Sensitivity Analysis of Value at Risk in over-parametrized Luciano-Semeraro model

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Abstract

In this document we consider the model proposed in [1]: the model is overparamtrized: is it possible that the Value at Risk shows a sensitivity to the choice of joint parameters $\boldsymbol{\theta} = (a, b, \rho)$ given that the market correlation of log-returns is matched?

1 Overview of the model

Consider the more general model for log-returns proposed in [1].

$$\mathbf{Y}^{\rho} = \mathbf{Y}^{X} + \mathbf{Y}^{Z}_{\rho}. \tag{1}$$

where

$$\begin{aligned} Y_1^{\rho} &= B_1(X_1) + \mu_1\alpha_1 Z + \sigma_1\sqrt{\alpha_1}B_1^{\rho}(Z) \\ Y_2^{\rho} &= B_2(X_2) + \mu_2\alpha_2 Z + \sigma_2\sqrt{\alpha_2}B_2^{\rho}(Z) \end{aligned}$$

where:

- B_1 and B_2 are indipendent brownian motion, with drift μ_i and standard deviation σ_i.
- B_1^{ρ} and B_2^{ρ} are correlated standard brownian motion with correlation ρ
- $X_j \sim \Gamma\left(\frac{b}{\alpha_j} - a, \frac{b}{\alpha_j}\right)$
- $Z \sim \Gamma(a, b)$

It can be shown [1] that:

$$corr\left(Y_1^{\rho}, Y_2^{\rho}\right) = \frac{\rho\sigma_1\sigma_2\sqrt{\alpha_1}\sqrt{\alpha_2}ab + \mu_1\mu_2\alpha_1\alpha_2 a}{b\sqrt{\left(b\sigma_1^2 + \mu_1^2\alpha_1\right)\left(b\sigma_2^2 + \mu_2^2\alpha_2\right)}} \tag{2}$$

2 Calibration and Sensitivity Analysis

In this section we first sketch the problem arising when we want to fit the parameters set $\boldsymbol{\theta}$ and then we provide a sensitivity analysis of multi asset options and Value at Risk indicator to varing $\boldsymbol{\theta}$.

2.1 Calibration issues

Usually in the market are quoted vanilla options written on the single asset. So the idea is to fit marginals' parameters $\boldsymbol{\zeta} = (\alpha_i, \mu_i, \sigma_i)$ from these single asset options and common parameters $\boldsymbol{\theta} = (a, b, \rho)$ from the history of asset log-returns such that the market log-returns correlation ρ^{mkt} is fitted.
Here the problem is that exist infite $\boldsymbol{\theta}$ such that the market log-return correlation ρ^{mkt} is matched. Since the value of $\boldsymbol{\theta}$ introduce the dependence between the proces we are worried about different value of $\boldsymbol{\theta}$ leads to different dependancy structures in log-returns giving different prices of multiasset options on in risk-metrics such as Value at Risk (VaR).

2.2 Premium Sensitivity Analysis

The idea is to price different exotic derivatives which depends on two asset whos log-returns are modelled as in (1). We fix $\boldsymbol{\zeta}$ and suppose a market correlation ρ^{mkt}. Then we choose a different sets of of prameters $\boldsymbol{\theta} = (a, b, \rho)$ such that ρ^{mkt} is matched. Then we price derivatives. If the choice of $\boldsymbol{\theta}$ doesn't not impact we can deduce that the only influencing parameter is ρ^{mkt}. Otherwise if different choices of $\boldsymbol{\theta}$, providing the same ρ^{mkt}, produce different premiums then we can deduce that the choice of a, b and ρ is not so arbitrary.

