The portfolio composition effect of wealth taxation

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Abstract

In this paper I study how the introduction of a wealth tax imposed on the households at the top of the wealth distribution impacts their portfolio choices and as a consequence the capital allocation in the economy, GDP and GDP growth. In order to do that I develop a household portfolio choice model which is able to replicate some key features of US households' choices in terms of private equity, public equity and safe assets investments, beside the aggregate investment in each of these investment opportunities. Then, I use this model as a metering device to quantify the effect of wealth taxation on the households' investment choices. I show that those at the top of the wealth distribution reduce their investment in private equity, in a lower extent they also reduce public equity investment, while they increase investment in safe assets. As a consequence wealth taxation induces a capital reallocation from private (and also public) equity investments to safe assets. Afterwards, I present some evidence on US capital allocation across industries, suggesting that private equity investments are directed towards very productive and high-growth sectors. As a consequence I show that the capital reallocation induced by a wealth tax determines not only a GDP reduction, which is quantified, but also a GDP growth decrease.

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1 Introduction

The long-standing debate on the desirability of wealth taxation has largely focused its attention on the long-run distortions on saving decisions induced by those taxes. The advocates of wealth taxation, instead, have overlooked those distortionary effects (Saez and Zucman (2019)), claiming that wealth taxation is a powerful tool to restore the progressivity of the tax system at the top of the wealth distribution (Saez and Zucman (2022)). However, the introduction of a wealth tax also generates other effects beside those on household saving choices. When taxed households become poorer: hence they may have access to fewer investment opportunities, face liquidity constraints or change their risk taking behaviour. Those effects induce households to change their portfolio choices, generating effects on the process of capital supply in the economy. The consequence of capital moving across sectors characterized by more or less productive firms, induces changes in GDP. Furthermore, the capital flows across sectors characterized by different productivity growth rates induces changes in GDP growth too.

The contribution of this paper is to analyze the effects of wealth taxation on household portfolio choices and the implied capital allocation effect across sectors. This will be done assuming away the distortionary effect of taxes on the household saving decisions. In this way I will shed light on some mechanisms that have been largely disregarded by the existing literature, such as how wealth taxes affect household portfolio choices and the induced capital allocation across different sectors of the economy.

To analyze the aggregate effects of wealth taxation it is necessary, first of all, to understand how households invest their wealth. In Section 2, indeed, I will explore the portfolio choices of US households across the whole wealth distribution. While those at the bottom and the middle of the wealth distribution invest essentially in real estate and safe assets, those at the top of the wealth distribution are more exposed towards risky assets. In particular, those at the top 1% of the wealth distribution invest significant shares of their wealth into private equity investments. Those are very risky but also very rewarding investment opportunities, since private equity funds have the capabilities of selecting projects with great potential in terms of growth and innovation. Venture capital funds, which invest in extremely promising and innovative start-ups are only an example of that kind of investment. On the contrary, public equity investments, convey funds to less innovative, more traditional firms and sectors.

As a consequence, understanding whether households invest in private, public equity or

other investment opportunities becomes crucial to understand to which kind of firms they supply their capital to. As a matter of fact, supplying capital to more or less productive firms has immediate consequences on GDP.

This reasoning explains the choice of introducing in Section 3 a portfolio choice model which captures how households invest their wealth in terms of public equity, private equity and other (safer) assets. In particular, I will consider a two period portfolio choice problem in which households differ for their initial wealth only (and as a consequence risk aversion). Each household at time 0 allocates his initial (after-tax) wealth among public equity, private equity and a safe asset. At time 1 each one consumes the wealth resulting from investment. Notice that in this simple framework households do not make consumption-saving choices. This is consistent with my objective of studying the effects of wealth taxation, assuming away the distortionary effect taxes generate on saving decisions. Once appropriately calibrated the model will be able to reproduce the aggregate investment of households in public equity, private equity and safe assets, as well as some relevant features of the portfolio choices across the wealth distribution.

In Section 4 I will introduce wealth taxation into the framework. In particular, the tax analyzed will be a proportional wealth tax on the wealth in excess of a given threshold (e.g. the 99th percentile). All the tax revenues will be uniformly redistributed across the whole wealth distribution through lump-sum transfers. I will then decompose the effects of the introduction of this tax-transfer schedule into two effects:

1. "Quantity effect": it captures the effect of the wealth tax on investment in the different investment opportunities, under the assumption of unchanged portfolio shares before and after the tax introduction. The larger the amount of an asset detained by the households who experience a reduction in their wealth, the larger the investment drop in that kind of asset.

2. "Portfolio composition effect": it captures how much the wealth tax affects investment in the different investment opportunities, due to the change in household portfolio shares induced by the tax introduction.

The two combined effects will induce a reduction in private equity supply, a smaller reduction in public equity investment and an increase, instead, in the household investment in safe assets.

In Section 5 I will analyze how the introduction of the wealth tax affects capital allocation across different industries.

In order to do that first of all I will analyze US data on how private equity, public

equity and safe assets capital is allocated across different sectors (software, pharma-biotechnology, utilities...). This evidence, together with household endogenous choices of private equity, public equity and safe assets allows me to obtain the capital allocation across the different sectors. Furthermore, I also collect data on TFP and TFP growth across those industries. In this way I show that the firms in the sectors towards which private equity investments are directed have the largest productivities and productivity growth, followed the firms in the sectors which receive more public equity capital. Hence, the reduction of public and private equity investments in favor of safe assets will induce a reallocation of capital towards less productive sectors, determining a GDP reduction, which will be precisely quantified. Not only this, but also capital will be reallocated towards sectors of lower TFP growth, showing a negative impact of the wealth tax on economic growth. The next step of this research project will be the introduction of employment into the framework. An appropriate modelling of the production side of the economy will allow to quantify the employment flows across sectors induced by capital reallocation.

Section 6 will conclude the work.

1.1 Literature review and contribution

One of the main objectives of this work is that of building a household portfolio choice model able to capture some salient features of households investment choices. This is crucial to correctly quantify how the introduction of a wealth tax will affect capital supply in the economy. In this respect, this work relates to the stream of literature which documents heterogeneity of investment behaviour (and hence returns) across the wealth distribution. Two very important contributions on this topic are Bach et al. (2020) and Fagereng et al. (2020). Using administrative Swedish and Norwegian data they both report that individuals at the very top of the wealth distribution tilt their portfolio allocations toward very risky assets (especially private equity). Consistently with that, they also show that households' returns on wealth are increasing across the wealth distributions. Both Bach et al. (2020) and Fagereng et al. (2020) argue that the mechanisms at play in determining this return heterogeneity are: 1- "scale dependence" (larger wealth scale allows households to obtain larger returns) and 2- "type dependence" (individuals at the top of the wealth distribution have personal traits, e.g. investment abilities, which allows them to get higher returns). Since the contribution of Gabaix et al. (2016) several papers have exploited scale and type dependence mechanisms to retrieve heterogeneous

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returns on wealth across the wealth distribution. For example, Cioffi (2021) builds a dynamic portfolio choice model in which households can choose across three assets (housing, stocks, bonds) and households' relative risk aversion is decreasing in wealth (capturing scale dependence). This allows him to reproduce some relevant features of the empirical household portfolio choices across the wealth distribution. However, he mentions that the choice of not including private equity separately from public equity (as instead I do) does not allow him to precisely capture households behaviour at the very top of the wealth distribution. Gaillard and Wangner (2021), instead, build a static and dynamic version of a portfolio choice model in which households can only choose between a safe and a risky asset. However, they model household preferences so to capture both scale and type dependence mechanisms in their household portfolio choices. Remarkably, they notice that different calibrations of the parameters governing the extent of type and scale dependence can provide observationally equivalent household portfolio choices across the wealth distribution. My portfolio choice problem, apart from being static, differs from the mentioned papers in choice of introducing three assets (one of which is specifically private equity). Beside this, I specify household preferences so to capture scale dependence of returns (and hence increasing shares of private and public equity across the wealth distribution). Furthermore, differently from the previously presented papers, I also introduce an intermediation fee for investing in private equity, which generates heterogeneous net returns in private equity investments (this allows me to replicate some features of households private and public equity choices across the wealth distribution).

Another stream of literature to which this work talks to is that on wealth taxation. Many recent empirical papers on the issue are focused on the estimation of the elasticity of taxable wealth with respect to wealth tax rate, for example Brülhart et al. (2019), Seim (2017), Zoutman (2018) Jakobsen et al. (2020). While the first two focus mainly on the role of tax evasion, the third on the effect of the tax on savings, the latter builds a life-cycle model with utility of bequests. Once the model is appropriately calibrated using evidence from the introduction of a wealth tax reform in Denmark, the authors simulate the long-run elasticity of taxable wealth with respect to the tax rate. The recent work of Akcigit et al. (2018) which shows that corporate and income taxes have reduced the quantity of innovation in the US throughout the 20th century, suggests the need of analyzing deeper the effects of wealth taxation which go beyond the mere effect on capital accumulation. A novel contribution of my work, indeed, is to focus on the distortions that wealth taxation may induce in terms of capital reallocation across pro-

duction sectors characterized by different productivity and growth. Distortionary effects of wealth taxation (beside those on capital accumulation) have been studied for example by Cagetti and De Nardi (2006), who show that taxing entrepreneurs' wealth, may reduce the number of entrepreneurs. Guvenen et al. (2019), argue that wealth taxation is able to induce capital reallocation from the least to the most productive entrepreneurs. Thus, in his setting wealth taxation (compared to capital income taxation) boosts aggregate productivity and output. Gaillard and Wangner (2021) simulate the effect of the introduction of a wealth tax on GDP. Their contribution is to discuss optimal wealth taxation in presence of scale and type dependence mechanisms together with returns which may not reflect the real productivity of the investment, but the presence of some forms of rent-extraction.

