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Lab 03: ABS

03/11/2025

Abstract

This report evaluates the sensitivity of Antilock Braking System (ABS) performance to variations in tire parameters and thermal conditions using a 7-DOF Simulink model. The study quantifies the influence of peak grip, longitudinal stiffness, shape factor, curvature factor, and relaxation length on some defined KPIs.

Contents

1. Introduction

The aim of this laboratory was to investigate the effect of varying different tire parameters on the performance of the Antilock braking system.

1.1 Antilock braking system (ABS)

The Antilock braking system is used in vehicles to avoid wheel lockup, maintaining the drivability and generally the handling of the vehicle and limiting the yaw moment build up in case of μ −split condition.

The ABS works by limiting the pressure in the braking system to avoid the tire slipping.

Control logic

The slip of the tire is very difficult to measure, since it is too expensive to obtain a fast and precise enough measure on the longitudinal speed of the vehicle. From the braking wheel dynamics model (here reported), the ABS can be implemented by looking at the wheel angular acceleration. As shown in Figure 1, it decreases abruptly after the peak is reached.

$$m\dot{V} = -F_x$$

$$J\dot{\omega} = F_x R - M_{br}$$

Equation 1

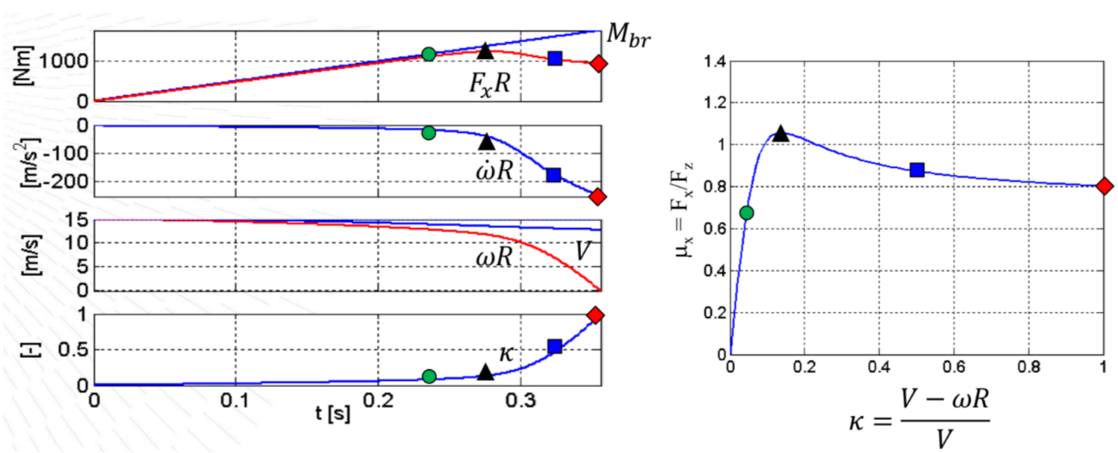

Figure 1

1.2 Model equations

The main equations considered in the model are the following

- The tire was considered to have 2 longitudinal and 2 torsional dofs. Therefore, the equations of motion are:

$$m_q\ddot{x}_m + r_b(\dot{x}_m - \dot{x}_c) + c_b(x_m - x_c) = F_{in}$$

$$m_c\ddot{x}_c + r_b(\dot{x}_c - \dot{x}_m) + c_b(x_c - m) = -F_x$$

$$J_m\ddot{\phi}_m + r_{bt}(\dot{\phi}_m - \dot{\phi}_c) + c_{bt}(\phi_m - \phi_c) = -M_{br}$$

$$J_c\ddot{\phi}_c + r_{bt}(\dot{\phi}_c - \dot{\phi}_m) + c_{bt}(\phi_c - \phi_m) = F_x R_e$$

Equation 2

- The Pacejka longitudinal MF was implemented considering also the effect of the relaxation length:

$$F_{xSS} = D_x sin\{C_x \text{atan}\,[B_x\epsilon_x - E_x[B_x\epsilon_x - \text{atan}\,(B_x\epsilon_x)]]\} + S_{vx}$$

$$\frac{\sigma_x}{\dot{\phi}_c R_e}\dot{F}_x + F_x = F_{xSS}$$

Equation 3

- Vertical load transfer was implemented as:

$$F_{zA} = m_q g + \frac{m_v}{2}\frac{h_g}{l}\ddot{x}_m$$

$$F_{in} = \frac{\Delta F_{zA}}{g}\ddot{x}_m$$

Equation 4

- The dynamics of the hydraulic braking system are represented by a second order system:

$$\frac{P_{out}}{P_{in}} = \frac{1}{T_1^2 s^2 + T_2 s + 1}$$

Equation 5

1.3 KPI definition

Some parameters were identified to evaluate the performance of the ABS system. They were defined as follows.

Stopping distance

(ISO 21994): distance from initial brake pedal contact until vehicle comes to a standstill (V $\leq$ 5km/h)

Build-up distance

(ISO 21994): distance during deceleration build-up (first 10km/h)

ABS braking distance

(ISO 21994): distance travelled during ABS-controlled braking

Longitudinal acceleration

(ISO 21994): mean value of acceleration during ABS-controlled braking

Normalized energy distribution

The grip is plotted against the speed and the longitudinal slip, and a band of slip is identified in which from 12.5% to 87.5% of the energy is dissipated.

1.4 Sensitivity analysis of the tire parameters

The following tire parameters are varied in the Simulink model to assess their impact on ABS performance:

- **Longitudinal grip:** maximum friction coefficient between the tire and road (variation: $\mu_{x,max} \pm 5\%$) – Section 2
- **Longitudinal stiffness:** initial slope of the longitudinal force/slip curve (variation: $K_x \pm 50\%$) – Section 3
- **Asymptotic factor:** determines the asymptotic behavior of the MF curve (variation: $C \pm 15\%$) – Section 4
- **Curvature factor:** controls the curvature after the peak of the MF curve (variation: $E \pm 15\%$) – Section 5
- **Relaxation length:** distance required for the tire to develop steady-state longitudinal force (variation: $\lambda_x \pm 20\%$) – Section 6

1.5 Thermal model

The thermal model used in this lab is similar to the one adopted in lab03 but with a few significant changes.

The following assumptions hold:

- Pure longitudinal slip is considered
- All the thermal power (input/output) is exchanged simultaneously at the contact area between the tire and the ground
- Tire thermal properties are uniform across all different regions

Under these assumptions, the tire surface temperature is given by:

$$W\frac{dT}{dt} = q - \lambda A(T - T_0)$$

Equation 6

- $W\left[\frac{J}{K}\right]$ → heat capacity
- $q[W]$ → heat flux
- $\lambda\left[\frac{W}{m^2K}\right]$ → heat transfer coefficient
- $A[m^2]$ → contact area
- $T_0[K]$ → initial temperature

Equation 6 represents a power balance: the difference between the input power due to sliding ($q \approx |F_x \cdot \varepsilon_{belt} \cdot V_{wheel}|$) and the output thermal power exchanged through the contact area ($-\lambda A(T - T_0)$) results in an increase of the tire surface temperature. W represents the thermal inertia of the tire (it is the energy needed to increase the temperature by $1\ K$.

The dependence of the lateral force on the temperature is included in the computation of the D_y, K_y terms of the MF-Tire model used.

- D_y accounts for the variation of the peak lateral force with temperature:

$$D_y = D_{y0}\left\{1 + \frac{\partial\mu}{\partial T}|T - T_{ref}|\right\}$$

Equation 7

  - The term $\frac{\partial\mu}{\partial T}$ (differently from lab02) is a fixed negative constant
  - **$\Longrightarrow$ $\boldsymbol{D_y}$ has a monotonic decreasing trend with increasing temperature**
- K_y accounts for the variation of the cornering stiffness with temperature:

$$K_y = K_{y0}\left\{1 + \frac{\partial C_p}{\partial T}\left(T - T_{ref}\right)\right\}$$

Equation 8

  - The term $\frac{\partial C_p}{\partial T}$ (as it was in lab02) is a fixed negative constant
  - **$\Longrightarrow$ $\boldsymbol{K_y}$ has a monotonic decreasing trend with increasing temperature**

Note that the thermal model is included only Section 7 where the effect of temperature on the ABS performance is investigated.

2. Effect of $\mu_{x,max}$

2.1 μ_x increase (+5%)

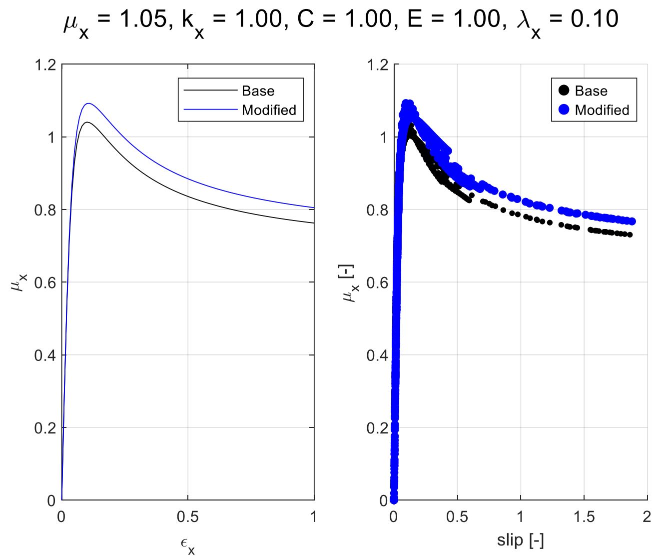

Figure 2

- The peak grip is effectively modified

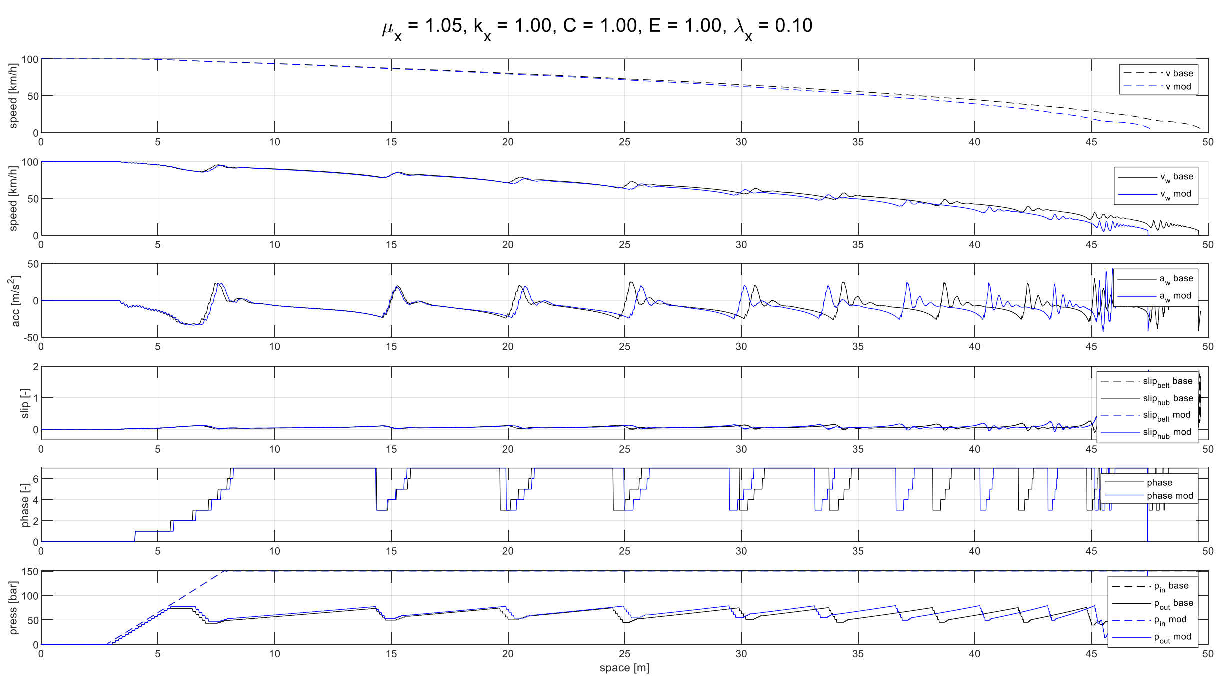

Figure 3

KPI evaluation	Baseline	Modified	Delta [%]
Stopping distance [m]	46.8679	44.7055	-4.6139
Build-up distance [m]	10.2579	9.8884	-3.6019
ABS braking distance [m]	36.6101	34.8171	-4.8975
Mean long. Acceleration [m/s^2]	-7.6677	-7.8638	2.5570