We indicate with $\Phi(T) = \Phi(S_1(T), S_2(T))$ the payoff and we choose the following derivatives:

- A Spread option: $\Phi(T) = (S_1(T) - S_2(T) - K_{sp})^+$
- A Basket option: $\Phi(T) = (\beta S_1(T) + \gamma S_2(T) - K_{bsk})^+$
- A Reverse option: $\Phi(T) = \left(\frac{S_1(T)}{S_2(T)} - K_{rev}\right)^+$
- A Everest option: $\Phi(T) = \min\left(\frac{S_1(T)}{S_1(t)}, \frac{S_2(T)}{S_2(t)}\right)$

If our experiment we fix the following set of parameters dispayed in Table 1.

Now we generate 50 different random starting point $\boldsymbol{\theta}_0$ and we solve the following optimization problem using the `fmincon` *MATLAB* function:

$$
\begin{aligned}
&\boldsymbol{\theta}^* = \min_{\boldsymbol{\theta}} |\rho^{mkt} - corr\left(Y_1^{\rho}, Y_2^{\rho}\right)| \\
&s.t. \\
&0 < \alpha_j < \frac{b}{a} \quad i = 1, 2.
\end{aligned}
$$

If Figure 1 are shown values of $\boldsymbol{\theta}^* = (a, b, \rho)$: as we expected, for the same value of ρ^{mkt} different values of a, b and ρ are possible. How this different sets of parameters impact on options' premia? In Figure 2 different contracts are priced. We can see that the options value remains the same for different choice of $\boldsymbol{\theta}$: we can desume that the only importan parameters in option pricing is log-returns correlation ρ^{mkt} and not the exact calibration of (a, b, ρ).

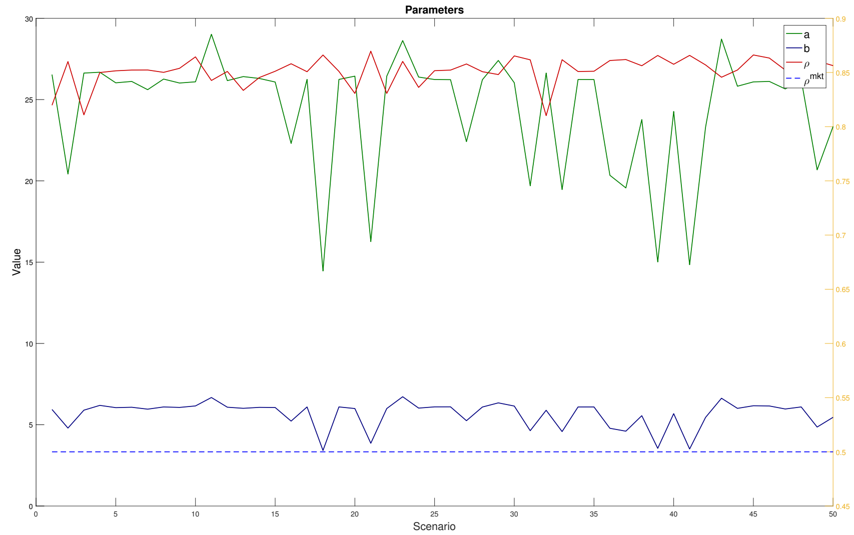

Figure 1: Some values calibrated. a in green, b in blue, ρ correlation between Brownian Motions in red and in dotted blue line the log-returns market correlation ρ^{mkt}.