2 Portfolio composition across the wealth distribution

In this Section I will analyze how US households allocate their wealth across different investment opportunities. Understanding the portfolio composition, especially of those at the top of the wealth distribution who will be taxed, is necessary to understand which kind of wealth will be taxed. This is a first step towards understanding how the introduction of the wealth tax will affect aggregate household capital supply to the production side of the economy.

2.1 Data and variables definitions

To obtain the portfolio composition of US households across the wealth distribution, the 2019 wave of the Survey of Consumer Finances (henceforth SCF) is used. The choice of the SCF over other surveys (for example the Panel Survey of Income Dynamics) has two reasons. The first one is the very detailed information on the household wealth composition, the second one is the over-sampling of the very wealthy households (for details on the sampling procedure see for example Kennickell (2008)). The latter motivation is crucial for my analysis: indeed, in order to evaluate how the introduction of a wealth tax on the very wealthy affects aggregate capital supply it is necessary to understand how much and which kind of wealth will be taxed.

Table 1 reports some features of the 2019 US net wealth distribution, showing that wealth is very unequally distributed. Only 1.5% of the overall wealth accrues to the

TABLE 1. *Summary statistics of the 2019 US wealth distribution*

Wealth percentile	Wealth share
0 - 49 percentile	1.49%
50 - 89 percentile	20.28%
Top 10%	76.46%
Top 5%	64.91%
Top 1%	37.20%
Top 0.5%	28.01%

Notes: *The Table represents some features of the US 2019 wealth distribution. The term wealth indicates net wealth = assets - debts of US households. Data are taken from the 2019 wave of the Survey of Consumer Finances.*

households in the bottom 50% of the wealth distribution, 76% of the overall wealth accrues to the top 10% and 37% to the top 1%. Those figures, hence, show that even introducing a wealth tax on the wealthiest 1% of households only, still means taxing a very sizable share of the aggregate US households' wealth.
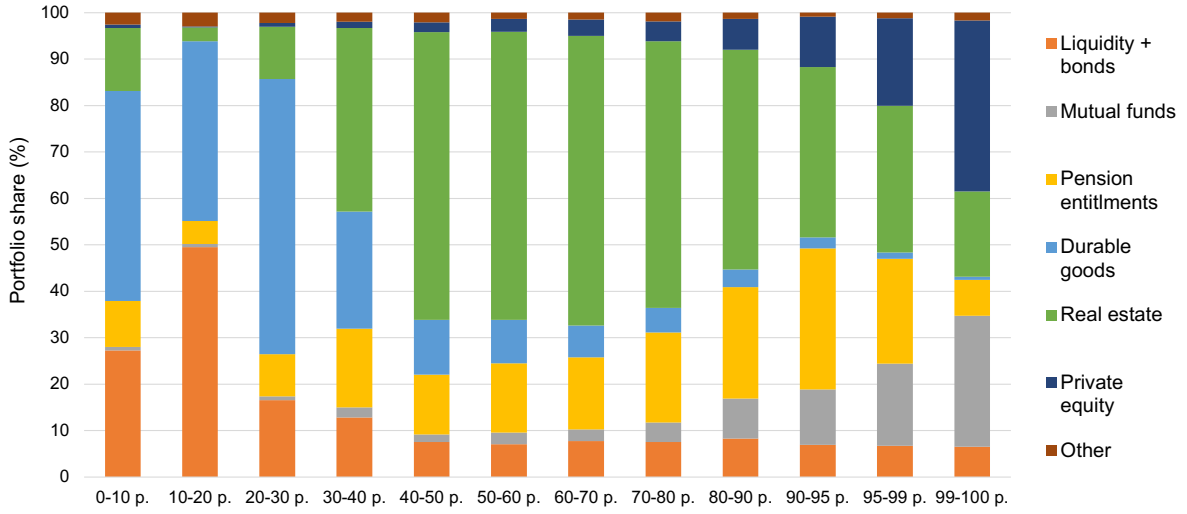
All the possible assets that US households hold can be categorized into 7 different groups: liquidity and bonds (all type of transaction accounts, certificates of deposit, directly held bonds, saving bonds, bonds funds), mutual funds (stock mutual funds or combination mutual funds), pension and insurance entitlements (quasi-liquid retirement accounts and cash value of life insurance), durable goods (e.g. vehicles...), real estate (residences and real estate investment), private equity (privately-held businesses, professional practices, limited partnerships, private equity investments, or any other business investments that are not publicly traded) and other assets (annuities, miscellaneous financial and non-financial assets). Those asset categorization will be used to provide an overview of household portfolio choices across the wealth distribution.

2.2 Portfolio choices across the wealth distribution

Figure 1 shows the portfolio choices of US households among the assets categories described above, across the entire wealth distribution.

It is worth noticing that the majority of assets held by the households at the bottom of the wealth distribution are essentially durable goods and liquidity. Instead, at the middle of the wealth distribution housing becomes by far the most important asset households hold, together with their insurance and pension entitlements. As long as we move toward the top of the wealth distribution the portfolio shares households invest in private equity and stock mutual funds increase, while the importance of real estate is

FIGURE 1. *US households portfolio composition across the wealth distribution*

diminished. In particular, at the top 1% of the wealth distribution mutual funds and private equity make up around 65% of the whole portfolio. Furthermore, notice that even if the portfolio shares invested in private equity are very small across almost the entire wealth distribution, it becomes the most chosen investment opportunity at the very top of the wealth distribution (almost 39% at the top 1%).

As argued in the introduction, private equity is a form of investment which is substantially different from other investment opportunities not only for its high riskiness and potential profitability. Indeed, private equity investments are directed towards extremely promising and innovative firms and start-ups, which do not have the possibilities of obtaining the necessary funds through more "traditional" channels such as bank loans, the emission of corporate bonds or new stocks. This substantial difference in terms of productivity between the firms to which private and public equity investments are directed to, makes necessary to distinguish how much households invest into each of these investment opportunities. This will allow me to study how capital allocation choices made by the US households affect the aggregate US production.

To analyze US households wealth allocation choices in terms of private equity, public equity and safe assets I collapse all the assets classes described in Section 2.1 (and shown in Figure 1) into the three previously mentioned categories:

- **Private equity:** illiquid, very risky and very profitable form of investment. It coincides with the private equity class reported in Figure 1 and described in Section 2.1: privately-held businesses, professional practices, limited partnerships, private equity investments, or any other business investments that are not publicly traded.

- **Public equity:** liquid form of investment, with intermediate level of profitability and riskness. It comprises the total value of financial assets that are invested in stocks (stock mutual funds, combinational mutual funds, directly held stocks, stock investments of pension funds)[2].

- **Safe assets:** all the remaining assets are categorized as "safe" (liquidity, bonds, bond mutual funds, durable goods, life insurance, pension funds shares not invested in stocks, other assets).

Figure 2 reports the portfolio choices of households in terms of private and public equity (the remaining share is invested in safe assets) across the wealth distribution. For the sake of clarity the Figure reports portfolio choices for the households above the 50th percentile only. This choice allows me to better focus on the portfolio decisions of the households who hold most of the aggregate wealth (the wealth held by the top 50% of households in the wealth distribution is 98.5%, see Table 1). Figure 2 shows that across almost the entire wealth distribution people hold a larger portfolio share invested in public equity than in private equity. However, the portfolio share invested in private equity starts increasing very steeply from the 95th percentile onward, overcoming the share invested in public equity around the 97th percentile. This is not a US peculiarity, for example similar empirical evidence has been found for Sweden and Norway by Bach et al. (2020) and Fagereng et al. (2020) using administrative records. This feature of the data can be explained by the own peculiarities of private equity investments. Indeed, investing in private equity funds entails committing very large amounts of money for long periods of time in very risky (but potentially very profitable) projects. Furthermore, the

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[2]Combinational mutual funds are included in the "public equity" class for 1/2 of their value only, the rest of value is attributed to the category "safe assets". Pension funds are included in the "public equity" class for 1/2 of their value only if the investment of the fund is split between stocks and other interest earning assets. If that is the case the remaining 1/2 of pension fund value is attributed to the category "safe assets". If the investment of the pension fund is mainly directed toward stock the value of the pension fund is fully included in the category "public equity" Those assumptions are chosen so that the "public equity" variable in my analysis matches the "EQUITY" variable directly provided in the SCF 2019 database, which exactly captures the total value of financial assets that households invest in stocks.

FIGURE 2. *US households portfolio composition: public vs private equity investments*
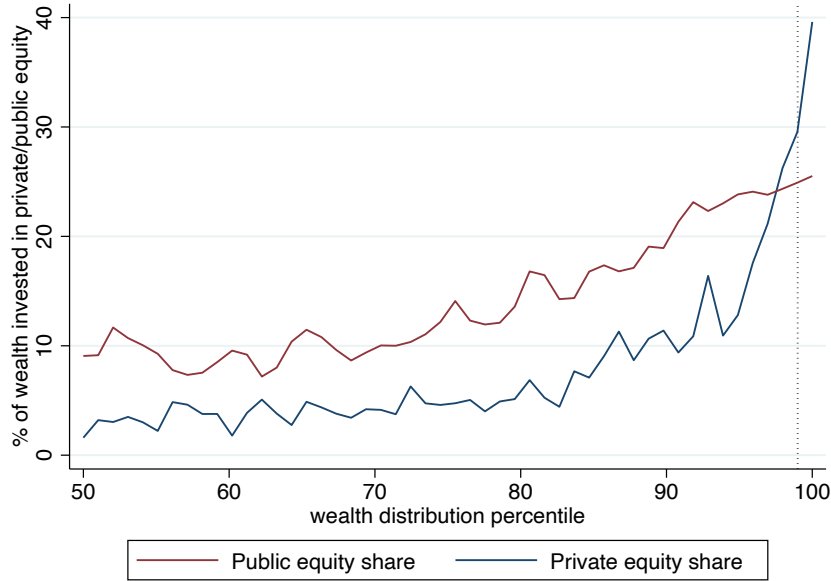
Notes: *The Figure plots the share of wealth US households invest in private equity and public equity, from the 50th wealth percentile onwards. For each wealth percentile, the share invested in asset class $j \in \{private\ equity,\ public\ equity\}$ is computed dividing the aggregate investment in asset j of households in a given wealth percentile by the total wealth owned by households in that percentile. The composition of public and private equity asset classes is described in Section 2.2. Data are taken from 2019 SCF.*

private equity market is not an easily accessible market, as the stock market is. All those features drawn together suggest that only the very wealthy have the resources, capabilities and enough propensity to risk, in order to invest significant shares of their wealth into private equity investments.

