# Multilayer Network Analysis of Milan Public Transport Network

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## 1 Introduction

In recent decades, analyzing the robustness of Public Transport Networks (PTNs) has become increasingly critical for ensuring their reliability and efficiency. As urban populations grow and transportation systems become more interconnected, understanding the resilience of PTNs to disruptions is essential for effective planning, optimization, and sustainable development. Complex network theory provides a powerful framework for investigating the structural and functional properties of spatial networks like PTNs, offering insights into their vulnerability and capacity to withstand failures.
Our analysis examines the multilayer PTN of Milan, Italy, focusing on three key aspects:

i) The spatial distribution of centrality measures, specifically betweenness centrality (BC) and closeness centrality (CC).

ii) The small-worldness properties of the network.

iii) Its robustness through what-if scenarios simulating node removal. These features help identify critical nodes that underpin the network's resilience.

The network comprises three interconnected layers: metro, tram, and bus systems. Each layer is represented as a graph in *L-space* topology [1], where stops and stations are nodes, and their connections are edges (e.g., a bus traveling between two stops). Data for this study were sourced from Open Street Map, and the full analysis, including a Python notebook, is publicly available at GitHub.

## 2 Problem and Motivation

We want to address the robustness of the network through a what-if analysis by simulating the removal of nodes in our network with both random failures as well as targeted attacks. Moreover, we also want to analyze which are the most important nodes (stops) in the network and their spatial position inside the area of Milan. These problems are important since urban transportation networks have significant implications for urban policies and urban planning chain.

In fact, intermodal transportation efficiency and simulations have been extensively studied in the transportation engineering literature [2].

One goal in this project is to shift the focus onto this topological coupling aspect of transportation network design: we show this to be extremely relevant and suggest that the multilayer network view of these systems should be integrated into elaborated models of urban planning. The multilayer can capture a broad perspective on the entire system rather than focusing on each transport mean in isolation.

Sienkiewicz and Holyst [3] collected and analyzed PTN data from 22 cities in Poland and found that degree distributions in L-space follow a power law. In addition, small-world behavior was observed in this topology.

Shanmukhappa, Wu and Dong [4] published an article which describes how they modeled the PTN structure of the London city, using the "supernode" (set of geographically closely associated nodes) graph structure representation. The bus transport and the metro transport network structures are analyzed by treating them as independent mono-layer or multi-layer network structures, using a method of spatial amalgamation to integrate the two layers.

# 3 Multilayer Networks

An undirected graph $\mathbf{G}$ is defined as a set of nodes $V$ and edges $E$, such that:

$$\mathbf{G} = (V, E), \tag{1}$$

where $E \subseteq \{\{n_i, n_j\} \mid n_i, n_j \in V,\ i \neq j\}$.
Our work extends this representation to incorporate spatial coordinates, defining the graph as:

$$\mathbf{G} = (V(x,y), E), \tag{2}$$

where:

$$V = \{n_i(x_i, y_i) \mid x_i = \text{latitude},\ y_i = \text{longitude}\}, \tag{3}$$

and edges are represented as:

$$E = \{e_{ij} \mid e_{ij} \to \{n_i(x_i, y_i), n_j(x_j, y_j)\}\}. \tag{4}$$

Each node $n_i(x_i, y_i)$ is uniquely identified by its latitude and longitude coordinates, and it is further weighted by the length of the line segment. This spatial representation is crucial in multilayer transport networks, where nodes belonging to different layers (e.g., metro, tram, or bus) may exist within close proximity near strategic urban locations. To address this, we introduce a threshold distance $d_{th}$ below which nodes from different layers are considered overlapped, based on the WGS84 - World Geodetic System (EPSG:4326). If the distance $d_{ij}$ between two nodes $n_i$ and $n_j$ satisfies:

$$d_{ij} \leq d_{th}, \tag{5}$$

then $n_i$ and $n_j$ are connected by an inter-layer edge. Consequently, the graph structure is updated to:

$$\mathbf{G} = (V(x,y), E, E_L), \tag{6}$$

where $E_L$ represents the set of inter-layer edges.

