A Schwartz-Smith model with jumps

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11 March 2019

Abstract

The aim of this short paper is to show how to combine a jump-diffusion model[1] with the well known Schwartz-Smith [6] model in order to obtain a model that shows a long term and short term behaviour mixed togheter. Calibration is implemented and numerical simulations are performed. In the end limitations and possible extension are briefly discussed.

1 Introduction

In this paper we introduce the well know Schwartz-Smith model [6]. The original model was presented to model oil market but can be extended to simulate other derivatives. The key idea of the model is that the spot price S_t has the form

$$S_t = \exp(\chi_t + \xi_t)$$

where χ_t is a short-term component and ξ_t is a long-term one. Schwartz and Smith assumed the following dynamic for the short-run deviation

$$d\chi_t = -k\chi_t dt + \sigma_k dW_\chi,$$

while the equilibrium level ξ_t follow a brownian motion

$$d\xi_t = \mu_\xi dt + \sigma_\xi dW_\xi.$$

where $dW_\chi dW_\xi = \rho_{\chi\xi} dt$.

In their article they shows that exist a closed formula for future prices and for option on futures. Moreover they show an interesting methodology for parameter estimation based on Kalman-Filter tecnique. Anyway, we do not follow their approach because is considerably more complicated in presence of jumps but we follow what was proposed in [8]. This approach is easies to implement and conducts to good results.

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[1] See January PhD report for details.

2 Modelling

The idea is to mix the model presented in previous paper with Schwartz-Smith model in order to fix one of the previous drawback of the previous model. Let's resume the previous model. The spot-price S_t was of the form

$$S_t = \exp\left(f(t) + g(t) + X_t\right)$$

where $f(t)$ is the macro-seasonality component, $g(t)$ is a micro-seasonality component and X_t follow the stochastic dynamic:

$$dX_t = -kX_t dt + \sigma dW_t + dJ_t \tag{1}$$

where the jump size J is supposed to be normal: $Y \sim \mathcal{N}\left(\mu_J, \sigma_J^2\right)$.
This model was easy to calibrate and flexible but it has a foundamental problem: long-term volatility is not present. Tha main problem is that, if we use this model to price long term derivatives, we understimate the value of the contract. To avoid this fact we can define X_t as:

$$X_t = \chi_t + \xi_t$$

where:

$$d\chi_t = -k\chi_t dt + \sigma_\chi dW_\chi + dJ_t,$$

and

$$d\xi_t = \sigma_\xi dW_\xi.$$

where $dW_\chi dW_\xi = \rho_{\chi\xi} dt$.

Doing so we add the long-term volatility σ_ξ to the process and this solve the problem for long-term contracts valuation. The drowback is that we have two more parameters to hestimate, $\rho_{\chi\xi}$ and σ_ξ.

3 Model Calibration

To perform model calibration we do an assumption: we say that the correlation between long and short process are incorrelated, i.e. we assume that $\rho_{\chi\xi} = 0$. Doing so the calibration is easier because the parameters of the two processes χ_t and ξ_t can be stimated in a separated way. The choice is to calibrate the parameter of ξ_t on long terms contract (i.e. option on calendar or on futures) and the parameters of χ_t on spot prices. We follow the procedure of *Maximum Likelihhod Estimation* presented in previous article applied to jump-diffusion model and to brownian motion. To calibrate σ_ξ. we follow what is presented in [4]. It can be showed that if you have a proces of the form:

$$dF_t = \mu F_t dt + \sigma F_t dW_t$$

and you have a series of observation F define $Y(t_i) = \log F(t_i) - \log(t_{i-1})$ then the parameters that maximize the Likelihood can be found explicitly. In particular we have that:

$$m = \left[\hat{\mu} - \frac{1}{2}\hat{\sigma^2}\right]\Delta t \qquad v = \hat{\sigma^2}\Delta t$$

