NB.07.F2

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1 Configuration (C_4)

[1]:
```
load("basic_functions.sage")
```

We define 6 points in a (C_4) configuration. In the following block we study the ideal $\delta_1(P_1, P_2, P_4), \delta_1(P_2, P_1, P_4), \delta_1(P_4, P_1, P_2)$ and we see it is, after admissible saturations, the ideal generated by s_{12}, s_{14}, s_{24}

[ ]:
```
P1 = vector(S, (A1, B1, C1))
P2 = vector(S, (A2, B2, C2))
P3 = u1*P1+u2*P2
P4 = vector(S, (A4, B4, C4))
P5 = v1*P1+v2*P4
P6 = w1*P2+w2*P4
```

If we have configuration (4), then we must have:

$\delta_1(P_1, P_2, P_4) = 0$, $\delta_1(P_2, P_1, P_4) = 0$, $\delta_1(P_4, P_1, P_2) = 0$

[ ]:
```
J = S.ideal(delta1(P1, P2, P4), delta1(P2, P1, P4), delta1(P4, P1, P2))
```

We saturate J w.r.t. some ideals which say that the points P_1, P_2, P_4 do not have all the coordinates zero and P_1, P_2, P_4 are not aligned:

[ ]:
```
J = J.saturation(S.ideal(A1, B1, C1))[0]
J = J.saturation(S.ideal(A2, B2, C2))[0]
J = J.saturation(S.ideal(A4, B4, C4))[0]
J = J.saturation(S.ideal(matrix([P1, P2, P4]).det()))[0]
```

Hence we have that $J = (\langle P_1, P_2 \rangle, \langle P_1, P_4 \rangle, \langle P_2, P_4 \rangle)$

[ ]:
```
assert(
    J ==
    S.ideal(
        scalar_product(P1, P2),
        scalar_product(P1, P4),
        scalar_product(P2, P4)
    )
)
```

The result is that the conditions defining J imply that all the possible δ_2 are zero, so the points are in a (C8) configuration.

[3]:
```
assert(J.reduce(delta2(P1, P2, P3, P4, P5))==S.ideal(S(0)))
assert(J.reduce(delta2(P2, P1, P3, P4, P6))==S.ideal(S(0)))
assert(J.reduce(delta2(P4, P1, P5, P2, P6))==S.ideal(S(0)))
```

CONCLUSION:

Configuration (C_4) is not possible: it gives configuration (C_8).