The distance $d_{ij}$ is computed using the Great-circle distance [5], and the threshold $d_{th}$ is set to 100 meters, assuming that is a walkable distance between two nodes. This assumption is consistent with the Moovit Global Public Transport Report 2022 [6], which reports that the average walking distance during a commute in Milan is 472 meters.
In this multilayer network, each node can exist in at most one layer (e.g., a stop cannot belong to both the metro and tram layers). This property ensures the network is layer-disjoint, defined as:

$$(n_i)_\alpha, (n_i)_\beta \in V_M \implies \alpha = \beta, \tag{7}$$

where a node $n_i$ belongs to either layer $\alpha$ or layer $\beta$, but not both. Layer-disjointness implies that there are no direct edges connecting the same node across layers; instead, layers are connected through spatial proximity using inter-layer edges weighted by the length of the line segment.
Considering the above structure, the multi-layer network is represented as:

$$\mathbf{M} = (V_M, E_M, L), \tag{8}$$

where:

$$\mathbf{L} = \{L_\alpha\}_{\alpha=1}^{d}, \tag{9}$$

and $\mathbf{L}$ is the set of $d$ layers. For $d = 1$, the network is a mono-layer network. In this study, we analyze a multi-layer network with $d = 3$ (metro, tram, and bus).

# 4 Datasets

Using data from OpenStreetMap (OSM), we construct the metro, tram and bus networks for Milan (Italy). We downloaded data in geo-referenced vectorial format from Open Street Map under the Open Database License (ODbL) v1.0. Specifically, the datasets were extracted using the Overpass Turbo tool by invoking its API. All data used in this study is publicly available, ensuring transparency and accessibility.
For storage and manipulation, the data was structured as a graph using the NetworkX library, a Python package designed for creating, analyzing, and visualizing complex networks. NetworkX offers a rich suite of tools for computing various graph measures and performing network analysis. Additionally, other Python libraries, such as OSMnx and GeoPandas, were employed to streamline the processing pipeline and enhance code readability while maintaining high development efficiency. Moreover, a series of automatic and manual topological cleaning operations were needed in order to extract consistent and usable graphs.
Three distinct datasets were collected for this study, representing different modes of public transport in Milan: metro, tram, and bus (including both stops and routes). The bus dataset is the most complex due to the extensive network. For consistency and relevance, only bus stops operated by Azienda Trasporti Milanesi S.p.A (ATM) were included. The datasets were further categorized as follows:

Table 1: Layers with their properties

| Layer | N | M | Lines |
|---|---|---|---|
| Metro | 125 | 258 | 5 |
| Tram | 308 | 680 | 17 |
| Bus | 1370 | 3788 | 113 |

The public transport system in Milan, as analyzed in the Moovit Global Public Transport Report 2022 [6], highlights the following key characteristics:

- The mean duration of a weekday commute on public transit is 44 minutes with approximately 1.97% of commuters spend more than 2 hours traveling daily.
- Passengers typically wait 10 minutes at stops or stations, with 7.65% experiencing waiting times exceeding 20 minutes.
- The average distance traveled per trip is 8.57 km, and 17.06% of riders journey more than 12 km in a single direction.

# 5 Validity and Reliability

## Validity

The validity of the datasets is robust, as they represent real-world transport stations and stops mapped by contributors to OpenStreetMap. This ensures that the data closely aligns with the physical infrastructure of Milan's public transport system. However, the accuracy of the data depends upon the timeliness of the OSM contributions, meaning that while the dataset reflects reality with high fidelity, there is potential for certain elements to be outdated due to delayed updates.

## Reliability

The preprocessing of the dataset involved converting raw OSM data into a graph format compatible with the NetworkX library. The resulting graph was further simplified to create a compact representation of each mode of transport. This simplification included merging bidirectional routes (e.g., A $\rightarrow$ B and B $\rightarrow$ A) into a single undirected edge. This approach prioritizes the topological structure of the network over detailed route directionality, allowing the analysis to focus on network connectivity and properties.