On the other hand, stock market investments are very liquid kind of investments which are easily accessible and not too risky. This could be the reason why in the top 50% of the wealth distribution every household has at least 10% of its portfolio invested in public equity. This share is steadily increasing with wealth. Differently to what happens for private equity, however, the share invested in public equity does not grow very steeply at the top of the wealth distribution. Being both public and private equity shares increasing across the wealth distribution the share of wealth invested in safe assets is instead strictly decreasing across the entire wealth distribution, with a big drop at the very top.

In the next Section I will model how agents make their portfolio choices across the wealth distribution. This will allow me to simulate how the introduction of a wealth tax will affect individual portfolio choices and as a consequence the aggregate capital supply to the production side of the economy.

3 A model of portfolio choice

In this Section I introduce a portfolio choice model whose aim is that of capturing how households supply capital to the production side of the economy. Appropriately calibrated the model will be able to reproduce some salient features of the empirical evidence presented in the previous Section. The objective of this model is that of serving as metering device to quantify how much households change their capital supply choices in response to the introduction of a wealth tax. This aim will be accomplished under the modelling assumption that households do not make saving choices. This will allow me to shed light on how the wealth tax affects household portfolio choices and the induced capital allocation, effects often overlooked by the literature aimed at computing the overall elasticity of taxable wealth with respect to the tax rate (e.g.Jakobsen et al. (2020), Zoutman (2018), Brülhart et al. (2019), Seim (2017))

3.1 Setup

I will consider a partial equilibrium model.

The economy is populated by a unit mass of heterogeneous households indexed by $i \in [0, 1]$, who differ only for their initial wealth endowment a_i and absolute risk aversion $\alpha(a_i)$ (which is a deterministic function of the initial wealth level a_i). Each household i solves the following two period problem: in period 0 he has to allocate his initial after-tax disposable wealth $d(a_i)$ (initial wealth endowment a_i minus taxes plus transfers) between a safe asset, a public equity investment and a private equity investment. In period 1 each household i does not make any further choice, he simply uses the wealth resulting from the investment to consume. Notice that the only choice households make in this framework is to choose the composition of their portfolio. No saving decisions are made. This modelling choice is consistent with the aim of this model to be used as a tool for quantifying the effects of wealth taxation on capital supply, assuming away the tax distortions on saving decisions.

Wealth endowment and preferences: Households $i \in [0, 1]$ differ for their value of initial wealth endowment a_i. Their wealth is assumed to be drawn from a Pareto distribution $Pa(1, \eta)$, where $\eta > 0$ is a shape parameter which captures the thickness of the right tail of the distribution. This choice is consistent with the literature analyzing how wealth is distributed. Indeed, since wealth distributions are skewed to the right and display thick upper tails, Pareto distributions, which have exactly those features, are

particularly appealing to model wealth (Benhabib and Bisin (2018), Vermeulen (2018)).

Many recent empirical studies show that wealthier households earn larger returns on wealth than poorer households (e.g. Xavier (2021), Bach et al. (2020), Fagereng et al. (2020)). A potential explanation for this phenomenon is that if households hold more wealth, they may have easier access to better investment opportunities or simply dare to take more risky behaviours. This mechanism is named "scale dependence" and it captures why households endowed with larger wealth make riskier portfolio choices (obtaining higher returns) than those holding lower wealth. I will specify household preferences of my portfolio choice problem so to capture exactly this mechanism. Household preferences over consumption (consumption of household i is denoted as usual with c_i) are specified through the following utility function:

$$u(c_i, a_i) = -\frac{1}{\alpha(a_i)} e^{-c_i \alpha(a_i)}$$

where the coefficient of absolute risk aversion for household i with initial wealth a_i (and disposable wealth $d(a_i)$) is:

$$\alpha(a_i) = \kappa \cdot d(a_i)^{-\gamma}$$

with $\kappa > 0$ and $\gamma > 1$. First of all notice that absolute (and also relative) risk aversion negatively depends on disposable wealth. This is consistent with the "scale dependence" mechanism, capturing that wealthier households have more risk taking behaviours. κ is a scale parameter whose role is that of governing the average level of absolute risk aversion of households. γ, instead, captures the extent in which household choices are affected by "scale dependence". In other words, the parameter γ governs the sensitivity of households absolute risk aversion to the initial value of household disposable wealth. The larger γ, the larger the reduction in absolute (and also relative[3]) risk aversion of a household when his wealth increases.

Assets and returns: Let's remind that each household i can invest in three possible assets: a safe asset, in public equity or in private equity. Moskowitz and Vissing-Jørgensen (2002) and Kartashova (2014) contributions (using SCF data) have shown

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[3]The coefficient of relative risk aversion for household i with initial wealth level a_i is: $r(a_i) = \kappa \cdot d(a_i)^{1-\gamma}$, which is decreasing in disposable wealth $d(a_i)$ as long as $\gamma > 1$.

that apart from the period 1990-2000, private equity generally earns a premium over public equity. More recent empirical evidence (for US Xavier (2021) and Gaillard and Wangner (2021), for Sweden and Norway Bach et al. (2020) and Fagereng et al. (2020)), has confirmed this result, highlighting the higher level of riskiness of private equity investments.

Coherently with the mentioned evidence I assume that the private equity investment is very risky but its expected return is also very high. The public equity investment, instead, has an intermediate level of profitability and riskiness, while the safe asset is risk-free but it is characterized by a low return.

To be specific, I will denote with δ_i the fraction of disposable wealth of household i invested in private equity, with ω_i the share of disposable wealth invested by household i in public equity, while $1 - \delta_i - \omega_i$ will be the disposable wealth share invested by household i in the safe asset.

The returns of private and public equity are assumed to be normally distributed random variables: $R_v \sim N(\phi_v, \sigma_v^2)$ is the return of the private equity investment, while $R_r \sim N(\phi_r, \sigma_r^2)$ the one of the public equity investment[4]. The return of the safe asset is a non-negative scalar $R_s \in \mathbb{R}_+$. As already argued, I assume that the expected return of private equity investment is the highest and the one of the safe asset is the lowest, that is: $\phi_v > \phi_r > R_s$. Furthermore, being the private equity investment a riskier kind of investment than the public equity one, I assume $\sigma_v^2 > \sigma_r^2$. Furthermore, I also allow private and public equity returns to have a non-zero covariance, namely: $Cov(R_v, R_r) = \theta$.

Private and public equity investments not only differ for their level of riskiness, indeed private equity investments are also less accessible (and less liquid) than public equity ones. To capture this feature of private equity investments I assume that to invest in private equity households have to pay a variable intermediation fee. This induces higher perceived private equity returns at the top of the wealth distribution. In particular, household i which invests a fraction δ_i of his disposable wealth $d(a_i)$ in private equity has to pay:

$$C(d(a_i), \delta_i) = \lambda_1 \delta_i d(a_i)^{1-\lambda_2}$$

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[4]I use the subscript "r" to refer to the private equity investment because it is a "risky" kind of investment, while I use the subscript "v" to refer to the public equity investment because it is a "very risky" kind of investment. This choice is to avoid the use of the letter "p" which may generate confusion between "private" and "public"

with $\lambda_1 > 0$, $0 < \lambda_2 < 1$.

The specific functional form chosen (hence the two parameters λ_1 and λ_2) is needed in order to allow the model to match a peculiarity of the empirical evidence on household portfolio choices previously presented. In particular, it allows to replicate the crossing between the lines representing private and public equity portfolio shares across the wealth distribution (see Figure 2). This is not surprising, indeed this parametrization of the fee households have to pay induces higher private equity net perceived returns (gross return R_v minus unitary investment cost) for households with higher disposable wealth.

Household problem:

To sum up: each household $i \in [0,1]$, with initial wealth a_i drawn from a Pareto distribution $Pa(1, \eta)$, solves the following problem:

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\begin{aligned}
\max_{\omega_i, \delta_i} \quad & \mathbb{E}\left[-\frac{1}{\alpha(a_i)} e^{-c_i \alpha(a_i)} \right] \\
\text{s.t.} \quad & c_i = (R_s(1 - \omega_i - \delta_i) + R_r \omega_i + R_v \delta_i) d(a_i) - \lambda_1 \delta_i d(a_i)^{1-\lambda_2} \\
& \omega_i \geq 0, \quad \delta_i \geq 0, \quad 1 - \omega_i - \delta_i \geq 0 \\
& R_v \sim N(\phi_v, \sigma_v^2), \quad R_r \sim N(\phi_r, \sigma_r^2), \quad R_s \in \mathbb{R}_+ \text{ given}, \quad Cov(R_v, R_r) = \theta
\end{aligned}
\tag{P}
$$

where $\alpha(a_i) = \kappa \cdot d(a_i)^{-\gamma}$, as already discussed.