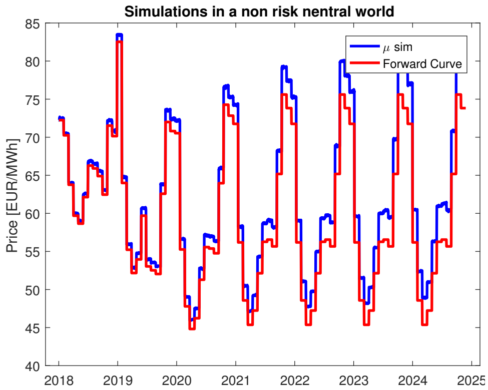

Figure 1: Simulation with real-world parameters against forward curve. One can see that there is no convergence.

where

$$m = \sum_{i=1}^{n} y_i/n \qquad v = \sum_{i=1}^{n} (y_i - m)^2/n$$

where y_i is the i observation and n is the size of observation. Knowing everithing $\hat{\mu}$ and $\hat{\sigma}$ can be computed.
The other parameters $k, \sigma_\chi, \mu_J, \sigma_J$ and λ can be estimated using MLE estimator for jump-diffusion process[2]. Once again the calibration of jump-diffusion part is very fast while the parameters of brownian motion can be estimated using explici formula.
The parameters we stimated for the spot price were obtained by an historical calibration, so they are valid under the real-work measure $\mathbb{P}$. If we are dealing with risk-management tasks that's okay but, if the aim is to price we must switch to risk-netural probability $\mathbb{Q}$.

4 Risk-Neutral simulations

In an imprecise but intuitive way, we are in risk-neutral world when we have the simulations converge to the forward curve. What happen if we don't simulate with risk-neutral parameters? A typical example of what can happen is shown in Figure 1.

So, we have to find a way to find parameters is risk-neutral world. For a very good introduction of what risk-neutral and real-world measure means the reader can refer to [1].
Here we follow what propesed in [8]. The idea is to modify the drift using a market price of risk that must be stimated and that can allow us to switch to risk-neutral world. In risk-neutral world we have that model prices will be consistent with forward prices observed in the market.
The main challenge with risk-neutral valuation in electricity markets is to obtain the market price of risk. Due to the non storability of eletricity, the market is incomplete. Thus the market price of risk ar not simpky ascernitable, but it

[2]See January PhD report for details.

offers flexibility to adjust the models to osbserved derivatives prices. In ordert to simplify the task we abstract for market prices of jump intensity risk, jums size risk and so on and we focus only on a varying market price of long term risk γ_t. To estimate the time-varying market price of long-term risk $gamma_t$ assume that at time t a cross-section of forward contracts with daily maturity is avaiable. Given there forward prices $F(t,T)$ it is possible to extract the time varying market price of long term-risk by the following relationship:

$$
\begin{aligned}
F(t,T) &= \mathbb{E}^{\mathbb{Q}}\left[P_T|\mathcal{F}_t\right] \\
&= \mathbb{E}^{\mathbb{Q}}\left[e^{f(t)+g(t)+\xi(T)+\chi(T)}|\mathcal{F}_t\right] \\
&= \mathbb{E}^{\mathbb{Q}}\left[e^{f(t)+g(t)+\xi(T)+\int_t^T \gamma_s ds+\sigma_\chi W_T^\chi}|\mathcal{F}_t\right] \\
&= \mathbb{E}^{\mathbb{Q}}\left[e^{\int_t^T \gamma_s ds} e^{f(t)+g(t)+\xi(T)+\sigma_\chi W_T^\chi}|\mathcal{F}_t\right] \\
&= e^{\int_t^T \gamma_s ds}\mathbb{E}^{\mathbb{P}}\left[e^{f(t)+g(t)+\xi(T)+\sigma_\chi W_T^\chi}|\mathcal{F}_t\right]
\end{aligned}
$$

Now we can discretize the equation and we get:

$$
\gamma_{T-1} = \frac{\log\left(\frac{F(t,T)}{\mathbb{E}^{\mathbb{P}}\left[e^{f(t)+g(t)+\xi(T)+\sigma_\chi W_T^\chi}|\mathcal{F}_t\right]}\right)}{\Delta t} - \sum_{s=t}^{T-2}\gamma_s \tag{2}
$$

and we can now successively derive the deterministic function γ_t by applying next days forward price and previosly derived market prices of risk. With γ_t the spot process can then be trasformed to the risk-neutral measure. Note that solving (2) means to solve a linear system and this can be do very efficienty using the most common softwares, such as *MATLAB*.
Once you computer γ_t you can simulate prices using the following long-trm dynamics:

$$
d\xi_t = \gamma_t dt + \sigma_\xi dW_\xi.
$$

In Figure **??** we can see that we have consistency with observed forward prices.