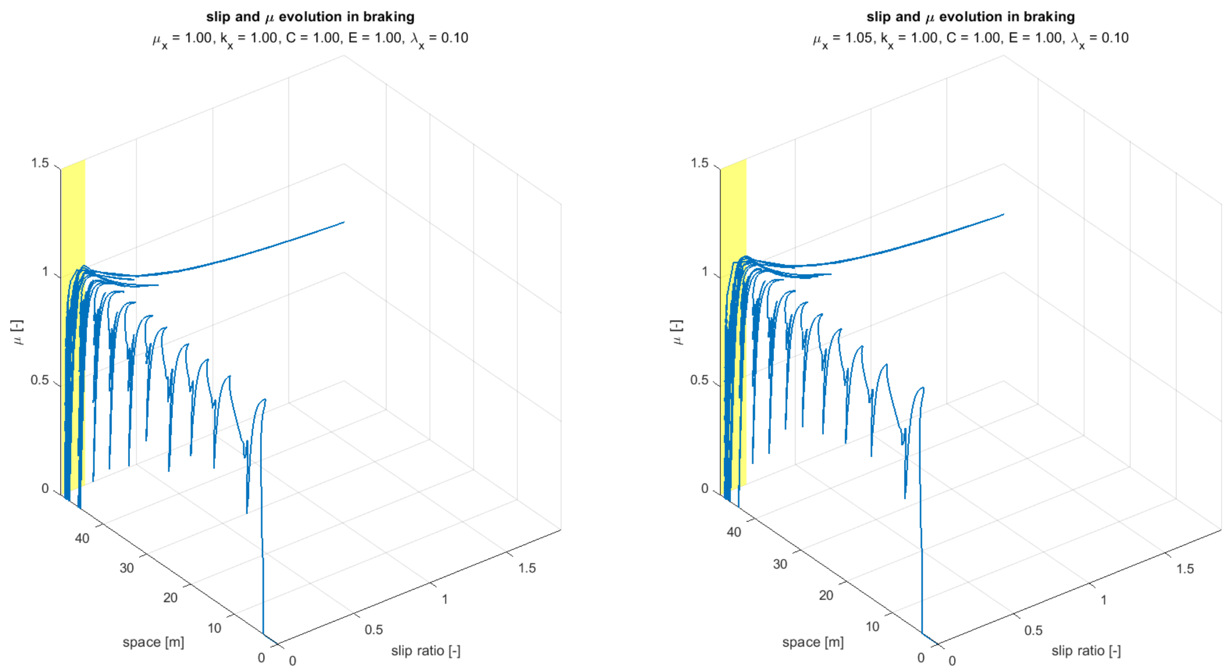

Figure 4

- Increasing the maximum grip allows the ABS system to "identify" the peak better (if the curve was flatter this would be harder) and to remain close to it; this is confirmed by Figure 4, where it is observed that the slip variations around the optimal value are smaller
- With increased max grip, most of the energy is dissipated in a narrower band around the peak
- Since the difference between the plots in Figure 4 is small, it is much more useful to compare the numerical values of the KPI computed
    - An increase of peak grip of 5% leads to a reduction of the braking distance of 4.6%

2.2 μ_x decrease (-5%)

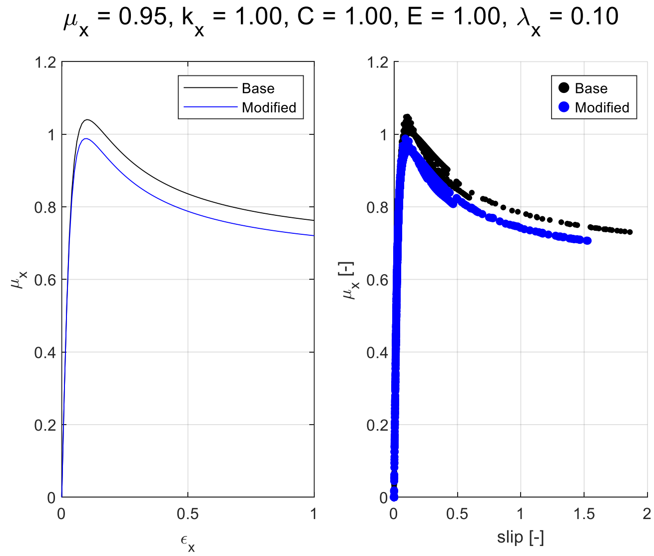

Figure 5

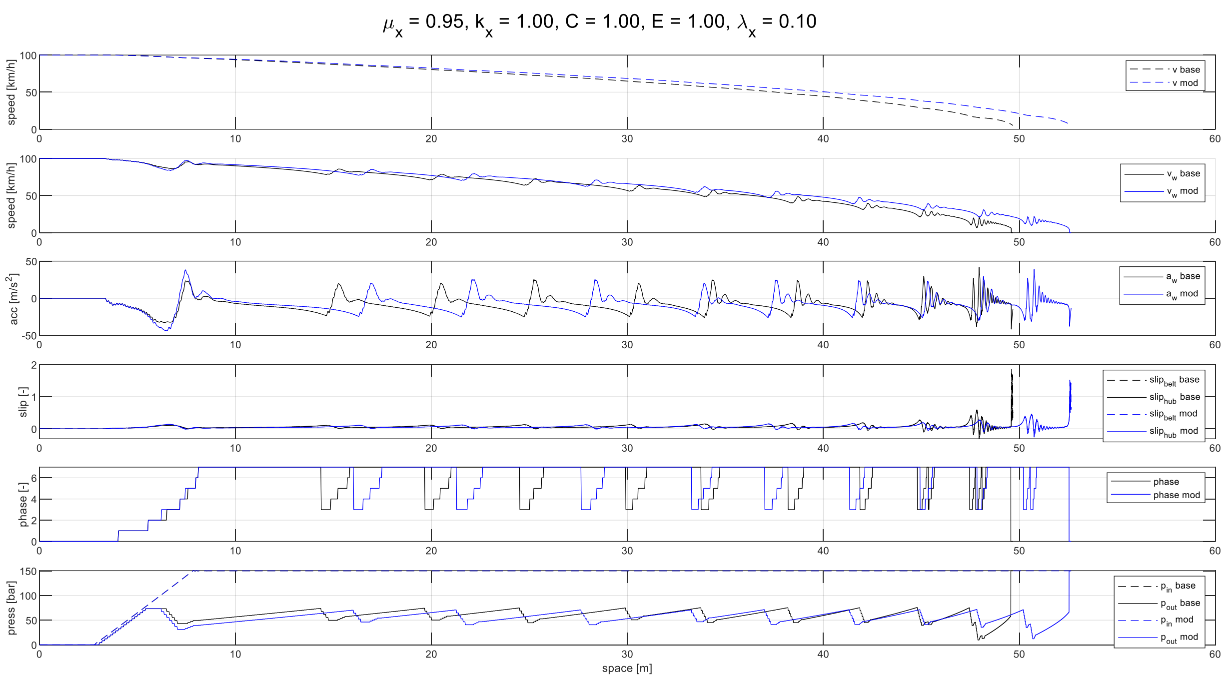

Figure 6

KPI evaluation	Baseline	Modified	Delta [%]
Stopping distance [m]	46.8679	49.8387	6.3386
Build-up distance [m]	10.2579	11.2293	9.4702
ABS braking distance [m]	36.6101	38.6094	5.4611
Mean long. Acceleration [m/s²]	-7.6677	-7.4224	-3.1992

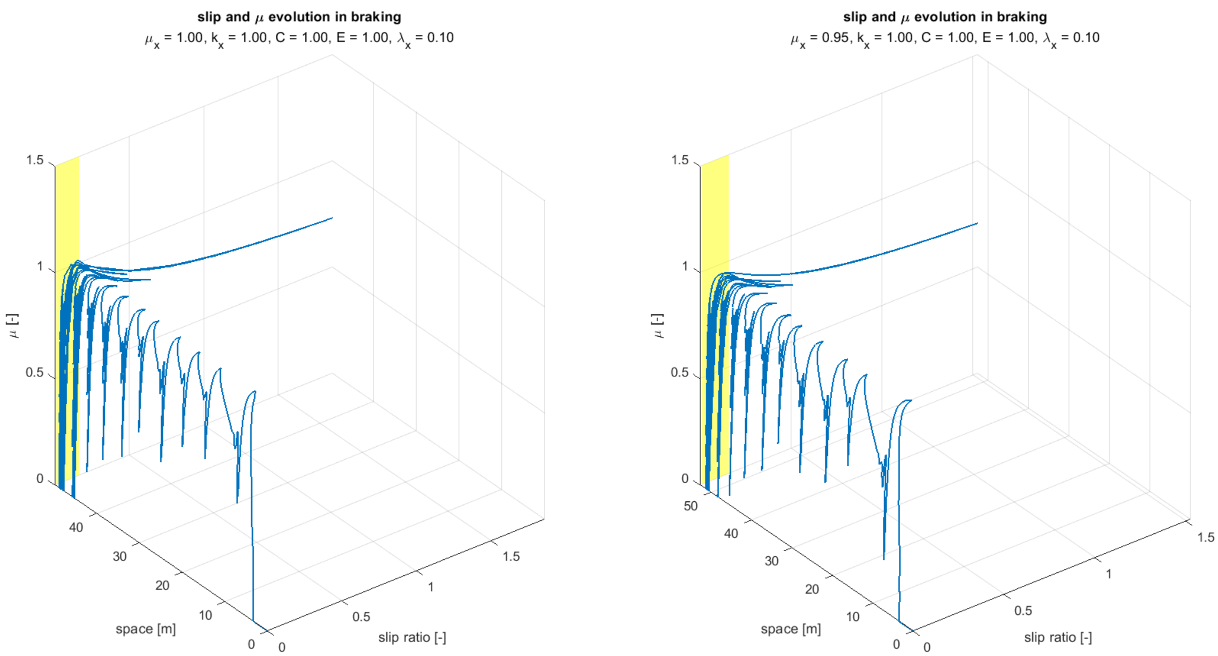

Figure 7

- Intuitively, the effects observed for a decrease of the peak grip are exactly opposite compared to the ones observed for an increase of that same parameter
- It is important to notice that in this case the effect on the stopping distance is higher in absolute value compared to the previous case:
  - $\mu_{x,max} + 5\% \rightarrow -4.6\%$ on stopping distance
  - $\mu_{x,max} - 5\% \rightarrow +6\%$ on stopping distance

