NB.04.F4

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1 Remark

Example of five points for which the matrix of conditions has rank 8

[3]:
```
load("basic_functions.sage")
```

[4]:
```
P1 = vector(S, (1, 0, 0))
P2 = vector(S, (A2, B2, C2))
```

We want $\delta_1(P_1, P_2, P_4) = 0$. Hence $\langle P_4, s_{11}P_1 - s_{12}P_2\rangle = 0$, so:

[5]:
```
Ptmp4 = vector(S, (A4, B4, C4))
Q4 = scalar_product(P1,P1)*P2 - scalar_product(P1,P2)*P1
aa4, bb4, cc4 = (
    scalar_product(Ptmp4, Q4).coefficient(A4),
    scalar_product(Ptmp4, Q4).coefficient(B4),
    scalar_product(Ptmp4, Q4).coefficient(C4)
)
```

Two alternative definitions of P_4 (solving $\delta_1 = 0$ w.r.t. A_4 or C_4)

[6]:
```
P4 = vector(S, (cc4*A4, cc4*B4, -aa4*A4-bb4*B4))
P4 = vector(S, (bb4*A4, -aa4*A4-cc4*C4, bb4*C4))
```

[7]:
```
assert(delta1(P1, P2, P4) == 0)
```

[8]:
```
P3 = (
    (scalar_product(P1, P2)^2+scalar_product(P1, P1)*scalar_product(P2, P2))*P1
    - 2*scalar_product(P1, P1)*scalar_product(P1, P2)*P2
)
assert(delta1b(P1, P2, P3) == 0)
```

[9]:
```
P5 = (
    (scalar_product(P1, P4)^2+scalar_product(P1, P1)*scalar_product(P4, P4))*P1
    - 2*scalar_product(P1, P1)*scalar_product(P1, P4)*P4
)
assert(delta1b(P1, P4, P5) == 0)
```

[10]:
```
M = condition_matrix([P1, P2, P3, P4, P5], S, standard="all")
```

[12]:
```
M1 = M.matrix_from_rows([0, 1, 3, 4, 6, 7, 9, 11, 12, 13])
assert(M1.rank() == 8)
```

[13]:
```
m9 = M1.minors(9)
assert(Set(m9) == Set([0]))
```