3.2 Solution

Throughout this Section, for notational convenience, I will denote disposable wealth of household i as d_i, rather than $d(a_i)$.

To understand how the introduction of a wealth tax affects household portfolio choices (and as a consequence capital supply) it is crucial to understand how household portfolio choices depend on their disposable wealth. This is the result of Proposition 1, which provides closed-form expressions for the solution of the household problem (P) previously described.

Proposition 1. *The internal solution to the portfolio choice problem (P) of household*

$i \in [0, 1]$ *is:*

$$\omega_i = \frac{1/\kappa}{\sigma_r^2 - \frac{\theta^2}{\sigma_v^2}} \left[\left(\phi_r - R_s - \frac{\theta}{\sigma_v^2}(\phi_v - R_s) \right) d_i^{\gamma-1} + \frac{\theta}{\sigma_v^2} \lambda_1 d_i^{\gamma-\lambda_2-1} \right] \qquad (1)$$

$$\delta_i = \frac{1/\kappa}{\sigma_v^2 - \frac{\theta^2}{\sigma_r^2}} \left[\left((\phi_v - R_s) - \frac{\theta}{\sigma_r^2}(\phi_r - R_s) \right) d_i^{\gamma-1} - \lambda_1 d_i^{\gamma-\lambda_2-1} \right] \qquad (2)$$

where ω_i is the fraction of wealth that household i, with disposable wealth d_i invests in public equity. Analogously, δ_i represents the fraction of wealth invested in public equity.

Proof: the derivation of the expressions is provided in Appendix A (note to the reader: algebra still to be reported).

To grasp some intuition on the previously derived expressions let's assume for the moment that $Cov(R_v, R_r) = \theta = 0$. Expressions (1)-(2) simplify to:

$$\omega_i = \frac{\phi_r - R_s}{\kappa \sigma_r^2} d_i^{\gamma-1} \qquad (3)$$

$$\delta_i = \frac{1}{\kappa \sigma_v^2}(\phi_v - R_s - \lambda_1 d_i^{-\lambda_2}) d_i^{\gamma-1} \qquad (4)$$

First of all, let's notice that both the wealth shares invested in private and public equity positively depend on their respective the Sharpe ratios. In other words, the larger the excess return of an investment opportunity (with respect to the safe asset) and the lower its returns volatility, the larger will be the share invested in that kind of asset. Second of all, assuming for a while that $\lambda_1 = 0$, notice that both portfolio shares positively depend on disposable wealth d_i. This is due to the fact that under the assumption that $\gamma > 1$ households relative risk aversion is decreasing in d_i[5], hence the wealth share households invest in risky assets is larger for wealthier households. Now, let's go back to the case in which $\lambda_1 > 0$. The third term in parenthesis in (4) shows that the share invested in private equity not only depends on the private equity Sharpe ratio and the relative risk aversion of the household, but it is also affected by the costly nature of private equity investments. In particular, wealthier households will experience larger net private equity returns and hence they will invest more into this investment

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[5]Remind that, as remarked in Section 3.1, the coefficient of relative risk aversion for household i with initial wealth level a_i is $r(a_i) = \kappa \cdot d_i^{1-\gamma}$.

TABLE 2. *Externally calibrated parameters*

Parameter	Description	Value	Source
η	shape parameter wealth distr.	1.35	SCF data
ϕ_v	expected private equity return	0.156	Gaillard and Wangner (2021)
ϕ_r	expected public equity return	0.058	Gaillard and Wangner (2021)
R_s	safe asset return	0.004	Gaillard and Wangner (2021)
σ_v^2	private equity returns variance	0.377	Gaillard and Wangner (2021)
σ_r^2	public equity return variance	0.174	Gaillard and Wangner (2021)
θ	cov. public/private equity returns	0.064	S&P500 returns, PE buyout index

Notes: *The Table presents the externally calibrated parameters for the household portfolio choice problem presented in Section 3.1-3.2. The first column indicates the symbol used to identify the parameter in the model, the second column the parameter description, the third column the chosen value for the parameter, the fourth column the data sources used to obtain the parameter value.*

opportunity, with respect to poorer households.

Now, let's go back to the expressions of Proposition 1. When the two risky assets (private equity and public equity) have a positive correlation, $\theta > 0$ agents tend to tilt more their portfolio choices towards the safe asset. The reason is the following. Call "composite risky asset" the asset with expected return $\mathbb{E}(\omega_i R_r + \delta_i R_v)$ and variance $Var(\omega_i R_r + \delta_i R_v)$. When the covariance between private and public equity increases, the variance of the "composite risky asset" goes up, leaving its expected return unaffected. This is the reason why, a household with a given level of risk aversion, if observes a higher covariance between private and public equity investments, decides to shifts its portfolio choices towards the safe asset. Indeed, investing in risky assets has become more dangerous, without extra rewards on the their return.

3.3 Calibration

The model has to be carefully calibrated in order to reproduce the aggregate investment behaviour of households in safe assets, private equity and public equity. This will allow, when introducing a wealth tax, to compute how capital supply will change in response to the wealth tax. Beside this, the calibration aims at capturing, as close as possible, the household portfolio choices across the wealth distribution (especially at the top). This will allow to identify how and how much changes in household portfolio composition will affect the aggregate capital supply (portfolio composition effect).

Table 2 reports the parameters which are externally calibrated.

The first externally calibrated parameter is the shape parameter of the wealth dis-

tribution $\eta = 1.35$. The value for η is chosen so to match as close as possible the shape of actual US wealth distribution (SCF 2019 data) at the top 1%. The focus on the top 1% of the wealth distribution is due to the fact that I am interested in simulating the effect of wealth taxes imposed at the very top of the wealth distribution. Furthermore, the case in which a tax is imposed on the wealthiest 1% of households will be an exemplifying policy often analyzed in the rest of the paper. The detailed procedure employed to obtain the value $\eta = 1.35$ is described in Appendix B.

The values for the expected returns of private, public equity and safe assets, together with their variances, are taken from Gaillard and Wangner (2021). They split household wealth into private equity, public equity and safe assets in the same way as I do, and compute mean returns and variances using PSID data from 1998 to 2018 (details of their procedure can be found in Appendix B).

Finally, the covariance between public and private equity returns θ is obtained by computing the covariance between the yearly returns of the S&P500 index (as a proxy for private equity returns) and the "Refinitive Private equity buyout index"[6] (as a proxy for private equity returns), from 2003 to 2021.

Table 4 shows in the first two columns the values of the parameters which are internally calibrated. The other columns, instead, present the chosen targets for the calibration of those parameters and the ability of the model to match them. The targets for calibration chosen are: first, the ratio between aggregate households investments in private equity and in safe assets, second the ratio between aggregate households investments in public equity and in safe assets. Those moments are easily computed using SCF 2019 data. As already argued, the choice of those targets relies in the need of having a model which precisely replicates the aggregate capital supply choices of US households in terms of private equity, public equity and safe assets. Indeed, a crucial use of this model will be that of measuring how much those quantities will change when a wealth tax will be introduced.

Beside this, I require the model to match the top 1% median portfolio shares US households invest in public and private equity. The reason why I need the model to replicate those moments is that of having a tool which performs well in replicating the portfolio

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[6]The index tracks the gross performance of the U.S. PE buyout industry through a comprehensive aggregation of company values. It is obtained by analyzing over 8,000 U.S. private equity companies. For details see https://www.refinitiv.com/en/financial-data/indices/private-equity-index.

TABLE 3. *Internally calibrated parameters*

Parameter	Value	Target	Model	Data
κ	1.95	ratio aggregate priv.eq./aggregate safe (SCF)	0.419	0.415
γ	1.2	ratio aggregate pub.eq./aggregate safe (SCF)	0.428	0.422
λ_1	0.13	median top 1% priv.eq. portfolio share (SCF)	29.17%	28.61%
λ_2	0.294	median top 1% pub.eq. portfolio share (SCF)	24.29%	24.58%

Notes: *Column 1-2 of the Table report the parameters which are internally calibrated and the chosen values for them. The meaning of the symbols is reported in Section 3.1, where the model is fully described. Columns 3-4-5, instead, present the targets for the calibration and how well the model is able to match the targeted moments.*

FIGURE 3. *Simulated portfolio choices vs empirical portfolio choices (SCF 2019 data)*
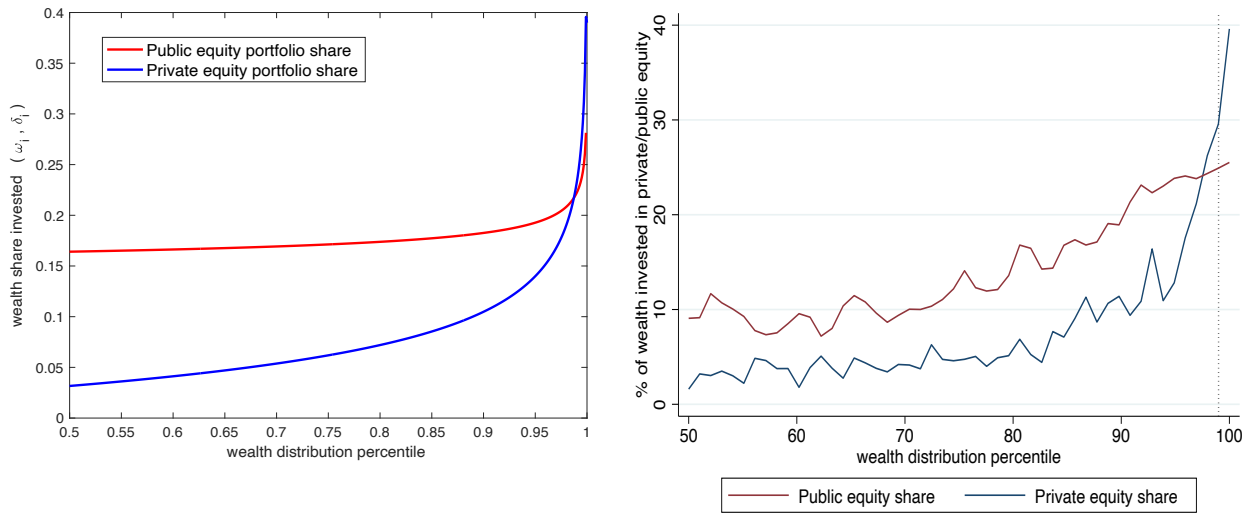
Notes: *The left panel of the Figure shows the household portfolio choices across the wealth distribution obtained calibrating the model as described in Section 3.3. The right panel reports Figure 2 of this paper, showing the empirical portfolio choices across the wealth distribution in terms of private and public equity.*

choices of the individuals at the top of the wealth distribution. This will allow me, when introducing a wealth tax on those agents, to identify which fraction of changes in capital allocation will be due to changes in household portfolio choices.