  $\Longrightarrow$ **the effect of peak grip is nonlinear**

3. Effect of K_x

3.1 K_x increase (+50%)

μ_x = 1.00, k_x = 1.50, C = 1.00, E = 1.00, λ_x = 0.10

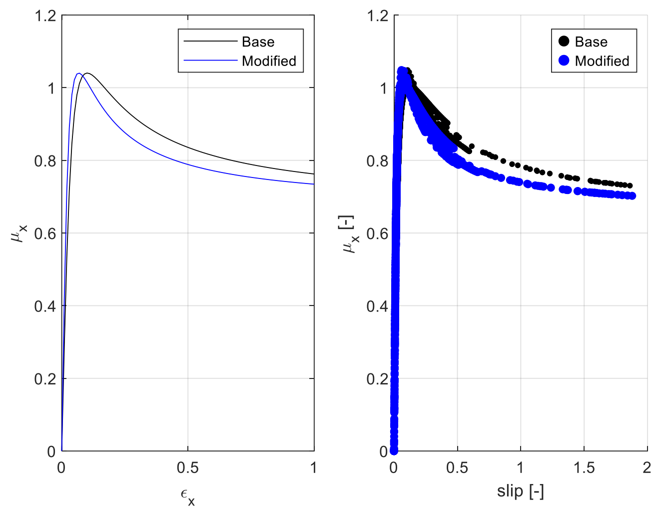

Figure 8

μ_x = 1.00, k_x = 1.50, C = 1.00, E = 1.00, λ_x = 0.10

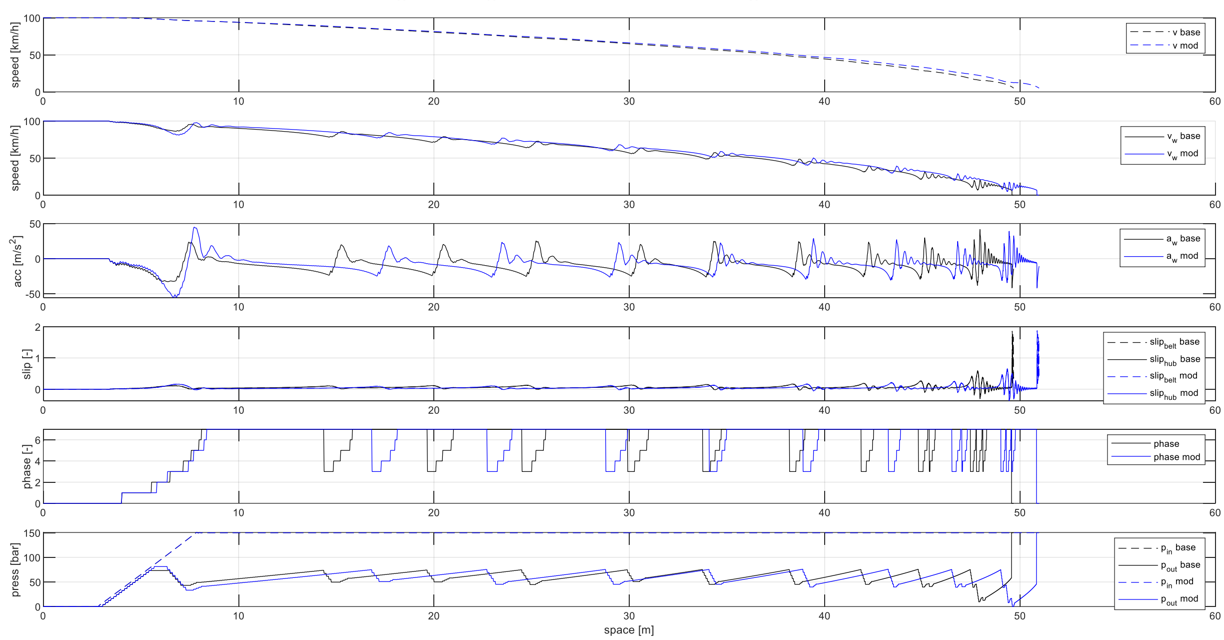

Figure 9

KPI evaluation	Baseline	Modified	Delta [%]
Stopping distance [m]	46.8679	48.1710	2.7803
Build-up distance [m]	10.2579	10.8522	5.7945
ABS braking distance [m]	36.6101	37.3187	1.9357
Mean long. Acceleration [m/s²]	-7.6677	-7.3172	-4.5711

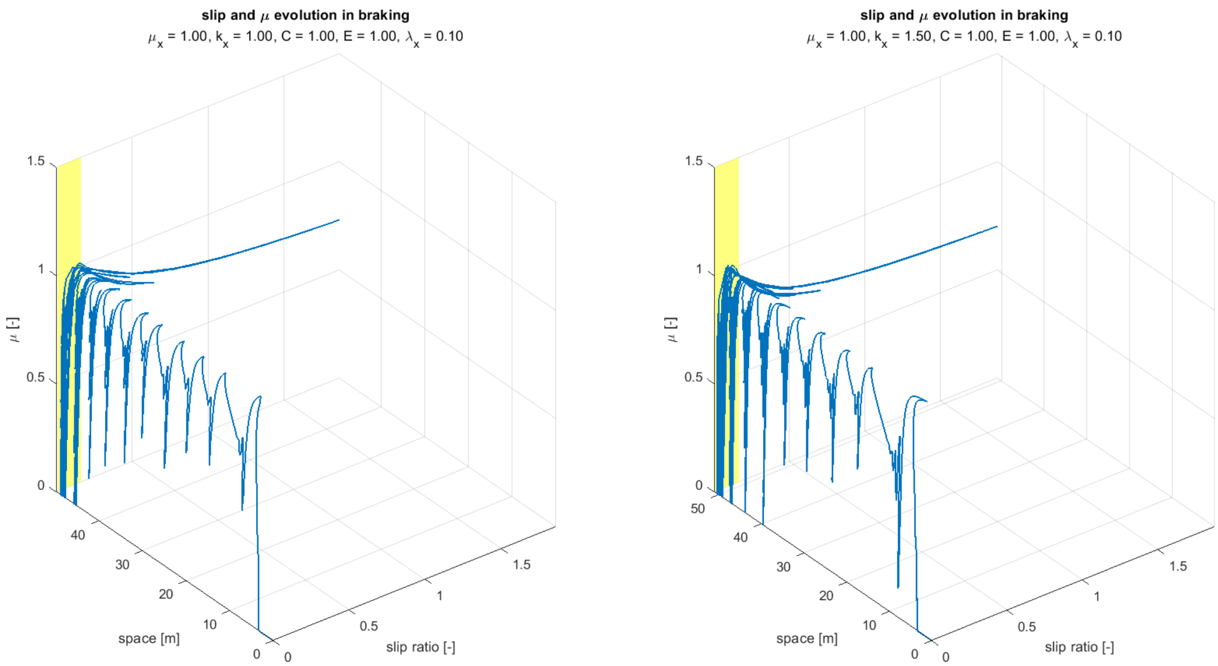

Figure 10

- Modifying the braking stiffness primarily affects the transient, this is confirmed by the fact that the largest % variation is observed in the build-up distance
- Increasing the braking stiffness makes the tire more responsive
    - The initial average deceleration obtained is higher compared to the baseline case (Figure 9)
- Increasing the braking stiffness may have counteracting effects
    - On one side it makes it easier for the ABS control system to identify the peak and stay closer to it → beneficial effect on the braking performance
    - On the other hand, the curvature after the peak is higher (Figure 8); as a consequence, small variations of longitudinal slip with respect to the optimal value lead to a significant reduction of the longitudinal force → compromising effect on the braking performance

- Overall, increasing the braking stiffness has a negative impact on the stopping distance

3.2 K_x decrease (-50%)

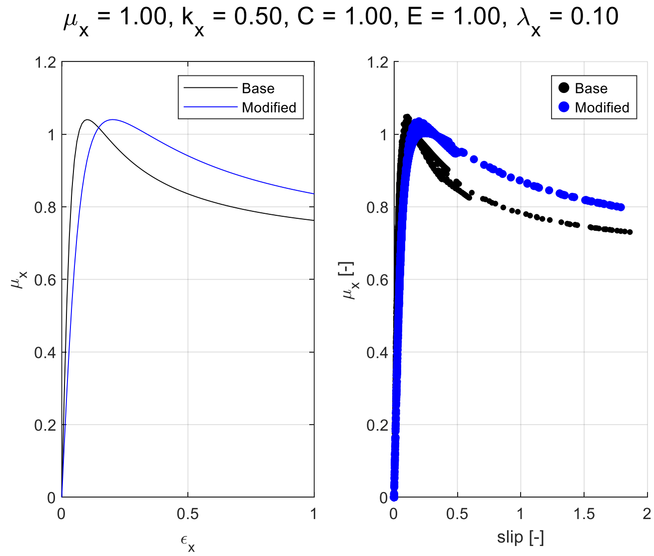

Figure 11

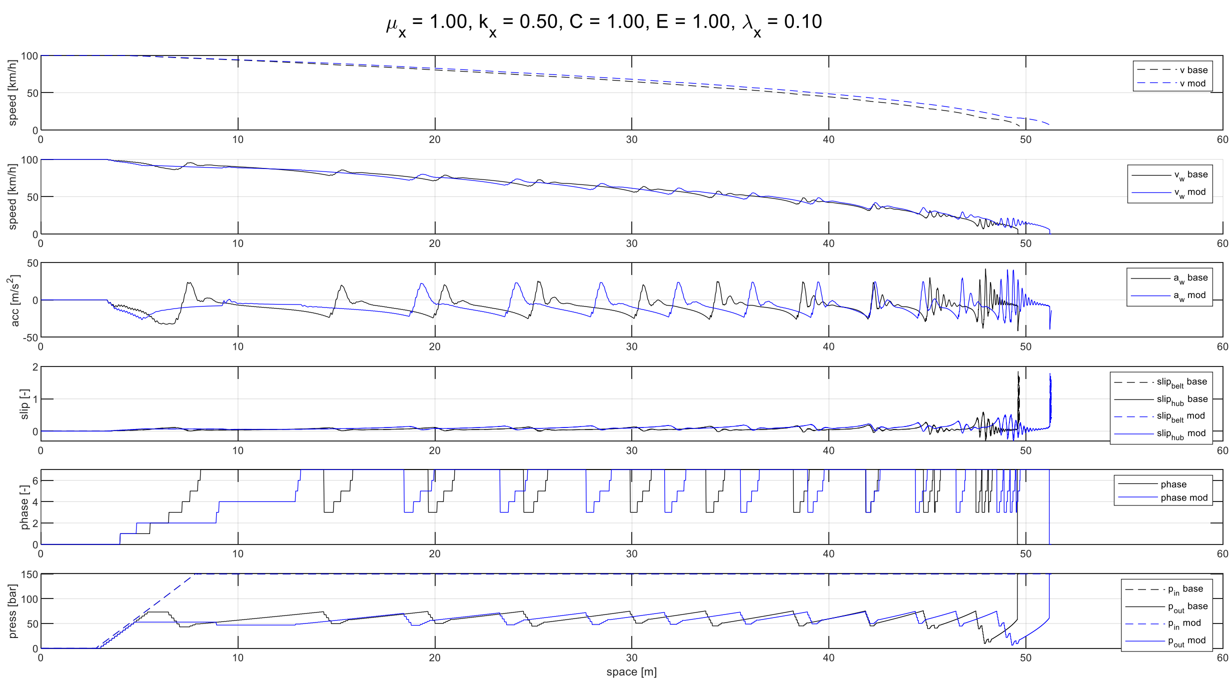

Figure 12

KPI evaluation	Baseline	Modified	Delta [%]
Stopping distance [m]	46.8679	48.4851	3.4506
Build-up distance [m]	10.2579	11.4593	11.7122
ABS braking distance [m]	36.6101	37.0259	1.1357
Mean long. Acceleration [m/s^2]	-7.6677	-7.5176	-1.9578