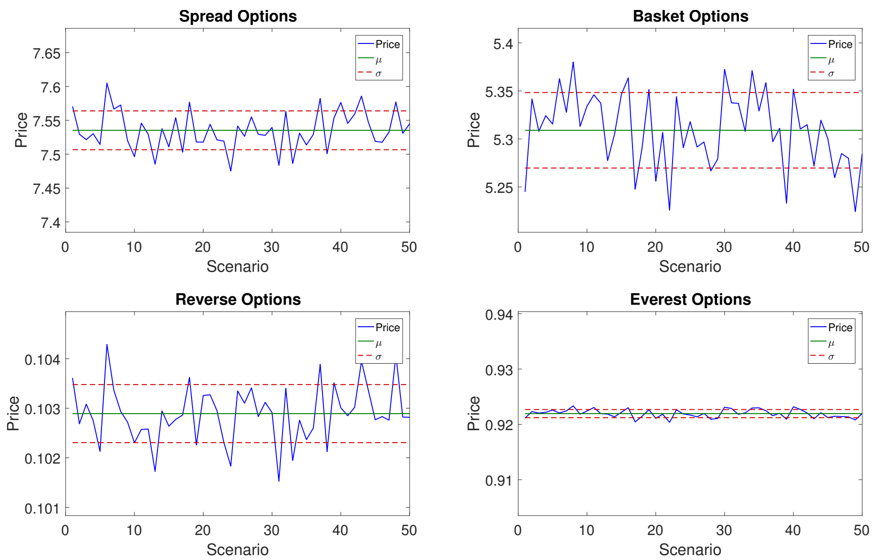

Figure 2: Premia for different derivatives varying the set of parameters $\boldsymbol{\theta}$. In blue the option price, in green the mean, in red the standard deviation of premia.

Parameter	**Value**
α_1	0.125
α_2	0.149
μ_1	0.03
μ_2	0.06
σ_1	0.2
σ_2	0.19
r	0.02
T	1
N_{sim}	$5 \cdot 10^4$
$S_1(t)$	59.82
$S_2(t)$	49.42
K_{sp}	5
K_{bsk}	$\beta \cdot S_1(t) + \gamma \cdot S_2(t)$
β	0.5
γ	0.8
K_{rev}	$\frac{S_1(t)}{S_2(t)}$
ρ^{mkt}	0.5

Table 1: Parameters

2.3 VaR Sensitivity Analysis

In this section we compute VaR sensitivity to a different set of parameters $\boldsymbol{\theta} = (a, b, \rho)$ that match the market correlation ρ^{mkt}. To do so, as in the previous section, we get 50 different values for the set of parameters $\boldsymbol{\theta}$ and we comput the VaR of a simple portfolio with a time horizon of 5 days. In VaR computation we use a fulle evaluation approach, since the porfolio is very small and, moreover, we are interested in stability of VaR varying parameters and we do not what introduce other approximations during computation. A $\Delta - \Gamma$ approach for practical reasons could be considered.
In this section we consider $N_{sim} = 10^4$. We consider a long portfoglio $\Pi(t)$ of three options: a Spread Option, a Reverse Option and a Basket Option. The parameters are the same of the previous experiment and are reported in Table 1. If Figure 4 we plot the quantity $\frac{VaR}{\Pi(t)}$ where $\Pi(t)$ is the value of portfolio. In Figure 3 we show different calibrated value for a, b, ρ and the corresponding value of the objective funcion $|\rho^{mkt} - corr(Y_1^\rho, Y_2^\rho)|$. We can conclude that the VaR computation is not sensible to the choice of a, b, ρ and that the only relevant parameter is ρ^{mkt}. We see again in Figure 5 that the value of Portfoglio and its VaR are very stable.

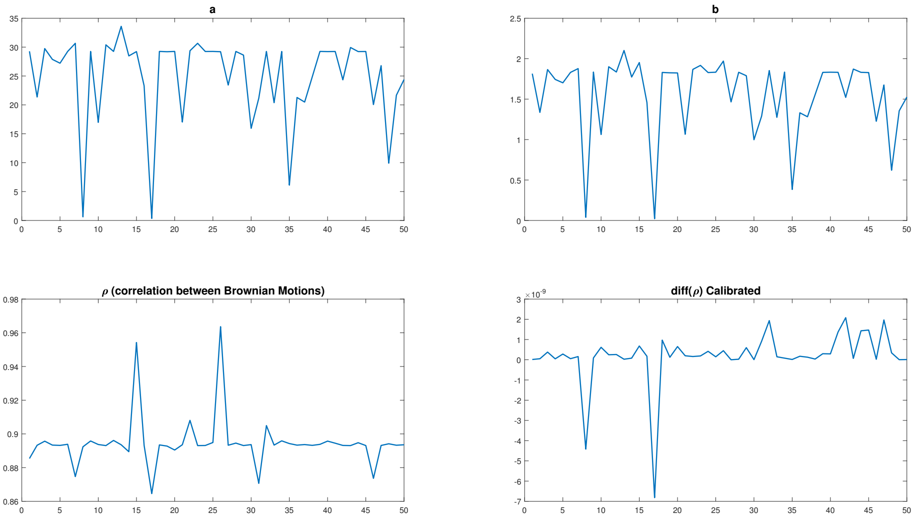

Figure 3: Differente calibrated value for a,b and ρ giving the same ρ^{mkt} value.