In the next paragraph I will also evaluate whether the calibrated model is able to match other relevant empirical moments, which have not been explicitly targeted.

3.4 Portfolio choices across the wealth distribution

Figure 3 (left panel) shows the household portfolio choices across the wealth distribution obtained through the calibration procedure described in the previous Section. The right panel of Figure 3, instead, is a copy of Figure 2, which is provided to facilitate the

comparison between the simulated model and the empirical evidence.

The blue lines of Figure 3 represent the share of disposable wealth that US households invest in private equity, while the red ones the fraction invested in public equity. Both lines are increasing since wealthier households have lower relative risk aversion. Furthermore, the share invested in public equity is larger than the one invested in private equity across almost the entire wealth distribution. The two lines cross at the 98th percentile (in the SCF 2019 data 97th, see Figure 3, right panel) and private equity is steeply increasing at the very top of the wealth distribution. Those features are consistent with the empirical evidence that extremely wealthy people invest large shares of their wealth (up to 39%) in very risky investment opportunities such as private equity. Instead, the majority of US households prefer public equity investments which have an intermediate level of riskiness and return.

The comparison of the left and right panel of Figure 3 shows that the model performs very well in replicating the shares households invest in private equity across the wealth distribution. The performance is slightly worse, instead, for public equity choices. Indeed, while the model predicts a pretty steep increase of the share invested in public equity at the top of the wealth distribution, this does not seem to be the case in the data. I leave to future research the analysis of this discrepancy between the model and the data.

4 The introduction of wealth taxation

In this Section I will introduce into the framework wealth taxation at the top of the wealth distribution. In particular, I will study:

(a) How the introduction of wealth taxation affects the household *portfolio composition*

(b) How the wealth tax affects *aggregate capital supply*.

Tax system: only 5 OECD countries currently have a tax on overall net wealth (or some kind of wealth only) which is still implemented (Colombia, France, Norway, Spain, Switzerland, see OECD (2018)). All those wealth taxes share a common feature: households have to pay a proportional tax on the wealth which exceeds a given threshold. The wealth tax I will consider will have the exact same features.

In particular, I will introduce a wealth tax above a given wealth threshold $\underline{a}$. Households who have wealth larger or equal to $\underline{a}$ have to pay a proportional tax (with tax rate

$\tau > 0$) on the wealth exceeding $\underline{a}$. Formally, after the introduction of this wealth tax the expression of disposable wealth of household i, with initial wealth level a_i becomes:

$$d(a_i, \tau) = a_i - \tau \mathbb{1}_{a_i \geq \underline{a}} (a_i - \underline{a}) + T$$

where T denotes a lump-sum transfer (and $\mathbb{1}_{a_i > \underline{a}}$ denotes the indicator function, $=1$ if $a_i > \underline{a}$). Notice that if the government decides to impose the wealth tax on the wealthiest $x\%$ of the population then the threshold $\underline{a}$ is such that $Prob(A > \underline{a}) = x/100$, where A is the Pareto distributed random variable $A \sim Pareto(1, \eta)$, from which wealth realizations a_i are drawn.

Furthermore, the tax-transfer schedule also included a lump-sum transfer T which is set so that all the tax revenues are uniformly redistributed across the whole wealth distribution. Formally, denoting with $G(\cdot)$ the distribution function of the random variable $A \sim Pareto(1, \eta)$ from which wealth realizations a_i are drawn, the lump-sum transfer T must satisfy:

$$\int_{a_i > 1} T dG(a_i) = \int_{a_i > \underline{a}} \tau (a_i - \underline{a}) dG(a_i)$$

integrating and rearranging the terms it is possible to obtain a closed form expression for the lump-sum transfer T:

$$T = \frac{\tau}{\underline{a}^{\eta - 1}(\eta - 1)} \tag{5}$$

Decomposing the aggregate effect of wealth taxation: in the model considered, the aggregate effect of the introduction of a wealth tax on capital supply can be decomposed into two effects: a "quantity effect" and a "portfolio composition effect".

First of all, suppose that after the introduction of the wealth tax the fraction of wealth each agent invests in each asset remains unaffected. Being taxed, hence poorer, households will have lower wealth to invest in the different investment opportunities. The "quantity effect" captures exactly this phenomenon, i.e. the change in the level of investment in the different investment opportunities, keeping the portfolio choices of the households unchanged.

The second effect, which will be extensively analyzed in the next paragraph is the "portfolio composition effect". The households, when taxed, become poorer and due to the "scale dependance effect" change their risk-taking behaviour, altering their portfolio choices. This phenomenon affects the aggregate investment of households in private equity, pub-

lic equity and safe assets. This is the "portfolio composition effect".

4.1 Portfolio composition effect

Suppose to introduce the wealth wealth tax previously described on the wealthiest 1% of households. This means that the threshold $\underline{a}$, above which the wealth tax is applied satisfies $Pr(A > \underline{a}) = 0.01$, with $A \sim Pareto(1, \eta)$. Assuming a tax rate $\tau = 1\%$, the disposable wealth of household i with initial wealth level a_i becomes:

$$d(a_i, 0.01) = a_i - 0.01 \times \mathbb{I}_{a_i \geq \underline{a}}(a_i - \underline{a}) + T$$

where T is the lump-sum transfer satisfying equation (5). How does the introduction of this wealth tax affects the individual portfolio choices across the wealth distribution? Figure 4 answers to this question.

In particular, the left panel of Figure 4 plots the percentage change of public equity, private equity and safe asset portfolio shares, caused by the introduction of the wealth tax. Notice that the focus of the Figure is on the top 10% of the wealth distribution.

First of all, let's notice that the percentile of the wealth distribution in which taxes paid by the household exactly compensate the lump-sum transfer received is the 99.03^{th} wealth percentile. On the right of that percentile it is possible to observe an increase in the share households invest in safe assets and a decrease in the shares invested in public and private equity. Instead, on the left of the 99.03^{th} percentile the behavior is the opposite. The cause of this portfolio composition effect is double. The main reason is the "scale dependence effect": when become poorer households pursue a more risk-averse behaviour. What happens in the model, indeed, is that when the disposable wealth of a household is reduced, its absolute (and relative) risk aversion increases, inducing households to prefer safe assets over risky ones. Furthermore, when poorer, households may face more troubles in accessing private equity investment opportunities, making them less convenient. In the the model this effect is captured by the intermediation fees to be paid by households which invest in private equity. In particular, when households become poorer investing in private equity becomes more costly, or in other terms the net return of this investment opportunity goes down. This is another reason behind the private equity investment drop observed in Figure 4.

The previously described changes get larger and larger as long as we move toward the very top of the wealth distribution for two reasons. The first reason is that the wealth tax introduced is progressive, hence the wealthier the household, the larger the taxes to

FIGURE 4. *The portfolio composition effect induced by a wealth tax imposed on the wealthiest 1% of households*
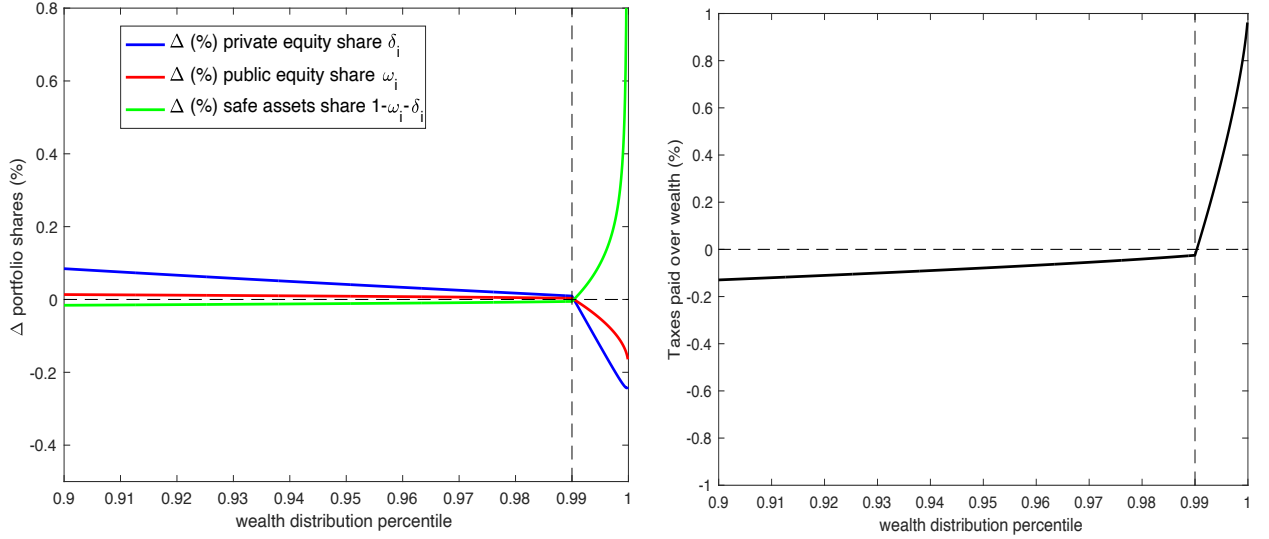
Notes: *The left panel of the Figure shows the percentage change in private equity, public equity and safe asset portfolio shares induced by the introduction of a wealth tax. The tax considered is a proportional wealth tax with tax rate $\tau = 1\%$, applied on the wealth in excess of the 99th percentile of the wealth distribution. All the tax revenues are uniformly redistributed through a lump-sum transfer. The right panel of the Figure, instead, shows the ratio between the amount of taxes paid by each household (according to the previous tax-transfer schedule) and the total amount of household's wealth.*