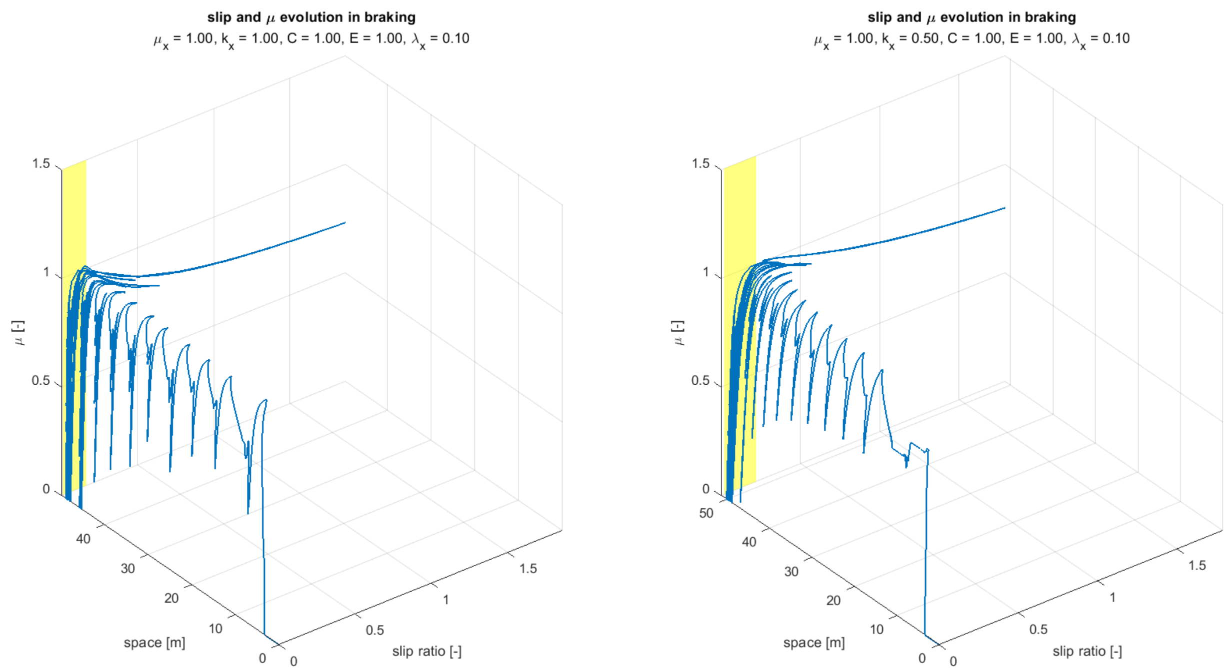

Figure 13

- Intuitively, the effects observed for a decrease of the longitudinal stiffness are exactly opposite compared to the ones observed for an increase of that same parameter
  - The tire is less responsive, this is confirmed by the lower (in absolute value) longitudinal acceleration obtained in the initial phase (Figure 12)
  - The grip curve is flatter around the peak, and this makes it harder for the ABS to identify the peak and stay close to it. Energy is dissipated in a wider slip bandwidth (Figure 13)
- The effect on the build-up distance is higher in absolute value compared to the previous case

4. Effect of C

4.1 C increase (+15%)

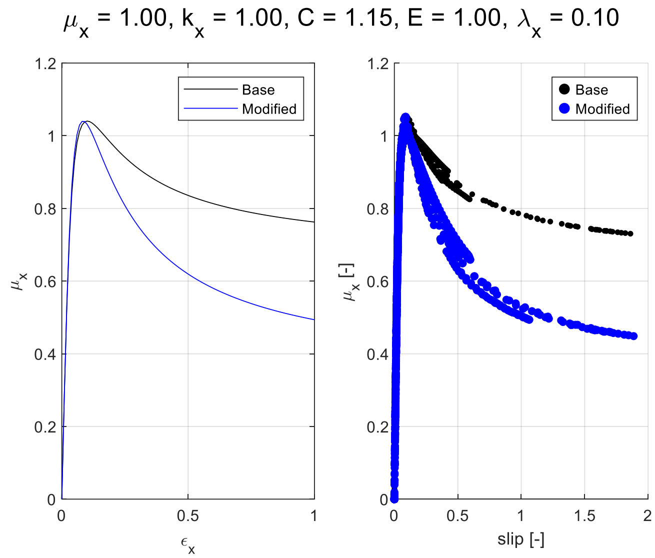

Figure 14

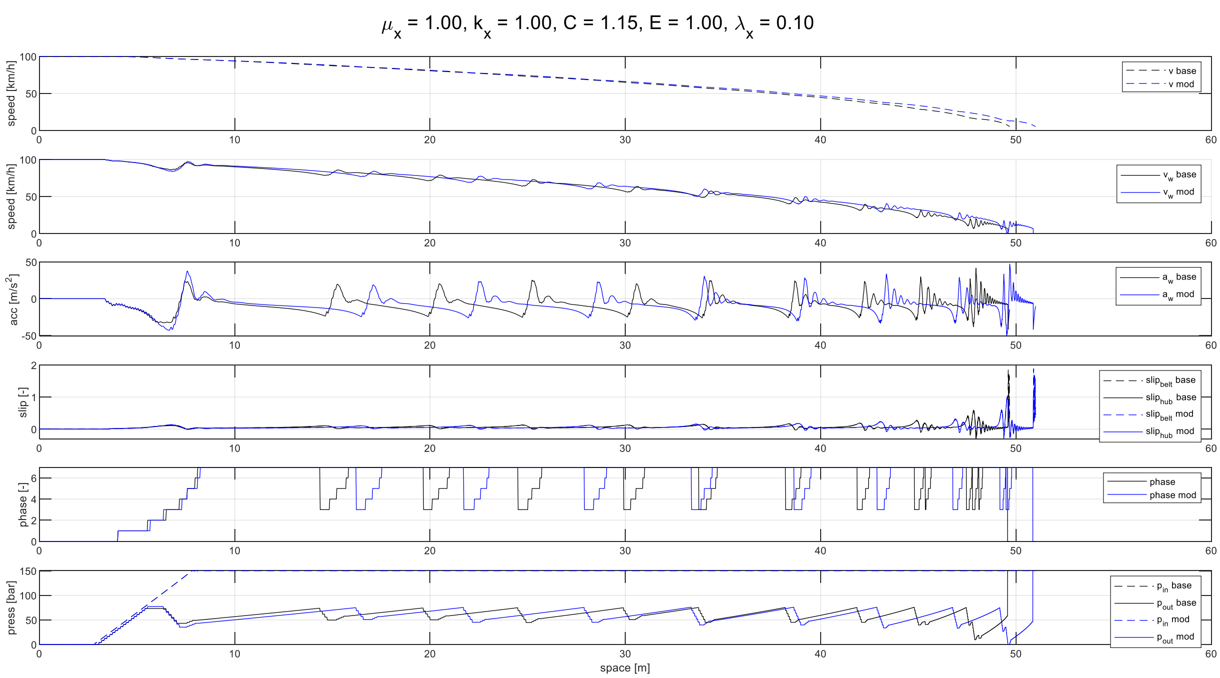

Figure 15

KPI evaluation	Baseline	Modified	Delta [%]
Stopping distance [m]	46.8679	48.1944	2.8302
Build-up distance [m]	10.2579	10.7519	4.8158
ABS braking distance [m]	36.6101	37.4425	2.2739
Mean long. Acceleration [m/s²]	-7.6677	-7.3511	-4.1296

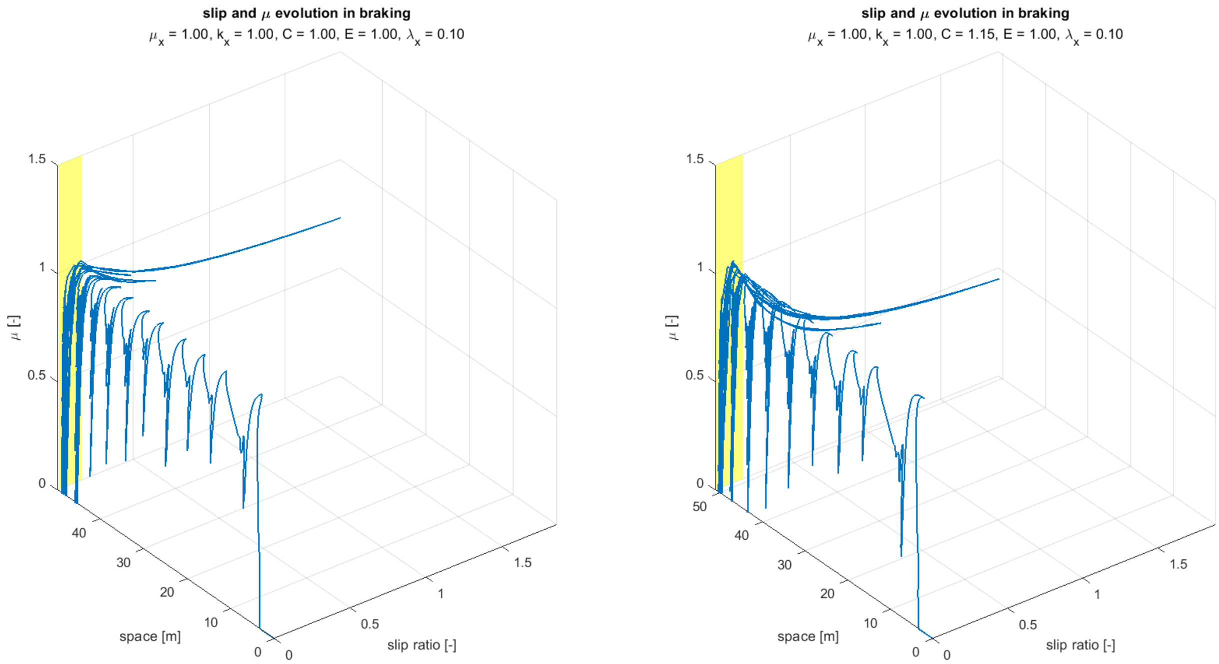

Figure 16

- As seen in Figure 14 the shape of the MF curve changes, the asymptotic value gets lower, and the curve is pointier. This reduces the effectiveness of the ABS control logic, since it will be more difficult for the ABS to stay near the optimum point. In this case the same small variation of slip ratio will bring to a larger reduction of braking force compared to the baseline case.
  - All the braking KPI are worse than the baseline
  - Therefore, it is possible to see in Figure 16, that in this case there is a higher slip distribution than the baseline

