Minimum Weight Vertex Cover Problem

Artificial Intelligence Project

a.y. 2024/25 - made by Matteo Galletta - assigned by professor M. Pavone

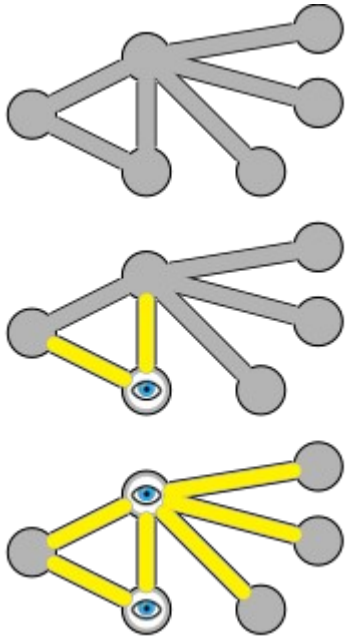
The problem

Given a problem instance (G, ω), where G is a undirected graph $G(V, E)$ and $\omega : V \to \mathbb{R}^+$ a function that associates a positive weight value $\omega(v)$ to each vertex $v \in V$, the Minimum Weight Vertex Cover can formally be defined as follows:

$$\textbf{minimize} \quad \omega(S) = \sum_{v \in S} \omega(v), \quad S \in V$$

such that $\forall (v_i, v_j) \in E, v_i \in S \vee v_j \in S$.

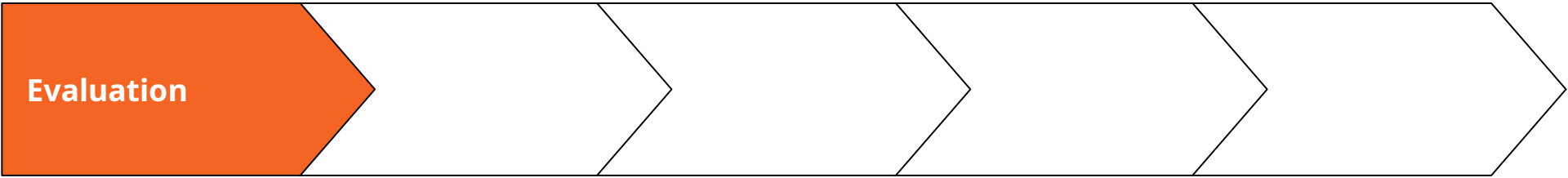
Note that the MWVC is a NP-complete problem.



The solution

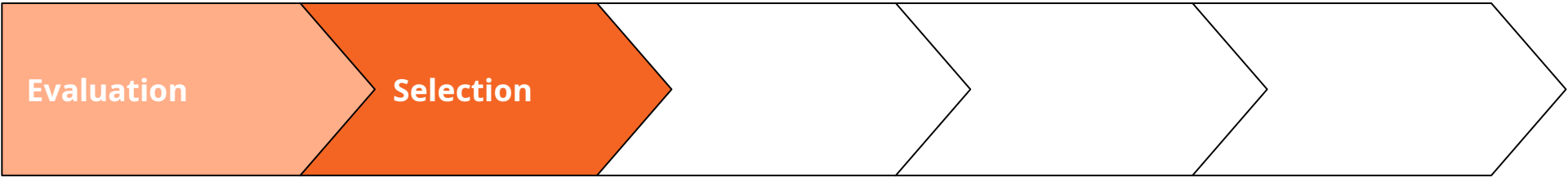
1. Tabu Search (TS);
2. Branch-and-Bound;
3. Genetic Algorithm (GA) with uniform crossover and roulette-wheel selection;
4. **Genetic Algorithm (GA) with one-point crossover and k-tournament selection;**

Genetic Algorithm



Each individual in the population is evaluated based on a predefined fitness function

Genetic Algorithm



A subset of individuals are selected from the populations, based on fitness.