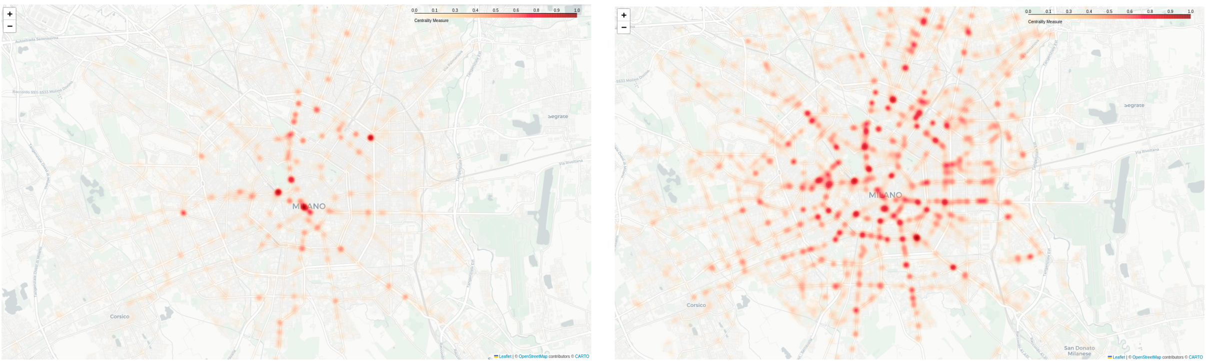
# 6 Measures and Results

In this section we will discuss the characteristics of the analyzed network. For each measurements we will briefly explain our hypothesis, the measures applied and the results obtained.

## Eigenvector centrality

Instead of analysing the Degree centrality which considers only the quantity of connections, we use eigenvector centrality which includes the quality of connections. A station may not be directly connected to many stations, but it could be connected to highly influential ones. To address this issue, we use Eigenvector centrality.
Eigenvector centrality is computed by assigning relative scores to all nodes in the network, where a node's score is determined by the scores of its connected nodes. These values are computed iteratively, with each node's score being proportional to the sum of the scores of its neighboring nodes, emphasizing connections to highly scored nodes. However, results show that the top nodes for this metric are Sesto San Giovanni FS, Via Zucchi - Via Italia (Cusano Milanino), Via Monfalcone - Via Manzoni (Cinisello Balsamo).

## Betweenness centrality



(a) Betweenness centrality spatial distribution in Milan

(b) Closeness centrality spatial distribution in Milan

Figure 1: Spatial distribution of the betweenness centrality (a) and closeness centrality (b) in the multilayer network

By applying the betweenness centrality (BC) we can verify which nodes of our network represents junctions, since betweenness centrality measures the extent to which a node lies on paths between other nodes. Our expectation is that these nodes will be the ones inside the city where there is the possibility to switch line. Results show that top nodes for this metric are Duomo M1 M3, Cadorna FN (M2) and Orefici/Cordusio (16), which are junctions between different transport means. The map of the spatial distribution of the metric is in figure 1a.

## Closeness centrality

We aim to identify the stations that are the most reachable from all possible starting points within the network. Identifying these stations provides insight into the most accessible locations, helping to determine the optimal stations for new services or facilities, such as shops. Additionally, locations near these identified stations are ideal for accommodations, as they allow fast and easy access to all possible destinations in the city.

Closeness centrality is the appropriate measurement for identifying these stations because it quantifies how quickly a node can be reached from all other nodes in the network. This metric is computed by taking the reciprocal of the sum of the shortest path distances from a given node to all other nodes.

We expect high values of this measure primarily at city center stations, as their location places them on many shortest paths between nodes. Results show that the top four nodes for this metric represent the same area (even if the nodes are different but still very close to each other) which is Cordusio, the fifth node for closeness centrality is Duomo M1 M3. The map of the spatial distribution of the metric is in figure 1b

## Average Shortest Path Length, Diameter, and Density

To assess the overall reachability of the network, we analyze three complementary metrics: average shortest path length (APL), network diameter, and density. These metrics collectively provide insight into the network's typical connectivity, its extreme distances, and the distribution of its stations and connections.