4.2 C decrease (-15%)

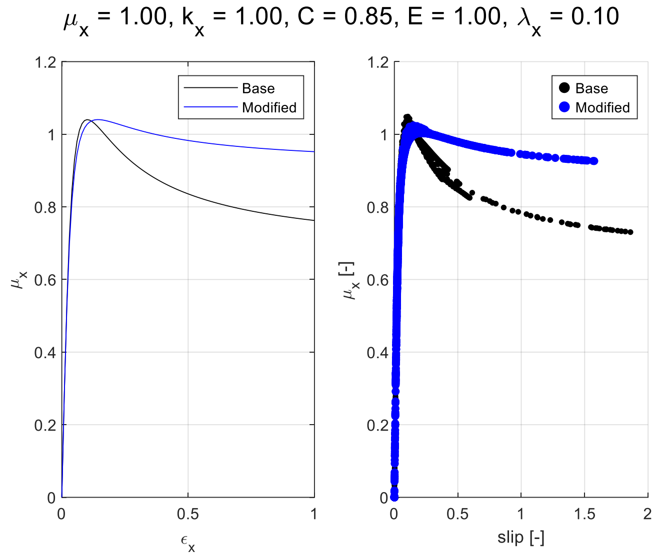

Figure 17

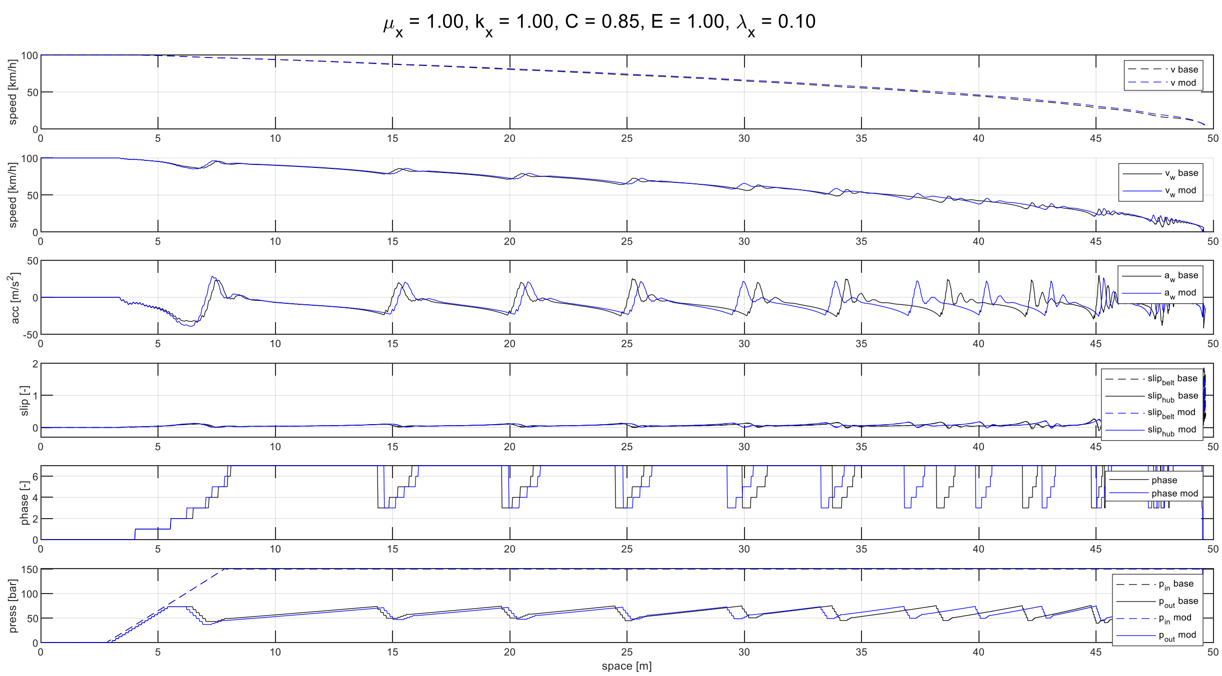

Figure 18

KPI evaluation	Baseline	Modified	Delta [%]
Stopping distance [m]	46.8679	46.8296	-0.0817
Build-up distance [m]	10.2579	10.6542	3.8634
ABS braking distance [m]	36.6101	36.1755	-1.1871
Mean long. Acceleration [m/s²]	-7.6677	-8.0163	4.5454

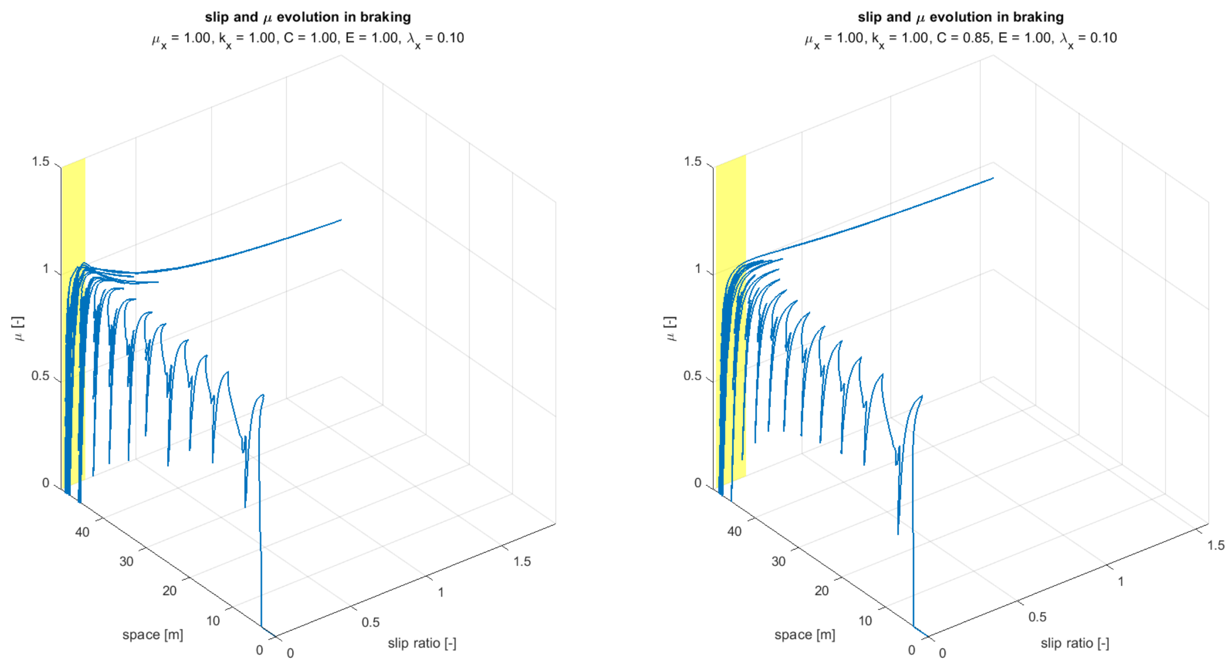

Figure 19

- As expected, the results in this case are the opposite compared to the previous case. The smoother peak, shown in Figure 17, allows the ABS to work better since the peak in consequently larger.
  - All the KPI are better than the baseline, except form the build-up distance.
    - This happens because the peak is also shifted towards higher slip ratio values
- The energy distribution across the slip axis has a larger band compared to the baseline.

5. Effect of E

5.1 E increase ($+15\%$)

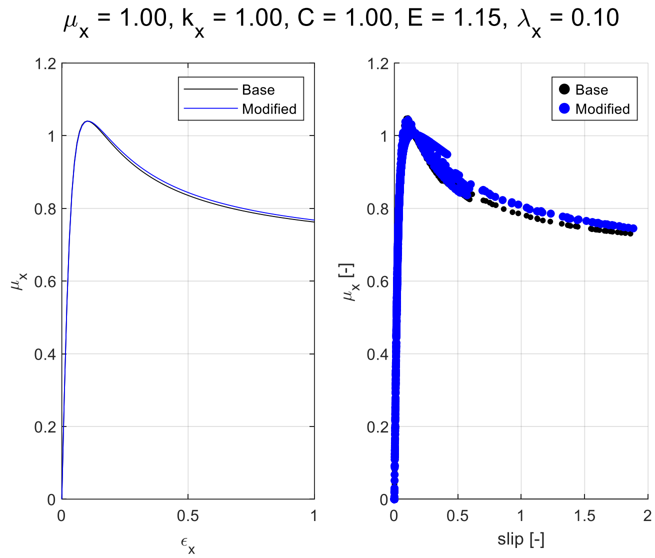

Figure 20

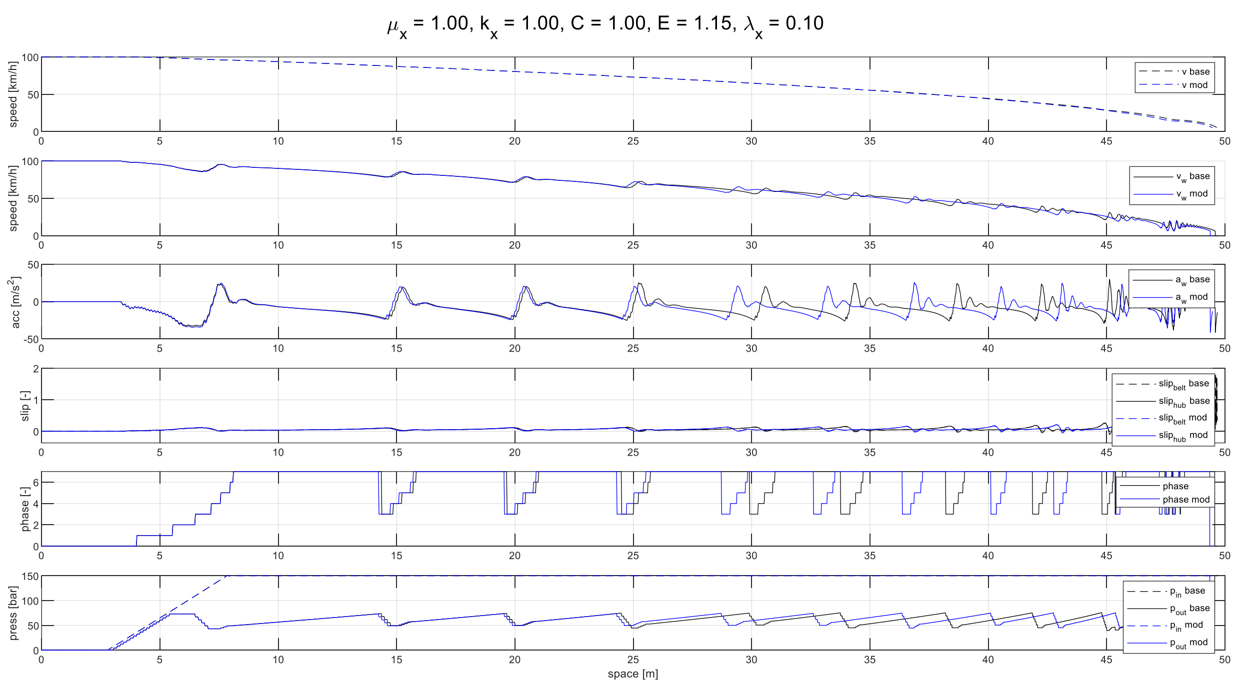

Figure 21

KPI evaluation	Baseline	Modified	Delta [%]
Stopping distance [m]	46.8679	46.6649	-0.4332
Build-up distance [m]	10.2579	10.2580	0.0018
ABS braking distance [m]	36.6101	36.4068	-0.5551
Mean long. Acceleration [m/s^2]	-7.6677	-7.6022	-0.8543