be paid relative to his wealth (this is shown in the right panel of Figure 4). However, even if the tax introduced was proportional, the left panel of Figure 4 would have been qualitatively (but not quantitatively) similar. This is due to the fact that in the model the larger the *absolute* change in disposable wealth, the larger the change in relative (and absolute) risk aversion that households experience. Remind that the magnitude of this effect is captured by the parameter γ [Note for the reader: here I should introduce an appropriate experiment which allows me to show the role of γ]. What happens is that those households who lose more wealth due to the introduction of the wealth tax are those who recalibrate more their portfolio choices in order to limit their risk exposure. Indeed, having fewer resources they prefer to be more cautious in their investment behaviour: this is to avoid the risk of consuming too little if their investment results are extremely bad.

4.2 Wealth taxation effects on aggregate capital supply

The aggregate effect of wealth taxation on the different household investment opportunities is obtained by combining the "quantity effect" together with the "portfolio compo-

sition effect". The aggregate amount of households investments in private equity, public equity and safe assets can be computed as[7]:

$$K_v(\tau) = \int \delta_i(a_i, \tau) d(a_i, \tau) dG(a_i) \qquad K_r(\tau) = \int \omega_i(a_i, \tau) d(a_i, \tau) dG(a_i)$$

$$K_s(\tau) = \int (1 - \omega_i(a_i, \tau) - \delta_i(a_i, \tau)) d(a_i, \tau) dG(a_i)$$

where $K_v(\tau)$, $K_r(\tau)$ and $K_s(\tau)$ denote respectively the aggregate investment in private equity, public equity and safe assets and $G(\cdot)$ denotes the distribution function of the random variable (Pareto distributed) from which wealth realizations a_i are drawn. Figure 5 presents the aggregate effect of wealth taxation on capital supply, for different tax rates $\tau > 0$. While the solid lines represent the total effects, the dotted lines represent the "quantity effects". Hence the difference between each solid and dotted line represents the "portfolio composition effect" of wealth taxation. In particular, the introduction of the wealth tax on the wealthiest 1% of households described in Section 4.1, with $\tau = 1\%$ induces:

- A reduction of 0.92% of private equity capital supply. The 14% of this change is due to the "portfolio composition effect", the remaining to the "quantity effect".

- A reduction of 0.35% of public equity capital supply. The 19% of this change is due to the "portfolio composition effect", the remaining to the "quantity effect".

- An increase of 0.55% of investment in safe assets. The 19% of this change is due to the "portfolio composition effect", the remaining to the "quantity effect".

5   The wealth taxation effect on GDP and growth

The previous Sections have shown that the introduction of wealth taxation affects household capital allocation among private equity, public equity and safe assets. The focus of this Section will be on how this effect does have an impact on GDP and growth.

Private equity, public equity, and safe assets investments not only differ for their level of riskiness and returns, but also they are used for financing very different kind of

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[7]To highlight the dependence of the portfolio choices of household i, δ_i and ω_i, on the wealth endowment a_i and the tax rate τ I have chosen the notation $\delta_i(a_i, \tau)$ and $\omega_i(a_i, \tau)$.

FIGURE 5. *The aggregate effects of wealth taxation on capital supply: decomposition*
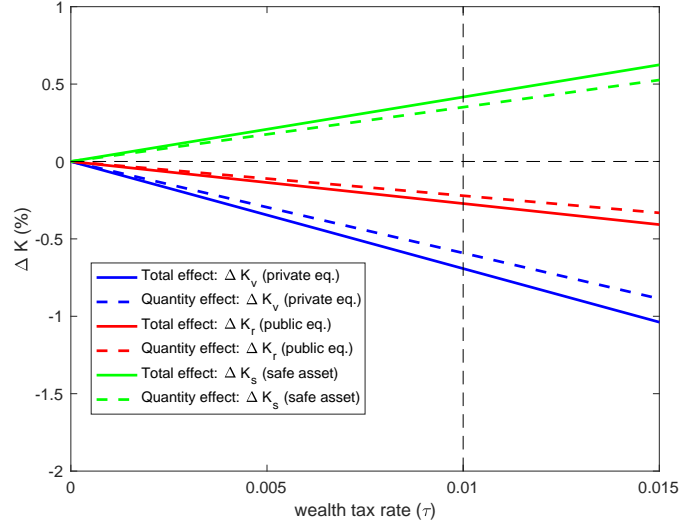
Notes: *The Figure shows the percentage change in private equity, public equity and safe asset capital supply induced by the introduction of a wealth tax. The tax considered is a proportional wealth tax with tax rate $\tau > 0$, applied on the wealth in excess of the 99th percentile of the wealth distribution. All the tax revenues are uniformly redistributed through a lump-sum transfers. The solid lines represent the aggregate effects of wealth taxation, while the dotted lines represent the "quantity effects". The difference between the two lines is the "portfolio composition effect". The detailed explanation of the "quantity effect" and "portfolio composition effect" is provided in Section 4 of this work.*

TABLE 4. *Empirical US capital allocation across industries*

	Private equity (ψ_v^j)	Public equity (ψ_r^j)	Safe assets (ψ_s^j)
Software	30%	10%	2.6%
Pharma and biotechnology	15%	9%	1.1%
Media and communication	3%	8%	3.9%
Utilities	2%	9%	2.3%
IT hardware	4%	11%	0.3%
Healthcare	6%	4%	1.6%
Commercial prod. and services	18%	12%	1.3%
Consumer prod. and services	12%	18%	7.1%
Finance and insurance	5%	14%	7%
Housing real estate	0%	2%	64.3%
Other (includes government)	5%	1%	8.5%
Total	100 %	100%	100%

Notes: *Column 1 of the Table reports private equity capital allocation across sectors, data are taken from PitchBook-NVCA Venture Monitor 2022. Column 2 of the Table reports public equity allocation across sectors. Data taken from Fidelity Investments Research. Column 3 reports safe assets capital allocation across sectors. Households holdings of safe assets have been disaggregated into housing, government bonds and corporate bonds using SCF data. The investment into corporate bonds has been split among different sectors by using S&P Global data on the US corporate debt market in 2019.*

projects and firms. This is shown in Table 4 which represents how the capital collected through each asset class is allocated across different sectors.

Empirical US capital allocation: in order to obtain the figures of Table 4 the entire US economy has been disaggregated into 12 sectors: software, pharma and biotechnologies, media and communication, utilities, IT hardware, healthcare, commercial products and services, consumer products and services, finance and insurance, housing and real estate and other (which includes government). The capital allocation across those sectors has been obtained by combining several data sources. First of all, private equity capital allocation across sectors has been obtained by using PitchBook data (2022 PitchBook-NVCA Venture Monitor[8]), which provide information on the value of private equity deals by sector in the past 15 years. To compute the allocation across sectors of private equity stock, the investment flows data (i.e. value of yearly private equity deals) have been aggregated assuming an yearly depreciation rate of 5%.

Second of all, to obtain the public equity allocation across sectors, I have retrieved data on US public equity market capitalization by industry. This is the sum of the market value of each listed US company, assigned to the applicable GICS (Global Industry Classification Standard) sector or industry.

Finally, using SCF 2019 data, households holdings of safe assets have been disaggregated into housing, government bonds and corporate bonds[9]. Furthermore, the investment into corporate bonds has been split among different sectors by using S&P Global data on the US corporate debt market in 2019[10].

Table 4 shows that the great majority of investment in private equity is directed toward the software sector (30%), followed by commercial products and services and pharma and biotechnology (15%). Public equity investments, instead, are mainly directed towards the production of consumer products and services (18%), financial and insurance services (14%), commercial products and services (12%) and IT hardware sector (11%). Finally, capital collected through safe assets investment is mainly allocated to the housing sector (64.3%), followed by the government sector (8.5%) and the sector devoted to the production of consumer products and services (7.1%).

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[8]https://pitchbook.com/news/reports/q1-2022-pitchbook-nvca-venture-monitor

[9]For the sake of completeness notice that liquidity has been added to the financial sector, durable goods to the housing sector.

