer function G_{22} between torque and motor angular speed

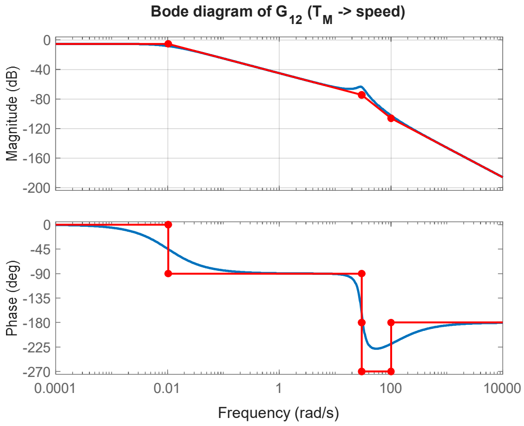
$$G_{22}(s) = \frac{\theta_M(s)}{T_M(s)}$$

$$G_{22}(s) = \frac{D_{11}}{\det(D)} = \frac{\left[M + 4\left(\frac{J_w}{R_w^2}\right)\right]s^2 + \left[\left(\frac{\tau_2}{R_w}\right)^2 (r_2 + c_t) + \rho A_f C_x (v_0 + w_0) + C_{rr} M g \cos(\alpha)\right]s + k_t \left(\frac{\tau_2}{R_w}\right)^2}{\det(D)}$$

4.b) Bode of TF G_{22} between torque and motor angular speed



4.c) Transfer function G_{12} between torque and vehicle speed

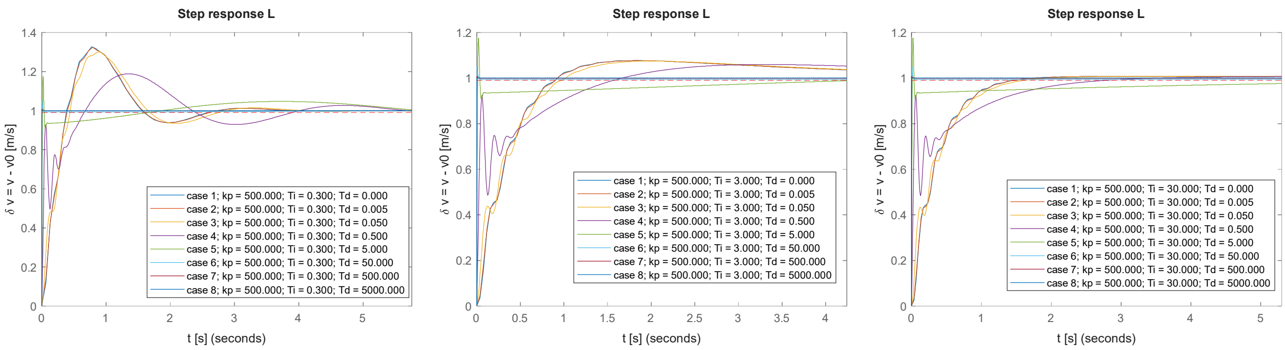


$$G_{12}(s) = \frac{v(s)}{T_M(s)}$$

$$G_{12}(s) = -\frac{D_{12}}{\det(D)} = -\frac{\left(-c_t \frac{\tau_2}{\tau_1 R_w}\right) s - k_t \frac{\tau_2}{\tau_1 R_w}}{\det(D)}$$

- NB: IT HAS ALREADY BEEN MULTIPLIED BY s TO GET THE SPEED AS OUTPUT, THEREFORE THIS TF ONLY HAS 3 POLES (THE ONE IN ZERO HAS ALREADY BEEN SIMPLIFIED)
- Note that for increasing kp

4.d) Preliminary stability analysis: step(L)



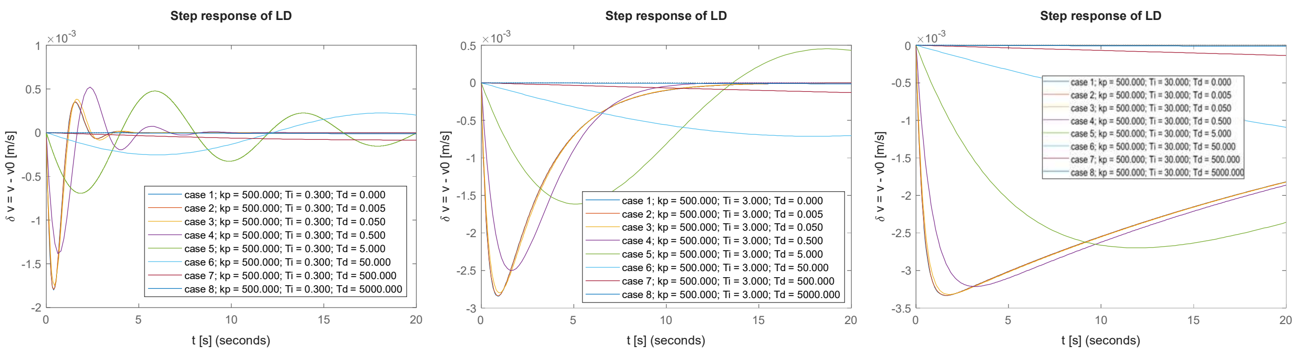
- Effect of Ti:
  - Ti affects low frequency oscillations and settling time (higher Ti better/lower settling time)
  - Small Ti implies higher overshoot of those oscillations