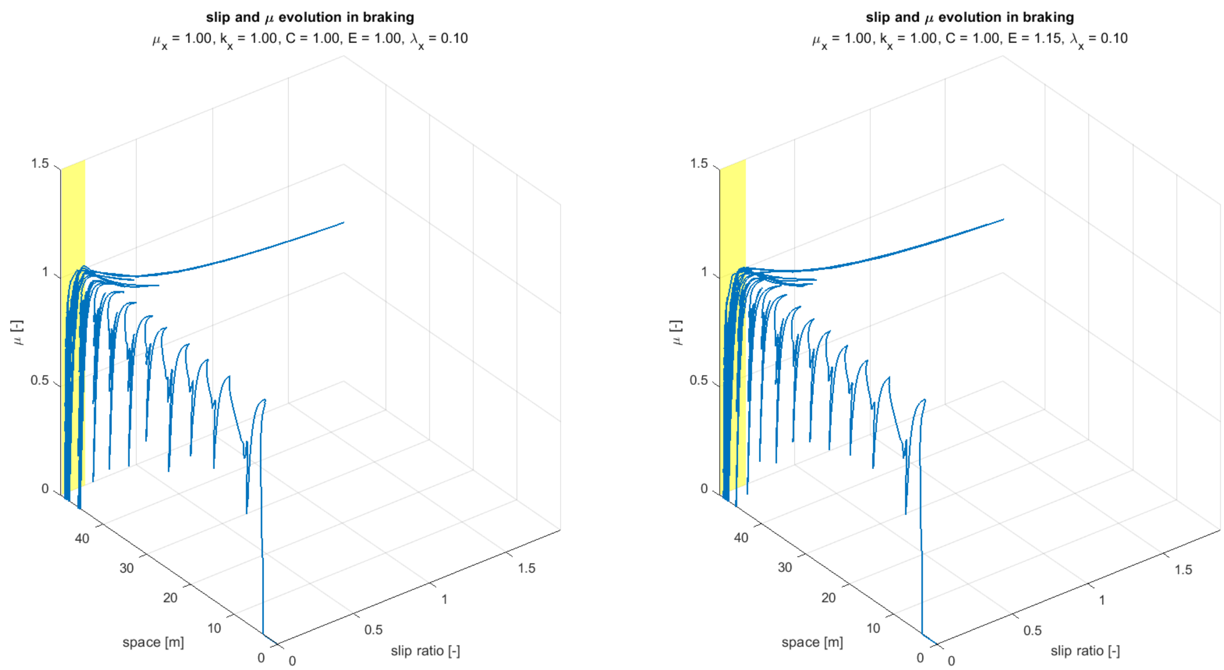

Figure 22

- Theoretically, the effect on the ABS of this parameter should be like parameter C. This happens because the ABS is only working in the near-peak region, so the most significant change is the peak shape change.
- In this case the change is so small that the results are almost equal
  - Stopping distance is reduced compared to the baseline
  - Mean long acceleration is lower than the baseline

5.2 E decrease (-15%)

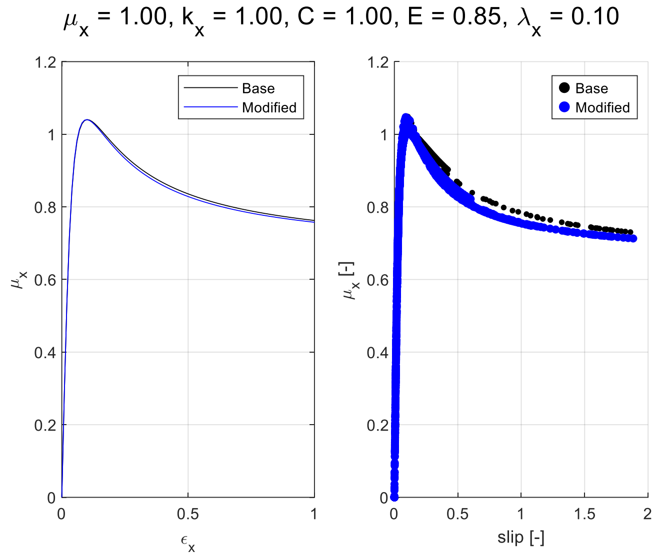

Figure 23

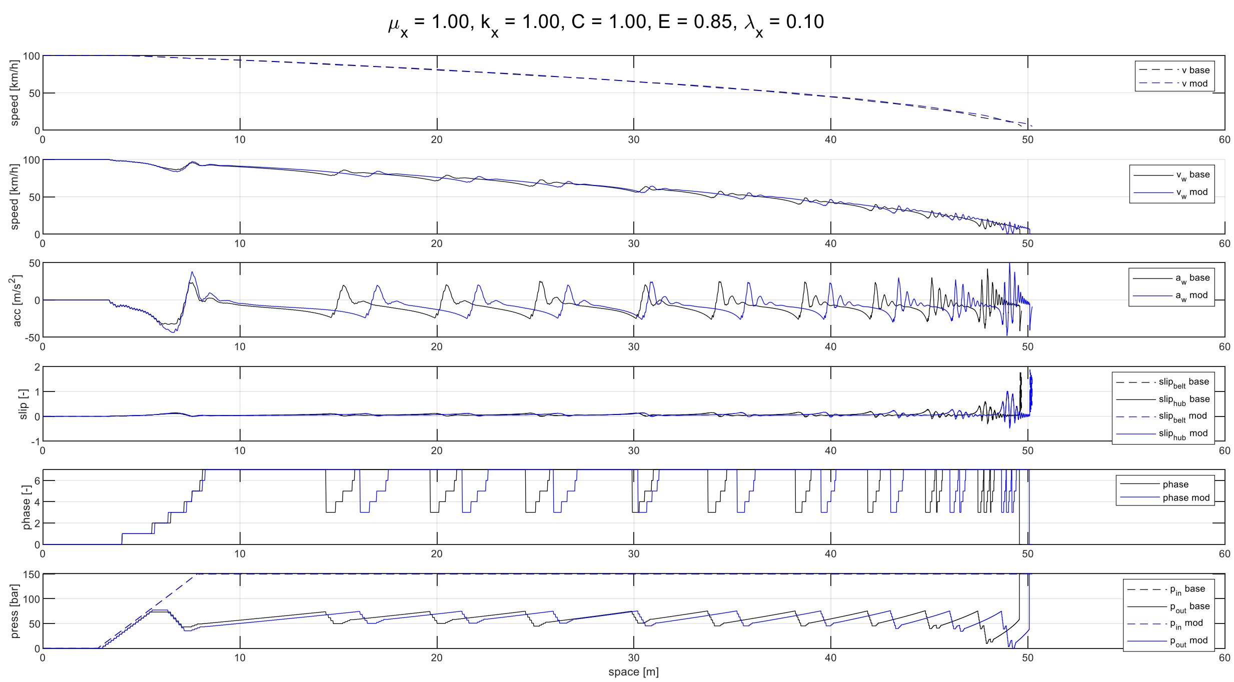

Figure 24

KPI evaluation	Baseline	Modified	Delta [%]
Stopping distance [m]	46.8679	47.4107	1.1581
Build-up distance [m]	10.2579	10.7521	4.8180
ABS braking distance [m]	36.6101	36.6586	0.1326
Mean long. Acceleration [m/s²]	-7.6677	-7.3935	-3.5769

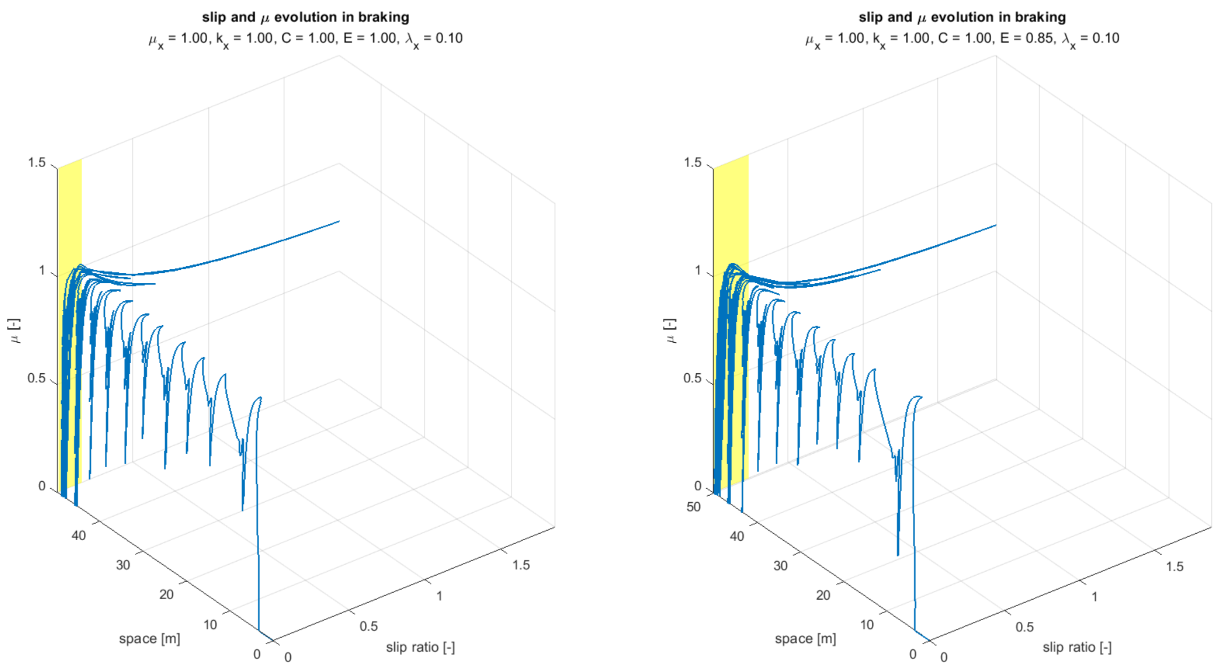

Figure 25

- Same considerations that hold for the opposite change
    - Stopping distance is incremented
    - Mean acceleration is reduced