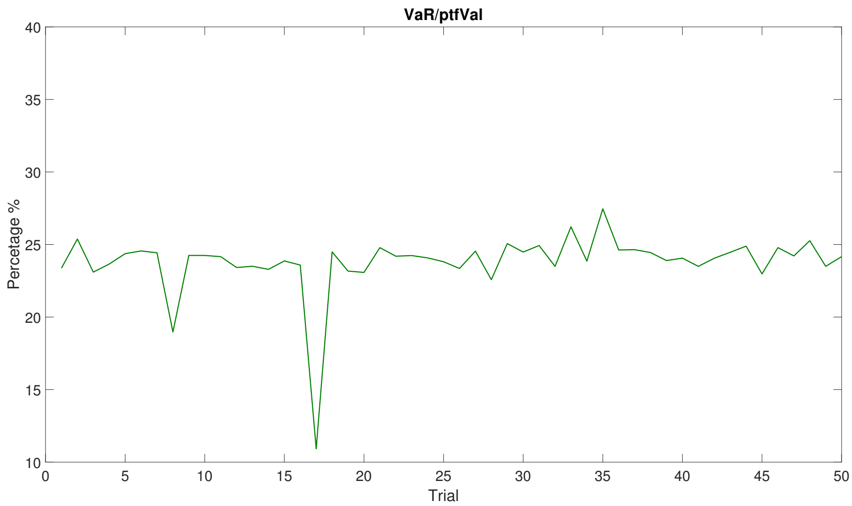

Figure 4: In green is plotted the quantity $\frac{VaR}{\Pi(t)}$: we see that it is very stable and 5 days VaR is roughtly the 25% of Porfolio Value.

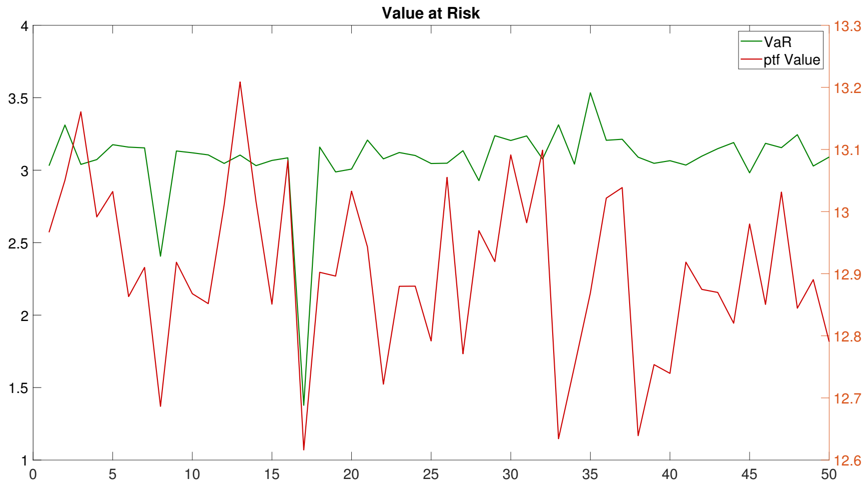

Figure 5: Value of Portfolio in red and 5 days VaR of portfolio in green. We see that both are very stable.

References

[1] P. Semeraro and E. Luciano. Mutivariate time changes for Lévy asset models: Characterization and calibration. *Journal of Computational and Applied Mathematics*, (233):1937–1953, 2010.