5 Numerical Results

In this section we show the results of numerical tests performed using this model. At the end of the section we show how this model can be used to price a simple plain-vanilla option using a Monte Carlo tecnique. The Monte Carlo technique can be adatapted to others type of contracts whit a relative little effort. One can easly price European option such as Asian, Look-Back, binary options and so on. If one have to deal with american option can try to use Longstaff-Schwartz algorithm presented in [9].

First of all we can analyze the path of the simulation. In Figure 3 we observe that price paths produced by the model are comparable to the real ones.

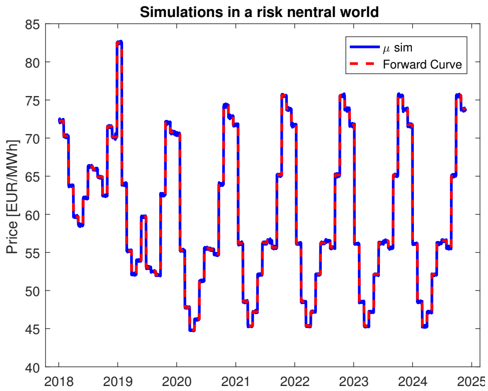

Figure 2: Simulation with risk-neutral-world parameters against forward curve. One can see that convergence is obtained.

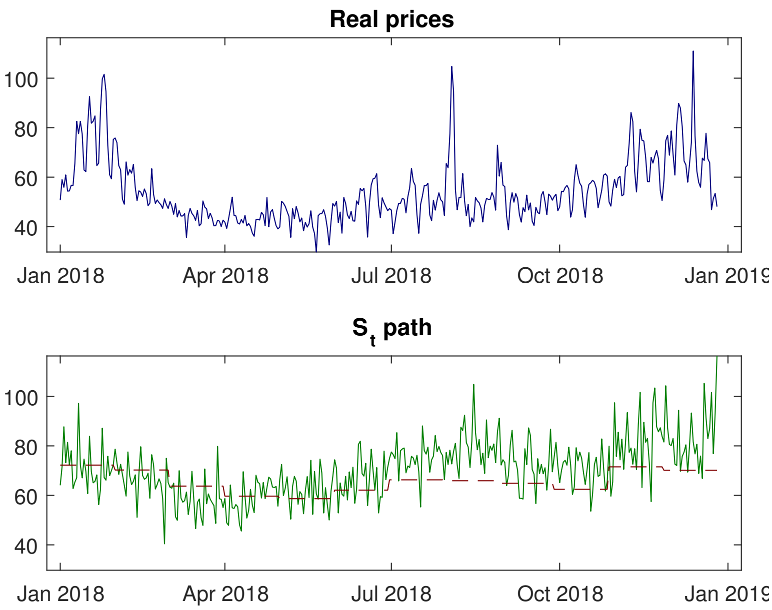

Figure 3: A possible path generated by the model.

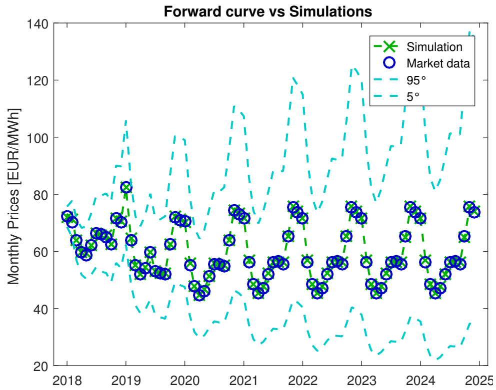

Figure 4: Monlty prices of simulation compared to market forward prices.