6. Effect of λ_x

6.1 λ_x increase (+20%)

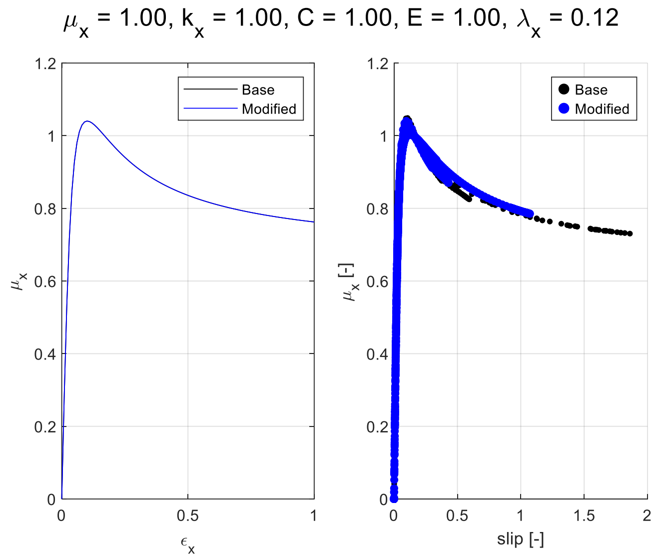

Figure 26

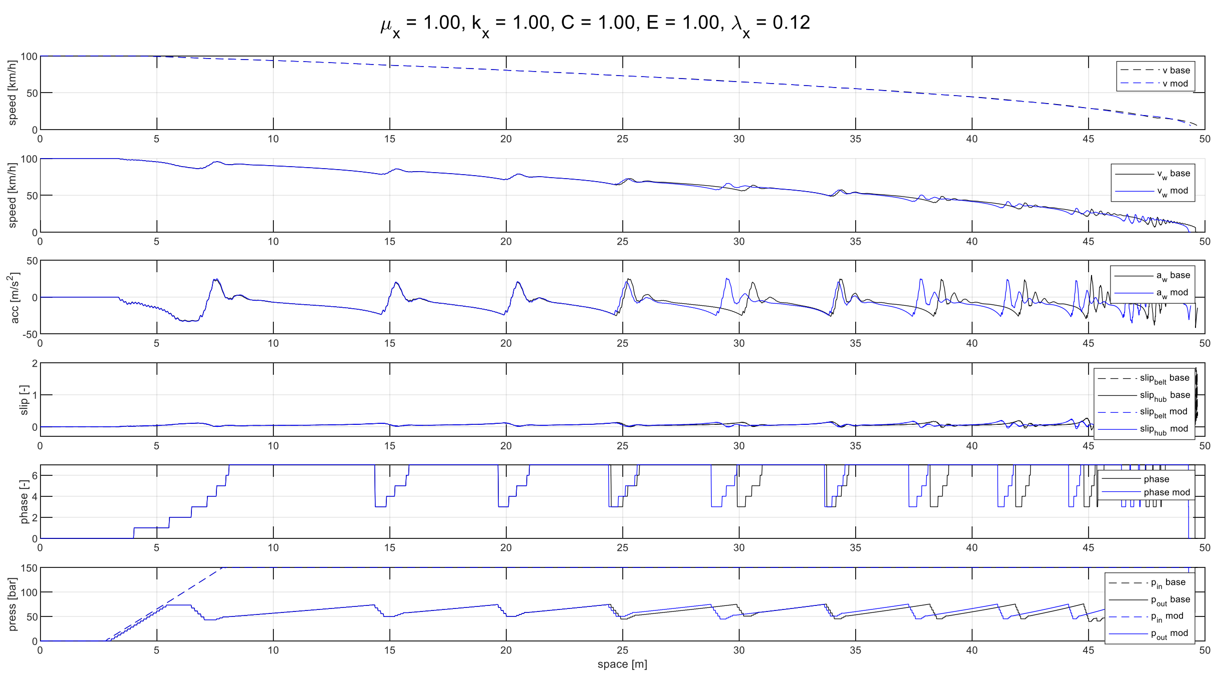

Figure 27

KPI evaluation	Baseline	Modified	Delta [%]
Stopping distance [m]	46.8679	46.5790	-0.6164
Build-up distance [m]	10.2579	10.2580	0.0015
ABS braking distance [m]	36.6101	36.3210	-0.7896
Mean long. Acceleration [m/s²]	-7.6677	-7.8707	2.6467

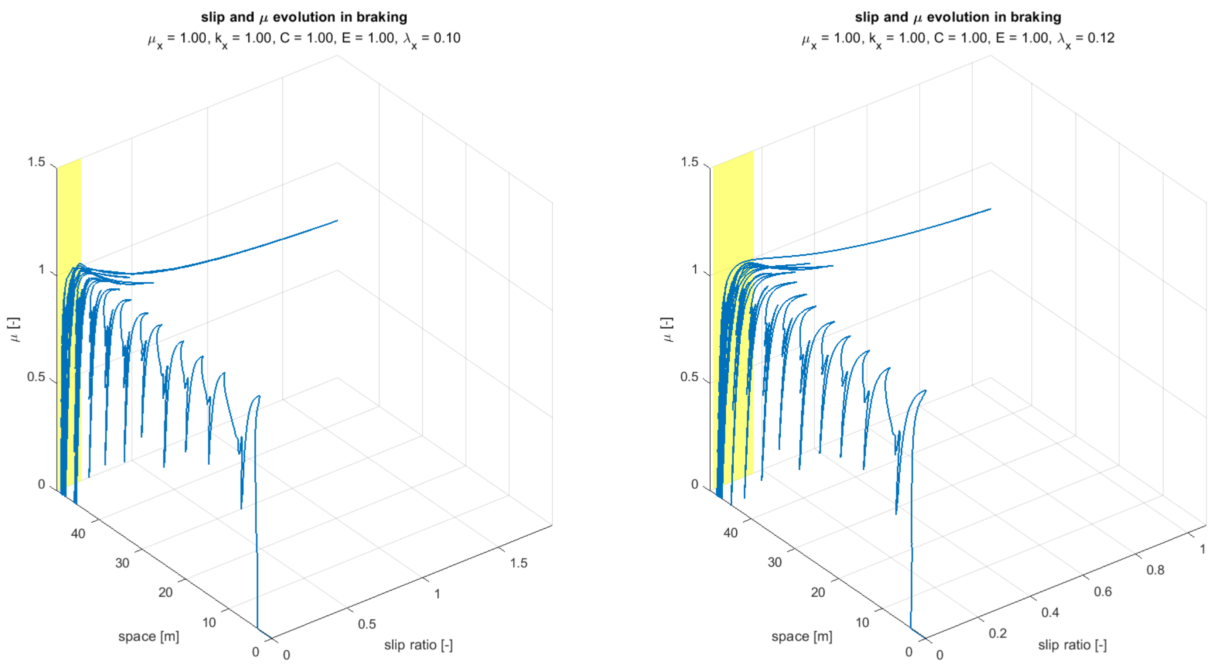

Figure 28

The relaxation length represents the distance the tire must run from the application of the input in order to see a change in the force applied in the contact patch.

Generally, having a higher relaxation length worsens the performance of the system since more time occurs between the input and the actual generation of the force. However, in this case, due the ABS control logic, having a higher relaxation length allows the force to stay longer over the peak (but not too far from it) before the ABS action is applied to the tire.

Moreover, what can be observed is that the lower responsiveness is also linked in Figure 28 to a larger area where most of the energy is dissipated.

6.2 λ_x decrease (-20%)

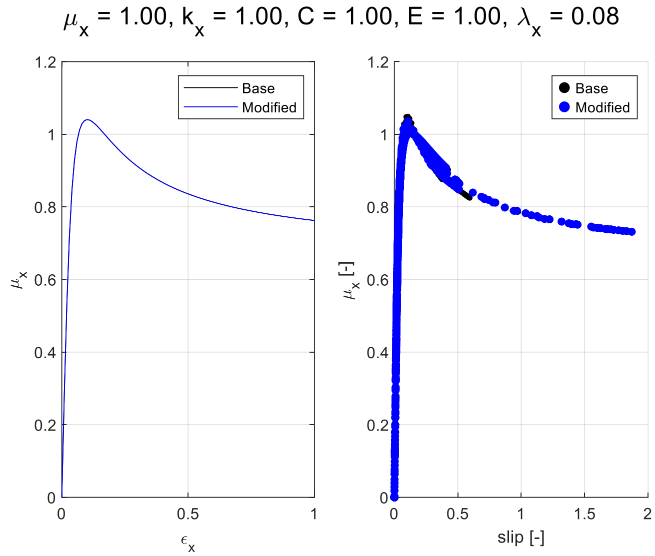

Figure 29

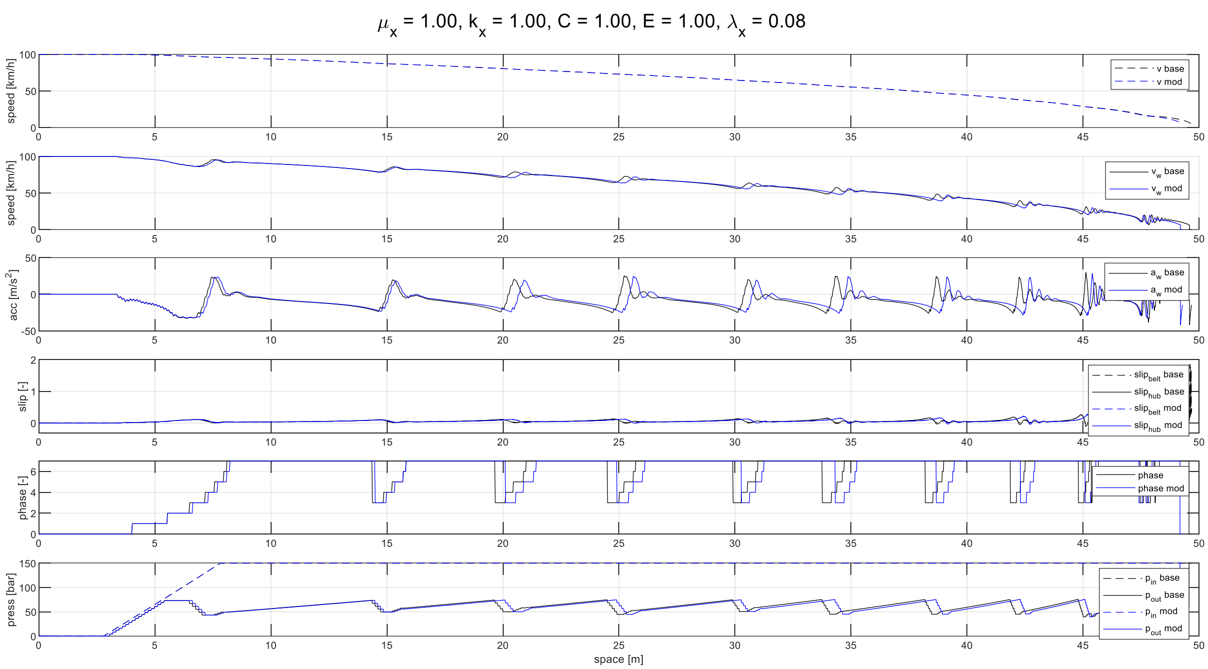

Figure 30

KPI evaluation	Baseline	Modified	Delta [%]
Stopping distance [m]	46.8679	46.4796	-0.8286
Build-up distance [m]	10.2579	10.2555	-0.0233
ABS braking distance [m]	36.6101	36.2241	-1.0542
Mean long. Acceleration [m/s²]	-7.6677	-7.8725	2.6701