[10]https://www.spglobal.com/en/research-insights/articles/u-s-corporate-debt-market-the-state-of-play-in-2019

Capital allocation, TFP, TFP growth: the portfolio choice model described in the previous Sections allows to endogenously derive the households choices in terms of private equity (K_v), public equity (K_r) and safe assets (K_s). I now assume that those endogenous quantities will be allocated to the different production sectors of the economy according to the proportions shown in Table 4. Formally, let's denote with ψ_v^j the fraction of investment in private equity that is supplied to sector $j \in J = \{$software, pharma, ... , housing, other$\}$ (first column of Table 4). Analogously let ψ_r^j and ψ_s^j denote, respectively, the fraction of investment in public equity and safe assets that is supplied to sector $j \in J$ (second and third column of Table 4). The total amount of private equity capital supplied to sector $j \in J$ (K_v^j) is computed as $K_v^j = \psi_v^j K_v$. Analogously are computed the total amount of public equity capital and safe assets capital supplied to sector $j \in J$: $K_r^j = \psi_r^j K_r$ and $K_s^j = \psi_s^j K_s$.

The capital allocation obtained in this way is represented in Figures 6-7 (for readability of the Figures the "housing and real estate sector" has not been reported. It accounts for 36% of the aggregate capital in the economy, its TFP is 98 while the TFP growth of the sector 0.5%). Figures 6-7 show that the sectors which receive the largest amounts of private equity capital are those characterized by the highest TFP (Figure 6) and highest TFP growth (Figure 7). The TFP and TFP growth measures reported in the two Figures have been computed by using the 2020 U.S. Bureau of Labor Statistics data, which provide official industry-level total factor productivity statistics. In particular, TFP growth has been computed as an average of the 2010-2020 yearly TFP growth rates.[11] In particular, notice that the three sectors to which the largest quantities of private equity capital is supplied (Software, Pharma-Bio., Commercial products and services) are among those those sectors characterized by the highest TFP and TFP growth rates.

5.1 Wealth taxation and capital allocation across industries

In Section 4.2 I have shown the effects of the introduction of a wealth tax on private equity, public equity and safe assets investment aggregates. Combining those results with the above described evidence on capital allocation across industries it is possible

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[11]The results would have been qualitatively similar by taking the average of the TFP growth rates from 2015 to 2020. The main difference would have been a significant reduction in the TFP growth rate in the "IT hardware" sector, from 0.94 to 0.5 and an increase of TFP growth in "utilities" sector from 0.56 to 0.9.

FIGURE 6. *Capital allocation across industries (excluding housing) ordered (decreasingly) by TFP*
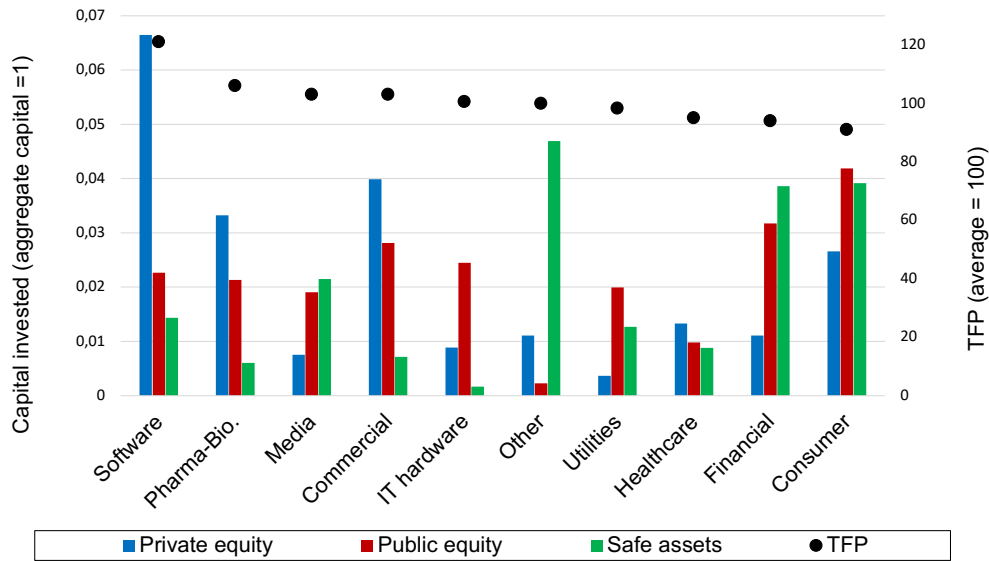
Notes: *The Figure shows sector by sector capital allocations (excluding housing, only for readibility reasons. Housing accounts for 36% of the aggregate capital in the economy, its TFP is 98). The capital allocation represented are determined by: 1- the endogenous household choices in terms of private equity, public equity and safe assets; 2- the exogenous capital allocation across sectors presented in Table 4. Sectors have been ordered (decreasingly) by TFP of the sector. Data on TFP have been taken from 2020 Industry level TFP statistics provided by US Bureau of Labor Statistics.*

FIGURE 7. *Capital allocation across industries (excluding housing) ordered (decreasingly) by TFP growth rate*
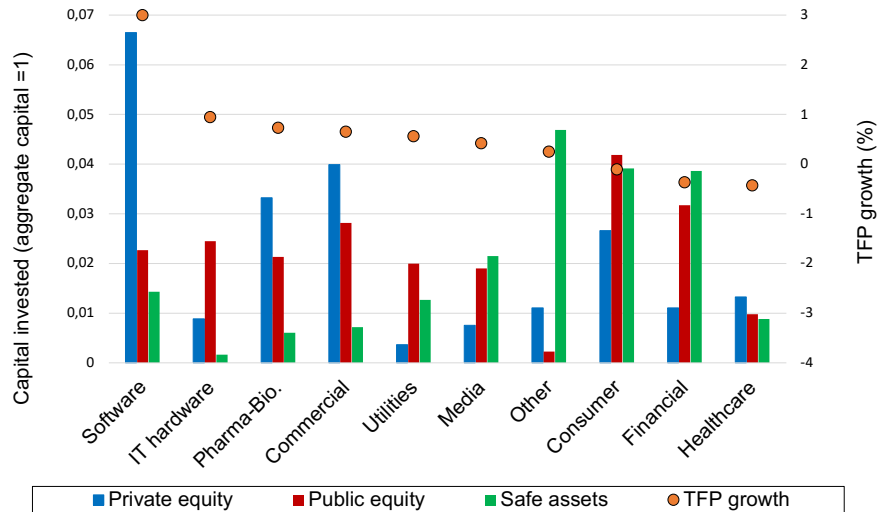
Notes: *The Figure shows sector by sector capital allocations (excluding housing, only for readibility reasons. Housing accounts for 36% of the aggregate capital in the economy, its TFP growth is 0.5%). The capital allocation represented is determined by: 1- the endogenous household choices in terms of private equity, public equity and safe assets; 2- the exogenous capital allocation across sectors presented in Table 4. Sectors have been ordered (decreasingly) by average TFP growth between 2010-2020 of the sector. Data on TFP growth have been taken from 2020 Industry level TFP statistics provided by US Bureau of Labor Statistics.*

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to obtain the effect of the introduction of a wealth tax on capital allocation across US industries.

Let's assume perfect substitutability between private equity, public equity and safe assets capital, the aggregate amount of capital supplied to sector $j \in J = \{$software, pharma, ..., housing, other$\}$ will be:

$$K^j = K^j_s + K^j_r + K^j_v$$

How does the introduction of the wealth tax affects this aggregate? Figure 8 shows the effect on K^j, $j \in J$ of the introduction of a proportional wealth tax (with proportional tax rate $\tau = 1\%$) on the wealth in excess of the 99th percentile of the wealth distribution. The results of Figure 8 have been obtained by assuming that all the revenues collected via the imposition of the tax are uniformly redistributed via a lump-sum transfer. The industries which experience the largest drops are those characterized by the largest amount of private equity investments, namely commercial products and services (-0.6%), pharma-biotechnology (-0.59%) and software (-0.58%). Also the IT hardware sector experiences a significant capital reduction (-0.49%). This is due to the fact that this sector is almost exclusively financed via public equity capital, whose supply, due to the introduction of the wealth tax is reduced (although much less than private equity, see Figure 5). The drops of capital in those sectors are partially compensated by the increase in capital supplied to the housing sector, and the "other" sector which comprises production activities mainly financed via households buying safe assets (e.g. government production of goods and services).

Effects on GDP and GDP growth: the introduction of the previously described wealth tax on the wealthiest 1% of households determines a shift of capital supply from sectors of higher productivity to those of lower productivity. In this way households at the top 1% of the wealth distribution obtain lower returns on their wealth investments, and as a consequence a reduction of their incomes.

In the economy analyzed in this paper the change of GDP can be simply computed as the change in aggregate households consumption (in the considered model there are no savings and no government expenditures). Hence, the GDP reduction due to the introduction of the wealth tax on the wealthiest 1% of households with $\tau = 1\%$ described above, amounts to -0.41%.