- Effect of Td:
  - Td affects high frequency oscillations
  - Higher Td tends to increase the frequency of those oscillations and the slope of the response (higher slope = faster response)

- Looking at the step(L), the best choice seems to be low Td, and high Ti

- There two range of interest for the frequency:
  - the higher one is controlled by the Td: as Td decreases the oscillations tend to disappear
  - The lower one is the same as always

4.d) Preliminary stability analysis: step(LD)



Effect of Ti:

- As Ti increases:
  - The frequency of the osclillations decreases
  - The overshoot increases

Effect of Td:

- As Td inceases
  - The overshoot decreases
  - Settlilng time increases

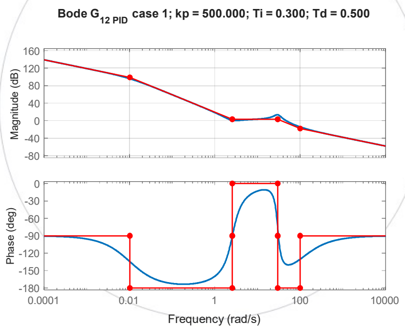
There are no more high frequency oscillations
The best choice seems to be having a low Td to have lower settling time, and to compensate the overshoot we would use a low Ti

4.d) Stability: Open loop transfer function G_{12}

Case 1.1:

Kp=500, Ti = 0,3, Td =0,5

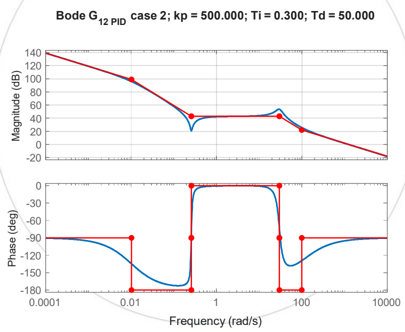
- P1 = 0 (PID); P2,3 = -4,5105 ±29,5072i (el-mech sys); P4 = -0,0103(el-mech sys);
- Z1 = -100 (el-mech sys) Z2,3= -1 ±2,3805i(PID)
- Pm = 40.624
- Gm = ∞



Case 1.2:

Kp=500, Ti = 0,3, Td = 50

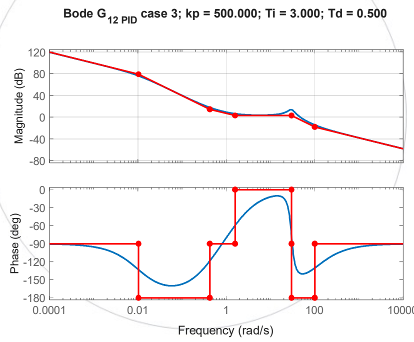
- P1 = 0 (PID); P2,3 = -4,5105 ±29,5072i(el-mech sys); P4 = -0,0103(el-mech sys);
- Z1 = -100 (el-mech sys) Z2,3= -0,01 ±0,25801i(PID)
- Pm = 85.831
- Gm = ∞



Case 1.3:

Kp=500, Ti = 3, Td = 0,5

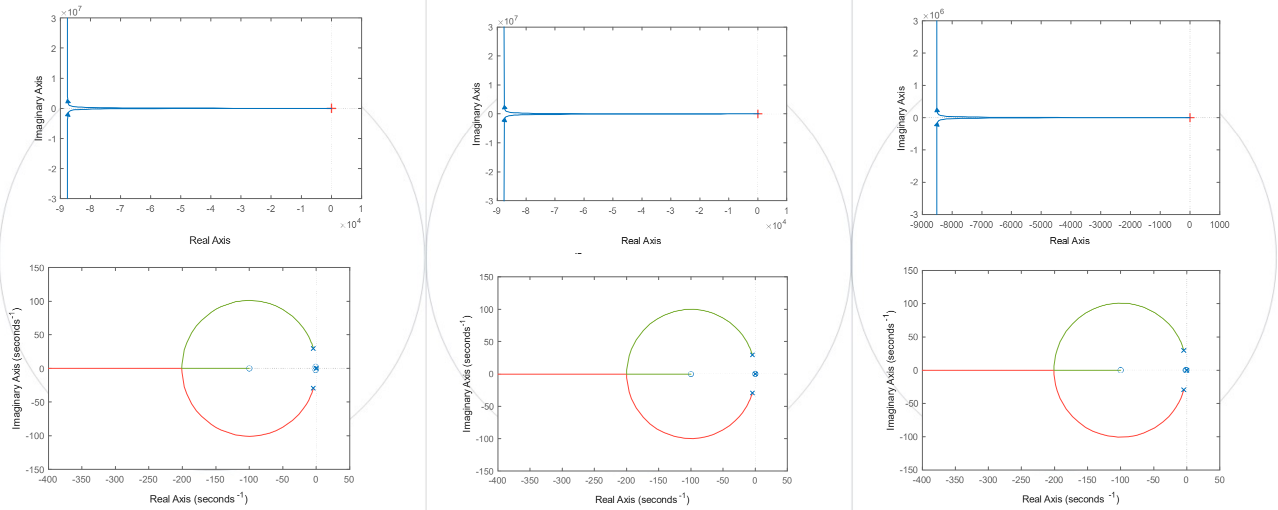
- P1 = 0 (PID); P2,3 = -4,5105 ±29,5072i(el-mech sys); P4 = -0,0103(el-mech sys);
- Z1 = -100 (el-mech sys) Z2=-1,5774(PID) Z3=-0,42265 (PID)
- Pm 40.617
- Gm = ∞



- G_m is the open loop TF of the controlled elettromechanic system.
- We change Td to change the position of the zeros introduced by the PID
- The system is minimum phase, looking at Gm and Pm the system is stable for each case of Td

4.d) Stability: Open loop TF G_{12} – Nyquist Diagram and Root Locus

Case 1.1:	Case 1.2:	Case 1.3:
Kp=500, Ti = 0,3, Td =0,5	Kp=500, Ti = 0,3, Td = 50	Kp=500, Ti = 3, Td = 0,5

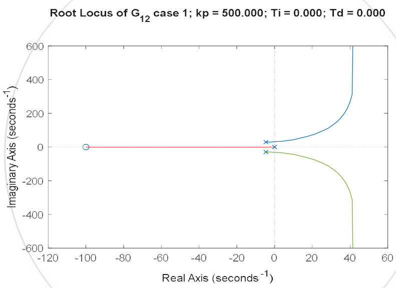


- The system is stable for the Td selected : this result is confirmed by the nyquist diagram and the root locus