Let's now observe if we have convergence to the forward curve and how the simulation behaves on long-term period. Remember that one of the limit of the previous model was that long-term simulations did not spread: that was a problem if you have to price derivatives with long maturities because you have to consider long-term volatility. Let's se what happens in Figure 4. We can observe that montly mean of simulated prices converges to forward montly prices. Moreover adding the term $\chi(t)$ to the model we got that the spread between the 5^{th} and the 95^{th} opens as the time goes on. We can deduce that this model can be used to price long term contracts.

Now, as we did in the previous report, let's analyze the QQ-plot of log-returns. In Figure 5 we compared percentiles of log-returns of real prices agaist percentiles of simulated paths. We note that the percentiles of the two distributions are mostly the same and so we can conclude that real distributions' log-returns and simulated ones are comparable.

6 Conclusion and future efforts

In this short paper we solved the two problems of the previous work. Now we have convergence to the forward curve and we added long-term volatility to our simulation. The resulting model can be used to valuate long-term derivative contracts. One of the main benefit of the model is that every single parameter has a very simple economical meaning. For this reason, the model lend itself to the so called *expert calibration*: if an analyst thinks that the calibration procedure has not been very accurate, can simpy decide to use another way to select a parameters of the model and impose the obtained value for the parameter. This can be dangerous but somethimes useful and it's mandatory to perform what-if analysis.

Note that the technique prosed in Section [**?**] is very general and can be applied to force the convergence to forward curve for a large number of models.

What are the drawbacks of the model? The main drowback of the model is that in order to perform calibration we assumed that the correlation between spot-prices and future prices $\rho_{\chi\xi}$ is zero. If one thinks that this is not true, the proposed methodology can not be used. In this situation one can use the

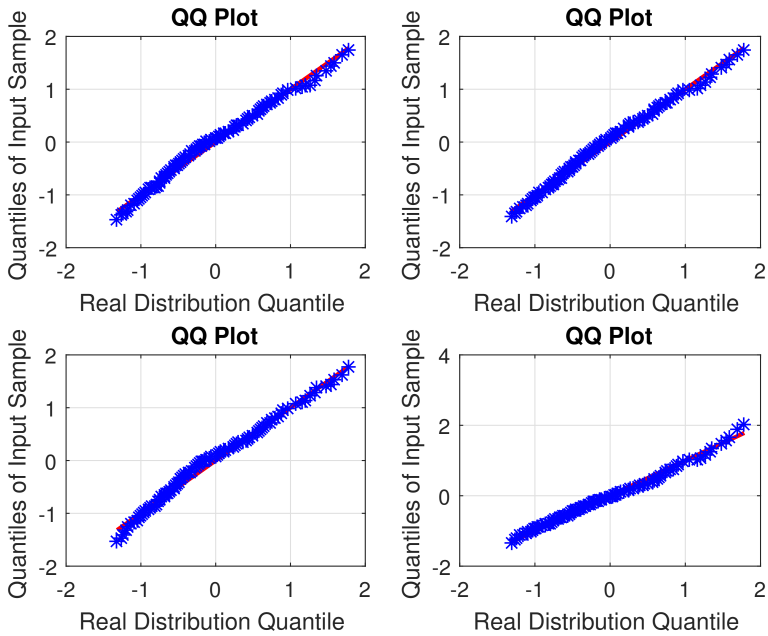

Figure 5: QQ-plot of log-returns.

calibration approach proposed in [6] based on Kalman-Filter.
One can make an objection: the long term dynamics of prices follows a geometric brownian motion S_t, which has a variance given by:

$$Var\left[S_t\right] = S_0^2 e^{2\mu t}\left(e^{\sigma^2 t} - 1\right)$$

where μ is the drift and σ is the volatility. One can easly observe that $\lim_{t\to\inf} Var\left[S_t\right] = \inf$ and this can be unrealistic. The solution in quite easy: you can decide to use a mean-reverting process for long-term component such as a Ornstein-Uhlenbeck or a CIR process. Anyway, we have't yet studied what is the most correct dynamic for long-term component, so we can't select the most appropriate model. The next challenge is to extend this model to more than one commodity introducing the concept of correlation between prices. Another issue is to find a way to scale the model from daily prices to hourly prices. This topics will be studied in the near future.

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