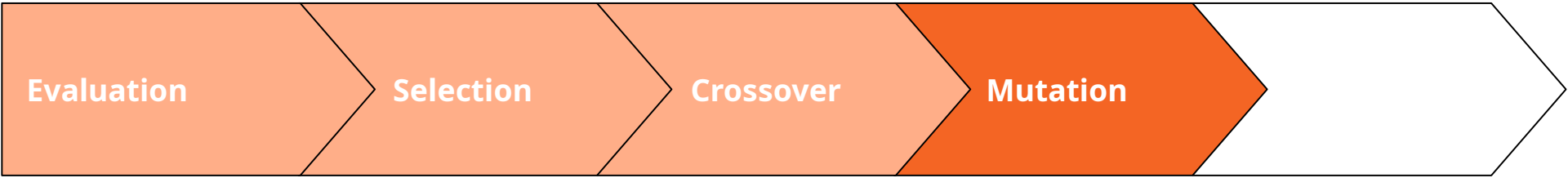
In case of the k-tournament selection algorithm, it involves running several ”tournaments” among k individuals chosen at random from the population, until the desired amount of population is reached. Each tournament selects the best amongst the k selected individuals

Genetic Algorithm

Genetic material of the individuals are combined to create offspring

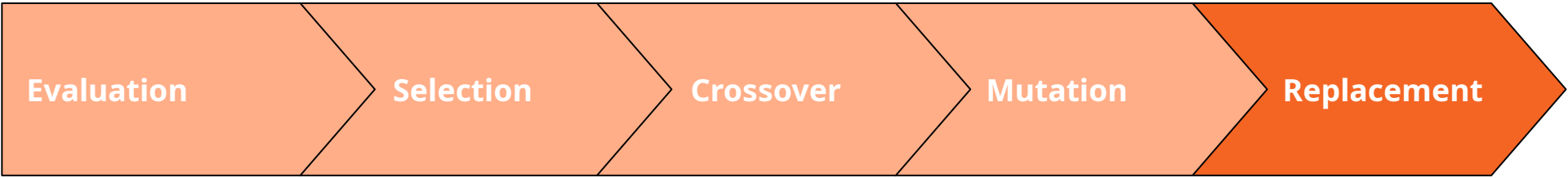
In case of single-point crossover, a random crossover point is selected and the genetic material is exchanged between the parents at that point

Genetic Algorithm



With a certain probability, random mutations are introduced into the offspring, helping to maintain diversity within the population

Genetic Algorithm



This newly generated population replaces the old one.

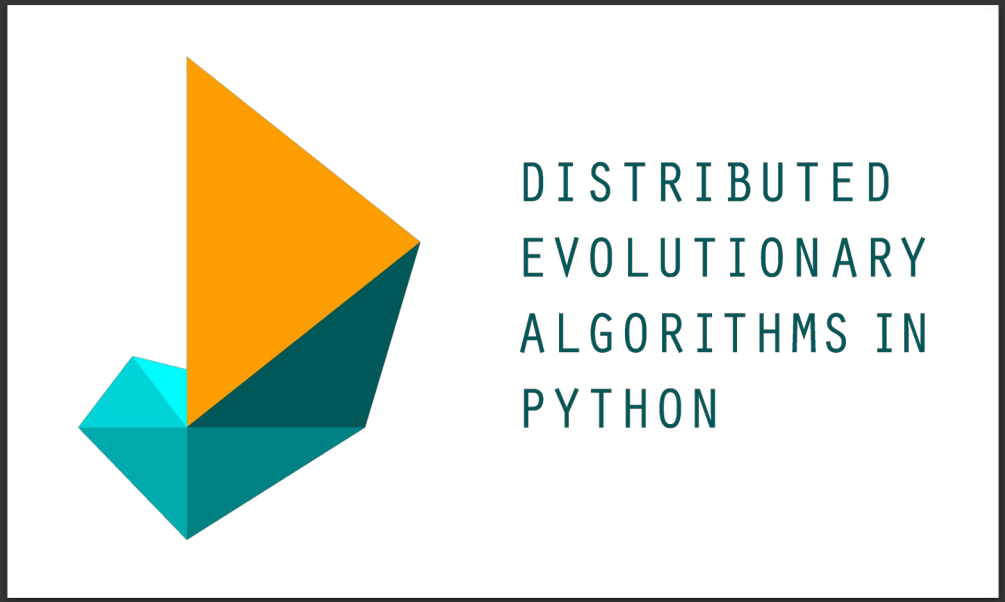
The process repeats until a stopping criteria is satisfied.