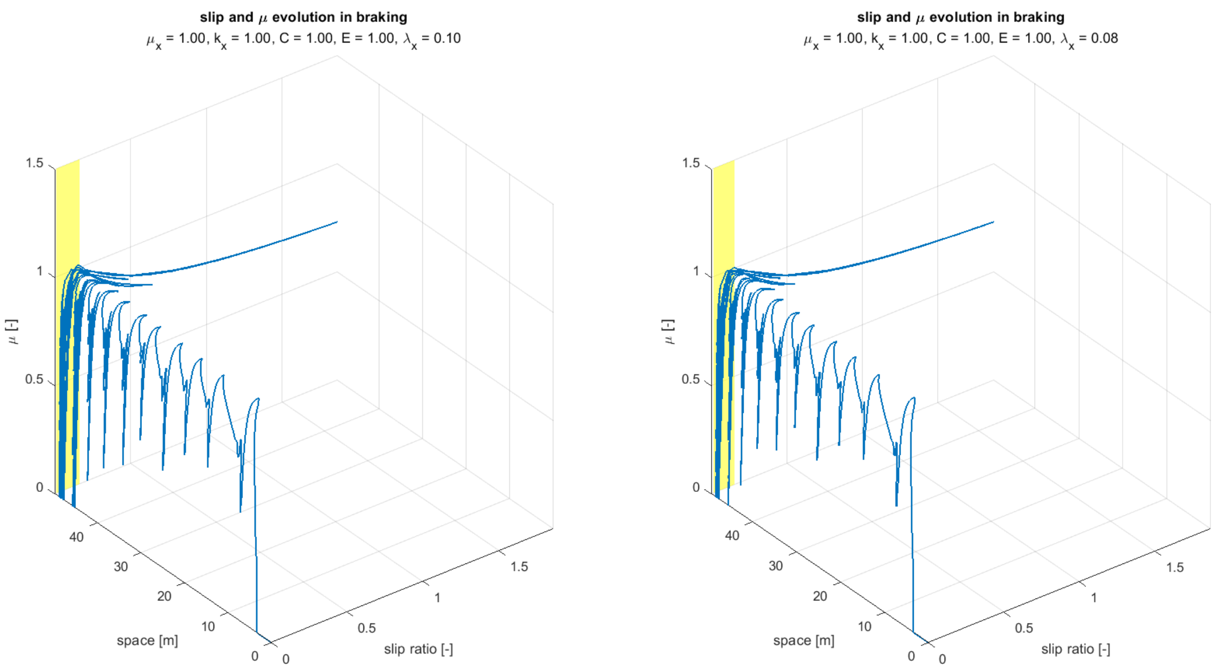

Figure 31

Counterintuitively from what observed before, it could have been expected that since increasing the relaxation length brings to an improvement in braking performances, reducing it would have decreased them. But by analysing the previously reported graphs it can be understood that it is not what happens. In fact, reducing the relaxation length brings to a further improvement in the ABS performance. For sure the tire becomes more responsive, as shown in Figure 29: in fact, the distance covered between imposing a slip and the relative force formation is smaller.

Accordingly, by looking at Figure 30, it can be observed that the modified system holds the pressure longer with respect to the baseline scenario, meaning that the deceleration can be kept higher for a longer time.

7. Effect of temperature

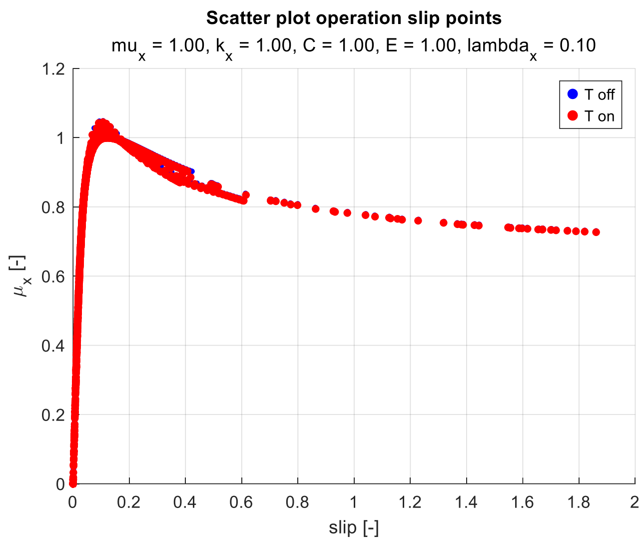

Figure 32

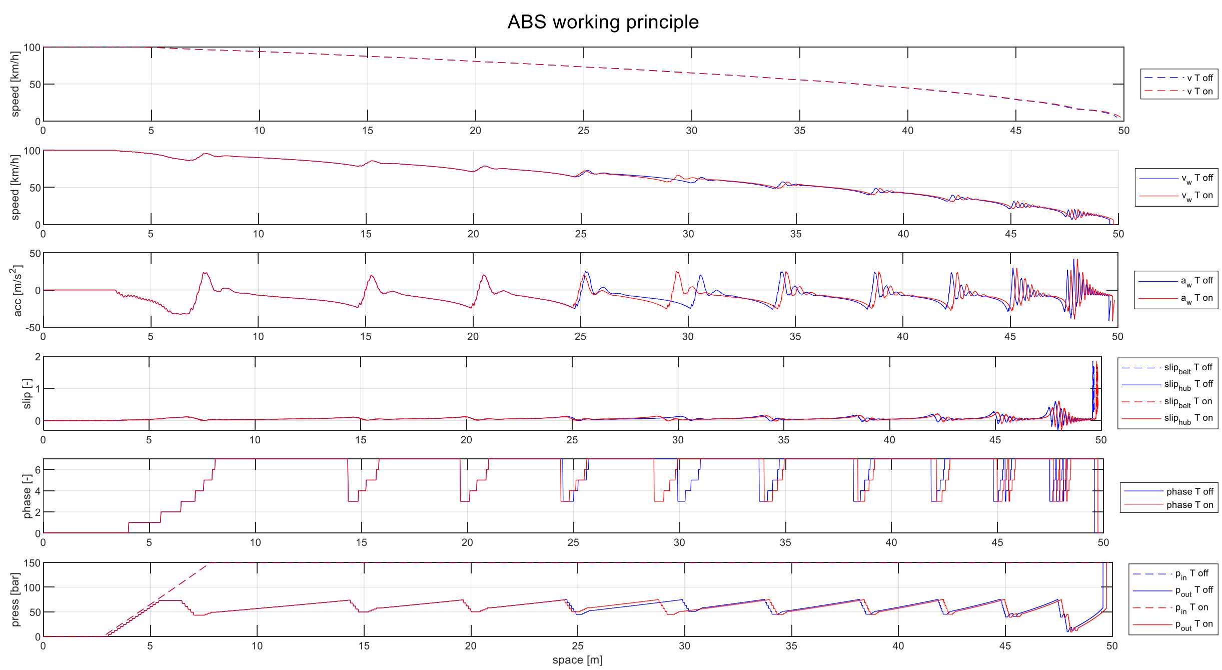

Figure 33

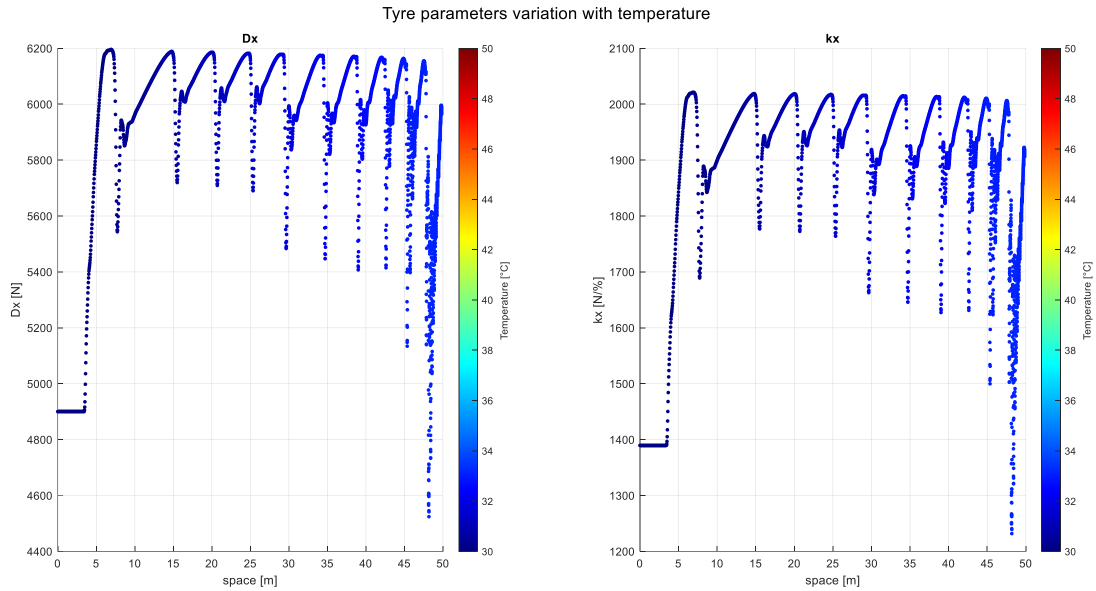

Figure 34

Temperature evaluation	T off	T on	Delta [%]
Stopping distance [m]	46.8679	47.0397	0.3665
Build-up distance [m]	10.2579	10.2579	0.0001
ABS braking distance [m]	36.6101	36.7818	0.4692
Mean long. Acceleration [m/s^2]	-7.6677	-7.6282	-0.5152

Switching on the thermal model, as expected the ABS performance decreases.

This is mainly due to the fact that as the manoeuvre goes on, the temperature of the tires increases and the grip lowers (being the maximum of the grip at 30°C), leading to a lower longitudinal force. The influence of the temperature in this model is as explained before in the D_x and K_x and as shown in Figure 34, the general trend shows that as the temperature increases the peak values tend to decrease with respect to the previous ones. Since the developed longitudinal force is lower, the effectiveness of the ABS becomes lower and as expected the stopping distance increases and the mean long deceleration is lower.

8. Conclusion

The investigation successfully characterized the interactions between tire dynamics and ABS control logic. The sensitivity analysis revealed that peak grip is the dominant factor affecting stopping distance. Specifically, a 5% reduction in friction resulted in a performance loss of 6.3% compared to the 4.6% gain from an equivalent increase. While increasing longitudinal stiffness and the shape factor enhances initial tire responsiveness, these changes complicate peak detection for the controller and can lead to longer stopping distances. Finally, the thermal simulation confirmed that heat generation from longitudinal sliding leads to a decrease in braking efficiency as tire grip degrades during the manoeuvre, at least for the specific compound simulated. These results show the necessity of robust control strategies that account for the evolving thermal and mechanical properties of tires during braking.