The APL measures the mean shortest distance (in meters) that passengers must travel between all pairs of stations, reflecting the average ease of movement. The diameter represents the longest shortest path in the network, indicating the maximum distance between two stations along the shortest route. Finally, density quantifies the ratio of actual connections (edges) to the total possible connections between stations (nodes). Higher density suggests a more interconnected system, while lower density often reflects sparse connections characteristic of geographically extensive or dispersed networks.
These metrics, taken together, reveal the network's structural efficiency under both typical and extreme conditions, as well as its spatial organization. For instance, a network with a high APL or diameter and low density is expected in systems that span vast geographic areas, where stations are more widely distributed. In contrast, dense networks generally have lower APL and diameter values, reflecting their compact and highly connected nature.
For this analysis, we anticipate the metro system to exhibit broad coverage while maintaining a low APL, moderate diameter, and high density. Similarly, the tram network is expected to behave like the metro but with less breadth due to its integration with street networks. In contrast, the bus network, covering extensive and dispersed urban and suburban areas, is likely to have a higher APL and diameter, stemming from its lower density and dependence on road infrastructure. Moreover, the multilayer network is expected to align with the bus layer in terms of diameter, reflecting the inclusion of extensive and dispersed connections. However, it is anticipated to have a lower APL due to the combined effect of overlapping metro, tram, and bus networks, which provide multiple redundant and shorter paths between stations. This integration increases connectivity and reduces the average travel distance across the network.
A summary of the results for these metrics is presented in Table 2.

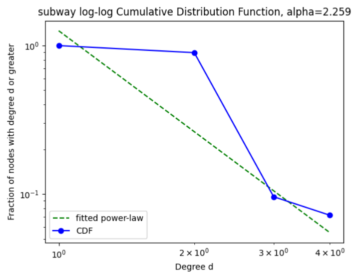
Table 2: APL, diameter, and density

| Layer | APL (mt) | Diameter (mt) | Density |
|---|---|---|---|
| Metro | 8906 | 33061 | 0.0166 |
| Tram | 6214 | 23106 | 0.0064 |
| Bus | 10958 | 39584 | 0.0011 |
| Multilayer | 9463 | 37995 | 0.0010 |

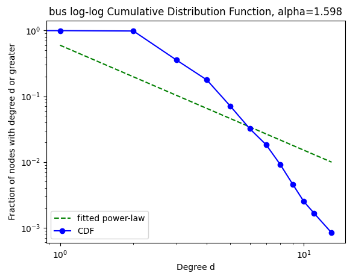
## Small-worldness

A network with a small world effect is a network where the distance $L$ between two random nodes grows proportionally to the logarithm of the number of nodes $N$ in the network.
As distance $L$ we consider the number of steps required to connect the two random nodes. The small world effect measures how easily two nodes are connected with a small number of steps. Sometimes two nodes are not directly connected, but for example they could both be connected to a third node, that allows a route between the previous two. This measure is related with the APL and the Clustering coefficient.
The latter highlights that small world networks usually contains cliques, i.e. highly connected subnetworks, whereas the APL points out that two random nodes are connected by at least one shortest path.
Networks with small-worldness effect are efficient in terms of reaching different destinations "quickly". They are also resilient against random failure of single nodes. The small-worldness coefficients are $\omega_m \approx 0.18$ for the metro layer, $\omega_t \approx -8.68$ for the tram layer, $\omega_b \approx -45.73$ for the bus layer, and $\omega_{mult} \approx -270.30$ for the multilayer network.

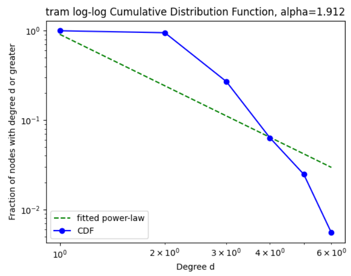
## Scale-freeness



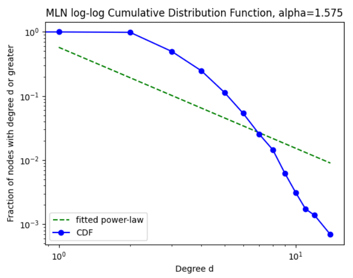

(a) CDF metro




(b) CDF bus




(c) CDF tram




(d) CDF multilayer

Figure 2: CDFs of different layers and multilayer network

A scale-free network is a network with power-law distribution of the degrees of nodes. Meaning that most of nodes have a small degree, while a small number of nodes (usually called hubs) have a disproportionately large number of connections. Scale-free networks are highly robust to node failures. In our case of public transportation networks this is particularly interesting, since failures of stations happen frequently and can greatly affect life of citizens if the whole network loses important properties such as connectedness.