Implementation

- ➔ DEAP Framework
- ➔ Gene Representation
- ➔ Fitness Function
- ➔ Performance Metrics
- ➔ Parameters
- ➔ Benchmarking Flow

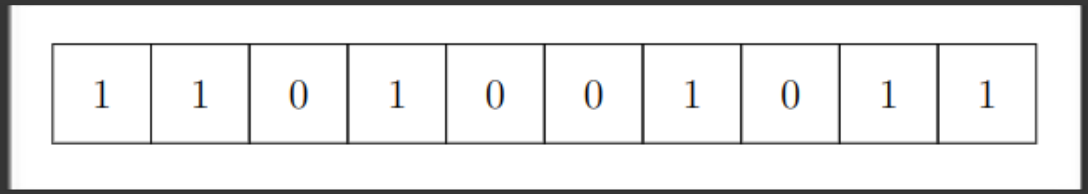
DEAP Framework

DEAP is a Python library that excels at rapid prototyping and testing of ideas



Gene Representation

Each gene is a binary string where each bit represents the presence of a vertex in the solution



Fitness Function

It calculates the weight of the solution by summing the weights of the vertices present in the solution.

Additionally, it imposes a penalty for each edge that is not covered by the vertices in the solution.

Performance Metrics

- → Fitness Function
- → Objective Function
- → Number of Evaluations

Parameters

- → Population Size
- → Crossover Probability
- → Mutation Probability
- → K–Tournament Selection Size
- → Number of Generations

Benchmarking Flow

The best parameters are found using an iterative process.

1. **Test instances are loaded**
2. **The algorithm is run**
3. **The results are saved**
4. **The results are analyzed using a Jupyter Notebook**

Results

1st Iteration

The algorithm is run with defaults parameters, to see how it performs

2nd Iteration

The population size is tuned.

The size of **150** is found to be the best.

3rd Iteration

Crossover and Mutation parameters are analyzed.

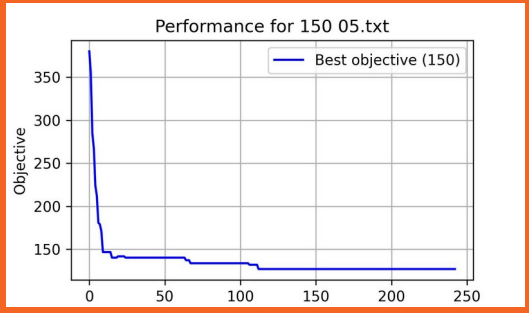
These are studied together since they’re tightly coupled.

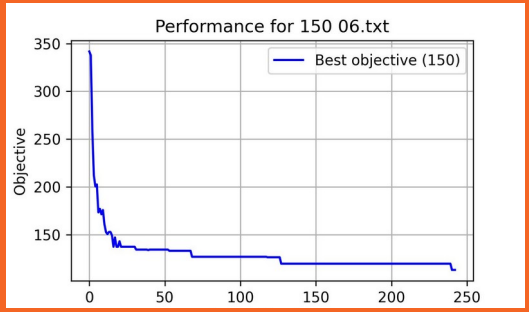
The best parameters found are **0.4** and **0.25** for crossover and mutation probability, respectively.

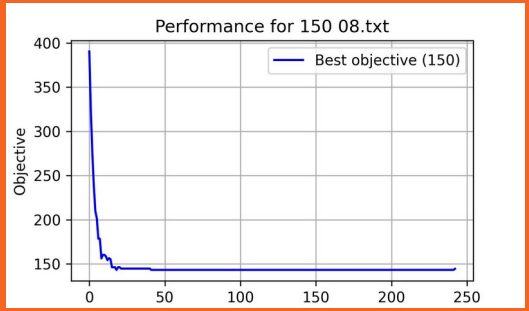
4th Iteration

Tournament size is modified.

Using k=**2** gave the highest results in most the instances, so this was chosen.







5th Iteration

To improve efficiency, the number of generations must be tuned.

Analyzing the plots, there's no correlation between optimal number of generation and problem size.

The chosen stopping criteria is to stop the algorithm after a fixed **100 generations of no improvement**.

Thanks