4.d) Stability: Open loop TF G_{12} – Root Locus with P, PI, PD

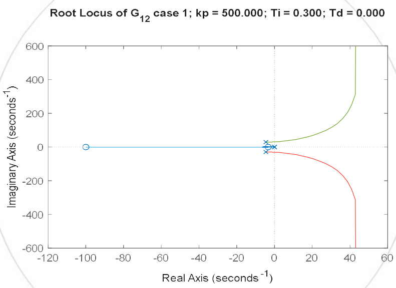
P

Kp=500



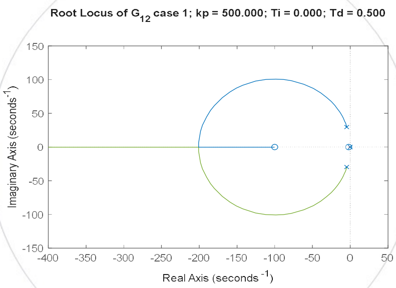
PI

Kp=500, Ti = 0,3

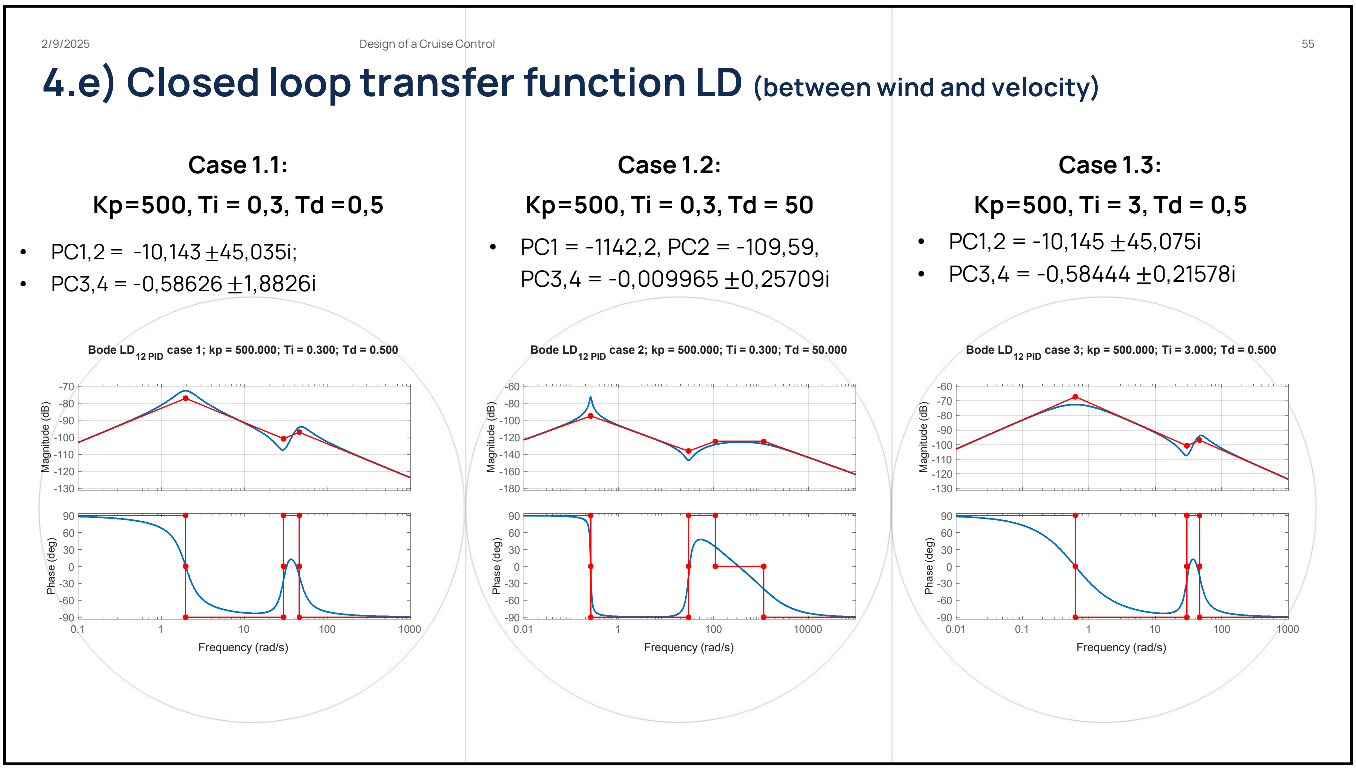


PD

Kp=500 Td = 0,5

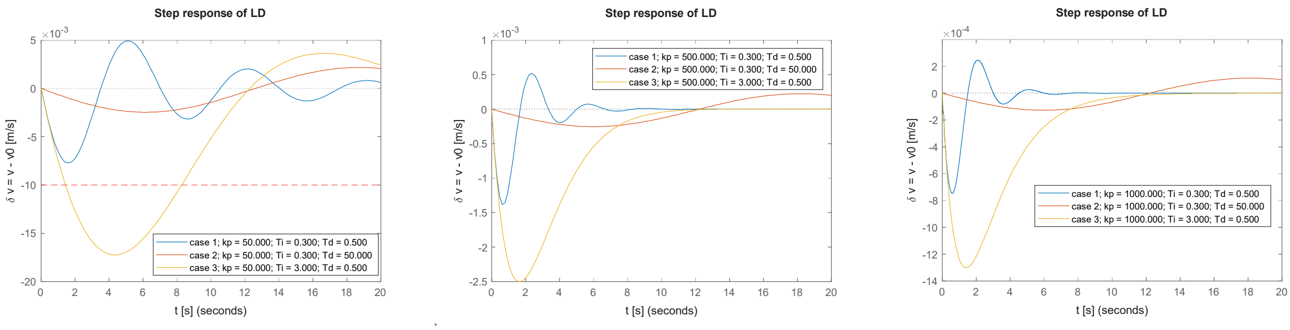


- In this slide it is evaluated the influence of the type od control system to the root locus shape
  - P controller: the system can become unstable over a value of Kp, which is typical of a non-collocated control system
  - PI controller: the general shape of the RL isn't influenced
  - PD controller: applying a derivative controller the system becomes always stable and the root locus is changed, this is due to the zero added by the derivative controller



This slide helped us understand what could have been the effect of Kp, because we have complex coniugate poles (looking at the root locus we see that they may become real).

4.e) Performance: step response of LD for different Kp



- Higher Kp the amplitude of oscillation decreases
- Higher Kp the response is faster