However, beside affecting GDP, the introduction of the wealth tax also affects GDP growth. This is caused by capital shifting away from sectors characterized by very high

FIGURE 8. *Capital reallocation across industries induced by the introduction of a wealth tax ($\tau = 1\%$) imposed on the wealthiest 1% of households*
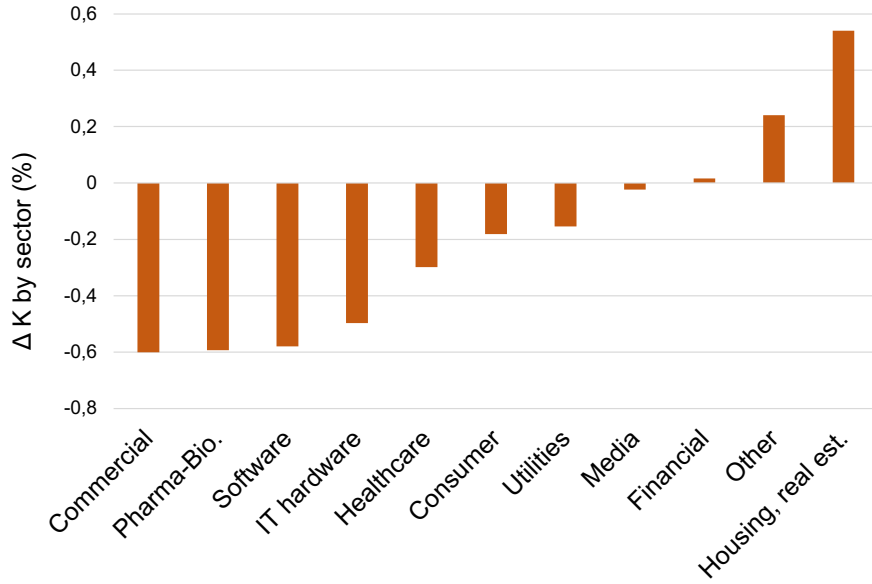
Notes: *The Figure shows the percentage change in capital supplied to each sector induced by the introduction of a wealth tax. The tax considered is a proportional wealth tax with tax rate $\tau = 1\%$, applied on the wealth in excess of the 99th percentile of the wealth distribution. All the tax revenues are uniformly redistributed through lump-sum transfers.*

growth (especially software), to sectors of lower growth (e.g. housing and real estate sector). Remember that in the household portfolio choice model introduced in the previous Sections households do not make savings decisions. Hence, the introduction of a wealth tax, even without inducing a distortionary effect on capital accumulation, is able to generate a GDP growth reduction. The mechanism driving this effect is only the capital misallocation from higher growth to lower growth sectors.

Further research, introducing employment: the next step to be accomplished in this research project will be that of introducing employment into the framework. While the relationship between changes in income inequality (e.g. induced by income taxes) and employment, has been investigated by Lee et al. (2022), analyzing the relationship between wealth taxation and employment would be a new contribution to the literature[12]. Suppose that in each sector a representative firm whose technology is described by a

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[12]Bjorneby et al. (2020) show a positive relationship between household's wealth tax and employment growth in the firms controlled by them. The reason is that higher wealth taxes induce households to invest in non-traded firms (hence hard-do-evaluate) to leave scope for tax elusion behaviours

Cobb-Douglas function is responsible for production. Furthermore, also assume that the labor force composition in terms of skilled and unskilled workers employed is different across sectors: then the capital allocation effect induced by the wealth tax will affect labor demand of skilled and unskilled workers. Consider, for example, the big drop in capital supply in the software sector: the representative firm of this industry will reduce employment in the same proportion as capital. However, if software sector's production mainly requires very skilled workers, in this sector there will be a big drop of skilled workers demand and only a very little reduction of that of unskilled workers. Notice that since in the model considered throughout the paper the aggregate amount of capital does not change after the introduction of the wealth tax (but it's only reallocated), neither employment does. What will happen on the employment side, instead, is going to be a change in the demand of skilled and unskilled workers. If sectors with the largest private equity investments are also those characterized by the largest shares of skilled workers employed, the aggregate labor demand for skilled workers will decrease and that of unskilled workers will increase.

6   Conclusion

In this work I have developed an analytically tractable portfolio choice model which has allowed me to investigate and quantify the effects of a wealth tax, imposed at the top of the wealth distribution, on households' portfolio choices. Differently from the existing literature which aims at capturing the household investment behaviour across the wealth distribution, I introduce in my portfolio choice problem the possibility for households of investing in private equity (besides in public equity and safe assets). This choice is crucial for capturing the behaviour of the individuals at the very top of the wealth distribution (which will be taxed), who invest large fractions of their wealth in private equity.

Handled with this tool I have derived the effects of wealth taxation on aggregate capital allocation across different sectors of the economy.

First of all, the introduction of a wealth tax on the wealthiest households induces the taxed agents to significantly reduce the share of wealth they invest in private equity. Furthermore, they also reduce (in a smaller extent) their portfolio share invested in public equity, while they increase their investment in safe assets. Given the concentration of wealth at the top of the wealth distribution this determines a reduction in aggregate investment in private equity and (in a smaller extent) in public equity, together with an increase in the investment in safe kind of assets. Since private equity investments

are directed towards more productive sectors, this capital misallocation effect induces a reduction of GDP which is quantified in -0.41%. Not only this, but it is shown that the sectors to which private equity investments are directed to (especially the software sector) are characterized by the highest level of TFP growth. Hence, the capital allocation effect induced by the wealth tax results in being detrimental to the economic growth.

The next step of this project will be that of introducing employment into the framework, in order to explore the implications of the wealth tax imposition on labor demand across sectors.

Appendix

A - Proof of Proposition 1

The problem to be solved is:

$$\max_{\omega_i,\delta_i} \quad \mathbb{E}\left[-\frac{1}{\alpha(a_i)}e^{-c_i\alpha(a_i)}\right]$$

$$\text{s.t.} \quad c_i = (R_s(1-\omega_i-\delta_i) + R_r\omega_i + R_v\delta_i)d(a_i) - \lambda_1\delta_i d(a_i)^{1-\lambda_2} \tag{P}$$

$$\omega_i \geq 0, \quad \delta_i \geq 0, \quad 1-\omega_i-\delta_i \geq 0$$

$$R_v \sim N(\phi_v,\sigma_v^2), \quad R_r \sim N(\phi_r,\sigma_r^2), \quad R_s \in \mathbb{R}_+ \text{ given}, \quad Cov(R_v,R_r) = \theta$$

The normality assumption of returns guarantees that consumption is a normally distributed random variable, with expectation and variance respectively:

$$\mathbb{E}(c_i) = (R_s(1-\omega_i-\delta_i) + \phi_r\omega_i + \phi_v\delta_i)d(a_i) - \lambda_1\delta_i d(a_i)^{1-\lambda_2} \tag{6}$$

$$Var(c_i) = d(a_i)^2\sigma_r^2\omega_i^2 + d(a_i)^2\sigma_v^2\delta_i^2 + 2\theta\omega_i\delta_i d(a_i)^2 \tag{7}$$

Since consumption c_i is normally distributed then the random variable $e^{-c_i\alpha(a_i)}$ is log-normally distributed. This observation allows to compute[13]:

$$\mathbb{E}\left[-\frac{1}{\alpha(a_i)}e^{-c_i\alpha(a_i)}\right] = -\frac{1}{\alpha(a_i)}e^{-\alpha(a_i)\mathbb{E}(c_i)+\frac{1}{2}\alpha(a_i)^2 Var(c_i)} \tag{8}$$

At this point it is possible to combine equations (6)-(7)-(8) and compute the derivates of the obtained expression with respect to δ_i and ω_i. Those will be the first order conditions of problem (P):

$$\delta_i: \quad -\alpha(a_i)\left[(\phi_v - R_s)d(a_i) - \lambda_1 d(a_i)^{1-\lambda_2}\right] + \frac{1}{2}\alpha(a_i)^2(2\delta_i\sigma_v^2 d(a_i)^2 + 2\theta\omega_i d(a_i)^2 = 0$$

$$\omega_i: \quad -\alpha(a_i)(\phi_r - R_s)d(a_i) + \frac{1}{2}\alpha(a_i)^2(2\omega_i\sigma_r^2 d(a_i)^2 + 2\theta\delta_i d(a_i)^2) = 0$$

Since those expressions are two equations in two unknowns (ω_i and δ_i), it is possible to combine them and solve in order to obtain the closed-form expressions for δ_i and ω_i reported in Proposition 1. $\square$

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[13]If a random variable $X \sim N(\mu,\sigma^2)$, then the random variable $Y = e^X$ has expectation $\mathbb{E}(Y) = e^{\mu+\sigma^2/2}$

B - Model calibration

In this Section I will go through the details of the procedures used to calibrate the port-folio choice model described in Section 3. The chosen values for all the parameters are reported in Section 3.3.

Shape parameter of wealth distribution η: When wealth is Pareto distributed with shape parameter η, then the share of wealth accruing to the top $q\%$ is:

$$\text{wealth share top q\%} = s_{q\%} = \left(\frac{q}{100}\right)^{\frac{\eta-1}{\eta}} \quad \Rightarrow \quad \eta = \frac{\ln\left(q/100\right)}{\ln\left(q/(100 s_{q\%})\right)} \tag{9}$$

The theoretical wealth shares that go to the top $q\%$, where $q \in \{1, 0.9, 0.8, ..., 0.1\}$ are computed for different η using equation (9). $\eta = 1.35$ is chosen to minimize the difference between the theoretical wealth shares that go to the top $q\%$ ($q \in \{1, 0.9, 0.8, ..., 0.1\}$) and the empirical ones, computed using the 2019 SCF data.

Assets returns and variances: Gaillard and Wangner (2021) use 1998-2018 PSID data to compute returns and variances of assets categorized as private equity, public equity and safe assets. Those asset classes are defined in the same way as I do in Section 2. In particular, they compute the return for household i, of asset $j \in \{$private equity, public equity, safe asset$\}$, at time t, in the following way:

$$r_{i,j,t} = \frac{R^K_{i,j,t} + R^I_{i,j,t} - R^D_{i,j,t}}{a_{i,j,t-1} + I_{i,j,t}/2}$$

where where $a_{i,j,t-1}$ is the beginning-of-period amount of asset of class j held by household i, and $I_{i,j,t}$ is net investment in the asset. $R^K_{i,j,t}$, $R^I_{i,j,t}$, $R^D_{i,j,t}$ indicate respectively capital gains, income derived from the asset and the cost of debt used to obtain the asset. The mean of $r_{i,j,t}$ across time and households is the expected return of asset class j. Analogously, the variance of $r_{i,j,t}$ across time and households is the variance of asset class j.

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