Monte Carlo Simulation of 2D Ising Model

André Melo 4519302
Matteo Domenighini 4512154

Delft University of Technology

Abstract

The aim of this report is to present the results obtained using Monte Carlo simulation for the Ising model. Two different algorithms have been implemented to simulate the behaviour of the system: the Metropolis Monte Carlo algorithm and the Hoshen-Kopelman cluster finding algorithm. Both algorithms have been used to extrapolate relevant physical quantities, such as the magnetization, the magnetic susceptibility and the specific heat. Finite-size scaling has also been used in order to calculate the critical exponents. For the Metropolis algorithm, we tried to find a proof of the critical exponent universality by considering second neighbour interaction. The algorithms have been compared: the Hoshen Kopelman revealed itself to be better suited for (...)

I. Introduction

The Monte Carlo algorithms are a type of algorithms in which "random" numbers play an essential role [1]. This method has been widely and successfully implemented in the past decades to simulate the behaviour of physical systems in order to extrapolate information regarding their static properties.
The Hoshen Kopelman algorithm is a cluster finding algorithm that has been implemented as a non-recursive alternative to the Swendsen-Wang algorithm. It is based on the more famous Union-Find algorithm.

In section II the interaction model and the working principles of the two algorithms are outlined. In section III the numerical results obtained for specific heat, magnetization, magnetic susceptibility, Binder cumulant and critical exponent are reported. In section IV, the accuracy and speed of the two algorithms are compared.

II. Methods

While implementing the models and algorithm outlined in this section, we have considered $k_B = 1$, $J = 1$ and a unitary distance between the lattice sites.

I. Ising Model

The Ising Model consister in a lattice of size $L \times L$ in which every lattice site is associated with a spin. The Hamiltonian of the system in absence of an external magnetic field is described by

$$H = -J \sum_{\langle ij \rangle} \mathbf{s}_i \mathbf{s}_j \tag{1}$$

where $\mathbf{s}_i$ represents the spin associated with the ith site. We consider $J > 0$, which means that the system favours the parallel alignment of adjacent spins. The partition function for the system is given by

$$\mathcal{Z} = \sum_{\mathbf{s}_i} e^{-\beta H(\mathbf{s}_i)} \tag{2}$$

which is dependent on the temperature of the system, contained in β.

II. Metropolis Monte Carlo Algorithm

The Metropolis Monte Carlo algorithm is an algorithm in which the information regarding different distributions of a system is stored in a Markov chain.

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[1] J.M.Thijssen, *Computational Physics*, Cambridge University Press, 2nd Edition, 2007.