To measure whether a network is scale-free, we use a statistical method to fit the degree distribution to a power-law model. An important parameter of the power law fit is $\alpha$, which denotes a scaling exponent of the distribution. Empirically, $2 < \alpha < 3$ means that the network is scale-free. $\alpha < 2$ means that network is dominated by a few highly connected nodes, while $\alpha > 3$ means that the tail of the distribution is lighter and the hubs are less dominant. The results indicate that the metro network has $\alpha = 2.25$, the tram network has $\alpha = 1.91$, and the bus network has $\alpha = 1.59$. The log-log plots of the Cumulative Distribution Functions (CDFs) and the fitted power-law distributions are presented in Figure 2.

## Connectedness / Fragmentation

We analyze the reliability of Milan's public transportation network using a "what if" analysis. We systematically remove nodes and assess the resulting changes in network connectedness. Specifically, we conduct three types of node removal: random node removal, removal of nodes with the highest degree, and removal of nodes with the highest betweenness. Random node removal simulates random network failures, while removing nodes based on degree and betweenness scores simulates targeted attacks. Our goal is to verify the resilience of the multilayer network, simulating different kinds of attacks. This analysis provides valuable insights into the network's robustness and potential vulnerabilities, contributing to the development of more resilient public transportation systems. We choose to analyze the network's connectedness, which measures its cohesion and the proportion of node pairs that can reach each other via any path. If two nodes are not in the same component, there is no path connecting them, meaning it is impossible to reach one station from another within different components.

$$\text{connectedness} = \frac{\sum_{i \neq j} r_{i,j}}{n(n-1)} \tag{10}$$

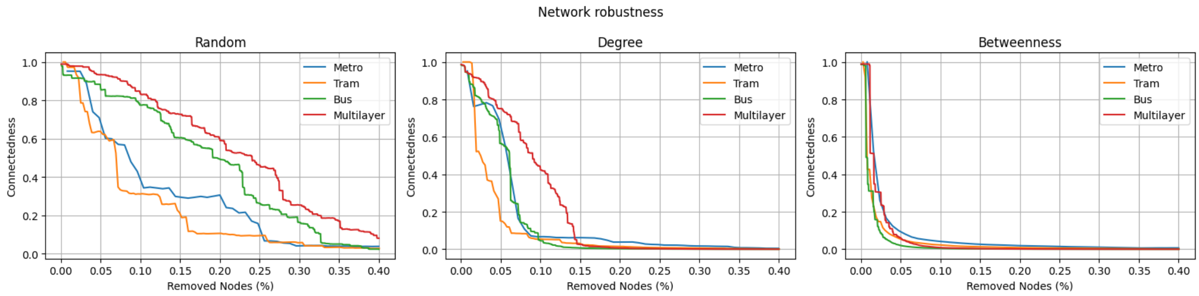



Figure 3: Network connectedness as a function of the percentage of nodes removed for random failure and targeted attacks

# 7 Conclusion

Given all the previous discussions about the computed metrics, we can interpret the findings from both qualitative and quantitative perspectives. Regarding the centrality measures, we observe that eigenvector centrality is not particularly informative for our analysis due to the influence of depots and parking areas of transport means. In contrast, betweenness and closeness centrality prove to be highly informative and offer valuable insights into identifying the most crucial nodes in our network. Furthermore, the overlap between the top nodes for closeness centrality and those for betweenness centrality suggests a strong relationship between these two measures, as evidenced by the map of the spatial distribution of betweenness and closeness centrality in Figure 1a. The limitations of degree and eigenvector centrality likely arise from the inter-layer edge creation process, where depots and parking areas, predominantly those associated with buses, significantly increase the degree and influence of nearby nodes, thereby biasing these metrics toward such locations.

For the average path length (APL), as expected, the multilayer network exhibits a lower APL compared to the bus layer but remains higher than those of the metro and tram layers. This indicates that the multilayer network strikes a balance between the localized efficiency of individual layers and the broader connectivity across the entire structure. Despite this, the multilayer network retains a broad structure in terms of diameter while having a lower overall density.

From a robustness perspective, the small-worldness coefficient and associated structural properties provide insights into the resilience of the metro layer compared to the tram, bus, and multilayer networks. The metro small-worldness coefficient suggests a structure that lies between a random graph and a small-world graph. This implies that the metro network benefits from the hallmark features of small-world graphs, such as short path lengths and high local clustering, which are conducive to robustness against random failures. In a small-world network, the redundancy of connections and the presence of alternative paths help maintain connectivity even if some nodes or edges fail randomly. However, the metro layer's scale-free nature ($\alpha = 2.25$) indicates that it relies heavily on a few highly connected hubs. While this property enhances its efficiency and connectivity, it makes the network more vulnerable to targeted attacks. The deliberate removal of these hubs could lead to a rapid fragmentation of the network, severely disrupting connectivity and efficiency. In contrast, the tram and bus layers, with small-worldness coefficients closer to lattice-like structures, have a more uniform distribution of connectivity. This means that they are less reliant on central hubs, making them more robust against targeted attacks. However, their lattice-like structure and longer path lengths make them less resilient to random failures, as the removal of even a few nodes can significantly impact overall connectivity. The multilayer network, exhibits an even more negative small-worldness coefficient, indicating a highly complex and interconnected structure. While this might offer robustness against random failures due to the redundancy provided by multiple layers, it also implies significant dependence on key inter-layer connections, which could be critical failure points in the event of targeted attacks. Regarding scale-free properties, the metro network's alpha value indicates that the metro network exhibits scale-free characteristics. On the other hand, the tram network and bus network do not exhibit scale-free properties, as their alpha values fall below the threshold typically associated with such behavior. For the multilayer network, the alpha value is even smaller, further reinforcing the observation that the addition of layers reduces the scale-free tendency. These coefficients suggest that the metro layer relies on a few highly connected hubs to maintain its connectivity, while the tram and bus networks, and by extension the multilayer network, follow a more uniform connectivity pattern with less reliance on hubs.

Regarding the connectedness of the multilayer network and its individual layers, it is evident that the three node removal strategies produce distinct degradation patterns, yet certain trends are consistent. Across all three graphs, the metro and tram networks fragment significantly faster than the bus network. This behavior is likely because the bus network has greater redundancy, while the removal of a single node from the tram or metro networks often results in an immediate split into at least two disconnected components. When considering random node removal, the degradation of network connectedness is notably slower compared to targeted strategies. Even after the removal of 10% of nodes, the public transportation multilayer network remains relatively robust, with only a minor loss in connectedness (approximately 0.2). This suggests that buses play a vital role in preserving the multilayer network's overall functionality under random failure, thanks to their widespread and redundant connections. In contrast, targeted attacks show a far more severe impact on the network. Removing nodes with the highest degree fragments the network significantly, resulting in a nearly 0.5 loss in connectedness with just 10% of nodes removed. Targeted removal based on betweenness centrality is even more destructive. Under this strategy, the removal of 10% of nodes renders the multilayer network highly fragmented and almost entirely non-functional. The heightened disruption caused by betweenness-based attacks stems from the fact that these nodes are often located in the city center or along major roads, serving as critical connectors between different parts of the network. By comparison, attacks targeting high-degree nodes are somewhat less effective in the multilayer context. This is because high-degree nodes, such as depots and parking areas around Milan, primarily represent localized hubs rather than crucial transit points. In contrast, nodes with high betweenness centrality lie on key shortest paths, making their removal much more impactful on overall connectivity. Thus, in the multilayer scenario, targeting high-betweenness nodes poses a significantly greater threat to network robustness than targeting high-degree nodes.

# 8 Critique

Our work provides valuable insights into the resilience of Milan's public transportation network but has several limitations.
First, we did not consider time and speed factors, assuming perfect synchronization across layers. In reality, different modes operate on distinct schedules, and transfers involve waiting times. Additionally, our shortest-path computations were based on distance rather than travel time, neglecting the varying speeds of buses, trams, and metros. Incorporating schedule and speed data in future studies would enable a more realistic assessment of network resilience.
Second, we ignored passenger volumes and capacity constraints. Overcrowding, especially during peak hours, can significantly impact network performance and amplify the effects of node failures. Future analyses should account for passenger flow and capacity to better model real-world conditions and understand how disruptions affect the system under varying demand.

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